EINDHOVEN UNIVERSITY OF TECHNOLOGY

WARM BENT GLASS

Development of a Flexible Mould

Master of Science Thesis

Arjen Seffinga
Warm Bent Glass: Development of a Flexible Mould

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The undersigned hereby certify that they have read and recommend to the Faculty of Architecture Building and Planning for acceptance of the thesis entitled “Warm Bent Glass Panels: Development of a Flexible Mould” by Arjen Seffinga in fulfilment of the requirements for the degree of Master of Science.

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Preface

This document contains the Master of Science thesis report: Warm Bent Glass: Development of a Flexible Mould. The research is carried out at the Faculty Architecture, Building and Planning of Eindhoven University of Technology.

My first word of thanks goes to Arno Pronk, my supervisor at the University. We often met in Amsterdam instead of at the University, which saved me a lot of travelling time.

Second, I would like to thank the companies Brakel Atmos, VDNDP Bouwingenieurs, SolidRock and Prodim for their financial and material support.

Third, I would like to thank my employer, VDNDP Bouwingenieurs, for giving me the needed freedom and trust while working there part-time.

Fourth, I would like to say a word of thank to my parents for their financial and motivational support.

A final thanks in this preface goes out to my friends and girlfriend (Anneke) who helped me and supported me during the long and intense days working on this thesis. Thank you for your continued support and patience.

Amsterdam, 30 October 2015

Arjen Seffinga
Abstract
The present work provides the necessary tool for the use of the 3DFlexMould and a finite element method (FEM) model to analyse the behaviour of glass while submitted to high temperatures and deformation. The numerical modelling of this kind of problems within the finite-element context encounters three main difficulties: the setup of the FEM, the multiple loads in the simulation and the validation of the outcome.

The 3D Flex Mould is a mould with a flexible top-layer that is deformed by the use of 25 pistons that can move up and down. The flexible top-layer is made of a spring steel mesh, with a plain woven pattern, which is the key feature of the mould. The spring steel can be deformed (in)finite times without losing its original shape.

With the use of Rhinoceros, Grasshopper\(^1\) and Kangaroo\(^2\) the Mould Design Tool (MDT) uses a real time physic engine to analyse the behaviour of mesh. The engine will calculate the optimal settings for the mould to bend the glass panel in the right shape.

Using the 3DFlexMould, the Mould Design Tool, the Finite Element Method, the Radius test and the Repetition test made it possible to build a double-curved-isolated-glass with a high accuracy and fast production time. Combining different double-curved-isolated-glass panels made it possible to create a self-supporting dome.

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\(^1\) Rhinoceros Plug-In
\(^2\) Grasshopper Plug-In
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$C$ specific heat (J·kg$^2$·K$^{-1}$)

$E$ total energy (J)

$F_{gs}$ view factor in the simplified domain

$g$ gravity acceleration (ms$^{-2}$)

$I$ radiation intensity (Wm$^2$)

$I_e$ compatibility index

$L_g$ glass thickness (m)

$k$ thermal conductivity (W·m$^{-1}$·K$^{-1}$)

$p$ pressure (Pa)

$R$ volumetric heat source due to radiation (W·m$^3$)

$T$ temperature (K)

$u(\cdot)$ standard uncertainty

$v$ velocity (ms$^{-1}$)

$\alpha$ absorption coefficient

$\varepsilon$ emissivity

$\theta$ time (s)

$\rho$ density (kg/m$^3$)

$\psi$ fluid stress tensor (Pa)

Subscripts

$\text{eq}$ equivalent temperature

$\text{exp}$ experimental

$g$ glass

$num$ numerical

$\text{sh}$ shields

$\text{st}$ steel
<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Description</th>
</tr>
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<tbody>
<tr>
<td>CAD</td>
<td>Computer Aided Drafting</td>
</tr>
<tr>
<td>CNC</td>
<td>Computer Numerical Control</td>
</tr>
<tr>
<td>SCC</td>
<td>Self Compacting Concrete</td>
</tr>
<tr>
<td>FEM</td>
<td>Finite Element Method</td>
</tr>
<tr>
<td>WLF</td>
<td>William Landal Ferry</td>
</tr>
<tr>
<td>SLSG</td>
<td>Soda Lime Silica Glass</td>
</tr>
<tr>
<td>BSG</td>
<td>Borosilicate Glass</td>
</tr>
<tr>
<td>NURBS</td>
<td>Non-Uniform Rational B-Splines</td>
</tr>
<tr>
<td>PCAP</td>
<td>Principal Components Analysis Regressions Plan</td>
</tr>
<tr>
<td>BREP</td>
<td>Boundary REPresentation</td>
</tr>
<tr>
<td>DOF</td>
<td>Degrees Of Freedom</td>
</tr>
<tr>
<td>FBX</td>
<td>Filmbox</td>
</tr>
</tbody>
</table>
Physical Constants

Newtonian Constant of Gravity \( G \) = \( 6.673 \times 10^{-11} \text{ m}^3\cdot\text{kg}^{-1}\cdot\text{s}^{-2} \) (exact)
Dad, you were like a father to me.

Dedicated to my father
PART I – INTRODUCTION
1. Introduction

With the introduction of Computer Aided Design (CAD) software it is possible to create complex shapes. The designer is no longer limited by its own capabilities. Fluid architecture has no limitation in material usage, however concrete is an often used material since it is cheap and quite easy to use. Glass is another well-known and used material in fluid architecture. These new free-form glass structures give new dimension to fluid architecture.

Double curved glass panels is still a rare seen phenomenon. This is due to the high production cost and labour intensity. The rapid upcoming of the modelling tools have led to an increasing gap between the designing party and the building industry (Vollers, 2002).

The value of a building design is often judged by the appearance of its exterior, the facade. Commonly, a large portion of the facade is occupied by transparent parts made of glass. Considering free form designs often exhibit curved surfaces, the glass parts must somehow be formed to a prescribed shape. An approved method to achieve this is 'hot bending of glass panels' (Pottmann, 2008).

1.1. Background

Modern architecture shapes are complex and are made possible due to the increasing use of advanced 3D Computer Aided Design (CAD) applications. These CAD applications accelerated the development of free form design, where the use of curved surfaces is an important aspect.

This new form of designing increased the gap between the designer and the building industry (Pottmann, 2008). Designs are limited to what the building industry can deliver and made to measure solutions are expensive. The value of a building design is often judged by the appearance of its exterior, the facade. The use of glass in fluid architecture is significant and cannot be ignored.

However, these glass panels are flat and the facade is divided in segments in order to make it look like a fluid surface. It is possible to bent glass slightly without thermal treatment, cold bending of glass panels. This method, has some limitations regarding the amount of stress allowed inside the glass panels. When bent the glass is under permanent stress and needs counterforce to force into the window frame.

1.2. State of the Art in Fluid Architecture

The roots of fluid architecture is Art Nouveau. This architecture movement can be catheterized by decorations based on patterns of elegance. Art Nouveau was a counter movement of Academic Art in the 19th century. The decorations of Art Nouveau found their origin in nature, e.g. flowers and plants. Art Nouveau buildings where build in the period from 1890 till 1910 (Duncan, 1994).
An example of Art Nouveau is the Casa Milà, designed by Antoni Gaudi (see Figure 1.1 a). After this era modern architecture started and fluid architecture started to show up. An example of early fluid architecture is the Philips-pavilion by Le Corbusier (see Figure 1.1 b). Due to the oil crisis in 1973 there was no money for fluid architecture. It disappeared for twenty years before showing up again. New possibilities came with the rise of the computer and gave new life to fluid architecture. Fluid architecture has no limitation in material usage, however concrete is an often used material since it is cheap and quite easy to use.

![Figure 1.1: (a) Casa Milà by Antoni Gaudi, 1910 (b) Philips Pavilion by Le Corbusier, 1958](image)

Nevertheless, glass is an upcoming material in the fluid architecture. These new free-form glass structures give new dimension to fluid architecture. Double curved glass panels is a rare seen phenomenon. An example of the use of double curved glass panels are the train stations in Innsbruck, Austria by Zaha Hadid (see Figure 1.2). Not surprisingly, the design pushed advanced glass technology to its limits. Each glass panel had to be moulded precisely to its final double-curved shape, while softened by heat at the glassworks. A total of 850 glass panels totalling 2,500m² in area were used to cover all four stations, and each panel was unique (Spring, 2007).
Lucio Blandini did research on glass as a structural material and built a glass dome (see Figure 1.3). Blandini created a frameless structural glass shell with a span of 8.5 meters. The dome has been designed and built with the aim of exploiting the structural efficiency as well as the aesthetical quality that can be achieved by combining structural use of glass with adhesives as joining system. According to Blandini, the Glass Dome is the first glass shell in the world which makes no use of structural frame (Blandini, 2008).

1.3. Problem Description

The complexity of building shapes increased by the capabilities of the CAD software and the lack of new innovative productions methods increased the gap between designing and building industry.
Manufacturing double curved glass is not new, think of drinking glasses, optical lenses and car windshields. There is much to be found on these topics, however the manufacturing method of drinking glasses and optical lenses is slightly different from car windshields. Producing car windshields use a static mould and the sagging method (Corre, 2013). However, every type of glass panels requires a different mould.

Due to the increasing demand for fluid surfaces, there is a demand to increase the efficiency of the production process of double curved glass panels. The problem statement which arises is, that there is no feasible flexible mould available for manufacturing double curved glass panels.

1.4. Objective

The main purpose of this study is to explore the possibilities with a flexible top layer mould since there is no flexible production method for creating double curved glass panels. Furthermore a self-supporting glass dome will be constructed as result of the exploration of the flexible mould. More extensive research in this topic will make it able to develop an accurate and a cheaper way to produce double curved glass panels.

There is need for more research of warm bent glass panels regarding the behaviour of glass by sagging with a flexible mould. The automotive industry uses the sagging method to create the complex windshields with static moulds. This research will help to reduce the cost of the manufacturing process of double curved glass panels and stimulate the use of fluid architecture.

1.5. Research Questions and Objectives

In order to manufacture double curved glass panels, the main research question is: Creating a flexible mould and optimal surface prediction. The main research objective is to create a self-supporting Glass Dome.

1.5.1. Partial research questions:
PRQ1: Creating a flexible mould;
PRQ2: Analyse the behaviour of a mesh;
PRQ2: Glass behaviour during thermal loads;
PRQ3: Simulating the glass bending process from 2D to 3D; and
PRQ4: Validating the simulation process with the manufactured glass panel.
1.5.2. Research objectives:

RO1: Create glass panels based on Finite Element Analysis (FEA) studies, which comply with the precision requirements.

RO2: Construct a glass dome with the manufactured glass panes from the own made flexible mould.

1.6. Research Outline

The research is a follow up based on several other studies and runs parallel to another master thesis research by Hicham el Ghazi and Niek Schuijers (El Ghazi & Schuijers, 2015). The communal object is to facilitate fluid architecture. The base is the production of the glass panels needed for the glass dome. In order to create these glass panels with a high accuracy simulation and testing needs to be done.

The research model (see Figure 1.4) is based on the approach of G. Pahl and W. Beitz (Pahl & Beitz, 1984). This consists of several steps to research and to systematically go through these steps. In the Figure the steps are shown on the right hand side of the research model. The problem analysis is the first step of the research which contains the goal and the research questions are being established. In the second step the topics; flexible moulds, bending glass, finite element (computational modelling) and mesh deformation behaviour are being elaborated by the research questions developed by the literature study.

The next step starts the design process and there will be an iterative process placed by the cyclical iterative design process by S. Thomke added. By this the problems can be focused and solved with the experimental trial and error process that consists of a four step cycle: design experiment, produce models, test models and evaluate results. The iterative cycle can be applied to the prototype until this is optimized.

In order to arrive at the proposed objective, some restrictions have to be introduced. In this section the three most prominent restrictions will be dealt with separately. The first restriction concerns the geometry. Examples are a shapes from the Master II project. This restriction brings forth that the geometry results into a glass panel that needs to be made in order to create the glass dome.

The second restriction concerns the material model. Glass exhibits almost perfect elastic behaviour up to a point where it fails abruptly. This is mainly caused by the non-crystalline composition of Si-O bonds and Na-O bonds. Further, the fabrication process by floating molten glass on a tin-bath assures the material to be isotropic. These properties would justify the use of a physically linear relation. Therefore, the type of analysis can be restricted to a pure geometrical non-linear analysis. At this point no assumptions are made with respect to the exact strength and stiffness values. Primarily because different types of glass (e.g. annealed glass, heat strengthened glass, toughened
glass) are available which all have different strength and stiffness properties (Haldimann, 2008). A user of the program should be able to specify the specific material properties.

The third restriction concerns the mould itself. In order to create a glass panel as accurate as possible the mould should be modelled and simulated as well. This is due to the coefficient of expansion.

Figure 1.4: Research model
PART II – LITERATURE
2. Existing Mould Systems

Creating a curved glass panel is done by the use of a mould. There are different mould and moulding techniques available. This chapter addresses the different types of moulds and techniques used to create a curved glass panel. J. Helvoirt divided the mould systems in three categories: Static-, reusable- and flexible moulds (Helvoirt, 2003).

1. The static mould: The most common mould of this kind is the EPS/PS (polystyrene) mould which has been 3D formed using a CNC cutter. This technique has a high accuracy, but produces a lot of waste material;

2. The reusable mould: For example a mould made of clay or sand. There is no waste material and therefore the method is a lot more environmental friendly. It is labour intensive and cannot be used in all circumstances; and

3. The flexible mould. This type of mould can be used for a lot of differently shaped elements, because the mould will form itself to the elements that will be produced. Examples of this kind of mould are the FlexiMould by Boers (Boers, 2006), and the adjustable mould by Rietbergen & Vollers (Rietbergen V. &., 2007). The mould contains a field of height-adjustable pins. Each pin can be set into height individually using a computer-automated machine. The pins are covered with a polymer to smoothen the surface. It is important to avoid that the pins give a local distortion of the surface. The downside of these flexible moulds is the high investment. These expanses can only be gained back when a large amount of elements will be produced using this particular mould.

First the static- and reusable moulds will be discussed. Next will be the flexible moulds. Finally a summary will hold the selection for the most suitable solution for construction a flexible mould.

2.1. Static Mould

2.1.1. CNC Mould

The CNC milled method originates from the by hand crafted moulds from different materials. Milling the mould with the use of CNC technology is less labour intensive than creating it by hand and cheaper. Most moulds made with CNC are made of a foam like material. The advantage of foams in comparison to heavier materials is that they are cheaper compared on volume, they allow fast milling, and they are easy to manually alter and fair after the milling process, that leaves a grooved surface texture. The main strength of the method is that it can be used for very advanced geometry as long as it is possible to de-mould the casted object. Further there is almost no curvature or detailing level limitations besides that of the milling tool. The weakness of this method is waste
material and requires post processing if a perfect smooth surface is required. The mould can only be used for one unique shape and has to be thrown away, which results in more waste (Raun, 2010).

2.1.2. Steel Frame Mould
The steel frame mould involves glass being cut into the ‘unfold’ shape and placed on the steel mould. The glass is heated to 600°C, at which point the glass becomes viscous and can be formed. The glass and mould are placed in the furnace together. The glass sags in the correct shape and is supported by the mould. The drawback of this system is that the steel frame mould can be used for only one particular shape. Double curved surfaces can be achieved with this method, meaning there is no real limitation to the form that can be created with glass panels. The only drawback is, is that if you make something too complex you will have to make a different mould for any unique shape (Spring, 2007) (Technology: Glass-shaping Techniques in Building Construction, n.d.).
2.2. Reusable Moulds

2.2.1. Sand Mould

A sand or clay mould is one of the oldest techniques to create unique shapes. The sand is formed in the desired shape, sometimes with a harder or coating. This technique is still used for producing metal parts, but does not have the right capabilities for bending glass. The sand mould takes a lot of time for creating the desired shape.

Figure 2.3: Sand Hill Pavilion (Pronk, 2007)

2.2.2. Flex Rod Mould

The flex rod mould developed by Vollers and Riebergen consists of four parts: a base, sheet supports for the rods, a lifting system with two main resting bars, and mould surface rods. The surface of the mould is created by a number of stainless steel support plates that are made with CNC cutting and are connected to a frame. The rods and plates are made out of stainless steel, because this material can withstand high temperatures.

The flexibility comes from the rods when they are lowered onto the stainless steel support plates. The Flex Rod Mould was used for bending glass and started in a horizontal position. The glass pane is heated in this horizontal position, which reduces the breaking while heating. When the pane reaches the annealing temperature the rods are lowered onto the supporting plates creating the desired shape. The panel will slump into the correct shape. No cutting to size is necessary anymore after transforming. This implies no risk of breaking the expensive transformed pane, no time lost in cutting to size and no cost for producing a mould on which to cut the pane to size (Rietbergen, 2006).
2.3. Flexible Mould

2.3.1. Single Curve Bending Machine

A semi-flexible mould called the ‘single curve tempered bending’ is able to create cylindrical glass with any given radius. The glass panel moves in the furnace and the machine uses a quick heating furnace, within 90 seconds the glass heats up to 590°C. The glass plate comes out of the oven and moves in the mould. The mould consist out of two main parts, the lower part and the upper part (see Figure 2.5 a and b). The lower part of the mould supports the glass and the upper part pushes the glass in its final shape. The glass is controlled cooled after it is bend in the correct shape to temper the glass. This method is cheap and fast producing, but only produces mono-clastic glass panels.

![Figure 2.4: Flexrod Mould Ran by Software (Vollers, 2004)](image)

![Figure 2.5: (a) Single Curve Bending Machine and (b) Tempering Furnace](image)
2.3.2. Pin-Bed Mould
David Hardt from MIT researched the use of a reconfigurable pin bed mould. The mould consists of 2,688 moveable pins, each pin measuring 30 millimetres in diameter and 500 millimetres long. One motor moves all the pins and a mechanism disconnects the pin when it reaches its designated height. The pins are grouped in clusters of eight and can be replaced easily. A modular design is wise for a tool such as this as a vast number of moving components all need to work in unison for proper function. The eight motor modular arrays, seen in Figure 2.6, allows for quick replacements of damaged sections and limits the down time of the valuable forming tool (Peters, 2011).

![Figure 2.6: Single Curve Bending Machine](image)

Hardt’s pin bed mould uses a hydraulic clamp to fixate the pins to prevent them from slipping under the large forces involved in stretch forming (Munro, 2007).

2.3.3. Flexible Top Layer Mould
Rietbergen and Vollers (2007) developed a prototype of a flexible surface on an adjustable pin-bed (i.e. a grid of actuators). The pin bed is originally designed for producing double curved glass panels. The actuators are simultaneously adjusted in height by a computer (see Figure 2.7).
On top of the actuators a flexible layer was placed which was the formwork for the concrete. In order to assure a constant thickness of an element, concrete was poured with the mould in horizontal position. Advantages of starting with a horizontal mould are that no contra mould is needed, the thickness of the element can be controlled easily and the pouring is relatively easy and quick. Casting concrete in the deformed situation of the formwork increases the range of element thickness with less accurate dimensions as a consequence (Schipper, Grünewald, Eigenraam, Raghunath, & Kok, 2015).

Students of the Technical University of Eindhoven researched the possibilities of a flexible top-layer mould for double curved glass panels. This prototype is made from a steel frame with 25 adjustable sliders on each side of the frame (see Figure 2.8). The 25 points control a steel mesh\(^3\), which deforms to the desired shape. This type of mesh showed great potential for flexibility. However the control points of the mould were insufficient to achieve the desired shape. Furthermore the sagging method showed that the used glass had insufficient mass to achieve the desired shape.

---

\(^3\) WNIO5 1.40 × 0.4 (opening × diameter)
2.4. Summary

The flexible top layer type of mould has the biggest potential for manufacturing a low-cost flexible mould. This type of mould could produce many different shapes without creating a new mould for each unique shape. The design of the new flexible mould will be elaborated by looking at the two flexible mould techniques of Schipper and the M2 project. The mould manufactured by students (see Figure 2.8). The steel mesh shows great potential and combining this with a grid of pistons to control the surface (Schipper, Grünewald, Eigenraam, Raghunath, & Kok, 2015) this could lead to a usable and potential prototype.

However, to improve the accuracy the control of the surface needs to be improved this should result in a smoother and less deviation in the surface. The pistons can be controlled by motors to achieve a higher accuracy and quicker production time, but this leads to higher development costs. This is in equilibrium with the wishes for the development of a flexible mould. Another step is to validate and verify the mould and the glass geometry.

The previous described moulds all have advantages over the others, nonetheless there is some significant difference in control versus price. Figure 2.9 shows the price versus control ratio. The more pistons are applied the more control is gained. There is an optimal ratio between pistons and cost.
Figure 2.9: Amount of Pistons versus the Cost of the Mould
3. Glass Deformation Submitted by Thermal Load

In this chapter the behaviour of glass submitted to a high temperature and a short introduction to Finite Element Analysis (FEA) will be presented. Glass as material will be discussed, second is the bending method by the use of gravity will be discussed, followed by the behaviour of glass submitted to high temperatures. Furthermore the Finite Element Analysis (FEA) for glass will be introduced.

3.1. Glass Properties

It is generally accepted that molten glass is in a viscous state when being heated to high temperatures. This is not completely true, but it is considered sufficiently accurate for practical purposes. Complex issues arise in the relationship between temperature and viscosity, which is the most important physical consideration in modelling glass bending. Firstly, there is the phenomenon of relaxation and equilibrium viscosity, and secondly the viscosity of a molten glass is highly temperature dependent.

3.1.1. Composition and Chemical Properties

Glass is a fusion product, which has been cooled to a rigid condition without crystallization. The most common used type of composition for glass is the soda lime silica glass or SLSG. Another common used type of glass is borosilicate glass, which is used for fire protection and heat resistance. Table 3.1 shows the chemical composition of both of the glass types. The composition of glass does not consist of a geometrically regular network of crystals, but an irregular network of silicon and oxygen atoms with alkaline parts in between (see Figure 3.1).

<table>
<thead>
<tr>
<th></th>
<th>Soda lime silica glass</th>
<th>Borosilicate glass</th>
</tr>
</thead>
<tbody>
<tr>
<td>Silica sand</td>
<td>SiO₂</td>
<td>69 – 74%</td>
</tr>
<tr>
<td>Lime (calcium oxide)</td>
<td>CaO</td>
<td>5 – 14%</td>
</tr>
<tr>
<td>Soda</td>
<td>Na₂O</td>
<td>10 – 16%</td>
</tr>
<tr>
<td>Boron-oxide</td>
<td>B₂O₃</td>
<td>-</td>
</tr>
<tr>
<td>Potassium oxide</td>
<td>K₂O</td>
<td>-</td>
</tr>
<tr>
<td>Magnesia</td>
<td>MgO</td>
<td>0 – 6%</td>
</tr>
<tr>
<td>Alumina</td>
<td>Al₂O₃</td>
<td>0 – 3%</td>
</tr>
<tr>
<td>others</td>
<td></td>
<td>0 – 5%</td>
</tr>
</tbody>
</table>
Figure 3.1: Schematic view of the irregular network of soda lime silica glass.

