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Hydrodynamics and pressure drop of isopropanol/nitrogen flows in microchannels

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Hydrodynamics and pressure drop of isopropanol/nitrogen flows in microchannels

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Graduation report
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# Table of contents

Table of contents .............................................................................................................................. I

Summary ......................................................................................................................................... II

1. Introduction ..............................................................................................................................1

2. Two-phase flow patterns and flow maps .................................................................................4
   2.1 Two-phase flow patterns .................................................................................................4
   2.2 Flow maps .......................................................................................................................5

3. Taylor flow ..............................................................................................................................6
   3.1 Taylor flow .............................................................................................................................6
   3.2 Taylor flow model ..................................................................................................................6
   3.3 Previous research ...................................................................................................................9

4. Two-phase pressure drop .......................................................................................................10
   4.1 Homogeneous flow model ...................................................................................................15
   4.2 Lockhart-Martinelli ..............................................................................................................15
   4.3 Unit cell model .....................................................................................................................17
   4.4 The Kreutzer model ..............................................................................................................14

5. Experiments ...........................................................................................................................16
   5.1 Experimental set-up .............................................................................................................16
   5.2 Experimental procedure .......................................................................................................17

6. Results and discussion ...........................................................................................................19
   6.1 Taylor flow model ................................................................................................................19
   6.2 Pressure estimation ...............................................................................................................20
   6.3 Pressure drop .......................................................................................................................21
   6.4 Comparison with pressure drop models in literature ...........................................................22

7. Conclusions and recommendations ....................................................................................26
   7.1 Conclusions ..........................................................................................................................26
   7.2 Recommendations ................................................................................................................28

Literature ........................................................................................................................................29

Nomenclature ...................................................................................................................................31
Summary

A hydrodynamic study is done of a two-phase flow in a rectangular micro channel (100x50 µm) as part of the MiRAACS project. The aim of this project is to design a micro reactor with a micro porous catalytic coating on the channel walls as an alternative for conventional reactors. The hydrogenation of citral will be used as model reaction. Because the solvent used in this reaction is isopropanol and hydrogen will be diluted with, isopropanol is used as liquid phase and nitrogen as gas phase. In previous research on this subject, water and nitrogen were used in order to compare the results with literature.

The hydrodynamic study consists of making a flow pattern map in order to identify the various flow regimes obtained, determining the gas hold-up and the pressure drop in the channel. The flow regimes obtained in the micro channel using isopropanol are bubbly flow, annular flow, Taylor flow, churn flow, ring flow and Taylor-ring flow. The flow pattern map differs for isopropanol/nitrogen compared to water/nitrogen. A larger area of Taylor ring flow is obtained. This is due to a higher viscosity compared to water and a lower surface tension compared to water. From all flow regimes, Taylor flow is of most interest for multi-phase reactions. Taylor flow consists of a sequence of gas bubbles and liquid slugs. A thin liquid film is formed between the channel wall and the gas bubbles and liquid slugs.

At six different locations in the channel, movies are made using a microscope in combination with a high speed camera (10,000 frames per second). These movies are analyzed using Matlab to determine the bubble length ($L_b$), the slug length ($L_s$), the bubble frequency ($F_b$) and the bubble velocity ($u_b$). The gas hold-up, pressure and pressure drop at every location in the channel is determined from the experimentally obtained parameters using a mass balance based Taylor flow model.

The experimental pressure drop is compared to three different pressure drop models. First, the Lockhart-Martinelli pressure drop model is compared. This model is flow regime independent and is based on pressure drop of single gas and liquid flow which are connected through the so-called two-phase multiplier. Second, the unit cell model, which is based on one unit cell consisting of a gas bubble and a liquid slug. This model considers a frictional pressure drop over the liquid phase and a pressure drop over the bubble phase due to the velocity profile in the bubble. Third, the Kreutzer model which states that an additional pressure drop on top of the pressure drop according to Hagen-Posseuille flow in the liquid phase. This is due to the bubbles which disturb the Hagen-Posseuille velocity profile in the liquid slug.

From the experiments and the Taylor flow model, it is found that the liquid film thickness remains constant along the channel length and is independent on the bubble velocity. The gas hold-up increased as a function of the channel length. This indicates that expansion of the gas phase occurs. The pressure decreases as a function of the channel length due to frictional pressure losses. The pressure drop is successfully estimated from video analysis and the Taylor flow model. The Lockhart-Martinelli model overpredicts most of the experimental data. The model is flow pattern independent and therefore it does not take the specifications of Taylor flow into account. The unit cell model underpredicts the experimental data. The unit cell model considers the pressure drop over the gas bubble according to a velocity profile in the bubble. It does not take the disturbances of the Hagen-Posseuille velocity profile of the liquid slugs into account. The model of Kreutzer predicts the experimental data well. The same trend is found plotting the friction factor times the dimensionless slug length, it however a small overprediction is obtained. This is because Kreutzer neglects the film thickness for describing the bubble velocity.
1. Introduction

Micro reactors have lately been discussed in many publications on multiphase reactors. Due to their small size (per channel, a hydraulic diameter varying between 50 µm and 4 mm), micro reactors can have great advantages compared to conventional reactors (i.e. trickle bed reactor). A micro reactor is e.g. easy to scale up by increasing the amount of micro channels and fast mass transfer due to the short diffusion distance in the micro channels. A higher specific surface area is obtained in micro reactors which causes increasing heat and mass transfer from and to the reactor walls. The use of micro reactors has a wide applicability in the chemical process engineering. For a multiphase system, a gas and liquid are fed into the micro channels and the catalyst is mostly deposited onto the inside reactor wall.

This study is part of the Microstructured Reaction Architectures for Advanced Chemistry Synthesis (MiRAACS) project done at the Chemical Reactor Engineering group at the Eindhoven University of Technology. The final goal of this project is to design a micro reactor with a micro porous catalytic coating on the channel walls as an alternative for conventional reactors. The chosen model reaction is the hydrogenation of citral which is a commonly used reactant in the production of vitamins and fragrance compounds. The project is split into two parts; the first part is producing a bimetallic catalyst. This catalyst will be used to ensure the hydrogenation of the C=O bond. The second part is studying the hydrodynamics and mass transfer. In order to determine the mass transfer in the micro reactor, the hydrodynamics need to be studied first. In this study the focus is on the hydrodynamics.

A hydrodynamic study includes flow mapping and obtaining various hydrodynamic parameters such as gas hold-up ($\varepsilon_g$) and superficial gas and liquid velocities ($U_g$ and $U_l$). Flow mapping is a way of indicating what flow pattern occurs at a given combination of superficial gas and superficial liquid velocities. Various two-phase flow patterns are formed in the micro channels viz. annular flow, churn flow, ring flow, Taylor flow and bubbly flow. For studying the hydrodynamics, a horizontal micro channel of rectangular cross-section (100x50 µm) is used.

Taylor flow is of most interest for multiphase reactions in small channels (< 1 mm). It can be obtained at relative low superficial gas and liquid velocities (0.01 - 1 m/s). Liquid is flowing in slugs between the gas bubbles, which prevent the bubbles from coalescence and a thin liquid film is formed between the bubbles and the reactor wall. The gas bubbles are a few times longer then the diameter of the channel (~5 times). The thickness of the liquid film is an important parameter for describing the hydrodynamics of Taylor flow. The liquid film thickness is dependent on the capillary number ($Ca=\eta u/\sigma$) and Weber number ($We=\rho u^2D_{Hy}/\sigma$) and thus on the bubble velocity ($u_b$). For higher liquid velocities (order of m/s), the film thickness is less dependent on the bubble velocity. Another important parameter is the gas hold-up. The film thickness and gas hold-up are both difficult to determine using image analysis, because with image analysis only a 2-dimensional image is obtained. This 2-dimensional image does not predict the cross-sectional geometry of the bubble. The 3-dimensional image is thus not known. Therefore a mass balance based model was developed by Warnier. This Taylor flow model describes the gas hold-up ($\varepsilon_g$) as a function of the film thickness, the bubble and liquid slug lengths ($L_b$ and $L_s$), number of bubbles formed per unit time ($F_b$), liquid superficial velocity ($U_l$) and the ratio between the cross-sectional areas of the bubble and the channel ($A/A_b$). Knowing $A/A_b$, the exact geometry of the cross-sectional bubble area and the exact film thickness are not needed. The film thickness is also an important parameter for mass transfer since in Taylor flow three different regions for mass transfer are formed. The first region is from the bubble to the film layer, the second from the slug to the film layer and the third from the bubble into the slug.
Due to recirculation in the liquid slugs, constant refreshing of product and reactant is obtained from the film to the slug and vice versa.

For reaction design, the two-phase frictional pressure drop is an important parameter. Several correlations between the hydrodynamics and the pressure drop are used in literature. These correlations predict the two-phase frictional pressure drop. Most correlations are based on either the homogeneous flow model, which assumes no slip boundary between the gas and liquid. The Lockhart-Martinelli model estimates the two-phase pressure drop from the separate pressure drops of the gas and liquid phases. A so called two-phase multiplier is used to describe the relation of the pressure drop over the single liquid and single gas phases. However, these correlations are flow pattern independent and therefore do not always describe the experimental results for Taylor flow. More detailed models specific for Taylor flow, are the unit cell model and the Kreutzer model. The unit cell model is based on a single liquid slug and gas bubble, forming a unit cell. Circular channels are used with diameters varying between 50 µm and 530 µm. The superficial gas velocities vary between 0.02 m/s and 73 m/s and superficial liquid velocities between 0.01 m/s and 5.77 m/s. The Kreutzer model considers pressure differences at the front and the back of the bubble due to inertial forces. These pressure differences cause differences in curvature at the front and the back of the bubble. However, it neglects the film layer thickness in the channel. The results are obtained using a circular channel with a diameter of 2.3 mm and superficial gas and liquid velocities between 0.04 m/s and 0.3 m/s.

In previous work done on this project, nitrogen was chosen as gas phase and water as liquid phase. In literature nitrogen and water are the most common gas and liquid phases used. The superficial gas velocities are varied between 0.5 m/s to 2.8 m/s and superficial liquid velocities between 0.1 m/s to 0.6 m/s. From this work, a design of the mixing part of the micro reactor was made. Flow patterns were compared to literature. The ratio of the cross-sectional areas of the bubble and the channel (A/A_b) as a function of slug length (L_s) and liquid velocity (U_l) divided by the bubble frequency (F_b) was investigated. It was concluded the liquid film thickness is not a function of the bubble velocity for a nitrogen/water system. The pressure is calculated for the nitrogen/water system. However, no comparison to the pressure drop models in literature is made.

