MASTER

Influence of a step and a slope on the behaviour of a dipole in a shallow fluid layer

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Title: Influence of a step and a slope on the behaviour of a dipole in a shallow fluid layer

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Colour
Many figures in this report require a colour print to be better understandable for the reader. If you received a black-and-white copy of this report, please e-mail theunissenron@gmail.com to receive a digital version (pdf).
Summary

In this research project the behaviour of an electromagnetically forced dipolar vortex that propagates in a shallow fluid layer towards an up-going step or slope in the fluid depth was investigated, by experiments and simulations. Close to and just in front of the step compact patches of oppositely signed vorticity are produced as a result of the no-slip boundary condition. In case of a sloping bottom, these patches of oppositely signed vorticity are produced too, be it less compact and more diffuse. In both configurations, a dipole rebounding process takes place, but due to the large damping in the shallow layer only a small part of it is observed.

The trajectory of the vortex centre of a dipole half depends on the depth position in the fluid. The shape of the central axis of the vortex column in the cases with a step or slope deviates most from that in the case of a flat bottom at fluid depths where the oppositely signed vorticity is created. In all three configurations qualitatively the same vertical flow patterns are observed. In the step configuration the amount of vertical kinetic energy is bigger compared to the configuration with the flat bottom. In the simulations with a slope this differs barely from those without a step or slope.
Summary

Contents

Voorwoord

1. Introduction

2. Theory
   2.1 The Navier-Stokes equation
   2.2 Vorticity and circulation
   2.3 Vortex models
   2.4 Electromagnetically forced dipolar vortex in a shallow fluid layer
   2.5 Dipole rebounding
   2.6 Three-dimensional behaviour of dipoles in shallow fluid layers

3. Setup of experiments and simulations
   3.1 Experiments
   3.2 Particle Image Velocimetry
   3.3 Numerical simulations

4. Results: step configuration
   4.1 Experimental results
   4.2 Simulations of the flow at the free surface of the fluid layer
   4.3 Three-dimensional behaviour of the numerically obtained flow
   4.4 Vertical flows

5. Results: slope configuration
   5.1 Experimental results
   5.2 Simulations of the flow at the free surface of the fluid layer
   5.3 Three-dimensional behaviour of the numerically obtained flow
   5.4 Vertical flows

6. Conclusions and recommendations
   6.1 Conclusions
   6.2 Recommendations for further research

7. Bibliography

Appendix
   Appendix A: Derivation of the magnetic field
   Appendix B: Determination of the multiplicative constant A
   Appendix C: Vortex profile in experiments and simulations
   Appendix D: Trajectory at various depths in the flat configuration
Voorwoord

Toen familieleden zich onlangs afvroegen waarom de beroemde aswolk van de IJslandse vulkaan Eyjafjallajökull in “zo’n rare vorm” boven Europa hing, kon ik ze mooi aanwijzen waar ik de afgelopen tijd veel mee bezig ben geweest. Een dipool: een quasi-tweedimensionale wervelstructuur die bestaat uit twee tegengestelde roterende stromingen.

Gedurende mijn studie Technische Natuurkunde ben ik, hoewel mijn interesses vooral heel breed zijn, langzaam gegroeid richting het vakgebied van de stromingsleer. Dit leidde tot een afstudeerproject bij de vakgroep Werveldynamica, waarvan het officiële resultaat nu voor u ligt. Maar naast dit verslag zijn er nog meer resultaten geboekt. Zo heb ik veel geleerd over de moeilijkheden van het onderzoek doen. Heb ik uitgebreid kennis mogen maken met numeriek werk. En weet ik inmiddels een stuk beter wat ik in de toekomst wil gaan doen.

Dit verslag zou niet tot stand gekomen zijn zonder de hulp van een aantal personen, die ik bij dezen dan ook graag zou willen bedanken. Leon Kamp, voor de begeleiding en zijn geduld. Gert-Jan van Heijst, voor de soms broodnodige motivatie. Rinie Akkermans, voor het op weg helpen met de experimenten. Mijn kamergenoten, voor de gezelligheid op de werkplek. En mijn familie en vrienden, voor alle steun die ik de afgelopen maanden van ze heb gehad.

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1. Introduction

Flows in shallow fluid layers are generally idealized as being quasi-two-dimensional. The main reason for that is the ratio between the horizontal and vertical length scales: if this ratio is large enough it is commonly assumed that the vertical velocities can be neglected because they are much smaller than the horizontal velocities. Due to a no-slip condition at the bottom of the fluid layer the vertical profiles of the horizontal velocities can be modelled like a Poiseuille flow.

The most well-known examples of such quasi-two-dimensional flows are geophysical flows: large-scale flows in the atmosphere, oceans and seas. Studying the dynamics of geophysical flow systems is important to gain a better understanding of the climate as well as the transport of heat, temperature, salinity and pollutants. Besides the dimensions (the ocean’s depth is much smaller than the horizontal length-scales) there are two extra reasons for the quasi-two-dimensionality: the rotation of the earth (according to the Taylor-Proudman theorem) and the density stratification (due to temperature and, in the ocean, salinity differences). Such flows are usually described in terms of vorticity. Due to the two-dimensionality only one component, in vertical direction, of the vorticity is of importance. A vortex is a rotating flow of fluid. Oppositely signed vortices are frequently observed to pair up and propagate further as dipoles. Figures 1.1 and 1.2 show two examples of dipolar vortices observed in the atmosphere and oceans. The estimated diameter of a half of the dipole in the atmosphere is 500 km. The vortices in the ocean have diameters between 10 and 100 km. The diameter of the dipole halves in figure 1.2 is approximately 70 km.

Figure 1.1: Satellite image of the Gulf of Alaska. The clouds show a dipolar vortex in the atmosphere, the vortices in the sea are made visible by the seawater temperature. (Source of this figure: NASA SeaWiFS Project.)
Figure 1.2: Dipolar vortices in the Labrador Sea, between the south of Greenland and Canada. The picture is a satellite image of the European Space Agency showing the sea surface temperature at the 10th of July 1992.

Also at much smaller length-scales such dipolar vortices can be observed. In the so-called ‘surf zone’ of a sea, the breaking of ocean waves can lead to flow currents and the formation of dipolar vortices with typical length scales of the order of 10 to 100 m. An example is shown in figure 1.3. In all cases, such a vortex dipole will move forward and can experience fluctuations in the fluid depth or approach a wall or coastline. This will influence the behaviour of the vortices.

Figure 1.3: Dipoles formed by rip currents in the surf zone. In the picture two dipoles are indicated by arrows. The drawing at the right shows how these dipoles are formed by longshore flows and the presence of variations in the fluid depth (for example due to the presence of breakwaters). (Source of the picture: [Per97]. Source of the drawing: SLSA.)
The research project reported in this thesis was focussed on dipolar vortices that travel towards the coastline. This coast is simplified by two different models: a step in the depth of the fluid layer and a constant slope. This is not a fully new research object: several researchers examined these configurations earlier. For example, experiments and simulations on a dipole that crosses a step were performed by Hinds et al [Hin07], who studied the relation between the angle of incidence and the refracted angle of the dipole path after crossing the step. Lejeune [Lej08] investigated the path of a dipole that propagates over a sloping bottom, as a function of the angle between the initial dipole’s symmetry axis and the shoreline. In Lejeune’s experiments the dipole was created above the slope. Centurioni [Cen02] studied the same configuration, but with a dipole created above a flat bottom that travels towards the slope.

There are several methods for modelling geophysical flows in laboratory experiments, see for example [Hei09]. Zavala Sansón et al. and Tenreiro et al. created dipolar vortices in a rotating tank and investigated the influence of a step [Ten06] and a topographic slope [Zav05] on it. Due to the rotation of the tank, the path of such a dipole differs from that in a shallow fluid layer without background rotation. Dipolar vortices in a shallow fluid layer can be created in different ways. Centurioni for example produced his dipolar vortex by moving a plate towards the beach, Hinds used a turbulent jet that forms a dipolar vortex, and Lejeune created a dipole by a magnetic field and an electric current pulse through salty water. Some properties of the dipolar vortex depend on the way it is created. In the experiments and simulations described in this report the last method is used: the creation of a dipolar vortex in a shallow fluid layer by electromagnetic forcing.

![Figure 1.4: An electromagnetically forced dipolar vortex in a shallow fluid layer experiment, visualized by fluorescent dye.](image)

Most of the work done on flows in shallow fluid layers focussed on the planar motions, usually at the fluid surface. This is due to the expectation that such a flow behaves in a quasi-two-dimensional manner. However, factors like the influence of the bottom boundary layer and the variation in forcing of the dipole with depth (due to the three-dimensional structure of the magnetic field) can be a source of vertical motion. For example Lacaze et al. [Lac09] characterized the three-dimensional dynamics of a vortex dipole that was created by the rotation of vertical flaps. Akkermans et al. [Akk08] investigated this three-dimensional behaviour of an electromagnetically generated dipole. Ciešlik [Cie09] also performed experiments and simulations on it, and focussed among other things on the three-dimensional behaviour when the dipole approaches a wall or moves above a sloping bottom. In the research described in this report also some three-dimensional properties of the dipole will be investigated.
This report presents the results of experiments and numerical simulations for a dipolar vortex in a shallow fluid layer, which approaches an up-going step or slope in the fluid depth. The organization of this report is as follows. In the next chapter some theoretical aspects of vortex dynamics and flows in shallow fluid layers are discussed. Also some relevant results of previous research on dipolar vortices in shallow fluid layers are shown. Chapter 3 describes the setup of the experiments and the numerical simulations. Chapter 4 shows the results of these experiments and simulations for the case in which a dipole travels towards a step. The results of experiments and simulations with a slope are presented in chapter 5. Finally, in chapter 6, the conclusions of this research are presented and some recommendations for future research are given.
2. Theory

2.1 The Navier-Stokes equation

Fluid dynamics is the physics of motions in gasses and liquids. These flows can be described by the equations for conservation of mass (the continuity equation) and momentum. If the fluid is Newtonian and incompressible, the continuity equation is:

\[ \nabla \cdot \vec{v} = 0 \quad (2.1) \]

Under the same conditions the equation of conservation of momentum becomes the Navier-Stokes equation:

\[ \frac{\partial \vec{v}}{\partial t} + (\vec{v} \cdot \nabla)\vec{v} = \frac{1}{\rho} \nabla p + \frac{\mu}{\rho} \nabla^2 \vec{v} + \vec{f} \quad (2.2) \]

In these equations, \( \vec{v} \) is the velocity vector, \( t \) the time, \( \rho \) the fluid density, \( p \) the pressure and \( \mu/\rho \) the kinematic viscosity of the fluid. The law for conservation of momentum is based on the idea that the sum of all forces working on the fluid particle is the source of the momentum of the flow. In the Navier-Stokes equation the forces are represented at the right hand side by a pressure term, a viscous term and a collection of other forces per unit of mass \( \vec{f} \). At this moment, only for simple specific flow configurations and with some premises the Navier-Stokes equation can be solved exactly. In other situations only a numerically obtained result is possible.