Figure 3.2: Schematic comparison of the column’s dependence on temperature for a glass and a crystalline material.

Table 3.2: Typical viscosities and corresponding temperatures for soda lime silica glass (SLSG) and borosilicate glass (BSG).

<table>
<thead>
<tr>
<th>Viscosity (Pas)</th>
<th>State</th>
<th>SLSG Temperature (°C)</th>
<th>BSG Temperature (°C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$10^7$</td>
<td>Working Point</td>
<td>1040</td>
<td>1280</td>
</tr>
<tr>
<td>$10^{8.6}$</td>
<td>Softening Point</td>
<td>720</td>
<td>830</td>
</tr>
<tr>
<td>$10^{14}$</td>
<td>Annealing Point</td>
<td>540</td>
<td>570</td>
</tr>
<tr>
<td>$10^{14.3}$</td>
<td>Transformation temperature $T_g$</td>
<td>530</td>
<td>560</td>
</tr>
<tr>
<td>$10^{15.5}$</td>
<td>Strain Point</td>
<td>506</td>
<td>530</td>
</tr>
</tbody>
</table>

During the process the glass does not crystallize but freezes. This phenomenon called the ‘supercooled liquid’ nature of glass means that, unlike most solids, the electrons in glass molecules are strictly confined to particular energy. This means the molecules cannot alternate between different
states of excitement by absorbing radiation in the bandwidths of visible and near infrared, they do not absorb or dissipate those forms of radiant energy.

3.1.2. **Physical Properties**

The most important physical properties of soda lime silica and borosilicate glass are summarized in table 3.3. The optical properties of the glass depend strongly on the thickness of the plate, the chemical composition and the applied coatings. Due to the interaction with O$_2$-ions in the glass, a large percentage of UV radiation is absorbed. The long wave infrared ($\lambda > 5000$nm) is absorbed by the Si-O groups in the composition of the glass. This phenomenon is called the greenhouse effect. The greenhouse effect allows the visual light to pass through the glass and heat up the interior, while long-wave thermal radiation is unable to escape. The refractive index of glass is around 1.5. This means that the reflection value of visual light with common soda lime glass is around 4% per surface, which means 8% of total per glass pane. The transparency is reduced but can be avoided by applying special coatings.

<table>
<thead>
<tr>
<th><strong>Table 3.3: Physical properties of soda lime silica glass (SLSG) and borosilicate glass (BSG).</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>Property</td>
</tr>
<tr>
<td><strong>Density</strong></td>
</tr>
<tr>
<td>Knoop hardness</td>
</tr>
<tr>
<td>Young’s modulus</td>
</tr>
<tr>
<td>Poisson’s ratio</td>
</tr>
<tr>
<td>Coefficient of thermal expansion</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>Specific thermal capacity</td>
</tr>
<tr>
<td>Thermal conductivity</td>
</tr>
<tr>
<td>Average refractive index within the visible spectrum</td>
</tr>
<tr>
<td>Emissivity (corrected)</td>
</tr>
</tbody>
</table>

At room temperature (20°C) the viscosity of glass is about $10^{20}$ Pa s (for comparison, the viscosity of water is $10^{1}$ Pa s and of honey, $10^{5}$ Pa s). The extreme high viscosity at room temperature would take more than earth’s age for a flow effect. This effect can be found in old church glass windows where the glass is thicker at the bottom. This phenomenon can be addressed to a poor production process and surface effects caused by condensed water accumulating at the bottom of glass panels; this leads to an increase of volume.

Glass behaves almost perfectly elastic, isotropic behaviour and exhibits brittle fracture. Local stress concentration occurs because it does not yield plastically. Glass is a strong material, which in some
cases can reach up to 32 GPa. However, the actual tensile strength is much lower. This is due to the brittleness of the material, it strongly depends on the flaws of the surface. These flaws are not always visible for the eye and can be found by advanced technology. The surface of the glass always contains a large number of flaws. Figure 3.3 shows an overview of typical strength values for various flaws depths.

![Tensile Strength Behaviour of Glass](image)

A glass panel fails as the stress intensity due to tensile stress at the tip of one flaw reaches its critical value. Flaws grow with time when the glass panel undergoes an external load. The tensile strength of glass is not a material constant, but depends on many aspects. The condition of the surface is the most important aspect. The higher the load, the longer the load duration and the deeper the initial surface flaw, the lower the effective tensile strength. Tensile stresses develop because of buckling in the case of stability problems and because of the Poisson’s ratio effect at load introduction points. In both cases, an element’s tensile strength is exceeded long before a critical compressive stress is reached.

3.2. Sagging Method

The glass sagging method consists of bending a glass panel by heating it in a furnace up to 600°C. The body temperature of glass rises and reaches a temperature at which the viscosity is low enough to allow glass to sag under its own weight due to gravity.
This method is mostly used in the automotive industry and for some applications in architecture. Obtaining curved glass requires a flat glass panel in order to acquire a complex shape with a good optical surface. Using the sagging method requires a counter-mould on which the glass panel will form. The process ends when the glass panel fits the desired shape of the counter-mould (see Figure 3.4 a and b).

The glass sagging method is a very simple looking operation, but in practice, process parameters need to be assigned and adjusted to get an acceptable outcome. Three of the parameters that affect the conditions are the viscoplastic behaviour of glass (Parsa, Rad, & Shahhosseini, 2005), optical properties after reheating (Wang & Wang, 2014) and mechanical stress (Heynderickx, Potze, & Maten, 1994). These three subjects will be discussed later in this chapter.

![Figure 3.4: (a) Initial Geometry (b) Final Geometry](image)

3.2.1. Slumping
When the viscosity level is higher, the change in temperature takes a finite time to be reflected in the corresponding change in viscosity. This effect is known as relaxation. When the deformation reaches its final shape and thereby its final viscosity level this is called the equilibrium viscosity. Experimental results show that slumping will occur when the viscosity level is L=11 (Schmidt, Scholze, & Tünker, 1986). However very little slumping will occur with this level, when the furnace temperature rises and the viscosity level drops. When the viscosity reaches the level 7 ≤ L ≤ 8 slumping will occur (Stokes, 2000).

The slumping stage during the sagging process occurs when the glass plate is soaking. The viscosity of the glass is low enough to allow the glass to slump under its own weight by gravity. As mentioned before the viscosity depends strongly on the embodied temperature of the glass. One of the common models describing the relation between temperature and viscosity is the Williams-Landal-Ferry or WLF model. This model was presented in 1955 and addressed the slumping phenomenon, it was presented as an empirical formula with a shift function AT, which can be expressed in equation 3.1.

\[
\log \alpha_T = \log \frac{\eta(T_{ref})}{\eta(T)} = -c_1(T - T_{ref}) \quad c_2 + (T - T_{ref})
\]

(3.1)
Where: \( c_1 \) and \( c_2 \) are the constants for selected glass material.

Research shows that the WLF is still accurate, when the viscosity is applied at 490°C, 575°C and 720°C to fit the shift function. Figure 3.5 and shows the data from the equation compared to the WLF fitting curve. Figure 3.6 shows insufficient slumping in a static mould by the sagging method.

![Figure 3.5: WLF curve](image)

![Figure 3.6: Insufficient slumping](image)

3.2.2. Surface Curvature
The criterion of the surface is that the surface of the deformed glass panel meets the specification within some tolerance. For the asymmetrical example under consideration, the height \( z \) of the top glass surface is a function of radius \( r \) only and curvature \( \kappa \) is given by equation (3.2) where primes denote differentiation with respect to \( r \). The coordinates \( r \) and \( z \) are located on top of the glass
surface and may be extracted from the finite element method output, from which first and second derivatives must be determined so as to compute the surface curvature profile.

\[ K = \frac{z''}{(1 + z'^2)^{3/2}} \]  

(3.2)

The difficult task of accurately obtaining the non-exact data from the FEA model is an simplification of the physical world.

3.2.3. Temperatures and Soaking Time

Once the furnace and especially the glass panel reaches the desired temperature and becomes viscous enough to bend under its own weight, the furnace needs to stay at this constant temperature in order to reach the desired curvature. Some studies show that a higher temperature and a longer soaking is required to fully slump thin glass plates. This can be done with vacuum during the bending process, or by increasing the soaking temperature to make sure the glass plate deforms completely. As discussed in paragraph 3.2.1. the viscosity depends strongly on the temperature. By using the WLF equation it is possible to determine the soaking time for bending the glass. See Figure 3.7 for the stoke flow for bending glass. However the true viscosity of glass at the soaking temperature might be higher than the calculated value from the WLF equation.

![Figure 3.7: Firing schedule](image-url)
3.2.4. Optical Properties after Reheating

Soda-lime float glass, the most common used float glass, shows a haze after a reheating process. Studies show the surface changes slightly from compositions. Scanning the glass plate with microscopes show that the haze indices is due to the formation of roughness on the bottom face of the glass, which has been described as ‘wrinkling’. The tin enriched bottom layer, which arises from the float bed during production, migrates to the top layer through the glass itself during the reheating process. Besides the tin contained in the glass this phenomenon happens with iron as well (Hayashi, Akiyama, & Kudo, 2001).

3.3. Heat Transfer Numerical Solution

The heat transferred to the glass from the furnace depends on the time of heating element and surrounding combination of gasses. It is very difficult to measure the temperature inside the glass itself; especially the change of temperature inside the glass. Therefore, modelling of heat transfer phenomenon in glass moulding is very important for working out heat balance. J. Yan et al studied this phenomenon and constructed an equation to address this problem (Yan, Zhou, Masuda, & Kuriyagawa, 2009).

\[
\rho C_p \frac{\partial T}{\partial t} = k \nabla^2 T
\]  

(3.3)

Where:

- \( \rho \) is the density;
- \( C_p \) is the specific heat;
- \( K \) is the thermal conductivity of glass; and
- \( T \) is the heating time.

As glass is a compound material, the specific heat of glass is known to vary with its composition and temperature. An empirical equation, namely, Sharp–Ginther equation (Sharp & Ginther, 1951), has been proposed to express the mean specific heat (\( C_m \)) of glass, as shown in equation (3.4).

\[
C_m = \frac{\alpha_3 T + C_0}{0.00146T + 1} \times 4.186 \times 10^3
\]  

(3.4)

Where:

- \( \alpha_3 \) is a constant of a glass material;
- \( C_0 \) is the true specific heat at 0°C; and
• $T$ is the temperature in °C (Celcius).

The properties of the glass are different with the change of composition on atom level. The table below (table 3.4) shows different types of glass options. This value can be used to calculate the constants $a_3$ and $C_0$ in equation 3.4. According to the method, the calculated values were respectively $a_3 = 0.000408$ J/°C² and $C_0 = 0.144$ J/°C. It was reported that oxides BaO and ZnO influence the thermal properties of glass.

<table>
<thead>
<tr>
<th>Glass Composition</th>
<th>Content (wt%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>SiO₂ (Silicon Dioxide)</td>
<td>40-50 (~45)</td>
</tr>
<tr>
<td>BaO (Barium Oxide)</td>
<td>20-30 (~25)</td>
</tr>
<tr>
<td>B₂O₃ (Boron Trioxide)</td>
<td>2-10 (~6)</td>
</tr>
<tr>
<td>Al₂O₃ (Aluminium Oxide)</td>
<td>2-10 (~6)</td>
</tr>
<tr>
<td>ZnO (Zinc Oxide)</td>
<td>2-10 (~6)</td>
</tr>
<tr>
<td>Others</td>
<td>0-2 (~2)</td>
</tr>
</tbody>
</table>

The actual specific heat $C_p$ in equation 3.5 then can be easily calculated from the differential equation as follows:

$$C_p = \frac{d(TC_m)}{dT}$$

(3.5)

For simplicity, many researchers treated the high-temperature irradiation in glass as an equivalent thermal conductivity problem. A temperature-dependent thermal conductivity ($k(T)$) was proposed for glass materials by (Mann, Field, & Viskanta, 1992).

$$-k \frac{dT}{dn} = h_M(T - T_M)$$

(3.6)

$$-k \frac{dT}{dn} = h_N(T - T_N)$$

(3.7)

Where:

• $h_M$ is the interface heat transfer coefficient between the mould and glass;

• $h_N$ is the heat transfer coefficient between the surrounding atmosphere and glass; and

• $T_M$ and $T_N$ are the temperatures of the mould and the surrounding atmosphere;
3.3.1. Simulation of Creep

When the glass plate slumps, the thickness of the glass plate changes as well. M.H. Parsa et al has studied this phenomenon in the automobile industry. Studies show that the accuracy versus prediction was as small as $10^{-3}$ millimetre. The simulation results also predicted that the maximum thinning of the glass plate is concentrated at the centre of surface (see Figure 3.8 and 3.9) (Parsa, Rad, & Shahhosseini, 2005).

![Figure 3.8: Insufficient Slumping](image)
![Figure 3.9: Distance Creep from the Edge](image)
3.4. Glass Strain

Flat soda-lime glass is manufactured using a molten tin bed. The molten glass is poured onto the tin surface, the glass cools down and solidifies. This type of production process can create a typical plate thickness in the range of 2 to 10 millimetres. The surface has a tin side and an air side. After the glass plate is solidified it undergoes a controlled cooling process to eliminate any residual stress before being post processed. The air side of the glass plate does not come in contact with any other material during the process. The tin side however has been in contact with the tin the whole process. This creates a small tin layer on the tin side of the glass plate. The tin side is also in contact with the roller during the cooling down process. The two sides of the glass panel undergo different histories during the process and both have different mechanical behaviour (Parsa, Rad, & Shahhosseini, 2005).

It is a known fact that there is some incorporation of tin in the glass network structure. The highest concentrations of Sn occur in the top 100 – 200 nanometre of the tin side. The Sn can penetrate up to 40 nanometre into the surface of float glass during processing where it exists in both stannous (Sn2+) and stannic (Sn4+) forms, with Sn2+ predominating near the surface and Sn4+ predominating in the subsurface region.

The composition of glass is a frozen disordered state of local lattice strains. Glass has some unique features, including frequency dispersion in its dynamic mechanical anomalies (Sarkar, Ren, & Otsuka, 2005) and breaking down of ergodicity (Wang, Bo, & Ben, 2007).

The thermal history of the glass has two sides (pun intended). The air side has only been in contact with air and the tin side has come in contact with the tin and rollers (ceramic). This has influence on the mechanical behaviour of the glass. The fictive temperature of the glass is defined as the effective glass transition temperature of a glass for its particular thermal history. Glass with the exact identical composition can have a significantly different physical properties. Vacher et al studied the properties of crown glass as a function using Brillouin scattering and found a longitudinal sound velocity $c_l = 5400 \text{ ms}^{-1}$ at $T_f = 740^\circ \text{ C}$ and $c_l = 5600 \text{ ms}^{-1}$ at $T_f = 590^\circ \text{ C}$ (Vacher, et al., 1974). Another study, the influence of fictive temperature on specimen density was measured and the elastic modulus computed at two fictive temperatures for silica glass, with $c_l = 5960 \text{ ms}^{-1}$ and $E = 78 \text{ GPa}$ at $T_f = 1200^\circ \text{ C}$ and, $c_l = 5980 \text{ ms}^{-1}$ and $E = 79 \text{ GPa}$ at $T_f = 1500^\circ \text{ C}$. Thus if the air and tin sides of the glass have different values of $T_f$, they could have a measurable difference in their mechanical properties (Tang, et al., 2015).

3.4.1. Mechanical Behaviour Laws

The continuity equation gives the displacement vector $u$ as a function of the stresses tensor $\sigma$. 


\[ \rho \dot{u} = \nabla \sigma + f \]  

(3.8)

Where:
- \( \rho \) is the density; and
- \( f \) is the body forces vector.

Then, in case of deformation, displacements are linked with the strain tensor.

\[ \varepsilon = \frac{1}{2} (\nabla(u) = \nabla'(u)) \]  

(3.9)

The assumption is made that the glass behaves like a Maxwell theological material (F. Richter, 2006). This fits in the assumption if the connection in series of a dashpot behave like a Newtonian fluid with a spring-mass connection obeying Hooke’s law. The strain tensor formula (equation 3.10) can be written as the sum of elastic, vicious and thermal strains:

\[ \varepsilon = \varepsilon^e + \varepsilon^v + \varepsilon^f \]  

(3.10)

3.4.2. **Elastic Strain Tensor**

\[ \varepsilon^e = \frac{1}{2G} \sigma + \frac{1}{3} \left( \frac{1}{3B} - \frac{1}{2G} \right) tr(\sigma) I \]  

(3.11)

\[ G = \frac{E}{2(1 + \nu_p)} \]  

(3.12)

\[ B = \frac{E}{3(1 - 2\nu_p)} \]  

(3.13)

Where:
- \( tr \) is the matrix trace;
- \( I \) is the identity matrix;
- \( G \) is the shear modulus;
- \( B \) is the bulk modulus;
- \( E \) is Young’s Modulus; and
- \( \nu_p \) is the Poisson’s ratio.

3.4.3. **Viscous Strain Tensor**

The viscous strain tensor is linked to the stress tensor \( \sigma \) using the relation:

\[ \varepsilon^v = \frac{1}{2\eta} \sigma + \frac{1}{3} \left( \frac{1}{3\eta_b} - \frac{1}{2\eta_s} \right) tr(\sigma) I \]  

(3.14)
\[ \eta_s = \frac{\eta}{2(1 + \nu_n)} \tag{3.15} \]

\[ \eta_b = \frac{\eta}{3(1 + 2\nu_n)} \tag{3.16} \]

Where:
- \( \eta_s \) is the viscous shear modulus;
- \( \eta_b \) is the viscous bulk modulus;
- \( \nu_\eta \) is the viscous Poisson’s ratio (assumed to be equal to the Poisson’s ratio); and
- \( \eta \) is the key parameter during the glass sagging process.

Which results into:

\[ \log \frac{\eta(T)}{\eta T_0} = \frac{-C_1(T - T_0)}{C_2 + (T - T_0)} \tag{3.17} \]

Where:
- \( \eta T_0 \), \( C_1 \) and \( C_2 \) are empirical coefficients.

3.4.4. Thermal Strain Tensor

\[ \varepsilon^t = \frac{\alpha}{3} \Delta T I \tag{3.18} \]

Where:
- \( \alpha \) is the thermal expansion coefficient; and
- \( \Delta T \) is the temperature variation.

The above described equations simulate the deformation of glass while submitted to high temperatures. These equations are implemented in most FEM software packages (Boubaker, Corre, Meshaka, & Jeandel, 2014).
3.5. Summary

Glass behaves as an isotropic material and depends strongly on the composition of the crystals. The temperature of the bending process cannot exceed over ± 640 °C or else the tin side will draw inside the glass panels instead of being an outside layer. Once this tin layer is inside the glass panel the glass panel will show a white dull haze, which lowers the amenity of the glass.

The insufficient slumping could be an issue while producing double curved glass panels with the sagging method, however this is the simplest and easiest way to create the desired shape. Creating a counter mould eliminates the low production of a flexible mould and an oven that could fit over the mould. To produce a flexible mould the sagging method is the most accessible method and requires less investment.

Chapter four (4) will introduce the finite element method used to analyse the behaviour of the glass and how the simulation of the glass will be set up in order to predict the desired shapes. The properties and formulas described in this chapter will be used to conduct a finite element analysis.
4. Finite Element Method

Finite Element Analysis (FEA) is a computational method for solving and making an approximation of real world physical problems. This type of computational method is mainly used for problems for which no exact solution is available. Therefore it is a numerical analysis instead of an analytical method. This type of method can compute more complicated problems that are met within engineering, typical analytical analysis cannot cope with this complicated elements.

For example, engineering strength of materials or the mathematical theory of elasticity can be used to calculate analytically the stresses and strains in a bent beam, but both will not succeed in finding what happens in a part during dynamic conditions.

The origin of FEA applications was to find stresses and strains in engineering components under load. FEA, when applied to any realistic model of an engineering component, requires an enormous amount of computation and the development of the method has depended on the availability of suitable digital computers for it to run on. The method is now applied to problems involving a wide range of phenomena, including vibrations, heat conduction, fluid mechanics and electrostatics, and a wide range of material properties, such as linear-elastic (Hookean) behaviour and behaviour involving deviation from Hooke's law (for example, plasticity or rubber-elasticity).

This chapter will discuss different methods for analysis and boundary conditions that should be met in setting up the simulation to conduct a thoroughly simulation.

4.1. Linear Static Stress Analysis

The linear static analysis is the most basic type of analysis in FEA software. The term ‘linear’ means that the computed response, displacement, stress or strain, is linearly related to the applied force. The term ‘static’ means that the forces do not vary with time, or that the time variation is insignificant and can be therefore ignored.

An example of static forces is a building’s dead load, which is comprised of the building’s weight plus the weight of the furniture. These loads are often defined using a maximum expected load with some factor of safety. Figure 4.1 shows the difference between a linear and non-linear analysis.
Furthermore a time invariant dead load, as described above, another example of static load is an enforced displacement. Figure 4.2 shows a foundation slab of a building with a constant static load of the building.

![Figure 4.2: Static Load of a Load on a Foundation](image)

E.G.: The foundation of a building may settle somewhat, including static loads.

The static analysis equation is:

\[
[K]{u} = {f}
\]  

(4.1)

Where \([K]\) is the system stiffness matrix, \(f\) is the vector of applied forces, and \(u\) is the vector of displacements that.

When the linear static analysis is completed the outcome, displacement, stress and strain, can be used to interpret the behaviour of the engineering part under a static load. (Petyt, 2010).
4.2. Non-Linear Analysis

A non-linear analysis allows for nonlinear stress-strain relationships and material properties that are temperature dependent. In addition, non-linear analysis allows for the accurate modelling of structures that undergo large deformation.

Many common structural features exhibit nonlinear behaviour that is *status-dependent*. For example, a tension-only cable is either slack or taut; a roller support is either in contact or not in contact. Status changes might be directly related to load (as in the case of the cable), or they might be determined by some external cause.

If in a situation contact occurs it is common to different non-linear applications. If contact occurs in the simulation it forms an important subset to the category of changing-status nonlinearities.

