Sound absorption by perforated walls
experimental study of the influence of geometry and bias/grazing flow

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Abstract

Perforated walls are encountered in many acoustic dampers (car engine mufflers, liners in combustion chambers or liners at the inlet of aircraft engines). For most of these wall perforations, the main cause of damping is the interaction of a flow with the acoustical field at the perforations. In systems that are operating in polluted flow and/or hot/aggressive gases, the use of these perforations is preferable. Sound absorbing foams or micro-perforates cannot be used in these cases, because they would plug.

The acoustic performance of such perforations depends on many geometrical parameters such as the dimensions of the perforations, the shape of the perforation, the perforation angle with respect to the wall and the sharpness of the edges. Also the flow to which the perforations are subjected and the spectrum of the sound that should be damped, determines the efficiency of the sound reduction. The combination of these three parameter groups (geometry, flow and frequency) make it very difficult to find the conditions of optimal acoustic performance. The flow can have both a component grazing along the wall and a bias component through the perforations. Experimental studies are available for either pure grazing flows or pure bias flows. Theoretical models are only available for pure bias flows. There is no systematic study of the acoustic performance of wall perforations in the presence of a combined bias-grazing flow, which are relevant in many technological applications. We provide a first step of such a study. The study is limited to slit-shaped perforations for which we investigate the effect of the angle of perforation and the sharpness of the edges.

Experimental data are obtained by means of impedance tube measurements based on a multi-microphone method. This means that we measure the acoustic response of different wall perforations and flow configurations to excitation by an external sound source. The incoming sound waves are provided by a loudspeaker. In some cases a negative acoustic resistance is observed, which indicates a possibility of self-sustained harmonic oscillation (whistling) at critical frequencies. We did not consider broad-band self-noise due to turbulence. Furthermore we provide for two limit cases an analytical quasi-steady model that is valid at low Strouhal numbers. We explain qualitatively the effect of edge geometry at higher Strouhal numbers by using the vortex sound theory.

We concluded that the most safe region of performance is at low Strouhal numbers. This means that the typical velocity of the flow should be large enough or that the perforations must be small in order to damp incoming sound waves over a larger frequency range. At low Strouhal numbers the bias flow enhances absorption. For higher Strouhal numbers, the interaction of the flow and the acoustic field can result in a very good damping, but the performance is very sensitive to the geometry and velocity of the flow. Ranges of Strouhal numbers with negative acoustic resistance are observed. High Strouhal numbers should therefore be avoided in industrial applications.
Chapter 1

Introduction

Cars, air-crafts, ventilation systems, etc. form part of daily life and make life more comfortable. But at the same time, most people consider noise coming from those systems as not pleasant. Noise pollution is a source of stress and there are therefore strong legal regulations forcing engine manufacturers to reduce noise: either by designing sufficiently silent engines and systems or by reducing the amount of noise that is emitted to the environment by the use of acoustical dampers. Perforated walls are encountered in many acoustic damping systems. Absorption of sound at perforated walls can be due to viscosity which is the dominant mechanism in so-called "micro-perforated plates". We consider here the case of larger perforations. For larger perforations the sound absorption is due to the interaction of the sound field with a steady flow. The study of this process is the subject of this thesis.

1.1 Technical applications

1.1.1 Car muffler

A muffler is a kind of acoustical damper that is inserted between the sound producing system and the environment. It allows the flow of gas from the system through the muffler but at the same time it reduces the sound emanating from its end. Mufflers can be categorized in two groups: reactive mufflers and dissipative mufflers. Reactive mufflers reduce the sound due to reflection of the acoustic waves at geometrical discontinuities. The sound is not attenuated but sent back to the source. The disadvantage of this kind of muffler is their effectiveness only within certain frequency ranges. Furthermore, the reflections may lead to high pulsations in the upstream section. This may lead for example to increased combustion instabilities. The second kind of mufflers are dissipative mufflers. They dissipate the sound by sound absorbing material or by aeroacoustic interaction at wall perforations. The disadvantage of the use of sound absorbing material such as foam is that it will get plugged and burned when the damper operates in circumstances with polluted or hot exhaust gas. The perforated walls are more robust which makes their use favourable.

A well known acoustic damper is a car muffler which is a combination of a reactive and a dissipative muffler (including absorbing foam and perforated pipe segments). The reactive features are due to the expansion chamber and the internal division of the muffler in chambers [1]. A sketch of an empty expansion chamber is shown in Fig.1.1a.). In Fig.1.1c.) a typical curve of the transmission loss for an expansion chamber geometry is given by the dashed line.
1.1 Technical applications

Figure 1.1: Car muffler. An expansion chamber geometry is sketched in (a.) where $p'_{in}$ and $p'_{out}$ are the incoming respectively outgoing acoustic pressure disturbance. $U$ represents the exhaust flow. A sketch of a simple plugged perforated-pipe muffler is given in (b.) From (c.) where a typical graph of the transmission loss as a function of the Helmholtz number ($He = fL/c_0$) is plotted for an expansion chamber (dashed line). Maximal transmission loss occur at $He_L = 1/4, 3/4, \ldots$ and for an expansion chamber with plugged perforated pipe, is seen that the flow at the edges enhances the damping.

The transmission loss is defined as $TL = 10 \log_{10} \left( \frac{P'_{in}^2}{P'_{out}^2} \right)$. It is a measure of the acoustic performance. From this graph, we see that the transmission of sound is lowest if the length of the expansion chamber corresponds with $1/4\lambda, 3/4\lambda, \ldots$ (at the maxima of the curve). Around $\lambda/2, 3\lambda/2, \ldots$, the performance of the muffler is very low.

A sketch of a simple muffler, consisting of an expansion chamber and a perforated main pipe that is plugged, is shown in Fig.(1.1 b.). Due to the plug in the main pipe, the flow is forced to pass through the perforations. Little jets are formed behind every perforation. Because of the main flow and the plug, the flow at every perforation is in fact a combination of a flow parallel to the pipe and a flow through the perforation (orifice). The flow parallel to the pipe axis is called the grazing flow with velocity $u_g$ and the flow through the perforation is called the bias flow with velocity $u_b$. The bias flow can be either towards the pipe (outflow) or towards the expansion chamber (inflow).

It was shown by measurements on a muffler that the combination of bias and grazing flow has
an increased sound absorption by 7 dB compared with an empty chamber muffler. Note that the bias flow is negligible if the plug is removed. It was shown that pure grazing flow has lower sound absorption than the combined bias-grazing flow. The enhanced acoustic performance due to the flow is shown in Fig.1.1c by the triangles 2. It was demonstrated earlier by experiments of by Bechert et al. [3] and the vortex sound theory of Howe [4], that a bias flow through a perforation indeed absorbs sound. Acoustic energy is converted into energy of fluctuating vorticity that is shed at the perforations. However the use of a bias flow through the perforations has a drawback: it increases the back pressure. Back pressure refers to the extra static pressure exerted by the muffler on the engine [1]. A high pressure restricts the outflow of exhaust gases and forms an extra load to the engine. This is unwanted for the overall performance of the engine. Therefore the designer of a muffler should find a good balance between engine performance and acoustic damping. There is no systematic study of the acoustic performance of wall perforations in presence of a combined bias-grazing flow. We provide a first step for such a study. In particular the effect of rounding the edges of the perforations, which make part of a real mufflers due to corrosion or low-cost production techniques, will be discussed in this report.

1.1.2 Aircraft engine liner

Another example of sound absorption by vorticity shedding can be found in aircrafts. Acoustic liners that consist of a layer of Helmholtz resonators, cover the walls around the inlet of the engines. Their position in the engine is indicated by the number 50 in Fig. 1.2(a) [5]. In conventional liners, there is a grazing flow along the opening of each resonator. Barthel (1958) proposed to blow or suck air through the Helmholtz resonators to increase the sound absorption. A sketch of this configuration is given in Fig. 1.2(b). The horizontal arrow represents the grazing flow, the vertical arrow the blowing bias flow. Two little jets will be formed for each resonator; one due to the pure bias flow (lowest perforation) and one due to the combination of bias and grazing flow (upper perforation). During take-off and landing, pressurized air could be used to blow air through the perforations. This interesting application is not yet understood well. It is an open question whether it is worth to manufacture these “more complicated Helmholtz resonators” and whether there is an optimal ratio of bias to grazing velocity.

1.1.3 Combustion chamber liner

A further example where a combined bias-grazing flow is a potential sound absorber, is in the combustion chambers of turbine engines. Film-cooling is a widely applied technique that prevent the walls of the combustion chamber from melting. Due to the Coanda effect these jets remain attached to the wall. Here cold air is blown into the chamber through oblique orifices. A film of cold air between the wall and the hot combustion gases is generated. This is shown in Fig. 1.3. Besides this cooling effect, well designed orifices could attenuate sound coming from the flame pulsations. But the acoustic performance strongly depends on the flow conditions and the geometry of the orifices. Despite recent numerical investigations [6], [7], the sound absorption at an oblique perforation in a bias-grazing flow configuration, is not yet clearly understood.
1.2 Literature

From these three examples it is clear that a good understanding of the effect of a bias-grazing flow on the acoustic properties of wall perforations is essential for the optimal design of acoustic dampers. And indeed, the effect of flow has been the subject of several studies. However in most of them the effect of either pure grazing flow or pure bias flow are considered separately. Only few studies have been carried out on the effect of a combination of both flows. It was found that the effect is not just a simple summation of both separate effects. The paper of Rogers and Hersh (1975) is a first attempt to consider a combined bias-grazing flow combination. They presented the influence on the steady-state resistance of square-edged perforations. The discharge coefficient, defined as the ratio of the actual flow rate through the perforation to the ideal one-dimensional flow rate through the perforation, was used to describe the steady-state resistance of the perforation itself. Based on this work Cummings
Chapter 1. Introduction

proposed a quasi-steady model to relate the acoustic resistance to the discharge coefficient of the perforation. Recently, Sun et al. proposed a quasi-steady model to evaluate the acoustic resistance based on the work of Cummings. Above mentioned studies concentrate on low Strouhal numbers. Recently, Heuwinkel et al. investigated the effect of several geometrical parameters on the damping performances of perforated liners subjected to a bias-grazing flow for larger Strouhal numbers.

1.3 Objective and scope of the project

The objective of the research that is presented in this master thesis is to provide systematic experimental data. Furthermore we provide for limit cases a quasi-steady model that is valid at low frequencies. We also explain qualitatively the effect of edge geometry using vortex sound theory. The aeroacoustic behaviour of different slit-shaped wall perforations is studied by experimental work in combination with a quasi-steady analytic model.

For geometry we limit ourself to the study of different slit-shaped perforations. These correspond to geometries commonly encountered in technical applications like oblique orifices or orifices with chamfered edges. Slit-shaped orifices are suitable for a possible future comparison with two-dimensional numerical simulations.

If one is familiar with the fact that jet flows produce broadband noise (self-noise), it can be surprisingly that we investigate the acoustic damping features of a jet. Also the whistling of a jet flow through an aperture with a definite pitch, is a kind of self-noise (this occurs also during human whistling). But as is illustrated by above mentioned technical applications and as it will be shown in the succeeding chapters, the damping is a fact, under certain circumstances. Exactly this apparently contradiction shows one of the restrictions of the research.

Since we measure the linear response to acoustic forcing (by means of an impedance tube set-up in combination with a multi-microphone method), we do not measure broad-band self-noise of the single slit-shaped perforation under bias-grazing flow conditions. The problem of broad-band self-noise is an important issue in the design of a damper, it is not included in this research. The potential of the generation of whistling tones corresponds to the occurrence of a negative acoustic resistance (real part of the impedance). This will be considered.

1.4 Thesis outline

Chapter provides the definitions of the necessary acoustic quantities in order to facilitate the following discussion of the subject in later chapters. This chapter also includes the characterisation of the steady flow regimes in order to give the reader an intuitive idea of the flow pattern for an orifice subjected to a bias-grazing flow. Measurements of the one-sided orifice impedance are carried out for perforations with different shapes. The instrumentation, measurement procedure and the orifice geometries that are used, are described in Chapter. The focus of the experiments is on both the Strouhal number (dimensionless frequency) dependency of the acoustic impedance and the low Strouhal (quasi-steady) behaviour. The quasi-steady behaviour of on orifice subjected to a bias inflow is discussed in Chapter by means of a model for some limit cases and the experimental results at low frequency. In the intermediate Chapter, the vortex sound theory is introduced by means of an energy corollary. The vortex sound theory will be used to explain qualitatively the experimental
results at higher Strouhal numbers. Chapter 6 provides the experimental results for a pure bias inflow and outflow and we explain the effect of the edge geometry using the vortex sound theory. Similarly Chapter 7 deals with the experimental results for a pure grazing flow and the effect of turbulence in the approaching grazing flow boundary layer. An extension of the quasi-steady model for a combination of bias inflow and grazing flow is presented in Chapter 8. As a final topic we discuss in Chapter 9 the acoustic performance of the orifices subjected to a combined bias-grazing flow by means of the experimental results of the acoustic resistance. The conclusions are summarized in Chapter 10.
Chapter 2

General theory

A brief review of the general theory, necessary to deduce the (acoustic) properties and quantities that will be used further in the thesis, is provided. We also introduce the dimensionless numbers that are relevant. The definition of acoustic flow resistance $\tilde{\rho}$ and reactance $\tilde{\delta}$ are given and the use of these quantities for this research subject is motivated. The last section of the chapter deals with the characterisation of the flow through an orifice and the terminology to denote the type of flow is introduced.

2.1 The conservation of mass

Consider a fixed control volume $V$ in a moving fluid. Conservation of mass requires the time rate of change of the mass that is contained in the volume to be equal to the net mass per unit time that enters volume $V$ through the surface $S$ enclosing the volume. This leads to the integral representation of conservation of mass:

$$\frac{d}{dt} \int \int \int_V \rho dV = - \int \int_{S} \rho (\vec{u} \cdot \vec{n}) dS$$ (2.1)

where $\rho$ is the density, $\vec{u}$ is the velocity of the fluid and $\vec{n}$ is the unit vector pointing outwards the control volume enclosed by $S$. By using Gauss theorem and the arbitrariness of $V$, the differential equation for conservation of mass in a fluid follows:

$$\frac{\partial}{\partial t} \rho = -\vec{\nabla} \cdot (\rho \vec{u})$$ (2.2)

2.2 The conservation of momentum

From the second law of Newton applied to material elements within a control volume $V$ in a fluid, it follows that the time rate of change of momentum plus the rate of momentum transport by convection out of the volume, must be equal to the total force on the volume:

$$\frac{d}{dt} \int \int \int_V \rho \vec{u} dV + \int \int_{S} \rho \vec{u} (\vec{u} \cdot \vec{n}) dS = \int \int_{S} \vec{f}_s dS + \int \int \int_V \vec{f}_b dV$$ (2.3)

where $f_s$ represents the surface force per unit area on $S$ and $f_b$ is the density of the force field that acts on the bulk of the fluid within $V$ (body force). For a frictionless flow, $\vec{f}_s$ is
directed normally to the surface and is written as: \( \vec{f}_s = -\bar{n}p \) where \( p \) is the pressure and \( \bar{n} \) the outer unit-vector normal to surfaces. Similarly as for the conservation of mass, we find using Gauss’ theorem and the mass conservation law, the differential form of the inviscid momentum equation (Euler equation):

\[
\rho \left( \frac{\partial \vec{u}}{\partial t} + (\vec{u} \cdot \vec{\nabla}) \vec{u} \right) = -\nabla p + \vec{f}_h. \tag{2.4}
\]

## 2.3 Constitutive equations

Additional information about the fluid is provided by the so-called constitutive equations. In our experiments we use air at room conditions. We will assume an ideal gas behaviour. For an ideal gas, we have the relation:

\[
p = \rho RT \tag{2.5}
\]

with \( R \) the specific gas constant.

Two intrinsic state variables are sufficient to specify the thermodynamic state of a fluid. So we can write \( p = p(\rho, s) \) where \( s \) is the entropy. Since we neglect friction in the gas, we should also neglect heat transfer. Hence we will assume an isentropic flow. It follows that for an isentropic flow holds:

\[
dp = \left( \frac{\partial p}{\partial \rho} \right)_s d\rho \tag{2.6}
\]

The speed of sound is defined as the thermodynamic variable:

\[
c^2 = \left( \frac{\partial p}{\partial \rho} \right)_s. \tag{2.7}
\]

It can be shown that for an ideal gas:

\[
c^2 = \gamma RT \tag{2.8}
\]

where \( T \) is the temperature and \( \gamma = \frac{c_p}{c_v} \) is the ratio of specific heat. For an ideal gas with constant \( c_p \) are \( c_v \) (perfect gas), the isentropic equation of state is:

\[
\frac{p}{p_{\text{ref}}} = \left( \frac{\rho}{\rho_{\text{ref}}} \right)^\gamma. \tag{2.9}
\]

## 2.4 Bernoulli equation

The stationary Bernoulli equation can be derived by integration from the conservation of mass Eq. 2.2 and momentum Eq. 2.4 for an inviscid and incompressible flow (constant density) in the absence of external forces:

\[
\frac{1}{2} U^2 + \frac{p}{\rho} = \text{Constant} \quad \text{along a streamline} \tag{2.10}
\]

with \( U^2 = (\vec{u} \cdot \vec{u}) \).

If the flow is compressible and the gas is a perfect gas, the stationary Bernoulli equation becomes:

\[
\frac{1}{2} U^2 + \frac{\gamma}{\gamma - 1} \frac{p}{\rho} = \text{Constant} \quad \text{along a streamline} \tag{2.11}
\]
2.5 Dimensionless numbers

We introduce a few dimensionless numbers which will be used in this report.

**Mach number** \( M = \frac{u}{c} \) where \( u \) is the characteristic velocity of the flow and \( c \) the local speed of sound. From the compressible Bernoulli equation, it can be shown that a flow is in good approximation incompressible if \( M^2 << 1 \).

**Strouhal number** \( Sr = \frac{fL}{u} \) where \( f \) is the frequency of the sound waves and \( L \) a characteristic length in the flow region. For low Strouhal numbers, the characteristic time (given by \( 1/f \)) that a flow particle needs in order to feel the effect of the acoustic wave propagation is much longer than the time that the particle needs to pass along an obstacle \((L/u)\). Hence for \( fL/u << 1 \), the perturbation of the flow due to the acoustic field can be assumed to be quasi-steady.

**Reynolds number** \( Re = \frac{uL}{\nu} \) where \( \nu \) is the kinematic viscosity. The Reynolds number gives the ratio of non-linear convective inertial forces to viscous forces. A high Reynolds number flow becomes turbulent. When \( Re >> 1 \), the bulk of the flow may be considered as inviscid.

**Helmholtz number** \( He = k_0L \) where \( k_0 = \frac{2\pi f c_0}{c} \) is the wave number. If the characteristic length scale \( L \) is much smaller than the wavelength \( k_0^{-1} \), the propagation of waves can be neglected in the wave equations and the flow region with length \( L \) is called compact.

**Prandl number** \( Pr = \frac{\nu}{\alpha} \) where \( \nu \) is the kinematic viscosity and \( \alpha \) the thermal diffusivity.

So \( Pr \) gives the ratio of momentum diffusivity to thermal diffusivity. A typical value for air at room conditions is \( Pr = 0.713 \) \cite{12}.

Note that all these dimensionless numbers are not independent. The relation \( He = 2\pi Sr M \) shows that a quasi-stationary incompressible flow is always compact.

2.6 Plane waves in a duct

For frequencies below the cut-off frequency \( f_c \) of a duct, the only wave modes which propagate in axial direction are plane waves. For a cylindrical duct, \( f_c = \frac{1.844 c}{2\pi a} \) where \( a \) is the radius of the duct cross-section \cite{13}. For the experiments, a cylindrical impedance tube is used with radius 3.5 cm so that \( f_c \approx 2870 \text{ Hz} \). The maximum frequency at which the measurements are performed is 850 Hz. The wave propagation in the impedance tube is dominated by plane waves.

