A new model for the MOSFET operating in saturation suitable for distortion analysis

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A NEW MODEL FOR THE MOSFET OPERATING IN SATURATION SUITABLE FOR DISTORTION ANALYSIS

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Abstract

For the purpose of circuit simulation, many different MOS models have been developed over the years. These models can be divided in models for digital applications and models for analog applications.

None of these models is suitable for distortion analysis in analog circuits.

At the Philips laboratories and factories, there exists a need for a model that is suitable for distortion analysis in circuit simulators. Therefore a new MOS model, suitable for circuit simulation, has to be developed.

The aim of this report is to describe a new physically based model for the MOSFET operating in saturation and suitable for distortion analysis in circuit simulators.

This report describes some different ways of modelling the MOSFET in saturation and gives a new model that includes channel-length modulation and describes the drain-voltage dependence of the threshold voltage and is suitable for distortion analysis in circuit simulators. The new model describes the drain-voltage dependence of the MOSFET operating in saturation very well. Howere the model lacks a physical base for describing the gate-voltage dependence, the model describes the gate-source voltage dependence well.

In the future, the model has to be tested for the source-bulk-voltage dependence and for the channel-length dependence.
Acknowledgements

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At the last place I would like to thank all my friends with whom I spend my free time the last year.
Contents

Chapter 1  Introduction  1

1.1 MOS models  1
1.2 Aim and organisation of this report  2

Chapter 2  Introduction to MOSFET physics  3

2.1 The two terminal MOS structure  3
  2.1.1 The ideal two terminal MOS structure  3
  2.1.2 The real two terminal MOS structure  6
  2.1.3 Potential balance  8

2.2 The four terminal MOSFET  9
  2.2.1 The four terminal MOS structure  9
  2.2.2 The linear region  10
  2.2.3 The saturation region  12
  2.2.4 Definition of the surface potential  13

Chapter 3  Channel-length modulation  14

3.1 The effect of channel-length modulation  14
3.2 Physics of the MOSFET in saturation  15
3.3 Different ways of modelling channel-length modulation  17
3.4 The saturated velocity model  18
3.5 The new model  23
3.6 The saturation voltage  25

Chapter 4  The drain voltage dependence of $V_T$  27

4.1 The effect of the drain-voltage dependence of $V_T$  27
4.2 Explanation of the drain-voltage dependence of $V_T$  27
4.3 Different methods of modelling $V_T$  29
  4.3.1 Charge sharing models  29
  4.3.2 Solving $V_T$ by calculation of the surface potential  30
Chapter 5  The Philips MOS9 model
   5.1 Channel-length modulation  34
   5.2 Static feedback  35

Chapter 6  Measurements  36
   6.1 Measurement set-up  36
   6.2 Test devices  37

Chapter 7  Testing the validity of the model  38
   7.1 The test method  38
      7.1.1 The new model  38
      7.1.2 Parameter estimation  39
      7.1.3 Measured coefficients  41
   7.2 The drain-voltage dependence  42
      7.2.1 The MOS9 model  42
      7.2.2 The new model  42
   7.3 The gate-voltage dependence  45
      7.3.1 The MOS9 model  45
      7.3.2 The new model  46
      7.3.3 The modified new model  48

Chapter 8  Conclusions and recommendations  53
   8.1 Conclusions  53
   8.2 Recommendations  54

Literature  55

Appendix A  Values of the fit parameters
# List of most important symbols

<table>
<thead>
<tr>
<th>symbol</th>
<th>description</th>
<th>unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_{ox}$</td>
<td>oxide capacitance per unit area</td>
<td>$\text{F/cm}^2$</td>
</tr>
<tr>
<td>$E$</td>
<td>electric field</td>
<td>$\text{V/cm}$</td>
</tr>
<tr>
<td>$E_{sat}$</td>
<td>electric field at which the velocity of the carriers saturates</td>
<td>$\text{V/cm}$</td>
</tr>
<tr>
<td>$E_{ox}$</td>
<td>electric field in the oxide layer</td>
<td>$\text{V/cm}$</td>
</tr>
<tr>
<td>$E_{ex}$</td>
<td>lateral field at the saturation point</td>
<td>$\text{V/cm}$</td>
</tr>
<tr>
<td>$E_F, E_{Fi}$</td>
<td>Fermi level and intrinsic Fermi level resp.</td>
<td>$\text{eV}$</td>
</tr>
<tr>
<td>$E_{c}, E_{v}$</td>
<td>energy levels of the conduction and valence band resp.</td>
<td>$\text{eV}$</td>
</tr>
<tr>
<td>$I_d$</td>
<td>drain current</td>
<td>$\text{A}$</td>
</tr>
<tr>
<td>$I_{dsat}$</td>
<td>saturated drain current</td>
<td>$\text{A}$</td>
</tr>
<tr>
<td>$L$</td>
<td>distance between source and drain</td>
<td>$\text{cm}$</td>
</tr>
<tr>
<td>$\Delta L$</td>
<td>under diffusion of the drain and source</td>
<td>$\text{cm}$</td>
</tr>
<tr>
<td>$l_d$</td>
<td>channel shortening term</td>
<td>$\text{cm}$</td>
</tr>
<tr>
<td>$l_{c}, l_{t}$</td>
<td>characteristic length defined as in (3-20) and (4-7) resp.</td>
<td>$\text{cm}$</td>
</tr>
<tr>
<td>$N_A$</td>
<td>acceptor concentration in the bulk</td>
<td>$\text{cm}^{-3}$</td>
</tr>
<tr>
<td>$n$</td>
<td>electron concentration</td>
<td>$\text{cm}^{-3}$</td>
</tr>
<tr>
<td>$n_{inv}$</td>
<td>amount of electrons per unit area in the inversion layer</td>
<td>$\text{cm}^{-2}$</td>
</tr>
<tr>
<td>$Q_{inv}$</td>
<td>charge per unit area in the inversion layer</td>
<td>$\text{C/cm}^2$</td>
</tr>
<tr>
<td>$Q_{ox}$</td>
<td>charge per unit area in the oxide layer</td>
<td>$\text{C/cm}^2$</td>
</tr>
<tr>
<td>$Q_B$</td>
<td>charge due to ionised bulk atoms</td>
<td>$\text{C/cm}^2$</td>
</tr>
<tr>
<td>$Q_{Bb}$</td>
<td>part of $Q_B$ induced by the source</td>
<td>$\text{C/cm}^2$</td>
</tr>
<tr>
<td>$Q_{bd}$</td>
<td>part of $Q_B$ induced by the drain</td>
<td>$\text{C/cm}^2$</td>
</tr>
<tr>
<td>$Q_G$</td>
<td>charge per unit area on the gate</td>
<td>$\text{C/cm}^2$</td>
</tr>
<tr>
<td>$V_x$</td>
<td>potential at point x with reference to ground</td>
<td>$\text{V}$</td>
</tr>
<tr>
<td>$V_{xy}$</td>
<td>potential at point x with reference to point y</td>
<td>$\text{V}$</td>
</tr>
<tr>
<td>$V_{FH}$</td>
<td>flat band voltage</td>
<td>$\text{V}$</td>
</tr>
<tr>
<td>$V_{Tx}$</td>
<td>threshold voltage with reference to point x</td>
<td>$\text{V}$</td>
</tr>
<tr>
<td>$V_{dysat}$</td>
<td>saturation voltage at the drain with reference to point y</td>
<td>$\text{V}$</td>
</tr>
<tr>
<td>$V_{bi}$</td>
<td>built-in voltage</td>
<td>$\text{V}$</td>
</tr>
<tr>
<td>$\Delta V_T$</td>
<td>change in the threshold voltage as function of $V_{ds}$</td>
<td>$\text{V}$</td>
</tr>
<tr>
<td>$W$</td>
<td>gate width</td>
<td>$\text{cm}$</td>
</tr>
<tr>
<td>$X_{dep}$</td>
<td>depletion layer width</td>
<td>$\text{cm}$</td>
</tr>
<tr>
<td>$X_{av}$</td>
<td>mean depletion layer width in the velocity saturated region</td>
<td>$\text{cm}$</td>
</tr>
<tr>
<td>$x$</td>
<td>distance in the direction perpendicular to the gate</td>
<td>$\text{cm}$</td>
</tr>
<tr>
<td>$y$</td>
<td>distance in the direction of the channel</td>
<td>$\text{cm}$</td>
</tr>
<tr>
<td>symbol</td>
<td>description</td>
<td>unit</td>
</tr>
<tr>
<td>--------</td>
<td>-------------------------------------------------------------</td>
<td>---------------</td>
</tr>
<tr>
<td>β</td>
<td>current gain</td>
<td>C/V²s</td>
</tr>
<tr>
<td>γ</td>
<td>body constant as defined in equation (2-6)</td>
<td>V¹/²</td>
</tr>
<tr>
<td>δ</td>
<td>body factor as defined in equation (2-7)</td>
<td>1</td>
</tr>
<tr>
<td>θ₁</td>
<td>parameter to bring the gate-source-voltage dependence of μ</td>
<td>V⁻¹</td>
</tr>
<tr>
<td></td>
<td>and the series resistances into account</td>
<td></td>
</tr>
<tr>
<td>θ₂</td>
<td>parameter to bring the source-bulk-voltage dependence of μ</td>
<td>V⁻¹/²</td>
</tr>
<tr>
<td></td>
<td>and the series resistances into account</td>
<td></td>
</tr>
<tr>
<td>θ₃</td>
<td>parameter to bring velocity saturation and the series</td>
<td>V⁻¹</td>
</tr>
<tr>
<td></td>
<td>resistances into account</td>
<td></td>
</tr>
<tr>
<td>μ</td>
<td>carrier mobility at low electric fields</td>
<td>m²/Vs</td>
</tr>
<tr>
<td>Φ₉₈</td>
<td>workfunction difference between the semiconductor</td>
<td>V</td>
</tr>
<tr>
<td></td>
<td>and the metal/polysilicon gate</td>
<td></td>
</tr>
<tr>
<td>φ_F</td>
<td>difference between the intrinsic Fermi level and the Fermi</td>
<td>V</td>
</tr>
<tr>
<td></td>
<td>level</td>
<td></td>
</tr>
<tr>
<td>φₛ</td>
<td>workfunction of the semiconductor bulk</td>
<td>eV</td>
</tr>
<tr>
<td>φ_M</td>
<td>workfunction of the metal/polysilicon gate</td>
<td>eV</td>
</tr>
<tr>
<td>φ_Sb</td>
<td>surface potential with reference to the bulk</td>
<td>V</td>
</tr>
<tr>
<td>φ_ox</td>
<td>potential drop across the oxide</td>
<td>V</td>
</tr>
<tr>
<td>Δφ_ox</td>
<td>change in φ_ox as result of a change in V_gb</td>
<td>V</td>
</tr>
<tr>
<td>Δφ_Sb</td>
<td>change in φ_Sb as result of a change in V_gb</td>
<td>V</td>
</tr>
</tbody>
</table>

### Physical constants

<table>
<thead>
<tr>
<th>symbol</th>
<th>description</th>
<th>value</th>
</tr>
</thead>
<tbody>
<tr>
<td>q</td>
<td>elementary charge</td>
<td>1.6x10⁻¹⁹ C</td>
</tr>
<tr>
<td>εₛᵣₛ</td>
<td>permittivity of Si</td>
<td>11.7ε₀ F/cm</td>
</tr>
<tr>
<td>εₒₓ</td>
<td>permittivity of SiO₂</td>
<td>3.9ε₀ F/cm</td>
</tr>
<tr>
<td>εₒ</td>
<td>permittivity of free space</td>
<td>8.85x10⁻¹⁴ F/cm</td>
</tr>
</tbody>
</table>
1. Introduction

1.1 MOS models

Since the beginning of MOS technology, a lot of different MOSFET models, suitable for circuit analysis with the help of a circuit simulator, have been developed. These models can be divided into models suitable for digital circuit analysis and models suitable for analog circuit analysis.

