Scheduling multiple products
having sequence dependent setups
in a no wait batch production
environment with shared equipment
at a food factory

by
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BSc Industrial Engineering and Management Science – TU/e (2011)
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in partial fulfilment of the requirements for the degree of

Master of Science
in Operations Management and Logistics

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Subject headings: production scheduling, multiple products, sequence dependent setups, no wait batch production, shared equipment.
I. Abstract

This master thesis describes the development of an mathematical scheduling model and an heuristic for a production environment that is characterized by many scheduling constraints including no wait batch production, share of equipment and long sequence dependent setups. The mathematical model provides production schedules for small problem instances, whereas the heuristic can provide schedules for large problem instances. The starting point of the research was to combine this environment with cyclic production, but it was found that cyclic production in this environment is not advisable.
II. Preface

This report is the result of my master graduation project as the final phase of the master program Operation Management and Logistics at Eindhoven University of Technology. This master graduation project was carried out at a company in the food industry, Company A.

My master thesis is the end of a 5 year study period, during which I learned a lot, in particular during the last year, the project period. I would like to thank several people who gave me the opportunity to perform this project and who supported me during the project.

First, I would like to thank Ir. dr. Flapper, my first supervisor from the TU Eindhoven, for his enthusiasm and overall support during my project. In particular, I appreciated his remarks and advices which helped me to continue with the next phase of my project. Next, I would like to thank Prof. dr. de Kok, my second supervisor from TU Eindhoven. By looking at the project from a broader perspective and asking critical questions he provided new directions for the research.

From Company A I would like to thank my supervisor for his guidance during my project. He supported me and gave me advices how to continue. I also would like to thank all other employees of Company A who were always willing to give me information which I needed for my project.

Last, but not least I want to thank my family and friends for their support during my study period. Especially, I would like to thank my parents and sister for always believing in me. Finally, I want to thank Giel for always being there for me and cheering me up.

Nicole Aarts
III. Management summary

The production area X of Company A’s factory Y is characterised by many scheduling constraints including no wait batch production, share of equipment and long sequence dependent setups. The production area X produces around 100 different SKUs. The main goal of this study is to answer two research questions concerning this production area:

1. *Is cyclic production a good scheduling policy for production area X?*
2. *Relaxing which constraints lead to the highest (cost) benefit at production area X?*

The combination of two constraints impact the scheduling opportunities heavily. These constraint characterize the production area X. First, equipment is shared in production area X. Different Stock Keeping Units (SKUs) require the same equipment when produced. Secondly, the production is classified as no wait and the equipment should be reserved simultaneously when producing an SKU, see figure III.1.

To answer the first research question, previous projects done at Company A considering cyclic production and literature about cyclic production were studied. The projects done at Company A did not match in terms of production area. From the literature study followed that a model which combines cyclic production with the other problem characteristics of the production area was not found in literature. Desired was an cyclic scheduling model were decisions on how often to produce a product and in which order to produce it are fixed and only the decision on how much to produce is not fixed. Two major problems occurred. Determining a fixed cycle in an environment where equipment is shared is very difficult because the cycles interact (1). Next, the flexibility in the quantity to produce is very limited, because cycles would have to be adapted all the time which is undesired. Concluding cyclic production for production area X is very challenging and combining it with research question 2 in one scheduling model is not possible. Therefore the research was focused on research question 2: ‘Relaxing which constraints lead to the highest (cost) benefit at production area X?’.

To investigate this second question a conceptual model for the production area X is developed. The following constraints were included in the conceptual model: equipment availability, SKUs having different production routings, the size switch on filling equipment 2 which can only be performed in the weekend, the sequence dependent setup times, the fact that equipment is shared, the fact that the production is no wait and equipment is reserved simultaneously, equipment SPM clean, and the fact that certain SKUs should start or end on their equipment due to extensive cleaning and/or contamination issues. Three types of costs are included, setup costs, penalty costs and equipment D costs. Setup costs
occur when changing from one SKU to another. Penalty costs occur when an SKU that is asked to be produced cannot be produced. Equipment D costs occur when the equipment D is enabled for production, but not used for the production of SKUs. Also some additional assumptions are made: one period is modelled, there is only one production routing per SKU, there are no setups at the beginning of a period and order splitting is not allowed.

The mathematical scheduling model Z corresponds to the above described conceptual model and its objective is to minimize costs. The model is verified and validated and works well for small problem instances (approximately 8 SKUs). For large problem instances (approximately 30 SKUs per period), the heuristic is developed. The heuristic is capable of creating production schedules for these large problem instances. The heuristic also corresponds to the conceptual model, only a few adaptations are made. The model assumption ‘one routing per SKU’ is relaxed. It is allowed to deviate from the requested routing if this can improve the schedule. Moreover the objective of the heuristic isn't minimizing costs, but minimizing makespan. However, by prioritizing the minimum makespan for equipment D, equipment D costs are also minimized. This heuristic provides an initial solution. Due to limited time, no further optimization of the heuristic could take place.

To investigate the functioning of the heuristic, the schedules created with the heuristic are compared to the schedules created with the mathematical model as well as with the actual schedules created by the schedulers of production area X. For every schedule associated setup and Equipment D costs can be calculated. These costs are used to compare the schedules, the lower the costs the better.

To perform the comparison between the schedules created with the heuristic and the schedules created with the mathematical model, a small data set is used since the scheduling model Z cannot be solved for a large problem instance. The schedules created with the heuristic performs better in terms of makespan (i.e. they finish earlier) compared to the schedules created with the scheduling model Z. As the objective of the heuristic was to minimize makespan and the objective of the scheduling model to minimize costs, this is a logical consequence. Remarkable is that the total costs are the same for both methods, despite the fact that the heuristic did not focus on minimizing costs.

To perform the comparison between the heuristic and the schedulers of production area X, the actual schedules created by the schedulers of production area X are used. These schedules contain input information about a large problem instance (approximately 30 SKUs). This input is used to perform the heuristic and come up with an production schedule. The heuristic performs better in terms of costs, but noted that the schedule of the heuristic does not include all SKUs in two of the four cases. One or two SKUs could not be included in the schedule. The makespan is comparable for the schedulers’ and heuristic’s schedule.

Despite the fact that the heuristic only provides an initial solution, the heuristic provides good executable production schedules. It is recommended to Company A to further investigate the usage of the heuristic. When this is further investigated and the heuristic is computerized, it can be used as an scheduling tool. Furthermore it is recommended to take into account that the schedulers and supply planners have a lot of knowledge about all the constraints. It should be considered that there is a risk when these people change their jobs.
Finally as a recommendation to literature, it should be further investigated if a mathematical model (dealing with simultaneous reserved equipment) can be developed which can deal with large problem instances. This should then be compared to schedules derived via an heuristic, as it can be concluded that the heuristic is able to deal with the large problem instances.
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## V. Definitions & Abbreviations

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<th>Description</th>
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<tr>
<td>Asset calendar</td>
<td>Excel file indicating the available hours in a given week for a given equipment.</td>
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<td>Case</td>
<td>A case is containing a certain amount of units (e.g. 12 units, 24 units).</td>
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<td>Cost of goods sold</td>
<td><strong>COGS</strong> Costs that go into making the goods that are sold by Company A.</td>
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<td>Customer service level</td>
<td><strong>CSL</strong> $CSL (%) = \frac{\text{# of cases ordered}}{\text{# of cases delivered}}$</td>
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<td>Planned average shift output</td>
<td><strong>PASO</strong> Amount of units produced per shift (in thousands)</td>
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<td>Quick served meals</td>
<td><strong>QSM</strong></td>
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<tr>
<td>Recipe</td>
<td>Represents the ingredients and their appropriate proportions for a product.</td>
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<td>Replenishment Frequency Index</td>
<td><strong>RFI</strong> The average time between production runs for a particular SKU.</td>
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<td>Production Routing</td>
<td>Indicates which equipments should be used to produce a given SKU.</td>
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<td>Stock Keeping unit</td>
<td><strong>SKU</strong> Goods produced by Company A. Sometimes abbreviated to unit.</td>
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<tr>
<td>Supply and operations planning</td>
<td><strong>S&amp;OP</strong> Department of Company A</td>
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\(^1\) monthly performance newsletter December 2012
VI. Introduction

This master thesis has been conducted for Company A in collaboration with the Supply and Operations Planning (S&OP) department and Company A’s factory Y. Company A manufactures and markets food products all over the world. The Supply and Operations Planning department is responsible for the demand and supply planning of all Company A’s factories in Europe. At the factory Y, Quick served Meals (QSM) are produced. Throughout the years, factory Y became rather complex because of the increase in equipment and the increase in variety of products (Tiemersma, 2012). This master thesis focuses on a production area characterised by many scheduling constraints including no wait batch production, share of equipment and long sequence dependent setups. The production area is called: production area X.

Since the sequence dependent setups are long, it was suggested to look into cyclic production. This resulted in the first research question:

1. Is cyclic production a good scheduling policy for production area X?

Previous to this research an internal research (bachelor project) was conducted by Linda Tiemersma. She focussed on the actual workload and compared it to the theoretically available time at production area X. Linda Tiemersma (2012) came with the following recommendation: relaxing some constraints may lead to more flexibility within planning and scheduling. This resulted in the second research question:

2. Relaxing which constraints lead to the highest (cost) benefit at production area X?
VII. Research Model and Report Structure

This paragraph discusses the Research Model followed in this master thesis (paragraph VII.I). In paragraph VII.II the Report Structure is discussed.

VII.I. Research Model

The Research Model developed by Mitroff, Betz, Pondy and Sagasti (1974) is used as a guideline for this master thesis, see figure VII.1.

![Conceptual Model Diagram]

Figure VII.1: Mitroff et al. (1974) research model (Source: Bertrand & Fransoo, 2002:252)

The Research Model consists of four phases, (1) conceptualization (yellow), (2) modeling (red), (3) model solving (green) and implementation (blue) (Bertrand & Fransoo, 2002). In the conceptualization phase a conceptual model of the studied problem is developed. “The researcher makes decisions about the variables that need to be included in the model, and the scope of the problem and model to be addressed” (Bertrand & Fransoo, 2002:253). Next, in the modeling phase a quantitative model is built, all causal relationships are defined. Hereafter the model is solved. In the last phase (implementation) the results are implemented. (Bertrand & Fransoo, 2002)

This master thesis covers the first 3 phases of Mitroff et al.’s (1974) model: conceptualization, modeling and model solving.
VII.II. Report Structure

The report structure corresponds to the Research Model described above. In figure VII.2 the Report Structure is given.

As can be seen in figure VII.2 the report is structured in four parts, Conceptualization, Modeling, Model Solving and Conclusions and Recommendations.

PART 1 Conceptualization
This part corresponds to the conceptualization phase in Mitroff’s (1974) model. In chapter 1, Company A and Factory Y Analysis, background information about Company A and in particular production area X is discusses. Next chapter 2, Cyclic Production, focusses on the first research question: “Is cyclic production a good scheduling policy for production area X?”. Previous projects conducted at Company X relating to cyclic production and literature about cyclic production are discussed here. It is discussed why cyclic production is very difficult for production area X and why cyclic production is not further pursued in this research. Finally the conceptual model is presented in chapter 3, Conceptual Model. The included constraints and costs for the model are discussed here.

PART 2 Modeling
This part corresponds to the modeling phase in Mitroff’s (1974) model. Chapter 4, Mathematical Model, presents an initial model, which takes into account important design requirements. Next, the scheduling model Z is presented. This model corresponds to the conceptual model described in chapter 3.

PART 3 Model Solving
This part corresponds to the model solving phase in Mitroff’s (1974) model. In chapter 5, Verification and Validation, the mathematical models are verified and validated and it is discusses why the scheduling model Z cannot be solved for a large data set. Chapter 6, Heuristic, presents a scheduling heuristic for production area X that is able to deal with the large data set. Next in chapter 7, Comparison, the schedules created with the heuristic are compared to the schedules created with the mathematical model as well as to the schedules created by the schedulers of production area X.
**PART 4 Conclusions and Recommendations**
This part contains the conclusion and the recommendations for business and literature.
In Chapter 8, Conclusions & Recommendations, conclusions and recommendations that can be derived based on the research presented for Company A are discussed here. Next, recommendations regarding future research will be given.
PART 1 Conceptualization

Chapter 1. Company X and Factory Y Analysis

1. Company A and Factory Y Analysis

This chapter discusses the key performance indicators that Company A manages paragraph 1.1. Next factory Y production process is explained in paragraph 1.2. Further the production control is discussed paragraph 1.3. Finally in paragraph 1.4 the production environment and constraints are discussed. Y produces a variety of Stock Keeping Units (SKUs). An SKU represents a finished product. Factory Y produces over 300 different SKUs. The production area in scope, produces about 100 different SKUs. Production takes place during the week, the factory is open during the weekend for maintenance.

1.1 Key Performance Indicators

In this paragraph Company A’s Key Performance Indicators are discussed. Company A uses the following Key Performance Indicators (KPI’s): Customer Service Level (paragraph 1.1.1), Inventory (paragraph 1.1.2) and Costs per SKU (paragraph 1.1.3) (Tiemersma, 2012). In paragraph 1.1.4 the relations between these indicators are discussed.

1.1.1 KPI Customer Service Level

Company A sells their products to retail companies. These retail companies order cases. A case contains a certain amount of (the same) Stock Keeping Units. Company A uses the following Customer Service Level (CSL)²:

\[
CSL \text{ (%) } = \frac{\text{# of cases delivered}}{\text{# of cases ordered}}.
\]

CSL is measured per week. According to the current target CSL needs to be at least xx%. In figure 1.1 the Supply chain from Company A to the end customers is shown. Inventory is kept at Company A to be able to serve the retail companies. Here the CSL of Company A is measured (see figure 1.1). The more inventory is kept, the higher the CSL, because the probability that Company A is out of stock is reduced. If Company A is unable to deliver the retail company this does not mean that this results in a lost sale at the end customer (see figure 1.1), since the retail companies keep inventory at their warehouses and stores. A lost sale might occur if the end customer decides to substitute the product or to not buy it at all. If the end customer decides to buy it at another retail company, it is a lost sale for the retail company but not for Company A. Therefore an incidental low CSL does not immediately lead to loss of income for Company A. However if the CSL is low for a longer period, inventory in the supply chain may fade out and lost sales may occur. Furthermore if the service of Company A to its customers (retail companies) is poor, the retail companies can decide to leave Company A as a client or delist some of its products. This could lead to a long term loss of income.

![Figure 1.1: Supply Chain, from Company A to customers.](image)

² monthly performance newsletter December 2012
1.1.2 KPI Inventory

Key Performance Indicator Inventory relates to the costs made by keeping SKUs in stock (storage costs and interest loss). The inventory costs are different per SKU, because the value of the raw materials and its necessary operations are different. This value is called cost of goods sold (COGS) per SKU. The COGS presents all the costs made by Company A (material costs, labour costs, overhead costs etc.) to make the SKU. Therefore as long as an SKU is not sold and kept in inventory it represents money. This money is ‘stuck’ in inventory and cannot be used for other purposes. The money could be used for investment and create interest. Furthermore when SKUs are stored this leads to storage costs (i.e. maintaining and operating the warehouse).

Recently a performance indicator is introduced to decrease inventory and therefore inventory costs. This performance indicator is the Replenishment Frequency Index (RFI). RFI indicates average time between production runs for a particular SKU. If the time between runs is small the SKU is more frequently replenished, which leads to a lower RFI value as is depicted in figure 1.2 and 1.3. If RFI is long, more inventory is created to meet demand (figure 1.2). When RFI is small less inventory is created to meet the demand (figure 1.3).

![Figure 1.2: Inventory versus Time (long RFI).](image)

![Figure 1.3: Inventory versus Time (short RFI).](image)

1.1.3 KPI Costs per SKU

The Costs per SKU are divided in material costs (e.g. raw material) and conversion costs (e.g. labour, utilities) (figure 1.4).
Costs per SKU could be reduced when producing more SKUs, but this is only beneficial if it is also possible to sell these SKUs for a profitable price. Company A is a demand driven company. This means that the production is planned according to forecasted demand. There is no gain in producing more than the forecasted demand, because it would not be possible to sell these SKUs. The costs per SKU are also influenced by how the production is scheduled, because when changing from one SKU to another setup costs.

### 1.1.4 Relations between Key Performance Indicators

As shown in figure 1.5 the key performance indicators interact with one another. The objective is that CSL is as high as possible, costs per SKU is as low as possible and Inventory is as low as possible. But because the indicators interact, the optimum lies somewhere in between. If inventory is low, customer service level is low as well. When there is less inventory, the probability to be out of stock increases. Next, to keep inventory low RFI should be decreased, but when RFI is decreased more setups will occur. This leads to an increase in setup costs, which increases the costs per SKU.
1.2 Production Process

At Company A’s factory Y, quick severed meals (QSM) are produced. The production process consists of 4 steps: Manufacturing, Filling, Sterilising and Packaging. In figure 1.6 the production steps are presented.

Manufacturing
In the manufacturing step ingredients are prepared for the filling step. The preparation consists of different activities.