When the Navier-Stokes equation is made dimensionless, some dimensionless parameters can be defined. One that is used very often is the Reynolds number:

\[ Re = \frac{\rho V^2}{\mu} = \frac{\rho V L}{\mu} \quad (2.3) \]

Here, \( V \) is a characteristic flow velocity, \( L \) the length scale and \( \mu \) the dynamic viscosity of the fluid. The Reynolds number gives the ratio of inertial forces to viscous forces in the flow, so it is a measure of the importance of the viscous term in the Navier-Stokes equation. The number also indicates whether the flow is laminar or turbulent. Flows with \( Re < 2300 \) are mostly considered to be laminar, but the Reynolds number on which the flow becomes turbulent depends on the flow geometry.

2.2 Vorticity and circulation

For studying vortices it is convenient to use the vorticity \( \vec{\omega} \) (in \( s^{-1} \)), which is defined as:

\[ \vec{\omega} \equiv \nabla \times \vec{v} \quad (2.4) \]

Vortices can be described by the vorticity as a function of space and time. The flux of vorticity through a surface with normal vector \( \vec{n} \) is equal to the circulation of the flow \( \Gamma \) around the contour of the surface:

\[ \Gamma = \oint \vec{\omega} \cdot \vec{n} \, dA \quad (2.5) \]
The circulation $\Gamma$ is a measure of the strength of a vortex. It is defined as the integral of the velocity vector over the closed contour of the surface:

$$\Gamma \equiv \oint \vec{v} \cdot d\vec{s} \quad (2.6)$$

With this equation, using equation 2.4 and Stokes’ theorem, equation 2.5 can be proven. The circulation around a closed contour of a vortex can be considered as a measure of the strength of it. In an inviscid, barotropic flow with only conservative body forces the circulation around a closed curve that moves with the fluid is constant in time. This is known as Kelvin’s circulation theorem:

$$\frac{d\Gamma}{dt} = 0 \quad (2.7)$$

A vortex line is a line in the fluid at which the vorticity vector is directed tangentially at any place, see figure 2.1. A collection of vortex lines passing through a surface forms a vortex tube, see figure 2.2. Important properties of such a vortex tube are known as Helmholtz’s vortex theorems, and can be proved from the equations above. In absence of body forces, the strength of a vortex tube is constant along its length and remains constant in time.

Figure 2.1: Example of a vortex line.

Figure 2.2: Example of a vortex tube that exists of the vortex lines that go through the surface A.
If the flow is inviscid, the vortex lines always move with the fluid flow. When a vortex tube is stretched due to the flow, like indicated in figure 2.3, the mass conservation law and the assumption that the flow is incompressible show that the volume of the tube remains constant:

$$H_1 A_1 = H_2 A_2$$  \hspace{1cm} (2.8)

The circulation remains constant too. From equation 2.5 follows:

$$\Gamma = \omega_1 A_1 = \omega_2 A_2$$  \hspace{1cm} (2.9)

Combining these two equations gives:

$$\omega_1 / H_1 = \omega_2 / H_2$$  \hspace{1cm} (2.10)

So the stretching of a vortex tube increases the vorticity.

Figure 2.3: Stretching of a vortex tube.

In a viscous flow the diffusion of vorticity plays a role. This causes the vorticity to spread out during time. The volume of the vortex column increases and the maximum vorticity in the column decreases due to the diffusion.

In this report the behaviour of a dipolar vortex in a quasi-two-dimensional flow is investigated. A dipolar vortex consists of two localized adjacent patches of oppositely signed vorticity. Each region is a vortex column. Figure 2.4 shows an example of the vorticity profile of a dipolar vortex. Each region of vorticity makes the other region move, so a dipolar vortex will translate in the fluid. If the absolute value of the vortex strength of each region is equal, the dipole is symmetric. In that case it translates in a straight line. The path of an asymmetric dipolar vortex is curved. Figure 2.5 gives two examples.
2.3 Vortex models

Due to the complexity of most vortex structures it is difficult (if not impossible) to give an exact mathematical description of the vortices. There are many models created to describe the behaviour of vortices. Here, only two simple models that help to understand the behaviour of the dipolar vortex in the experiments of this report are described briefly. In the models the flow is assumed to be inviscid.

In the point vortex model all vorticity of the vortex is concentrated in one singular point: the centre of the vortex. A symmetric vortex dipole can be represented as a configuration of two point vortices with oppositely-signed strengths $\gamma$ and $-\gamma$, which are separated a distance $d$ as shown in figure 2.6. Each point vortex sets the other point vortices in the plane in motion. Due to this, the dipole translates with velocity $V$:

$$V = \frac{\gamma}{2\pi d}$$  \hspace{1cm} (2.11)
Figure 2.6: Sketch of a symmetric point vortex dipole.

The point vortex model is useful to roughly determine the translational motion and the trajectories of the centres of the dipole. The effect of the presence of a solid wall can be described by the ‘image principle’ [Heij92]. The wall can be considered as a mirror, the original vortex experiences a motion induced by its own mirror image. Figure 2.7 shows what this means for a dipolar vortex translating towards a wall: the original vortices bend away from each other and translate further parallel to the wall.

Figure 2.7: A point vortex dipole translates towards a solid wall and is split up into two vortices that translate parallel to the wall, due to the image dipole.

For the vortices experiencing a step or slope in the fluid depth, the point vortex model is not sufficient. An option is to model the vortex as consisting of several smaller vortex tubes each with their own vorticity. Figure 2.8 shows a monopolar cylindrical vortex with $\Gamma > 0$. In this figure, two vortex tubes with vorticity $\omega_1$ and $\omega_2$ are indicated. They both have the same cross-sectional area and their distance to the vortex centre is equal.
Figure 2.8: A vortex column on a flat bottom, divided into vortex tubes of which two are shown. At the left side the front view, at the right side the top view.

On a flat bottom (so the fluid depth is constant) it is assumed that $\omega_1 = \omega_2$. The mutually induced velocities of both vortex tubes are identical: $V_2 = V_1$. Here, $V_2$ is the velocity of vortex tube 2 induced by $\omega_1$. When the vortex experiences a step in the fluid depth, one of the vortex tubes is squeezed, see figure 2.9. According to section 2.3 the vorticity of that tube, $\omega_2$, decreases. So now $\omega_2 < \omega_1$. Due to this inequality of the vortices in the full monopole, the induced velocities are also not equal anymore: $V_2 > V_1$. The result of these two velocities is a net velocity $V_{\text{res}}$ of the complete monopole, parallel to the step.

Figure 2.9: A vortex column on a stepped bottom, divided into vortex tubes of which two are shown. At the left side the front view, at the right side the top view.

For a dipolar vortex that travels towards a step this means that, at the step, the distance between the two dipole halves increases, see figure 2.10. When the step is too high, or $\omega_2$ is too low, the dipole is not able to cross the step. When a dipolar vortex experiences a slope (a gradually decrease of depth) the phenomenon that is described above also occurs. From this model it can be concluded that if a dipole propagates into shallower water the distance between the vortex cores increases.
2.4 Electromagnetically forced dipolar vortex in a shallow fluid layer

As will be described in section 3.1, the dipolar vortex in the experiments presented in this report is created in a shallow fluid layer by a Lorentz force. This is the result of an electric current through the fluid and a magnetic field of a circular permanent magnet. The Lorentz force per unit of mass $\mathbf{f}_L$ is:

$$\mathbf{f}_L = \frac{1}{\rho} \mathbf{j} \times \mathbf{B}$$  \hspace{1cm} (2.12)

with $\mathbf{j}$ the current density and $\rho$ the fluid density. The electric current is assumed to be uniform:

$$\mathbf{j} = j_0 \hat{e}_x$$  \hspace{1cm} (2.13)

From equation 2.12 follows then that $\mathbf{f}_L$ has only components perpendicular to the direction of the current:

$$f_{L,y} = -\frac{1}{\rho} j_0 B_x$$  \hspace{1cm} (2.14)

$$f_{L,x} = \frac{1}{\rho} j_0 B_y$$  \hspace{1cm} (2.15)

The magnetic field of a single uniformly magnetized circular magnet can be derived with the Biot-Savart law, which relates the magnetic field to the magnetization current. This derivation can be found in Appendix A. The result contains an unknown constant, which can be determined by matching the results of experiments with numerically obtained results using the derived magnetic field. The component of the Lorentz force in the $y$-direction sets the fluid in a planar motion: two oppositely signed vortices with equal strength as in figure 2.11 originate. As explained by, for example, the point vortex model (figure 2.6) the two dipole halves force each other to propagate in positive $y$-direction.
Due to the shallow fluid layer the bottom friction plays an important role. The absolute value of the vorticity decreases by damping, which is caused mostly by the no-slip condition at the bottom. The time scale in which this damping takes place can be estimated using the Rayleigh problem theory [Kun02]: the typical thickness of the layer in which the velocity is influenced by the presence of the bottom is \( \delta = 4\sqrt{\nu H} \) (with \( \nu \) the kinematic viscosity of the fluid). The damping time scale \( \tau_d \) can be regarded as the time at which \( \delta \) is equal to the fluid depth \( H \):

\[
H = 4\sqrt{\nu \tau_d}
\]  
(2.16)

So the damping time scale can be estimated by:

\[
\tau_d = \frac{H^2}{16\nu}
\]  
(2.17)

In a water layer of \( H = 8 \) mm the damping time scale is approximately 4 seconds.

### 2.5 Dipole rebounding

In section 2.3 the effect of a wall on a dipolar vortex in the point vortex model for an inviscid flow has been discussed. In practice, the flow is viscous and so-called ‘dipole rebounding’ takes place. In previous researches of a dipole propagating towards a wall or a slope the rebounding process plays an important role.

For example Orlandi [Orl90] investigated this process for a dipole travelling towards a no-slip vertical wall. When the dipole approaches the wall a thin viscous boundary layer is created close to the wall. This layer consists of vorticity with a sign opposite to the sign of the original dipole half. This region of oppositely signed vorticity grows in strength when the dipole comes closer to the wall. It is responsible for the separation of the dipole halves during their collision with the wall. The dipole halves and the created oppositely signed vorticity form new asymmetric dipolar vortices which will propagate along a curved path towards the original symmetry axis of the dipole. For an example of
this, see figure 2.12. Depending on the strength of the new dipole halves, the new dipoles can collide to the wall another time or they can show ‘partner exchange’ in a head-on collision.

