A game-theoretical approach to the Dutch health insurance market
investigating the effects of the level and announcement sequences of price on key performance indicators of health insurers

Schlicher, Loe

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A game-theoretical approach to the Dutch health insurance market

- Investigating the effects of the level and announcement sequence of price on key performance indicators of health insurers -

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Abstract

In this thesis a mathematical model is developed to investigate the effects of the level and the announcement sequence of price on the key performance indicators of health insurers in the Dutch health insurance market. Herewith differential equations and Game Theory are used as key elements for the modelling of customer and health insurer behavior, respectively.
Preface

-In het Nederlands.-


Dank aan Marco Slikker, mijn eerste begeleider, dat hij mij de kans gegeven heeft een afstudeerrichting op te gaan die ver weg staat van logistiek, maar ook dank, omdat Marco mij heeft laten inzien dat een PhD traject iets voor mij is. Daarnaast dank aan Arun Chockalingam, mijn tweede begeleider, voor zijn constructieve feedback (thank you). Verder een dank aan Niels Van der Laan, mijn begeleider bij Milliman en vooral bedankt voor de leuke gesprekken over workouts, bootcampen, mooie auto’s (zoals bijvoorbeeld de beetle) en wielrennen. Daarnaast Ji Kwen NG bedankt voor de hulp in de laatste weken en Michiel Hochstenbach, omdat hij altijd tijd vrij maakt als ik met (wiskundige) vraagstukken zit. Als laatste wil ik ook pap en mam en Janneke bedanken. Het moet zwaar zijn geweest om al mijn gezwets te moeten aanhoren en dat voor minstens vijf lange jaren.

Loe Schlicher
Management Summary

In 2006 due to health reform a new Zorgverzekeringswet (Zvw) is introduced in the Netherlands. In this Zvw everyone in the Netherlands is required to purchase basic health insurance from private health insurers on a yearly basis. Currently health insurers get compensated through a risk equalization system to cover unfavorable risk profiles (i.e. people with high medical expenses). The idea of the government is to reduce this equalization system in the upcoming years. As a result, health insurers will take more risk, given that unfavorable risk profiles are not completely compensated anymore and as a consequence key performance indicators become more important for them. Key performance indicators are mainly effected in the yearly time period that people renew their policy. There are mainly two instruments that health insurers can use in this time period to effect key performance indicators, namely the level and the announcement sequence of price. According to Milliman, there is an insufficient knowledge of these instruments, which had lead to the following research question:

Research Question What is the effect of the level and announcement sequence of price on key performance indicators of health insurers in the Dutch health insurance market?

In the first part of the thesis attention is paid to general aspects of the Dutch health insurance market. Moreover, general information is given over the level and the announcement sequence of price and the important key performance indicators of health insurers.

In the second part of the thesis a mathematical model is developed that captures the choice behavior of policy holders and connects this with the key performance indicators. A system of differential equations, which is based on the level of prices, is used to model the choice behavior of policy holders and outcomes are used to determine the key performance indicators. For situations with more than two health insurers, it was not possible to derive exact solutions for the system of differential equations and as a consequence approximations are used. We have analyzed how well the mathematical model performs on historical data, whereafter a sensitivity analysis is executed regarding to fictive data. Finally some interesting cases, formulated by Milliman, are investigated. These cases are formulated as optimization problems and premiums that optimize on profit or market share, with other key performance indicators as constraints, are determined.
In the third part of the thesis a game theoretical approach is introduced to investigate, next to the level of price, the effect of the announcement sequence of price by considering health insurers as players of a game. Well-known solution concepts as Nash equilibria and Stackelberg equilibria are used to investigate the effects of the level and the announcement sequence of price. Pricing to a Nash equilibria results into stable profits and market share and increasing solvency ratios. Stackelberg equilibria have showed that announcing price before the others (which all announce simultaneously) is beneficial for all health insurers from a profit maximization perspective.

In the last part conclusions and recommendations are drawn. The mathematical tool gave insights in the effects of the level of price when profit or market share are optimized, while other key performance indicators are used as constraints. Moreover, the game theoretical approach gave insights in how key performance indicators behave when health insurers set prices regarding to a Nash or Stackelberg equilibrium. However, not all effects of the level and announcement sequence of price on key performance indicators are (and can be) investigated. Fortunately, the mathematical model is developed as an Excel tool, which gives Milliman the possibility to determines the effects of the level and announcement sequence of price on key performance indicators for any kind of situation in the future.
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Introduction
1 Introduction

During the exploration for a graduation project into the field of game theory and actuarial science, Milliman, an international consultancy company specialized in actuarial products and services, came into the picture. Milliman was founded in 1947 by Wendell Milliman and Stuart Robertson and nowadays has offices in major cities around the whole world. They operate at 54 offices worldwide with 2,500 employees, including more than 1,300 consultants and actuaries and owned by approximately 350 principals. Their main activity is consultancy of actuarial aspects such as health care, investment, life insurance, financial services and casualty insurance. The office of interest is the one located in Amsterdam, supervised by Laurens Roodbol. One of the topics of interest of this department is health care. At the moment there are interesting cases into this field of area.

1.1 Problem motivation

In 2006 due to health reform a new Zorgverzekeringswet (Zvw) is introduced in the Netherlands. In this new Zvw everyone in the Netherlands is required to purchase basic health insurance from private health insurers on a yearly basis. The Zvw imposes on health insurers an obligation to accept all who apply for basic insurance. Every year, health insurers set their own premium rate, which does not vary by policy holder, health status, or other risk characteristics. Then, policy holders have one and a half month to renew their policy. Afterwards, health insurers get compensated through a risk equalization system to cover unfavorable risk profiles (i.e. people with high medical expenses). The idea of the government is to reduce this equalization system in the upcoming years. As a result, health insurers will take more risk, given that unfavorable risk profiles are not completely compensated anymore and as a consequence key performance indicators become more important for them. This forms the motivation of this thesis.

1.2 Problem definition

As health insurers are more subject to risk, key performance indicators become more important for them. These key performance indicators are mainly affected in the time period that people renew their policy. Hence, this yearly time period is crucial for health insurers. There are mainly two instruments that health insurers can use in this time period to effect key performance indicators\(^1\). The first one is the level of price. Charging a too high price results in profitable insurances but against a low market share, while charging a too low price results in a higher market share but against

\(^1\)Health insurer can also control by advertisement and promotion, but this is not considered in this thesis.
non profitable (loss making) insurances. The second one is to control on the 
anouncement sequence of price. Announcing price too early could give 
competitors the opportunity to anticipate on your price, while announcing 
too late could give competitors the opportunity to set market price. With 
these instruments, health insurers can control and as a consequence affect 
key performance indicators. According to Milliman, there is an insufficient 
knowledge of the effects that the level and announcement sequence of price 
have on key performance indicators of health insurers. In a natural way, the 
problem of this thesis can be formulated:

**Problem** There is an insufficient knowledge of the effects of the level and the 
announcement sequence of price on the key performance indicators of health 
insurers in the Dutch health insurance market.

### 1.3 Research objectives

A natural approach for solving the problem is to investigate how the level 
and the announcement sequence of price effects key performance indicators 
of health insurers. This forms the research question of this thesis:

**Research Question** What is the effect of the level and announcement 
sequence of price on key performance indicators of health insurers in the 
Dutch health insurance market?

The first step of answering this research question is to investigate which key 
performance indicators are relevant and important for health insurers. This 
results into the first sub question:

**Sub-Question 1** What are the important and relevant key performance 
indicators of health insurers?

Afterwards, a mathematical model is developed, that captures the effect of 
the level and announcement sequence of price. Besides deciding how the 
level and the announcement sequence of price should be included, we also 
invent how the choice behavior of policy holders should be included in the 
mathematical model. This leads to the last two sub questions:

**Sub-Question 2** How should the choice behavior of policy holders be incor-
porated in the mathematical model?

**Sub-Question 3** How should the level and the announcement sequence of 
price be incorporated in the mathematical model?

Results of the sub questions will conduct to the mathematical model. With 
this mathematical model and the result(s) of the other sub research questions 
it is possible to answer the research question.

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2Note that health insurers can freely choose their announcing moment as long as it is announced before the 15th of November.
1.4 Deliverables

Before this thesis, there was insufficient knowledge of the effects of the level and the announcement sequence of price on key performance indicators of health insurers. Therefore a (game theoretical) mathematical model is developed, that determines these effects. The mathematical model is built in Excel and forms a deliverable for Milliman. Besides the Excel tool, this report, with all necessary information of the effects of the level and announcement sequence of price, forms the second deliverable for Milliman.

1.5 Outline thesis

The stated problem and research question will form the content of this thesis. First, an overview is given of the Dutch health insurance market to make the reader familiar with the subject. The possible health insurers are introduced and information over the level and the announcement sequence of price is given. Moreover, the important key performance indicators are discussed. Then, a mathematical model is introduced that captures the choice behavior of policy holders (based on the level of price) and connects this with key performance indicators. Results are presented in the analysis part and effects of the level of price on key performance indicators of health insurers are discussed in more detail. Then, a game theoretical approach is used to investigate, next to the level of price, the effects of the announcement sequence of price by considering health insurers as players of a game. Results are presented in the analysis part and discussed in more detail. In the last part of this thesis conclusions and recommendations are drawn over the effects of the level and announcement sequence of price on key performance indicators, related with useful insights for Milliman.
Situation
2 Health insurance in the Netherlands

In 2006 due to a health reform a new Zorgverzekeringswet (Zvw) is introduced in the Netherlands. In this new Zvw everyone in the Netherlands is required to purchase basic health insurance from private health insurers on a yearly basis. The Zvw imposes on health insurers an obligation to accept all who apply for basic health insurance. Every year, health insurers set their own prices and then policy holders have one and a half month to renew their policy. This implies that policy holders can decide to renew their current health insurance(s) or to switch to another health insurer. Besides the basic health insurance, policy holders can insure themselves for elements which are not included in the basic health insurance, called supplementary health insurance. The total package per person, including basic and supplementary insurance(s), forms the health insurance policy of a policy holder. In what follows, basic health insurance and supplementary health insurance will be discussed in more detail. Moreover, the current health insurers in the Netherlands are discussed.

2.1 Basic health insurance

The Zorgverzekeringswet obliges every health insurer in the Netherlands to offer at least the basic health insurance. This basic health insurance will meet the minimum standard level of coverage set by the government. In 2013 the minimum standard includes medical care including care provided by general practitioners, medical specialists and obstetricians, hospital treatment, medication, dental care up to the age of 18, postnatal care, limited physiotherapy, exercise therapy, speech therapy, occupational therapy, dietary and advice to help stop smoking.

Health insurers have the possibility to offer basic health insurance in one or more of the following forms: natura, restitution or a combination of both. The natura form obliges policy holders to take out health care at predefined health care providers, while the restitution form liberates policy holders in choosing health care providers. The third form is a combination of both. Since 2010 a deductible (‘eigen risico’ in Dutch) is also part of the basic health insurance and equals €350 in the year 2013. This implies that the first €350 spent on health care, except general practitioners, obstetric, dentist care up to the age of 18 and postnatal care, has to be paid by the policy holder of the basic health insurance itself. Health insurer have the possibility to offer basic health insurance with a higher deductible than €350, but they should at least offer the one with a deductible of €350.
The price of basic health insurance is not set by the government and health insurers are free in choosing their own prices. However, the price may not vary by policy holder, health status, or other risk characteristics. Given that the health insurer have some freedom in forming their own basic health insurance (i.e. natura, restitution and amount of extra own risk) the prices between them will in general differ. In Chapter 3 the prices of the basic health insurances will be discussed in more detail.

Despite that basic health insurance is obliged for everyone in the Netherlands, the amount of basic health care needed per person is not the same for everyone. In general elderly or sick people will need more basic health care than healthy people. Given that it is prohibited for health insurers to charge prices regarding to risk characteristics, the government introduced in 2006 the so called risk equalization system. This system will compensate for that fact that some health insurers will have more unfavorable risk profiles. These health insurers receive for every unfavorable risk profile a compensation amount. In this way health insurers will be treated more fairly. The idea of the government is to reduce this risk equalization system.

2.2 Supplementary health insurance

Not all health care (e.g. more extensive physiotherapy or the cost of dental care) will be covered by basic health insurance. Therefore, one can take out supplementary health insurance. In 2012 more than 74 % (see Vektis (2012)) of the policy holders in the Netherlands took out some supplementary health insurance. The variation in the kind of supplementary insurances per health insurer is large. For example, there exist more than 8 different supplementary health insurances for the dental care (see Vektis (2012)). Given that policy holders can form their own supplementary insurances package, the total number of possible combinations of basic health insurance combined with one or more supplementary health insurances is enormous. Health insurers are not obliged to accept policy holders on their supplementary part. Fortunately, policy holders are not obliged to take out basic and supplementary health insurance at the same health care insurance, which allow them to look for another health insurer if they were not accepted at some health insurer. On average, the supplementary insurance(s) will cover 15 % of the total price of the health insurance policy, while basic health insurance will cover 85 % of the total (see Vektis (2012)).
2.3 Current health insurers

In 2006, the total health insurance market existed of 33 different health insurers, while the health insurance market nowadays (2013) exist of 26 health insurers. These health insurers are coordinated by concerns, as for example Achmea, Menzis and CZ. In January 2012 four of these concerns separately dealt with more than two million health insurance policies and in total serve more than 14 million policy holders. These four concerns are Achmea with 4.80 million health care insurance policies, CZ with 3.42 million health insurance policies, Menzis with 2.03 million health insurance policies and UVIT with 4.26 million health insurance policies. In Table 1 an overview is given of all health insurers coordinated per concern.

<table>
<thead>
<tr>
<th>Concern</th>
<th>Health insurers</th>
</tr>
</thead>
<tbody>
<tr>
<td>Achmea</td>
<td>Agis Zorgverzekeringen N.V.</td>
</tr>
<tr>
<td></td>
<td>Avéro Achmea Zorgverzekeringen N.V.</td>
</tr>
<tr>
<td></td>
<td>De Friesland Zorgverzekeringen N.V.</td>
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<tr>
<td></td>
<td>FBTO Zorgverzekeringen N.V.</td>
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<tr>
<td></td>
<td>Interpolis Zorgverzekeringen N.V.</td>
</tr>
<tr>
<td></td>
<td>OZF Achmea Zorgverzekeringen N.V.</td>
</tr>
<tr>
<td></td>
<td>Zilveren Kruis Achmea Zorgverzekeringen N.V.</td>
</tr>
<tr>
<td>ASR</td>
<td>ASR Basis Ziektekostenverzekeringen N.V.</td>
</tr>
<tr>
<td>CZ</td>
<td>Delta Lloyd Zorgverzekeringen N.V.</td>
</tr>
<tr>
<td></td>
<td>OHRA Zorgverzekeringen N.V.</td>
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<tr>
<td></td>
<td>OHRA Zorgverzekeringen N.V.</td>
</tr>
<tr>
<td></td>
<td>OWM CZ Zorgverzekeraar U.A.</td>
</tr>
<tr>
<td>DSW-SH</td>
<td>OWM DSW Zorgverzekeraar U.A.</td>
</tr>
<tr>
<td></td>
<td>OWM Stad Holland Zorgverzekeraar U.A.</td>
</tr>
<tr>
<td>Eno</td>
<td>Eno Zorgverzekeraar N.V.</td>
</tr>
<tr>
<td>Menzis</td>
<td>AnderZorg N.V.</td>
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<tr>
<td></td>
<td>AZIVO Zorgverzekeraar N.V.</td>
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<tr>
<td></td>
<td>Menzis Zorgverzekeraar N.V.</td>
</tr>
<tr>
<td>ONVZ</td>
<td>ONVZ Ziektekostenverzekeraar N.V.</td>
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<tr>
<td>VGZ</td>
<td>IZA Zorgverzekeraar N.V.</td>
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<td>IZZ Zorgverzekeraar N.V.</td>
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<td></td>
<td>N.V. Univé Zorg</td>
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<td>N.V. Zorgverzekeraar UMC</td>
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<td></td>
<td>Coöperatie VGZ U.A.</td>
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<tr>
<td></td>
<td>Zorgverzekeraar Cares Goude N.V.</td>
</tr>
<tr>
<td>Zorg en Zekerheid</td>
<td>OWM Zorgverzekeraar Zorg en Zekerheid U.A.</td>
</tr>
</tbody>
</table>

Table 1: Health insurers and concerns
3 Price and Announcement sequence

In this chapter general information is given on prices of health insurances, called premiums, and the sequence in which health insurers announce these premiums. First premiums are discussed, whereafter a description of the announcement sequence is given.

3.1 Premium

In general, health insurers consider two types of premiums, namely the (1) technical premium and (2) the commercial premium. In what follows, these two types of premiums will be discussed in more detail. The discussion of the premium will be restricted to the basic health insurance.

3.1.1 Technical premium

The technical premium of a basic health insurance is based on three components, namely a burden of losses component, a costs component and a solvency raise component. The first component is based on the losses (e.g. claims) per policy holder. With information of last year(s), health insurers can predict the losses per policy holders on average. Only considering the expected losses is not enough, given that some variation exist in the number of policy holders and the total claims for upcoming year. Therefore, health insurers will cover the burden of losses component as the sum of expected losses and some uncertainty component. The second component is the costs component. The costs component can be divided into variable and fixed costs. The fixed costs consists for instance of salary costs of employees and hires of buildings, while variable costs are administrative costs per policy holder. Where fixed costs are independent of the number of policy holders, variable costs are dependent. This plays an important role when the number of policy holders is changing rapidly. The third and last component is the solvency component. Given that health insurers should hold a minimal required capital, no investments on this capital can be made. As a consequence, health insurer charge a solvency component to policy holders to compensate for this (indirect) loss.

3.1.2 Commercial premium

Finally, health insurers set their commercial premium as the price that is offered to policy holders. The commercial premium is based on two components, namely the technical premium, which is discussed in previous section

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3 Note that supplementary health insurances will not be discussed here.
4 A health insurer should hold a minimal required capital to be solvable, following the Solvency II norm. For more information see QIS5 (2012) or section 5.4.
and a commercial component. It is not necessarily the case that the commercial component is positive, but often this will be the case. A positive commercial component will in general result in a profit per policy holder, while a negative commercial component will in general result in a loss per policy holder. The choice of this component is a decisive factor in how health insurers perform on their key performance indicators. Charging a too high commercial premium results in profitable insurances but against a low market share, while charging a too low price results in a high market share but against non profitable (loss making) insurances. Both situations are in general not acceptable ones. In the remainder of this thesis, the commercial premium is for convenience denoted by \textit{premium}.

3.2 Announcement sequence

In the Netherlands, health insurers are free in choosing their time moment to announce the new commercial premium for the upcoming year. The only requirement is to announce the commercial premium before the 15th of November. Not every health insurer announces its commercial premium on the 15th of November. Normally DSW, a relative small health insurer in the Netherlands, is the first one that announces its commercial premium (see Vektis (2012)). When health insurer announce their commercial premium more than six weeks before the 15th of November, they have the possibility to readapt their premium. In the time period of six weeks before the 15th of November, this is not possible anymore. In Figure 1 a graphical (sequence) overview is given of the time schedule of 2013 wherein the four largest health insurers of the Netherlands and DSW have announced their premium.

![Figure 1: Announcement sequence in 2013](image.png)

Observe that DSW was the first one that announces its premium and that Menzis reacted at the end of October. Why did Menzis announce their premium way before the other ones? Was it beneficial for them (e.g. key performance indicators) to announce the premium early? Or did CZ and Achmea perfectly react on this situation and charged low(er) prices to compete against Menzis? In Chapter 8, the effects of the announcement sequence on key performance indicators are discussed in more detail.
4 Key Performance Indicators

A common approach for measuring the continuity of a health insurer is to measure its key performance indicators (see Taiber (2008)). In Kasturi (2006) key performance indicators are defined as:

**Definition** A Key performance indicator is a management instrument that measures variables to analyze performance of a company.

In literature a lot is written about key performance indicators. Despite that, the number of articles that discuss key performance indicators in a health insurance setting is small. Exceptions are Kasturi (2006) and Taiber (2008). They have in common that (1) profit, (2) market share and (3) solvency ratio are discussed as the most important key performance indicators. Profit is an important key performance indicator, given that it represents the absolute benefit for a company year by year, while market share is an important key performance indicator, given that it indicates the dynamics in the number of policy holders. When, for example, market share drops down rapidly, (fixed) costs cannot be covered anymore. Finally, the solvency ratio is an important key performance indicator, given that it measures the ratio between a health insurer buffer (e.g. own funds) and the solvency required capital obliged by the government. Taiber (2008) and Kasturi (2006) mentioned that it is possible to measure the performance (e.g. continuity) well with these three key performance indicators. The three key performance indicators are formulated as:

- **Profit**: the difference between the total revenues and the (variable and fixed) costs of a health insurer.

- **Market Share**: The percentage of policy holders of a health insurer in relation to the total number of policy holders in the health insurance market.

- **Solvency ratio**: The fraction of the own funds of a health insurer and its solvency required capital (SCR)\(^5\)

Calculations of these key performance indicators will be discussed alongside the introduction of the mathematical model in Chapter 5.

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\(^5\)SCR : Solvency capital Requirement. A new restriction of the government to hold a health insurer solvable. For more concrete information, see section 5.4 for Calculations of the SCR level or QIS5 (2012).
Model & Analysis
5 Mathematical model

In this chapter a mathematical model is introduced that captures the effects of the level of price on key performance indicators as profit, market share and solvency ratio. The announcement sequence is not considered in this model, but will be discussed later on. In the first section the underlying assumptions of the mathematical model are discussed and in the second section a formal introduction of all variables is given. In third section the first part of the model, that describes how policy holders switch, is introduced whereafter in the fourth section the second part of the model, that describes how key performance indicators are affected, is introduced.

5.1 Assumptions

This section unites all assumptions of the mathematical model. The first assumption is related to the number of health insurers in the mathematical model. In the Dutch health insurance market, only four large health concerns dominate the health insurance market with a total market share of 95.2 % (see Vektis (2013)). This was enough motivation to model the market with only four health insurers (e.g. concerns):

- The health insurance market exists of four health insurers.

The second assumption is related to the health insurances that are offered by health insurers. In section 2.1 one has discussed the difference(s) between basic health insurances and supplementary health insurances. The number of possible combinations of basic and supplementary health insurances is enormous. For simplicity, it is assumed to only consider basic health insurance and also excluding any variation of this form. This implies that all health insurers offer only one type of health insurance:

- All health insurers offer only one type of basic health insurance in the Dutch health insurance market.

In the Dutch health insurance market around 67 % of the health insurers announce their premium on the final date (see Vektis (2013)), while others announce it on an earlier date. For simplicity, it is assumed that all health insurers announce their premium at the same moment. This leads to the following assumption:

- All health insurers announce their premium at the same moment.

The fourth assumption considers the number of policy holders. In the Netherlands everyone is obliged to take out basic health insurance, while

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6This assumption is relaxed later on (in Chapter 8).
only people older than 17 pay premium. This part of the Dutch population is relative stable. For example, in 2012 the Dutch population (> 17 years) increased with 0.29 % (see CBS (2012)). This was enough motivation to consider the number of policy holders as a constant number:

- **The number of policy holders in the Dutch health insurance market is a constant.**

In the health insurance market, policy holders make their decision based on several factors. In Gopfert (2002), price is often highlighted as the key factor of choice of customers. Representative interview based studies underline this assumption in Anderson (1998). Alongside price, also the price sensitivity plays a role in how policy holders choose. When policy holders are more sensitive for price, policy holders will switch more (see McFadden (1992)). Finally, the reputation of health insurers plays a role in the choice of policy holders. A good or bad reputation of a health insurer can influence the decision of policy holders (see Cretu (2007)). Next to these factors, several others factors exist, but will not be taken into account in this thesis. The three factors price, price sensitivity and reputation form together the next assumption:

- **The choice of policy holders is based on the price of health insurances, the price sensitivity and the reputation of health insurers.**

Next to the fifth assumption a strict sixth assumption is made. Based on the second assumption, that all health insurers offer the same health insurance, it is assumed that no policy holder will choose a health insurer that charges a higher price than the price of the current health insurer. This leads to the sixth assumption:

- **Policy holders will never choose for a health insurer that charges a higher price than the price of the current health insurer.**

The seventh assumption is related to the time span of the health insurance market. Every year, policy holders have one and a half month to renew their policy. In the Netherlands, policy holders have the possibility to choose and re-choose within this time span (see Kleine gids (2013)). In the mathematical model it is assumed that policy holders can only choose once. This leads to the seventh assumption:

- **All policy holders can only choose once in the time period of 15th November till the 31th of December of that year.**

Based on these made assumptions, it is possible to introduce the mathematical model. We will start with a formal introduction of all variables and parameters, followed by a description of the structure of the model.
5.2 Notation and structure

In our mathematical model a distinction is made between a situation and an outcome. A situation is denoted by $\theta$, which forms the necessary (input) information of the mathematical model, while $\omega$ is denoted as the outcome of a given situation $\theta$. An operator $\Gamma$ connects these two with each other, such that for a given situation $\theta$ an outcome $\omega$ is obtained:

$$\Gamma : \Theta \to \Omega,$$

where $\Theta$ is the set of all situations $\theta$ and $\Omega$ the set of all outcomes $\omega$. In what follows, situations $\theta$ and outcomes $\omega$ are introduced in more detail.

5.2.1 Situation $\theta$

A situation $\theta$ is defined as a tuple. The tuple consists of the year $t$, the time moment that policy holders start to choose, denoted by $t^*$, the number of policy holders per health insurer at year $t$, denoted by $N = (N_1, \ldots, N_k)$, the premiums charged per health insurer in year $t+1$ denoted by $X = (x_1, \ldots, x_k)$, the own equity per health insurer in year $t$, denoted by $E$, the fixed costs and variable costs per health insurer in year $t+1$, denoted by $c$ and $C$ respectively and finally a price sensitivity factor, denoted by $\alpha$. Hence, a situation $\theta \in \Theta$ is denoted by the following tuple:

$$\theta = (t, t^*, N, X, E, c, C, \alpha).$$

In what follows, only $\theta$ is denoted when the model is considered. Furthermore, the total number of policy holders is denoted by $\overline{N}$, where the set of health insurer is denoted by $\overline{N} = (1, 2, \ldots, k)$.