Filling
In this step cans are filled with prepared ingredients. Thereafter the cans are sealed with a lid. From now on no ingredients can be added. This filling step can consist a number of filling actions when a recipe contains several prepared ingredients.

Sterilising
The cans are heated up to a certain temperature to sterilise the can and its ingredients. Then they are cooled down.

Packaging
In the packaging steps the cans are provided with a label and sealed together in a certain format (e.g. 12 cans 3 x 4, 24 cans 2 x 3 x 4). The sealed packages are placed upon pallets and these are moved to the nearby distribution centre.

1.3 Production Control

In figure 1.7 the total production control for Company A (focusses on production area X) is given. It is divided into (Bertrand, Wortmann & Wijngaard, 1990):

- Good flow control
- Production Unit Control

The Good flow control releases work orders to the production unit (Bertrand et al, 1990), see figure 1.7. The production unit is responsible to execute the work order. The Good flow control is responsible for the inventories. At Company A multiple SKUs are produced. This is represented by the various stock points (triangles) of finished goods. The production unit refers to the factory Y. “A production unit may consist of one large installation or machine, or it may consist of an entire production hall with many different types of machinery and personnel” (Bertrand et al. 1990:13). In production area X many types of machinery is used to execute the work orders.
Chapter 1. Company X and Factory Y Analysis

Figure 1.7: Total production control Company A (focuses on production area X).

Linking figure 1.7 to the production control at Company A: the goods flow control corresponds to the S&OP department, in particular Supply Planning where they make inventory decisions and decide on a supply plan (work orders). Scheduling corresponds to operations within the boundaries of the production unit. In figure 1.8 an overview is given of the decision making process for production at Company A.

Figure 1.8: Decision-making process for production (Tiemersma, 2012:20)
1. The demand planners are located in the countries for which they plan the demand. The demand planners provide the supply planners with weekly demand information at the beginning of each month for a period of approximately 52 weeks. If necessary during a month a demand planner can change the demand plan via an exception form, which has to be approved by the supply planner.

2. The supply planners are located at the S&OP department. They plan production for a horizon of 6 weeks based on the demand and certain factory constraints (e.g. capacity, equipment availability). The supply plans are on week level and are forwarded to the schedulers.

3. The schedulers are located in the factory, Y. They make schedules for a horizon of 6 weeks based on the supply plan and factory constraints. These schedules state in detail at which day and what time which SKU needs to be produced.

4. The schedule is forwarded to the production employees and material planners. They are also located in the factory and make sure that the proposed schedules are executed.

This research focusses on scheduling (red in figure 1.8). Supply planners and schedulers frequently deliberate on the supply plans and schedules. Plans and schedules are adapted if demand information changes, a CSL risk appears, or a scheduling problem (requested supply plan cannot be scheduled) occurs. Sometimes less SKUs are made, mostly caused by equipment failure. Sometimes the factory produces more of an SKU, this happens when production runs more quickly than expected.

1.4 Production Environment and Constraints

An overview of the entire factory is shown in Appendix A. This overview provides insight in Y complexity, although the presented factory layout is not up to date. The production area in scope produces Quick Served Meals. In figure 1.9 an up to date overview of the production area in scope is shown. It shows that a lot of equipment is shared. This has a large impact on the planning and scheduling of the production area. In Appendix B is described what every equipment does. As can be seen in figure 1.9, various equipment is used to produce the SKUs. A production routing indicates which equipment is necessary to produce a given SKU. Some SKUs have only one production routing and some have several production routings. The production routings are further explained in chapter 3, paragraph 3.1.3.
As indicated before, the production process consists of 4 steps (Manufacturing, Filling, sterilising and packaging). Every produced SKU has to pass these steps, but the equipment that is used can differ. The production is classified as no wait. This means that there are no intermediate stock points between the production steps. Further if an SKU is produced, all equipment required has to be reserved simultaneously. This is clarified in figure 1.10.
When an equipment is reserved for production of an SKU, no other SKU can use this equipment. When producing SKU 1: Manufacturing 1, Filling 1, Steriliser 1 and Packager 1 are reserved simultaneously. When reserved the equipment cannot be used to produce another SKU.

There are more constraints that supply planners and schedulers have to take into account. A list of all the constraints of production area X is displayed in Appendix C. It is indicated whether the constraints are covered by the supply planners and or schedulers. The supply planners use the tool, in which certain constraints are built in. Some constraints are taken into account within planning but not built into the system. These constraints are dealt with manually. The schedulers use a tool to schedule the SKUs. Furthermore they use additional information like the Factory Process sheet (stating which SKU can be produced via which routing) and the outage plan (to check labour availability). Some constraints are taken into account both in planning and in scheduling, but this does not necessarily mean that they are taken into account in the same way. If constraints are taken into account by both, the schedulers usually possess and use the most accurate information.
2. Cyclic Production

This chapter focuses on the first research question: “Is cyclic production a good scheduling policy for production area X?” First, previous projects done at Company A concerning cyclic production are discussed (paragraph 2.1). Next, the use of cyclic production in the literature is discussed (paragraph 2.2). In paragraph 2.3 the conclusion of the first research question is discussed.

2.1 Previous Projects

Previous projects at Company A that involve cyclic planning and scheduling are discussed here. The goal of these projects was to reduce RFI and by this inventory. These projects can provide valuable information for the investigation of cyclic production of production area X. First the RFI project Y is discussed (paragraph 2.1.1). Within this project the implementation of cyclic production on planning level is investigated. Next in paragraph 2.1.2 a scheduling wheel project at another factory of Company A is discussed. It is indicated why the work done in these projects cannot (completely) be followed up.

2.1.1 RFI project factory Y

This project considered all the paste SKUs (products containing paste) at Y factory. These SKUs are produced in the following production area X. Besides paste SKUs also non-paste SKUs are produced in this area. The project presents a cyclic production plan for the paste SKUs. The plan states via which routings and how often SKUs should be produced. Furthermore it states how much should be produced per cycle. The plan was generated by an employee of Company A’s department S&OP, together with colleagues at the S&OP department, and the Y scheduler of the paste SKUs.

The SKUs are categorized as A, B, C or D SKUs based on their percentage of the yearly demand (in amount of units). The volumes of all (paste) SKUs are added up. For every SKU the percentage of volume it contributes to the total amount is calculated. An SKU is classified as an A, B, C or D SKU based on cumulative percentages.

- A SKUs: to 70%
- B SKUs: from 70% to 85%
- C SKUs: from 85% to 95%
- D SKUs: from 95% to 100%

Within Company A the objective is that A SKUs should be produced every week, B SKUs every other week, C SKUs every four weeks and D SKUs every 8 weeks. This distinction is based on the assumption that large volume SKUs have a bigger influence on the inventory and corresponding inventory costs. Therefore a large volume SKU (A SKU) should be produced more often than a lower volume SKU (D SKU). This replenishment objective (A once a week, B every other week, C every four weeks and D every 8 weeks) is used as a starting point for the RFI project.

Next, all the SKUs are allocated to routings. They are allocated to the routing via which the SKUs are produced most regularly (i.e. SKU X is 90% of the time produced on line Z, then SKU X is allocated to line Z). This information is provided by the scheduler of the paste SKUs.

It is assumed that the equipment is available for an on-going period of 32 weeks and that every week is the same in terms of time available in a week and demand that occurs in a week. Based on the replenishment objective (A once a week, B every other week, C every four weeks and D every 8 weeks) and yearly demand, it is calculated how much should be produced of each SKU every cycle. Note that the cycle lengths depend on the replenishment objective (e.g. yearly demand 1,000,000
(SKU level) for an A item, weekly production 1,000,000/52. Table 2.1 represents an example of first 8 weeks of the production plan.

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<th>Table 2.1: Example of production plan for 8 weeks</th>
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The production plan takes into account several constraints which are listed in Table 2.2.

<table>
<thead>
<tr>
<th>Table 2.2: Constraints used in RFI Project</th>
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<tbody>
<tr>
<td>Name</td>
</tr>
<tr>
<td>------</td>
</tr>
<tr>
<td>Available hours</td>
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<tr>
<td>SKU routings</td>
</tr>
<tr>
<td>Setup times</td>
</tr>
<tr>
<td>Speed</td>
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<tr>
<td>Equipment D run</td>
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<td>MOQ (minimum order quantity)</td>
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</table>

As seen in Table 2.1, the production plan is given per weeks. This means that it is ‘only’ determined what should be produced in a week, disregarding the order in which SKUs are produced. Sequence dependent setup times are not included, but averages are used.

The conclusion was that not enough time was available to implement this plan, due to the increase in setup time. Solutions suggested to solve the time issue were, moving (non-paste) SKUs to other areas, improve (equipment) output and usage of overtime. Furthermore within the project it was assumed that a changing from 200 gram cans to 400 gram cans on filling equipment 2 is possible during the week, but in practice this is a difficult issue. The filling equipment 2 can fill both 200 gram cans as 400 gram can, but the equipment can only be setup for one of them. This setup is done by qualified engineers. These engineers are always available during the weekends to perform the setup. During the week these engineers aren’t available for this setup. In the past exceptions were made: the engineers were taken away from other activities to perform the setup during the week. This was only done when the setup was prioritized to the other activities of the qualified engineers (e.g. due to CSL issues).
The project provides insight in how cyclic planning in this production area could be realized. Drawbacks of the project are: Costs are not included (1), only the expectation that reduction in RFI leads to reduction in inventory and corresponding costs represents the influence on the costs. An increase in costs due to more lost material is not included. Furthermore a ‘small’ selection of the constraints is incorporated (2). Finally the MOQ (which are provided by the factory) included are partly based on non-technical constraints (3). Some MOQs are based on costs consideration and not on technical possibilities. A production plan might appear to be very different when disregarding these MOQs and base the production plan on technical constraints and costs (e.g. setup costs), because the MOQs do not represent the current technical constraints.

2.1.2 Scheduling Wheel

The introduction of a scheduling wheel was another project within Company A to decrease the RFI and therefore inventory. This project was executed at another factory of Company. The scheduling wheel was developed for 2 production lines. These production lines each produce a given set of SKUs, which cannot be produced at other lines. The production lines do not share equipment. When using a scheduling wheel, all SKUs are allocated to a line and the SKUs are produced in a fixed sequence (given by the wheel). The decisions how often to produce an SKU and in which order to produce are fixed. Only the decision how much to produce (could also be zero) is not fixed.

The scheduling wheels have been developed to decrease setup time. First a distinction between major and minor setups is made. The major setups are long setups that occur for a whole group of products. The minor setups are short setups that occur between products of the same group (for example label changes). In the scheduling wheel the major setups are scheduled first and sequence in the order that minimizes setup time. Next the minor setups are considered. By using the same cycle every period, the number of major setups are minimized. This decreases overall setup time. When decreasing the setup time, more time is available to perform other setups (produce more frequently). This decreases the Replenishment Frequency Index (RFI).

The biggest difference between the production area in scope for this master project is sharing of equipments which does not exist in this scheduling wheel project (1). Further this scheduling wheel project doesn’t consider the trade-off between constraints and costs to optimize planning or scheduling (2). Finally the idea to distinguish major and minor setups is difficult to apply to the production area in scope (3), because the major and minor setups are difficult to define here. The production sequence is more influenced by the constraints and the different (shared) equipment that SKUs need to use to be produced.

2.2 Literature Review

This research focusses on scheduling multiple products in a no wait batch production environment where equipment is shared. The objective is to combine the 2 model requirements (cyclic production and cost-benefit analysis) into one scheduling problem. In this chapter it is investigated if such a model (with all or a selection of the problem characteristic) already exists in literature. In paragraph 2.2.1 the problem characteristics are presented. Further in paragraph 2.2.2 it is discussed which articles match the problem characteristics to the greatest extend and whether these articles are useful for this research.
2.2.1 Problem Characteristics

To target the search for literature, the problem characteristics have been identified. The problem characteristics are a combination of the objective of the business and the technical constraints of the production environment. The two research questions are:

(1) Is cyclic production a good scheduling policy for production area X?

(2) Relaxing which constraints lead to the highest (cost) benefit at production area X?

are taken into account. Seven problem characteristics are distinguished.

1. Multi-line (multiple routings)
   The problem contains multiple non identical lines in production area X. This can also be addressed in literature by multiple non identical lines. Characteristic for the production area X is that equipment is shared among the ‘lines’.

2. Multi-product
   Multiple SKUs are produced in production area X.

3. Sequence dependent setup
   The setup times are sequence dependent.

4. Inventory costs
   Inventory costs are included as a problem characteristic, related to key performance indicator inventory.

5. Setup costs
   The setup costs are different per product.

6. Shared equipment
   SKUs have different routings where equipment is shared.

7. Cyclic production
   A cyclic schedule should be developed. This corresponds to research question 1 (“Is cyclic production possible at production area X?”). Company A would like to see that decisions on how often to produce a product and in which order to produce is fixed. Only the decision on how much to produce (could also be zero) is not fixed.

2.2.2 Related Articles

Paragraph 2.2.1 provides the problem characteristics. In the literature, articles have been searched that deal with these characteristics. In Appendix D per article is indicated what the article is about in general. Further it indicates if a problem characteristic is included and what is interesting about it. Concluding from the overview, cyclic production is common for single production lines. Production area X is characterized by multiple lines that share equipment, in other words; multiple routings. However Bollapragada and Rao (1999) and Pesenti and Ukovich (2003) do consider multiple lines and cyclic production. The models they (Bollapragada and Rao (1999) and Pesenti and Ukovich (2003)) present match with the problem characteristics to a large extend. However Bollapragada and Rao (1999) do not consider that share equipment, which is an important problem characteristic that effects the scheduling heavily. Pesenti and Ukovich (2003) do consider multiple lines but they are considered to be identical, whereas in production area X the equipment used to produce SKUs is not identical. Finally sequence dependent setups are not included in both models. Concluded is that a model that combines cyclic production with the other problem characteristics was not found in literature.
2.3 Conclusion Cyclic Production

As can be seen in the overview of production area X (figure 1.9) and the overview of constraints at production area X (Appendix C), a lot of constraints have to be handled. It became clear that the share of equipment is an important constraint.

The current production planning and scheduling is flexible since the production decisions; via which routing, which SKU, when and how much to produce, are determined every period (week) all over again. In meetings with the head of schedulers materials management and packaging (at Y factory) and the head of the supply planners for Y (S&OP department) it became clear that they are looking for a more fixed environment by introducing cyclic production on scheduling level (see paragraph 2.1.2). Decisions on how often to produce SKUs and in which order to produce them are fixed. Only the decision on how much to produce (could also be zero) is not fixed. However as became clear from the literature review a model that combines all the problem characteristics does not exist yet. Especially cyclic production schedules are difficult to construct, since equipment is shared. Two issues play an important role.

1. Determining a fixed cycle in an environment where equipment is shared.

To be able to find a fixed production cycle, where decisions on how often and in which order production takes place are fixed, is difficult for production area X. The most important reason is the fact that equipment is shared. If according to sequence dependent setup times cycles are determined for every equipment, it can be found that these cycles conflict.

For example:
Best cycle on equipment A (in terms of sequence dependent setup time) is: SKU 1, SKU2, SKU 5, SKU 6, SKU 8,
best cycle on equipment B (in terms of sequence dependent setup time) is: SKU 3, SKU 4, SKU 6, SKU 1, SKU 9,
see, figure 2.1:

![Figure 2.1: Conflicting cycles equipment A and B (SKU 1 and 6).](image)

As can be seen in figure 2.1 as far as sequence dependent setup times are considered, production for SKU 1 and SKU 6 isn’t possible.

2. Flexibility in quantity to produce.

When it is managed to determine ‘optimum’ cycles for all equipment taken into account that they are shared among various SKUs, another issue occurs. The flexibility in quantity to produce is very limited, since adapting the quantity causes the cycle to shift, which affects another, and another etc.

For example:
Cycle on equipment A is: SKU 1, SKU2, SKU 6, SKU 5, SKU 8,
cycle on equipment B is: SKU 1, SKU 4, SKU 6, SKU 3, SKU 9,
see, figure 2.2:

![Figure 2.2: Matching cycles equipment A and B.](image)
Changing the quantity of SKU 2 affects cycle A and therefore cycle B, see, figure 2.3:

![Figure 2.3: Interacting cycles equipment A and B, when quantity is changed.](image)

As shown in figure 2.3 changing the quantity of SKU 2 does not only affect equipment A but also equipment B, because the cycles interact with one another.

Cyclic production, where decisions on how often and in which order to produce are fixed and quantity is variable, is very difficult for production area X. Cycles would have to be adapted all the time which is undesired. The first research question, ‘Is cyclic production possible at production area X?’ is answered.

Cyclic production at planning level (corresponding to the RFI project, paragraph 2.1.1) could be possible. Note that the decision ‘in which order to produce’ will be relaxed and the benefits of avoiding sequence dependent setups will be reduced.

Concluding cyclic production on scheduling level for production area X is very challenging and combining it with research question 2 in one scheduling model is not possible. Therefore the focus will be on research question 2: ‘Relaxing which constraints lead to the highest (cost) benefit at production area X?’. To investigate this question a mathematical scheduling model for the production area X is developed, starting with a conceptual model in chapter 3.
3. Conceptual Model

This chapter presents a conceptual model for scheduling with multiple products having sequence dependent setups in a no wait batch production environment with shared equipment for production area X at Company A’s factory Y.