Figure 2.12: Example of dipole rebounding at a vertical no-slip wall. Edited contour plots of vorticity from numerical simulations from Orlandi [Orl90].

When the dipole is strong enough, so it has sufficient circulation, and the influence of the bottom friction is small the rebounding process can take place more than once. This leads to more complex flow structures. Also in experiments and simulations with a sloping bottom instead of a wall the rebounding process has been observed, see for example [Car97] and [Cen02]. In this case, there is more damping of the dipole strength due to the decreasing fluid depth. So mostly less than one complete rebounding process can be seen. In the experiments and simulations that are described in this report oppositely signed vorticity plays an important role and the beginning of a rebounding process is observed.

2.6 Three-dimensional behaviour of dipoles in shallow fluid layers
The motion of vortices in shallow fluid layers is often regarded as a quasi-two-dimensional problem. The vortices are seen as columnar structures, in which vertical velocities are absent. As already mentioned in chapter 1 there is a no-slip condition at the bottom of the fluid layer. There, a boundary layer exists, which is often modelled by the assumption of a Poiseuille-like profile in the vertical direction.

Some investigations are performed on the three-dimensional character of flows in shallow fluid layers. For dipoles, see for example [Akk08], [Cie09] and [Lac09]. Here, some three-dimensional characteristics of the dipolar vortex are identified. Figure 2.13 shows typical vertical flows in a dipole. There is an upward motion in the vortex cores (I), surrounded by a band of negative vertical velocity (II). These change sign a few times during the dipole propagation. In the symmetry plane (III), there is upward motion, which also forms the tail of the dipole. At the front of the dipole significant negative vertical motion can be found (IV). In front of that the so-called frontal circulation can be observed (V and VI). This is a roll-like flow structure with a significant horizontal vorticity $\omega_x$. It consists of an upward motion in front of the dipole and a weaker downward motion in front of that. All these features are not only observed for electromagnetically forced dipoles. Also dipoles in shallow fluid layers that are created in another way have these vertical flow structures.
Figure 2.13: Example of a dipolar vortex, from numerical simulations from Akkermans et al [Akk08]. The arrows indicate the horizontal velocities, the colour scale shows the vertical velocities at approximately half depth.

The presence of the vertical velocities in the vortex can be explained by the $z$-dependence of the vertical vorticity component $\omega_z$. Due to the forcing of the dipole and boundary conditions $\omega_z$ is a function of height. According to the cyclostrophic balance (which involves the centrifugal force and the pressure gradient force) the pressure $p$ must also be a function of height. This vertical pressure gradient drives a vertical motion.

A conclusion of Akkermans et al. [Akk08] is that the three-dimensionality of the flow is mainly caused by the flow dynamics itself and the impermeability of the boundaries. Factors like the bottom friction, free-surface deformations and the three-dimensionality of the forcing mechanism are less important in the generation of vertical motions. In this report will be paid attention to the vertical motions in the flow and the influence of the step or slope on it.
3. Setup of experiments and simulations

3.1 Experiments

Experiments on the behaviour of a dipolar vortex are performed in a 52\times 52 \text{ cm}^2 water tank. A dipole vortex is created in a shallow layer of salty water with a thickness of 8 mm and travels by itself towards a step or slope. The dipole is created by electromagnetic forcing, which is the result of an electric current in the fluid and the magnetic field of a circular permanent magnet. Figures 3.1 and 3.2 show the experimental setup. The $x$, $y$- and $z$-direction are defined as in figure 3.1. The centre of the magnet is positioned under $(x,y) = (0,0)$; $z = 0$ corresponds to the bottom of the water layer.

The permanent magnet has a diameter of 25 mm, a thickness of 5 mm and is positioned directly under the very thin bottom of the water tank. Its magnetic flux density close to the magnet is in the order of 1 T. The magnetic field has components in all directions. The component in the $z$-direction is the most important one in creating the dipole.

The flow is forced by a constant electric current of 2 or 3 A during one second in the $x$-direction. For this purpose, the fluid is a salt solution of around 25% Brix. Two electrodes are constructed at the side of the tank, in such way that the formation of gasses due to the electrolysis does not influence the flow in the tank. The construction is customized so that the electrical current goes only through the deep part of the water (where the dipole is created), and not the part where the step or slope exists. This is to avoid differences in the dipole strength due to differences in current density as a result of the various depths. The current density $J$ in the deep part of the fluid is around $2.2 \times 10^3 \text{ A/m}^2$.

A power supply delivers the constant current during exactly one second. This choice is made to get a dipole vortex that is strong enough to propagate towards the step or slope, but is not elongated too much due to the forcing time.

![Image of experimental setup](image.png)

*Figure 3.1: Sketch of the experimental setup, with the top view (a), side view (b) and front view (c).*
Figure 3.2: Pictures of the experimental setup.

The electric current in the x-direction and the magnetic field in the z-direction result in a Lorentz force in the y-direction. This force creates two equal but oppositely signed vortices above the edges of the magnet: a vortex dipole. After the forcing this dipole propagates towards the step or slope. At time $t = 1$ s, when the forcing is switched off, the maximum vorticity of the dipole is $\omega_z \approx 5 \, \text{s}^{-1}$.

The flow is made visible by small particles on the water surface, see figure 3.3 and section 3.2. The particles are illuminated by two slide projectors. The bottom of the tank is black to avoid unwanted light reflections. Experiments are performed in different configurations as shown in table 3.1, with a step or slope in the water depth. The steps are made from black PVC plates with different thickness. The slope is a 1 mm thick black PVC plate that is mounted in the tank under an angle. Before the slope the vortices experience a step from 1 mm, experiments show that the influence of that is very small. In all experiments the movement of the dipole vortex is perpendicular to the step or slope. In this report only the experiments with the forcing current $I = 3 \, \text{A}$ are discussed. The experiments with $I = 2 \, \text{A}$ show the same qualitative phenomena, but they are mostly less pronounced.

Table 3.1: Overview of performed experiments, with $I$ the forcing current, $y_{\text{edge}}$ the distance between the centre of the magnet and the beginning of the step or slope, $h$ the step height and $\alpha$ the slope angle.

<table>
<thead>
<tr>
<th>Configuration</th>
<th>$I$ (A)</th>
<th>$y_{\text{edge}}$ (mm)</th>
<th>$h$ (mm)</th>
<th>$\alpha$ (°)</th>
</tr>
</thead>
<tbody>
<tr>
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<td>-</td>
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<td>slope</td>
<td>2</td>
<td>45</td>
<td>-</td>
<td>3.4</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>60</td>
<td>-</td>
<td>6.0</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>6.8</td>
</tr>
</tbody>
</table>
Figure 3.3: Tracer particles on the water surface visualize the flow. During the preparation of the experiments, the tracer particles are divided homogenously over the water surface. When time progresses they clot together in line patterns. The picture shows a dipolar vortex close to a step in the fluid depth (dark shadow line).

In the experiments with a slope, the distance between the start of the slope and the shoreline is smaller than would be expected geometrically assuming a slope like in figure 3.4 (left). This is due to the 1 mm step at the beginning of the slope and the surface tension of the fluid which rounds off the shoreline, see figure 3.4 (right).

Figure 3.4: The position of the shoreline in an ideal situation (left) and in the performed experiments (right). When $\alpha$ and the fluid depth remain constant, the thickness of the sloping bottom and the round end of the fluid layer (indicated by arrows) cause a change in the shoreline position $y_{\text{shore}}$.

3.2 Particle Image Velocimetry

Particle Image Velocimetry (PIV) is a technique to obtain quantitative velocity information of a two-dimensional flow field. The fluid is seeded with small tracer particles, which are illuminated. The moving particles are recorded by a camera. Evaluation of the recorded images gives the velocity information.
PIV determines the displacement of the particles between two subsequent images, with $\Delta t$ the time between the two images. For this, the images are divided into rectangular sectors. A sector of the image at time $t$ is compared with the corresponding sector at time $t+\Delta t$, as visualized in figure 3.5. The sector at $t+\Delta t$ is shifted in such way that the particle pattern matches the best with the sector at $t$, so that there is maximum correlation. This results in a displacement vector $\Delta \vec{x}$ which is the mean particle displacement in the sector. When the sectors are small enough, this is a good estimate for the local particle displacement in that sector. When $\Delta \vec{x}$ is divided by $\Delta t$ the mean velocity of the particles in the sector is found, which is a good estimate for the flow velocity. For more information on PIV see [Pla03].

![Sectored image frame at $t$ and $t+\Delta t$](image)

**(a)** The dots are seeding particles. With **(a)** the position of the particles in one sector at time $t$, **(b)** the position of the particles in the corresponding sector at time $t+\Delta t$, **(c)** the result when the two previous pictures are put on top of each other and **(d)** the situation when one sector is shifted a small distance $\Delta \vec{x}$, so that the position of the particles has maximum correlation. Figure adapted from [Pla03].

The seeding particles that are used in the experiments of this report are polystyrene particles with a mean diameter of 200 $\mu$m. The density of these small particles is only a little smaller than the density of the salty water, so that most of the particles stay at the water surface. The dimensionless Stokes number $S$ is a measure of the influence of solid particles on the fluid flow. If $S \ll 1$ the particles will follow the flow closely. The Stokes number is the ratio between the relaxation time of the particle in the flow $\tau_p$ and the flow time scale $\tau_f$:

$$S = \frac{\tau_p}{\tau_f}$$
\[ St = \frac{\tau_p}{\tau_f} \]  

(3.1)

For planar vortices, the flow time scale can be estimated by

\[ \tau_f = \frac{1}{\omega_z} \]  

(3.2)

The relaxation time of the particle is the time that the particle needs to respond on flow accelerations. This is a function of the particle diameter \( d_p \), the density of the particle \( \rho_p \) and the fluid \( \rho_f \), and the kinematic viscosity of the fluid \( \nu_f \) (see [Akk08]):

\[ \tau_p = \frac{\rho_p d_p^2}{18 \rho_f \nu_f} \]  

(3.3)

So the Stokes number becomes:

\[ St = \frac{1}{18} \frac{\rho_p d_p^2 \omega_z}{\rho_f \nu_f} \]  

(3.4)

In the flow, the Stokes number is maximum \( 10^4 \), which indicates that the seeding particles follow the flow passively.

