Experiments on human swimming: passive drag experiments and visualization of water flow

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Experiments on human swimming:
Passive drag experiments and
visualization of water flow.

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Abstract

Better insight in the physics of human swimming is obtained by performing three types of experiments. The drag force on towed objects is examined by exploring two setups: towing experiments on human swimmers in the swimming facility and tethered sphere experiments in a water channel are performed. Furthermore the flow of the surrounding water during swimming is visualized and quantified. Footage obtained in experiments at the swimming facility is analyzed by a PIV routine in order to create and interpret the flow field of the water, induced by active swimming.

The average drag on a human swimmer is validated with previous research (the observed drag force $F$ is linear with the squared velocity $v^2$ and the passive drag coefficient $K$ is $29.3 \, [\text{kg/m}]$). However more insight is gained in the fluctuations of the drag force. Analysis of frequency spectra, imply that vorticity is shed from the frontal area (probably the head or hands) of a swimmer, which enhances fluctuations of the drag force. Furthermore, towing experiments on human swimmers are recommended to be performed at high towing velocities, since the relative error in the drag force is the smallest for large towing velocities.

Fluctuations of the drag force are investigated in more detail during tethered sphere experiments. The drag coefficient ($C_D = 0.69$) and Strouhal numbers ($St = 0.14$ and $St = 0.28$) for the tethered sphere are comparable to values obtained in previous research. Uniquely, this study combines synchronized information of the force on the tethered sphere and the position of this sphere in the flow. The fluctuating force is observed to be closely related to the streamwise position of the sphere as well as the radial distance from the original position to the actual position of the sphere.

By using the PIV routine, some interesting flow fields created during human swimming are examined. Moments after the downkick of the legs during butterfly swimming, two opposite-signed vortices are observed. These patches are created by the forceful downkick, Newton’s third law implies a reaction force exerted on the swimmer. This force is in forward and upward direction. For this particular swimmer the magnitude of the force is calculated to be 62 N.

Just before the downkick with the legs (again during butterfly swimming), a large vortex is created underneath the swimmer. In this case the force exerted on this particular swimmer is 63 N in the upward direction.

New notions in the interpretation of results of drag force experiments on human swimmers are obtained. Fluctuations in this drag force are most probably created due to vortex shedding, as also observed during tethered sphere experiments in the laboratory. Furthermore, some large-scale structures (vortices) are created by active swimming. These vortices exert a force on the swimmer, thereby contributing to the propulsion of the swimmer.
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1 Introduction

Although a vast amount of physiological research on swimming has already been performed, the amount of research on physical aspects of swimming is limited. Therefore in this study, swimming is regarded from a physical point of view. Experiments on swimmers have been performed at the InnoSportlab, located in a swimming facility in Eindhoven. Complementary experiments were performed in the fluid dynamics laboratory facility of the University of Eindhoven.

During swimming, the velocity of the swimmer is limited because of drag forces. This drag can be divided into two parts. When a body is passively towed through the water, the body is subject to 'passive drag'. During active swimming, the drag will be enhanced because of the swimmer’s extra movement, hence this is called 'active drag'. In the present study, towing experiments have been performed in which passive drag is investigated. Literature on this phenomenon contains only the averaged passive drag force and up to now no research has been performed on the time dependency of the drag force obtained during such experiments. The present research project however, is aimed at finding time dependencies in passive drag measurements and it is attempted to explain their origin. To further investigate these time dependent drag forces, experiments on a more simple geometry were performed as well: time dependent drag forces on a towed sphere were measured in experiments performed in a water channel. The goal of these experiments for swim sciences is clear: a better understanding of the drag force could lead to a reduction of drag, which can lead to improvement of swimming performances.

The fluid dynamics of water surrounding a swimmer is also an area of interest which has hardly been investigated; the present project aims at mapping the velocity field of the fluid around a swimmer. For this purpose the 'Bubble System' is used: small air bubbles are injected at the bottom of the swimming pool, these bubbles will rise towards the water surface in the form of a 'bubble curtain'. While a swimmer passes through these curtains of bubbles, it is tried to capture the disturbed movement of the bubbles and thereby reconstruct the flow field of the water around a swimmer. This task is performed by different visualization techniques, namely Particle Imaging Velocimetry and Optical Flow. Better insight in the flow field of the surrounding water will increase the knowledge about propulsion of swimmers, which again can result in improvement of the swimming performances of athletes.

Some basic theoretical concepts necessary to analyse the experiments is provided in section 2. A brief and general description of the towing experiments and the results of these experiments are provided in section 3, while section 4 discusses the experiments regarding the Bubble System. All findings will be gathered in the conclusion (section 5).
1. Introduction
2 Theory

Since this study consists of two different subjects concerning swimming, the theory will be divided into two main sections as well. Firstly, the theory corresponding to the towing of bodies through fluids will be reviewed. Afterwards techniques to visualize water flows around swimmers will be discussed.

2.1 Towing a body through a fluid

In this section, the basic theoretical concepts related to the case of a body towed through a fluid will be reviewed. An important dimensionless number to characterize a flow is the Reynolds number \( \text{Re} \):

\[
\text{Re} = \frac{vL}{\nu}
\]

(1)

In which \( v \) and \( L \) are the typical velocity and length scale of the flow and \( \nu \) is the kinematic viscosity of the fluid. The Reynolds number represents the ratio between advective and viscous forces.

2.1.1 Drag on a body

When a body is towed through a fluid, it experiences a drag force. Generally, the drag force \( F_D \) is represented as:

\[
F_D = \frac{1}{2} C_D \rho A v^2
\]

(2)

in which \( C_D \) is the dimensionless drag coefficient, \( \rho \) the density of the medium in which the drag is applied onto the body, \( A \) is the area of the body on which the drag is active and \( v \) is the velocity of the body. \( A \) is mostly regarded as the frontal area, which is the maximum projection of the body onto a plane normal to the direction of the flow. In swim sciences, equation (2) is often presented as:

\[
F_D = K v^2
\]

\[
K = \frac{1}{2} C_D \rho A
\]

(3)

in which \( K \) [kg/m] is the passive drag coefficient.

In the case of a sphere, the drag coefficient is a well documented property. Multiple experiments show that the \( C_D \)-values of a stationary sphere, calculated for a wide range of values of the Reynolds number. The result of different experimental results is presented in figure 1(a) [Schlichting and Gersten, 2004].

However, while experiments on stationary spheres show an asymptotic value of \( C_D \approx 0.4 \) (observed in figure 1(a)), the drag coefficient for tethered spheres is
significantly larger [Williamson and Govardhan, 1997]. Contrary to experiments on stationary spheres, in tethered sphere experiments, the sphere has more degrees of freedom: the sphere is able to move in a plane perpendicular to the streamwise direction. In this case the drag coefficient almost doubles, as can be seen in figure 1(b).

This indicates the difficulty of defining one absolute drag coefficient for a certain object. In the case of human swimmers, no distinction is made in literature be-
between a stationary or tethered (towed) situation of a swimmer while performing experiments on the drag coefficient.

In comparison to a sphere, the geometry of a swimmer is more complex, therefore the drag coefficient is more difficult to determine and hence less documented. In the recent past research has been performed on the drag on human swimmers. This drag is divided into two types of drag: active and passive drag. The drag force created while towing a swimmer passively through the water is called 'passive drag', while the drag induced by active swimming is coined 'active drag'. This study focusses on passive drag. In previous research a linear relation between the drag force and the velocity is observed, in comparison with equation (3). However, some inconsistency in the passive drag coefficient $K$ is present. In two comparable studies $K$ is measured to be 29 [Toussaint et al., 2000] [Kjendlie and Stallman, 2008]. Although by using similar experimental techniques, other results have been observed as well: $K = 20$ [Zamparo et al., 2009]. The differences can possibly be attributed to the fact that different swimmers are used during these experiments. On top of that, passive drag measurements are extremely dependent on the body position and shape during the measurement: drag differences of about 100% were observed in one set of measurements, caused by small variations in the position of the swimmer’s head [Miyashita and Tsunoda, 1978].

### 2.1.2 Added mass

By pulling an object through a fluid, not only the object moves, also the fluid has to move around the object. When the object is accelerated, the fluid around it must be accelerated too, thereby inducing a new force. So by accelerating a body through a fluid, next to the drag force on the body and the force to accelerate the body, also the acceleration of the fluid moving backwards induces a force: this force is commonly associated with an imaginary ’added mass’ of the body. The total drag force can then be expressed as:

$$F = \frac{1}{2}C_D \rho A v^2 + m \frac{dv}{dt} + C_a \rho V \frac{dv}{dt}$$

(4)

In this expression, the first term represents the drag force, as given in (2); the second term represents the force needed to accelerate a body with mass $m$ and acceleration $dv/dt$. Furthermore, the force to accelerate the fluid around the body is determined by the volume of the body $V$ and the dimensionless added mass coefficient $C_a$. It is possible to analytically derive the added mass for some bodies. For a sphere with radius $R$ the added mass is $M_a = \frac{2}{3} \pi R^3 \rho$ [Kundu et al., 2012]. However, for a more complicated body, as a human body, no exact analytical derivation is available. Statistical analysis [Caspersen et al., 2010] has provided a relation of the added mass for a human body. In this study it was found that the
2.1 Towing a body through a fluid

added mass $M_a$ of a body, as a function of the body mass ($BM$), frontal area ($FA$) and reaching height ($RH$), is reasonably well described by the following empirical relationship:

$$M_a = C_1 + C_2 BM + C_3 FA + C_4 RH$$  \hspace{1cm} (5)

in which $C_1$, $C_2$, $C_3$ and $C_4$ are constants determined by Caspersen et al. (2010): $C_1 = 16.0 \,[\text{kg}], \, C_2 = 0.24 \,[-], \, C_3 = 22.0 \, [\text{kg}/\text{m}^2] \, \text{and} \, C_4 = -0.05 \, [\text{kg}/\text{m}]$.

The coefficients in (5) are statistically determined, some seem a bit odd though. For example $C_1$: for small values of $FA$, $RH$ and $BM$, the added mass is totally determined by the constant value of $C_1$. According to (5), there would even be a significant value of added mass if $FA$, $RH$ and $BM$ are equal to zero.

Therefore equation (5) should be tested. For example by towing experiments in which swimmers are towed with a constant acceleration. The difference in the measured force and the ‘normal’ drag on a swimmer (equation (2)) can than be contributed to added mass.

2.1.3 Shedding frequency

When certain bodies are kept in a uniform flow, well-known von Kármán vortex streets arise. For example in figure 2 vortices of opposite sign are observed to be created in the wake of a circular cylinder [Van Dyke, 1982]. This phenomenon is called vortex shedding. Vortices are shed with a certain frequency: the shedding frequency.

Figure 2: Von Kármán vortex street behind a circular cylinder, created by a fixed cylinder in a uniform water flow. Photograph by Sadatoshi Taneda [Van Dyke, 1982].
2. Theory

2.1 Towing a body through a fluid

An important parameter characterizing the vortex shedding is the Strouhal number $St$, defined as

$$St = \frac{fd}{v}$$

in which $f$ is the shedding frequency, $v$ the velocity of the fluid and $d$ a characteristic length. Research on the shedding frequency (Strouhal number) by studying the wake flow of stationary objects has been performed at an extensive range of $Re$-values, for the wake flows of circular cylinders [Williamson, 1996], square cylinders [Sohankar et al., 1998], air foils [Ausoni et al., 2006] and spheres [Sakamoto and Hanui, 1990]. Results obtained for the case of a stationary sphere are presented in figure 3(a).

As observed in figure 3(a), relatively small values of the Strouhal number ($St = 0.2$) are observed in the entire range of Reynolds numbers. Simultaneously high shedding frequencies are observed. The low Strouhal number is associated with periodic shedding of vortices, while the high Strouhal number corresponds with the instability of the boundary layer and therefore changes by varying the Reynolds number. Similar to this result, in the studies on the wake flow of circular and square cylinders also a constant (low) Strouhal number of $St \approx 0.2$ is observed for a wide range of $Re$, while for air foils a constant Strouhal number between 0.18 and 0.24 was obtained.

The drag coefficient of a sphere differs in a static and a tethered situation, as was observed in section 2.1.1. In figure 3 a difference in Strouhal numbers for a sphere in static and tethered case is observed. Measured Strouhal numbers for a static sphere are presented in figure 3(a). The Strouhal numbers for a tethered sphere can be derived from figure 3(b). The streamwise position of the tethered sphere oscillates with a characteristic Strouhal number of 0.28, while the sphere oscillates with $St = 0.14$ in the transverse direction (perpendicular to the stream direction) [Williamson and Govardhan, 1997], as displayed in figure 3(b). This is again an indication of the subtle difference in the behaviour of flows interacting with a static and a tethered sphere.

The Strouhal number is an often used property in swim sciences as well, although in that case the Strouhal number is determined by the stroke frequency $f$, stroke amplitude $d$ and the swimming velocity $v$. This Strouhal number is in no way comparable to the Strouhal numbers corresponding to vortex shedding. No literature is found on frequency analysis for passively towed swimmers.