First of all the production area X is considered. In paragraph 1.4, figure 1.9, an overview of the production area X is given. This overview contains all equipment used for production. In order to simplify the problem, plausible assumptions are made. Some equipment can be ignored and for other equipment such assumptions cannot be made. The justification of these assumptions is given in Appendix E. The simplified overview of the production area X is shown in figure 3.1.
Next this conceptual model elaborates on which constraints (paragraph 3.1) and costs (paragraph 3.2) to include and which additional model assumptions are made (paragraph 3.3).

### 3.1 Included Constraints

In this paragraph the included constraints are given.

- Availability
- Size switch filling equipment 2
- SKU routings
- Setup times
- Speed
- Sharing equipment
- No wait batch production and simultaneous use of equipment
- S clean
- Start week, End week

These constraints correspond to constraints in production area X see Appendix C. Some constraints are not included, this is displayed (and justified) in Appendix C as well.

#### 3.1.1 Availability

The equipment is available for 103 hours per week in the Summer period (March - mid July) and 113 hours a week in the Winter period (mid July - February). Some equipment is not available this entire time (see justification equipment availability Appendix E).

1. **Equipment available for a percentage of the time**

   \[
   T_{\text{start} 1} \leq T_{\text{end} 1} \leq T_{\text{start} 3} \leq T_{\text{end} 3} \leq t \cdot T
   \]

   Tend 1 – Tstart 1 + Tend 3 – Tstart 3 is less or equal to t * T, where t is the percentage of time the equipment is allowed to be available.

   - B Is available a percentage of the time
   - P Is available a percentage of the time

2. **Equipment available from and to a certain moment in time**

   Equipment being available between 2 time points: \( T_{\text{start} \ e} \) indicates moment equipment \( e \) is available and \( T_{\text{end} \ e} \) indicates moment equipment \( e \) is not available anymore).

   - S Is available until a certain moment in time
   - J Is available until a certain moment in time
   - D Is available from a certain moment in time until a certain moment in time
3.1.2 Size switch filling equipment 2
The filling equipment 2 fills either 200 gram or 400 gram products. When changing from 200 gram to 400 gram products a setup takes place. This setup is in principal performed during the weekend, since the qualified engineers that perform the setup are only available during weekends. Sometimes an exception is made and the setup is performed during the week, but it cannot be assumed that is always possible. Therefore assumed is that the filling equipment 2 can either fill 200 gram or 400 gram products during a week. So there is either a 200 gram week or a 400 gram week for filling equipment 2.

3.1.3 SKU Routings
As mentioned before, SKUs can be produced via multiple routings, but all have to pass the manufacturing, filling, sterilising and packaging step. A new production routing code is introduced to indicate which set of equipments is used. Within Company A a production routing code already existed for the SKUs in scope, but this new code is specific for the production area X. It only includes the relevant equipment (as in the simplified overview figure 3.1). The production routing code provides information over the manufacturing, filling, sterilising and packaging equipment required for production. The filling and sterilising step are combined, because the filling and sterilising equipment are connected. (see figure 3.1). The production routing code is explained in Appendix F, an example of an SKU with corresponding routings is given also given in Appendix F. Every SKU has at least one possible routing. In Appendix G an overview of all SKUs produced in the production area X is given. Per SKU the possible routings are given. Some SKUs have also a routing outside of the production area X.

3.1.4 Setup Times
The setup time is the time required for changing a particular equipment (technical settings, connecting pipelines, cleaning activities). Setups occur when changing from one SKU to another on a particular equipment. The setup times for the SKUs in production area X are sequence dependent (i.e. the setup time when changing from SKU a to SKU b is different from when changing from SKU a to SKU c or when changing from SKU b to SKU a). The setup times are mainly caused by setup on filling equipment. The setup times for the SKUs produced in production area X for the filling equipment are shown in Appendix H. Within manufacturing additional setups occur on some of the equipment. These times are also given in Appendix H. For sterilising no additional setups occur. For packaging 30 minutes setups can occur when changing label or package format. The setups can be performed in parallel. This means that on different equipment setups for the same SKU can take place. The moment that the production of an SKU can start is determined by the setup on the necessary equipment that is finishes latest.

<table>
<thead>
<tr>
<th>Time</th>
<th>Manufacturing 1</th>
<th>Manufacturing 2</th>
<th>Filler 1/ Steriliser 1</th>
<th>Filler 2/ Steriliser 2</th>
<th>Packager 1</th>
<th>Packager 2</th>
</tr>
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<tbody>
<tr>
<td>1</td>
<td>Producing SKU 1</td>
<td>Producing SKU 2</td>
<td>Producing SKU 1</td>
<td>Producing SKU 2</td>
<td>Producing SKU 1</td>
<td>Producing SKU 2</td>
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<td>2</td>
<td>Setup</td>
<td>Setup</td>
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**Figure 3.2: Parallel setups**

As can be seen in figure 3.2 the setup for SKU 4 on manufacturing 1 takes place from time point 4 to 6, parallel to the setup for SKU 4 on Packager 1 time point 4 to 5. The setup on Filler 2/Steriliser 2 finishes latest (time point 7), and this determines the start of the production of SKU 4.
Further, combining the sequence dependent setups with the shared resources influence scheduling. In Appendix I an example is included to clarify this. The conclusion is that allowing longer setup times can lead to a better production schedule. Therefore sequence dependent setup times cannot be ignored and sequence dependency should be included in the model.

3.1.5 Speed
The speed by which SKUs can be produced is reflected in the PASO (Planned Average Shift Output). PASO is measured by package outage\(^3\), the amount of units coming out of packaging per shift. A shift takes 8 hours. All related processes (manufacturing, filling, sterilising and packaging) are executed with the same speed. The PASO measurement does not include setup times. The PASO value is based on historical data, so when the production of a certain SKU via a certain routing performs bad in terms of speed this is reflected in the PASO. Total failed production (i.e. nothing produced in a run) is not included. The speed for every possible routing is given in 1000 of units per shift (8 hours). The speed per production routing per product is given in Appendix G (via the last characters of the production routing code).

3.1.6 Sharing Equipment
The fact that equipment is shared is taken into account in the model. As described in Appendix I this is an important characteristic of the production environment. As shown in figure 3.3 where equipment M is shares by SKUs also using equipment G1 and G2. When equipment M is used in combination with equipment G1, M is not available for G2 and vice versa.

![Figure 3.3: shared equipment](image)

3.1.7 No wait batch production and simultaneous use of equipment
The whole routing is reserved when an SKU is produced. This characteristic of the production area is taken into account in the model. This specification corresponds to paragraph 1.4, figure 1.10. When a product is produced all the equipment in the routing is reserved simultaneously.

3.1.8 S clean
Whenever the equipment S is used continuously for 24 hours, a 5 hour cleaning should take place. Currently a maximum run of 24 hours can be requested to be produced of one SKU in one week. The constraint is included by demanding a 5 hour clean after every run.

3.1.9 Start week, End week
Certain SKUs have to start and others have to end the week on their filling equipment when they are produced. This is done to avoid contamination issues or extensive cleaning. The SKUs that have to

\(^3\) PASO is measured by package outage, except for multipack in combination with single pack at filling equipment 4 then the fill rate is taken.
PART 1 Conceptualization

Chapter 3. Conceptual Model

start and end the week are given in Appendix C, constraints 14 and 15. If more than one SKU should start or end on the same equipment one SKU is chosen. It is assumed that if more than one SKUs should start that they have always the same recipe.

3.2 Included Costs

Three types of costs are included: setup costs, penalty costs and equipment D costs. These 3 costs are minimized together.

3.2.1 Setup Costs

If an SKU is made, not all the ingredients used to make the product can be retraced in the end product. When starting to produce material is lost. At production start ‘material’ coming out of manufacturing machines or filling machines is not yet at the right quality (e.g. pipes are still full of cleaning liquid (water) and therefore the first ‘material’ is diluted). When the ‘material’ is at the right quality, the actual production can start. Further at the end of production material is also lost (caused by e.g. material left that is the pipes or in the manufacturing tanks).

Setup costs are different per recipe produced. Different SKUs may contain the same ingredients, but carry a different label. The recipe for these SKUs is the same. A recipe is a code that represents the ingredients and their appropriate proportions for a SKU. Some recipes contain multiple ingredients fed by different manufacturing machines. These recipes result in more lost material than recipes that use only one manufacturing machine. Also the fact that some ingredients are more expensive than others influences the setup costs. Further the setup costs are sequence dependent. When changing from an SKU with recipe x to an SKU with recipe y setup costs occur, due to loss of material. When changing from an SKU with recipe x to an SKU with recipe y the setup costs do not occur. This happens when only the label changes.

3.2.2 Penalty Costs

Whenever an SKU cannot be produced there must be a penalty in terms of costs. This is necessary because the objective in the mathematical model is to minimize costs. If there is no penalty for not producing an SKU, the model will concluded that nothing should be made. Since this only results in setup costs and Equipment D costs. The penalty costs should be sufficiently high assuming that producing the SKUs is most important to Company X, more important than the costs made by producing them.

3.2.3 Equipment D Costs

Assumed is that the manufacturing equipment D can only be setup for production ones a week. When the machine is setup and not used for actual production of an SKU it produces waste. This waste is lost material and a cost for Company A. Therefore it should be calculated for how long the machine is setup for production but not used for production of SKUs.

For example when producing SKU 1, 2 and 3 (see figure 3.4) equipment D is producing waste when it is not used for production of an SKU but enabled (yellow).

4 It takes at least 36 hours to set up the machine again when it is stopped and is it is only available 77 hours a week. Furthermore performing the setup will require a lot of labour.
Figure 3.4: Equipment D is creating costs

Equipment D is enabled between time points Tstart1 and Tend 3, but not used for production this entire time. It should be quantified in the model how much time equipment D is enabled and not used for production of SKUs (yellow parts in figure 3.4). Every 100 minutes equipment D is not used for production of an SKU, [redacted] kg of material is lost which is worth [redacted].

3.3 Additional Model Assumptions

The following additional model assumptions are made: one period, one routing per SKU, no setups at the start and no order splitting.

3.3.1 One Period

One period is modelled. I.e. for one period it is determined when production for an SKU starts and when its production end. One period represent one production week. The factory produces from Monday to Friday. In the model one period is described as an available capacity of T hours.

3.3.2 One Routing per SKU

It is assumed that all SKUs only have one production routing. This means that upfront it is decided which routing an SKU has, this is an input value. This assumption is made to restrict the decision possibilities of the model, therefore it is easier to solve the model.

3.3.3 No Setups at the Start

Sequence dependent setups are taken into account in the model. Changing from one product to the another takes a certain amount of time. Assumed is that the first product can start at the beginning of the period (at time point 0), because the setup for this product is performed in the weekend.

3.3.4 No order splitting

It is assumed that SKUs are produced at most once a week. Order splitting is not allowed.
4. **Mathematical Model**

First an initial model is presented that deals with some of the important constraints (paragraph 4.1). Then this model is extended to the scheduling model Z which is corresponding to the conceptual model described in chapter 3 (paragraph 4.2).

4.1 **Initial model**

The initial model (see figure 4.1) focuses on the following design requirements:

- Sharing equipment
- No wait continuous process
- Sequence dependent setup times and costs.

**Figure 4.1: Initial model (visual)**

Assumptions:

- One period is modelled (a week of T hours).
- Every SKU has one routing
  - Possible routings in see figure 4.2.

**Figure 4.2: Production routings**

- Possible routings in code
  - \((m, g_1)\)
  - \((m, g_2)\)
  - \((g_1)\)
  - \((g_2)\)

- There is no setup required for the first SKU produced.
- No order splitting.

When producing an SKU the whole routing is reserved see figure 4.3 and 4.4:
Figure 4.3 SKUs and their routing

Figure 4.4 Routings reserved

Sets

I SKUs

All SKUs that are asked to be produced in a week.

E Equipment

All equipment considered (here m, g1 and g2).

Parameters

\( c_i^p \) Penalty costs per unit of SKU \( i \) not delivered at the end of the period

\( c_i^{s,e} \) Setup costs when changing from SKU \( i \) to SKU \( i^* \) on equipment \( e \)

\( M \) Number bigger than the available hours in a period \( (T+1) \)

\( M' \) Number bigger than the maximum amount of SKUs produced on an equipment in a period

\( \rho_i \) Processing speed of SKU \( i \) (units per hour)

\( s_i^{e} \) Setup time when changing from SKU \( i \) to SKU \( i^* \) on equipment \( e \) (in hours)

\( T \) Hours available per week

\( U_i^e \) Binary value, 1 if SKU \( i \) requires equipment \( e \), 0 otherwise

\( WO_i \) Demand for SKU \( i \) in a period (in units)

Continuous decision variables

\( AP_i \) Produced quantity of SKU \( i \) in a period (in units)

\( T_i^{Start} \) Start time: Time that the production of SKU \( i \) starts

Other continuous variable

\( T_i^{End} \) End time: Time that the production of SKU \( i \) ends

Binary decision variables

\( X_i \) Binary value, 1 if SKU \( i \) is produced in a period, 0 otherwise

\( Y_{it}^{s,e} \) Binary value, 1 if SKU \( i \) is produced before\(^5\) SKU \( i^* \) on equipment \( e \) in a period, 0 otherwise

\( \overline{Y}_{it}^{s,e} \) Binary value, 1 if SKU \( i \) is produced exactly before\(^6\) SKU \( i^* \) on equipment \( e \) in a period, 0 otherwise

\(^5\) Before refers here to the fact that SKU \( i \) is produced before SKU \( i^* \) and between product \( i \) and SKU \( i^* \) other SKUs could be produced.

\(^6\) Before refers here to the fact that SKU \( i \) is produced before SKU \( i^* \) and between product \( i \) and SKU \( i^* \) other SKUs could be produced.
\( \bar{Y}_{iE} \) Binary value, 1 if SKU \( i \) is produced the last on equipment \( e \) in a period, 0 otherwise

\( \bar{Y}_{iF} \) Binary value, 1 if SKU \( i \) is produced the first on equipment \( e \) in a period, 0 otherwise

**Objective**

\[
\min \sum_{i} c_{i}^{p} \cdot (W_{i} - A_{i}) + \sum_{i} \sum_{t} \sum_{e} e_{iE}^{e} \cdot \bar{Y}_{iE}^{e}.
\]

The penalty costs are calculated as follows: Subtracting the amount produced of SKU \( i \) \( (A_{i}) \) from the amount asked of SKU \( i \) \( (W_{i}) \). This number is multiplied with \( c_{i}^{p} \); the penalty costs for SKU \( i \). Next for all the SKUs this number is summed \( \sum_{i} c_{i}^{p} \cdot (W_{i} - A_{i}) \).

The setup costs are different per equipment and per SKU, therefore these are summed for every SKU and every equipment. The setup costs occur whenever a the setup occurs, therefore the setup costs \( (c_{i}^{s} \cdot e) \) are multiplied by the either 0 or 1 \( (\bar{Y}_{iE}^{e}) \), depending on if the setup occurred.