In the acoustic approximation, we are interested in small perturbations from a steady flow. Considering plane waves, this means:

\[
\begin{align*}
p &= p_0 + p' \\
u &= u_0 + u' \\
\rho &= \rho_0 + \rho'
\end{align*}
\]
We assume a quiescent (uniform stagnant) reference flow $u_0 = 0$, so $u = u'$. We perturb and linearise the continuity equation Eq. 2.2 and the Euler equation Eq. 2.4 in the absence of external forces:

\[
\frac{\partial \rho'}{\partial t} = -\rho_0 \frac{\partial u'}{\partial y} \quad (2.12)
\]

\[
\rho_0 \frac{\partial u'}{\partial t} = -\frac{\partial p'}{\partial y} \quad (2.13)
\]

where $y$ is the direction of the plane wave propagation. From Eq. 2.6 with $dp = p'$ and $d\rho = \rho'$, it follows:

\[
\rho' = \frac{1}{c_0^2} p'. \quad (2.14)
\]

We subtract the time derivative of Eq. 2.12 from the divergence of Eq. 2.13. Substitution of above relation, gives:

\[
\frac{1}{c_0^2} \frac{\partial^2 p'}{\partial t^2} - \frac{\partial^2 p'}{\partial y^2} = 0 \quad (2.15)
\]

This is the one-dimensional homogeneous wave equation. The general solution for this wave equation is the solution of d’Alembert:

\[
p'(y, t) = F(y - c_0 t) + G(y + c_0 t) \quad (2.16)
\]

where $F$ represents a wave travelling in positive $y$–direction and $G$ in negative $y$–direction. For harmonic waves the solution is written in complex notation as:

\[
p'(y, t) = \Re \left\{ p^+ e^{i(\omega t - k^+ y)} + p^- e^{i(\omega t - k^- y)} \right\} \quad (2.17)
\]

where $p^+, p^-$ are the complex amplitudes at time $t = 0$ and position $y = 0$ of respectively a wave travelling in positive and negative $y$–direction. $\omega = 2\pi f$ is the angular frequency and $k_0 = \omega/c_0$ is the wave number.

When the reference flow is not stagnant, one can derive that the acoustic perturbations propagate with the speed of sound $c_0$ with respect to the moving fluid. If the flow is uniform with magnitude $u_0$ and directed along the pipe axis, the effect of the flow on the wave number is: $k_0 = \frac{\omega}{c_0 - u_0}$, where $k_0 = k^+ = k^-$ is the wave number for the wave travelling to the right respectively to the left. For low Mach number flow, the convective effect is negligible. It will be ignored in the experiments.

Substituting of Eq. 2.17 in the linearised momentum equation (Eq. 2.13) and using the pressure/density relation Eq. 2.14, we find the expression for the acoustic velocity perturbations:

\[
u'(y, t) = \frac{1}{\rho_0 c_0} \Re \left\{ e^{i\omega t} (p^+ e^{-ik^+ y} - p^- e^{ik^- y}) \right\} \quad (2.18)
\]

which appears also to be valid for $u_0 \neq 0$

### 2.6.1 Reflection coefficient

We define the pressure reflection coefficient $R$ as the ratio between the amplitude of the waves travelling in negative and positive $y$–direction:

\[
R = \frac{p^- e^{ik^- y}}{p^+ e^{-ik^+ y}} \quad (2.19)
\]

At $y = 0$ this simplifies to $R = \frac{p^-}{p^+}$. 

2.6.2 Orifice impedance

An often used quantity to describe the acoustic behaviour of a wall perforation is the orifice impedance $Z_h$. It measures the acoustic motion due to an applied pressure difference over the cross sectional surface of the orifice. It is made dimensionless by the characteristic impedance of air $Z_0 = \rho_0 c_0$:

$$
\frac{Z_h}{Z_0} = \frac{1}{\rho_0 c_0} \frac{p_{in} - p_{out}}{(u_b' \cdot \hat{n})}
$$

(2.20)

where $u_b'$ is the amplitude of the acoustic velocity perturbation through the orifice. $p_{in}$ and $p_{out}$ are the amplitude of the acoustic pressure perturbation on the outer respectively inner side of the orifice surface (with respect to the chosen coordinate system and \(\hat{n}\) along positive $y-$direction (see Fig. 2.1). Following Golliard [14], we are only interested in the contribution of the flow on the acoustic performance, therefore we subtract the impedance in the absence of flow: $Z_h - Z_{h, no \, flow}$.

The acoustic pressure above the orifice $p_{out}$ equals the radiation pressure. Ingard and Singal [15] showed that the influence of a main flow along a steady source is small for Mach numbers below 0.2. Because of this and because we subtract the no-flow impedance from the impedance, it is sufficient to calculate the one-sided orifice impedance $Z_{h,in}$ in which the radiation pressure is neglected:

$$
Z_{h,in} \equiv \frac{p_{in}}{u_b' \cdot \hat{n}}
$$

(2.21)

Using the plane wave equations for a duct having cross section $S_p$ that is closed with an orifice having cross section $S_o$, we have:

$$
p_{in} = p^+ + p^-
$$

(2.22)

$$
\vec{u}_b' \cdot \hat{n} = \frac{S_p}{S_o} \frac{p^+ - p^-}{\rho_0 c_0}
$$

(2.23)

where $p^+$ and $p^-$ are the amplitude of the waves travelling in respectively positive and negative $y-$direction. Substituting these expressions in Eq. (2.21) we find the one-sided dimensionless orifice impedance:

$$
\frac{Z_{h,in}}{Z_0} = \frac{S_p}{S_o} \frac{p^+ + p^-}{p^+ - p^-}
$$

(2.24)
Further, the orifice impedance can be decomposed in the resistance (real part) and the reactance (imaginary part). For the one-sided flow impedance, this becomes:

\[ r_{flow} = r - r_{u=0} = \Re \{ Z_{h,in} - Z_{h,in,u=0} \} \]  
\[ \delta_{flow} = \delta - \delta_{u=0} = \Im \{ Z_{h,in} - Z_{h,in,u=0} \} \]  

(2.25) \hspace{1cm} (2.26)

Subsequently dividing \( r_{flow} \) by the flow Mach number \( M \) and \( \delta_{flow} \) by \( k_0L \) with \( L \) the width of the orifice, and both dividing by the characteristic impedance, we find the non-dimensional scaled resistance and reactance based on the one-sided orifice impedance:

\[ \tilde{r} = \frac{1}{M \rho_0 c_0} \Re \{ Z_{h,in} - Z_{h,in,u=0} \} \]  
\[ \tilde{\delta} = \frac{1}{k_0L \rho_0 c_0} \Im \{ Z_{h,in} - Z_{h,in,u=0} \} \]  

(2.27) \hspace{1cm} (2.28)

The quantities \( Z_{h,in} \) and \( Z_{h,in,u=0} \) will be measured by means of the impedance tube set-up. How this is performed will be explained in the next chapter (Chapter 3).

### 2.7 Characterisation of the steady-flow regimes

An important issue in the investigation of the aeroacoustic performance of a wall perforation is the characterisation of the steady flow. Tonon [16] summarizes the findings of the steady flow behaviour of Baumeister and Rice [17] and Rogers and Hersh [8] for a perforation subjected to a bias and grazing flow. The summary is presented in this chapter section.

Baumeister and Rice [17] as well as Rogers and Hersh [8] used for their visualisation dye injection in a water flow system. A “resonator cavity” was attached to a duct (water channel) with grazing by a squared orifice hole. A piston supplied oscillation flow in the resonator cavity. See Fig. 2.22 for the notation. The oscillating flow through the squared orifice corresponds to a bias flow. This cavity-duct system explains their definition of inflow and outflow that is widely adapted in literature: inflow corresponds with the flow during one half of the piston cycle that is directed from the main flow channel into the cavity. Outflow refers to flow outwards the cavity, into the main channel. This definition of inflow and outflow is used in what follows. In the chosen coordinates frame, inflow has a negative bias velocity component and outflow has a positive bias component.

Consider a sharp edged single wall perforation drilled orthogonally to a plate. The orifice width is of the same order as the thickness of the plate. On one side of the plate, referred as duct, the fluid is either at rest or is flowing along a direction normal to the perforation axis. There are eight possible flow regimes which are presented schematically in Fig. 2.3 the pure grazing flow (a), low inflow (b), high inflow (c), pure bias inflow (d), low outflow (e), intermediate outflow (f), high outflow (g) and pure bias outflow (h).
Figure 2.2: Schematic representation of the water-flow experiment and the notation for bias inflow and outflow according to the coordinate frame. The words between brackets link to the experimental set-up that is used in this report. The water channel corresponds with the room with grazing flow supplied by the open-jet wind tunnel. The cavity corresponds with the impedance tube.

**Pure grazing flow (Fig. 2.3(a))** For this regime there is no bias flow $u_b = 0 \text{ m/s}$ through the perforation. The grazing flow $u_g$ induces a recirculating flow within the perforation driven by the shear exerted by the fluid flowing along the opening. This recirculation region fills the perforation. For high grazing flow, this secondary flow can extend down into the cavity.

**Low inflow (Fig. 2.3(b))** The bias flow is directed from the duct into the cavity and $u_b < 0$. In this regime, the ratio between the bias and the grazing flow velocities $|u_b|/u_g$ is low. The bias flow induces a jet flow into the cavity. A shear layer originates due to flow separation at the upstream edge with respect to the grazing flow. It extends through almost the full width of the perforation. The second shear layer bounding the jet flow is tangentially from the downstream side wall with respect to the grazing flow of the perforation. Separation occurs at the edge on the cavity side of the perforation. As the bias flow increases the cross sectional area of the separated region decreases and the cross sectional area of the jet formed by the flow through the perforation increases. The jet flow contracts further because of the vena contracta effect ([13], p. 317). The flow in the jet region is directed along the direction of the downstream wall of the perforation. An important feature of the inflow case is the position of the stagnation point (zero velocity) of the grazing flow. For the low inflow this stagnation point regime is on the downstream side wall of the perforation (within the perforation). Increasing the ratio...
|u_b|/u_g it moves up, toward the duct (u_b < 0).

High inflow (Fig. 2.3(c)) The bias flow is from the duct to the cavity (u_b < 0). This regime is characterized by high values of the ratio |u_b|/u_g. Separation occurs tangentially at both edges of the perforation on the side of the duct. The direction of the flow at the vena contracta of the jet depends on the ratio of the bias and grazing flow |u_b|/u_g. The jet moves toward the centreline of the perforation as |u_b|/u_g increases.

Pure bias inflow (Fig. 2.3(d)) This regime occurs when there is no grazing flow along the perforation u_g = 0. Only the bias flow, directed from the duct into the cavity, is present. This regime represents the interesting limit case when the effect of the grazing flow is negligible compared to the effect of the bias flow |u_b|/u_g >> 1. Separation occurs symmetrically at both edges of the perforation at the duct side. The direction of the flow at the vena contracta of the jet is along the centreline of the perforation.

Low outflow (Fig. 2.3(e)) The bias flow is directed from the cavity to the duct so u_b > 0. In this regime, the ratio between the bias and the grazing flow velocities u_b/u_g is low. The bias flow emerging from the cavity to the duct through the perforation is deflected in the duct in a direction parallel to the wall and the grazing flow by pressure and shear forces exerted by the grazing flow. There is a tangential separation of the flow at the upstream edge of the perforation on the duct side. A small jet is formed downstream of the orifice in the cavity.

Intermediate outflow (Fig. 2.3(f)) The bias flow is directed from the cavity to the duct (u_b > 0). As the ratio u_b/u_g increases, the bias flow leaving the perforation penetrates further into the grazing flow before bending towards the downstream wall in grazing flow direction. There is separation of the bias flow at the inner edges of the orifice. After some distance the flow reattaches to the wall of the duct downstream of the orifice. The flow is essentially two-dimensional [19].

High outflow (Fig. 2.3(g)) The bias flow is directed from the cavity to the duct (u_b > 0). For high values of the ratio between the bias and the grazing flow velocities u_b/u_g the penetration of the bias flow emerging the perforation is large. The interaction of this jet with the main grazing flow is essential three-dimensional [19].

Pure bias outflow (Fig. 2.3(h)) The bias flow is from the cavity toward the duct (u_b > 0). It is the same as pure bias inflow for a symmetric orifice.
Chapter 2. General theory

Figure 2.3: Schematic representation of the different regimes of the steady flow through a single wall perforation.
2.7 Characterisation of the steady-flow regimes
Chapter 3

Experimental set-up

In order to retrieve a quantitative description of the effect of bias, grazing or grazing-bias flow on the acoustic properties of an orifice, impedance measurements are performed. The experimental set-up is based on the set-up that has been used by Kooijman [20], [21]. In this chapter, a description of the instrumentation, the measurement procedure and the post-processing of the data, is provided.

3.1 Set-up and instrumentation

The measurements are performed in a semi-anechoic room. An open jet wind tunnel has its exit with a cross section of $20 \times 20 \text{cm}^2$ in the room with a cut-off frequency of 300 Hz. A 70 cm long smooth cylindrical tube with an inner radius of $R = 35 \text{mm}$ and a wall thickness of 15 mm forms the basis of the impedance tube. An overview of the set-up is given in Fig. 3.1. A loudspeaker is connected with a latex collar to one end of the impedance tube. The collar avoids vibrational coupling between the loudspeaker and the impedance tube. The back end of the tube, between the loudspeaker and the first microphone, is filled with porous acoustical damping material to avoid high acoustic amplitudes due to resonances. The orifice test plates are mounted on the front end of the tube. The test plates are fixed to the nozzle of the silent open jet wind tunnel, which provides the so called grazing flow along the orifice. To reduce leaks, o-rings are placed between the microphones and the tube, as well as between the test plates and the tube. Fig. 3.2 is a picture of the set-up with a view on the wind-tunnel exit and the mounting of the test plate. Seven piezoelectric dynamic pressure sensors (PCB series 116A) [22] are flush-mounted in the wall of the impedance tube. The distances between each microphone is chosen so that they are not simply related, by not being a multiple of a basic distance. Each microphone is connected to a charge amplifier (Kistler 5011). An harmonic signal from the signal generator (NI PXI-5411) is send via the amplifier (Toellner TOE 7608) to the loudspeaker. The signals of the microphone’s charge amplifiers as well as the signal from the function generator are digitally sampled by an 8 channel dynamic signal acquisition (DSA) card (NI PXI-4472) at a sample rate of 10490 Sa/s). Both the DSA card and the signal generator module are driven by an embedded controller (NI PXI-8176). The three units are housed in the 8 slot chassis of the computer system.
Figure 3.1: Schematic layout of the impedance tube setup.
Figure 3.2: Picture of a test plate mounted at the front end of the impedance tube and the wind tunnel exit. The 15 mm thick aluminium plate has a rectangular slit shaped orifice.

A silent organ pump (*Ventola GmbH & Co type 3/80*) is connected to the impedance tube between the loudspeaker and the damping material, to provide the flow through the orifice. This is the bias flow. For measurements at high bias flow, the organ pump is replaced by a vacuum cleaner (*AEG Electrolux*). A turbine gas meter (*Dresser IMTM-CT G65 DIN PN16 DN50*) is mounted between the pump and the impedance tube to measure the bias flow rate. It is connected to the computer system via a switching amplifier *Turk MK-15*, a 24 V − to − 5 V converter, a shielded I/O connector block (*NI SCB-68*) and a timing and digital I/O module (*NI PXI 6602*). The static pressure in the impedance tube is measured by means of a pressure hole in the wall of the impedance tube, positioned 2 cm from the front end of the impedance tube and a Betz manometer (*Nonius Delft*) with an accuracy of 2 Pa. On a similar manner the static pressure in the settling chamber of the wind tunnel is measured with an accuracy of 1 Pa.

The temperature of the air is measured with an uncertainty of 0.1° C by means of a digital thermometer (*Omega HH309A*). The sensor is positioned in the room close to the impedance tube.

### 3.2 Microphones

We take the origin $y = 0$ of our coordinate axis along the tube, at the front end of the impedance tube. This front end is at the same position as the inner of the test plate. The negative direction of the $y$− axis points towards the loudspeaker. The position of the micro-
phones based on the chosen coordinate system and the microphone sensitivities are listed in Table 3.1.

The amplitude of the acoustic pressure disturbance is supplied by the charge amplifier as a voltage in V, which is converted to a pressure in Pa by:

\[
\text{amplitude[Pa]} = \text{amplitude[V]} \cdot \frac{T[pC/psi] \cdot S[psi/V]}{g[pC/psi]} \cdot 6.894 \cdot 10^3 [Pa/psi] \tag{3.1}
\]

where \(T\) and \(S\) are respectively the transducer sensitivity and scale of the amplifiers. \(g\) is the sensitivity of the microphone as provided by the manufacturer. 1 psi corresponds to \(10^5 \text{Pa}/14.50377 = 6.894 \cdot 10^3 \text{Pa/psi}\). 

<table>
<thead>
<tr>
<th>microphone number</th>
<th>microphone code</th>
<th>position y [mm]</th>
<th>microphone sensitivity [pC/psi]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1511</td>
<td>-20</td>
<td>9.48</td>
</tr>
<tr>
<td>2</td>
<td>990</td>
<td>-70</td>
<td>10.26</td>
</tr>
<tr>
<td>3</td>
<td>991</td>
<td>-170</td>
<td>9.95</td>
</tr>
<tr>
<td>4</td>
<td>1512</td>
<td>-310</td>
<td>10.38</td>
</tr>
<tr>
<td>5</td>
<td>1871</td>
<td>-365</td>
<td>10.30</td>
</tr>
<tr>
<td>6</td>
<td>1987</td>
<td>-410</td>
<td>9.54</td>
</tr>
<tr>
<td>7</td>
<td>1988</td>
<td>-485</td>
<td>9.66</td>
</tr>
</tbody>
</table>

3.3 Measuring the flow velocity

3.3.1 Grazing velocity

The wind tunnel provides a uniform flow that is bounded by a thin boundary layer along the orifice test plate. It is called the grazing flow with velocity \(u_g\). We choose the \(x\)-axis to have the same direction as the grazing flow and so \(u_g\) is positive. The flow outside the viscous boundary layer is assumed to be adiabatic, irrotational, incompressible and frictionless. The static pressure in the main flow equals the atmospheric pressure \(p_{atm}\). We apply Bernoulli’s equation along a streamline from the settling chamber of the wind tunnel (with pressure \(p_{wt}\)) to a point in the grazing flow at atmospheric pressure \(p_{atm}\). This allows us to estimate the grazing velocity \(u_g\) from the pressure difference measurement:

\[
u_g = \sqrt{\frac{2(p_{wt} - p_{atm})}{\rho}} \tag{3.2}
\]

with \(\rho = 1.2 \text{kg/m}^3\) (standard atmospheric density).

Due to the grazing flow along the orifice and the quasi-stagnant flow in the wall perforation, a shear layer is formed at the opening as it was described at the flow characterisation in Section 2.7. The boundary layer profile just upstream the orifice has an important effect on the behaviour of the shear layer. The measurements are performed with either a laminar, a transitional (from laminar to turbulent) or a turbulent boundary layer profile. The turbulent
boundary layer is established by tripping the flow just before the wind tunnel exit by means of a strip of sandpaper (3 cm width in flow direction and roughness no. 24). Details concerning the boundary layer velocity profile and a study of its effect on the shear layer is provided for pure grazing flow by Kooijman [25]. The distance from the upstream edge of the orifice to the outlet of the wind tunnel, which is an important parameter determining for the boundary layer thickness, is listed in Table 3.2.

3.3.2 Bias velocity and jet velocity

The silent organ pump provides the bias flow. By reversing the direction in which it is mounted, it blows (outflow) or sucks (inflow) air through the impedance tube. We define the direction of the flow that goes outwards the impedance tube toward the grazing flow as “outflow”. According to our chosen y-axis, the outflow has positive velocity in y-direction. Bias flow in the negative y-direction, flowing into the impedance tube, is referred to as “inflow”. The definition of inflow and outflow were introduced in Section 2.7 by means of a sketch Fig. 2.2. The maximum volume flow rate provided by the organ pump is around 7.7 dm³/s. For higher inflow, the vacuum cleaner is used. The jet flow (only for the inflow configuration) is estimated as follows: the jet velocity is taken much lower than the speed of sound so that the flow is assumed to be incompressible with density \( \rho \). Further we assume the flow to be adiabatic and frictionless, except for the turbulent mixing region downstream the jet. We assume the static jet pressure \( p_j \) equal to the static pressure \( p_p \) at the position of the pressure hole in the impedance tube (pipe). Because the pressure hole is close to the orifice, the turbulent mixing region is downstream and we apply Bernoulli:

\[
\begin{align*}
  u_j &= \sqrt{\frac{2(p_{atm} - p_p)}{\rho}} + u_y^2, \quad \text{(3.3)} \\
  &= \sqrt{2(p_{atm} - p_p)} + u_y^2, \quad \text{(3.4)}
\end{align*}
\]

The mean velocity of the bias flow through the orifice \( u_b \) is deduced from the volume flow rate \( \dot{Q} \) measured by means of the turbine meter:

\[
  u_b = \frac{\dot{Q}}{S_o} \quad \text{(3.5)}
\]

where \( S_o \) is minimum cross section of the orifice. The turbine meter is not certified for flow lower than 10 m³/h. We calibrated it for this low flow region against a Rota-meter with a range at its maximum scale of 10.8 m³/h. The calibration curve is given in Appendix A, Fig. A.1.

