MASTER

Progressive collapse of reinforced concrete structures
a study to the dynamic load factor used to include the dynamic nature of progressive collapse

Couwenberg, R.P.P.

Award date:
2014

Link to publication

Disclaimer
This document contains a student thesis (bachelor's or master's), as authored by a student at Eindhoven University of Technology. Student theses are made available in the TU/e repository upon obtaining the required degree. The grade received is not published on the document as presented in the repository. The required complexity or quality of research of student theses may vary by program, and the required minimum study period may vary in duration.

General rights
Copyright and moral rights for the publications made accessible in the public portal are retained by the authors and/or other copyright owners and it is a condition of accessing publications that users recognise and abide by the legal requirements associated with these rights.

• Users may download and print one copy of any publication from the public portal for the purpose of private study or research.
• You may not further distribute the material or use it for any profit-making activity or commercial gain
Progressive Collapse of Reinforced Concrete Structures

A study to the dynamic load factor used to include the dynamic nature of progressive collapse
Preface

The report in front of you is the result of my graduation project "progressive collapse of reinforced concrete structures". During this graduation project a detailed literature study to the state of the art with respect to alternate load paths and catenary action was performed. Further a numerical model was developed that is able to investigate the behavior of reinforced concrete structures subjected to sudden column removal. This model was used for the derivation of a dynamic load factor to include the dynamic nature of the load caused by the sudden loss of a load bearing element in static analysis.

I would like to express my gratitude toward the supervisor’s prof.dr.ir. Theo Salet from Eindhoven University of Technology, prof.dr.ir. Dick Hordijk form Delft University of Technology and dr.ir. Sander Zegers from Vericon Ingenieurs for their support and advice during my graduation project. Special thanks go to Vericon Ingenieurs for offering the possibility to do my graduation project within their company. I would also like to thank dr. ir. Paul Teeuwen from Witteveen+Bos for his support during the development of the finite element model. Further I would like to thank Thomas Paus and Patrick Marinus for their contribution to the development of the multilayer model.

Finally, I want to thank my family and friends for their support and interest during my entire study. Without their support it would be a lot more difficult to complete this study.

Reno Couwenberg

Tilburg, December 2013
Summary

For this study two research goals were specified. The first goal was defined as the performance of a detailed literature study to the state of the art with respect to alternate load paths and catenary action for reinforced concrete structures. The second goal was the development of a dynamic load factor that includes the dynamic nature of the load caused by the sudden loss of a corner column by means of the finite element method.

During this literature study some interesting findings were done, especially with respect to catenary action. It was observed that catenary action can develop after the sudden failure of intermediate column. It was shown that catenary action can really increase the progressive collapse resistance as long as the beams are able to deform sufficiently. It was found that partly debonding the reinforcement bars highly increased this deformation capacity. Further the collapse of a corner column was considered in a few researches. Here no catenary action was observed in the beams which indicate that it is doubtful if this mechanism can develop. However, including the influence of the floor to redistribute the loads highly increased the progressive collapse resistance in case of failure of a corner column. Further some researchers investigated the progressive collapse resistance of existing reinforced concrete structures. It was interesting to note that all the tested structures did not collapse after a column was suddenly removed, most of the considered structures did not even show large deformations.

The second goal specified for this thesis was the determination of a dynamic load factor for a reinforced concrete frame structure to include the dynamic nature of the load caused by the sudden failure of a corner column by means of the finite element method. In order to determine such a dynamic load factor it was important that the finite element model was able to simulate the behavior of the frame as realistic as possible. Therefore the model must fulfill some requirements. Considering the behavior of concrete it was important that the material model was able to describe the full nonlinear material behavior of the concrete including strain softening in tension and strain hardening in compression. Further the material model should be able to deal with cyclic loads since progressive collapse is a dynamic phenomenon, this was also valid for the finite elements. Further it was also important that the flexural reinforcement could be included in the finite elements since a 2D representation of a reinforced concrete frame was studied. In order to determine a dynamic load factor for a reinforced concrete frame structure it was sufficient to study the overall response of the reinforced concrete frame. Therefore mainly the deflections and bending moments were of interest for this study. Local stress distributions and crack patterns were not necessary to consider.

For the current thesis the finite element package Abaqus was used to perform the analysis. The material model and the elements used for the finite element model were selected on the previously discussed requirements. The concrete model used for this thesis was the concrete damage plasticity model, this was the only model available in Abaqus that meets the requirements as specified above. Within this model a smeared crack approach was used to model the tensile behavior of the concrete including strain softening. The concrete compressive behavior was modeled by a parabolic stress strain relation. For the finite elements it was found that the Euler-Bernoulli beam elements available in Abaqus fulfill the requirements as stated above the best way. The flexural reinforcement could be added within the beam elements by means of a fictive layer of reinforcement. A plasticity material model was used to specify the reinforcement properties.

Due to a lack of experimental data available to benchmark the developed finite element model simple analytical models were developed to benchmark both the static and dynamic behavior obtained with the finite element model. To benchmark the cross sectional response of the beams for static loads a multilayer model was developed. With this model
it was possible to derive the moment curvature response for various materials including reinforced concrete. Further the moment-area method was used as a benchmark model for the deflection of a beam on two supports. The multilayer model was used here to provide the cross sectional response of the beam at various locations in the beam so that the deflection could be determined for a beam with varying flexural rigidity like reinforced concrete.

The first benchmark tests with respect to static loads were performed on statically determined structures. After comparing the results obtained with both the benchmark tests and the finite element model for statically determined structures it was found that the models show good agreement. This led to confidence in the proper functioning of the finite element model with respect to the nonlinear behavior of reinforced concrete. Further a study was performed to the behavior of statically indeterminate structures under various loads and supports. Also the results obtained during this study led to sufficient confidence in the proper functioning of the developed finite element model.

As discussed previously also a simple analytical model was developed to benchmark the results obtained with the finite element model for dynamic loads. For this benchmark test a nonlinear single degree of freedom (SDOF) mass spring system was developed. For this benchmark test a beam on two supports was considered loaded by an instantaneously applied uniformly distributed load. In order to include the nonlinear behavior of the reinforced concrete in the spring a translation was made of the moment curvature diagram into a load deflection diagram. This load deflection diagram was used to derive the stiffness of the spring for different deformation stages.

Also for the dynamic loads a comparison was made between the benchmark test and the finite element model for various loads. Initially the results showed quite large deviations. It was found that Abaqus did not exactly describe the moment curvature relation as for static loads. However, small deviations in the moment curvature response caused larger deviations in the final deflection. After modifying the input data of the mass spring system to the moment curvature response obtained with Abaqus a better agreement between the models was observed. Further it was observed that Abaqus uses the uncracked stiffness for the unloading stage. This is not correct since the beam is cracked during the loading stage. However, this was not a major concern for the current study.

After confidence was obtained in the proper functioning of the finite element model with the benchmark tests a reinforced concrete reference frame was modeled and some analyses were performed regarding the instantaneous removal of a corner column. First a static analysis of the initial frame was performed to obtain an equivalent reaction force in the column that was removed in the dynamic analysis. Thereafter a static analysis was performed on the frame where the column was assumed to be removed. For this analysis no dynamic effects were included. This static analysis served as a reference for the analysis where dynamic effects were included. The third analysis was also a static analysis only now the load was multiplied with a dynamic load factor of 2 to include the dynamic nature of the progressive collapse load. This dynamic load factor of 2 is often used for design purposes with respect to progressive collapse. A very large deflection was found and all beams reached their plastic capacity. The final analysis was a full dynamic analysis. For this analysis the reaction force obtained with the first static analysis was used as an equivalent column force. This equivalent column force replaced the initial column and was instantaneously removed to simulate the sudden column loss. The dynamic analysis showed a much smaller deflection at the position where the column was assumed to fail than the static analysis where the load was multiplied with a factor 2. Since this load factor of 2 resulted in a very large deformation of the frame a load factor that better fits the real structural behavior was derived for the considered system. This load factor was set to 1,52 which is 24% smaller than the initial factor of 2.
### List of symbols

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>Cross sectional area</td>
</tr>
<tr>
<td>A_c</td>
<td>Cross sectional area concrete</td>
</tr>
<tr>
<td>A_s</td>
<td>Cross sectional area reinforcement</td>
</tr>
<tr>
<td>A_{sw}</td>
<td>Cross sectional area shear reinforcement</td>
</tr>
<tr>
<td>b</td>
<td>Width</td>
</tr>
<tr>
<td>b_c</td>
<td>Ratio between inelastic and plastic strain</td>
</tr>
<tr>
<td>b_l</td>
<td>Width of a layer</td>
</tr>
<tr>
<td>b_t</td>
<td>Ratio between cracking and plastic strain</td>
</tr>
<tr>
<td>c</td>
<td>Concrete cover</td>
</tr>
<tr>
<td>c</td>
<td>Damping factor</td>
</tr>
<tr>
<td>C_{RD;c}</td>
<td>Factor for shear resistance</td>
</tr>
<tr>
<td>d</td>
<td>Effective width</td>
</tr>
<tr>
<td>d_c</td>
<td>Compression damage parameter</td>
</tr>
<tr>
<td>d_t</td>
<td>Tensile damage parameter</td>
</tr>
<tr>
<td>E_{cl}</td>
<td>Secant modulus</td>
</tr>
<tr>
<td>E_{cm}</td>
<td>Youngs modulus concrete</td>
</tr>
<tr>
<td>E_{fictive}</td>
<td>Fictive youngs modulus</td>
</tr>
<tr>
<td>EI</td>
<td>Flexural rigidity</td>
</tr>
<tr>
<td>e_l</td>
<td>Eccentricity</td>
</tr>
<tr>
<td>E_s</td>
<td>Youngs modulus steel</td>
</tr>
<tr>
<td>f_c</td>
<td>Concrete compressive strength</td>
</tr>
<tr>
<td>f_{cd}</td>
<td>Concrete compressive design strength</td>
</tr>
<tr>
<td>f_{cm}</td>
<td>Average concrete compressive strength</td>
</tr>
<tr>
<td>f_{ctm}</td>
<td>Average concrete tensile strength</td>
</tr>
<tr>
<td>F_e</td>
<td>Equivalent load</td>
</tr>
<tr>
<td>F_t</td>
<td>Actual load</td>
</tr>
<tr>
<td>F_{tie}</td>
<td>Tie force</td>
</tr>
<tr>
<td>F_{tie,col}</td>
<td>Tie force in columns</td>
</tr>
<tr>
<td>F_{tie;fac}</td>
<td>Tensile force in facade</td>
</tr>
<tr>
<td>F_{tie;int}</td>
<td>Tensile force internal ties</td>
</tr>
<tr>
<td>F_{tie;per}</td>
<td>Tensile force peripheral ties</td>
</tr>
<tr>
<td>f_{yk}</td>
<td>Characteristic yield stress steel</td>
</tr>
<tr>
<td>f_{yk;max}</td>
<td>Maximum characteristic yield stress</td>
</tr>
<tr>
<td>G_c</td>
<td>Crushing energy</td>
</tr>
<tr>
<td>G_f</td>
<td>Fracture energy</td>
</tr>
<tr>
<td>G_k</td>
<td>Characteristic dead load</td>
</tr>
<tr>
<td>H</td>
<td>Story height</td>
</tr>
<tr>
<td>H</td>
<td>Horizontal reaction force</td>
</tr>
<tr>
<td>h</td>
<td>Cross sectional height</td>
</tr>
<tr>
<td>h</td>
<td>Crack band width</td>
</tr>
<tr>
<td>h_l</td>
<td>Layer height</td>
</tr>
<tr>
<td>k</td>
<td>Factor</td>
</tr>
<tr>
<td>k</td>
<td>Spring stiffness</td>
</tr>
<tr>
<td>k_1</td>
<td>Rotational stiffness at column top</td>
</tr>
<tr>
<td>k_2</td>
<td>Rotational stiffness at column bottom</td>
</tr>
<tr>
<td>Symbol</td>
<td>Description</td>
</tr>
<tr>
<td>--------</td>
<td>-------------</td>
</tr>
<tr>
<td>$K_L$</td>
<td>Transformation factor load</td>
</tr>
<tr>
<td>$K_M$</td>
<td>Transformation factor mass</td>
</tr>
<tr>
<td>$K_R$</td>
<td>Transformation factor resistance</td>
</tr>
<tr>
<td>$L$</td>
<td>Length</td>
</tr>
<tr>
<td>$L_0$</td>
<td>Initial length</td>
</tr>
<tr>
<td>$L_{0_e}$</td>
<td>Effective column length</td>
</tr>
<tr>
<td>$l_i$</td>
<td>End of span</td>
</tr>
<tr>
<td>$l_p$</td>
<td>Plastic hinge length</td>
</tr>
<tr>
<td>$m$</td>
<td>Component or element demand modifier</td>
</tr>
<tr>
<td>$m$</td>
<td>Mass</td>
</tr>
<tr>
<td>$M$</td>
<td>Bending moment</td>
</tr>
<tr>
<td>$M_0$</td>
<td>Maximum field moment</td>
</tr>
<tr>
<td>$M_{0,SL}$</td>
<td>Moment at half of beam span length</td>
</tr>
<tr>
<td>$m_e$</td>
<td>Equivalent mass</td>
</tr>
<tr>
<td>$M_{ed}$</td>
<td>Design value occurring bending moment</td>
</tr>
<tr>
<td>$M_{bd}$</td>
<td>Maximum bending moment resistance</td>
</tr>
<tr>
<td>$M_{sup}$</td>
<td>Moment at support</td>
</tr>
<tr>
<td>$m_t$</td>
<td>Actual mass</td>
</tr>
<tr>
<td>$M_y$</td>
<td>Yield moment</td>
</tr>
<tr>
<td>$n$</td>
<td>Number of layers</td>
</tr>
<tr>
<td>$N$</td>
<td>Normal force</td>
</tr>
<tr>
<td>$N_{ed}$</td>
<td>Design value occurring normal force</td>
</tr>
<tr>
<td>$n_s$</td>
<td>Number of stories</td>
</tr>
<tr>
<td>$P_1$</td>
<td>Elastic maximum load</td>
</tr>
<tr>
<td>$P_2$</td>
<td>Plastic maximum load</td>
</tr>
<tr>
<td>$P_{d;favorable}$</td>
<td>Load with favorable effect force distribution</td>
</tr>
<tr>
<td>$P_{d;unfavorable}$</td>
<td>Load with unfavorable effect force distribution</td>
</tr>
<tr>
<td>$q$</td>
<td>Uniform distributed load</td>
</tr>
<tr>
<td>$Q_{CE}$</td>
<td>Expected strength</td>
</tr>
<tr>
<td>$Q_{CL}$</td>
<td>Lower bound strength</td>
</tr>
<tr>
<td>$Q_k$</td>
<td>Characteristic live load</td>
</tr>
<tr>
<td>$R_m$</td>
<td>Maximum capacity</td>
</tr>
<tr>
<td>$s$</td>
<td>Distance between ties</td>
</tr>
<tr>
<td>$t$</td>
<td>Wall thickness</td>
</tr>
<tr>
<td>$t$</td>
<td>Time</td>
</tr>
<tr>
<td>$T_i$</td>
<td>Internal tie force</td>
</tr>
<tr>
<td>$T_p$</td>
<td>Peripheral tie force</td>
</tr>
<tr>
<td>$V$</td>
<td>Shear force</td>
</tr>
<tr>
<td>$v_1$</td>
<td>Factor for determination compressive diagonal</td>
</tr>
<tr>
<td>$V_{ed}$</td>
<td>Design value shear force</td>
</tr>
<tr>
<td>$V_{bd}$</td>
<td>Shear capacity</td>
</tr>
<tr>
<td>$w$</td>
<td>Crack opening</td>
</tr>
<tr>
<td>$w_0$</td>
<td>Maximum crack opening</td>
</tr>
<tr>
<td>$w_c$</td>
<td>Fracture process zone</td>
</tr>
<tr>
<td>$X_u$</td>
<td>Height concrete compressive zone</td>
</tr>
<tr>
<td>$y$</td>
<td>Displacement</td>
</tr>
<tr>
<td>$\dot{y}$</td>
<td>Speed</td>
</tr>
<tr>
<td>$\ddot{y}$</td>
<td>Acceleration</td>
</tr>
<tr>
<td>$\gamma_{dynamic}$</td>
<td>Dynamic displacement</td>
</tr>
</tbody>
</table>
\( Y_{\text{max}} \) : Maximum displacement
\( Y_{\text{static}} \) : Static displacement
\( z \) : Internal lever arm
\( z_i \) : Lever arm layer
\( \alpha_n \) : Factor
\( \beta_i \) : Factor
\( y_c \) : Concrete material safety factor
\( y_G \) : Partial factor dead load
\( y_Q \) : Partial factor live load
\( y_t \) : Area under stress strain diagram
\( \delta \) : Displacement
\( \delta_f \) : Relative displacement
\( \Delta t \) : Time step
\( \epsilon_{\text{el}} \) : Elastic compressive strain
\( \epsilon_{\text{et}} \) : Elastic tensile strain
\( \epsilon_c \) : Compressive strain
\( \tilde{\epsilon}_{\text{in}}^c \) : Inelastic compressive strain
\( \epsilon_{c;1} \) : Strain at maximum compressive stress
\( \epsilon_{c;\text{lim}} \) : Ultimate compressive strain
\( \epsilon_f \) : Accumulated strain
\( \epsilon_i \) : Strain in layer
\( \epsilon_{\text{slip}} \) : Strain caused by slip
\( \epsilon_t \) : Tensile strain
\( \tilde{\epsilon}_{\text{ck}} \) : Cracking strain value
\( \tilde{\epsilon}_{\text{pl}} \) : Plastic strain
\( \epsilon_{\text{uk}} \) : Ultimate yield strain steel
\( \theta_{\text{pra}} \) : Plastic rotation angle
\( \theta_y \) : Yield rotation
\( \kappa \) : Curvature
\( \lambda \) : Slenderness
\( \rho \) : Reinforcement ratio
\( \sigma_c \) : Compressive stress
\( \sigma_i \) : Stress in layer
\( \sigma_t \) : Tensile stress
\( \psi \) : Instantaneous factor
\( \omega \) : Natural frequency
\( \Omega_n \) : Dynamic increase factor
\( \phi \) : Reinforcement diameter
\( \phi \) : Strength reduction factor
## Contents