The images are recorded with a 12-bit CCD-camera. The frame-rate is 15 Hz and the image size is 1024×1024 pixels. For the evaluation of the images the software ‘PIVview’ is used. In this program the image frames can be improved, the size and overlap of the sectors can be chosen etcetera. Then it calculates the velocity field at time \( t \) out of the sequential frames at \( t \) and \( t+\Delta t \) as described above.

### 3.3 Numerical simulations

Besides the experiments also numerical simulations are performed. The results of the experiments can be compared with the numerical outcomes. The simulations give also information on quantities that cannot be measured in the experiments described earlier: for instance velocity information at every depth and information on vertical velocities. Numerical simulations also allow to examine conditions that are not possible in experiments, like using a stress-free bottom instead of a no-slip bottom of the tank. The numerical simulations were obtained with the software package ‘Comsol’.

Figure 3.5 shows the geometry of the computational domain used for the numerical simulations. It consists of two parts. Part I is the part where the dipolar vortex is created. The dipole moves by itself towards part II, which contains the step or slope. Figure 3.5 shows the flat geometry without any step or slope and in figure 3.6 the geometry with a step (with height \( h \)) and a sloping bottom (with slope \( \alpha \)) are shown. Under the assumption of symmetry with respect to the vertical plane \( x = 0 \), the computations were carried out for only one dipole half. A symmetry boundary condition was applied at the symmetry plane of the dipole. At the other side walls of the computational domain a stress-free-condition is assumed, to approach an infinite domain. At the bottom the no-slip condition is valid, but in some experiments it is taken to be stress-free in order to eliminate the influence of bottom friction in the dipole’s dynamics. The important length scales in the simulations are the same as in the experiments. The horizontal distance between the centre of the magnet and the beginning of the step or slope is 45 mm, the thickness of the fluid layer in part I is 8 mm.
In the numerical simulations the water surface is assumed to be stress-free and non-deformable. This is a difference with the experiments, where the free surface may show wavelike deformations. In the simulations also the surface tension, which deforms the water surface in the experiments as mentioned in section 3.1, is absent.

Figure 3.5: Basic geometry used for the numerical simulations in Comsol. The axes x, y and z are defined. The distances are in meters. The position of the centre of the magnet is \((x,y,z) = (0,0,-3.5 \text{ mm})\). The boundary surface \(x = 0\) is a symmetry boundary.

To perform simulations a mesh has to be created. The maximum element size in the simulations is 3 mm in the \(x\)- and \(y\)-direction and 2 mm in the \(z\)-direction. For the step geometry with \(h = 5 \text{ mm}\), this results in a mesh of approximately 80000 elements. Results of experiments with a finer mesh do not differ from results with the chosen mesh.

The numerical simulation consists of two steps. First, the electromagnetic forcing of the permanent magnet is calculated as a function of position, as demonstrated in section 2.4. After that, a multiplicative constant is needed to match the simulations with the experiments. This constant \(A\) is tuned by comparing the velocity of the vortex centre in the \(y\)-direction in simulations and experiments without the step or sloping bottom, see appendix B. As showed in appendix C, using the obtained value for \(A\), the widths of the vortex in the experimental and numerical case correspond.

In the second step of the simulation the three-dimensional incompressible Navier-Stokes equation including the forcing term (see section 2.1) is solved numerically. The result is a three-dimensional velocity field as a function of time. From this field other quantities can be calculated, like vorticity and kinetic energy. With these data, the dynamics of the dipole moving towards a step or slope can be examined. Table 3.2 shows an overview of the performed numerical simulations. In this report, only the simulations with \(A = 220\) are discussed. These simulations correspond best with the experiments with \(I = 3 \text{ A}\). There are no qualitative differences with simulations with a lower value for \(A\).
Figure 3.6: Examples of a step geometry and a slope geometry for the numerical simulations. The step height \( h \) and the slope \( \alpha \) are indicated.

Table 3.2: Overview of performed numerical simulations, with \( I \) the forcing current, \( y_{\text{edge}} \) the distance between the centre of the magnet and the beginning of the step or slope, \( h \) the step height and \( \alpha \) the slope angle.

<table>
<thead>
<tr>
<th>Configuration</th>
<th>( A )</th>
<th>( y_{\text{edge}} ) (mm)</th>
<th>( h ) (mm)</th>
<th>( \alpha ) (°)</th>
</tr>
</thead>
<tbody>
<tr>
<td>flat</td>
<td>120</td>
<td>220</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>step</td>
<td>120</td>
<td>220</td>
<td>45</td>
<td>1 3 5 7</td>
</tr>
<tr>
<td>slope</td>
<td>120</td>
<td>220</td>
<td>45</td>
<td>-</td>
</tr>
</tbody>
</table>
4. Results: step configuration

This chapter presents the results of the performed experiments and numerical simulations in which a dipolar vortex travels towards a step in the fluid depth. First, the flow at the free surface of the fluid is described. Then the flow is examined through numerical simulations at various depths in the fluid layer. The three-dimensionality of the flow is also described in the last section on vertical velocities.

4.1 Experimental results

In the experiments the flow at the fluid surface was measured as described in section 3.1, where also an overview of the performed experiments can be found. Steps at various positions $y_{edge}$ and with different heights $h$ were used. Figure 4.1 shows an example of the measured flow field at the water surface after the forcing of a dipolar vortex, with $y_{edge} = 45$ mm and $h = 4$ mm that is half of the depth of the deep part of the fluid. In figure 4.2 this situation is compared with experiments with a larger step height ($h = 5$ mm) and without a step. The maximum Reynolds number in the horizontal flow at $t = 1$ s is $Re \approx 1500$. To improve the visibility of structures with low vorticity, the range of the colour scale is chosen small. All vorticity values that exceed the colour scale are coloured according to the maximum or minimum of the scale. The frayed shape of the vortices at $t = 2$ s, $t = 3$ s and $t = 4$ s is caused by the inaccurate calculation of the flow by PIVview at large velocities.

The vorticity diagrams with a step show that the vortex is not able to completely cross that step. Some vorticity (indicated by I in the figures) is transferred over the step, but the main part of the vortex (II) stays in front of it. The part of the vortex column above the step height ($z > h$) is not able to drag the complete vortex over the step. This is only the case when the step is high enough.

Most pronounced of the flow dynamics in these situations is the formation of oppositely signed vorticity (III) just before the step. The formation of it starts together with the creation of the dipole, but gets stronger when the dipole comes closer to the step. This is caused by a no-slip condition at the step, which is the same as at a vertical wall as described by Cieślik [Cie09]. This oppositely signed vorticity causes a broadening of the vortex halves and a movement of them parallel to the step. However, due to the fact that the strength of this vortex patch is much smaller than that of the dipole half it is wrapped around the dipole half as a sort of band. This prevents the dipole half to continue travelling in the $x$-direction.
Figure 4.1: Velocity field (small arrows) and $\omega_z$ (colours) at the fluid surface of a dipole moving towards and over a step with $h = 4$ mm, at different times. The step, at $y_{edge} = 45$ mm, is indicated with a black horizontal line. The forcing current is 3 A and $t = 1$ s corresponds the end of the forcing.
Figure 4.2: Velocity field (small arrows) and $\omega_z$ (colours) at the fluid surface at different times of a dipole moving over a flat bottom ($h = 0$ mm, left column), and a dipole moving towards and over a step with $h = 4$ mm (middle column) and a step with $h = 5$ mm (right column). The step, at $y_{edge} = 45$ mm, is indicated with a black horizontal line.
The maximum created oppositely signed vorticity depends on the step height and the position of the step. Figure 4.3 shows the approximate maximum value of the created oppositely signed vorticity and the vorticity in the centre of the original dipole half as a function of time, for the configurations of figure 4.2. The vorticity in the centre of the original dipole half decreases by damping and the widening of the vortex. The created oppositely signed vorticity reaches a maximum when the centre of the original dipole comes close to the step, and decreases again by damping. The damping is caused mostly by the no-slip condition at the bottom. With \( H = 8 \) mm the damping time scale (see equation 2.18) is approximately 4 seconds, so the friction at the bottom surely contributes to the decrease of vorticity of the dipole half. As can be seen in, for example, figure 4.3 the vorticity of the part of the vortex that passes the step (I) decreases much faster. This can be explained by the damping time scale: for example \( H = 4 \) mm implies \( \tau_d = 1 \) second.

In the diagrams that show \( \omega_z \) at the fluid surface a remarkable gap (IV) is observed in the cases where (a part of) the dipole passes the step. This might be an intrusion of the oppositely signed vorticity that, as will be showed in the simulations in section 4.3, is formed just only before the step at \( z < h \). At the same time that the upper part of the dipole passes the step, the oppositely signed vorticity that is created between the lower part of the dipole and the step diffuses up and intrudes into the original vortex patch.

Figure 4.4 shows the situation at \( t = 7 \) s, for different step heights and \( y_{edge} = 60 \) mm, where it is clear that the amount of vorticity that is able to pass the step decreases when the step height increases. There is only little difference between the experiments with the step at \( y_{edge} = 45 \) mm and the step at \( y_{edge} = 60 \) mm. In the latter case it lasts a little longer until the movement of the dipolar vortex is clearly influenced by the step. The created oppositely signed vorticity is a little weaker because of the decrease of the vortex strength of the original dipole half during time. But the qualitatively behaviour of the dipole moving towards the step is the same.
Figure 4.4: The dipole at time $t = 7\, s$, with $y_{\text{edge}} = 60\, \text{mm}$ and various step heights $h$.

Figure 4.5 shows the $y$-position of the vortex centre as a function of time for various step heights. The vortex centre is defined as the position inside the dipole half where the horizontal velocity, which is the component of the velocity in the $x,y$-plane, is minimal. In the graph of step height $h = 0\, \text{mm}$ (no step) a velocity decrease of the dipole propagation is visible, which is due to a decrease of the vortex strength predominantly by friction at the bottom. The line for $h = 1\, \text{mm}$ shows that the propagation of the vortex centre at the surface is nearly not influenced by such a small step. When the step height $h = 3\, \text{mm}$ the vortex centre slows down before the step at $t = 4\, s$ but accelerates again from $t = 5\, s$ to pass the step. The line of step height $h = 4\, \text{mm}$ shows the same, but the centre of the dipole half is not able to pass the step at all. In the case that the step height $h = 5\, \text{mm}$ the created opposite vorticity is bigger, so the centre of the vortex is stopped earlier. After $t = 6\, s$ the
distance between the vortex centre and the step increases again, which is a rebounding process like in the case where the dipolar vortex travels towards a wall (see section 2.5), but now much weaker. This is caused by the created oppositely signed vorticity, which forms a new a-symmetric dipole with the original dipole half. The rebounding process is not very pronounced in these experiments: after $t = 5$ s the dipolar vortex does barely propagate anymore. Only a strong decay of its strength is observed.