3.4 Orifice geometries

The influence of the geometry of a wall perforation (orifice) on the aeroacoustic behaviour is investigated by mounting test plates with different orifice geometries. A cross section in the \((x, y)\)-plane of the orifices is shown in Fig. 3.3. In the graph, grazing flow is directed from left to right, bias inflow is from top to bottom, bias outflow from bottom to top. The bottom part of the orifice is mounted to the impedance tube. The minimal cross sectional area \( S_o \)
of all orifices is the same. All slits are rectangular but have different edges or directions with respect to the bias and grazing flow. The slits are made by spark erosion or by milling. Special attention is paid to the smoothness of the surface and the sharpness of the edges (not de-flash). The test plates can be mounted in different directions with respect to the flow. In that case, we give the orifice a different name (letter). An overview of the orifice’s name, their description referring to the edges and their dimensions are given in Fig. 3.3 and Table 3.2. The slits have equal height \( h = 50 \text{ mm} \) (not shown in the graph). The thickness of the plates is called \( t \). The orifice length \( d \) refers to the distance that the bias flow follows along one straight edge. The centre of the plates is 100 mm from the exit nozzle of the silent jet wind tunnel. \( L \) is the distance from the exit of the silent wind tunnel to the upstream edge of each wall perforation depends on the geometry of the slit. \( w \) is the width of the orifice in the direction of the grazing flow.

### Table 3.2: Orifice dimensions for the different rectangular orifices as presented in Fig. 3.3

<table>
<thead>
<tr>
<th>Orifice</th>
<th>Edge Description</th>
<th>Width</th>
<th>Plate Thickness</th>
<th>Orifice Length</th>
<th>Min. Width</th>
<th>Dist. to Upstream Edge</th>
<th>Min. Cross Section</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>sharp edges</td>
<td>10</td>
<td>15</td>
<td>-</td>
<td>10</td>
<td>95</td>
<td>500</td>
</tr>
<tr>
<td>B</td>
<td>orthogonal edges</td>
<td>10</td>
<td>15</td>
<td>10</td>
<td>95</td>
<td>500</td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>upstream sharp edge</td>
<td>10</td>
<td>15</td>
<td>10</td>
<td>100</td>
<td>500</td>
<td></td>
</tr>
<tr>
<td>D</td>
<td>downstream sharp edge</td>
<td>10</td>
<td>15</td>
<td>15</td>
<td>10</td>
<td>90</td>
<td>500</td>
</tr>
<tr>
<td>E</td>
<td>oblique in grazing flow direction</td>
<td>20</td>
<td>15</td>
<td>30</td>
<td>10</td>
<td>7.6</td>
<td>500</td>
</tr>
<tr>
<td>F</td>
<td>oblique opposite to grazing flow direction</td>
<td>20</td>
<td>15</td>
<td>30</td>
<td>10</td>
<td>10.4</td>
<td>500</td>
</tr>
<tr>
<td>G</td>
<td>chamfered edges</td>
<td>16</td>
<td>15</td>
<td>12</td>
<td>10</td>
<td>9.2</td>
<td>500</td>
</tr>
</tbody>
</table>

### 3.5 Impedance measurements

To measure the impedance over a certain frequency range for different flow conditions, a single impedance measurement is done as follows: The test plate is mounted at the front end of the impedance tube. The bias and grazing flow are set and are measured using the Bets manometers. The amplitude of the sinusoidal signal that is sent to the loudspeaker, is set. Then there are two ways of increasing the frequency.

1. Manually: the time signal of the 7 microphones is recorded for 120 seconds for each
frequency. The temperature is measured each time the frequency is increased. In most
cases, the lowest frequency is taken at 31 Hz and the frequency is increased in steps of
31 Hz up to 869 Hz. This long measuring time is used for some measurements as will be
discussed later.

2. Automatic sweep: the time signal of the microphones is recorded for only 3 seconds and
an interruption of recording of 1 second between each step in frequency ensures that
the loudspeaker has reached steady oscillation. The temperature is measured once at
the begin of the sweep. Mostly the same frequency sequence is used as for the manual
measurements (steps of 31 Hz starting at 31 Hz up to 869 Hz). For the lowest frequency,
recording the pressure signal during 3 seconds implies a measurement of at least 91 full
oscillation periods.

Then, the time signal for each frequency of the microphones and the reference signal from
the frequency generator are post-processed using an implementation of a lock-in amplifier
and a multi-microphone method. In the last stage, the dimensionless one-sided scaled orifice
resistance and reactance are calculated as a function of the frequency for the chosen flow and
orifice.

3.5.1 Lock-in method

The lock-in amplification method is implemented in Matlab to evaluate the amplitude and
phase with respect to the reference signal generated by the function generator. It is performed
on an integer number of oscillation periods. If \( f_s = 10490 \text{ Hz} \) is the sample rate and \( N \) the
number of samples in the time signal, then \( t_k \) with \( k \) from 1 to \( N \) refers to the discretisation
of the time. For each microphone, we write the time signal of the pressure as \( p_{mic,i}(t_k) \).
Similar we write for the sinusoidal reference signal \( p_{ref}(t_k) \). By using the detection of the
zero crossing of the sinusoidal reference signal, we obtain a signal consisting of an integer
number of periods. A Hilbert transform \( [26] \) is used to generate a complementary reference
signal. Now the Fourier coefficients are calculated as:

\[
a_{1i} = \frac{1}{2N} \sum_{k=1}^{N} p_{mic,i}(t_k) p_{ref}(t_k)
\]

\[
b_{1i} = -\frac{1}{2N} \sum_{k=1}^{N} p_{mic,i}(t_k) \cdot \text{Im}\{\text{hilbert}(p_{ref}(t_k))\}
\]

The amplitude and phase of the microphone signal and the phase become:

\[
\hat{p}_i = \sqrt{a_{1i}^2 + b_{1i}^2}
\]

\[
\phi_i = \arctan \frac{b_{1i}}{a_{1i}}
\]

It is found that an offset in the reference signal or in the microphone signal leads to a decreased
performance of the lock-in method. The error due to the offset is independent of the measuring
time. In order to reduce this error, the mean value of the time signal is subtracted for the
reference signal as well as the microphone signal before applying the lock-in method. After
doing this, the points corresponding with the zero-crossing of the reference signal are searched.
The effectiveness of this remedial measure is confirmed in Appendix A, Fig. A.2. Please note
that its effectiveness can not be understood intuitively. However it deserves further research.
3.5.2 Multi-microphone method

The maximum frequency used in our experiments is much lower than the cut-off frequency (Section 2.6) so that only plane waves propagate. Hence Eq. 2.17 applies for the microphone signals:

\[ p'(y_i) = \text{Re} \left\{ p^+ e^{-iky_i} + p^- e^{iky_i} \right\} \]  

(3.10)

where \( y_i \) is the position of the \( i \)th microphone as listed in Table 3.1. \( p'(y_i) \) is known from the lock-in method:

\[ p'(y_i) = \hat{p}_i e^{i\phi_i} \]  

(3.11)

Accounting for visco-thermal damping of the acoustic wave at the walls of the tube, the complex wave number is given, in the low frequency approximation \( k_0 R_{\text{tube}} << 1 \) and for high shear numbers \( Sh = \sqrt{R_{\text{tube}} \omega/\nu} >> 1 \), by [20], neglecting convective effects:

\[ k^\pm = \frac{\omega}{c_0} \left[ 1 + \frac{1 - \frac{1}{2} \gamma}{Sh} \left( 1 + \frac{\gamma - 1}{\sqrt{Pr}} \right) \right] \]  

(3.12)

where \( \nu = 1.51 \times 10^{-5} \text{m}^2/\text{s}, \gamma = 1.4, \) and \( Pr = 0.71 \) for air at 20°C [12]. The effect on the damping due to visco-thermal losses in the bulk of the flow appears to be two order of magnitudes smaller than damping due to wall losses. Therefore, bulk losses are neglected in the wave propagation [20], [13].

Eq. 3.10 holds for every microphone. The set of equations is written as:

\[
\begin{bmatrix}
  p'(y_1) \\
  \vdots \\
  p'(y_7)
\end{bmatrix}
= 
\begin{bmatrix}
  e^{-iky_1} & e^{iky_1} \\
  \vdots & \vdots \\
  e^{-iky_7} & e^{iky_7}
\end{bmatrix}
\begin{bmatrix}
  p^+ \\
  p^-
\end{bmatrix}
\]  

(3.13)

In order to determine \( p^+ \) and \( p^- \), the overdetermined system of equations Eq. 3.14 is solved by means of the least square method:

\[
\begin{bmatrix}
  p^+ \\
  p^-
\end{bmatrix}
= 
(M_{\text{exp}} M_{\text{exp}}^T)^{-1} M_{\text{exp}}^T p'_{\text{exp}}
\]  

(3.15)

where the superscript \( T \) indicates the complex conjugate transpose. Now the one-sided orifice impedance or reflection coefficient at the orifice can be calculated using Eq. 2.21 respectively Eq. 2.19 which we repeat here:

\[
\frac{Z_{in}}{Z_0} = \frac{S_o}{S_p} \frac{p^+ + p^-}{p^+ - p^-}
\]  

(3.16)

\[
R = \frac{p^-}{p^+}
\]  

(3.17)

where \( S_o \) is the minimum cross section of the perforation and \( S_p \) the cross section of the impedance tube.
3.6 Accuracy of the acoustical measurements

The accuracy of the set-up is estimated on basis of a reflection coefficient measurement at a closed end wall (a 20 mm thick aluminium plate). The theoretical reflection coefficient is 1 if we neglect heat transfer. In Fig. 3.4 we present the difference between the theoretical and measured reflection coefficient in terms of the absolute value of $R$ and the phase $\phi$ scaled to $2\pi$ for a measurement time of 120 seconds for each frequency. The deviation of the experimental reflection coefficient (the amplitude as well as the phase) from the theoretical value is $O(10^{-2})$ for frequencies up to 869 Hz.

All the impedance measurements have been carried out at low sound pressure levels so that the impedance of each orifice is linear independent of the amplitude of the acoustic velocity through the orifice. A detailed characterisation of the linearity of the acoustic impedance in the set-up has been reported for grazing flow by Kooijman [20]. The linearity of our measurements has been verified by performing the same experiments at higher or lower amplitudes. The influence of the amplitude appears to be negligible, for the amplitude that we consider (see Appendix A, Fig. A.3).

The influence of the measuring time on the accuracy is checked. We concluded that a measurement time of 3 s is sufficient when the signal to noise ratio for the microphone signals is not too small. For example, when the vacuum cleaner is used, the noise level is higher and a longer measuring time is required.

Note furthermore that the value of the speed of sound is not critical to calculate the orifice impedance by means of the lock-in method and the multi-microphone method, as can be seen from Eq. 3.10 and the dimensionless one-sided orifice Eq. 2.21. Nevertheless, an error in the speed of sound will make a difference for the dimensional impedance and the scaling of the dimensionless one-sided orifice impedance with the Mach number. We assume for the calculations the air to be dry, neglecting the effect of moisture. However a relative moisture level of 50% is expected. According to Cramer [27], [28] and approximating the gas by an ideal gas, the speed of sound is calculated for the dry as well as for humidified air. We use the values of the constituent of standard dry air at $p_{atm} = 101325 \text{ Pa}, t = 20^\circ \text{C}$ and assume the composition of the air is constant except for the water vapour [29]. The molar mass of dry air becomes $M_{dry} = 0.0290 \text{ kg/mol}$ and the molar mass of air at 50% relative humidity is $M_{RH=50} = 0.0288 \text{ kg/mol}$. The relative change in speed of sound for 50% humidified air and dry air becomes 0.21%, which leads to an equal error in the calculation of the characteristic impedance of air $Z_0 = \rho_0 c_0$. We neglect the effect of humidity in the measurements.
Figure 3.3: Geometries of the different rectangular wall perforations (orifices) used in the experiments.
Figure 3.4: Difference between the theoretical and the measured (impedance tube measurement) reflection coefficient of an end wall, in terms of absolute value $|R|$ (left graph) and phase $\phi$ scaled to $2\pi$ radians (right graph)
3.6 Accuracy of the acoustical measurements
Chapter 4

Quasi-steady model for inflow through an orifice in a pipe

In this chapter we introduce a quasi-steady model to predict the aeroacoustic behaviour of an orifice placed at the end of a pipe and flow entering the pipe (inflow). This corresponds to a pure bias inflow in our set-up and the pure bias inflow regime as described in Section 2.7. Bechert [3] presented a simple model to describe the reflection of plane acoustic waves travelling at a pipe termination with an open orifice plate. His model is valid for low frequencies and assumes that the flow is incompressible. The model of Bechert deals with a pipe outflow. However, the basic idea is the same for the pipe inflow which is the subject of this chapter.

4.1 Incompressible inflow model

We assume a pipe with cross section $S_p$ that is ended on one side by a plate in which an orifice is present. Air is uniformly sucked at the other end of the pipe so that flow enters the pipe from the free space through the orifice with cross section $S_o$. Inside the pipe, a free jet is formed downstream the orifice. An illustration of the configuration is given in Fig. 4.1. When sucking air the flow outside the pipe at the orifice side follows the plate walls until it reaches the orifice (we assume a potential flow). Then, flow separates at the sharp edges. Assuming flow separation implicitly takes viscosity into account because separation is due to viscous effects. In order to pass the orifice, the fluid flowing from outside the pipe has to bend towards the direction of the axis of the pipe. This bending results in an additional contraction of the flow. This is called the vena contracta effect. The jet area cross section $S_j$ is defined at the position where the tangent lines to the borders of the jet are parallel to each other. At this point the jet cross section reaches its minimum. The fluid that surrounds the jet is stagnant. Downstream the vena contracta the jet flow is assumed to mix with the surrounding air. After the mixing region we have a pipe flow which we assume for simplicity uniform. Due to turbulence, viscous bulk dissipation occurs in the mixing region. We estimate this dissipation from integral conservation laws across the mixing region. Again we do not take viscous effects explicitly into account.

We assume the flow to be incompressible with uniform density $\rho$. In Fig. 4.1 $\rho_0 = \rho_p = \rho$. The pressure downstream the turbulent mixing region is called $p_p$. At the position of highest
Figure 4.1: Schematic representation of a steady flow through a wall perforation in the pure bias inflow regime. Curve C1 represents the region in which the stagnant flow far from the orifice is accelerated isentropically into the vena contracta of the jet. The vena contracta of the jet referees to the part of the free jet flow downstream the orifice and before the flow mixes with the surrounding pipe flow. Curve C2 represents the turbulent mixing region of the jet flow and the transition in a pipe flow. We assume that the pipe flow has uniform velocity $u_p$.

contraction with jet cross section $S_j$, the pressure in the free jet $p_j$ is uniform and equals the pressure of its surrounding. The velocity in the jet flow is $u_j$.

The vena contracta factor is defined as $\Gamma = \frac{S_j}{S_o}$. It is an important parameter for the aeroacoustic response, because it gives the ratio of the velocity in the jet and the mean velocity through the orifice in a plate and so it is a measure of the steady flow resistance. For a rectangular orifice in a thin plate, the theoretical vena contracta factor is calculated by Kirchoff for an incompressible flow: $\Gamma = \frac{S_j}{S_o} = \frac{\pi}{\pi + 2} \approx 0.61$ [18]. Experimentally, this high contraction is hardly reached, mainly because edges are not infinitely sharp. In the model we assume the vena contracta factor $\Gamma$ to be constant. We determine its value experimentally. However, in real flow, $\Gamma$ also depends on the Mach number [25]. This Mach number dependency is significant for $M_j = u_b/c_0 > 0.4$ or $M_b > 0.3$.

As will be discussed in the Chapter [4] the acoustic flow is by definition a potential flow. Also the steady flow is a potential flow (frictionless and isentropic), it is justified to use the
stationary incompressible Bernoulli equation along a streamline from the stagnant free space to a point in the jet upstream the turbulent mixing region (region C1):

\[ p_0 = p_j + \frac{1}{2} \rho u_j^2 \quad (4.1) \]

where \( p_0 \) is the stagnation pressure outside the pipe, far from the orifice and \( p_j \) equals the pressure in the region surrounding the vena contracta jet flow.

In the turbulent mixing region (region C2), the jet structure decays and kinetic energy is dissipated. The flow is still assumed to be adiabatic, but not isentropic. The continuity equation and the integral conservation of momentum in axial pipe direction, are applied to the mixing region:

\[ S_p u_p = S_j u_j \quad (4.2) \]
\[ p_j S_p + \rho u_j^2 S_j = p_p S_p + \rho u_p^2 S_p. \quad (4.3) \]

We are interested in the small perturbations of the uniform reference flow which propagate with the speed of sound \( c_0 \). Therefore we perturb the equations for the pressure fluctuations and associated velocity fluctuations. We assume that the free space pressure is constant so that \( p_0 = 0 \) and find:

\[ p_j = (p_j + p_j') + \frac{1}{2} \rho (u_j + u_j')^2 \]
\[ S_p (u_p + u_p') = S_j (u_j + u_j') \]
\[ (p_j + p_j') S_p + \rho (u_j + u_j')^2 S_j = (p_p + p_p') S_p + \rho (u_p + u_p')^2 S_p \]

We linearise the perturbed equations:

\[ p_j' = -\rho_j u_j' \quad (4.4) \]
\[ u_j' = u_p' \frac{S_p}{S_j} \quad (4.5) \]
\[ p_j' S_p + 2 \rho u_j' u_j S_j = p_p' S_p + 2 \rho u_p' u_p S_p \quad (4.6) \]

Substitution of Eq. 4.5 and Eq. 4.6 in Eq. 4.6 and dividing by \( \rho \), \( S_p \) and \( u_p' \):

\[ -u_j' \frac{S_p}{S_j} + 2u_j = \frac{1}{\rho} \frac{p_p'}{u_p'} + 2u_p \quad (4.7) \]

It is convenient to define the orifice impedance for a surface pointing outwards the pipe, from the orifice to the free space: \( Z = \frac{p_p'}{u_p'} \). The one-sided orifice impedance was introduced in Chapter 2 to study the influence of flow on the resistance of an orifice. With above notations and coordinate frame, the one-sided orifice impedance is: \( Z_{h,in} = \frac{p_p'}{u_p'} = \frac{p_p'}{u_p' S_p / S_o} \) where \( u_p' \) is the amplitude of the acoustic velocity through the orifice. Note that the value of velocities \( u_j, u_b \) and \( u_p \) are negative for inflow.

Multiply Eq. 4.7 by \( \frac{1}{c_0} \frac{S_p}{S_o} \) and using the definition of the vena contracta factor \( \Gamma = \frac{S_j}{S_p} \), we obtain an expression for the orifice impedance as a function of the jet Mach number \( M_j \):

\[ \frac{Z_{h,in}}{Z_0} = -\frac{M_j}{\Gamma} + 2M_j \frac{S_j}{S_p \Gamma} - 2 \left( \frac{S_j}{S_p} \right) \frac{1}{\Gamma} M_j \quad (4.8) \]
where $Z_0 = \rho_0 c_0$ is the characteristic impedance of the air. As a function of the Mach number for flow through the orifice $M_b = \Gamma M_j$, holds:

$$\frac{Z_{h,\text{in}}}{Z_0} = -\frac{M_b}{\Gamma^2} + 2M_b \frac{S_o}{S_p \Gamma} - 2 \frac{S_o^2}{S_p^2} M_b$$  \hspace{1cm} (4.9)

In contrast to $M_j$, $M_b$ is directly determined by measuring the bias volume flow. We therefore prefer Eq. 4.9 to Eq. 4.8. Rewriting above equation to an expression for the reflection coefficient, one obtains the result for the absolute value of the reflection coefficient:

$$R = \left(\frac{S_p}{S_o}\right) Z_{h,\text{in}} - Z_0 \left(\frac{S_p}{S_o}\right) Z_{h,\text{in}} + Z_0$$  \hspace{1cm} (4.10)

It is clear that the vena contracta factor $\Gamma$ plays an important role for the acoustic reflection. In Fig. 4.2 the reflection coefficient is plotted for different values of the vena contracta factor. The ratio of pipe cross section to orifice cross section is taken as $\frac{S_p}{S_o} = \frac{\pi 0.035^2}{0.01 0.05} = 7.70$ for this graph. It is found that there is a critical orifice Mach number for which the reflection coefficient vanishes and that this depends on $\Gamma$ as well as the ratio of pipe to orifice cross section $\frac{S_p}{S_o}$.