5.2.2 Outcome $\omega$

With a situation $\theta$, operator $\Gamma$ determines an outcome $\omega$. An outcome is also denoted by a tuple. This tuple consists of the new year $t+1$ and the key performance indicators of that year. Hence, the profit per health insurer, denoted by $P = (P_1, \ldots, P_k)$, the market share per health insurer, denoted by $M = (M_1, \ldots, M_k)$ and the solvency ratio per health insurer, denoted by $S = (S_1, \ldots, S_k)$. In mathematical terms, an outcome $\omega$ of a given situation $\theta$ is denoted by the following tuple:

$$\omega = (t + 1, P, M, S).$$

In the rest of this chapter the structure of $\Gamma$ will be discussed in more detail. We explain how policy holders behave (i.e. choose) under a given situation $\theta$ and how this effect key performance indicators (e.g. outcome $\omega$).
5.3 Policy holder behavior

As discussed in section 3.2, health insurers have to announce their premium at latest on the 15th of November. Then, policy holders have a time interval of one and a half month to choose their new basic health insurance for the upcoming year. After this time interval, the new configuration of the number of policy holder per health insurer is determined. In this section one will explain how this principle (e.g. how policy holders choose) is translated into the model.

5.3.1 Mathematical introduction

In mathematical terms, one will look for an operator $L(\theta)$ that determines a new situation $\hat{\theta}$, which coincides with $\theta$, except that $N$ is replaced by the updated number of policy holders, denoted by $\hat{N}$. In mathematical terms the operator is defined as:

$$L : \Theta \rightarrow \hat{\Theta}, \quad (4)$$

with $\Theta$ the set of situations $\theta$ and $\hat{\Theta}$ the set of updated situations $\hat{\theta}$.

Our task is not only to develop an operator $L$ that determines an updated situation with a new configuration of policy holders, but also to develop a new situation with a realistic prediction of the new configuration of policy holders. In Figure 2 an operator $L$ is presented that determines for a situation $\theta$ a new configuration of policy holders. Company green changes from a large company into a small company, while company yellow undergoes the opposite direction. The operator indeed determines a new configuration of policy holders, but is the prediction also a realistic one?

![Figure 2: A specific operator with input and output](image)

In general, it is not easy to develop an operator, nor to investigate if its prediction is realistic. For a better understanding, it is decided to start doing a literature study into the direction of operators that capture switching behavior of customers in a general setting. Obtained results of this literature study are presented in the upcoming section.
5.3.2 Literature

The most basic operators that are discussed in literature are originating from basic economic models, in where the switching behavior of customers only relies on price (see Thaler (1987)). In such models, customers will switch to the company with the lowest price in the market. This results in a situation in where the company with the lowest price receives all customers. In practice, customers do not move from one company to a cheaper one as quickly as economic models anticipate. In Smith et al. (2000) a model is introduced that prevents customers to always look for the cheapest company when their price is slightly higher than the cheapest price. Smith mentioned that the inclusion of price differences into an operator will improve its performance regarding to operators in basic economic models.

In a working paper of Dutang (2013) an interesting operator is introduced in a French motor insurance setting. In French, motor insurance contracts are on a yearly basis, which makes them quite similar to the Dutch health insurance situation. Despite that, in the Netherlands everyone is obliged to take out health insurance, while in French motor insurance is not obliged. Nevertheless, part(s) of the operator could be of interest for this thesis. The operator that Dutang (2013) uses is based on a multi-logit model, introduced by McFadden (1981), which relies on probability theory. Their situation set is based on the current configuration of customers per insurer, prices, price sensitivity and lapse rate of customers.

One drawback of most operators in literature is the non-dynamic approach of customer behavior during the switching period. They all assume that customers make their decision at the same moment in time. In reality customers can switch on different moments in time. Health insurers that announce their premium on an early date could benefit from this, given that customers can already decide (in mind) to switch to them. On the other side, the choice of customers will be influenced by all kind of information through time. Customers that switch at the end of a switching period will be more effected by information from media (television, internet) than customers that switch in the beginning of the period. Hence, the dynamic focus of customer behavior through time cannot be neglected.

A new operator \( \mathcal{L} \) will be developed that combines elements of Dutang (2013) with dynamic elements of customers. Given that no relevant literature is found for the dynamic approach, this part will be developed by ourselves. In the next section the operator will be introduced.
5.3.3 Operator $\mathcal{L}$

In this section the new operator will be introduced, which originates from the operator as defined by Dutang (2013), combined with several new insights. Markov chains will form the basis of this approach.

Define $N_i(t)$ as the number of policy holders of health insurer $i \in \tilde{N}$ at time $t$. The continuous state space of $N_i(t)$ is defined by $S_i = [0, \overline{N}]$, with $\overline{N}$ the total number of policy holders in the Netherlands\footnote{Note that the number of policy holders is approximated with a continually process, despite that the number of policy holders is discrete.}. A process $\{N_i(t), t \geq 0\}$ is called a continuous-time stochastic process.

We consider the process $\{N_i(t'), t^* \leq t' \leq t + 1\}$, where $[t^*, t + 1]$ is defined as the time interval in where policy holders can switch. Within this time period, policy holders have the possibility to switch from one health insurer to another one. Consider transition rate $r_{i,j} \geq 0$ as the intensity of change between state $N_i(t)$ and $N_j(t)$. A stochastic process with transition rates belongs to the class of Continuous-Time-Markov-Chains (CTMC):

**Definition (CTMC)** A stochastic process $\{N_i(t), t \geq 0\}$ on state space $S_i$ is called a CTMC if, for all $i$ and $j$ in $S_i$,

$$
\begin{align*}
P(N_i(s + t) = a | N_i(s) = b, N_i(u), 0 \leq u \leq s) = \\
P(N_i(s + t) = a | N_i(s) = b) \quad t, s \geq 0.
\end{align*}
$$

A proof that a stochastic process with transition rates belongs to the class of CTMC’s is given in Kulkarni (1999). The transition rates $\{r_{i,j}\}_{i,j \in \tilde{N}}$ incorporates the effects of price, the price sensitivity and the reputation of health insurers. These effects are captured by the following elements:

- Premiums of health insurers;
- Price sensitivity of policy holders.
- Number of policy holders per health insurer;

The price component is defined as the difference in premium $(x_i - x_j)$ between health insurer $i$ and $j$. When the premium $x_i$ is higher than the premium $x_j$ the transition rate $r_{i,j}$ is positive and some policy holders might switch from $i$ to $j$. When $x_i < x_j$, the transition rate $r_{i,j}$ is negative and some policy holders might switch from $j$ to $i$. Next to the price component, also the number of policy holders per health insurer influences the transition rate. The component is defined as the product $N_i \cdot N_j$; the number of policy holders of health insurer $i$ and $j$ respectively. This component translates (1) how many policy holders of company $i$ can potentially switch and (2)...
how strong company $j$ can attract policy holders. A small $N_i$ implies that only a few policy holders can switch, while a large $N_i$ implies that a large amount of policy holders can switch. A small $N_j$ (unknown, small company) will not attract that many policy holders, while a large $N_j$ (known, large company) will attract more policy holders. The last component is a price sensitivity factor $\alpha$ of policy holders. The price sensitivity factor translates how sensitive policy holders are for differences in price. The higher the value of this factor, the more policy holders will switch. With these elements, the transition rate of the policy holders can be constructed.

**Definition (Transition rate)** The transition rate $r_{i,j}$ from state $i$ to state $j$ is given by:

$$r_{i,j} = \alpha \beta (x_i - x_j) N_i N_j$$

with $\alpha$ the price sensitivity factor and $\beta = \frac{1}{k} \sum_{i=1}^{k} N_i$ a relativity factor.

The relativity factor $\beta$ is included to ensure transition rates that have the same order of magnitude for different sizes of policy holders.

Observe that policy holders of health insurer $i$ only switch to company $j$ if $x_i > x_j$. Define the set of health insurer with a premium $x_j > x_i$ as $W_i$ and define the set of health insurers with a premium $x_j \leq x_i$ as $W_i'$. Then, $W_i \cup W_i' \cup \{i\} = \hat{N}$. The flow of policy holders from and to state $i$ can then be visualized by nodes and arcs (Figure 3), where the node defines the state of health insurer $i$ and the arcs the transition rates from and to state $i$.

![Figure 3: Flow of policy holders from and to state $i$](image)

Note that $r_{i,j} \Delta t$ is the number of policy holders that switch between time $t$ and $t + \Delta t$ from company $i$ to company $j$. Hence, the net change in the number of policy holders of health insurer $i$ between time $t$ and $t + \Delta t$ depends on the difference between the ingoing transition rates $\{r_{j,i}\}_{j \in W_i}$ and outgoing transition rates $\{r_{i,k}\}_{k \in W'_i}$:

$$N_i(t + \Delta t) - N_i(t) = \sum_{j \in W_i} r_{j,i} \Delta t - \sum_{k \in W'_i} r_{i,k} \Delta t,$$

(7)
or in differential form:

\[
\frac{dN_i}{dt} = \sum_{j \in W_i} r_{j,i} - \sum_{k \in W'_i} r_{i,k}. \tag{8}
\]

The following lemma deals with the relationship between \( r_{i,j} \) and \( r_{j,i} \).

**Lemma 1 (Relationship transition rates)** Let \( \theta \in \Theta \). Then for all \( i, j \in \tilde{N} \) the transition rate \( r_{i,j} \) has the following relationship:

\[
r_{i,j} = -r_{j,i}. \tag{9}
\]

**Proof** For any \( i, j \in \tilde{N} \) the transition rate \( r_{i,j} \) can be rewritten to:

\[
r_{i,j} = \alpha \beta (x_i - x_j)N_iN_j = -\alpha \beta \cdot (x_j - x_i)N_jN_i = -r_{j,i}. \quad \blacksquare
\]

By making use of Lemma 1 one can rewrite equation (8) into:

\[
\frac{dN_i}{dt} = \sum_{j \in W_i} r_{j,i} - \sum_{k \in W'_i} r_{i,k} = \sum_{j \in \tilde{N} \setminus \{i\}} r_{j,i} \quad \blacksquare
\]

Substituting equation (6) into the last equation of (10) results into the general form of the differential equation:

\[
\frac{dN_i}{dt} = \sum_{j \in \tilde{N} \setminus \{i\}} \alpha \beta (x_j - x_i)N_iN_j. \tag{11}
\]

In a similar way one can derive the \( k - 1 \) differential equations for all other health insurers. Then a system of differential equations is obtained, which is denoted by:

\[
\frac{dN_i}{dt} = \sum_{j \in \tilde{N} \setminus \{i\}} \alpha \beta (x_j - x_i)N_iN_j \quad \forall i \in \tilde{N}. \tag{12}
\]

Solving this system of differential equations with initial starting vector \( N_i(t^*) = N_i \) of \( \theta \) and substituting \( t + 1 \) in the solutions of the system of differential equations results into the updated number of policy holders \( \hat{N}_i \). These values will then form vector \( \hat{N} = N(t + 1) \) as an element of the new situation \( \hat{\theta} \). Now, it is also possible to define the model operator \( \mathcal{L} \) in words.

---

*For a gentle introduction into (systems of) differential equation, see Appendix B.*
**Definition (Model Operator)** The model operator $\mathcal{L}$ determines for a given situation $\theta$ a new situation $\hat{\theta}$ that only deviates from $\theta$ in the new number of policy holders, which are obtained by substituting $t + 1$ in the solutions of the system of differential equations.

In general, it is hard to obtain the new configuration of policy holders $\hat{N}$ of $\hat{\theta}$ in an analytical way. However, for $k = 2$ health insurers it is possible to find an analytical solution. In section 5.3.4 this derivation is given.

### 5.3.4 Exact solution for $k = 2$

The system of differential equations for a situation $\theta \in \Theta$ with $k = 2$ health insurers is given by the follows two differentials:

$$
\frac{dN_1}{dt} = \alpha \beta (x_2 - x_1) N_1 N_2, \\
\frac{dN_2}{dt} = \alpha \beta (x_1 - x_2) N_2 N_1,
$$

where the total number of policy holders $\overline{N}$ is a constant. Observe that $N_2$ can be rewritten as the difference between $\overline{N}$ and $N_1$. Substituting this into the differential equation of $N_1$ results into:

$$
\frac{dN_1}{dt} = \alpha \beta (x_2 - x_1) N_1 (\overline{N} - N_1) dt.
$$

A method for solving this type of differential equation(s) is separation of variables (see Adams (2000)). This results into:

$$
\frac{dN_1}{N_1 (\overline{N} - N_1)} = \alpha \beta (x_2 - x_1) dt \\
\left[ \frac{1}{\overline{N}} \left( \frac{1}{N_1} + \frac{1}{\overline{N} - N_1} \right) \right] dN_1 = \alpha \beta (x_2 - x_1) dt \\
\frac{1}{\overline{N}} \ln |N_1| - \frac{1}{N} \ln |\overline{N} - N_1| = \alpha \beta (x_2 - x_1) t + c \\
\ln \left| \frac{N_1}{\overline{N} - N_1} \right| = \overline{N} \alpha \beta (x_2 - x_1) t + \overline{N} c \\
\frac{N_1}{\overline{N} - N_1} = c_1 e^{2 \alpha (x_2 - x_1) t},
$$

where $c_1 = e^{\overline{N} c}$ and 2 is obtained from $\overline{N} \cdot \beta = \frac{N_1 + N_2}{\overline{N} (N_1 + N_2)} = 2$.

It follows from the last equation that:

$$
N_1(t) = \frac{\overline{N} \cdot c_1 e^{2 \alpha (x_2 - x_1) t}}{1 + c_1 e^{2 \alpha (x_2 - x_1) t}} = \frac{\overline{N} c_1}{e^{-2 \alpha (x_2 - x_1) t} + c_1}.
$$
Given that we are only interested in the time interval \( t^* \leq t \leq t + 1 \) the solution \( N_1(t) \) will be shifted in time. Hence, we rewrite \( t \) as \( t - t^* \), such that at \( t = t^* \) the process starts. Substituting \( t = t^* \) in last equation results, after some rewriting, into:

\[
c_1 = \frac{N_1(t^*)}{N - N_1(t^*)},
\]

and therefore:

\[
N_1(t) = \frac{N \cdot N_1(t^*)}{N_1(t^*) + (N - N_1(t^*)) \left( e^{-2\alpha(x_2-x_1)(t-t^*)} \right)}, \tag{13}
\]

with \( t \in [t^*, t + 1] \).

In a similar way, the equation of \( N_2(t) \) can be derived (see Appendix C1).

The final equation for \( N_2(t) \) and initial condition \( N_2(t^*) \) is given by:

\[
N_2(t) = \frac{N \cdot N_2(t^*)}{N_2(t^*) + (N - N_2(t^*)) \left( e^{-2\alpha(x_1-x_2)(t-t^*)} \right)}, \tag{14}
\]

with \( t \in [t^*, t + 1] \). The reader can check that it holds that:

\[
N_1(t) + N_2(t) = N \quad \text{for} \quad t \in [t^*, t + 1].
\]

For the derivation of the last equation, see Appendix C2.

As discussed before, the updated situation \( \hat{\theta} \) can be formulated, given that vector \( \hat{N} = N(t + 1) \) is obtained by Substituting \( t + 1 \) into equations (13) and (14) and implementing all other input variables of a given situation \( \theta \).

Remark that the system of two differential equations is analytically solvable, given that \( N_1 + N_2 = N \) can eliminate one of the two variables. In a situation with more than two variables the substitution of equation \( N_1 + N_2 = N \) in the system of differential equation is not enough anymore. Given the time constraints to this thesis, it is decided to solve systems with more than two differential equations in a numerical way. In the next section this approach will be discussed in more detail.
5.3.5 Numerical techniques

As discussed in section 5.3.4 one will not obtain \( \hat{N} = N(t+1) \) by deriving analytic solutions for systems with more than two differential equations. In this section, numerical techniques will be introduced that approximate the updated configuration of policy holders \( N(t+1) \) for systems with more than two differential equations.

By using the program Matlab, it is a straightforward exercise to approximate the updated configuration of policy holders \( N(t+1) \) for a system of more than two differential equation. In this thesis, not Matlab, but Excel VBA will be used for this, given that Milliman has no license for Matlab.

In Excel VBA an algorithm is developed that considers the system of differential equations as a discrete system of equations with some initial starting values. The concept is to update the initial starting values (e.g. the number of policy holders at time \( t^* \)) by adding differentials for small time interval up to time \( t+1 \). The algorithm script is depicted below.

**Algorithm:** Approximating differential Equations

1. Input: \( \theta, \Delta t \)
2. Output: \( N(t+1) \)
3. \[ t' = t^* \]
4. Compute \( \frac{dN_i}{dt} = \alpha \beta \sum_{j \in N\setminus\{i\}} (x_i - x_j)N_iN_j \quad \forall i \in N \)
5. \[ N_i = N_i + \frac{dN_i}{dt} \cdot \Delta t \quad \forall i \in N \]
6. \[ t' = t' + \Delta t \]
7. If \( t' \leq t + 1 \rightarrow [2] \text{ else stop} \]

In the algorithm first a situation \( \theta \) and \( \Delta t \) is chosen. Then, the time variable \( t' \) is chosen equal to the starting time \( t^* \) that policy holders can switch. Then, the differentials of all companies are calculated at time \( t' \). Afterwards, the number of policy holders is updated by adding the value of the differential multiplied by \( \Delta t \). Then, time variable \( t' \) is updated. As long as this value is below threshold \( t + 1 \), this process is executed again.

Observe that in step [3] differentials are calculated for small time intervals \( \Delta t \). Relative large errors can occur when the differentials increase rapidly by time. For this reason, the choice of \( \Delta t \) should be related to the order of magnitude of the differentials. In Appendix C3 tests are executed that determine how well the algorithm performs and which \( \Delta t \)'s are acceptable. It turned out that the algorithm performs well with a \( \Delta t \) of \( \Delta t = 10^{-6} \).
5.3.6 Properties of system of differential equations

The system of differential equations have some interesting properties. These properties will be investigated below. First, one shows that the differentials can be ordered based on price, when the number of policy holders are equal to each other. Then, one shows that the outcomes of the solutions of the system of differential equations are restricted, given that the sum of the differential is equal to zero. Moreover, the behavior of the differential equations in the limit is investigated, whereafter equilibrium points and phase planes of the system of differentials are discussed. Finally, some results related to the proportionality of the differential equations are investigated.

Ordered differentials

The first theorem shows that differentials can be ordered based on price, when the number of policy holders are equal to each other.

**Theorem 1** Let $\theta \in \Theta$ and $i, j \in \tilde{N}$ with $N_i = N_j$ and $x_i \leq x_j$. Then:

$$\frac{dN_i}{dt} \geq \frac{dN_j}{dt}.$$ 

**Proof** From Lemma 3 it is known that:

$$r_{i,j} = -r_{j,i}.$$ 

Given that $x_i \leq x_j$ and thus $r_{j,i} \geq 0$ it follows that

$$r_{j,i} \geq r_{i,j}.$$ 

Moreover, for $k \in \tilde{N}\{i,j\}$ it holds that:

$$r_{k,i} \geq r_{k,j},$$

given that $N_i = N_j$ and $x_i \leq x_j$. So:

$$r_{j,i} + \sum_{k \in \tilde{N}\{i,j\}} r_{k,i} \geq r_{i,j} + \sum_{l \in \tilde{N}\{i,j\}} r_{l,j} \Rightarrow \sum_{k \in \tilde{N}\{i\}} r_{k,i} \geq \sum_{l \in \tilde{N}\{j\}} r_{l,j}. \quad (15)$$

The last inequality of (15) can be rewritten to:

$$\frac{dN_i}{dt} \geq \frac{dN_j}{dt},$$

which completes the proof. \ \blacksquare
Stability of number of policy holders

In this section it is shown that the sum of the differential equations is equal to zero. When the sum of the differential equations is equal to zero, the stability of total number of policy holders is guaranteed.

**Theorem 2** Let \( \theta \in \Theta \) be a situation, then:

\[
\sum_{i \in \tilde{N}} \left( \sum_{j \in \tilde{N} \backslash \{i\}} r_{i,j} \right) = 0
\]

**Proof** Summating \( r_{i,j} \) over all \( j \in \tilde{N} \backslash \{i\} \) and over all \( i \in \tilde{N} \) gives:

\[
\sum_{i \in \tilde{N}} \sum_{j \in \tilde{N} \backslash \{i\}} r_{i,j}. 
\tag{16}
\]

As \( r_{i,i} = 0 \) for every \( i \) and \( r_{i,j} = -r_{j,i} \) for every \( i \) and \( j \) (Lemma 1), equation (16) can be rewritten to:

\[
\sum_{i \in \tilde{N}} \sum_{j > i} r_{i,j} = \sum_{i \in \tilde{N}} \sum_{j > i} r_{i,j} + \sum_{i \in \tilde{N}} \sum_{j > i} r_{j,i} \\
= \sum_{i \in \tilde{N}} \sum_{j > i} r_{i,j} - \sum_{i \in \tilde{N}} \sum_{j > i} r_{i,j} \\
= \sum_{i \in \tilde{N}} \sum_{j > i} (r_{i,j} - r_{i,j}) \\
= \sum_{i \in \tilde{N}} \sum_{j > i} 0 = 0,
\]

which concludes the proof. \( \blacksquare \)

Limit behavior

In this sub section the limit behavior of the system of differential equations is considered. More concrete, one investigates the behavior of number of policy holders for a health insurer with the lowest set premium. First a theorem is formulated for the situation with \( k = 2 \) health insurer, followed by a theorem that considers \( k \) health insurers.

**Theorem 3** Let \( \theta \in \Theta \) be a situation with \( k = 2 \) and \( x_i < x_j \), then:

\[
\lim_{t \to \infty} N_i(t) = \overline{N}.
\tag{17}
\]
Proof\ Let $\varepsilon > 0$. Take $R = \frac{N^2}{\alpha(x_j - x_i)N_i\varepsilon} + t^*$. Then for all $t > R$:

\[
|N_i(t) - \overline{N}| = \left| \frac{NN_i - \overline{N}N_i - \overline{N}(\overline{N} - N_i)e^{-2\alpha(x_j - x_i)(t - t^*)}}{N_i + (\overline{N} - N_i)e^{-2\alpha(x_j - x_i)(t - t^*)}} \right|
\]
\[
= \frac{N_i e^{2\alpha(x_j - x_i)(t - t^*)} + (\overline{N} - N_i)}{N_i (\alpha(x_j - x_i)(t - t^*))}
\]
\[
\leq \frac{\overline{N}^2}{N_i (\alpha(x_j - x_i)(t - t^*))}
\]
\[
= \frac{\overline{N}^2}{N_i \left( \frac{N^2}{\alpha(x_j - x_i)N_i\varepsilon} \right)}
\]
\[
= \frac{\overline{N}^2}{\overline{N}^2} = \varepsilon.
\]

where the first equality is obtained by (13), the second equality by multiplying the fraction by the exponent and the first and second inequality by using that $e^x > x$ for $x > 0$ and $\overline{N}(\overline{N} - N_i) < \overline{N}(\overline{N}) = \overline{N}^2$. ■

Now some other propositions will be introduced that are needed for generalizing Theorem 3 to the case with $k$ health insurers.

**Proposition 1** Let $\theta \in \Theta$ be a situation with $k > 2$ and $i \in \tilde{N}$ with $x_i < x_j$ for all $j \neq i$. Consider a situation $\theta' \in \Theta$ that is equal to $\theta$, but now with $k = 2$ such that $\overline{N'} = (N_i, \overline{N} - N_i)$. Then it holds that:

\[
\frac{dN_i}{dt} \geq \frac{dN_i'}{dt},
\]
at time $t^*$.

**Proof** Let $\varphi_i$ defined as:

\[
\varphi_i = \min_{j \neq i} (x_j - x_i).
\]

Then, the differential of $N_i(t)$ can be rewritten to:

\[
\frac{dN_i}{dt} = \alpha \beta \sum_{j \in \overline{N} \setminus \{i\}} (x_j - x_i)N_iN_j
\]
Taking $N' = (N_i, N - N_i)$ forms situation $\theta'$. It immediately follows that:

$$\frac{dN_i}{dt} \geq \frac{dN'_i}{dt},$$

at time $t^*$.  

**Proposition 2** Let $\theta \in \Theta$ be a situation with $k > 2$ and $i \in \tilde{N}$ with $x_i < x_j$ for all $j \neq i$. Then there exists a situation $\theta^0 \in \Theta$ that is equal to $\theta$, but now with $k = 2$ and $N^0 = (N_i, N - N_i)$. Then it holds that:

$$N_i(t') \geq N^0_i(t') \quad \forall \ t' \geq t^* \quad (20)$$

**Proof** Situation $\theta^0$ satisfy all conditions of Proposition 1 and as a consequence it holds that:

$$\frac{dN_i}{dt} \geq \frac{dN^0_i}{dt} \quad (21)$$

at time $t^*$.

This implies that at time $t + \Delta t$ with $\Delta t \ll 1$:

$$N_i(t + \Delta t) \geq N^0_i(t + \Delta t). \quad (22)$$

Suppose now that $N_i(t'') < N^0_i(t'')$ for some $t'' > t + \Delta t$. This implies that there exists a time $\Delta t + t < t''' < t''$ such that $N_i(t''') = N^0_i(t''')$ and

$$\frac{dN^0_i}{dt} > \frac{dN_i}{dt}. \quad (23)$$

Given that $N^0_i(t''')$ and $N_i(t''')$ both satisfy continuity and differentiability and the condition of proposition 1, it immediately follows that:

$$\frac{dN_i}{dt} \geq \frac{dN^0_i}{dt}, \quad (24)$$

which contradicts with (23). Regarding to the contradiction, it holds for all $t' \geq t^*$ that $N_i(t') \geq N^0_i(t')$ and the proof is given.

Now, it is possible with use of proposition 1 and 2 to generalize Theorem 3 to the case with $k$ health insurers. This theorem states that a health insurer $i$ with $x_i < x_j$ for all $j \neq i$ receive the total number of policy holders.
Theorem 4  Consider a situation $\theta \in \Theta$ and $i \in \tilde{N}$ with $x_i < x_j$ for all $j \neq i$. Then

$$\lim_{t \to \infty} N_i(t) = \mathcal{N}. \quad (25)$$

Proof  Given that $N_1(t) + N_2(t) + ... + N_k(t) = \mathcal{N} < \infty$ it follows that $\mathcal{N}$ is an upper bound for $N_i(t)$:

$$N_i(t) \leq \mathcal{N} \quad \forall t.$$  

From propositions 2 it follows that there exists a situation $\theta^\circ$ such that:

$$N_i(t') \geq N_i^\circ(t') \quad \forall t' \geq t,$$

Hence, for all $t' \geq t$ it holds that:

$$N_i^\circ(t') \leq N_i(t') \leq \mathcal{N}.$$

From Proposition 3 it is known that

$$\lim_{t \to \infty} N_i^\star(t) = \mathcal{N},$$

and by using Squeeze theorem (see Zill (1992)) it follows that

$$\lim_{t \to \infty} N_i(t) = \mathcal{N},$$

and the proof is given. $\blacksquare$

According to the theorem above, it could also occur that more than one health insurer sets the same lowest price, such that

$$x_i = x_j = ... = x_s < x_k \quad \forall k \in \tilde{N}\backslash\{i, k, ..., s\}.$$  

In this situation the distribution of the number of policy holders on the long run depends on the proportion of the starting number of policy holders to the total number of policy holders.