2.1.4 Force fluctuations

Both the effects of added mass and vortex shedding give rise to fluctuations of the force on bodies towed through a fluid. In the laboratory, the towing of spherical objects is often mimicked by holding a tethered sphere in a uniform flow [Williamson
2.1 Towing a body through a fluid

Figure 3: Panel (a) presents the measured $St$-values of a stationary sphere as a function of an extensive range of $Re$ [Sakamoto and Haniu, 1990]. The energy spectrum of the velocity in the wake of a towed sphere as a function of dimensionless frequency ($St$) is presented in (b) [Williamson and Govardhan, 1997].

and Govardhan, 1997] [Lee et al., 2013]. The drag force of these tethered spheres is determined and force fluctuations are observed in both the streamwise and the
2. Theory

2.1 Towing a body through a fluid

In figure 4 the in-plane trajectories (plane perpendicular to the direction of flow) of a tethered sphere are presented [Lee et al., 2013]. Clearly different behaviour is observed at different Reynolds numbers. Lee et al. (2013) observed seven different regimes, based on these in-plane trajectories of the sphere, these are assembled in table 1 [Lee et al., 2013].

The drag force of a tethered sphere is almost twice as large as that of a stationary sphere, as already discussed in section 2.1.1. Besides, fluctuations in the drag force arise while towing a body through a fluid. So a body behaves rather different while towed, compared to a stationary situation. Next to the average (drag) force on a body, it is therefore of interest to investigate force fluctuations in time as well.

![Figure 4: In plane trajectories of a tethered sphere, obtained at different Reynolds numbers.](image)

<table>
<thead>
<tr>
<th>Regime</th>
<th>Re</th>
<th>In plane trajectory</th>
</tr>
</thead>
<tbody>
<tr>
<td>i</td>
<td>50≤Re&lt;210</td>
<td>N/A</td>
</tr>
<tr>
<td>ii</td>
<td>210≤Re&lt;270</td>
<td>N/A</td>
</tr>
<tr>
<td>iii</td>
<td>270≤Re&lt;300</td>
<td>Radial, shifting to azimuthal</td>
</tr>
<tr>
<td>iv</td>
<td>300≤Re&lt;332</td>
<td>Primarily azimuthal</td>
</tr>
<tr>
<td>v</td>
<td>332≤Re&lt;550</td>
<td>Linear radial</td>
</tr>
<tr>
<td>vi</td>
<td>550≤Re&lt;3000</td>
<td>Irregular</td>
</tr>
<tr>
<td>vii</td>
<td>3000≤Re&lt;12000</td>
<td>Quasi-circular</td>
</tr>
</tbody>
</table>
2.1.5 Wavelet transform

In order to study shedding frequencies of a passively towed swimmer, a frequency analysis is necessary. A well-known method to analyze frequency spectra is the Fourier transform. For this analysis, a trade-off in the choice of window size has to be made. A longer time window improves the frequency resolution, but it results in a poorer time resolution. On the other hand, a short time window results in a poor frequency resolution, while the time localization improves. Therefore, in some cases it is more convenient to use a wavelet transform. Wavelet analysis uses long time intervals when low frequencies need to be analyzed and short intervals where high frequencies have to be investigated.

The continuous wavelet transform (CWT) uses inner products to measure the similarity between a signal and an analyzing function. In wavelet analysis this function is a wavelet \( \psi \). The CWT compares the signal to shifted and compressed or stretched versions of a wavelet. The comparison between the analyzed signal and the wavelet is performed at various scales and positions, therefore a function of two variables is obtained. For a scale parameter \( a > 0 \), and a position parameter \( b \), the definition of the CWT is:

\[
C(a, b, f(t), \psi(t)) = \int_{-\infty}^{\infty} f(t) \frac{1}{\sqrt{a}} \psi^*(\frac{t - b}{a}) dt
\]

(7)

In this study, the Morlet wavelet is used as the analyzing wavelet, the Morlet wavelet is defined as:

\[
\psi(t) = e^{-t^2/2} \cos(5t)
\]

(8)

By continuously varying the values of \( a \) and \( b \), the CWT coefficients \( C(a, b) \) are obtained. The coefficient \( C \) is directly related to the degree of correlation between the signal and the wavelet: a high correlation is present for high values of \( C \), while the resemblance between the signal and the wavelet is small for small values of \( C \). For example, a high coefficient at scale 10 and position 40, implies a large resemblance between the wavelet and the analyzed function at a scale of 10 and \( t = 40s \). The scale values correspond to the inverse frequency. So a high scale value, corresponds to a low frequency and the other way around.

2.2 Visualizing fluid flows

Two techniques used for visualizing water flows around a swimmer are considered in this study, namely Particle Image Velocimetry and Optical Flow. The algorithm of both will be treated theoretically in this section, while experiments will also be performed with Particle Image Velocimetry (section 4).
2.2.1 Particle image velocimetry

Particle Image Velocimetry (PIV) is a technique aiming at visualizing different flows. There are many different types of PIV, although the core of the technique is always similar: particles are placed in the fluid and the trajectories of these particles in time are being measured. These trajectories characterize the flow of the fluid.

A small number of particles is easily followed, since it is not likely that they will overlap one another. When large amounts of tracing particles are used, cross-correlation techniques have to be used to trace the particles [Adrian and Westerweel, 2010]. Different frames of a film will be investigated and by cross-correlating parts of these frames the particles can be traced. These parts of the frames are also known as interrogation windows, see figure 5.

![Figure 5: Interrogation window (red box) in a frame (black box).](image)

Frames are obtained at instant times. In between different frames, particles will move. Some particles will move inside an interrogation window. By cross-correlating the two interrogation windows, the displacement of these particles can be tracked. However, some particles will have moved out of the interrogation window, which results in so-called ‘in-plane losses’. ’Out of plane losses’ occur when particles have moved out of the interrogation window in tangential direction. Lastly, particles which are initially not in the interrogation window can move in the interrogation window. So only the particles that are both in the first and second interrogation window help to visualize the flow. This is schematically shown in figure 6, where eventually only the blue and yellow particle are useful.

The interrogation method as described is the standard interrogation method, usually called ’single pass interrogation’: the interrogation window has similar size and position in both the first and second frame. The next interrogation window will then be placed next to the previous one or with a certain overlap. The overlap is used in order to obtain more interrogation windows in the total frame and thereby reducing the number of in-plane losses.

Another interrogation technique is called ’multipass interrogation’. The same image pair is interrogated multiple times using the same dimensions of the interrogation domain. In this technique, results of the first pass are used in the following
2.2 Visualizing fluid flows

2.2.1 Multipass interogations

Standard cross-correlation is applied at the first step. Secondly, the interrogation window of the second image is shifted, based on the predictor (displacement vector obtained by the previous process of standard cross-correlation), whereafter cross-correlation is applied again. The smaller displacement is obtained this time. After applying this operation several times iteratively, the resulting displacement becomes nearly zero. The final displacement is the sum of all displacements in previous processes. By this routine, in-plane losses are reduced [Adrian and Westerweel, 2010].

Next to the multipass method, another way of obtaining more refined information is the 'multigrid interrogation'. The main difference between the two methods lies in the dimensions of the interrogation windows. Where the interrogation window in the multipass method is similar for every pass, the dimension of the interrogation window reduces in each subsequent pass in the multigrid method. The first pass has the largest dimensions e.g. 64x64 pixels, the second pass has the dimensions 32x32 pixels etc. The use of larger interrogation windows in previous passes ensures that in-plane losses are small.

2.2.2 Optical flow

Another technique used for visualizing flows is Optical Flow. The optical flow is defined as the 'flow' of gray values at the image plane [Jähne, 1991]. A central assumption of this technique is the conservation of irradiance, in that case the
optical flow is equal to the motion field. The conservation law is written as:

\[ \frac{\partial g}{\partial t} + \mathbf{f} \cdot \nabla g = 0 \]  

(9)

in which \( \mathbf{f} \) is the velocity field and \( g \) the intensity. Clearly, the aim of Optical Flow is to find \( \mathbf{f} \). In one dimension equation (9) becomes:

\[ \frac{\partial g}{\partial t} + f_x \frac{\partial g}{\partial x} = 0 \]  

(10)

and \( f_x \) is simply determined by the ratio of the partial derivatives of the intensity in time and in the \( x \)-direction:

\[ f_x = -\frac{\partial g}{\partial t} \frac{\partial g}{\partial x} \]  

(11)

However for higher dimensions the calculation of \( \mathbf{f} \) becomes more complicated. Hence \( \mathbf{f} \) is often estimated, for example by the least-squares method. In two dimensions this weighted averaging procedure looks like:

\[ \|e\|_2^2 = (f_x \frac{\partial g}{\partial x} + f_y \frac{\partial g}{\partial y} + \frac{\partial g}{\partial t})^2 \]  

(12)

in which \( f_x \) and \( f_y \) are the components of the velocity field in the \( x \) and \( y \) direction, respectively. Equation (12) can be solved by calculating the partial derivatives of the error \( \|e\|_2^2 \) with respect to \( f_x \) and \( f_y \) and setting these equal to zero:

\[ \frac{\partial \|e\|_2^2}{\partial f_x} = 2 \frac{\partial g}{\partial x} (f_x \frac{\partial g}{\partial x} + f_y \frac{\partial g}{\partial y} + \frac{\partial g}{\partial t}) = 0 \]  

(13a)

\[ \frac{\partial \|e\|_2^2}{\partial f_y} = 2 \frac{\partial g}{\partial y} (f_x \frac{\partial g}{\partial x} + f_y \frac{\partial g}{\partial y} + \frac{\partial g}{\partial t}) = 0 \]  

(13b)

From equation (13) the following linear equation system can be obtained:

\[ \begin{bmatrix} \frac{\partial g}{\partial x} & \frac{\partial g}{\partial x} \\
\frac{\partial g}{\partial x} & \frac{\partial g}{\partial y} \\
\frac{\partial g}{\partial y} & \frac{\partial g}{\partial y} \\
\frac{\partial g}{\partial y} & \frac{\partial g}{\partial y} \\
\end{bmatrix} \begin{bmatrix} f_x \\
f_y \end{bmatrix} = - \begin{bmatrix} \frac{\partial g}{\partial x} \\
\frac{\partial g}{\partial y} \\
\frac{\partial g}{\partial x} \\
\frac{\partial g}{\partial y} \\
\end{bmatrix} \]  

(14)

in which the overlines again denote the average values. In a more compact notation, (14) becomes:

\[ \mathbf{Gf} = \mathbf{d} \]  

(15)
The linear system given in equation (14), can only be solved when the determinant of \( G \) is non-zero, so:

\[
\det G = \frac{\partial g}{\partial x} \frac{\partial g}{\partial y} \frac{\partial g}{\partial y} \frac{\partial g}{\partial x} - \frac{\partial g}{\partial x} \frac{\partial g}{\partial y}^2 \neq 0
\] (16)

In a two-dimensional case, the solution for the optical flow \( f \) can be written down explicitly, since it is relatively easy to invert the 2x2 matrix \( G \):

\[
G^{-1} = \frac{1}{\det G} \begin{bmatrix}
\frac{\partial g}{\partial y} & -\frac{\partial g}{\partial x} \\
-\frac{\partial g}{\partial x} & \frac{\partial g}{\partial y}
\end{bmatrix}
\] (17)

And therefore the optical flow \( f \) becomes:

\[
f = G^{-1} d = \begin{bmatrix} f_x \\ f_y \end{bmatrix} = -\frac{1}{\det G} \begin{bmatrix}
\frac{\partial g}{\partial y} & \frac{\partial g}{\partial x} & -\frac{\partial g}{\partial t} \\
-\frac{\partial g}{\partial x} & \frac{\partial g}{\partial y} & \frac{\partial g}{\partial t}
\end{bmatrix}
\] (18)

In three dimensions this calculation is similar but much more extensive, since the inversion of the 3x3 matrix \( G \) is an elaborate calculation. Although equation (18) gives an explicit solution for the optical flow, it is not trivial to calculate the optical flow, since in general it is very hard to obtain the gradients of the intensity \( g \).