**Constraints**

Constraints I2 and I3 guarantee that the quantity produced of SKU \( i \) can only be greater than zero if it is decided to produce SKU \( i \) \( (X_{i} = 1) \). The produced quantity can upmost be the demand for SKU \( i \) \( (W_{i}) \).

\[
\begin{align*}
A_{i} & \leq W_{i} \cdot X_{i} & \forall i \in I \quad (I2) \\
A_{i} & \geq 0 & \forall i \in I \quad (I3)
\end{align*}
\]

Constraint I4 and I5 (based on Kopanos et al., 2010:647) together guarantee that when two SKUs are produced \( (X_{i} = 1 \) and \( X_{j} = 1) \) and they both use equipment \( e \) \( (U_{i}^{e} = 1 \) and \( U_{j}^{e} = 1) \) a decision is made about in which order the SKUs are produced \( (\text{either } Y_{iE}^{e} \text{ or } Y_{jE}^{e} \text{ is equal to 1}) \). On the other hand when SKU \( i \) is not produced or does not use equipment \( e \) the constraints I4 and I5 guarantee that both \( Y_{iE}^{e} \) or \( Y_{jE}^{e} \) are equal to 0.

\[
\begin{align*}
2(Y_{iE}^{e} + Y_{jE}^{e}) & \leq X_{i} \cdot U_{i}^{e} + X_{j} \cdot U_{j}^{e} & \forall, e \in E, i \in I, i^{*} \in I: i \neq i^{*} \quad (I4) \\
X_{i} \cdot U_{i}^{e} + X_{j} \cdot U_{j}^{e} & \leq 1 + Y_{iE}^{e} + Y_{jE}^{e} & \forall, e \in E, i \in I, i^{*} \in I: i \neq i^{*} \quad (I5)
\end{align*}
\]

Constraint I6 guarantees that the start time of an SKU is equal or bigger than zero. Constraint I7 guarantees that the end time of an SKU that is produced is at or before \( T \).

\[
\begin{align*}
T_{i}^{Start} & \geq 0 & \forall i \in I \quad (I6) \\
T_{i}^{End} & \leq T \cdot X_{i} & \forall i \in I \quad (I7)
\end{align*}
\]

The end time of SKU \( i \) is the start time of SKU \( i \) plus the time spent on producing SKU \( i \) (Constraint I8).

\[
T_{i}^{End} = T_{i}^{Start} + \frac{A_{i}}{\rho_{i}} & \forall i \in I \quad (I8)
\]

Constraints I9, I10 and I11 are related to the exact sequence \( \bar{Y}_{iE}^{e} \). When SKU \( i^{*} \) has exactly one predecessor more than SKU \( i \), SKU \( i \) is produced exactly before SKU \( i^{*} \) and \( \bar{Y}_{iE}^{e} \) should be 1. The other way around is also true, when \( \bar{Y}_{iE}^{e} \) is equal to 1 SKU \( i^{*} \) has exactly one predecessor more than SKU \( i \). An example \( \bar{Y}_{iE}^{e} = 1 \), \( \sum_{i} e_{iE}^{e} = 4 \) and \( \sum_{i} e_{iE}^{e} = 3 \rightarrow 1 \geq 1 - 0 \) and \( 1 \leq 1 + 0 \)

\[6 \text{ Exactly before refers here to the fact that between SKU } i \text{ and SKU } i^{*} \text{ no SKUs are produced. For example } \bar{Y}_{iE}^{e} = 1 \text{ means that SKU 1 is produced before SKU 2 and no products can be produced in between.} \]
(combining constraints I9 and I10). Further the exact sequence \( \hat{Y}_{it}^e \) can only be 1 if the ‘general’ sequence \( \tilde{Y}_{it}^e \) is 1. This is ensured by constraint I11.

\[
\sum_{i \neq i^*} Y_{it}^e - \sum_{i \neq i^*} Y_{it}^e \geq 1 - M'(1 - \tilde{Y}_{it}^e) \quad \forall e \in E, i \in I, i^* \in I: i \neq i^* \quad (I9)
\]

\[
\sum_{i \neq i^*} Y_{it}^e - \sum_{i \neq i^*} Y_{it}^e \leq 1 + M'(1 - \tilde{Y}_{it}^e) \quad \forall e \in E, i \in I, i^* \in I: i \neq i^* \quad (I10)
\]

\[
\tilde{Y}_{it}^e \leq Y_{it}^e \quad \forall e \in E, i \in I, i^* \in I: i \neq i^* \quad (I11)
\]

Constraints I12, I13, I14 and I15 determine the first and last SKU to be produced on equipment \( e \).

Constraint I12 ensures that every SKU has only one successor or is the last SKU on equipment \( e \). Either \( \sum_{i \neq i^*} \hat{Y}_{it}^e \) or \( \hat{Y}_{it}^e \) is equal to 1 when \( \hat{Y}_{it}^e \) is produced and it uses equipment \( e (X_t \cdot U_t^e) \).

Constraint I13 ensures that every SKU has only one predecessor or is the first SKU on equipment \( e \). Either \( \sum_{i \neq i^*} \tilde{Y}_{it}^e \) or \( \tilde{Y}_{it}^e \) is equal to 1 when \( \tilde{Y}_{it}^e \) is produced and it uses equipment \( e (X_t \cdot U_t^e) \).

Constraints I14 and I15 ensure that there is only one SKU the last respectively the first on equipment \( e \).

\[
\sum_{i \neq i^*} \hat{Y}_{it}^e + \tilde{Y}_{it}^e = X_t \cdot U_t^e \quad \forall e \in E, i \in I \quad (I12)
\]

\[
\sum_{i \neq i^*} \tilde{Y}_{it}^e + \hat{Y}_{it}^e = X_t \cdot U_t^e \quad \forall e \in E, i \in I \quad (I13)
\]

\[
\sum_{i \neq i^*} \hat{Y}_{it}^e \leq 1 \quad \forall e \in E \quad (I14)
\]

\[
\sum_{i \neq i^*} \tilde{Y}_{it}^e \leq 1 \quad \forall e \in E \quad (I15)
\]

Constraint 16 (based on Kopanos et al., 2010:647) guarantees that if \( \hat{Y}_{it}^e \) is produced exactly before \( \hat{Y}_{it^*}^e \) on equipment \( e \), the start time of \( \hat{Y}_{it}^e \) is bigger or equal to the end time of \( \hat{Y}_{it^*}^e \) plus the setup time from \( \hat{Y}_{it}^e \) to \( \hat{Y}_{it^*}^e \).

\[
T_{i^*}^{\text{End}} + M(1 - \hat{Y}_{i^*}^e) \geq T_{i}^{\text{End}} + s_{i^*} \cdot \tilde{Y}_{i^*}^e, \quad \forall e \in E, i \in I, i^* \in I: i \neq i^* \quad (I16)
\]

Constraint 16 is further clarified using an example:

**SKU 1** is produced exactly before **SKU 2** on equipment **A** \( (\hat{Y}_{12}^A = 1) \) and **SKU 3** is produced exactly before **SKU 2** on equipment **B** \( (\hat{Y}_{32}^B = 1) \). The start time of **SKU 2** should be bigger or equal to the end time of **SKU 1** plus the setup time changing from **SKU 1** to **SKU 2** on equipment **A**. In formula:

\[
T_2^{\text{Start}} + M(1 - \hat{Y}_{12}^A) \geq T_1^{\text{End}} + s_{12}^A \cdot \tilde{Y}_{12}^A = T_2^{\text{Start}} \geq T_1^{\text{End}} + s_{12}^A.
\]

Further the start time of **SKU 2** should also be bigger or equal to the end time of **SKU 3** plus the setup time changing from **SKU 1** to **SKU 2** on equipment **B**. In formula:

\[
T_2^{\text{Start}} + M(1 - \hat{Y}_{32}^B) \geq T_1^{\text{End}} + s_{32}^B \cdot \tilde{Y}_{32}^B = T_2^{\text{Start}} + 0 \geq T_3^{\text{End}} + s_{32}^B.\]

Note that it is assumed here that the setups can be performed in parallel. In figure 4.5 the example is visualized.
The initial model covers the following constraints: SKU routings (3.1.3), Setup Times (3.1.4), Speed (3.1.5), Sharing equipment (3.1.6) and No wait batch production and simultaneous use of equipment (3.1.7). Further the following costs are included: setup costs (3.2.1) and penalty costs (3.2.2). In paragraph 4.2 the initial model is extended and the remaining constraints and costs are modelled.

4.2  Scheduling model Z

Adaptations to initial model:

- The work orders and actual production times are given in hours. The possible routings are known in advance so when an SKU is asked it can be determined what the production time should be.
- All the SKUs are represented in set $I$. Subset set $Id$ contains the SKUs that are asked by the supply planners, the so called work orders for a week.

$I$  
All SKUs produced in production area X.

$E$  
equipment

All equipment considered (B, D, G, J, P, S, Y, 1, 2, 3, 4, 2A, 2B, P3 and P4, see figure 3.)

$Id$  
SKUs asked

All SKUs that are asked by the supply planners for a period (work orders)

### Parameters

- $c_{n,e}^e$: Penalty costs when equipment $e$ is setup for production, but not used for actual production.
- $c_{i}^e$: Penalty costs per hour not producing SKU $i$.
- $c_{i}^{\delta,e}$: Setup costs when changing from SKU $i$ to SKU $i^*$ on equipment $e$.
- $c_{i}^{\gamma,e}$: Setup costs for SKU $i$ when it is last on equipment $e$.
- $FP_{i}$: Binary value, 1 if SKU $i$ should start the week if produced, 0 otherwise.
- $LP_{i}$: Binary value, 1 if SKU $i$ should end the week if produced, 0 otherwise.
- $M$: Number bigger than the available hours in a period (T+1).
- $M'$: Number bigger than the maximum amount of SKUs produced on an equipment a period.
- $s_{i}^{\delta,*}$: Setup time when changing from SKU $i$ to SKU $i^*$ on equipment $e$ (in hours).
- $T$: Hours available per week.
- $\tau^{end,e}$: Time until equipment $e$ is available.
- $\tau^{start,e}$: Time from which equipment $e$ is available.
- $t^e$: Percentage of the time that equipment $e$ is available.
- $U_{i}^e$: Binary value, 1 if SKU $i$ requires equipment $e$, 0 otherwise.
- $WO_{i}$: Work order for SKU $i$ in a period (hours).

---

\[\text{This parameters has only a value for equipment D, for the rest of the equipments the value is zero, because for those equipments no costs of downtime are made.}\]

\[\text{Actual production refers to production time for an SKU}\]
Continuous decision variables

$AP{T_i}$: Actual Production Time for SKU $i$ in a period (in hours)

$T_{i}^{\text{Start}}$: Start time: time that the actual production of SKU $i$ starts

Other continuous variable

$T_{i}^{\text{End}}$: End time, time that the actual production of SKU $i$ ends

$T_{\text{END} - e}^{\text{e}}$: End time of equipment $e$: the time when the production of the last SKU on equipment $e$ finishes

$T_{\text{START} - e}^{\text{e}}$: Start time of equipment $e$: the time when the production of the first SKU on equipment $e$ starts

Binary decision variables

$X_i$: Binary value, 1 if SKU $i$ is produced in a period, 0 otherwise

$V_{i}^{e}$: Binary value, 1 if SKU $i$ is produced before SKU $i^*$ on equipment $e$ in a period, 0 otherwise

$\overline{V}_{i}^{e}$: Binary value, 1 if SKU $i$ is produced exactly before SKU $i^*$ on equipment $e$ in a period, 0 otherwise

$\overline{V}_{i}^{e}$: Binary value, 1 if SKU $i$ is produced last on equipment $e$ in a period, 0 otherwise

Objective

Minimize

\[
\sum_{i} \sum_{l}^{\text{id}} c_{i}^{p} \cdot (WO_{l} - A{P}_l) + \sum_{i} \sum_{l}^{l} \sum_{e}^{E} e_{i}^{e} \cdot \overline{V}_{i}^{e} + \sum_{i} \sum_{l}^{l} \sum_{e}^{E} c_{i}^{E} \cdot \overline{V}_{i}^{e} + \sum_{e}^{E} \left( T_{\text{END} - e}^{\text{e}} - T_{\text{START} - e}^{\text{e}} - \sum_{i}^{l} A{P}_{i} \cdot U_{l}^{e} \right)
\]  

(E1)

Objective function (E1) is extended, equipment D costs are added. When equipment D is setup and not used for production it produces waste. Therefore it should be calculated for how long the machine is enabled but not used for production. The end time of equipment D ($T_{\text{END} - D}^{\text{D}}$) minus the start time of equipment D ($T_{\text{START} - D}^{\text{D}}$) represent the time equipment D is used. When the total time of production on equipment D ($\sum_{i}^{l} A{P}_{i} \cdot U_{l}^{D}$) is subtracted, the time spent not producing while the Equipment D is enabled is given. This time is multiplied the costs per hour ($c_{D}^{\text{n DEMACO}}$) equipment D is enabled but not used for production of SKUs. To keep the modelling consistent, for all equipment the ‘time enabled but not used for production’ is calculated, where $c_{D}^{\text{n DEMACO}} \neq 0$, only for equipment D ($e = \text{equipment D}$). For all the other equipment this parameter is 0.

Further to objective function the setup costs for the SKU produced last on its equipment is added. Setup costs occur when changing from one SKU to the other and these costs are sequence dependent. The last SKU on the equipment also creates setup costs, since after this production the equipment has to be cleaned as well and material is lost. This setup cost occurs whenever an SKU is produced last on equipment $e$, therefore the setup costs ($c_{i}^{S} \cdot e$) are multiplied by the either 0 or 1 ($\overline{V}_{i}^{e}$) depending on if the SKU is produced last on equipment $e$.

\[\text{Before refers here to the fact that product } i \text{ is produced before SKU } i^* \text{ and between product } i \text{ and SKU } i^* \text{ other SKUs could be produced.}

\[\text{Exactly before refers here to the fact that between SKU } i \text{ and SKU } i^* \text{ no products are produced. For example } \overline{V}_{12}^{2} = 1 \text{ means that SKU 1 is produced before SKU 2 and no SKUs can be produced in between.}\]
Note that for the constraints that remained the same as in the initial model no additional explanation is given.

**Constraints**

Constraints E2 and E3 correspond to constraints I2 and I3 of the initial model.

\[
\begin{align*}
\text{E2:} & \quad APT_i \leq \text{WO}_i \cdot X_i \\
\text{E3:} & \quad APT_i \geq 0 
\end{align*}
\]

\(\forall i \in \text{Id} \)  \hspace{1cm} (E2)

\(\forall i \in \text{Id} \)  \hspace{1cm} (E3)

Constraint E4 and E5 correspond to constraints I4 and I5 of the initial model.

\[
\begin{align*}
\text{E4:} & \quad 2(Y_{1i}^e + Y_{2i}^e) \leq X_i \cdot U_i^e + X_{i'} \cdot U_{i'}^e \\
\text{E5:} & \quad X_i \cdot U_i^e + X_{i'} \cdot U_{i'}^e \leq 1 + Y_{1i}^e + Y_{2i}^e 
\end{align*}
\]

\(\forall e \in E, i \in \text{Id}, i' \in \text{Id}: i \neq i' \)  \hspace{1cm} (E4)

\(\forall e \in E, i \in \text{Id}, i' \in \text{Id}: i \neq i' \)  \hspace{1cm} (E5)

The end time of SKU $i$ is the start time of SKU $i$ plus the time spent on producing SKU $i$ (Constraint E6).

\[
T_{i}^{\text{End}} = T_{i}^{\text{Start}} + \text{AP}_i 
\]

\(\forall e \in E, i \in \text{Id}, i' \in \text{Id}: i \neq i' \)  \hspace{1cm} (E6)

Constraints E7, E8 and E9 correspond to constraints I9, I10 and I11 in the initial model.

\[
\begin{align*}
\sum_{i \neq i'}^\text{Id} Y_{1i}^e - \sum_{i \neq i'}^\text{Id} Y_{2i}^e & \geq 1 - M'(1 - Y_{1i}^e) \\
\sum_{i \neq i'}^\text{Id} Y_{1i}^e - \sum_{i \neq i'}^\text{Id} Y_{2i}^e & \leq 1 + M'(1 - Y_{1i}^e) \\
\bar{Y}_{1i}^e & \leq Y_{1i}^e
\end{align*}
\]

\(\forall e \in E, i \in \text{Id}, i' \in \text{Id}: i \neq i' \)  \hspace{1cm} (E7)

\(\forall e \in E, i \in \text{Id}, i' \in \text{Id}: i \neq i' \)  \hspace{1cm} (E8)

\(\forall e \in E, i \in \text{Id}, i' \in \text{Id}: i \neq i' \)  \hspace{1cm} (E9)

Constraints E10, E11, E12 and E13 correspond to constraints I12, I13, I14 and I15 in the initial model.

\[
\begin{align*}
\sum_{i \neq i'}^\text{Id} \bar{Y}_{1i}^e + \bar{Y}_{le}^e & = X_i \cdot U_i^e \\
\sum_{i \neq i'}^\text{Id} \bar{Y}_{1i}^e + \bar{Y}_{le}^e & = X_i \cdot U_i^e \\
\sum_{i \neq i'}^\text{Id} \bar{Y}_{le}^e & \leq 1 \\
\sum_{i \neq i'}^\text{Id} \bar{Y}_{le}^e & \leq 1
\end{align*}
\]

\(\forall e \in E, i \in \text{Id} \)  \hspace{1cm} (E10)

\(\forall e \in E, i \in \text{Id} \)  \hspace{1cm} (E11)

\(\forall e \in E \)  \hspace{1cm} (E12)

\(\forall e \in E \)  \hspace{1cm} (E13)

Constraint E14 corresponds to constraint I16 in the initial model.

\[
T_{i}^{\text{Start}} + M(1 - \bar{Y}_{1i}^e) \geq T_{i}^{\text{End}} + \bar{Y}_{le}^e \cdot \bar{Y}_{le}^e 
\]

\(\forall e \in E, i \in \text{Id}, i' \in \text{Id}: i \neq i' \)  \hspace{1cm} (E14)

Constraints E15 ensures that equipment ends in time. Constraint E16 ensures that equipment can only start at the time it is available.

\[
T_{i}^{\text{End}} \leq T - \bar{Y}_{le}^e(T - T^{\text{End}}e) 
\]

\(\forall e \in E, i \in \text{Id} \)  \hspace{1cm} (E15)
Constraints E15 and E16 relate to the ‘sub’ period in which SKUs can be produced (see figure 4.6).

Figure 4.6: Availability of equipment e

Constraint E17 ensures that an equipment is only occupied the percentage of the time it is allowed to be available per period. When an equipment is available for the entire period, \( t^e = 1 \).

\[
\sum_{i}^{id} Q_i \cdot U_i^e \leq t^e \cdot T \quad \forall e \in E,
\]

Constraint E17 is further explained using an example. SKU 1 and 2 are produced on equipment A (see figure 4.7). The sum of the production time for SKU 1 and the production time for SKU 2 can at most use \( t^e \) percentage of the total time in period \( T \). In formula: \( Q_1 \cdot U_1^A + Q_2 \cdot U_2^A \leq t^A \cdot T \).