<table>
<thead>
<tr>
<th>Section</th>
<th>Pages</th>
</tr>
</thead>
<tbody>
<tr>
<td>Preface</td>
<td>II</td>
</tr>
<tr>
<td>Summary</td>
<td>III</td>
</tr>
<tr>
<td>List of symbols</td>
<td>V</td>
</tr>
<tr>
<td><strong>1 Introduction</strong></td>
<td>1</td>
</tr>
<tr>
<td>1.1 Background</td>
<td>1</td>
</tr>
<tr>
<td>1.2 Problem area</td>
<td>2</td>
</tr>
<tr>
<td>1.3 Problem statement</td>
<td>2</td>
</tr>
<tr>
<td>1.4 Research goal</td>
<td>3</td>
</tr>
<tr>
<td>1.5 Research approach</td>
<td>3</td>
</tr>
<tr>
<td>1.6 Structure of the report</td>
<td>4</td>
</tr>
<tr>
<td>1.7 Reference case study</td>
<td>5</td>
</tr>
<tr>
<td><strong>2 General</strong></td>
<td>6</td>
</tr>
<tr>
<td>2.1 Progressive collapse</td>
<td>6</td>
</tr>
<tr>
<td>2.2 Design strategies</td>
<td>6</td>
</tr>
<tr>
<td>2.2.1 Direct design strategy</td>
<td>6</td>
</tr>
<tr>
<td>2.2.2 Indirect design approach</td>
<td>8</td>
</tr>
<tr>
<td>2.3 Structural properties</td>
<td>8</td>
</tr>
<tr>
<td>2.4 Standardization</td>
<td>8</td>
</tr>
<tr>
<td>2.4.1 NEN</td>
<td>9</td>
</tr>
<tr>
<td>2.4.2 Eurocode</td>
<td>9</td>
</tr>
<tr>
<td>2.4.3 (Old) British standard</td>
<td>12</td>
</tr>
<tr>
<td>2.5 Catenary action</td>
<td>12</td>
</tr>
<tr>
<td>2.6 Alternate path method</td>
<td>13</td>
</tr>
<tr>
<td><strong>3 State of the art</strong></td>
<td>16</td>
</tr>
<tr>
<td>3.1 Collapse resistance existing structures</td>
<td>16</td>
</tr>
<tr>
<td>3.2 Collapse of peripheral columns</td>
<td>17</td>
</tr>
<tr>
<td>3.3 Collapse of corner columns</td>
<td>20</td>
</tr>
<tr>
<td>3.4 Catenary action</td>
<td>22</td>
</tr>
<tr>
<td>3.5 Influence of infill walls</td>
<td>25</td>
</tr>
<tr>
<td><strong>4 Finite element modeling</strong></td>
<td>27</td>
</tr>
<tr>
<td>4.1 Solution sequence</td>
<td>27</td>
</tr>
<tr>
<td>4.2 Element formulation</td>
<td>28</td>
</tr>
<tr>
<td>4.3 Concrete model</td>
<td>29</td>
</tr>
<tr>
<td>4.4 Material properties</td>
<td>31</td>
</tr>
<tr>
<td>4.4.1 Concrete in tension</td>
<td>31</td>
</tr>
<tr>
<td>4.4.2 Concrete in compression</td>
<td>34</td>
</tr>
<tr>
<td>4.4.3 Reinforcing steel</td>
<td>36</td>
</tr>
<tr>
<td>4.4.4 Strain rate effect</td>
<td>37</td>
</tr>
<tr>
<td>4.4.5 Material properties current thesis</td>
<td>37</td>
</tr>
</tbody>
</table>
Contents

4.5 Model geometry ........................................................................................................ 37
4.6 Loads ......................................................................................................................... 38
4.7 Cross sectional dimensions ..................................................................................... 39
4.8 Column loss approach ........................................................................................... 39

5 Benchmark procedures ............................................................................................ 41
  5.1 Static analysis ......................................................................................................... 41
    5.1.1 Multilayer model ............................................................................................. 41
    5.1.2 Statically determined RC beam ...................................................................... 42
    5.1.3 Statically indeterminate RC beam ................................................................. 44
  5.2 Dynamic analyses .................................................................................................. 48
    5.2.1 SDOF system ................................................................................................... 48
    5.2.2 Linear elastic material .................................................................................... 49
    5.2.3 Elastic plastic material .................................................................................... 51
    5.2.4 Reinforced concrete ....................................................................................... 53
    5.2.5 Sudden column loss simulation ...................................................................... 55

6 Application FE framework ......................................................................................... 57
  6.1 Static analysis initial frame .................................................................................... 57
  6.2 Static analysis without corner column ................................................................. 58
  6.3 Static analysis DLF of 2 ....................................................................................... 59
  6.4 Dynamic analysis .................................................................................................. 61
  6.5 Dynamic load factor ............................................................................................ 62

7 Concluding remarks and recommendations ............................................................ 65

Bibliography .................................................................................................................. 67

A Appendix: progressive collapse properties ............................................................... 71
B Appendix: determination ties EC ............................................................................... 75
C Appendix: catenary action ......................................................................................... 77
D Appendix: alternate path method ............................................................................... 79
E Appendix: ULS design ............................................................................................... 82
F Appendix: transformation factors SDOF system ...................................................... 89
G Appendix: linear elastic SDOF system ....................................................................... 91
H Appendix: central difference method ....................................................................... 92
I Appendix: input files Abaqus .................................................................................... 94
1 Introduction

1.1 Background

Society expects that the structure of a building is sufficient safe. This safety should be the result of robust design, proper execution and good material choice. A safe structure should be able to bear the loads acting on it and may not collapse completely when a structural element fails due to an accident or unforeseen action. Unfortunately, there are a number of instances where this was not the case.

The one that is most referred to is the 22-story Ronan Point apartment tower in Newham, east London. When an occupant on the 18th floor of the tower struck a match in her kitchen, she set off a gas explosion that knocked out the load bearing precast concrete panels near the corner of the building. The loss of the load bearing precast concrete panel at the 18th floor caused the floors above to collapse. The impact of these collapsing floors set off a chain reaction of collapses all the way to the ground (see Figure 1). This phenomenon is known as progressive collapse. Progressive collapse can be defined as a situation where local failure of a primary structural component leads to the collapse of adjoining members, which in turn leads to additional collapse.

![Figure 1; collapse of a part of the Ronan Point apartment](image)

In particular, the collapse of Ronan Point has served as an encouragement for the development of new regulations in the UK. The one which is most important is the Fifth Amendment from 1970. These regulations set additional requirements to the robustness of a structure. One of the main items of these additional requirements is that the removal of a single element of the structure may not lead to a disproportionate collapse.
Technical institutes and building authorities in several countries developed design guidelines and criteria that would reduce or eliminate the susceptibility of buildings to progressive collapse. One of the standards that provide design guidelines with respect to progressive collapse is the Eurocode. With the introduction of the Eurocode in the Netherlands it is ensured that there are more tools available to design a robust structure. However, the introduction of the Eurocode also leads to new discussions about and researches to progressive collapse. This study will continue the research to progressive collapse of reinforced concrete structures.

A remark should be made with respect to the research to progressive collapse. The phenomenon of progressive collapse is mainly studied for in-situ-cast buildings. However, precast structures (like Ronan Point) are more vulnerable to progressive collapse than cast in situ buildings. This enhanced vulnerability to progressive collapse of precast structures is mainly caused by a smaller degree of cohesion between the structural elements. The connections between precast elements are less rigid than the connections between in-situ-cast elements, therefore a lower amount of cohesion can be achieved with precast structures. Due to a lack of time it was not possible to investigate the complex behavior of the connections within precast structures in the current study. This thesis will therefore focus on in-situ-cast structures only.

1.2 Problem area

With the introduction of the Eurocode in 2012 in the Netherlands, new requirements with regard to progressive collapse were introduced. These new requirements are more extensive than the requirements as specified in the previous Dutch standard NEN. However, the new regulations also lead to a discussion regarding the subject progressive collapse. This discussion is mainly caused by the cable mechanism that the Eurocode prescribes for the prevention of progressive collapse. There are doubts about the functioning of this cable mechanism. Especially for the prevention of progressive collapse in case of failure of a corner column there are doubts if the mechanism can develop. Therefore alternatives are necessary to prevent progressive collapse. An alternative for the cable mechanism is the development of an alternate load path by means of the flexural capacity of the beams. For the assessment of this alternate load path it is important to consider the dynamic nature of the load caused by the sudden failure of, for instance, a corner column.

In order to include this dynamic effect in static analysis often a Dynamic Load Factor (DLF) is used. The load above the removed structural element can be multiplied with this dynamic load factor when a static progressive collapse analysis is performed. In the literature this DLF is often set to two. However, according to various researchers, a DLF of two will in many cases lead to a too conservative structure. Especially for reinforced concrete structures were highly nonlinear material behavior can be expected this factor will in mainly cases be smaller than two. Therefore, more research is needed to this dynamic load factor with respect to progressive collapse.

1.3 Problem statement

After identifying the problem area with respect to progressive collapse, a problem statement was defined in which the problem to be investigated was specified. For the current study the problem statement consists of two parts, one based on performing a literature study to the state of the art with respect to alternate load paths and catenary action, and one based on the development of a dynamic load factor to included the dynamic nature of the loads caused by the sudden failure of a structural element in static analysis. This resulted in the following two problem statements:

Is it possible to conduct a detailed overview about the current state of the art with respect to alternate load paths and catenary action for reinforced concrete structures?
Is it possible to derive a dynamic load factor for a reinforced concrete frame structure in case of the sudden failure of a corner column by means of the finite element method?

1.4 Research goal

After the stipulation of the problem statement a research goal was defined. Also now two goals were specified for this graduation project. The next two goals were specified:

Performing a detailed study to the current state of the art with respect to alternate load paths and catenary action for reinforced concrete structures.

The determination of a dynamic load factor for a reinforced concrete frame structure to include the dynamic nature of the load caused by the sudden failure of a corner column by means of the finite element method.

1.5 Research approach

In the initial phase of this research project a literature study was conducted to form a general view on the considered problem area. With the information obtained during this phase a problem statement and research goal were derived for the current study. After identification of the problem statement and research goal an in depth study was performed to the state of the art with respect to alternate load paths and catenary action. This study forms the first part of this graduation thesis about progressive collapse. Simultaneously, a reference case study was designed that was used as a basis for the finite element model developed in this thesis. A flow chart that includes the steps that were taken for the study to the dynamic load factor with the finite element method is shown in Figure 2. The steps shown in Figure 2 are explained in more detail in the subsequent paragraphs.

Due to a lack of experimental data to benchmark the results obtained with the finite element model for static loads a simple analytical benchmark test was developed. This benchmark test was based on a multilayer model and the moment-area method. After the development of this benchmark test the finite element model was established with the finite element package Abaqus. Statically applied loads on a statically determined structure were benchmarked first, thereafter statically indeterminate structures were tested. The results obtained with the finite element model were compared to the results obtained with the multilayer model and the moment area method. Further, for the statically indeterminate structures, the bending moment equilibrium was checked for each load configuration and also a comparison was made of the bending distribution according to the linear elasticity theory.

Since progressive collapse is a dynamic phenomenon also a simple analytical benchmark model was developed to compare the results obtained with the finite element model regarding dynamic loads. This model was based on a single degree of freedom mass spring system. Here the stiffness of the spring is adapted to the stiffness of the reinforced concrete beam for different deformation stages.

Finally, the reference frame was modeled with the developed finite element model. With this reinforced concrete frame structure some analyses with respect to the sudden loss of a corner column were performed. For this simulation use was made of the sudden column loss approach as often used in the literature. A treat independent progressive collapse analysis was performed since the cause of the sudden column removal was left out of consideration.
1.6 Structure of the report

This thesis is divided into two main goals as discussed in paragraph 1.4. The first part of the study is included in chapter 2 and 3 and the second part in chapter 4 to 6. Chapter 2 contains a study about the current designs strategies and regulations with respect to progressive collapse. In chapter 3 a study to the state of the art with respect to alternate load paths and catenary action for reinforced concrete structures is presented. Chapter 4 contains a description on how the finite element model is developed for this thesis. Chapter 5 includes a discussion about the benchmark process used to benchmark the developed finite element model. In chapter 6 the simulations performed on the reference frame are discussed. The findings and recommendations obtained during this study are discussed in chapter 7.
1.7 Reference case study

As discussed in the previous paragraph a reference case study was developed that served as a basis for the finite element model. This structure is a fictive reinforced concrete building as shown in Figure 3. The structure consists of a 5 story beam-column structure with a regular grid of 7,2 x 7,2 m. The stability of the structure is provided by a concrete core in the middle of the building (see Figure 3 and Figure 4). It is assumed that the building will be used as an office building.

For the current research only a part of the building was considered as shown with the red lines in Figure 3. Here it can be seen that only the frame at the front of the building was analyzed. The influence of the floors and flexural capacity of the columns was not considered in the current research. This is a very unfavorable assumption since both the floors and columns have a large influence on the progressive collapse resistance. However, with respect to precast structures it might be useful to analyze what the resistance of the beams only is. In precast structures the floors and columns cannot contribute to the progressive collapse resistance due to the very low flexural capacity of the floors above the supports and the often pinned connections between the beams and columns.