Better control over noise is provided by a variational approach [Jähne, 1991]. In this approach, an error functional for \( f \) is introduced:

\[
\epsilon(f) = \int L(f, \nabla f, x) dx
\] (19)

where \( x \) is the position parameter. A minimum for the error functional will be sought. A perturbation \( \delta \) is applied to the velocity field \( f \), which vanishes on the boundary. For brevity a 1D field is assumed. The variation of \( \epsilon \) is obtained by Taylor expansion:

\[
\epsilon(f + \delta) - \epsilon(f) = \int \left( \frac{\partial L}{\partial f} \delta + \frac{\partial L}{\partial f_x} \frac{d \delta}{dx} \right) dx = 0
\] (20)

The last term can be integrated by parts, which results in:

\[
\epsilon(f + \delta) - \epsilon(f) = \int \left( \frac{\partial L}{\partial f} - \frac{d}{dx} \left( \frac{\partial L}{\partial f_x} \right) \right) \delta dx = 0
\] (21)
Equation (21) should hold for arbitrary values of \( \delta \), therefore:
\[
\frac{\partial L}{\partial f} - \frac{d}{dx} \left( \frac{\partial L}{\partial f_x} \right) = 0
\] (22)

In more dimensions, equation (22) can be written as:
\[
\frac{\partial L}{\partial f_i} - \frac{d}{dx_j} \left( \frac{\partial L}{\partial f_{ij}} \right) = 0
\] (23)
in which the first subscript of \( f_{ij} \) represents the direction of the corresponding parameter, while the second subscript represents the derivative of that corresponding parameter (e.g. \( f_{xy} = \frac{\partial f_x}{\partial y} \)). Equation (23) represents the Euler-Lagrange equation. In this case the Lagrangian \( \| \mathbf{f} \cdot \nabla g + \frac{\partial g}{\partial t} \| \) is used as a first attempt. The first term of equation (23) is easily calculated, since the Lagrangian does not (yet) depend on the gradients of \( f \), the second term of equation (23) vanishes.
\[
\frac{\partial L}{\partial f_i} - \frac{d}{dx_j} \left( \frac{\partial L}{\partial f_{ij}} \right) = \frac{\partial L}{\partial f_i} = 2 \left( (\mathbf{f} \cdot \nabla) g + \frac{\partial g}{\partial t} \right) \frac{\partial g}{\partial x_i}
\] (24)

This result is similar to the result obtained via the least-squares method, represented in equation (13) and little has been gained. However, extra conditions can be added to \( L \), for example requirements on the smoothness of the velocity field. In order to obtain a smooth velocity field, the spatial gradients should be small, therefore a suitable Lagrangian can be represented as:
\[
L = (\mathbf{f} \cdot \nabla g + \frac{\partial g}{\partial t})^2 + \lambda^2 (|\nabla f_x|^2 + |\nabla f_y|^2)
\] (25)
in which \( \lambda \) is the smoothness-term [Jähne, 1991]. This Lagrangian is definitely dependent on the spatial derivatives of the velocity field. The Euler-Lagrange equation (23) therefore leads to:
\[
\frac{d}{dx_j} \frac{\partial L}{\partial f_{ij}} = 2\lambda^2 \left( \frac{\partial^2}{\partial x_j^2} f_x + \frac{\partial^2}{\partial x_j^2} f_y \right)
\] (26a)
\[
((\mathbf{f} \cdot \nabla) g + \frac{\partial g}{\partial t}) \frac{\partial g}{\partial x_i} - \lambda^2 \nabla^2 f_i = 0
\] (26b)

Starting from a trial field \( \mathbf{f}(x) \), equation (26) can be solved as a diffusion equation.
\[
\frac{\partial \mathbf{f}}{\partial t} = \lambda^2 \Delta \mathbf{f} - ((\mathbf{f} \cdot \nabla) g + \frac{\partial g}{\partial t}) \nabla g
\] (27)
The regularization factor \( \lambda^2 \) is a diffusion constant. Multiple solutions and algorithms exist to tune the diffusion. In this study, the main interest concerns small
vorticity fields, hidden in flows dominated by constant vertical strain. Therefore it is desired to highlight the vorticity component while the strain components of the optical flow are regularized. Hence, the gradient term of equation (25) is rewritten as

\[
|\nabla f_x|^2 + |\nabla f_y|^2 = \frac{1}{2} \left\{ \left( \frac{\partial f_x}{\partial x} + \frac{\partial f_y}{\partial y} \right)^2 \right. \\
\left. + \left( \frac{\partial f_x}{\partial y} - \frac{\partial f_y}{\partial x} \right)^2 + \left( \frac{\partial f_x}{\partial x} + \frac{\partial f_y}{\partial y} \right)^2 \right. \\
\left. + \left( \frac{\partial f_x}{\partial y} - \frac{\partial f_y}{\partial x} \right)^2 \right\}
\]

(28)

The first term in equation (28) represents dilation, which should be zero anyhow because of \(\nabla \cdot f = 0\). The second and third term correspond with strain and the last term represents the rotation of a velocity field. Again, in this study rotation is the most interesting component of the velocity field and therefore the regularization should only be applied on the first three terms of equation (28). Coefficients for the different terms are introduced, \(\alpha\) corresponding to dilation, \(\beta\) for strain and rotation is represented by \(\gamma\). The regularization term of the Lagrangian becomes:

\[
|\nabla f_x|^2 + |\nabla f_y|^2 = \frac{1}{2} \left\{ \alpha\left( \frac{\partial f_x}{\partial x} + \frac{\partial f_y}{\partial y} \right)^2 \\
+ \beta\left[ \left( \frac{\partial f_x}{\partial x} - \frac{\partial f_y}{\partial y} \right)^2 + \left( \frac{\partial f_x}{\partial y} + \frac{\partial f_y}{\partial x} \right)^2 \right] \\
+ \gamma\left( \frac{\partial f_x}{\partial y} - \frac{\partial f_y}{\partial x} \right)^2 \right\}
\]

(29)

This Lagrangian will be implemented in the Euler-Lagrange equation. After some extensive mathematics (Appendix A.1) the Euler-Lagrange equation evolves into:

\[
\frac{\partial L}{\partial f_i} - \frac{d}{dx_j} \frac{\partial L}{\partial f_{ij}} = ((f \cdot \nabla)g + \frac{\partial g}{\partial t}) \frac{\partial g}{\partial x_i} - \left\{ (\alpha + \beta)(\frac{\partial^2 f_x}{\partial x^2} + \frac{\partial^2 f_y}{\partial y^2}) + \\
(\alpha - \gamma)(\frac{\partial^2 f_x}{\partial x \partial y} + \frac{\partial^2 f_y}{\partial x \partial y}) + (\beta - \gamma)(\frac{\partial^2 f_x}{\partial y^2} + \frac{\partial^2 f_y}{\partial x^2}) \right\} = 0
\]

(30)

Clearly, it is possible to tune the regularization term in order to highlight vorticity and ignore strain. Other options for the regularization term could be worth investigating as well: the div-curl method [Gupta and Prince, 1996], also focusses on capturing rotating motion from a picture.
2. Theory

2.2 Visualizing fluid flows

2.2.3 Force calculation

Next to the visualization of the velocity and vorticity fields, the Bubble System aims at measuring the force acting on the swimmer. The force can be calculated by using only these velocity \( (v) \) and vorticity \( (\omega) \) fields. Multiple algorithms performing this task are already present [Noca et al., 1999]. Generally the derivations start with the momentum balance equation [Graziani and Bassanini, 2002]:

\[
\frac{\partial \rho v_i}{\partial t} + \frac{\partial \rho v_i v_j}{\partial x_j} = \frac{\partial T_{ij}}{\partial x_j}
\]  

(31)

in which \( v_i \) is the \( i \)th component of the flow velocity \( v(x, t) \) in a space-fixed inertial coordinate system, \( \rho \) the fluid density and \( T_{ij} \) the stress tensor. With the pressure \( p \) and the viscosity \( \mu \), the stress tensor is denoted as:

\[
T_{ij} = -p\delta_{ij} + 2\mu E_{ij}
\]

(32)

An arbitrary, time-dependent, body-fixed control volume \( V(t) \) is considered, bounded internally by the body surface \( \partial B(t) \) and externally by a smooth compact connected surface \( S(t) \). The normal \( n \) is defined outwards on the external surface \( S(t) \). The control volume translates rigidly with the body \( B \) at a translation velocity \( U(t) \). Integrating (31) over the volume \( V(t) \) and applying Gauss’s theorem, results in:

\[
\int_{V(t)} \frac{\partial v_i}{\partial t} dV + \int_{S(t)} v_i n_j v_j dS - \int_{\partial B(t)} v_i n_j v_j dS = \int_{S(t)} n_j T_{ij} dS - \int_{\partial B(t)} n_j T_{ij} dS
\]

(33)

The last term of (33) is regarded as the \( i \)th component of the resultant force \( F = F(t) \) exerted by the fluid on the body, which is exactly the quantity of interest. By lengthy algebraic calculations [Graziani and Bassanini, 2002], the expression can be written as:

\[
F = -\frac{1}{N-1} \frac{d}{dt} \left[ \int_{V(t)} x \times \omega + \int_{\partial B(t)} x \times (n \times v) dS \right]
\]

(34)

\[
+ \int_{S(t)} n \cdot G dS
\]

in which the tensor \( G \) is denoted as:

\[
G = 2\mu E - vv + \frac{1}{2}|v|^2 I
\]

\[
+ \frac{1}{N-1} \left\{ x \times v \omega - (v - U) x \times \omega - \mu (x \cdot \nabla \times \omega \text{I} - x \nabla \times \omega) \right\}
\]

(35)
2.2 Visualizing fluid flows

2. Theory

In which all vector products result in dyadic variables. In (35), \( \mathbf{E} = \frac{1}{2}(\nabla \mathbf{v} + \nabla \mathbf{v}^T) \) and \( \mathbb{I} \) is the unity tensor.

This method provides an exact algorithm for the resulting force of the fluid on a moving body, by only using the velocity and vorticity field far away from this body. However, the mathematics is rather complex and by using a large control volume the force calculated is the resulting force on the entire swimmer. Although forces on parts of the swimmer are interesting for swim performances as well, e.g. the force on the legs during the downkick phase of swimming.

In analogy with experimental biological research on the propulsion of swimming fish, in this study forces corresponding to one stroke of a full swimming cycle (for example downkick with the legs) have been investigated [Drucker and Lauder, 1999]. Drucker & Lauder (1999) performed PIV experiments on swimming fish. Propulsive forces on the fin were associated with the near-fin circulation of vortices in the wake. The momentum of the wake, which is initiated by the fin stroke, will produce a propulsive force on the fin. The near-wake of the fin is characterized by a vortex ring, shed into the wake after pectoral fin abdution. The momentum \( (M) \) of a vortex ring created in a non-rotating environment, may be estimated from a planar section by using [Milne-Thomson, 1966]:

\[
M = \rho \Gamma A
\]

where, \( \rho \) is the fluid density, \( \Gamma \) the circulation of the vortex ring and \( A \) the area of the cross-section of the vortex ring. The instantaneous force \( F \), necessary to create the vortex ring, is defined by the time derivative of the momentum \( M \):

\[
F = \frac{dM}{dt}
\]
3 Towing Experiments

Towing experiments are both performed in the swimming facility and in the laboratory facility. In the first case, the drag is monitored while a human swimmer is passively towed through the water. In the latter case, the swimmer is mimicked by a simple, but well-defined object: a sphere.

For the towing experiments it is important to note that different swimmers are used in previous researches. In contrast with a sphere, every swimmer is unique. Hence forces on that particular swimmer will be unique as well. Therefore, differences with previous swim research will always be present.

3.1 Setup towing system

The towing system available at the Innosportlab consists of a rope attached to an electromotor and a dynamometer. Swimmers can be towed through the water with a constant velocity while laying still; this is called ‘passive towing’. While passively towing a swimmer, the force exerted on the swimmer is measured. This force is equal to the drag force. Towing is performed by constant velocities \( v \) in a range of 1.0 - 2.8 m/s. The force is captured as a function of time, with a sample frequency of 1000 Hz. The experiments were performed at a water depth of 1 m, with the swimmer regarded to be fully immersed. A schematic view of the towing system is given in figure 7.

![Figure 7: Schematic representation of the towing system. Swimmers are passively towed with the rope attached to the electromotor. Forces are measured with a dynamometer (grey ellips) and stored on a computer memory.](image)

A snapshot of a swimmer during a towing experiment is shown in figure 8. The swimmer is holding the yellow dynamometer, while clearly she intends to keep her body in a horizontal position, although this will never be accomplished perfectly.
3.2 Results towing system

Multiple experiments on the towing system of the Innosportlab are performed. In these experiments, the towing velocity $v$ is varied from 1 m/s to 2.8 m/s, while all other parameters were kept constant throughout all experiments. Prior to the towing, the swimmer has to dive underneath the water surface in order to reach the desired water depth. It is almost impossible to perform this movement similarly for all experiments.

3.2.1 Force calibration

The direct output of the experiment is a towing force, which is equal to the drag force on the swimmer. The registered force signal in Volts has to be converted in Newtons. Therefore the system is statically calibrated above the water surface. The force in Newton relates to the force signal in Volt as:

$$F[N] = AF[V] + B_1$$

with constants $A = 100.4 \ [N/V]$ and $B_1 = -208.9 \ [N]$ determined by the calibration. By applying equation (38) on a typical experimental result, the drag force on the swimmer as a function of the experimental time is obtained.

However, this results in negative drag forces on the swimmer. It seems that the offset of the calibration in equation (38) is not accurate. The force on the swimmer should be equal to zero at $t = 0$, since the towing has not yet started.
Therefore a new offset is introduced:

\[ F[N] = AF[V] + B_2 \]  

(39)

in which \( A = 100.4 \text{ [N/V]} \) and \( B_2 = -81.6 \text{ [N]} \). With the use of equation (39), the drag force in all experiments starts around zero at \( t = 0 \). Note, however, that no experiment will start exactly with zero drag force, since the swimmer has to dive to the starting depth, thereby exerting forces on the towing system. In figure 9 an example of the drag force \( F(t) \) on the swimmer during a towing experiment is presented.

\[ \text{Figure 9: Recording of the drag force } F(t) \text{ on a swimmer while towed at } v = 1.6 \text{ m/s. The measured signal was converted to Newtons by using (39).} \]

Startup phenomena are clearly seen in figure 9. During the first 2 to 6 seconds the towing system has to adjust its speed from zero to the desired speed, which is clearly seen as force fluctuations during this time interval. Afterwards the drag force is more constant, although some clear, but smaller, fluctuations are still visible.

\subsection{3.2.2 Force velocity diagram}

Once a constant towing velocity of the swimmer is established, the average (passive) drag force on a swimmer can be determined by simply averaging the drag force in time. For the experiment presented in figure 9, the force is averaged over the last 10 seconds of the time interval. This routine is performed for all
experiments: an overview of the average drag forces corresponding to the towing velocities for all experiments is displayed in figure 10.