Isopropanol is now chosen as the liquid phase because isopropanol is used as a solvent for the hydrogenation of citral. The objective of this graduation work is to characterize the hydrodynamics including making a flow pattern map of the nitrogen/isopropanol system and determining the gas hold-up, the local gas velocity and the pressure drop in the channel. The approach used is to first characterize Taylor flow through flow mapping (verifying different flow patterns going with fixed gas and liquid velocities) for the nitrogen/isopropanol system. Within the Taylor flow regime movies are made (using a high speed camera and a microscope) at different locations in the channel. The hydrodynamic parameters (bubble and slug lengths, bubble velocity and number of bubbles formed per unit time) are obtained from image processing. The gas hold-up and A/A_b are calculated using the Taylor flow model of Warnier (2006). Measuring the pressure drop with the use of sensors in micro channels is difficult due to their small sizes (D_h = 67 µm). In this study, the pressure at a certain location in the channel is calculated. The ideal gas law and the superficial local gas velocity (U_g) obtained from image analysis and the Taylor flow model is used to calculate the pressure. Finally, the pressure drop can be estimated along the channel length using the calculated pressures at various locations in the channel. The estimated pressure drop is compared to the Lockhart-Martinelli model, the unit cell model and the Kreutzer model.
Outline

Chapter 2 discusses the different flow patterns which are observed in micro channels. Flow maps obtained in previous research within this project using water/nitrogen are compared to flow maps for this work using isopropanol/nitrogen.

Chapter 3 describes Taylor flow and the Taylor flow model by Warnier.

Chapter 4 is dedicated to pressure drop. The pressure drop models and the application of these models on this work will be described.

Chapter 5 describes the experiments done and the experimental set-up.

Chapter 6 gives an overview of the results obtained on the hydrodynamics and pressure drop and the discussion of these results.

Chapter 7 contains the conclusions made and gives some recommendations for further research.
**Figure 2.1:** Flow patterns in 1 mm round channel using air and water. Bubbly flow is obtained at $U_i=3$ m/s and $U_g=0.08$, annular at $U_i=0.08$ and $U_g=73$ m/s, Taylor flow at $U_i=0.2$ m/s and $U_g=0.15$ m/s, churn flow at $U_i=1.2$ m/s and $U_g=4.6$ m/s and ring flow at $U_i=0.04$ m/s and $U_g=4$ m/s$^3$. 
2. Two-phase flow patterns and flow maps

Various flow patterns can be obtained in micro channels. This chapter describes the different flow regimes obtained using isopropanol as liquid phase and nitrogen as gas phase. An overview is given using a flow map for this system which is compared to the water and nitrogen system used in previous research\(^9\),\(^10\).

2.1 Two-phase flow patterns

The flow patterns obtained, using isopropanol and nitrogen, are annular flow, churn flow, ring flow, Taylor flow and bubbly flow. In the transition region a combination of flow patterns can be obtained e.g. Taylor/ring flow.

**Bubbly flow**

Bubbly flow, shown in figure 2.1(a), consists of small bubbles with a diameter smaller than the channel diameter. The flow occurs at high liquid velocities \((U_l > 2 \, \text{m/s})\) and low gas velocities \((U_g = 3 \, \text{m/s})\). In previous work done, bubbly flow was only observed at the beginning of the channel\(^10\).

**Annular flow**

At relatively low liquid velocities \((U_l = 0.01 \, \text{m/s})\) compared to gas velocities \((U_g = 2 \, \text{m/s})\), annular flow occurs. Annular flow is characterized by a continuous gas core surrounded by a liquid film between the gas core and capillary wall as shown in figure 2.1(b). The diameter of the gas core changes with varying gas and liquid velocities. With increasing gas velocity or decreasing the liquid velocity the diameter will increase, while with decreasing gas velocity and increasing liquid velocity the diameter will decrease.

**Taylor flow**

Taylor flow, shown in figure 2.1(c), consists of a sequence of gas bubbles and liquid slugs which prevent the gas bubbles to coalescence. A thin liquid film is formed between the channel wall and the gas bubbles and liquid slugs. Recirculation patterns occur in the liquid slugs at \(Ca<0.5^5\). Taylor flow is also called slug flow, bubble-train flow or plug flow.

**Churn flow**

At both large gas velocities \((U_g = 50 \, \text{m/s})\) and liquid velocities \((U_g = 0.8 \, \text{m/s})\), churn flow occurs which is shown in figure 2.1(d). A wavy structure is formed at the gas-liquid interface and small gas bubbles appear in the liquid film. Churn flow can originate from two different processes: by disturbing the end of the gas bubbles in Taylor flow or by disturbing the continuous gas core in annular flow\(^11\).

**Ring flow**

When the gas velocity is decreased from churn flow, ring flow appears as shown in figure 2.1(e). Large amplitude liquid waves occur and in the liquid film constricting the continuous gas core. Ring flow has only been observed in micro channels.

**Taylor-ring flow**

The combination of Taylor flow and ring flow is called Taylor-ring flow. Gas bubbles separated by liquid slugs occur at one moment, while at the other moment gas bubbles coalescence and Taylor/ring flow occurs. This phenomenon occurs at irregular time intervals. Because this
**Figure 2.2:** Mixer designs: smooth mixer and cross-shaped mixer.

**Figure 2.3:** Flow maps for nitrogen-water in cross shaped mixer at the beginning and end of the channel.

**Figure 2.4:** Flow patterns obtained in 100x50µm channel using isopropanol/nitrogen including annular flow, churn flow, ring flow, Taylor flow and bubbly flow.
Two-phase flow patterns and flow maps

A combination of flow does not become either Taylor flow or ring flow it has been classified in a separate flow regime.

2.2 Flow maps

Flow maps are an indication what flow pattern occurs at any combination of liquid gas velocities. The gas velocities are given at the x-axes and the liquid velocities at the y-axes. These flow maps are used to determine the flow regime at a certain combination of gas and liquid velocities.

In previous research done in the MiRAACS project, flow maps were made for a nitrogen/water system. Different mixer geometries were used, the smooth mixer and cross shaped mixer (see figure 2.2). It was concluded that mixer geometry has an influence on the flow map. At low liquid velocities the flow maps do not differ much because surface tension forces are dominating. At higher velocities, especially liquid velocities, inertial forces are dominating which causes a difference in transition boundaries. Therefore, the direction of the liquid flow onto the gas flow is important since that produces the gas bubble. For the cross shaped mixer, flow maps are made at the beginning and the end of a 50x100µm channel (figure 2.3). The difference observed was the additional flow regime (bubbly flow) at the beginning of the channel. This is due to the pressure drop along the channel which causes expansion of the gas bubble. Gas bubbles grow and Taylor flow occurs instead of bubbly flow.

Since in this graduation work isopropanol and nitrogen are used, a flow map for this system is made. At various combinations of gas and liquid velocities video images are made. A 10.000 frames per second camera, a microscope and Redlake MiDAS software are used. The images are played back to visually determine the flow pattern at the combination of gas and liquid velocities. From previous research it is concluded that with nitrogen/water the flow maps at the beginning and the end of the channel do not differ much. Due to the small differences in flow maps at the end and beginning of the channel, in this study a flow map is made at the end of the channel. Figure 2.4 shows the flow patterns observed in the set-up used, and figure 2.5 shows the flow map made. The gas velocities given at the x-axes are given at standard conditions (20°C and 1 bar) since the exact gas velocity at the measurement location is dependent on pressure and thus unknown. All flows patterns described in section 2.1 are observed using isopropanol and nitrogen. At relatively low liquid velocities ($U_l<0.04$ m/s) and gas velocities ($U_g<4$ m/s) annular flow occurs. Increasing the gas velocity, ring flow occurs. At liquid velocities above 0.2 m/s and gas velocities above 20 m/s churn flow occurs. Decreasing the gas velocity below 3 m/s Taylor flow is observed. At higher liquid velocities than the Taylor flow regime ($U_l>1$ m/s), bubbly flow appears. Taylor/ring flow occurs as transition flow between every flow pattern with increasing gas velocity except for the transition of bubbly flow to churn flow and annular flow to ring flow.

Comparing the flow map of isopropanol/nitrogen (iso-pOH/N$_2$) to the flow map of water/nitrogen (H$_2$O/N$_2$), differences are observed. This is caused by the differences in viscosity and surface tension of water and isopropanol (table 2.1). Bubbly flow was not observed at the beginning of the channel for H$_2$O/N$_2$ but is observed for iso-pOH/N$_2$. At lower liquid velocities ($U_l<0.2$ m/s) increasing the gas velocity leads to ring flow, which is easier to obtain from a Taylor/ring flow then an annular or churn flow. This leads to a larger Taylor/ring flow region compared to the flow map of H$_2$O/N$_2$. Due to this large range of gas and liquid velocities for which occurs Taylor/ring flow, the area of Taylor flow and annular flow become smaller. Churn flow occurs in the same range for iso-pOH/N$_2$ and H$_2$O/N$_2$ since at higher velocities inertia becomes more important then surface tension. The flow map for isopropanol/nitrogen is used within this graduation work to determine the ranges of gas and liquid velocities for which Taylor flow occurs. The Taylor flow regime is used for further research on hydrodynamics and pressure drop.
Figure 2.5: Flow map for nitrogen-isopropanol in cross shaped mixer at the end of the channel.

<table>
<thead>
<tr>
<th>Viscosity m(Pa·s)</th>
<th>Surface tension (mN/m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Water</td>
<td>1</td>
</tr>
<tr>
<td>Isopropanol</td>
<td>2.27</td>
</tr>
</tbody>
</table>

Table 2.1: Viscosity and surface tension of water and isopropanol.

Figure 3.1: Three regions for gas component mass transfer with Taylor flow. (1) directly through the film layer, (2) through the bubble cap into the liquid slug, (3) from the liquid slug through the film layer.

Figure 3.2: Transforming of the bubble interface at three different regions: a cap region ab, a transition region bc and a uniform film region cd. Capillary numbers displaced are a: 0.15, b: 0.3 and c: 0.6².
3. Taylor flow

Taylor flow is the most applied flow pattern for multiphase reactions in micro channels. Therefore in this chapter the details of Taylor flow are described and is discussed why it is more advantageous over other regimes. To obtain the hydrodynamic parameters e.g. gas hold-up \((\varepsilon_g)\) and the local gas velocity \((U_g)\), a Taylor flow model is used. The Taylor flow model is described in the second section of this chapter. Previous results on the Taylor flow model are described in the last section of this chapter.

3.1 Taylor flow

Taylor flow consists of a sequence of gas bubbles and liquid slugs which prevent the gas bubbles to coalescence. The bubbles almost fill the cross-sectional channel area and only a thin liquid film separates the channel wall from the gas bubbles. The length of the bubbles is several times the channel diameter (2 to 15 times) depending on the ingoing gas and liquid velocities and pressure.

For mass transfer limited three-phase reactions, Taylor flow is the most suitable regime due to its characteristics. Often the catalyst is deposited on the channel wall. In Taylor flow there are three regions for gas component mass transfer due to its configuration of gas bubbles, liquid slugs and the liquid film, as shown in figure 3.1: first directly through the film layer to the catalyst, secondly through the bubble cap into the liquid slug and third from the liquid slug through the film layer to the catalyst. When the capillary number, which describes the ratio of viscous forces versus surface tension, is smaller then 0.5 recirculation in the liquid slugs occurs\(^5\) (figure 3.2). This recirculation causes a constant refreshing of the liquid film with reactant and diffusion of the product into the liquid slug takes place. Due to the small size of the micro channels and the small thickness of the liquid film, a short diffusion distance to the catalyst is obtained.