For digital applications, the model, describing the DC current behaviour, does not need to be very accurate, since only two signal levels are of importance and a short simulation time is desired.

The models for circuit simulation of analog circuits have to be more accurate than the models for digital applications.

In analog circuits there exist near the desired ground harmonic also higher-order harmonics. These higher-order harmonics result in harmonic distortion.

The higher-order harmonics in an analog circuit can be expressed in the higher-order derivatives of the currents through the devices in the circuit to the terminal-voltages. So the distortion in a circuit can be analysed if the model used for circuit analysis gives a good description of the higher-order derivatives of the device currents.

The current models for MOSFETs in analog circuit analysis, are suitable for. Since there is a need for models that can be used for distortion analysis, a new MOSFET model has to be developed, suitable for distortion analysis in circuit simulations.
1.2 Aim and organisation of this report

The aim of this work is to develop a new model, suitable for circuit simulation, which predicts the drain current and the first, second and third-order derivative (\( \frac{\partial I_d}{\partial V_{ds}} \), \( \frac{\partial^2 I_d}{\partial V_{ds}^2} \) and \( \frac{\partial^3 I_d}{\partial V_{ds}^3} \)) of a MOS device operating in the saturation region and in strong inversion. For modelling the drain current in the saturation region, channel-length modulation and the dependence of the threshold voltage on the drain-voltage are very important effects that have to be taken into account in the model. Therefore, an extensive literature study on these effects has been carried out. For both effects a new model has been developed that can be implemented in the basic equation of the Philips MOS9 model.

This report starts with an introduction to MOSFET physics in chapter 2. Chapter 3 consists of a short description of different models for channel-length modulation found in the literature, followed by a description of the new model for channel-length modulation. Next, in chapter 4, a short overview of different methods for modelling the threshold voltage will be given followed by a description of the new threshold voltage model. To make a comparison of the new model with the Philips MOS9 model possible, a description of the concerning part of the MOS9 will be given in chapter 5. Chapter 6 gives a description of the measurement set-up. In chapter 7 the validity of the new model for channel-length modulation and the drain-voltage dependence of the threshold voltage will be tested with the help of the basic equation of the MOS9 model. Also a comparison of the complete MOS9 model with the new model will be given. The conclusions and some recommendations are finally given in chapter 8.
2. Introduction to MOSFET physics

In this section we will give a short description of MOSFET physics and the relationship between the DC voltages and currents. For more detailed information on MOS transistors, the reader is referred to the more comprehensive text books on MOSFETs [1]

2.1 The two terminal MOS structure

The two terminal MOS (Metal-Oxide-Semiconductor) structure is formed by a semiconductor (Si) bulk with on top of it an oxide (SiO₂) layer with a metal or poly silicon contact as shown in figure 2-1 (a). This contact is usually called the gate contact. The bulk is contacted at the bottom with a contact called bulk. The terminal voltages are called gate-voltage ($V_g$) and bulk-voltage ($V_b$) respectively.

2.1.1 The ideal two terminal MOS structure

In the ideal two terminal MOS structure there is no charge contained in the oxide layer and there is no difference between the workfunction of the bulk material and the workfunction of the gate material.

Figure 2-1 (b) shows the energy band diagram of the ideal two terminal MOS structure with zero gate-bulk-voltage ($V_{gb}=V_g-V_b=0$). Here a p type substrate was assumed.
By applying a non-zero voltage $V_{gb}$ between the gate and the bulk, the energy levels will, depending on the sign of the voltage, bend up or down at the interface between the semiconductor and the oxide ($\text{SiO}_2$-interface).

Depending on the direction and the strength of the bending we can distinguish three cases:

- accumulation
- depletion
- inversion

Accumulation occurs when a negative gate-bulk-voltage is applied. This negative voltage results in a negative charge on the gate metal, which has to be counteracted by a positive charge in the semiconductor, to maintain charge neutrality. This charge is formed by the accumulation of holes at the Si-$\text{SiO}_2$ interface which come from the bulk. The accumulation of these positive charged holes results in an upward bending of the energy levels at the interface. Since in accumulation the layer just below the Si-$\text{SiO}_2$ interface contains more holes than the bulk does, this layer behaves more p-type like than the bulk.

For the functioning of the MOS transistor the accumulation condition is of no importance.
When a small positive gate-voltage is applied, a small positive charge is formed on the gate metal. To maintain charge neutrality again, now a small negative charge has to be formed in the semiconductor just below the Si-SiO$_2$ interface.

At small gate-voltages, this charge is formed by depleting the acceptor atoms of the semiconductor just below the Si-SiO$_2$ interface, so that ionised acceptor atoms are left behind, forming a depletion layer. For this reason, this condition is called depletion.

Figure 2-2 MOS structure in depletion (a) structure (b) band diagram

Figure 2-2 shows the structure (a) and the band diagram (b) of the MOS structure in depletion. As we can see in this figure, the intrinsic Fermi level bends towards the Fermi level at the surface.

At higher gate-voltages, the bending of the energy bands can become so strong, that the Fermi level at the interface lays below the intrinsic Fermi level (see Figure 2-3). Now an n-type like layer is formed just below the oxide layer. This layer is called inversion layer or channel. When this happens, one speaks about inversion.
At even higher gate-voltages, the band bending is so strong that at the Si-SiO₂ interface the intrinsic Fermi level lays as far below the Fermi level as it lays above the Fermi level far away from the surface, see also figure 2-4. Now one speaks about strong inversion. If the gate-voltage further increases, the banddiagram will stay the same, since the inversion layer shields the semiconductor bulk from the influence of the gate-voltage. Only the charge in the inversion layer increases with increasing gate-voltage in strong inversion. The depletion charge and with it the depletion layer width approximately stay the same.

2.1.2 The real two terminal MOS structure

In a real MOS structure there is a difference between the workfunctions of the gate material and the bulk material. This workfunction difference results in a storage of charge in the semiconductor at the Si-SiO₂ interface with zero applied gate bias. This charge can be positive or negative, depending on the materials used. In terms of the banddiagram, this means that there is some bandbending with zero applied gate-bulk bias.
Furthermore, in a real structure there exists some charge in the oxide layer. This charge has to be counteracted by an induced charge in the semiconductor, also resulting in some bandbending.

So in a real MOS structure there is in general some bandbending with zero applied gate-bulk-voltage.

At a certain gate-voltage, the nett charge in the semiconductor will be zero. Then the banddiagram will look the same as that of the ideal structure with zero applied gate-voltage, as shown in figure 2-1 (b). Therefore this case is called the flatband condition and the gate-voltage at which this happens is called the flatband voltage, with symbol $V_{FB}$.

This voltage may be expressed as [1]:

$$V_{FB} = \Phi_{MS} - \frac{Q_{OX}}{C_{OX}},$$

(2-1)

where $Q_{OX}$ is the charge in the oxide layer per unit area, $C_{OX}$ the oxide capacitance per unit area and $\Phi_{MS} = \phi_s - \phi_m$ is the widely used symbol for the difference between the workfunction of the bulk material ($\phi_s$) and that of the gate material ($\phi_m$). Depending on the materials used, this voltage may be positive or negative.
2.1.3 Potential balance

A gate-voltage different from the flatband voltage will cause a nett charge in the semiconductor as discussed in section 2.1.1 and 2.1.2. Practically all this charge will be contained in the hatched regions in figure 2-5 (a). Outside this region, the semiconductor will be neutral. If we define $\phi$ as the potential somewhere in the MOS structure, we may make a picture as in figure 2-5 (b).

\[ V_{gb} = \phi_{ox} + \phi_{sb} + \Phi_{MS} \]  

where $\phi_{ox}$ and $\phi_{sb}$ are the potential drops across the oxide and across the depletion layer respectively. $\phi_{sb}$ is called the surface potential. In figure 2-4 we can see that this potential equals $2\phi_F$ in strong inversion.

Since $\Phi_{MS}$ has a fixed value for a certain structure, a change in $V_{gb}$ will result in a change in $\phi_{sb}$ and $\phi_{ox}$ as:

\[ \Delta V_{gb} = \Delta \phi_{sb} + \Delta \phi_{ox} \]
### 2.2 The four terminal MOSFET

#### 2.2.1 The four terminal MOS structure

A MOS transistor (or MOSFET = Metal-Oxide-Semiconductor-Field-Effect-Transistor) is formed by adding two \( n^+ \) doped regions to the two terminal MOS structure, as shown in figure 2-6.

![Figure 2-6 four terminal MOSFET](image)

These two regions are called source and drain with terminal voltages \( V_{sb} \) and \( V_{db} \) respectively (\( V_{ab} \) means voltage at point x with respect to the bulk).