Theorem 5  Consider a situation $\theta \in \Theta$ with $A$ the set of players with $x_i = x_j = ... x_s$ and $B$ the complementary set, such that $A \cup B = \tilde{N}$. Then it holds for company $i \in A$ that:

$$\lim_{t \to \infty} N_i(t) = \frac{N_i(t^\star)}{\sum_{i \in A} N_i(t^\star)} \cdot \mathcal{N} \quad (26)$$
Proof The number of policy holders \( N_i(t) \) at time \( t + \Delta t \) is given by

\[
N_i(t + \Delta t) = N_i(t) + \frac{dN_i(t)}{dt} \cdot \Delta t
= N_i(t) \cdot \left( 1 + \alpha \beta \left( \sum_{k \in \tilde{N} \setminus \{i\}} (x_k - x_i)N_k \right) \Delta t \right).
\]  

(27)

The second term of the second equation of (27) is equal for all \( i \in \mathcal{A} \), given that \( x_i = x_j = \ldots x_s \). Hence, it holds for all \( i \in \mathcal{A} \) that:

\[
N_i(t + \Delta t) = N_i(t) \cdot \vartheta(\Delta t)
\]

with \( \vartheta(\Delta t) = \left( 1 + \alpha \beta \left( \sum_{i \in \tilde{N} \setminus \{i\}} (x_k - x_i)N_k \right) \Delta t \right) \), and also that:

\[
N_i(t + \Delta t) = N_i(t - \Delta t) \cdot \vartheta(\Delta t) \vartheta(\Delta t)
\]

\[
\vdots
\]

\[
= N_i(t^*) \cdot (\vartheta(\Delta t))^g \Delta t.
\]

(28)

with \( g \in \mathbb{N} \). Hence, \( N_i(t) \) depend for all \( t \) on the starting value \( N_i(t^*) \). Now, combine all \( i \in \mathcal{A} \), such that:

\[
\sum_{i \in \mathcal{A}} N_i(t + \Delta t) = \sum_{i \in \mathcal{A}} N_i(t) \vartheta(\Delta t)
= \vartheta(\Delta t) N_A(t),
\]

with \( N_A(t) = \sum_{i \in \mathcal{A}} N_i(t) \). Hence, the combination of all \( i \in \mathcal{A} \) results into a new situation \( \theta \) with one health insurer that sets the lowest price. Hence, Theorem 4 can be used to state that:

\[
\lim_{t \to \infty} N_A(t) = \sum_{i \in \mathcal{A}} N_i(t) = \overline{N}.
\]

Using the first equation (28) and multiplying \( N_i(t) \) by 1 results into:

\[
N_i(t) = N_i(t^*) \cdot (\vartheta(\Delta t))^g \Delta t \cdot \sum_{i \in \mathcal{A}} \frac{N_i(t^*)}{\sum_{i \in \mathcal{A}} N_i(t^*)} \sum_{i \in \mathcal{A}} N_i(t + \Delta t).
\]

Now, the limit of \( N_i(t) \) is equal to:

\[
\lim_{t \to \infty} N_i(t) = \frac{N_i(t^*)}{\sum_{i \in \mathcal{A}} N_i(t^*)} \sum_{i \in \mathcal{A}} N_i(t + \Delta t) = \frac{N_i(t^*)}{\sum_{i \in \mathcal{A}} N_i(t^*)} \cdot \overline{N},
\]

and the proof is given.
Relation (multiple) inputs and outcomes

In this sub section it is shown that a multiple of any initial number of policy holders of \( \theta \) results into the same multiple of the original outcome.

**Theorem 6** Let \( \theta, \theta' \in \Theta \) be two identical situations, except that \( N' = c \cdot N \) with \( c \in \mathbb{R}^+ \). Then it holds that \( \hat{N}'_i = c \cdot \hat{N}_i \) for all \( i \in \hat{N} \).

**Proof** Let \( \{N_i(t)\}_{i \in \tilde{N}} \) be a solution of the differential equations of situation \( \theta \) and define \( \{N'_i(t)\}_{i \in \tilde{N}} \) as a possible solution of the differential equations of situation \( \theta' \), with:

\[
N'_i(t) = c \cdot N_i(t) \quad \text{for all } i \in \tilde{N} \text{ and } t \geq t^*.
\]

Now, let \( i \in \tilde{N} \) and fix some \( t \geq t^* \). Then take the derivative of \( N'_i(t) \) to \( t \), which gives:

\[
\frac{d}{dt} N'_i(t) = c \cdot \frac{d}{dt} N_i(t)
\]

(29)

Now, substituting (11) in the right side of (29) gives:

\[
\frac{d}{dt} N'_i(t) = c \cdot \left( \sum_{j \in \tilde{N} \setminus \{i\}} \alpha \beta (x_j - x_i) \hat{N}_i \hat{N}_j \right)
\]

\[
= c \cdot \left( \sum_{j \in \tilde{N} \setminus \{i\}} \alpha' \cdot c' \cdot \beta' \cdot (x'_j - x'_i) \left( \frac{N'_i}{c} \right) \left( \frac{N'_j}{c} \right) \right)
\]

\[
= \left( \sum_{j \in \tilde{N} \setminus \{i\}} \alpha' \cdot \beta' (x'_j - x'_i) \hat{N}'_i \hat{N}'_j \right),
\]

where the second equality is obtained by substituting \( \alpha = \alpha' \), \( x_j = x'_j \), \( x_i = x'_i \), \( \beta = \frac{1}{\sum_{j \in \tilde{N}} (N_i)} = \frac{c}{\sum_{j \in \tilde{N} \setminus \{i\}} (cN_i)} = c \cdot \beta' \), \( N_i = N'_i/c \) and \( N_j = N'_j/c \) (which holds for all \( t \)) and the last equality by eliminating \( c \).

We conclude that \( \{N'_i(t)\}_{i \in \tilde{N}} \) is indeed a solution for \( \theta' \). Then, it also holds that \( N'_i(t+1) = c \cdot N_i(t+1) \) for all \( i \in \tilde{N} \) and thus \( \hat{N}_i = c \cdot \hat{N}_i \), which completes the proof. \( \blacksquare \)
Combination of situations

In previous sub section it is shown that a multiple of any initial number of policy holders results into the same multiple of the original outcome. It is also possible to find a new situation with an initial number of policy holders equal to the sum of the two other situations that has an outcome equal to the sum of outcomes of the other situations.

**Theorem 7** Let $\theta, \theta', \theta'' \in \Theta$ be three identical situations, except that $N' = c \cdot N$ with $c \in \mathbb{R}^+$ and $N'' = N' + N$. Then:

$$\hat{N}'' = \hat{N}' + \hat{N}. \quad (30)$$

**Proof** Using that $N' = c \cdot N$ we can write

$$N'' = N' + N = c \cdot N + N = (c + 1) \cdot N.$$

Situation $\theta$ and $\theta''$ satisfy the conditions of Theorem 6 with constant $(c+1)$. Hence $\hat{N}''$ is equal to:

$$\hat{N}'' = (c + 1) \cdot \hat{N}.$$

Now, observe that:

$$(c + 1) \cdot \hat{N} = c \cdot \hat{N} + \hat{N}.$$

Using that $c \cdot \hat{N} = \hat{N}'$ by Theorem 6 again, it follows that:

$$(c + 1)\hat{N} = \hat{N}' + \hat{N},$$

which completes the proof. ■

Equilibrium solutions

For finding the equilibrium points of the system of differential equations, one will set the differentials in the system equal to zero:

$$\frac{dN_i}{dt} = \sum_{j \in \tilde{N} \setminus \{i\}} \alpha \beta (x_j - x_i)N_i N_j = 0 \ \forall i \in \tilde{N}. \quad (31)$$

A trivial equilibrium point is $\alpha = 0$. If the price sensitivity is equal to zero, no policy holder has the intention to switch and the differential equations will be equal to zero as well. Observe that $\beta = (N_1 + N_2 + ... + N_k)^{-1}$
which implies that $\beta \neq 0$. Hence, $\beta = 0$ is no equilibrium point. Another equilibrium point that satisfies the condition of (31) is $x_i = x_j = \ldots = x_k$. When all health insurers charge the same premium, no policy holder has the intention to switch and as a result the differential equations will be equal to zero again. Finally, there are $k$ (similar) equilibrium points, namely $N = (N, 0, \ldots, 0), N = (0, N, 0, \ldots, 0), \ldots, N = (0, \ldots, N)$. These equilibrium points imply that only one health insurer has all policy holders, while other health insurers have no policy holders. This results into differential equations that are equal to zero and thus to an equilibrium point.

Now, one will investigate if the last bunch of equilibrium points are stable ones. Consider a situation $\theta$ with $k = 2$ and $x_i < x_j$. A phase portrait with $N_i$ and $N_j$ on the axes is depicted below.

![Phase portrait differential equation](image)

Observe that the graph is only of interest in the area of $[0, 5]$. The arcs in Figure 4 depict how many and to which health insurer policy holders switch. Given that $x_i < x_j$, the point $(0, 5)$ has only outgoing arcs. This implies that if health insurer $j$ has (nearly) all policy holders, they will not stay at this company, but switch to company $i$. The equilibrium point is thus not stable. However, the point $(5, 0)$ has only ingoing arcs. This implies that this point is stable and thus attracts all policy holders. More general, for the $k$ equilibrium points, only the $i$-th equilibrium point, with $x_i < x_j$ for all $j \neq i$ is a stable equilibrium point. All other equilibrium points are unstable, given that arcs exist with a direction opposite to the point.

---

10 An equilibrium point is called stable if the direction of the arcs in a phase diagram around the equilibrium point are directed to this point.
5.4 Key Performance Indicators

As discussed in previous section, operator $L$ determines for a given situation $\theta$ a new situation $\hat{\theta}$. With this new situation $\hat{\theta}$ it is possible to determine an outcome $\omega$. An operator $D$ is used to determine for year $t + 1$ outcomes as profit, market share and solvency ratio. In mathematical terms, the operation between $\theta$ and $\omega$ is denoted by:

$$D : \hat{\Theta} \rightarrow \Omega,$$

where $\hat{\Theta}$ is the set of situations $\hat{\theta}$ and $\Omega$ the set of outcomes $\omega$. In the upcoming sections, the elements profit, market share and solvency ratio of outcome $\omega$ will be discussed in more detail.

5.4.1 Profit

As discussed in Chapter 3, profit of health insurers depends on the commercial premium, technical premium and the number of policy holders. The commercial premium is the premium set to policy holders, where the technical premium is a cost covering premium. This implies that charging the technical premium is in general enough for not going bankrupt.

We assume that the technical premium only consists of variable costs and fixed costs, where the main variable costs are the costs for the claims and the main fixed costs are the salaries of the employees, hire and loans. The variable costs depend on the number of policy holders, where the fixed costs are independent. Hence, the technical premium per policy holder of health insurer $i$ is of the form:

$$x_{tec,i} = \left( c_i + \frac{C_i}{\hat{N}_i} \right),$$

The difference between the commercial premium and the technical premium results in the profit per policy holder. In a natural way, one can obtain the total profit as the profit per policy holder multiplied by the number of policy holders. Hence, profit is defined as:

$$P_i = \left( x_i - \left( c_i + \frac{C_i}{\hat{N}_i} \right) \right) \cdot \hat{N}_i = \left( x_i - c_i \right) \cdot \hat{N}_i - C_i.$$  

\[\text{Note that operator } \Gamma : \Theta \rightarrow \Omega \text{ is a combination of } L : \Theta \rightarrow \hat{\Theta} \text{ and } D : \hat{\Theta} \rightarrow \Omega.\]
\[\text{Note that health insurers can go bankrupt when claims exceed the threshold of the expected losses and the uncertainty component of the technical premium.}\]
\[\text{Normally the claim component and costs component are separated, but now collected together. Moreover, the solvency raise is excluded, given that it is often sufficient low.}\]
\[\text{Note that profit is the difference between contribution margin and fixed costs.}\]
5.4.2 Market share

In literature, market share is defined as the percentage of policy holders of a health insurer in relation to the total number of policy holders. With this key performance indicator, one can measure the percentage of the market that a specific health insurer has. In mathematical terms, the market share is defined as:

\[
M_i = \frac{\hat{N}_i}{\sum_{j \in \hat{N}} \hat{N}_j} \cdot 100\%
\] (35)

5.4.3 Solvency ratio

The last key performance indicator is the solvency ratio. The solvency of a health insurer is measured by the solvency ratio, given that it calculates the ratio of the own equity and the SCR level. In Solvency II\(^{15}\), the solvency ratio is used as a measurement to indicate how solvable a health insurer is.

In QIS 5 (2012) the SCR level is defined as a sum of reserve risk and lapse risk. A large amount of input parameters are needed for determining these risk levels. For this reason, an simplified calculation of the SCR level is used. The simplification of the SCR level is given by:

\[
SCR_i = \rho(\sigma) \cdot \hat{N}_i \cdot x_{tec,i},
\] (36)

where \(\rho(\sigma)\) is approximately equal to \(\rho(\sigma) \approx 0.0871\) in the Dutch health insurance market\(^{16}\). The idea is that the SCR level of a health insurer is at least 8.7% of the total amount of costs (see QIS5 (2012)).

Next to the SCR level, also the own equity \(E_t^{+}\) of year \(t + 1\) is needed for calculating the solvency ratio. The own equity is defined as the sum of the own equity of last year \(E_t\) and the profit of the current year \(P_t\):

\[
E_t^{+} = E_t + P_t.
\] (37)

With both the SCR level and the own equity, it is possible to define the solvency ratio. Dividing the own equity by the SCR level results into the solvency ratio. The higher the ratio, the better a health insurer is protected against extreme situations. In mathematical terms, the solvency ratio of health insurer \(i\) in year \(t + 1\) is defined as:

\[
S_i = \frac{E_t^{+}}{SCR_i}.
\] (38)

\(^{15}\)Solvency II is a new European regulation that concerns the amount of capital that European insurers must hold to reduce the risk of insolvency.

\(^{16}\)For more detailed information of \(\rho(\sigma)\) and related calculations, see QIS 5 (2012).
6 Analysis of the model

In this Chapter the mathematical model is analyzed. First, historical data is used to back test the policy behavior part of the model. Afterwards, fictive situations \( \theta \) are used for analyzing the effect of the level of price on the key performance indicators of health insurers.\(^{17}\)

6.1 Back testing: Policy holder model

For back testing the policy behavior part of the mathematical model, some historical data is needed. It is decided to consider historical data of the four largest health insurers, namely Achmea, CZ, Menzis and Univé. In what follows an explanation is given about how historical data is obtained.

6.1.1 Number of Policy holders

The number of policy holders \( N \) is obtained from annual reports of health insurers. In Table 2, the number of policy holders of the four different health insurers of the last four years is depicted.

<table>
<thead>
<tr>
<th>Company</th>
<th>2009</th>
<th>2010</th>
<th>2011</th>
<th>2012</th>
</tr>
</thead>
<tbody>
<tr>
<td>Univé</td>
<td>4,276,470</td>
<td>4,258,003</td>
<td>4,223,823</td>
<td>4,104,733</td>
</tr>
<tr>
<td>Achmea</td>
<td>4,789,832</td>
<td>4,799,676</td>
<td>4,817,676</td>
<td>4,910,765</td>
</tr>
<tr>
<td>Menzis</td>
<td>2,052,713</td>
<td>2,109,388</td>
<td>2,134,388</td>
<td>2,147,797</td>
</tr>
<tr>
<td>CZ</td>
<td>3,354,000</td>
<td>3,370,900</td>
<td>3,425,000</td>
<td>3,348,900</td>
</tr>
</tbody>
</table>

Table 2: Number of policy holders 2009 - 2012

6.1.2 Premiums

The premiums \( X \) of the health insurers are obtained from annual reports as well.\(^{18}\) In Table 3, an overview of these premiums is given.

<table>
<thead>
<tr>
<th>Company</th>
<th>2009</th>
<th>2010</th>
<th>2011</th>
</tr>
</thead>
<tbody>
<tr>
<td>UVIT</td>
<td>1,126</td>
<td>1,171</td>
<td>1,273</td>
</tr>
<tr>
<td>Achmea</td>
<td>1,103</td>
<td>1,154</td>
<td>1,255</td>
</tr>
<tr>
<td>Menzis</td>
<td>1,125</td>
<td>1,135</td>
<td>1,241</td>
</tr>
<tr>
<td>CZ</td>
<td>1,063</td>
<td>1,094</td>
<td>1,284</td>
</tr>
</tbody>
</table>

Table 3: Premiums of health insurers 2009 - 2011

\(^{17}\)The mathematical model is built in Excel, where all calculations of this chapter are executed. For a guideline of this Excel tool, see Appendix H.

\(^{18}\)Health insurers (often) exist of labels, with different set premiums. The proportional average of these premiums regarding to the number of policy holders per label is taken.
6.1.3 Price sensitivity parameter

In this section the price sensitivity parameter $\alpha$ is estimated. We discuss how this estimation is executed and results are presented.

Consider a situation $\theta \in \Theta$ where $N$ and $X$ are obtained from the field and $\alpha$ not fixed. Moreover, define $\hat{N}^e$ as a vector of number of policy holders, obtained from the field as well. Then, the price sensitivity parameter $\alpha$ is estimated by minimizing the sum of squared differences of $\hat{N}_i$ and $\hat{N}^e_i$ for all $i \in \tilde{N}$. Hence in mathematical terms, this is denoted by:

$$\min_{\alpha} \sum_{i \in \tilde{N}} (\hat{N}^e_i - \hat{N}_i)^2.$$  \hspace{1cm} (39)

In this thesis, the price sensitivity values are only estimated for the years 2009, 2010 and 2011. When these price sensitivity values are stored in one graph, the following figure is obtained.

![Estimated $\alpha$ for 2009 - 2010 - 2011](image)

Figure 5: Estimated $\alpha$ for 2009 - 2010 - 2011

Observe that the price sensitivity $\alpha$ has an increasing behavior during the years. This supports Vektis (2012), namely that policy holders become more price sensitive during the years. Despite that the price sensitivity values minimize the sum of the squared differences, the relative error between $\hat{N}$ and $\hat{N}^e$ could be sufficient large. It turns out that the maximal relative error of the estimated price sensitivity parameter is 2.80 % whereas the minimal relative error is 0.28 %. The relative errors of the other two price sensitivity parameters lies within this range. This implies that the price sensitivity values predict the new configuration of policy holders with a minimal difference of 0.28 %, but with a maximal difference of 2.80 %.

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19 Empirical data is obtained from section 6.1.1 and 6.1.2.
20 The price sensitivity of 2012 is not calculated, given that $\hat{N}^e$ is not published yet.
21 The relative error is defined as $\frac{\hat{N}_i - \hat{N}^e_i}{\hat{N}^e_i} \cdot 100\%$
6.1.4 Back testing

In this section we test how well the outcome $\hat{N}$ of the mathematical model performs against the empirical data $\hat{N}^e$. First, a situation $\theta \in \Theta$ is considered wherein everything is known beforehand. This implies that we can use the estimated price sensitivities of previous section. The premiums $X$ are originating from Table 3 and the outcomes $\hat{N}$ of the model of year $t$ form the input for year $t + 1$. Moreover, $t^* = 0.87$ which stands for the 15th of November is used. In Figure 6 the trajectory for the years 2009 - 2012 is depicted, where the punctate lines stand for the outcomes of the model and the full lines stand for the empirical data.

![Figure 6: Model configuration and empirical data with changing $\alpha$](image)

Observe that the outcomes of the model are close related to the empirical data (maximal relative error of 0.23 %). Hence, when everything is known on beforehand, the mathematical model is able to predict the outcome well. However, in reality it is not possible to know everything on beforehand, given that the number of policy holders of year $t + 1$ are needed for the determination of the price sensitivity of year $t$. For this reason, the model is also tested against fixed price sensitivities obtained by historical data.

In total, we consider three different situations, the first one starting in 2009 with $\alpha = 0.18 \cdot 10^{-5}$, the second one starting in 2010 with $\alpha = 0.62 \cdot 10^{-5}$ and the last one starting in 2011 with $\alpha = 2.75 \cdot 10^{-5}$. The (graphical) results of each of those tests are presented in Appendix D1. We observed that outcomes of the model become less accurate during the years (with a maximal relative error of 10.4 %). Hence, when price sensitivity increases in reality (see Figure 5) the model performance becomes worse during the years. For this reason, it is best to determine the price sensitivity year by year. Our advise is to first determine price sensitivity with historical data as discussed before and then to use forecasting techniques (see Hamilton (1994)) and expert judgement to make the price sensitivity value more accurate.
6.2 Sensitivity Analysis

In this section the impact of the input parameters is being investigated. First, the premium vector $X$, vector of policy holders $N$ and price sensitivity $\alpha$ are discussed. Later on, the costs $c$, $C$ and own equity $E$ in relation with the key performance indicators are discussed. In what follows, one general situation $\tilde{\theta} \in \Theta$ is considered, with the following parameters:

<table>
<thead>
<tr>
<th>Company</th>
<th>$N$</th>
<th>$X$</th>
<th>$E$</th>
<th>$c$</th>
<th>$C$</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1) Univé</td>
<td>100</td>
<td>32</td>
<td>200</td>
<td>23</td>
<td>600</td>
</tr>
<tr>
<td>(2) CZ</td>
<td>100</td>
<td>35</td>
<td>200</td>
<td>23</td>
<td>600</td>
</tr>
<tr>
<td>(3) Menzis</td>
<td>100</td>
<td>34</td>
<td>200</td>
<td>23</td>
<td>600</td>
</tr>
</tbody>
</table>

Table 4: Input parameters for situation $\tilde{\theta}$

Moreover, the price sensitivity parameter is equal $\alpha = 0.05$, the year is $t = 0$ and the starting moment to switch is $t^* = 0.87$. All values are fictive.

6.2.1 Price, policy holders and price sensitivity

The first input parameter that is considered is price. In situation $\tilde{\theta}$, CZ is the one with the highest price, while Univé is the one with the lowest price. With the current price sensitivity parameter $\hat{N}$ becomes:

$$\hat{N} = (126, 80, 94).$$

Observe that most policy holders switch to Univé (+26), while most policy holders leave CZ (-20). Menzis is the one that loses policy holders to Univé (-12), while it receives policy holders from Menzis (+6). It is of interest to investigate what happens with $\hat{N}$ if Univé deviates from its current premium. In Figure 7 an overview for this premium deviation is given.

Figure 7: Configuration of policy holders by varying premium Univé
Observe that for \( x_1 \gg x_2 > x_3 \) all policy holders of Univé decide to switch to CZ and Menzis. In the opposite direction, when \( x_1 \ll x_3 < x_2 \) all policy holders from health insurers CZ and Menzis switch to Univé. Moreover, observe that for \( x_1 \gg x_2 > x_3 \) there is a stable allocation of policy holders between CZ and Menzis. Policy holders of Univé switch to health insurer \( i \) with rate \( \alpha \beta (x_1 - x_i) N_1 N_i \). Given that \( x_2 > x_3 \) there is a higher intensity of policy holders going to Menzis than to CZ. Furthermore, also policy holders of CZ switch to Menzis, given that \( x_2 > x_3 \). This forms the stable allocation between these two health insurers. Last, observe that the graph of Univé intersects at \( x_1 = 34 \) with the graph of Menzis. This phenomena is logical, given that Menzis’ premium is equal to 34 as well. When Univé charges a premium of 34, no policy holders of Univé and Menzis will switch between each other. Only policy holders of CZ will switch to Univé and Menzis and against the same intensity rate, given that the initial number of policy holders is equal to 100 for both Menzis and Univé. This argument also holds for the intersection of Univé with CZ.\(^{22}\)

The second input parameter that is considered is the price sensitivity parameter \( \alpha \). Consider again situation \( \tilde{\theta} \), but now with \( \alpha = 0.07 > 0.05 \). Now, we investigate how the configuration of policy holders for a varying premium of Univé deviates from the original situation \( \tilde{\theta} \). The results of \( \alpha = 0.07 \) are included as punctate lines in Figure 8.

![Figure 8: Configuration of policy holders by \( \alpha = \{0.05, 0.07\} \)](image)

Observe in Figure 8 that policy holders are more sensitive in regarding to the original situation \( \tilde{\theta} \) with \( \alpha = 0.05 \). For example, when Univé charges a premium of 32 euro, the number of policy holders become 144, which is higher than 126 as obtained in the original situation. In the opposite direction, when Univé charges a high premium of 35 euro, the number of policy holders is equal to 100 for both Menzis and Univé.

\(^{22}\)In this situation, Univé and CZ both charge \( x_1 = x_3 = 35 \) and have the same number of policy holders.
holders become 118, which is lower than 122 of the original situation. Hence, when \( \alpha \) increases, policy holders switch more by same price differences. It is of interest to investigate what happens when a relative high price sensitivity parameter (\( \alpha = 1 \)) is chosen. In Figure 9 this situation is depicted.

Observe in Figure 9 that policy holders react on price heavily. When the premium of Univé is lower than 34 euro, nearly all policy holders immediately switch to them, while in the opposite direction, when the premium of Univé is higher than 34 euro, nearly all policy holders switch to Menzis. Moreover, when Univé and Menzis charge the same premium (\( x_1 = x_3 = 34 \)) they nearly split the whole market (\( N_1 = N_2 \approx 150 \)). Hence, CZ always loses is policy holders, given that they have a (slightly) higher price.

Last, also a configuration with a relative low price sensitivity parameter is considered. With \( \alpha = 5 \cdot 10^{-4} \) the following figure is obtained.

Observe in Figure 10 that policy holders react on price heavily. When the premium of Univé is lower than 34 euro, nearly all policy holders immediately switch to them, while in the opposite direction, when the premium of Univé is higher than 34 euro, nearly all policy holders switch to Menzis. Moreover, when Univé and Menzis charge the same premium (\( x_1 = x_3 = 34 \)) they nearly split the whole market (\( N_1 = N_2 \approx 150 \)). Hence, CZ always loses is policy holders, given that they have a (slightly) higher price.

Last, also a configuration with a relative low price sensitivity parameter is considered. With \( \alpha = 5 \cdot 10^{-4} \) the following figure is obtained.
Observe that policy holders are less sensitive for price differences, given that only a small number of policy holders switch to Univé when price decreases. Note that these results are based on the (best) estimated price sensitivity parameter of 2012.