The flow patterns described in chapter 2.1 are less suitable for gas-liquid reactions in micro channels compared to Taylor flow. Bubbly flow has a longer diffusion path from the gas bubble to the catalyst. Annular flow has less contact area of the gas and liquid. This can cause saturation of the liquid film and no reaction can take place anymore. Churn flow and ring flow have both not the advantage of refreshing of the liquid film and diffusion of the product. High liquid and gas velocities are required to obtain these flow patterns. These large velocities require more power input and less residence time.

3.2 Taylor flow model

The thickness of the liquid film is an important parameter for mass transfer. The liquid film determines the diffusion distance to the wall. Another important parameter is the gas hold-up. It determines the mean residence times of both phases in the micro channel. However, these parameters are difficult to obtain using rectangular channels since the cross-sectional area of the gas bubble in the channel is unknown. Due to the liquid film, both gas bubbles and liquid slugs flow through a smaller cross-sectional area. A different gas hold-up is then obtained with respect to the cross sectional area of the channel. For large channels, the gas hold-up is normally described as a function of the gas flow quality. It is assumed that no slip occurs between the gas and liquid phase.

\[
\beta_g = \frac{U_g}{U_l + U_g}
\]

(3.1)
**Figure 3.3:** Recirculation cells formed in the liquid slug surrounded by a thin liquid film at $\text{Ca}<0.5^{25}$.

**Figure 3.4:** Unit cell consisting of a gas bubble and a liquid slug.
However, if slip does occur, the gas hold-up is not equal to the gas flow quality. In many models, the gas flow quality is then multiplied by a fitting parameter $C$.

$$
\varepsilon_g = C \frac{U_g}{U_l + U_g}
$$

(3.2)

Armand (1946) found a value of 0.833 for $C$ using water and nitrogen in horizontal cylindrical tubes of 26 mm in diameter.

To obtain the film thickness and the gas hold-up, a mass balance based Taylor flow model for horizontal channels was made by Warnier. The model describes the cross-sectional area of the gas bubble relative to the channel cross-sectional area ($A/A_b$), the gas hold-up ($\varepsilon_g$) as a function of the bubble ($L_b$) and liquid slug lengths ($L_s$), liquid superficial velocity ($U_l$) and the number of bubbles formed per unit time ($F_b$). Measuring a pressure using a pressure sensor in micro channels (100x50 µm) with a two-phase system can cause artifacts due to in and outgoing effects of the sensor. Therefore, with the parameters obtained from the Taylor flow model, a pressure at a certain location in the channel is calculated.

Three main assumptions have been made for using the Taylor flow model:

- A uniform continuous liquid film surrounds the gas bubbles and the liquid slugs throughout the channel length. As described before, at Ca<0.5 circulation cells occur in the liquid slugs. With regard to these recirculation cells and velocity profiles, a thin liquid film separates the recirculation cells from the wall shown in figure 3.3.
- There is no flow in the liquid film for horizontal channels. The boundary conditions for this flow are that there is no slip at the channel wall and no shear stress between the liquid and the gas phase. No pressure gradient in the liquid film in the uniform bubble region was reported which eliminates another source for flow in the liquid film.
- There is no variation in gas bubble and liquid slug sizes at a certain location in the channel. Due to the compressibility of the gas phase and pressure drop along the channel length, gas bubbles will vary in volume along the channel length. However, for a certain location in the channel, all bubbles and slugs passing that location have the same volume.

The train of gas bubbles and liquid slugs is divided into unit cells consisting of one gas bubble, one liquid slug and the liquid film surrounding both gas bubble and liquid slug, shown in figure 3.4. A mass balance is made over one unit cell. The total volume of one unit cell ($V_{uc}$) is equal to the cross-sectional are of the channel ($A$) times the sum of the bubble length ($L_b$) and the slug length ($L_s$):

$$
V_{uc} = A(L_s + L_b)
$$

(3.3)

The volume of the liquid film is then:

$$
V_f = (A - A_b)(L_s + L_b)
$$

(3.4)

where $A_b$ is the area of the bubble cross-section.

The volume of the liquid slug is equal to the volumetric flow rate ($U_l A$) divided by the bubble frequency ($F_b$):
Figure 3.5: Liquid around the nose and tail of the bubble presented as $\delta^{10}$. 
Taylor flow

\[ V_t = \frac{U_t A}{F_b} \]  \hspace{1cm} (3.5)

Now the volume of the gas bubble can be described as the volume of the unit cell minus the volume of the liquid slug and the liquid film:

\[ V_b = V_{uc} - V_l - V_p = A(L_s + L_b) - \left( \frac{U_t A}{F_b} \right) - (A - A_b)(L_s + L_b) \]

\[ V_b = A_b(L_s + L_b) - \left( \frac{U_t A}{F_b} \right) \]  \hspace{1cm} (3.6)

The gas hold-up can be described as the volume of the gas bubble divided by the volume of the unit cell:

\[ \varepsilon_g = \frac{V_b}{V_{uc}} = \frac{A_b}{A} - \frac{U_t}{F_b(L_s + L_b)} \]  \hspace{1cm} (3.7)

The liquid and the gas in one unit cell move with the bubble velocity \((u_b)\), which is equal to the length of all unit cells passing per unit time for a certain location \((u_b = F_b(L_s + L_b))\). Therefore, the gas hold-up can be given as:

\[ \varepsilon_g = \frac{A_b}{A} \frac{U_t}{u_b} \]  \hspace{1cm} (3.8)

The unknown parameter here is \(A_b/A\). This parameter can be obtained by making a mass balance for the liquid entering the unit cell which is equal to the liquid in the liquid slug plus the liquid around the nose and the tail of the gas bubble. To describe this latter volume, length \(\delta\) is introduced which is the length added to the liquid slug presenting the liquid volume around the nose and tail of the gas bubble as shown in figure 3.5.

\[ \frac{A U_l}{F_b} = A_b L_s + A_b \delta \]  \hspace{1cm} (3.9)

Re-arranging equation 3.7 gives a function for the liquid slug length as a function of the superficial liquid velocity \((U_l)\) divided by the bubble frequency \((F_b)\):

\[ L_s = \frac{A}{A_b} \frac{U_l}{F_b} - \delta \]  \hspace{1cm} (3.10)

Plotting the liquid slug length \((L_s)\) against the superficial liquid velocity divided by the bubble frequency \((U_l/F_b)\) gives \(A/A_b\) and \(\delta\) by curve fitting. Once \(A/A_b\) is known, the gas hold-up can be calculated using equation 3.7.
Figure 3.6: Results of previous research on $A/A_b$ and $\delta$ using two different mixer geometries in a rectangular micro channel of 100x50 $\mu$m$^2$.

Figure 3.7: Results of previous research on gas hold-up using two different mixer geometries in a rectangular micro channel of 100x50 $\mu$m$^2$.

Table 3.1: Results of $A/A_b$ and $\delta$ for various locations in the 100x50 $\mu$m channel$^{10}$. 

<table>
<thead>
<tr>
<th>Channel location (mm)</th>
<th>$A/A_b$ (-)</th>
<th>$\delta$ (µm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.3-3.3</td>
<td>1.14 ±0.1</td>
<td>40</td>
</tr>
<tr>
<td>3.3-6.3</td>
<td>1.13±0.1</td>
<td>38</td>
</tr>
<tr>
<td>6.3-9.3</td>
<td>1.12 ±0.1</td>
<td>36</td>
</tr>
<tr>
<td>9.3-12.3</td>
<td>1.11 ±0.1</td>
<td>35</td>
</tr>
<tr>
<td>12.3-15.3</td>
<td>1.11 ±0.1</td>
<td>35</td>
</tr>
<tr>
<td>15.3-18.3</td>
<td>1.12 ±0.1</td>
<td>36</td>
</tr>
</tbody>
</table>
Knowing the gas hold-up from the Taylor flow model, the local superficial gas velocity \( U_g \) can then be calculated using the bubble velocity \( u_b \) which is obtained experimentally.

\[
U_g = \varepsilon_g u_b
\]  

(3.11)

Assuming that the total ingoing gas flow is present as gas bubbles, the local pressure can be calculated using the local superficial gas velocity \( U_g \) and the ideal gas law \((PV=nRT)\). The gas velocity at the mass flow controller \( U_{g,MFC} \) is known at standard conditions \((P_{MFC}=1\ \text{bar})\).

\[
P_{MFC} U_{g,MFC} = P_L U_g
\]  

(3.12)

### 3.3 Previous research

In previous work done on the MiRAACS project regarding the hydrodynamics of Taylor flow, movies were made at the end of the channel (100x50 \( \mu \text{m} \)). Water was used as liquid phase and nitrogen as gas phase. For the end of the channel, the liquid slug length is plotted against \( U_l/F_b \) for the smooth mixer and the cross-shaped mixer and is shown in figure 3.6. For both mixers the data show a linear relationship between the slug length and \( U_l/F_b \) which means that \( A/A_b \) is not dependent on bubble velocity. \( A/A_b \) is equal to 1.19 for the smooth mixer and 1.22 for the cross-shaped mixer. The gas hold-up has been calculated using the Taylor flow model. This is done for mixer designs, the smooth mixer and cross-shaped mixer. The gas hold-up was plotted as a function of the gas quality \( (U_g/(U_l+U_g)) \) shown in figure 3.7. It is found that the gas hold-up is 0.833 times the gas quality. This is in agreement with the Armand correlation\(^{14}\). Armand (1946) showed that the gas hold-up is 0.833 times the gas quality using water and air. The hold-up was determined by weighting the used channel. Using this method no physical background was given on the correlation. In previous literature it was also found that the Armand correlation holds for water-nitrogen Taylor flow\(^1\).

For various locations in the channel the relationship between the slug length and \( U_l/F_b \) was also found to be linear\(^{10}\). The values for \( A/A_b \) and \( \delta \) are shown in table 3.1. The error shown on \( A/A_b \) is due to the liquid mass flow controllers which showed oscillations in the read out. Considering the values for \( A/A_b \) it is expected that the film thickness is also independent of channel location. For all locations the calculated gas hold-up from the Taylor flow model was plotted against the gas quality \( (U_g/(U_l+U_g)) \). These results showed a maximum deviation of 0.833±8% for the Armand correlation for all locations using water-nitrogen.
<table>
<thead>
<tr>
<th>Author</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Owens (1961)</td>
<td>$\eta_H = \eta_L$</td>
</tr>
<tr>
<td>McAdams (1954)</td>
<td>$\eta_H = \left(\frac{x}{\eta_G} + \frac{1-x}{\eta_L}\right)^{-1}$</td>
</tr>
<tr>
<td>Cicchittie et al. (1960)</td>
<td>$\eta_H = x\eta_G + (1-x)\eta_L$</td>
</tr>
<tr>
<td>Lin et al. (1991)</td>
<td>$\eta_H = \frac{\eta_L \eta_G}{\eta_G + x^{1.4}(\eta_L - \eta_G)}$</td>
</tr>
</tbody>
</table>

Table 4.1: Two phase homogeneous viscosity models with homogeneous Reynolds number in the laminar flow regime ($Re < 2000$).
4. Two-phase pressure drop

For the reactor design, pressure drop is an important parameter. For this, various models are available in literature, like the homogeneous flow model\(^{14}\) and the Lockhart-Martinelli model\(^{15}\). However, these models are flow pattern independent and can not describe all experimental results in micro channels specific for Taylor flow\(^{15}\). Models based on Taylor flow are the unit cell model\(^{7}\) and the Kreutzer model\(^{16}\). These models and the application of these models on this work are described in more detail in this chapter.