Figure 4.7: Percentage availability example.

Constraints E18 to E26 involve with determining the actual start (\( T_{\text{START-}e} \)) and end (\( T_{\text{END-}e} \)) times of an equipment. The actual end and the start time of an equipment should be bigger or equal to zero (constraints E18 and E22). The actual end and start time of an equipment should be less or equal to the allowed end time (\( T_{\text{END-}e} \)) of that equipment (constraints E19 and E23). The actual end time of an equipment should be bigger or equal to all individual SKU end times that uses that equipment (constraint E20). The end time of the last SKU on the equipment should be bigger or equal to the actual end time of that equipment (constraint E21). The actual start time of an equipment should be less or equal to all the individual start times of SKU that uses that equipment (constraint E24). The start time of the first SKU on the equipment should be less or equal to the actual start time of that equipment (constraint E25). Finally the actual start time of an equipment should be less or equal to the actual end time of the equipment (constraint E26).

\[
\begin{align*}
0 \leq T_{\text{END-}e} & \quad \forall e \in E, \\
T_{\text{END-}e} & \leq T_{\text{end} e} \\
T_{\text{END-}e} & \geq T_i^e \cdot U_i^e \\
T_i^e \cdot U_i^e + (1 - \tilde{Y}_i^e) T_{\text{end} e} & \geq T_{\text{END-}e} \\
0 \leq T_{\text{START-}e} & \quad \forall e \in E, i \in Id, \\
T_{\text{START-}e} & \leq T_{\text{end} e} \\
T_{\text{START-}e} & \leq T_i^e \cdot U_i^e + (1 - \tilde{Y}_i^e) T_{\text{end} e} \\
T_i^e \cdot U_i^e - (1 - \tilde{Y}_i^e) T_{\text{END-}e} & \leq T_{\text{START-}e} \\
T_{\text{END-}e} & \geq T_{\text{START-}e} \\
\end{align*}
\]
Constraints E27 and E28 ensure that SKUs that have to start production is a period on their equipment are actually scheduled at the start. Constraints E29 and E30 ensure that SKUs that have to end the week on their equipment are actually scheduled at the end.

\[
V_{Fi}^e - \sum_{i' \neq I}^{ld} FP_{i'} \cdot X_{i'} \cdot U_{i'}^e \quad \forall e \in E, i \in Id \\
\sum_{i' \neq I}^{ld} FP_{i'} \cdot X_{i'} \cdot U_{i'}^e \quad \forall e \in E, i \in Id, i^* \in Id \\
V_{Li}^{e} - \sum_{i' \neq I}^{ld} LP_{i'} \cdot X_{i'} \cdot U_{i'}^e \quad \forall e \in E, i \in Id \\
\sum_{i' \neq I}^{ld} LP_{i'} \cdot X_{i'} \cdot U_{i'}^e \quad \forall e \in E, i \in Id, i^* \in Id
\]
5. Verification and Validation

The mathematical model given in chapter 4 has been solved using software package GUSEK. It can deal with mixed integer linear programming problems. This program can be used for free. GUSEK uses the General Modelling Programming Language (GMPL). Therefore the mathematical expressions in chapter 4 have been translated into GMPL. In Appendix J the initial model is given in GMPL, as used in GUSEK. In Appendix K the scheduling model Z is given in GMPL, as used in GUSEK.

According to Sargent (2003) model verification is defined as “ensuring that the computer program of the computerized model and its implementation are correct” (Sargent 2003:1). The definition of model validation is “substantiation that a computerized model within its domain of application possesses a satisfactory range of accuracy consistent with the intended application of the model” (Schlesinger, 1979).

First the models are verified (paragraph 5.1), this can be done for both models: the initial model and the scheduling model Z. Next the models are validated (paragraph 5.2), some examples are included which represent various situations, it is checked if the model behaves as expected.

Finally in paragraph 5.3 the experiences with solving the scheduling model Z are discussed.

5.1 Verification

As been said the mathematical model is translated into GMPL so that GUSEK can read the optimization problem. GUSEK has a built-in debugger. This debuggers displays an error whenever a set, parameter, variable, objective function or constraints isn’t modelled correctly. GUSEK displayed which statement contains an error and what kind of error it is. For example there could be displayed ‘mod:57: y must have 3 subscripts rather than 2’. This means that the error appears in line 57. Previously it is defined that variable y should have 3 subscripts. If thereafter in a constraint variable y only has 2 subscript, GUSEK observes an error. This way GUSEK performs the model verifications.

5.2 Validation

First the initial model is validated by investigating if the model behaves at it is intended. It is checked if the model meets the important design requirements, no wait batch production and share of equipment. Next the costs are calculated and it is checked if they match the costs that are calculated by GUSEK. Hereafter three scenarios are presented and checked is if the model (in GUSEK) reacts as expected. It is assumed that if the initial model can be validated the scheduling model Z is also reliable.

To be able to solve the model, data has to be provided. This data is given in Appendix J. In the data there are work order for 6 SKUs. These SKUs are given in set I, the work orders per SKU are given by parameter WO. Further for all parameters the values are given.

Next GUSEK runs and output is generated. Depending on which variables and or parameters are requested they will be displayed. For the standard output following variables and parameters are requested:

\[ WO_i \quad \text{Demand for SKU } i \text{ (in units)} \]
\[ AP_i \quad \text{Produced quantity of SKU } i \text{ (in units)} \]
\[ X_i \quad \text{Binary value, 1 if SKU } i \text{ is produced, 0 otherwise} \]
\[ T_i^{start} \quad \text{Start time, time that the production of SKU } i \text{ starts} \]
\[ T_i^{end} \quad \text{End time, time that the production of SKU } i \text{ ends} \]
\( P_{it}^e \) is a binary value, 1 if SKU \( i \) is produced exactly before \( i^* \) on equipment \( e \), 0 otherwise.

**Costs** The value of the objective function.

The output is displayed in Appendix J. The output states that 6 SKU are to be produced (all \( X_i \) values are 1). Below in figure 5.1 the scheduling plan is visualized. In the visualisation, the following colours are used to indicate that an SKU is produced (see table 5.1). Further in table 5.1 it is indicated which pieces of equipment are necessary to be able to produce an SKU. In the initial model in GUSEK this is input information, see parameter \( U_f^e \).

<table>
<thead>
<tr>
<th>SKU</th>
<th>Colour Indication</th>
<th>Equipment Usage</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>g1</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>g2</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>g1</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
<td>g2</td>
</tr>
<tr>
<td>5</td>
<td>5</td>
<td>m1, g2</td>
</tr>
<tr>
<td>6</td>
<td>6</td>
<td>g1</td>
</tr>
</tbody>
</table>

As follows from the start and end times given and the visualisation in figure 5.1, equipment is reserved at the same time when an SKU is produced. This corresponds to the design requirements *sharing equipment* and *no wait batch production*, equipment should be reserved at the same time. Next the scheduling plan takes into account that when the equipment is reserved for an SKU it cannot be reserved for another SKU at the same time. Checked if the costs are calculated correctly, this is displayed in Appendix J as well. The total costs correspond to the cost value calculated by GUSEK.

Next, various scenarios are created. It is known in advance how the model should react, checked is if this was indeed the case.

### 5.2.1 Scenario 1, difference in penalty costs

The penalty costs \( (Cp[i]) \) are different per product, the ‘less costly’ product should be cut. In other words if not all work orders can be produced in total, the SKU of which the penalty costs are the lowest should be cut first.

Output:

\begin{align*}
\text{Cp}[1] & = 1 \quad \text{WO}[1] = 150 \quad X[1].val = 1 \quad \text{AP}[1].val = 30 \quad Tstart[1].val = 16 \quad Tend[1].val = 19 \\
\text{Cp}[2] & = 2 \quad \text{WO}[2] = 200 \quad X[2].val = 1 \quad \text{AP}[2].val = 200 \quad Tstart[2].val = 0 \quad Tend[2].val = 10 \\
\text{Cp}[3] & = 3 \quad \text{WO}[3] = 300 \quad X[3].val = 1 \quad \text{AP}[3].val = 300 \quad Tstart[3].val = 20 \quad Tend[3].val = 50 \\
\text{Cp}[4] & = 3 \quad \text{WO}[4] = 200 \quad X[4].val = 1 \quad \text{AP}[4].val = 200 \quad Tstart[4].val = 12 \quad Tend[4].val = 32 \\
\text{Cp}[5] & = 1 \quad \text{WO}[5] = 200 \quad X[5].val = 1 \quad \text{AP}[5].val = 200 \quad Tstart[5].val = 40 \quad Tend[5].val = 50 \\
\text{Cp}[6] & = 2 \quad \text{WO}[6] = 150 \quad X[6].val = 1 \quad \text{AP}[6].val = 150 \quad Tstart[6].val = 0 \quad Tend[6].val = 15
\end{align*}

\( ^{11} \) Exactly before refers here to the fact that between product \( i \) and product \( i^* \) no products are produced. For example \( P_{12}^m = 1 \) means that product 1 is produced before product 2 and no products can be produced in between.
PART 3 Model Solving

Chapter 5. Verification and Validation

Note that the penalty costs for SKU 1 and SKU 5 are the lowest.

Visual output (figure 5.2)

|   | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 | 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40 | 41 | 42 | 43 | 44 | 45 | 46 | 47 | 48 | 49 | 50 |
|---|---|---|---|---|---|---|---|---|---|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|
| v1 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 |
| w1 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 |
| g2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 |

Figure 5.2: Visual output scenario 1.

SKU 1 is the ‘less costly’ SKU and of this SKU less units are produced. This corresponds to the desired output.

5.2.1 Scenario 2, WO is zero for an SKU

If there is no work order (WO = 0) for an particular SKU, nothing should be produced of this SKU. In other words: if WO[i]=0 → X[i], AP[i], Tstart[i] and Tend[i] should be 0.

Output:


Note that the work order for SKU 5 is 0.

Visual output (figure 5.3):

|   | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 | 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40 | 41 | 42 | 43 | 44 | 45 | 46 | 47 | 48 | 49 | 50 |
|---|---|---|---|---|---|---|---|---|---|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|
| v1 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 |
| w1 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 |
| g2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 |

Figure 5.3: Visual output scenario 2

Work order for SKU 5 is zero. WO[5]=0 → X[5], AP[5], Tstart[5] and Tend[5] are 0. This corresponds to the desired output.

5.2.2 Scenario 3, penalty costs versus setup cost

Say the penalty costs (CP[i]) of an SKU are much less than setup costs. Decided should be to not produced the ‘cheap’ SKU. If Cp[i] << St [ix], X[i], AP[i], Tstart[i] and Tend[i] should be 0.

Output:

Cp[1]=0.01 WO[1] = 150 X[1].val = 0 AP[1].val = 0 Tstart[1].val = 0 Tend[1].val = 0

Note that the penalty costs for SKU 1 are very low.

Visual output (figure 5.4):
The penalty costs for SKU 1 are much less than the setup costs, therefore SKU 1 should not be produced. $Cp[1] \ll St[1x] \rightarrow X[1]$, $AP[1]$, $Tstart[1]$ and $Tend[1]$ are 0. This corresponds to the desired output.

It can be concluded that the initial model behaves as it is expected to. Since the scheduling model $Z$ is built in the same way, it is assumed that this model will also behave as expected. For small instances (approximately 8 SKUs) this model can be solved and provides an executable schedule.

5.3 Experiences solving the scheduling model $Z$

To test the usage of the scheduling model $Z$, the handover scheduling documents are used. These documents contain the production plan of the supply planners. Or as in paragraph 1.3.2 the work orders for the production area $X$. The supply planners try to come up with a schedulable plan. They consider some of the constraints:

- Production time for every product (speed per routing)
- Setup times (historical averages)
- If the load per equipment (production time and changeover), isn’t exceeding the capacity.

This production plan is used as an input for the model. As been said in Appendix K the scheduling model $Z$ is given in GMPL, as used in GUSEK. The parameter data for all SKU is added in GUSEK. All SKUs are united in set $I$ and the selection of SKUs corresponding to the handover schedules are united in set $Id$. This set $Id$ and its corresponding work orders (WO) are input for the scheduling model. About 30 SKU are asked in a week. When running the scheduling model $Z$ in GUSEK using the data giving in the handover schedules no solution is found (runs over 12 hours). To simplify the optimization constraint 2 is adapted (see below).

Before

$$AP_i \leq WO_i \cdot X_i \quad \forall \ i \in Id$$

After

$$AP_i = WO_i \cdot X_i \quad \forall \ i \in Id$$

The actual production time ($AP_i$) cannot be varied anymore. However GUSEK could still not come up with an integer solution. GUSEK did come up with an initial ‘optimal’ solution, but in this solution the requirement of integer variables is ignored. Therefore this solution is not feasible. Furthermore this initial ‘optimal’ solution is used as a starting point in the search for an integer solution. Expected is that this initial ‘optimal’ solution is so poor that GUSEK is not able to find an integer solution and therefore no feasible solution. Unfortunately the solver is not able to solve a total set of work orders as they are given by the supply planners via the handover scheduling documents. Therefore a heuristic is proposed, which is presented in chapter 6.
6. **Heuristic**

In this chapter a heuristic is presented that comes up with a production schedule for production area X. The *handover scheduling documents* provided by the supply planners, serve as input for the heuristic/production schedule. These documents contain the following information:

- Which SKUs should be produced
- Via which routing these SKUs should be produced
- How much of these SKUs should be produced.
- Production hours per SKU (this information is obtained combining the routing via which an SKU should be produced and amount of an SKU that should be produced. When the routing of an SKU is known, the corresponding speed is known as well. Then, production hours can be calculated.

Further additional information has to be collected: should an SKU be scheduled first or last on their filling equipment, what are the setup times per SKU and from what time to what time is the different equipment available.

The output of the heuristic is a production schedule indicating which SKUs should be produced, at what time and in which order.

This heuristic slightly deviates from the conceptual model presented in chapter 3. The model assumption 'one routing per SKU' (paragraph 3.3.2) is relaxed. It is allowed to deviate from the requested routing if this can improve the schedule. Moreover the objective of the heuristic isn't minimizing costs, but minimizing makespan. This and the idea behind the heuristic will be explained in paragraph 6.1. In paragraph 6.2 the execution steps are given.

### 6.1 Idea behind the heuristic

The objective of the heuristic is to minimize makespan, i.e. trying to finish the total set of work orders as soon as possible. Makespan is chosen as the objective for two reasons. It is expected that by minimizing the total makespan the heuristic is able to come up with a feasible production schedule that includes as many SKUs as possible (1). Furthermore by scheduling the SKUs as close as possible to each other the amount of idle capacity is limited. Further it can be investigated if there is lots of idle capacity left? Which is an indication that the equipment isn’t fully utilized (2). Finally, by prioritizing the minimum makespan for equipment D, equipment D costs are also minimized.

In the heuristic (steps), the SKUs are classified by the filling equipment to be used. This is done to take into account the sequence dependent setups.

Here an explanation per execution step is given.

1. The SKUs that have to be produced first due to contamination and extensive cleaning issues on their filling equipment are considered, since their only possible position is first in the sequence. It could be that none of these SKUs have this requirement. If more than one SKU should be produced first, they should have the same the recipe. If the recipe is not the same, a choice has to be made which SKUs to include in the production schedule for that week. This should be consulted with the supply planners, since they have insight in which SKU are more at risk to go out of stock.

2. The SKUs that have to be produced last due to contamination and extensive cleaning issues on their filling equipment are considered. For these SKUs only their position relative to other
SKUs to produced needs to be determined. Their actual start time on the equipment cannot be
determined yet, because their predecessors aren’t known yet. It could be that none of these
SKUs have this requirement. If more than one SKU should be produced last, the recipes of
these SKUs should be the same. If the recipe is not the same, a choice has to be made which
SKUs to include in the production schedule for that week. This should be consulted with the
supply planners, since they have inside in which SKU are more at risk to go out of stock.
3. Next, the SKUs requiring equipment D and filling equipment 4 are considered, because:
   1. These SKUs should be produced together as close as possible, because otherwise costs
      are created. The costs appear when equipment D is setup but not used for the actual
      production of an SKU (see paragraph 3.2.3).
   2. The available time on equipment D is less than the time available on other equipment.
      (80 hours instead of 103/113)
   3. The SKUs requiring filling equipment 4 are chosen first (priority over SKUs requiring
      filling equipment 3) since they use on average most time of equipment D (more than
      the SKUs requiring Equipment D and filling equipment 3).
4. Hereafter the SKUs requiring Equipment D and filling equipment 3 are considered, see reason
   1 and 2 of explanation 3.
5. Following the remaining SKUs requiring filling equipment 3 are considered, because after
   step 4 an idle period of time on filling equipment 3 may appear, in contrary to the situation for
   the SKUs on filling equipment 4 after step 3.
6. Next, the remaining SKUs requiring filling equipment 4 are considered, because these may
   clash with SKUs requiring filling equipment 3.
7. Hereafter the SKUs requiring filling equipment 1 are considered, because filling equipment 1
   is mostly fully loaded.
8. Then the SKUs requiring filling equipment 2 are considered.
9. Next for the SKUs positioned last in the sequence the start and end times are determined. This
    step is performed now because the start and end times of all their predecessors are known.
10. It is checked if the production schedule fits in the available time per equipment. If the
    schedule doesn’t fit, it is checked if there are SKUs that can be produced using one of their
    alternative routings, based on the time left at the required equipment. If the schedule still
    doesn’t fit it is decided to take some SKUs out (not produce them). This is consulted with the
    supply planners, since they have insight in which SKUs are more at risk to go out of stock. If
    there is time left, one should make a decision if more of a particular SKU or additional SKUs
    should be produced. This should also be consulted with the supply planners.