The third input parameter that is considered is the number of policy holders. Consider again situation $\tilde{\theta}$, but now with $N = (150, 75, 75)$. The graph with the new configuration of policy holders for a deviating premium of Univé, together with the original situation is given in Figure 11 below.

![Graph showing two different starting number of policy holders](image)

**Figure 11: Two different starting number of policy holders**

Observe that the punctate line of Univé is above the line of the original situation. This is mainly caused by the higher starting number of policy holders of 150. Moreover, observe that the intersection of the graph of Univé and Menzis is shifted to the right. It is logical that this intersection is shifted. When $x_1 = x_2 = 34$, the starting number of policy holders of Univé and Menzis is not the same (150 against 75). Hence, the intensity whereby Univé attracts policy holders is higher than that of Menzis. As a consequence, the number of policy holders cannot be the same. Therefore, the price of Univé should be higher, to compensate for this starting advantage. It turns out that for $x_1 = 41.70$ the same number of policy holders is obtained. In the same line for $x_1 = 42.70$ the graph of Univé and CZ intersects.

We mention that the effect of the interaction term $N_iN_j$ is also investigated. Results are depicted in Appendix D2. Moreover, also the the effect of considering more than one group of policy holders is investigated and results are depicted in Appendix D3.
6.2.2 Profit, market share and solvency ratio

The first key performance indicator that is considered is profit. Remember that profit per health insurer is defined as:

\[ P_i = (x_i - c_i) \cdot \hat{N}_i - C_i. \]  

(40)

Hence, profit depends on the multiplication of profit per policy holder, the number of policy holders and the fixed costs \( C_i \). Where the profit per policy holder \( (x_i - c_i) \) is an increasing linear function in \( x_i \), the number of policy holders \( \hat{N}_i \) is a non-linear decreasing function (see for example Figure 7). The fixed costs \( C_i \) is a constant function and only decreases the profit function on the whole profit-axes. Thus, the shape of the profit function only depends on the combination of the profit per policy holder and the number of policy holders. Given that profit per policy holder is linear and the number of policy holders is non linear, it is not immediately clear how the product of these two will behave. For this reason, some situations \( \theta \) are considered to investigate the shape of the profit function in more detail.

Consider again situation \( \tilde{\theta} \in \Theta \). Then, the profit of Univé behaves for a deviating premium as the graph in Figure 12.

Figure 12: Profit of Univé for a deviating premium

Observe that the profit function first increases, then crosses the price axis at \( x_1 = 28.7 > c_1 \), then reaches the top at \( x_1 = 33.52 \) and finally decreases asymptotically to \(-600 = -C_1\); the fixed costs. Apparently, for small \( x_1 \) the linear increase of the profit per policy holder is stronger than the non-linear decrease of the number of policy holders, while for large \( x_1 \) the non-linear decrease is stronger than the linear increase of the profit per policy holder. Moreover, for large \( x_1 \), the product of the profit per policy holder and the number of policy holders tends to zero and as a consequence the profit...
decreases to the fixed costs. This asymptotical behavior is denoted by the red punctate line in Figure 12. When price sensitivity $\alpha = 0.04$ is chosen, the profit function of Univé changes into the punctate line of Figure 13.

Observe that the curve of the graph is quite similar to the original graph, but it is shifted to the right. This result is logical. Given that price sensitivity parameter is lower now, Univé can charge a higher premium before the same amount of policy holders switch. As a consequence, the extreme of the function is shifted to the right as well. In the opposite direction, when $\alpha = 0.06$ is chosen higher, the following graph is obtained.

Figure 13: Profit of Univé for a deviating premium with $\alpha = \{0.04, 0.05\}$

Figure 14: Profit of Univé for a deviating premium with $\alpha = \{0.05, 0.06\}$

Observe that the profit function is now shifted to the left. This means that Univé has to charge a lower premium to lose the same amount of policy holders. As a consequence, the extreme of the function is shifted to the left as well. Moreover, observe that the maximal profit is a little bit higher than
the original profit. When considering again $\alpha = 0.05$, but now with higher variable costs $c_1 = 25$ for Univé, the following graph is obtained.

Figure 15: Profit of Univé for a deviating premium with $c_1 = \{23, 25\}$

Hence, the profit function is shifted to the right and downwards. Apparently, when variable costs increase, it is beneficial for Univé to charge a higher premium, despite that more policy holders switch. The profit is decreased by the fact that variable costs are increased. Last, with decreased fixed costs $C_1 = 400$, the following graph is obtained.

Figure 16: Profit of Univé for a deviating premium with $C_1 = \{400, 600\}$

Remark that the profit function is exactly shifted upwards. As discussed, the fixed costs will not effect the shape of the profit function, but in this case only increases the profit function on the whole profit - axis.

Alongside profit, the second key performance indicator is market share. Given that market share is a ratio of the number of policy holders and
the total number of policy holders, the analysis of the market share is similar to paragraph 6.2.1. When considering situation $\hat{\theta}$ the market share of Univé behaves for a deviating premium as depicted in Figure 17.

![Figure 17: Market share of Univé for a deviating premium](image)

The shape of the function is (indeed) the same as the number of policy holders of Univé. The only difference with the number of policy holders is the re-scaling factor of $\left(\sum_{i=1}^{k} N_i\right)^{-1}$. For this reason we will not investigate market share in more detail.

The third and last key performance indicator is the solvency ratio. Remark that the Solvency ratio is formulated as:

$$S_i = \frac{E_i^+}{SCR_i}.$$  \hspace{1cm} (41)

The ratio depends on the funds of the health insurer and the required SCR level. Remark that the own funds are defined as:

$$E_i^+ = E_i + (x_i - c_i)\hat{N}_i - C_i.$$  \hspace{1cm} (42)

and the SCR level by:

$$SCR_i = \rho(\sigma) \left(\hat{N}_i c_i + C_i\right).$$  \hspace{1cm} (43)

It is hard to directly analyze how the solvency function behaves under a given situation. For this reason, we rewrite the function. Substituting equation (42) and (43) into equation (41) gives:

$$S_i = \frac{E_i + (x_i - c_i)\hat{N}_i - C_i}{\rho(\sigma) \left(\hat{N}_i (c_i + C_i)\right)}.$$
\[
= -\left( c_i \hat{N}_i + C_i \right) + E_i + x_i \hat{N}_i \\
\rho(\sigma) \left( \hat{N}_i c_i + C_i \right)
\]
\[
= -\frac{1}{\rho(\sigma)} + \frac{E_i + x_i \hat{N}_i}{\rho(\sigma) \left( \hat{N}_i c_i + C_i \right)}
\]
\[
= \frac{1}{\rho(\sigma)} \left( \frac{x_i \hat{N}_i + E_i}{\hat{N}_i c_i + C_i} - 1 \right).
\]

With this new expression, it is easier to explain the behavior of the solvency function. When considering situation \( \tilde{\theta} \in \Theta \) and a varying premium of Univé, the graph of Figure 18 is obtained.

![Figure 18: Solvency level of Univé for a deviating premium](image)

When premium is relative low, Univé receives the whole market, while profit per policy holder becomes negative. As a consequence, \( (x_i \hat{N}_i + E_i) \) approximates to \( E_i \), while \( (\hat{N}_i \cdot c_i + C_i) \) approximates to \( N \cdot c_i + C_i \). Hence, the solvency function approximates for relative low premiums to:

\[
S_i \approx \frac{1}{\rho(\sigma)} \left( \frac{E_i}{N \cdot c_i + C_i} - 1 \right)
\]

Implementing the parameter values of Univé of situation \( \tilde{\theta} \) results into \( \frac{1}{0.05} \cdot \left( \frac{200}{300 \cdot 23 + 600} - 1 \right) \approx -20 \). The solvency function approximates -20, which is visible in the graph of Figure 18. Then, for an increasing premium, the solvency function increases as well, until a premium of \( x_1 = 36.02 \). From this premium on, the number of policy holders decrease rapidly and as a consequence the solvency function begins to decrease as well. For a relative large premium, the solvency function will mainly depend on the the fixed costs, own funds and \( \rho(\sigma) \). The solvency function then approximates to:
\[ S_i \approx \frac{1}{\rho(\sigma)} \left( \frac{E_i}{C_i} - 1 \right) . \]  

(45)

Implementing the parameters values of Univé of situation ̂\( \theta \) results into 1.05 \cdot ( \frac{200}{600} - 1 ) \approx -13.33. The solvency function approximate -13.33, which is visible in the graph of Figure 18. Hence, the behavior of the function for relative low and relative high premiums depends on the parameters as denoted in equations (44) and (45). Given that more parameters can effect the behavior of policy holders and as a consequence the solvency function, it is of interest to investigate the function for other kind of parameter values.

When again considering ̂\( \theta \), but now with relative low fixed costs \( C_1 = 120 < E_1 \), the following graph is obtained.

![Figure 19: Solvency level of Univé with \( C_1 = \{10, 200\} \)](image)

Where the behavior of the solvency function is the same for relative low premium, the function increases more for high premiums. For high premiums, the function approximates \( \frac{1}{\rho(\sigma)} \left( \frac{E_i}{C_i} - 1 \right) \). In our situation, \( E_1 = 200 \) and \( C_1 = 120 \). As a consequence, the ratio stays positive regarding to the original situation and approximates \( \frac{2}{3} \cdot 20 = 13\frac{1}{3} \) (see red punctate line). In the oppositive direction, when the fixed costs are relative high, a graph is obtained in where the solvency level drops down the premium - axis for some premium value, given that the ratio becomes negative.

Moreover, it is of interest to investigate the effect of the variable costs. The variable costs also influence the behavior of the solvency function, but only for low prices. When premium is high, the term \( c_i \hat{N}_i \) tends to zero, while the terms \( c_i N \) influences the function when premium is low. Consider again situation ̂\( \theta \), but now with variable costs \( c_1 = 16 < 23 \). Then, the following graph for a deviating premium of Univé is obtained.
In Figure 20 it is shown that a decrease in the variable costs leads to a solvency function that crosses the premium axis earlier, obtains a higher value for a higher premium and crosses the premium axes later on again. Hence, when variable costs decrease, there are more possibilities to obtain a positive solvency level. Last, the own funds can be adapted. When having own funds of $E_1 = 300 > 200$ the following graph is obtained.

Figure 21: Solvency level of Univé with $E_1 = \{200, 300\}$

Observe that more own funds results into a solvency function that crosses the premium axis earlier, reaches a higher value (at a higher premium) and decreases the premium-axis at a later premium. This result is logical, given that for large premiums, the function approximates equation (45) and thus $20 \cdot \left(\frac{300}{600} - 1\right) = -10$, while the behavior for low premiums is quite similar to the original situation, given that $N c_i + C_i \gg E_1$ still holds, which implies that the function approximates $\frac{1}{0.05} \cdot \left(\frac{300}{300+23+600} - 1\right) \approx -20$ as well.
7 Case study Milliman

In chapter [6] one has analyzed how price can effect the key performance indicators of health insurers for several kinds of situations. In these situations everything was fixed on beforehand. For Milliman it is more of interest to consider situations in where premium is an unknown for a health insurer and to investigate which premium is best for them. In what follows, such situations with related questions (e.g. cases) are considered with a time horizon of one, three and five year(s). All values are fictive. 23

7.1 One year cases

In this section we discuss three interesting cases with a time horizon of one year. In all cases, Milliman has formulated one question related to the predefined situation. In the first case, health insurers are comparable with each other and a best price regarding to a maximal profit is obtained, while in the second case health insurers differ from each other. In the last situation, health insurers differ from each other as well, while now a best price regarding to a maximal market share is obtained.

7.1.1 Case I

We consider a situation $\theta \in \Theta$ with $k = 3$ health insurers, a price sensitivity of $\alpha = 0.05$ and the following parameters:

<table>
<thead>
<tr>
<th>Company</th>
<th>$N$</th>
<th>$X$</th>
<th>$E$</th>
<th>$c$</th>
<th>$C$</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1) Univé</td>
<td>150</td>
<td>?</td>
<td>200</td>
<td>23</td>
<td>1400</td>
</tr>
<tr>
<td>(2) CZ</td>
<td>120</td>
<td>35</td>
<td>150</td>
<td>21</td>
<td>1000</td>
</tr>
<tr>
<td>(3) Menzis</td>
<td>180</td>
<td>34</td>
<td>220</td>
<td>22</td>
<td>1500</td>
</tr>
</tbody>
</table>

Table 5: Input parameters for Case I

Hence, Univé, Menzis and CZ all have a different number of policy holders and different variable and fixed costs. Moreover, the premium of Univé is unknown for the upcoming year. For Milliman it is of interest to investigate which premium should be charged to maximize profit. Alongside profit, also the key performance indicators market share and solvency ratio are of importance. In this case, Univé wants to hold a minimal market share of 30 % and a solvency ratio of 1.5. The question related to this case is:

Q1. Which premium should Univé charge to optimize profit, without violating the market share and solvency ratio constraints?

23 All values used in this case study are fictive and are not directly related to any existing health insurer in the Netherlands.
This question can be formulated as a mathematical optimization problem. The objective function is profit, while premium is the decision variable. The key performance indicators market share and solvency ratio play the role as constraints. The optimization problem is formulated as:

\[
\max_{x_i \in \mathbb{R}^+} (x_1 - c_1)N_1 - C_1
\]

\[
\text{s.t. } \frac{E_1^+}{SCR_1} - 1.5 \geq 0
\]

\[
\frac{N_1}{\sum_{i=1}^3 N_i} - 0.30 \geq 0.
\]

Calculating the profit for a premium range of \(x_1 \in [1, 57]\) results in the following profits, as depicted in Figure 22 below.

**Figure 22: Profit of Univé for a deviating premium**

Observe that the maximal profit is obtained at the highlighted point in Figure 22. By using Newton’s iterative method (see Papalambros (2000)) it is possible to solve the optimization problem and thus to find a related premium \(x_i^\star\). The premium related to the maximal profit is \(x_1^\star = 33.48\). Note that key performance indicators market share and solvency ratio become \(M_1 = 36.40\%\) and \(S_1 = 2.00\) respectively. These values both satisfy the constraints, which makes \(x_1^\star = 33.48\) a feasible solution. Hence, for Univé it is best to charge a premium of \(x_1^\star = 33.48\) to maximize profit for the upcoming year without violating the market share and solvency constraints.

---

\(\text{Note that the constraints } g(x) \text{ are formulated in the positive null-form } (g(x) \geq 0), \text{ which makes the optimization problem numerically more stable.}\)
7.1.2 Case II

We consider a situation $\theta \in \Theta$ with $k = 3$ health insurers, a price sensitivity of $\alpha = 0.05$ and the following parameters:

<table>
<thead>
<tr>
<th>Company</th>
<th>$N$</th>
<th>$X$</th>
<th>$E$</th>
<th>$c$</th>
<th>$C$</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1) DSW</td>
<td>30</td>
<td>?</td>
<td>200</td>
<td>19</td>
<td>500</td>
</tr>
<tr>
<td>(2) CZ</td>
<td>120</td>
<td>35</td>
<td>800</td>
<td>21</td>
<td>800</td>
</tr>
<tr>
<td>(3) Menzis</td>
<td>180</td>
<td>34</td>
<td>800</td>
<td>22</td>
<td>800</td>
</tr>
</tbody>
</table>

Table 6: Input parameters for Case II

The difference with the first situation is the introduction of a small health insurer (DSW) with relative low fixed and variable costs. For Milliman it is of interest to investigate which premium DSW should charge to optimize profit in the upcoming year. Alongside this, DSW also wants to hold a minimal solvency ratio of 1.5 and only accepts a minimal market share of 30% for upcoming year. The question related to this case is:

**Q2. Which premium should DSW charge to optimize profit, without violating the market share and solvency ratio constraints?**

Again a (similar) optimization problem can be formulated. Regarding to the first case, it is most likely that one of the constraints is now active, given that the market share constraint of DSW is relative high regarding to its current market share. When an overview of the profit is given against a premium range of $x_1 \in [1, 57]$, the following figure is obtained.

![Figure 23: Profit of DSW for a deviating premium](image)

---

A constraint $g(x)$ is called active, if $g(x) = 0$. 

48
Observe that a part of the graph in Figure 23 is shaded red. In this area, the premium is that high, that market share drops down the minimal required level of 30%. Hence, premiums in this area are not part of the feasible set of possible premiums. When optimizing on profit, the premium on the bound of the shaded area forms the (feasible) premium that maximizes profit. By using Newton’s iterative method a premium of $x^\star_1 = 24.67$ is found. Note that the first constraint, regarding to market share is indeed active. The solvency ratio constraint is not active, given that a solvency ratio of 2.20 is obtained. To conclude, for DSW is it best to charge a premium of $x^\star_1 = 24.67$ to maximize profit for the upcoming year without violating the market share and solvency constraints.

7.1.3 case III

In this last case, we consider a situation $\theta \in \Theta$ with $k = 3$ health insurers, a price sensitivity of $\alpha = 0.05$ and the following parameters:

<table>
<thead>
<tr>
<th>Company</th>
<th>$N$</th>
<th>$X$</th>
<th>$E$</th>
<th>$c$</th>
<th>$C$</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1) TakeCare</td>
<td>30</td>
<td>?</td>
<td>50</td>
<td>17</td>
<td>300</td>
</tr>
<tr>
<td>(2) CZ</td>
<td>120</td>
<td>35</td>
<td>600</td>
<td>23</td>
<td>850</td>
</tr>
<tr>
<td>(3) Menzis</td>
<td>180</td>
<td>34</td>
<td>600</td>
<td>23</td>
<td>850</td>
</tr>
</tbody>
</table>

Table 7: Input parameters for Case III

TakeCare\(^{26}\) is a new health insurer in the health insurance market and has for this reason less own funds, but also low fixed and variable costs. For Milliman it is of interest to investigate which premium TakeCare should charge to optimize market share in the upcoming year. The only requirement is to hold a solvency ratio of 1.5. The question related to this case is:

Q3. Which premium should TakeCare charge to optimize market share, without violating the solvency ratio constraint?

Again an optimization problem can be formulated, which is quite different to the first two optimization problems. In mathematical terms, this optimization problem is given by:

$$
\max_{x_i \in \mathbb{R}^+} \left( \frac{\hat{N}_1}{\sum_{i=1}^3 \hat{N}_i} \right)
$$

subject to

$$
\frac{E^+_{i}}{SCR_1} - 1.5 \geq 0.
$$

\(^{26}\)TakeCare is a fictive health insurer, that enters the health insurance market, with relative low own funds, low variable and fixed costs.
When an overview of the market share is given against a premium range of \( x_1 \in [1, 57] \), the following figure is obtained.

![Figure 24: Market share of TakeCare for a deviating premium](image)

Observe that again a part of the graph in Figure 24 is shaded red. In this area, the premium is that low, that the solvency ratio drops down the minimal required level of 1.5. Hence, premiums in this area are not part of the feasible set of possible premiums. When optimizing on market share, the premium on the bound of the shaded area forms the (feasible) premium that maximizes market share. By using Newton’s iterative method a premium of \( x_1^* = 20.07 \) is found with a maximal market share of \( M_1 = 46.08\% \). Note that the solvency ratio constraint is active, such that \( S_1 = 1.5 \). To conclude, for TakeCare is it best to charge a premium of \( x_1^* = 20.07 \) to maximize market share for the upcoming year without violating the solvency ratio constraint.

### 7.1.4 Discussion

In all our three cases we have seen that charging a premium below the premiums of the others is optimal from a profit or market share perspective. In the case where the health insurer is quite similar to the others (e.g. in the number of policy holders and costs), the optimal premium is close related to the others (but still lower), while in the other two cases, where the health insurer is quite different from the rest (e.g. small number of policy holders and low costs), it is optimal to charge a premium far below the premiums of the others. We mention that we cannot conclude any general results from these cases. Probably, the results depend on the chosen parameters and different results can be obtained by other input parameters. For further research, it is of interest to investigate how sensitive the outcomes as profit, market share and solvency ratio are for deviating parameters.
7.2 Many years cases

Only considering scenarios with a one year horizon is sometimes too short-sighted, given that effects on the key performance indicators for upcoming years are not taken into account. For this reason, two many years cases are discussed. In the first case a time horizon of five years is considered, while in the second case a time horizon of three years is chosen.

7.2.1 Case IV

Consider a situation $\theta \in \Theta$ with $k = 3$ health insurers, a price sensitivity of $\alpha = 0.01$ and the following parameters for a time horizon of five years:

<table>
<thead>
<tr>
<th>Company</th>
<th>$N$</th>
<th>$X$</th>
<th>$E$</th>
<th>$c$</th>
<th>$C$</th>
<th>year</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1) Unive</td>
<td>150</td>
<td>?</td>
<td>1000</td>
<td>12.1</td>
<td>120</td>
<td></td>
</tr>
<tr>
<td>(2) CZ</td>
<td>120</td>
<td>35</td>
<td>1000</td>
<td>11.8</td>
<td>130</td>
<td>1</td>
</tr>
<tr>
<td>(3) Menzis</td>
<td>180</td>
<td>34</td>
<td>1000</td>
<td>12.8</td>
<td>150</td>
<td></td>
</tr>
</tbody>
</table>

Table 8: Input parameters for Case IV

Observe that the variable and fixed costs are the same per health insurers during the years, while the own funds and number of policy holders are unknown (i.e. marked with $\diamond$), given that they depend on the chosen premiums of Unive. For Milliman it is of interest to investigate the premium(s) that Unive should charge to maximize the sum of the profits on a five year horizon. Moreover, Unive wants to hold a minimal market share of 30 % and a solvency ratio of 1.5 for the upcoming five years. The question related to this case is:

Q4. Which premiums should Unive charge to optimize the sum of profits on a five year horizon, without violating the market share and solvency ratio constraints in the upcoming five year?

This question can be formulated as a mathematical optimization problem as well. The objective function is the sum of the profits, while premiums are the decision variables. The key performance indicators market share and solvency ratio play the role as constraints. Before stating the optimization problem, some new notation is introduced. We define $x_i(t)$, $c_i(t)$ and $C_i(t)$ as the premium, variable and fixed costs in year $t$. Furthermore, we denote $N_i(t)$, $SCR_i(t)$ and $E_i^{+}(t)$ as the updated configuration of policy holders, solvency ratio and own funds respectively in year $t + 1$ regarding to year $t$. Note that for optimizing, the mathematical model should be executed five
times. This implies that the outcome $\omega$ of the mathematical model of year $t$ forms the input $\theta$ for the mathematical model in year $t+1$. In mathematical terms, this optimization problem can be formulated as:

$$\max_{(x_1(1), x_1(2), \ldots, x_1(5)) \in \mathbb{R}^+} \sum_{t=1}^{5} \left( (x_1(t) - c_1(t)) \hat{N}_1(t) - C_1(t) \right)$$

s.t. \[
\frac{E_{1}^+(t)}{SCR_1(t)} - 1.5 \geq 0 \text{ for } t = 1, 2, \ldots, 5 \\
\frac{\hat{N}_1(t)}{\sum_{i=1}^{3} \hat{N}_i(t)} - 0.30 \geq 0 \text{ for } t = 1, 2, \ldots, 5.
\]

Solving this optimization problem by using Newton’s iterative method\(^{27}\) results into the following premiums and SCR levels.

<table>
<thead>
<tr>
<th>year</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>premium</td>
<td>28.14</td>
<td>27.47</td>
<td>27.96</td>
<td>28.45</td>
<td>40.80</td>
</tr>
<tr>
<td>SCR</td>
<td>1.5</td>
<td>1.5</td>
<td>1.5</td>
<td>2.3</td>
<td>4.2</td>
</tr>
</tbody>
</table>

Table 9: Premiums for Univé for five year horizon

Observe that in the first four years, the premium is lower than the premiums of Menzis and CZ, while in the fifth year the premium increases to 40.80. Hence, the strategy behind this solution is to first attract many policy holders and then to charge them (finally) a relative high premium. Moreover, observe that the SCR constraint is active for the first three years. This result confirms that in the first three years a minimal premium is charged.

A weakness of this optimization problem is that the focus relies on the premium of year five. In the optimization problem, no attention is paid to the effects that the premium of year five can have on year six and so on. In our opinion this is not realistic and for this reason the optimization problem should be reformulated. We consider a time horizon of 30 years, wherein only the premiums for the first five years are changeable. The premiums from year six on are equal to the premium of year five. This implies that charging a (too) high premium in year five year effects the rest of the years substantial. Remark that the market share and solvency ratio constraints are also used up to year 30. Solving this new optimization problem results into the following table of premiums and SCR levels.

\(^{27}\)Note that Newton’s iterative method works well when considering more than one variable. For more information, see Papalambros (2000).
Observe that the premiums and SCR levels are similar to the original situation for the first three years, while they deviate in year four and five. Again, the first three years are used to increase the number of policy holders. Then in year four a relative high premium is charged to obtain a large profit. Contrary to the original situation, the premium in year five is quite similar to the premiums of Menzis and CZ. Hence, it is best for Univé to first charge a low price to attract policy holders, then charge a high price in year four to obtain a large profit and then charge a premium that is similar to the others (to survive in the future).