4.1 Homogeneous flow model

The homogeneous flow model assumes that there is no slip between the liquid phase and the gas phase. The model treats the two phases together as a pseudo-single-phase and is flow pattern independent. It was shown to describe data well for ammonia-steam flow and for air-water for micro channels with a hydraulic diameter \(D = 1.09-3.15 \text{ mm}\)\(^{14}\).

The two-phase pressure drop in the homogeneous flow model can be calculated by:

\[
\left( \frac{\Delta P}{\Delta z} \right) = f \frac{1}{D^2} \frac{G^2}{2 \rho_h} \tag{4.1}
\]

Where \(G\) is the total mass flux and \(\rho_h\) is the homogeneous mixture density which is defined by:

\[
\frac{1}{\rho_h} = \frac{x}{\rho_g} + \frac{1-x}{\rho_l} \tag{4.2}
\]

with \(x\) is the mass gas quality and \(f\) is the two-phase Darcy friction factor which is a function of the homogeneous Reynolds number:

\[
Re_h = \frac{GD}{\eta_h} \tag{4.3}
\]

where \(\eta_h\) is the homogeneous mixture viscosity. Several models were developed to predict this two-phase homogeneous viscosity. The models for \(\eta_h\) that cause the homogeneous Reynolds number \((Re_h)\) in the laminar flow regime \((Re_h < 2000)\) are given in table 4.1. Yue et al. used two rectangular micro channels with a hydraulic diameter of 528 \(\mu\text{m}\) and 333 \(\mu\text{m}\)\(^{15}\). A comparison of the experimental data with the homogeneous flow model was made using the homogeneous viscosity correlations given in table 4.1. It was shown that all correlations over-predict the two-phase frictional pressure drop except the McAdams correlation. This was also concluded by Triplett et al. who used circular micro channels with diameters of 1.1 mm and 1.45 mm and semi-triangular channels with hydraulic diameters of 1.09 mm and 1.49 mm\(^{17}\).

4.2 Lockhart-Martinelli

The Lockhart-Martinelli correlation is based on pressure drop over the both gas and liquid phases. Single gas velocities \((U_g)\) and single liquid velocities \((U_l)\) are used in stead of the total velocity \((U_g + U_l)\). A so called two-phase friction multiplier \((\Phi_t)\) is used to combine these single phase pressure drops in order to predict the two-phase frictional pressure drop. This model is flow pattern independent and is based on a separate liquid and gas flow.
<table>
<thead>
<tr>
<th>Author</th>
<th>Hydraulic diameter $(D_H)$ (µm)</th>
<th>C value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kawahara et al.(^{14})</td>
<td>100</td>
<td>0.24</td>
</tr>
<tr>
<td>Mishima and Hibiki's(^{25})</td>
<td>1000-4000</td>
<td>$C = 2(1 - e^{-0.319D_H})$</td>
</tr>
<tr>
<td>Lee and Lee(^{14})</td>
<td>780-6670</td>
<td>$C = A x^q y^r Re_{LO}^r$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\lambda = \frac{\eta L}{\rho L \sigma D_H}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\psi = \frac{\eta L U_{\text{slug}}}{\sigma}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$Re_{LO} = \frac{GD_H}{\eta_L}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>For both laminar flow: $A=6.833 \times 10^{-8}$, $q=-1.371, r=0.719, s=0.557$</td>
</tr>
<tr>
<td>Yue et al.(^{15})</td>
<td>528 and 333</td>
<td>$C = a x^b Re_{LO}^c$</td>
</tr>
</tbody>
</table>

**Table 4.2:** Predicted C values for the Lockhart-Martinelli model by different authors using various channel diameters.

**Figure 4.1:** Two-phase frictional multiplier plotted against the Lockhart-Martinelli parameter to obtain the value for $C^{14}$.

**Figure 4.2:** Cross section of the flow pattern assumed in the unit cell model\(^{7}\).
Two-phase pressure drop

\[
\frac{\Delta p_F}{\Delta z}_{TP} = \Phi^2 \frac{\Delta p_F}{\Delta z}_l, \tag{4.4}
\]

where \((\Delta p_F/\Delta z)_{TP}\) is the two-phase frictional pressure drop and \((\Delta p_F/\Delta z)_l\) the frictional pressure drop based on Hagen-Poiseuille flow for liquid through the channel. The friction multiplier has been correlated in terms of the Lockhart-Martinelli parameter \((X)\), which is equal to the frictional pressure drop by single liquid phase divided by the frictional pressure drop by single gas phase.

\[
X^2 = \frac{\left(\frac{\Delta p_F}{\Delta z}\right)_{TP}}{\left(\frac{\Delta p_F}{\Delta z}\right)_g}, \tag{4.5}
\]

The correlation used to predict the two-phase friction multiplier \((\Phi_l)\) was proposed by Chisholm and Laird (1958).

\[
\Phi^2 = 1 + \frac{C}{X} + \frac{1}{X^2}, \tag{4.6}
\]

where \(C\) is a constant between 5 and 20 dependent on the flow regimes of the gas and liquid flow. When both gas and liquid flow are laminar, which is common in micro channels, \(C\) is equal to 5\(^{15}\). This value is independent of the channel diameter. However, it is known that with decreasing channel diameter, the constant \(C\) drops\(^8\). For hydraulic diameters smaller then 50 µm the value of \(C\) would be zero, which corresponds to separate laminar flow with minimal momentum coupling between the phases. Various values for \(C\) were predicted in literature, as shown in table 4.2. Kawahara et al. predicted a value for \(C\) by fitting the two-phase friction multiplier against the Lockhart-Martinelli parameter, as shown in figure 4.1. Mishima et al. and Yue et al. predicted a correlation based on their data also by fitting the two-phase multiplier plotted against the Lockhart-Martinelli parameter. Lee and Lee\(^{18}\) proposed a correlation for the \(C\) value for Taylor flow based on their own data. The correlation is depending on whether the liquid phase and gas phase are laminar or turbulent and is valid for the Reynolds number for the liquid phase between 175 and 17700.

4.3 Unit cell model

Garimella et al. developed a model based on Taylor flow for circular and noncircular channels with hydraulic diameters varying between 0.42 mm and 4.91 mm\(^{7,19}\). A cooling agent (R-134) is used and the velocity is varied between 28.4 m/s and 142 m/s. The model is based on a single unit cell composed of one gas bubble and one liquid slug. It is assumed that all unit cells remain the same in the channel. One unit cell is divided into a single phase flow of liquid and a two-phase flow of liquid and vapor (figure 4.2). In contrast to the Taylor flow model used in this graduation work, the vapor bubble surrounded by the uniform liquid film is assumed to flow faster than the liquid slugs. The film is assumed to be uniform along the channel length and to flow very slowly compared to the bubble and the slug. Furthermore, it is assumed that the bubbles are uniform and no vapor is present in the liquid slugs. The pressure drop is equal to the sum of the frictional pressure drop and the pressure losses due to flow between the film and the slug.
Two-phase pressure drop

The frictional pressure drop is then equal to the sum of the contributions of the film/bubble region and of the liquid slugs.

\[
\frac{\Delta p_f}{L} = \left[ \left( \frac{dp}{dz} \right)_{f/b} \left( \frac{L_h}{L_s + L_b} \right) + \left( \frac{dp}{dz} \right)_{s} \left( 1 - \frac{L_h}{L_s + L_b} \right) \right]
\]  

(4.7)

The slug length is calculated using a control volume analysis for non-circular tubes. A turbulent velocity profile in the liquid slugs is considered in the work of Garimella et al. The frictional pressure drop contribution of the liquid slugs \((\frac{dp}{dz})_s\) is therefore described using the Darcy-Weisberg equation \((\Delta p = f L/D \ (\rho V^2/2))\) and the Blasius equation \((f=0.3164/Re^{0.25})\) for turbulent flow:

\[
\left( \frac{dp}{dz} \right)_s = \frac{0.3164 \ \rho U_s^2}{Re_s^{0.25} \ 2D_H}
\]  

(4.8)

For the contribution of the film/bubble region, it is assumed that the ratio between the bubble and the slug velocity is approximately 1.2 \((U_b/U_s=1.2)\) and that the flow of the gas is driven by the pressure gradient only. Based on the Reynolds number, the flow in the film was considered turbulent in the bubble by Garimella et al. for their data. The same equation is used as for the liquid slugs (equation 4.8) for the film/bubble region.

\[
\left( \frac{dp}{dx} \right)_{f/b} = \frac{0.3164 \ \rho_g (U_b - U_{film})^2}{Re_b^{0.25} \ 2D_b}
\]  

(4.9)

where \(U_{film}\) is the velocity of the film/bubble interface. The bubble diameter is equal to 0.9D which is typical for channels having a hydraulic diameter in the order of mm. The slug length in equation 4.7 is calculated using the correlation of Fukano et al. (1989). Instead of the Blasius equation, also the Churchill equation can be used to predict the friction factor \(f\). The Churchill equation holds for the entire range from laminar to turbulent flow.

\[
f(Re, \varepsilon / D_H) = 8 \left\{ \frac{8}{Re} \right\}^{12} \left[ 2.457 \ln \left( \frac{1}{(7 / Re)^{0.9} + 0.27 \varepsilon / D_H} \right) \right]^{16} + \left( \frac{37530}{Re} \right)^{16} \left( \frac{\varepsilon}{D_H} \right)^{15} \right\}^{1/12}
\]  

(4.10)

where \(D_H\) is the hydraulic diameter. The contribution of the pressure loss from film-to-slug flow is associated with the acceleration and subsequent mixing of the liquid film and the liquid slug at the front of the liquid slug. This pressure drop contribution in one unit cell \((\Delta p_{one \ transition})\) is described as a dynamic pressure drop:

\[
\Delta p_{one \ transition} = \rho_l \left( \frac{U_s - U_f}{2} \right)^2
\]  

(4.11)

where \(U_f\) is assumed to have a parabolic velocity profile and is driven by the pressure gradient in the film/bubble region. The interface flow rate is then:
Two-phase pressure drop

\[ U_j = \frac{-\left( \frac{dP}{dz} \right)_{f/b}}{8 \eta_l} (R^2_{\text{channel}} - R^2_b) \]  

(4.12)

Multiplying this dynamic pressure drop by the number of unit cells gives the overall contribution of the pressure drop due to film-to-slug flow.

For the condensation of the refrigerant R134a in cylindrical channels, 88% of the predicted results are within 25% of the experimental data of Garimella et al. For the non-cylindrical channels, 90% of the predicted results are within 28% of the experimental data.