When a sufficient high gate-voltage is applied, the semiconductor becomes strongly inverted at the Si-SiO\(_2\) interface, as discussed in section 2.1. The inversion layer is of the same type as the source and drain regions, so electrons can flow from source to drain if a positive drain-source-voltage (\( V_{ds}=V_{db}-V_{sb} \)) is applied. This results in an electric current \( I_d \) flowing from drain to source.
2.2.2 The linear region

If the transistor operates in strong inversion, and \( V_{ds} \) is positive and small, the drain current \( I_d \) can be expressed as [2,4]

\[
I_d = \beta \frac{(V_{gs} - V_{Th}) V_{ds} - \frac{1+\delta}{2} V_{ds}^2}{(1 + \theta_1 (V_{gs} - V_{Th}) + \theta_2 (\sqrt{V_{sb} + 2\phi_F} - \sqrt{2\phi_F}))(1 + \theta_3 V_{ds})}.
\]  

(2-4)

In this equation

\[
\beta = \frac{\mu W C_{OX}}{L},
\]

(2-5)

where \( \mu \) is the electron mobility in the channel at low electric field strengths, \( C_{OX} \) is the oxide capacitance per unit area and \( W \) and \( L \) are the width of the gate and the distance between the source and drain as depicted in figure 2-6 respectively.

Furthermore in equation (2-4) \( \theta_1 \) is a parameter representing the influence of the drain and source series resistances and the effective gate-voltage dependence of the mobility, \( \theta_2 \) is a parameter to take into account the series resistances and the source bulk-voltage dependence of the mobility respectively and \( \theta_3 \) is a parameter to take velocity saturation and the influence of the series resistances into account.

Further

\[
\delta = \frac{0.3\gamma}{\sqrt{V_{sb} + 2\phi_F}}
\]

(2-6)

is the body constant, where the body factor \( \gamma \) equals

\[
\gamma = \frac{\sqrt{2\varepsilon_s q N_A}}{C_{OX}}.
\]

(2-7)

with \( N_A \) the acceptor concentration in the bulk.
One of the most important quantities in equation (2-4) is the threshold voltage $V_{Ts}$.

This voltage can be expressed with respect to the source as [2]

$$V_{Ts} = V_{T0} - \Delta V_T,$$ (2-8)

where

$$V_{T0} = V_{FB} + 2\phi_F + \frac{qN_A X_{dep}}{C_{OX}}.$$ (2-9)

In this equation $X_{dep}$ is the width of the depletion layer formed by the ionised acceptor atoms in the semiconductor just below the oxide. This depletion width can be written as [3]

$$X_{dep} = \sqrt{\frac{2e\phi_{sb}}{qN_A}},$$ (2-10)

where with a non-zero source-bulk-voltage

$$\phi_{sb} = V_{sb} + 2\phi_F.$$ (2-11)

Combining equations (2-7) - (2-11) the threshold voltage can be written as

$$V_{Ts} = V_{FB} + 2\phi_F + \gamma \sqrt{2\phi_F + V_{sb}} - \Delta V_T.$$ (2-12)

With respect to the bulk, the threshold voltage reads

$$V_{Tb} = V_{sb} + V_{FB} + 2\phi_F + \gamma \sqrt{2\phi_F + V_{sb}} - \Delta V_T.$$ (2-13)

The quantity $\Delta V_T$ is due to a reduction of the threshold voltage with increasing drain-source-voltage and decreasing channel-length and is an important quantity in short channel MOSFETs.

This quantity will be discussed in detail in chapter 4.

The threshold voltage is an important quantity for estimating the inversion charge.

With zero drain-source-voltage the inversion charge per unit area can be estimated as

$$Q_{inv} = -C_{OX} (V_{gs} - V_{Ts})$$ (2-14)
To come to the expressions used in this section, the gradual channel approximation has been used.

The gradual channel approximation implies that the inversion layer thickness is infinitesimally thin compared to the depletion layer width.

Further this approximation implies

$$\frac{\partial^2 \phi}{\partial x^2} / \frac{\partial^2 \phi}{\partial y^2} = \infty, \quad (2-15)$$

where $\phi$ is the electrostatic potential in the semiconductor and $x$ and $y$ are the directions perpendicular to the oxide and from source to drain respectively.

### 2.2.3 The saturation region

Equation (2-4) only applies to small drain voltages. Above a certain drain-source-voltage $V_{dssat}$ the current saturates. The mechanism that causes this saturation will be discussed in detail in the next chapter.

In the ideal case the current will remain constant with value $I_{dsat}$. This current can be calculated from equation 2-4 by simply replacing $V_{ds}$ with $V_{dssat}$, so

$$I_{dsat} = \beta \frac{(V_{gs} - V_{Ts})V_{dssat} - \frac{1+\delta}{2} V_{dssat}^2}{(1+\theta_1 (V_{gs} - V_{Ts}) + \theta_2 (\sqrt{V_{sb} + 2\phi_F} - \sqrt{2\phi_F}))(1+\theta_3 V_{dssat})} \quad (2-16)$$

In a real MOSFET, the $I_d-V_{ds}$ characteristic has a small slope beyond saturation. The cause of this slope is amongst others the reduction of the threshold voltage as mentioned in the previous section. Another very important cause of this slope is channel-length modulation. Channel-length modulation will be discussed in detail in chapter 3.
2.2.4 Definition of the surface potential

For calculations in the next chapters, it is necessary to have a good definition of the surface potential. The surface potential with respect to the bulk $\phi_{SB}$ is defined as the potential difference between the semiconductor at the Si-SiO$_2$ interface and the neutral bulk.

Figure 2-7 shows a two dimensional energy band diagram of a short channel MOSFET with positive drain and gate-voltages. In this figure, the definition of the surface potential with respect to the bulk has been given.

*Figure 2-7 two dimensional energy band diagram*
3. Channel-length modulation

In this chapter we will discuss the effect of channel-length modulation and give a description of the physics typical for the MOSFET in saturation. Further more a short overview of the different methods for modelling channel-length modulation will be given. In the last section of this chapter, a qualitative and quantitative description of channel-length modulation will be given.

3.1 The effect of channel-length modulation

As mentioned in chapter 2, the drain current saturates at high drain voltages. In the ideal case, the slope of the current-voltage ($I_D-V_{DS}$) characteristic equals zero in the saturation region. In the real case, the characteristic has a small non-zero slope.

One of the main causes of this slope is channel length-modulation. Channel-length modulation is due to the fact that at high drain voltages the conducting channel pinches off at a point close to the drain, so that the gradual channel approximation no longer applies in the region at the drain side of the pinch-off region. Since the derivation of equation (2-16) is based on the gradual channel approximation, this equation is in fact only valid in the region at the source side of the pinch-off point. This implies that for the validity of this equation the channel-length has to be replaced by an effective channel-length

$$L' = L - I_D(V_{DS})$$

where $L$ is the distance between the source and drain junction and $I_D$ is a channel shortening term, so that in saturation the amplification $\beta$ in equation (2-16) has to be rewritten as

$$\beta' = \beta \frac{L}{L - I_D} = \beta \frac{1}{1 - \frac{I_D}{L}}$$

So in order to describe the current beyond saturation, an expression for $I_D$ has to be found.
3.2 Physics of the MOSFET in saturation

In order to explain some effects for the MOSFET operating in saturation, it is convenient to have some insight in the behaviour of the drain dependence of the surface potential. Figure 3-1 shows the surface potential versus the location in the channel for different drain voltages in a qualitative way. The behaviour of the surface potential as shown in the figure was found by simulating a MOSFET.

As we can see in this figure the surface potential increases with increasing distance from the source and with increasing drain voltage. The increase of the surface potential is strongest close to the drain.

When the drain-source-voltage is high enough, the surface potential at a point \( y_p \) close to the drain junction has a value equal to the effective gate voltage \( V_{gb} \),

\[
V_{gb} = V_{gs} - V_{sd}.
\]

This implies that the potential drop across the oxide equals zero at this point. This also implies that vertical electric field in the semiconductor at the Si-SiO\(_2\) interface \( E_{OX} \) equals zero at \( y=y_p \) [6,7]. \( E_{OX} \) can be written as

\[
E_{OX} = \varepsilon_{ox} \frac{1}{t_{ox}} \{V_{eb} - V_{FB} - \phi_{sb}(y)\},
\]  

**Figure 3-1 surface potential versus distance**
From equation (3-3) it becomes clear that at the source side of \( y_p \), where the surface potential is smaller than the effective gate-source-voltage, \( E_{OX} \) is directed downward and at the drain side of \( y_p \), where the surface potential is larger than the effective gate-source-voltage \( E_{OX} \) is directed upward. At \( y=y_p \) the \( E_{OX} \) changes sign.

The upward directed electric field, at the drain side of \( y_p \) pushes the electrons away from the surface, so that the electrons don’t move near to the surface anymore in this region. To clarify this, figure 3-2 shows in a qualitative way the current path near the drain of a MOSFET with \( V_{ds}>V_{dsp} \).

![Figure 3-2 current path of a MOSFET with high drain source voltage](image)

When the drain-source-voltage increases, \( y_p \) moves in the direction of the source.

The second important effect in the MOSFET operating beyond saturation is velocity saturation.

The high surface potential near the drain results, especially for short channel MOSFETs, in a high longitudinal electric field near the drain.

At a certain drain source voltage \( V_{dssat} \) this field may reach such a high value that the velocity of the electrons very close to the drain junction saturates. This is the onset of saturation. As the drain voltage increases beyond this value, the point at which the electrons reach the saturated velocity moves towards the source.

Figure 3-3 shows in a qualitative way the electron velocity versus location in the channel for different drain voltages.
Numerical calculations [8] show that the point $y_s$ where the electrons reach the saturated velocity lays closer to the source than $y_p$, so $y_s < y_p$.

### 3.3 Different ways of modelling channel-length modulation

The past few decades, a lot of different methods to model channel-length modulation have been published.

First the channel shortening term $l_d(V_{ds})$ was modelled as the distance from drain junction to the point $y_p$ where the voltage drop across the oxide equals zero, since in this area the electrons don't move near to the Si-SiO$_2$ interface anymore.

Later on most authors used the so called two section models to calculate $l_d(V_{ds})$ [7,9-21]. In these models, the channel is divided into two regions when the transistor is biased in saturation. One region of length $l_d$ at the drain side of the channel and a region of length $L-l_d$ at the source side of the channel. In the region near the source, the gradual channel approximation applies. To calculate $l_d$, the region near the drain is treated in different ways by different authors.

Some authors considered the region near the drain as a depletion region (as in a normal pn junction) [7,9]. This implies that the free carriers have to move with an infinite velocity in this region, to maintain a non zero current flow through the channel. From a physical point of view this is impossible.
El-mansy et. al. and Popa [5,10,11] used

$$\frac{\partial^2 V}{\partial x^2} = k \frac{\partial^2 V}{\partial y^2}$$

(3-4)

with $k$ a constant much larger than 1, as boundary between the two regions. In fact this is the definition for the gradual channel approximation if $k$ is infinity.