Figure 3; reference structure

Figure 4; floor plan
2 General

2.1 Progressive collapse

Progressive collapse is defined as a situation where local failure of a primary structural component leads to the collapse of adjoining members, which in turn leads to an additional collapse. Local failure is always caused by an accidental action. An accidental action can be expressed as a design situation involving exceptional conditions of the structure or its exposure to explosion, impact or local failure [1]. Examples of accidental actions are gas explosions, vehicle impact or car bomb attacks, but can also be the result of design and construction errors. In case of an accidental action, the sudden unexpected load is typically concentrated on one or two key elements in a structure. Hereby it is possible that a structural element has its load pattern or boundary conditions changed such that it will be loaded beyond its static or dynamic capacity. Because of these changes, progressive collapse can occur. There are various types of progressive collapse. A description of some of the different forms of progressive collapse is given in appendix A.

2.2 Design strategies

There is no state of absolutely safety, as well as there is no approach available that meet simultaneously the wide range of disastrous abnormal environmental events. Therefore it is necessary to emphasize some broad strategic rules which are suitable for the mitigation of hazardous situations. Through time, different methods are developed to cope with accidental actions. These methods have resulted in design strategies [2] [3] [4]. For one building, different design strategies can be used. Mainly, a distinction is made between two design strategies; direct design strategy and indirect design strategy. The direct design strategy is based on project dependent factors. So, for each project a new strategy needs to be developed. The indirect strategy is more generally applicable for different projects. When a building remains within certain standard specifications, the indirect design strategy can lead to a fast and safe result. This doesn’t mean that this strategy unconditionally lead to a building with sufficient safety against progressive collapse. An overview of design strategies is given in Table 1.

<table>
<thead>
<tr>
<th>Design strategies</th>
<th>Direct</th>
<th>Indirect</th>
</tr>
</thead>
<tbody>
<tr>
<td>Active → Prevent local failure</td>
<td>Passive → assume local failure</td>
<td>Prescriptive design rules</td>
</tr>
<tr>
<td>1. Non structural protective measures</td>
<td>1. Alternative load paths</td>
<td></td>
</tr>
<tr>
<td>2. Specific local resistance (key elements)</td>
<td>2. Isolation by segmentation</td>
<td></td>
</tr>
</tbody>
</table>

2.2.1 Direct design strategy

As presented in Table 1, the direct design approach can be divided into an active and a passive approach [3]. The active approach includes those preventive measures which are directed to minimize the probability of incidents or to filter out sources of abnormal inputs. The passive approach can be summarized as the acceptance of a limited local failure of non critical size.

Active approach

Considering the minimizing of the probability of incidents, there are two ways in which this can be achieved. The first one consists of the elimination of the possible abnormal inputs, such as the use of gas in a building. Eliminating the possible source of an incident will also ensure that the risk of an incident due to this source is reduced. The disadvantage of this method is that it is not possible to predict all of the risks in advance. The second method
consists of measures that are able to avert a possible risk, e.g. barriers to resist vehicle impact, sacrificial elements to mitigate blast loading and other protective measures. However, also for this method it is not possible to predict all risks that might occur.

Another method is called specific local resistance method. Here, some structural elements are identified as key elements and therefore their safety against initial failure is increased. This increase of safety can be achieved by designing the key elements for the incidental actions resulting from possible occurring hazards.

**Passive approach**
This method is based on limiting the extend of local failure, and not on preventing or reducing the probability of local failure. In other words, this approach involves the acceptance of limited local failure due to an accidental action. Limiting the extend of local failure can be achieved by the application of an alternative load path or through isolation by segmentation.

When using the alternative load path method one should assume various alternative locations of local damage. At each location where local damage is assumed a redistribution of loads should be possible to the remaining undamaged structural elements. To achieve this redistribution of loads, also the connections and ties should be designed to resist the resulting actions. An important property that needs be charged is the influence of dynamic effects due to the sudden loss of a load bearing element. To take the influence of the dynamic nature of the load into account often the load is multiplied with a Dynamic Load Factor (DLF).

There are various ways in which an alternative load path can develop, in some cases the flexural and, if necessary, plastic reserves of the structure are mobilized and in other cases an alternative load path can form through the mobilization of axial or torsional resistance. When using a passive designs approach, and accept local damage, the extend of accepted damage should not exceed certain requirements which are prescribed in the standards.

Another important structural property that needs to be considered is the stability of the structure in its whole, it need to be proven that the structure is sufficiently stable within the local damage under the relevant load combinations.

For some structures it is not desirable or possible to create an alternative load path. Starossek [4] gives an example of the Confederation Bridge between Prince Edward Island and the Canadian mainland (see Figure 5). Here, the main spans of the girder are 250 meter each. When this structure should be made collapse resistant by providing an alternative load path, the notional failure of a bridge pier should be considered. This would result in a double span length of 500m which is a very undesirable design situation, and arguably a vain endeavor even if resorting to catenary action. Therefore, another approach is used for this structure. The bridge is divided into segments. By isolating collapsing sections, the extend of collapse will be limited. This method also allows a limited extend of local failure. The segmentation borders can be carried out by different modes of effectiveness:

1. Local reduction of the stiffness or by discontinuities in the structure
2. High local resistance
3. High ductility and large energy dissipation capacity
2.2.2 **Indirect design approach**

The design methods discussed above are quite devious. Especially for small and medium sized structures it might be disproportionate to use these methods. However, it is still desirable to achieve at least a certain degree of collapse resistance by following prescriptive design rules, which is an indirect design approach. With this method, resistance against progressive collapse is considered indirectly by providing tension ties, enabling catenary action and ensuring ductility through the whole structure. To achieve this, the main structural elements are required to be tied together horizontally and vertically by continuous tension ties.

For this design approach it is assumed that the tension ties can provide in catenary action. Enabling catenary action may avert progressive collapse because of its capacity to bridge a damaged field by tensile load transfer, for example in the event of failure of a column. Thereafter, catenary action may also prevent falling of failed elements on underlying structural elements. These ties also contribute to increasing the ductility of the structure. By ensuring ductility, the plastic reserves of the structure will be activated in case of failure of a structural element, and will therefore be able to dissipate energy.

However, some caution is required to these prescriptive design rules since they may increase the number of potential alternative load paths but do not guarantee their quality, i.e. sufficient resistance against the occurring loads. Also for large, unique and expensive structures, the indirect design method will not unconditionally lead to a safe and cost efficient building, therefore it is desirable to use a direct design approach for these structures.

2.3 **Structural properties**

For regular structural design mainly stiffness in the initial state and maximum resistance are the most important properties. For design with regard to accidental action and prevention of progressive collapse also parameters such as the maximum deformation, total strain energy and ductility are important parameters to consider [2] [4]. Appendix A contains an overview of the most important structural properties with respect to progressive collapse.

2.4 **Standardization**

This chapter provides an overview of the regulations with regard to progressive collapse for some key standards. First, a brief description of the NEN will be given and thereafter the Eurocode will be treated so that insight can be obtained in the differences between the old and new standards with regard to progressive collapse. Then the British standard will be considered. The reason for considering the British standard is that the rules regarding progressive collapse used in the Eurocode are for a large part derived from the British standard.
2.4.1 NEN

The NEN contains not much information about the prevention or mitigation of progressive collapse. The most important requirement is given by NEN6700 [5], here it is state that the structure of a building must be designed in such way that failure of one element will not lead to disproportionately extensive damage. This means that the structure needs certain resilience against progressive collapse. Another requirement is that the system behavior of the entire structure must be considered and that the structure must be designed in such way that:

- After failure of one component, the damage of collapse is limited to the adjacent rooms or adjacent load bearing elements of the construction under consideration, regardless the cause of failure, or;
- The probability of failure of key elements is reduced by:
  - Taking preventive measures or creating sufficient resistance to accidental actions and
  - Specific attention to the quality of the design and execution process

Especially the second part of this requirement can provide discussion. In the explanatory notes it is argued that the accidental load that needs to be taken into account can be determined by Chapter 9 of NEN 6702 [6]. In this article the next few loads are treated:

- Fire
- Gas explosion
- Vehicle impact
- Impact load on floors and roofs
- Loads by extreme ground water levels
- Loads by earthquakes

The loads shown above do not cover the wide spectrum of abnormal loads. Therefore it does not unconditionally provide a building with sufficient resistance against progressive collapse. In addition, NEN6702 indicates that it is very difficult to predict the occurring forces in case of an incidental action. Therefore, it is indicated here that the occurrence of damage should be taken into account, and that the structural measures should aim on limiting the extent of damage.

Which or how structural measures should be applied to prevent or limit progressive collapse is not further specified and is left to the designer. It can be concluded that the requirements described in this standard are indirect. For each project a new strategy needs to be developed since no prescriptive design rules are given with respect to progressive collapse in this standard.

2.4.2 Eurocode

Since April 2012 the Eurocode is into force in the Netherlands, which means that all new structures should be designed according to this standard. This has influence on the design against progressive collapse since both Eurocode 1 [7] and Eurocode 2 [8] contain rules for the prevention of progressive collapse. This standard contains an indirect design strategy since it provides project independent requirements, for instance the application of horizontal ties. The requirements in the Eurocode are quite different in comparing to the previous Dutch standard NEN. This chapter gives an overview on the most important requirements which are included by the Eurocode regarding progressive collapse.
Eurocode 1: General actions

In NEN-EN 1991-1-7 (Eurocode 1 – Actions on structures, General actions – Accidental actions due to impact and explosions) [7] aspects such as progressive collapse and the resilience of constructions are discussed. Also design approaches are recommended for the design against accidental actions. Eurocode 1 is for a large part based on the British standard, only for a few topics some adjustments were made. Figure 6 presents a flow chart for the design strategies in case of accidental actions [7].

![Figure 6; strategies for Accidental Design according to EN 1991-1-7 [7]](image)

The design strategies for accidental actions may be based on consequence classes. Every building can be classified into a different consequence class. There are three consequence classes:

- Consequences class 1: low (limit consequences)
- Consequences class 2a and 2b: (medium consequences)
- Consequences class 3: (high consequences)

In which class a building belongs primary depends on the size of the negative effects. An example of classification is given in Table 2. This table is taken from Eurocode EN 1991-1-7, and provides a recommendation for the division of buildings into consequence classes where the influence of the building type and degree of occupation is considered.

<table>
<thead>
<tr>
<th>Consequence class</th>
<th>Type of building and occupancy</th>
</tr>
</thead>
<tbody>
<tr>
<td>CC1</td>
<td>Single occupancy houses not exceeding 4 storey’s, Agricultural buildings, Buildings into which people rarely go, provided no part of the building is closer to another building, or areas where people do go, than a distance of 1 1/2 times the building height</td>
</tr>
<tr>
<td>CC2a Low Risk Group</td>
<td>5 story single occupancy houses, Hotels not exceeding 4 storey’s, Flats, apartments and other residential buildings not exceeding 4 storey’s, Offices not exceeding 4 storey’s, Industrial buildings not exceeding 3 Storey’s, Retailing premises not exceeding 3 storey’s of less than 1 000 m² floor area in each storey, Single storey educational buildings, All buildings not exceeding two storey’s to which the public are admitted and which contain floor areas not exceeding 2000 m² at each storey</td>
</tr>
<tr>
<td>CC2b Upper Risk Group</td>
<td>Hotels, flats, apartments and other residential buildings greater than 4 storey’s but not exceeding 15 storeys, Educational buildings greater than single storey but not exceeding 15 storeys, Retailing premises greater than 3 storey’s but not exceeding 15 storeys.</td>
</tr>
</tbody>
</table>
| **CC3** | Hospitals not exceeding 3 storeys.  
|         | Offices greater than 4 storeys but not exceeding 15 storeys.  
|         | All buildings to which the public are admitted and which contain floor areas exceeding 2000 m² but not exceeding 5000 m² at each storey. Car parking not exceeding 6 storeys.  
| **All buildings defined above as Class 2 Lower and Upper Consequences Class that exceed the limits on area and number of storeys.** |  
|         | All buildings to which members of the public are admitted in significant numbers.  
|         | Stadia accommodating more than 5000 spectators.  
|         | Buildings deemed to be high-risk targets. Buildings containing hazardous substances and processes. |

**Note 1:** For buildings of more than one type of use the “consequences class” should be that relating to the most onerous type.  
**Note 2:** In determining the number of storeys, basement storeys may be excluded provided such basement storeys fulfill the requirements of “Consequences Class 2b Upper Risk Group”.  
**Note 3:** Consequences class for building types not specifically covered should be taken as the closest similar type. 

Depending on the consequence class a design strategy is recommended. The following design strategies are recommended for the various consequence classes:

- **Consequence class 1:** Provided a building has been designed in accordance with the rules given in national or international standards for satisfying stability in normal use, no further specific consideration is necessary with regard to accidental actions from unidentified causes.
- **Consequence class 2a:** Buildings should be designed in accordance with the requirements of the indirect approach. Effective peripheral and internal ties should be provided, respectively for framed and load-bearing wall constructions. Vertical ties are not strictly required but always recommended.
- **Consequence class 2b:** Horizontal and vertical ties should be provided. As an alternative, the building should be designed in accordance with the requirements of the alternative path approach. The building should be checked to ensure that upon the notional removal of each supporting column and each beam supporting a column, or any nominal section of load bearing wall, the building remains stable and that any local damage does not exceed a certain limit. Where the notional removal of such columns and sections of walls would result in an extent of damage in excess of the agreed limit, then such elements should be designed in accordance with the specific load resistance method. In the case of wall frame buildings, the notional removal of sections of wall, one at a time, is likely to be the most practical strategy to adopt.
- **Consequence class 3:** Distinction is made between buildings with normal occupancy and buildings with a high occupancy or significant consequence of an accidental action.
  - Buildings exceeding the limits of Class 2a and 2b. This category of building should be either
    - Designed in accordance with the requirements of the alternative path approach
    - A systematic qualitative risk assessment of the building should be performed and the required improvements based on this assessment implemented
  - Buildings with high occupancy and stadia with a capacity of more than 5000 persons. For this category of building the consequences of accidental actions can be significant and systematic risk assessment of the building should be
undertaken and the required improvements based on this assessment implemented.

- Buildings deemed to be high risk or buildings containing dangerous substances, or where dangerous processes are carried out. For this category of building a systematic risk assessment of the building should be undertaken and the required improvements based on this assessment implemented.

Eurocode 1 also provides a limit for the extent of damage. The recommended value for the area of damage equals 15% of the floor surface or 100m² on each storey of two adjacent floors (see Figure 7). The procedure to determine the cross sectional dimensions of the ties are discussed in appendix B.

![Figure 7; damage area according to Eurocode](image)

2.4.3 (Old) British standard

Shortly after the collapse of Ronan Point in 1968 there was a need for developing prescriptive design rules for the mitigation of progressive collapse. Research to progressive collapse resulted in the publication of new standards in the UK. These new standards provided design rules for limiting progressive collapse, among other by recommending horizontal and vertical ties. The recommendations included in the British standard still form the basis for our current design standards in relation to progressive collapse. Only a few minor adjustments were made in the Eurocode. This section does not contain a detailed description of the requirements as included in the British Standard due to the similarity with the Eurocode.

2.5 Catenary action

As described in chapter 2.4.2 the Eurocodes base their methodology for the prevention of progressive collapse mainly on catenary action. In this paragraph the basics of catenary action will be discussed.

Before catenary action occurs another mechanism can develop called a strut mechanism. A strut mechanism will only develop when the deflection is less than the depth of the beam section. The principle behind this mechanism is that the diagonal length exceeds the free space between the support sections \((\sqrt{h^2+L^2} > L)\), see Figure 8), this will cause a compression force along the diagonal which makes it possible to carry a vertical force. The compression force in the diagonal should be resisted by bracing units in the structural system. The size of the compression force that can be resisted depends on the stiffness of the structural system and is difficult to predict and rely on. For instance stiffness can be drastically reduced by joint gaps, crack openings and soft connection details. It is doubtful if this mechanism can develop [9]. These doubts come primarily from the fact that the horizontal forces resulting from this mechanism will be that large that they cannot be
resisted by the remaining structure. Therefore the beam will snap through and catenary action will develop.