Chung and Kawaji\(^8\) used the same unit cell model of Garimella et al. for describing the pressure drop in a micro channel with a diameter of 100\(\mu\)m. Nitrogen was used as gas phase and water as liquid phase. The contribution of film-to-slug flow to the pressure drop is negligibly small in channels with a \(D_H<100\ \mu\)m and is therefore neglected by Chung and Kawaji. The Darcy-Weisberg equation is used to predict the frictional pressure drop contribution of the liquid slugs.

\[ \left( \frac{dp}{dz} \right)_s = f_l \rho l U^2_s \frac{2}{D} \]  

(4.13)

where \(f_l\) is equal to \(64/Re_l\) for laminar flow in circular channels. For the contribution of the two-phase regions, the similar Darcy-Weisberg correlation is used.

\[ \left( \frac{dp}{dz} \right)_{f/b} = f_b \rho_b (U_h - U_l)^2 \frac{2}{2D_b} \]  

(4.14)

Where \(f_b\) is equal to \(64/Re_b\). Since the Fukano et al. correlation for the slug length only holds for channel diameters in the order of mm, the slug length was estimated from the measured gas hold-up.

\[ \varepsilon_g = \frac{\pi D_b^2 L_h / 4}{\pi D^2 L_{uc} / 4} \]  

(4.15)

The relative lengths of the gas bubbles \((L_b)\) and the liquid slugs \((L_s)\) are described as:

\[ \frac{L_b}{L_{uc}} = \varepsilon_g \left( \frac{D}{D_b} \right)^2 \quad \frac{L_s}{L_{uc}} = 1 - \left( \frac{L_h}{L_{uc}} \right) \]  

(4.16)
**Figure 4.3**: Effect of inertia on the shape of the gas-liquid interface for the Re=1, 10, 100, 200 at Ca=0.04. The x-axes and y-axes are the lengths divided by the channel diameter\(^\text{\textsuperscript{16}}\).

**Figure 4.4**: Pressure at the wall versus the channel length obtained from the numerical results of Kreutzer et al.\(^\text{\textsuperscript{16}}\). A pressure drop over the gas bubble is caused by the disruption of the Hagen-Poiseuille flow of the liquid phase.
Two-phase pressure drop

The frictional pressure drop can then be given by:

\[
\left( \frac{dP_F}{dz} \right)_{TP} = \left( \frac{dP}{dz} \right)_{f/b} \frac{L_{b^*}}{L_{uc^*}} + \left( \frac{dP}{dz} \right)_{s} \frac{L_{s^*}}{L_{uc^*}} \tag{4.17}
\]

The pressure drop prediction lies within 25\% of the experimental pressure drop using the experimental gas hold-up.

4.4 The Kreutzer model

Kreutzer et al.\textsuperscript{16} uses a semi-emperical equation to describe the frictional pressure drop for Taylor flow in a circular channel with a diameter of 2.3 mm. The calculations are done for a single bubble trapped between two half liquid slugs. The bubble moves at an average velocity \( U \), which is equal to the sum of the gas and liquid superficial velocities, so therefore the liquid film is neglected. The viscosity and the density of the gas phase are negligible compared to the liquid phase, therefore only the pressure drop over the liquid phase is considered. It is assumed that the liquid flow is laminar and far away from the bubble a Hagen-Poiseuille velocity profile is developed.

From the experimental results it is found that the pressure drop over the bubble is not negligible. This is due to the effect of inertia on the bubble shape as is shown in figure 4.3. The gas bubbles disrupt the Hagen-Poiseuille flow in the liquid phase. The pressure at the nose of the bubble differs from the tail of the bubble and caused a pressure drop over the bubble as shown in figure 4.4. The total frictional pressure \( \Delta P^* \) is described as the sum of the pressure drop of the liquid slug \( (\Delta P_{s^*}) \) and the pressure drop over the bubble \( (\Delta P_{b^*}) \).

\[
\Delta P^* = -32L_{s^*}^* + \Delta P_{b^*}^* (Re, Ca) \tag{4.18}
\]

\[
\Delta P_{b^*}^* = \frac{\Delta p \, Re}{\beta_{l^*} \rho U^2} \]

\[
L_{s^*}^* = \frac{L_{s^*}}{D_H}
\]

\( \Delta P_{b^*}^* \) is a function of Reynolds \( (Re) \) and the capillary number \( (Ca) \) and describes the effects near the bubble. The effects of flow of the gas bubbles on pressure drop are negligible compared to the pressure drop in the slugs. The pressure drop over the bubble is caused by the disruption of the Hagen-Poiseuille flow. Therefore, the pressure drop contributed by the gas bubble is modeled as part of the pressure drop in the liquid slugs.

\[
\Delta P^* = -32[ 1 + \xi(Re, Ca, L_{s^*}) ]L_{s^*} \tag{4.19}
\]

where \( \xi \) describes the pressure drop occurring due to the presence of bubbles. When the liquid slugs become infinitely long, the effect of the presence of bubbles vanishes and equation 4.18 is simplified to \( \Delta P^* = -32L_{s^*}^* \). The latter equation corresponds to single liquid flow having a Hagen-Poiseuille velocity profile.
**Figure 4.5:** Experimental friction factor multiplied with the Reynolds number plotted against the dimensionless slug length\(^{16}\). With increasing slug lengths the value for \( f_{Re} \) goes to 16.

**Figure 4.6:** Experimental friction factor multiplied with the Reynolds number plotted against the dimensionless group \( L_{\text{slug}}^*(Ca/Re)^{0.33} \) \(^{16}\).
Two-phase pressure drop

Within this model, the pressure drop over the liquid slugs can be given as:

\[
\frac{\Delta P}{L} = \beta_l f \left( \frac{1}{2} \rho_l U^2 \right) \frac{4}{D_H} \tag{4.20}
\]

where $\beta_l$ is the liquid hold-up in the channel and $f$ is the slug friction factor defined as

\[
\beta_l = \frac{L_s}{L_s + L_b} \tag{4.21}
\]

\[
f = \frac{16}{Re} \left[ 1 + 16 \frac{\xi(Re, Ca, L_s^*)}{Re} \right] \tag{4.22}
\]

It should be noted that the liquid hold-up as described in equation 4.21 neglects the liquid film layer.

Kreutzer et al. measured the pressure drop in a vertical circular channel with a diameter of 2.3 mm. Gas and liquid superficial velocities are varied from 0.04 to 0.3 m/s. Air is used as gas phase and water, decane and tetradecane are used as liquid phases. A friction factor based on the experimental results $f_{obs}$ is obtained.

\[
f_{obs} = \frac{\left( \frac{\Delta P}{L} \right)_{tot} - \beta_l \rho_l g L}{\left( \frac{1}{2} \rho_l U^2 \right) \left( \frac{4}{D_H} \right) \beta_l} \tag{4.23}
\]

For horizontal channels the static pressure can be neglected. The experimental friction factor $f_{obs}$ multiplied by the Reynolds number ($Re$) is plotted against the dimensionless slug length shown in figure 4.5. The single phase limit is equal to $16/Re$. The friction factor is independent of bubble velocity but is dependent on liquid properties. The contribution of the presence of the gas bubbles in equation 4.19 is described using the ratio of $Ca$ and $Re$.

\[
\xi = a \left\{ \frac{Re}{L_s^* (Ca)} \right\}^b \tag{4.24}
\]

The friction factor multiplied by the Reynolds is plotted against the dimensionless group $L_s^*/(Ca/Re)$ to determine $a$ and $b$ by nonlinear regression (figure 4.6). An expression for the friction factor including the presence of bubbles was found by Kreutzer et al. describing all their data for $Re<50$:

\[
f = \frac{16}{Re} \left[ 1 + 0.17 \left( \frac{Re}{L_s^* (Ca)} \right)^{0.33} \right] \tag{4.25}
\]

Equation 4.25 can then be used in equation 4.20 to calculate the overall pressure drop including the gas bubbles.
**Figure 5.1:** The experimental set-up.

**Figure 5.2:** PID of the flow system used for the experiments.

**Figure 5.3:** The rectangular micro channel etched in the glass chip.
5. Experiments

In the first part of this chapter the experimental setup is described. The second part describes the experiments done to obtain the hydrodynamic parameters and the pressure drop.

5.1 Experimental set-up

The set-up consists of three parts: the flow system, the micro channel and the imaging system (figure 5.1), which are described hereafter.

Flow system

In this work, isopropanol is used as liquid phase and nitrogen as gas phase. The isopropanol is introduced into the micro channel using an Isco 100 DM syringe pump. The syringe pump has a flow rate range of 0.01µl/min to 25 ml/min. The nitrogen flow is regulated by a mass flow controller (Bronkhorst F-200C) which covers a range from 0.03 ml/min to 1.5 ml/min. An overview of the flow system is given in figure 5.2.

Micro channel

The micro fluidic structure is etched in a glass chip by Deep Reactive Ion Etching and the entrance and exit holes for the gas and liquid are made by powder blasting. The micro chip consists of a cross mixer to mix the gas and liquid followed by a 20x0.1x0.05 mm channel as shown in figure 5.3. In the chip, the liquid stream is split up into two separate liquid streams which recombine perpendicular to the gas stream at the entrance of the micro channel. In order to connect the capillaries to the chip and to place the chip under the microscope, a chip holder is used. The holder has an open part so the chip can be visualized in the holder. Furthermore, the holder has two inlet capillaries, for the gas and liquid, and one outlet capillary. O-rings are used between the capillaries and the in- and outlets of the chip to prevent leaking. An overview of the holder is given is figure 5.4.

In order to check the dimensions of the cross-sectional area of the micro channel, the chip is broken at several places and scanning electron microscope (SEM) pictures are made. The pictures of the cross-sectional area and the dimensions are shown in figure 5.5. The average dimension of the cross-sectional area for various locations in the channel is 98x50 µm. These inaccuracies have only minor influences on the measured data.

Imaging system

In order to visually analyze the flow in the micro channel a Zeiss Axiovert 200 MAT inverted microscope is used. A Redlake MotionPro CCD camera is connected to the microscope and images are recorded at 10.000 frames per second. The images have a resolution of 1280 x 48 pixels where 1 pixel corresponds to 3.57 µm of the channel. Transmitted light is used from a 100W halogen lamp. The exposure time is set to 12 µs which is enough to prevent image blurring.
Figure 5.4: Overview of the holder in which the glass chip is placed.

Figure 5.5: SEM picture of cross-sectional area of the 2x0.1x0.05 mm silicon channel. The average dimension determined by various SEM pictures at various locations is 98x50 µm.

<table>
<thead>
<tr>
<th>Gas velocity (m/s)</th>
<th>Liquid velocity</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
<td>0.5</td>
</tr>
<tr>
<td>1.5</td>
<td>0.5</td>
</tr>
<tr>
<td>2.0</td>
<td>0.5</td>
</tr>
<tr>
<td>1.0</td>
<td>0.6</td>
</tr>
<tr>
<td>2.0</td>
<td>0.6</td>
</tr>
<tr>
<td>2.8</td>
<td>0.6</td>
</tr>
<tr>
<td>0.5</td>
<td>0.3</td>
</tr>
<tr>
<td>1.0</td>
<td>0.3</td>
</tr>
<tr>
<td>2.0</td>
<td>0.3</td>
</tr>
<tr>
<td>0.5</td>
<td>0.2</td>
</tr>
<tr>
<td>1.0</td>
<td>0.2</td>
</tr>
<tr>
<td>0.5</td>
<td>0.1</td>
</tr>
<tr>
<td>1.0</td>
<td>0.1</td>
</tr>
</tbody>
</table>

Table 5.1: Overview of the combination of gas and liquid velocities used.