Most analysis software use the "Newton-Raphson" approach to solve nonlinear problems. The Newton-Raphson approach subdivides the load into smaller series of loads. The load is then applied over a *x* amount steps related to time. Figure 4.3 shows the Newton-Raphson equilibrium iterations in a non-linear analysis (Anstee, 2006).

![Figure 4.3: Newton-Raphson Approach](image)

The Newton-Raphson method evaluates the out-of-balance load vector during the analysis.

The Newton-Raphson method evaluates on forehand the out-of-balance load vector. It checks the difference between the restoring forces and the applied forces. After checking forces and loads, a linear solution is done to check for convergence. If the criteria are not met the out-of-balance load vector is re-evaluated, the stiffness matrix is updated, and a new solution is obtained. This iterative procedure continues until the problem converges.
The arc-length method causes the Newton-Raphson equilibrium iterations to converge along an arc, this prevents the arc from divergence. This phenomenon even prevents is when the slope of the load versus the deflection curve becomes zero or negative. Figure 4.4 shows the schematically behaviour of the arc-length method with the Newton-Raphson method (Anstee, 2006).

The non-linear analysis is based on three steps:

- The "top" level consists of the load steps that are defined by the user explicitly over a "time" span. Loads are assumed to vary linearly within load steps (for static analyses);
- Within each load step, you can direct the program to perform several solutions (sub steps or time steps) to apply the load gradually;
- At each sub step, the program will perform a number of equilibrium iterations to obtain a converged solution.

Figure 4.5 shows a typical load history for a non-linear analysis. (Katoh & Nomoto)

4.2.1 Creep

Creep calculation depends on the material characteristics, in which the material continues to deform under a constant or variable load. If a displacement is imposed, the reaction force (and stresses) will be smaller over time and eventually disappear. Creep consists out of three stages (see Figure 4.6 a
The first stage, or primary stage, the strain rate is relatively high, but slows as time is increasing. Due to hardening the strain slows over time.

In the second stage the strain eventually reaches a minimum value and becomes near constant. This is due to the balance between work hardening and annealing, also known as thermal softening.

The third and last stage the strain rate exponentially increases with stress because of the necking phenomena. Necking is a mode of tensile deformation where relatively large amounts of strain localize disproportionately in a small region of the material (Bridgman, 1964).

Creep is important in high temperature stress analyses.

E.G. A preload is applied to some part in a nuclear reactor to keep adjacent parts from moving. Over a period of time at high temperature, the preload would decrease (stress relaxation) and potentially let the adjacent parts move.

Creep can also be significant for some materials such as pre-stressed concrete. Typically, the creep deformation is permanent.

4.2.2. Viscoplasticity

Viscoplasticity is a time-dependent plasticity phenomenon, where the development of the plastic strains are dependent on the rate of load. The primary application are high-temperature glass forming process.

An example is forming glass with high temperature by rolling it. This involves large plastic strains and displacement with small elastic strains. Figure 4.7 shows the behaviour of glass during this process.
Viscoplasticity is defined by unifying plasticity and creep via a set of flow and evolutionary equations. A constraint equation is used to preserve volume in the plastic region (Chaboche, 2008).

4.2.3. Contact

The contact between elements or surfaces consist of two parts. The first part is the part that moves or initiates the movement and the second part is the element or surface where it will make contact with. When a contact pair is created the real number of constants should be applied both to the target and the contact element.

The most common contact of collision type is surface-to-surface. The surface-to-surface have several advantages over the node-to-surface contact set. This is due to surfaces colliding with a surfaces and not a node colliding with a surface. This means that the node behaves different from the surface contact (ANSYS Structural Analysis Guide, 2009).

4.3. Mesh Definition

The mesh density in a finite element model is a key component, because of its relationship to accuracy and cost. The finer the mesh is defined the better and more accurate the result will be. However, this means that the model needs more time to calculate the model. Figure 4.8 shows two different mesh densities.
The minimum number of elements is set by topological considerations. An element per member in a space frame or one element per panel in stiffened shell structure. With computers becoming faster and cheaper, the current trend is to represent all major components individually in the finite element model.

If the minimum topological requirements are easily satisfied, the question remains as to how fine to subdivide the major components. The question is particularly relevant for elastic continua, such as slabs and unreinforced shells. In general, as the mesh density increases, you can expect the results to become more accurate. The mesh density required can be a function of many factors. Among them are the stress gradients, the type of loadings, the boundary conditions, the element types used, the element shapes, and the degree of accuracy desired. The grid point spacing should typically be the smallest in regions where stress gradients are expected to be the highest.

In a study of mesh densities, elements and output options, three different mesh densities were used in the example as shown in Figure 4.9. The first one is a coarse mesh model with the elements evenly distributed. The second model consists of double the amount of elements. The third model
consists of an ever denser mesh with the elements evenly distributed. The theoretical circumferential stress $\sigma_\theta$ is given by the following equation:

$$\sigma_\theta = \frac{p_i a^2 (1 + \frac{b^2}{r^2})}{(b^2 - a^2)}$$ (4.2)

Where:
- $a =$ inner radius;
- $b =$ outer radius
- $r =$ radial distance as measured from the centre of the disk
- $p_i =$ pressure applied at the inner radius
- The stresses are then plotted as a function of the radius in a non-dimensional fashion: stress $p_i$ versus $r/a$. The results are summarized in table 4.1.

![Figure 4.9: Square with Different Meshes Density](image)

**Table 4.1**: Stresses close to $r=a$ for circular disk

<table>
<thead>
<tr>
<th>Case No.</th>
<th>Element type</th>
<th>DOF (L-Set)</th>
<th>Description</th>
<th>Stress theoretical</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>CQUAD4</td>
<td>194</td>
<td>Coarse, even mesh</td>
<td>0.594</td>
</tr>
<tr>
<td>2</td>
<td>CQUAD4</td>
<td>194</td>
<td>Coarse, Biased mesh</td>
<td>0.881</td>
</tr>
<tr>
<td>3</td>
<td>CQUAD4</td>
<td>868</td>
<td>Fine, Even mesh</td>
<td>0.801</td>
</tr>
<tr>
<td>4</td>
<td>CQUAD8</td>
<td>550</td>
<td>Coarse, Even mesh</td>
<td>0.929</td>
</tr>
<tr>
<td>5</td>
<td>CQUAD8</td>
<td>550</td>
<td>Coarse, Biased mesh</td>
<td>1.041</td>
</tr>
<tr>
<td>6</td>
<td>CQUAD8</td>
<td>2538</td>
<td>Fine, Even mesh</td>
<td>1.015</td>
</tr>
<tr>
<td>7</td>
<td>CQUAD4 with Corner Option</td>
<td>194</td>
<td>Coarse, Even mesh</td>
<td>1.142</td>
</tr>
<tr>
<td>8</td>
<td>CQUAD4 with Corner Option</td>
<td>194</td>
<td>Coarse, Biased mesh</td>
<td>1.047</td>
</tr>
<tr>
<td>9</td>
<td>CQUAD4 with Corner Option</td>
<td>868</td>
<td>Fine, Even mesh</td>
<td>1.127</td>
</tr>
<tr>
<td>10</td>
<td>Theoretical</td>
<td>--</td>
<td>--</td>
<td>1.000</td>
</tr>
</tbody>
</table>

How fine a mesh you want depends on many factors. Among them is the cost you are willing to pay versus the accuracy you are receiving. The cost increases with the number of Degrees of Freedom (DOF). The definition of cost has changed with time. In the past, cost is generally associated with computer time. With both hardware and software becoming faster each day, cost is
probably associated more with the time required for you to debug and interpret your results. In
general, the larger the model is, the more time it takes you to debug and interpret your results.

In general, if you can visualize the form of the solution beforehand, you can then bias the grid point
distribution. However, this type of information is not necessarily available in all cases. If a better
assessment of accuracy is required and resources are available (time and money), you can always
establish error bounds for a particular problem by constructing and analysing multiple mesh
spacing’s of the same model and observe the convergences (Benes & Kruis, 2015).

4.4. Analysis/Simulation Setup

4.4.1. Type of analysis

Nature is non-linear. That means linear analysis can only approximate the real nonlinear behaviour
of parts and assemblies. Most of the time, such an approximation is acceptable, and linear analysis
can provide valuable insight into product characteristics. In many cases, however, linear
assumptions differ too much from reality and provide crude or misleading information. Using the
results of linear analysis to decide if a part will fail under its operating loads may lead to overdesign.

Since the simulation will exists from two different stages. First the heating schedule of the glass
panel and the second the contact between the mould and the heated glass panel which are both time
conducting a non-linear analysis should be used.

4.4.2. Glass simulation rules

Haldimann and Luibe made list of rules that should be taken into account when glass is simulated
with FEA software (Haldimann, 2008).

However, some general rules for the modelling of glass elements are given in the following:

- The mesh density should endeavour to match the expected stress concentrations, i.e. a finer
  mesh should be adopted around bolt holes and other geometric discontinuities in the glass;
- The results for any given mesh density should be verified by carrying out convergence tests
to ensure that any further mesh refinement does not affect the magnitude of the stresses
obtained from the analysis;
- Contact between glass and hard materials, such as steel, is normally prevented by using a
  liner, gasket or bushing that has a lower modulus of elasticity than that of glass (e. g. Nylon,
POM, aluminium, EPDM). One important consideration when modelling a fixing region
is, therefore, to ensure that the contact surfaces and releases are modelled such that forces
are transmitted in compression only and that no tension is transmitted through the gap. This
can normally be achieved by using contact elements or by prescribing contact and non-
contact surfaces. This approach requires a non-linear analysis; and
• Details must be modelled with care. In a point fixing, for instance, the rotational stiffness assigned to the model should match that of the specified bolt, i.e. whether the bolt is free to rotate as in fully articulated bolts or allows only partial rotation as in spring-plate type fixings.

4.5. Software Selection

In order to use the finite element method correctly for a glass deformation simulation all the above criteria should be met in selecting the software. The viscoplastic relations for simulation of creep have been formulated earlier discussed equation. Explicit finite element software has been used with both kinds of formulations for creep forming simulation of glass sheet at constant temperature of 630°C. Therefore, MSC Marc & Mentat has been selected of having the capability of working with the desired requirements (MSC-Software-Co, 2008).

4.6. Summary

The FEA should be non-linear in order to simulate the bending process of the glass. This is due the time dependent and temperature dependent setup. Nonetheless because of the glass panel moving by gravity. The mesh density of the glass panel should be as fine as possible to have a high accuracy of the panel prediction. Not only should the edge be defined fine but the middle as well. This is due to the creep and insufficient slumping that may occur during the bending process.

The FEA setup should be a simplification of the reality. The glass panel will have an internal temperature instead of the heat transfer via the actual furnace. The glass panel will sag onto the deformed mould instead of the mould deforming when the glass is on the mould.
5. Mesh Deformation Behaviour

In this chapter the behaviour of a steel woven mesh is introduced by a representation of formulas. First a short overview of different types of meshes and a detailed description of one of the chosen meshes for the mould will be presented. Further the shear behaviour of a plain woven mesh will be analysed. Finally the material Spring Steel will be introduced for using in the flexible mould.

Generally, the term “wire mesh” is used to describe a matrix of individual wires arranged at ninety degree angles, held in place by a weaving process. Woven wire mesh is more difficult and expensive to produce, as the individual wires must be arranged over and under one another (interlocked) and mechanically crimped into place by the weaving loom ram, resulting in superior strength and reliability. Offering superior precision of opening size woven mesh is preferred in applications requiring precise openings.

Warp threads are the vertical threads that run along the length of mesh. Weft threads are the horizontal threads that run from side to side.

5.1. Different Types of Woven Mesh

There are four common types of woven wire mesh. Three types will be covered in this paragraph. The fourth type of woven mesh, will be covered in the next paragraph. The mechanical properties of the mesh depends strongly on the type of weave and material used. In the last paragraph the used material will be introduced.

5.1.1. Dutch Woven Mesh

Dutch weave mesh has the weft wires close together and no readily discernible apertures. The warp wires are thicker than the weft wires. Dutch weave mesh is woven in both plain Dutch weave and twilled Dutch weave. This method of weaving enables mechanically strong meshes with tiny apertures to be produced. The mesh size is difficult to calculate as there are no square apertures, so the term ‘retention’ is used in the case of Dutch weave mesh, indicating that the mesh retains globules with a diameter larger than the retention value (see Figure 5.1 (a)).

5.1.2. Twilled Woven Mesh

Twilled Dutch Weave is produced by a combination of the features of the Dutch Weave and the Twilled Weave. Fill wires are passed alternately over and under two warp wires forming a fine mesh in one direction and a coarse mesh (mesh wire, woven wire mesh, wire mesh basket) in the other. This type of weave is capable of supporting greater loads than the Dutch Weave, with finer openings than the Twilled Weave. It is used in applications where the filtering of heavy material is necessary (see Figure 5.1 (b)).
5.1.3. Twilled Dutch Woven Mesh

Twill Dutch weave wire combines Dutch and twill weaving to provide a fine mesh filtering. Weft wires are passed over and under two warp wires, providing a tight, fine filter with tapered openings. The weaving process allows for the use of very fine wires, to micron size, producing filtering mesh for gas and liquid applications (see Figure 5.1(c)).

![Figure 5.1: Four types of meshes (a) Dutch weave (b) twilled weave (c) twilled Dutch weave (d) plain weave](image)

5.2. Plain Woven Mesh

The plain woven mesh is the most commonly used weave. Each weft wire passes alternately over and under each warp wire and vice versa. Warp and weft wire diameters are generally the same (see Figure 5.1 (d)). Wires are crimped in the weaving operation. Two of the most important wire parameters are the wire diameter, \(d\) and the mesh number, \(M\). Mesh number is defined as number of openings in a linear inch measured from the centre of one wire to the centre of a parallel wire in the direction parallel to the centre of the other group of wires. The plain woven steel mesh has identical mechanical properties in both weft wire and warp wire directions. The weft and warp wires are spatially orthogonal to each other’s. Assuming the wires have a circular cross section shape and have tangent contact at the crossing (Li, Li, & Liao, 2009).

Plain weave wire mesh are formed by interlacing or weaving two sets of orthogonal wires. The wires in the longitudinal direction are known as warp wires. The tows in the transverse direction are known as the fill wires or weft. The interlacing causes bending in the wires, called wire crimp. The deformation analysis of a plain weave mesh can be categorized in two categories: analytical models and numerical models. In de next part the analytical method will be discussed (Zhao, Peles, & Jensen, 2013).

5.2.1. Shear Deformation of a Plain Woven mesh

The plain woven mesh consist of squares created by wraps and wefts by the steel wires. Shearing in a plain-weave steel occurs by the relative moment of two sets of steel wire, warp and weft, which
are interlaced in a one-up and one-down. As the mesh is deformed the square starts to shear. The forces acting on the bar linkage can be resolved into shear and tensile force components acting perpendicular and parallel to the steel wire. At a small deformation the acting tensile force has a low magnitude which can be ignored.

Creating a computer simulation of the behaviour of the mesh requires a few adjustments and additions. To understand the behaviour the mesh is divided in $m \times n$ virtual masses (see Figure 5.2). Each mass is linked to its neighbours by massless springs of natural length non equal to zero. The link between each neighbours is achieved in three different ways.

- Springs linking masses $[i, j]$ and $[i + 1, j]$, and masses $[i, j]$ and $[i, j + 1]$, will be referred to as “structural springs”;
- Springs linking masses $[i, j]$ and $[i + 1, j + 1]$, and masses $[i + 1, j]$ and $[i, j + 1]$ will be referred to as “shear springs”;
- Springs linking masses $[i, j]$ and $[i + 2, j]$, and masses $[i, j]$ and $[i, j + 2]$, will be referred to as “flexion springs”.

When there is pure shear stress inside the mesh only the “shear springs” are constrained; under pure flexion stresses only the “flexion springs” are constrained.; whereas under pure compression or contraction stresses only the “structural springs” are constrained (Barbero, Trovillion, Hayugo, & Sikkil, 2006).

![Figure 5.2: Mesh Model Consisting of Mass and Springs](image)

5.2.2. Dynamic and forces

Computing the deformation of the mesh $m \times n$ at point $Pi, j$ over time $t$, where $i = 1, ..., m$ and $j = 1, ..., n$. The evolution of the system is governed by the fundamental law of dynamics:

$$F_{i,j} = \mu a_{i,j}$$  \hspace{1cm} (5.1)
Where $\mu$ is the mass of the points $i$ and $j$ and $a_{i,j}$ is the acceleration caused by the force $F_{i,j}$. Where $F_{i,j}$ can be divided between the internal and external forces. We can address the internal forces to the tension of the springs linking $P_{i,j}$ to its direct neighbours:

$$F_{int}(P_{i,j}) = -\sum_{(k,l) \in \mathcal{R}} K_{i,j,k,l} [1_{i,j,k,l} - l_{i,j,k,l}^0 \|1_{i,j,k,l}\|]$$  

(5.2)

Where:

- is the set regrouping all couples $(k, l)$ such as $P_{k,l}$ is linked by a spring to $P_{i,j}$;

- $l_{i,j,k,l}$ is the natural length of the spring linking $P_{i,j}$ and $P_{k,l}$; and

- $K_{i,j,k,l}$ is the stiffness of the spring linking $P_{i,j}$ and $P_{k,l}$.

The external forces acting on the mesh can be of various nature. The viscous damping will be given by:

$$F_{dis}(P_{i,j}) = -C_{dis} v_{i,j}$$  

(5.3)

Where:

- $C_{dis}$ is a damping coefficient and $v_{i,j}$ is the velocity of point $P_{i,j}$.

The role of this damping is in fact to model in first approximation the dissipation of the mechanical energy of our model. It is considered as an external force, but it could actually be considered as an internal force as well.

We can then compute the for $F_{i,j}(t)$ applied on point $P_{i,j}$ at any time $t$. The fundamental equation of dynamics can therefore be explicitly integrated through time by a simple Euler method.

$$a_{i,j}(t + \Delta t) = \frac{1}{\mu} F_{i,j}(t)$$

$$v_{i,j}(t + \Delta t) = v_{i,j}(t) + \Delta t a_{i,j}(t + \Delta t)$$

$$P_{i,j}(t + \Delta t) = P_{i,j}(t) + \Delta t v_{i,j}(t + \Delta t)$$  

(5.4)

5.2.3. Increasing the Stiffness of the Mesh

Increasing the stiffness of the mesh Simulating deformation in the mesh can counter the “super-elastic” effect. This effect can be reduced by increasing the stiffness in the springs. Xavier Provot created a simple mathematical equations to increase the stiffness if $\Delta t$ is greater than the natural period of the system.

$$T_0 = \sqrt{\frac{\mu}{K}} \Rightarrow K_e \approx m \frac{T_0^2}{\pi^2}$$  

(5.5)
If we want to increase the stiffness, $\Delta t$ needs to be decreased until the value is below the new value of $T_0$.

5.3. Spring Steel Mesh

Spring steel has been developed from the search for a weight and cost savings in the automobile industry. Weight reduction has been an important issue to reduce the fuel consumption. The last decade the automobile industry has been improving the strength, which results in weight reduction in cars and high-end springs with a 2350 MPa grade tensile. Spring Steel is typically used because of its’ high yield strengths, resistance to deformation and its ability to return to its original shape.

5.3.1. Decarburization by Thermal Load

Temperatures higher than 500 °C affect the properties of spring steel. When the spring steel is submitted to these high temperatures the spring steel will start to decarburise. This means that the percentile of carbon (CO) in the steel will drop. This has large effects on the spring steel. The tensile strength of a heat-treated steel depends primarily on the carbon content. The maximum stress on a component in bending occurs at the surface. Clearly, if excessive failure of steel component under reversing bending stresses is to be avoided, the specified carbon content must be maintained within the surface layers of the component.

Decarburization is a loss of carbon atoms from the surface of the spring steel wire, thereby producing a surface with lower carbon content than at some other distance beneath the surface. The mechanisms by which the decarburization of steels occurs are well understood. The phenomenon of decarburization usually takes place at temperatures above about 700 °C, with the following reaction.

$$C_{Fe} + \frac{1}{2}O_2 = CO$$

$$C_{Fe} + H_2O = CO + H_2$$

$$C_{Fe} + FeO = Fe + CO$$

Decarburization occurs when the reaction goes from left to right in the given chemical equation. When the reaction goes from the left to right, decarburization occurs. Decarburization of the spring steel leads to four reactions (Prawoto, Yajid, & Lee, 2013).

1: Diffusion of carbon atoms from the bulk to the spring steel surface;

2: Gas adsorption (water vapour and hydrogen) on the spring steel surface;

3: Oxidation of carbon; and

4: Formation of iron oxides.
5.4. NURBS Surfaces

The mesh mass point’s \( m \) must be seen as points in general. Recreating the surface after deformation is done by using NURBS splines and surfaces. NURBS are Non-Uniform Rational B-Splines. NURBS is a geometry that is easily manipulated by computer, allowing for great flexibility in modelling. NURBS can create smooth curves and surfaces with a small amount of points. The NURBS curves can be used to build a surface, define motion paths and control deformations.

5.4.1. Bézier Splines

To understand the work of NURBS we can find the base in the Bézier curve. Bézier was an engineer with the Renault car company and set out in the early 1960’s to develop a curve formulation which would lend itself to shape design. We can think of the Bézier curve in terms of the centre of mass of a set of point masses. Figure 5.3 shows the centre of mass of four points, where: the four masses \( m_0, m_1, m_2 \) and \( m_3 \) located at points \( P_0, P_1, P_2 \) and \( P_3 \). The centre of mass of these four point masses is given by equation (5.6).

![Figure 5.3: Points used to Construct a Bézier Curve](image)
The next step in creating the Bézier curve is letting go of the fixed constant values of each mass and introduce the function that the mass varies for each point given by parameter \( t \). Let \( m_0 = (1 - t)^3 \), \( m_1 = 3t(1 - t)^2 \), \( m_2 = 3t^2(1 - t) \) and \( m_3 = t^2 \). For each value of \( t \), the masses adopts new weights and their centre of mass changes continuously. As \( t \) varies between the domain 0 and 1, a curve is swept out by the centre of masses. This curve is a cubic Bézier curve – cubic because the mass equations are cubic polynomials in \( t \). Notice that, for any value of \( t \), \( m_0 + m_1 + m_2 + m_3 \equiv 1 \), and so we can simply write the equation of this Bézier curve as:

\[
P = \frac{m_0P_0 + m_1P_1 + m_2P_2 + m_3P_3}{m_0 + m_1 + m_2 + m_3}
\]  

(5.6)

If a straight line is drawn between points \( A, B, C \) and \( D \), as in a dot to dot puzzle, the resulting Figure is known as a control polygon. The blending functions, in the case of Bézier curves, are known as Bernstein polynomials.