![Figure 10: The average drag force on a specific swimmer, towed at different velocities. Calculated by equation (39).](image-url)

A linear relation between the drag force $F$ and the squared velocity $v^2$ is obtained: $F = Kv^2$. The numerical value of the passive drag coefficient $K$ for this particular swimmer is 29.3 $[\text{kg/m}]$. A similar passive drag coefficient was observed in previous research ($K = 29 [\text{kg/m}]$) [Toussaint et al., 2000] [Kjendlie and Stallman, 2008], although different swimmers were used.

### 3.2.3 Drag coefficient

The drag coefficient $C_D$ is directly related to the average drag force, as observed in equation (2). In order to calculate $C_D$, the frontal area must be known. However, this frontal area depends on the angle of the swimmer with the direction of the flow. If this angle is zero, the frontal area is minimized and therefore the drag force will be minimal as well. During these experiments, the swimmers intended to remain in this position, although this will never be exactly the case. Assuming the frontal area to be $A = 0.08 \text{ m}^2$. By using the fluid density of water at a temperature of $T = 300 \text{ K}$ ($\rho = 0.997 \cdot 10^3 \text{ kg/m}^3$), the corresponding value of $C_D$
3. Towing Experiments

3.2 Results towing system

is calculated: $C_D = 0.73$. This is in accordance with earlier research, in which the $C_D$ of a human body is calculated to be in the range of 0.58 - 1.04 [Zatsiorsky et al., 2000] [Toussaint et al., 2000].

3.2.4 Force fluctuations

In all of the experiments fluctuations of the drag force were observed, as also seen in figure 9. Therefore it is useful to investigate the time dependency of the force, with the use of wavelet transforms (see section 2.1.5).

An example of the result of such a wavelet transform is given in figure 11.

![Wavelet Transform Example](image)

Figure 11: Result of the wavelet transform, for an experiment with towing velocity 1.3 m/s. Red colors correspond with a high CWT coefficient, while blue represents low CWT coefficients

In this particular experiment, the towing velocity was 1.3 m/s. Clearly some high correlations between the wavelet and the drag force throughout the entire time domain are seen as red areas.

By summing all coefficients $C$ in time, the scale values with the highest coefficients over the entire time domain become much more clear, as can be seen in figure 12. Three distinct peaks are distinguished, at scale values of 1210, 830 and 360, these scales correspond to frequencies of 0.26, 0.37 and 0.86 Hz. These frequencies are most present in this analyzed time interval of this particular experiment. By performing this routine for all experiments at different towing velocities, an overview is obtained. This overview shows the frequency of the drag force fluctuations as a function of the towing velocity of the particular experiment, it is presented in figure 13.

Data points presented in figure 13(a) correspond to frequencies in the drag force which are most present during analysis. Some less present frequencies are also found, but adding these points to the graph would make the representation less clear. Multiple small frequencies are observed: for the sake of clarity, they are omitted in figure 13(a), while they are present in figure 13(b).
3.2 Results towing system

Figure 12: Result of the wavelet transform, for an experiment with towing velocity 1.3 m/s (see figure 11). Obtained by summing all coefficients $C$ in time.

From figure 13(a), a few linear relations between frequency data points and the corresponding velocities are obtained, represented by the black lines. The frequencies represented by the red lines in figure 13(b) are constant frequencies, which are present throughout nearly all experiments. Probably these frequencies are owing to the set-up, for example an eigen frequency of the towing system. However, the black lines in figure 13(a) must have another nature, since they depend on the towing velocity of the experiment. Three linearly increasing relations between the observed frequencies in the force and the towing velocity are obtained: $f = 1.85v$, $1.0v$ and $0.75v$. The linear relation between the frequency and the velocity suggests an analysis based on Strouhal numbers $St$ (see equation (6)). In order to calculate $St$ for all three relations, a characteristic length scale is necessary. However it is hard to define one definite characteristic length scale, since it is not sure where these frequencies exactly originate from. Vortex shedding from a sphere or a cylinder is characterized by a constant Strouhal number, $St = 0.2$ (section 2.1.3). Since this value of $St$ is observed for multiple different bodies, it is plausible to assume that $St$ corresponding with vortex shedding on a human body is also around 0.2. In that case, the corresponding length scales are 0.1, 0.2 and 0.27 m. Plausible length scales are the width of a hand or the width of the head. These body parts are also likely to contribute to vortex shedding. So vortex shedding by the hands and head of a swimmer is hypothesized to contribute to fluctuations in passive drag measurements.

Testing of different bathing suits on the influence of drag is an important application of the towing system. Therefore the magnitude of the force fluctuations is
3. Towing Experiments

3.2 Results towing system

Figure 13: Overview of the frequencies most present in towing experiments at different towing velocities. The black lines in (a), represent linear fits, while red lines in (b) are constant frequency bands, present during all experiments.

of importance. For that purpose, the standard deviation $\sigma$ of the force (calculated once the force is approximately constant) is examined. In figure 14, the relative fluctuation ($\chi$) of $F$ is presented as a function of the towing velocity $v$, with $\chi$
defined as the ratio between the standard deviation of $F$ and the average of $F$:

$$\chi = \frac{\sigma(F)}{<F>} \quad (40)$$

The relative fluctuation is observed to decrease for increasing values of $v$. So for testing different bathing suits, it is desirable to perform experiments at relative high towing velocities. Although $\chi$ decreases for increasing values of $v$, the relative fluctuation is still approximately 5% for the largest velocity measured. While comparing drag forces on different bathing suits, differences in drag forces will probably be small and the fluctuations can still be rather large.

Figure 14: The relative fluctuation of $F$, defined as the ratio between the standard deviation $\sigma(F)$ and the average value of the drag force $< F >$, presented as a function of the towing velocity $v$. Clearly, the relative error becomes smaller for larger values of $v$.

3.3 Setup water channel

A simple representation of a swimmer is a sphere. In order to mimic towing experiments on human swimmers, tethered sphere experiments are performed. Instead of towing a body through a fluid at a constant velocity, a body is kept in a uniform flow.
Experiments have been performed in a water channel, with a cross-sectional area of 30 cm by 35 cm and a maximum flow velocity of 25 cm/s. A hollow sphere is made neutrally buoyant by filling it with water. The sphere has a diameter of 3.7 cm. By placing the origin of the sphere in the middle of the cross-section of the channel, the influence of the water channel solid side walls and bottom is minimized. The origin of the sphere is placed at half the water depth, thereby the possible creation of waves at the free surface is also minimized. The tether which is used has a length of \( L = 87 \) cm and a negligible diameter compared to the sphere diameter. The tethered sphere is attached to a piece of metal, which is used as a spring balance. The deflection of the spring is measured in order to calculate the force on the sphere parallel to the flow. A mirror is attached to the metal and illuminated by a laser beam, which reflects the beam onto a CCD sensor. While a force is applied on the sphere, the metal bar will bend, which is observed as a translation of the laser beam on the sensor. During all experiments, \( x \) is the direction parallel with the flow, \( y \) is the transverse horizontal direction, while \( z \) is the upwards direction; a schematic representation of the water channel and the coordinate system is shown in figure 15. Simultaneously with the force measurements, the position of the sphere is monitored by a camera, capturing the position in the \( yz \)-plane. Both the force measurements and the camera recordings are triggered by a pulse generator in order to synchronize the two measurements.

![Schematic representation of the tethered sphere experiment in the water channel.](image)

A more detailed view of the spring system configuration is presented in figure 16, showing the metal mass-spring system, an aluminium rod with a diameter of 3 mm, bent into a v-shape and an inclination system. The aluminium rod is assumed to be stiff, so it does not contribute to the deflection of metal bar. Because of pressure differences above and underneath the water surface it is useful to keep...
the dynamical part of the setup (the metal bar), above the water surface during all experiments. Therefore only the aluminium rod will be underneath the water surface. Three different values of the spring constant $k$ of the metal bar are investigated during the experiments. For all values of $k$, measurements at different flow velocities were performed. By inclining the bar at different heights, the value $x$ (see figure 16) is varied, hence $k$ is changed. Spring-mass systems with a small, medium and large spring constant $k$ are obtained.

$Re$-values reached during the experiments are in the range of 2500 to 8000. An overview of the experiments performed in the water channel is provided in table 2. In order to test reproducibility, all experiments in table 2 have been performed twice (noted by (i) and (ii)). During experiments I, II and III (i) the force measurements were performed while simultaneously the position of the sphere was monitored. During the other experiments (ii) only force measurements were performed. By increasing the spring constant, while keeping all other length scales (distance from mirror to sensor etc.) constant, the range of $Re$-values was increased. However, this could result in less accurate results, since the deflections become smaller as well. An additional photographic recording of the setup is provided in figure 17.

![Figure 16: Detail view of the spring system used in the tethered sphere experiments.](image)

By using a spring in the experimental setup, a damped mass-spring system is introduced. This system will give a response on the measurements. The equation of motion of a damped mass-spring system, with mass $m$, damping coefficient $\lambda$ and spring constant $k$ is given by:

$$m\ddot{x}(t) + \lambda\dot{x}(t) + kx(t) = F(t)$$

(41)

By using this system, the measured signal is based on two effects: the force on
3. Towing Experiments

3.3 Setup water channel

Figure 17: Picture of the setup for the water channel experiments.

Table 2: Overview of tethered sphere experiments in the water channel. Each experiment has been performed twice.

<table>
<thead>
<tr>
<th>Experiment</th>
<th>x (cm)</th>
<th>k (N/V)</th>
<th>v (cm/s)</th>
<th>Re</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>9.4</td>
<td>0.0022±0.0001</td>
<td>11 - 19</td>
<td>4100 - 6900</td>
</tr>
<tr>
<td>II</td>
<td>7.4</td>
<td>0.0031±0.0001</td>
<td>10 - 22</td>
<td>3800 - 8200</td>
</tr>
<tr>
<td>III</td>
<td>5.4</td>
<td>0.0054±0.0001</td>
<td>6 - 25</td>
<td>2350 - 9400</td>
</tr>
</tbody>
</table>

the sphere and the response of the mass-spring system. The resulting signal $S$ in frequency space is defined as:

$$S(\omega) = H(\omega)F(\omega)$$  \hspace{1cm} (42)

The response of the mass-spring system is given by its transform function $H$, while the force on the ball is given by $F$. The transform function $H(\omega)$ is obtained by Fourier analysis:

$$H(\omega) = \frac{m(\omega^2_0 - \omega^2) - i\lambda \omega}{m^2(\omega^2_0 - \omega^2)^2 + \lambda^2 \omega^2}$$  \hspace{1cm} (43)
in which $i$ is the imaginary number equal to $\sqrt{-1}$. The force on the sphere in frequency space is therefore given by the ratio of the measured signal and the transform function of the mass-spring system:

$$F(\omega) = \frac{S(\omega)}{H(\omega)}$$

A more extensive derivation of the response of the mass-spring system is provided in appendix A.2.

### 3.4 Results water channel

Multiple force fluctuations are observed in the towing experiments on human swimmers, as discussed in section 3.2. It is hypothesized that part of these fluctuations originate from vortex shedding by the hands and head of the swimmer. However, those experiments were performed in a swimming facility and therefore no visual data is present to check if this assumption is valid. Therefore it is desirable to perform experiments in a laboratory facility as well, with better optical access. Furthermore, the complex and instationary (although the swimmer attempts to keep a stationary position during the experiment) geometry of the swimmer gives rise to force fluctuations. Therefore, a more simple but well-defined geometry is used during experiments in the laboratory: force measurements on tethered spheres are performed. In order to mimic a towed swimmer, a tethered sphere is kept in a uniform flow in a water channel as described in section 3.3. The goal of these experiments is to investigate whether or not similar force fluctuations arise as observed for towed swimmers and to examine these fluctuations in more detail.

#### 3.4.1 Force calibration

The calibration of the force measurements in the water channel experiments was performed with the setup presented in figure 18.

The calibration setup is obtained by minor adjustments of the experimental setup used for the force measurements on a sphere, as presented in figure 15. The spring system, which deflection is to be calibrated, is connected to a scale by a similar wire as during the experiments on a sphere. The wire is led from the spring system to the scale via a stainless steel cylinder of length 10 cm and diameter 1 cm. Thereby the horizontal force on the bar is transferred into a vertical force, which can be read out on the scale. The friction between the cylinder and the wire is minimized by attaching a teflon pulley to the cylinder, see figure 19. The wire is now lead to the water surface via the pulley. A typical calibration is given in figure 20.
3. Towing Experiments

3.4 Results water channel

Figure 18: Setup for the calibration of the force measurements in the water channel.

Figure 19: Teflon pulley being used in order to remove the observed hysteresis.

3.4.2 Force velocity diagram

Numerous tethered sphere experiments have been performed, in which the stream velocity $v$ was varied, while the spring constant of the mass-spring system was kept constant on one of the three values (see table 2). A typical force signal, observed during experiment II(i) for a stream velocity of 17 cm/s, is presented in figure 21.

One observes a periodic force signal. The average force obtained from these force signals will be used (see section 3.4.3) in order to calculate the drag coefficient of the sphere. Clearly, the signal contains many force fluctuations as well, which will be further examined in section 3.4.4. An interesting part of the signal is observed around $t = 55s$. The fluctuations of the force are significantly smaller.
3.4 Results water channel

Figure 20: Typical calibration of the force measurements by using a teflon pulley.

Figure 21: Typical overall force signal, observed during experiment II(i) with \( v = 17 \) cm/s.
than observed during other time periods of this experiment. This phenomenon will be examined by investigating the spheres motion simultaneous to these force measurements (section 3.4.5).