<table>
<thead>
<tr>
<th>Length (µm)</th>
<th>Width (µm)</th>
<th>Cross sectional area (µm²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>47</td>
<td>4.7e-9</td>
</tr>
<tr>
<td>101</td>
<td>48</td>
<td>4.9e-9</td>
</tr>
<tr>
<td>92</td>
<td>49</td>
<td>4.5e-9</td>
</tr>
<tr>
<td>98</td>
<td>53</td>
<td>5.2e-9</td>
</tr>
<tr>
<td>101</td>
<td>51</td>
<td>5.2e-9</td>
</tr>
</tbody>
</table>
5.2 Experimental procedure

Flow pattern mapping
The pressure changes with changing the set velocities and is dependent on the flow pattern. Since the gas phase is compressible, the flow pattern is dependent on the pressure. It takes about 10 minutes to be in a steady state situation. Therefore, video images are recorded after waiting 10 minutes at the end of the channel. The video images are played back at a lower frame rate in order to visualize the flow pattern corresponding to the set gas and liquid velocity. This process is repeated for various combinations of gas and liquid velocities until a flow pattern map is obtained.

Hydrodynamic parameters and pressure drop estimation
Within the Taylor flow regime of the flow pattern map shown in figure 2.5, various combinations of gas and liquid velocities are chosen as shown in table 5.1. The channel is divided into six different locations given in table 5.2. At all six locations, the combinations in table 5.1 are set. Movies of 0.5 seconds at a frame rate of 10,000 frames per second are made.

Each movie is analyzed using a series of Matlab scripts written by Warnier. These scripts are used to determine the hydrodynamic parameters like bubble length, slug length, bubble velocity and bubble frequency. Each frame in the movie is analyzed; a typical example of one frame is given in figure 5.6. The background, see figure 5.7, is subtracted from the original image of each frame to isolate the bubbles from the channel wall. To get rid of the non-uniformly lit areas at the beginning and at the end of the frame, a measurement area of 800x48 pixels is chosen. Since the edge of the bubble appears to be black and the liquid white, tress holding is used to identify the boundary of the bubble. Within this boundary all pixels are made white and outside the boundary black to reach a binary image as shown in figure 5.8. From this binary image various parameters are obtained, like bubble area, coordinates of the centre of mass, bubble length, slug length (distance between bubble nose of one bubble and the tail of the other) and a 2-dimensional gas hold-up. For the 2-dimensional gas hold-up the amount of white pixels from the bubbles are divided by the total amount of pixels of the channel \(N_{\text{pixels,white}}/N_{\text{pixels,channel}}\). In order to determine the bubble velocity, each bubble in the first frame is numbered as shown in figure 5.9. Each bubble appears in more frames. Only bubbles that are completely in the frame are considered. When a bubble is identified in a frame, then the first bubble downstream in the consecutive frame is the same bubble. This only holds when the frame rate is high enough. The bubbles are identified and numbered. The first time a bubble appears and the last time the bubble appears is used to determine the bubble velocity using the z-coordinates. The distance obtained from subtracting the z-coordinates is divided by the amount of frames divided by the frame rate. This is done for all bubbles identified in the movie. The limitation for this procedure is that the slug length plus the bubble length cannot be greater than 800 pixels.

The average of all individual bubble lengths over all frames in one movie is the average bubble length \(L_b\) at a certain location. The average slug length \(L_s\) is obtained in the same way. The bubble frequency \(F_b\) is obtained by dividing the number of tracked bubbles by the measured time (0.5 seconds). The bubble velocity for each bubble is averaged in order to get the average bubble velocity \(u_b\) at a certain location. These parameters are used in the Taylor flow model described in section 3 to obtain the 3-dimensional gas hold-up \(\varepsilon_g\) and the pressure \(P\).
<table>
<thead>
<tr>
<th>Letter</th>
<th>Location</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0.3-3.3 mm</td>
</tr>
<tr>
<td>B</td>
<td>3.3-6.3 mm</td>
</tr>
<tr>
<td>C</td>
<td>6.3-9.3 mm</td>
</tr>
<tr>
<td>D</td>
<td>9.3-12.3 mm</td>
</tr>
<tr>
<td>E</td>
<td>12.3-15.3 mm</td>
</tr>
<tr>
<td>F</td>
<td>15.3-18.3 mm</td>
</tr>
</tbody>
</table>

**Table 5.2:** Locations of the measurement sections, starting with A at 0.3 mm from the entrance of the micro channel.

**Figure 5.6:** Image of one frame of Taylor flow in a 100x50µm channel.

**Figure 5.7:** Image of background.

**Figure 5.8:** Binary image after thresholding.

**Figure 5.9:** Numbering of bubbles. The bubbles in frame 1 are linked to the bubbles in frame 2.
The bubble velocity is also determined from the first two appearances of a bubble \((u_{b, \text{begin}})\) and from the last two appearances \((u_{b, \text{end}})\). The distance \((dz)\) is the distance between mean position of the first two appearances and the mean position of the last two appearances. With the use of the Taylor flow model, the gas hold-up \((\varepsilon_g)\) and the pressure \((P)\) are determined at the beginning and the end of the channel part under consideration in one movie. The experimental pressure drop \(((\Delta P/\Delta z)_{\text{exp}})\) is obtained for every location.
\[ L_s = \frac{A U_l}{A_b F_b} - \delta \]

**Figure 6.1:** Slug length plotted against the liquid velocity divided by the bubble frequency for location B.

**Table 6.1:** Results of \( A/A_b \) and \( \delta \) at all locations.

<table>
<thead>
<tr>
<th>Channel location (mm)</th>
<th>( A/A_b ) (-)</th>
<th>( \delta ) (µm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A: 0.3-3.3</td>
<td>1.48 ±0.03</td>
<td>44 ± 3</td>
</tr>
<tr>
<td>B: 3.3-6.3</td>
<td>1.56 ±0.04</td>
<td>48 ± 3</td>
</tr>
<tr>
<td>C: 6.3-9.3</td>
<td>1.55 ±0.02</td>
<td>41 ± 4</td>
</tr>
<tr>
<td>D: 9.3-12.3</td>
<td>1.57 ±0.06</td>
<td>53 ± 7</td>
</tr>
<tr>
<td>E: 12.3-15.3</td>
<td>1.56 ±0.05</td>
<td>53 ± 6</td>
</tr>
<tr>
<td>F: 15.3-18.3</td>
<td>1.42 ±0.08</td>
<td>39 ± 12</td>
</tr>
</tbody>
</table>

\[ \varepsilon_g = \frac{C U_g}{U_g + U_l} \]

\( C = 0.65 \)

**Figure 6.2:** The gas hold-up obtained from the Taylor flow model plotted against the flow quality in order to determine the constant \( C \) in equation 3.2 for location B.

**Figure 6.3:** Bubble velocity (left) and gas hold-up (right) plotted against the channel length using isopropanol and nitrogen in a micro channel of 50x100µm cross sectional area. In the legend, ’N’ indicates the nitrogen velocities and ’I’ the isopropanol velocities.
6. Results and discussion

In this chapter the results obtained from the experiments described in section 5 are given. The results regarding to the Taylor flow model, i.e. film thickness and gas hold-up, are described in the first part. In the second part the pressure drop results are given. The applied pressure drop models given in section 4 are discussed in the last part.

6.1 Taylor flow model

From the Taylor flow model, the ratio between the cross-sectional area of the channel and the bubble \((A/A_b)\), the additional slug length \((\delta)\) and the gas hold-up \((\varepsilon_g)\) are obtained.

\(A/A_b\) and \(\delta\)

From analyzing the data, the average slug length \((L_s)\) and the bubble frequency \((F_b)\) are obtained at each measurement location. The average slug length \((L_s)\) is plotted against the liquid velocity \((U_l)\) divided by the bubble frequency \((U_l/F_b)\) for each location according to equation 3.10. The ratio between the cross-sectional area of the channel and the bubble \((A/A_b)\) and the additional slug length \((\delta)\) are determined by fitting the plot. An example at location B is given in figure 6.1. For all locations in the channel, a linear relationship is found between \(L_s\) and \(U_l/F_b\). This means that \(A/A_b\) is a constant value for all data points at that location. Table 6.1 gives the results of \(A/A_b\) and \(\delta\) for all locations in the channel.

The value for \(A/A_b\) is equal to 1.56±0.06 for locations B, C, D and E. For location A and F, a lower value of \(A/A_b\), 1.48 and 1.42 respectively, is obtained. This is probably due to entrance-effects at the entrance and the end of the channel. From these results it can be concluded that the film thickness remains constant over the channel length and is independent on the bubble velocity \((u_b)\). This is in agreement with Aussilous and Quere (2000). They found that the film thickness is less dependent on the bubble velocity for high velocities (order of 1 m/s).

Comparing the results for \(A/A_b\) using isopropanol with the results using water, a larger value for \(A/A_b\) is found for isopropanol. For water, \(A/A_b\) is found to be 1.2 and for isopropanol 1.56. This difference is due to the larger viscosity and lower surface tension of isopropanol \((\eta=2.27 \text{ mPa}\cdot\text{s}, \sigma=23 \text{ mN/m})\) compared to water \((\eta=1 \text{ mPa}\cdot\text{s}, \sigma=73 \text{ mN/m})\). The bubble pushes against the liquid in the direction of the channel wall. Due to the larger viscosity of isopropanol, less liquid is pushed away. And due to the lower surface tension fewer forces are performed between the gas and liquid phase.

\(\varepsilon_g\)

The gas hold-up is calculated using equation 3.8. The gas hold-up can be described as a function of the flow quality \((\varepsilon_g=C*U_g/(U_l+U_g))\). The gas hold-up against the flow quality is plotted against the flow quality. As example, the plot is given in figure 6.2 for location B. The constant \(C\) is derived from the Taylor flow model.

The values of \(C\) for locations B, C, D and E are equal to 0.65. For locations A and F the values for \(C\) are equal to 0.68 and 0.69 respectively. These values correspond to \(A/A_b\).

The local superficial gas velocity is dependent on the bubble velocity \((u_b)\) and the gas hold-up according to equation 3.11 of the Taylor flow model. The bubble velocity is experimentally determined, from the data analysis described in section 5.2. The average bubble velocity \((u_b)\) plotted against the channel length is given in figure 6.3. The gas hold-up \((\varepsilon_g)\) is also plotted against the channel length as shown in figure 6.3. Both the bubble velocity \((u_b)\) and the gas hold-up...
Figure 6.4: Pressure plotted against the channel length using isopropanol and nitrogen in a micro channel of 50x100µm cross sectional area. In the legend, ‘N’ indicates the nitrogen velocities and ‘I’ the isopropanol velocities.
up ($\varepsilon_g$) are increasing along the channel length. This increase along the channel length indicates that expansion of the gas phase occurs.