6.2 Execution Steps

Definitions used in the execution steps:
Fix position: determine the position the SKU, i.e. determine the sequence.
Fix start and end time: determine at which time the production of an SKU will start and at which
   time it will end.
Current situation: After every executed step the situation changes. This changed situation is the
 starting point for following step. See the following example for equipment A and B.
Before a certain step, none of the equipment is reserved and the current situation is shown in figure
6.1.

| Time | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 |
|------|---|---|---|---|---|---|---|---|---|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|
| Equipment A |   |   |   |   |   |   |   |   |   |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |
| Equipment B |   |   |   |   |   |   |   |   |   |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |

Figure 6.1: Current situation, before any steps are performed
After this step is performed the situation changes. The new situation is shown in figure 6.2.

This new situation is the ‘current situation’ for the next step that is performed.

**INPUT DATA**

Gather the following information.

1. The SKUs that are asked to be produced (provided in the handover schedule document). In the following set \( I_d \) is \( i \) an SKU in the set. \( i \in I_d \).
2. Which equipment should be used to produce the SKUs in set \( I_d \).
3. Which SKUs in set \( I_d \) should be produced first on their filling equipment?
4. Which SKUs in set \( I_d \) should be produced last on their filling equipment?
5. What are the sequence dependent setup times on every equipment for the SKUs in set \( I_d \).
6. From when to when is an equipment available for production.

Every step has an name and they are divided in sub steps.

**STEP 0. Start times equipment**

i. Find at what time every equipment may start.

ii. Reserve the time before the equipment may start. (this command is clarified in an example)

   Original current situation (see figure 6.3)

   | Time | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 |
   |------|---|---|---|---|---|---|---|---|---|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
   | Equipment A | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
   | Equipment B | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |

   Figure 6.3: Original current situation

   Say start time for equipment A is 4 and the start time for equipment B is 0.

   Situation after reserving the time before the equipment may start (see figure 6.4)

   | Time | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 |
   |------|---|---|---|---|---|---|---|---|---|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
   | Equipment A | Producing SKU 1 | Producing SKU 2 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
   | Equipment B | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |

   Figure 6.4: Current situation after consideration of start time points per equipment.

**STEP 1. SKUs produced first at their required filling equipment.**

i. Find in set \( I_d \) all SKUs that should be produced first on their filling equipment.

ii. Remove these SKUs from set \( I_d \).

iii. Is there more than one SKU that should start on the same filling equipment?

   - NO, continue with the next sub step (iv).
   - YES, are the recipes the same?
     - YES, continue with the next sub step (iv).
     - NO, consult with supply planning which SKUs should be scheduled.

   iv. Find the sequence that minimizes the total setup time and fix the position of the SKUs accordingly.
v. Assume that the time available is infinite and fix the start and end times of the SKUs as close as possible to time 0 considering the current situation.

**STEP 2. SKUs produced last at their required filling equipment.**

i. Find in the remaining set \( Id \) all SKUs that should be produced last on their filling equipment?

ii. Remove these SKUs from set \( Id \).

iii. Is there more than one SKU that should finish on the same filling equipment?

- NO, continue with the next sub step (iv).
- YES, are the recipes the same?
  - YES, continue with the next sub step (iv).
  - NO, consult with supply planning which SKUs should be scheduled.

iv. Find the sequence that minimizes the total setup and fix the position of the SKUs accordingly.

**STEP 3. Equipment D, filling equipment 4 SKUs.**

i. Find in the remaining set \( Id \) all SKUs that use equipment D and filling equipment 4.

ii. Remove these SKUs from set \( Id \).

iii. Find the sequence that minimizes the total setup time considering the situation at the end of step 2 and fix the position of the SKUs accordingly.

iv. Assume that the time available is infinite and fix the start and end times of the SKUs as close as possible to time 0 considering the situation at the end of step 2.

*An example to clarify this step:*

Say the remaining set \( id \) consist of SKU 1, 2, 3, 4, 5, 6, 7, and 8. SKU 1, 2 and 3 require filling equipment 4 and equipment D, SKU 4 and 5 require filling equipment 3 and equipment D and SKU 6, 7 and 8 only require filling equipment 3.

i. SKU 1, 2 and 3 are found,

ii. SKU 1, 2 and 3 are removed from set \( id \).

iii. The sequence that minimizes setup is SKU 1, 2, 3.

iv. The start time and end times are fixed as close as possible to 0 for SKU 4 and 5, see Figure 6.5 (grey blocks represent setup time, colored blocks represent the production of SKUs).

![Figure 6.5: SKUs on Equipment D and filling equipment 4](image)

**STEP 4. Equipment D, filling equipment 3 SKUs.**

i. Find in the remaining set \( Id \) all SKUs that use equipment D and filling equipment 3.

ii. Remove these SKUs from set \( Id \).

iii. Find the sequence that minimizes the total setup time considering the situation at the end of step 3 and fix the position of the SKUs accordingly.

iv. Assume that the time available is infinite and fix the start and end times of the SKUs as close as possible to time 0 considering the situation at the end of step 3.

*An example to clarify this step (following the example at step 3)*

Say the remaining set \( id \) consist of SKU 4, 5, 6, 7, and 8. SKU 4 and 5 require filling equipment 3 and equipment D and SKU 6, 7 and 8 only require filling equipment 3.

i. SKU 4 and 5 are found,
ii. SKU 4 and 5 are removed from set id.

iii. the sequence that minimizes setup is SKU 4, 5.

iv. the start time and end times are fixed as close as possible to 0 for SKU 4 and 5, see figure 6.6 (grey blocks represents setup time, colored blocks represent the production of SKUs).

<table>
<thead>
<tr>
<th>Equipment D</th>
<th>SKU 1</th>
<th>SKU 2</th>
<th>SKU 3</th>
<th>SKU 4</th>
<th>SKU 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Filling equipment 4</td>
<td>SKU 1</td>
<td>SKU 2</td>
<td>SKU 3</td>
<td>SKU 4</td>
<td>SKU 5</td>
</tr>
<tr>
<td>Filling equipment 3</td>
<td>SKU 1</td>
<td>SKU 2</td>
<td>SKU 3</td>
<td>SKU 4</td>
<td>SKU 5</td>
</tr>
</tbody>
</table>

Figure 6.6: SKUs on equipment D and filling equipment 3

STEP 5. Remaining SKUs on filling equipment 3.

i. Find in the remaining set Id all SKUs that use filling equipment 3.

ii. Remove these SKUs from set Id.

iii. Find the sequence that minimizes the total setup time considering the situation at the end of step 4 and fix the position of the SKUs accordingly.

iv. Is this the best sequence in terms of makespan?
   - Yes, assume that the time available is infinite and fix the start and end times of the SKUs as close as possible to time 0 considering the situation at the end of step 4.
   - No, change the sequence and go back to step iv.

An example to clarify this step (following the example at step 3)

Say the remaining set id consist of SKU 6, 7, and 8. SKU 6, and that 7 and 8 only require filling equipment 3.

i. SKU 6, 7, and 8 are found.

ii. SKU 6, 7, and 8 are removed from set id.

iii. the sequence that minimizes setup is SKU 4, 5, 7, 8, 6.

iv. (1) Is this the best sequence in terms of makespan?
   - No, change the sequence to 7, 8, 4, 5, 6

iv. (2) Is this the best sequence in terms of makespan?
   - Yes, the start time and end times as close as possible to 0 for SKU 6, 7 and 8, see figure 6.7 (grey blocks represents setup time, colored blocks represent the production of SKUs).

<table>
<thead>
<tr>
<th>Equipment D</th>
<th>SKU 1</th>
<th>SKU 2</th>
<th>SKU 3</th>
<th>SKU 4</th>
<th>SKU 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Filling equipment 4</td>
<td>SKU 1</td>
<td>SKU 2</td>
<td>SKU 3</td>
<td>SKU 4</td>
<td>SKU 5</td>
</tr>
<tr>
<td>Filling equipment 3</td>
<td>SKU 1</td>
<td>SKU 2</td>
<td>SKU 3</td>
<td>SKU 4</td>
<td>SKU 5</td>
</tr>
</tbody>
</table>

Figure 6.7: SKUs on equipment D filling equipment 3.

STEP 6. Remaining SKUs on filling equipment 4.

i. Find in the remaining set Id all SKUs that use filling equipment 4.

ii. Remove these SKUs from set Id.

iii. Find the sequence that minimizes the total setup time considering the situation at the end of step 5 and fix the position of the SKUs accordingly.

iv. Is this the best sequence in terms of makespan?
   - Yes, assume that the time available is infinite and fix the start and end times of the SKUs as close as possible to time 0 considering the situation at the end of step 5.
   - No, change the sequence and go back to step iv.
STEP 7. **SKUs on filling equipment 1.**

i. Find in the remaining set $Id$ all SKUs that use filling equipment 1.

ii. Remove these SKUs from set $Id$.

iii. Find the sequence that minimizes the total setup time considering the situation at the end of step 6 and fix the position of the SKUs accordingly.

iv. Is this the best sequence in terms of makespan?
   - Yes, assume that the time available is infinite and fix the start and end times of the SKUs as close as possible to time 0 considering the situation at the end of step 6.
   - No, change the sequence and go back to step iv.

STEP 8. **SKUs on filling equipment 2.**

i. Find in the remaining set $Id$ all SKUs that use filling equipment 2.

ii. Remove these SKUs from set $Id$.

iii. Find the sequence that minimizes the total setup time considering the situation at the end of step 7 and fix the position of the SKUs accordingly.

iv. Is this the best sequence in terms of makespan?
   - Yes, assume that the time available is infinite and fix the start and end times of the SKUs as close as possible to time 0 considering the situation at the end of step 7.
   - No, change the sequence and go back to step iv.

STEP 9. **Fix the SKUs to be produced last.**

i. Find the SKUs of which the position is already fix but not the start and end time (see step 2).

ii. Assume that the time available is infinite and fix the start and end times of the SKUs as close as possible to time 0 considering the situation at the end of step 8.

STEP 10. **Check availability.**

i. For every equipment find when this equipment should be finished

ii. Check if every equipment is finished in time.
   - YES, continue with next sub step (iii).
   - NO, see if there are SKUs that have an alternative routing and see if a routing change will make the plan feasible.
     - YES, continue with next sub step (iii).
     - NO, consult supply planning which SKU should be taken out.

iii. Is there space left?
   - NO, ready.
   - YES, consult supply planning if production of a certain SKU should be added and if so add these SKU’s.

This heuristic provides an initial solution. Due to limited time, no further optimization of the heuristic could take place. However, as is shown in the next chapter (chapter 7 comparison) this initial solution performs quite well.
7. Comparison

In order to test the functioning of the heuristic, the schedules created by the heuristic are compared to schedules created by the mathematical scheduling model Z model (paragraph 7.1). Hereafter, the schedules created by the heuristic are compared to actual schedules created by schedulers of Company A (paragraph 7.2).

7.1 Comparison between scheduling model Z and heuristic

To perform the comparison between the schedules created with the heuristic and the schedules created with the mathematical model, fictional data is used since the scheduling model Z cannot be solved for a large number of SKUs (explained in paragraph 5.3). The scheduling model Z, as it is used in the program GUSEK, is displayed in Appendix K. The heuristic is displayed in paragraph 6.2.

To compare the two methods, the scheduling model and the heuristic are executed for four periods. Per period 8 SKUs should be produced. The routings and amounts to be produced of these SKUs are specified on forehand. The input data for the periods are different in terms of routings and amounts to be produced. Further, it is indicated if an SKU should start or end on its required filling equipment. The other input information setup times, setup costs, equipment D costs and availability of the equipment is the same for all periods.

In Appendix L the results of the comparison are shown, the start and end times per SKU and the total costs are given. For all periods the total costs of the schedule created by the scheduling model Z is equal to the total costs of the schedule created using the heuristic. The schedules created with the heuristic performs better in terms of makespan (i.e. they finish earlier) compared to the schedules created with the scheduling model Z. As the objective of the heuristic was to minimize makespan and of the scheduling model to minimize costs, this is a logical consequence. Remarkable is that the total costs are the same for both methods, despite the fact that the heuristic did not focus on minimizing costs.

7.2 Comparison between schedulers and heuristic

To perform this comparison it was thought to use the handover scheduling documents and come with an solution via the heuristic and compare this solution to the actual scheduling plan provided by the schedulers. However the handover scheduling documents do not correspond to the schedules created by the schedulers at the factory Y. The handover scheduling documents are provided four weeks before the actual production takes place. The supply plan provided in the handover scheduling documents corresponds to the actual schedules roughly (in terms of SKUs produced and amounts to produce). However in those four weeks, between releasing the handover scheduling documents and the actual production, some adaptation may take place, because in those four weeks planners are allowed to pass on changes (see figure 7.1). For example an SKU appears to be sooner out of stock than expected and it is requested to make that SKU instead of another. This affects the actual schedule (see figure 7.1).

<table>
<thead>
<tr>
<th>Handover scheduling document (submitted)</th>
<th>Adaptations</th>
<th>Adaptations</th>
<th>Actual schedule completed (also available in a document)</th>
<th>Actual production</th>
</tr>
</thead>
<tbody>
<tr>
<td>Week 1</td>
<td>Week 2</td>
<td>Week 3</td>
<td>Week 4</td>
<td>Week 5</td>
</tr>
</tbody>
</table>

Figure 7.1: process of handing down production plans
A comparison between the outcome of the heuristic, based on the handover scheduling documents, with the actual schedules is not fair. Therefore, the available information available of the actual schedules developed by the schedulers is used.

The actual schedules provide the following information:

- Which SKUs should be produced?
- Via which routing these SKUs should be produced
- Production hours per SKU (this information is obtained combining the routing via which an SKU should be produced and the amount of an SKU that should be produced. When the routing of an SKU is known the corresponding speed is known, this way the production hours can be calculated).

This is the same information as is provided in the handover scheduling documents.

For four weeks in a row the heuristic is performed and production schedules are created. For every production schedule the associated setup costs (paragraph 3.2.1) and Equipment D costs (paragraph 3.2.3) are calculated as well. The setup costs are sequence dependent. When changing from one SKU to another and both SKUs have the same recipe no costs occur. When the recipe is different a material loss occurs at the filling equipment. Equipment D costs occur when equipment D is enabled but not used for actual production.

The production schedules created by the schedulers are also available and for these schedules the related costs (Setup and Equipment D costs) are also calculated. Then these production schedules and related costs are compared with one another. Both schedules and according costs are shown in Appendix N.

In table 7.1 the results of this comparison are summarized.

<table>
<thead>
<tr>
<th></th>
<th>Schedulers</th>
<th></th>
<th>Heuristic</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>WEEK 1</td>
<td>All</td>
<td>7675.94</td>
<td>All, but 1 hour less can be produced of an SKU</td>
<td>6852.425</td>
</tr>
<tr>
<td>WEEK 2</td>
<td>All</td>
<td>8906.18</td>
<td>All</td>
<td>8510.74</td>
</tr>
<tr>
<td>WEEK 3</td>
<td>All</td>
<td>8128.135</td>
<td>2 SKU cannot be produced</td>
<td>7620.735</td>
</tr>
<tr>
<td>WEEK 4</td>
<td>All</td>
<td>8962.37</td>
<td>All</td>
<td>8962.37</td>
</tr>
</tbody>
</table>

As shown in table 7.1 the heuristic performs better in terms of costs for week 1, 2 and 3. For week 4 the costs of the schedulers’ schedule and the heuristic’s schedule is the same. Also following from table 7.1 when the heuristic’s schedule is used, in week 1 and week 3 less can be produced in comparison to the schedulers’ schedule. Following from Appendix N the time spent producing is for all weeks comparable for the schedulers’ and heuristic’s schedule.

Per week the main similarities and differences of the schedules are discussed.

For week 1 the sequence of the SKUs is similar for all filling equipment except for filling equipment 4. On this equipment the sequence created by the heuristic is different because the heuristic sequences SKUs requiring Equipment D first. Equipment D is available for production from time point 4, therefore, there are 4 hours idle time at the beginning of filling equipment 4. This causes shortage of time at the end of filling equipment 4 and 1 hour less can be produced of the last SKU on filling equipment 4. On the other hand the total costs of the schedule created via the heuristic are lower than...
the total costs of the schedulers’ schedule. The heuristic ensured that the SKUs requiring equipment D are scheduled together as close as possible. This resulted in lower costs.