The force signal presented in figure 21 represents the raw data of the force on the system. It contains not only the force on the sphere, but also the force exerted on the spring system itself. Therefore measurements of the force on the spring system only are performed for every experiment as well. The force on the metal bar corresponding with the same experiment as in figure 21 is presented in figure 22.

![Image](image.png)

*Figure 22: Force on the spring system, observed during experiment II with v=17 cm/s.*

The average drag force on the sphere is now determined by extracting the average force on the spring system from the average force of the overall force signal. For this particular experiment the average overall force is 0.023 N. While the average force on the spring system during this experiment is 0.012 N. Combination of these two values leads to the average force on the sphere: 0.011 N. This calculation was performed for all experiments. In figures 23, 24 and 25 the average force on the sphere for all experiments are presented as a function of the squared stream velocity.

Linear relations between the force on the tethered sphere and the squared velocity are obtained. This is observed for all experiments performed during this study.
3.4 Results water channel

**Figure 23:** Average force on the sphere as a function of the squared stream velocity, observed during experiment I(i).

**Figure 24:** Average force on the sphere as a function of the squared stream velocity, observed during experiment II(i).
3. Towing Experiments

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Figure 25: Average force on the sphere as a function of the squared stream velocity, observed during experiment III(i).

3.4.3 Drag coefficient

The quadratic relation between the force and the stream velocity, indicates a constant drag coefficient, see equation (2). The drag coefficient of the sphere will be determined for all experiments. An overview of the calculated drag coefficients and their corresponding experiments, is given in table 3. As also noted in section 3.3, every experiment is performed twice, hence also two drag coefficients are calculated for each type of experiment.

Table 3: Results of experiments in water channel: drag coefficient.

<table>
<thead>
<tr>
<th>Experiment</th>
<th>k (N/V)</th>
<th>$C_D$ (i)</th>
<th>$C_D$ (ii)</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>0.0022±0.0001</td>
<td>0.87</td>
<td>0.63</td>
</tr>
<tr>
<td>II</td>
<td>0.0031±0.0001</td>
<td>0.67</td>
<td>0.74</td>
</tr>
<tr>
<td>III</td>
<td>0.0054±0.0001</td>
<td>0.65</td>
<td>0.63</td>
</tr>
</tbody>
</table>

Drag coefficients in the range of 0.63 to 0.87 were measured, which are of the same order as values measured in previous studies [Williamson and Govardhan, 1997]. The two drag coefficients observed during the separate experiments with
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$k = 0.0022 \text{ N/V}$ are rather different from each other. By increasing the spring constant, the two values of the drag coefficient become more similar to one another. Combining all the data points from the six experiments results in one average drag coefficient, as shown in figure 26. The approximately linear relationship observed in figure 26, leads to the overall drag coefficient: $C_{D_{overall}} = 0.69$.

![Figure 26: All data points obtained during the six tethered sphere experiments: Average force on the sphere as a function of the squared stream velocity.](image)

Williamson and Goverdhan (1997) observed an increasing behaviour of the drag coefficient on tethered spheres with increasing values of the Reynolds number, as presented in figure 1(b). This behaviour is only observed during one experiment of type I. In figure 27 the drag coefficient $C_D$ is presented as a function of the Reynolds number $Re$, great similarity with the result of Williamson and Goverdhan (1997) is present (figure 1(b)). The drag coefficient increases until a critical value of the Reynolds number is reached, after which the drag coefficient increases less with increasing $Re$. This critical Reynolds number is observed near $Re = 5000$ in both this study (figure 27) as in previous research [Williamson and Govardhan, 1997]. However, this phenomena is only observed during one out of six experiments.

Once all measurements are combined, a rather different behaviour is observed. In figure 28, again values of the drag coefficient as a function of the Reynolds number are presented, but in this case for all experiments. The red line indicates the overall drag coefficient $C_{D_{overall}}$. In the lower Reynolds range, now actually
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Figure 27: The drag coefficient $C_D$ as a function of the Reynolds number $Re$, obtained during experiment I.

larger values of the drag coefficient are observed. Figure 26 already implied a larger error in the drag force for small stream velocities (and hence small Reynolds numbers) compared to large stream velocities, so these inconsistencies for small Reynolds numbers could be experimental errors as well.

3.4.4 Force fluctuations

Before investigating frequency spectra of the force signals, firstly the influence of the mass-spring system has to be examined. Equation (43) outlines the transform function of a damped mass-spring system. In order to investigate its influence, two experiments at similar stream velocities are performed for different spring constants. By investigating the Fourier spectra, peaks were determined at similar frequencies. Also after performing the transformation summarized in equation (44), peaks were observed at similar frequencies, in appendix A.3 some results of this analysis are displayed. Therefore the influence of the spring constant is assumed to be negligible while investigating frequencies of the force on the sphere. Results obtained during the rest of the study confirm this assumption to be correct, since no significant differences caused by a different spring constant will be observed in frequency spectra.

As observed in figure 21, the typical force signal contains multiple fluctuations.
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Figure 28: The drag coefficient $C_D$ as a function of the Reynolds number $Re$, gathered from all experiments.

These fluctuations seem very periodic. Therefore, the frequency of these fluctuations is determined by Fourier analysis. These frequencies in the force fluctuations are determined for all measurements and highly dependent of the stream velocity. For experiment III(i), the frequency of the force fluctuations is presented as a function of the stream velocity in figure 29. Similar results were obtained in all other experiments.

The linearity between the force fluctuation frequency and the stream velocity, leads to a Strouhal number (see equation (6)). By using the sphere diameter as characteristic length scale, in this example $St = 0.27$. The $St$-values calculated for all six experiments are given in table 4.

Table 4: Results of experiments in water channel: Strouhal number analysis.

<table>
<thead>
<tr>
<th>Experiment</th>
<th>$k$ (N/V)</th>
<th>$St$ (i)</th>
<th>$St$ (ii)</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>0.0022±0.0001</td>
<td>0.25</td>
<td>0.30</td>
</tr>
<tr>
<td>II</td>
<td>0.0031±0.0001</td>
<td>0.25</td>
<td>0.27</td>
</tr>
<tr>
<td>III</td>
<td>0.0054±0.0001</td>
<td>0.27</td>
<td>0.26</td>
</tr>
</tbody>
</table>
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Figure 29: Fluctuation frequency of the force signals obtained during experiment III(i) as a function of stream velocity. Clearly a linear relation is obtained.

In all six experiments a Strouhal number between 0.25 and 0.30 is observed. In order to obtain an overall Strouhal number, force fluctuation frequencies of all experiments are summarized as a function of the corresponding stream velocities, see figure 30. Again a linear relation is obtained, corresponding with the overall Strouhal number: $St_{\text{overall}} = 0.25$.

No previous studies on the frequency of the force fluctuations during tethered sphere experiments seem to have been reported in the literature. However in section 2.1.3 the Strouhal numbers of a tethered sphere based on the sphere’s position are discussed. The streamwise position of the sphere oscillates with a characteristic Strouhal number of 0.28 [Williamson and Govardhan, 1997], which is close to the Strouhal number corresponding to the force fluctuations obtained in the present study. We can therefore conclude that the force on the sphere is closely related to the movement of the sphere in the streamwise direction. In section 3.4.5, further examination of Strouhal numbers corresponding to the sphere position will be displayed.

3.4.5 Sphere motion analysis

In order to further investigate the fluctuations in the force signal, it is of interest to examine the position of the sphere simultaneous with the force on the sphere.
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Figure 30: Fluctuation frequency of the force signals obtained during all experiments as a function of stream velocity. Clearly a linear relation is obtained.

Therefore the position of the sphere is monitored by making video recordings of the $yz$-plane. In this video footage, the sphere was found by correlating a simple disk-like kernel with each frame, and finding the maximum intensity in the correlated images. The size of the disk was made equal to the size of the sphere. It was ensured that the sphere was illuminated evenly, in which case the center of the sphere image could be located to sub-pixel accuracy. Thereby the $y$- and $z$-coordinate are well determined. By using the $y$- and $z$-coordinate in combination with the fixed tether length, the $x$-coordinate is a known property as well.

Firstly, the trajectory of the sphere in the $yz$-plane will be investigated and compared with observations reported in the literature (figure 4 and table 1). Afterwards, the time dependency of the different coordinates will be investigated by creating frequency spectra. Eventually, the correlation between the force on the sphere and the position of this sphere is examined.

Trajectories in the $yz$-plane
In order to verify the different regimes based on the tethered spheres trajectory in the $yz$-plane [Lee et al., 2013], the behaviour of the sphere recorded during experiments with different Reynolds numbers is investigated. In line with the work of Lee et al. (2013), presented in figure 4, all position coordinates will be normalized
with the sphere diameter $D$ and all positions are relative to the mean position of
the sphere, in other words: the origin of the coordinate system is placed at the
initial position of the sphere, $(x_0, y_0, z_0) = (0, 0, 0)$.

In figures 31(a) and 31(b), trajectories of the sphere in the $yz$-plane are pre-
sented by increasing values of $Re$ for two different experiments.

According to Lee et al. (2013), the trajectories in the $yz$-plane at $Re < 3000$
are irregular while above the critical Reynolds number of 3000, the trajectories
are quasi-circular. Since no circular movement is distinguished in figure 31, both
trajectories seem to belong in the irregular regime. For experiment III(i) (figure
31(a)) the Reynolds number is very near the critical value, so irregularities can
be observed. However, figure 31(b) represents an experiment for $Re = 7115$
and should therefore definitely be quasi-circular, according to table 1. The trajectory
of experiment II(i) will be investigated more thoroughly by examining the trajectory
at certain time intervals and investigating the corresponding force signal during
this particular experiment.

The force on the sphere measured during experiment II(i) is presented in figure
32. Clearly, around $t = 40s$ an interesting phenomenon occurs: the force fluctua-
tions become significantly smaller compared to other time intervals during the
experiment. This sudden decrease of the force is a result of the trajectory of the
sphere during this time interval, as displayed in figure 33(b).

During this short time period, the trajectory of the sphere is quasi-circular as
observed in figure 4 as well. After this period of time, the trajectory becomes
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Figure 32: Force $F(t)$ on the sphere during experiment II(i).

Figure 33: Trajectory in $yz$-plane from experiment II(i). (a) again represents the trajectory during the entire experiment, while (b) only considers the time interval $t = 39$ s to $t = 45$ s. Here $(0, 0) = (y_0, z_0)$.

irregular again. During this quasi-circular movement, the force on the sphere is significantly smaller than during the irregular trajectory. While irregular vortex shedding patterns were observed in the irregular regime, Lee et al. (2013) observed helix-shaped vortex formation without shedding in the quasi-circular regime [Lee et al., 2013]. So, while following the quasi-circular path, less vortices are shed which can imply a force on the sphere. Hence, the force fluctuations on the sphere
A new question arises: why does the trajectory become irregular again, while the quasi-circular state seems energetically more favourable? The answer lies probably in the limitations of the current experimental setup. The hollow sphere being used is filled with water in order to become approximately neutrally buoyant; however, the sphere will not be exactly neutrally buoyant. Furthermore, the sphere is attached to the wire with a tiny screw. These two effects create a density gradient in the sphere. Hence, a small disruption, for example a small amount of vorticity being shed, can make the trajectory of the sphere unstable. Therefore the trajectory will become irregular again. A recommendation for future studies on this subject is to reduce the density gradient in the sphere as much as possible.

Position fluctuations
As mentioned, the force on the sphere is a rather periodic and regular signal, while the trajectories in the $yz$-plane are mostly irregular. There must be a connection between the regular force signal and the irregular trajectory. Therefore the three position coordinates are investigated independently. The trajectory in $yz$-plane of experiment III(i) at a stream velocity of 24 cm/s is presented in figure 34, while the individual position coordinates and the force on the sphere are shown in figure 35. Also the radial distance $r = \sqrt{(x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2}$ is presented.

![Figure 34: Trajectory in the $yz$-plane of experiment III(i) at a stream velocity of 24 cm/s.](image)

Clearly there is some order in the irregular looking trajectory of figure 34. Both the radial distance from the origin and the individual coordinates seem to be more organized, as would be assumed from the $yz$-trajectory. As also observed in the $yz$-trajectory, the amplitude of both the $y$- and $z$-coordinate is about half the sphere
diameter. In this example, the $y$-coordinate is more periodic and smooth than the $z$-coordinate. However, during other experiments the opposite was observed. Often one of these coordinates is periodic and smooth, while the other during that time period is more irregular. For example in figure 36 the $z$-coordinate is periodic, while simultaneously the $y$-coordinate behaves more irregularly. This behaviour can be observed as a periodic movement in $y$- or $z$-direction, while small
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Figure 35: Experiment III(i) at $v=24$ cm/s: (a) the force on the sphere, (b) the radial distance $r$ as a function of time and the individual $x$, $y$ and $z$ coordinates (c), (d) and (e). All position coordinates are normalized by the diameter $D$ of the sphere.

Perturbations are present in the other direction. In order to create a quasi-circular trajectory in the $yz$-plane, both have to be periodic and smooth. The fluctuation in the $x$-position is very small, since this position is set by the balance between the drag force on the sphere and the force in the wire.