![Figure 4.5](image)

**Figure 4.5:** Position of the vortex centre in the y-direction as a function of time for several step heights, from experiments with $y_{\text{edge}} = 60$ mm. The dots are data points.

The paths of the centre of the dipole half from the experiments with $h = 3$ mm and $h = 5$ mm are shown in figure 4.6. For the first one, which is able to pass the step, the effect described in section 2.3 is present: the distance between the vortex cores of the dipole increases when the dipole propagates into shallower water. This can also be observed in figure 4.7 that shows the distance $d$ between the vortex centres as a function of time. The line of $h = 5$ mm in both figures shows that there has to be another reason for the increasing of $d$. When the dipole does not pass the step, the presence of the oppositely signed vorticity leads to a decrease of $d$ again. This is a property of the rebounding process that is described in section 2.5. Compared to the situation with a vertical wall the oppositely signed vorticity that is created at the step is less intense. Also the influence of the bottom friction is very large in the shallow fluid layer. This can explain why only a weak first part of the dipole rebounding process can be observed in the experiments.
Figure 4.6: Path of the vortex centre, from two experiments with $y_{\text{edge}} = 60$ mm and different step heights. The path is shown from $t = 1$ s (the end of the forcing stage) to $t = 9$ s. The y-axis ($y = 0$) is the symmetry axis of the dipole.

Figure 4.7: Distance $d$ between the vortex centres of the dipole halves as a function of time, from the same experiments as in figure 4.6. Due to the symmetry axis at $x = 0$ this distance is twice the x-position of the vortex centre.
4.2 Simulations of the flow at the free surface of the fluid layer

The experiments in the water tank provide only data for the velocity at the free surface of the fluid layer. The results that are provided in this section are numerically computed flows at \( z = 7.5 \) mm, a half mm below the water surface. The simulations that are performed are listed in section 3.3. Figure 4.8 shows an example of the numerically calculated flow field after the forcing of a dipolar vortex, with \( y_{\text{edge}} = 45 \) mm and \( h = 5 \) mm. In figure 4.9 the result of this simulation is compared with simulations with a smaller step (\( h = 3 \) mm) and no step at all (\( h = 0 \) mm). In the figures not all vectors are shown, in order to make the representation more clear. In the simulations only one dipole half is shown, the vertical surface \( x = 0 \) is a symmetry plane.

After the forcing, the vorticity patches have a kidney-like shape, which is typical for the magnetic field produced by a cylindrical magnet. Around the negative dipole half two patches of oppositely signed vorticity arise. A band of positive vorticity is observed at the symmetry plane (A), for simulations with and without a step. This implies that the presence of this band of vorticity is not caused by the step. Cieślik \cite{Cie09} demonstrated that this vorticity band originates from non-uniform forcing by the magnet, which implies production of vorticity. This band is not visible in the experimental results (see for example figure 4.1). A second band of positive vorticity is observed at the step. This is also seen in the experiments and is caused by the no-slip condition at the step. This oppositely signed vorticity is responsible for a movement of the dipole half in positive \( x \)-direction. Hence the dipole halves separate from each other. The oppositely signed vorticity forms a band around the dipole half, as is also observed in the experiments. At later stages in the vortex evolution (see figure 4.9) some fragmentation of the vorticity is visible, which becomes less pronounced for increasing step size.
Figure 4.8: Velocity field (arrows) and $\omega_z$ (colours, in s$^{-1}$) at the fluid surface of a dipole moving towards a step with $h = 5$ mm, at different times. Positions are given in mm. The step, at $y_{edge} = 45$ mm, is indicated with a black horizontal line. The black dot denotes the centre of the magnet.
Figure 4.9: Velocity field (arrows) and $\omega_z$ (colours, in s$^{-1}$) at the fluid surface with no step (left column), step $h = 3$ mm (middle column) and step $h = 5$ mm (right column). The step, at $y_{\text{edge}} = 45$ mm, is indicated with a black horizontal line. The black dot denotes the centre of the magnet.
Figure 4.10 shows the situation with larger step heights, and with a wall instead of a step. At higher step heights the positive vorticity created at the step is stronger. Also a second patch of oppositely signed vorticity (C), now negative, is observed. It is caused by the advection of the positive vorticity (B) around the original vortex away from the step and, again, the no-slip condition at the step. This negative vortex patch fragmentises the positive patch and influences in that way also the form of the original dipole half.

The \( y \)-position of the vortex centre as a function of time is given in figure 4.11, for several step heights \( h \). Just like in the experiments the dipole half travels towards the step. The height of the step and the velocity of the dipole are factors that define if the dipole passes it or not. For the small step, \( h = 1 \) mm, the vortex centre is able to pass the step. For step heights \( h \geq 3 \) mm the vortex centre is not able to pass the step, but the lower the step the closer the centre can get to the step. Later on in the evolution, the distance from the vortex centre to the step increases again. This is caused by the oppositely signed vorticity created in front of the step, which forms a new a-symmetric dipole with the original dipole half.

In figure 4.12 the experiments and the numerical simulations are compared, for the case without a step and with step height \( h = 5 \) mm. The results match very well, which is also the case for other step heights. It can be concluded that the performed numerical simulations are in good agreement with the experiments.
Figure 4.11: Position of the vortex centre in the y-direction as a function of time for several step heights. In these numerical simulations \( y_{\text{edge}} = 45 \text{ mm} \).

Figure 4.12: Position of the vortex centre in the y-direction as a function of time. Experimental and numerical results are compared for a flat bottom \((h = 0 \text{ mm})\) and a step \((h = 5 \text{ mm})\) with \( y_{\text{edge}} = 45 \text{ mm} \).
4.3 Three-dimensional behaviour of the numerically obtained flow

A dipolar vortex in a shallow fluid layer has three-dimensional properties that are not negligible (see section 2.6). In numerical simulations the flow differs as a function of the z-position. Contrary to the two-dimensional approach, the flow velocities and the x,y-position of the vortex centre are not equal at different depths. This is also the case for simulations with a step. Figure 4.13 shows the trajectory of the centre of the vortex evaluated at different heights z. There is a smooth transition around z ≈ h, but there is a clear difference in the trajectory of the vortex centre between z < h and z > h. The vortex column is not a straight cylinder in vertical direction but bends during the propagation.

In the right diagram of figure 4.13 the trajectories of the centre of the dipole half at two different depths are compared for the case without a step (dashed lines) and with a step (solid lines). It can be concluded that the difference in the trajectories in the first part of the dipole propagation is not caused by the step, but is a property of the vortex itself. As well with as without the step more broadening of the dipole is observed when z increases. The deflection away from the dashed path is caused by the step. This deflection takes place earlier for z < h (where, as will be showed later, the oppositely signed vorticity is formed) then for z > h.

Figure 4.13: Sketch of the trajectory of the vortex centre of the right dipole half at various z-positions, when the dipole propagates towards a step (left). The trajectories at two fluid depths are compared with the case without a step (right). The thickness of the fluid layer at y < y_edge is 8.0 mm, the step height h = 5.0 mm and the step position y_edge = 45 mm. The grey area denotes the position of the magnet.
The dipole halves are not split up into an upper (above the step) and lower (beneath the step) part due to the step. Figure 4.13 as well as figure 4.14 show that the difference between the y-position of the vortex centres grows, but after a longer time the centres come together again. The latter figure shows the y-position of the vortex centre as a function of time, at different heights z. When the dipolar vortex approaches the step, the lower part of the vortex column is stopped by the step. The upper part goes slightly further, but is drawn back by the vortex column. This is visualized more clearly in figure 4.15, which shows the central axis (the line of vortex centres at each z-position) of the vortex column at different times t. The central axis is decelerated by the step. At t = 2.2 s it is clearly visible that for z > h this deceleration, compared with the case without a step, is smaller than for z < h.

![Figure 4.14: The y-position of the centre of the planar vortex half as a function of time, for various heights z in the fluid. The step height h = 5.0 mm and the step position y_{edge} = 45 mm (indicated with the thick black line).](image)

Figure 4.16 shows the vorticity $\omega_y$ in a vertical slice in the y-direction taken at x = 12.5 mm (the position of the edge of the magnet). Figure 4.17 shows top view plots of $\omega_y$ at various depths in the fluid. With help from these plots, the form of the central axis of the vortex column in figure 4.16 at, for example, t = 3.4 s can be made comprehensible.
Figure 4.15: The y-position of the centre of the planar vortex half as a function of the z-position in the fluid, for various times. The thickness of the fluid layer is 8.0 mm, the step height $h = 5.0$ mm and the step position $y_{edge} = 45$ mm (indicated with the thick black line). The dashed lines show the result of simulations without a step, at $t = 1.0$ s, $t = 2.2$ s and $t = 3.4$ s.

At $z > h$ (in this case $z > 5$ mm) no oppositely signed vertical vorticity is created, simply because there is no no-slip wall in front of the dipole at that height. The vortex half is able to propagate forward there, and is able to pass the step a little. Near the bottom of the tank the horizontal velocities are very small, so there is also no oppositely signed vertical vorticity created. This explains why the y-position of the vortex centre is greater near the bottom than in the middle of the fluid: the movement of it is stopped later. Between $z = 1$ mm and $z = 4$ mm the horizontal velocities are greatest, so at these heights most of the oppositely signed vorticity is created, which holds back the original vortex.

Figure 4.16: Vorticity $\omega_z$ (colour scale, see figure 4.17, in s$^{-1}$) in the y,z-plane at $x = 12.5$ mm and $t = 1.7$ s (top figure) and at $x = 17$ mm and $t = 3.4$ s (lower figure). The dipole half travels in this simulation towards a step with height $h = 5$ mm. The frontal circulation is referred to in section 4.4.
Figure 4.17: Vorticity $\omega_z$ (colour scale, in s$^{-1}$) in the x,y-plane at different heights $z$, for $t = 1.7$ s (left column) and $t = 3.4$ s (right column). The dipole half travels in this simulation towards a step with height $h = 5$ mm.
A similar visualization of the vortex column, but now in the x,z-plane, is given in figure 4.18. The vortex centres move to the right because of the broadening of the dipole half and the presence of the oppositely signed vorticity. At \( t = 1.0 \) s and \( t = 2.2 \) s the x,z-profile is nearly not influenced by the step. Later, the difference between these profiles with and without the step is most pronounced at \( z < h \) (here: \( z < 5 \) mm). The propagation in the x-direction of the vortex centre close to the bottom is mostly caused by the oppositely signed vorticity. At \( z = 7.5 \) mm it can be observed that the x-position of the vortex centre is, at every instance, almost equal for the cases with and without the step. So the propagation in the x-direction of the vortex centre close to the surface is not caused by the presence of the step. This is a property of the dipolar vortex in a shallow fluid layer itself. In the middle of the fluid, the increase of the distance between the centres of the dipole halves is caused by a combination of the two mentioned properties.