### 7.2.2 Case V

Last, consider a situation $\theta \in \Theta$ with $k = 3$ health insurers, a price sensitivity of $\alpha = 0.01$ and the following parameters:

<table>
<thead>
<tr>
<th>Company</th>
<th>$N$</th>
<th>$X$</th>
<th>$E$</th>
<th>$c$</th>
<th>$C$</th>
<th>year</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1) TakeCare</td>
<td>30</td>
<td>?</td>
<td>300</td>
<td>17</td>
<td>250</td>
<td>1</td>
</tr>
<tr>
<td>(2) CZ</td>
<td>120</td>
<td>35</td>
<td>1000</td>
<td>23</td>
<td>500</td>
<td>1</td>
</tr>
<tr>
<td>(3) Menzis</td>
<td>180</td>
<td>34</td>
<td>1000</td>
<td>23</td>
<td>500</td>
<td>1</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Company</th>
<th>$N$</th>
<th>$X$</th>
<th>$E$</th>
<th>$c$</th>
<th>$C$</th>
<th>year</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1) TakeCare</td>
<td>◊</td>
<td>?</td>
<td>◊</td>
<td>17/23</td>
<td>250/500</td>
<td>1</td>
</tr>
<tr>
<td>(2) CZ</td>
<td>◊</td>
<td>35</td>
<td>◊</td>
<td>23</td>
<td>350</td>
<td>2.3</td>
</tr>
<tr>
<td>(3) Menzis</td>
<td>◊</td>
<td>34</td>
<td>◊</td>
<td>23</td>
<td>350</td>
<td>1</td>
</tr>
</tbody>
</table>

* If market share > 30%

Table 11: Input parameters for Case V

Observe that a relative small health insurer is considered with relative low variable and fixed costs and low own funds. However, observe that the variable and fixed costs of TakeCare can increase in year two and three, when the market share increases to 30%. For Milliman, it is now of interest to investigate which premiums TakeCare should charge to obtain the maximum sum of market share on a three years horizon, without exceeding the solvency ratio of 1.5. The question related to this case is:

**Q5. Which premiums should TakeCare charge to optimize the sum of market shares on a three years horizon, without violating the solvency constraint?**

This question can be formulated as a mathematical optimization problem. The objective function is the sum of the market shares, while premiums are
the decision variables. The key performance indicator solvency ratio plays the role as constraint. The optimization problem is formulated as:

$$\max_{(x_1(1), x_1(2), ..., x_1(3)) \in \mathbb{R}^+} \sum_{t=1}^{3} \sum_{i=1}^{3} \frac{\hat{N}_i(t)}{\hat{N}_i(t)} \sum_{t=1}^{3} E_i^+(t) - 1.5 \geq 0 \text{ for } t = 1, 2, 3$$

Solving this optimization problem by using Newton’s iterative method results into the following premiums:

<table>
<thead>
<tr>
<th>x_1(1)</th>
<th>x_1(2)</th>
<th>x_1(3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>29.22</td>
<td>27.67</td>
<td>29.86</td>
</tr>
</tbody>
</table>

Table 12: Premiums for Univé for five year horizon

Observe that in the first year, a relative low premium regarding to Menzis and CZ is charged to obtain more policy holders. However, given that variable and fixed costs can increase heavily when market share increases to 30% a sufficient high premium is charged to not exceed this threshold. Then in year two, premium is set higher, to restrain the increase in market share to protect against the increased fixed and variable costs. In last year, a quite similar premium is charged. The final market share in year three is 28.56%. If we compare this with the first and second year, the market share increased from 10% to 21.34% and finally to 28.56%. The underlying idea of this case is, to show that maximizing market share is first priority, but exceeding the barrier of 30% is only possible when enough buffer (e.g. own funds) is built in to capture the increase in fixed and variable costs. For TakeCare this was not the case, given that own funds were too low.

### 7.2.3 Discussion

We have seen that it is beneficial from a profit perspective to first charge a premium below the premiums of the others, then above the the others and finally relative equal to the others and that deviation in costs can constraints insurers in increasing market share. Despite that it doesn’t give us general results, it gives insights in what is optimal from a profit perspective. Finally, we mention that we missed an important aspect in our analysis. In all cases, only one decision maker (e.g. health insurer) was considered at a time. In reality, many decision makers exist, which makes the outcomes of the discussed cases less useful. A discipline in mathematics that deals with multiple decision makers at the same time is game theory. In what follows, we will approach the model from a game theoretical point of view.

---

28 Observe that we don’t need to formulate the optimization problem for 30 years again, given that the deviation in costs (> 30%) already constraints it in a natural way.
A game theoretical approach
8 A Game theoretical approach

As discussed in Chapter 7, the mathematical model has some shortcomings regarding to the one decision maker view. In reality, many decision makers exist which all want to optimize on key performance indicators simultaneously. Therewith, the effect of announcement sequence on price has not yet been investigated. In what follows, a game theoretical approach is introduced that deals with these aspects. We consider our health insurance market as a model with players (health insurers) that have specific strategies (premiums and an announcement sequence). Well-known solution concepts of game theory are used to determine the effects of the level and the announcement sequence of price on key performance indicators.

8.1 The health insurance game

In this section, one fits the mathematical model, as introduced in Chapter 5, into a (strategic) game theoretical model. The game is called the health insurance game and is defined by the following elements:

- **Player set**: The health insurance game exists of a player set \( \tilde{N} = \{1, 2, ..., k\} \) with \( i \in \tilde{N} \) the \( i \)-th health insurer.

- **Strategy set**: The strategy set of player \( i \) is denoted by \( S_i = [ \underline{x}_i, \overline{x}_i ] \) with \( \underline{x}_i \) and \( \overline{x}_i \) the minimal respectively maximal charged premium.

- **payoff set**: The payoff set of player \( i \) consists of its key performance indicators, namely profit, market share and solvency ratio, by \( u_i = S_1 \times S_2 \times ... \times S_k \rightarrow \mathbb{R}^3 \).

- **alpha**: The price sensitivity \( \alpha \) of policy holders.

- **Cost set**: The cost set of player \( i \), denoted by \( K_i = (E_i, c_i, C_i) \) as the collection of the own funds, variable and fixed costs.

For convenience, the calculations of the health insurance game are similar to the ones in the original mathematical model. Hence, with the number of policy holders \( N \), premiums in vector space \( S_1 \times S_2 \times ... \times S_k \) and price sensitivity \( \alpha \) it is possible to determine the new number of policy holders. Thereafter, with \( (K_i)_{i \in N} \) it is possible to determine the key performance indicators \( u_i \) of health insurer \( i \). Observe that the payoff set of this game consists of three criteria, which makes the game a so called multiple criteria game (Voorneveld, 1999). In this thesis, we call our multiple criteria game a health insurance game, which is defined by the following tuple:

\[(\tilde{N}; (S_i)_{i \in N}; (u_i)_{i \in \tilde{N}}, (K_i)_{i \in \tilde{N}}, \alpha). \quad (48)\]

---

29 In Appendix E, a gentle introduction into (non-cooperative) game theory is given.
Observe that the health insurance game is played on a one year horizon. As discussed in Chapter 7 also long time horizons are of interest. For this reason, we also introduce a health insurance game with a $T$ years horizon. This game is a collection of $T$ one year health insurance games, where the outcomes of the health insurance game of year $t$ form the input for the health insurance game in year $t+1$. The health insurance game sequence for a time horizon of $T$ years is given below:

**Game Sequence:** Health insurance game $T$ years

**Input :** $\theta$

1. Set premiums $x = (x_i)_{i \in N}$.
2. Update configuration of policy holders by $L$
3. Update Profit, Market share and Solvency ratio by $D$
4. **If** $t < T$ **Then**: Update $\theta : N = \hat{N}$ and
   
   $E_i = E_i + P_i$ for all $i \in \hat{N}$ and go to [1]
   
   **Else**: stop.

Observe that the $T$ year game starts with a situation $\theta \in \Theta$. Then, the original mathematical model is used to calculate the new configuration of policy holders $\hat{N}$ by operator $L$, whereafter operator $D$ is used to calculate the key performance indicators. Then, the new configuration of policy holders $\hat{N}$ and the updated own funds $E$ form the input for a new situation, where the cycle starts again (until year $T$ is reached).

### 8.2 Nash equilibrium

As discussed in Chapter 7 the original mathematical model has some shortcomings regarding to the one decision maker view. In reality, many decision makers exist which all want to optimize on key performance indicators simultaneously. For example, health insurers want to maximize on profit.

31 The key performance indicator profit of health insurer $i$ is denoted by $u_{i,1}$ and the market share and solvency ratio by $u_{i,2}$ and $u_{i,3}$ respectively. Observe they they can also fungate as constraints. However, this is not taken into account over here.

30 For example, health insurers want to maximize on profit.
as well, but over a given situation $x_{-j}$. Hence, we are in a situation wherein health insurer $i$ is optimizing on profit, taking into account the premiums of all health insurers $j \neq i$, while all others health insurers $j \neq i$ are also optimizing on profit, taking into account the premiums of health insurers $k \neq j$. From a game theoretical perspective, we are looking for a set of premiums such that no health insurer can benefit by changing their premium, while other players keep their premiums unchanged. Nash (1950) discussed this issue first, which is called a Nash equilibrium later on.

**Definition** A Nash equilibrium is defined as a vector $x^*=(x^*_i)_{i \in \tilde{N}} \in \Pi_{i=1}^k S_i$ of a game $(\tilde{N}; (S_i)_{i \in \tilde{N}}; (u_i)_{i \in \tilde{N}}; (K_i)_{i \in \tilde{N}}; \alpha)$ if for each player $i \in \tilde{N}$ and each $x'_i \in S_i$ it holds that

$$u_{i,k}(x^*) \geq u_{i,k}(x'_i, x^*_{-i}),$$

where $x^*_{-i} = (x^*_j)_{j \in \tilde{N} \setminus \{i\}}$ denotes the premiums of the players other than $i$ and $(x'_i, x^*_{-i}) \in \Pi_{i=1}^k S_i$ denotes the vector of premiums, in which player $i$ sets premium $x'_i$ and each player $j \in \tilde{N} \setminus \{i\}$ sets premium $x^*_j$.

We assume that all health insurers know everything from each others, except their premium. This is not necessarily true in the Dutch health insurance market, given that competitors will not share information on a usual basis. Despite this, it is of interest to investigate Nash equilibria in more detail, given that new insight(s) on the effect of the level of price on the key performance indicator profit can be obtained.

### 8.2.1 How to find a Nash equilibrium

In general, it is no easy exercise to obtain a Nash equilibrium. Mathematical packages as Matlab or Maple don’t even provide any add-ins for finding Nash equilibria for some predefined games. In R, a statistic package, an add-in exist for finding Nash equilibria, but this one is very restricted and is not bug free at the moment (see Dutang (2011)). For this reason, it is decided to find an own way for finding Nash equilibria. Unfortunately, in our situation Nash equilibria are really hard too find, given that it requires an optimization over $k$ health insurers simultaneously. By trial and error, we fortunately found a relative easy method that usually calculates a Nash equilibrium in a quite reasonable time ($< 0.1$ seconds on average). Given time constraints to this thesis, it was not possible to investigate carefully why this algorithm works well. Despite that, some intuitive explanation is given below for a situation with $k = 2$ health insurers and a discrete strategy set.

Assume that only two premiums can be set, namely $S_i = \{10, 20\}$ and that obtained profits are as the ones as depicted in table 13 below.
Table 13: Strategic form game with two health insurers

<table>
<thead>
<tr>
<th></th>
<th>Menzis 10</th>
<th>Menzis 20</th>
</tr>
</thead>
<tbody>
<tr>
<td>Univé</td>
<td>(400,400)</td>
<td>(500,200)</td>
</tr>
<tr>
<td>20</td>
<td>(200,500)</td>
<td>(300,300)</td>
</tr>
</tbody>
</table>

Our algorithm starts with choosing a fixed premium for Univé, for example $x_1 = 20$. The second step is to find a best reply for player 2, based on $x_1 = 20$, which is in our example $x_2 = 10$. Then, we fix this new premium of player 2 and find a best reply for player 1, based on $x_2 = 10$. In our example, this is $x_1 = 10$. Then, again the premium of Univé is fixed and a best reply is found for player 2, based on $x_1 = 10$. It follows that the best responds is $x_2 = 10$ again and we are in a stable situation, which implies that $x = (10, 10)$ is a Nash equilibrium. The idea of this algorithm is, that when a Nash equilibrium exist, it will be attracted. A weakness of this algorithm is, that when more than one Nash equilibrium exist, it is not clear which Nash equilibrium will be found. Moreover, when no nash equilibrium exist, the algorithm will go on without finding a nash equilibrium.

The idea of the algorithm above is also applied in the health insurance game with more than two health insurers and a continuously strategy set. The algorithm for the health insurance game is formal defined by:

**Algorithm Nash equilibrium**

<table>
<thead>
<tr>
<th>Input : $\theta, x = (x_1, x_2, ..., x_k) \in \mathbb{R}^n, \varepsilon \ll 1, i = 1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1) Let $\hat{x} = x$</td>
</tr>
<tr>
<td>2) Solve $\max_{x_i} u_i(x_i, x_{-i})$</td>
</tr>
<tr>
<td>3) Let $x_i = x_i^<em>$ with $x_i^</em>$ solution of [2]</td>
</tr>
<tr>
<td>4) If $i &lt; k$ Then : $i = i + 1 \rightarrow [2]$</td>
</tr>
<tr>
<td>Else : If $\sum_{j=1}^k</td>
</tr>
<tr>
<td>Then : stop</td>
</tr>
<tr>
<td>Else : $i = 1 \rightarrow [1]$</td>
</tr>
</tbody>
</table>

Observe that the idea of the algorithm is quite similar, namely to start with an initial premium vector, then to find a best reply for health insurer 1, whereafter the premium vector is updated and a best reply for health insurer 2 is found. This is executed in a recursive way for all $k$ health insurers. Then the difference is measured between the outcome of the initial premium vector and outcomes of the new premium vector. When the difference is small enough, the algorithm stops, otherwise the initial premium vector is updated by the new premium vector and the procedure starts again.
8.2.2 Existence of Nash equilibrium

As discussed in previous section, a weakness of the algorithm is that it will go on when no Nash equilibrium exists. For this reason, it is acceptable to investigate the existence of a Nash equilibrium in our health insurance game. In Literature, the existence of a nash equilibrium is discussed by Debreu et al (1954). They formulated the following theorem:

**Theorem 8** (Existence Pure Nash Equilibrium) Consider a strategic form game \((\tilde{N}, (S_i)_{i \in \tilde{N}}, (u_i)_{i \in \tilde{N}})\) where \(\tilde{N}\) is a finite set. Assume that the following holds for each \(i \in \tilde{N}\):

1. \(S_i\) is a non-empty, convex and compact subset of a finite-dimensional Euclidean space
2. \(u_i(x_i, x_{-i})\) is continuous in \(x_i\)
3. \(u_i(x_i, x_{-i})\) is quasi-concave in \(x_i\)

Then, the game \((N, (S_i)_{i \in \tilde{N}}, (u_i)_{i \in \tilde{N}})\) has a pure Nash equilibrium.

For proving the existence of a pure nash equilibrium, the health insurance game should satisfy these three sufficient conditions of theorem 8. In what follows we proof the existence of a Nash equilibrium is the case of \(k = 2\) health insurers. First, some necessary Lemmas are introduced.

**Lemma 2** Consider health insurance game \((\tilde{N}; (S_i)_{i \in \tilde{N}}; (u_i)_{i \in \tilde{N}}, (K_i)_{i \in \tilde{N}}, \alpha)\), then \(S_i\) is a non-empty, convex and compact subset of a finite-dimensional Euclidean space.

**Proof** The strategy set \(S_i = [\underline{x}, \bar{x}]\) is a convex set, given that it is an interval on the real line \(\mathbb{R}\). The strategy set is also non-empty, given that it always consists of the elements \(\underline{x}\) and \(\bar{x}\). Furthermore, the set is also closed, given that the complement \(\mathbb{R} \setminus S_i\) is open. The strategy set is also bounded, namely by \(\underline{x} - 1\) and \(\bar{x} + 1\). By theorem of Heine-Borel (see Jeffreys (1988)) the set is compact, which makes the first condition true.

**Lemma 3** Consider health insurance game \((\tilde{N}; (S_i)_{i \in \tilde{N}}; (u_i)_{i \in \tilde{N}}, (K_i)_{i \in \tilde{N}}, \alpha)\), then \(u_i(x)\) is continuous in \(x_i\).

**Proof** The function \(u_i(x)\) is a (product) combination of well-known continuous functions. As long as the nominator of any part of the combination is not equal to zero, the function is continuously. Given that \(N_i > 0\) for any \(i\), this is satisfied and the proof is given.

For the upcoming Lemma an additional proposition is introduced.

---

32 For the reader who is unfamiliar with terms as convex, compact, quasi concavity and continuity, see Appendix F1.
Proposition 3 Consider a continuously and differentiable function \( f(x) \) with \( f'(x) \geq 0 \) for all \( x \leq a \) and \( f'(x) < 0 \) for all \( x > a \) with \( a \in \mathbb{R} \), then \( f(x) \) is so called quasi-concave.

**Proof** See Appendix F2.

Lemma 4 Consider health insurance game \( (\tilde{N}; (S_i)_{i \in \mathbb{N}}; (u_i)_{i \in \mathbb{N}}, (K_i)_{i \in \mathbb{N}}, \alpha) \) with \( k = 2 \), then \( u_i(x, x_{-i}) \) is quasi-concave in \( x_i \).

**Proof** The derivative of \( u_i(x, x_{-i}) \) in \( x_i \) is given by:

\[
\frac{d}{dx_i} \left( (x_i - c_i) \cdot \tilde{N}_i \right) = \left( \frac{N N_1 \left( e^{-2\alpha(x_2-x_1)t} \left( (\bar{N} - N_i)(1 - 2\alpha(x_i - c_i)) \right) + N_1 \right)}{(N + (\bar{N} - N_i)e^{-2\alpha(x_j-x_i)t})^2} \right)
\]

Observe that \( (1 - 2\alpha(x_i - c_i)) \) is the only term that becomes negative for \( x_i \) large enough, while all other terms remain positive. Given that \( e^{-2\alpha(x_2-x_1)t} \) becomes sufficient large when \( x_i \) increases, we can conclude that there exist a value \( a \in \mathbb{R} \) such that for all \( x_i \leq a \) the derivative is positive, while for all \( x_i > a \) the derivative is negative. Hence, \( u_i(x_i, x_{-i}) \) is quasi-concave.

Remark that the first sufficient condition is being satisfied for any health insurance game, while the second and third sufficient condition are only satisfied for the health insurance game with \( k = 2 \) health insurers. Hence, a health insurance game with two health insurers has a pure Nash equilibrium.

Theorem 9 A health insurance game \( (\tilde{N}; (S_i)_{i \in \mathbb{N}}; (u_i)_{i \in \mathbb{N}}, (K_i)_{i \in \mathbb{N}}, \alpha) \) with \( k = 2 \) health insurers has a least one pure Nash equilibrium.

**Proof** All three sufficient conditions of Debreu et al. (1958) are satisfied by Lemma (2), Lemma (3) and Lemma (4). As a consequence, a health insurance game has at least one pure Nash equilibrium.

Observe that nothing is said about the uniqueness of a Nash equilibrium. In the next section some attention is paid to uniqueness.

### 8.2.3 Uniqueness of Nash equilibrium

Proving the uniqueness of the Nash Equilibrium of the health insurance game \( (\tilde{N}; (S_i)_{i \in \mathbb{N}}; (u_i)_{i \in \mathbb{N}}, (K_i)_{i \in \mathbb{N}}, \alpha) \) with \( k = 2 \) health insurers is (too) hard. Despite that proving is too hard, there is some numerical evidence that the Nash equilibrium is unique. A (short) numerical analysis is executed to give some evidence that a Nash equilibrium is unique.

**Best reply functions**

Consider situation \( \theta' \in \Theta \) (of Chapter 6) with only the first two health insurers (Univé and CZ). When the best reply functions (i.e. function that determines the optimal premium for a given situation) of both health insurers are depicted together, Figure 25 is obtained.
Observe that both graphs intersect with each other at (41.3,41.3), which forms a Nash equilibrium. The function shapes give evidence to only intersect once, which implies that uniqueness is guaranteed. For this reason, we also investigated some close related situations. First, we considered a situation with $c_1 > c_2$, then a situation with a higher price sensitivity $\alpha$ and finally a situation with adapted number of policy holders $N_1 = 20$ and $N_2 = 100$. The (graphical) results are depicted in Appendix G1. We observed that the shapes of these graphs (again) give evidence that they only intersect once. Remark, that if we can find a function $f_r(x_1) = rx_1 + b$, that can separate both best reply functions from each other such that $x'_2(x_1) < r$ for all $x_1$ larger than the first intersection and $x'_1(x_2) < \frac{1}{r}$ for all $x_2$ larger than the first intersection, then uniqueness can be guaranteed. In Figure, such possible (red) function is drawn. Most likely, it is possible to find such function by using techniques as implicit differentiation. We challenge the reader to find such function $f_r(x_1)$.

Figure 25: Best reply functions

Figure 26: Best reply functions with function $f_r(x_1)$
In this section an analysis regarding to Nash equilibria is executed. We consider several situations on a one year and a five year horizon and determine for both horizons the Nash equilibria and related outcomes of key performance indicators as profit, market share and the solvency ratio. In what follows, the general (starting) situation $\tilde{\theta} \in \Theta$ of Chapter 6 is used.

### One year

In the general starting situation $\tilde{\theta}$, the variable costs for Univé, Menzis and CZ are equal to each other. When the Nash equilibrium for this situation is calculated, the following vector is obtained:

$$x^* = (33.00, 33.00, 33.00).$$

Hence, in a situation with similar variable costs and number of policy holders the premiums of the Nash equilibrium are equal to each other as well. So, when variable costs and number of policy holders are similar, it is optimal to charge the same premium. This result is logical, given that all three health insurers have exactly the same profit function. It is of interest to investigate how the key performance indicators as profit, market share and solvency ratio are effected. We will compare two situations, the first situation with premiums equal to the Nash equilibrium and the second situation with premiums equal to $X = (34, 33, 33)$. The results are depicted in Table 14 below.

<table>
<thead>
<tr>
<th></th>
<th>$x = (33, 33, 33)$</th>
<th>$x = (34, 33, 33)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>profit</td>
<td>(400.00, 400.00, 400.00)</td>
<td>(392.88, 44.69, 448.69)</td>
</tr>
<tr>
<td>market share</td>
<td>(33.33, 33.33, 33.33)</td>
<td>(49.48, 23.37, 27.15)</td>
</tr>
<tr>
<td>solvency ratio</td>
<td>(4.14, 4.14, 4.14)</td>
<td>(3.18, 3.99, 4.01)</td>
</tr>
</tbody>
</table>

Table 14: Key performance indicators for varying premiums

Observe that the profit decreases for Univé in the second situation. This is logical, given that any deviation from the Nash equilibrium leads to a decrease in profit (when others hold the same premium). However, we see that it is beneficial for Menzis and CZ if Unive changes its premium. However, knowing that CZ and Menzis set premium 33, Univé would never set premium 34, but 33. This would also hold for any deviation of Menzis or CZ. Hence, the idea of a Nash equilibrium is strong, namely that no one has an incentive to deviate from its premium. Finally, observe that it is beneficial for Univé to charge a higher premium related to market share or to the solvency ratio. To conclude, regarding to profit, the Nash equilibrium finds

\[{}^{33}\text{The game theoretical aspects are included as well in the Excel tool of Appendix H.}\]
the best premium, while market share and solvency ratio are not optimized in general.

In what follows, we investigate what happens with the Nash equilibrium if variable costs change for one or more health insurers, together with a change in the price sensitivity parameter $\alpha$. The results are depicted in table 15 below.

<table>
<thead>
<tr>
<th>variable costs vector</th>
<th>Nash equilibrium $\alpha$</th>
</tr>
</thead>
<tbody>
<tr>
<td>(23,23,23)</td>
<td>(33.00, 33.00, 33.00)</td>
</tr>
<tr>
<td>(23,23,24)</td>
<td>(33.15, 33.15, 33.73) 0.05</td>
</tr>
<tr>
<td>(23,28,24)</td>
<td>(33.93, 36.89, 34.44)</td>
</tr>
<tr>
<td>(23,23,23)</td>
<td>(73.00, 73.00, 73.00)</td>
</tr>
<tr>
<td>(23,23,24)</td>
<td>(37.14, 73.14, 73.71) 0.01</td>
</tr>
<tr>
<td>(23,28,24)</td>
<td>(73.92, 76.86, 74.44)</td>
</tr>
<tr>
<td>(23,23,23)</td>
<td>(28.00, 28.00, 28.00)</td>
</tr>
<tr>
<td>(23,23,24)</td>
<td>(28.16, 28.16, 28.75) 0.1</td>
</tr>
<tr>
<td>(23,28,24)</td>
<td>(28.94, 31.88, 29.42)</td>
</tr>
</tbody>
</table>

Table 15: Nash equilibria for varying variable costs and $\alpha$

Observe that when the variable costs of Menzis increase to 24, the Nash equilibrium changes as a consequence. The premiums for Univé and CZ are equal to each other, given that both have similar costs (and thus similar profit functions). The premium of Menzis is higher than the other two. Apparently, it is optimal from a Nash equilibrium perspective to charge a higher premium when variable costs are higher. The third row strengthens this observation, given that CZ, the one with the highest variable costs, should charge the highest premium as well. When another price sensitivity is chosen, the Nash equilibria changes again. A careful look to Table 15 tells us that an increase in the price sensitivity of $\alpha = 0.1 > 0.05$ let decrease the the Nash equilibrium $(28, 28, 28) < (32, 32, 32)$. In the opposite direction, when the price sensitivity decreases, the Nash equilibrium becomes $(73, 73, 73) > (32, 32, 32)$. Hence, increasing price sensitivity decreases a Nash equilibrium and decreasing price sensitivity increases a Nash equilibrium. When looking to the other two Nash equilibria for $\alpha = 0.01$ and $\alpha = 0.1$ we see similar things, namely that an increase in price sensitivity decreases the Nash equilibrium (with a fixed constant) and an decrease in price sensitivity increases the Nash equilibrium (with a fixed constant). The phenomena that a Nash equilibrium only shifts with a fixed constant for all premiums is logical. Our idea is that when price sensitivity becomes large, optimal premiums converge to the variable costs. (e.g. extremes of the profit functions are more located around the variable costs) and as a consequence, the Nash equilibrium follows. In the opposite direction, when
price sensitivity decreases, the extremes of the profit functions are located at higher premiums and as a consequence, Nash equilibrium increases too.

Last, we also consider what happens if the starting number of policy holders are effected. Consider, for example that Univé has a relative low number of policy holders, while the rest stay remain, such that \( N = (10, 100, 100) \). Then, we obtain for the same set of input parameters the following outcomes.

<table>
<thead>
<tr>
<th>variable costs vector</th>
<th>Nash equilibrium</th>
<th>( \alpha )</th>
</tr>
</thead>
<tbody>
<tr>
<td>(23,23,23)</td>
<td>(30.36, 35.19, 35.19)</td>
<td></td>
</tr>
<tr>
<td>(23,23,24)</td>
<td>(30.40, 35.44, 35.84)</td>
<td>0.05</td>
</tr>
<tr>
<td>(23,28,24)</td>
<td>(30.65, 38.77, 37.08)</td>
<td></td>
</tr>
<tr>
<td>(23,23,23)</td>
<td>(59.80, 83.96, 83.96)</td>
<td></td>
</tr>
<tr>
<td>(23,23,24)</td>
<td>(59.83, 84.21, 84.61)</td>
<td>0.01</td>
</tr>
<tr>
<td>(23,28,24)</td>
<td>(60.06, 87.44, 85.83)</td>
<td></td>
</tr>
<tr>
<td>(23,23,23)</td>
<td>(26.67, 29.09, 29.09)</td>
<td></td>
</tr>
<tr>
<td>(23,23,24)</td>
<td>(26.72, 29.35, 29.75)</td>
<td>0.1</td>
</tr>
<tr>
<td>(23,28,24)</td>
<td>(27.08, 32.79, 30.99)</td>
<td></td>
</tr>
</tbody>
</table>

Table 16: Nash equilibria for varying variable costs and \( \alpha \)

When we compare the outcomes of Table 16 with the outcomes of Table 15 we see some interesting results. For example, when variable costs are the same, the premium of Univé is lower (30.36 < 33) while the premiums of Menzis and CZ has increased similarly regarding to the original situation. Hence, it is optimal for Menzis and CZ to charge a higher premium when Univé becomes small and opposite for Univé to charge a lower premium. For the other outcomes, we see similar results. The premium of Unive is lower regarding to the original situation, where the premiums of CZ and Menzis are higher regarding to the original situation.