Figure 36: Transverse coordinates observed during experiment III(i) at $v = 15$ cm/s. In this example the $y$-coordinate (a) seems more irregular than the $z$-coordinate (b).
The time dependency of the position coordinates shows some interesting features. By performing Fourier analysis on all signals of figure 35, the frequency of the fluctuations can be deducted. These Fourier spectra are presented in appendix A.4. The two most common frequencies of the fluctuations in the force on the sphere \( F \), the positions \( x, y, \) and \( z \) and the radial distance \( r \) are presented in table 5.

Table 5: Results of tethered sphere experiments in the water channel: Fluctuation frequencies obtained during Experiment III(i) with \( v=24 \) cm/s.

<table>
<thead>
<tr>
<th>Variable</th>
<th>( f_1 ) (Hz)</th>
<th>( f_2 ) (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( F )</td>
<td>1.51</td>
<td>-</td>
</tr>
<tr>
<td>( r )</td>
<td>1.51</td>
<td>0.72</td>
</tr>
<tr>
<td>( x )</td>
<td>1.51</td>
<td>0.72</td>
</tr>
<tr>
<td>( y )</td>
<td>0.76</td>
<td>-</td>
</tr>
<tr>
<td>( z )</td>
<td>0.75</td>
<td>-</td>
</tr>
</tbody>
</table>

By examining the fluctuation frequency of the different spectra, most fluctuations in both the \( x \)-coordinate as the radial distance \( r \) are observed to have the same frequency as the fluctuations of the force on the sphere. While the fluctuation frequency in the \( y \)- and \( z \)-coordinate are half this frequency. Furthermore it is noted that the fluctuation frequency of the \( y \)- and \( z \)-coordinate is also observed in the signals of \( x \) and \( r \), although less often. Gathering all these fluctuation frequencies of the streamwise \((x)\) position and the transverse \((y)\) position and combining them with the corresponding stream velocity, provides figure 37.

This result shows resemblance with previous work by Williamson and Govardhan (1997), who also found Strouhal numbers of \( St = 0.28 \) and \( St = 0.14 \) for the streamwise and transverse position of a tethered sphere, as displayed in figure 3(b). The fluctuation frequency of the streamwise coordinate \( x \) and radial distance \( r \) is twice the fluctuation frequency of the transverse coordinates as a trivial result of the geometry of the coordinate system. The variables \( x \) and \( r \) are defined by the squared values of the primary variables \( y \) and \( z \). For example the coordinate \( x \) is defined as:

\[
x = (L^2 - y^2 - z^2)^{1/2}
\]

in which \( L \) is the tether length. Since \( L >> y, z \), (45) can be simplified by performing a Taylor expansion:

\[
x \cong L(1 - \frac{1}{2L^2}y^2 - \frac{1}{2L^2}z^2 + \mathcal{O}(y^4, z^4))
\]
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Figure 37: Fluctuation frequencies for the streamwise ($x$) and transverse ($y$) position coordinate plotted as a function of the stream velocity. Black lines represent linear fits corresponding with $St = 0.28$ and $St = 0.14$.}

When $y$ and $z$ are assumed to be oscillatory with a frequency $\omega$: $y, z \sim e^{i\omega t}$, one obtains: $x \sim e^{2i\omega t}$. Hence, the fluctuation frequency of $x$ is twice the fluctuation frequency of $y$ and $z$. For the radial distance $r$ similar steps lead to the exact same result.

The fluctuation frequency of the streamwise position $x$ (black line in figure 37) and the fluctuation frequency of the force on the sphere, presented in figure 30 are alike. For the force on the sphere an overall Strouhal number of $St = 0.25$ is obtained while for the streamwise position $St = 0.28$. This is an expected result, since the force on the sphere is determined via the streamwise movement of the sphere. More remarkable is the correspondence between the fluctuation frequency of the radial distance $r$ and the force on the sphere $F$.

The Strouhal number corresponding with the fluctuations of the radial distance $r$ is, again, similar to the fluctuations observed in the force signal. Apparently, the force on the sphere is not only strongly related to the streamwise position of the sphere, but also to the radial distance from the initial position. The relation between the force on the sphere and its position is further diagnosed by making phase diagrams in the following section.
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Phase diagrams
In order to show the general trend of phase diagrams a selection is displayed in figure 38. In this figure, the force signal on the sphere is outlined as the blue line, while the red line represents one of the other coordinates. All signals in figure 38 are normalized in the vertical axis for displaying purposes.
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Figure 38: Phase diagrams: all blue lines represent the force on the sphere, while the red line represents (a) the radial distance $r$, (b) the $x$-coordinate, (c) the $y$-coordinate and (d) the $z$-coordinate. All signals are scaled in the vertical direction, for displaying purposes, thereby non arbitrary units are presented on the vertical axis. Figure (a), (b) and (c) are obtained from experiment III(i) at $v=24$ cm/s while figure (d) is observed during experiment III(i) at $v=15$ cm/s.

First of all, figure 38 provides a confirmation of the different Strouhal numbers observed during this study. The fluctuation frequency of the force on the sphere, the streamwise position coordinate $x$ and the radial distance $r$ are observed to be similar, while the fluctuation frequency of the transverse position coordinates $y$ and $z$ is half this frequency. Some interesting information is observed in the phases of the signals as well. The $x$-coordinate is in phase with the force on the sphere, which is a consequence of the coordinate system, as it is chosen to be positive in streamwise direction. The radial distance $r$ is 180 degrees out of phase with the force on the sphere. Since the fluctuation frequency of the $y$- and $z$-coordinate is half the fluctuation frequency of the force, the two signals are alternating in and 180 degrees out of phase with each other. These trends do not seem to be influenced by the Reynolds number of the flow, since they are observed for all experiments. Again the streamwise position component $x$ and the radial distance $r$ show the most resemblance with the force on the sphere.

Lee et al. (2013) characterized different flow regimes of a tethered sphere, based on their trajectories in the $yz$-plane, see table 1. Perhaps a more quantitative way would be to base different regimes on the phase differences of the individual coordinates with the force. Further analysis of the present data of all experiments is needed in order to make such distinction in regimes. Regimes based on vortex configurations shed by moving cylinders [Morse and Williamson, 2010], and oscillating airfoils [Schnipper et al., 2009] have been charted. However, in both studies
the regimes are based on observations of the wake flow. In present study, the wake flow is not examined. Therefore, it is again recommended to investigate the wake flow of the tethered sphere in more detail in future research.

By combination of the force signal on the sphere and the position coordinates of the sphere, some remarkable phenomena are observed. For high Reynolds number flows, quasi-circular $yz$-trajectories are expected, in present study they are only obtained during short periods of time. Therefore it is recommended to investigate the influence of the (small) density gradient of the sphere, experiments at even larger Reynolds numbers could be useful as well.

During this quasi-circular movement of the sphere in the $yz$-plane, a huge reduction in the magnitude of the force fluctuations is observed. The impact of this result for swim sciences is not directly clear, since the geometry of a swimmer is asymmetric and complex. However, the quasi-circular movement (and therefore the huge reduction in the magnitude of the force fluctuations) occurs only for high Reynolds number flows, hence it is recommended to perform towing experiments on swimmers at large towing velocities. Section 3.2.4 showed that indeed smaller relative fluctuations were present during towing experiments at large towing velocities.

The force on a tethered sphere is concluded to be mostly defined by the radial distance $r$ of the sphere. However the exact nature of this relation is still unclear. Further research on the wake flow of a tethered sphere is therefore recommended. PIV measurements could provide some useful intelligence on the wake characteristics of such flows. However, some new notions on the origin of the force fluctuations of tethered spheres are gained. Nevertheless, more research will be necessary to completely understand the exact nature of these fluctuations. For swim research purposes it is useful to understand the origin of the force fluctuations in order to decrease them. Since the force fluctuations relate greatly with the radial distance $r$ of the body, the importance of a stable body position during towing experiments has yet again become clear.
4 Bubble Experiments

Experiments with the Bubble System have been performed and analyzed. Both the setup and the results of these experiments are described in this section. The best results were obtained after analyzing the frames with PIV analysis instead of Optical Flow measurements. Therefore, only the results of the PIV routine are shown.

4.1 Setup bubble system

The Bubble System consists of one long tube and about 20 shorter tubes, perpendicularly attached on the core tube, placed at the bottom of the swimming pool. The entire system is about 8 m long and has a width of 1 m. The long tube is attached to a compressor which pumps air into the tubes, via small holes in all the short tubes, air bubbles will enter the swimming pool from the bottom. Swimmers will change the movement of these rising bubbles while they swim over the tube system. Underwater cameras are placed to capture the disturbed movement of the rising bubbles and thereby study the flow field around swimmers. Cameras are placed at the side of the swimmer, so a swimmer will translate through the field of view horizontally. Images with a resolution of 1920x1080 pixels are captured at a frame rate of 50 Hz. The recorded images are stored on a computer memory, after which they will be processed further. A photographic representation of the Bubble System is given in figure 39(a), while a typical frame of a swimmer going through the curtain of rising bubbles is shown in 39(b).

4.2 Pre-experimental problems

In laboratory studies, all important parameters are controlled during experiments. Thereby experimental conditions are optimized. However, in this study experiments are performed in a swimming facility and the experimental conditions are far from perfect.

The setting of the experiment is important in order to create footage, that can be used for data analysis such as PIV or Optical Flow. For example lighting, light scattering and the background of the experimental setup are important experimental conditions. In a laboratory these conditions can be adjusted, however, in the swimming facility this is harder to accomplish.

Since the camera is mounted underwater, the background of the pictures consists of the tiles and joints of the sides of the swimming pool. This non-uniform background will create disturbances in the velocity field during data analysis. This problem is solved by averaging the intensity of multiple frames. Since the bubbles move upwards, the average intensity of a lot of frames will mostly contain the
Figure 39: A photographic representation of the Bubble System available at the Innosportlab Eindhoven is shown in (a). The tube system is placed on the bottom of the swimming pool. A snapshot of a swimmer moving through the ‘curtain’ of rising bubbles is given in (b).
background intensity. The average intensity is subtracted from each frame. In this manner, the background is filtered out of each frame of the footage.

The second problem is lighting. In order to vary the contrast of the experimental setup, it can be convenient to adjust the amount of light. In the swimming facility this is not a possibility. Since the camera is situated underwater, part of the image consists of the top surface of the water, which is filled with scattered light. By aiming the camera and focusing on the swimmer and the rising bubbles, the part of the top surface in the footage is minimized. The remaining part of the top surface in the frames is masked before further analysis.

Next to these imperfections of the surroundings of the experimental setup, also the bubble system itself creates some problems. First of all, the amount of bubbles is not adjustable. In the current setup, the bubble system has a width of about 1 m and a length of 8 m, hence a rather large area will be filled with rising bubbles. The majority of the bubbles will be rising, while a minority of bubbles will contain information about the flow structures created by the swimmer. Often rising bubbles are observed in front of, or behind the swimmer and the bubbles interacting with the swimmer. Therefore the interesting parts of the flow field are less visible. Frames will be analyzed in two dimensions (since only one camera is used), while the input of the footage is three dimensional. In laboratory studies fluid tracers are illuminated by a laser sheet, in order to obtain a two-dimensional image of the fluid flow. For safety reasons this is not suitable in a swimming facility. Hence there will be depth in the images obtained during the experiments, which will be regarded as noise during image processing. For future research an adjustable bubble system is recommended, in which bubbles only rise underneath the swimmer, therefore only (quasi) two-dimensional information is present. In theory, by adding a second camera underneath or in front of the swimmer, 3D PIV is a possibility as well.

As often stated before, air bubbles will rise in water because of the difference in density. This leads to another problem. In typical fluid flow experiments, tracers are always very small and passively following the flow of the fluid. In this case, the bubbles will always rise and therefore they do not a priori follow the flow field of the water.

Finally, the object in these experiments is a human being, which gives rise to experimental errors. It is desirable to make constant and similar movements throughout different experiments, however the swimmer will not always succeed in this task.

It is possible to optimally solve the problems described above, by performing these experiments in a laboratory facility [Hochstein et al., 2012]. Hochstein et al. (2012), performed PIV experiments on human swimmers by filling a swimming pool with small tracers and illuminate them with a laser sheet to obtain a two
4.3 Results bubble experiments

In order to gain insight in the flow field of the water a multitude of footage of swimmers passing over the bubble system is analyzed. Firstly the global features of the velocity field are investigated and the software is tested. Subsequently, the footage is analyzed in more detail and the force on the swimmer is determined. All footage will be pre-processed as explained in section 4.2. The multipass method (section 2.2.1) is used during the PIV analysis: in the first step, the rising movement of the bubbles will be traced, while following passes will track the small deviations from this general movement. Generally, 3 or 4 passes are sufficient to optimally observe all important aspects of the flow induced during the experiments. Firstly, an image without swimmer is investigated (figure 40(a)). Once analyzed with the PIV routine, clearly only rising bubbles are observed, as displayed in figure 40(b).

The velocity obtained by the PIV analysis is disturbed by the bubbles breaking at the top surface and scattering from light on the top surface. Hence, no useful results can be found in the top water surface. Therefore the top surface is masked during the analysis, which is visible in figure 40(b) as the top blue surface. An average vertical velocity of 0.21 m/s is obtained. Bubbles observed during the experiments had radii of about 0.5 cm. An extensive study on the rise velocity of air bubbles in a fluid is performed by Rodrigue (2004). For bubbles with a similar radius, a rise velocity of 0.22 m/s is obtained in previous study [Rodrigue, 2004]. In appendix A.5 an overview of Rodrigue’s (2004) work is given.