Most models assume that the free carriers move with the saturation velocity in the region near the drain [12-21]. Since the most recent models are based on this assumption and these models have a good physical base, we will discuss this kind of model thoroughly in the next section, and derive a new expression for $l_d$ based on this assumption.

### 3.4 The saturated velocity model

As mentioned in the last section, the channel can be divided in two regions (some authors use three regions). In the region near the source the gradual channel approximation applies and in the region near the drain the electrons move with the saturation velocity.

In the drain region the electrons in the semiconductor do not necessarily move close to the Si-SiO$_2$, so the problem for solving $l_d$ becomes two dimensional.

Most authors have attempted to solve this problem by applying Gauss's law to a pillbox in the drain region. Various shapes for this pill box and various boundary conditions have been used by different authors. Now we will discuss some of these pillbox shapes and boundary conditions.

Dejentfelt [14] suggested to approximate the shape of the edge of the drain diffusion as circular with radius $r$ and used a Gaussian pillbox to calculate the surface potential in the drain region as shown in figure 3-4. The circular edge of the drain junction implies that all field lines coming from the drain junction are directed in the direction of the radius.
With this pillbox Dejentfelt assumed that the electric field perpendicular to boundary c-d equals zero, since boundary c-d is the extension of the radius of the circle formed by the edge of the drain diffusion, so that the all the field lines are parallel with it. Applying Gauss’s law to the pillbox in the figure results in a relation between the drain voltage and $I_d$, which cannot be solved in an analytical way for $I_d$, so that this method of modelling channel-length modulation is unsuitable for implementing in a computer program for circuit simulation.

El-Banna and El-Nokali [19] used a rectangular Gaussian surface as shown in figure 3-5 to find $I_d$. They assumed that the electrons are spread over a mean depth $X_{av}$.
They used boundary conditions as shown in the figure. $E_{\text{sat}}$ is the electric field at which the free carriers reach the saturation velocity, $E_{\text{ox}}$ is the oxide field, $E_y$ is the electric field perpendicular to boundary $b'-c'$ and $E(y)$ is the field perpendicular to boundary $c'-d$. $E(y)$ is used to take the shape of the depletion edge $x_d(y)$ into account. It was assumed that this edge varies linearly with $y$, so that $E(y)$ can be expressed as

$$E(y) = \frac{qN_A}{\varepsilon_s} (ky + X_d - X_{av}).$$

(3-5)

where $X_d$ is the width of the depletion layer at $y=0$ and $k$ is the slope of the depletion edge $c'-d$. Simulations showed that this assumption is not as bad.

Applying Gauss’s law, solving for the surface potential and putting $\phi_{\text{sat}}(y=ld)=V_{\text{dsat}}+2\phi_F$ yields

$$V_{ds} = V_{\text{dsat}} + \frac{C}{A^2} \left\{ \cosh(AL_d) - 1 \right\} + \frac{1}{A} \left\{ E_{\text{sat}} + \kappa \right\} \sinh(AL_d) - \kappa l_d.$$  

(3-6)

where $C, A$ and $\kappa$ are functions of device parameters, $X_{av}$, saturation current and $k$.

In spite of the simple Gaussian pillbox used, equation (3-6) results in a solution for $l_d$ that has to be solved in an iterative way too and thus cannot be used for circuit simulation.

Another method to bring the shape of the depletion edge into account is to take the Gaussian pillbox a-b-c-d as shown in figure 3-6.
Here, it is assumed, just as El-Banna did, that the depletion edge varies linearly with distance $y$, so that

$$x(y) = X_s + \frac{X_d - X_s}{l_d} y. \quad (3-7)$$

Now the field perpendicular to boundary c-d equals zero.

By applying Gauss's law to the surface we get

$$-\varepsilon \int_{x=0}^{x+x_d-x_s} dx + \varepsilon \int_{y=0}^{y} E_y dx - \varepsilon \int_{x=0}^{x+x_d-x_s} dx + = -q \int_{y=0}^{y} (n + N_A) dx dy, \quad (3-8)$$

Here $n$ is the free electron concentration in the velocity saturated region and $N_A$ is the ionised acceptor concentration. $E_{ox}$ and $E_{sat}$ are the electric fields perpendicular to boundary a-b and a-d respectively and $E_y$ is the electric field perpendicular to b'-c'. The other symbols are as defined in figure 3-6.

Differentiating (3-8) with respect to $y$ and using

$$E_{ox} = \frac{1}{t_{ox}} \{V_{gb} - V_{FHV} - \phi_{sb}(y)\} \quad (3-9)$$

yields

$$\varepsilon \frac{\partial E_y}{\partial y} + \varepsilon \frac{X_d - X_s}{l_d} E_y + \varepsilon \frac{X_d - X_s}{l_d} y \frac{\partial E_y}{\partial y} - C_{ox} \{V_{gb} - V_{FHV} - \phi_{sb}(y)\} =$$

$$-q(N_A X_s + N_A \frac{X_d - X_s}{l_d} y) - C_{ox} (V_{gb} - V_{Th} - V_{dssat}) \quad (3-10)$$
To come to equation (3-10)

\[ x_s \int_{x_s}^{d} \frac{x_d - x_s}{l_d} \frac{qndx}{0} \]  

(3-11)

in equation (3-8) is considered as the total amount of charge in a sheet that extends in the x and the z direction somewhere in the velocity saturated region. Since the total current has to be the same everywhere in the velocity saturated region, this charge is the same at every \( y>0 \), so that it can be calculated at \( y=0 \).

At \( y=0^- \), the gradual channel approximation still applies, so that the total charge in a sheet at \( y=0^- \) (denoted as \( q_{ninv} \) here) can be approximated as (see also equation (2-13))

\[ q_{ninv} \approx C_{ox} (V_{gb} - V_{tB} - V_{dsmat}). \]  

(3-12)

Since the charge at \( y=0^- \) has to be equal to that at \( y=0^+ \), equation (3-12) is equal to (3-11)

Using

\[ E_y = -\frac{\partial \phi_{sb} (y)}{\partial y} \]  

(3-13)

in equation (3-10) yields

\[ \varepsilon_{si} \left( X_s + \frac{X_d - X_s}{l_d} y \right) \frac{\partial^2 \phi_{sb} (y)}{\partial y^2} + \varepsilon_{si} \frac{X_d - X_s}{l_d} \frac{\partial \phi_{sb} (y)}{\partial y} - C_{ox} \phi_{sb} (y) - qN_A \frac{X_d - X_s}{l_d} y = +qN_A X_s - C_{ox} (V_{tB} + V_{dsmat}). \]  

(3-14)

Equation (3-14) is a complicated differential equation that can not be solved for \( l_d \) in an analytical way, so another Gaussian surface has to be used.
3.5 The new model

As follows from the previous methods, it is difficult to derive an analytical solution for $l_d$ that takes the shape of the drain diffusion or the shape of the depletion edge into account. So some simplifications have to be made.

If we use the same Gaussian surface as in figure 3-5, but with the assumption that the electric field perpendicular to boundary c-d equals zero, then we get by applying Gauss’s law to the surface

$$-\varepsilon_s \int_0^{X_w} E_{sat} dx + \varepsilon_{si} \int_0^{X_w} E_x dx - \varepsilon_{ox} \int_0^{y} E_{ox} dy = -q \int_0^{y} \int_0^{X_w} (n + N_A) dx dy. \quad (3-15)$$

$X_{av}$ has to be considered as a fitting parameter now to get a good solution.

Intuitively we can see that this is a good approximation if $X_{av}$ large enough, since than the amount of field lines crossing edge c'-d is much larger then the field lines crossing boundary a-b'.

In fact this is the same as Chow and Feng [20] did, but we will use some modifications here.

In equation (3-15)

$$q \int_0^{X_w} ndx \quad (3-16)$$

is equal to equation (3-12). Differentiating (3-15) with respect to $y$ and using equation (3-9) yields

$$X_{av} \varepsilon_{si} \frac{\partial E_y}{\partial y} - C_{ox} \{V_{gb} - V_{FB} - \phi_{sb} (y)\} = -qX_{av}N_A - C_{ox} (V_{gb} - V_{Tb} - V_{dsat}). \quad (3-17)$$

Using (3-13) and some rearrangements yields

$$\frac{\partial^2 \phi_{sb} (y)}{\partial y^2} - \frac{C_{ox}}{X_{av} \varepsilon_{si}} \phi_{sb} (y) = \frac{qN_A}{\varepsilon_{si}} - \frac{C_{ox}}{X_{av} \varepsilon_{si}} (V_{Tb} - V_{FB} + V_{dsat}), \quad (3-18)$$

which can be written as

$$\frac{\partial^2 \phi_{sb} (y)}{\partial y^2} - \frac{1}{l_{sc}^2} \phi_{sb} (y) = C, \quad (3-19)$$
where

\[ l_c = \frac{\sqrt{X_n \varepsilon_{si}}}{C_{ox}} \quad \text{and} \quad C = \frac{qN_A}{\varepsilon_{si}} - \frac{1}{l_c^2} \left( V_{Th} - V_{FB} + V_{dsat} \right). \quad (3-20) \]

At the onset of saturation, we assume that the channel is still in strong inversion, so that \( \phi_{sb} = V_{dsat} + 2\phi_F \) at \( y = 0 \). Solving (3-18) subjected to the boundary conditions \( \phi_{sb} = V_{dsat} + 2\phi_F \), and \( E_y = E_{sat} \) at \( y = 0 \) yields

\[ \phi_{sb}(y) = \gamma_y^2 \left( V_{dsat} + 2\phi_F + C_{f} + E_{sat} l_c \right) e^{\gamma_y y} + \gamma_y^2 \left( V_{dsat} + 2\phi_F + C_{f} - E_{sat} l_c \right) e^{-\gamma_y y} - C_{f}^2. \quad (3-21) \]

By substituting the value of \( \phi_{sb} \) at \( y = l_d \) now this equation can be solved for \( l_d \).

Beyond saturation, the channel is not in strong inversion any more, so that the difference between the intrinsic Fermi level in the bulk and at the Si-SiO\(_2\) interface equals no longer \( 2\phi_F \).

We define the difference between the intrinsic Fermi level in the bulk and at the Si-SiO\(_2\) interface as \( \phi_m \), where \( \phi_F \leq \phi_m \leq 2\phi_F \).