Since a cable only provides axial stiffness it is not capable to bear any load perpendicular to its axis in the initial undeformed state. However, the axial stiffness will be activated when the cable starts to deform and therefore the cable is able to transfer loads to the supports. This process can be observed in the load deflection diagram as shown in Figure 9. Due to this axial force in the cable also a horizontal reaction force will develop at the supports in the opposite direction as for the strut mechanism as shown in Figure 8. To reach a state of equilibrium the deformation capability of the cable is decisive since the lever arm of the horizontal reaction force will increase with an increased deformation of the cable. By considering the bending moment equilibrium it can be determined how much a cable must deform to reach a state of equilibrium. For a more detailed description about the development of catenary action reference is made to appendix C.

![Figure 8; phases development catenary action](image)

![Figure 9; load deflection curve development catenary action](image)

### 2.6 Alternate path method

NEN-EN 1991-1-7 gives as an alternative for the tie approach the notional removal of load bearing elements to provide an alternate load path. However, there are no additional requirements given on how to design this alternate load path. As mentioned before, dynamic and nonlinear effects are important properties to consider when designing against progressive collapse. To include nonlinearity and dynamic effects when designing against progressive collapse, often a dynamic load factor (DLF) of two is used. This factor is based on the fact that the maximum dynamic deflection for an instantaneous applied load is twice as large as the deflection of the same load when it is applied statically when a structure behaves in a perfect linear elastic manner [10]. Among others Ellingwood et al. [11] and Kai et al. [12] showed that this load factor will often result in an over conservative design. Only when the structure is designed to remain elastic a factor of two would be appropriate. However, when designing against progressive collapse it is often allowed that the structure responds nonlinear.

The Eurocode does not give a procedure to take dynamic and nonlinear effects into account, however both GSA [13] and UFC [10] provide some procedures to determine factors that take these effects into account. According to, among others, UFC 4-023-03, the alternate path analysis can be performed by three different procedures; linear static, nonlinear static and nonlinear dynamic. All procedures have some advantages and disadvantages. A nonlinear dynamic analysis will approach reality the best way but is labor intensive and a linear static analysis is quite simple to perform but will usually result in an over conservative design. In 2009 the new UFC requirements were introduced which propose some new procedures for dealing with dynamic and nonlinear effects to avoid over
conservative structures. For all these procedures a three dimensional model of the structure should be build, two dimensional analysis are not permitted. Next a brief description of the design procedures provided by the UFC guidelines will be given.

**Linear static procedure**

For performing a linear static procedure, the UFC guidelines give a procedure to take the influence of dynamic and nonlinear effects into account. For an alternate load path analysis the UFC distinguish two types of elements; primary and secondary elements. If an element increase the capacity of the structure to resist collapse due to removal of a load bearing element, then it will be classified as primary, if not, the element is classified as secondary. For example, a steel gravity beam may be classified as secondary if it is assumed to be pinned at both ends and the designer chooses to ignore any flexural strength at the connection [10].

Further the UFC distinguish deformation and force controlled actions. Deformation controlled actions represents ductile behavior and force controlled actions represents brittle behavior. The deformation controlled action consists of two factors; a Load Increase Factor (LIF) and a capacity increase factor (called m-factor). The load increase factor takes dynamic and nonlinear effects into account and the capacity increase factor accounts for the expected ductility. For a linear static analysis the enhanced loads will be compared to the enhance capacity. A general formulation is presented in [2.8].

\[
\phi \cdot m \cdot Q_{CE} \geq Q_{UD} \quad [2.8]
\]

Here, $\phi$ is the strength reduction factor, $m$ the component or element demand modifier (m-factor) to account for expected ductility, $Q_{CE}$ the expected strength of the component or element for deformation controlled actions and $Q_{UD}$ the enhanced deformation controlled action from the linear static model.

For a force controlled action the load will be enhanced with a different LIF which only accounts for inertia effects and the capacity of the element will not be modified. For this situation the general formulation is presented in [2.9].

\[
\phi \cdot Q_{CL} \geq Q_{UF} \quad [2.9]
\]

Where $\phi$ is the strength reduction factor, $Q_{CL}$ the lower bound strength of a component and $Q_{UF}$ the force controlled action. The procedure to determine the strength reduction factor $\phi$ and the component demand modifier $m$ is given in appendix D.

**Nonlinear static**

For the nonlinear static procedure only the loads are magnified with a dynamic increase factor to take inertia effects into account. Again only a three dimensional model is permitted for this procedure. The model should be able to included material- and geometrical nonlinear behavior. So also effects like strength degradation should be included. After the model is build a dynamic increase factor must be derived to include the dynamic nature of the load. The UFC provides different dynamic increase factors for different materials. Appendix D also contains a procedure to determine the dynamic increase factor for the nonlinear static procedure.

**Nonlinear dynamic procedure**

Again a three dimensional assembly of elements and components must be modeled just as for the previously discussed procedures. When developing the model also the structural components that are assumed to be removed need to be modeled. All primary elements need to be considered. Also secondary elements might be included in the analysis but then they also need to be checked in the analysis. Further the same recommendations as used for the nonlinear analysis are valid.
Additionally a procedure is given on how the loads should be applied on the structure. First the gravity and lateral loads needs to be applied monotonically starting from zero until the full load is applied and equilibrium is reached. After a state of static equilibrium is reached the component that is assumed to fail must be removed. The duration of removal of the load bearing wall or column should be less than one tenth of the period associated with the structural response mode for the vertical motion of the bays above the removed column. The analysis must continue until the maximum displacement is reached or one cycle of vertical motion occurs at the position where the column or wall is assumed to be removed.
3 State of the art

As mentioned in the introduction, the collapse of Ronan Point in 1968 has played an important role in the inducement for research to the phenomenon progressive collapse. More recent collapses like the collapse of the Alfred P. Murrah Federal Building and collapse of Terminal 2E from the Paris Charles de Gaulle Airport emphasize the importance of well functioning requirements to prevent this form of collapse. Despite that there has been done a lot of research on progressive collapse, it still is difficult to collect information about this topic. This is mainly caused by the fact that the information that is available on progressive collapse is very fragmented. This chapter tries to provide an overview on important researches that have been conducted to the topic progressive collapse.

3.1 Collapse resistance existing structures

To investigate how susceptible existing reinforced concrete structures are for progressive collapse, some full scale tests have been performed by Sasani et al. [14] and Al-Ostaz et al [15]. The tested structures were old buildings that would be demolished and therefore suitable for doing experiments with local element removal. The first experiment was done on the hotel San Diego [14], a six story frame structure from 1914. For this experiment two adjacent columns, a corner and an edge column, were removed at the same time by explosion (see Figure 10). It turned out that the structure had sufficient capacity to prevent progressive collapse. After analyzing the deformed shape that developed after removal of the columns they concluded that the major mechanism that resists the loads consists of Vierendeel action. Vierendeel action can be described as a truss beam without diagonals where the connections between the structural elements are rigid and must resist bending forces.

![Figure 10; locations removed columns hotel San Diego [14]](image1)

The second test on an existing structure was a two story apartment building as shown in Figure 12 [15]. Because there were more almost identical buildings it was possible to investigate the behavior of the structure for different positions of column removal. The first one was the removal of a corner column. After the column was removed no cracks were observed in the concrete and the deformation of the corner was nearly 4 mm. This denotes that the remaining structure has sufficient resistance in the linear elastic range. Secondly an interior column was removed. Also for this position of column removal the structure had sufficient reserve capacity. The deflection found for this column removal position was about 6 mm. Based on the experimental result it can be concluded that the structure has sufficient reserve capacity to resist local element removal. This is mainly caused by over designed members [15]. It is interesting to see that even when both structures were not designed for seismic loads or alternate paths they are still able to prevent progressive collapse.
3.2 Collapse of peripheral columns

Different experimental studies are performed on the progressive collapse behavior of reinforced concrete frame structures in case of removal of an intermediate column. Yi et al. [16] did research to a four bay three story one third scale model as shown in Figure 14. The frame under consideration represents a segment of an eight story RC frame structure. The frame was designed according to the concrete design code of China which is generally similar to the American concrete code. The main purpose of this experiment was to investigate the force-deflection response after column removal. After analyzing the test results, Yi et al. divided the increase in the vertical displacement into four stages: elastic stage, elasto plastic stage, formation of plastic hinges and catenary mechanism. Before testing, an analytical approximation was made based on a plastic mechanism which was 70% of the tested failure capacity if catenary effects were taken into account. Failure of the frame occurred due to rupture of the reinforcing steel bars in the beams. However, Yi et al. concluded that the structure possesses sufficient resistance to avoid progressive collapse in case of failure of an intermediate column. Further, they expect that when the strain in the tensile bars can be distributed more uniform over the length of the bar also the deformation capacity of beam will be enhanced. The enhance of the deformation capacity will also increase the load carrying capacity of the beam by catenary action, this might be achieved by partly debonding the reinforcement bars.

Another research to collapse resistance mechanisms for reinforced concrete frame structures after removal of an intermediate column was executed by Stinger [17]. He investigated three almost identical reinforced frame structures as shown in Figure 15. The first frame was executed with discontinuous reinforcement the second frame with continuous reinforcement and the third frame with discontinuous reinforcement and infill walls with openings. The considered frames are specimen from a six story building designed according to the ACI code. In this research it appears that the frame with discontinuous reinforcement had a higher capacity under compressive arch action than predicted with the flexural theory. As can be expected, the frame with continuous reinforcement had a higher flexural capacity than the frame with discontinuous reinforcement. However, due to a lack of ductility, the frame with continuous reinforcement did not exhibit significantly better in catenary action that the frame with...
discontinuous reinforcement. Further, the frame with partially infill walls did not perform better than the frame without these walls. The flexural capacity of the frame with infill walls increased but preformed worse in the catenary action phase.

To investigate the influence of different amounts and types of reinforcement He et al. [18] tested five beam-column substructures as presented in Figure 16. Al beams are designed according the Chinese design code and provided different types and amounts of reinforcement. In order to ensure the rotation at the supports all beams are provided with 80 mm tubes at the ends that are connected with steel hinge supports by steel pins (see Figure 16). One of the main conclusions of this experiment is that the development of catenary action is mainly related to the uniform elongation and strength of the steel. Uniform elongation improves the deformation capacity and the steel strength improves the load carrying capacity. It is therefore concluded that round bars are favorable for increasing the progressive collapse resistance because round bars deform more evenly and had a longer elongation than the ribbed bars. An important overall conclusion is that catenary action can only play an important role in increasing the load bearing capacity when a large deformation of the beam is possible. Further, this research also showed that a beam under column removal experienced elastic deformation stage, plastic deformation stage and catenary action stage.

Another research that focused on the deformation capacity of joints is executed by Yu et al. [19]. For this research also a mid column removal scenario is considered where catenary action is mobilized. To achieve catenary action the beams must be able to undergo large deflections and rotations. Two frames were tested, one frame with non-seismic detailing in accordance with ACI 318-05 and one frame designed by debonding the bottom bars in the joint regions with plastic sleeves to release the strain concentration of bars at the joint interfaces and to increase rotation capacity of beam column connections as shown in Figure 17.

Tests showed that for conventional detailing catenary action will develop but fail to significantly increase the structural resistance due to excessively premature fracture of top bars at the side joint interfaces. The second frame allowed catenary action to be mobilized to significantly enhance structural resistance. The corresponding experimental results showed that the structural mechanisms changed from flexure and compressive arch action at small deformations to catenary action at large deformations. The eventual failure was mainly concentrated on the beam-column connections near the middle joint interfaces. The test results of the frame with and without partial debonding are presented in Figure 18 and Figure 19. Here, F-CD is the frame without debonding and F-PD with debonding. It can be seen that debonding the bars will increase the catenary action resistance of the frame. In this study it was found that the maximum resistance due to catenary action was about 80% higher for the frame with debonded reinforcement compared with the conventional frame.
Besides tests on sub-structures as discussed previously also studies on the behavior of complete reinforced concrete frame structures are performed by Sasani et al [20]. The main objective of this research was studying the potential progressive collapse and the dynamic load redistributions of a seven story reinforced concrete structure after column removal. The first step in this research was experimentally testing the beam which arises immediately after the removal of a peripheral column (see Figure 20). This experiment is done on a 3/8 scale model of the beam which is bridging over the removed column. Also this experiment showed the effectiveness of catenary action. After the bottom bars fractured the top reinforcement provided catenary action. The beam end rotation in the end had a slope of about 20%. After this experimental test a detailed finite element model is developed of the experimentally tested beam. The experimental results are used to verify the finite element model.

After the finite element model was developed for the beam directly above the removed column a second analytical model was made which captures the behavior of the total structure. The finite element model for the beams above the removed column is integrated in the model for the total structure to avoid a too detailed finite element model. The two models are integrated by a hybrid simulation which is a combination of two simulation methods. For this research an integration is made with the finite element programs OpenSEES and ANSYS.

With the hybrid simulation model different (dynamic) analysis were performed. The dynamic analysis showed that 95% of the axial force of the removed column was transferred to column C-3 and E-3 and only 5% to column D-2 (see Figure 21). After removal of one column also an analysis is performed of the simultaneously removal of two adjacent columns. The linear dynamic analysis showed that the structure will not collapse when both columns are removed.
3.3 Collapse of corner columns

Much less studies are performed to the situation that a corner column fails. Only one experimental research is found that focus on the collapse of a corner column. Further some researchers performed a finite element simulation for the situation that a corner column fails.

Mohamed [21] did research to the progressive collapse resistance of an 8 story cast in situ reinforced concrete frame structure in case of removal of a corner column (see Figure 22). For this research the finite element program SAP2000 was used. First an examination was conducted to a structure designed for regular loads (like wind). The reinforcement needed to resist these loads is used as a starting point for the further research. For the second analysis notional column removal is assumed and the mechanism that should provide progressive collapse is cantilever beam action (see Figure 22). It turned out that preventing progressive collapse with this mechanism leads to a large demand of reinforcement. Comparing the reinforcement needed for regular loads with reinforcement needed for cantilever beam action showed an increase of reinforcement of about 216%. However, according to Mohamed, this mechanism can met the requirements as included in UFC 4-023-03.

Further Mohamed investigated other mechanisms that could prevent progressive collapse in case of removal of a corner column. To achieve this he studied the application of steel bracing under different positions as shown in Figure 24.

All configurations of steel bracing had a positive effect on the amount of flexural reinforcement. However, for all the positions of steel bracing failure of the edge beams occurred as a result of shear failure caused by a combination of shear and torsion. According to Mohamed it is therefore important to do three dimensional analyses to account torsional shear forces.

To verify the analytical test results like the research that is treated previously, Kai et al. [12] conducted an experimental test to the progressive collapse resistance of a reinforced concrete beam-column sub structure in the event of a removed corner support (see Figure 25). The tested sub structure consists of the two corner beams at the first floor as shown in Figure 26. A total of six specimens were tested varying in span length, detailing and amounts of steel. A few specimens were designed with non-seismically requirements
according to the Singapore Standard and the main part of the specimen was designed with seismically requirements according to the ACI 318-08. To investigate the dynamic behavior of the structure a heavy hammer is used to simulate sudden column removal. Test results showed that the extra axial load on the adjacent columns due to the removal of the corner support was larger than the load on the corner support in the initial state. Further, there were no horizontal tensile forces measured at the fixed supports which indicate that catenary action did not develop. This could be due to insufficient deformation capacity for catenary action to develop, or that the corner column could not provide enough horizontal constraint to develop catenary action in the beams. The flexural capacity of the beams was the main mechanism that prevented progressive collapse. However, the actual bending moment at the fixed supports increased with more than 670% compared to the initial state. The research also focused on the dynamic behavior of the system due to sudden column removal. Analyzing the test results showed that a Dynamic Load Factor (DLF) of 2 which is often recommended by current standards is over-conservative. However, further studies are needed to refine the guidelines.