To conclude, we see that a Nash equilibrium leads to a situation wherein no health insurers has the incentive to change premium to obtain higher profit. However, it is not in general true that also market share and solvency ratio are maximized. Moreover, we have seen that a Nash equilibrium is influenceable by price sensitivity, variable costs and number of policy holders. Next to these results, it is of interest to investigate how the premiums of a Nash equilibrium effect the key performance indicators on a five year horizon.
Many years

For the analysis of the five years horizon a health insurance game with a time horizon of $T = 5$ years is considered. Moreover, the same starting situation $\tilde{\theta} \in \Theta$ is chosen. For every year the Nash equilibrium is determined and outcomes of year $t$ form the input for year $t + 1$. The obtained Nash equilibria for the five years are depicted in Table 17.

<table>
<thead>
<tr>
<th>year</th>
<th>$x^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(33.00, 33.00, 33.00)</td>
</tr>
<tr>
<td>2</td>
<td>(33.00, 33.00, 33.00)</td>
</tr>
<tr>
<td>3</td>
<td>(33.00, 33.00, 33.00)</td>
</tr>
<tr>
<td>4</td>
<td>(33.00, 33.00, 33.00)</td>
</tr>
<tr>
<td>5</td>
<td>(33.00, 33.00, 33.00)</td>
</tr>
</tbody>
</table>

Table 17: Nash equilibria for five years

Observe that the Nash equilibria are the same for every year. This result is logical, given that after the first year, no policy holder has switched to another health insurer. As a consequence, the situation in year two is similar to the situation of year one and thus results into the same Nash equilibrium. Hence, the key performance indicators profit and market share will remain the same. The solvency ratio increases, given that (positive) profit is added to the capital year by year, while the SCR level, that depends on the number of policy holders and the technical premium, remains the same.

Now, consider again situation $\tilde{\theta}$, but with other variable costs $c = (23, 23, 29)$. The obtained Nash equilibria for the upcoming five year are depicted in Table 22 below.

<table>
<thead>
<tr>
<th>year</th>
<th>$x^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(33.89, 33.89, 37.58)</td>
</tr>
<tr>
<td>2</td>
<td>(34.45, 34.45, 36.98)</td>
</tr>
<tr>
<td>3</td>
<td>(34.80, 34.80, 36.66)</td>
</tr>
<tr>
<td>4</td>
<td>(35.04, 35.05, 36.47)</td>
</tr>
<tr>
<td>5</td>
<td>(35.21, 35.21, 36.34)</td>
</tr>
</tbody>
</table>

Table 18: Nash equilibria for five years

Observe that the premiums of Univé and Menzis are the same every year and have an increasing behavior, while the premium of CZ deviate from them and has a decreasing behavior. In the first years CZ has relative higher premium, which results into a decrease of their market share, while it results in an increase of market share for Univé and Menzis. It is of interest to observe what happens with this market share of all health insurers during
the years. When the market share for Univé, Menzis and CZ are depicted for the five years in one figure, the following graphs of Figure 27 are obtained.

Figure 27: Deviation of the number of policy holders during the years

Observe that the number of policy holders for Menzis and Univé increase similarly, while the number of policy holders of CZ decreases rapidly in the first years. However, after the third year, it looks like if the number of policy holders stabilize for all health insurers. This is caused by the fact that premiums (of Table 22) tend to one fixed premium vector. When the model is executed for 20 years, a Nash equilibrium of $x^* = (35.89, 35.89, 35.89)$ is obtained, with a market share of $M = (48.1\%, 48.1\%, 3.8\%)$. Hence, premiums become closer and closer to each other, and market shares become stable as a consequence. Hence, also profit becomes stable and solvency ratio will increase year by year (given that profit is positive, while the SCR level remains the same). Now, it is of interest to investigate what happens when another price sensitivity parameter is chosen.

We consider two different price sensitivity parameters, namely $\alpha = 0.1$ and $\alpha = 0.01$. In Table 19 below the obtained Nash equilibria are depicted.

<table>
<thead>
<tr>
<th>year</th>
<th>$x^*(\alpha = 0.01)$</th>
<th>$x^*(\alpha = 0.1)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(73.87, 73.87, 77.34)</td>
<td>(28.88, 28.88, 32.84)</td>
</tr>
<tr>
<td>2</td>
<td>(74.38, 74.38, 76.38)</td>
<td>(29.31, 29.31, 32.52)</td>
</tr>
<tr>
<td>3</td>
<td>(74.68, 74.68, 75.93)</td>
<td>(29.51, 29.51, 32.41)</td>
</tr>
<tr>
<td>4</td>
<td>(74.87, 74.87, 75.63)</td>
<td>(29.59, 29.57, 32.36)</td>
</tr>
<tr>
<td>5</td>
<td>(74.98, 74.98, 75.40)</td>
<td>(29.65, 29.26, 32.34)</td>
</tr>
<tr>
<td>20</td>
<td>(75.12, 75.12, 75.12)</td>
<td>(30.01, 30.01, 32.02)</td>
</tr>
<tr>
<td>50</td>
<td>(75.12, 75.12, 75.12)</td>
<td>(31.44, 3.44, 31.44)</td>
</tr>
</tbody>
</table>

Table 19: Nash equilibria during the years
Observe that for $\alpha = 0.1$ a stable Nash equilibrium is obtained after 20 years, while for $\alpha = 0.01$ it takes more than 50 years before the Nash equilibrium become stable. If we investigate the market shares of both scenarios, we observe that nearly all policy holders are switched. For $\alpha = 0.01$ a stable market share of $M = (49.99\%, 49.99\%, 0.02\%)$ is obtained. Despite that this market share is stable, it is no acceptable situation for Menzis. Also in these situations, profit becomes stable (not necessarily positive) and solvency ratio increases or decreases, depending on profit.

Finally, we are interested in what happens on the time horizon if the starting number of policy holders deviates. We consider situation $\tilde{\theta}$, but now with $N = (30, 100, 100)$. The obtained Nash equilibria for the five years are depicted in table 20 below.

<table>
<thead>
<tr>
<th>year</th>
<th>$x^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(31.23, 34.20, 34.20)</td>
</tr>
<tr>
<td>2</td>
<td>(31.77, 34.75, 34.75)</td>
</tr>
<tr>
<td>3</td>
<td>(32.20, 34.45, 34.45)</td>
</tr>
<tr>
<td>4</td>
<td>(32.51, 33.26, 33.26)</td>
</tr>
<tr>
<td>5</td>
<td>(32.70, 33.15, 33.15)</td>
</tr>
<tr>
<td>20</td>
<td>(33.00, 33.00, 33.00)</td>
</tr>
</tbody>
</table>

Table 20: Nash equilibria during the years

Observe that premiums of Univé are lower than the (similar) premiums of Menzis and CZ. Hence, the premium of Univé in the Nash equilibrium is such that policy holders are attracted in next year. This is repeated in the upcoming years as long as the market share is lower than the market shares of the other two. When the number of policy holders are depicted against the years, the following Figure is obtained.

Figure 28: Deviation of number of policy holders during the years
Note that after 20 years, market share approximates $M = (33\%, 33\%, 33\%)$. Hence, when the number of policy holders is not equal to each other, but variable costs are equal to each other, setting prices to a Nash equilibrium year by year leads to an equal division of the market share. As a consequence, also profit becomes stable and the solvency ratio increases (given that profit is positive) or decreases (when profit is negative).

8.2.5 Discussion

From the analysis it can be concluded that pricing regarding to a Nash equilibrium has some benefits. Profit and market share become stable and solvency ratio increases (when profit is positive). However, it depends on the properties of the health insurers if it beneficial for them to follow a Nash equilibrium. Despite that a Nash equilibrium gives the best premium regarding to profit, it tends to an equilibrium in the future in where the number of policy holders can become sufficient low. Moreover, the key performance indicators market share and solvency ratio are not optimized in this chapter, which forms a weakness. In future research, one should also try to maximize on market share and the solvency ratio simultaneously. For more detailed information about multi critera games optimization, see Voorneveld (1999).
8.3 Stackelberg Equilibrium

As discussed in the introduction of this thesis, also the sequence in which health insurers announce their premium can effect key performance indicators of health insurers. In this chapter, we investigate the possible advantage of announcing premium before the others. We assume that all health insurers optimize on profit and that only one health insurer announces its premium earlier. All other health insurers announce their premium on the 15th of November simultaneously. In Figure 29, a possible situation is depicted.

Figure 29: Possible announcement sequence

In Figure 29, DSW is the one that announces its premium first, while the others simultaneously follow at the 15th of November. Given that all health insurers want to optimize on profit, we assume that the $k-1$ health insurers set their premiums equal to a Nash equilibrium. From this perspective, we are interested in how DSW should charge its premium such that they optimize on profit too. In mathematical terms, we are looking for a premium $x_i^\triangledown$ that satisfies the outcome of the following optimization problem:

$$\max_{x_i \in \mathbb{R}} \{u_i, k(x_i, x_{-i}^\star(x_i))\}$$ (50)

with $x_{-i}^\star(x_i)$ a Nash equilibrium for $k-1$ health insurers under a given premium $x_i$. Hence, we obtain a vector $x^\triangledown = (x_i^\triangledown, x_{-i}^\star)$ with $x_i^\triangledown$ the premium that maximizes profit for all possible combination of $x_i$ with related premiums $x_{-i}^\star(x_i)$ as a best reply on $x_i$ obtained from a Nash equilibrium. In Meyerson (1992), such vector is called a Stackelberg Equilibrium. More formally, a Stackelberg equilibrium is defined by:

**Definition** A Stackelberg equilibrium with leader $i$ is a vector $x^\triangledown = (x_i^\triangledown, x_{-i}^\star)$ such that $x_i^\triangledown$ solves the problem

$$\max_{x_i \in \mathbb{R}} \{u_i, k(x_i, x_{-i}^\star(x_i))\},$$ (51)

where $x_{-i}^\star(x_i)$ a Nash equilibrium for the subgame with $k-1$ followers and given action $x_i$ for health insurer $i$. 
In our situation it is more complex to determine a Stackelberg equilibrium than only identifying a Nash equilibrium, given that for every $x_i$ a Nash equilibrium should be determined. In what follows, our approach for finding a Stackelberg equilibrium is discussed, whereafter we analyze the effect(s) of the announcement sequence in more detail.

### 8.3.1 How to find a Stackelberg equilibrium

As discussed in previous section, it is relative hard to find a Stackelberg equilibrium, given that for every $x_i$ a Nash equilibrium is determined. For this reason, an algorithm is introduced that approximates the stackelberg equilibrium. The idea of the algorithm is discussed below.

The algorithm starts with an premium $x_i \ll 1$ and for this $x_i$ also the Nash equilibrium (by using Algorithm Nash equilibrium) for $k - 1$ health insurers is calculated. Then, the profit function of health insurer $i$ is determined, whereafter $x_i$ is increased a little bit and again the related Nash equilibrium and profit function is determined. As long as the updated $x_i$ results into a higher profit, the procedure is executed again, while a decrease in the updated profit function immediately stops the algorithm. The underlying assumption for this approach is that the function that determines the profit per $x_i$ is unimodal, which means that only one extreme exist. More formal, the algorithm that is used, is given by:

**Algorithm Stackelberg equilibrium**

**Input**: $\theta$, $x_i \ll 1$, $\varepsilon \ll 1$

**Output**: $x_i$

1. Let $\hat{x} = x$
2. Find Nash equilibrium for $k - 1$ and $x_{i} \in x$ fixed.
3. Let $x = (x_1^*, x_2^*, ..., x_{i}^*, ..., x_k^*)$ solution of [2].
4. If $u_i(x) \geq u_i(\hat{x})$ **Then**: $\hat{x} = x$ and $x_i = x_i + \varepsilon$ and $\rightarrow$ [2].
   *Else*: stop.

In the next section, we will analyze the effect of announcement sequence on key performance indicators in more detail, by making use of the Stackelberg equilibrium algorithm.

---

34Regarding to time constraints of this thesis is was not possible to proof this statement. However, in Appendix G2 a graphical representation is given for the function for some $\theta \in \Theta$ to give some evidence that it is unimodal.
8.3.2 Analysis

In this section an analysis regarding to the effect of announcement sequence on key performance indicators, by using Stackelberg equilibria, is executed. We only consider situations on a one year horizon and investigate outcomes of key performance indicators as profit, market share and the solvency ratio. Consider again the general (starting) situation \( \tilde{\theta} \in \Theta \) of chapter 6.

In the general starting situation \( \tilde{\theta} \), the variable costs for Univé, Menzis and CZ are equal to each other. We assume that Univé is the health insurer that announces its premium first, while Menzis and CZ simultaneously react on a later time moment. When the Stackelberg equilibrium for this situation is calculated, the following vector is obtained:

\[
x^* = (34.72, 33.38, 33.38)
\]

When we compare this vector with the Nash equilibrium of last section we see an increase in the premiums. When the key performance indicators profit, market share and solvency ratio are depicted for both the Nash and the Stackelberg equilibrium the following table is obtained.

<table>
<thead>
<tr>
<th></th>
<th>( x = (33, 33, 33) )</th>
<th>( x = (34.72, 33.38, 33.38) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>profit</td>
<td>(400.00, 400.00, 400.00)</td>
<td>(416.35, 513.63, 513.63)</td>
</tr>
<tr>
<td>market share</td>
<td>(33.33, 33.33, 33.33)</td>
<td>(28.47, 35.77, 35.77)</td>
</tr>
<tr>
<td>solvency ratio</td>
<td>(4.14, 4.14, 4.14)</td>
<td>(4.81, 4.64, 4.65)</td>
</tr>
</tbody>
</table>

Table 21: Key performance indicators for varying \( x \)

Observe that profit of Univé in the Stackelberg equilibrium increased compared to the Nash equilibrium. Hence, when Univé decides to announce a premium of 34.72 before the others, they obtain a higher profit than in the Nash equilibrium, which makes announcing earlier a beneficial action. However, observe that profits of Menzis and CZ are increased as well. So, it could also be beneficial to announce simultaneously later, given that one can anticipate on the first one. Last, observe that the other key performance indicators are not optimized in general. The solvency ratio of Univé increases to 4.81, while market share of Univé decreases to 28.47 %.

Moreover, it is of interest to investigate situations where variable costs of Univé are higher or respectively lower compared to the others. When for example the variable costs changes to \( c = (25, 23, 23) \) or \( c = (21, 23, 23) \) respectively the following Stackelberg equilibria are obtained.

\[
x^*_{\text{high}} = (36.10, 33.61, 33.61)
\]
\[
x^*_{\text{low}} = (33.30, 33.06, 33.06)
\]

\(^{35}\) Also this game theoretical aspect is included in the Excel tool of Appendix H.
When we compare the key performance indicators of the obtained Stackelberg equilibria with the key performance indicators of the related Nash equilibria, the following table is obtained.

<table>
<thead>
<tr>
<th>Stackelberg</th>
<th>$x = (36.10, 33.61, 33.61)$</th>
<th>$x = (33.30, 33.06, 33.06)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>profit</td>
<td>(253.06, 584.34, 584.34)</td>
<td>(600.66, 418.00, 418.00)</td>
</tr>
<tr>
<td>market share</td>
<td>(25.62, 37.19, 37.19)</td>
<td>(32.54, 33.73, 33.73)</td>
</tr>
<tr>
<td>solvency ratio</td>
<td>(3.59, 4.95, 4.95)</td>
<td>(6.04, 4.22, 4.22)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Nash</th>
<th>$x = (34.46, 33.29, 33.29)$</th>
<th>$x = (31.61, 32.72, 32.72)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>profit</td>
<td>(238.92, 487.59, 487.59)</td>
<td>(582.03, 316.57, 316.57)</td>
</tr>
<tr>
<td>market share</td>
<td>(29.55, 35.22, 35.22)</td>
<td>(37.15, 31.43, 31.43)</td>
</tr>
<tr>
<td>solvency ratio</td>
<td>(3.12, 4.54, 4.54)</td>
<td>(5.43, 3.73, 3.73)</td>
</tr>
</tbody>
</table>

Table 22: Key performance indicators of Stackelberg and Nash equilibria

Observe that for both situations (e.g. with lower and higher variable costs), the profit of Univé increases regarding to the original Nash equilibria. Hence, it is beneficial for Univé to announce premium earlier. However, the key performance indicator market share decreases for both situations, while the solvency ratio increases for both situations. Moreover, observe that the profit of Menzis and CZ increases as well. It is of interest to investigate wether the increase of profit also holds for other variable costs of Univé. In Figure 30 below, an overview is given of the profit of Univé and CZ (Menzis is equal to CZ) against different variable costs of Univé for both the Nash (full lines) and the Stackelberg equilibria (punctate lines).

Figure 30: Profit functions for different variable costs of Univé

Observe that the profit functions regarding to the Stackelberg equilibria are all above the profit functions of the Nash equilibria. So, the one that announces premium first has an advantage regarding to the Nash equilibrium. However, the ones that announce later on have an advantages regarding to
profit as well. A careful look to Figure 30 tells us, that the benefit of Menzis and CZ is higher than the benefit of Univé. So, despite that announcing premium first is beneficial for Univé, it is also (indirectly) non-beneficial, given that Menzis and CZ benefit more, which could have negative consequences on the long (competitive) horizon. Based on these results, we conclude that announcing premium first is beneficial regarding to a Nash equilibrium, while it is non-beneficial as well, given that others can benefit more, which can have negative effects on the long time horizon. More research should be executed to investigate these effects in more detail.

Finally, we also investigate what happens if the number of policy holders is not equal to each other. Suppose two situations, wherein Univé has a relative low and a relative high number of policy holders such that \( N = (60, 100, 100) \) and \( N = (160, 100, 100) \). For both situation, we consider two sub situations, one with relative low variable costs for Univé and the other one with relative high variable costs. It turned out that the profit of all health insurers is higher in a Stackelberg equilibrium than in a Nash equilibrium and that for all considered situations. Hence, the benefit for all health insurers is still there when input parameters are adapted.

8.3.3 Discussion

In our analysis, we have seen that announcing premium before the others (while others announce simultaneously) is beneficial compared to a Nash equilibrium. Also for different variable costs or number of policy holders, the effect is still there. However, it is also beneficial for the other health insurers that announce their premium on a similar (later) moment. To conclude, a time difference between announcing premiums is beneficial for the profits of all health insurers compared to a Nash equilibrium. Future research should be executed to proof these results. We will also draw some negative comments to the benefits from a more realistic perspective. When a health insurer decides to announce premium before the others, the others can relatively benefit more (see Figure 30) which can have negative effects on the long horizon. On the other side, announcing premiums together with the others is also dangerous, given that market price is already set. When a health insurer has, for example, low own funds, is can occur that they cannot charge the same low premium regarding to the market. As a consequence, they can get in trouble. For such situations, it is probably beneficial to announce premium before the other health insurers, given that they (maybe) follow that premium. This is also of interest for future research.
Conclusions & Recommendations
9 Conclusions and Recommendations

9.1 Conclusions

At the beginning of this thesis, an insufficient knowledge of the effects of the level and the announcement sequence of price on the key performance indicators of health insurers was noticed. Therefore a related research question was formulated:

*What are the effects of the level and the announcement sequence of price on the key performance indicators of health insurers?*

Before we could answer this research question, the important and relevant key performance indicators of health insurers were determined. It turned out that profit, market share and the solvency ratio are the important and relevant key performance indicators for health insurers. Then, a mathematical model was built that (1) captures the choice behavior of policy holders and (2) connects this with key performance indicators. Differential equations, based on the level of price, are used to describe the choice behavior of policy holders and their outcomes are used to determine the key performance indicators of health insurers. With use of the mathematical model, several interesting cases from the perspective of one health insurer were discussed. In the one year horizon cases, we have seen that it is optimal to charge a premium below the premiums of the others when considering a profit and market share perspective, while in the long time horizon cases, it is optimal from a profit perspective to (first) charge a premium below the others, then to charge a premium above the premiums of the others and then finally to charge a premium relative equal to the others. Despite these results, we could not conclude any general results from these cases. Although, it gave us useful insights in how key performance indicators are effected by the level of price when one health insurer wants to optimize on key performance indicators. Next to the one health insurer perspective, also the perspective with many health insurers (that all have their own interests) was considered. We used a game theoretical approach and simultaneously investigated the effect of the announcement sequence of health insurers. We discussed several cases and we saw that profit maximizing health insurers obtain stable profits, market shares and increasing solvency ratios (when profit is positive) when premiums are charged along a Nash equilibrium. Moreover, we have seen that a time difference between announcing premiums is beneficial for all health insurers compared to a Nash equilibrium when a profit perspective is considered. The health insurers that announce their premiums simultaneously benefit more than the one who announces premium first. The results obtained by using game theory are subject to a strict assumption, namely the all health insurers know everything from each other. In reality this assumption is controversial, which makes the game
theoretical results not of direct practical use. However, it gives Milliman (new) insights in how key performance indicators are effected when health insurers price against a Nash equilibrium or Stackelberg equilibrium. To conclude, in this thesis some interesting situation are discussed that gave insights in the effect of the level and the announcement sequence of price on key performance indicators of health insurers. However not all effects are (and can be) exposed and determined. Fortunately, the mathematical model is developed as an Excel tool, which gives Milliman the possibility to determines (other or new) effects of the level and announcement sequence of price on key performance indicators for any kind of situation in the future.

9.2 Recommendations and future research

A first recommendation regarding to Milliman is to investigate how well the tool performs on realistic data of the year 2012 and 2013. When it predicts the situation well, there will be more evidence that the tool performs well. Next to this, it is of interest to investigate more cases as discussed in Chapter 6 and try to proof that some results hold in general. In this way more insight(s) in the effects of the level and announcement sequence can be obtained. Finally, we recommend Milliman to use the mathematical model as support in advise to health insurers. It gives them, for example, the opportunity to develop new ideas and thus opportunities to go to health insurers and advise them in setting prices for upcoming year.

For future research, we advise to investigate the other system of differential equations (without the interaction term) in more detail, given that exact solutions exist. Next to this, it is also recommendable to make the mathematical model more realistic. Now, it is assumed that only one type of health insurance exist. In reality several kind of health insurances with different prices exist. It is plausible to assume that policy holders also make their decisions based on these aspects and as a consequence to include this into the model as well. Moreover, it is of interest to investigate how policy holders behave (in their mind) when health insurers announce their premium (before the 15th of November). Maybe, a lot of policy holders already decide to switch when one health insurers announces its price. Next to this, it is of interest to investigate how the announcing sequence can influence the optimal premium. In this thesis, both aspects are discussed separately, but not combined and investigated together. Finally, it is of interest to extend the game theoretical approach by optimizing over all possible key performance indicators in stead of optimizing over only profit. this could coincide with a study into multi criteria games (see Voorneveld (1999)).
10 Appendix

A. Overview of Excel scripts

A1. Algorithm script system of differential equations

Function differential equation(N1 As Double, N2 As Double, N3 As Double, N4 As Double, N5 As Double, alpha As Double, p1 As Double, p2 As Double, p3 As Double, p4 As Double, p5 As Double)

n = 1000

For i = 1 To n

Change12 = alpha * 1 / n * (p2 - p1) * N1 * N2 * ((N1 + N2 + N3 + N4 + N5 + 10 ^ (-120)) / 5) ^ (-1)

Change13 = alpha * 1 / n * (p3 - p1) * N1 * N3 * ((N1 + N2 + N3 + N4 + N5 + 10 ^ (-120)) / 5) ^ (-1)

change14 = alpha * 1 / n * (p4 - p1) * N1 * N4 * ((N1 + N2 + N3 + N4 + N5 + 10 ^ (-120)) / 5) ^ (-1)

change15 = alpha * 1 / n * (p5 - p1) * N1 * N5 * ((N1 + N2 + N3 + N4 + N5 + 10 ^ (-120)) / 5) ^ (-1)

Change23 = alpha * 1 / n * (p3 - p2) * N2 * N3 * ((N1 + N2 + N3 + N4 + N5 + 10 ^ (-120)) / 5) ^ (-1)

change24 = alpha * 1 / n * (p4 - p2) * N2 * N4 * ((N1 + N2 + N3 + N4 + N5 + 10 ^ (-120)) / 5) ^ (-1)

change25 = alpha * 1 / n * (p5 - p2) * N2 * N5 * ((N1 + N2 + N3 + N4 + N5 + 10 ^ (-120)) / 5) ^ (-1)

change34 = alpha * 1 / n * (p4 - p3) * N3 * N4 * ((N1 + N2 + N3 + N4 + N5 + 10 ^ (-120)) / 5) ^ (-1)

change35 = alpha * 1 / n * (p5 - p3) * N3 * N5 * ((N1 + N2 + N3 + N4 + N5 + 10 ^ (-120)) / 5) ^ (-1)

change45 = alpha * 1 / n * (p5 - p4) * N4 * N5 * ((N1 + N2 + N3 + N4 + N5 + 10 ^ (-120)) / 5) ^ (-1)
N1 = N1 + Change12 + Change13 + change14 + change15
N2 = N2 - Change12 + Change23 + change24 + change25
N3 = N3 - Change13 - Change23 + change34 + change35
N4 = N4 - change14 - change24 - change34 + change45
N5 = N5 - change15 - change25 - change35 - change45

Next i

Nreken = (N1,N2,N3,N4,N5)

End Function

A2. Solver Nash equilibria script
Sub updater()

Dim year As Integer

year = Range("L7")

If year = 0 Then

For i = 1 To 5

Range("D13").Offset(i - 1, 0) = Range("D6").Offset(i - 1, 0)
Range("P13").Offset(i - 1, year) = Range("D15").Offset(i - 1, 0)

Next i

End If

For i = 1 To 5

Range("D21").Offset(i - 1, 0) = Range("H6").Offset(i - 1, 0)
Range("P21").Offset(i - 1, year) = Range("E15").Offset(i - 1, 0)

Next i
'update
For i = 1 To 5
Range("H6").Offset(i - 1, 0) = Range("E15").Offset(i - 1, 0)
Next i
For i = 1 To 5
Range("D6").Offset(i - 1, 0) = Range("P13").Offset(i - 1, year)
Next i
End Sub

Sub solver1()
SolverReset
SolverAdd CellRef:="$D$30:$M$30", Relation:=3, FormulaText:="1"
SolverSolve True
' report on success of analysis
If Result <= 3 Then
' Result = 0, Solution found, optimality and constraints satisfied
' Result = 1, Converged, constraints satisfied
' Result = 2, Cannot improve, constraints satisfied
' Result = 3, Stopped at maximum iterations
MsgBox "Solver found a solution", vbInformation, "Solution found"
Else
' Result = 4, Solver did not converge
' Result = 5, No feasible solution
Beep
MsgBox "Solver was unable to find a solution.", vbExclamation,
"Solution not found"
End If
End Sub

Sub Nashsolver()
SolverReset
SolverOk SetCell:="H$15", MaxMinVal:=1, ValueOf:="0", ByChange:="$E$6"

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SolverSolve True

SolverReset
SolverOk SetCell:="H$16", MaxMinVal:=1, ValueOf:="0", ByChange:="$E$7"
SolverSolve True

SolverReset
SolverOk SetCell:="H$17", MaxMinVal:=1, ValueOf:="0", ByChange:="$E$8"
SolverSolve True

SolverReset
SolverOk SetCell:="H$18", MaxMinVal:=1, ValueOf:="0", ByChange:="$E$9"
SolverSolve True

SolverReset
SolverOk SetCell:="H$19", MaxMinVal:=1, ValueOf:="0", ByChange:="$E$10"
SolverSolve True

End Sub

Sub optimizeoneyear()

SolverReset
SolverOk SetCell:="D$22", MaxMinVal:=1, ValueOf:="0", ByChange:="$C$6"
SolverSolve True

SolverReset
SolverOk SetCell:="E$22", MaxMinVal:=1, ValueOf:="0", ByChange:="$D$6"
SolverSolve True

SolverReset
SolverOk SetCell:="F$22", MaxMinVal:=1, ValueOf:="0", ByChange:="$E$6"
SolverSolve True

SolverReset
SolverOk SetCell:="G$22", MaxMinVal:=1, ValueOf:="0", ByChange:="$F$6"
SolverSolve True
SolverReset
SolverOk SetCell:="H$22", MaxMinVal:=1, ValueOf:="0", ByChange:="$G$6"
SolverSolve True

End Sub

A3. Next year script

Private Sub CommandButton1_Click()

If Range("L7") = 10 Then

Antwoord = MsgBox("Het maximale aantal simulatie jaren is bereikt. Wilt u terugkeren naar de begin situatie? (Alle gegevens worden gereset)", vbOKCancel + vbInformation, "Maximale aantal simulaties")

If Antwoord = vbOK Then

CommandButton3_Click

Exit Sub

Else: Exit Sub

End If

End If

updater

Range("L7").Value = Range("L7").Value + 1

End Sub

Private Sub CommandButton2_Click()

For i = 1 To 10

Nashsolver

CommandButton1_Click

Next i
End Sub

Private Sub CommandButton3_Click()

Range("O13:Y17").ClearContents

Range("O21:Y25").ClearContents

Range("L7") = 0

End Sub

A4. Lay out script

Private Sub CommandButton1_Click()

solver1

End Sub

Private Sub CommandButton2_Click()

optimizeoneyear

End Sub

A5. Layout script

Private Sub Workbook_Open()

Dim wSheet As Worksheet

For Each wSheet In Worksheets
    wSheet.Protect UserInterFaceOnly:=True
Next wSheet

Sheets(1).ScrollArea = "A1:Y42"
Sheets(2).ScrollArea = "A1:AB42"

End Sub
B. Differential equations

An equation containing the derivatives or differentials of one or more dependent variables, with respect to one or more independent variables, is said to be a differential equation. In general, differential equations are classified according to three properties, namely type, order and linear or nonlinear. In the upcoming paragraphs these three properties will be discussed. Alongside them, also some interesting results of differential equations will be introduced.