4.3.1 Observed flow fields

Some features of the velocity field created during butterfly swimming are observed: after the downkick of the legs, some large-scale vortices are created in the wake of the swimmer. In figure 41 the flow field of that instant time is presented: figure 41(a) shows the displacement field of the bubbles, while in figure 41(b) the average displacement (rising bubbles) is subtracted from the flow field. The foot of the swimmer is visible as the dark blue area and it is masked during the PIV analysis.

After the downkick during butterfly swimming, two opposite signed vortices arise as observed in figure 41(b). These vortices will translate through the fluid in
Figure 40: An example of a frame without a swimmer (a) and the resulting velocity field in an initial cross-sectional plane (b). Obviously, only rising bubbles are tracked. NB. the scale of figures (a) and (b) is not similar for displaying purposes.
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Figure 41: Flow field created just after the downkick during butterfly swimming. Plane (a) presents the actual displacement field, obviously rising bubbles are observed. Two opposite-signed vortices become visible when the average displacement is removed, see (b).

time, because of their induced velocity on one another. They will move towards the bottom left corner, parallel to the direction of the leg after the downkick. As seen in figure 41(b), the observed vortices are neither perfectly smooth or symmetric. These imperfections are caused by a combination of factors: partly because of the turbulent creation of the vortices just after a powerful leg kick and partly because of
4. Bubble Experiments 4.3 Results bubble experiments

the imperfections of the experimental setup (as presented in section 4.2). Despite of the imperfect results, the main aspects of the vortices are clearly visible. Similar vortex patterns were observed after the downkick during the laboratory study of Hochstein et al. (2012).

Just before the downkick during butterfly swimming, a swimmer will lift his thighs towards its torso, while bending his knees as well (making a scissor like movement with his legs). During this movement a large anticlockwise vortex is created underneath the body. In figure 42, this vortex is visible, again the displacement field of the bubbles is presented in figure 42(a), while the mean displacement is subtracted from the flow field in figure 42(b). In the latter case, the vortex is clearly visible. The blue area represents the swimmer’s body, in this case his thighs and stomach, and will be masked during the PIV analysis.

Both phenomena (figures 41(b) and 42(b)) were observed during the analysis of multiple experiments.

4.3.2 Force calculations

Two methods for calculating forces in a flow were drafted in section 2.2.3. Firstly the method summarized by Graziani & Bassanini (2002). This method provides an exact algorithm for the resulting force of the fluid on a moving body, by only using the velocity and vorticity field far away from this body. However, it is a rather complicated algorithm. Secondly the method described by Drucker & Lauder (1999), originally used in experimental biology, which calculates the momentum generated when vortex rings are created in a flow.

The available information in the frames is gathered as two-dimensional velocity and vorticity fields at certain instants of time. 3D PIV measurements and a high frame rate will be necessary to calculate the force by using the full three-dimensional and time-dependent 'Graziani algorithm'. In present study, the 'Graziani method' will only be considered in the two-dimensional stationary case, for which the algorithm (34) is greatly simplified:

\[
\mathbf{F} = \int_{S(t)} \mathbf{n} \cdot \mathbf{G} dS
\]

where \( \mathbf{G} \) is the tensor defined in (35). The tensor \( \mathbf{G} \) contains multiple vectors, which are only regarded in the two-dimensional case. During their analysis, Graziani & Bassanini (2002) generally used large contours, distantly surrounding the object on which the force is calculated. Since the frames have a limited size, this is not possible in our present study. At a certain time instant a vortex is created. At this time instant the swimmer is not necessarily entirely in the frame, and even if he is, the edges of the contour cannot be very distant from the swimmer, since the frame size is limited. The contour cannot cut through the swimmer,
4.3 Results bubble experiments

Figure 42: Flow field created just before the downkick during butterfly swimming. Plane (a) presents the actual displacement field, obviously rising bubbles are observed. One anticlockwise vortex is visible when the average displacement is removed, see (b).

it either has to surround the swimmer, or not surround it at all. Hence, relatively small contours have to be used. The influence of the size and position of the contour is not investigated during this study, however this is highly recommended for future work.
In figures 41(b) and 42(b) vortices are observed in two-dimensional frames. These vortices are assumed to be part of (imperfect) three-dimensional vortex rings. By using the ‘Drucker method’, the momentum of such a vortex ring will be determined. Hence the force on the swimmer is determined over a certain time interval. For convenience we restate equations (36) and (37) here:

\[
M = \rho \Gamma A \\
F = \frac{dM}{dt}
\]  \hspace{1cm} (48)

in which \( \rho \) is the fluid density, \( \Gamma \) the circulation of the vortex ring, \( A \) the cross-sectional area of the vortex ring and \( F \) the force defined by the time derivative of the momentum \( M \). The circulation \( \Gamma \) is calculated by a surface integral over the vortex ring cross-sectional area: \( \Gamma = \int \omega dA \). The cross-sectional area \( A \) and hence \( \Gamma \) are relatively easily obtained for perfect vortex rings. In case of human swimming, imperfect vortices arise and determining \( A \) is therefore rather subjective. Hence multiple outcomes of \( M \) are observed. The time period \( dt \) in which the impulse of the swimmer is transferred to the water is subjective as well. In line with Drucker & Lauder (1999), no vortex is assumed to be present before a certain movement and after the full cycle of that movement, the vortex ring is present (e.g., during butterfly swimming: before the downkick the legs are completely upwards, while after the downkick the legs are completely stretched). The force \( F \) necessary to create the vortex ring during this movement, will therefore be an averaged force during this period of time. Because of these practical problems a range of possible forces will be calculated with this method and these will only be used as an estimate of the force necessary to create the vortex ring. By using Newton’s third law, it is also an estimate of the force exerted on the swimmer.

The flow structures observed in section 4.3.1 exert a force on the swimmer, which will be calculated by both methods.

*After downkick*

From the velocity field displayed in figure 41(b), the vorticity field as well as the enstrophy can be constructed. In figure 43, both the enstrophy and the vorticity field created just after the downkick of the butterfly stage are presented.

Accumulation of vorticity is observed near the water surface. A possible explanation for the formed vorticity is vortex shedding from the head or shoulders of the swimmer. However, PIV analysis near the top surface is difficult, since all bubbles break at the water surface. The region near the top surface will therefore not be taken into account.

The entire flow domain is filled with small patches of vorticity caused by the overall motion of the swimmer. Nevertheless, most of the vorticity is generated near the feet of the swimmer, which corresponds to the area in which the vortices
4.3 Results bubble experiments

Figure 43: Results obtained from the 2D velocity field observed just after the down-kick during butterfly swimming: (a) vorticity field and (b) enstrophy field.

are observed (figure 41(b)). This area will be investigated further, and the force caused by these vortices will be calculated.

By using the contour specified in figure 44 the force on the swimmer is calculated by using equation (47). The calculated force in the contour is:

\[
F[N] = F_x \hat{e}_x + F_y \hat{e}_y = -57\hat{e}_x - 24\hat{e}_y
\]

\[
F_t[N] = |F| = 62N
\]

This force is exerted by the swimmer on the surrounding water. By using Newton’s third law, the reaction force of the water on the swimmer will be similar but opposite signed, so in upper right direction. The direction of the force calculated
in equation (49) is similar to the direction of movement of the dipolar structure (as outlined in section 4.3.1).

By using the method of Drucker & Lauder (1999), a range of possible values for the force on the swimmer is calculated by considering both the smallest and the largest possible solution. The smallest possible value of \( F \) is calculated, by assuming the cross-sectional area to be as small as possible and the time step as large as possible. Exactly the opposite is assumed for \( A \) and \( \Delta t \) in the case of the largest possible \( F \). In this way \( F \) is determined to be in the range of: \( F = 30 - 70 \) N. This is a rather coarse estimation. Nevertheless the value of \( F \) obtained by the method Graziani, lies in this range of possible values for the force on the swimmer.

**Before downkick**

As seen in section 4.3.1, a remarkable vortex arises just before the downkick of the legs as well. The calculated vorticity and enstrophy field obtained at an instant time before the downkick are displayed in figure 45.

Again some accumulation of vorticity is observed near the top surface of the water. For similar reasons as before, no further investigation of these vortices will be performed. In between the thighs and the calf a significant amount of vorticity is created as well. Since the movement of the calf is toward the thighs, this vorticity will be destroyed a few moments after this time instant. Therefore the only area of interest is beneath the stomach of the swimmer. In this area a large patch of positive signed vorticity is created. For this area the force exerted on the swimmer will be calculated.
4.3 Results bubble experiments

Figure 45: Results obtained from 2D velocity field observed just before the downkick during butterfly swimming: (a) vorticity field and (b) enstrophy field.

The contour used for this calculation is displayed in figure 46. The force calculated by applying equation (47) on the specified contour is:

\[
F[N] = F_x \hat{e}_x + F_y \hat{e}_y = -6 \hat{e}_x - 63 \hat{e}_y
\]
\[
F_t[N] = |F| = 63 \text{N}
\]

(50)

This force is almost purely in the downwards direction, created by the swimmer and exerted on the surrounding water. The corresponding force from the water on the swimmer will therefore be mostly in upward direction, creating lift on the swimmer. The direction of this force is in agreement with our expectations based on the Kutta-Joukowski theorem [Kundu et al., 2012].
4. Bubble Experiments

4.3 Results bubble experiments

Figure 46: Contour being used in order to calculate the force exerted on the swimmer with the algorithm formulated in equation (47) [Graziani and Bassanini, 2002].

For this situation, the force is calculated according to equation (48) as well. Again, a largest and a smallest possible value for $F$ is determined by following similar steps. The range of $F$-values found for this situation is: $F = 15 - 70$ N. Again a large range of possible values for the force on the swimmer is obtained, but the value of $F$ calculated with the method Graziani is again consistent with this range.

In both situations the Drucker method and the Graziani method provide in a force which is consistent with one another. However both methods have large drawbacks.

The method described by Drucker & Lauder (1999) is based on the formation of symmetric and smooth vortex rings. No such three dimensional rings are created during active swimming. Therefore the cross-sectional area of the imperfect vortex ring has to be estimated, which leads to a large range of possible outcomes for the force on a swimmer. However, the method can be useful as an estimation of the force on a swimmer, corresponding to a very specific aspect of the induced flow. In future work it is desired to obtain footage with less noise in order to acquire a smoother vorticity field so that the cross-sectional area is better determined. For that purpose, both the bubble system itself and the data processing methods have to be improved.

Graziani & Bassanini (2002) have derived an algorithm for the calculation of the force on a moving object, by only using the distant velocity and vorticity field in three dimensions. However, in the present study only stationary two-dimensional information is available. Therefore either the method by Graziani & Bassanini (2002) has to be tested and verified in a well known stationary two-dimensional
case or three-dimensional time dependent information has to be gathered.

Very little is known about the forces acting on a human body during swimming, therefore it is hard to compare the values obtained during the present study. For butterfly swimming, no studies on the forces on swimmers are present. However, forces on the legs during a human dolphin kick are measured in a numerical study [von Loebbecke et al., 2009]: the total work performed during a submerged dolphin kick is 199 J. By using the values of the work performed by the legs and the distance over which the legs traveled, an estimation of the force on the legs during a dolphin kick is made. On average a force of $F \approx 80$ N is obtained. This value is comparable to the force on the legs during the downkick in butterfly swimming, obtained in present study.

Furthermore, the direction of the forces obtained in present study by the method of Graziani & Bassanini (2002) are as expected. Also the magnitude of these forces coincide with the values obtained by the method Drucker. These are all positive signs. Nevertheless, it is strongly recommended to verify the method described by Graziani & Bassanini (2002) in a well known two-dimensional situation in future research, in order to provide exact forces on swimmers during active swimming.
5 Conclusion

This study focusses on the physical aspects of human swimming. For that aim the drag force of swimmers was examined and visualizations of the flow around swimmers were created and interpreted. Two experiments were performed in a swimming facility, in cooperation with the Innosportlab.

The first experiment concerns the drag force on humans: swimmers were towed through the water in order to measure the passive drag working on their bodies, influences of different bathing suits are tested by this routine.

Secondly, experiments with the Bubble System are performed in the swimming facility. Small air bubbles are created at the bottom of the swimming pool. By swimming over this Bubble System and through the curtains of bubbles, the rising bubbles will change their paths. This disturbed motion of the bubbles represents the water flow and is captured with underwater cameras. Interesting features of the flow field are observed by performing PIV analysis.

Thirdly, experiments in the fluid dynamics laboratory were performed to further investigate fluctuations in the drag force on towed objects. A tethered sphere was kept in a uniform flow, while the position of the sphere and the force on the sphere were monitored simultaneously.

Swimmers were towed through the swimming pool at varying velocities, during which the swimmer intends to keep a streamlined and constant position. Towing velocities in a range of 1 m/s to 2.8 m/s were examined. Forces obtained in these towing experiments fluctuate heavily. By averaging in time the mean drag force on the swimmer is calculated.

The average drag force on human swimmers is quadratic with the towing velocity, as is a generally accepted relation for a multitude of different objects. The passive drag coefficient \( K \) is calculated for this particular swimmer: \( K = 29.3 \, \text{[kg/m]} \). This result is in agreement with previous research on swimming. In order to calculate the drag coefficient of a human being, the frontal area of this person has to be known (projected area of the swimmer in the plane perpendicular to the stream direction). On average the position of the swimmer will be parallel to the stream direction, the frontal area over all experiments is therefore averaged to be 0.08 m\(^2\), correspondingly a drag coefficient of \( C_D = 0.73 \) is calculated. This is again in agreement with previous research. In future research it is recommended to make photographic or video recordings of each experiment in order to calculate the frontal area and hence \( C_D \) precisely for all individual experiments.