Another research performed by Kai et al. [22] investigated the progressive collapse behavior of flat slab structures in the event of a removed corner column (see Figure 27 and Figure 28). Two series of flat slabs where tested, one series without drop panels and one series with drop panels. An earlier numerical research showed that also for a structure build up out of floor slabs Vierendeel action will develop, which cause a redistribution of loads over the floors above the removed column. To simulate this behavior in the experiment the rotation capacity of the corner column is partially restrained. After testing the two series of specimen it was shown that one of the potential failure modes of the slabs without drop panels was caused by punching shear in the corner column-slab connection. The slabs with drop panels significantly mitigate the like hood of this failure mode. Further it was shown that the drop panels increase the overall progressive collapse resistance of the slabs. It was also noted that tensile membrane action developed in the slab. All experiments showed a yield line between the two penultimate columns (see Figure 29). Another important issue was that the predicted dynamic effects for the test specimen range from 1,13 to 1,23 which is much less then the factor two as often used in guidelines.
In addition to the two previous researches, Kai et al. [23] also performed an experimental study to the interaction between the slabs and beams when a corner column is removed (see Figure 30 and Figure 31). Again, first a finite element model was built to predict the behavior of the structure after removal of the corner column. They concluded that neglecting the influence of the floor slabs is very conservative, especially for cast in situ structures. The contribution of the slabs resulted in an increase of the ultimate load carrying capacity up to 63%.

3.4 **Catenary action**

The tie force method is one of the most common techniques to prevent progressive collapse, this method is among others used in the British Standard and Regulation, Eurocode, ASCE, ACI, GSA and DoD. Due to the large impact that the collapse of Ronan Point had on society, one needed a fast alternative for designing against progressive collapse. Therefore, shortly after the collapse of Ronan Point, the tie force method was recommended in the British Standard. The current tie force method should ensure structural integrity and capacity of alternate load paths by means of a simplified analytical model.

Li et al. [24] investigated the reliability of the current tie force method as included in many standards. They used a finite element model to examine the behavior of two different reinforced concrete frame structures under different amounts of reinforcement and different positions of column removal. The analysis was conducted on a structure with three stories and a structure with eight stories as shown in Figure 32 and Figure 33 respectively.
Figure 32; studied three story structure [24]

Figure 33; studied eight story structure [24]

Figure 34; locations removed columns [24]

Table 3 and Table 4 give an overview of the test results. The abbreviation C stands for collapse and NC for not collapsed. Here it can be seen that according to their analysis the basic tie strength recommended by the current standards would lead to the collapse of the frame.

<table>
<thead>
<tr>
<th>Location of removed column</th>
<th>Corner</th>
<th>Middle (X-direction)</th>
<th>Middle (Y-direction)</th>
<th>Interior</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st floor</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Normal seismic design</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Current $F_t = 20 + 4n_0$</td>
<td>C</td>
<td>C</td>
<td>NC</td>
<td>C</td>
</tr>
<tr>
<td>TF design $F_t = 60$</td>
<td>C</td>
<td>C</td>
<td>NC</td>
<td>NC</td>
</tr>
<tr>
<td>2nd – 3rd floor</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Normal seismic design</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Current $F_t = 20 + 4n_0$</td>
<td>C</td>
<td>C</td>
<td>NC</td>
<td>-</td>
</tr>
<tr>
<td>TF design $F_t = 60$</td>
<td>C</td>
<td>C</td>
<td>NC</td>
<td>-</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Location of removed column</th>
<th>Corner</th>
<th>Middle (X-direction)</th>
<th>Middle (Y-direction)</th>
<th>Interior</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st floor</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Normal seismic design</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Current $F_t = 20 + 4n_0$</td>
<td>NC</td>
<td>NC</td>
<td>NC</td>
<td>C</td>
</tr>
<tr>
<td>TF design $F_t = 60$</td>
<td>NC</td>
<td>NC</td>
<td>NC</td>
<td>NC</td>
</tr>
<tr>
<td>2nd – 3rd floor</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Normal seismic design</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Current $F_t = 20 + 4n_0$</td>
<td>NC</td>
<td>NC</td>
<td>NC</td>
<td>-</td>
</tr>
<tr>
<td>TF design $F_t = 60$</td>
<td>NC</td>
<td>NC</td>
<td>NC</td>
<td>-</td>
</tr>
<tr>
<td>4th floor</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Normal seismic design</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Current $F_t = 20 + 4n_0$</td>
<td>C</td>
<td>NC</td>
<td>NC</td>
<td>-</td>
</tr>
<tr>
<td>TF design $F_t = 60$</td>
<td>C</td>
<td>NC</td>
<td>NC</td>
<td>-</td>
</tr>
<tr>
<td>5th – 8th floor</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Normal seismic design</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Current $F_t = 20 + 4n_0$</td>
<td>C</td>
<td>C</td>
<td>NC</td>
<td>-</td>
</tr>
<tr>
<td>TF design $F_t = 60$</td>
<td>C</td>
<td>C</td>
<td>NC</td>
<td>-</td>
</tr>
</tbody>
</table>

Based on their test results, Li et al. [24] concluded that the current tie force method does not provide sufficient resistance against progressive collapse. Therefore, they tried to develop a new approach for calculating the necessary tie strength needed for i.e. catenary action. The difference of this new approach in comparison to the current approach is that the tie force is based on the actual structural properties of the beam under consideration (like load and length of the beam) while the current basic tie strength is mainly based on the number of stories. After developing this new approach they examined it with the finite element model to see if the new approach provides better results, which seems to be true.

Further, they investigated what type of “action” the accidental load resists for different positions of damage and what the distribution of the load is carried by each individual beam through displacement compatibility. For this analysis they used three zones; interior zone, two edge zones and a corner zone as presented in Figure 35. For all zones an examination is made if the beams are able to provide tie forces.
In case of collapse of an interior column there will develop an axial tie force in both directions as shown in Figure 36. The beam in the Y-direction contains the shortest beam, fracture will therefore be more likely to occur in this direction first (which corresponds to the analysis). When the beam in Y direction failed the load will be distributed to the beam in X direction until also here fracture occurs.

In case of failure of an edge support it seems that only the periphery beams develop axial tie forces as shown in Figure 37. As can be observed in the graph besides, the beams perpendicular to the periphery beams do not develop any axial tie forces. The only way these beams can contribute to an alternate load path is by beam action. For the edge zone, the resistance should be determined by the greater of catenary and beam action.

In case of the removal of a corner column it is clear that there will not develop any axial tie force as can be observed in Figure 38. Therefore, the collapse resistance for this situation should be obtained by beam action.
3.5 Influence of infill walls

A lot of buildings are equipped with infill walls which are usually neglected during the structural design. However, they do have influence on the behavior of a structure after sudden column removal. A few researchers investigated the influence of infill walls to the progressive collapse resistance of a building. Research performed by Tsai et al. [25] investigated the influence of three different types of infill walls; wing type, panel type and parapet type (see Figure 39). The investigated frame is a ten story moment resisting frame designed according to the GSA guidelines. An examination is made in case of removal of a mid column on the short side, mid column on the long side and a corner column. For this research both linear and non-linear static analyses were carried out. Catenary action is not considered in this study, only the flexural capacity. The research showed that infill walls increase the stiffness as well as the strength of the structure. A remark should be made on the influence of parapets. They enlarge the progressive collapse resistance but the extend of increase is not significantly.

Another research to the influence of infill walls on the progressive collapse resistance of reinforced concrete frame structure is performed by Lupoae et al. [26]. They used a finite element model to examine the behavior of a six story reinforced concrete building. They first examined the behavior of the bare frame (see Figure 40), then the behavior of the frame with full infill walls (see Figure 41) and thereafter the behavior of the frame with walls with openings. Different column removal scenario’s are considered; a column at the corner of the building, a column at the middle of the short side of the building and a column at the middle of the long side of the building. The amount of reinforcement is in accordance with Bucharest seismic demand. Research showed that the full masonry infill wall reduced the vertical displacement for about 40% to 70% for different column removal scenarios. Research also showed that the impact of a blast explosion is larger than due to column removal by a demolition scenario. However, according to this research a disadvantage of infill walls is that they increase the effect of a blast wave. Because of the large surface of the wall also adjacent columns are damaged in this experiment. When infill walls are used, more columns are damaged than without infill walls resulting in progressive collapse.
4 Finite element modeling

As discussed in chapter 1 the finite element method was used to derive a dynamic load factor for a reinforced concrete frame structure after the sudden loss of a corner column. The finite element model discussed in this study is relatively simple compared to the numerical studies as shown in chapter 3. The models discussed in chapter 3 all represent 3D simulations of reinforced concrete frame structures with more complex elements (like shell or solid elements). For this thesis a simple 2D representation with beam elements is considered, so it is relatively easy to obtain information from the finite element model like the bending moments or curvature in the beams. Therefore it is also easier to check the results obtained with the finite element model with simple analytical models. For this study use was made of the commercial available finite element package Abaqus. This chapter includes the main ingredients which were necessary to build a well functioning finite element model for a reinforced concrete structure.

4.1 Solution sequence

When using a commercial available finite element program usually three main steps need to be carried out. These three steps are: pre-processing, processing and post-processing. This paragraph contains a brief description of each step as used for the current study.

During the pre-processing phase the data needed to set up the structural equations need to entered. Amongst others the following items are required as input data for this phase [27]:

- Element type
- Geometry properties of the element
- Material properties
- Model geometry
- Loads

In Abaqus the pre-processing phase can be executed by different modeling procedures. One of these procedures is the Abaqus/CAE environment which stands for Complete Abaqus Environment. With this procedure all steps can be performed within one interface and is therefore quite user friendly. When modifications are made in the model they are immediately visible in the Abaqus viewer. However, Abaqus/CAE does not support all modeling features available in Abaqus. One of the features that is not supported by Abaqus/CAE is the ability to add reinforcement to beam elements [28]. Therefore the pre-processing for this thesis is performed with the second modeling capability provided by Abaqus, namely the use of input files. A sample of such an input file is shown in Figure 42. In appendix I the complete input files used for this thesis are added.

![Figure 42](image)

During the processing phase the element and structural equations are set up and solved. In Abaqus two finite element analyzers are available to solve the problem under consideration; Abaqus/Standard and Abaqus/Explicit. Abaqus/Standard uses an implicit integration scheme and Abaqus/Explicit uses an explicit integration scheme.
Abaqus/Explicit is a more efficient solver than Abaqus/Standard especially for dynamic problems. However, Abaqus/Explicit offers fewer element types than Abaqus/Standard and most important for this thesis is that Abaqus/Explicit is not able to deal with reinforcement within beam elements. Therefore, the Abaqus/Standard analyzer is used. For more information about the Abaqus solvers reference is made to [28].

In Abaqus/Standard the type of analysis the user wants to perform must be specified (static analysis, eigenvalue analysis, dynamic analysis, etc.). Since reinforced concrete behaves highly nonlinear an ordinary static solution technique is not sufficient to obtain accurate results with Abaqus. Therefore the modified Riks method (arc length technique) is used to predict the response of the reinforced concrete under static loads.

Since progressive collapse is a dynamic phenomenon also a Dynamic solver is used which also includes inertia effects. The Dynamic solver is also able to obtain static solutions as long as the load is applied sufficient slow so that inertia effects will be avoided.

The last step is the post-processing phase. During this step the results obtained during the processing step can be viewed. The user must specify the results he wants to view in advance. Further the user must specify in which form he wants to view the results (graph, table or graphically). In Abaqus the results can be viewed with Abaqus/Viewer or in Abaqus/CAE.

4.2 Element formulation

The main goal of the finite element simulation performed for this thesis is the development of a dynamic load factor. Therefore the elements used for this thesis must meet certain requirements to achieve this goal without providing too much or less information. In order to obtain the dynamic load factor it sufficient to study the overall structural behavior of the reference frame without considering local stress distributions and crack patterns. Further the following requirements are specified for the elements:

- The elements must be able to describe the nonlinear response of the concrete without considering crack patterns and local stress distributions
- The elements must be able to deal with flexural reinforcement
- The elements must be able to provide deflections, bending moments, curvatures, stresses and strains at various locations on the beams
- The elements must be able to deal with uniformly distributed loads
- The elements must be able to deal with dynamic/cyclic loads

Based on these requirements it was found that Euler-Bernoulli beam elements are well suited for this thesis. The Euler-Bernoulli beam elements are included in the Abaqus library as B23 elements. With beam elements is was possible to study the overall structural behavior of the frame without taking crack patterns and local stress distributions into account. Further are, according to the Abaqus user’s manual, Euler-Bernoulli beam elements able to deal with uniformly distributed loads that can be applied statically and/or dynamically [28].

![Figure 43; Euler Bernoulli beam element](image)
It is important to note that the Euler Bernoulli beam theory assumes that plane sections remain plane and do not include shear deformation. The latter assumption is only valid for slender beams, which are considered in this research project. Slender beams are beams where the cross-sectional dimensions are small compared to their length. The Euler-Bernoulli beam elements contain three degrees of freedom at each node, two translational and one rotational degree of freedom as shown in Figure 43.

The Euler-Bernoulli beam elements used in Abaqus contain three integration points over the length of the element as indicated with A, B and C in Figure 44 which are fixed and cannot be changed by the user. Further, the number of integration points over the height of a beam section can be specified by the user (see Figure 45). Within Abaqus it is possible to obtain various output parameters at each integration point over the length and height of the element, for instance stresses, strains and curvatures.

4.3 Concrete model

Abaqus provides three different models to simulate the mechanical properties of the concrete; smeared cracking model, brittle cracking model and damage plasticity model. For this study the concrete model most fulfills the next requirements:

- Describe the full nonlinear behavior of concrete including tension softening and strain hardening
- The model must be able to deal with cyclic loading were also effects like stiffness degradation and strain rates could be included
- The model must be able to be analyzed by the Abaqus/Standard solver
- An application with beam elements must be possible

The only model that can fulfill all these requirements is the concrete damage plasticity model (CDPM). The CDPM is the most suitable model for this study since it is designed to deal with cyclic loading and allows the definition of both strain softening in tension and strain hardening in compression. Further this model can be used within beam elements and solved by the Abaqus/Standard solver. The Concrete Damage Plasticity model is developed by Lubliner and elaborated by Lee [29]. The model assumes cracking of the concrete in tension and crushing of concrete in compression as the two main failure mechanisms.

For concrete in tension the CDP model assumes a linear stress strain relation up till the maximum tensile stress [28]. To simulate the complete tensile behavior of the concrete the youngs modulus \( E_{cm} \), the tensile stresses \( \sigma_t \), cracking strain values \( \varepsilon_t^c \) and damage parameters \( d_t \) must be specified. The cracking strain can be obtained with [4.1]. The damage variables can be obtained with [4.2] [29]. Here \( b_t \) indicates a damage parameter which gives the ratio between the cracking stain and plastic strain as shown in Figure 46. For this project a damage variable \( b_t \) of 0.9 is used to let the model run properly. Further research to the influence of this damage parameter is needed.
Finite element modeling

Also for compression first a linear elastic stress strain relation is assumed up till about 0.4·f_cm, thereafter the inelastic stress strain relation must be specified. The user must specify the stresses $\sigma_c$, inelastic strains $\varepsilon^{pl}_c$ and additionally the damage parameter $d_c$. This damage parameter is not specified for this project since stiffness degradation due to cyclic loading is not considered in this thesis. Further research to this topic is needed. The inelastic strain can be calculated with [4.3]. If the damage variable would be included it could be determined with [4.4].

For the CDP model different parameters can be specified with respect to the flow potential, yield surface and viscosity parameters. Most of these parameters are assumed to be the default values. Only the viscosity parameter differs from the default settings. This viscosity parameter helps to improve the rate of convergence in the softening regime. For this project a viscosity parameter of 0.0004 is used which is empirically determined. For more information about the concrete damage plasticity model reference is made to the Abaqus user’s manual [28].
4.4 Material properties

The definition of the material properties of the reinforced concrete is one of the most important topics for this thesis. This is caused by the complex behavior of reinforced concrete due to among others a nonlinear compressive stress strain relation, tensile cracking, tension softening and the interaction between the reinforcing steel and the concrete. For this thesis full bond between the reinforcement and concrete was assumed. The next paragraphs discuss the material definitions used in this thesis. All the material properties are assumed to be the mean strength properties so the material safety factors were not included in the material definition. The mean material properties were considered since this thesis aims at studying the behavior of the reinforced concrete in a more realistic way than when material safety factors would be included. In the subsequent paragraphs first the material models used for this thesis are discussed, thereafter the material properties used for this thesis are specified.