A Bézier curve has \( n + 1 \) control points, which is called the Bézier degree (see Figure 5.5). This function can be put in an equation denoted \( B_i^n(t) \).

\[
B_i^n(t) = \binom{n}{i} (1 - t)^{n-i} t^i, \quad i = 0, 1, \ldots, n
\]  

(5.8)
Recall that \( \binom{n}{i} \) is called a binomial coefficient, sometimes spoken “\( n \)-choose-\( i \)”, and is equal to \( \frac{n!}{i!(n-i)!} \). In our introductory example, \( n = 3 \) and \( m_0 = B_3^0 = (1 - t)^3 \), \( m_1 = B_3^1 = 3t(1 - t)^2 t \), \( m_2 = B_3^2 = 3t^2(1 - t) \) and \( m_3 = B_3^3 = t^3 \). \( B_i^n(\tau) \) is also referred to as the \( i \)th Bernstein polynomial of degree \( n \). The equation of a Bézier curve is thus:

\[
P(\tau) = \sum_{i=0}^{n} \binom{n}{i} (1 - \tau)^{n-i} \tau^i P_i
\]

(5.9)

5.4.2. Three Dimensional Bézier Curves

The control points are defined in three dimensional space, the resulting Bézier curve is also three dimensional. Such a curve is called space curve, referring to the three dimensional location of the points. A two dimensional Bézier curve is call a planar curve. Note that since a degree two Bézier curve is defined using three control points, every degree two curve is planar, even if the control points are in a three dimensional coordinate system.

5.4.3. Degree Elevation

The degree elevation algorithm is used to calculate degree two curves exactly. In fact, any Bézier curve can be represented as a Bézier curve of higher degree. We illustrate by raising the degree of a cubic Bézier curve to degree four, the cubic Bézier can be put in an equation:

\[
x(\tau) = P_0 B_3^0(\tau) + P_1 B_3^1(\tau) + P_2 B_3^2(\tau) + P_3 B_3^3(\tau)
\]

(5.10)

Note that this curve is not changed at all if we multiply it by \([\tau + (1 - \tau)] \equiv 1\). However, multiplication in this way does serve to raise the degree of the Bézier curve by one. Its effect in the cubic case is to create five control points from the original four. Those five control points thus define a degree four Bézier curve which is precisely the same as the original degree three curve. The new control points \( P_i' \) are easily found as follows:

\[
\begin{align*}
P_0' &= P_0 \\
P_1' &= -\frac{1}{4} P_0 + \frac{1}{4} P_1 \\
P_2' &= \frac{1}{2} P_1 + \frac{1}{4} P_2 \\
P_3' &= \frac{3}{4} P_2 + \frac{1}{4} P_3 \\
P_4' &= P_3
\end{align*}
\]

(5.11)

This works for any degree \( n \) as follows:

\[
P_i' = \alpha_i P_{i-1} + (1 - \alpha_i) P_i, \quad \alpha_i = \frac{i}{n+1}
\]

(5.12)
5.4.4. Constructing NURBS Surfaces

A collection of points, either organized or unorganized (called a point cloud), can be split in two different types of NURBS surfaces. The first one regularizes and fits the NURBS surface. The second one optimizes the NURBS surface fitting and trims the surface.

The first step to create the NURBS surface from the point cloud that takes place in a two-dimensional space. This space refers to the Principal Components Analysis Regressions Plane (PCAP), which is a projection of the point cloud onto the PCAP plane (see Figure 5.6).

The regularization process for creating the new surface can be divided in four steps.

1. Project the point cloud on the PCAP and obtain extreme projected points;
2. Construct a grid from the extreme points with density $\rho$ calculated according to equation 5.13.
   \[ \rho = \frac{P}{A} \]  
   (5.13)

   Where $P$ is the number of points and $A$ is the area of the ellipse determined by the extreme points.
3. If no point is present, insert one point in its centroid, if the inserted point is inner to the cloud (Figure 5.6) of if the inserted point is outer to the cloud (Figure 5.7).

   \[ P_{\text{ins}} = AVG_{\text{Neigh}}P_{\text{grid}} - P_j(AVG_{\text{Neigh}}P_{\text{planePCA}}) \]  
   (5.14)

   \[ P_{\text{ins}} = P_{\text{ext}} + \lambda\vec{B} \]  
   (5.15)
Where $AVG_{Neigh}$ is the neighbourhood mean of the point for inserting, $P_{grid}$ is the centroid of the empty element analysed, $Plane_{PCA}$ is the PCAP, $P_j$ is the projection function, $P_{ext}$ is the point of the cloud closest to the point for inserting, $\lambda$ is the distance between them and $B$ is the PCAP basis vector in the inserting direction; and

4. If there are two or more points in the element, remove points located farthest from the centroid.

Once this process is completed a NURBS surface can be constructed from the point cloud. The NURBS surface process creates a smooth surface where there might occur a fitting error. To reduce the fitting error of the NURBS surface an optimization process is carried out for preserving the sharp features of the original point cloud.

![Diagram](image)

**Figure 5.7:** (a) Insertion of a point inside the cloud and (b) Insertion of a point outside the cloud

5.4.5. Surface Optimization

The influence of the NURBS surface smoothening optimization can create an incorrect model. The NURBS surface optimization process is devised to maintain the influence of the original point cloud stronger than the points inserted. The point cloud is the leading data in creating the NURBS surface. The optimization process is carried by the following equation:

$$ (\mu + \lambda) - ES $$  \hspace{1cm} (5.16)

This is described as follows.

Let $P = \{p_1, p_2, ..., p_n\}$ be a set of 3D points sampled from a real object, e.g. the regularized points, and $S = \{s_1, s_2, ..., s_3\}$ be a NURBS surface that approximates $P$. This should result in minimizing the error shown in equation 5.17.

$$ E(S) = d_{p,S} < \delta $$ \hspace{1cm} (5.17)
Where $d_{P,S}$ is the total distance between $P$ and the NURBS approximation surface $S$. The parameter $\delta$ is a given user error tolerance. The evolutionary strategy obtains control point weights of $S$, so that equation 5.17 is true.

This developed strategy will obtain the weights of the points that belong to the original point cloud. The points will be inserted according to weight as follows: i) if the point was inserted inside the cloud, its weight will be mean of the weights of the neighbour points. ii) If the point was inserted outside the cloud, its weight will be zero.

The influence of a single point $P$ is only local, the sampled points $P$ will be divided into clusters where a local optimization process will be carried out, which reduces the computational cost of the proposed method.

The optimization process starts with creating a cluster of the point cloud $P$, the clustering is done by the k-means method (Ursem). By using the k-means method it is possible to find a homogeneous region where the optimization process can be run without distorting the local shape of the surface. The last run of the algorithm will find the most homogeneous region where the optimization process was run, see Figure 5.7.

Once $P$ is clustered with the use of the k-means method the next process will optimize the local fitting of the NURBS in each cluster with the use of equation 5.18.

$$(\mu + \lambda) - ES$$

(5.18)

The boundaries of the cluster

This process configuration is as follows:

i) Individuals: the individuals of the strategy are confirmed by the weights of the cluster points and the mutation steps $\sigma$, see table 5.1, where $w_i$ are the control point weights and $\sigma_i$ are the mutation step sizes;

ii) Mutation operator: uncorrelated mutation with $n$ mutation step sizes $\sigma$ is applied to the individuals, according to equation 5.19 and 5.20.

$$\sigma'_i = \sigma_i e^{(\tau_0 N(0,1) + \tau_i N_i(0,1))}$$

(5.19)

$$x'_i = x_i + \sigma'_i \cdot N_i(0,1)$$

(5.20)
Recombination operator: the recombination operator is different for object variables \( w_i \) and parameters \( \sigma_i \). A global intermediary recombination is applied to object variables, according to equation 5.21, whereas a local intermediary recombination is applied to mutation step sizes \( \sigma_i \), according to equation 5.22.

\[
\begin{align*}
    b'_i &= \frac{1}{\rho} \sum_{k=1}^{\rho} b_{k,i} \\
    b'_i &= u_i b_{k_1,i} + (1 - u_i) b_{k_2,i}
\end{align*}
\] (5.21) (5.22)

Where \( i \) is the allele of the individual \( b_i \) is the value of the allele, \( \rho \) is the size of the recombination pool and \( \mu \) is a random number uniformly distributed in \([0,1]\).

Selection operator: the best individuals according to the aptitude function given in equation 5.22. In order to perform a fast computation of the distance between the points \( P \) and the NURBS surface \( S \), the points of \( S \) are stored in a kd-tree structure, so that the searching process for finding the nearest points between \( P \) and \( S \) is of \( \log(n) \) order.

Table 5.1: Individual of the evolutionary strategy.

<table>
<thead>
<tr>
<th>( w_1 )</th>
<th>( w_2 )</th>
<th>( \ldots )</th>
<th>( w_n )</th>
<th>( \sigma_1 )</th>
<th>( \sigma_2 )</th>
<th>( \ldots )</th>
<th>( \sigma_n )</th>
</tr>
</thead>
</table>

The algorithm in table X.X summarizes the optimization process. After the optimization process, the optimized surface is trimmed to eliminate the points inserted outside the cloud.

Table 5.2: Optimization process

```
Perform a clustering of \( P \) by using k-means
For each cluster do
    Set individual size = cluster size
    Set population size = \( \mu \)
    Initialize the population randomly
    Evaluate the population in the aptitude function
While the stop criterion \( \delta \) has not been reached do
    For \( i = 1 \) to \( \lambda \cdot 0.9 \) do
        \( Ind_i = \text{mut(Population}_{\text{rand}(1,\mu)} \) \)
    End for
    For \( i = 1 \) to \( \lambda \cdot 0.9 \) do
        \( Ind_i = \text{rec(Population}_{\text{rand}(1,\rho)} \) \)
    End for
Population = select from(\( \mu + \lambda \))
End while
```

Perform a clustering of \( P \) by using k-means
For each cluster do
    Set individual size = cluster size
    Set population size = \( \mu \)
    Initialize the population randomly
    Evaluate the population in the aptitude function
While the stop criterion \( \delta \) has not been reached do
    For \( i = 1 \) to \( \lambda \cdot 0.9 \) do
        \( Ind_i = \text{mut(Population}_{\text{rand}(1,\mu)} \) \)
    End for
    For \( i = 1 \) to \( \lambda \cdot 0.9 \) do
        \( Ind_i = \text{rec(Population}_{\text{rand}(1,\rho)} \) \)
    End for
Population = select from(\( \mu + \lambda \))
End while
5.5. Summary

A plain woven constructed mesh behaves like a NURBS controlled surface, which makes it easier to handle and produce in the simulation of the mould. The surfaces representation of the mould could be controlled via NURBS and then will show a large similarity with the original surface.

The mesh used for the mould is the WNFE3 1.75 \times 0.8 millimetre. This mesh consists of 180 \times 180 up and down weave mesh. The spring steel mesh has a thickness 0.8 millimetre and space between the wires of 1.75 millimetre.
PART III – METHOD
6. **Design of the Mould Design Tool**

The previous chapter described the behaviour of the spring-steel mesh. Simulating the spring steel mesh is one of the most important parts of the tool. The prediction of the mould should be as accurate as possible in order to create glass as precise as possible. This real time physical engine will simulate the behaviour of the mesh and create a ‘counter-mould’ of the original shape. This chapter deals with the implementation and behaviour analysis of the spring-steel mesh in the program Rhinoceros-Grasshopper-Kangaroo.

6.1. **Purpose of the Mould Design Tool**

The main purpose of this software is to predict the settings for the physical mould. The physical mould needs the amounts of steps each motor should make in order to reach the desired height. The speed of the stepper motors is crucial in the bending process. The speed is determined by each piston. Each piston has its own height difference and speed. All the pistons will move a different height in the same amount of time.

The software will be based on the mould designed by el Ghazi and Schuijers. In appendix II – the design of the mould can be found (El Ghazi & Schuijers, 2015).

6.2. **Physic Behaviour Engine**

The Mould Design Tool uses a physic behaviour engine from the software Kangaroo (Piker, 2011). This real time physical behaviour is based on the research done by Attar et al (Attar, et al., 2009).

6.2.1. **Dynamics**

The dynamics of a simplicial complex is defined by the motion of the vertices \(N\). Vertices are computer graphic objects often represented as triangulated polyhedrons. The object vertices are associated with three spatial coordinates, but also with other graphical information necessary to render the object correctly. Three vertices create a triangle. We can compact this description in a \(3N\) vector (see equation 6.1). In this software model the vertices represent the mass points or knots of the spring steel mesh. Figure 6.1 shows the vertices of a triangle.

\[
x(t) = (x_1(t), ..., x_N(t))
\]  

(6.1)
The vertices, \( N \), evolve due to external forces and internal deformations. The edges that lie between the vertices will change in angle once points \( N \) will move in space. The laws that govern the motion of the vertices are based on the theory of Newton. The second law states that:

\[
\mathbf{x}(t) = -\nabla f(\mathbf{x}) + f_e \\
\mathbf{x}(0) = \mathbf{x}_0 \\
\mathbf{x}(0) = \mathbf{v}_0
\]  \hspace{1cm} (6.2)

Where:

- \( f(\mathbf{x}) \) is the internal energy due to deformations; and
- \( f_e \) is the model external forces like gravity.

The initial state is defined by the position in space of the vertices \( N \). This basic equation describes the real time engine process for moving the spring steel into the right position (Stam, 2009).

6.2.2. Spring Behaviour

The behaviour of the linear spring is:

\[
\mathbf{x}(t) = -\mathbf{x}(t) \\
\mathbf{x}(0) = \mathbf{x}_0 \\
\mathbf{v}(0) = \mathbf{x}(0) = \mathbf{v}_0
\]  \hspace{1cm} (6.3)

Visualizing the motion of the spring is shown in Figure 6.0. When this is put into an equation we get:

\[
E = \frac{1}{2} v^2 + \frac{1}{2} x^2 + \frac{1}{2} v_0^2 + \frac{1}{2} x_0^2
\]  \hspace{1cm} (6.4)

Where:

- \( E \) is the energy; and
- \( x \) and \( v \) is the phase space.
Figure 6.2: Trajectory of the spring I tangent to the vector field

This proves that the motion proceeds clock-wise along the trajectories. The equation in the complex domain also shows that the trajectories are tangent to the vector field as shown in Figure 6.3.

Figure 6.3: Trajectories of the spring in phase space

There are two ways to let the spring behave. The first is an explicit integration, which means the spring will spins outward and gains energy over time which is unstable. The second is an implicit integration, which is unconditionally stable and the motion is that of an inward spiral. See Figure 6.4 for the explicit and implicit difference of a spring integration. The implicit equation of the spring is as follows (Stam, 2009):

\[ z^1 = (1 + h)^{-2} e^{ih} z^0 \]  

(6.5)
6.2.4. Collision
Collision refers to the calculation of forces at the point of contact among various elements in the simulation. It involves the momentum transfer at the point of contact interacting with material properties to deform and displace objects. We describe draping, wrapping, and bounded growth as prime examples of collision physics-based results. Collisions differ from the other constraints as they are unilateral, which means that they are expressed as a quality constraint:

\[ C(x) \geq 0 \]  

(6.6)

6.2.5. Drapery
Drapery is the computer simulation of draping a surface over an object. For example putting a towel over an apple. The software simulates the behaviour towel over the apple. The surface consist of a mesh. The mesh is constructed of masses and springs that represent the weave. The software allows the user to adjust the parameters assigned to the mesh behaviour. As discussed in chapter five the mesh consists of a spring steel mesh woven in a plain pattern. Figure 6.5 shows a towel draped over a circle.
The software uses the drape collision with the use of a negative force upwards. How this is used in the software will be discussed later in this chapter.

6.2.6. Time Integration

There is a time dependent involved, to simulate the dynamic deformation, this because the world coordinates \( \mathbf{x}(\mathbf{m}, t) \) of all point in \( M \). With \( \mathbf{x}(\mathbf{m}, t) \) given the configurations can be displayed as followed. \( \mathbf{x}(0), \mathbf{x}((\Delta t), \mathbf{x}(2\Delta t), \ldots \), which results in the animation of the object. \( \Delta t \) is a fixed time step of the simulation and \( \mathbf{x}(t) \) represents the entire vector field a time \( t \). The unknown vector fields \( \mathbf{x}(t) \) are not given, this can be achieve by the use of Newton’s second law of motion of the form given in equation 6.7.

\[
\mathbf{x} = F(\mathbf{x}, \mathbf{v}, t) \quad (6.7)
\]

The simplest scheme is forward Euler integration, where the time derivatives are replaced by finite differences. This yields in the following equation:

\[
\mathbf{x}(t + \Delta t) = \mathbf{x}(t) + \Delta t \mathbf{v}(t) \quad (6.8)
\]

\[
\mathbf{v}(t + \Delta t) = \mathbf{v}(t) + \Delta t F(\mathbf{v}(t), \mathbf{x}(t), t) \quad (6.9)
\]

Time integration are evaluated by two main criteria: first the stability and second the accuracy. Their accuracy is measured by their convergence with respect to the time step size \( \Delta t \), i.e. first order \( O(\Delta t) \), second order \( O(\Delta t^2) \), etc. In the field of physically based animation in Computer Graphics, stability is often much more important than accuracy (Nealen, Müller, Keiser, Boxerman, & Carlson, 2005).

6.3. Mould Design Tool Setup (step by step)

The Mould Design Tool (MDT) needs a surface as input and generates an Arduino-script\(^4\) (with the piston height), the mould surface as a separate drawing (used for verifying the tool) and a model of the complete mould in its final position.

To extract the desired data, the input surface will be used in a model of the mould (Figure 6.6) to get the settings for the mould.

\(^4\) Arduino is an open-source computer hardware microcontroller for building digital devices and interactive objects that can sense and control the physical world.
The mould is designed and build by Hicham el Ghazi and Niek Schuijers (El Ghazi & Schuijers, 2015): The 3DFlexMould. The flowchart below (Figure 6.7, appendix I for a detailed view) shows the proceedings and steps of the software tool.

6.3.1. Input Data for Mould Tool
The input for the MDT is a free-form surface in a 3DS file format. The 3DS file format is the geometry format used to insert the file from a different location. A 3DS extension is used by Autodesk 3D image3D Studio. It contains mesh data, material attributes, bitmap references, smoothing group data, viewport configurations, camera locations, lighting information and object
animation data. 3DS files consist of chunks of data that contain an ID and length description. Chunks store the shapes, lighting, and viewing information that together represent a three-dimensional scene.

Once the surface is loaded in the tool, it will automatically rotate and place it in the correct way. This means that the centre of gravity of the surface will be aligned with the centre point of the mould. The four, or more, corners of the surface will be aligned in the same horizontal plane. See Figure 6.3 for the positioning of the surface above the mould.

![Figure 6.3: Surface positioning above the mould](image)

6.3.2. Counter-mould

The repositioned surface will be extruded to create a solid, called a BREP. This is done to create a counter-mould surface, the bottom side of the BREP, where the mesh will drape onto. See Figure 6.4 for the construction of the BREP.

Boundary representation models are composed of two parts: topology and geometry (surfaces, curves and points). The main topological items are faces, edges and vertices. A face is a bounded portion of a surface; an edge is a bounded piece of a curve and a vertex lies at a point. Other elements are the shell (a set of connected faces), the loop (a circuit of edges bounding a face) and loop-edge links (also known as winged edge links or half-edges) which are used to create the edge circuits. The edges are like the edges of a table, bounding a surface portion.
6.3.3. Mesh setup and size
The density of the mesh is critical in the simulation process. The finer the mesh is divided, the better and more accurate the result is, but the more calculations the computer has to do. This means that the mesh setup of $18 \times 18$ will fulfill the required accuracy. Within a reasonable amount of time.

Table 6.1: Dimension of the mesh versus the simulation time (same surface is used in these tests)

<table>
<thead>
<tr>
<th>Dimension of the mesh</th>
<th>Simulation time</th>
<th>Iterations</th>
<th>Accuracy</th>
</tr>
</thead>
<tbody>
<tr>
<td>$6 \times 6$</td>
<td>00:02:50</td>
<td>380</td>
<td>73 %</td>
</tr>
<tr>
<td>$12 \times 12$</td>
<td>00:06:32</td>
<td>500</td>
<td>82 %</td>
</tr>
<tr>
<td>$18 \times 18$</td>
<td>00:15:48</td>
<td>680</td>
<td>95 %</td>
</tr>
<tr>
<td>$24 \times 24$</td>
<td>00:32:14</td>
<td>1200</td>
<td>98 %</td>
</tr>
<tr>
<td>$30 \times 30$</td>
<td>00:58:12</td>
<td>2360</td>
<td>99 %</td>
</tr>
<tr>
<td>$36 \times 36$</td>
<td>01:52:16</td>
<td>4800</td>
<td>&gt;99 %</td>
</tr>
</tbody>
</table>

In Figure 6.10 the six different types of meshes are shown used in the test.

6.3.4. Spring Mesh Settings
As discussed in Chapter 5, the construction of the mesh in the software should be done via the Mass-Spring connection. The software allows the user to adjust the strength of the springs used to connect the masses, $m$, with each other. The higher the value of the spring, the stiffer the spring and
the mesh will be. The spring value is set to default as the deviation of the mould is acceptable, see Paragraph 6.3.9.

**Figure 6.11:** Spring Strength Definition ‘n’

6.3.5. Pistons

The pistons or actuators that control the mesh are constructed as they behave in the physical world. This means that the software uses the outer contact circle as bend point (see Figure 6.12). However, practice shows that this is not the actual point of bending. The spring inside the steel tubes has a smoother arc curve as bending point (see Figure 6.13). The pistons are controlled by stepper motors. The height is derived from the outcome of the simulation and the screw thread of 1.25 millimetre per rotation. This means that the stepper motor need 200 steps per rotation and the piston will move 1.25 millimetre. The calculation of the angle of the piston, the 3D ball joint, is shown in Figure 6.14.

**Figure 6.12:** Software Bend Point
6.3.6. Physical Simulation

The software combination of Rhinoceros, Grasshopper and Kangaroo delivers a real time engine of the physical world. The operation of the physical engine is described in the previous paragraphs. By starting the simulation the mould surface, or mesh, will move towards the counter mould created by the original geometry. The original geometry will float above the mesh. Once the mesh makes contact with the counter mould the middle point will be constrained and can only move in the z-direction. The mesh will drape against the counter mould to find to most optimal shape for the mould. The masses of the mass-spring setup will collide against the counter mould. Once all the masses made contact with the counter mould the simulation will stop and produce the export data. The masses are used to calculate the height difference of each piston.