For week 2 the sequence of the SKUs is different for all filling equipment except for filling equipment 4. For filling equipment 1 the sequence is different, because the heuristic sequences SKUs with the same recipe together to minimize total setup time. As a side effect this also positively influences the setup costs, since the setup costs occurs when changing from one recipe to another. For filling equipment 2 the sequence is different, but this does not affect the total setup time or setup costs. This also applies to filling equipment 3; the sequence is different and this does not affect the total setup time or setup costs. In this week no SKUs are included that require equipment D, therefore no equipment D costs are created. The total costs of the heuristic’s schedule are a bit lower than the total costs of the schedulers’ schedule. This is caused by the positive side effect occurring at filling equipment 1 as mentioned before.

For week 3 the sequence of the SKUs is different for all filling equipment except for filling equipment 1. The sequence of the SKUs on filling equipment 2 is different for the heuristic compared to the schedulers’ sequence. The heuristic minimizes the total setup time, but because of this an SKU cannot be made on equipment 2. This SKU requires equipment that is also used by the SKUs on filling equipment 1. For filling equipment 4 the sequence is different. On this equipment the sequence created by the heuristic is different because the heuristic sequences SKUs requiring Equipment D first. Equipment D is available for production from time point 4, therefore, there are 4 hours idle time at the beginning of filling equipment 4. This causes that one SKU cannot be made. Further it affects the sequence on filling equipment 3. One SKU requires both filling equipment 3 and equipment D and this SKU is scheduled as close as possible to the other SKUs that use Equipment D (these are already scheduled). The start and end times of this SKU are fixed before the remaining SKUs requiring filling equipment 3 are scheduled. This creates an extra ‘constraint’, the remaining SKUs should be scheduled around this SKU. This caused some idle time on filling equipment 3 for the heuristic’s schedule. It also caused additional setup costs for the heuristic’s schedule, because SKUs of the same recipe weren’t scheduled together. This was better in terms of makespan, as this was the objective of the heuristic. Equipment D costs were the same for the schedulers’ schedule and the heuristic’s schedule. Although additional setup costs occur at filling equipment 3 the total costs were lower for the heuristic. This is caused by the savings on filling equipment 2, because an SKU cannot be made no setup costs occur for this SKU.

For week 4 the sequence of the SKUs is similar for all filling the equipment except for filling equipment 4. This difference in sequence does not affect the costs or time spent on the equipment.

Concluded is that the heuristic comes up with good executable production schedules. Its strengths are: Minimizing equipment D costs by scheduling the SKUs that require equipment D first (1). Minimizing sequence dependent setup times and therefore also sequence dependent setup costs (2). Its weaknesses are: By scheduling equipment D first, idle time may appear on filling equipment 4 and or 3(1). By fixing the start and end time of SKUs after each step, flexibility in scheduling is decreased.
8. Conclusions & Recommendations

This chapter gives an overview of the conclusions and recommendations that can be derived based on the research presented in the previous chapters. The first section gives the conclusions of the research and the recommendations for Company A (paragraph 8.1) and the second section explains the recommendations for literature (paragraph 8.2).

8.1 Conclusions & Recommendations for Company A

The first research question was: “Is cyclic production a good scheduling policy for production area X?”. Concluded is that cyclic production is not a good scheduling policy for production area X. First of all determining a fixed cycle in an environment characterized by shared equipment and no wait batch production is very difficult, because the cycles interact. Secondly, the flexibility in the quantity to produce is very limited. Whenever a cycle of an equipment is adapted this affects the cycle of another equipment. As a result cycles would have to be adapted all the time which is undesirable for Company A. It is recommended to Company A not to invest in cyclic production schedules for production area X or comparable environments (i.e. where equipment is shared and the production is no wait and equipment is reserved simultaneously).

The second research question was: “Relaxing which constraints lead to the highest (cost) benefit at production area X?”. To answer this research question a scheduling model and a heuristic were developed for production area X. It was found that the mathematical scheduling model could not be solved for large problem instances. The heuristic is able to handle large problem instances, SKUs to be produced in production area X. When comparing the two methods, the scheduling model Z and the heuristic, for a small data set it was found that the heuristic performed evenly good in terms of costs and even better considering makespan. Comparing the heuristic to the current scheduling at production area X, it was found that the heuristic came with similar schedules. The heuristic performed well in terms of costs, despite that the costs are not objected to be minimized. However it should be indicated that some SKUs could not be included in the schedules created with the heuristic.

Next, following from the comparison between the schedulers and the heuristic the focus on equipment D is important. Equipment D creates costs when it is enabled, but not used for the production of SKUs. The heuristic focuses on equipment D and therefore ensures that these equipment D costs, which are equivalent to the setup costs, are minimized. It shows that if equipment D was able to stop without losing products, this would have a (huge) impact on the costs. Further, it is indicated that by minimizing the setup times also the setup costs are minimized. Long setups occur when changing from recipe, also higher costs occur when changing from recipe.

It can be concluded that the production area X is a complex environment where the characteristics: sharing equipment and no wait batch production influence the scheduling opportunities heavily. Due to limited time and the fact the mathematical scheduling model could not be solved for large instances, a larger survey of the costs and the constraints cannot be performed. Therefore to have a complete answer of the second research question additional research should be done. Although it is recommended to further investigate the usage of the heuristic. The initial solution of the heuristic already comes up with a decent schedules. When this is further investigated and the heuristic is computerized, it can be used as an scheduling tool.

Lastly, it is recommended to take into account that the schedulers and supply planners have a lot of knowledge about all the constraints. They use their experience to come up with executable schedules.
Not all of this knowledge is documented. Also the steps they take to come up with an executable schedule is not. It should be considered that there is a risk when these people change their jobs.

8.2 Recommendations for Literature

The production area X is characterized by sharing equipment and equipment that is reserved simultaneously. For this production area X a mathematical scheduling model is developed. For development of the mathematical scheduling model the model presented by Kopanos et al. 2010 was used as a guidance. The model of Kopanos et al. 2010 presents a multi-stage environment, in contrast to the model presented in this research where equipment is reserved simultaneously. This presents a gap in the literature, as a model for a comparable environment was not found. The mathematical model can be solved for small problem instances. It should be further investigated if a mathematical model (dealing with simultaneous reserved equipment) can be developed which can deal with large problem instances.

Further it can be concluded that the heuristic is able to deal with the large problem instances in an environment where equipment is shared and equipment is reserved simultaneously.
References


Other resources:
Company X annual report 2012
Monthly performance news letter 2012
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<table>
<thead>
<tr>
<th>Article</th>
<th>Year</th>
<th>General</th>
<th>Multi-line</th>
<th>Multi-product</th>
<th>Sequence dependent setup</th>
<th>Inventory costs</th>
<th>Setup costs</th>
<th>Shared equipment</th>
<th>Cyclic production</th>
<th>NOTE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Alle, Papageorgiou, Pinto</td>
<td>2004</td>
<td>&quot;A mixed-integer programming (MINLP) model based on continuous time representation is proposed that can simultaneously optimize the production and cleaning scheduling&quot; (p 3)</td>
<td>X</td>
<td>This article considers multiple products.</td>
<td>Sequence dependent setups are included using a time component that matches the time that is used to change from product i to j in the model</td>
<td>Inventory costs for finished goods are included in the model</td>
<td>Sequence dependent setup costs are included in the model</td>
<td>X</td>
<td>Since the problem in the article is a multistage problem, sub cycles are developed.</td>
<td>The problem addressed in the article is a multistage problem using intermediate stock, this in contrast to Company A’s problem.</td>
</tr>
<tr>
<td>Ashayeri, Heuts, Lansdaal, Strijbosch</td>
<td>2006</td>
<td>The paper shows how cyclic production can improve an organization. Advantage and disadvantages are presented</td>
<td>X</td>
<td>Multiple products, high-volume/low-value are considered</td>
<td>Sequence dependent setup are included, in particular by introducing families of which article have short setup times</td>
<td>Inventory costs are a combination of several costs and included</td>
<td>Setup costs are split into major setup costs and minor setup up costs</td>
<td>X</td>
<td></td>
<td>The presented solution is in a cycle form</td>
</tr>
<tr>
<td>Bohnen, Buhl and Deuse</td>
<td>2013</td>
<td>The paper &quot;presents a systematic procedure of leveling of low volume and high mix production&quot;(p53). It concerns clustering techniques.</td>
<td>X</td>
<td>Multiple products, low value and high mix production</td>
<td>Between families there is no sequence dependence. Within setups are minimal (in terms of time and yield losses)</td>
<td>Mentions is that leveling ensured customer demand without large volumes of inventory. Inventory costs though are not included</td>
<td>If raw material, part or components are shared, this can be used as grouping criteria</td>
<td>X</td>
<td></td>
<td>This article concerns with clustering products into families could be interesting considering Company A objective to lower setup times.</td>
</tr>
<tr>
<td>Bollapragada, Croce, Chirardi</td>
<td>2011</td>
<td>This paper deals with real time allocation and lot sizing considering multiple lines, multiple products each with different costs (production, set up, inventory)</td>
<td>x</td>
<td>Multiple non identical production lines are included. These products characteristics (e.g. production rate, demand, setup times/costs etc.) are different when produced on another line</td>
<td>Multiple products are included.</td>
<td>Inventory cost are object to be minimized. Inventory of each item at the end of a period is included in the model</td>
<td>Setup costs are object to be minimized. Setup costs are different per product per line (not sequence dependent)</td>
<td>X</td>
<td>X</td>
<td>The setup times and costs are included in the models, but they are not considered to be sequence dependent.</td>
</tr>
<tr>
<td>Authors</td>
<td>Year</td>
<td>Description</td>
<td>Relevant Costs</td>
<td>Cyclic Production</td>
<td>Cyclic Schedule Considered</td>
<td>Cyclic Schedule Developed</td>
<td>Schedule Used as Input</td>
<td>Relevant Constraints</td>
<td></td>
<td></td>
</tr>
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<td>----------------------------------</td>
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<td>------------------------</td>
<td>----------------------</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bollapragada, Rao</td>
<td>1999</td>
<td>The paper &quot;focuses on simultaneous resource allocation, lot sizing, and scheduling in a multimachine, deterministic ELSP environment. They consider multiple manufacturing lines with different costs and capabilities. Average production, setup, inventory and shortages costs are minimized.&quot; (p889)</td>
<td>I number of products are considered in this article. Setup costs are object to be minimized. Inventory costs (in this article holding costs) are included and objected to be minimized.</td>
<td>X</td>
<td>X</td>
<td>Both multiple non identical lines and cyclic production is considered.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Brander, Forsberg</td>
<td>2006</td>
<td>Deals with the stochastic version of the economic lot size scheduling problem. They show how to determine safety stock and order up to levels for fixed cyclic sequence.</td>
<td>Multiple products are considered, each having random demand.</td>
<td>Inventory costs are included and to be minimized. Set up costs are included and to be minimized (not sequence dependent)</td>
<td>X</td>
<td>Cyclic production is considered but the cycle is not developed it is assumed to be known.</td>
<td>The article doesn't show how to develop a cyclic schedule. A cyclic schedule is used as input.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Broecke, Landeghem, Aghezzaf</td>
<td>2008</td>
<td>The paper describes &quot;the implementation and operational results of a cyclical master production-scheduling model&quot;(p121). The production process consists of intermediate stock</td>
<td>The article deal with the production of multiple photographic film products (large units).</td>
<td>The objective is to minimize set up costs. (not sequence dependent)</td>
<td>X</td>
<td>This article considers a multi-stage problem with intermediate stock, this doesn't coincide with Company A's problem.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Kallrath</td>
<td>2005</td>
<td>The paper gives an overview of recent activities (progress and current state) of planning/scheduling in the process industry (hereby focusing on mixed integer and global optimization)</td>
<td>The paper mentions problems in the process industry (of which multi-purpose, multi-mode). Specifically 5.3 reference to a MILP model considering</td>
<td>Addressed as relevant cost in production plan</td>
<td>X</td>
<td>Interesting because it gives an overview of several modelling techniques. Furthermore the article presents options on how to model certain constraints</td>
<td></td>
<td></td>
<td></td>
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Appendices

<table>
<thead>
<tr>
<th>Article</th>
<th>Year</th>
<th>Database</th>
<th>Key words</th>
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<tr>
<td>Alle, Papageorgiou, Pinto</td>
<td>2004</td>
<td>Science direct</td>
<td>cyclic AND production</td>
</tr>
<tr>
<td>Ashayeri, Heuts, Lansdaal, Strijbosch</td>
<td>2006</td>
<td>Science direct</td>
<td>“cyclic production”</td>
</tr>
<tr>
<td>Bohnen, Buhl and Deuse</td>
<td>2013</td>
<td>Via consultant at Company A</td>
<td></td>
</tr>
<tr>
<td>Bollapragada, Croce, Chirardi</td>
<td>2011</td>
<td>Advice S.D.P. Flapper</td>
<td></td>
</tr>
<tr>
<td>Bollapragada, Rao</td>
<td>1999</td>
<td>Scopus</td>
<td>Multiple lines AND ELSP</td>
</tr>
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<td>Brander, Forsberg</td>
<td>2006</td>
<td>Via Reference Ashayeri</td>
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<td>Broecke, Landeghem, Aghezzaf</td>
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<td>Science direct</td>
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<td>Lukac, Soric, Rosenweig</td>
<td>2008</td>
<td>Advice S.D.P. Flapper</td>
<td></td>
</tr>
<tr>
<td>Pesenti, Ukovich</td>
<td>2003</td>
<td>Scopus</td>
<td>Multiple lines AND ELSP</td>
</tr>
</tbody>
</table>

In this article the objective is to minimize the set up times and minimize "production storage and set up costs" by allocation products to lines. Multiple products are produced on two machines and both machines can produces all products. N products are considered to be produced and meet the demand. The time to switch from one product to the other is considered to be sequence dependent. Inventory costs are objected to be minimized. Setup costs are different per product per line (not sequence dependent). X X Only two lines are considered and both lines can produces all products

The article provides cyclic production schedules and shared equipment is included in the model.

The table below depicts where the articles where and by the use of which search words the articles where found.

<table>
<thead>
<tr>
<th>Article</th>
<th>Year</th>
<th>Database</th>
<th>Key words</th>
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<tbody>
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<tr>
<td>Pesenti, Ukovich</td>
<td>2003</td>
<td>Scopus</td>
<td>Multiple lines AND ELSP</td>
</tr>
</tbody>
</table>
Appendix J Initial model GMPL, data, output and costs

set I;
/* SKUs */
set E;
/* equipment*/

param Cp {i in I};
param Cs{e in E,i in I, x in I};
param M;
param m;
param rho {i in I};
param St{e in E,i in I, x in I};
param T;
param U {e in E,i in I},binary;
param WO {i in I};

var AP {i in I}>=0;
/* constraint 3, APT is bigger or equal to 0*/
var Tstart {i in I}>=0;
/* constraint 6, Tstart is bigger or equal to 0*/
var Tend {i in I};
var P {i in I};
/*additional*/

var X {i in I},binary;
/*SKU i is produced*/
var Y {e in E, i in I, x in I:i!=x},binary;
/* Y[e,i,x] --> SKU i is preceding SKU x on equipment e*/
var y {e in E, i in I, x in I:i!=x}, binary;
/* y[e,i,x] --> SKU i is produced exactly before SKU x on equipment e*/
var yL {e in E, i in I},binary;
/* Last SKU on equipment e*/
var yF {e in E, i in I},binary;
/* First SKU on equipment e*/

minimize cost: sum {i in I} Cp[i]*P[i]+sum {e in E,i in I, x in I:i!=x}Cs[e,i,x]*y[e,i,x];

s.t. rx {i in I}: P[i]=WO[i]-AP[i];
/* production demand minus actual production*/

s.t. r2{i in I}: AP[i] <= WO[i] * X[i];
/* production quantity, constraint 2*/

s.t. r4 {e in E,i in I, x in I:i!=x}: 2*(Y[e,i,x] + Y[e,x,i]) <= X[i]*U[e,i]+X[x]*U[e,x];
/* possible sequence, constraint 4*/
s.t. r5 \{e \in E, i \in I, x \in I: i \neq x\}: X[i] \cdot U[e, i] + X[x] \cdot U[e, x] \leq 1 + Y[e, i, x] + Y[e, x, i];

/* possible sequence, constraint 5*/

s.t. r7 \{i \in I\}: Tend[i] \leq T \cdot X[i];

/* Tend is 0 if SKU i is not produced, constraint 7*/

s.t. r8 \{i \in I\}: Tend[i] = Tstart[i] + AP[i]/\rho[i];

/* production start plus production time, constraint 8 */

s.t. r9 \{e \in E, i \in I, x \in I: i \neq x\}: \sum_{c \in I: c \neq x} Y[e, c, x] - \sum_{c \in I: c \neq i} Y[e, c, i] \geq 1 - m \cdot (1 - y[e, i, x]);

s.t. r10 \{e \in E, i \in I, x \in I: i \neq x\}: \sum_{c \in I: c \neq x} Y[e, c, x] - \sum_{c \in I: c \neq i} Y[e, c, i] \leq 1 + m \cdot (1 - y[e, i, x]);