4.4.1 Concrete in tension

In this paragraph the behavior of concrete in tension will be discussed. First the main characteristics of concrete in tension will be discussed and thereafter the model used for this study to describe the uniaxial tensile behavior of the concrete is presented. It should be noted that only mode I cracks are considered in this thesis. Mode I cracks are cracks that have no shear stress at their front [30]. Since cracking of the concrete was taken into account to study the overall structural behavior of a concrete frame, it was sufficient to consider mode I cracks only. The shear stress at the front of a crack mainly influences the direction of the crack, however local crack patterns are not considered in the current study.

A typical load displacement curve for concrete subjected to uniaxial tension is shown in Figure 48. Here it can be seen that the load displacement curve shows a gradual descending branch after the peak load is reached. When a plain concrete bar is loaded in uniaxial tension a crack will appear after the ultimate tensile strength $f_{ctm}$ is reached somewhere in the bar. After the concrete reaches its ultimate tensile strength the stress will not immediately drop to zero. Ahead of a crack opening the concrete is still able to transfer some stresses after reaching the tensile strength $f_{ctm}$. This process causes the gradual descending branch shown in Figure 48.

In order to model the behavior of concrete in tension it might be useful to translate the load deflection diagram into a stress strain diagram. However, this translation cannot be performed unconditionally. Due to the fact that strain softening is a highly localized phenomenon it is not possible to derive the stress versus strain relation in a simple way [31]. Consider the behavior of a tensile bar subjected to a deformation controlled tensile test. The bar will behave uniform and linear elastic until somewhere in the cross-section the ultimate tensile strength $f_{ctm}$ is reached. Due to the linear elastic behavior of the specimen up till the ultimate tensile strength it is possible to derive a stress versus strain relation. However, after the ultimate tensile strength is reached somewhere in the specimen a strain localization will develop at a relative narrow zone of micro cracks which
is known as the process zone. These micro cracks will form a continuous crack after the deformation will be increased. The load deformation diagram of the test will depend on the position at which these parameters are measured which is illustrated in Figure 49. If the load deformation response is measured at position I a different response will be obtained than for position II. This is caused by the load reduction at the process zone which leads to unloading of the zone outside this process zone.

Because the overall load deformation response of the specimen is built up out of strains and crack openings it is not correct to describe the behavior just by means of a stress strain relation through dividing the deformation by the measuring length [32]. The load deformation behavior must be separated in to a stress strain relation and a stress crack opening relation (see Figure 50) by which the former contains real material behavior and the latter a fictitious crack model [32]. It should be noted that the length over which the strain localization is measured also affects the results.

As discussed above a fictitious crack model can be used to describe the behavior of plane concrete subjected to uniaxial tension. This model assumes that a crack propagates when the tip of a crack reaches the tensile strength [33]. After a crack opens it is assumed that the stress will not suddenly drop to zero but decrease with increasing crack width w (see Figure 51). The zone over which this process takes place is known as the fracture process zone. The amount of stress that can be transferred in this zone depends thus on the width of the crack opening.

To obtain the stress versus crack width relation the fracture energy \( G_f \) is used. The fracture energy is defined as the amount of energy required to form one unit of area of a pure mode I continuous crack. It is assumed that the fracture energy is a fixed material constant [31] that can be obtained by dividing the area under the load displacement curve through the total cross section that is cracked.

Consider the stress crack width diagram from the fictitious crack model as shown in Figure 50. If a crack opens energy is absorbed. The amount of energy absorbed per unit crack area in widening the crack from zero to beyond \( w_0 \) can be expressed with [4.5] [33]. It should be noted that the softening curve shown in Figure 50 can be expressed in different ways as long as the stored energy remains the same.
The previously discussed fictitious crack model is a discrete crack model since it describes the behavior of one single crack (see Figure 52). However, for this thesis a smeared crack concept was used (see Figure 53). Smeared cracking is a continuum approach for fracture mechanics in which the cracks are smeared out over a certain length. Since the main aim of this research is studying the overall behavior of reinforced concrete without concerning among others crack patterns the smeared cracking model seems to be an appropriate model.

For the smeared crack model it is assumed that the relative displacement caused by a crack must remain constant for both smeared and discrete cracks. Thus, if a relation is assumed between the stress and relative displacement $\delta_f$ this must be identical to the relation of the stress and the relative displacement expressed as $\delta_f = \varepsilon_f W_c$ [35]. If for instance the crack $w$ in Figure 52 is defined as the relative displacement $\delta_f$ than this expression becomes $w = \varepsilon_f W_c$. Here $\varepsilon_f$ are the accumulated strains due to micro cracking over width $W_c$ and the relative displacement is the sum of the openings of individual micro cracks. The width $W_c$ is a characteristic width which expresses the width of the fracture process zone. The fracture process zone is often assumed to be several times the maximum aggregate size [35]. With the definition of the relative displacement $w = \varepsilon_f W_c$ it is now possible to rewrite [4.5] to [4.6].

\[
G_f = w_c \int \sigma \cdot d\varepsilon_f
\]  

[4.6]

According to Bažant and Oh [35] it should not matter if the cracks are smeared out over the fracture process zone width $W_c$ or over a fictive crack band $h$ as long as the stored energy remains constant. The term $w_c$ in [4.6] now becomes $h$. It is assumed that the cracks are uniformly distributed over this fictive crack band. In relation to finite element modeling this crack band width $h$ is related to the finite element formulation [35]. For beam elements as used in this thesis this crack band can be assumed to be equal to the distance between the integration points of the beam element. In order to avoid snap back behavior of the stress strain diagram this crack band width should not exceed the criterion as specified in [4.7] [35].

\[
h \leq \frac{G_f \cdot E_{cm}}{f_{cm}^2}
\]

[4.7]

So if the tensile strength, fracture energy and the shape of the softening curve are known a stress strain relation based on the finite element characteristics can be obtained. For this thesis an exponential relation is assumed for the tension softening branch of the stress strain relation as shown in [4.8] [36]. An exponential curve is chosen because it is easier for Abaqus to reach convergence with a smooth curve than one based on a linearized diagram.
$$\sigma(\varepsilon) = f_{ctm} \cdot e^{(\varepsilon - \varepsilon_c)/\gamma_t}$$ \hspace{1cm} [4.8]$$

The parameter $\gamma_t$ represents the area under the stress strain diagram and can be determined with [4.9].

$$\gamma_t = \frac{G_f}{h} \cdot f_{ctm} - \frac{f_{ctm}}{2 \cdot E_{cm}}$$ \hspace{1cm} [4.9]$$

To obtain the stress strain relation as shown in Figure 53 a crack band width $h$ of 180 mm was assumed. This crack band is equal to half of the beam length which is the distance between the integration points as discussed previously. The beam element length used for this thesis was 360mm which was mainly based on the tensile behavior of the concrete. Finally the stress strain relation as shown in Figure 54 was derived and used for this study. The values used to obtain this stress strain relation are presented in Table 5.

![Figure 54; stress strain relation concrete in tension used in this thesis](image)

A lower bound limit for the concrete tensile behavior was considered for the current thesis. Due to safety considerations only tension softening was considered so the influence of the reinforcement on the tensile capacity is left out of consideration (tension stiffening). Including tension stiffening might overestimate the tensile capacity of the concrete. This overestimation might be the result of the procedure used to describe the influence of the reinforcement steel which is often a uniaxial loaded tensile bar. When a beam in bending is considered the tensile force will not be uniform over the beam length. Due to this non uniform tensile force the cracks will appear closer to each other than for a uniform tensile force [37]. This effect causes a decrease of the tension stiffening effect.

### 4.4.2 Concrete in compression

For concrete in compression a similar approach could be used as for tension. When concrete is loaded in compression micro cracks will form when the maximum compressive stress is reached. As mentioned for tension also now the stress will not immediately drop to zero but will gradually decrease with increasing deformation. However, also here it states that this process of strain softening is a highly localized phenomenon and therefore a simple stress strain relation cannot be derived. To overcome this problem the same procedure as discussed for tension can be used based on the crushing energy of the concrete as shown in Figure 55.
With this approach it is possible to describe the full strain softening behavior of concrete in compression where the softening branch depends on the chosen finite element type. However, in this thesis effects like stiffness degradation due to cyclic loading are not considered. When the full softening branch would be used this stiffness degradation must be included with respect to progressive collapse analysis to obtain reliable results. Therefore a parabolic stress strain relation with a maximum compressive strain of 3,5‰ was used for this thesis to obtain a safe approximation of the ultimate capacity. This 3,5‰ boundary was used as a failure criterion for the beams. For future research to progressive collapse it is recommended to consider the full strain softening behavior of concrete in compression including stiffness degradation.

To approach the uniaxial concrete compressive behavior a parabolic stress strain relation was assumed which is described by the International Federation for Structural Concrete (fib) in its Bulletin 42 [39]. The fib bulletin 42 proposes a stress strain relation for the short-term as shown in [4.10].

\[
\frac{\sigma_c}{f_{cm}} = - \left( \frac{k \cdot \eta - \eta^2}{1 + (k - 2)\eta} \right) \text{ for } \left| \varepsilon_c \right| < \left| \varepsilon_{c,lim} \right| \tag{4.10}
\]

In [4.10] the following terms are specified \( \eta = \frac{\varepsilon_c}{\varepsilon_{c1}} \) and \( k = \frac{E_{cm}}{E_{c1}} \), where \( \varepsilon_c \) is the actual compressive strain, \( \varepsilon_{c1} \) is the strain at the ultimate compressive stress, \( E_{cm} \) the modulus of elasticity and \( E_{c1} \) the secant modulus from the origin to the peak compressive stress as shown in Figure 56. The material properties as defined in Table 5 are used to define the uniaxial compressive stress strain diagram. The stress strain relation as shown in Figure 57 was used for this study. All values used to obtain the stress strain diagram for concrete in tension are shown in Table 5.

![Figure 55; stress strain relation and stress interpenetration relation [38]](image)

![Figure 56; proposed stress strain diagram concrete in compression](image)

![Figure 57; stress strain relation concrete in compression used for this thesis](image)
4.4.3 Reinforcing steel

The definition of the reinforcement properties used for this thesis were based on the material definition as included in the Eurocode. Eurocode 2 proposes two types of stress strain curves that can be used for design calculations with reinforcement steel. One curve contains ideal plastic behavior with a horizontal yield plateau. The second shows a slightly increasing branch up to maximum stress $f_t$. Both curves are shown in Figure 58. For this thesis the stress strain diagram with the increasing branch was used. A slightly increasing stress strain diagram after the yield stress is reached makes it easier for Abaqus to reach convergence.

![Figure 58; idealized stress strain diagram for reinforcing steel](image)

The ultimate stress $f_t$ can be determined with \[4.11\]. According to Eurocode 2 the factor $k$ should be larger than or equal to 1.08 for steel class B. The ultimate strain $\varepsilon_{uk}$ for steel class B should be larger than or equal to 5%. Considering the aforementioned parameters for the reinforcement this resulted in a stress strain curve as shown in Figure 59.

$$f_t = k \cdot f_y$$  \[4.11\]

![Figure 59; stress strain diagram reinforcing steel used for this thesis](image)

In Abaqus the reinforcement was modeled as an element property within the beam elements with the *REBAR command. With this option it is not possible to visualize the reinforcement [28]. The position of the reinforcement must be entered relative to the center of the beam and is equal for both the top and bottom reinforcement. For the specification of the material properties a regular elastic-plastic material is entered. Here the youngs modulus, yield stress, ultimate stress and plastic strain must be specified. The plastic strain can be determined with the same procedure as for the concrete.
4.4.4 Strain rate effect

The strain rate is the rate of change in strain of a material with respect to time. Since progressive collapse is a highly dynamic process also strain rate effects are involved. Materials like concrete and steel are strain rate sensitive materials [40] which mean that the material properties change due to these strain rates. Both the compressive and tensile strength of concrete increase with the strain rate [41]. Strain rate effects might affect the overall response of a structure with respect to progressive collapse because the load redistribution depends on the stress redistribution. So if the stress distribution is changed also the structural behavior changes.

According to [41] the strength enhancement due to the strain rate effect increased the resistance against progressive collapse in most of the studied cases. Here, the strain rate effect of the reinforcement steel played a major role in the structural response. Based on the results provided by among others [41] it might be useful to consider the strain rate effect with respect to progressive collapse. Due to a lack of time the influence of strain rate effects is not considered in this thesis. However, for further research it is recommended to take these effects into account.

4.4.5 Material properties current thesis

For this thesis a C30/37 concrete class was assumed which is a standard type of concrete. The fracture energy $G_f$ for concrete in tension is assumed to be $135\ \text{Nmm/mm}^2$ which is in accordance with [41] for concrete class C30/37. For the reinforcement a S500 class was assumed. All material properties were based on the mean strength material properties as defined in EC2 [42]. Table 5 provides an overview of the material properties used for this thesis.

<table>
<thead>
<tr>
<th>Table 5: material properties used for this thesis</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Concrete compressive properties</strong></td>
</tr>
<tr>
<td>$f_{cm}$</td>
</tr>
<tr>
<td>$\varepsilon_{c,lim}$</td>
</tr>
<tr>
<td>$\varepsilon_{c,1}$</td>
</tr>
<tr>
<td>$E_{cm}$</td>
</tr>
<tr>
<td>$E_{c1}$</td>
</tr>
<tr>
<td><strong>Concrete tensile properties</strong></td>
</tr>
<tr>
<td>$f_{ctm}$</td>
</tr>
<tr>
<td>$G_f$</td>
</tr>
<tr>
<td>$h$</td>
</tr>
<tr>
<td><strong>Reinforcement properties</strong></td>
</tr>
<tr>
<td>$k$</td>
</tr>
<tr>
<td>$E_s$</td>
</tr>
<tr>
<td>$f_y$</td>
</tr>
<tr>
<td>$k \cdot f_y$</td>
</tr>
<tr>
<td>$\varepsilon_{cu}$</td>
</tr>
</tbody>
</table>

4.5 Model geometry

As discussed in paragraph 1.7 a reference structure was designed for the development of the finite element model. The frame consists of five stories with a story height of 3.6 meters. A regular grid of 7.2 meters was used. In order to reduce the analyze time of the finite element model only half of the frame was modeled. To imply that the beams are continuous the rotational degree of freedom at the right ends of the beams were fixed. Further, all beam column connections were assumed to be pinned connected. In this study only the flexural capacity of the beams was taken into consideration and therefore the flexural capacity of the columns was neglected. All columns were modeled by a single beam element. Some modifications on the initial frame as discussed in chapter 1.7 were necessary. These modifications were needed since the capacity of the beams used for the
initial frame was not sufficient to resist the load of the cantilever that arises after the corner column is removed, even when the load is applied statically. Therefore an additional column was added at half of the span length of the last bay as shown in Figure 60.

4.6 Loads

The loads and load combinations which act on the structure were determined in accordance with the requirements as mentioned in EC2. For the assessment of the resistance against progressive collapse it is not necessary to consider live loads like wind and therefore they were not considered for this thesis [7]. Appendix E contains an overview of the considered loads. In Table 6 a summary is given of the load combinations used for the assessment of the ultimate limit state.