![Figure 4.18: The x-position of the centre of the planar vortex half as a function of the z-position in the fluid, for various times. This is the same simulation as in among others figure 4.15. The results are compared with the results from a simulation without a step (dashed lines in the corresponding colours).](image)

### 4.4 Vertical flows

As shown in the previous section, the vertical vorticity \( \omega_z \) varies with height significantly. According to the cyclostrophic balance, this means that the pressure must also vary with \( z \). This vertical pressure gradient will drive a vertical motion. So, despite of the quasi-two-dimensionality of the flow, there will be vertical velocities \( w \). Figure 4.19 shows these vertical velocities for two step heights \( (h = 3 \) mm and \( h = 5 \) mm) and for the case without a step. These are results of the same numerical simulations as the results in figure 4.9, where the vertical vorticity at the surface is shown.
Figure 4.19: Vertical velocity $w$ (colour scale, in m/s) and horizontal velocity (arrows) of a propagating dipolar vortex, from the numerical simulations. These are the vertical velocities at $z = 4$ mm, so in the middle of the fluid layer. The left column shows the case without a step, the middle column with a step with $h = 3$ mm and the right column with a step with $h = 5$ mm. The position of the step ($y_{edge} = 45$ mm) is indicated with a black line. The position of the centre of the magnet is denoted by a black dot.
After the electromagnetic forcing has stopped, at \( t = 1 \) s, an upward motion in the vortex cores (A, in figure 4.19) is observed. It is surrounded by a band of negative vertical velocity (B), which is especially strong at \( y < y_{\text{centre}} \). Also a flow structure with an upward motion in front of the dipole (C) and a, much weaker, downward motion in front of that (D) is observed. This is referred to as the “frontal circulation”, and acts like a vortex roll with a horizontal vorticity \( \omega_x \). This circulation is also visible in the velocity field in figure 4.16. There is also a strong positive vertical motion in the symmetry plane of the dipole (E), which corresponds with the tail with oppositely signed vorticity indicated in figure 4.8. This tail is not visible in the experiments. When the time progresses the sign of the vertical velocity in the vortex centres switches a few times. These properties of the dipolar vortex in a shallow fluid layer correspond to the properties described in section 2.6 and are extensively discussed in [Akk08]. In regions with strong vertical velocity its magnitude turns out to be 25% of that of the horizontal velocities, signifying that vertical motions are of importance in such shallow layer flows.

Cieślik [Cie09] observed an increase of vertical velocities during the collision of a dipole with a lateral wall. In the simulations described above with a step instead of a wall similar behaviour is observed, although in a weaker form. As can be seen at \( t = 1 \) s in figure 4.19, the magnitude of all vertical velocities increases when the step height increases. This is already the case immediately after the forcing of the vortices, so before the dipole meets the step. In time, especially the frontal circulation becomes stronger (relative to other velocities), because it is narrowed between the step and the dipolar vortex. There is also production of vertical velocity at the step (F), caused by the fact that the production of oppositely signed vorticity varies with height. This vertical velocity is weak compared with the vertical motions in the dipole and the increase of it by the step. The downward and upward motion inside the dipole are rather unaffected by the interaction with the step.

The amount of vertical motion can be described by the diagrams in figures 4.20 and 4.21. The first shows the amount of vertical kinetic energy per unit mass

\[
T_{\text{vert}} = \iiint\frac{u^2}{2} \, dx \, dy \, dz \tag{4.3}
\]

computed over the full domain of the simulation, as a function of time. The amount of vertical kinetic energy increases not only during the creation of the dipole \( (0 < t < 1 \) s), but also in the first approximately half second after the forcing. After that, it decreases gradually predominantly due to bottom friction. The peak of vertical kinetic energy is twice as high for the situation with a wall at \( y_{\text{edge}} = 45 \) mm as for the situation with a flat bottom. The results of simulations with a step are in between these situations. There, the amount of vertical velocity is larger when the step is bigger. The step height \( h = 7 \) mm is an unexpected exception on that.

Figure 4.18 shows the ratio of \( T_{\text{vert}} \) and the horizontal kinetic energy

\[
T_{\text{hor}} = \iiint\frac{u^2 + v^2}{2} \, dx \, dy \, dz \tag{4.4}
\]

as a function of time for the various step heights and taken over the full domain of simulation. This kinetic energy ratio

\[
q = \frac{T_{\text{vert}}}{T_{\text{hor}}} \tag{4.5}
\]
is bigger for bigger step heights. Its maximum value is approximately \( q = 0.02 \), which means that the vertical components of the complete velocity field are approximately 15\% of the horizontal components. The amount of vertical kinetic energy decreases faster than the amount of horizontal kinetic energy. In both diagrams a strange peak at \( t = 4 \) s is shown, which does not appear when there is a step in the fluid layer depth. It corresponds with a sudden large increase of the vertical velocity in front of the dipole (see figure 4.22) at \( t = 4 \) s, which disappears gradually in a few seconds. The cause of this is unknown. In the configurations with a step this vertical movement is not observed.

![Figure 4.20: Total vertical kinetic energy per unit mass \( T_{\text{vert}} \) (in m\(^2\)/s\(^2\)) as a function of time \( t \) (in s), for various step heights.](image)

**Figure 4.20**: Total vertical kinetic energy per unit mass \( T_{\text{vert}} \) (in m\(^2\)/s\(^2\)) as a function of time \( t \) (in s), for various step heights.
Figure 4.21: Ratio $q$ of vertical and horizontal kinetic energy as a function of time $t$ (in s), for various step heights.

Figure 4.22: Sudden large increase of the vertical velocity $w$ (figures left, colour scale, in m/s) for the flat configuration ($h = 0$) around $t = 4$ s. The right figure shows the vertical vorticity $\omega_z$ (colour scale, in s$^{-1}$) for $t = 4.0$ s at $z = 4.0$ mm, in which disturbances are visible at the same position in the flow.
5. Results: slope configuration

This chapter presents the results of experimental and numerical simulations of a dipolar vortex that travels towards a slope in the fluid dept. First, the flow at the free surface of the fluid is described. Then the numerically obtained flow is examined at various depths in the fluid layer. The three-dimensionality of the flow is also described in the last section on vertical velocities.

5.1 Experimental results

In the experiments the flow at the fluid surface was measured just like in the experiments with a step (section 4.1). In section 3.1 an overview of the performed experiments can be found. Slopes at various positions $y_{edge}$ and with different sloping angles $\alpha$ were used. Only results with $y_{edge} = 45$ mm are reported. The qualitative results of experiments with $y_{edge} = 60$ mm are equal but the effects of the sloping bottom on the flow are less distinct, because the dipolar vortex is less strong when it reaches the slope due to damping. Figure 5.1 shows an example of the measured flow field at the water surface after the forcing of a dipolar vortex, with $y_{edge} = 45$ mm and $\alpha = 6.8^\circ$. In figure 5.2 this case is compared with experiments with smaller slopes ($\alpha = 6.0^\circ$ and $\alpha = 3.4^\circ$). The maximum Reynolds number in the horizontal flow at $t = 1$ s is $Re \approx 1500$. Not all velocity vectors are shown, in order to make the representation more clear. To improve the visibility of structures with low vorticity in the figures, the range of the colour scale is chosen small. All vorticity values that exceed the colour scale are coloured according to the maximum or minimum of the scale. The frayed shape of the vortices at $t = 2$ s, $t = 3$ s and $t = 4$ s is caused by the inaccurate calculation of the flow by PIVview at large velocities.

Most pronounced in the flow dynamics as illustrated in figure 5.1 and 5.2 is the formation of oppositely signed vorticity above the sloping bottom, in front of the dipole half. This vorticity is not spatially confined to a patch-like region, but spread out over the area between the dipole half and the shoreline. The formation of it starts together with the creation of the dipole, but gets stronger when the dipole comes closer to the shoreline. This is caused by a no-slip condition at the sloping bottom. The oppositely signed vorticity induces a movement of the original dipole halves parallel to the step and away from the symmetry plane $x = 0$. Due to the fact that the strength of the area of oppositely signed vorticity is much smaller than that of the dipole half, it is wrapped around the dipole half as a sort of band. This prevents the dipole half to continue travelling in the positive $x$-direction.

The creation of oppositely signed vorticity was also observed in the experiments with a step. The shape of it differs. In the case with a step (see for example figure 4.3) it is a compact region of concentrated positive vorticity that arises only close to and just in front of the step. In the case with a slope this region is more diffuse: it arises above the complete sloping bottom. As observed in experiments with a step and described in investigations with a wall (for example [Orl90] and [Cie09]), oppositely signed vorticity in the $z$-direction is formed at a vertical no-slip boundary. Between $y_{edge}$ and $y_{shore}$ the slope has a vertical component, which leads to the creation of the oppositely signed vorticity in that area. The maximum magnitude of the oppositely signed vorticity created in the experiments with a slope is less than in the experiments with a step.
Figure 5.1: Velocity field (arrows) and $\omega_z$ (colours) at the fluid surface of a dipole moving towards and over a slope with $\alpha = 6.8^\circ$, at different times. The horizontal lines indicate the start of the slope ($y_{\text{edge}} = 45$ mm) and the shoreline ($y_{\text{shore}} = 101$ mm). The forcing current is 3 A.
Figure 5.2: Velocity field (arrows) and $\omega_z$ (colours) at the fluid surface at different times of a dipole moving towards and over a slope with $\alpha = 3.4^\circ$ (left column), $\alpha = 6.0^\circ$ (middle column) and $\alpha = 6.8^\circ$ (right column). The beginning of the slope and the shoreline are indicated with black horizontal lines. The shoreline of experiments with $\alpha = 3.4^\circ$ is out of the measurement area.
Figure 5.3 shows the approximate maximum value of the created oppositely signed vorticity and the vorticity in the original dipole half as a function of time, for the configurations of figure 5.2. The vorticity in the centre of the original dipole half decreases by damping and the broadening of the vortex. The damping is caused mostly by the no-slip condition at the bottom, leading to bottom friction. As shown in section 2.4 the effect of the bottom friction increases when the fluid depth decreases, so the propagating dipole experiences damping that increases due to the sloping bottom. This implies a strong decay of the vortex flow. The oppositely signed vorticity increases gradually when the dipole approaches the shoreline.