Now \( \phi_{sb}(y = l_d) = V_{db} + \phi_m \). Substituting this in equation (3-21) and solving for \( l_d \) yields

\[ l_d = l_c \ln \left( \frac{V_{db} + \phi_m + C_{f} + \sqrt{(V_{db} + \phi_m + C_{f})^2 + (E_{sat} l_c)^2 - (V_{dsat} + 2\phi_F + C_{f})^2}}{E_{sat} l_c + V_{dsat} + 2\phi_F + C_{f}} \right). \quad (3-22) \]

Substituting \( C \) back into equation (3-22) yields

\[ l_d = l_c \ln \left( \frac{V_{ds} + k - V_{Ts} - V_{dsat} + \sqrt{(V_{ds} + k - V_{Ts} - V_{dsat})^2 + (E_{sat} l_c)^2 - (k - V_{Ts} + \gamma_F)^2}}{E_{sat} l_c + k - V_{Ts} + \gamma_F} \right). \quad (3-23) \]

Here

\[ k = \frac{qN_A X_n}{C_{ox}} + V_{FB} + \phi_m \quad (3-24) \]

and \( \gamma_F = 2\phi_F - \phi_m \).

We have to notice that equation (3-23) does not equal zero for \( V_{ds} = V_{dsat} \). This is caused by introducing \( \gamma_F \), which has a small value.
This need not to be a problem, since smoothing functions has to be used to connect the equations describing the saturation region with the linear region. These functions can be used in such a way that equation (3-23) is first 'switched' on at a drain-source-voltage somewhat higher than the saturation voltage.

### 3.6 The saturation voltage

So far we have assumed that the saturation voltage is a known constant. In reality, the saturation voltage is a function of the gate voltage.

In our model we will use the same expression for the saturation voltage as the one used in the Philips MOS 9 model [2,4]. For completeness sake, we will give a short description of the derivation of $V_{dsat}$ with some comments to the approximations used.

In the MOS 9 model the saturation voltage is derived as follows:

The inversion charge at the saturation point is approximated by [2,4]

$$Q_{inv} = C_{ox}(V_{gs} - V_{Ts} - (1 + \delta)V_{sat}). \quad (3-25)$$

With this charge, the saturation current can be calculated as

$$I_{dsat} = \beta(V_{gs} - V_{Ts} - (1 + \delta)V_{sat}) \frac{LE_{s}}{1 + \frac{E_{s}}{E_{c}}} \quad (3-26)$$

Where $E_{s}$ is the lateral field at the saturation point and $E_{c}$ is the critical field [2] and in general $E_{s} > 3E_{c}$ [8], so that (3-26) can be approximated as

$$I_{dsat} = \beta(V_{gs} - V_{Ts} - (1 + \delta)V_{sat})LE_{c}. \quad (3-27)$$

At the onset of saturation, this equation is equal to equation (2-4), where $V_{ds}$ has to be replaced by $V_{dsat}$. Solving then for $V_{dsat}$ yields

$$V_{dsat} = V_{c}\left(\frac{2a(V_{gs} - V_{Ts})}{(2a - 1)(1 + \delta)V_{sat}^{2}} - 1\right). \quad (3-28)$$
In equation (3-28)

\[ V_c = \left[ \frac{a}{(2a-1)\theta_3} + \frac{(1-a)(V_{gs} - V_{Ts})}{(1+\delta)(2a-1)} \right]. \]  \hspace{1cm} (3-29)

and

\[ a = [1 + \theta_1 (V_{gs} - V_{Ts}) + \theta_2 (\sqrt{V_{ib} + 2\Phi_F} - \sqrt{2\Phi_F})] \]  \hspace{1cm} (3-30)

and \( \theta_3 = (LEc)^{-1} \).

Equation (3-28) can be rewritten as

\[ V_{dssat} = \frac{2a(V_{gs} - V_{Ts})}{(2a-1)(1+\delta)V_c \theta_3} \left( \sqrt{\frac{2a(V_{gs} - V_{Ts})}{(2a-1)(1+\delta)V_c^2 \theta_3}} + 1 \right)^{-1}. \]  \hspace{1cm} (3-31)

From fitting in the linear region it follows that \( \theta_3 < 0.1 \), so that the second term in equation (3-29) can be neglected.

Substituting equation 3-29 with this neglect into equation 3-31 finally yields

\[ V_{dssat} = \frac{2(V_{gs} - V_{Ts})}{(1+\delta)} \left[ \sqrt{1 + \frac{2\theta_3 (V_{gs} - V_{Ts})}{a (1+\delta)}} + 1 \right]^{-1}. \]  \hspace{1cm} (3-32)

If \( \theta_1 \) is small enough, the term \( (2a-1)/a \) can be approximated by one. Then equation (3-32) becomes

\[ V_{dssat} = \frac{2(V_{gs} - V_{Ts})}{(1+\delta)} \left[ \sqrt{1 + \frac{2\theta_3 (V_{gs} - V_{Ts})}{(1+\delta)}} + 1 \right]^{-1}. \]  \hspace{1cm} (3-33)

With our test devices it turned out that \( \theta_1 = 0.352 \). Strictly speaking \( (2a-1)/a \) cannot be approximated by 1 then, but the difference for the saturation voltage with or without this approximation is negligible.

Equation (3-33) is the same expression as used in the Philips MOS 9 model.
4. The drain voltage dependence of $V_T$

In this chapter we will discuss the effect of the drain voltage dependence of the threshold voltage $V_T$ and give a physical explanation of it. Furthermore, a new equation will be derived to describe the drain voltage dependence of $V_T$.

4.1 The effect of the drain-voltage dependence of $V_T$

In section 2.2.2 we mentioned that the threshold voltage with respect to the source can be written as

$$V_{Ts} = V_{T0} - \Delta V_T(V_{ds}),$$  \hspace{1cm} (4-1)

where $\Delta V_T$ has a positive value.

Looking at equation (2-16) in section 2.2.3, we see that in saturation this results in an increasing drain current with increasing drain voltage. So qualitatively, we can conclude that the decreasing threshold voltage with increasing drain voltage has the same result as channel-length modulation. This also means that these two effects cannot be measured apart, which makes the modelling of these effects more complicated.

4.2 Explanation of the drain-voltage dependence of $V_T$

The easiest way of explaining the drain voltage dependence of the threshold voltage is with the help of energy banddiagrams. Figure 4-1 shows the energy banddiagram of the npn junction formed by the source-bulk-drain junction of the MOSFET with zero applied drain voltage (a) and with a moderately high ($V_{dsm}$) and a high drain voltage ($V_{dsh}$) (b).
As becomes clear from figure 4-1 (b), electrons coming from the source have to surmount an energy barrier when they enter the channel in the bulk region.

If we deal with small channel lengths, this energy barrier may be affected by the drain voltage as the drain voltage becomes larger, as shown in figure 4-1 (b). So an increase of the drain voltage results in a decrease of the energy barrier, which makes it easier for the electrons to enter the channel, so that the actual current becomes larger than expected.

This effect also strongly depends on channel length, since for short channel MOSFETs the drain voltage has much more influence on the source region as is the case for long channel MOSFETs.

An other way of explaining the drain voltage dependence of the threshold voltage can be obtained with the charge sharing principle. This will be discussed in the next section.
4.3 Different methods for modelling $V_T$

4.3.1 Charge sharing models

The first models for $\Delta V_T$ were based on the principle of charge sharing. Though these models give no good results for very short channel lengths [23], the charge sharing principle gives a good insight into what happens with the inversion layer as the drain voltage increases. The charge sharing principle is based on the fact that a part of the field lines ending on the bulk charge below the gate begins in the drain and source depletion regions instead of at the metal gate [24]. Figure 4-2 shows a qualitative distribution in an n-channel MOSFET with zero source-bulk voltage.

![Figure 4-2 qualitative field line distribution in an n-channel MOSFET](image)

The central idea behind the charge sharing principle is that the depletion charge belonging to the drain and source region has to be neglected in the charge balance to calculate the threshold voltage. So that the charge balance can be written as

$$Q_G + Q_{OX} + Q_{inv} + (Q_B - Q_{Bs} - Q_{Bd}) = 0,$$

(4-2)

where $Q_G$ and $Q_{OX}$ are the charge per unit area on the gate and in the oxide respectively. $Q_B$ is the depletion charge under the gate, $Q_{Bs}$ is the part of $Q_B$ belonging to the source and $Q_{Bd}$ is the part of $Q_B$ belonging to the drain.
Now the threshold voltage may be written as equation (2-9), where $Q_B = -qN_A X_{dep}$ has to be replaced by $(Q_B - Q_{Bs} - Q_{Bd})$, so that

$$V_T = V_{FB} + 2\phi_F - \frac{(Q_B - Q_{Bs} - Q_{Bd})}{C_{OX}}, \quad (4-3)$$

where $-(Q_{Bs} + Q_{Bd})/C_{OX} = \Delta V_T$.

An increase of the drain voltage results in an increase of the depletion width of the drain, which results in turn in an increase of $Q_{Bd}$, so that $V_T$ decreases as the drain voltage increases.

### 4.3.2 Solving $V_T$ by calculation of the surface potential

Another method for the calculation of the drain voltage dependence of the threshold voltage is by solving the surface potential and putting the minimum of the surface equal to $\phi_{Sbmin} = 2\phi_F + V_{sb}$ [25,26,27,28] at $V_{gb} = V_{Th}$.

Now we will give a derivation for the threshold voltage which is based on that of Liu et. al. [28] with some modifications to take the source bulk voltage into account.

The first thing to do is to calculate the surface potential. This can be done in the same way as in the case of channel-length modulation in chapter 3, but now we have to consider the whole channel and not only the region near the drain.

Let us start with the transistor operating in the linear region and at the onset of strong inversion for a transistor with uniform substrate doping.

![Figure 4-3 Gaussian pillbox to calculate the surface potential](image-url)
Using the Gaussian pillbox with the boundary conditions as shown in figure 4-3, we get

\[
-\varepsilon_{si} \int_0^{X_{dep}} E_0 \, dx - \varepsilon_{ox} \int_0^{y} E_{ox} \, dy + \varepsilon_{si} \int_0^{X_{dep}} E_s (y) \, dx = -q \int_0^{y} N_A \, dy \, dx. \tag{4-4}
\]

Here \(X_{dep}\) is the mean depletion depth. This means that the field perpendicular to boundary cd equals zero. We will consider \(X_{dep}\) as a fitting parameter. Further the mobile charge in the channel has been neglected. This is justified since the transistor operates at the onset of strong inversion.