Table 6; design loads peripheral beam ultimate limit state

<table>
<thead>
<tr>
<th>Design loads</th>
<th>Floor Load</th>
<th>Roof Load</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P_{d;\text{unfavorable;floor}} )</td>
<td>1,35 x 38,9 kN/m + 1,5 x 3,6 kN/m</td>
<td>57,6 kN/m</td>
</tr>
<tr>
<td>( P_{d;\text{unfavorable;floor}} )</td>
<td>1,2 x 38,9 kN/m + 1,5 x 7,2 kN/m</td>
<td>57,5 kN/m</td>
</tr>
<tr>
<td>( P_{d;\text{favorable;floor}} )</td>
<td>0,9 x 38,9 kN/m</td>
<td>35,0 kN/m</td>
</tr>
<tr>
<td>( P_{d;\text{unfavorable;roof}} )</td>
<td>1,35 x 23,7 kN/m + 1,5 x 0 kN/m</td>
<td>32,0 kN/m</td>
</tr>
<tr>
<td>( P_{d;\text{unfavorable;roof}} )</td>
<td>1,2 x 23,7 kN/m + 1,5 x 0 kN/m</td>
<td>28,4 kN/m</td>
</tr>
<tr>
<td>( P_{d;\text{favorable;roof}} )</td>
<td>0,9 x 23,7 kN/m</td>
<td>21,3 kN/m</td>
</tr>
</tbody>
</table>

For the design against progressive collapse other load combinations are valid than for the ultimate limit state [7]. For the assessment of the progressive collapse resistance only 30% of the live load needs to be applied on the structure. Table 7 gives an overview of the loads needed for the progressive collapse assessment.

Table 7; design loads peripheral beam for progressive collapse assessment

<table>
<thead>
<tr>
<th>Design loads</th>
<th>Floor Load</th>
<th>Roof Load</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P_{d;\text{unfavorable;floor}} )</td>
<td>38,9 kN/m + 7,2 kN/m</td>
<td>41,1 kN/m</td>
</tr>
<tr>
<td>( P_{d;\text{unfavorable;roof}} )</td>
<td>23,7 kN/m</td>
<td>23,7 kN/m</td>
</tr>
</tbody>
</table>
4.7 Cross sectional dimensions

For the reference frame discussed in paragraph 4.5 a design calculation was made to obtain the cross sectional dimension of the beams and columns which were used as a starting point for the finite element model. The loads as previously determined were applied on the considered structure. The complete calculation of the cross sectional dimensions and the reinforcement are given in appendix E. The cross sectional dimension with regard to the ultimate limit state for the beams and columns are given in Figure 61 and Figure 62. For sake of simplicity it was assumed that the reinforcement is continuous in all beams and columns. The shear reinforcement was not modeled within the finite element model.

![Figure 61; cross section beams](image1)

![Figure 62; cross section columns](image2)

4.8 Column loss approach

Because a treat independent progressive collapse analysis was performed for this thesis the instantaneous removal of a load bearing element must be simulated. Since Abaqus is, according to the author's knowledge, not able to delete beam elements or boundary conditions during the analysis another procedure to simulate the sudden column loss was used. This procedure is based on an equivalent column force which replaces the column that will be removed. This procedure is often used in the literature by among others [41], [43] and [44]. The sudden column loss approach used for this thesis can be divided into three stages. First, a static analysis must be performed to obtain the reaction force in the column that is assumed to be removed. Next, the previously determined equivalent column force must be applied at the position where the column is assumed to be removed and again a static analysis must be performed. The last step is the sudden removal of the equivalent column force to simulate the sudden loss of a column. A schematic representation of the proposed procedure is shown in Figure 63 and Figure 64.

![Figure 63; initial static analysis](image3)

![Figure 64; equivalent loads which replace the removed column](image4)
In Figure 64 two forces are shown with a dashed line which indicates that, for this thesis, no transfer of bending moments was assumed between the beams and columns. Therefore only a vertical equivalent force was applied on the frame. However, when the connection between the beams and columns are assumed to be restrained also the other reaction forces should be applied as equivalent loads on the frame.

For the progressive collapse analysis a dynamic solver was used. Therefore it was important to apply the initial static loads sufficient slow to avoid inertia effects. Both the equivalent column force and the regular gravity loads were applied within the same time. The time by which the loads were applied will be discussed in more detail in section 6. After a static equilibrium state was reached the loads were kept constant over a certain time to be sure that the structure was at rest before unloading. Thereafter the equivalent column force was removed within 5ms. This unloading time is often used in the literature by among others [45] and [41]. A schematic representation of the load history of the applied loads is shown in Figure 65. For further research it is recommended to investigate the influence of the column removal time on the progressive collapse behavior.

![Figure 65; time history applied loads](image)
5 Benchmark procedures

When using a finite element model one must be sure that the model provides reliable and sufficient accurate results. It is therefore important to perform benchmark tests that show this reliability and accuracy. A flowchart of the performed benchmark tests for this thesis is shown in Figure 66. Here it can be seen that the benchmark process is divided in three main components: statically determined, statically indeterminate and dynamic systems. For each system different material models were considered. The flowchart as shown in Figure 66 also contains a reference to the paragraph numbers where the considered system is discussed. First linear elastic material was considered for each system since it was quite easy to check if the results found by Abaqus are correct for this material model. Only in situations where strongly nonlinear response was expected also elastic plastic material was considered. Elastic plastic material is less complicated then the behavior of reinforced concrete and therefore very useful to check how Abaqus deals with nonlinear material models. The next paragraphs will explain in more detail the used benchmark procedures and the results of the benchmark tests.

![Figure 66; flowchart benchmark procedures](image)

5.1 Static analysis

This paragraph will describe the benchmark tests performed to check the accuracy of the material model used in Abaqus for static analysis. Only the results of the reinforced concrete will be discussed here since the behavior of linear elastic and elastic plastic material is quite obvious. First the multilayer model and moment-area method are discussed since they were used as benchmark procedures for the finite element model. Thereafter a comparison is made between the benchmark models and the finite element model.

5.1.1 Multilayer model

Since there was no experimental data available to benchmark the Abaqus model some simple analytical models were developed to check the results obtained by Abaqus. First the multilayer model will be discussed here. This model is based on the multilayer model as described by Hordijk [46]. The basic idea behind this model is that the cross-section under consideration is divided into n layers over the height of the beam. The number of layers used depends on the accuracy that the user wants to obtain. Each layer is connected to a predefined stress strain relation who depends on the material model used. The main principle of this model is that the strain at the top of the cross-section is increased step wisely. For each strain found in a particular layer the corresponding stress can be obtained
by interpolation from the predefined stress strain relation. For each increment of strain at the top of the cross-section a corresponding strain at the bottom needs to be found to achieve horizontal equilibrium. This state of equilibrium is achieved with an iterative solver. A linear relation is assumed between the strain at the top and the strain at the bottom. This is in accordance with the beam element formulation used in this thesis. When the strains at the top and bottom of the cross-section are known also the curvature can be determined. The bending capacity can be determined by the summation of the horizontal force \( N_i \) found for each layer multiplied with the corresponding lever arm \( z_i \), measured from the top of the cross-section to the centre of the considered layer. A schematic representation of the model can be found in Figure 67. With this model it was possible to determine the moment – curvature response for different types of cross-sections and different types of materials.

![Figure 67: schematic representation multilayer model](image)

With the developed multilayer model it is also possible to determine the deflection for a beam on two supports which was used as a second benchmark test for the developed finite element model. In order to obtain the deflection of the beam use was made of the moment-area method. The beam under consideration is divided into a number of segments over the length of the beam for this method. Thereafter the bending force can be determined for each segment by means of the linear elasticity theory. The curvature can now be obtained for each segment with the previously discussed multilayer model, only now the bending moment is used as an input value and the iterative solver derives the strains at both the top and bottom of the beam. If the strains at the top and bottom of the beam are known also the curvature can be derived for each segment. Since the curvature \( \kappa \) is known also the flexural rigidity \( EI \) is known because \( \kappa = \frac{M}{EI} \). Now the \( M/EI \) diagram is applied on the beam as a fictitious load. The deflection of the beam can now be derived by double integration of the \( M/EI \) diagram.

5.1.2 Statically determined RC beam

The first step in the benchmark procedure was an analysis of a statically determined structure. To obtain the moment curvature behavior with Abaqus a deformation controlled three point bending test was performed as shown in Figure 68. Here it can be seen that only half of the beam was modeled due to symmetry of the beam. At the middle of the beam the rotation around the z-axis and the translation in the x-direction was fixed to simulate this symmetry. The cross section as shown in Figure 69 and discussed in paragraph 0 was considered here and the material model as discussed in paragraph 4.4 was used. The moment curvature response found with Abaqus was obtained at the element which is located in the middle of the beam since the beam reaches its ultimate capacity here first. However, the moment curvature response of the other sections was also checked.
The analysis was performed for the capacity of both the top and bottom reinforcement. The results found with the Abaqus and multilayer model are shown in Figure 70. Here it can be seen that the results obtained with both models show quite good agreement. However, the ultimate curvature obtained with Abaqus is slightly lower than that found with the multilayer model. This is caused by small deviations in the strains found by Abaqus compared to the multilayer model which have quite a large effect on the final curvature. The deviations in the strains are caused by the fact that, in Abaqus, it is not possible to abort the analysis if a certain strain is exceeded. Therefore it is not possible to reach the ultimate strain of the concrete exactly. Further the stresses and strains are checked at various points in the beam to get confidence in the proper functioning of the model.

Also the deflection of the beam previously discussed was compared with the deflection found with the moment-area method. A uniform distribute load was applied on the beam as shown in Figure 71. Two loads were considered, a load of 20N/mm and a load of 40 N/mm. The first load resulted in a deflection where the concrete was not cracked and for the second load the concrete was cracked. The two loads were applied on the beam and analyzed with the Abaqus model and the moment-area method.
The deflections found with both methods are shown in Figure 72. Here it can be observed that they find the same deflection shapes and also the ultimate deflection found for the different loads show good agreement. The results discussed in this section gave confidence in the proper functioning of the Abaqus model with respect to statically determined structures. Thereafter some tests were performed on statically indeterminate structures.

5.1.3 Statically indeterminate RC beam

The load distribution of a continuous reinforced concrete beam highly depends on the stiffness distribution of the beam. The stiffness in turn depends mostly on the amount of reinforcement that is applied in the cross section. A lower amount of reinforcement will also cause a lower stiffness. If for instance more reinforcement is applied at the support than in the field also a redistribution of loads will be observed to the support due to the higher stiffness there. The opposite is also true. Especially when the reinforcement starts to yield the stiffness of the beam will decrease rapidly and the amount of redistribution will be larger.

With respect to the developed finite element model it is important that the model is able to simulate this redistribution of loads. Especially when the ultimate resistance of a structure is studied, like the progressive collapse resistance, it is important that the model predicts the amount of redistribution accurate. To test if the finite element model developed in this study was able to simulate this behavior two different amounts of top reinforcement were considered. So, different redistribution paths should be observed. First a low amount ($A_s=804 \text{ mm}^2$) and thereafter a high amount ($A_s=1885 \text{ mm}^2$) of top reinforcement was considered. The latter is in accordance with the amount as determined for the ULS design. The field reinforcement is equal for both analysis ($A_s=1257 \text{ mm}^2$). The beam as shown in Figure 73 was considered for this analysis.
Since the developed multilayer model was not capable to analyze a statically indeterminate structure other test procedures were needed. The test procedure used for this system is explained next.

The first step in this process was the derivation of the bending distribution according to the linear elasticity theory for each applied load. This served as a reference to study the amount of redistribution. Thereafter different analyses were performed with the finite element model (indicated with RC) for different uniformly distributed loads. For each analysis the bending moment equilibrium was checked. This check was performed at half of the span length. If a beam on two supports is considered with a uniformly distributed load the maximum bending moment at the mid span is \( \frac{1}{8}qL^2 \). For a statically indeterminate structure this is also valid only now the bending moments at the beam ends are moved. Thus for the system shown in Figure 73 the check of the bending moment equilibrium is performed with [5.1].

\[
M_0 = M_{0.5L} + \frac{1}{2}M_{\text{support}} = \frac{1}{8}qL^2 \quad [5.1]
\]

As discussed previously first the lower amount of reinforcement \((A_s=804\text{mm}^2)\) was considered. The results obtained with a regular linear elastic calculation and the finite element model are shown in Figure 74. As can be seen in the figure the loads are varied from 45 N/mm up till 70 N/mm. Because the bending capacity above the support is relatively low a large amount of redistribution to the field is observed which is in accordance with what was expected. Table 8 contains the check of the bending moment equilibrium. It can be observed that after the reinforcement starts to yield a small deviation from the bending moment equilibrium occurs. These small deviations can be caused by various parameters like the number of elements, number of integration points or the convergence criterion specified in Abaqus.
Next the amount of top reinforcement was set to $A_s=1885 \text{ mm}^2$ and the same procedure just as performed previously was repeated. The results of this analysis are shown in Figure 75. It can be observed that for this situation a small redistribution takes place in the direction of the support. This is also in accordance with what was expected. Again a check of the bending equilibrium was performed, the results are shown in Table 9. It can be seen that a smaller deviation from the bending equilibrium is found for this case. Since the reinforcement did not yield due to the applied loads it is easier for Abaqus to reach convergence and therefore smaller deviations in the bending equilibrium were found.

The last test for a statically indeterminate structure was performed with the multilayer model. After the bending equilibrium was checked the force distribution in the beam was assumed to be known. Now the multilayer model was slightly modified by introducing a bending moment at the beam end of the statically determined beam as shown in Figure 76. Thus the multilayer model now contains the same bending distribution as found with the finite element model. Since the multilayer model is able to calculate the curvature corresponding to a certain bending moment this quantity can be used as a reference for the finite element model.
For both the low and high amount of reinforcement the results obtained with the multilayer model and the finite element model were compared. For this test only a load of 45 N/mm was considered because it was quite cumbersome to perform this analysis with the multilayer model. The results for the amount of top reinforcement $A_s=804$ mm$^2$ and $A_s=1885$ mm$^2$ are shown in Figure 77 and Figure 78 respectively. As can be seen the results obtained with both models show relative good agreement. Since the curvature obtained with the finite element model is plotted at the beam element ends a quite coarse pattern was found for the Abaqus results. The largest difference was found for the curvature directly above the support for the lower amount of reinforcement. Here the multilayer model found a relative large curvature compared to the curvature found with Abaqus. This might also have various reasons. Because the reinforcement above the support yields it is harder to achieve equilibrium for both the multilayer and Abaqus model. This might be a cause of the differences between the two models.
Since the beam considered in the framework will be modeled as a beam on multiple supports a last test was performed with a beam on five supports as shown in Figure 79. The bending moment distribution found for this analysis is shown in Figure 80. In this graph the result found with Abaqus are compared to the bending moment distribution according to the linear elasticity theory. It can be observed that the results obtained with Abaqus seem to be correct and that a small redistribution takes place at the two mid fields.

5.2 Dynamic analyses

Since progressive collapse is a dynamic phenomenon it is important to check how Abaqus deals with dynamically applied loads. Therefore some simple benchmark tests were performed to study the accurateness of the results found with Abaqus. In contrast to the previous paragraph also the results of the elastic and elastic plastic material behavior are included here.