B1. Classification : Type

The classification by type is ordinary or partial. An ordinary differential equation contains only ordinary derivatives of one or more dependent variables with respect to a single independent variable. For example

$$\frac{dy}{dx} - 5y = 1,$$

is a ordinary differential equation. Equations involving the partial derivatives of one or more dependent variables of two or more independent dependent variables id called a partial differential equation. For example

$$\frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x},$$

is a partial differential equation. Alongside the classification of type, also the order is of importance, which will be discussed in the next section.

B2. Classification : Order

The classification of order is based on the highest derivative in a differential equation. For example

$$\frac{d^2 y}{dx^2} + 5x - 5y = 0,$$  \hspace{1cm} (52)

is a second order differential equation, while for example

$$\frac{\partial^4 u}{\partial x^4} + \frac{\partial^2 u}{\partial t^2} = 0,$$  \hspace{1cm} (53)

is a fourth order partial differential equation. For the remaining part of this chapter only the ordinary differential equations will be concerned. A general nth-order ordinary differential equation is often represented by the symbolism:

$$F(x, y, \frac{dy}{dx}, \ldots, \frac{d^n y}{dx^n}) = 0.$$  \hspace{1cm} (54)
B3. Classification: (non)Linear

The last classification, which is a special case of equation \[54\] is based on linear and nonlinear. A differential equation is said to be linear if it has the form

\[
a_n(x) \frac{d^n y}{dx^n} + a_{n-1}(x) \frac{d^{n-1} y}{dx^{n-1}} + ... + a_1(x) \frac{dy}{dx} + a_0(x)y = g(x)
\] (55)

Observe that the dependent variable \(y\), and all its derivatives of a linear ordinary differential equation are linear and coefficients depends only of the independent variables \(x\). For example,

\[y'' - 2y'' + y = 0,\] (56)

is a linear second order ordinary differential equation. On the other hand,

\[y'' + y^3 = 0,\] (57)

is a nonlinear second order ordinary differential equation. In general, one want to solve, or find solutions, of differential equations. In the next sections solutions of differential equations will be discussed.

B4. Solutions differential equation

As mentioned before, solving or finding a solution of a differential equation is a commonly asked exercise. A solution of a differential equation is defined as any function \(f\) defined on some interval \(I\) which, when substituted into a differential equation, reduces the equation to an identity. Formally:

**Definition** A solution \(y = f'(x)\) that possesses at least \(n\) derivatives and satisfies the equation that os,

\[F(x, f(x), f'(x), ..., f^{(n)}(x)) = 0 \quad \forall x \in I.\] (58)

An example will be used to illustrate this definition of a solution into more detail.

**Example 1** Consider the function \(y(x) = e^{\frac{1}{2}x^2}\) as the solution of the nonlinear equation

\[
\frac{dy}{dx} - xy = 0,
\]
on \(x \in \mathbb{R}\). Since

\[
\frac{dy}{x} = 2 \cdot \frac{1}{2} x \cdot e^{\frac{1}{2}x^2} = x \cdot e^{\frac{1}{2}x^2}
\]
one can see that

\[ \frac{dy}{dx} - xy = x \cdot e^{\frac{1}{2}x^2} - x \cdot e^{\frac{1}{2}x^2} = 0 \quad \forall x \in \mathbb{R}. \quad (59) \]

Sometimes, it is not possible to obtain a closed form solution of any kind of differential equation. Then programs as Matlab or Mathematica could be useful to find numerical solution(s) of the outcomes of differential equation. Often, there exist no closed forms for differential equations.

**B5. System of differential equations**

Alongside the single differential equations, also systems of differential equations exist. These can be divided into linear and nonlinear systems of differential equations. When the system of differential equations is not linear, it is usually not possible to find solutions in terms of elementary functions. Despite that, valuable information can be obtained, by first analyzing special constant solutions called critical points and then searching for periodic solution called limit cycles. For more detailed information see Adams (2000).
C. Collection of proofs, derivations and tests

C1. Derivation $N_2(t)$ for $k = 2$

The differential equation for $N_2(t)$ is given by:

$$\frac{dN_2}{dt} = \alpha \beta (x_2 - x_1) N_2 N_1 dt,$$

where the total number of policy holders $N$ is a constant. Observe that $N_1$ can be rewritten as the difference between $N$ and $N_2$. Substituting this into the differential equation of $N_1$ results into:

$$\frac{dN_2}{dt} = \alpha \beta (x_2 - x_1) N_2 (N - N_2) dt.$$

A method for solving this type of differential equation(s) is separation of variables (see Adams (2008)). This results into:

$$\frac{dN_2}{N_2(N - N_2)} = \alpha \beta (x_1 - x_2) dt$$

$$\left[ \frac{1}{N} \left( \frac{1}{N_2} + \frac{1}{N - N_2} \right) \right] dN_2 = \alpha \beta (x_1 - x_2) dt$$

$$\frac{1}{N} \ln |N_2| - \frac{1}{N} \ln |N - N_2| = \alpha \beta (x_1 - x_2) t + c$$

$$\ln \left| \frac{N_2}{N - N_2} \right| = N \alpha \beta (x_1 - x_2) t + N c$$

$$\frac{N_1}{N - N_2} = c_2 e^{2 \alpha (x_1 - x_2) t},$$

where $c_2 = e^{N c}$ and 2 is obtained from $N \cdot \beta = \frac{N_1 + N_2}{2(N_1 + N_2)} = 2$.

It follows form the last equation that:

$$N_2(t) = \frac{N \cdot c_2 e^{2 \alpha (x_1 - x_2) t}}{1 + c_2 e^{2 \alpha (x_1 - x_2) t}} = \frac{N c_2}{e^{-2 \alpha (x_1 - x_2) t} + c_2}.$$

Again, the solution $N_2$ is shifted in time. Hence, $t = t - t^*$ with initial starting value $t^*$. Hence, the initial condition is $N_2(t^*)$ and substituting $t = t^*$ in the last equation results, after some rewriting, into:

$$c_2 = \frac{N_2(t^*)}{N - N_2(t^*)},$$

and therefore:

$$N_2(t) = \frac{N \cdot N_2(t^*)}{N_2(t^*) + (N - N_2(t^*)) e^{-2 \alpha (x_1 - x_2)(t - t^*)}}, \quad (60)$$

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C2. Proof of $N_1(t) + N_2(t) = \overline{N}$

In this paragraph, we show that $N_1(t) + N_2(t) = N$ holds for $t \in [t^*, t + 1]$.

**Proof** Consider a situation $\theta \in \Theta$ with $k = 2$ and $N_1(t) + N_2(t) = \overline{N}$ for $t = t^*$. Take a $t \in [t^*, t + 1]$ and substitute $N_1(t)$ and $N_2(t)$ of equation (13) and (14), which gives:

$$N_1(t) + N_2(t) = \frac{\overline{N} \cdot N_1(t^*)}{N_1(t^*) + (\overline{N} - N_1(t^*)) e^{-2\alpha(x_2-x_1)t}} + \frac{\overline{N} \cdot N_2(t^*)}{N_2(t^*) + (\overline{N} - N_2(t^*)) e^{-2\alpha(x_1-x_2)t}}.$$

Multiplying both fractions by its complement denominator and some rewriting of the obtained fraction gives:

$$= \frac{\overline{N} \left[ N_1(t^*) (N_2(t^*) + (\overline{N} - N_2(t^*)) e^{-2\alpha(x_1-x_2)t}) + N_2(t^*) (N_1(t^*) + (\overline{N} - N_1(t^*)) e^{-2\alpha(x_2-x_1)t}) \right]}{(N_1(t^*) + (\overline{N} - N_1(t^*)) e^{-2\alpha(x_2-x_1)t}) (N_2(t^*) + (\overline{N} - N_2(t^*)) e^{-2\alpha(x_1-x_2)t})}$$

$$= \frac{\overline{N} \left[ 2N_1(t^*)N_2(t^*) + N_1(t^*) (\overline{N} - N_2(t^*)) e^{-2\alpha(x_1-x_2)t} + N_2(t^*) (\overline{N} - N_1(t^*)) e^{-2\alpha(x_2-x_1)t} \right]}{(N_1(t^*) + (\overline{N} - N_1(t^*)) e^{-2\alpha(x_2-x_1)t}) (N_2(t^*) + (\overline{N} - N_2(t^*)) e^{-2\alpha(x_1-x_2)t})}.$$

Now, using that $N_1(t) + N_2(t) = \overline{N}$ at $t = t^*$ it follows that:

$$= \frac{\overline{N} \left[ 2N_1(t^*)N_2(t^*) + N_1(t^*) (\overline{N} - N_2(t^*)) e^{-2\alpha(x_1-x_2)t} + N_2(t^*) (\overline{N} - N_1(t^*)) e^{-2\alpha(x_2-x_1)t} \right]}{2N_1(t^*)N_2(t^*) + N_1(t^*) (\overline{N} - N_2(t^*)) e^{-2\alpha(x_1-x_2)t} + N_2(t^*) (\overline{N} - N_1(t^*)) e^{-2\alpha(x_2-x_1)t}}$$

$$= \overline{N},$$

which completes the proof.
C3. Test of algorithm differential equations

In this appendix tests are executed that determine how well the algorithm performs and which $\Delta t$’s are acceptable.

**Test 1** We consider three situations $\theta_1, \theta_2, \theta_3 \in \Theta$ all with $k = 2$ health insurers, but with an order of magnitude of $\mathcal{O}(10^3)$, $\mathcal{O}(10^6)$ and $\mathcal{O}(10^9)$ respectively. For this three different situations, the relative error of $N(t+1)$ compared to the exact solution for a varying $\Delta t$ is calculated.

<table>
<thead>
<tr>
<th>order $\mathcal{O}(10^x)$</th>
<th>relative error (%)</th>
<th>$\Delta t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>0.0017</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>0.0038</td>
<td>$10^{-3}$</td>
</tr>
<tr>
<td>9</td>
<td>0.0089</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>0.0001</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>0.0003</td>
<td>$10^{-6}$</td>
</tr>
<tr>
<td>9</td>
<td>0.0011</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>0.0000</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>0.0000</td>
<td>$10^{-9}$</td>
</tr>
<tr>
<td>9</td>
<td>0.0001</td>
<td></td>
</tr>
</tbody>
</table>

Table 23: Relative errors for different order of magnitudes and different $\Delta t$’s

Observe that the relative error increases when the order of magnitude increases. Although, for every order, the relative error decreases for a decreasing $\Delta t$. In our opinion, $\Delta t = 10^{-9}$ is the one that is most accurate. Despite that, the computation time related to $\Delta t = 10^{-9}$ is significant, namely 5.34 seconds on average. This is not acceptable for the rest of our thesis. The computation time of $\Delta t = 10^{-6}$ is 0.016 seconds on average, while the computation time of $\Delta t = 10^{-3}$ is 0.001 seconds on average. Both have an acceptable computation time and therefore $\Delta t = 10^{-6}$ is chosen, given that it is more accurate.

As the algorithm performs well in situations with $k = 2$ health insurers, it is not immediately clear that it also performs well for more than two health insurers. Given that no explicit expressions are found, Matlab is used to compare the results. In the next test, the outcomes of the algorithm are compared with the outcomes of matlab and this is done for $k = 3, 4$ and 5 health insurers, with $\Delta t = 10^{-6}$.

**Test 2** We consider three situations $\theta_1, \theta_2, \theta_3 \in \Theta$ with $k = 3, 4$ and 5 health insurers, different order of magnitudes $\mathcal{O}(10^3)$, $\mathcal{O}(10^6)$ and $\mathcal{O}(10^9)$.

\footnote{The relative error is calculated as $\frac{N_{\text{exact}}(t+1) - N_{\text{exact}}}{N_{\text{exact}}} \times 100\%$ for every health insurers. The average of all relative errors is taken as the final relative error.}

\footnote{In Chapter 8 some hard computations are executed. With an average of 5.34 seconds per calculation, the computation time will be unacceptable long.}
and $\Delta t = 10^{-6}$. The relative errors are determined by taking the average of the relative errors (between the algorithm and the outcomes of matlab) of all three, four or respectively five health insurers. The relative errors for all of these combinations are depicted below.

<table>
<thead>
<tr>
<th>order $O(10^k)$</th>
<th>relative error (%)</th>
<th>$k$</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>0.0001</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>0.0002</td>
<td>3</td>
</tr>
<tr>
<td>9</td>
<td>0.0004</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>0.0003</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>0.0005</td>
<td>4</td>
</tr>
<tr>
<td>9</td>
<td>0.0007</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>0.0009</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>0.0011</td>
<td>5</td>
</tr>
<tr>
<td>9</td>
<td>0.0012</td>
<td></td>
</tr>
</tbody>
</table>

Table 24: Relative errors for different order of magnitudes and different number of health insurers

Observe that the relative error increases by the number of health insurers increases and the order of magnitude. However, the relative error are still sufficient small for the given $\Delta t = 10^{-6}$, which implies that our developed algorithm performs also well in the case with $k = 3, 4$ and $5$ health insurers.

Based on the results of Test 1 and Test 2, we conclude that our developed algorithm works well. As a consequence it is used in the rest of this thesis with a $\Delta t$ of $\Delta t = 10^{-6}$.

C4. Derivation adapted system of differential equations

In this paragraph, a derivation for the adapted system of differential equations with only $N_i$ as the intensity, is given. The new transition rate of the system of differential equations is given by:

$$r_{i,j} = \alpha \beta (x_j - x_i) N_i.$$  \hspace{1cm} (61)

Remark that $r_{i,j}\Delta t$ is the number of policy holders that switch between time $t$ and $t + \Delta t$ from company $i$ to company $j$. Hence, the net change in the number of policy holders of health insurer $i$ between time $t$ and $t + \Delta t$ depend on the difference between the ingoing transition rates $\{r_{j,i}\}_{j \in W}$ and outgoing transition rates $\{r_{i,k}\}_{k \in W'}$:

$$N_i(t + \Delta t) - N_i(t) = \sum_{j \in W_i} r_{j,i} \Delta t - \sum_{k \in W'_i} r_{i,k} \Delta t,$$  \hspace{1cm} (62)

or in differential form:
\[
\frac{dN_i}{dt} = \sum_{j \in W_i} r_{j,i} - \sum_{k \in W'_i} r_{i,k}.
\] (63)

Substituting equation (61) into the last equation of (63) results into the general form of the differential equation:

\[
\frac{dN_i}{dt} = \sum_{j \in W_i} \alpha \beta (x_j - x_i)N_j - \sum_{k \in W'_i} \alpha \beta (x_i - x_k)N_i
\]

\[
\frac{dN_i}{dt} = \sum_{j \in W_i} \alpha \beta (x_j - x_i)N_j + \sum_{k \in W'_i} \alpha \beta (x_k - x_i)N_i.
\]

It is not possible to rewrite this differential equation shorter. In a similar way one can derive the differential equations for all other health insurers. The system of differential equations that will be obtained is given by:

\[
\frac{dN_i}{dt} = \sum_{j \in W_i} \alpha \beta (x_j - x_i)N_i - \sum_{k \in W'_i} \alpha \beta (x_i - x_k)N_k. \quad \forall i \in N. \quad (64)
\]

The \( k \)-th differential is easy to solve, given that it only depends on a multiple of \( N_k \). Then, solving the \( k-1 \)-th differential is also possible. This differential depends on \( N_k(t) \), for which the solution was already obtained. Hence, with this solution it is also possible to solve the \( k-2 \)-th solution and so on. Hence, the system of differential equations is solvable for any \( k > 2 \) health insurers. For \( k \geq 3 \) it is possible to solve the system of differential equation by hand within a reasonable time range. For \( k > 3 \) programs as Matlab are very useful. In what follows, a derivation for a system with \( k = 3 \) health insurers is given.

**C5. Proof for \( k = 3 \)**

Consider a situation \( \theta \in \Theta \) with \( k = 3 \). Assume without loss of generality that \( x_1 \leq x_2 \leq x_3 \). The system of differential equations is given by:

\[
\frac{dN_1}{dt} = (x_2 - x_1)N_2 + (x_3 - x_1)N_3
\]

\[
\frac{dN_2}{dt} = (x_1 - x_2)N_2 + (x_3 - x_2)N_3
\]

\[
\frac{dN_3}{dt} = (x_1 - x_3)N_3 + (x_2 - x_3)N_3.
\]
The third differential equation only depends on \( N_3 \) and is easy to solve:

\[
\frac{dN_3}{dt} = (x_1 - x_3)N_3 + (x_2 - x_3)N_3 \\
\frac{dN_3}{N_3} = (x_1 + x_2 - 2x_3)dt \\
\ln |N_3| = (x_1 + x_2 - 2x_3)t + c.
\]

It follows from the last equation that:

\[ N_3(t) = c_1 e^{(x_1 + x_2 - 2x_3)t}, \quad (65) \]

and with initial condition \( N_3(t^*) \) it follows that:

\[ N_3(t) = N_3(t^*) e^{(x_1 + x_2 - 2x_3)t}. \quad (66) \]

Now, it is possible to also solve the second differential equation, by substituting (66) into the second differential equation:

\[
\frac{dN_2}{dt} = (x_1 - x_2)N_2 + (x_3 - x_2)N_3 \\
\frac{dN_2}{dt} = (x_1 - x_2)N_2 + (x_3 - x_2)N_3(t^*) e^{(x_1 + x_2 - 2x_3)t} \\
\frac{dN_2}{dt} - (x_1 - x_2)N_2 = (x_3 - x_2)N_3(t^*) e^{(x_1 + x_2 - 2x_3)t}.
\]

Multiplying with factor \( e^{-(x_1 - x_2)t} \) gives:

\[
\frac{dN_2}{dt} e^{-(x_1 - x_2)t} - (x_1 - x_2)e^{-(x_1 - x_2)t}N_2 = (x_3 - x_2)N_3(t^*) e^{(x_1 + x_2 - 2x_3)t} e^{-(x_1 - x_2)t},
\]

and results into:

\[
\frac{d}{dt} \left[ N_2(t) e^{-(x_1 + x_2)t} \right] = (x_3 - x_2)N_3(t^*) e^{2(x_2 - x_3)t} \\
N_2(t) e^{-(x_1 - x_2)t} = \frac{x_3 - x_2}{2(x_2 - x_3)} N_3(t^*) e^{2(x_2 - x_3)t} + c_2 \\
N_2(t) = -\frac{1}{2} N_3(t^*) e^{x_1 + x_2 - 2x_3t} + c_2 e^{(x_1 - x_2)t}.
\]

With initial condition \( N_2(t^*) \) it follows that:

\[ N_2(t) = -\frac{1}{2} N_3(t^*) e^{x_1 + x_2 - 2x_3t} + \left( N_2(t^*) + \frac{1}{2} N_3(t^*) \right) e^{(x_1 - x_2)t}. \quad (67) \]

Now, the solution for \( N_1(t) \) follows immediately, namely as the difference between \( N \) and \( N_2(t) \) and \( N_3(t) \):

\[ N_1(t) = N - \left( N_2(t^*) + \frac{1}{2} N_3(t^*) \right) e^{(x_1 - x_2)t} - \frac{1}{2} N_3(t^*) e^{x_1 + x_2 - 2x_3t}. \quad (68) \]
D. Analysis mathematical model

D1. Back testing results

In this section, we consider three different situations, the first one starting in 2009 with $\alpha = 0.18 \cdot 10^{-5}$, the second one starting in 2010 with $\alpha = 0.62 \cdot 10^{-5}$ and the last one starting in 2011 with $\alpha = 2.75 \cdot 10^{-5}$. In Figure 32 the result of the first situation is depicted.

Figure 31: Model configuration with $\alpha = 0.18 \cdot 10^{-5}$ and empirical data

Observe that the model configuration of policy holder deviates from the empirical data with a maximal error of 10.4%. Especially in the year 2011 and 2012 the deviation is large. Probably, the price sensitivity value is too low, to let policy holders switch as the empirical data shows.

When the estimated price sensitivity parameter $\alpha = 2.75 \cdot 10^{-5}$ of the year 2011 is used, the following figure is obtained.

Figure 32: Model configuration with $\alpha = 0.62 \cdot 10^{-5}$ and empirical data

Observe that the model configuration of policy holders in 2012 deviates again from the empirical data (maximal error of 3.4%). Where the real number
of policy holders of UVIT and Achmea increase respectively decrease, the model configuration remains stable. Probably, the price sensitivity parameter is too low again.

When the estimated price sensitivity parameter $\alpha = 2.75 \cdot 10^{-5}$ of the year 2011 is used, the following figure is obtained.

![Figure 33: Model configuration with $\alpha = 2.75 \cdot 10^{-5}$ and empirical data](image)

The model configuration of policy holders is quite similar to the empirical data (max error of 0.7 %). Unfortunately, it is not possible to compare this price sensitivity with empirical data of 2012 and 2013. Probably, the price sensitivity parameter is too low again, given that the price sensitivity of policy holders has an increasing trend (Vektis, 2012). As discussed in section 6.1.4, it is our advice for the determination of price sensitivity to first obtain a price sensitivity with historical data and then to use forecasting techniques (see Hamilton (1994)) and expert judgement to make the price sensitivity value more accurate.
D2. Another system of differential equations

One of the important elements of the system of differential equations is the interaction of \( N_i \) and \( N_j \) in the form of \( N_i \cdot N_j \). The idea of this interaction is that a larger health insurer attracts more policy holders than a smaller health insurer when they charge the same premium. It is of interest to analyze what happens if this interaction is not taken into account and both health insurers attract policy holders independent of their own number of policy holders. In this new situation, the transition rate \( r_{i,j} \) is given by:

\[
r_{i,j} = \alpha (x_i - x_j) N_i
\]

Observe that the transition rate now only depends on number of policy holders of company \( i \). Moreover, the weighting factor \( \beta \) is excluded, given its property of being a scaling factor of the interaction term.

With the transition rates a (new) system of differential equations can be formed. This system of \( k \) differential equations is given by:

\[
dN_i \over dt = \alpha \left( \sum_{k \in W_i} (x_k - x_i) N_k + \sum_{s \in W'_i} (x_k - x_i) N_i \right) \quad \forall i \in \tilde{N},
\]

with \( W \) the set of health insurers with property \( x_j < x_i \) and \( W' \) the set of health insurers with property \( x_j \geq x_i \).