**Figure 4.2:** Absolute value of the reflection coefficient as a function of the orifice Mach number $M_b$ for different values of the vena contracta factor $\Gamma$ and $\frac{S_p}{S_o} = 7.7$. $u_b$ is the area averaged velocity of the flow through the orifice.
4.2 Compressible inflow model

We modify the model to take into account the compressibility of the flow. This is a better approximation for high Mach number flow. By using the compressible Bernoulli equation we take into account density variations in the (acoustic) flow. The equation of state for a perfect gas (ideal gas with constant ratio of specific heat $\gamma$) is used. The steady-flow equations are:

$$\rho_j u_j S_j = \rho_p u_p S_p$$

(4.11)

$$\rho_j u_j^2 S_j + p_j S_p = \rho_p u_p^2 S_p + p_p S_p$$

(4.12)

$$p_j + \frac{1}{2} \rho_j u_j^2 \left( \frac{\gamma - 1}{\gamma} \right) = \frac{p_0}{\rho_0} \rho_j$$

(4.13)

$$p_p + \frac{1}{2} \rho_p u_p^2 \left( \frac{\gamma - 1}{\gamma} \right) = \frac{p_0}{\rho_0} \rho_p$$

(4.14)

$$\left( \frac{\rho_j}{\rho_0} \right) ^\gamma = \frac{p_j}{p_0}$$

(4.15)

The unknowns are: $\rho_j, u_j, u_p, p_j, p_p$ where $\rho_j, \rho_0$ and $\rho_p$ are respectively the density in the jet flow, the stagnant room and in the pipe downstream the turbulent mixing region. The steady flow equations are perturbed and solved with the Newton-Rapson method because of its fast and reliable convergence. The incompressible solution is used as an initial value for the iterations.

In order to calculate the acoustic response to small acoustic perturbations of the steady reference flow, we perturb the equations numerically. A perturbation $\Delta p_p$ of the pipe pressure is added and the equations are solved for $p_p + \Delta p_p$. The unperturbed solution is subtracted from the perturbed solution, resulting in the perturbation of the velocity in the pipe: $\Delta u_b = u_b(p + \Delta p_p) - u_b(p)$. The ambient room pressure is $p_0 = 1.013 \cdot 10^5$ Pa, $p_p$ and the density $\rho_0$ are known. The dimensionless orifice impedance $Z_{h,in}/Z_0$ is found:

$$\frac{Z_{h,in}}{Z_0} = \frac{1}{\rho_0 c_0} \frac{S_o}{S_p} \frac{\Delta p_p}{\Delta u_b}$$

(4.16)

and the absolute value for the reflection coefficient is calculated using Eq. 4.10.

$$|R| = \frac{S_p/S_o Z_{h,-} - Z_0}{S_p/S_o Z_{h,-} + Z_0}$$

(4.17)

The obtained reflection coefficient and impedance for the compressible model are compared with the results of the incompressible model (Eq. 4.10 and Eq. 4.19 in Fig. 4.3). We use as the values of the parameters in the model: $S_p = \pi 0.035^2$ m$^2$, $S_o = 0.01 \cdot 0.05$ m$^2$, the vena contracta factor $\Gamma = 0.71$, the density of air $\rho_0 = 1.2$ kg/m$^3$ and the ratio of specific heat capacities $\gamma = 1.4$. For this geometry, we see that the Mach number at which the pipe end becomes anechoic, is higher for the compressible approximation as for the incompressible one. It is also found that at $M_b = 0.15$, the difference between the compressible and incompressible $Z_{h,-}/Z_0$ is around 10% of the incompressible value.
4.3 Vena contracta factor

4.3.1 Steady flow resistance

The vena contracta factor $\Gamma$ is a measure for the resistance of the steady flow through the orifice. If we define the flow resistance $R_{\text{flow}}$ as:

$$ R_{\text{flow}} = \frac{\Delta p}{|u_b|} \quad (4.18) $$

where $\Delta p$ is the pressure difference over the orifice and $u_b$ is the mean velocity of the flow through the orifice. Using the equation of Bernoulli, we estimate the pressure difference by means of the jet velocity $u_j$:

$$ \Delta p = \frac{1}{2} \rho_0 u_j^2 \quad (4.19) $$

For an incompressible flow and using the definition of the vena contracta factor we have $\Gamma = u_b/u_j$, hence:

$$ R_{\text{flow}} = \frac{\rho_0 u_j}{2\Gamma} \quad (4.20) $$

We scale the flow resistance by the Mach number $M_b$ and divide by $Z_0 = \rho_0 c_0$:

$$ \tilde{R}_{\text{flow}} = \frac{R_{\text{flow}}}{Z_0 M_b} = \frac{1}{2\Gamma^2} \quad (4.21) $$

From Eq. (4.9) we see that for an incompressible flow and if the pipe cross section is large compared with the orifice cross section, the one-sided dimensionless orifice impedance increases as $M_b^2$. Using the notation of scaled acoustic resistance $\tilde{r}$ (see Eq. (2.27)), we see that it roughly scales with the inverse of $\Gamma^2$. But it is a factor 2 larger than the scaled steady-flow resistance $\tilde{R}_{\text{flow}}$, so:

$$ \tilde{r} \approx 2 \tilde{R}_{\text{flow}}. \quad (4.22) $$

Figure 4.3: Comparison of the incompressible and compressible model by means of the absolute value of the reflection coefficient and the one-sided dimensionless orifice impedance $Z_{h,\text{in}}/Z_0$, both as a function of the orifice Mach number $M_b$ and for $\Gamma = 0.71$ and $\frac{S}{S_o} = 7.7$. 

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4.3 Vena contracta factor
4.3.2 Experimental vena contracta factor

Experimentally, the vena contracta factor $\Gamma$ is deduced from the ratio of the jet velocity and the bias velocity (see Section 3.3.2): $\Gamma_{exp} = \frac{u_b}{u_j}$. For incompressible flow, this conforms to the definition of $\Gamma$. The results are plotted in Fig. 4.4 for all orifices. It is seen that sharp edges indeed lead to a high flow contraction (orifice A, $\Gamma = 0.71$). For the orifice with orthogonal edges (orifice B), $\Gamma$ increases with increasing $M_b$ and approaches the asymptotic value of $\Gamma = 0.85$. The fact that orifice B has a higher vena contracta factor than orifice A can be due to partial reattachment of the flow at the walls of the orifice. This effect is expected to be strongly dependant on the Reynolds number. Reattachment will be more pronounced in a turbulent flow than in a laminar flow. The orifice with one sharp edge and one orthogonal (C and D) has a slightly higher vena contracta factor than orifice A, which is plausible because of its geometry. The slanted orifice (E and F), has a constant vena contracta around 0.78 for $M_b < 0.04$. The vena contracta factor of orifice G, (chamfered edges), approaches $\Gamma = 0.95$ for inflow. This could be due to reduced flow separation at the edges and reattachment of the flow. The fact that we find a vena contracta factor larger than unity for $M_b \approx 0.12$ is expected to be a measurement error which is dominated by the uncertainty in estimating the jet velocity.

![Graph](image)

**Figure 4.4:** Experimentally obtained vena contracta factor as a function of the orifice Mach number $M_b$, for bias inflow through different orifices.
4.4 Comparison of quasi-steady model and measurements

The inflow model explained in the previous paragraph is valid in the limit of small Helmholtz numbers, so that the source region is compact ($He << 1$ and wave propagation can be neglected). Because $He = 2\pi SrM$, these approximation is reasonable if we are interested in a quasi-steady theory for $M < 1$. In what follows, we will use the compressible model and compare it with the experimental results at low frequency for which $M_b < 0.12$ so that $M_j << 1$. Hence is meaningful to compare the model with the experiments.

The comparison is performed by means of the one-sided orifice impedance for three orifices: the sharp edged orifice A, ($\Gamma = 0.71$) the orthogonal edged orifice B ($\Gamma = 0.85$) and the chamfered edged orifice G ($\Gamma = 0.95$). The one-sided orifice impedance is measured as explained in Section 3.5 Eq. 3.16. The value of $Z_{h,in}/Z_0$ is taken at low frequency (mainly at $f = 31$ Hz and in some cases at 62 or 93 Hz). The measured vena contracta factor is used as model parameter. The ratio $S_p/S_o$ is taken equal to the ratio given by the set-up. The result is shown in Fig. 4.5.

A reasonable agreements is found for orifice A but not for orifice B. This disagreement indicates the limitations of the model, in which friction is neglected. For orifice B and G, it is expected that the flow reattaches to the walls of the orifice, giving also the higher value of the vena contracta factor. Experimentally we observed an increase of the vena contracta factor $\Gamma$ with the orifice Mach number.

The experimental result of the scaled acoustic resistance $\tilde{r}_b$ are now presented for the orifices in Table 4.1 for a bias inflow. The last column includes the results for outflow through orifice G (chamfered edges). The resistance is scaled with the Mach number. Note that the data for orifice B are obtained using a frequency of 93 Hz and so differ from the data that are presented in Fig. 4.5.

For orifice G, we observe a higher scaled acoustic resistance for outflow than for inflow. Outflow corresponds with downstream chamfered edges while inflow with upstream chamfered edges. For orifice E and F (oblique orifices), the increase in the scaled acoustic resistance as a function of the bias velocity, is larger compared with the increase for the other orifices. Earlier we found an almost constant value of the vena contracta factor Fig. 4.4.
0.02 0.04 0.06 0.08 0.1 0.12
0 0.02 0.04 0.06 0.08 0.1 0.12 0.14 0.16
\[ \frac{M_b}{c_0} = |u_b| / c_0 \]

Figure 4.5: Experimental results for the acoustic resistance at low Strouhal number as a function of the orifice Mach number for the orifice with sharp edges (A), the orifice with orthogonal edges (B) and the orifice with chamfered upstream edges (G). The lines represent the results for the compressible model where the vena contracta factor is constant over the range and equals the experimental asymptotic value for the different orifices.

Table 4.1: Scaled acoustic resistance measured at a frequency of 31 Hz for orifices A, C-D, E-F and G. The data for orifice B are measured at 93 HZ. For each orifice, the first column lists the bias velocity \( u_b \), the second row lists the scaled acoustic resistance \( \tilde{r}_b \), scaled with the bias Mach number \( M_b \). \( u_b \) is the mean velocity of the flow through the orifice and is given in m/s.

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4.5 Chapter conclusion

We presented an incompressible model that is valid at low Strouhal number, for the inflow through an orifice. We found a relation between the acoustic impedance as a function of the vena contracta factor. The scaled acoustic resistance \( \tilde{r}_b \) is found to be about two times
larger than the steady-flow resistance $\tilde{R}_{\text{flow}}$. The acoustic resistance as well as the flow resistance scale with the inverse of the square of the vena contracta factor. It is found that the compressibility has a significant effect on the damping, at even relatively high Mach numbers $M_b < 0.4$. Taking the compressibility effect into account, the quasi-steady model agrees well with the measured acoustic resistance.
Chapter 5

Vortex sound theory

In the previous chapter we dealt with the acoustic flow resistance at low Strouhal number and presented a quasi-steady theory. For higher Strouhal number, the propagation of acoustic waves and the unsteadiness of the flow dominates. There are many daily examples where vorticity is a source of sound as for example the singing electrical transmission lines and the human whistling. In this chapter we will deduce a few aspects that are necessary to understand qualitatively how vorticity can be a sound source. We will provide an energy corollary allowing a more quantitative discussion. This theory will be used in the next chapter.

5.1 Potential flow and acoustic definition

We start from a general flow field and a useful definition of the acoustic flow. Generally, each flow can be decomposed in a rotational and an irrotational (potential flow) component (the Helmholtz decomposition):

\[ \vec{u} = \nabla \phi + \nabla \times \vec{\psi}. \]  

(5.1)

As \( \vec{u} \) has three components the description by means of the scalar and vector potential is not unique. By using boundary conditions for the potential flow we will solve this problem. Howe defines the acoustic velocity as the time dependant part of the potential flow [30]:

\[ \phi = \phi_0 + \phi'(t), \]  

(5.2)

where \( \phi' = \nabla \phi'(t) \) is the acoustic velocity and \( \phi_0 \) is the potential function of the steady reference flow. This definition implies that the acoustic streamlines follow the walls even when the edges are sharp. A sketch of the streamlines for a potential flow through an orifice is given in Fig. 5.1. For a flow through an orifice with sharp edges the flow is locally singular. This implies an infinitely large velocity at the edges. In the quasi-steady model presented in the previous chapter, the flow was modelled as a potential flow, except for the edges (Kutta condition) [31] where flow separated and a jet is formed. The shear layers delimiting the jet compensates the non-physical large velocity of the potential flow. It is known from literature that the shear layer flow is a good representation of the actual flow [17]. Also, the acoustic flow was defined as the perturbation of this reference flow. In terms of vortex theory, the vorticity in the shear layers is time dependant and compensates the non-physical singularity of the acoustic (potential) flow.
5.2 Non-uniform force as sound source

As a second step, the acoustic wave equation is derived for a stagnant reference flow. From the linearised mass equation (Eq. 2.2) and the linearised momentum equation (Eq. 2.4), we become:

\[
\frac{\partial p'}{\partial t} + \rho_0 (\nabla \cdot \vec{u}') = 0 \quad (5.3)
\]

\[
\rho_0 \frac{\partial \vec{u}'}{\partial t} + \nabla p' = \vec{f} \quad (5.4)
\]

where \( \vec{f} \) is the force density of an external force field acting on the fluid. Take the time derivative of the mass equation and the divergence of the momentum equation, use \( p' = c_0^2 \rho' \) and \( c_0 = \text{Cte} \):

\[
\frac{1}{c_0^2} \frac{\partial^2 p'}{\partial t^2} + \rho_0 \frac{\partial}{\partial t} (\nabla \cdot \vec{u}') = 0 \quad (5.5)
\]

\[
\rho_0 \nabla \cdot \frac{\partial \vec{u}'}{\partial t} + \nabla^2 p' = \nabla \cdot \vec{f} \quad (5.6)
\]

By subtracting those equations, we find the wave equation:

\[
\frac{1}{c_0^2} \frac{\partial^2 p'}{\partial t^2} - \nabla p' = -\nabla \cdot \vec{f}, \quad (5.7)
\]

The left hand part represents the propagation of acoustical waves. The right hand side, shows that a non-uniform external force density \( \vec{f} \) acts as a source of acoustic energy.

5.3 Generation of acoustic power

As a third step we consider the acoustic energy (energy corollary of Kirchoff, 1876). Multiply the linearised mass conservation equation with \( \frac{p'}{\rho_0} \) and take the inner product of the linearised
momentum equation with the velocity:

$$\frac{1}{\rho_0 c_0^2} p' \frac{\partial p'}{\partial t} + p' \nabla \cdot \vec{u}' = 0$$  \hspace{1cm} (5.8)$$

$$\rho_0 \left( \vec{u}' \cdot \frac{\partial \vec{u}'}{\partial t} \right) + \vec{u}' \cdot \nabla p' = \vec{u} \cdot \vec{f}$$  \hspace{1cm} (5.9)$$

Adding both equations and using the product rule for derivatives, we find an expression for the acoustic energy:

$$\frac{\partial}{\partial t} \left( \frac{1}{2} \rho_0 (\vec{u}')^2 \right) + \vec{u}' \cdot \nabla p' - \vec{u}' \cdot \vec{f} + \frac{1}{\rho_0 c_0^2} \frac{\partial}{\partial t} \left( \frac{1}{2} p'^2 \right) + p' \nabla \cdot \vec{u}' = 0$$  \hspace{1cm} (5.10)$$

With the definition of acoustic energy $E$ and the acoustic intensity $\vec{I}$:

$$E \equiv \frac{1}{2} \rho_0 (|\vec{u}'|^2) + \frac{1}{2} \frac{p'^2}{\rho_0 c_0^2}$$  \hspace{1cm} (5.11)$$

$$\vec{I} \equiv p' \vec{u}'$$  \hspace{1cm} (5.12)$$

we can write Eq. (5.10):

$$\frac{\partial E}{\partial t} + \nabla \cdot \vec{I} = \vec{u}' \cdot \vec{f}$$  \hspace{1cm} (5.13)$$

We conclude from above energy expression, that there is only generation of acoustic power by an external force in the presence of an acoustic flow.

Note that until here, we assumed a stagnant reference flow. When dealing with vortex shedding due to a flow along a geometry, this is obviously not exact. However, it is found that the resulting prediction of acoustic power is satisfactory at very low Mach numbers. A better description of the energy is obtained if the reference flow is not stagnant and the acoustic intensity is defined as the product of variations of the total enthalpy and variations in the mass flux[32].

### 5.4 Vorticity as external force for potential flow

Consider now the Euler equation Eq. (2.4) for a general flow (velocity $\vec{u}$). Use the vector identity

$$(\vec{u} \cdot \nabla) \vec{u} = \nabla \left( \frac{|\vec{u}|^2}{2} \right) + \vec{\omega} \times \vec{u}$$  \hspace{1cm} (5.14)$$

where $\vec{\omega} = \nabla \times \vec{u}$ is the vorticity, to rewrite the Euler equation:

$$\frac{\partial \vec{u}}{\partial t} + \nabla \left( \frac{|\vec{u}|^2}{2} \right) + \frac{\nabla p}{\rho} = -\vec{\omega} \times \vec{u} + \frac{\vec{f}_b}{\rho}$$  \hspace{1cm} (5.15)$$

In above equation, the left hand side represents a potential flow. The analogy of Howe is based on the assumption that the term $-\vec{\omega} \times \vec{u}$ acts as an external force $\vec{f}_c = -\rho (\vec{\omega} \times \vec{u})$ on the potential flow.
5.5 Bias flow: edge and shear layer

We will now apply above findings on a flow through a sharp edged orifice (bias inflow) to have a qualitative understanding of vortex sound generation.

A free jet is formed downstream the orifice subjected to a bias flow. Flow continues tangentially along the upstream wall rather than turning at the edge as for a potential flow. The shear layer surrounding the jet separates the quasi stagnant fluid downstream the edges of the orifice, from the jet flow through the centre of the orifice. This shear layer is unstable and the instability mechanism is inviscid [33]. Sato [33] found that a hydrodynamic perturbation due to sound from a loudspeaker initially grows exponential with distance along the shear layer. Further downstream the disturbed shear layer rolls up in discrete vortices. This also occurs in the free jet downstream the orifice.

Assume a bias flow in negative $y$-direction through an orifice with sharp edges. According to the notation that is used earlier in this report, we refer to the direction of the bias flow from a duct into a pipe through an orifice in negative $y$-direction, as inwards and outwards flow has a positive velocity (see Fig. 5.2). A vortex that is formed due to the shear layer instability will travel with an almost constant convection velocity $u_c = \frac{1}{2} u$ [34]. This is the average of the velocity $u_j$ in the jet flow and the low flow velocity outside the jet. This low flow region exists downstream the edges as shown in Fig. 5.2. According to the energy equation Eq. 5.13 with $\dot{f} = \dot{f}_c = -\rho (\vec{\omega} \times \vec{u}_c)$, we find the approximation of the time averaged acoustic power $< P >$ for a source region of volume $V$, proposed by Howe [35]:

$$< P > = \int_V < \dot{f}_c \cdot \vec{u}^\prime > dV = -\rho \int_V < (\vec{\omega} \times \vec{u}_c) \cdot \vec{u}^\prime > dV$$  \hspace{1cm} (5.16)

From the definition of the acoustic velocity (Eq. 5.2), its value is largest at the edges of the orifice. For an harmonic oscillation, the acoustic flow through the orifice changes sign every half period. This means for the acoustic power that there is an alternation of production and absorption assuming that the vorticity remains constant. The vortex is shed from the edge of the orifice when the acoustic field start to flow inwards. The triple product $-\rho (\vec{\omega} \times \vec{u}_c) \cdot \vec{u}^\prime$ is then negative and hence there is sound absorption. Intuitively we can understand that the formation of vortices due to an acoustic perturbation do absorb energy from the acoustic energy. After an half period, there is sound production because the Coriolis acceleration $-\vec{\omega} \times \vec{u}$ and the convective velocity $\vec{u}_c$ are in the same direction while the acoustic velocity has changed sign and is now directed outwards. This production of sound is not easy to understand but does occur as we will verify from our experiments, as we observe negative acoustic resistance.