### 6.2 Pressure estimation

The local pressure ($P_L$) is calculated according to equation 3.12 using the ideal gas law and the bubble velocity. When using the ideal gas law for calculating the pressure at a certain location in the channel, the implicit assumption is made that no gas is dissolved in the liquid during operation. In order to check this, the maximum amount of nitrogen dissolved in isopropanol is calculated. First $k_{GL}a_{GL}$ is calculated\(^{20}\):

\[
k_{GL}a_{GL} = \frac{0.133u_b^{1.2}}{L_s^{0.5}}
\]

where for $u_b$, the average bubble velocity is chosen ($u_b=1.2$ m/s) and for $L_s$ the average slug length ($L_s=0.128x10^{-3}$ m). $k_{GL}a_{GL}$ is equal to 14.6 l/s. In order to calculate the maximum amount of nitrogen that can dissolve into isopropanol, some assumptions are made. First the liquid film is not taken into account since it is assumed that the liquid film is saturated with nitrogen. Furthermore, it is assumed that the pressure is 5 bar over the whole channel and a minimum concentration of nitrogen in the liquid slug. The Henry coefficient is used ($H=2170$ atm) to calculate the maximum concentration of nitrogen $C_{N2}^0$ into isopropanol which is equal to $2.3x10^{-3}$ mole N$_2$/mole isopropanol ($x=p/H$). To calculate the amount of N$_2$ that is dissolved into isopropanol, the following equation is used:

\[
\frac{\text{mole N}_2}{\text{s}} = k_{GL}a_{GL} \tau C_{N2}^0 F_{iso}
\]

where $\tau$ is the residence time in the micro channel ($1.62x10^{-2}$ s) and $F_{iso}$ the mole flow of isopropanol. The mole flow of isopropanol $F_{iso}$ is calculated using the maximum liquid velocity ($0.6$ m/s) and is equal to $3.92x10^{-5}$ mole isopropanol/s. The maximum mole flow dissolved nitrogen into isopropanol is then $2.13x10^{-8}$ mole nitrogen/s. The nitrogen flow is equal to $2.23x10^{7}$ mole/s ($U_g=1$ m/s). So the amount of nitrogen that is dissolved into isopropanol is 10% of the total mole flow of nitrogen. It should be noted that the maximum liquid velocity and a minimum gas velocity are used. Furthermore, the concentration nitrogen in the liquid slugs increases and the pressure decreases. This is thus a worse case scenario. Therefore, it can be concluded that the amount of nitrogen dissolved into isopropanol is a few percent and is therefore not taken into account. The estimation gives an acceptable error in the pressure estimation.

The pressure is plotted against the channel location in figure 6.4. The local pressure ($P_L$) is decreasing along the channel. This is due to frictional pressure loses. At 0.3 mm and 18.3 mm, locations A and F respectively, inaccurate data points are observed. This is due to entrance effects at the beginning and at the end of the channel.

The influence of pressure on the viscosity of isopropanol is checked using the following equation\(^{21}\):

\[
\frac{\eta}{\eta_{SL}} = \frac{1 + D \left( \frac{AP_c}{2.118} \right)^4}{1 + C_0AP_c}
\]
Results and discussion

\[ \Delta P_r = \frac{(P - P_{vp})}{P_c} \]  \hspace{1cm} (6.4)

\[ A = 0.9991 - \frac{4.674 \times 10^{-4}}{1.0523T_r^{-0.03877} - 1.0513} \]  \hspace{1cm} (6.5)

\[ C = -0.07921 + 2.1616T_r - 13.4040T_r^2 + 14.1706T_r^3 - 0.84829T_r^4 + 96.1209T_r^5 - 59.8127T_r^6 + 15.6719T_r^7 \]  \hspace{1cm} (6.6)

\[ D = \frac{0.3257}{(1.0039 - T_r^{2.575})^{0.2906}} \]  \hspace{1cm} (6.7)

where \( \omega, T_r, P_{vp} \) and \( P_c \) are physical properties of isopropanol. As reference for \( P \) is taken 6 bar with a viscosity \( \eta = 2.27 \text{ mPa s} \) at 20°C. The ratio of viscosity using these parameters is 1.0038. It can therefore be concluded that the pressure has no influence on viscosity in this case.

6.3 Pressure drop

The experimental pressure drop is calculated as described in section 5.2. To plot the experimental pressure drop, the friction factor is used, which expresses the linear relation between the pressure drop and the kinetic energy. The friction factor is based on Hagen-Poiseuille flow in the liquid slugs.

\[ f = \frac{\left( \frac{\Delta P}{\Delta z} \right)_{TP}}{\left( \frac{1}{2} \rho U^2 \right) \left( \frac{4}{D_H} \right) \varepsilon_l} \]  \hspace{1cm} (6.8)

Only the liquid flow in the slugs is assumed, however part of the liquid does not flow because this is present as a liquid film layer between the gas bubbles and the liquid slugs. Therefore, there are two ways to describe the velocity \( (U) \) and liquid hold-up \( (\varepsilon_l) \) in equation 6.8.

- Describing the velocity \( (U) \) and liquid hold-up \( (\varepsilon_l) \) regarding to the cross sectional area of the channel \( (A) \). The velocity \( (U) \) is then described by the local superficial gas velocity \( (U_g) \) plus the liquid velocity \( (U_l) \). The liquid hold-up is obtained from the experimental gas hold-up \( (\varepsilon_g) \) calculated using the Taylor flow model.

\[ U = U_g + U_l \]  \hspace{1cm} (6.9)

\[ \varepsilon_l = 1 - \varepsilon_g \]  \hspace{1cm} (6.10)

The hydraulic diameter is then also based on the cross sectional area of the channel \( (A) \).

- Describing the velocity \( (U) \) and liquid hold-up \( (\varepsilon_l) \) regarding to the bubble cross-sectional area \( (A_b) \). This assumes that the liquid film is considered as a stagnant wall. Then the velocity \( (U) \) is equal to the bubble velocity \( (u_b) \) and the liquid hold-up is described using the slug length \( (L_s) \) and the bubble length \( (L_b) \).

\[ U = u_b \]  \hspace{1cm} (6.5)
Figure 6.5: Friction factor times Reynolds plotted against the slug length, (a) based on the cross sectional area of the channel \((U_g + U_l)\) and \(L_s\), and (b) based on the cross sectional area of the bubble \((u_B)\) and \(L_s/(L_s+L_B)\). \(L_s\) is the slug length and \(D_h\) the hydraulic diameter of the channel.

Figure 6.6: Visual analysis of the movies to determine the pressure drop and bubble diameter\(^{23}\). The black area is the radius \(r\) of the cross sectional bubble area.

Figure 6.7 Applying the Cubaud method (2004) on this graduation work, \(r\) is the radius of the inner circle of the bubble geometry and \(\alpha\) the film thickness. It is assumed that the liquid film thickness is the same around the bubble top, bottom and sides.
Results and discussion

\[ \varepsilon_L = \frac{(L_s + \delta)}{L_s + L_b} \]  \hspace{1cm} (6.11)

The hydraulic diameter is then based on the cross-sectional bubble area \((A_b)\).

Both methods are used to determine the friction factor described in equation 6.2. The friction factor times the Reynolds number is plotted against the slug length. The single liquid phase limit is equal to \(fRe=15.2\) for rectangular channels and with the presence of bubbles, \(fRe\) increases since the bubbles interrupt the Hagen-Poiseuille flow in the liquid phase\(^{16}\). Due to entrance effects at the entrance end of the channel, which influences \(A\) and \(F\), only locations \(B\) to \(E\) are considered.

The friction factor, calculated using the first method based on the cross-section of the channel \((A)\) times the Reynolds number is given in figure 6.5a. 61\% of all data points are below the single liquid phase limit \((fRe=15.2)\). In figure 6.5b the friction factor calculated using the second method based on the cross-section of the bubble \((A_b)\) times Reynolds number is plotted. With increasing slug length (less bubbles) the value for \(fRe\) decreases to the constant value of 15.2. This is in agreement with literature\(^{16}\). From all data points 8.3\% is below 15.2.

From these results it can be concluded that using the second method based on \(A_b\), the same trend is found as in literature\(^{16}\). From these results it can be said that for describing the pressure drop by the friction factor and Reynolds number, the liquid film should be considered as a stagnant wall.

**6.4 Comparison with pressure drop models in literature**

The experimental pressure drop is compared to the Lockhart-Martelli model, the unit cell model and the Kreutzer model described in section 4. Because this study only considers Taylor flow, the unit cell model and the Kreutzer model are studied in more detail since these are based on Taylor flow. The homogeneous model has not been studied, because the model assumes no slip between the gas and the liquid phase. In this graduation work, slip is assumed to occur due to the liquid film.

Adjustments have to be made to use the unit cell model and the Kreutzer model for this work. The following rearrangements of the equations describing the model are done:

- The friction factor is equal to \(15.2/Re\) since in this work rectangular channels are used\(^{22}\).
- Chung and Kawaji and Kreutzer both use an empirical gas hold-up which is shown to underestimate the gas hold-up for this work in previous research\(^3\). The gas hold-up determined by the Taylor flow model is used for this work.
- The diameter of the bubble is equal to 90\% of the channel diameter \((D_b=0.9D)\) in the unit cell model. In this work, rectangular channels are used and therefore this assumption cannot be made. Cubaud and Ho (2004)\(^23\) determine their gas hold-up and geometry of the bubble by analyzing their movies shown in figure 6.6. It is assumed that the width of the black area is equal to the radius of the curvature of the part of the bubble in the corner of the channel. The same method is applied on our data. The geometry of the cross-sectional area of the bubble is determined by visually analyzing the length of the black area as shown in figure 6.7. From the analysis it was found that the geometry at the sides of the bubble is equal to half a circle (radius=20 \(\mu m\)). It is assumed that the film thickness is the same at the top, bottom and the sides of the bubble. For various movies, the black area is analyzed and it is obtained that the black area remains the same with varying bubble velocity.
Figure 6.8: The pressure drop calculated using the Lockhart-Martinelli model plotted against the experimental results. The Lockhart-Martinelli model over-predicts the pressure drop for 75% and under-predicts the pressure drop for 25%. The experiments were done in a 50x100 µm channel using isopropanol and nitrogen.
Results and discussion

When the geometry is known, an assumption can be made on the film thickness, which is calculated using the average ratio between the cross-sectional channel area and cross-sectional bubble area \( \frac{A}{A_b} = 1.54 \).

\[
\frac{A}{A_b} = \frac{50 \times 10^{-6} \cdot 100 \times 10^{-6}}{\frac{1}{4} \pi (d - 2\alpha)^2 + (d - 2\alpha)^2}
\]  

(6.12)

Where \( \alpha \) is the film thickness and \( d \) is equal to the diameter of the circle in the bubble area, which is equal to the width of the channel minus two times the film thickness \( (d - 2\alpha) \); \( \alpha \) is then equal to 3.57 \( \mu \)m.