![Figure 5.3](image)

*Figure 5.3: Indication of the absolute value of the vorticity \( \omega \), in the centre of the original dipole half (blue) and the maximum value of the created oppositely signed vorticity (red), as a function of time.*

In figure 5.4 the \( y \)-position of the vortex centre is shown as a function of time for three different slope angles. The propagation in the \( y \)-direction is, in the first seconds, approximately equal for the three cases. At \( t = 4 \) s the vortex centre is at the beginning of the slope, where it starts to experience a decrease of the fluid depth. The velocity in the \( y \)-direction decreases due to the increasing bottom friction at the no-slip bottom and the presence of oppositely signed vorticity in front of the dipole half. When the slope \( \alpha \) is larger, the influence of the bottom is larger and (as can be seen in figure 5.2) there is more production of oppositely signed vorticity. This explains that the propagation of the vortex half ends earlier and at a lower \( y \)-position when \( \alpha \) is bigger.

The presence of the oppositely signed vorticity causes a process similar to the dipole rebounding, which is described in section 2.5. Due to the very small amount of vertical oppositely signed vorticity and the strong decrease of the strength of the original dipole half, only a small part of the rebounding process is observed. Figure 5.5 shows the path of the centre of the dipole half. Only in experiments with the steeper slope (for example \( \alpha = 6.8^\circ \)) an increase of the distance \( d = 2x \) between the centres of the dipole halves that is caused by the oppositely signed vorticity is observed. The increasing \( d \) in the experiment with \( \alpha = 3.4^\circ \) is mainly caused by the broadening of the vortex due to diffusion, not by oppositely signed vorticity.
Figure 5.4: Position of the vortex centre in the y-direction as a function of time for three slope angles, from experiments. The dots are data points. The black horizontal line indicates the start of the slope.

Figure 5.5: Path of the vortex centre (left), from two experiments with different slopes. The path is shown from $t = 1$ s (the end of the forcing stage) to $t = 10$ s. The y-axis ($x = 0$) is the symmetry axis of the dipole. The decrease of the fluid depth is shown in the right diagram.
5.2 Simulations of the flow at the free surface of the fluid layer

The results that are provided in this section are numerically computed flows at \( z = 7.5 \) mm, a half millimetre below the fluid surface. The simulations that are performed are listed in section 3.3. Figure 5.6 shows an example of the numerically calculated flow field after the forcing of a dipolar vortex, with \( \gamma_{\text{edge}} = 45 \) mm and \( \alpha = 6.5^\circ \). In figure 5.7 this situation is compared with simulations with smaller slope angles \( \alpha \). In order to make the representation more clear, not all vectors are shown in the figures.

Just like in the simulations with a step (see section 4.2), two patches of oppositely signed vorticity arise after the forcing. First a tail of positive vorticity at the symmetry plane is present (A), which is a property of the dipole itself and is not caused by the sloping bottom. Second, a region of oppositely signed vorticity between the dipole half and the shoreline is observed (B). The position and the shape of it correspond to the experimental results in the previous section. Most vorticity is produced close to the shoreline. An explanation for the spreading of the oppositely signed vorticity will be given in the next section. Just like in the experiments, the created oppositely signed vorticity forms a new asymmetric dipole with the original dipole half. This causes propagation in the x-direction, which corresponds to the beginning of the rebounding process that is described in section 2.5 and is also observed in experiments and simulations with a step in the fluid depth. Besides the rebounding also broadening of the vortex can be observed. This is caused by diffusion and the squeezing of the vortex column (see section 2.3).

To create vertical vorticity \( \omega_z \) with an opposite sign, a horizontal flow velocity and a vertical component in the no-slip wall are needed. The sloping bottom can be regarded as a wall with a horizontal and a vertical component. When the slope is steeper, the vertical component of it is bigger and more vertical oppositely signed vorticity can be created. This can be seen in as well as the experiments (figure 5.2) as the simulations (figure 5.7): the bigger \( \alpha \), the more pronounced is the created oppositely signed vorticity above the slope.
Figure 5.6: Velocity field (arrows) and $\omega_z$ (colours, in $s^{-1}$) at the fluid surface of a dipole moving towards and above a slope with $\alpha = 6.5^\circ$, at different times. The start of the slope, at $y_{\text{edge}} = 45$ mm, is indicated with a black horizontal line. The shoreline is at $y_{\text{shore}} = 115$ mm. The black dot denotes the centre of the magnet. The line $x = 0$ mm is a symmetry line.
Figure 5.7: Velocity field (arrows) and \( \omega_z \) (colours, in \( s^{-1} \)) at the fluid surface with three different slopes: \( \alpha = 3.4^\circ \) (left column), \( \alpha = 5.0^\circ \) (middle column) and \( \alpha = 6.5^\circ \) (right column).
The y-position of the vortex centre as a function of time is given in figure 5.8, for all three slope angles of figure 5.7 and the case without a slope. Just like in the experiments the propagation of the dipole half in the y-direction is decelerated when the dipole experiences the slope. As expected, and as follows from the previous figures, this deceleration is faster when $\alpha$ is bigger. When the velocity of the vortex centre in the y-direction decreases an increasing velocity in the x-direction is observed, which is induced by the oppositely signed vorticity. At $t = 8$ s, the end of the simulation, the dipole half practically does not propagate anymore. As in the experiments the vortex strength decreases fast, mainly caused by the bottom friction. This corresponds to the results of simulations with a step instead of a slope.

![Graph](image)

*Figure 5.8: Position of the vortex centre in the y-direction as a function of time for several slope angles.*

In figure 5.9 the propagation of the vortex centre in the y-direction are compared for the experiments and the numerical simulations. The results match reasonably.
Figure 5.9: Position of the vortex centre in the y-direction as a function of time. Experimental and numerical results are compared for two different slope angles.

5.3 Three-dimensional behaviour of the numerically obtained flow

Already in section 4.3 it is shown that the horizontal components of the flow differ as a function of the z-position. Figure 5.10 shows this for a numerical simulation with a sloping bottom. The trajectory of the centre of the dipole half is evaluated at different heights z. As observed earlier, the vortex column is not a straight cylinder in the vertical direction but bends during the propagation. Just like in the simulations with a step (section 4.3) and a flat bottom (dashed lines) the distance between the centres of both dipole halves increases with increasing z. Then, when the dipole experiences the slope, the vortex deviates more from the path of the flat bottom case at low z-values than at high z-values.
Figure 5.10: Sketch of the trajectory of the vortex centre of the right dipole half at various z-positions, when the dipole propagates towards and above a slope (left). The trajectories at two fluid depths are compared with the case without the slope (right). The thickness of the fluid layer at $y < y_{\text{edge}}$ is 8.0 mm and the slope angle is $\alpha = 6.5^\circ$. The grey area denotes the position of the magnet.

Figure 5.11 shows that the propagation of the centre of the dipole half in the y-direction is decelerated more close to the bottom, which is also observed in simulations with a step. At the end of the simulation, where the dipole centre nearly does not propagate anymore, the difference between the positions of the centre at different heights becomes smaller again. This is visualized more clearly in figure 5.12, which shows the central axis (the line of vortex centres at each z-position) of the vortex column at different times $t$. It shows that, also when there is no slope, the dipole experiences less deceleration at the free surface than at the bottom. This is due to the influence of the bottom friction, which is stronger close to the bottom. In the case with a slope, besides the deceleration caused by the bottom friction, extra deceleration takes place at all fluid depths due to the oppositely signed vorticity.
Figure 5.11: The $y$-position of the centre of the planar vortex half as a function of time, for various heights $z$ in the fluid. The slope angle $\alpha = 6.5^\circ$ and the start of the slope $y_{\text{edge}} = 45$ mm.

Figure 5.12: The $y$-position of the centre of the planar vortex half as a function of the $z$-position in the fluid, for various times. This is the same simulation as in figure 5.10. The results are compared with the results from a simulation without a slope (dashed lines in the corresponding colours).

Figures 5.13 and 5.14 show where the oppositely signed vorticity is created. When the fluid layer is thought of as being composed of several horizontal slices, this vorticity is created at the ‘shoreline’ (the maximum value of $y$) of each such a slice. This line can be regarded as a sort of no-slip wall for that slice. As was seen in the simulations with a step (section 4.3) the oppositely signed vorticity is created just before that wall. This results, for the case with a sloping bottom, in a sheet of vorticity on the sloping bottom (see figure 5.14). When the dipole propagates above the slope, this vorticity is advected forward towards the much shallower area before the shoreline. There, it is also found at the fluid surface and it is able to participate in the rebounding process that is described earlier.
Figure 5.13 (please rotate the page): Vorticity $\omega_z$ (colour scale, in $s^{-1}$) in the $x,y$-plane at different heights $z$, for $t = 2.5$ s (upper row) and $t = 5.0$ s (lower row). The dipole travels in this simulation towards and above a slope with $\alpha = 6.5^\circ$.

$z = 1.0\ mm$
$z = 2.5\ mm$
$z = 4.0\ mm$
$z = 5.5\ mm$
$z = 7.0\ mm$
Figure 5.14: Vorticity $\omega_z$ (colour scale, see figure 5.13, in s\(^{-1}\)) in the y,z-plane at $x = 12.5$ mm and $t = 1.5$ s (top figure), $x = 14.0$ mm and $t = 2.5$ s (middle figure) and $x = 20.2$ mm and $t = 5.0$ s (lower figure). The dipole travels in this simulation towards and above a slope with $\alpha = 6.5^\circ$.

As already mentioned in section 5.2 the oppositely signed vorticity at the fluid surface is spread out much more than for example in experiments with a step. This vorticity is most intense close the no-slip sloping bottom.

A visualization of the projection of the central axis of the vortex column in on the x,z-plane is given in figure 5.15. Besides the propagation in the y-direction the vortex centres also propagate in positive x-direction. The increasing distance $d = 2x$ between the dipole halves in the higher z-areas for $t < 3$ s is equal to the case without a slope (dashed lines). After $t \approx 3$ s the oppositely signed vorticity starts to influence the propagation in the x-direction at all depths in the fluid layer. Contrary to the case with a step (figure 4.18) no clear difference between the upper and lower areas is observed.