6.3.7. Arduino Script

The simulation software creates an Arduino-script, which is used to control the mould, automatically as output data. The Arduino mini-controllers control the twenty-five stepper motors.
6.3.8. Output
The output consist of two types of data. The first is the Arduino script as discussed in the previous paragraph. The second is a geometry file of the mould surface. The geometry file is exported as a FBX\(^5\) file type. This file type is needed for the verification of the mould. This geometry file is used in the software CloudCompare to compare the original surface, the outcome of the MDT, with the surface of the physical mould.

6.4. Summary
The MDT and the physical mould were tested and the deviation of the mould and measuring device can be used to determine to the first overall deviation that should be taken into account for. The glass test showed an average deviation of ± 2.56 millimetre. The Prodim Proliner, see Appendix VII for the calibration certificate, has a deviation of ± 0.019 millimetre.

To determine the tolerance of the mould there are four main factors involved. The first part is the stepper motors, the second is the calibration of the mould, the third is the simulation and the fourth, the one with the greatest influence, is human error. The stepper motors has the smallest tolerance factor of all four factors, 5% per rotation. Each rotation consist out of 200 steps. Every rotation moves the piston 1.25 millimetre in the Z-direction. This results in a ± 0.03 millimetre deviation.

The Prodim Proliner has a tolerance of ± 0.019 millimetre. If the mould is calibrated correctly this is the smallest factor. In addition there is a 5% extra tolerance factor on top of the 0.019 millimetre due to reading errors. However the tool does not measure after the decimal point, which means that the tolerance factor is set to 1 millimetre.

---

\(^5\) FBX or Filmbox is used to provide interoperability between digital content creation applications.
Since the simulation has not been tested in this setting and there are no references we can assume there is a tolerance factor of ± 10 millimetre. The fourth tolerance factor has the most influence on the complete process. However, there is no factor to assign to human errors. These four tolerance factors result in a total tolerance factor of ± 2 millimetre. Which means the surface of the mould can deviate 2 millimetre in the Z-direction (Associates, Grasshopper - Generative Modeling for Rhino, 2011) (Associates, Modeling Tools for Designers, 2011) (Piker, 2011) (Associates, Galapagos, 2011).
7. Finite Element Method Glass Behaviour Analysis

The validation or prediction of the physical bending method with the 3DFlexMould is performed with MSC.Marc to see how the glass behaves during the deformation process. Stress of the glass panel will also be explored within the glass panel itself. The first paragraph will summarise the experiment and its model. The second paragraph will discuss the model itself and the model properties. The third paragraph will discuss the result of the run Marc simulation.

7.1. Model setup

The FEA used to simulate the behaviour of the glass while submitted to high temperatures and deformation is a simplified model from the physical world. There are no external variables present in the model and the glass panel sag onto the deformed mould rather then slowly being pushed in the right shape. The temperature is set as an internal boundary condition instead of the heat being transferred from the heat source to the glass.

The simulation has been done with the use of the software Marc Mentat. The ‘unfolded’ glass panels, produced with the software of Prodim, is used as original surface. The outcome of the surface, the bended glass panel, is compared with the original designed geometry. The comparison between the two geometries is done with the software CloudCompare.

7.1.1. Material Properties

The glass in the model has the characteristics from table 7.1. The surface mould are assigned as a ceramic material. This is done to eliminate any influence and deformation of the high temperature of the glass that will make contact with the mould.

| Table 7.1: Physical properties of soda lime silica glass (SLSG). |
|-----------------|-----------------|-----------------|
| Density         | $\rho$          | $\text{Kg/m}^3$ | 2500 |
| Knoop hardness  | $GPa$           | $\text{Knoop hardness}$ | 6 |
| Young’s modulus | $E$             | $\text{MPa}$ | 70 000 |
| Poisson’s ratio | $\nu$           | -               | 0.23 |
| Coefficient of thermal expansion | $\alpha_T$ | $10^{-6} \text{ K}^{-1}$ | 9 |
| Specific thermal capacity | $c_p$ | $\text{Kg}^{-1} \text{ K}^{-1}$ | 720 |
| Thermal conductivity | $\lambda$ | $\text{W m}^{-1} \text{ K}^{-1}$ | 1 |
| Average refractive index within the visible spectrum | $n$ | - | 1.52 |
| Emissivity (corrected) | $\xi$ | - | 0.837 |

7.1.2. Model Loads

Table 7.2 shows the firing schedule for the glass panel. The glass panel will start to sag once the internal temperate has reached 630°C. Apart from the firing schedule the glass panel has a gravity load applied. Figure 7.1 shows the gravity load applied to the glass panel.
Table 7.2: Firing Schedule

<table>
<thead>
<tr>
<th>Step</th>
<th>Temperature (°C)</th>
<th>Time (min)</th>
<th>Total time (min)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Start</td>
<td>24 °C</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Initial heat</td>
<td>630 °C</td>
<td>90</td>
<td>90</td>
</tr>
<tr>
<td>Soaking time</td>
<td>630 °C</td>
<td>20</td>
<td>110</td>
</tr>
<tr>
<td>Anneal cool</td>
<td>300 °C</td>
<td>45</td>
<td>155</td>
</tr>
<tr>
<td>Rapid cool</td>
<td>24 °C</td>
<td>20</td>
<td>175</td>
</tr>
</tbody>
</table>

Figure 7.1: Gravity Load Applied to the Glass Panel

7.1.3. Model Constraints

The mould surface is constraint in the three dimension. Figure 7.3 shows the constraints applied to the mould surface, the surface will not move in any direction.
7.1.4. Contact Set
To deform the glass panel a contact set is created between the glass panel and the mould surface. The mould surface is completely constraint as shown in the previous paragraph. The glass panel sags by the use of a gravity load and will make contact with the mould surface.

7.1.5. Stress Prediction
The software Prodim makes it possible to make a stress prediction plot. Figure 7.5 shows the stress prediction of panel 1.
Figure 7.5: Stress Prediction Plot

7.2. Results
The results of the FEA simulation are promising, there are some insufficient slumping at the edges and the middle. Appendix IV shows the simulation result of the six unique panels. The results are more or less the same as the glass dome outcome (Appendix III), however the domain of the deviation is less.

7.3. Summary
The analysis shows a very large comparison with the original surface, but shows a large deviation compared to the physical bended glass panel. The insufficient slumping is not noticeable in the simulation, but it is to note that the physical model shows insufficient slumping. The deviation is comparable with the deviation of the original geometry compared with the physical model. The deviation between the original geometry and the physical model is comparable with the comparison between the finite element analysis and physical model. This concludes the bending process differs from the finite element analysis.
8. Verification of the 3DFlexMould

This chapter discusses the first physical tests of the 3DFlexMould and eliminates teething problems. The 3DFlexMould is controlled via the Mould Design Tool, as explained in chapter 6. The mould surface is measured with a measuring machine, the Prodim Proliner. The first paragraph will discuss the verification process with and how it is working. The second paragraph will discuss the results and improvements for a better result.

8.1. Verification Process

The company Prodim provided a measuring machine, the Proliner. The Proliner is used to measure double curved glass panels and can be used to verify the mould ansich.

8.1.1. Prodim Proliner

The Prodim Proliner is a digital measuring device that creates a point cloud of the measured data. The device calculates two angles and the length of the cord. The cord is attached to ‘pen’ and controlled by the user. The planes that are measured are the XY-plane and XZ-plane. The machine registers one point every second. The two angles, distance and time create a point cloud which can be imported in most CAD software. Figure 8.1 shows the basic principle of the Prodim Proliner. The device has an accuracy of less than 0.02 millimetre. Prodim claims an accuracy of > 0.19mm. The machine is calibrated according to the ISO 17025 by the company VSL, the Dutch Metrology Institute. See Appendix VII for the certificate of the machine used.

![Figure 8.1: Basic principle of the Prodim Proliner](image-url)
Table 8.3: Test Results Prodim Proliner

<table>
<thead>
<tr>
<th>Measure point</th>
<th>Distance</th>
<th>Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average uncertainty of a reference length (HBT)</td>
<td>2.5 m</td>
<td>± 0.027 mm</td>
</tr>
<tr>
<td>Standard deviation (HBT)</td>
<td>2.5 m</td>
<td>± 0.055 mm</td>
</tr>
<tr>
<td></td>
<td>5.0 m</td>
<td>± 0.037 mm</td>
</tr>
<tr>
<td>Average uncertainty of a reference length (LBT)</td>
<td>2.5 m</td>
<td>± 0.018 mm</td>
</tr>
<tr>
<td>Standard deviation (LBT)</td>
<td>2.5 m</td>
<td>± 0.041 mm</td>
</tr>
<tr>
<td></td>
<td>5.0 m</td>
<td>± 0.108 mm</td>
</tr>
<tr>
<td>Single point repeatability (SPR)</td>
<td>2.5 m</td>
<td>± 0.019 mm</td>
</tr>
<tr>
<td></td>
<td>5.0 m</td>
<td>± 0.029 mm</td>
</tr>
</tbody>
</table>

8.2. Verification results

The mould is tested in different settings. Table 8.1 shows different types of panels with the number of measured points and the standard deviation measured. There is a side note to be made to the standard deviation values. The edges of the surface measured show the largest deviation. However the panels will never (in this prototype) exceed the dimensions of 400 × 400 millimetre (L × W). The inside dimensions, 400 × 400 millimetres of the mould measurements should be used for verifying the mould. Table 8.2 shows the adjusted dimensions of the surfaces. The standard deviation is significant lower than the earlier measured data. These adjusted surfaces show an average deviation of ± 2.56 millimetre.

Table 8.1: Mould accuracy, measured with using six different settings

<table>
<thead>
<tr>
<th>Panel nr.</th>
<th>Numbers of Points in Point Cloud</th>
<th>Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>446</td>
<td>± 8.87 mm</td>
</tr>
<tr>
<td>2</td>
<td>882</td>
<td>± 7.17 mm</td>
</tr>
<tr>
<td>3</td>
<td>531</td>
<td>± 1.08 mm</td>
</tr>
<tr>
<td>4</td>
<td>400</td>
<td>± 4.46 mm</td>
</tr>
<tr>
<td>5</td>
<td>622</td>
<td>± 4.30 mm</td>
</tr>
<tr>
<td>6</td>
<td>882</td>
<td>± 5.37 mm</td>
</tr>
<tr>
<td>Average Deviation</td>
<td></td>
<td>± 5.20 mm</td>
</tr>
</tbody>
</table>

Table 8.2: Corrected Mould Surface

<table>
<thead>
<tr>
<th>Panel nr.</th>
<th>Numbers of Points in Point Cloud</th>
<th>Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>446</td>
<td>± 5.99 mm</td>
</tr>
<tr>
<td>2</td>
<td>882</td>
<td>± 3.63 mm</td>
</tr>
<tr>
<td>3</td>
<td>531</td>
<td>± 0.74 mm</td>
</tr>
<tr>
<td>4</td>
<td>400</td>
<td>± 2.19 mm</td>
</tr>
<tr>
<td>5</td>
<td>622</td>
<td>± 1.41 mm</td>
</tr>
<tr>
<td>6</td>
<td>882</td>
<td>± 1.39 mm</td>
</tr>
<tr>
<td>Average Deviation</td>
<td></td>
<td>± 2.56 mm</td>
</tr>
</tbody>
</table>

These settings were used for creating the Glass Dome constructed by Hicham el Ghazi and Niek Schuijers.
### Table 8.3: MDT Outcome versus Physical Mould

<table>
<thead>
<tr>
<th>Number</th>
<th>Measured</th>
<th></th>
<th>Corrected</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Average</td>
<td>Max. Deviation</td>
<td>Average</td>
<td>Max. Deviation</td>
</tr>
<tr>
<td></td>
<td>Deviation</td>
<td></td>
<td>Deviation</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>5.27</td>
<td>23.93</td>
<td>1.09</td>
<td>7.6</td>
</tr>
<tr>
<td>2</td>
<td>4.29</td>
<td>17.57</td>
<td>0.94</td>
<td>7.54</td>
</tr>
<tr>
<td>3</td>
<td>3.93</td>
<td>15.99</td>
<td>0.15</td>
<td>5.11</td>
</tr>
<tr>
<td>4</td>
<td>2.64</td>
<td>16</td>
<td>0.43</td>
<td>5.47</td>
</tr>
<tr>
<td>5</td>
<td>3.01</td>
<td>15</td>
<td>0.07</td>
<td>4.32</td>
</tr>
<tr>
<td>6</td>
<td>2.64</td>
<td>25</td>
<td>0.97</td>
<td>6.2</td>
</tr>
</tbody>
</table>

#### 8.3. Summary

The mould has a deviation of ± 2.56 millimetre. The calculated deviation in chapter six was ± 2 millimetre. The mould is very accurate.
PART IV – RESULTS
9. Results

The results of the test consist of four different parts. The first part is the glass produced for the Glass Dome, this glass will be verified with the original geometry.

The second part is the outcome of the FEA simulation, where the bending process is simulated to check the behaviour of the glass panels while deforming. This is done in a simplified physical world.

The third is repetition test, where one geometry is produced five times in order to check the if the 3DFlexMould produces accurate glass and if the slumping phenomenon occurs as predicted.

The final part is the “radius test” to find out if the glass slumps sufficient enough and if not to find a correction factor.

9.1. Glass Panel Production of the Glass Dome

The Glass Dome is developed in Rhinoceros in combination with Grasshopper. The Glass Dome consists out of six unique glass panels, these six are used to validate the bending process. Figure 9.1 shows a render of the Glass Dome as it should be produced, Figure 9.2 shows the actual constructed Glass Dome.

The Glass dome consist out of twenty insulated glass panels glued together by 3M DP490. The dimension of the Glass Dome are: 2000 × 1500 × 350 millimetre. Figure 9.3 shows the division of the six different glass panels that are measured and validated. Table 9.1 shows the values of the average deviation of the panels and the deviation domain. Figure 9.4 shows a graph of the measured values. Appendix III shows a detailed report of the results of the Glass Dome bending.

![Figure 9.1: Glass Dome Render](image)
**Figure 9.2:** Constructed Glass Dome

**Figure 9.3:** Glass Dome Glass Panel Division

**Table 9.1:** Original surface versus bended glass panel

<table>
<thead>
<tr>
<th>Panel nr.</th>
<th>Negative Deviation</th>
<th>Positive Deviation</th>
<th>Average Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-2.1659</td>
<td>6.111208</td>
<td>2.2188</td>
</tr>
<tr>
<td>2</td>
<td>-3.5107</td>
<td>6.3669</td>
<td>2.5045</td>
</tr>
<tr>
<td>3</td>
<td>-3.11837</td>
<td>4.972077</td>
<td>2.9711</td>
</tr>
<tr>
<td>4</td>
<td>-4.19052</td>
<td>5.166499</td>
<td>1.8764</td>
</tr>
<tr>
<td>5</td>
<td>-9.33821</td>
<td>6.80685</td>
<td>3.3849</td>
</tr>
<tr>
<td>6</td>
<td>-2.76729</td>
<td>2.021958</td>
<td>0.71962</td>
</tr>
</tbody>
</table>
Overall it is to conclude that with an average deviation of 2.279 millimetre the outcome of the panels is acceptable. Panel 1, 2 and 3 show insufficient slumping in the middle of the panel. Panel 4, 5 and 6 show insufficient slumping at the edges. It is difficult to make a statement about insufficient slumping in the middle of the panel. However the domain of the deviation lies between -9.33 and 6.80 millimetre, which is unacceptable.

9.2. FEA Outcome

The glass panels used to create the Glass Dome are simulated with the use of Marc. The six unique panels are tested in a ‘simplified’ physical world to look for the behaviour of the glass while deforming. A detailed report can be found in Appendix IV. This report discusses the six unique panels FEA setup and outcome. The glass panels are compared via cloud compare software to validate the simulation.

Table 9.2 shows the negative, positive and average deviation of the six unique panels. The negative and positive deviation domain is big and lies between -8 and +7 millimetre. The average deviation is less than 2 millimetres, 1.89 millimetre to be exact. Figure 9.5 shows the graph of the deviation domain and average deviation. It shows a trend-line overall, except for panel 3 which has a very low negative deviation. The positive and average deviation show a more horizontal trend-line, the negative value shows steeper trend-line. Figure 9.6 shows the domain where the measured values or points lay within. Read this graph as the amount of measured values that are within the domain.

The Appendix shows the predicted stress in the bended glass panel. Figure 9.7 shows an example of the predicted stress in the glass panel.

![Figure 9.4: Original surface versus bended glass panel](image-url)

Table 9.2: Original surface versus FEA outcome
<table>
<thead>
<tr>
<th>Panel nr.</th>
<th>Negative Deviation</th>
<th>Positive Deviation</th>
<th>Average Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-8.02867</td>
<td>6.871416</td>
<td>0.62335</td>
</tr>
<tr>
<td>2</td>
<td>-6.74504</td>
<td>6.15687</td>
<td>1.6969</td>
</tr>
<tr>
<td>3</td>
<td>-0.74574</td>
<td>5.156783</td>
<td>1.5051</td>
</tr>
<tr>
<td>4</td>
<td>-7.897</td>
<td>6.012875</td>
<td>2.2101</td>
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<tr>
<td>5</td>
<td>-5.93192</td>
<td>4.50851</td>
<td>4.3551</td>
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<tr>
<td>6</td>
<td>-5.21987</td>
<td>6.111447</td>
<td>0.97352</td>
</tr>
</tbody>
</table>

**Figure 9.5:** Original Geometry versus Finite Element Analysis Outcome

**Figure 9.6:** Measured Value within the Domain
The analysis shows a very large comparison with the original surface, but shows a large deviation compared to the physical bended glass panel. The insufficient slumping is not noticeable in the simulation, but it is to note that the physical model shows insufficient slumping. The deviation is comparable with the deviation of the original geometry compared with the physical model. The deviation between the original geometry and the physical model is comparable with the comparison between the finite element analysis and physical model. This concludes the bending process differs from the finite element analysis.

9.3. Slumping process test

The slumping test is set up to see how the slumping accrue in the 3DFlexMould. As discussed in chapter 3.2.1, insufficient slumping accrue when the glass sagging method is used. This test uses five different radiuses to investigate the phenomenon and improve the bending process accuracy.

Table 9.4 shows the result of the five slumping tests. Figure 9.8 and 9.9 show the graphs that complement table 9.4. The results show that the actual radius converges from the original radius the larger the radius gets. Even the domain of the deviation shows the diverging line. The larger the radius gets, the bigger the domain becomes.

Appendix V discusses the complete test and addresses each panel indvial.
The radius test shows a very large deviation domain. Furthermore it shows over-slumping instead of insufficient slumping, what was expected. The edges show an acceptable result, however the middle, or the centre, of the panel shows a smaller radius. Previous test, for example the Glass Dome panels, show insufficient slumping. Figure 9.8 and 9.9 show the diverging lines in the graphs. The bigger the radius, the bigger the deviation domain an smaller the radius.

9.4. Repetition process

The repetition is carried out to validate the process, will every test have the same result. Table 9.5 shows the result of this test. Five panels were bend with a single radius, mono clastic, of 1000 millimetre. Table 9.5 shows that the radius is smaller than intended. This shows over-slumping, the glass panel sags deeper than it should be.
This could be occurred by the insulation top-layer or the setting of the mould. The average deviation falls within the domain of the mould deviation, which is ±3.63 millimetre. Figure 9.10 shows the negative, positive and average deviation where there is no constant value that shows a trend in the deviation that could lead to a statement about the repetition. However, if we look at Figure 9.11 we can see that the overall measured radius shows similarity. The average radius is 861.6 millimetre shows a continuous trend line.

Appendix VI discusses the comparison of the designed geometry and outcome of the test. It also discusses the over-slumping issue and where the slumping is sufficient, insufficient and over-slumping.

Table 9.5: Repetition Test Result

<table>
<thead>
<tr>
<th>Panel nr.</th>
<th>Measured Radius</th>
<th>Negative Deviation</th>
<th>Positive Deviation</th>
<th>Average Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1000</td>
<td>858</td>
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<td>6.902</td>
<td>1.4472</td>
</tr>
<tr>
<td>1000</td>
<td>810</td>
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<td>1000</td>
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<td>-2.463899</td>
<td>0.101106</td>
<td>0.94497</td>
</tr>
<tr>
<td>1000</td>
<td>897</td>
<td>-1.029375</td>
<td>8.615161</td>
<td>2.859</td>
</tr>
<tr>
<td>1000</td>
<td>883</td>
<td>0</td>
<td>6.63387</td>
<td>2.5187</td>
</tr>
</tbody>
</table>

Figure 9.10: Deviation per Glass Panel
The Repetition Test shows that the average radius is close to each other. However the domain of deviation differs from each other. The bending process shows pleasing results however the accuracy should be improved in order to be more pleasing. It does not show insufficient slumping at the five panels. Some have insufficient slumping in the middle, others on the edges and panel 1 shows the best overall result but with a very big domain. The average deviation is 1.85 millimetre which is acceptable, however the domain of deviation is too big.

9.5. Summary

The four different tests show different results. The glass panels used to construct the Glass Dome show insufficient slumping in the middle, but the edges show promising results with an acceptable accuracy.

The Finite Element Analysis shows very acceptable results that match the original geometry. There is some insufficient and over slumping but this can be addressed to the set-up of the simulation, the simplification of the physical world.

The slumping test with five different radii show over slumping. The radius is smaller than it should be. This could be an error of the mould setting itself or the use of the insulation top-layer. The deviation domain lies within the margin measured in chapter 7, which makes it acceptable.

The repetition test shows five different deviation domains, but when it comes to the overall radius, Figure 9.11, it shows consistency with an average radius of 861.6 millimetre. This is still less than it should be but still acceptable. Further investigation could lead to a correction factor applied on the MDT to control the mould more accurate for the given geometry.

The radius test shows a different result than expected. It shows over-slumping instead of insufficient slumping what was expected. The ratio factor is approximately around 1.3 times the desired radius.

![Figure 9.11: Radius Deviation per Glass Pane](image-url)
PART V – CONCLUSION AND RECOMMENDATION
10. Conclusion

The goal of this research was to analyse, predict and manufacture double curved glass panels at low cost within a certain fail margin. The production of the double curved glass panels was successful by producing and constructing the Glass Dome.

The Finite Element Analysis shows only the behaviour of the glass but does not gives a view of the actual behaviour of the glass. Glass is a material that is difficult to control and predict.

Creating double curved glass was successful, however the slumping process showed to be insufficient. This means that the glass does not sag deep enough or too much to adapt the correct radius. The edges however sag sufficient enough, which has the highest priority for fitting it in a frame and such. The radius of the double curved glass is sufficient.