When integrating over one acoustic period, there can be either net sound absorption or production. This will depend on the geometry of the orifice and the Strouhal number $Sr_b = \frac{ft}{u_b}$ based on the orifice thickness $t$ and bias velocity through the orifice $u_b$. 
Figure 5.2: Principle of sound production by vortex shedding. The vorticity $\vec{\omega}$ is directed into the plane for the vortex on the left.
5.5 Bias flow: edge and shear layer
Chapter 6

Experimental results of the high Strouhal behaviour for a pure bias inflow and outflow

The result of the impedance measurements are presented for different orifice geometries subjected to a pure bias flow \( \vec{U} = (0, u_b) \). In this chapter, the data are presented by the dimensionless scaled resistance \( \tilde{r}_b \). The definition of \( \tilde{r}_b \) is given by Eq. (6.1). The scaling is based on the orifice Mach number \( M_b = u_b/c_0 \). This leads to the following definition that is used in this chapter:

\[
\tilde{r}_b = \frac{1}{M_b \rho_0 c_0} \text{Re}\{Z_{h,in} - Z_{h,in,u=0}\} \quad (6.1)
\]

The use of a Strouhal number based on the bias velocity is meaningful when there is a significant length scale in the flow direction. For example the Strouhal number for an orifice with orthogonal edges (orifice B) is defined as \( Sr_b = \frac{fd}{u_b} \) where \( d \) equals the thickness of the orifice plate as shown in Fig. 3.3. The Strouhal number is a measure of the ratio between the travel time of fluid particles through the orifice and the period of oscillation of the acoustic field:

\[
Sr_b = \frac{fd}{u_b} \quad (6.2)
\]

where \( d \) is the length of the orifice in bias flow direction. We use in the graphs the short notation for the pure bias flow: \( \vec{U} = (0, u_b) \). In the legend of the graphs, the arrow of \( \vec{U} \) is not shown. We continue this chapter first with a few typical results to explain the principle of sound generation by vortex shedding.

6.1 Principle of sound production by vortex shedding

Consider the orifices A, B and C. Orifice A has two sharp edges, orifice C has one sharp edge and one orthogonal edge, and orifice B has two orthogonal edges. In Fig. 6.1 the dimensionless scaled resistance \( \tilde{r} \) is plotted as a function of the Strouhal number based on the orifice length \( d \). Note that \( d \) is in fact not defined for orifice A since it has sharp angled edges. Nevertheless we take \( d \) equal to the plate thickness \( t \) in order to compare the behaviour of this orifice with B and C. When the resistance \( \tilde{r} \) is positive this means that perturbations from an external sound source are absorbed due to the flow. A negative resistance means an
amplification of the perturbation.

For orifice A, we observe a positive resistance over the measured frequency range (0 to 869 Hz). This means that there is a net sound absorption. From the vortex sound theory (Chapter 5) we understood that a net absorption is due to a higher sound absorption during the first part of the acoustic period than the production during the remaining part. We explain now why this happens for this particular geometry by means of a sketch of the situation in Fig. 6.2. Upon vortex shedding the bias flow is in negative $y$-direction. The dashed lines represent the left part of the acoustic field (streamlines). Vortices are shed at the moment that the acoustic flow through the orifice changes direction and start flowing in the same direction as the main flow $\vec{u}_b$. This shedding happens at the edges which we refer to as the separation point. Assume that the vortex travel time is $\vec{u}_c$. On the right part of the graph, the horizontal arrows pointing in positive $x$-direction represent the cross product $\vec{\omega} \times \vec{u}_c$ for each distinct vortex. We consider for simplicity the vortices as point vortices for which their vorticity $\vec{\omega}$ is concentrated in one point. The dashed arrows represent the local direction and magnitude of the acoustic field at the position of the vortex.

Because of the sharp edge, the acoustic velocity $\vec{u}'$ is locally large and directed along the walls of the orifice. The cross product $\vec{\omega} \times \vec{u}_c$ is almost in the same direction as the acoustic field. Therefore, the triple product $- (\vec{\omega} \times \vec{u}_c) \cdot \vec{u}'$ is very large and negative in the first half period and so there is a large initial sound absorption. In the second half period, the acoustic field is reversed and the shed vortex has travelled downstream over a distance $\frac{1}{2} f \vec{u}_c$. During this time the circulation of the vortex has grown. But on the other hand, the amplitude of the acoustic field has decreased because we moved away from the sharp edge. Also the direction of the acoustic velocity field has deviated and is not any more parallel to $\vec{\omega} \times \vec{u}_c$. The growth of the vortex is not large enough to compensate for the reduction of the acoustic field amplitude. Hence the production of sound during the second half period is lower than the absorption during the first half period. This explains qualitatively net absorbing that is measured for orifice A.

For orifice B that has orthogonal edges, we observe a clear frequency dependency of the acoustic power production. At small Strouhal number, $\tilde{r}$ is lower for orifice B than for A. This can be explained by a reduction of the singularity of the acoustic field at the separation point. The acoustic flow around an orthogonal edge is less singular than for a sharp edge. Furthermore, around $Sr_b \approx 0.25$, we observe a reduced sound absorption for orifice B. This behaviour is related to whistling and is discussed in the next section.

For orifice C that has one sharp edge and one orthogonal edge, we find again a frequency range (now around $Sr_b = 0.45$) where the absorption is reduced. This could be explained as a similar whistling behaviour as orifice B. Above $Sr_b = 0.5$ the absorption increases. At very low Strouhal number, orifice A and C display similar absorption.
Figure 6.1: Acoustic resistance as a function of the Strouhal number $Sr_b = fd/u_b$ where $u_b$ is the flow velocity through the orifice and $d = t = 15$ mm the thickness of the plate. Orifice A has two sharp edges, orifice B has orthogonal edges and orifice C has one sharp edge and one orthogonal edge. We use the notation $U = (0, u_b)$.

Figure 6.2: The acoustic field (dashed lines) and the shedding of discrete vortices due to a bias flow for an orifice with sharp edges. $\omega$ is the concentrated vorticity at the discrete vortices and is directed out of the plane for the vortices on the right and into the plane for the vortices at the left. $\vec{u}_c$ is the velocity at which the vortices are convected. The dashed arrows at the right represent the local acoustic field.
6.2 Whistling behaviour of an orthogonally edged orifice (B)

In Fig. [6.3] the dimensionless scaled real part of the one-sided orifice impedance $\tilde{r}_b$ is given as a function of the Strouhal number $Sr_b$ for orifice B. The measurements, at four different bias flow rates, display the same global behaviour with a minimum of the resistance around $Sr_b = 0.25$. There is a clear tendency of the orifice to whistle. A sketch of the orifice and the flow is given in Fig. [6.4] where the bias velocity is in positive $y-$direction.

The net absorption is lower for orifice B than for orifice A because downstream the orifice, for both orifice the vortex grows in time but for orifice B the amplitude of the acoustic field near the downstream edge is large. Also the acoustic field near the downstream edge turns away from the direction of the path of the vortices. This means a locally better alignment of $-(\vec{\omega} \times \vec{u}_c)$ and $\vec{u}'$ during the second half acoustic period, hence increased sound production if the vortex reaches the downstream edge after half an oscillation period. Call $\tau$ the travel time of a vortex to reach the downstream edge. A Strouhal number $Sr_b = f_{\text{d}}/u_b = 0.25$ corresponds to half an oscillation period if $\tau = \frac{d_{\text{c}}}{u_{\text{c}}} = \frac{1}{2f}$ and $u_{\text{c}} = \frac{u_b}{2}$. The whistling of an orifice has been extensively studied in [36].

At higher Strouhal number, around 0.75, there is a second peak of sound production. This is due to the occurrence of two travelling vortices inside the orifice. This happens if the time between the shedding of two subsequent vortices, given by the frequency of the acoustic forcing, is smaller than the time needed for a vortex to travel through the orifice. This is what is called the second hydrodynamic mode.

![Figure 6.3](image)

**Figure 6.3:** Acoustics resistance scaled with the bias Mach number as a function of the Strouhal number based on the orifice length $d$ that equals for this orifice with the plate thickness $t$. Results for different values of the bias velocity $u_b$ are shown.
Chapter 6. Experimental results of the high Strouhal behaviour for a pure bias inflow and outflow

Figure 6.4: The acoustic streamlines (dashed lines on the right) and the shedding of discrete vortices due to a bias flow for an orifice with orthogonal edges (B). The bias flow is in positive $y$-direction. $\omega$ is the concentrated vorticity at the discrete vortices and is directed into the plane for the vortices on the right and out of the plane for the vortices at the left. The dashed arrows at the right represent the local acoustic field.

6.2.1 Rounding the upstream edges

The effect of rounding the upstream edges on the acoustic damping is investigated by comparing the orthogonally edged orifice (B) with an orifice for which the upstream edges are chamfered. The latter we denote as orifice G. The angle of the edges is now 135$^\circ$ so that each of the two edges of orifice B is replaced by two new edges. In Section 4.3 we have seen that rounding the edges or increasing their angle implies an increase in vena contracta factor $\Gamma = S_o/S_j$ from $\Gamma = 0.85$ for orifice B to $\Gamma = 0.95$ for orifice G. This means that the vena contracta effect is less pronounced.

In Fig. 6.5, the dimensionless scaled resistance of both orifices are compared. Note that the orifice length $d$ is 0.012 m for orifice G instead of 0.015 m for orifice B. Both orifices display a local minimum of absorption around $Sr_b = 0.25$.

We observe also that in general the absorption is lower for orifice G than for B. A reduction of the vena contracta effect implies a lower jet velocity. Following the quasi-steady theory, absorption increases as $\frac{M_b}{\Gamma^2}$ if the orifice is small compared to the pipe (see Eq. 4.9). The absorption was related to a reduction of energy dissipation by the jet. We observe this behaviour in the low Strouhal limit by taking into account that $\Gamma = 0.95$ for G and $\Gamma = 0.85$ for B. However it also seems to be valid at higher Strouhal numbers. Besides this, we expect that the reduction of the acoustic field singularity at the upstream chamfered edge also has an effect on the reduced absorption. In Fig. 6.6, the dimensionless acoustic resistance is scaled with $\frac{M_b}{\Gamma^2}$ and plotted as a function of $Sr_b$ for both orifices. We obtain a surprisingly
6.2 Whistling behaviour of an orthogonally edged orifice (B)

reasonable collapse of the two data sets.

![Graph](image1)

**Figure 6.5:** Acoustics resistance scaled with the bias Mach number as a function of the Strouhal number $Sr_b$ for orifice B and G. The chamfered edges on orifice G are upstream, attained by blowing.

![Graph](image2)

**Figure 6.6:** Acoustic resistance scaled with $M_B/\Gamma^2$ where $\Gamma = S_j/S_o$ is the vena contracta factor of the jet flow. The chamfered edges on orifice G are upstream.
6.2.2 Rounding the downstream edge

Consider now chamfering of the downstream edge, now air is blown through orifice G instead of sucked. Chamfering the downstream edges does have a strong effect on the sound production as can be seen in Fig. 6.7. The scaled acoustic resistance \( \tilde{r} \) is plotted as function of the Strouhal number \( Sr_b \). Similar as the whistling of the human lips at a critical Strouhal number, chamfering enhances the net production. We observe a negative resistance around \( Sr_b = 0.2 \).

In order to understand this, we include the secondary vortex shedding at the downstream edge. Note that we did not considered these is the discussion of chamfering the upstream edges in the previous section. The situation is shown in Fig. 6.8. The secondary vortices stick to the edge rather than being carried away with the main flow. They imply enhanced absorption, similar as the absorption at the upstream edge. However, the absorption is lower for chamfered edges (orifice G) than for orthogonal edges (orifice B). This is because the shedding of secondary vortices is more pronounced if the acoustic field is larger in amplitude, as it is the case at an orthogonal edge and less for a chamfered edge. Hence the net production for chamfered edges is expected to be due to a reduction of absorption.

![Figure 6.7: Acoustics resistance scaled with the bias Mach number as a function of the Strouhal number \( Sr_b \) for orifice B and G. The chamfered edges on orifice G are now downstream, attained by blowing.](image-url)
6.3 Whistling behaviour of an oblique orifice

The orifices E and F are drilled oblique under an angle of 30° with respect to the grazing flow direction. This implies both orifices have one edge with an obtuse angle of 150° with respect to the grazing flow and one edge with an acute angle 30° with respect to the grazing flow. The results for the acoustic resistance as a function Strouhal numbers $Sr_b$ based on the orifice length $d$ are plotted in Fig. 6.9. $d$ is related to the plate thickness $t$ by the angle of drilling (30°). The tendency to whistling is similar to that of the orthogonal edged orifice B: we observe again maxima and minima of absorption at a critical Strouhal number. But there are a few differences. We see that the peaks in sound absorption increase with increasing bias velocity. For large bias flow, we also notice that the peak around $Sr_b = 1.5$ is larger than the absorption at low Strouhal number. The same occurs for orifice C (see Fig. 6.1). This was not observed for the other orifices (A,B,G). The sound absorption at low frequency is higher than those for orifice B and G. Note that the vena contracta factor for orifice E and F was found to be lower than for orifice B and G. Generally the hydrodynamic behaviour of oblique orifices is more complex that orifices with orthogonal edges due to the complex jet flow and flow separation. This make the understanding of the acoustic response to external forcing even more difficult.

Figure 6.8: Representation of the vortex shedding at an orifice with downstream (and up-stream) orthogonal edges or downstream chamfered edges (and orthogonal up-stream edges). Flow is directed from the bottom of the page to the top. Notice the secondary vortex shedding at the downstream edge. These vortices stick to the edge.
Chapter 6. Experimental results of the high Strouhal behaviour for a pure bias inflow and outflow

6.4 Chapter conclusion

While the orifice with orthogonal edges showed only a lower sound absorption around a critical Strouhal number based on the orifice length $S_{rb} = f_d/u_b = 0.25$, negative resistance was observed for the orifice with chamfered downstream edges. This is an indication for the potential of such perforations to drive whistling.

For most industrial applications, thick orifices are used rather than orifices with very sharp edges because of the difficulties in the production method of sharp edges and their reduced robustness. Rounding the edges occur due to corrosion. For the application of a pure bias flow through an orifice, the range of Strouhal numbers in which the orifice is sensitive to whistling, should be avoided. This can be done by making the perforations to operate at low Strouhal number. In order to cover a large frequency range, the thickness of the wall in which the orifices are drilled can be decreased in accordance with the bias flow $u_b$. This is a more practical approach than the use of the “ideal” sharp edged orifices.

Oblique orifices have a similar tendency to whistle as orifices with orthogonal edges. But they show a more complex behaviour in which a good understanding of the hydrodynamics is not yet available.

According to the quasi-steady theory, the low Strouhal number absorption was related to an energy dissipation by the jet. We observed this behaviour in the low Strouhal limit. However it also seems to be also valid at higher Strouhal numbers. This was concluded from the comparison of two orifices (B and G) with different vena contracta factor $\Gamma = S_j/S_o$. 

Figure 6.9: Acoustics resistance scaled with the bias Mach number as a function of the Strouhal number $S_{rb}$ for orifice E and F (oblique).
6.4 Chapter conclusion
Chapter 7

Experimental results of the high Strouhal behaviour for a pure grazing flow

In this chapter, we consider an orifice that is placed between two flow regions: a region with a uniform flow tangential along the orifice (grazing flow) and a region of stagnant fluid at the other side of the fluid. In the opening of the orifice, a shear layer will be formed that separates the moving fluid from the stagnant plate. This corresponds with flow regime (a) in Fig. 2.2. Similar as for a jet flow, the shear layer is unstable and for sufficiently strong perturbation amplitudes it breaks up into discrete vortices that travel in the direction of the grazing flow. This instability forms the mechanism of sound absorption or production which will be illustrated in this chapter by experimental results.

In the previous chapter, we discussed the experiments for a pure bias flow through the orifice. Now we consider a purely grazing flow with velocity $u_g$ in positive $x$–direction along the orifice. The flow is driven by the wind tunnel. Acoustic perturbations are provided by the loudspeaker at the end of the impedance tube, placed behind the orifice plate (see Fig. 3.1). We measure the dimensionless one-sided scaled orifice impedance. Based on the grazing flow, the scaled resistance and reactance that are used in this chapter are defined as:

\[
\tilde{r}_g = \frac{1}{M_g \rho_0 c_0} \text{Re} \{Z_{h, in} - Z_{h, in, u=0}\} \tag{7.1}
\]

\[
\tilde{\delta}_g = \frac{1}{k_0 \rho_0 c_0} \text{Im} \{Z_{h, in} - Z_{h, in, u=0}\} \tag{7.2}
\]

where $M_g = \frac{u_g}{c_0}$ and $w$ is the width of the orifice in grazing flow direction. $\tilde{r}_g$ and $\tilde{\delta}_g$ are plotted as function of the Strouhal number based on the grazing flow velocity $u_g$ and the width $w$:

\[
Sr_g = \frac{f w}{u_g} \tag{7.3}
\]

We use in the graphs the short notation for the pure grazing flow: $\vec{U} = (u_g, 0)$. In the legend of the graphs, the arrow of $\vec{U}$ is not shown.
7.1 Principle of sound generation by unstable shear layer

Similar as for pure bias flow, sound generation is a result of the hydrodynamic instability of the shear layer. Since we limited the perturbation amplitude such that the measured impedance is independent of the amplitude of the acoustic field, the shear layer response is linear (see Fig. 7.2). However, we have a spatially growth of the instability of the shear layer. The mechanism of sound production due to an unstable shear layer or due to the shedding of discrete vortices are similar. When considering pure bias flow we have already gained some insight into sound production by discrete vortices. Therefore we assume in what follows that the vorticity is confined in discrete vortices that are shed at the sharp upstream edge.

Assume a uniform flow $\vec{U}$ along the orifice. This causes a convective velocity of the vortices $\vec{u}_c = \frac{1}{2} \vec{u}_g$, which is a compromise between the stagnant fluid downstream the orifice and the uniform grazing flow. If the orifice width is $w$, the time that a vortex needs to travel along the orifice is $\frac{2w}{u_g}$. A sketch of the situation is given in Fig. 7.1. Consider now Fig. 7.2, in which the resistance is plot for an orifice with sharp edges and $u_g = 12.8 \text{ m/s}$. Minima and maxima of $\tilde{r}$ as a function of the Strouhal number are observed. This is qualitatively explained by means of the vortex sound theory.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{shear_layer.png}
\caption{Shear layer due to the grazing flow $\vec{u}_g$ for an orifice with sharp edges (A).}
\end{figure}

Vortices are formed periodically with period $T$ each time the acoustic flux through the orifice changes direction from outflow (in positive $y$–direction) to inflow (in negative $y$–direction). This occurs at time $t = nT$, with $n = 0, 1, 2, \ldots$ where $T = 1/f$ is the period of oscillation of the acoustic field. The vorticity of the shear layer is directed into the paper (negative $z$–direction). The product $\vec{\omega} \times \vec{u}_c$ points in the negative $y$–direction. So this is aligned with the acoustic velocity (having large amplitude at the edge due to the sharp edge) during the first half period ($nT < t < 1/2T + nT$). The triple product $-(\vec{\omega} \times \vec{u}_c) \cdot \vec{u}'$ is negative. The vortex shed at the upstream edge absorbs sound during half an oscillation period.