Figure 5.15: The x-position of the centre of the planar vortex half as a function of the z-position in the fluid, for various times. This is the same simulation as in among others figure 5.10. The results are compared with the results from a simulation without a slope (dashed lines in the corresponding colours).
5.4 Vertical flows

The variation of the vertical vorticity $\omega_z$ with height leads, as explained in section 2.6, to vertical motions in the flow. Figure 5.16 shows the velocity in the $z$-direction for two different slopes ($\alpha = 3.4^\circ$ and $\alpha = 6.5^\circ$) and for the case without a slope. After the electromagnetic forcing has stopped, at $t = 1$ s, the flow structures that are described in section 2.6 can be observed. These were also visible in the simulations with a step (section 4.4). The vertical velocities in the configurations with a sloping bottom differ only slightly from those in the flat bottom configuration ($\alpha = 0^\circ$). Contrary to the case with a step the vertical velocities at the front of the dipole decrease when $\alpha$ increases, due to the decreasing fluid depth. The frontal circulation, which was mentioned in section 4.4 and 2.6, is strongly damped due to the increased shallowness of the fluid layer that the dipole is entering. This corresponds with simulations of Cieślik [Cie09] that showed that the frontal circulation does not fully develop as the dipole propagates up-slope. The strong vertical velocity that is shown at $t = 5$ s in the flat bottom simulation can be observed at the slope $\alpha = 3.4^\circ$ in a much weaker form, and is not visible anymore at the slope $\alpha = 6.5^\circ$.

In the diagrams of figure 5.17 and 5.18 the amount of vertical kinetic energy $T_{vert}$ and the kinetic energy ratio $q$ (see equations 4.3 and 4.5) are plotted for the configurations of figure 5.16. The diagrams show, in the first three seconds, very little difference between the cases with and without a slope. The peak (A) for the case without a slope at $t = 4.2$ s corresponds with the strong vertical velocity visualized in figure 5.16 at $t = 5$ s. For the case with slope $\alpha = 3.4^\circ$ this peak (B) arises later and its amplitude is much smaller, due to the decreased fluid depth. Figure 5.19 shows the vertical velocity at the moments that these peaks arise. The increase of the vertical kinetic energy is mainly caused by the locally increasing vertical velocity (positive and negative) indicated in the figures (A and B). In the simulation with slope $\alpha = 6.5^\circ$ this peak has fully disappeared (as was also shown in figure 5.16), just like in the simulations with a step.
Figure 5.16: Numerically obtained vertical velocity $w$ (colour scale, in m/s) and horizontal velocity (arrows) at $z = 4$ mm for a propagating dipolar vortex. Shown are the cases without a slope (left column), slope $\alpha = 3.4^\circ$ (middle column) and slope $\alpha = 6.5^\circ$ (right column). The position of the start of the slope ($y_{\text{edge}} = 45$ mm) is indicated with a black line. The position of the centre of the magnet is denoted by a black dot.
Figure 5.17: Total vertical kinetic energy per unit mass $T_{\text{vert}}$ (in m$^2$/s$^2$) as a function of time $t$ (in s), for the case without a slope and two different slope angles.

Figure 5.18: Ratio $q$ of vertical and horizontal kinetic energy as a function of time (in s), for the case without a slope and two different slope angles.
Figure 5.19: Horizontal (arrows) and vertical (colour scale, in m/s) velocity at $z = 4$ mm, for a dipolar vortex propagating above a flat and a sloping bottom. Shown are the instances that a strong vertical velocity appears in the front of the dipole half.
6. Conclusions and recommendations

6.1 Conclusions
In this research project the behaviour of a dipolar vortex in a shallow fluid layer that propagates towards an up-going step or slope in the fluid depth was investigated, by experiments and numerical simulations. Both configurations have in common that the fluid depth decreases in the direction of propagation of the dipole.

Due to the increase of the bottom friction the deceleration of the propagation of the dipole halves is increased. The maximum vorticity in the z-direction decreases faster in experiments and simulations with a step or slope than with a flat bottom. For the step configuration this is observed in that part of the vortex that passes the step: this part is damped rapidly.

More pronounced is the formation of oppositely signed vorticity that is created at a no-slip wall. In the experiments and simulations this is just before the step or above the slope. The oppositely signed vorticity forms, with the original dipole half, a new asymmetric dipole that propagates parallel to and later on away from the step or shoreline. Due to the large damping in the shallow fluid layer, only a small part of this rebounding process is observed.

For the step configuration the created oppositely signed vorticity is a small compact patch that originates between the step and the original dipole half, and is wrapped around the primary dipole half. In the slope configuration it is a diffuse area of vorticity that originates between the original dipole half, and is wrapped around it as a band.

The trajectory of the vortex centre of a dipole half depends on the z-position in the fluid. In the case with a step the broadening of the dipole close to the fluid surface is equal to that for the case of a flat bottom. At depths beneath the step height, a propagation of the vortex centre in the x-direction is caused by the oppositely signed vorticity, as in the dipole rebounding process. In the slope configuration oppositely signed vorticity is created at all depths, which causes a rebounding process at all depths. In general, the shape of the central axis of the vortex column deviates most from the flat bottom situation at the fluid depths where the oppositely signed vorticity is created.

In all three configurations (flat, step and slope) qualitatively the same vertical flow patterns are observed, for $t < 4 \text{ s}$. In the step configuration the ratio of vertical to horizontal kinetic energy is larger compared to the flat configuration. The simulations with a slope show no significant difference with the simulations without a slope. In the case with a flat bottom a disturbance around $t = 4 \text{ s}$ takes place. This disturbance is damped away by the step and slope.
6.2 Recommendations for further research

In connection with this research, some recommendations for further research can be given.

- In the experiments and numerical simulations only a small part of the rebounding process is observed, due to the damping of the vorticity by the no-slip bottom. In order to investigate the influence of the no-slip bottom on the flow, it is recommended to perform numerical simulations with a free-slip boundary condition at the bottom.

- In order to validate the numerically obtained flow at various depths, it is recommended to perform experiments in which the horizontal velocities are measured at different vertical positions in the fluid.

- The numerical simulations for the configuration without a step or slope show a disturbance in the flow in which the vertical velocity in the front of the dipole increases rapidly. The cause of this is not well known and may be a subject for later research.

- It would be interesting to perform experiments and simulations with more step heights, smaller than half of the fluid depth, in order to obtain information on the transition between the cases in which the dipole passes the step and in which it does not.

- Configurations with steeper slope angles in a full range between 0 and 90° are worth to investigate, for example to examine cases with a sloping bottom in which the kinetic energy ratio increases compared to the flat bottom configuration.
7. Bibliography


Appendix

Appendix A: Derivation of the magnetic field

The dipolar vortices in the experiments are created by a Lorentz force, which is the result of an electrical current and the magnetic field of a permanent magnet. The magnetic field of a single uniformly magnetized circular magnet can be derived with the Biot-Savart law, which relates the magnetic field to the magnetization current. The magnet with radius $r$ can be seen as a stack of circular current loops each having radius $r$, as sketched in figure A.1. Each current loop produces a magnetic field $\vec{B}_l$.

The $y$- and $z$-component of $\vec{B}_l$ are, according to for example [Cie09], in a Cartesian coordinate system $(x,y,z)$:

$$B_{l,y} = -B_0 \frac{y(z+z_0)}{x^2+y^2} \left[ K(n) - \frac{r^2+x^2+y^2+(z+z_0)^2}{(r-\sqrt{x^2+y^2})^2+(z+z_0)^2} E(n) \right] \quad (A.1)$$

$$B_{l,z} = B_0 s \left[ K(n) + \frac{r^2-x^2-y^2-(z+z_0)^2}{(r-\sqrt{x^2+y^2})^2+(z-z_0)^2} E(n) \right] \quad (A.2)$$

with:

$$s = \left[ \left( r + \sqrt{x^2 + y^2} \right)^2 + (z + z_0)^2 \right]^{-\frac{1}{2}} \quad (A.3)$$

The constant $B_0$ is unknown, because the magnitude of the loop current $I_l$ is not known a priori. The $z$-position of the current loop with respect to the origin is $z = -z_0$. $K(n)$ and $E(n)$ are, respectively, the so-called complete elliptic integral of the first and second kind, defined as:

$$K(n) \equiv \int_0^\pi \left( 1 - n^2 \sin^2 \varphi \right)^{-\frac{1}{2}} d\varphi \quad (A.4)$$

$$E(n) \equiv \int_0^\pi \left( 1 - n^2 \sin^2 \varphi \right)^{\frac{1}{2}} d\varphi \quad (A.5)$$

with $n$ the elliptic modulus.
The magnetic field $\vec{B}$ at a position in the fluid can be calculated by integrating the fields $\vec{B}_i$ from all current loops:

$$\vec{B}(x, y, z) = \int_{h_1}^{h_2} \vec{B}_i(x, y, z, z_0) dz_0$$  \hspace{1cm} (A.7)

The result can be put into equation 2.12 to calculate the forcing. The unknown constant $B_0$ can be determined by matching the results of experiments with numerically obtained results using the above magnetic field.

Appendix B: Determination of the multiplicative constant $A$

As mentioned in section 3.3 a multiplicative constant is needed to match the numerical simulations with the experiments. This constant $A$ is tuned by comparing the propagation of the vortex centre at the fluid surface in the $y$-direction during the creation of the dipole and the first second after that, in simulations and experiments. The constant fluid depth is 8 mm. Figure B.1 shows the result of the experiment and some simulations with different $A$. With $A = 220$ the results match the best.

![Figure B.1](image-url)

*Figure B.1: The $y$-position of the vortex centre as a function of time, for a dipole that is created in the first second and propagates above a flat bottom. The constant $A$ was tuned to match the simulations with the experiments.*
Appendix C: Vortex profile in experiments and simulations
Figure C.1 shows the profile of the vertical vorticity $\omega_z$ and the velocity $v$ in $y$-direction taken along a horizontal line through the vortex centre at the fluid surface, in the experimental and the numerical case. Regarding the amplitude of $\omega_z$ and $v$ these do not correspond. This is probably caused by the fact that at high velocities, which is especially the case at the end of the forcing, the flow is calculated more inaccurate by PIVview. Regarding the width of the vortices the diagrams do correspond, so the numerical and experimental dipoles are comparable.

![Figure C.1: Profile of the vertical vorticity $\omega_z$ (upper diagram) and the velocity in the $y$-direction $v$ (lower diagram) of a dipole in the experiments and a numerically obtained dipole, at $t = 1$ s (at the end of the forcing). The profile is taken parallel to the $x$-axis, at the fluid surface and through the centre of the vortex.](image)

Appendix D: Trajectory at various depths in the flat configuration
Figure E.1 shows the trajectory of the dipole centre at various fluid depths in the configuration without a step or slope. It can be concluded that the vortex column is not, as expected using the quaso-two-dimensional approach, a pure vertical cylinder.
Figure E.1: Propagation trajectory of the centre of the right dipole half at various depths $z$ in the fluid. The constant fluid depth is 8 mm.