Remark that the system of differential equations is a homogenous linear system of differential equations. (see Adams (2008)) A derivation of the solutions of the system with \( k = 3 \) health insurers is given in Appendix C4 and C5. Solving the system of differential equations by hand for more than three health insurers is a quite intensive task. For \( k = 3 \) health insurers, the solution of differential equations, with \( x_1 \leq x_2 \leq x_3 \) is given by:

\[
\begin{align*}
N_1(t) &= N - \left( N_2(t^*) + \frac{1}{2} N_3(t^*) \right) e^{\alpha(x_1 - x_2)t} - \frac{1}{2} N_3(t^*) e^{\alpha(x_1 + x_2 - 2x_3)t} \\
N_2(t) &= -\frac{1}{2} N_3(t^*) e^{\alpha(x_1 + x_2 - 2x_3)t} + \left( N_2(t^*) + \frac{1}{2} N_3(t^*) \right) e^{\alpha(x_1 - x_2)t} \\
N_3(t) &= N_3(t^*) e^{\alpha(x_1 + x_2 - 2x_3)t}.
\end{align*}
\]

Substituting \( t + 1 \) into equation (71) results into vector \( \tilde{N} = N(t + 1) \). In what follows, we investigate the outcomes of this system against the original system. Again situation \( \theta \in \Theta \) is considered. For a deviating price of Univé, the following graphs are obtained (punctate lines belong to the new system).
The punctate line of Univé intersects at \( x_1 = 34 \) with the graph of Menzis. This phenomena is logical, given that Menzis’ premium is equal to 34 as well. When Univé charges a premium of 34, no policy holders of Univé and Menzis will switch between each other. Only policy holders of CZ will switch to CZ and Menzis and against the same intensity rate, given that the initial number of policy holders is equal 100 for both Menzis and Univé. This argument also holds for the intersection of Univé with CZ. Note that the number of policy holders of both intersections are not exactly equal to the interaction of the original model, but close related. This is also a logical result. When \( x_1 = x_3 \) the differential of \( N_1 \) and \( N_3 \) becomes:

\[
\frac{dN_i}{dt} = \alpha(x_2 - x_i)N_2 \text{ for } i = 1, 3.
\]  

(72)

These differentials deviate not that much from the differentials of the original model, given that only \( \beta N_i \) is added. For \( t = t^* \) this value is equal to one, given that \( N_1(t^*) = N_3(t^*) = 100 \) and \( \beta = \frac{1}{100} \). For \( t > t^* \) the differentials of the original model increase more, given that \( \beta > 1 \). But given the relative small price differences regarding to CZ, the differences between the two models stays small till \( t + 1 \). The number of policy holders of the new model is \( \hat{N} = (104.75, 90.50, 104.75) \) where the number of policy holder of the original model is \( \hat{N} = (104.87, 90.26, 104.87) \).

When \( x_1 \ll x_3 \) or \( x_1 \gg x_3 \), the difference between \( (x_2 - x_i) \) increases. As a result, \( \beta \) increases more and as a consequence also the number of policy holders. This is also visible in Figure 34, where the punctate line of Univé deviates less than the full line of Univé. To conclude, the interaction term \( N_iN_j \) let policy holders switch more if price differences are larger regarding to situation without the interaction term and the number of policy holders is the same for all companies. Last, we mention that if \( N_1 = N_2 = N_3 \) holds no longer, other effects can occur! These effects are because of time constraints of this thesis not investigated.

Figure 34: New model and original model for deviating price of Univé
D3. More groups of policy holders

In the original mathematical model it is assumed that the behavior of policy holders is captured by only one parameter $\alpha$. In reality, there exist groups of policy holders that have different price sensitivities. In this sub section we consider a situation with more than one group of policy holders with different price sensitivities.

First, some additional notation is introduced. The general situation $\tilde{\theta} \in \Theta$ is copied into $m$ sub situations denoted by $\theta_m$. The only difference with the original situation is that the price sensitivity is denoted by $\alpha_m$ and that $N_m = (N_{1,m}, N_{2,m}, \ldots, N_{k,m})$ is the vector of policy holders, where $N_{i,k}$ is the number of policy holders of health insurer $i$ with price sensitivity $\alpha_m$. Note that it holds that $N_i = \sum_{j=1}^{m} N_{i,j}$.

For all sub situations $\theta_m$ a new situation $\tilde{\theta}_m$, with the updated number of policy holders, is formed. The updated vectors of policy holders are collected and added pairwise such that $\hat{N}_i = \sum_{j=1}^{m} \tilde{N}_{i,j}$.

For our analysis, we consider situation $\tilde{\theta} \in \Theta$ and two related sub situations $\theta_1$ and $\theta_2$. The sub situations have a price sensitivity parameter of $\alpha_1 = 0.05$ and $\alpha_2 = 0.01$ respectively. Moreover $N_{1,1} = N_{2,1} = 70$ and $N_{1,2} = N_{2,2} = 30$. When the updated price vector $\hat{N}$ of this model is depicted against the updated price vector of the original model, the following figure is obtained.

![Figure 35: Two groups with different price sensitivity parameters](image)

Observe that the interaction of the graph of Menzis and Univé is at the same point again. This result is logical, given that no policy holders switch between Menzis and Univé and they both attract policy holders of CZ at the same rate. Moreover, 70% of the policy holders is less sensitive and as
a consequence, the switching number of policy holders is lower for deviating prices of Univé. When, for example, 90 % of all policy holders is less sensitive, the following figure is obtained.

Figure 36: Two groups with different price sensitivity parameters

We see that the price differences influence the switching number of policy holders less. Hence, how larger the percentage of policy holders with a lower price sensitivity, the less influence price has. Last, we will consider a special situation, where \( \alpha_1 = \alpha_2 \) and \( N_{1,1} = N_1, 2 = 60 \ N_{2,1} = N_{2,2} = 40 \). Now, the sum of these two groups of policy holders can be seen as one group of policy holders with \( \alpha = 0.05 \). So, \( N_{i,1} + N_{i,2} = N_i \) and \( N_{i,1} = c \cdot N_{i,2} \). This is the result of Theorem 7, given that one sub scenario is a multiple of the other. In Figure 37 this result is shown (both graphs follows the shape path).

Figure 37: Two groups with same price sensitivity parameter
E. Non cooperative Game Theory

Game theory consists of two parts, namely (1) non-cooperative game theory and (2) cooperative game theory. The first one primarily deals with analysis of conflict situations between several players. The second one primarily deals with the analysis of dividing benefits among players that decide to coordinate their actions. The analysis of conflict situations between several players is of large interest for this thesis and will therefore be discussed in more detail. Non-cooperative games can be divided into situations where all decisions are made simultaneously (strategic form) and situations where decisions are made sequentially (extensive form). Both situations will be discussed.

E1. Strategic games

A game in strategic form is a static and condensed description of a situation, in which the players submit their strategies, which are then consistently followed and lead to a unique outcome with corresponding payoffs. A formal description of a strategic form game is given by:

Definition A game in strategic form is a tuple $(N; (S_i)_{i \in N}; (f_i)_{i \in N})$ with $N = \{1, 2, \ldots, n\}$ the set of players, $S_i$ the set of strategies of player $i \in N$ and $f_i : S_1 \times S_2 \times \cdots \times S_n \to \mathbb{R}$ the payoff of player $i \in N$.

In what follows, an example, related to the health insurance market is introduced, to illustrate a game in strategic form.

Example 2 Consider a set of health insurers (player) of $N = \{1, 2\}$. Each health insurer has a strategy set $S_i$ consisting of two premiums $S_i = \{10, 20\}$. The payoff (profit) function for the game in strategic form are given in Table 25:

<table>
<thead>
<tr>
<th></th>
<th>10</th>
<th>20</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>(400,400)</td>
<td>(500,200)</td>
</tr>
<tr>
<td>20</td>
<td>(200,500)</td>
<td>(300,300)</td>
</tr>
</tbody>
</table>

Table 25: Strategic form game

Hence, if player 1 decides to set premium 10, while player 2 decides to set premium 10 as well, the profit of player 1 and 2 are both 400. In the opposite direction, when player 2 decides to set premium 30, the payoffs will be 500 for player 1 and 200 for player 2.

Observe from Example 2, that if player 1 plays A, player 2 can better play A than B (given that 4 > 3). But, if for example, player 1 plays B, player 2 can better play B. Given that both players determine their strategy at the
same moment (and as a consequent don’t know the concurrent strategies), it is hard to determine the right strategy. In strategic form games, there is a well-known solution concept that deals with such situations, namely the Nash equilibrium. (see Nash (1950)).

E2. Nash Equilibrium

The Nash equilibrium is a solution concept in which each player is assumed to know the equilibrium strategies of the other players, and no player has anything to gain by changing only his own strategy unilaterally. In the situation that each player has chosen a strategy and no player can benefit by changing strategies while the other players keep theirs unchanged, then the current set of strategy choices and the corresponding payoffs forms a Nash equilibrium. In mathematical terms, a Nash Equilibrium is defined as:

\[
\text{Definition} \quad \text{The vector } s = (s_i)_{i \in N} \in S \text{ is a Nash equilibrium of the game } (N; (S_i)_{i \in N}; (f_i)_{i \in N}) \text{ if for each player } i \in N \text{ and each } s'_i \in S_i \text{ it holds that } f_i(s) \geq f_i(s'_i, s_{-i}),
\]

where \( s_{-i} = (s_j)_{j \in N \setminus \{i\}} \) denotes the (fixed) strategies of the players other than \( i \) and \( (s'_i, s_{-i}) \in S \) denotes the strategy profile in which player \( i \) plays \( s'_i \) and each player \( j \in N \setminus \{i\} \) plays \( s_j \).

Example 3 Consider the situation of Example 2. The strategy set \((B, B)\) is a Nash equilibrium, given that a deviation of player 1 from strategy \( A \) to \( B \) decreases its payoff from 2 to 1 and a deviation of player 2 from strategy \( B \) to \( A \) decreases its payoff from 3 to 2. Hence, \((B, B)\) is a Nash equilibrium.

E3. Extensive games

A game in extensive form is a dynamic description of a situation that explicitly specifies all its details, such as which player is on turn, what are his options and at which times he is called upon to move. More formal, an extensive game consists of:

- A set of players \( N = \{1, 2, \ldots, k\} \)
- A set of sequences \( \mathcal{H} \), also called terminal histories, such that no sequence is a proper subsequence of another sequence
- A decision maker function \( \mathcal{D} \) that assigns a player to every proper subsequence of a terminal history.
- A payoff function \( f(h) \) with \( h \in \mathcal{H} \) that assigns a payoff to each player for every possible terminal history
In what follows, an example related to the health insurance market is used to illustrate extensive games in more detail.

**Example 4** Consider the situation of example 3 and the following additional properties:

- **Terminal histories:** \( (10, 10), (10, 20), (20, 10), (20, 20) \)
- **Decision maker functions:** \( D(\emptyset) = 1, D(10) = 2, D(20) = 2 \)
- **Payoff functions:** \( f(10, 10) = (400, 400), f(10, 20) = (500, 200), f(20, 10) = (200, 500), f(20, 20) = (300, 300) \)

A visualization of the game in extensive form is given in Figure 38.

![Figure 38: Extensive forms game](image)

In Example 4, player 1 is on turn first, followed by player 2. If player, for example, sets premium 10, players two can choose between premium 10 and 20 as well. Given that profit 400 > 200, player 2 will rationally set premium 10 as well. Hence, the decision for player 2 is easier regarding to the game in strategic form, given that the profit of player 1 is known beforehand.

A well-known solution concept is subgame-perfect Nash equilibrium, which is found through backward induction.

**E4. Backward induction**

By backward induction, the extensive game is considered upside down. First the best strategies are determined for the players that decides last. Based on these results, the best strategies of the second last player are determined. Hence, in a recursive way the best strategies are determined up to the first player. Then the best strategy of the first player completes the recursion.
The best strategy of the first player in combination with the consequently best strategies for the other players forms a subgame-perfect Nash equilibrium. Now, example 5 is used to illustrate the backward induction method.

**Example 5** Consider again the situation of example 4. Using backward induction, the best strategies for player 2 are premium 10 when player 1 plays 10 (400 > 200), and premium 10 when player 1 plays premium 10 (500 > 300). Based on these results, the best strategy for player 1 is 10, given that 400 > 200. In Figure below the best strategies are arced.

![Figure 39: Backward induction for Extensive form game](Image)
F. mathematical definitions

F1. Convexity, Closeness, Non-emptyness, Openness, Boundedness, Compactness, Quasi concavity

In this section some necessary definitions as non-emptyness, convexity, closeness, openness, boundedness and compactness are introduced. In what follows, $V$ will stand for a real vector space. The first definition that will be discussed is the non empty set:

**Definition** (non empty) Let $K \subset V$. Set $K$ is said to be non-empty if

\[ \{ \emptyset \} \subset K. \]  

(73)

**Example 6** Consider $V = \mathbb{R}$ and $K = [1, 3]$. Hence $\{ \emptyset \} \subset K$ which implies that set $K$ is non-empty.

The definition of a convex set is given by:

**Definition** (Convex set) Let $K \subset V$. Set $K$ is said to be convex provided that given two points $x, y \in K$, the set

\[ \{ \gamma \in V : \gamma = \lambda x + (1 - \lambda) y, \ 0 \leq \lambda \leq 1 \}. \]  

(74)

is a subset of $K$.

**Example 7** Consider $V = \mathbb{R}$ and $K = [1, 2]$. Let $x, y \in [1, 2]$ and assume without loss of generality that $x < y$. Let $\lambda \in (0, 1)$. Then:

\[ 1 \leq x = \lambda x + (1 - \lambda) x < \lambda x + (1 - \lambda) y \]
\[ < \lambda y + (1 - \lambda) = y \]
\[ \leq 2. \]  

(75)

and the set is convex.

The next definitions that will be introduced are necessary for defining a compact set. First an open and closed set will be defined:

**Definition** (Open set) Let $K \subset V$. Set $K$ is said to be open if

\[ \forall x \in A, \ \exists \varepsilon > 0 \ s.t. \ B_\varepsilon(x) \subseteq A. \]  

(76)

**Example 8** Consider $V = \mathbb{R}^2$ and $A = \{ x \in \mathbb{R}^2 : x_1^2 + x_2^2 \leq 2 \}$. Set $A$ is not an open set, given that for $x = (0, \sqrt{2})$ no $\varepsilon > 0$ exist, such that $B_\varepsilon(x) \subseteq A$. In Figure 49 it is shown that no $\varepsilon > 0$ exist such that the orange circle $B_\varepsilon(\sqrt{2})$ is in between area $A$. 

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Definition (Closed set) Let $K \subset V$. Set $K$ is said to be closed if the complement $V \setminus K$ is open.

Example 9 Consider $V = \mathbb{R}$ and $K = [0, 2]$. The complement $V \setminus K$ is open. Hence, $K$ is closed. In Figure 50 a graphical representation of the closed set is given.

The definition for a bounded set is given by:

Definition (Bounded set) Let $K \subset V$. Set $K$ is said to be bounded if

$$\exists \varepsilon > 0 \text{ and } x \in K \text{ s.t. } A \subseteq B_\varepsilon(x)$$

(77)

Example 10 Consider $V = \mathbb{R}^2$ and $K = \{x \in \mathbb{R}^2 : x_1^2 + x_2^2 \leq 3\}$ than choosing $\varepsilon = 5$ and $x \in K$ will bound set $K$. In figure 51 a graphical representation is given of the set $A$ and the bounded set (green) $B_5(x)$. 

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A compact set will now be introduced following the theorem of Heine-Borel.

**Definition (Heine-Borel)** Let $K \subset V$. Set $K$ is said to be compact if it is closed and bounded.

Next, to them also the definition of a continuous function will be given:

**Definition** A function defined on subset $S$ of a real vector space $V$ is continuous if

$$\lim_{x \to s} f(x) = f(s) \quad \forall x \in S$$

(78)

**Example 11** Consider $V = \mathbb{R}$ and $K = [2,6]$. Then the limits of every points is the points itself. Hence, the interval is continuous.

The last definition is quasi-concave. Quasi-concavity is defined as:

**Theorem 10** A function $f$ defined on a convex subset $S$ of a real vector space is quasi-concave if for all $x, y$ and $\lambda \in [0,1]$ one has:

$$f(\lambda x + (1 - \lambda)y) \geq \min \{f(x), f(y)\}.$$  

(79)

**Example 12** Consider the function $f = x$. Take two values $x, y \in \mathbb{R}$. Without loss of generality assume that $x < y = x + \varepsilon$. Then
\[ f(\lambda x + (1 - \lambda)y) = \lambda x + (1 - \lambda)y \]
\[ = \lambda x + (1 - \lambda)(x + \epsilon) \]
\[ = \lambda x + (x + \epsilon) - \lambda(x + \epsilon) \]
\[ = x + \epsilon(1 - \lambda). \]

Observe that \( x + \epsilon(1 - \lambda) > x = f(x) = \min \{ f(x), f(y) \} \); one can argue that the function is quasi concave.

It is also possible to see quasi-concavity by using a graphical representation.

**Example 13** Consider the function \( f = \frac{1}{1+x^2} \). Take two values \( x, y \in \mathbb{R} \). Then, the graph looks like:

![Graph of function \( f(x) \)](image)

All point in between the two green points \( x, y \) have an higher outcome \( f(x) \). One can check that this also holds for other configurations of \( x \) and \( y \). Hence, the function is quasi-concavity.

**F2. Proof of quasi concavity**

**Proof** Let \( x_1, x_2 \in \mathbb{R} \) such that \( x_1 \leq x_2 \leq a \). Then \( f(x_1) \leq f(x_2) \) given that \( f(x) > 0 \) and:

\[ f(x_1 \lambda + x_2(1 - \lambda)) \geq f(x_1 \lambda + x_1(1 - \lambda)) = f(x_1) \geq \min \{ f(x_1), f(x_2) \}. \]

Now, let \( x_3, x_4 \in \mathbb{R} \) such that \( a < x_3 \leq x_4 \). Then \( f(x_3) \geq f(x_4) \) given that \( f(x) < 0 \) and:

\[ f(x_3 \lambda + x_4(1 - \lambda)) \geq f(x_4 \lambda + x_4(1 - \lambda)) = f(x_4) \geq \min \{ f(x_3), f(x_4) \}. \]

Finally, let \( x_5, x_6 \in \mathbb{R} \) such that \( x_5 \leq a \leq x_6 \) and without loss of generality that \( f(x_5) \leq f(x_6) \) then:

\[ f(x_5 \lambda + x_6(1 - \lambda)) \geq f(x_5 \lambda + x_5(1 - \lambda)) = f(x_5) \geq \min \{ f(x_5), f(x_6) \}. \]

Hence, function \( f(x) \) satisfies for all possible combinations the quasi concavity condition and the proof is given.

\[ \blacksquare \]
G. Uniqueness and unimodality

G1. Evidence uniqueness Nash Equilibrium

In this appendix the best reply functions for adapted situations with $c_1 > c_2$, with a higher price sensitivity $\alpha$ and finally a situation with adapted number of policy holders $N_1 = 20$ and $N_2 = 100$ are considered. The (graphical) results are depicted below.

Figure 44: Best reply functions with $c_1 = \{23, 28\}$

Figure 45: Best reply functions with $\alpha = \{0.05, 0.1\}$
Figure 46: Best reply functions with $N_1 = \{20, 100\}$

G2. Unimodal function

In this appendix the function is depicted that determines the profit of the leader of a stackelberg equilibrium for a varying premium. Observe that the function has one extreme on the interval of $x_1 \in [0, 80]$. Note that this is absolutely no proof that the function is unimodal.

Figure 47: Profit Unive for varying $x_i$ when other set premium $x_{-i}^*(x_i)$
H. Excel tool

In this thesis, an Excel tool is developed to display the effects of the level and the announcement sequence of price on key performance indicators of health insurers in a clear and convenient way. The Excel tool is divided into two sheets, namely a year by year sheet and a ten year horizon sheet. In what follows, both sheets are discussed in more detail.

H1. Year by year sheet

The year by year sheet exists of two parts, namely the input part and the output part. In the input part, customers can fill in values for a situation \( \theta \in \Theta \). Hence, the number of policy holders, the premiums, the variable and fixed costs, the own funds and the price sensitivity of policy holders.\(^{38}\) In Figure 48 an overview of this input scheme is given.

![Figure 48: Input scheme Excel Tool](image)

With these input parameters, the Excel tool immediately calculates the outcome parameters for next year. Hence, the profit, the capital, the SCR level, Market share and solvency ratio are determined per health insurer. These values are depicted in the output field. The output field for the given situation in Figure 48 is depicted in Figure 49.

![Figure 49: Output scheme Excel Tool](image)

In this way, customers can immediately obtain the effects of the input parameters on the outcome parameters. When input parameters are changed, output parameters are updated as well. When a customer wants to store

\(^{38}\)Note that the technical premium and beta depend on other input variables and can for this reason not be filled in.
the outcomes of a specific situation, the button next year can be used. This button stores the values of the key performance indicators for a specific situation and the current situation is updated automatically with the new number of policy holders and the updated capital. Customers can then repeat this for ten independent years and simulate the health insurance market for ten years. Note that when negative values (e.g. profits or own funds) are obtained, cells become red. In Figure 50 an overview of stored outcome parameters of a time horizon of ten years are depicted.

<table>
<thead>
<tr>
<th>Year 0</th>
<th>Year 1</th>
<th>Year 2</th>
<th>Year 3</th>
<th>Year 4</th>
<th>Year 5</th>
<th>Year 6</th>
<th>Year 7</th>
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<th>Year 10</th>
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<td>113</td>
</tr>
</tbody>
</table>

Figure 50: Output scheme Excel tool

For convenience also graphs are included to visualize the outcome variables year by year. In Figure 51 a graph is depicted for the number of policy holders during the years.

Figure 51: Output graphs Excel tool

Alongside the next year button, it is also possible to determine the Nash and Stackelberg equilibrium for a given situation. Clicking on the button Nash Equilibrium solver or Stackelberg equilibrium solver makes it possible to determine such equilibrium. Last, also a reset button is included, to reset all settings and remove all outcomes.

Note that it is only possible to calculate the stackelberg equilibrium for three health insurers (with reference to computation times)
H2. Ten year horizon sheet

Alongside the year by year sheet also a ten year horizon sheet is included. With this sheet it is possible for a customer to set premium immediately for the upcoming years. Note that all others input parameters are obtained from the first sheet. So, the variable costs, fixed costs, number of policy holders, price sensitive and own funds originates from the first sheet. The advantage of this sheet is, that key performance indicators are obtained immediately for the upcoming ten years, where in the first sheet one has to simulate these ten years manually. In Figure 52 an overview of the input field of the ten year horizon sheet is depicted.

Figure 52: Input field ten year horizon sheet

With the input parameters of the first sheet and the premiums of the second sheet, the outcome parameters are determined. In Figure 53 these outcome parameters capital, market share (e.g. number of policy holders) and solvency ratio are depicted.

Figure 53: Output field ten year horizon sheet
Again, observe that when a negative capital is obtained, the cells become red. For convenience also graphs are included to visualize the outcome parameters. These are presented in Figure 54 below.

![Figure 54: Output graphs Excel tool](image)

Next to the advantage that the scenario for ten years is depicted in a glance, it is also possible with this sheet to determine the optimal premiums for a 5 year horizon. As discussed in Chapter 7, finding optimal premium on a year by year horizon is not always useful. With the button *optimal strategy* customers can calculate the optimal premiums for five years. To conclude, for the interested reader, please contact the writer of this report for the most recent Excel tool.
I. Poster of thesis

Game theoretical approach for The Dutch health insurance market
Investigation of the effects of price and announcement sequence on the key Performance indicators of Health insurers.

Dutch health insurance market
In the Netherlands it is required to purchase basic health insurance from private health insurers. Once per year, one has the possibility to switch from health insurer, which makes the Dutch health insurance market a competitive one. KPI’s as profit, market share and solvency ratio become important indicators is such markets.

Two important aspects that affect KPI’s are the level of price and the announcement sequence in which health insurers announce their premiums. In this project, the effects of these two aspects on KPI’s are investigated.

Mathematical model
A mathematical model is developed to determine for a given vector of prices $X$ the KPI’s for the concerning health insurers. A system of differential equations $L$ is developed to model the switching behavior of policy holders. With these outcomes, the KPI’s for the concerning health insurers are determined. A graphical representation of this model is given in Figure 1.

Figure 1. Graphical representation of the mathematical model, with price vector $X$, the differential equations $L$ and KPI’s.

In Figure 1, we can observe that health insurer blue is a relative large one, but becomes small in the upcoming year, while health insurer yellow is a relative small one and becomes large. Any change in the price vector $X$ of prices would immediately result into other outcomes.

Results
First, the effect of the level of price is investigated from the perspective of one health insurer. Optimization problems were formulated to determine the premiums that optimize one KPI, while other KPI’s were used as constraints.

Next to this, the model is considered as a multi criteria game (Voormevel, 1999) with health insurers as players, KPI’s as payoffs and the prices and announcement sequences as the strategy sets of players. With this approach it was possible to consider the perspective of more health insurers simultaneously and to investigate the effect of the announcement sequence of price.

![Figure 2. Premiums regarding Nash equilibrium during the years.](image)

Outcomes of a Nash equilibrium have lead to interesting results. From the perspective that every health insurer wants to optimize on profit, pricing to a Nash equilibrium results into stable premiums and as a consequence into stable profits and market shares and increasing solvency ratios. In Figure 2 an overview of the path of premiums regarding to A Nash equilibrium is given.

Outcomes of a Stackelberg equilibrium with one leader has given insights into the effects of announcement sequence. It turned out that it is beneficial for all health insurers when one health insurer decides to announce it premium first, regarding to a Nash equilibrium.

Conclusions
More insights in the effects of the level and announcement sequence of price on KPI’s are derived by developing a mathematical model. Game theory and optimization are used to obtain interesting results for specific situations.
J. Declaring Variables

\( \alpha \) : price sensitivity of policy holders
\( \beta \) : weighting factor
\( c_i \) : variable costs of health insurer i
\( c \) : vector of variable costs
\( C_i \) : fixed costs of health insurer i
\( C \) : vector of fixed costs
\( D \) : operator key performance indicators
\( \Gamma \) : operator of mathematical model
\( M \) : vector of market share
\( M_i \) : market share of health insurer i
\( L \) : operator policy holder behavior
\( N \) : vector of number of policy holders
\( N_i \) : number of policy holders of health insurer i
\( \hat{N}_i \) : updated number of policy holders of health insurer i
\( \hat{N} \) : vector of updated number of policy holders
\( \omega \) : outcome
\( \Omega \) : set of outcomes
\( P \) : vector of profit
\( P_i \) : profit of health insurer i
\( r_{i,j} \) : transition rate between health insurer company i and j
\( \rho(\sigma) \) : term in the SCR calculations
\( \tilde{S}_i \) : price space set
\( S \) : vector of solvency ratios
\( S_i \) : solvency ratio of health insurer i
\( \theta \) : situation
\( \Theta \) : set of situation
\( t \) : time moment (years)
\( t^* \) : starting time moment that policy holders can switch
\( x_i \) : premium of health insurers i
\( X \) : vector of premiums
\( x_{\min,i} \) : lowest possible premium of health insurer i
\( x_{\max,i} \) : highest possible premium of health insurer i
\( x_{\text{tec},i} \) : technical premium of health insurer i
\( W \) : set of health insurers with higher (or equal) premiums than company i
\( W' \) : set of health insurers with lower premiums than company i
References


[29] Zill, D., (1992), Advanced engineering mathematics, Boston