While the vortex is travelling along the orifice, the acoustic field changes direction. Suppose the vortex approaches the downstream edge at the moment the acoustic field is reversed, the product $-(\vec{\omega} \times \vec{u}_c) \cdot \vec{u}'$ is positive, so we have sound production. Since the vortex has grown due to the spatial growth of the hydrodynamic instability of the shear layer, and because the amplitude of the acoustic field around this edge is large, the production can be larger than the initial absorption. We can understand now the four Strouhal regions observed in Fig. 7.2.
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Chapter 7. Experimental results of the high Strouhal behaviour for a pure grazing flow

Figure 7.2: Sound production/absorption due to the hydrodynamic instability of the shear layer for an orifice with sharp edges (A).

Oscillations of the resistance $\tilde{r}_g$ as a function of the Strouhal number $Sr_g$ are observed for orifice A (sharp edges). Consider a fixed frequency and width $w$.

1. If the travel speed of the discrete vortices is very high ($Sr$ low), the vortex has passed the orifice before the acoustic field had time to reverse direction. The vorticity induces a net sound absorption (figure, case 1).

2. If the travel speed matches the acoustic period so that the shed vortex arrives at the downstream edge when the acoustic field is in the positive $y$–direction (outwards), there will be a net production of sound. The resistance is negative. This is called the first hydrodynamic mode at a critical Strouhal number $Sr = 0.4$ (figure, case 2). This critical Strouhal number implies that the convective velocity of the vortices is $0.4u_g$.

3. If the travel time of the vortices is larger than $T$, the first vortex is still travelling along the orifice, while a second is shed and starts to absorb sound (figure, case 3). This happens at a Strouhal number $Sr_w = 0.5$. Now there is a net absorption.

4. If the travel time of the vortices is twice the acoustic period, two vortices are found along the orifice width. They are shed, one after the other, and both travel along the orifice width during two acoustic oscillation periods $2T$ (figure, case 4). Similar as for
case 2, there is a net production of sound (negative resistance). This is called the second hydrodynamic mode.

Note that for the pure bias flow, the low Strouhal number limit (case 1) predicted absorption, corresponding to the quasi-steady theory. It was explained as the loss of acoustic power at the benefit of the jet flow which is dissipated by turbulence. For a pure grazing flow, a similar behaviour is expected.

For the orifice with sharp edges (A), measurements are performed for increasing grazing flow velocity. The result is plotted in Fig. 7.3. Earlier measurements with the same set-up and orifice were done by Kooijman [21]. The agreement between the results of Kooijman and our results is good. The effect of the boundary layer on the shear layer has been studied by Kooijman [21] and Golliard [14]. The shape of the boundary layer determines the growth rate of the instability of the shear layer and the convective velocity at which vorticity (shed vortices) travels along the orifice. Therefore it plays an important role in the acoustic performance. For Reynolds numbers $Re = \frac{u_g L}{\nu}$ below $2 \times 10^5$, the boundary layer will be laminar, if the plate surface upstream the orifice is smooth. This critical value of the Reynolds number implies a theoretical velocity $u_g = 30 \text{ m/s}$ below which we can assume a laminar boundary layer. The measurement that are shown in Fig. 7.3 are done at lower flow velocities. Measurements of the boundary layer velocity profile just upstream of the orifice by [21] confirm that the boundary layer remains laminar up to $u_g = 17 \text{ m/s}$. By placing a surface roughness we will later force turbulence.

We observe the oscillations of the scaled acoustic resistance $\tilde{r}_g$ as a function of the Strouhal number to occur around the same critical Strouhal number, for the different flow velocities. The peaks in the absorption as well as the peaks in the production are different in amplitude for different flow velocities. A higher Reynolds number implies a thinner and more unstable shear layer (as long as it remains laminar).

For low Strouhal numbers, the acoustic resistance (scaled with the grazing flow Mach number $M_g = u_g/c_0$) is presented in Table 7.1 for orifice A. The grazing velocity $u_g$ is listed in the first row. In the second row the corresponding value of the scaled resistance $\tilde{r}_g$ is listed. These values are obtained at 31 Hz. We observe a small increase in the scaled acoustic resistance for increasing grazing flow velocity.

Table 7.1: Acoustic resistance measured at a frequency of 31 Hz for orifice A (sharp edges) for increasing grazing flow velocity $u_g$. For velocities up to 17 m/s, we assume the approaching grazing flow boundary to be laminar.

<table>
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<th>$r_g$ [-]</th>
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</tr>
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</tr>
<tr>
<td>25.6</td>
<td>0.80</td>
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</table>
Chapter 7. Experimental results of the high Strouhal behaviour for a pure grazing flow

Figure 7.3: Dimensionless scaled resistance $\tilde{r}_g$ and reactance $\tilde{\delta}_g$ for the orifice A (sharp edges) at different flow velocities $u_g$. We expect a laminar boundary layer for grazing flow velocities lower than $17 \text{ m/s}$ and a transitional boundary layer for higher velocities.
7.2 Laminar boundary layer

The measured impedance of the orifices A (sharp edges), B (orthogonal edges), C (sharp upstream, orthogonal downstream), D (orthogonal upstream, sharp downstream), E (oblique in grazing flow direction) and F (oblique opposite to grazing flow direction) are presented in Fig. 7.4 for $U = (u_g, u_b) = (12.8, 0) \text{ m/s}$. For all orifices, we have a laminar boundary layer. We observe no significant difference in the general acoustical response of orifices A, B, C and D. The peak in the production around $Sr_g = 0.4$ is largest for orifice D. This could be due to the sharp downstream edge (local high amplitude of the acoustic field) in combination with the orthogonal upstream edge (local moderate amplitude of the acoustic field). Orifice D displays the largest peak in the absorption (around $Sr = 0.45$) and is followed by orifice A. The peaks in the production, seen from the resistance $\tilde{r}_g$, correspond reasonable well with a zero-crossing of the reactance $\tilde{\delta}_g$.

It is remarkable that orifice F (obtuse downstream angle) is only absorbing sound. The oscillation of the acoustic resistance as a function of the Strouhal number, noticed for orifice E (obtuse upstream angle) are lower than those for orifices A, B, C and D but happen around the same Strouhal number. At low Strouhal number, orifice E (obtuse upstream angle) is a very poor sound absorber, where the other orifices do clearly absorb sound.
Chapter 7. Experimental results of the high Strouhal behaviour for a pure grazing flow

Figure 7.4: Dimensionless scaled resistance $\tilde{r}_g$ and reactance $\tilde{\delta}_g$ for the orifices with different edge geometry in the case of a laminar boundary layer of the flow upstream the orifice. The grazing flow velocity $u_g = 12.8 \text{ m/s}$ and is equal for all orifices.
7.3 Turbulent boundary layer

In many technical applications we expect the approaching boundary layer to be turbulent. We therefore focus now on the acoustical response of the orifices for turbulent boundary layers. The overall measurements are performed at higher grazing flow velocity $\vec{U} = (u_g, u_b) = (16.8, 0)$ m/s and the turbulence in the boundary layer is triggered by the use of sandpaper (see Section 3.3.1). The turbulent velocity profile has been measured by the use of a Pitot-tube with a diameter smaller than 1 mm. It is in agreement with the more accurate hot-wire measurement of the profile is given by Kooijman [21].

Similarly as in Fig. 7.4 (laminar boundary layer), the impedance is plotted in Fig. 7.5 for the turbulent boundary layer for orifice A, B, E, F and G. The acoustical response of orifice A oscillates more strongly than that of orifices B, E, F and G. We explain this as a result of the sharp edges. The Strouhal number corresponding with the first hydrodynamic mode differs for the different orifices and is in the range of $0.3 < Sr_g < 0.4$.

Chamfering the edges (orifice G compared with orifice B) reduces the amplitude of the oscillations. This is shown in Fig. 7.5 for which we zoom in to lower Strouhal numbers. Notice also that in the limit of low Strouhal numbers (quasi-steady behaviour), orifice G displays a reduced absorption compared with the orifice with orthogonal edges.

We zoom in on orifices B, E and F in Fig. 7.7 to see the influence of the orifice angles. Orifice F is oblique opposite to the grazing flow. The upstream edge has an acute angle of 30° and the downstream edge an angle of 150°. This orifice is absorbing sound for Strouhal numbers up to 1. Orifice B with both orthogonal edges (90°) and orifice E having an upstream edge with an obtuse angle (150°) and an acute downstream angle (30°), are both producing and absorbing sound with the same extent. But for orifice E we observe for low Strouhal numbers a very low absorption. We do not have an explanation for this behaviour.
Chapter 7. Experimental results of the high Strouhal behaviour for a pure grazing flow

Figure 7.5: Dimensionless scaled resistance $\tilde{r}_g$ and reactance $\tilde{\delta}_g$ for the orifices with different edge geometry in the case of a turbulent boundary layer of the flow upstream the orifice. The grazing flow velocity $u_g = 16.8$ m/s and is equal for all orifices. The turbulence in the boundary layer is triggered by the use of sandpaper.
Figure 7.6: Dimensionless scaled resistance $\tilde{r}_g$ for the orifice B (orthogonal edges) and orifice G (chamfered edges) in the case of a turbulent boundary layer of the flow upstream the orifice and grazing flow velocity $u_g = 16.8 \text{ m/s}$. 

Figure 7.7: Dimensionless scaled resistance $\tilde{r}_g$ for the orifice E (obtuse upstream angle, acute downstream angle), orifice B (orthogonal edges) and orifice F (acute upstream angle, obtuse downstream angle) in the case of a turbulent boundary layer of the flow upstream the orifice. The grazing flow velocity $u_g = 16.8 \text{ m/s}$ and is equal for all orifices.
Chapter 7. Experimental results of the high Strouhal behaviour for a pure grazing flow

7.4 Compare impedance measurements for laminar and turbulent boundary layer

To show more explicitly the effect of the boundary layer on the acoustic performance, we focus on orifice A and E by plotting the data for the turbulent and laminar boundary layer, together in a single graph (Fig. 7.8). For a sharp edged orifice (A), it is seen that the oscillations in the sound absorption as a function of the Strouhal number have a smaller amplitude when the boundary layer is laminar.

For an oblique orifice in streamwise grazing flow direction (orifice E, Fig. 7.9), we observe for $Sr_g > 0.6$ that these oscillations are larger for a laminar boundary layer than for a turbulent boundary layer. For $Sr_g < 0.6$ we observe no pronounced increase for the peaks in the absorption but the peak of production is larger for a turbulent boundary layer than for a laminar boundary layer. Further we see that the shift in critical Strouhal number for sound production with different boundary layers is more pronounced for orifice F than for orifice A. We remark that the influence of the boundary layer is not explicitly noticeable for the oblique orifice in anti streamwise direction (orifice F). This can be seen from Fig. 7.4 and Fig. 7.5.

We compare the acoustic damping in the low Strouhal limit for a laminar and a turbulent boundary layer. Measurements of the low Strouhal scaled acoustic resistance $\tilde{r}_g$ are presented for the orifices in Table 7.2. The data for the laminar boundary layer are obtained for $u_g = 12.8$ m/s, the data for the turbulent boundary layer for $u_g = 16.8$ m/s and the turbulence in the boundary layer is triggered by the use of sandpaper.

We conclude that the characteristic of the boundary layer influences the acoustic resistance at low Strouhal. The effect depends on the geometry of the edge: orifice B and E seems to be more sensitive to the velocity profile of the boundary layer than orifice A and F.

<table>
<thead>
<tr>
<th>orifice</th>
<th>scaled acoustic resistance $\tilde{r}_g$ [-]</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0.6</td>
</tr>
<tr>
<td>B</td>
<td>0.52</td>
</tr>
</tbody>
</table>

Table 7.2: Scaled acoustic resistance measured at a frequency of 31 Hz for the different orifices. The first row lists the scaled acoustic resistance $\tilde{r}_g$ for a laminar boundary layer, the second row for a turbulent boundary layer.
7.4 Compare impedance measurements for laminar and turbulent boundary layer

Figure 7.8: Comparison of the effect on the acoustic response of a laminar or turbulent boundary layer by means of the dimensionless scaled resistance $\tilde{r}_g$ and reactance $\tilde{\delta}_g$ for the orifices with sharp edges (A). The turbulence in the boundary layer is triggered by the use of sandpaper.
Chapter 7. Experimental results of the high Strouhal behaviour for a pure grazing flow

Figure 7.9: Comparison of the effect on the acoustic response of a laminar or turbulent boundary layer by means of the dimensionless scaled resistance $\tilde{r}_g$ and reactance $\tilde{\delta}_g$ for the orifices that is oblique in streamwise grazing flow direction (E). The turbulence in the boundary layer is triggered by the use of sandpaper.
We have dealt with the acoustical response of the shear layer at a wall perforations that is subjected to a pure grazing flow. For different orifices, we observed an oscillation of the acoustic resistance as a function of the Strouhal number for an orifice with sharp edges (A), orthogonal edges (B), chamfered edges (G), one sharp upstream edge (C), one sharp downstream edge (D) and an oblique orifice in streamwise direction (E). No oscillations were noticed up to Strouhal numbers smaller than 1 for an orifice that is oblique in anti-streamwise direction (F). Critical Strouhal numbers for the oscillations were found to be related to the hydrodynamic modes of the instability of the shear layer.

At low Strouhal numbers the orifices, except for orifice E, displays an absorption of sound. We expect this dissipation of acoustic power to be similar as for the pure bias flow that was discussed in the previous chapter. However the absorption for the pure grazing flow is lower than for the pure bias flow for grazing Mach number $M_g$ equal to bias Mach numbers $M_b$.

We do not have an explanation for the different behaviour of orifice E. We will discuss this further in Chapter 9.

We concluded that chamfering the edges reduces the oscillation in the absorption. In particular it affect the absorption in the limit of low Strouhal numbers. Notice that in this limit, a similar effect was observed for chamfering the upstream edges for a pure bias inflow.

The difference of the effect of a laminar and a turbulent boundary layer on the aeroacoustic response of the orifices is complicated. At the moment we do not have a theoretical explanation of the dependency of the edge geometry and the effect of the boundary layer. However we observed an obvious shift in the critical Strouhal number to lower values for a turbulent boundary layer compared with a laminar boundary layer, indicating a reduction of convective velocity due to turbulence.
Chapter 8

Quasi-steady model for an orifice subjected to combined bias-grazing flow

The incompressible model for pure bias flow through an orifice in a pipe of Chapter 4 is extended to combined bias/grazing flow. We consider now a geometry that consists of a pipe and a duct that are perpendicular to each other. The duct is carrying the grazing flow (in positive \(x\)-direction). The bias flow enters the pipe through the orifice, away from the duct in negative \(y\)-direction (inflow, see Fig. 8.1). First we present the steady two-dimensional model that is the keystone to the quasi-steady model. The model is based on the model of Hofmans for the aeroacoustic behaviour of T-junction in a pipe [25].

8.1 Steady model

Consider Fig. 8.1 where a schematic sketch is given of the duct, orifice and pipe. At the duct inlet we assume a uniform grazing flow in positive \(x\)-direction having velocity \(u_g\). Flow leaves the duct at the outlet with uniform velocity \(u_2\) in positive \(x\)-direction. The remaining of the flow enters the orifice and eventually leaves the pipe with uniform velocity \(u_p\) in negative \(y\)-direction.

Flow separates at the upstream edge of the orifice and a jet flow is formed in the pipe. We assume the jet flow that leaves the region \(C_1\) (see graph), to have only a velocity component in the \(y\)-direction. Doing so, we imply that the model corresponds, within its limitations, to the flow regime that was presented in (b) of Fig. 2.3. The jet flows along the downstream side wall of the orifice in the negative \(y\)-direction.

We assume the flow in region \(C_1\) to be frictionless, except for the flow separation at the edges. By assuming flow separation, we implicitly take viscosity into account. The flow that leaves the region \(C_2\) downstream, is assumed to be uniform (ideal pipe flow). In the region \(C_2\), the jet flow mixes with the pipe flow and there is turbulent dissipation of the jet. Further, we assume that the jet flow entering region \(C_2\) has a cross sectional area \(S_j\). The vena contracta factor \(\Gamma\) is the ratio of \(S_j\) to the orifice cross sectional area \(S_o\): \(\Gamma = S_j/S_o\). We remark that \(S_j\) is the minimum cross sectional area of the jet and that it is reached after a certain distance. This is the position at which the vena contracta factor \(\Gamma\) is defined. Further downstream, the
jet structure will degrade which we assume in the model to occur in region $C_2$.

8.1.1 Equations

Assuming an incompressible flow, we apply the integral mass conservation equation (Eq. 2.1) over the whole geometry ($C_1$ and $C_2$):

$$u_g S_d - u_2 S_d = u_p S_p$$

where $S_d$ and $S_p$ are respectively the duct and pipe cross section. Similar we apply mass conservation over the turbulent mixing region:

$$u_j S_j = u_p S_p.$$  

From the definition of the vena contracta factor $\Gamma = \frac{S_j}{S_o}$ and mass conservation over the jet region, we have for an incompressible flow: $u_j = \frac{1}{\Gamma} u_b$ where $u_b = u_p \frac{S_p}{S_o}$, $S_o$ the orifice cross
section, $u_j$ the maximum velocity at the jet which occurs at the position where the jet flow reaches its minimum cross section $S_j$.

Bernoulli is applied along a streamline that enters the duct and leaves the duct:

$$ p_1 + \frac{1}{2}\rho u_j^2 = p_2 + \frac{1}{2}\rho u_2^2 $$

(8.3)

where $p_1$ and $p_2$ are the pressure at the inlet respectively the outlet of the duct.

Bernoulli is also applied along another streamline of flow entering the pipe through the orifice, until the position of minimum jet cross section (common border of region $C_1$ and $C_2$):

$$ p_1 + \frac{1}{2}\rho u_j^2 = p_j + \frac{1}{2}\rho u_j^2 $$

(8.4)

Note that this is the same equation as used to estimate the jet velocity during the experiments Eq. 3.4 with $p_1 = p_{atm}$. We apply integral momentum conservation in $y-$direction over the turbulent mixing region (region $C_2$):

$$ \rho u_j^2 S_p - \rho u_j^2 S_j + p_p S_p - p_j S_p = 0 $$

(8.5)

where we neglected friction at the walls. The flow in the pipe has only a component in pipe direction and the flow in the duct only in the perpendicular direction. We also apply integral momentum conservation in $y-$direction over $C_1$ but the integral’s $x-$domain is restricted in correspondence with the orifice cross sectional area.

$$ \rho u_j^2 S_j + p_j S_o = S_o \frac{1}{2}(p_1 + p_2) $$

(8.6)

The part $\frac{1}{2}(p_1 + p_2)$ is an estimate of the average pressure on the duct wall opposite to the orifice (upper wall). This is an ad hoc assumption which appears to be reasonable.

### 8.1.2 Pressure force correction

#### 8.1.2.1 Borda mouthpiece approximation

When applying the last mentioned integral equation of momentum in $y-$direction (Eq. 8.6) on control volume $C_1$, we neglect the pressure difference at the opposite (parts of the) walls in the duct upstream and downstream from the orifice. We only take into account the pressure force generated by the duct wall opposite to the orifice. This is an implicit implementation of the pressure conditions that holds for a Borda mouthpiece [37]. We explain this similarity. A Borda mouthpiece consists of an orifice in a plate that is surrounded by two long walls that are orthogonal to the plate (see Fig. 8.2 (a)). We consider first a pure bias flow. If flow is sucked through the orifice, the fluid at the left corner around point Q (see Fig. 8.2 (a)) and the fluid at the right corner around point R, has negligible velocity. This implies no pressure variation along the lower wall. But when we approach the orifice, the pressure variation is present at the lower duct wall (see Fig. 8.2 (b)). For a normal orifice in a duct with bias and grazing flow, the pressure variation left and right of the orifice at the bottom duct wall, is larger (because of the acceleration inwards the orifice) than the pressure drop at the opposite upper duct wall (caused by the grazing flow along the upper duct wall). A sketch of the streamlines for this case is shown in Fig. 8.2 (c). The difference between those pressure drops (in the presence of grazing flow) equals the difference between a Borda mouthpiece and a normal mouthpiece in the absence of grazing flow.
8.1.3 Momentum equation

The Borda mouthpiece model predicts for pure bias flow exactly $\Gamma_{Borda} = 0.5$. The present model also predicts $\Gamma = 0.5$ in the limit $M_b >> M_g$. To describe a flow through a real orifice with a higher vena contracta factor, we modify above presented model empirically by using the experimental obtained vena contracta factor $\Gamma_{bias}$ for a pure bias flow. Below we explain how.