The fluctuations of the drag force are investigated as well, therefore frequency spectra of all experiments were measured and analyzed. Fluctuations at some (small) frequencies appeared in the force signal during experiments performed at all towing velocities. These frequencies are probably the result of the used setup.
5. Conclusion (e.g. eigenfrequency of the rope system). However, some frequencies seemed to be linearly dependent on the towing velocity. Based on analysis of frequency spectra, vortex shedding by the frontal area (probably the hand or head) of a swimmer is hypothesized to contribute to these fluctuations in the passive drag measurements. Furthermore, it is recommended to perform towing experiments at large towing velocities, since the relative fluctuation (ratio between standard deviation and the average value) of the drag force is the smallest in that case. Nevertheless, the minimum relative error is still about 5%. While measuring the drag force on various bathing suits with the towing system, differences in drag will probably be very small, hence a relative error of 5% can still be rather large.

Fluctuations of the drag force were examined more thoroughly during tethered sphere experiments in the water channel. The drag coefficient ($C_D = 0.69$) for the tethered sphere was observed to be in comparison with previous research. The fluctuations of the sphere’s position in the streamwise and transverse direction are consistent with previous research as well. Strouhal numbers corresponding with these position are $St = 0.14$ for the transverse and $St = 0.28$ for the streamwise position coordinate. While the Strouhal number corresponding with the force fluctuations is $St = 0.25$.

The $yz$-trajectory of the sphere (in the plane perpendicular to the stream direction) seems very irregular, while simultaneously the force on the sphere looks periodic and regular. By decomposing the trajectory into the individual position coordinates ($x, y, z$) and calculating the radial distance $r$ from the initial position to the actual position of the sphere, one observes more regular looking patterns of the sphere’s position. Fluctuation frequencies in the streamwise coordinate and the radial distance are similar to the ones observed in the force on the sphere. Phase diagrams confirm the correlation between the streamwise coordinate and the radial distance with the force on the sphere as well.

Therefore the fluctuating drag force on the sphere is concluded to be determined mostly by the radial distance $r$ and the streamwise coordinate $x$. Further investigation on the wake flow of tethered spheres could provide more insight in this behaviour. For example, PIV measurements on the wake flow of a tethered sphere would be very useful.

Some interesting flow features arise in the PIV analysis of footage obtained during the experiments on the Bubble System. During this study, only butterfly swimming is investigated. Two large opposite-signed vortices are created by the forceful downkick of the legs. These patches translate away from the swimmer in a direction parallel with the line of the legs. These vortices are created by the swimmers legs, Newton’s third law therefore predicts a reaction force of the
water on the swimmer. This (resultant) force is calculated to be 62 N in the combined up- and forward direction, for this particular swimmer. Just before the downkick, while the swimmer pulls his calves towards its thighs and torso, a large anti clockwise vortex is created underneath his stomach. This vortex provides in a lift force of 63 N on this swimmer.

The force on the swimmer is calculated via two methods. The forces calculated by both methods are consistent with one another and the direction of the forces is as expected. However, both methods have drawbacks. The ‘method Drucker’ is an estimation of the force needed to create vortex rings. However, no perfect vortex rings arise during active swimming. An exact algorithm of the force on a moving object by using only velocity and vorticity data is provided by Graziani & Bassanini (2002). However, in present study only two-dimensional stationary data is available, while the ‘Graziani method’ is a three-dimensional time-dependent algorithm. Therefore verification of the Graziani method in a well-known two-dimensional case is highly recommended.

Although further research is necessary in order to provide in exact forces on swimmers, new insights of the interpretation of the flow field during active swimming are gained. In two different situations, vortices were created in the wake flow of a swimmer during butterfly swimming. These flow structures exert a force on the swimmer, which enhances the propulsion mechanism of the swimmer.
A Appendix

A.1 Euler-Lagrange

In section 2.2.2 a Lagrangian is used in order to solve the Euler-Lagrange equation. This Lagrangian consists of regularization term which contains dilation, strain and rotation terms. The Lagrangian is given by:

\[
((f \cdot \nabla)g + \frac{\partial g}{\partial t})^2 + |\nabla f_x|^2 + |\nabla f_y|^2 = \\
((f \cdot \nabla)g + \frac{\partial g}{\partial t})^2 + \frac{1}{2} \left\{ \alpha \left( \frac{\partial f_x}{\partial x} + \frac{\partial f_y}{\partial y} \right)^2 \\
\beta \left[ \left( \frac{\partial f_x}{\partial x} - \frac{\partial f_y}{\partial y} \right)^2 + \left( \frac{\partial f_y}{\partial x} + \frac{\partial f_x}{\partial y} \right)^2 \right] + \gamma \left( \frac{\partial f_x}{\partial y} - \frac{\partial f_y}{\partial x} \right)^2 \right\}
\]

(51)

The Euler-Lagrange equation is defined as:

\[
\frac{\partial L}{\partial f_i} - \frac{d}{dx_j} \frac{\partial L}{\partial f_{ij}} = 0
\]

(52)

Substituting the Lagrangian of (51) into (52) gives the desired optical flow algorithm. Since the regularization term depends only on the gradients of the velocity field, the first term of equation (52) gives the same result as before:

\[
\frac{\partial L}{\partial f_i} = 2((f \cdot \nabla)g + \frac{\partial g}{\partial t}) \frac{\partial g}{\partial x_i}
\]

(53)

However, the second term of (52) becomes:

\[
\frac{d}{dx_j} \frac{\partial L}{\partial f_{ij}} = 2\alpha \left( \frac{d}{dx} (f_{xx} + f_{yy}) + \frac{d}{dy} (f_{xx} + f_{yy}) \right)
\]

\[
+ 2\beta \left( \frac{d}{dx} (f_{xx} - f_{yy} + f_{yx} - f_{xy}) + \frac{d}{dy} (f_{xy} + f_{yx} + f_{yy} - f_{xx}) \right)
\]

\[
+ 2\gamma \left( \frac{d}{dx} (f_{yx} - f_{xy}) + \frac{d}{dy} (f_{xy} - f_{yx}) \right)
\]

\[
= 2\alpha \left( \frac{\partial^2 f_x}{\partial x^2} + \frac{\partial^2 f_y}{\partial y^2} + \frac{\partial^2 f_x}{\partial x \partial y} + \frac{\partial^2 f_y}{\partial x \partial y} \right)
\]

\[
+ 2\beta \left( \frac{\partial^2 f_x}{\partial x^2} + \frac{\partial^2 f_y}{\partial y^2} + \frac{\partial^2 f_x}{\partial y^2} + \frac{\partial^2 f_y}{\partial x^2} \right)
\]

\[
+ 2\gamma \left( \frac{\partial^2 f_y}{\partial x \partial y} - \frac{\partial^2 f_x}{\partial y \partial x} + \frac{\partial^2 f_x}{\partial y^2} - \frac{\partial^2 f_y}{\partial x \partial y} \right)
\]

(54)
Combining these two equations, results in the solution of the Euler-Lagrange equation as presented in section 2.2.2:

\[
\frac{\partial L}{\partial f_i} - \frac{d}{dx_j} \frac{\partial L}{\partial f_{ij}} = ((f \cdot \nabla)g + \frac{\partial g}{\partial t})\frac{\partial g}{\partial x_i} - \left\{ (\alpha + \beta)\left(\frac{\partial^2 f_x}{\partial x^2} + \frac{\partial^2 f_y}{\partial y^2}\right) + \\
(\alpha - \gamma)\left(\frac{\partial^2 f_x}{\partial x\partial y} + \frac{\partial^2 f_y}{\partial x\partial y}\right) + (\beta - \gamma)\left(\frac{\partial^2 f_x}{\partial y^2} + \frac{\partial^2 f_y}{\partial x^2}\right) \right\} = 0
\]

(55)

### A.2 Response mass-spring system

The governing equation of a mass-spring system with damping is given by:

\[
m\ddot{x}(t) + \lambda \dot{x}(t) + kx(t) = F(t)
\]

(56)

in which \(m\) is the mass of the system, \(\lambda\) the damping coefficient, and \(k\) the spring constant. The position of the mass as function of time is given by \(x(t)\), while the force on the system is given by \(F(t)\). This mass-spring system is used to measure the force on a tethered sphere. However, the resulting signal \(s(t)\) that is measured relates to the response function \(h(t)\) of the system and the actual force on the sphere \(f(t)\) as:

\[
s(t) = h(t) \ast f(t)
\]

(57)

in which \(\ast\) indicates a convolution between the two right sided terms. However, in Fourier Space the relation between the measured signal and its components is given by a normal multiplication [Phillips et al., 2003]:

\[
S(\omega) = H(\omega)F(\omega)
\]

(58)

Therefore it is useful to transform equation (56) into the frequency domain, by taking the Fourier Transform at both sides:

\[
-m\omega^2X(\omega) + i\lambda \omega X(\omega) + kX(\omega) = F(\omega)
\]

(59)

The transform function is then easily calculated as the ratio between the input function \(X(\omega)\) and the output function \(F(\omega)\) of the system:

\[
H(\omega) = \frac{X(\omega)}{F(\omega)} = \frac{1}{k - m\omega^2 + i\lambda \omega}
\]

(60)
By using $\omega_0 = \sqrt{\frac{k}{m}}$, equation (60) can be further simplified into the desired transform function $H(\omega)$ as used during this study:

$$
H(\omega) = \frac{1}{m(\omega_0^2 - \omega^2) + i\lambda\omega} = \frac{m(\omega_0^2 - \omega^2) - i\lambda\omega}{(m(\omega_0^2 - \omega^2) + i\lambda\omega)(m(\omega_0^2 - \omega^2) - i\lambda\omega)} = \frac{m(\omega_0^2 - \omega^2) - i\lambda\omega}{(m^2(\omega_0^2 - \omega^2)^2 + \lambda^2\omega^2)}
$$

(61)

A.3 Results transform function

For two experiments with different spring constants and (nearly) the same stream velocity, the Fourier spectra are analyzed. Also the spectra after the transformation function (equation (44)) is investigated. Clearly, the frequency peaks still appear at similar frequencies. The experiments examined in figure 47 are experiment I(ii) at $v = 13$ cm/s and experiment III(i) at $v = 12$ cm/s. Clearly, the spectra of the two different experiments look very similar. Small differences in the primary frequency observed are caused by the small difference in the stream velocity. After performing the transform, similar primary frequencies are observed.
Figure 47: Fourier spectra of the force on the sphere obtained during (a) experiment I(ii) at \( v = 13 \) cm/s and (b) experiment III(i) at \( v = 12 \) cm/s. The spectra corrected with the transform function are presented for (c) experiment I(ii) and (d) experiment III(i).
A.4 Fourier analysis: Fluctuations in position and force

During the analysis of the tethered sphere experiments, multiple frequency spectra were obtained by Fourier analysis. For experiment III(i) with \( v = 24 \) cm/s, all Fourier spectra are represented in this section: firstly the frequency spectra of the force \( F \) on the sphere, and the spectra of the \( x, y \) and \( z \) coordinates. Subsequently we present the Fourier analysis of the radial distance \( r \). The dotted red lines represent the frequency most present in the signal.

(a) 

(b) 

(c)
A.5 Rising bubbles

Rodrique (2004) performed research on the rise velocity of air bubbles in liquids. Thereby he gathered experimental data from multiple studies. Based on this experimental data, Rodrigue (2004) found a relation between the rise velocity of an air bubble and the surface tension $\sigma$ of the bubble, the density $\rho$ of the fluid, the dynamic viscosity $\mu$ of the fluid, the diameter $d$ of the bubble, the temperature $T$ of the water and the gravitational acceleration $g$. The velocity number $V$ is eventually given as a function of the Morton number $Mo$ and the flow number $Fl$:

$$V = \frac{Fl}{12} \left[ \frac{(1 + 1.31 \cdot 10^{-5} Mo^{11/20} Fl^{73/33})^{21/176}}{(1 + 0.020 Fl^{10/11})^{10/11}} \right]$$  \hspace{1cm} (62)$$

in which $Mo = \frac{\mu d^4}{\rho \sigma}$ and $Fl = \frac{g(\rho d^5)}{\sigma \mu^3}^{1/3}$. This algorithm is purely based on the experimental data, figure 49 shows that (62) indeed fits the experimental data for different values of $Mo$. 

Figure 48: Experiment III(i) at $v=24$ cm/s, frequency spectra of: (a) the force on the sphere, (b) the radial distance $r$ and (c), (d) and (e) the individual $x$, $y$ and $z$ coordinates.
For the bubbles created during the experiments, the Reynolds number is of the order $10^2$, and the Morton number is of the order $10^{-11}$. In this regime, Rodrigue (2004) has greatly simplified (62):

$$U = 1.08 Mo^{1/320} \sqrt{gR}$$  \hspace{1cm} (63)

in which $U$ is the rise velocity of the air bubbles, $g$ the gravitational acceleration, $R$ the radius of the bubbles and $Mo$ again the Molton number. In present study bubbles with a radius of about 0.5 cm are created. According to (63) the rise velocity of the air bubbles will be: $U = 0.22 \text{ m/s}$. 

---

Figure 49: The rise velocity of air bubbles as a function of the flow number $Fl$. Both the experimental data points as the numerical fit (62) is presented for different values of the Morton number.
References