During the radius test, see chapter 9.4, it showed that the actual radius has a large deviation of (this deviation is the radius deviation not the z-direction deviation). The warm bending method showed high potential to deliver the desired geometry. The glass used to create the double curved glass panels is soda lima glass. This is due the low price, its hardness and workable properties. The radius test gives a, this needs to be tested in further research, correction factor in order to find the correct radius.

The radius test shows a divergent graph opposite from what was assumed. The lower the radius the more accurate the panel is and the bigger the radius the lower it actually comes out of the bending process.

The glass panels for the Glass Dome showed an acceptable deviation of ± 3 millimetre. The feet of the Glass Dome show the biggest deviation because the slumping process was insufficient. The four feet panels show an average deviation of ± 6 millimetre. Once the radius gets smaller than 1000 millimetre the sagging gets insufficient.

Overall it is to conclude that the mould works, but needs further testing and professionalization. The double curved panels show insufficient slumping and the single curved panels shows over-slumping.
11. Recommendations

The 3DFlexMould proved itself but needs further fine-tuning and further research to be perfected. To compete with other flexible mould systems it is necessary to professionalise.

The software used for this thesis. The FEA could be extended and more variables could be taken into account when running the simulation. The simulation/prediction of the glass is simplified, but a better simulation should show a better result. The software used to control the mould and simulation could be upgraded into a stand-alone software so the user has better controls of the mould and simplifies the user experience.

Introducing self-calibration could reduce the error of calibrating by hand and setting the mould in its ‘zero’ position. And self-checking if the pistons and surface are at the right position. Furthermore if the surface could be checked if it is in the right position.

To make an actual difference on the market larger glass panels should be produced to test the accuracy and strength. Upscaling the principle of the 3DFlexMould is simple, the principle should work on a larger scale since the mesh and the pistons will behave the same way.

The used furnace in this research was an old pots oven modified to fit the 3DFlexMould, the open area in the bottom side of the furnace creates huge thermal bridges. The difference of temperature at local spots has influence of the body temperature of the glass panels and temperature inside the furnace. This difference in temperature could introduce unwanted stresses and insufficient temperature for the glass panel to slump enough.

In conclusion, this research contributes to the research topic about flexible moulding focused on low-cost production of double curved glass. The first tests with the 3DFlexMould show great opportunities for future fluid architecture. In addition of glass it is possible to use the system of the 3DFlexMould for different materials such as concrete and composites. The principle behind the 3DFlexMould is simple but could lead to a breakthrough in the flexible and low-cost production for double curved materials.
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APPENDIX I
MOULD DESIGN TOOL
FLOWCHART
APPENDIX II
MOULD DESIGN
APPENDIX III

GLASS DOME

VERIFICATION REPORT
Glass Dome Verification Report

Appendix III for:
Warm Bent Glass: Development of a Flexible Mould

Eindhoven University of Technology

October, 2015
1. Introduction

This document is the verification result of six unique panels used to construct the Glass Dome. The glass panels are deformed with the use of the Mould Design Tool and the 3DFlexMould. The physical glass panels are compared to the original glass geometry. The result of this report shows the accuracy of each glass panel. Figure 1 shows the division of the Glass Dome panels. The panels are aligned by picking the five corner points of the panels. The panels discussed in this report are: 1, 2, 5, 6, 9 and 10.

![Figure 1: Glass Dome Glass Panel Division](image)
2. Panel 1

Figure 3 shows the measured values and the approximate distances in millimetre. Table 1 shows the values of the deviation measured by CloudCompare. This comparison used 1133 points to verify the surface with the original glass panel.

Figure 3: Panel 1 Geometry Deviation Scalar Field

Figure 4: Panel 1 Approximate Distances in Millimetres
### Table 1: Deviation Domain and Amount of Measured Values

<table>
<thead>
<tr>
<th>Amount of Measured Points</th>
<th>Domain Start</th>
<th>Domain End</th>
</tr>
</thead>
<tbody>
<tr>
<td>126</td>
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</tr>
<tr>
<td>156</td>
<td>-0.983</td>
<td>0.197</td>
</tr>
<tr>
<td>191</td>
<td>0.198</td>
<td>1.380</td>
</tr>
<tr>
<td>121</td>
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<tr>
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<td>2.563</td>
<td>3.745</td>
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</tr>
<tr>
<td>166</td>
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</tr>
</tbody>
</table>

#### 2.1. Summary

The comparison shows insufficient slumping in the middle of the panel and over-slumping at one of the five edges. The domain of the deviation lies between -2.16 and +6.11 and has an average deviation of 2.21 millimetres. The edges show a very pleasing result, however the middle of the panel shows insufficient slumping. However connecting the panel together the edges are acceptable, but the middle of the panel should slump more in order to acceptable.
3. Panel 2

Figure 5 shows the measured values and the approximate distances in millimetre. Table 2 shows the values of the deviation measured by CloudCompare. This comparison used 883 points to verify the surface with the original glass panel.

Figure 5: Panel 2 Geometry Deviation Scalar Field

Figure 6: Panel 2 Approximate Distances in Millimetres
Table 2: Deviation Domain and Amount of Measured Values

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<tr>
<th>Amount of Measured Points</th>
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<tbody>
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</tr>
<tr>
<td>58</td>
<td>-2.099</td>
<td>-0.689</td>
</tr>
<tr>
<td>183</td>
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<tr>
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<td>128</td>
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<td>138</td>
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<td>103</td>
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</tr>
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<td>88</td>
<td>6.669</td>
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</tr>
</tbody>
</table>

3.1. Summary

The comparison shows insufficient slumping in the middle of the panel and over-slumping at one of the four edges. The domain of the deviation lies between -3.51 and +6.36 and has an average deviation of 2.50 millimetres. The edges show a very pleasing result, however the middle of the panel shows insufficient slumping. However connecting the panel together the edges are acceptable, but the middle of the panel should slump more in order to acceptable.
4. Panel 3

Figure 7 shows the measured values and the approximate distances in millimetre. Table 3 shows the values of the deviation measured by CloudCompare. This comparison used 2643 points to verify the surface with the original glass panel.

Figure 7: Panel 3 Geometry Deviation Scalar Field

Figure 8: Panel 3 Approximate Distances in Millimetres
Table 3: Deviation Domain and Amount of Measured Values

<table>
<thead>
<tr>
<th>Amount of Measured Points</th>
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<td>88</td>
<td>6.366</td>
<td>7.778</td>
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</tbody>
</table>

4.1. Conclusion

The comparison shows insufficient slumping at the edges of the panel. The domain of the deviation lies between -3.11 and +4.97 and has an average deviation of 2.97 millimetres. The edges show a disappointing result, however the middle of the panel shows sufficient slumping.
5. **Panel 4**

Figure 9 shows the measured values and the approximate distances in millimetre. Table 4 shows the values of the deviation measured by CloudCompare. This comparison used 440 points to verify the surface with the original glass panel.

**Figure 9: Panel 4 Geometry Deviation Scalar Field**

**Figure 10: Panel 4 Approximate Distances in Millimetres**
Table 4: Deviation Domain and Amount of Measured Values

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<td>22</td>
<td>5.166</td>
<td>6.503</td>
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</table>

5.1. Conclusion
The comparison shows an overall acceptable slumping, only one of the four edges show insufficient slumping. The domain of the deviation lies between -4.19 and +5.16 and has an average deviation of 1.87 millimetres. The edges and the middle show a very pleasing result with sufficient slumping.
6. **Panel 5**

Figure 11 shows the measured values and the approximate distances in millimetre. Table 5 shows the values of the deviation measured by CloudCompare. This comparison used 900 points to verify the surface with the original glass panel.

![Panel 4 Geometry Deviation Scalar Field](image1)

**Figure 11: Panel 4 Geometry Deviation Scalar Field**

![Panel 4 Approximate Distances in Millimetres](image2)

**Figure 12: Panel 4 Approximate Distances in Millimetres**
Table 5: Deviation Domain and Amount of Measured Values

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<tr>
<td>60</td>
<td>6.806</td>
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</table>

6.1 Conclusion
The comparison shows insufficient slumping in the middle of the panel and over-slumping at one of the four edges. The domain of the deviation lies between -9.33 and +6.80 and has an average deviation of 3.38 millimetres. The edges show insufficient slumping and the middle shows average slumping.
7. Panel 6

Figure 13 shows the measured values and the approximate distances in millimetre. Table 6 shows the values of the deviation measured by CloudCompare. This comparison used 506 points to verify the surface with the original glass panel.

Figure 13: Panel 4 Geometry Deviation Scalar Field

Figure 14: Panel 4 Approximate Distances in Millimetres
Table 6: Deviation Domain and Amount of Measured Values

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<td>13</td>
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<td>-1.398</td>
</tr>
<tr>
<td>30</td>
<td>-1.398</td>
<td>-0.713</td>
</tr>
<tr>
<td>800</td>
<td>-0.714</td>
<td>-0.029</td>
</tr>
<tr>
<td>155</td>
<td>-0.030</td>
<td>0.652</td>
</tr>
<tr>
<td>129</td>
<td>0.653</td>
<td>1.336</td>
</tr>
<tr>
<td>66</td>
<td>1.337</td>
<td>2.020</td>
</tr>
<tr>
<td>31</td>
<td>2.021</td>
<td>2.706</td>
</tr>
</tbody>
</table>

7.1. Conclusion
The comparison shows insufficient slumping in the middle of the panel and over-slumping at one of the four edges. The domain of the deviation lies between -2.76 and +2.02 and has an average deviation of 0.71 millimetres. The edges show a very pleasing result, however the middle of the panel shows insufficient slumping. However connecting the panel together the edges are acceptable, but the middle of the panel should slump more in order to acceptable.
8. Conclusion

Overall it is to conclude that with an average deviation of 2.279 millimetre the outcome of the panels is acceptable. Panel 1, 2 and 3 show insufficient slumping in the middle of the panel. Panel 4, 5 and 6 show insufficient slumping at the edges. It is difficult to make a statement about insufficient slumping in the middle of the panel. However the domain of the deviation lies between -9.33 and 6.80 millimetre, which is unacceptable. Table 7 shows the deviation domain of the six panels and the average measured deviation.

**Table 7: Original Geometry Compared to Physical Model**

<table>
<thead>
<tr>
<th>Panel Number</th>
<th>Negative Deviation</th>
<th>Positive Deviation</th>
<th>Average</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-2.1659</td>
<td>6.111208</td>
<td>2.21886</td>
</tr>
<tr>
<td>2</td>
<td>-3.5107</td>
<td>6.3669</td>
<td>2.50454</td>
</tr>
<tr>
<td>3</td>
<td>-3.11837</td>
<td>4.972077</td>
<td>2.97112</td>
</tr>
<tr>
<td>4</td>
<td>-4.19052</td>
<td>5.166499</td>
<td>1.87649</td>
</tr>
<tr>
<td>5</td>
<td>-9.33821</td>
<td>6.80685</td>
<td>3.38492</td>
</tr>
<tr>
<td>6</td>
<td>-2.76729</td>
<td>2.021958</td>
<td>0.719627</td>
</tr>
</tbody>
</table>
APPENDIX IV

FINITE ELEMENT ANALYSIS REPORT
Finite Element Analysis of Warm Bent Glass

Appendix IV for: Warm Bent Glass: Development of a Flexible Mould
Eindhoven University of Technology

October, 2015
1. Introduction

This document is a combined report of the Finite Element Analysis outcome of the six unique glass panels used to construct the Glass Dome.

The FEA used to simulate the behaviour of the glass while submitted to high temperatures and deformation is a simplified model from the physical world. There are no external variables present in the model and the glass panel sag onto the deformed mould rather than slowly being pushed in the right shape. The temperature is set as an internal boundary condition instead of the heat being transferred from the heat source to the glass.

The simulation has been done with the use of the software Marc Mentat. The ‘unfolded’ glass panels, produced with the software of Prodim, is used as original surface. The outcome of the surface, the bended glass panel, is compared with the original designed geometry. The comparison between the two geometries is done with the software CloudCompare.

This reports covers the six unique panels and shows the deviation per glass panel.

2. Method of Analysis

The simulation is run as a Non-Linear analysis since there is time and temperature dependent. A simple linear analysis is insufficient for a representative outcome.

3. Material Characteristics

Table 1: Physical properties of soda lime silica glass (SLSG)

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Density</td>
<td>$\rho$ Kg/m$^3$</td>
</tr>
<tr>
<td>Knoop hardness</td>
<td>$H_v$ GPa</td>
</tr>
<tr>
<td>Young’s modulus</td>
<td>$E$ MPa</td>
</tr>
<tr>
<td>Poisson’s ratio</td>
<td>$\nu$ -</td>
</tr>
<tr>
<td>Coefficient of thermal expansion</td>
<td>$\alpha_T$ 10$^{-6}$ K$^{-1}$</td>
</tr>
<tr>
<td>Specific thermal capacity</td>
<td>$c_p$ Jkg$^{-1}$ K$^{-1}$</td>
</tr>
<tr>
<td>Thermal conductivity</td>
<td>$\lambda$ W m$^{-1}$ K$^{-1}$</td>
</tr>
<tr>
<td>Average refractive index within the visible spectrum</td>
<td>$n$ -</td>
</tr>
<tr>
<td>Emissivity (corrected)</td>
<td>$\xi$ -</td>
</tr>
</tbody>
</table>
4. Panel 1

The following paragraphs describe the Finite Element Analysis of glass panel 1 used to construct the Glass Dome. The bended glass panel is compared with the original designed geometry and with the physical bended model.

4.1. Description of the Geometry

The glass panel is unrolled by using the Prodim software. The unfolded glass panel is used to bend it in the correct shape. Figure 1 shows the original geometry and the unfolded geometry.

![Figure 1: Unfolded Geometry of Glass Panel #](image)

4.2. Load Cases

There are three load cases introduced in the model. The first is the complete fixation of the mould surface, this load or constrain keeps the surface in place and does not move during the simulation.

The second load case is gravity on the glass panel. This lets the glass panel sag onto the mould, as it will do during the bending process.

The third load case is the internal temperature of the glass panel. The temperature is set to 630°C. The temperature has no influence on the mould surface, but only affects the glass panel itself. This temperature simulates the subjected temperature as in the bending process.
4.3. Stresses

Figure 4 shows the predicted stress levels inside the glass after the warm bending process\(^1\).

4.4. Analysis Results

Figure 4 shows the comparison between the original designed 3D geometry and the FEA outcome. The outcome shows a large similarity with the original geometry. The negative and positive deviation lies between -8 and +7 millimetre. Figure 5 shows that this deviation occurs at the edges and at one small point inside the analysis. The average deviation of the FEA outcome compared with the original geometry is 0.62 millimetre.

\(^1\) The stress is calculated in the software provided by Prodim.
Figure 4: CloudCompare with CAD Model

Figure 5: CloudCompare result Graphic

Table 2: FEA Outcome Compared with Original 3D Geometry

<table>
<thead>
<tr>
<th>Amount of Measured Values</th>
<th>Domain Start</th>
<th>Domain End</th>
</tr>
</thead>
<tbody>
<tr>
<td>12</td>
<td>-8.03</td>
<td>-5.90</td>
</tr>
<tr>
<td>15</td>
<td>-5.90</td>
<td>-3.77</td>
</tr>
<tr>
<td>223</td>
<td>-3.77</td>
<td>-1.64</td>
</tr>
<tr>
<td>627</td>
<td>-1.64</td>
<td>0.49</td>
</tr>
<tr>
<td>203</td>
<td>0.49</td>
<td>2.61</td>
</tr>
<tr>
<td>2</td>
<td>2.61</td>
<td>4.74</td>
</tr>
<tr>
<td>4</td>
<td>4.74</td>
<td>6.87</td>
</tr>
<tr>
<td>12</td>
<td>6.87</td>
<td>9.00</td>
</tr>
</tbody>
</table>
4.5. Comparison with Physical Model

Figure 6 shows the deviation in millimetre of the FEA outcome compared with the physical model. The major difference between the physical model is the insufficient slumping. It is clear that the highest deviation occurs in the middle of the panel.

The comparison between the two panels used 1008 points. The table below shows the values within the domain.

<table>
<thead>
<tr>
<th>Amount of Measured Values</th>
<th>Domain Start</th>
<th>Domain End</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-7.90</td>
<td>0.05</td>
</tr>
<tr>
<td>5</td>
<td>-5.92</td>
<td>2.04</td>
</tr>
<tr>
<td>10</td>
<td>-3.93</td>
<td>4.03</td>
</tr>
<tr>
<td>15</td>
<td>-1.94</td>
<td>6.02</td>
</tr>
</tbody>
</table>

Figure 6: CloudCompare with Physical Model

Figure 7: CloudCompare Result Graphic

Table 3: FEA Outcome Compared with Physical Model
4.6. Summary

The analysis shows a very large similarity compared with the original surface, but shows a large deviation compared to the physical bended glass panel. The insufficient slumping is not noticeable in the simulation, but it is to note that the physical model shows insufficient slumping in the middle of the panel. The predicted insufficient slumping, the notable insufficient slumping, occurs at the same location, see figure 4 and 6.
5. Panel 2

The following paragraphs describe the Finite Element Analysis of glass panel 2 used to construct the Glass Dome. The bended glass panel is compared with the original designed geometry and with the physical bended model.

5.1. Description of the Geometry

The glass panel is unrolled by using the Prodim software. The unfolded glass panel is used to bend it in the correct shape. Figure 8 shows the original geometry and the unfolded geometry.

![Figure 8: Unfolded Geometry of Glass Panel #](image)

5.2. Load Cases

There are three load cases introduced in the model. The first is the complete fixation of the mould surface, this load or constrain keeps the surface in place and does not move during the simulation.

The second load case is gravity on the glass panel. This lets the glass panel sag onto the mould, as it will do during the bending process.

The third load case is the internal temperature of the glass panel. The temperature is set to 630°C. The temperature has no influence on the mould surface, but only affects the glass panel itself. This temperature simulates the subjected temperature as in the bending process.
5.3. Stresses

Figure 10 shows the predicted stress levels inside the glass after the warm bending process\(^2\).

5.4. Analysis Results

Figure 11 shows the comparison between the original designed 3D geometry and the FEA outcome. The outcome shows a large similarity with the original geometry. The negative and positive deviation lies between -7 and +6.1 millimetre. Figure 10 shows that this deviation occurs at the

\(^2\) The stress is calculated in the software provided by Prodim.
edges and at one small point inside the analysis. The average deviation of the FEA outcome compared with the original geometry is 1.69 millimetre.

**Table 4:** FEA Outcome Compared with Original 3D Geometry

<table>
<thead>
<tr>
<th>Amount of Measured Values</th>
<th>Domain Start</th>
<th>Domain End</th>
</tr>
</thead>
<tbody>
<tr>
<td>14</td>
<td>-6.75</td>
<td>-4.90</td>
</tr>
<tr>
<td>16</td>
<td>-4.90</td>
<td>-3.06</td>
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<tr>
<td>372</td>
<td>-3.06</td>
<td>-1.22</td>
</tr>
<tr>
<td>228</td>
<td>-1.22</td>
<td>0.63</td>
</tr>
<tr>
<td>15</td>
<td>0.63</td>
<td>2.47</td>
</tr>
<tr>
<td>13</td>
<td>2.47</td>
<td>4.31</td>
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<tr>
<td>3</td>
<td>4.31</td>
<td>6.16</td>
</tr>
<tr>
<td>19</td>
<td>6.16</td>
<td>8.00</td>
</tr>
</tbody>
</table>

**Figure 11:** Cloudcompare with CAD Model

**Figure 12:** Cloudcompare result Graphic
5.5. **Comparison with Physical Model**

Figure 13 shows the deviation in millimetre of the FEA outcome compared with the physical model. The major difference between the physical model is the insufficient slumping. It is clear that the highest deviation occurs at the edges of the panel.

The comparison between the two panels used 1133 points. The table below shows the values within the domain.

![Cloudcompare with Physical Model](image1)

**Figure 13: Cloudcompare with Physical Model**

![CloudCompare Result Graphic](image2)

**Figure 14: CloudCompare Result Graphic**
Table 5: FEA Outcome Compared with Physical Model

<table>
<thead>
<tr>
<th>Amount of Measured Values</th>
<th>Domain Start</th>
<th>Domain End</th>
</tr>
</thead>
<tbody>
<tr>
<td>29</td>
<td>-6.96</td>
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<tr>
<td>30</td>
<td>-5.92</td>
<td>-3.93</td>
</tr>
<tr>
<td>145</td>
<td>-3.93</td>
<td>-1.94</td>
</tr>
<tr>
<td>254</td>
<td>-1.94</td>
<td>0.05</td>
</tr>
<tr>
<td>173</td>
<td>0.05</td>
<td>2.03</td>
</tr>
<tr>
<td>223</td>
<td>2.03</td>
<td>4.02</td>
</tr>
<tr>
<td>224</td>
<td>4.02</td>
<td>6.01</td>
</tr>
<tr>
<td>55</td>
<td>5.52</td>
<td>8.00</td>
</tr>
</tbody>
</table>

5.6. Summary

The FEA outcome shows great similarity with the original geometry. The stress prediction has a slight similarity with the insufficient slumping that is notable in figure 13. The FEA outcome shows insufficient slumping at one of the four edges, the physical glass panel shows insufficient slumping at the same side, but with less deviation.
6. Panel 3

The following paragraphs describe the Finite Element Analysis of glass panel 3 used to construct the Glass Dome. The bended glass panel is compared with the original designed geometry and with the physical bended model.

6.1. Description of the Geometry

The glass panel is unrolled by using the Prodim software. The unfolded glass panel is used to bend it in the correct shape. Figure 15 shows the original geometry and the unfolded geometry.

![Unfolded Geometry of Glass Panel #](image)

6.2. Load Cases

There are three load cases introduced in the model. The first is the complete fixation of the mould surface, this load or constrain keeps the surface in place and does not move during the simulation.

The second load case is gravity on the glass panel. This lets the glass panel sag onto the mould, as it will do during the bending process.

The third load case is the internal temperature of the glass panel. The temperature is set to 630°C. The temperature has no influence on the mould surface, but only affects the glass panel itself. This temperature simulates the subjected temperature as in the bending process.
6.3. Stresses

Figure 17 shows the predicted stress levels inside the glass after the warm bending process\(^3\).

6.4. Analysis Results

Figure 18 shows the comparison between the original designed 3D geometry and the FEA outcome. The outcome shows a large similarity with the original geometry. The negative and positive deviation lies between -0.7 and +5.15 millimetre. Figure 18 shows that this deviation occurs at the

\(^3\) The stress is calculated in the software provided by Prodim.
edges and at one small point inside the analysis. The average deviation of the FEA outcome compared with the original geometry is 1.5 millimetre.