Differentiating (4-4) with respect to \(y\) and making use of equation (3-8) results in

\[
\varepsilon_{si} X_{dep} \frac{\partial E_s (y)}{\partial y} - C_{ox} (V_{gb} - V_{FB} - \phi_{sb} (y)) = -q X_{dep} N_A \tag{4-5}
\]

Here \(\phi_{sb}(y)\) is the surface potential with reference to the bulk again.

Making use of equation (3-11) and some rearrangements, (4-5) yields

\[
\frac{\partial^2 \phi_{sb} (y)}{\partial y^2} - \frac{1}{l_t^2} \phi_{sb} (y) = \frac{q N_A}{\varepsilon_{si}} - \frac{1}{l_t^2} (V_{gb} - V_{FB}), \tag{4-6}
\]

where

\[
l_t = \sqrt{\frac{\varepsilon_{si} X_{dep}}{C_{ox}}}. \tag{4-7}
\]

Solving equation (4-6) with the boundary conditions \(\phi_{sb}(0)=V_{sb}+V_{bi}\) and \(\phi_{sb}(L)=V_{db}+V_{bi}\) yields

\[
\phi_{sb} (y) = V_{gb} - V_{T0} + 2\phi_F + (V_{db} + V_{bi} - V_{gb} + V_{T0} - 2\phi_F) \frac{\sinh \left( \frac{y}{l_t} \right)}{\sinh \left( \frac{L}{l_t} \right)} + \frac{\sinh \left( \frac{L-y}{l_t} \right)}{\sinh \left( \frac{L}{l_t} \right)} (V_{sb} + V_{bi} - V_{gb} + V_{T0} - 2\phi_F) \tag{4-8}
\]
For $L > l_t$, which is the case for $L > 0.5 \mu m$, when an implanted layer with $N_A = 10^{17}$ is used, which is the case with our test devices, the following approximations can be made:

$$\frac{\sinh \left( \frac{y}{l_t} \right)}{\sinh \left( \frac{L}{l_t} \right)} \approx \frac{\frac{y}{l_t} - e^{-\frac{y}{l_t}}}{\frac{L}{l_t} - e^{-\frac{L}{l_t}}} = e^{\frac{y}{l_t}} - e^{-\frac{y}{l_t}}$$

$$\frac{\sinh \left( \frac{L - y}{l_t} \right)}{\sinh \left( \frac{L}{l_t} \right)} \approx \frac{\frac{L - y}{l_t} - e^{-(L - y)/l_t}}{\frac{L}{l_t} - e^{-(L - y)/l_t}} = e^{\frac{L - y}{l_t}} - e^{-(L - y)/l_t}$$

so that equation (4-8) reduces to

$$\phi_{sb}(y) \approx V_{gb} - V_{T0} + 2\phi_F + (V_{db} + V_{bi} - V_{gb} + V_{T0} - 2\phi_F)(e^{\frac{y}{l_t}} - e^{-\frac{y}{l_t}})$$

$$+ (V_{sb} + V_{bi} - V_{gb} + V_{T0} - 2\phi_F)e^{\frac{-(L - y)}{l_t}}$$

Calculating $y_0$, at which the surface potential has its minimum by calculating

$$\frac{\partial \phi_{sb}}{\partial y} \bigg|_{y=y_0} = 0$$

and solving for $y_0$ yields

$$y_0 = \frac{L}{2} - \frac{l_t}{2} \ln \left( \frac{V_{db} + V_{bi} - V_{gb} + V_{T0} - 2\phi_F}{V_{sb} + V_{bi} - V_{gb} + V_{T0} - 2\phi_F} \right)$$

(4-10)

Inserting this in equation (4-9) yields

$$\phi_{sb\text{min}} = V_{gb} - V_{T0} + 2\phi_F - (V_{db} + V_{bi} - V_{gb} + V_{T0} - 2\phi_F)e^{\frac{-L}{l_t}}$$

$$+ 2\sqrt{(V_{db} + V_{bi} - V_{gb} + V_{T0} - 2\phi_F)(V_{sb} + V_{bi} - V_{gb} + V_{T0} - 2\phi_F)e^{\frac{-2L}{l_t}}}$$

(4-11)
Putting $q_{\text{Sbmin}}=2\phi_F + V_{sb}$ at $V_{gb}=V_{Th}$ (the onset of strong inversion) and solving for $V_{Th}$ yields

$$V_{Th} = V_{T0} + V_{sb} - \Delta V_T,$$

where

$$\Delta V_T = \frac{(V_{ds} + V_{bi} - 2\phi_F)(1 - e^{\frac{-L}{2l}}) + 2(V_{bi} - 2\phi_F) + \sqrt{(V_{bi} - 2\phi_F)(V_{bi} - 2\phi_F) + V_{ds}e^{\frac{L}{2l}} - (V_{bi} - 2\phi_F)V_{ds}}}{4 \sinh^2 \left( \frac{L}{2l} \right)}.$$

(4-13)

If we take the threshold voltage with respect to the source, equation 4-12 can be rewritten as

$$V_{Ts} = V_{T0} - \Delta V_T.$$  

(4-14)

Since we assumed that $L > l$, equation (4-13) reduces to

$$\Delta V_T = \frac{(V_{ds} + 3(V_{bi} - 2\phi_F))e^{\frac{-L}{2l}} + \sqrt{(V_{bi} - 2\phi_F)(V_{bi} - 2\phi_F) + V_{ds}e^{2l}}}{4 \sinh^2 \left( \frac{L}{2l} \right)}.$$

(4-15)

For deriving equation (4-15), we did not make use of the gradual channel approximation, since we did not include the electron charge in equation (4-4). So no distinction has to be made between the regions where the electrons do or do not move with the saturated velocity, as was the case with channel-length modulation. This implies that equation (4-15) can be used in the saturation region without modifications.
5. The Philips MOS9 model

To make it possible to compare the new model with the Philips MOS9 model, we will discuss channel-length modulation and the drain voltage dependence of the threshold voltage of the MOS9 model in this section.

5.1 Channel-length modulation

The derivation of the channel shortening term is based on solving the two dimensional Poisson equation

\[
\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = \frac{qN_A}{\varepsilon_s} + \frac{I_{\text{sat}}}{qWv_sX_{av}}.
\]  

(5-1)

where \(I_{\text{sat}}\) is the saturation current, \(v_s\) is the saturation velocity, \(\phi\) the potential somewhere in the semiconductor and \(X_{av}\) is the average depth of the electrons. The directions of \(x\) and \(y\) are as defined in chapter 3.

With the assumption

\[
\frac{\partial^2 \phi}{\partial x^2} \approx \frac{C_{OX}(V_{gb} - V_{FB} - \phi_{SB})}{\varepsilon_s X_{av}}
\]

(5-2)

for the whole region between the point of the onset of velocity saturation and the drain diffusion, and \(I_{\text{sat}}\) expressed in \(\phi_{SB}\), equation (5-1) can be written as[2]

\[
\frac{\partial E_s(\phi_{SB})}{\partial y} = A^2 (\phi_{SB}(y) - V_{\text{dssat}}) + B
\]

(5-3)

Solving this equation for the electric field yields

\[
E_s(\phi_{SB}) = \sqrt{\alpha^2 (\phi_{SB}(y) - V_{\text{dssat}})^2 + E_{\text{sat}}^2}
\]

(5-4)
and the channel shortening term can be written as

\[ I_d = \alpha^{-1} \ln \left[ \frac{\alpha(V_{ds} - V_{d_{sat}}) + E_s(V_{ds})}{E_{sat}} \right] \]  

(5-5)

where \( \alpha \) is considered as a fitting parameter. For \( V_{d_{sat}} \) expression (3-32) is used.

### 5.2 Static feedback

The MOS9 model [2, 4, 8] uses a complete other method to model the drain voltage dependence of the threshold voltage when the transistor operates in saturation. It is based on the fact that an increase of the drain voltage beyond saturation induces some excess charge in the channel when the average distance between source and drain becomes small. This effect is the strongest in the region near the saturation point.

The increase in mobile charge \( \Delta Q_i \) has been approximated as

\[ \Delta Q_i = \eta C_{OX} V_{ds} \]  

(5-6)

where

\[ \eta \propto \frac{1}{L^{3/2} N_{A}^{1/4} C_{OX}} \]  

(5-7)

For the drain dependent part of the threshold voltage in saturation this results in

\[ \Delta V_T = \eta V_{ds} \]  

(5-8)

In the MOS9 model the drain dependent part of the threshold voltage in saturation is modelled as

\[ \Delta V_T = \eta V_{ds}^{\eta_{ds}} \]  

(5-9)

where \( \eta_{ds} \approx 0.6 \).
6. Measurements

In this section we discuss the measurement setup and the test devices.

6.1 Measurement set-up

As mentioned in the introduction, the model must be suitable for describing the drain current and the first three derivatives of it to the drain-source-voltage.

For measuring the derivatives of the drain current to the drain-source-voltage the measurement set-up as depicted in figure 6-1 has been used.

![Figure 6-1 measurement set-up](image_url)

The used instruments are shown in table 6-1.

Table 6-1 used instruments

<table>
<thead>
<tr>
<th>instrument</th>
<th>type</th>
<th>make</th>
</tr>
</thead>
<tbody>
<tr>
<td>spectrum analyser, (V_i)</td>
<td>HP 35665A</td>
<td>Hewlett-packard</td>
</tr>
<tr>
<td>current amplifier</td>
<td>model 5182 pre. amp.</td>
<td>EG&amp;G instruments</td>
</tr>
<tr>
<td>(V_{b0})</td>
<td>HP 4140B</td>
<td>Hewlett-Packard</td>
</tr>
<tr>
<td>(V_{g0}, V_{s0})</td>
<td>HP 6625A</td>
<td>Hewlett-Packard</td>
</tr>
</tbody>
</table>
The drain current $I_d$ in figure 6-1, can be expanded in a Taylor series as:

$$I_d = C_0 + C_1 V_1 + C_2 V_1^2 + C_3 V_1^3 + C_4 V_1^4 + \ldots \quad (6-1)$$

where

$$C_i = \frac{1}{i!} \frac{\partial^i I_d}{\partial V_i^i} \bigg|_{V_{ds, V_{gs, V_{so}}}} \quad (6-2)$$

and

$$V_i = V_p \sin(\omega t) \quad (6-3)$$

After some rearrangements equation (6-1) can be rewritten as:

$$I_d = a_0 + a_1 \sin(\omega t) + a_2 \cos(2\omega t) + a_3 \sin(3\omega t) \ldots \quad (6-4)$$

where

$$a_0 = C_0 + \frac{1}{2} C_2 V_p^2 + \frac{1}{3} C_4 V_p^4 + \ldots$$
$$a_1 = C_1 V_p + \frac{1}{3} C_3 V_p^3 + \frac{1}{5} C_5 V_p^5 + \ldots$$
$$a_2 = -\frac{1}{2} C_2 V_p^2 - \frac{1}{2} C_4 V_p^4 - \frac{1}{12} C_6 V_p^6 + \ldots$$
$$a_3 = -\frac{1}{6} C_3 V_p^3 - \frac{1}{10} C_5 V_p^5 - \frac{1}{40} C_7 V_p^7 + \ldots \quad (6-5)$$

The factors $a_i$ are measured by the spectrum analyser in the measurement set-up, shown in figure 6-1.