Consider in the sketch in Fig. 8.3 the grey box around the orifice, representing the boundary for the integral $y-$momentum equation Eq. 8.6. The $x-$domain is limited to the orifice with cross sectional area $S_o$. It is clear that with this boundaries the duct walls left and right are not taken into account, causing the problem of the prediction of $\Gamma = 0.5$ that is too low. Call the experimental obtained value $\Gamma_{bias}$. For $u_g = 0$, we have $\Gamma_{bias} = S_j / S_o$. $S_o$ is the orifice cross section.

Consider again the sketch in Fig. 8.3 (a) of the Borda mouthpiece for which $S_j = \Gamma_{Borda} S_o$. This is what the model predicts. In order to translate the difference between the model and the experiments to a good prediction of $S_j$, we enlarge the apparent orifice cross section and call it $S_o^*$. The relation is:

$$S_o^* = S_o \frac{\Gamma_{bias}}{\Gamma_{Borda}} \quad (8.7)$$

We now use this cross section instead of the real orifice cross section that bounds the integral domain and find the “**” momentum equation:

$$\rho u_j^2 S_j + p_j S_o^* = S_o^* \frac{1}{2} (p_1 + p_2) \quad (8.8)$$

Since no fundamental change of the model based on this “**” equation is made, the model calculates the vena contracta factor $\Gamma^*$:

$$\Gamma^* = \Gamma_{Borda} = \frac{S_j^*}{S_o^*} \quad (8.9)$$

---

**Figure 8.2:** Representation of the flow through (a) a Borda mouthpiece, (b) an normal orifice subjected to pure bias flow and (c) a normal orifice with orthogonal edges subjected to a bias-grazing flow. Points Q and R are in the region of negligible flow velocities.
where $S_j^*$ is the jet cross section that must match with the experiments. Substitution of Eq. 8.7 in Eq. 8.9 gives:

$$\Gamma^* = \frac{S_j^* \Gamma_{Borda}}{S_o \Gamma_{bias}} = \Gamma_{Borda}$$

(8.10)

so that the model correctly calculates in the limit of $M_b >> M_g$:

$$S_j^* = \Gamma_{bias} S_o.$$  

(8.11)

Figure 8.3: In (a) the vena contracta for a Borda mouthpiece is shown with: $\Gamma_{Borda} = \frac{1}{2} = \frac{S_j}{S_o}$. In (b), the same is done for a normal orifice: $\Gamma_{bias} = \frac{S_j^*}{S_o}$. The apparent orifice cross section is $S_o^* = S_o \frac{\Gamma_{bias}}{\Gamma_{Borda}}$. Using $S_o^*$ instead of $S_o$, the model will calculate $S_j$ in accordance with the experimental obtained value in the limit $M_b >> M_g$.

8.2 Solving the equations

The set of six equations given in Section 8.1.1 are solved for the six unknowns $u_2, u_j, p_2, p_3$, $S_j$ and $p_j$. The parameters $p_1$ and $p_p$ are imposed by boundary conditions. As the model is incompressible, $\rho$ is a constant parameter. $S_o, S_p$ and $S_d$ are the geometrical parameters. The equations are solved by elimination. This leads to a quadratic equation which is solved analytically.

Consider now the same set of equations except for the integral momentum equation over the jet region and exchange this with Eq. 8.8. This new set is solved by means of the Newton-Rapson method. The solution that we obtained by elimination for the original set, is now used to obtain the initial values in the numerical calculation. We compare the model with the experiments, we take $S_o = 5 \times 10^{-4} \text{ m}^2$, $S_p = \pi 3.5^2 \times 10^{-4} \text{ m}^2$, $S_d = 0.4 \text{ m}^2$. The latter cross section corresponds with the cross sectional opening of the wind tunnel outlet. Further we use $\rho = 1.2 \text{ kg/m}^3$ and $p_1$ and $p_p$ are chosen in agreement with the bias and grazing velocity of the experimental results.
8.3 Vena contracta factor $\Gamma$

The vena contracta factor $\Gamma = S_j/S_o$ that is calculated by the model is plotted as a function of the absolute value of the bias Mach number $M_b$. The result is shown in Fig. 8.4 for $u_g = 12.8 \text{ m/s}$ (solid line) and $u_g = 16.8 \text{ m/s}$ (dashed line). We used for both curves $\Gamma_{bias} = 0.8$ and $\Gamma_{Borda} = 0.5$ as the empirical value for the correction factor in the momentum equation. Experimentally, the vena contracta is measured for the orifice with orthogonal edges (orifice B) by means of the ratio of the jet velocity and the bias velocity (see Section 3.3.2). For an incompressible flow, this the ratio gives the vena contracta factor: $\Gamma_{exp}(M_b) = \frac{u_b}{u_j(M_b)}$. The experimental steady state vena contracta factor is plotted for $u_g = 12.8 \text{ m/s}$ (symbol □ in Fig. 8.4) and $u_g = 16.8 \text{ m/s}$ (symbol △ in Fig. 8.4). It is found that for $\Gamma_{bias} = 0.8$, the empirical model fits well with the experimental results. We remark that we found experimentally for the pure bias flow, a vena contracta factor $\Gamma_{bias} = 0.85$. While in the wind tunnel experiments the duct is replaced by a free-jet (grazing flow), the model still provides a reasonable fit of the experimental data.

![Graph](image)

**Figure 8.4:** Comparison of the calculated vena contracta factor $\Gamma$ and the experimental obtained $\Gamma$ as a function of the bias Mach number $M_b$ for two different grazing flow velocities. The empirical value in the model $\Gamma_{bias} = 0.8$. The experimental results correspond with orifice B (orthogonal edges).
Chapter 8. Quasi-steady model for an orifice subjected to combined bias-grazing flow

8.4 Quasi-steady acoustic model

As we have seen for the pipe model with pure bias flow, there are several methods to obtain a quasi-steady model for the response of the flow to an acoustic perturbation. We assume plane wave propagation at the inlet (with pressure perturbation $p'_1$) and outlet ($p'_2$) of the duct and the outlet of the pipe ($p'_p$). In the model, valid for low Strouhal number and incompressible flow, the flow region is compact. We neglect the effect of wave propagation and the pressure perturbations are Eq. 2.17:

\[ p'_1 = p_{1+}^+ + p_{1-}^- \]  
\[ p'_2 = p_{2+}^+ + p_{2-}^- \]  
\[ p'_p = p_{p+}^+ + p_{p-}^- \]  

where the subscript $+$ and $-$ are the amplitude of the wave travelling in the positive respectively negative axis-direction. Eq. 2.18 gives the velocity perturbations:

\[ u'_y = \frac{p_{1+}^+ - p_{1-}^-}{\rho c_0} \]  
\[ u'_2 = \frac{p_{2+}^+ - p_{2-}^-}{\rho c_0} \]  
\[ u'_p = \frac{p_{p+}^+ - p_{p-}^-}{\rho c_0} \]

We are not specifically interested in the perturbation in the jet region. Moreover it is physically not correct to assume plane waves in the jet region. Therefore we had also the perturbation: $p'_j$, $u'_j$. This also implies that we must take into account the variation in jet cross section, by introducing the unknown $S'_j$.

We linearise the perturbed set of equations:

\[ p_{1+}^+ S_d - p_{1-}^- S_d - p_{2+}^+ S_d + p_{2-}^- S_d - p_{p+}^+ S_p + p_{p-}^- S_p = 0 \]  
\[ u'_j S_j + u'_j S'_j - p_{p+}^+ S_p - p_{p-}^- S_p = 0 \]  
\[ p_{1+}^+ (1 + \frac{u'_j}{c_0}) + p_{1-}^- (1 - \frac{u'_j}{c_0}) - p_{2+}^+ (1 - \frac{u'_j}{c_0}) - p_{2-}^- (1 - \frac{u'_j}{c_0}) = 0 \]  
\[ p_{p+}^+ (\frac{2 u'_p S_p}{c_0} + S_p) + p_{p-}^- (\frac{-2 u'_p S_p}{c_0} + S_p) - 2 \rho u_j S_j u'_j - \rho u'_j S'_j S_p - p_{1+}^+ S'_j = 0 \]

\[ p_{p+}^+ \left( \frac{2 u'_p S_p}{c_0} + 1 \right) + p_{p-}^- \left( \frac{-2 u'_p S_p}{c_0} + 1 \right) - p_{1+}^+ \left( 1 - \frac{S'_o}{S_p} \right) + p_{p+}^+ \left( 1 - \frac{S'_o}{S_p} \right) + \]

\[ p_{p+}^+ \frac{C_o}{S_p}(c-1) - c S'_o S'_o / S_p p_{1+}^+ - c S'_o S'_o / S_p p_{1-}^- = 0 \]

We assume no incoming waves towards the orifice from the duct inlet and the outlet: $p_{1+}^+ = p_{2+}^+ = 0$). We assume a plane wave in the pipe towards the orifice (positive $y$–direction) with knowns amplitude $p_p^+$ and $p_p^+ << p_p$. The aim is to calculate the response of the flow and the orifice due to this perturbation. In order to do so, we use the solution of
the steady model \((u_j, S_j, u_g, u_p u_2)\) and substitute it in above equations. The geometrical parameters are again \(S_d, S_p, S_o\). The empirical variable \(S_o^*\) corrects for the experimental vena contracta factor. The set of linearised perturbed equation can now be solved for the unknowns: \(S_j^*, p_j^+, p_p^+, p_j'\). The solution of the linear set of equations is obtained by elimination. The explicit expressions of the solution is complex. It does not give much insight in the problem and therefore we do not present them.

From the solution of the unknowns, we calculate the acoustical response by means of the dimensionless one-sided orifice impedance.

\[
Z_{h,\text{in}}/Z_0 = \frac{p_p^+ + p_p^-}{p_p^- - p_p^+} \left( \frac{S_o}{S_p} \right).
\] (8.24)

Note that above expression is in agreement with the measured one-sided orifice impedance by means of the multi-microphone method (Eq. 3.16).

The result obtained by linearisation is checked by means of an alternative method to obtain the linear acoustic response directly from the steady-flow model. Basically this is the same method (perturbing/subtracting of the steady solution), as used for the pure bias quasi-steady model Chapter 4. The agreement of above method and this alternative method show the absence of calculation errors due to the linearisation or the elimination method.

The result of the quasi-steady model is presented in Fig. 8.5 where the scaled dimensionless one-sided orifice impedance \(Z_{h,\text{in}}/(Z_0 M_g)\) is given as a function of the ratio of bias to grazing flow velocity \(u_b/u_g\) for \(u_g = 12.8\, \text{m/s}\) (solid line) and \(u_g = 16.8\, \text{m/s}\) (dashed line). Note that \(u_b = S_p/S_o u_p\) For the empirical parameter we take \(\Gamma_{bias} = 0.8\). To the limit \(u_b >> u_g\), the acoustic resistance that is given by the pure bias model and the bias-grazing model agree. There is no significant difference in the acoustic resistance for the different grazing flow velocities \(u_{g,1}/u_{g,2} = 0.76\). This leads to the presumption that for high velocity ratio \(u_b/u_g\), the response of the perforation to the flow is dominated by the bias flow.

We also see that at low bias flow, the model shows clearly the effect of grazing flow on the resistance. The correctness of this effect will be confirmed in the next chapter by the experiments. For the turbulent boundary layer, the experimental results of the scaled acoustic resistance agree well with the model at moderate ratio of bias-to-grazing flow velocity.

### 8.5 Chapter conclusion

We considered a wall perforation in a duct-pipe configuration. The orifice is subjected to a grazing flow (in the duct) and the bias flow (through the orifice). The steady flow is modelled by means of integral momentum equations, it is an extension to the pure-bias model. We used an empirical parameter based on the vena contracta factor of a pure bias flow which able to correct for the pressure variation along the lower duct wall. The results for the vena contracta factor compared reasonable with the experimental results for bias Mach numbers \(M_b < 0.07\).

The quasi-steady incompressible model is obtained from the response to perturbation of the steady flow. We concluded that the grazing flow has a significant increase in the resistance for low \(M_b\). For higher bias flow, the bias flow dominates in the acoustic resistance. We will confirm this by in the next chapter, based on the experimental results.
Figure 8.5: Experimental results for the scaled acoustic resistance at low Strouhal number as a function of the ratio of bias to grazing flow velocity $u_b/u_g$. The data are obtained for the orifice with orthogonal edges (B) at $u_g = 12.8 \text{ m/s}$ (squares) and at $u_g = 16.8 \text{ m/s}$ (triangles). The lines represent the results for the incompressible quasi-steady bias-grazing model at different grazing flow velocities (solid line and dashed line) with constant vena contracta factor $\Gamma_{bias} = 0.8$, in accordance with the experimental vena contracta factor found (see Fig. 8.4).
Chapter 9

Experimental result of high Strouhal behaviour for a combined bias-grazing flow

Where we discussed in the previous chapters the experimental results for a pure bias or a pure grazing flow, we here consider the combination of both flows. The results are presented for most graphs by means of the scaled resistance and reactance based on the grazing flow as defined in the introduction of Chapter 7. We use in the graphs the short notation for the pure grazing flow: \( \vec{U} = (u_g, u_b) \) where \( u_b \) is negative for inflow and positive for outflow. In the legend of the graphs, the arrow of \( \vec{U} \) is not shown.

9.1 Vena contracta

A first aspect is the effect of grazing flow on the contraction of the flow at the orifice and the direction of the jet. The vena contracta factor for the combination of grazing flow and bias inflow is measured on a similar way as for the pure bias. The direction of the jet will be influenced by the ratio of both flows. This effect is not studied in this research. However, a visual study of the grazing flow over a resonator orifice done by Baumeister [17] shows that for an orifice with orthogonal edges (orifice B), the jet flow is almost vertical and follows the downstream side wall of the orifice. The result of \( \Gamma \) as a function of \( u_g \) can be found in the appendix separately for the laminar and turbulent boundary layer (Fig. E.2 and Fig. E.3). We present in Fig. 9.1 the vena contracta factor \( \Gamma \) as a function of the ratio \( u_b/u_g \). We combine the results for different grazing flow velocities in one graph. This implies that the characteristics of the boundary layer velocity profile are not the same for all points. From the smooth increase of the curve, we conclude that \( \Gamma \) is mainly dependant on the ratio \( u_b/u_g \) and that the velocity profile of the boundary layer is of minor importance.
9.2 Effect of adding bias inflow

We show the influence of adding a bias inflow to a grazing flow by means of the measurements for orifice A. Orifice A is most suitable to show the effect of “sucking at the shear layer” because it has a more constant resistance for the pure bias flow compared with the other orifices, as we have seen in Fig. 6.1. In Fig. 9.1, the one-sided orifice impedance scaled with the grazing flow parameters, is plotted as a function of the Strouhal number for $u_g = 12.8\text{ m/s}$. In Fig. 9.3, similar measurements are plotted for the turbulent boundary layer ($u_g = 16.8\text{ m/s}$ and the turbulence in the approaching boundary layer is triggered by the use of sandpaper). It is shown that for the turbulent as well as for the laminar boundary layer, adding a bias
Chapter 9. Experimental result of high Strouhal behaviour for a combined bias-grazing flow

Figure 9.2: Acoustic resistance scaled with the grazing Mach number, $\tilde{r}_g$, as a function of the Strouhal number based on the grazing velocity $u_g$ and orifice width $w$ for orifice A (sharp edges) and for different velocities of the bias inflow $u_b < 0$. The approaching grazing flow boundary layer is laminar.

Figure 9.3: Acoustic resistance scaled with the grazing Mach number $M_g \tilde{r}_g$ as a function of the Strouhal number based on the grazing velocity $u_g$ and orifice width $w$ for orifice A (sharp edges) and for different velocities of the bias inflow $u_b < 0$. The approaching grazing flow boundary layer is turbulent.
flow increases the critical Strouhal number at which the resistance displays a minimum. This corresponds to the first hydrodynamic mode. Additionally it is seen that sucking at low bias velocity, \( u_b < u_g/2 \) (only measured for laminar boundary layer), the peaks of sound production \( (Sr \approx 0.4 – 0.5) \) also increase in amplitude. For higher bias velocity, the first mode almost vanishes and the orifice is absorbing sound over the whole frequency range. This effect holds for the laminar as well as for the turbulent boundary layer. For low bias velocity, we expect that the shear layer over the orifice due to the grazing flow is still dominating the acoustical response. For high bias velocity, the shear layer moves away from the downstream edge and a distinct jet flow is formed. The jet flow is dissipative as it is seen for pure bias flow.

In Chapter 6 we concluded that the low Strouhal resistance scales with the orifice Mach number \( M_b \). We notice that this is still valid for a combined flow.

### 9.3 Orifice with orthogonal edges (B)

The difference between blowing or sucking through an orifice with orthogonal edges (orifice B), in combination with a grazing flow, is shown in Fig. 9.4. Orifice B is subjected to a grazing flow with velocity \( u_g = 16.8 \text{ m/s} \). The bias flow is \( |u_b| \approx 0.4 \cdot u_g \).

Similarly as concluded for orifice A, adding a bias flow increases the absorption at low Strouhal number. For inflow this increase is larger than for outflow. Also, additional oscillations of the resistance as a function of the Strouhal number, appear for bias inflow as well as for outflow compared to pure grazing flow. This is due to interactions of the vortices with the edges on both sides of the orifice in bias flow direction. As it is known from the visual study done by Baumeister [17], there is for sufficiently high bias outflow a recirculation of flow along the lower duct wall (Fig. 2.3). It is clear that outflow and inflow results in a completely different behaviour. Note the drastic difference in critical Strouhal number for the maxima and minima in the resistance.

### 9.4 Orifice with chamfered edges (G)

Orifice G (chamfered edges at grazing flow side) is tested at \( u_g = 16.8 \text{ m/s} \). The bias flow is now of the same order as the grazing flow \( |u_b| \approx u_g \). The results is shown in Fig. 9.5. The resistance is small in amplitude for an orifice with chamfered edges at the grazing flow side, compared with orifice B with sharp or orthogonal edges. We concluded this in Chapter 7.3. We observe that chamfered edges in combination with a bias outflow increases the oscillations in the resistance. This is similar to what we found for a pure bias flow where rounding the edges at the bias downstream side (with respect to the bias flow) increases the whistling potential because of the reduction of absorption due to secondary vortex shedding. For bias inflow combined with grazing flow, chamfering the edges leads to a similar behaviour as an orthogonal edged orifice, subjected to the same flow configuration. From Appendix 1 Fig. 4.8 it can be seen that chamfered edges at lower velocity ratio of bias to grazing flow, display a similar behaviour. For the chamfered orifice G, the oscillations of the acoustical resistance for inflow and outflow appear at the same critical Strouhal numbers. Note that this was certainly not the case for orifice B (Fig. 9.4).
Chapter 9. Experimental result of high Strouhal behaviour for a combined bias-grazing flow

Figure 9.4: Acoustic resistance for an orifice with orthogonal edges (B), subjected to pure grazing flow, combined bias inflow-grazing flow ($u_b < 0$) and combined bias outflow-grazing flow. The ratio $|u_b|/u_g \approx 0.4$.

Figure 9.5: Acoustic resistance for an orifice with chamfered edges at grazing flow side (G), subjected to pure grazing flow, combined bias inflow-grazing flow and combined bias outflow-grazing flow. The ratio $|u_b|/u_g \approx 1$.

9.5 Orifice oblique in streamwise grazing flow direction (E)

I Fig. 9.6 similar data for orifice E are shown for a pure grazing flow with velocity $u_g = 16.8 \text{ m/s}$ or a combined bias-grazing flow with $|u_b| \cong u_g/4$. Sucking (inflow) does not change
the acoustic behaviour much. For blowing additional oscillations in the impedance are seen at low Strouhal numbers. But almost over the whole range of measured Strouhal numbers, the orifice is absorbing sound for this relatively low bias flow. Increasing the bias outflow, the oscillations vanish and a more or less constant absorption is observed. This can be seen in Fig. 9.9 or in Appendix F Fig. F.4.

Figure 9.6: Acoustic resistance for an oblique orifice in streamwise grazing direction (E), subjected to pure grazing flow, combined bias inflow-grazing flow and combined bias outflow-grazing flow $u_b > 0$. The ratio $|u_b|/u_g \approx 0.25$.