Figure 18: Cloudcompare with CAD Model

Figure 19: Cloudcompare result Graphic

Table 6: FEA Outcome Compared with Original 3D Geometry

<table>
<thead>
<tr>
<th>Amount of Measured Values</th>
<th>Domain Start</th>
<th>Domain End</th>
</tr>
</thead>
<tbody>
<tr>
<td>15</td>
<td>-0.75</td>
<td>0.10</td>
</tr>
<tr>
<td>128</td>
<td>0.10</td>
<td>0.94</td>
</tr>
<tr>
<td>230</td>
<td>0.94</td>
<td>1.78</td>
</tr>
<tr>
<td>156</td>
<td>1.78</td>
<td>2.63</td>
</tr>
<tr>
<td>76</td>
<td>2.63</td>
<td>3.47</td>
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<tr>
<td>2</td>
<td>3.47</td>
<td>4.31</td>
</tr>
<tr>
<td>7</td>
<td>4.31</td>
<td>5.16</td>
</tr>
<tr>
<td>32</td>
<td>5.16</td>
<td>6.00</td>
</tr>
</tbody>
</table>
6.5. Comparison with Physical Model

Figure 20 shows the deviation in millimetre of the FEA outcome compared with the physical model. The major difference between the physical model is the insufficient slumping. It is clear that the highest deviation occurs in the middle of the panel.

The comparison between the two panels used 1133 points. The table below shows the values within the domain.

Figure 20: Cloudcompare with Physical Model

Figure 21: CloudCompare Result Graphic
### Table 7: FEA Outcome Compared with Physical Model

<table>
<thead>
<tr>
<th>Amount of Measured Values</th>
<th>Domain Start</th>
<th>Domain End</th>
</tr>
</thead>
<tbody>
<tr>
<td>29</td>
<td>-5.65</td>
<td>-5.92</td>
</tr>
<tr>
<td>30</td>
<td>-5.92</td>
<td>-3.93</td>
</tr>
<tr>
<td>145</td>
<td>-3.93</td>
<td>-1.94</td>
</tr>
<tr>
<td>254</td>
<td>-1.94</td>
<td>0.05</td>
</tr>
<tr>
<td>173</td>
<td>0.00</td>
<td>2.03</td>
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<tr>
<td>223</td>
<td>2.03</td>
<td>4.02</td>
</tr>
<tr>
<td>224</td>
<td>4.02</td>
<td>6.01</td>
</tr>
<tr>
<td>55</td>
<td>5.43</td>
<td>8.00</td>
</tr>
</tbody>
</table>

### 6.6. Summary

The analysis shows a very large comparison with the original surface and with the physical outcome. The slumping occurs in the middle of the panel. However the domain of the deviation is different if the comparisons are compared with each other. The physical glass panel shows a bigger deviation than the outcome of the FEA.
7. Panel 4

The following paragraphs describe the Finite Element Analysis of glass panel 4 used to construct the Glass Dome. The bended glass panel is compared with the original designed geometry and with the physical bended model.

7.1. Description of the Geometry

The glass panel is unrolled by using the Prodim software. The unfolded glass panel is used to bend it in the correct shape. Figure 22 shows the original geometry and the unfolded geometry.

![Figure 22: Unfolded Geometry of Glass Panel #](image)

7.2. Load Cases

There are three load cases introduced in the model. The first is the complete fixation of the mould surface, this load or constrain keeps the surface in place and does not move during the simulation.

The second load case is gravity on the glass panel. This lets the glass panel sag onto the mould, as it will do during the bending process.

The third load case is the internal temperature of the glass panel. The temperature is set to 630°C. The temperature has no influence on the mould surface, but only affects the glass panel itself. This temperature simulates the subjected temperature as in the bending process.
7.3. Stresses

Figure 24 shows the predicted stress levels inside the glass after the warm bending process.\(^4\)

7.4. Analysis Results

Figure 25 shows the comparison between the original designed 3D geometry and the FEA outcome. The outcome shows a large similarity with the original geometry. The negative and positive

\(^4\) The stress is calculated in the software provided by Prodim.
deviation lies between -8 and +6 millimetre. Figure 25 shows that this deviation occurs at the edges and at one small point inside the analysis. The average deviation of the FEA outcome compared with the original geometry is 2.21 millimetre.

Figure 25: *Cloudcompare with CAD Model*

Figure 26: *Cloudcompare result Graphic*
### Table 8: FEA Outcome Compared with Original 3D Geometry

<table>
<thead>
<tr>
<th>Amount of Measured Values</th>
<th>Domain Start</th>
<th>Domain End</th>
</tr>
</thead>
<tbody>
<tr>
<td>18</td>
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<tr>
<td>22</td>
<td>-5.91</td>
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<td>31</td>
<td>-3.92</td>
<td>-1.94</td>
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<tr>
<td>94</td>
<td>-1.94</td>
<td>0.05</td>
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<tr>
<td>145</td>
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<td>77</td>
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<td>4.03</td>
</tr>
<tr>
<td>33</td>
<td>4.03</td>
<td>6.01</td>
</tr>
<tr>
<td>20</td>
<td>6.01</td>
<td>8.00</td>
</tr>
</tbody>
</table>

#### 7.5. Comparison with Physical Model

Figure 27 shows the deviation in millimetre of the FEA outcome compared with the physical model. The major difference between the physical model is the insufficient slumping. It is clear that the highest deviation occurs in the middle of the panel.

The comparison between the two panels used 1004 points. The table below shows the values within the domain.

![Cloudcompare with Physical Model](image)

**Figure 27: Cloudcompare with Physical Model**
7.6. Summary

The FEA outcome shows insufficient slumping at three sides. The comparison with the physical model shows a deviation at two sides. The deviation at the right sides of the panel, shows the largest negative deviation. This deviation is comparable to the outcome of the FEA.

Table 9: FEA Outcome Compared with Physical Model

<table>
<thead>
<tr>
<th>Amount of Measured Values</th>
<th>Domain Start</th>
<th>Domain End</th>
</tr>
</thead>
<tbody>
<tr>
<td>29</td>
<td>-4.56</td>
<td>-5.92</td>
</tr>
<tr>
<td>30</td>
<td>-5.92</td>
<td>-3.93</td>
</tr>
<tr>
<td>75</td>
<td>-3.93</td>
<td>-1.94</td>
</tr>
<tr>
<td>344</td>
<td>-1.94</td>
<td>0.05</td>
</tr>
<tr>
<td>24</td>
<td>0.00</td>
<td>2.03</td>
</tr>
<tr>
<td>223</td>
<td>2.03</td>
<td>4.02</td>
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<tr>
<td>224</td>
<td>4.02</td>
<td>6.01</td>
</tr>
<tr>
<td>55</td>
<td>4.53</td>
<td>8.00</td>
</tr>
</tbody>
</table>

Figure 28: CloudCompare Result Graphic

Approximate Distances in mm
8. Panel 5

The following paragraphs describe the Finite Element Analysis of glass panel 5 used to construct the Glass Dome. The bended glass panel is compared with the original designed geometry and with the physical bended model.

8.1. Description of the Geometry

The glass panel is unrolled by using the Prodim software. The unfolded glass panel is used to bend it in the correct shape. Figure 29 shows the original geometry and the unfolded geometry.

Figure 29: Unfolded Geometry of Glass Panel #

8.2. Load Cases

There are three load cases introduced in the model. The first is the complete fixation of the mould surface, this load or constrain keeps the surface in place and does not move during the simulation.

The second load case is gravity on the glass panel. This lets the glass panel sag onto the mould, as it will do during the bending process.

The third load case is the internal temperature of the glass panel. The temperature is set to 630°C. The temperature has no influence on the mould surface, but only affects the glass panel itself. This temperature simulates the subjected temperature as in the bending process.
8.3. Stresses

Figure 31 shows the predicted stress levels inside the glass after the warm bending process\(^5\).

---

\(^5\) The stress is calculated in the software provided by Prodim.
8.4. Analysis Results

Figure 32 shows the comparison between the original designed 3D geometry and the FEA outcome. The outcome shows a large similarity with the original geometry. The negative and positive deviation lies between -8 and +7 millimetre. Figure 32 shows that this deviation occurs at the edges and at one small point inside the analysis. The average deviation of the FEA outcome compared with the original geometry is 0.62 millimetre.

![Figure 32: CloudCompare with CAD Model](image)

![Figure 33: CloudCompare result Graphic](image)
<table>
<thead>
<tr>
<th>Amount of Measured Values</th>
<th>Domain Start</th>
<th>Domain End</th>
</tr>
</thead>
<tbody>
<tr>
<td>84</td>
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</tr>
<tr>
<td>111</td>
<td>-4.44</td>
<td>-2.95</td>
</tr>
<tr>
<td>100</td>
<td>-2.94</td>
<td>-1.46</td>
</tr>
<tr>
<td>133</td>
<td>-1.45</td>
<td>0.03</td>
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<td>88</td>
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<tr>
<td>51</td>
<td>1.53</td>
<td>3.02</td>
</tr>
<tr>
<td>64</td>
<td>3.02</td>
<td>4.51</td>
</tr>
<tr>
<td>269</td>
<td>4.51</td>
<td>6.00</td>
</tr>
</tbody>
</table>

8.5. Comparison with Physical Model

Figure 34 shows the deviation in millimetre of the FEA outcome compared with the physical model. The major difference between the physical model is the insufficient slumping. It is clear that the highest deviation occurs in the middle of the panel.

The comparison between the two panels used 532 points. The table below shows the values within the domain.

![CloudCompare with Physical Model](image_url)
### Table 11: FEA Outcome Compared with Physical Model

<table>
<thead>
<tr>
<th>Amount of Measured Values</th>
<th>Domain Start</th>
<th>Domain End</th>
</tr>
</thead>
<tbody>
<tr>
<td>40</td>
<td>-7.99</td>
<td>-5.99</td>
</tr>
<tr>
<td>61</td>
<td>-5.99</td>
<td>-4.00</td>
</tr>
<tr>
<td>57</td>
<td>-4.00</td>
<td>-2.00</td>
</tr>
<tr>
<td>56</td>
<td>-2.00</td>
<td>0.00</td>
</tr>
<tr>
<td>47</td>
<td>0.00</td>
<td>2.00</td>
</tr>
<tr>
<td>148</td>
<td>2.00</td>
<td>4.00</td>
</tr>
<tr>
<td>54</td>
<td>4.00</td>
<td>6.00</td>
</tr>
<tr>
<td>69</td>
<td>6.00</td>
<td>8.00</td>
</tr>
</tbody>
</table>

8.6. **Summary**

The FEA outcome and the physical model, both show large similarity in deviation compared to the original panel. The stress prediction shows the same location of insufficient slumping as the both outcomes of validation.
9. Panel 6

The following paragraphs describe the Finite Element Analysis of glass panel 6 used to construct the Glass Dome. The bended glass panel is compared with the original designed geometry and with the physical bended model.

9.1. Description of the Geometry

The glass panel is unrolled by using the Prodim software. The unfolded glass panel is used to bend it in the correct shape. Figure 36 shows the original geometry and the unfolded geometry.

9.2. Load Cases

There are three load cases introduced in the model. The first is the complete fixation of the mould surface, this load or constrain keeps the surface in place and does not move during the simulation.

The second load case is gravity on the glass panel. This lets the glass panel sag onto the mould, as it will do during the bending process.

The third load case is the internal temperature of the glass panel. The temperature is set to 630°C. The temperature has no influence on the mould surface, but only affects the glass panel itself. This temperature simulates the subjected temperature as in the bending process.
9.3. Stresses

Figure 38 shows the predicted stress levels inside the glass after the warm bending process\textsuperscript{6}.

9.4. Analysis Results

Figure 39 shows the comparison between the original designed 3D geometry and the FEA outcome. The outcome shows a large similarity with the original geometry. The negative and positive deviation lies between -5 and +6 millimetre. Figure 39 shows that this deviation occurs at the edges and at one small point inside the analysis. The average deviation of the FEA outcome compared with the original geometry is 0.97 millimetre.

\textsuperscript{6} The stress is calculated in the software provided by Prodim.
Figure 39: *Cloudcompare with CAD Model*

![Cloudcompare result Graphic](image)

**Figure 40: CloudCompare result Graphic**

**Table 12**: FEA Outcome Compared with Original 3D Geometry

<table>
<thead>
<tr>
<th>Amount of Measured Values</th>
<th>Domain Start</th>
<th>Domain End</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-7.11</td>
<td>-7.10</td>
</tr>
<tr>
<td>0</td>
<td>-5.22</td>
<td>-5.21</td>
</tr>
<tr>
<td>98</td>
<td>-3.33</td>
<td>-3.33</td>
</tr>
<tr>
<td>208</td>
<td>-1.44</td>
<td>-1.44</td>
</tr>
<tr>
<td>162</td>
<td>0.45</td>
<td>0.45</td>
</tr>
<tr>
<td>32</td>
<td>2.33</td>
<td>2.34</td>
</tr>
<tr>
<td>1</td>
<td>4.22</td>
<td>4.23</td>
</tr>
<tr>
<td>4</td>
<td>6.11</td>
<td>6.12</td>
</tr>
</tbody>
</table>
9.5. Comparison with Physical Model

Figure 41 shows the deviation in millimetre of the FEA outcome compared with the physical model. The major difference between the physical model is the insufficient slumping. It is clear that the highest deviation occurs in the middle of the panel.

The comparison between the two panels used 2480 points. The table below shows the values within the domain.

![Figure 41: Cloudcompare with Physical Model](image)

![Figure 42: CloudCompare Result Graphic](image)
Table 13: FEA Outcome Compared with Physical Model

<table>
<thead>
<tr>
<th>Amount of Measured Values</th>
<th>Domain Start</th>
<th>Domain End</th>
</tr>
</thead>
<tbody>
<tr>
<td>127</td>
<td>-4.99</td>
<td>-3.74</td>
</tr>
<tr>
<td>102</td>
<td>-3.74</td>
<td>-2.49</td>
</tr>
<tr>
<td>158</td>
<td>-2.49</td>
<td>-1.25</td>
</tr>
<tr>
<td>245</td>
<td>-1.25</td>
<td>0.00</td>
</tr>
<tr>
<td>394</td>
<td>0.00</td>
<td>1.25</td>
</tr>
<tr>
<td>303</td>
<td>1.25</td>
<td>2.50</td>
</tr>
<tr>
<td>182</td>
<td>2.50</td>
<td>3.75</td>
</tr>
<tr>
<td>1096</td>
<td>3.75</td>
<td>5.00</td>
</tr>
</tbody>
</table>

9.6. Summary

Panel 6 shows insufficient slumping at the edges of the panel. The average deviation is 3.22 millimetre. The deviation domain however is -5 to +3.75 millimetre. The edges show in both comparisons the most deviation. The FEA outcome shows less deviation than the physical glass panel.
Conclusion

The analysis shows a very large comparison with the original surface, but shows a large deviation compared to the physical bended glass panel. The insufficient slumping is not noticeable in the simulation, but it is to note that the physical model shows insufficient slumping. The deviation is comparable with the deviation of the original geometry compared with the physical model. The deviation between the original geometry and the physical model is comparable with the comparison between the finite element analysis and physical model. This concludes the bending process differs from the finite element analysis.
APPENDIX V
RADIUS SLUMPING REPORT
Radius Test
of Warm Bent Glass

Appendix V for:
Warm Bent Glass: Development of a Flexible Mould
Eindhoven University of Technology

October, 2015
1. Introduction

This document is a combined report of the Radius Test of five different panels with each a different radius. The test is conducted in order to determine the amount of slumping during the bending process.

The glass panels used are 35 × 35 centimetres and bended with a radius ranging from R1000 to R 1800. Each panel is measured after the bending process and the radius is measured of the complete surface. This average radius is used to compare it with its original given radius.

The outcome of the report should make a statement about the slumping process during the bending process.

This reports covers the five unique panels and shows the deviation from the original radius per glass panel.

2. Radius Test

Table 1 shows the deviation results of the five unique panels. The most negative and positive value is measured to show the domain of the deviation (see figure 1). Figure 2 shows the measured radius of the five unique panels, it shows an overall insufficient slumping. Furthermore it shows that the smaller the radius the better the slumping is and the domain of the deviation is smaller.

<table>
<thead>
<tr>
<th>Radius</th>
<th>Measured Radius</th>
<th>Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Negative</td>
</tr>
<tr>
<td>1000</td>
<td>844</td>
<td>-0.959717</td>
</tr>
<tr>
<td>1200</td>
<td>914</td>
<td>-7.401986</td>
</tr>
<tr>
<td>1400</td>
<td>992</td>
<td>-5.185419</td>
</tr>
<tr>
<td>1600</td>
<td>1175</td>
<td>-10.578926</td>
</tr>
<tr>
<td>1800</td>
<td>1326</td>
<td>-11.823936</td>
</tr>
</tbody>
</table>
Figure 1: Domain of Deviation in mm

Figure 2: Original Radius versus Measured Radius
3. Panel 1, R1000

Panel 1 is the smallest radius created, the radius of 1000 millimetres.

3.1. Comparison with original geometry

Figure 3 and 4 shows the scalar field outcome comparison with the original surface and the domain between the measured values and the deviation. Table 2 is part of Figure 4 which shows the exact domain versus the measured values.
Table 2: FEA Outcome Compared with Physical Model

<table>
<thead>
<tr>
<th>Amount of Measured Values</th>
<th>Domain Start</th>
<th>Domain End</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.00</td>
<td>-0.96</td>
<td>-0.27</td>
</tr>
<tr>
<td>4.00</td>
<td>-0.27</td>
<td>0.42</td>
</tr>
<tr>
<td>5.00</td>
<td>0.42</td>
<td>1.11</td>
</tr>
<tr>
<td>4.00</td>
<td>1.11</td>
<td>1.80</td>
</tr>
<tr>
<td>7.00</td>
<td>1.80</td>
<td>2.49</td>
</tr>
<tr>
<td>7.00</td>
<td>2.49</td>
<td>3.19</td>
</tr>
<tr>
<td>16.00</td>
<td>3.19</td>
<td>3.88</td>
</tr>
<tr>
<td>20.00</td>
<td>3.88</td>
<td>4.57</td>
</tr>
</tbody>
</table>

3.2. Summary

The overall measured radius is R844 this is smaller than the given R1000. The radius shows over slumping, this could also be caused by the insulation layer that pushes the glass more in the radius. The glass panels shows a positive value of almost 4 millimetre. The edges however only show a deviation of less than -1 millimetre.
4. Panel 2, R1200

Panel 2 has a radius of 1200 millimetres.

4.1. Comparison with original geometry

Figure 5 and 6 shows the scalar field outcome comparison with the original surface and the domain between the measured values and the deviation. Table 3 is part of Figure 6 which shows the exact domain versus the measured values.

![CloudCompare Result Scalar Field](image1)

![CloudCompare Result Graph](image2)

**Figure 5:** CloudCompare Result Scalar Field

**Figure 6:** CloudCompare Result Graph
Table 3: FEA Outcome Compared with Physical Model

<table>
<thead>
<tr>
<th>Amount of Measured Values</th>
<th>Domain Start</th>
<th>Domain End</th>
</tr>
</thead>
<tbody>
<tr>
<td>9</td>
<td>-7.40</td>
<td>-6.49</td>
</tr>
<tr>
<td>15</td>
<td>-6.49</td>
<td>-5.59</td>
</tr>
<tr>
<td>9</td>
<td>-5.59</td>
<td>-4.68</td>
</tr>
<tr>
<td>8</td>
<td>-4.68</td>
<td>-3.77</td>
</tr>
<tr>
<td>7</td>
<td>-3.77</td>
<td>-2.86</td>
</tr>
<tr>
<td>6</td>
<td>-2.86</td>
<td>-1.96</td>
</tr>
<tr>
<td>6</td>
<td>-1.96</td>
<td>-1.05</td>
</tr>
<tr>
<td>6</td>
<td>-1.05</td>
<td>-0.14</td>
</tr>
</tbody>
</table>

4.2. Summary

The overall measured radius is R914 this is smaller than the given R1200. The radius shows over slumping, this could also be caused by the insulation layer that pushes the glass more in the radius. The glass panels shows a positive value of almost -7 millimetre. The edges however only show a deviation of less than -1 millimetre.
5. Panel 3, R1400

Panel 3 has a radius of 1400 millimetres.

5.1. Comparison with original geometry

Figure 7 and 8 shows the scalar field outcome comparison with the original surface and the domain between the measured values and the deviation. Table 4 is part of Figure 8 which shows the exact domain versus the measured values.

**Figure 7:** CloudCompare Result Scalar Field

**Figure 8:** CloudCompare Result Graph
Table 4: FEA Outcome Compared with Physical Model

<table>
<thead>
<tr>
<th>Amount of Measured Values</th>
<th>Domain Start</th>
<th>Domain End</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>-5.19</td>
<td>-4.32</td>
</tr>
<tr>
<td>9</td>
<td>-4.32</td>
<td>-3.45</td>
</tr>
<tr>
<td>6</td>
<td>-3.45</td>
<td>-2.59</td>
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<tr>
<td>4</td>
<td>-2.59</td>
<td>-1.72</td>
</tr>
<tr>
<td>3</td>
<td>-1.72</td>
<td>-0.85</td>
</tr>
<tr>
<td>2</td>
<td>-0.85</td>
<td>0.01</td>
</tr>
<tr>
<td>3</td>
<td>0.01</td>
<td>0.88</td>
</tr>
<tr>
<td>1</td>
<td>0.88</td>
<td>1.75</td>
</tr>
</tbody>
</table>

5.2. Conclusion

The overall measured radius is R992 this is smaller than the given R1400. The radius shows over slumping, this could also be caused by the insulation layer that pushes the glass more in the radius. The glass panels shows a positive value of almost -5 millimetre. The edges however only show a deviation of less than -1 millimetre.
6. Panel 4, R1600

Panel 4 has a radius of 1600 millimetres.

6.1. Comparison with original geometry

Figure 9 and 10 shows the scalar field outcome comparison with the original surface and the domain between the measured values and the deviation. Table 5 is part of Figure 10 which shows the exact domain versus the measured values.