/* exact sequence, constraints 9 to 11*/

s.t. r11 \{e \in E, i \in I, x \in I: i \neq x\}: y[e, i, x] \leq Y[e, i, x];

s.t. r12 \{e \in E, i \in I\}: \sum_{x \in I: i \neq x} y[e, i, x] + yL[e, i] = X[i] \cdot U[e, i];

s.t. r13 \{e \in E, i \in I\}: \sum_{x \in I: i \neq x} y[e, x, i] + yF[e, i] = X[i] \cdot U[e, i];

s.t. r14 \{e \in E\}: \sum_{i \in I} yL[e, i] \leq 1;

s.t. r15 \{e \in E\}: \sum_{i \in I} yF[e, i] \leq 1;

/* last and first SKU on equipment, constraints 12 to 15*/

s.t. r16 \{e \in E, i \in I, x \in I: i \neq x\}: Tstart[x] + M \cdot (1 - y[e, i, x]) \geq Tend[i] + St[e, i, x] \cdot y[e, i, x];

/* production of SKU x can start if the previous SKU i is finished and the changeover is performed, constraint 16*/

solve;
display X, APT, y, Tstart, Tend, cost;
data;
set I:=1 2 3 4 5 6;
set E:=g1 g2;

param Cp:=
1 1
2 1
3 1
4 1
5 1
6 1;

param Cs:=
[m,*,*]: 1 2 3 4 5 6:=
1 2 3 1 2 1 2
2 1 2 1 3 1 2
3 2 1 3 2 1 2

53
\[
\begin{array}{cccccc}
4 & 1 & 2 & 3 & 2 & 1 \\
5 & 2 & 3 & 1 & 2 & 1 \\
6 & 1 & 2 & 1 & 3 & 1 \\
\hline
\end{array}
\]

\[
\begin{array}{cccccc}
[g1,*,*]: & 1 & 2 & 3 & 4 & 5 \\
1 & 3 & 2 & 1 & 2 & 3 \\
2 & 1 & 3 & 2 & 1 & 2 \\
3 & 2 & 3 & 1 & 3 & 3 \\
4 & 1 & 2 & 3 & 2 & 1 \\
5 & 2 & 3 & 1 & 2 & 1 \\
6 & 1 & 2 & 3 & 2 & 1 \\
\hline
\end{array}
\]

\[
\begin{array}{cccccc}
[g2,*,*]: & 1 & 2 & 3 & 4 & 5 \\
1 & 1 & 1 & 0 & 0 & 1 \\
2 & 1 & 2 & 3 & 2 & 1 \\
3 & 0 & 1 & 0 & 1 & 1 \\
4 & 1 & 2 & 3 & 2 & 1 \\
5 & 2 & 3 & 1 & 2 & 1 \\
6 & 1 & 2 & 1 & 3 & 1 \\
\hline
\end{array}
\]

\[
\text{param } M:=51; \text{ }
\text{param } m:=10; \text{ }
\text{param } \rho:=
\begin{array}{c}
1 \\
2 \\
3 \\
4 \\
5 \\
6 \\
\end{array}
\begin{array}{c}
10 \\
20 \\
10 \\
10 \\
20 \\
10 \\
\end{array}
\]

\[
\text{param } St:=
\begin{array}{cccccc}
[m,*,*]: & 1 & 2 & 3 & 4 & 5 & 6:=
1 & 2 & 3 & 1 & 2 & 1 \\
2 & 1 & 2 & 1 & 3 & 1 \\
3 & 2 & 1 & 3 & 2 & 1 \\
4 & 1 & 2 & 3 & 2 & 1 \\
5 & 2 & 3 & 1 & 2 & 1 \\
6 & 1 & 2 & 1 & 3 & 1 \\
\hline
\end{array}
\]

\[
\begin{array}{cccccc}
[g1,*,*]: & 1 & 2 & 3 & 4 & 5 \\
1 & 3 & 2 & 1 & 2 & 3 \\
2 & 1 & 3 & 2 & 1 & 2 \\
3 & 2 & 3 & 1 & 3 & 3 \\
4 & 1 & 2 & 3 & 2 & 1 \\
5 & 2 & 3 & 1 & 2 & 1 \\
6 & 1 & 2 & 3 & 2 & 1 \\
\hline
\end{array}
\]

\[
\begin{array}{cccccc}
[g2,*,*]: & 1 & 2 & 3 & 4 & 5 \\
1 & 1 & 1 & 0 & 0 & 1 \\
2 & 1 & 2 & 3 & 2 & 1 \\
3 & 2 & 3 & 1 & 2 & 1 \\
\hline
\end{array}
\]
param T := 50;
param U :=
  1  2  3  4  5  6 :=
m  1  1  0  0  0  0
g1 1  0  1  0  0  1
g2 0  1  0  1  1  0;
param WO :=
  1  150
  2  200
  3  300
  4  200
  5  200
  6  150;
end;

Output

y[m,1,2].val = 0  y[m,4,5].val = 0  y[g1,2,3].val = 0  y[g1,5,6].val = 0  y[g2,3,4].val = 0
y[m,1,3].val = 0  y[m,4,6].val = 0  y[g1,2,4].val = 0  y[g1,6,1].val = 1  y[g2,3,5].val = 0
y[m,1,4].val = 0  y[m,5,1].val = 0  y[g1,2,5].val = 0  y[g1,6,2].val = 0  y[g2,3,6].val = 0
y[m,1,5].val = 1  y[m,5,2].val = 0  y[g1,2,6].val = 0  y[g1,6,3].val = 0  y[g2,4,1].val = 0
y[m,1,6].val = 0  y[m,5,3].val = 0  y[g1,3,1].val = 0  y[g1,6,4].val = 0  y[g2,4,2].val = 0
y[m,2,1].val = 1  y[m,5,4].val = 0  y[g1,3,2].val = 0  y[g1,6,5].val = 0  y[g2,4,3].val = 0
y[m,2,3].val = 0  y[m,5,6].val = 0  y[g1,3,4].val = 0  y[g2,1,2].val = 0  y[g2,4,5].val = 1
y[m,2,4].val = 0  y[m,6,1].val = 0  y[g1,3,5].val = 0  y[g2,1,3].val = 0  y[g2,4,6].val = 0
y[m,2,5].val = 0  y[m,6,2].val = 0  y[g1,3,6].val = 0  y[g2,1,4].val = 0  y[g2,5,1].val = 0
y[m,2,6].val = 0  y[m,6,3].val = 0  y[g1,4,1].val = 0  y[g2,1,5].val = 0  y[g2,5,2].val = 0
y[m,3,1].val = 0  y[m,6,4].val = 0  y[g1,4,2].val = 0  y[g2,1,6].val = 0  y[g2,5,3].val = 0
y[m,3,2].val = 0  y[m,6,5].val = 0  y[g1,4,3].val = 0  y[g2,2,1].val = 0  y[g2,5,4].val = 0
y[m,3,4].val = 0  y[g1,1,2].val = 0  y[g1,4,5].val = 0  y[g2,2,3].val = 0  y[g2,5,6].val = 0
\text{Appendices}

\begin{align*}
y[m, 3, 5].\text{val} &= 0 & y[g1, 1, 3].\text{val} &= 1 & y[g1, 4, 6].\text{val} &= 0 & y[g2, 2, 4].\text{val} &= 1 & y[g2, 6, 1].\text{val} &= 0 \\
y[m, 3, 6].\text{val} &= 0 & y[g1, 1, 4].\text{val} &= 0 & y[g1, 5, 1].\text{val} &= 0 & y[g2, 2, 5].\text{val} &= 0 & y[g2, 6, 2].\text{val} &= 0 \\
y[m, 4, 1].\text{val} &= 0 & y[g1, 1, 5].\text{val} &= 0 & y[g1, 5, 2].\text{val} &= 0 & y[g2, 2, 6].\text{val} &= 0 & y[g2, 6, 3].\text{val} &= 0 \\
y[m, 4, 2].\text{val} &= 0 & y[g1, 1, 6].\text{val} &= 0 & y[g1, 5, 3].\text{val} &= 0 & y[g2, 3, 1].\text{val} &= 0 & y[g2, 6, 4].\text{val} &= 0 \\
y[m, 4, 3].\text{val} &= 0 & y[g1, 2, 1].\text{val} &= 0 & y[g1, 5, 4].\text{val} &= 0 & y[g2, 3, 2].\text{val} &= 0 & y[g2, 6, 5].\text{val} &= 0 \\
\text{Costs} &= 127
\end{align*}

\textbf{Calculating costs}

\[\text{Min} \sum_i c^p_i \cdot (WO_i - AP_i) + \sum_i \sum_{e} c^{s,e}_{ii'} \cdot \bar{y}_{ii'}^{e}\]

\begin{align*}
WO[1] &= 150 & \text{AP}[1].\text{val} &= 80 & \text{Cp}[1] &= 1 \\
WO[6] &= 150 & \text{AP}[6].\text{val} &= 100 & \text{Cp}[6] &= 1 \\
\sum_i c^p_i \cdot (WO_i - AP_i) &= (150 - 80) \cdot 1 + (150 - 100) \cdot 1 = 1
\end{align*}

\begin{align*}
y[m, 1, 5].\text{val} &= 1 & \text{Cs}[m, 1, 5] &= 1 \\
y[m, 2, 1].\text{val} &= 1 & \text{Cs}[m, 2, 1] &= 1 \\
y[g1, 1, 3].\text{val} &= 1 & \text{Cs}[g1, 1, 3] &= 1 \\
y[g1, 6, 1].\text{val} &= 1 & \text{Cs}[g1, 6, 1] &= 1 \\
y[g2, 2, 4].\text{val} &= 1 & \text{Cs}[g2, 2, 4] &= 2 \\
y[g2, 4, 5].\text{val} &= 1 & \text{Cs}[g2, 4, 5] &= 1 \\
\sum_i \sum_{e} c^{s,e}_{ii'} \cdot \bar{y}_{ii'}^{e} &= (1 \cdot 1) + (1 \cdot 1) + (1 \cdot 1) + (1 \cdot 1) + (2 \cdot 1) + (1 \cdot 1) = 7
\end{align*}

\textbf{Total costs}

\[\text{Min} \sum_i c^p_i \cdot (WO_i - AP_i) + \sum_i \sum_{e} c^{s,e}_{ii'} \cdot \bar{y}_{ii'}^{e} = 120 + 7 = 127\]

The total costs correspond to the cost value calculated by GUSEK (127).
Appendix K Scheduling model Z GMPL

set I; /* All SKU's */
set E; /* equipment*/
set Id; /*SKU's asked by the supply planners */

param Cn {e in E};
param Cp {i in I};
param Cs{e in E,i in I, x in I};
param CsL {e in E, i in I};
param FP{i in I},binary;
param LP{i in I}, binary;
param M;
param m;
param St{e in E,i in I, x in I};
param T;
param Tendeq{e in E};
param Tstarteq{e in E};
param t {e in E};
param U {e in E,i in I},binary;
param WO {i in Id};

var APT {i in Id}>=0;
/* constraint 3, APT is bigger or equal to 0*/
var Tstart {i in Id}>=0;

var Tend {i in Id};
var TENDVAR {e in E};
/* end time of equipment*/
var TSTARTVAR {e in E} >=0;
/*start time of equipment*/

var X {i in Id},binary;
/*SKU i is produced*/
var P {i in Id};
/*additional variable, production P*/

var Y {e in E, i in Id, x in I:i!=x},binary;
/* Y[e,i,x] --> SKU i is preceding SKU x on equipment e*/
var y {e in E, i in Id, x in I:i!=x}, binary;
/* Y[e,i,x] -- SKU i is produced exactly before SKU x on equipment e*/
var yL {e in E, i in Id}, binary;
/* Last SKU on equipment e*/
var yF {e in E, i in Id}, binary;
/* First SKU on equipment e*/

minimize cost: sum {i in Id} Cp[i]*P[i] + sum {e in E, i in Id, x in Id:i!=x} Cs[e,i,x]*y[e,i,x] + sum {e in E, i in Id} CsL[e,i] * yL[e,i] + sum {e in E} Cn[e]* (TENDVAR[e] - TSTARTVAR[e] - sum {i in Id} APT[i]*U[e,i]);

s.t. rx {i in Id}: P[i]=WO[i]-APT[i]; /* production demand minus actual production*/

s.t. r2 {i in Id}: APT[i] = WO[i] * X[i]; /* production quantity, constraint 2*/

s.t. r4 {e in E, i in Id, x in Id:i!=x}: 2*(Y[e,i,x] + Y[e,x,i]) <= X[i]*U[e,i]+X[x]*U[e,x]; /* possible sequence, constraint 4*/

s.t. r5 {e in E, i in Id, x in Id:i!=x}: X[i]*U[e,i]+X[x]*U[e,x]<=1+Y[e,i,x]+Y[e,x,i]; /* possible sequence, constraint 5*/

s.t. r6 {i in I}: Tend[i]= Tstart[i]+ APT[i]; /* production start plus production time, constraint 6*/

s.t. r7 {e in E, i in Id, x in Id:i!=x}: sum {c in Id:c!=x} Y[e,c,x] - sum {c in Id:c!=i} Y[e,c,i]>=1-m*(1-y[e,i,x]);

s.t. r8 {e in E, i in Id, x in Id:i!=x}: sum {c in Id:c!=x} Y[e,c,x] - sum {c in Id:c!=i} Y[e,c,i]<=1+m*(1-y[e,i,x]);

s.t. r9 {e in E, i in Id, x in Id:i!=x}: y[e,i,x] <= Y[e,i,x]; /* exact sequence, constraints 6 to 8*/

s.t. r10 {e in E, i in Id}: sum {x in Id:i!=x} y[e,i,x] + yL[e,i]=X[i]*U[e,i];

s.t. r11 {e in E, i in Id}: sum {x in Id:i!=x} y[e,x,i] + yF[e,i]=X[i]*U[e,i];

s.t. r12 {e in E}: sum {i in Id} yL[e,i]<=1;

s.t. r13 {e in E}: sum {i in Id} yF[e,i]<=1; /* last and first SKU on equipment, constraints 9 to 12*/

s.t. r14 {e in E, i in Id, x in Id:i!=x}: Tstart[x]+M*(1-y[e,i,x])>=Tend[i]+St[e,i,x]*y[e,i,x]; /*production of SKU x can start if the previous SKU is finished and the setup is performed*/

s.t. r15 {e in E, i in Id}: Tend[i] <= T - yL[e,i]*(T-Tendeq[e]);

s.t. r16 {e in E, i in Id}: Tstart[i] >= 0 + yF[e,i]*(Tstarteq[e]); /* production can takes place within certain time points*/
s.t. r17 \{e \in E\}: \sum_{i \in Id} (APT[i] \times U[e,i]) \leq t[e] \times T;
/* equipment percentage*/

s.t. r18 \{e \in E\}: 0 \leq \text{TENDVAR}[e];
s.t. r19 \{e \in E\}: \text{TENDVAR}[e] \leq \text{Tendeq}[e];
s.t. r20 \{e \in E, i \in Id\}: \text{TENDVAR}[e] \geq \text{Tend[i]} \times U[e,i];
s.t. r21 \{e \in E, i \in Id\}: \text{Tend[i]} \times U[e,i] + (1 - yL[e,i]) \times \text{Tendeq}[e] \geq \text{TENDVAR}[e];
s.t. r22 \{e \in E\}: 0 \leq \text{TSTARTVAR}[e];
s.t. r23 \{e \in E\}: \text{TSTARTVAR}[e] \leq \text{Tendeq}[e];
s.t. r24 \{e \in E, i \in Id\}: \text{TSTARTVAR}[e] \leq Tstart[i] \times U[e,i] + (1 - yF[e,i]) \times \text{Tendeq}[e];
s.t. r25 \{e \in E, i \in Id\}: Tstart[i] \times U[e,i] - (1 - yF[e,i]) \times \text{Tendeq}[e] \leq \text{TSTARTVAR}[e];
s.t. r26 \{e \in E\}: \text{TENDVAR}[e] \geq \text{TSTARTVAR}[e];
/* determining actual start and end time of an equipment*/

s.t. r27 \{e, i \in Id\}: yF[e,i] \geq FP[i] \times U[e,i] \times X[i] - \sum_{x \in Id: i \neq x} FP[x] \times U[e,x] \times X[x];
s.t. r28 \{e, i \in Id, x \in Id: i \neq x\}: 1 + yF[e,i] + yF[e,x] \geq FP[i] \times U[e,i] \times X[i] + FP[x] \times U[e,x] \times X[x];
s.t. r29 \{e, i \in Id\}: yL[e,i] \geq LP[i] \times U[e,i] \times X[i] - \sum_{x \in Id: i \neq x} LP[x] \times U[e,x] \times X[x];
s.t. r30 \{e, i \in Id, x \in Id: i \neq x\}: 1 + yL[e,i] + yL[e,x] \geq LP[i] \times U[e,i] \times X[i] + LP[x] \times U[e,x] \times X[x];
/* ensuring that the right SKU starts*/

solve;
display X, APT, WO, Tstart, Tend, cost;