So if the factors $a_i$ are measured and $V_p$ is small enough, the derivatives can be calculated from equations (6-2) and (6-5).

### 6.2 Test devices

The test devices are n-type LDD MOSFETs with a threshold implantation from the process TC150 DM2 from Philips semiconductor. The minimum mask length of this process equals 0.8μm.
7. Testing the validity of the model

In this chapter we will start a repetition of the necessary equation and give an estimation of the constants that are used as fitting parameters in the corresponding equations.
In the next section we will discuss the different ways of using the model and discuss the drain voltage dependence of the model.
Next we will discuss the gate voltage dependence of the model.

7.1 The test method

7.1.1 The new model

For convenience, we will first repeat the equations necessary for testing the new model.
To test the model for the channel-length modulation and the model for the threshold voltage, we use equation (2-16), where 3-2 is used instead of $\beta$, so that

$$I_{dsat} = \frac{1}{1 - \frac{1}{L}} \beta \frac{(V_{gs} - V_{Ts}) V_{dsat} - \frac{1 + \delta}{2} V_{dsat}^2}{(1 + \theta_1 (V_{gs} - V_{Ts}) + \theta_2 (\sqrt{V_{sb} + 2\phi_F} - \sqrt{2\phi_F}))(1 + \theta_3 V_{dsat})} \quad (7-1)$$

In this equation, $I_d$ and $V_{Ts}$ have to be replaced by equations (3-23) and (4-14) respectively in combination with (4-15).
Equation (3-23) reads:

$$I_d = I_c \ln \left( \frac{V_{ds} + k - V_{Ts} - V_{dsat} + \sqrt{(V_{ds} + k - V_{Ts} - V_{dsat})^2 + (E_{sat} l_c)^2 - (k - V_{Ts} + \gamma_F)^2}}{E_{sat} l_c + k - V_{Ts} + \gamma_F} \right), \quad (7-2)$$

where

$$k = \frac{qN_A X_{ev}}{C_{ox}} + V_{FB} + \phi_m \quad (7-3)$$
and

\[ I_c = \sqrt{\frac{X_{nr}E_{si}}{C_{ox}}} \]  \hspace{1cm} (7-4)

Equation (4-14) and (4-15) read:

\[ V_{Ts} = V_{10} - \Delta V_T \]  \hspace{1cm} (7-5)

\[ \Delta V_T = (V_{ds} + 3(V_{bi} - 2\phi_F))e^{\frac{-L}{L}} + \sqrt{(V_{bi} - 2\phi_F)[(V_{bi} - 2\phi_F) + V_{ds}]e^{2L}}. \]  \hspace{1cm} (7-6)

where

\[ I_t = \sqrt{\frac{X_{dep}E_{si}}{C_{ox}}} \]  \hspace{1cm} (7-7)

Furthermore in equation (7-1) \( V_{dssat} \) has to be replaced by (equation 7-33)

\[ V_{dssat} = \frac{2(V_{gs} - V_{Ts})}{(1 + \delta)} \left( \frac{2\theta_3(V_{gs} - V_{Ts})}{(1 + \delta) + 1} \right)^{-1} \]  \hspace{1cm} (7-8)

7.1.2 Parameter estimation

To test the validity of the new model, the combination of these equations is fitted to the measured characteristics with the help of the fitting program JANDEL. For optimal fitting, this program needs a start estimation for the constants which are used as fitting parameters. First we will discuss the start estimates of these fitting parameters and give the values of the parameters that are considered as constants.

In order to test the drain-source-voltage dependence of the model, \( \delta \) and \( \theta_1 \) are considered as constants with values 0.228 and 0.352 V\(^{-1}\) respectively, in the case of a device with a mask-channel-length of 1 \( \mu \)m. These values are found by fitting the MOS9 model to the measured current in the linear region. To test the gate-source-voltage dependence, \( \theta_1 \) is considered as a fitting parameter with a start value as mentioned above.

Furthermore, both for testing the drain-source-voltage dependence and testing the gate-source-voltage, \( \beta \) and \( \theta_3 \) are considered as fitting parameters with start values \( \beta = 1.63 \times 10^{-3} \) C/V\(^2\)s and \( 0.01 < \theta_3 < 0.1 \) again in the case of a device with a mask-channel-length of 1 \( \mu \)m. These values are also found by fitting the MOS9 model to the measured current in the linear region.
Since the source-bulk-voltage dependence has not yet been tested, $\theta_2$ is of no importance here.
The remaining constant $L$ in equation (7-1) is the electric channel-length, which equals the
mask-channel-length $L_{\text{mask}}$ minus the total underdiffusion of the source and drain diffusions $\Delta L$.
In the technology used for our test devices, $\Delta L = 0.32 \, \mu m$.

Estimating $X_{av}$ in equations (7-3) and (7-4) is difficult, since we don't have a complete device
description. We estimate that $X_{av}$ lays somewhere between 0.1 $\mu$m and 1 $\mu$m.
Furthermore $k$ in equation (7-3) contains $N_A$. Our test devices have threshold voltage
implantation, so that the value of $N_A$ depends on the fact whether $X_{av}$ is smaller or larger than
the threshold implantation layer, therefore we can not give an accurate start estimate for $N_A$.
We assume that $1 \times 10^{16} \, \text{cm}^{-3} < N_A < 1 \times 10^{17} \, \text{cm}^{-3}$.

$C_{OX}$ in (7-3) is given by $\varepsilon_{OX}/t_{OX}$, where $t_{OX} = 150 \, \AA$, so that $C_{OX} \approx 2.3 \times 10^{-7} \, \text{F.cm}^{-2}$
Furthermore in (7-3) $V_{FB}$ has a value of about -0.5 V and we estimate that $0.3 < \phi_m < 0.8$.
In worst case, the above values yields a value of $0 \, V < k < 7 \, V$. Here we assumed that $k$ can not be negative.
For $l_e$ the estimated values result in $10^{-6} \, \text{cm} < l_e < 5 \times 10^{-6} \, \text{cm}$.
The estimated value of the constant $\gamma_F$ lays somewhere between 0 and 0.4 V.

The value of the saturation field $E_{sat}$ is smaller than the saturation field in a piece of pure
silicon. In silicon, the value of $E_{sat}$ equals about $3 \times 10^4 \, \text{V/cm}$. This implies that the term $E_{sat} l_e$ in
equation 7-2 has to be smaller than 0.15 V. During the fitting procedure, it became clear that a
fixed value of 0.05 V for $E_{sat} l_e$ gives the best result.
In equations (7-6) and (7-7), the term $V_{br}-2\phi_F$ is estimated as 0.25 and $l_t$ is estimated
somewhere between $3 \times 10^{-6}$ and $7 \times 10^{-6}$.
7.1.3 Measured coefficients

For testing the model, the coefficients $a_1$ through $a_5$ are measured. The coefficients $a_4$ and $a_5$ are measured to check if the higher derivatives can be neglected. Figure 7-1 gives a typical plot of the measured coefficients. The coefficients were measured for a device with a mask length of 1 μm and at gate and bulk voltages of $V_{gs}=3$ V and $V_{sb}=0$ V respectively.

![Figure 7-1 typical measured coefficients]
7.2 The drain-voltage dependence

7.2.1 The MOS 9 model

For comparing the new model with the MOS 9 model figure 7-2 gives the results of fitting the MOS 9 model to the current and the first and second derivative at \( V_{gs} = 3 \text{ V} \) and \( V_{sb} = 0 \text{ V} \). In this figure (mes) stands for measured and (mod) stands for model. As we can see in this figure, the second and third derivative don’t fit very well to the measured derivatives.

![Figure 7-2: Result of fitting the MOS 9 model](image)

7.2.2 The new model

Figure 7-3 gives the modelled current together with the measured current as a function of the drain-source-voltage. The model was fitted to the current and the first and second derivatives at a gate-source-voltage of 3 V and a source-bulk-voltage of 0 V. As we can see in this figure, the modelled current fits very well to the measured current.
Figure 7-3 comparison of the modelled and the measured drain current

Figure 7-4 gives the first, second and third derivative belonging to measured and modelled current in figure 7-3.

As this figure shows, the modelled first derivative fits very well to the measured first derivative at drain-source-voltages from 3 V and higher. The modelled second and third derivatives fit very well to the measured ones at drain voltages higher than 3.5 V.

The values of the fit parameters belonging to the figures 7-3 and 7-4 are given in table 1 in appendix a. As we can see in this table, the values of k, I_c, β, θ, and V_br-2ϕ_F are in agreement with the expected values. The value of I_i is somewhat too high. This can be due to the fact that the value of X_dep is underestimated. Furthermore, the value of γ_F is too high. During the fitting procedure it became clear that γ_F is very important to fit the second and third derivative. The value of γ_F can be chosen smaller, but then the fit to the second and third derivative is not as good as with the given value.
Using the new model in a circuit simulator consumes a lot of computing time. To reduce the computing time of the model we also tried to fit the model with the neglect of the $\Delta V_T$ terms in equations (7-2) and (7-8). Figure 7-5 gives the result. As we can see, all modelled derivatives fit very well to the modelled ones.

The values of the fit parameters are given in table 2 in appendix a. The values are almost the same as in table 1, so no further comments are needed.
7.3 The gate-voltage dependence

7.3.1 The MOS 9 model

In order to compare the gate voltage dependence of the new model with the gate voltage dependence of the MOS9 model we tried to fit the drain current and its first derivative described by the MOS9 model to the measured current and first derivative. It appeared that automatic fitting to the current and the first derivative was impossible, therefore we tried to fit the model to the characteristics by hand.

Figures 7-6 and 7-7 give the result of this. The gate-source-voltage shown in these figures ranges from 2.5 V till 4.5 V with steps of 0.5 V. As we can see from this figure, the MOS9 model does not give a good description of the gate-voltage dependence if we try to fit the current and the first derivative, which is necessary to get a good description of the higher-order derivatives.