Figure 9: CloudCompare Result Scalar Field

Figure 10: CloudCompare Result Graph
**Table 5:** FEA Outcome Compared with Physical Model

<table>
<thead>
<tr>
<th>Amount of Measured Values</th>
<th>Domain Start</th>
<th>Domain End</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>-10.58</td>
<td>-9.28</td>
</tr>
<tr>
<td>8</td>
<td>-9.28</td>
<td>-7.99</td>
</tr>
<tr>
<td>3</td>
<td>-7.99</td>
<td>-6.69</td>
</tr>
<tr>
<td>4</td>
<td>-6.69</td>
<td>-5.39</td>
</tr>
<tr>
<td>4</td>
<td>-5.39</td>
<td>-4.10</td>
</tr>
<tr>
<td>1</td>
<td>-4.10</td>
<td>-2.80</td>
</tr>
<tr>
<td>4</td>
<td>-2.80</td>
<td>-1.50</td>
</tr>
<tr>
<td>4</td>
<td>-1.50</td>
<td>-0.21</td>
</tr>
</tbody>
</table>

6.2. **Summary**

The overall measured radius is R1175 this is smaller than the given R1600. The radius shows over slumping, this could also be caused by the insulation layer that pushes the glass more in the radius. The glass panels shows a positive value of almost -10 millimetre. The edges however only show a deviation of less than -1.5 millimetre.
7. Panel 5, R1800

Panel 5 has a radius of 1800 millimetres.

7.1. Comparison with original geometry

Figure 11 and 12 shows the scalar field outcome comparison with the original surface and the domain between the measured values and the deviation. Table 6 is part of Figure 12 which shows the exact domain versus the measured values.

![Figure 11: CloudCompare Result Scalar Field](image1)

![Figure 12: CloudCompare Result Graph](image2)
Table 6: FEA Outcome Compared with Physical Model

<table>
<thead>
<tr>
<th>Amount of Measured Values</th>
<th>Domain Start</th>
<th>Domain End</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value</td>
<td>-11.82</td>
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</tr>
<tr>
<td>72</td>
<td>-10.46</td>
<td>-9.10</td>
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<tr>
<td>71</td>
<td>-9.10</td>
<td>-7.74</td>
</tr>
<tr>
<td>42</td>
<td>-7.74</td>
<td>-6.38</td>
</tr>
<tr>
<td>38</td>
<td>-6.38</td>
<td>-5.02</td>
</tr>
<tr>
<td>31</td>
<td>-5.02</td>
<td>-3.66</td>
</tr>
<tr>
<td>43</td>
<td>-3.66</td>
<td>-2.30</td>
</tr>
<tr>
<td>40</td>
<td>-2.30</td>
<td>-0.94</td>
</tr>
</tbody>
</table>

7.2. Summary

The overall measured radius is R1326 this is smaller than the given R1800. The radius shows over slumping, this could also be caused by the insulation layer that pushes the glass more in the radius. The glass panels shows a positive value of almost -11 millimetre. The edges however only show a deviation of less than -2 millimetre.
8. Conclusion

The radius test shows a very large deviation domain. Furthermore it shows over-slumping instead of insufficient slumping, what was expected. The edges show an acceptable result, however the middle, or the centre, of the panel shows a smaller radius. Previous test, for example the Glass Dome panels, show insufficient slumping. Figure 1 and 2 show the diverging lines in the graphs. The bigger the radius, the bigger the deviation domain an the smaller the radius.
APPENDIX VI
REPETITION REPORT
Repetition Test
of Warm Bent Glass

Appendix I for:
Warm Bent Glass: Development of a Flexible Mould
Eindhoven University of Technology

October, 2015
1. **Introduction**

This document is a combined report of the Repetition Test of five panels with all the same radius. The test is conducted in order to determine the amount of slumping during the bending process and if the same test has the same outcome.

The glass panels used are $35 \times 35$ centimetres and bended with a radius of R1000. Each panel is measured after the bending process and the radius is measured of the complete surface. This average radius is used to compare it with its original given radius.

The outcome of the report should make a statement about the repetition process during the bending process.

This reports covers the five panels and shows the deviation from the original radius per glass panel.

2. **Repetition Test**

Table 1 shows the deviation results of the five panels. The most negative and positive value is measured to show the domain of the deviation (see figure 1). Figure 2 shows the measured radius of the five panels, it shows an overall insufficient slumping. Furthermore it shows that the smaller the radius the better the slumping is and the domain of the deviation is smaller.

<table>
<thead>
<tr>
<th>Radius</th>
<th>Measured Radius</th>
<th>Deviation</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1000</td>
<td>858</td>
<td>-8,5950</td>
<td>6,902</td>
<td>1,44723</td>
<td></td>
</tr>
<tr>
<td>1000</td>
<td>810</td>
<td>-3,491197</td>
<td>0,058941</td>
<td>1,52788</td>
<td></td>
</tr>
<tr>
<td>1000</td>
<td>860</td>
<td>-2,463899</td>
<td>0,101106</td>
<td>0,944975</td>
<td></td>
</tr>
<tr>
<td>1000</td>
<td>897</td>
<td>-1,029375</td>
<td>8,615161</td>
<td>2,8592</td>
<td></td>
</tr>
<tr>
<td>1000</td>
<td>883</td>
<td>0</td>
<td>6,63387</td>
<td>2,51877</td>
<td></td>
</tr>
</tbody>
</table>
Figure 1: Domain of Deviation in mm

Figure 2: Original Radius versus Measured Radius
3. Panel 1, R1000

3.1. Comparison with original geometry

Figure 3 and 4 shows the scalar field outcome comparison with the original surface and the domain between the measured values and the deviation. Table 2 is part of Figure 4 which shows the exact domain versus the measured values.

Figure 3: CloudCompare Result Scalar Field

Figure 4: CloudCompare Result Graph
### Table 2: FEA Outcome Compared with Physical Model

<table>
<thead>
<tr>
<th>Amount of Measured Values</th>
<th>Domain Start</th>
<th>Domain End</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>-8.60</td>
<td>-6.66</td>
</tr>
<tr>
<td>14</td>
<td>-6.66</td>
<td>-4.72</td>
</tr>
<tr>
<td>13</td>
<td>-4.72</td>
<td>-2.78</td>
</tr>
<tr>
<td>168</td>
<td>-2.78</td>
<td>-0.85</td>
</tr>
<tr>
<td>125</td>
<td>-0.85</td>
<td>1.09</td>
</tr>
<tr>
<td>26</td>
<td>1.09</td>
<td>3.03</td>
</tr>
<tr>
<td>6</td>
<td>3.03</td>
<td>4.97</td>
</tr>
<tr>
<td>2</td>
<td>4.97</td>
<td>6.90</td>
</tr>
</tbody>
</table>

3.2. **Summary**

The overall measured radius is R858 this is smaller than the given R1000. The radius shows over slumping, this could also be caused by the insulation layer that pushes the glass more in the radius. The glass panels shows a positive value of almost 7 millimetre. The edges however only show a deviation of less than 1 millimetre.
4. Panel 2, R1000

4.1. Comparison with original geometry

Figure 5 and 6 shows the scalar field outcome comparison with the original surface and the domain between the measured values and the deviation. Table 3 is part of Figure 6 which shows the exact domain versus the measured values.

Figure 5: CloudCompare Result Scalar Field

Figure 6: CloudCompare Result Graph
### Table 3: FEA Outcome Compared with Physical Model

<table>
<thead>
<tr>
<th>Amount of Measured Values</th>
<th>Domain Start</th>
<th>Domain End</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>-3.49</td>
<td>-2.98</td>
</tr>
<tr>
<td>6</td>
<td>-2.98</td>
<td>-2.48</td>
</tr>
<tr>
<td>14</td>
<td>-2.48</td>
<td>-1.97</td>
</tr>
<tr>
<td>7</td>
<td>-1.97</td>
<td>-1.46</td>
</tr>
<tr>
<td>13</td>
<td>-1.46</td>
<td>-0.96</td>
</tr>
<tr>
<td>7</td>
<td>-0.96</td>
<td>-0.45</td>
</tr>
<tr>
<td>7</td>
<td>-0.45</td>
<td>0.06</td>
</tr>
<tr>
<td>4</td>
<td>0.06</td>
<td>0.57</td>
</tr>
</tbody>
</table>

#### 4.2. Summary

The overall measured radius is R810 this is smaller than the given R1000. The radius shows over slumping, this could also be caused by the insulation layer that pushes the glass more in the radius. The glass panels shows a positive value of almost 0.5 millimetre. The edges however only show a deviation of less than 0.25 millimetre.
5. Panel 3, R1000

5.1. Comparison with original geometry
Figure 7 and 8 shows the scalar field outcome comparison with the original surface and the domain between the measured values and the deviation. Table 4 is part of Figure 8 which shows the exact domain versus the measured values.

![Figure 7: CloudCompare Result Scalar Field](image1)

![Figure 8: CloudCompare Result Graph](image2)
**Table 4:** FEA Outcome Compared with Physical Model

<table>
<thead>
<tr>
<th>Amount of Measured Values</th>
<th>Domain Start</th>
<th>Domain End</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>-2.46</td>
<td>-2.10</td>
</tr>
<tr>
<td>7</td>
<td>-2.10</td>
<td>-1.73</td>
</tr>
<tr>
<td>7</td>
<td>-1.73</td>
<td>-1.36</td>
</tr>
<tr>
<td>10</td>
<td>-1.36</td>
<td>-1.00</td>
</tr>
<tr>
<td>13</td>
<td>-1.00</td>
<td>-0.63</td>
</tr>
<tr>
<td>11</td>
<td>-0.63</td>
<td>-0.27</td>
</tr>
<tr>
<td>10</td>
<td>-0.27</td>
<td>0.10</td>
</tr>
<tr>
<td>1</td>
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5.2. Summary

The overall measured radius is 860mm, this is smaller than the given 1000mm. The radius shows over slumping, this could also be caused by the insulation layer that pushes the glass more in the radius. The glass panels show a positive value of almost 2.5mm. The edges however only show a deviation of less than 0.1mm.
6. Panel 4, R1000

6.1. Comparison with original geometry
Figure 9 and 10 shows the scalar field outcome comparison with the original surface and the domain between the measured values and the deviation. Table 5 is part of Figure 10 which shows the exact domain versus the measured values.

Figure 9: CloudCompare Result Scalar Field

Figure 10: CloudCompare Result Graph
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<td>0.09</td>
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</table>

6.2. Summary

The overall measured radius is R897 this is smaller than the given R1000. The radius shows over slumping, this could also be caused by the insulation layer that pushes the glass more in the radius. The glass panels shows a positive value of almost 8 millimetre. The edges however only show a deviation of less than 1 millimetre.
7. Panel 5, R1000

7.1. Comparison with original geometry

Figure 11 and 12 shows the scalar field outcome comparison with the original surface and the domain between the measured values and the deviation. Table 6 is part of Figure 12 which shows the exact domain versus the measured values.

Figure 11: CloudCompare Result Scalar Field

Figure 12: CloudCompare Result Graph
### Table 6: FEA Outcome Compared with Physical Model

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<td>4.20</td>
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</table>

#### 7.2. Summary

The overall measured radius is R883 this is smaller than the given R1000. The radius shows over slumping, this could also be caused by the insulation layer that pushes the glass more in the radius. The glass panels shows a positive value of almost 6.5 millimetre. The edges however only show a deviation of less than 0.1 millimetre.
8. Conclusion

The Repetition Test shows that the average radius is close to each other. However the domain of deviation differs from each other. The bending process shows pleasing results however the accuracy should be improved in order to be more pleasing. It does not show insufficient slumping at the five panels. Some have insufficient slumping in the middle, others on the edges and panel 1 shows the best overall result but with a very big domain. The average deviation is 1.85 millimetre which is acceptable, however the domain of deviation is to big.
APPENDIX VII

IASS CONGRESS PAPER

(INTernational Association for SHELL and SPATIAL STRUCTURES)
Flexible mould by the use of spring steel mesh

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Abstract
The present paper describes the development of a digitally controlled flexible mould that forms a double curved smooth surface directly from a CAD model. The controlled surface consists of a plain woven spring steel mesh which adopts the desired smooth shape. The need for a flexible mould is to reduce the cost of double curved and smooth architecture. Most elements are produced with a static mould or semi-flexible mould, which are time consuming and expensive. This technique is feasible simple, cost efficient and can be used with different materials such as concrete, glass and composites.

Keywords: flexible mould, adjustable surface, free-form, double curved, fluid architecture.

1. Introduction
Today’s architecture is more contemporary than before and has a higher demand for free-form shapes. However the manufacturers lack the capability to actually produce the desired products designed by the architects [1]. With the advent of 3D software designing complex shapes became much easier, however the translation to a real object can be hard and expensive. The designers are forced to
simplify their design because of the lack of available production methods. Producing free-form structures requires segmentation into panels, which are either flat, single- or double curved [2].

There is a need for a mould that can take on any shape required. The key of the concept presented in this paper is to have a flexible surface formed into any given shape using a CAD model. This happens faster than traditionally, almost fully automatic and without waste of materials. The produced elements do not need any further treatments after production. Limitations to the possibilities of the flexible form are limited curvature and limited level of detail, making it especially suited for larger, double curved surfaces like facades or walls, where the curvature of each element is relatively small in comparison to the overall shape.

The present paper describes the development of a flexible mould for production of glass and concrete, which can have any desired form. The mould consists of pistons fixed to a woven mesh which creates an interpolated surface. The pistons can move up and down the take on the desired shape created from a CAD model.

2. Mould techniques
Moulding techniques for this purpose can be divided into three different categories, as in Van Helvoirt, 2005 [2]:

1. The static mould. The most common mould of this kind is the EPS/PS (polystyrene) mould which has been 3D formed using a CNC cutter. This technique has a high accuracy but produces a lot of waste material;

2. The reusable mould, for example a mould made of clay or sand. There is no waste material and therefore the method is a lot more environmental friendly. It is labour intensive and cannot be used in all circumstances; and

3. The flexible mould. This type of mould can be used for a lot of differently shaped elements because the mould will form itself to the elements that will be produced. Examples of this kind of mould are the FlexiMould by Boers [3], and the adjustable mould by Rietbergen & Vollers [4]. The mould contains a field of height-adjustable pins. Each pin can be set into height individually using a computer-automated machine. The pins are covered with a polymer to smoothen the surface. It is important to avoid that the pins give a local distortion of the surface. The downside of these flexible moulds is the high investment. These expanses can only be gained back when a large amount of elements will be produced using this particular mould.

2.1. Flexible mould
Making use of flexible mould scan results is a significant cost reduction within the manufacturing process. Yet these flexible moulds are not feasible for commercial use because of the relatively high costs (Munro, 2007). These costs can be reduced by designing a simple and inexpensive flexible mould. The number of actuators required has a great impact on the costs as shown in figure 1. However, the amount of actuators has a direct influence on the form freedom of the flexible mould. With a higher density of actuators, more extreme curvatures and more complex fluid forms can be produced. For architectural use, the curvature is limited. The radius of existing double curved
Constructions varies between 0.75m and 45m (Schipper, 2015), so a high density pin bed is not necessary. The balance between costs and necessary curvature should determine the density of actuators.

![Figure 1: Relation between density of actuators and investment costs (F.Gard, 2013).](image)

The surface curvatures can be classified in four categories as shown in figure 2: zeroelastic, monoclastic, synclastic and anticlastic. When using a tensioned flexible layer only zeroelastic and anticlastic forms can be made which has a major limitation on the form freedom. With the use of a flexible top layer with a higher density of pistons, all different surface curvatures can be made. When considering the aspects of costs and its form freedom, the flexible top layer has the best balance between these aspects and is the most suitable method for the bending of curved panels.

![Figure 2: Zeroelastic, monoclastic, synclastic and anticlastic surface (Pronk, 2012).](image)

3. 3Dflexmould
Avoiding high development costs that are involved with a pin bed mould and to avoid the problems with local distortion of the surface, the authors developed an adjustable mould without pins in the surface of the mould. Instead of pins a membrane is used with the capability to curve in two directions. The membrane is deformed by 25 pistons that move up and down and have a ball joint like connection to put no further stress in the membrane. With the combination of these two parts makes it
possible to create any desired shape. In this way the benefits of a flexible mould (adjustable to a lot of different elements) are combined with the benefits of a reusable mould (no waste materials and lower investment costs).

The developed flexible mould uses a spring steel mesh membrane that can take on any desired shape without losing its original shape. The shape will be set automatically in a form. Through the automation, a continuous production process of curved panels can be achieved. The Mould Design Tool developed for predicting the shape for the mould helps setting the mould in the correct shape. Figure 4 shows the mould in 3D and physical.

![Flexible Mould](image)

**Figure 3: Flexible Mould**

### 3.1. Spring steel mesh

The spring steel mesh is the key feature of the design. Spring steel has the characteristics to maintain its original shape even after deforming. The plain woven mesh is the most commonly used weave. Each weft wire passes alternately over and under each warp wire and vice versa. Warp and weft wire diameters are generally the same (see figure 4). Wires are crimped in the weaving operation. Two of the most important wire parameters are the wire diameter, $d$, and the mesh number, $M$. Mesh number is defined as number of openings in a linear inch measured from the centre of one wire to the centre of another.

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1 The Mould Design Tool (MDT) is an own developed program that allows the user to set the mould in the right shape.
a parallel wire in the direction parallel to the centre of the other group of wires. The plain woven steel mesh has identical mechanical properties in both weft wire and warp wire directions.

The mesh is woven from a 0.8 millimetre wire with an opening of 1.75 millimetre. Each side has 180 wires or the mesh number $M$.

The plain woven mesh consist of squares created by wraps and wefts by the steel wires. Shearing in a plain-weave steel occurs by the relative moment of two sets of steel wire, warp and weft, which are interlaced in a one-up and one-down. As the mesh is deformed the square starts to shear. The forces acting on the bar linkage can be resolved into shear and tensile force components acting perpendicular and parallel to the steel wire. At a small deformation the acting tensile force has a low magnitude which can be ignored.

Creating a computer simulation of the behaviour of the mesh requires a few adjustments and additions. To understand the behaviour the mesh is divided in $m \times n$ virtual masses (see figure 5). Each mass is linked to its neighbours by massless springs of natural length non equal to zero. The link between each neighbours is achieved in three different ways.

- Springs linking masses $[i, j]$ and $[i + 1, j]$, and masses $[i, j]$ and $[i, j + 1]$, will be referred to as “structural springs”;
- Springs linking masses $[i, j]$ and $[i + 1, j + 1]$, and masses $[i + 1, j]$ and $[i, j + 1]$ will be referred to as “shear springs”; and

![Figure 4: Plain woven mesh](image)
Springs linking masses \([i, j]\) and \([i + 2, j]\), and masses \([i, j]\) and \([i, j + 2]\), will be referred to as “flexion springs”.

Figure 5: Mass-spring connection

When there is pure shear stress inside the mesh only the “shear springs” are constrained; under pure flexion stresses only the “flexion springs” are constrained; whereas under pure compression or contraction stresses only the “structural springs” are constrained [5].

3.2. Control of the surface

The pistons controlling the meshes shape are made of three parts. An upper- and lower tube and a bending spring. The joint of the piston has a different inflection point at each setting. The middle piston can only move in the \(z\)-direction. The mesh will deform and follow the constraint point in the middle. The pistons form after the desired shape. Figure 6 shows an operating piston.
The mesh mass point’s $m$ must be seen as points in general. Recreating the surface after deformation is done by using NURBS splines and surfaces. NURBS are Non-Uniform Rational B-Splines. NURBS is a geometry that is easily manipulated by computer, allowing for great flexibility in modelling. NURBS can create smooth curves and surfaces with a small amount of points. The NURBS curves can be used to build a surface, define motion paths and control deformations.

The new generated surface is reconstructed by B-splines. The surface is then divided in a rectangular mesh. The intersections or knots are used as points to calculate the height of the pistons.
5. Results and Discussion
Validating the mould and the tool to control the desired shape a 3D measuring machine was used. The Prodim Proliner creates a point cloud that can be compared via software to verify the deviation. The precision of the machine is 0.8 millimetre.

The mould needs to be validated before any shape can be produced with the mould. By adjusting the pistons manually and measuring the surface a complete horizontal surface can be achieved. After running a simulation of the desired shape the surface is validated.

The result of a measurement can be seen in figure 9.1 and 9.2. The blue area shows a deviation of ± 1 millimetre, which is the most important area used in creating a curved surface. The edges of the mesh show a larger deviation due to insufficient support of pistons. The biggest fault error is the human error itself at this moment.
6. Conclusion
This paper discusses a study on the development of a flexible mould with the use of spring steel mesh membrane. The mould was used for bending glass, but can be used for different materials. Based on the study the following conclusion can be drawn:

- It was possible to create an accurate mould surface with a deviation of ± 1 millimetre with smooth curves and surfaces;
- The latest test results with bending float glass show a deviation of ± 3 millimetre, where insufficient slumping is the main error in;
- There is a limitation is different shape, but the hardest and most difficult shapes can be made with a high accuracy;

7. Future possibilities
The new production method has been put into the test and proves that it is possible to economically produce unique elements. It is possible to produce double-curved elements using a membrane mould. The unique elements show a deviation of +/- 3 millimetre in a small scale. The main advantages of the technique are the low costs and the simplicity of the mould. It is also possible to optimize the process by the introduction of computer-controlled positioning and the introduction of more pistons to control the surface better.

Acknowledgement
The authors would like to thank Brakel Atmos, Prodim, Solidrocks and VDNDP Bouwingenieurs for their support in this research.

References
[1] Artikel dat productie achterloopt op ontwerpen
APPENDIX IIX
PRODIM PROLINER CERTIFICATE
Certificate

Product: Proliner 10 IS Measuring device

Serial No.: ###-#####

Calibration No.: ProlinerIS Evaluation ###-##### (dd-mm-yyyy)

Test Results:
Average uncertainty of a reference length (HBT) 2,5m +/-0.027 mm
Standard deviation (HBT) 2,5m +/-0.055 mm (σ)
5,0m +/-0.037 mm (σ)

Average uncertainty of a reference length (LBT) 2,5m +/-0.018 mm
Standard deviation (LBT) 2,5m +/-0.108 mm (σ)
5,0m +/-0.046 mm (σ)

Single point repeatability (SPR) 2,5m +/-0.019 mm
Single point repeatability (SPR) 5,0m +/-0.029 mm

Certificate
We herewith confirm that the product described has been tested and complies with the specifications as stated above.
The test equipment used is traceable to national standards or to recognized procedures. This is established by our Quality Management System. Prodim test methods are described in the “Proliner® IS – Certification procedure”.
(based on ASME B89.4.19-2006)

Prodim International BV
Date: March 20, 2014

Technical Director: Inspector:
Test results: <this is a copy of the excel proliner is certification results>

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<td>Distance</td>
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| Avg. Dev.:          | **0.027** 0.063 | **0.018** 0.041 |
| Std. Dev. (σ):      | 0.055 0.037 | 0.108 0.046 |

* Uncertainty of the measurement: 10 µm + 4,5x10^-6 x l.
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