Now the ratio between the hydraulic diameter of the cross-sectional bubble area and the hydraulic diameter of the cross-sectional channel area \( \frac{D_H^b}{D_H} = \frac{4A_b / p_b}{4A / p} \) is calculated.

\[
\frac{D_H^b}{D_H} = \frac{1}{1.1328} D_H
\]  

(6.13)

(6.14)

Lockhart-Martinelli

The frictional two-phase pressure drop based on the Lockhart-Martinelli is calculated using equation 4.11. The two-phase friction multiplier \( (\Phi) \) is calculated using the single liquid phase pressure drop \( (\Delta P_{f}/\Delta L) \) and the single gas phase pressure drop \( (\Delta P_{g}/\Delta L) \).

\[
\left( \frac{\Delta P_{f}}{\Delta z} \right)_l = \frac{-31.095 \eta_l U_l}{D_H^2}
\]  

(6.15)

\[
\left( \frac{\Delta P_{g}}{\Delta z} \right)_g = \frac{-31.095 \eta_g U_g}{D_H^2}
\]  

(6.16)

\( \eta_l \) and \( \eta_g \) are equal to 2.27 mPa s and 1.75 \( \times 10^{-2} \) mPa s respectively.

The pressured drop based on Lockhart-Martinelli is plotted against the pressure drop obtained from the experiments as shown in figure 6.8. The values for the experimental pressure drop at location A and F are not taken into account, since entrance effects may occur at these locations. 21\% is underpredicted within an error bound of -30\% of the experimental data and 63\% is over predicted within an error band of +70\% of the experimental data.

These results are not in agreement with literature. Yue et al.\textsuperscript{15} compared their data with the Lockhart-Martinelli model for various flow regimes. It was found that the Lockhart-Martinelli model under-predict the experimental data. The experiments were done in two circular channels with a hydraulic diameter of 333 \( \mu \)m and 528 \( \mu \)m using nitrogen and water. The superficial gas velocity is varied between 2 m/s and 23 m/s and the superficial liquid velocity is varied between 0.18 m/s and 0.91 m/s. Fukano and Kariyasaki\textsuperscript{24} found the same conclusion as Yue et al. based on their experimental pressure drop results for a circular channel with a diameter of 2.4 mm. It should be noted that the Lockhart-Martinelli model is independent of flow pattern. This means that no specific Taylor flow details are taken into account and therefore does not predict the experimental pressure drop well.
Figure 6.9: The pressure drop calculated using the unit cell model plotted against the experimental results. The unit model under-predicts the pressure drop obtained from the experiments.

Figure 6.10: The pressure drop calculated using the Kreutzer model plotted against the experimental results. The Kreutzer model shows the same trend as for the experimental pressure drop.
Results and discussion

Unit cell
The frictional pressure drop based on the unit cell model is calculated using equation 4.17. The experimental gas hold-up is used to calculate the relative gas bubble length \((L_b/L_{uc})\) in equation 4.16. The pressure drop obtained from the unit cell model, described in chapter 4.3, is plotted against the experimental pressure drop in figure 6.9. Again, the locations A and F are not taken into account due to entrance effects at the entrance and the end of the channel. It can be seen from figure 6.9 that the unit cell model under-predicts the experimental two-phase frictional pressure drop (96%) \((\Delta P/\Delta z)\).

The velocity of the film layer is equal to \((1-(A_b/A)^{2/3})\) of \(U_t\) and calculated using equation 4.12, is varying between 0.0012 m/s and 0.0026 m/s. Compared to the liquid velocities (0.1 m/s to 0.6 m/s) and the gas velocities (0.27 m/s to 1.04 m/s), it can be concluded that the velocity in the film layer is negligible compared to the gas and liquid velocities. The contribution of the two-phase region on the pressure drop is calculated using equation 4.14. The pressure drop contribution of the two-phase region is varying between 1.78 bar/m and 3.82 bar/m. These values are 2% to 10% of the two-phase experimental pressure drop (18 bar/m to 149 bar/m). The pressure contribution of the two-phase region is assumed to be negligible.

The unit cell model does not take the disturbances of the Hagen-Poiseuille flow due to the presence of the gas bubble. Due to the large velocities (order of 1 m/s) and a small micro channel \((D_{1F}=66.67e-6 m)\) used in this study, inertia do play an important role.

Kreutzer
The friction factor is calculated using equation 4.25 and used in equation 4.23 for \(f_{obr}\). The total frictional pressure drop is calculated using equation 4.23. The term for the contribution of gravity \((\beta\rho gL)\) is neglected because in this graduation work a horizontal micro channel is used. The pressure drop obtained by the Kreutzer model \(((dP/dz)_{Kreutzer\ model})\) is plotted against the experimental pressure drop \(((dP/dz)_{experimental})\) in figure 6.10. The pressure drop obtained from the model follows the same trend as the experimental pressure drop. However, most data are underpredicted by the Kreutzer model (76% within an error bound of 70%).

It must be noted that the model obtained by Kreutzer et al. is fitted from their own data. Therefore, a deviation can be noted using the data obtained from this graduation work. Some remarks are made on the pressure drop model by Kreutzer:

- For the friction factor described by equation 4.23, the velocity \((U_{TP})\) is described using the gas velocity plus the liquid velocity \((U_g+U_l)\), which is based on the cross sectional area of the channel. However, it is shown by Kreutzer et al. that the bubble velocity is not equal to the sum of the liquid velocity and the gas velocity as shown in figure 6.11a. This is also found using the data obtained in this graduation work as shown in figure 6.11b. This shows that the presence of the liquid film can not be neglected.

- The liquid hold-up is described by using the slug length and the bubble length \((\beta=L_s/(L_s+L_b))\), which is based on the cross sectional area of the bubble. This assumes that the volume of the liquid in the film is negligible. This is however in contradiction to the results on the bubble velocity shown in figure 6.11a.

Thus using the friction factor both the velocity \((U_{TP})\) and the liquid hold-up \((\beta_l)\) should be based on either the cross-section of the channel \((A_t)\) using \(U_g+U_l\) and \(1-\epsilon_g\) or the cross-section of the bubble \((A_{b})\) using \(u_b\) and \(\beta_l\). The pressure drop obtained from the Kreutzer model is now plotted against the experimental pressure drop is using \(u_b\) and \(\beta_l\) given in figure 6.12a. It can be seen that the pressure drop calculated using \(u_b\) and \(\beta_l\), is larger then the pressure drop obtained from the
Figure 6.11: a. the bubble velocity plotted against the liquid velocity plus the gas velocity by a. Kreutzer et al. and b. for the data obtained in this graduation work.

Figure 6.12: (a) The pressure drop calculated using the Kreutzer model plotted against the experimental results using $u_b$ and $\beta_l$. The pressure drop obtained from the Kreutzer model using $u_b$ and $\beta_l$ do not agree with the experimental pressure drop obtained in this graduation work. (b) $f_{obs}$ plotted against the slug length divided by the hydraulic diameter and the model plot by Kreutzer. It can be seen that the model over-predicts the data.
Results and discussion

Experiments (92%). In figure 6.12b, the friction factor times the Reynolds number ($fRe$) for the experimental data is plotted against the dimensionless slug length times ($Ca/Re)^{0.33}$. The friction factor according to the Kreutzer model in equation 4.25 times the Reynolds number is also plotted against the dimensionless slug length times ($Ca/Re)^{0.33}$. Here, $u_b$ and the hydraulic diameter of the bubble ($D_b$) are used.

The model follows the same trend as the experimental data, as shown in figure 6.12b. However, it over-predicts the experimental data. This is due to the over-prediction of $f_{obs}$ by using the sum of the liquid and gas velocities instead of the bubble velocity. Since the constants $a$ and $b$ are obtained by non linear regression using $U_g+U_l$, the model over-predicts the experimental data for this graduation work.
Conclusions and recommendations

7. Conclusions and recommendations

In the first part of this chapter, the conclusions are summarized of the results obtained in this graduation report. In the last part recommendations are given for further research on this subject.

7.1 Conclusions

Flow patterns
In the 50x100 µm channel using isopropanol and nitrogen, the following flow patterns are observed: bubbly flow, annular flow, Taylor flow, churn flow, ring flow and Taylor-ring flow. Compared to water and nitrogen used in previous study, the flow pattern differs at lower gas and liquid velocities. A larger area of Taylor-ring flow is obtained for isopropanol and nitrogen. This is due to the higher viscosity and lower surface tension of isopropanol compared to water. At high gas and liquid velocities churn flow occurs within the same range.

Taylor flow model
The ratio between the cross-sectional area of the channel and the cross-sectional area of the bubble \( \frac{A}{A_b} \) is for location B to E a constant value of 1.56±4% for bubble velocities used in this work. Therefore it can be concluded that the film thickness remains constant along the channel length and is independent on the bubble velocity \( u_b \). Aussilous and Quere (2000) found within the same range of gas and liquid velocities that the film thickness remains the same. From this graduation work, the same conclusion is made.

The gas hold-up \( \varepsilon_g \) can be described as the flow quality \( \varepsilon_g = C \frac{U_g}{U_l + U_g} \). The Taylor flow model described in section 3.3 is used to determine the gas hold-up \( \varepsilon_g \) and the superficial gas velocity \( U_g \). It is found that the constant \( C \) for isopropanol/nitrogen is equal to \( \frac{A}{A_b} \) (\( = 0.65 \) for locations B to E). The gas hold-up is increasing along the channel length and therefore it can be concluded that expansion of the gas phase occurs.

The pressure drop can be obtained from images of Taylor flow in the micro channel using the Taylor flow model.

Pressure drop
It is found that for describing the friction factor, the film layer should be seen as a stagnant wall.

The experimental pressure drop is compared to the Lockhart-Martinelli two-phase pressure drop model. For the experimental data of location B to E, 63% of the data is over-predicted within 70% and 21% of the data is under-predicted within 30%. This is in disagreement with the literature which concluded that the Lockhart-Martinelli model under-predicts the two-phase frictional pressure drop.

Next, the experimental pressure drop is compared to the unit cell model. For the experimental data of location B to E, 4% is overpredicted and 96% is under-predicted. The model does not take the disturbance of the Hagen-Poiseuille flow in the liquid phase by the gas bubbles into account which causes a larger pressure drop over the gas bubble.

The experimental pressure drop is also compared to the Kreutzer model. The pressure drop obtained from the model using, \( U_g + U_l \), follows the same trend as the experimental pressure drop. However, most data is under-predicted by the Kreutzer model (76% within 70%). Using \( u_b \), the
pressure drop obtained from the model, almost all data (92%) are overpredicted. This over-prediction is due to fitting the constants for the model using overpredicted fobs. It should be noted that Kreutzer neglects the larger bubble velocity as compared to the total flowrate.
7.2 Recommendations

In order to check the pressure estimations of the Taylor flow model, experiments could be done with the use of pressure sensors. A larger channel should be used so entrance effects for the sensors are minimal and a larger diameter so pressure sensors can be placed (250 µm). These experiments can also be used in order to check the validity of the pressure model.

From this study, the film thickness remains constant along the channel with varying the bubble velocity. This is in agreement with literature. However, what is unknown is why this film thickness remains a constant value. The model for the dimensionless film thickness of Aussillous and Quere could be updated by using experimental data.
Literature


<table>
<thead>
<tr>
<th>Reference</th>
<th>Author(s)</th>
<th>Title</th>
<th>Journal/Book Information</th>
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### Nomenclature

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### Greek letters

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### Dimensionless groups

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31