9.6 Effect of chamfering the edges

We discuss now in more detail the effect of chamfering the edges for combined grazing-bias outflow for a velocity ratio $|u_b|/u_g$ of the order unity. The acoustic resistance is plotted for orifice B and G as a function of the grazing flow parameters. The result is shown in Fig. 9.7. Apparently, there is no common $Sr_g$ for the first hydrodynamic mode (orifice B has width $w = 0.01\,m$, orifice G has width $w = 0.016\,m$). We plot now in Fig. 9.8 the scaled orifice impedance as a function of the Strouhal number based on the bias flow $Sr_b = fd/u_b$ where $d$ is given in Table 3.2. We scale the resistance with the bias Mach number. Using this dimensionless representation for both orifices, the critical Strouhal number at maxima and minima in the impedance coincide much better. A strong peak of sound production is observed around $Sr_b = 0.2$.

We conclude that for a sufficiently high bias flow $|u_b| \geq u_g$, the effect of the shear layer due to the grazing flow becomes negligible. The effect of the bias flow, as it is discussed in Chapter 6, dominates. The amplitude of the peaks in the production and absorption of sound are larger for a combination of bias outflow and grazing flow, than for a pure bias outflow (see Fig. 6.3).
Chapter 9. Experimental result of high Strouhal behaviour for a combined bias-grazing flow

\[ Sr_g = \frac{fw}{u_g} \]

Figure 9.7: Comparison of an orifice with orthogonal edges (B) and chamfered edges subjected to a bias outflow \( u_b > 0 \).
9.6 Effect of chamfering the edges

Figure 9.8: Comparison of the impedance of an orifice with orthogonal edges (B) and chamfered edges subjected to a bias outflow. The acoustic resistance is now scaled with the bias Mach number $M_b = |u_b|/c_0$ and given as a function of the Strouhal number based on the bias flow: $Sr_b = f d/u_b$. For orifice B, $d = 0.015$ m and for orifice G, $d = 0.012$. 
Chapter 9. Experimental result of high Strouhal behaviour for a combined bias-grazing flow

9.7 Effect of orifice angle, inflow

We compare the resistance for three values of the perforation angle with respect to the grazing flow direction and for bias inflow. The upstream edge with respect to the grazing flow of orifice E is obtuse (150°), its downstream edge makes an acute (30°) angle with respect to the grazing flow. Orifice F is the reversed of Orifice E and so F has an acute upstream edge and an obtuse downstream edge. Orifice B has two equal and orthogonal edges. Measurement results for these orifices are presented for an equal grazing flow and similar bias flow. The velocity ratio $|u_b|/u_g$ is of the order unity.

Orifice E displays an almost constant absorption of the order $\tilde{r} \approx 1$ for $Sr_g < 1$. Orifice F has a higher absorption over this Strouhal domain. The main difference with orifice E is that F has in the low Strouhal limit, a higher acoustic resistance than at moderate values of $Sr_g$, except for the first measure point (at 31 Hz).

Orifice B displays a higher low Strouhal absorption than orifice E and lower than orifice F. At higher Strouhal numbers, $Sr_g > 0.1$, orifice B displays strong oscillations in the resistance as a function of the Strouhal number.

![Figure 9.9: Comparison of acoustic resistance for orifices with different perforation angle with respect to the grazing flow and subjected to combined bias inflow-grazing flow. Orifice E has an upstream angle of 150°. Orifice F has an upstream angle of 30° and orifice B has an upstream angle of 90°.](image)

9.8 Effect of orifice angle, outflow

A similar graphs as Fig. 9.9 is presented for outflow $u_b > 0$ in Fig. 9.10. We observe that orifice E has an increased resistance at low Strouhal numbers. This resistance measured with outflow is of the same order as the low Strouhal resistance for orifice F with inflow. We expect that for orifice E with inflow and for orifice F with outflow, the bending of the flow
through the orifice that is perforated in the anti-streamwise direction, has a strong impact on
the acoustic resistance at low Strouhal numbers.
It is also clear that oscillations in the resistance due to the bias flow outflow are much more
important for an orthogonal edged orifice (B) than for an oblique orifice. For orifice B we
observe potential whistling around a critical Strouhal number. This does not occur for orifices
E and F.
Note that orifice E is absorbing sound over a relatively large Strouhal number range (0 to 1).
For this orifice we observe a distinct peak in the absorption at high Strouhal number around
$S_{r_g} = 0.8$. The maximal resistance is twice the low Strouhal number resistance. A similar
behaviour was observed for orifice A (sharp edges).

![Figure 9.10: Comparison of acoustic resistance for orifices with different perforation angle with respect to the grazing flow and subjected to combined bias outflow-grazing flow. Orifice E has an upstream angle of 150°. Orifice F has an upstream angle of 30° and orifice B has an upstream angle of 90°.](image)

**Figure 9.10:** Comparison of acoustic resistance for orifices with different perforation angle with respect to the grazing flow and subjected to combined bias outflow-grazing flow. Orifice E has an upstream angle of 150°. Orifice F has an upstream angle of 30° and orifice B has an upstream angle of 90°.

## 9.9 Chapter conclusion

For a combined bias inflow $u_b < 0$ and grazing flow, the critical Strouhal number correspond-
ing with the first hydrodynamic mode of the shear layer, increases with increasing magnitude
of the bias inflow. For a turbulent boundary layer and for a sufficiently high velocity ratio of
$|u_b|/u_g$, the peaks in the sound production decrease in amplitude. Bias inflow increases the
absorption of incoming sound in the limit of low Strouhal numbers. For bias outflow $u_b > 0$
this increase is strongly dependant on the geometry and the angle of perforation.
Adding a bias flow can result in additional oscillations in the resistance as a function of
the Strouhal number based on the grazing flow velocity. This is similar to the potential of
whistling for a pure bias flow.
There is a critical velocity ratio of bias to grazing flow, for which above the acoustic behaviour
at Strouhal number of order unity is dominated by the effect of the bias flow. This ratio depends on the geometry.
The vena contracta factor for inflow is observed to be independent of the velocity profile of the approaching grazing flow boundary layer.
In general blowing (bias outflow) results in much more complicate acoustical behaviour than sucking (bias inflow), in combination with a grazing flow
Chapter 10

Conclusions

10.1 Goal

Many acoustical silencers rely on sound absorption by perforated walls. For most wall perforations, the main cause of damping is the interaction of a flow with the acoustical field at the perforation. This interaction has been studies experimentally for seven slit shaped perforations. We have investigated the influence on the sound absorption of the ratio of bias flow to grazing flow as a function of the Strouhal number. A quasi-steady theoretical model is presented for pure bias flow. For one particular geometry the model is extended to a combined bias inflow and grazing flow at moderate bias-to-grazing velocity ratio.

10.2 Conclusions

Measurement method

The resistance, i.e. real part of the impedance, is measured in order to quantify the sound absorption. For this purpose we used a multi-microphone method in combination with the impedance tube measurements. The method has been tested on a closed wall pipe termination. We found in the frequency range $30 \, \text{Hz} \leq f \leq 850 \, \text{Hz}$ that the measurements reproduce an order of magnitude better than the systematic error that is observed using the closed-wall test measurement. Results for a perforation with sharp edges subjected to a pure grazing flow, agree with earlier measurements of Kooijman [21].

Effect of turbulence

The turbulence in the approaching grazing flow boundary layer has a strong effect on the acoustic performance and depends on the geometry of the perforation. The conclusions concerning this effect can be found in Chapter 7. For the study of perforations subjected to bias-grazing flow, we have focused on the turbulent case since turbulence will prevail for most technological applications. In our experiments, the low Strouhal number limit of the acoustical resistance is not strongly influenced by the transition from laminar to turbulent boundary layer.

Influence of bias flow

At low Strouhal numbers, adding a bias flow significantly enhances sound absorption
compared with a pure grazing flow. Depending on the geometry and for Strouhal numbers of the order unity (based on the orifice thickness or width), strong oscillations in the aoustical resistance as a function of the Strouhal number are observed. These oscillations can be an order of magnitude larger than the low Strouhal number resistance. In some cases, depending on the geometry and the ratio of bias to grazing velocity, negative resistance is observed. This is an indication for the potential of such perforations to drive whistling (self noise with pure harmonic oscillation).

Globally, sucking displays less complex behaviour than blowing through the perforation. The rough-and-ready rule for sucking is that the bias-to-grazing velocity ratio must be of the order of unity to retrieve a fair absorption over a large range of Strouhal number. The here mentioned influence of the bias flow is a summary of the conclusions formulated for a pure bias flow regime in Chapter 6 and for the combined bias-grazing flow in Chapter 9.

Influence of geometry

Perforations used in industry have a typical width-to-plate thickness ratio of the order of unity. Depending on the application, the perforation angle is right or oblique with respect to the plate. This angle is an important parameter in sound absorption. The edges can be rounded or chamfered, due to the manufacturing process, erosion or deposition of dirt. We observed that the sharpness of the edges is crucial. In particular, chamfering the downstream edges leads to an increased potential of whistling in the presence of a pure bias flow. The influence of the geometry is discussed and concluded for a pure bias flow regime in Chapter 6 for the combined bias-grazing flow in Chapter 9. Sharp edges and a thin plate can be used to reduce whistling potential induced by a high bias flow. Such plates are however fragile and will still display whistling for pure grazing flow or low bias-to-grazing velocity ratio. This was concluded in Chapter 7.

Quasi-stationary model

Low Strouhal number operation is the most safe condition for sound absorption at perforations. Quasi-steady models can be used to predict aeroacoustic performance for such conditions. We provide a quasi-steady model for a pure bias flow in which one empirical parameter should be determined. This parameter is associated with the vena- contracta phenomenon of the free jet that is formed in the perforation. In some cases the parameter can be determined from steady-flow measurements (flow resistance). In cases where this determination is not appropriate, the parameter can be determined by acoustical impedance measurements. The detailed conclusions concerning this model can be found in Chapter 4.

We have made a first step for extending such a model to a combination of bias inflow and grazing flow. This has been done for the particular case of wall perforation at right angle and sharp edges where the bias-to-grazing velocity ratio is smaller than 0.5. The results are encouraging. The detailed conclusions concerning the model for a combined bias-grazing flow can be found in Chapter 8.

10.3 Limitations and recommendations

While we have gained significant insight into sound absorption by perforated walls in the presence of a combined grazing/bias flow, some fundamental aspects are left for future research.
We provide here a short list of some of the most important aspects:

- We have limited the study to slit-shaped wall perforations where the thickness of the wall is of the same order as the width of the perforation. It is interesting to compare the aeroacoustic behaviour of the slit-shaped perforations with that of circular holes, which are more commonly used for industrial applications.

- We focused on the fundamental behaviour of one perforation so that we did not considered the possible hydrodynamic interaction between perforations.

- When considering pure bias flow, it appears that compressibility has a significant effect on the acoustical resistance at even moderately high Mach numbers ($M < 0.4$). We ignored compressibility for the combined bias-grazing model.

- Broad-band jet noise (self-noise) was not considered. However, sharp edges, that were found to be favourable for absorption of incoming sound waves, can produce noise of large amplitude. It is recommended to study this noise production and sound absorption in order to find a compromise between mechanical robustness of the perforations, broad-band self-noise production and absorption of incoming sound waves.

- A purely experimental approach is impossible. Analytical models are limited to simplified flow conditions and geometries. There is a need for a numerical model. The next step should be a quasi-steady numerical model.
Acknowledgement

I would like to thank Mico Hirschberg and Devis Tonon for their extensively educational counselling during this project. Also the cooperation with and the advise of the technician of the group is much appreciated. The companionship of the members of “Mesoscopic Transport Phenomena” make it a pleasure for working during the past year, thanks!
Appendices
Appendix A

Details concerning the set-up

Figure A.1: Calibration data of the volume flow measurements for the Dresser turbine flow meter by the use of the Rota-meter.
Figure A.2: Effect of the presence of a DC component in the microphone/reference signal. The remedial of subtracting this component is shown by means of the impedance of orifice A (sharp edges) in the absence of flow.
Figure A.3: Impedance measurements at different sound pressure level in order to see the linear independence of the acoustic velocity. Measured for orifice B, \( w = 10 \text{ mm} \).
Appendix B

Impedance measurements no flow

The no-flow impedance depends on the room and the orifice. It is measured for all orifices and will be subtracted from the impedance measured with flow, describing the effect of the flow to the acoustic behaviour.
Figure B.1: Reference signal (zero flow) for the different geometries
Appendix C

Measurements pure bias inflow and outflow

Measurements are presented for pure bias flow. For orifice A, the scaled resistance/reactance based on the one-sided orifice impedance are presented as a function of the frequency. For all orifices, this is presented as a function of the Strouhal number based on the bias velocity. The data files and dimensions which are listed below each figure.
Orifice A inflow

\[
\tilde{r} = \frac{1}{M b Z_0} \text{Re} \{Z_{h-} - Z_{h-\varnothing=0}\}
\]

\[
\tilde{\delta} = \frac{1}{k_0 w Z_0} \text{Im} \{Z_{h-} - Z_{h-\varnothing=0}\}
\]

\[w = 0.01 \text{ m}\]
\[u_g = 0 \text{ m/s}\]

No-flow measurement \(Z_{h-\varnothing=0}\):

100908_B00_G00_workspace_AC

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Figure C.1: Orifice A inflow
Orifice B inflow

\[ Sr_b = \frac{ft}{u_b} \]
\[ \tilde{r} = \frac{1}{M_b Z_0} \text{Re} \{ Z_{h-} - Z_{h-,u=0} \} \]
\[ \tilde{\delta} = \frac{1}{k_0 w Z_0} \text{Im} \{ Z_{h-} - Z_{h-,u=0} \} \]

\( w = 0.01 \) m  
\( t = 0.015 \) m  
\( u_g = 0 \) m/s

No-flow measurement \( Z_{h-,u=0} \):

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Figure C.2: Orifice B inflow
Orifice C inflow

\[ S_{rb} = \frac{ft}{u_b} \]

\[ \tilde{r} = \frac{1}{M_{b}Z_0} \text{Re}\{Z_{h-u=0} - Z_{h-u=0}\} \]

\[ \tilde{\delta} = \frac{1}{k_0 w Z_0} \text{Im}\{Z_{h-u=0} - Z_{h-u=0}\} \]

\( w = 0.01 \text{ m} \)

\( t = 0.015 \text{ m} \)

\( u_g = 0 \text{ m/s} \)

No-flow measurement \( Z_{h-u=0} \):

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Figure C.3: Orifice C inflow
Orifice D inflow

\[ S_{rb} = \frac{ft}{u_b} \]

\[ \tilde{r} = \frac{1}{M_b Z_0} \text{Re} \{ Z_h - Z_{h,u=0} \} \]

\[ \tilde{\delta} = \frac{1}{k_0 w Z_0} \text{Im} \{ Z_h - Z_{h,u=0} \} \]

\[ w = 0.01 \text{ m} \]
\[ t = 0.015 \text{ m} \]
\[ u_g = 0 \text{ m/s} \]

No-flow measurement \( Z_{h,u=0} \):

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Figure C.4: Orifice D inflow
Orifice E inflow

\[ Sr_b = \frac{fd}{u_b} \]
\[ \tilde{r} = \frac{1}{M_b Z_0} \text{Re} \{ Z_h - Z_{h,u=0} \} \]
\[ \tilde{\delta} = \frac{1}{k_0 w Z_0} \text{Im} \{ Z_h - Z_{h,u=0} \} \]

\[ w = 0.02 \text{ m} \]
\[ d = 0.03 \text{ m} \]
\[ u_g = 0 \text{ m/s} \]

No-flow measurement \( Z_{h,u=0}: \)

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Figure C.5: Orifice E-F inflow
Orifice G inflow

\[ Sr_b = \frac{ft}{u_b} \]
\[ \tilde{r} = \frac{1}{M_b Z_0} \text{Re} \{ Z_{h-} - Z_{h-,u=0} \} \]
\[ \tilde{\delta} = \frac{1}{k_0 w Z_0} \text{Im} \{ Z_{h-} - Z_{h-,u=0} \} \]

\( w = 0.016 \text{ m} \)
\( t = 0.012 \text{ m} \)
\( u_g = 0 \text{ m/s} \)

No-flow measurement \( Z_{h-,u=0} \):

110314_B0022_G000_sweeps_plateG_sp_workspace

List of data files

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<th>( u_b ) [m/s]</th>
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<th>note</th>
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Figure C.6: Orifice G inflow
Orifice G outflow

\[ Sr_b = \frac{ft}{u_b} \]
\[ \tilde{r} = \frac{1}{M_b Z_0} \text{Re} \{ Z_{h^-} - Z_{h-,u=0} \} \]
\[ \tilde{\delta} = \frac{1}{k_0 w Z_0} \text{Im} \{ Z_{h^-} - Z_{h-,u=0} \} \]

\[ w = 0.016 \text{m} \]
\[ t = 0.012 \text{m} \]
\[ u_g = 0 \text{m/s} \]

No-flow measurement \( Z_{h-,u=0} \):

110314_B00_G000_sweeps_plateG_sp_workspace

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Figure C.7: Orifice G outflow
Appendix D

Measurements pure grazing flow

Data files

The data files listed below are used for the measurements presented in this report.

\[
\tilde{r}_g = \frac{1}{M_g Z_0} \text{Re} \{ Z_{h-} - Z_{h-,u=0} \}
\]
\[
\tilde{\delta}_g = \frac{1}{k_0 w Z_0} \text{Im} \{ Z_{h-} - Z_{h-,u=0} \}
\]

\( w = 0.01 \text{ m} \)
\( u_g = 12.8 \text{ or } 16.8 \text{ m/s} \)

No-flow measurement \( Z_{h-,u=0} \):  
100908_B00_G00_workspace_AC

List of data files

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<th>note</th>
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The measurement results are shown in the report in Chapter 7.
Appendix E

Measurements combined bias inflow and grazing flow

E.1 Effect of boundary layer on Vena contracta

Figure E.1: Vena contracta factor for different orifices and boundary layer.
Figure E.2: Vena contracta factor for laminar boundary layer, combined bias-grazing flow

Figure E.3: Vena contracta factor for turbulent boundary layer, combined bias-grazing flow

E.2 Impedance measurements
Figure E.4: Orifice A laminar boundary layer, $u_g = 12.8 \text{ m/s}$
Figure E.5: Orifice A turbulent boundary layer, $u_g = 16.8 \text{ m/s}$
Figure E.6: Orifice B laminar boundary layer, $u_g = 12.8 \text{ m/s}$
Figure E.7: Plate B turbulent boundary layer, $u_g = 16.8 \text{ m/s}$
Figure E.8: Plate C laminar boundary layer, $u_g = 12.8 \text{ m/s}$
\[ r^* = \text{Real}(Z_{- Z_{\text{no flow}}} \div Z_0 M_g) \]

\[ d^* = \text{Im}(Z_{- Z_{\text{no flow}}} \div Z_0 k_0 w) \]

**Figure E.9:** Plate D laminar boundary layer, \( u_g = 12.8 \text{ m/s} \)
Figure E.10: Orifice E laminar boundary layer, $u_g = 12.8 \text{ m/s}$
Figure E.11: Orifice E turbulent boundary layer, $u_g = 16.8 \text{ m/s}$, new measurement, microphone at old position
Figure E.12: Orifice F laminar boundary layer, $u_g = 12.8 \text{ m/s}$
Figure E.13: Orifice F turbulent boundary layer, $u_g = 16.8 \text{ m/s}$
Figure E.14: Plate G turbulent boundary layer, $u_g = 16.8 \text{ m/s}$
Appendix F

Measurements combined bias outflow and grazing flow
Figure F.1: Orifice A outflow, turbulent boundary layer
Figure F.2: Orifice B, turbulent boundary layer
Figure F.3: Orifice G, turbulent boundary layer
Figure F.4: Orifice E, turbulent boundary layer
Figure F.5: Orifice F, turbulent boundary layer
Bibliography


[12] [http://www.engineeringtoolbox.com/air-properties-d_156.html](http://www.engineeringtoolbox.com/air-properties-d_156.html)


[24] [http://www.nist.gov/pml/process/pressure_vacuum/unit_conversions.cfm](http://www.nist.gov/pml/process/pressure_vacuum/unit_conversions.cfm)