Figure 7-6 comparison of the measured $I_d$-$V_{ds}$ characteristic with the MOS9 model at gate-source voltages ranging from 2.5 to 4.5 V with steps of 0.5 V.
Figure 7-7 comparison of the measured first derivatives and the first derivatives of the MOS9 model at gate-source voltages ranging from 2.5 to 4.5 V with steps of 0.5 V

7.3.2 The new model

Figure 7-7 and 7-8 show a plot of the measured and the modelled drain current and their first derivatives as function of the drain voltage respectively. The gate-source-voltages at which these characteristics are measured range from 2.5 V (border of strong inversion) to 4.5 V with steps of 0.5 V. The model was fitted to the drain current and the first derivative. Though the modelled drain current and first derivative fit better to the measured characteristics (the current within an accuracy of about 3%) than the MOS9 model does, the modelled first derivatives still do not fit to the measured characteristics very well. Additionally the values (shown in appendix a) for \( k \) and \( \Phi_F \) are higher than those belonging to figures 7-3, 7-4 and 7-5.
Figure 7-8 comparison of the measured $i_d-V_{ds}$ characteristic with the new model at gate-source voltages ranging from 2.5 to 4.5 V with steps of 0.5 V

Figure 7-9 comparison of the measured first derivatives and the new model at gate-source voltages ranging from 2.5 to 4.5 V with steps of 0.5 V
7.3.3 The modified new model

Many attempts have been made to improve the gate voltage dependence of the model. No real physical cause could be found why the gate voltage dependence of the model was not correct. From the picture in figure 7-7, it can be observed that, for the gate-source-voltages shown, the gate-source-voltage dependence of the modelled first derivative is overestimated, since the range of the modelled first derivative is much larger than the range of the measured ones. From this point of view the most obvious solution is to divide the channel shortening term (equation (7-2)) by a power term of \( V_{gs} \), since the first derivative contains a term proportional to its derivative. Dividing equation (7-2) by \( V_{gs}^2 \) turned out to give the best result.

Now equation (7-2) becomes

\[
I_d = \frac{l_f}{V_{gs}^2} \ln \left( \frac{V_{ds} + k - V_{Ts} - V_{dssat} + \sqrt{(V_{ds} + k - V_{Ts} - V_{dssat})^2 + (E_{sat} l_c)^2 - (k - V_{Ts} + \gamma_f)^2}}{E_{sat} l_c + k - V_{Ts} + \gamma_f} \right).
\]

(7-9)

In equation (7-9) \( l_f \) is used instead of \( l_c \), to denote that we are dealing with another constant used as fitting parameter here. As start value for \( l_f \) we will use \( 4^2 \) times the start value of \( l_c \). The result of fitting with equation (7-8) substituted for \( I_d \), in equation (7-1) is shown in figure 7-10 through 7-15. In equation (7-8) and (7-2) \( \Delta V_f(V_{ds}) \) has been neglected, since this consumes much less computing time as mentioned in the previous section.
Testing the validity of the model

Figure 7-10 comparison of the measured $i_d-V_{ds}$ characteristic with the modified new model at gate-source voltages ranging from 2.5 to 4.5 V with steps of 0.5 V.

Figure 7-11 comparison of the modelled and measured derivatives at $V_g=2.5V$
Testing the validity of the model

Figure 7-12 comparison of the modelled and measured derivatives at $V_g = 3.0V$

Figure 7-13 comparison of the modelled and measured derivatives at $V_g = 3.5V$
Testing the validity of the model

Figure 7-14 comparison of the modelled and measured derivatives at $V_g=4.0\, V$

Figure 7-15 comparison of the modelled and measured derivatives at $V_g=4.5\, V$
As follows from these figures, the modelled first derivatives fit very well for all gate-source-voltages shown. For gate-source-voltages $V_{gs}$ ranging from 2.5V till 4V, the modelled second derivative fit very well. The modelled second derivative at a gate voltage of 4.5 voltage does not fits to the measured data. The cause of this is that the second derivative is not yet in saturation in the drain voltage region shown, as discussed in section 7-2. Saturation for the second derivative occurs at a higher drain voltage, but testing the devices at higher drain voltage damages the devices.

The same is the case with the third derivatives at gate voltages of 4V and 4.5V. The values of the fitparameters belonging to the above figures are given in table 4 in appendix a. As can be seen, the value of $\gamma_F$ is still too high. Also the value of $V_{bi-2\phi_F}$ is too high. Since equation (7-8) has no real physical base any more, no conclusions can be drawn about the value of $V_{bi-2\phi_F}$.

A problem with the new model and the modified new model is that equations (7-2) and (7-9) only exist for drain-source-voltages a little higher then the saturation voltage, since the high value of $\gamma_F$ makes the sign of the term in the square root negative for lower drain-source voltages. But since the equations describing the linear region have to be connected to the equations describing the saturation region by smoothing functions, this does not need to be a problem. The smoothing functions can be made so that the equations describing the saturation region are first “switched on” for drain-source-voltages a little higher than the saturation voltage.

After all this was already necessary, since the equations describing the saturation region are only valid for the derivatives at drain-source-voltages drain-source-voltages higher than the saturation voltage.
8. Conclusions and recommendations

8.1 Conclusions

A new model for the drain current and the first, second and third-order derivative ($\partial I_d/\partial V_{ds}$, $\partial^2 I_d/\partial V^2_{ds}$ and $\partial^3 I_d/\partial V^3_{ds}$) of a MOSFET operating in the saturation region and in strong inversion has been developed. The model includes channel-length modulation and the drain-voltage dependence of the threshold voltage.

The model has been tested for the drain-source voltage dependence and the gate-source voltage dependence. The model describes the drain-source-voltage dependence of the drain current and the first three derivatives of the current very well.

The model fails to describe the gate-source-voltage dependence of the current and the derivatives, however by multiplying the model for channel-length modulation by $1/V_{gs}^2$ the gate-source-voltage dependence of the model is improved, so that the total model describes it well now.

In the future the model has to be tested for the source-bulk-voltage dependence and channel-length dependence.
8.2 Recommendations

Since the test devices are transistors with a threshold implantation, the model has to be extended for these types of transistors.

For the threshold voltage this can easily be done by using two body factors for describing the threshold voltage, as is done in the Philips MOS9 model [2,4].

For the model of the channel-length modulation the same sort of solution can be used, since the value of $k=k(N_A)$ strongly depends on whether the edge of the depletion layer lays inside or outside the implanted layer, as is the case with the body factor in the expression for the threshold voltage.

Furthermore it is important to investigate the temperature dependence of the current and the derivatives, since the threshold voltage also depends on temperature [2]. If the current and the derivatives strongly depend on temperature, self heating has to be taken be into account in the model.
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THRESHOLD VOLTAGE MODEL FOR DEEP SUBMICROMETER MOSFETS
## appendix A  Values of the fit parameters

### table 1 fitpar. belonging to fig. 7-3 and 7-4

<table>
<thead>
<tr>
<th>parameter</th>
<th>value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k$</td>
<td>2.87 V</td>
</tr>
<tr>
<td>$l_c$</td>
<td>$1.37 \times 10^{-6}$ cm</td>
</tr>
<tr>
<td>$\beta$</td>
<td>$1.56 \times 10^{-3}$ C/V$^2$s</td>
</tr>
<tr>
<td>$\theta_3$</td>
<td>0.077 V$^{-1}$</td>
</tr>
<tr>
<td>$V_{br-2\phi_F}$</td>
<td>0.168 V</td>
</tr>
<tr>
<td>$l_t$</td>
<td>$8.6 \times 10^{-6}$ cm</td>
</tr>
<tr>
<td>$\gamma_r$</td>
<td>1.022 V</td>
</tr>
</tbody>
</table>

### table 2 fitpar. belonging to fig. 7-5

<table>
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<th>parameter</th>
<th>value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k$</td>
<td>2.87 V</td>
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<tr>
<td>$l_c$</td>
<td>$1.40 \times 10^{-6}$ cm</td>
</tr>
<tr>
<td>$\beta$</td>
<td>$1.56 \times 10^{-3}$ C/V$^2$s</td>
</tr>
<tr>
<td>$\theta_3$</td>
<td>0.077 V$^{-1}$</td>
</tr>
<tr>
<td>$V_{br-2\phi_F}$</td>
<td>0.168 V</td>
</tr>
<tr>
<td>$l_t$</td>
<td>$8.7 \times 10^{-6}$ cm</td>
</tr>
<tr>
<td>$\gamma_r$</td>
<td>1.022 V</td>
</tr>
</tbody>
</table>

### table 3 fitpar. belonging to fig. 7-8 and 7-9

<table>
<thead>
<tr>
<th>parameter</th>
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</thead>
<tbody>
<tr>
<td>$k$</td>
<td>10.0 V</td>
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<tr>
<td>$l_c$</td>
<td>$1.8 \times 10^{-6}$ cm</td>
</tr>
<tr>
<td>$\beta$</td>
<td>$1.96 \times 10^{-3}$ C/V$^2$s</td>
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<tr>
<td>$\theta_3$</td>
<td>0.17 V$^{-1}$</td>
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<tr>
<td>$\theta_1$</td>
<td>0.36 V$^{-1}$</td>
</tr>
<tr>
<td>$V_{br-2\phi_F}$</td>
<td>0.64 V</td>
</tr>
<tr>
<td>$l_t$</td>
<td>$7.7 \times 10^{-6}$ cm</td>
</tr>
<tr>
<td>$\gamma_r$</td>
<td>0.98 V</td>
</tr>
</tbody>
</table>

### table 6 fitpar. belonging to fig. 7-10 - 7-15

<table>
<thead>
<tr>
<th>parameter</th>
<th>value</th>
</tr>
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<tbody>
<tr>
<td>$k$</td>
<td>6.33 V</td>
</tr>
<tr>
<td>$l_f$</td>
<td>$2.32 \times 10^{-5}$ cm</td>
</tr>
<tr>
<td>$\beta$</td>
<td>$1.97 \times 10^{-3}$ C/V$^2$s</td>
</tr>
<tr>
<td>$\theta_3$</td>
<td>0.12 V$^{-1}$</td>
</tr>
<tr>
<td>$\theta_1$</td>
<td>0.394 V$^{-1}$</td>
</tr>
<tr>
<td>$V_{br-2\phi_F}$</td>
<td>0.52 V</td>
</tr>
<tr>
<td>$l_t$</td>
<td>$7.0 \times 10^{-6}$ cm</td>
</tr>
<tr>
<td>$\gamma_r$</td>
<td>1.08 V</td>
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</table>