5.2.1 SDOF system

A single degree of freedom (SDOF) mass spring system was used to benchmark the time deflection response obtained with Abaqus. A beam on two supports with a uniform distributed load was considered and transformed into a single degree of freedom system as shown in Figure 81. Different analyses were performed with different loads and different types of material behavior. The mid span deflection of the beam was used as a reference for each analysis.
The load and mass used for the SDOF system can be determined by multiplying both the mass and load with the length of the continuous beam. The elastic stiffness of the spring is equal to the stiffness of the beam measure at mid span as shown in [5.2].

\[ k = \frac{384EI}{5L^3} \]  

[5.2]

Because the physically behavior of the continuous beam is different from that of the SDOF system some transformation factors are needed to obtain more accurate results [47]. These transformation factors convert the real system into the equivalent SDOF system. Here the load, mass, resistance and stiffness of the real system are multiplied with the corresponding transformation factors. The equation of motion for a system without damping is shown in [5.3]. If now the transformation factors are included the equation of motion can be written as [5.4].

\[ m \cdot \ddot{y}(t) + k \cdot y(t) = F(t) \]  

[5.3]

\[ \kappa_M \cdot m \cdot \ddot{y}(t) + \kappa_R \cdot k \cdot y(t) = \kappa_L \cdot F(t) \]  

[5.4]

According to [47] the resistance factor \( \kappa_R \) must always be equal to the load factor \( \kappa_L \). Therefore a load-mass factor can be derived that needs to be multiplied with the mass only as shown in [5.5]. For this thesis a load-mass factor of 0,787 was used for the elastic strain range and a factor of 0,64 for the plastic strain range. The derivation of these factors is shown in appendix F. For further information about among others transformation factors reference is made to [47].

\[ \frac{\kappa_M}{\kappa_L} \cdot m \cdot \ddot{y}(t) + k \cdot y(t) = F(t) \rightarrow \kappa_{LM} \cdot m \cdot \ddot{y}(t) + k \cdot y(t) = F(t) \]  

[5.5]

5.2.2 Linear elastic material

After the continuous beam on two supports was transformed into a SDOF system the first analyses was carried out. First a system with linear elastic material behavior was considered. For this situation it was possible to derive an exact mathematical solution of the time deflection behavior by solving the equation of motion as shown in [5.3]. The general solution of [5.3] for linear elastic material behavior is shown in [5.6]. The derivation of [5.6] is shown in appendix G. The maximum deflection of an instantaneously applied uniformly distributed load on a linear elastic system can be obtained by deriving the maximum of [5.6]. The result is shown in [5.7]. Here it can be seen that for this system the deflection due to a dynamic load is twice that of the deflection when the load is applied statically just as discussed previously.

\[ y(t) = \frac{F(t)}{k} \left(1 - \cos(\omega t)\right) \]  

[5.6]

\[ y_{\max} = \frac{2F}{k} \approx 2y_{\text{static}} \]  

[5.7]

After derivation of the time deflection response with [5.6] an analysis was performed with Abaqus for the continuous beam shown in Figure 81. The load that was applied on the beam for this analysis was 100 N/mm. The results of the exact mathematical solution and the Abaqus results are compared and shown in Figure 82. Here it can be seen that the result obtained with Abaqus and the exact mathematical solution are almost identical. Further, the graph contains a line which indicates the deflection when the load is applied statically. Again it can be seen that the dynamic deflection is about two times the deflection found for a static load.
The graph shown in Figure 82 also contains a line which indicates the time deflection behavior when the Central Difference Method (CDM) is used. When the material behavior becomes inelastic it will become quite cumbersome to derive an exact mathematical solution of the time deflection behavior. Therefore the central difference method was used to benchmark the time deflection response in case of inelastic material behavior. In order to monitor if it is possible to obtain sufficient accurate results with the CDM method the SDOF system with linear elastic material behavior was used. As can be seen in Figure 82 the results are very close to each other.

The main principle of the central difference method is that the two previously determined values $y(i)$ and $y(i-1)$ are used to determine the subsequent value $y(i+1)$. A schematic representation of the procedure followed for the central difference method is shown in Figure 83. Here $\Delta t$ indicates the time interval used to estimate the values of $y(t)$. Convergence is reached when the time interval $\Delta t$ is sufficiently small. A guideline for a sufficiently small time interval is one tenth of the natural period $T$ of the system [47]. The algorithm used for solving the second order differential equation as shown in [5.3] by means of the central difference method is shown in appendix H.
5.2.3 Elastic plastic material

When nonlinear material is considered the spring of the SDOF system will also behave nonlinear. Therefore some modifications were necessary to be able to solve the equation of motion accurate for the considered system. For this situation first elastic plastic material was considered as shown in Figure 84.

For the transformation of the elastic plastic material into a nonlinear spring the moment curvature diagram obtained with the multilayer model was used. The moment curvature diagram was transformed into a load deflection diagram so that the stiffness of the spring could be determined for different deformation stages. This procedure is proposed by [40]. The accuracy of the response obtained with the SDOF model is highly dependent on the way the behavior of the continuous system is transformed into the SDOF model. In Figure 85 the moment curvature response for the elastic plastic material is shown. Here it is assumed that the youngs modulus $E=32837 \text{ N/mm}^2$ and the yield strength $f_y=38 \text{ N/mm}^2$. Figure 85 also contains a line which represents a schematized moment curvature response used for the SDOF model. Here the moment curvature response is divided into three stages; a linear elastic stage, an elasto plastic stage and a plastic stage. The boundary of each stage was assumed at the transition points from one to another stage.
The schematized moment curvature diagram shown in Figure 85 was used to transform the behavior of the beam into a load deflection diagram as shown in Figure 86. This translation can be obtained by first deriving the stiffness EI of each segment of the schematized moment curvature diagram. Further, it is possible to calculate the load which corresponds to a certain moment found in the moment curvature diagram with [5.8], note that this load corresponds to the entire load \( F=qL \) applied on the beam. Now it is also possible to calculate the deflection at mid span corresponding to a certain load and stiffness EI, which is determined from the moment curvature diagram. This deflection can be calculated with [5.9]. The spring stiffness \( K \) of the different segments of the load deformation diagram can now be determined through dividing the increment in load by the increment in deformation as shown in [5.10] and [5.11].

\[
M = \frac{q \cdot L^2}{8} \quad F = q \cdot L = \frac{8 \cdot M}{L} \quad [5.8]
\]

\[
y = \frac{5 \cdot F \cdot L^2}{384 \cdot EI} \quad [5.9]
\]

\[
k_1 = \frac{p_1}{y_1} \quad [5.10]
\]

\[
k_2 = \frac{p_2 - p_1}{y_2 - y_1} \quad [5.11]
\]

The previous determined stiffnesses were used in the SDOF system to determine the time deflection behavior of the beam. The same beam as used for the linear elastic material was considered for this analysis. The load applied on the beam was 150 N/mm. The results of both the SDOF system and Abaqus are shown in Figure 87. It can be seen that the results are quite close to each other. This gave confidence in results obtained with both Abaqus and the SDOF model.

![Figure 87; time deflection behavior elastic plastic material](image)
5.2.4 Reinforced concrete

The last step of the benchmark process with respect to structural dynamics was analyzing the dynamic behavior of reinforced concrete. This process proceeded in the same way as for the elastic plastic material. Again the moment curvature diagram was specified with the multilayer model which was transformed into a load deflection diagram by which the stiffnesses for the different deflection stages were determined. Figure 88 shows the simplified moment curvature diagram used for the SDOF system.

![Figure 88; simplified moment curvature response](image)

With this moment curvature relation again a translation was made to a load deflection diagram as shown in Figure 89 and also the stiffnesses of the different deflection stages were determined.

![Figure 89; load deflection curve used for SDOF system](image)

Different loads were applied on the reinforced concrete beam. It was assumed that the loads were applied suddenly just like for the previous analyzes. The results of the different applied loads obtained with the SDOF model and the Abaqus model are shown in Figure 90. Here it can be seen that there are quit large differences between the SDOF system and the Abaqus results in the initial stage. These differences are caused by the fact the Abaqus does not exactly match the moment curvature response as obtained with the multilayer model which serves as a foundation of the input variables of the SDOF model. These
deviations from the moment curvature response are shown in Figure 91. Especially for higher loads a larger deviation from the initial moment curvature response was observed. It should be noted that the deviation from the initial moment curvature response obtained with Abaqus is probably caused by the convergence criterion specified in Abaqus.

To check if a more accurate deflection response could be derived with the SDOF system the input values of the SDOF system were slightly modified in such way that they approach the moment curvature response obtained with Abaqus in a better way. Also the results of these analyzes are shown in Figure 90. Here it can be seen that the time deflection response of both models shows better agreement.

A major concern with respect to the finite element model is the un- and reloading response after the full load is applied. As can be seen in Figure 91 Abaqus uses the uncracked stiffness of the reinforced concrete beam for this stage. However, due to the applied load the concrete is cracked and therefore the stiffness at the un- and reloading stage should be the cracked stiffness. For future research it is recommended to do more research to the cyclic behavior of reinforced concrete and how this can be modeled accurately within Abaqus. However, this incorrect unloading response does not affect the first peak deflection and therefore no problems were expected for the dynamic analysis of the frame.
5.2.5 **Sudden column loss simulation**

One procedure which is important to consider is the test procedure to simulate the sudden loss of a column. In order to check if the proposed procedure is able to simulate the sudden loss of a column correctly a simple test was performed. A beam fixed at one end and simply supported at the other was assumed for this test as shown in Figure 92. The material was assumed to behave linear elastic with a young's modulus $E=32837$ N/mm$^2$, which is equal to young's modulus of the concrete considered in this thesis.

A uniform distributed load of 40 N/mm was applied on the structure. Now first the reaction force at the simply supported right end of the beam was determined. The reaction force resulted from a static analysis was 108000N. Secondly the support at the right end of the beam was removed and replaced by the reaction force found with the static analysis (see Figure 92). Again a static analysis was performed so that the structure was in equilibrium before unloading takes place. The last step is the sudden removal of the applied equivalent column force at the right end of the beam. This initiates the sudden loss of the support. It should be noted that both the static analysis and the sudden loss of the equivalent column force are entered as separated steps in Abaqus. So the system is solved statically within the first step and dynamically in the second step.

![Figure 92; procedure to initiate progressive collapse](image)

The time deflection response of the previously discussed system is shown in Figure 93. The deflection shown here is measured at the right end of the beam where the support was assumed to be removed. Both the equivalent column force and uniformly distributed load are applied in 50 seconds and remain constant during 10 seconds. As can be seen in Figure 93 the deflection at the right end of the beam is zero at the initial (static) stage during the first 60 seconds. The system is at rest until the equivalent column force is suddenly removed at time $t=60$ seconds. The removal time is assumed to be 5ms.

As a reference a second analysis was performed on the system. Here no load was applied during the first 60 seconds of the analysis and thereafter the full uniformly distributed load was applied instantaneously on the cantilever beam. Also here the load was applied within 5ms. The equivalent column force was left out of consideration for this analysis. Also the results of this analysis are shown in Figure 93. Further this graph contains a line which represents the deflection when the load would be applied statically on the beam. The graph also contains a line which indicates the maximum dynamic displacement which is twice the static as discussed previously.

It can be seen in Figure 93 that the response of the instantaneous applied load and sudden column removal show a comparable deflection. The deflection is about the expected value of the maximum displacement. Only the deflection found for the sudden column loss approach is slightly lower. However, the result obtained during these analyzes give confidence in the proposed test procedure to simulate the sudden loss of a column.
Figure 93: time deflection response progressive collapse simulation
6 Application FE framework

In this chapter some analysis were performed that show the effectiveness of the model developed during this research project. The reference frame as discussed in paragraph 4.5 and shown in Figure 94 is analyzed here. As indicated in Figure 94 a corner column is assumed to fail in this study. The deflections shown in the subsequent paragraphs are measured at the blue circle in Figure 94.

For all the analysis a dynamic solver was used. The loads were applied in 300 seconds to obtain a static solution. This loading time was empirically determined. For the simulation of the sudden column removal the equivalent column load was applied in 300 seconds just as the regular loads. Thereafter the static equilibrium state was hold for 50 seconds to be sure that the structure is at rest before unloading (see Figure 95). Thereafter the equivalent column load was removed within 0.005 seconds which is in accordance with unloading times used in the literature (among others [45] and [41]).

6.1 Static analysis initial frame

The first analysis that was performed was a static analysis on the initial frame to obtain the equivalent column force at the position where the column is assumed to be removed. The time history response of the reaction force in the corner column for the initial state is shown in Figure 96. The reaction force measured at this location was used as an equivalent column load to simulate the sudden column loss. The reaction force in the column found with this analysis was 321336 N (see Figure 96).

Further, the bending moment distribution of the beams was analyzed. These bending moments served as a reference for the further analysis. The result of this analysis is shown in Figure 97. Here beam 1 represents the lower beam and beam 5 the upper beam.
of the frame shown in Figure 94. As can be seen in the graph the bending moment distribution is uniform for all the beams just as expected.

Figure 96; time history response reaction force at column loss location

Figure 97; bending moment distribution static load RC frame

6.2 **Static analysis without corner column**

The second analysis performed on the frame was a static analysis where the corner column was omitted. For this analysis no dynamic effects were considered. The design load for the assessment of progressive collapse as derived in section 4.6 was applied on the frame. The results obtained with this analysis served as a reference for the further analyses. The time history response of the deflection at the location where the column is removed is shown in Figure 98. Also here the load is applied in 300 seconds. It can be seen that a small vibration occurs after the full load is applied. However, this vibration is so small that it does not affect the purpose of this analysis (deriving the maximum deflection at the location of the removed column under a static load). The maximum deflection found with this analysis is 20.5 mm. Also for this analysis the bending moment distribution of the beams was determined as shown in Figure 99. The bending moment at the first support is 275713000 Nmm. This was the maximum bending moment found during the analysis.
The third analysis was a static analysis where the loads above the removed column were multiplied with a dynamic load factor of 2. This factor is often used to include dynamic effects on for instance progressive collapse analysis. Because a static analysis was performed the loads were applied in 300 second just as for the previous analysis.

The time history response of the deflection at the position of the removed column is shown in Figure 100. Here it can be seen that the applied load caused a very large deflection. Especially when the reinforcement starts to yield at $t \approx 290$ seconds the deflection rapidly increased. The beams are loaded quite far within their plastic zone as can be seen in Figure 101. Here the moment curvature response at the first support of the lowest beam is shown. Also the moment curvature response obtained with the multilayer model is shown as a reference in this graph. It is doubtful if the beam is able to deform that much. However the maximum deformation capacity of the reinforced concrete beams is not considered in the current research. For future research it is recommended to investigate the maximum deformation capacity of the beams. The last graph shown in Figure 102 contains the bending moment distribution of the beams of the frame. The maximum moment found was 509937000 Nmm which is quite near the ultimate capacity of the beams.

### 6.3 Static analysis DLF of 2

The time history response deflection at location of removed column is shown in Figure 98. The bending moment distribution static analysis DLF=1 is shown in Figure 99.
Figure 100; deflection response statically applied load DLF=2

Figure 101; moment curvature response beam1 at support

Figure 102; bending moment distribution static analysis DLF=2
6.4 Dynamic analysis

The last analysis was a full dynamic analysis. For the dynamic analysis it was assumed that if the beams were able to resist the first peak displacement that they have sufficient resistance against progressive collapse. According to [40] it is unlikely that the structure collapse after unloading. However, [40] focused on the blast resistance of reinforced concrete beams. Further research is needed to check if this assumption is also valid with respect to progressive collapse of reinforced concrete structures. A material damping of 5% was used for the analysis which is often used for dynamic analysis with respect to reinforced concrete [41]. As mentioned in chapter 5.2.4 the unloading response obtained with the Abaqus model was incorrect and therefore no conclusion could be drawn on the results after the first peak displacement.

The loading time as discussed in the introduction of this chapter was used for the current analysis. Again the deformation at the column removal location was measured. The time history response of the deflection is shown in Figure 103. It can be seen in this figure that a zero displacement was found during the first 350 seconds of the analysis which was expected. Thereafter a sudden jump in the displacement can be observed when the equivalent column force was instantaneously removed. The peak deflection found was 39,75 mm. This deflection was found at time t=353,04 seconds. A more detailed view of the time history response after the instantaneous column removal is shown in Figure 104.

Also for this analysis the bending distribution of the beams is plotted as shown in Figure 105. The moment distribution shown in this graph is obtained at t=353,04 seconds and includes the maximum occurring bending moment. This is in accordance with the time at which the maximum deflection was found. The maximum bending moment amounts 419252000 Nmm. During this analysis no plastic deformations were observed due to yielding of the reinforcement.

![Figure 103; time deflection response instantaneous removed column](image-url)
Figure 104; zoom at maximum deflection due to dynamically applied load

Figure 105; moment distribution after dynamically applied load

6.5 Dynamic load factor

Four analyses were performed on the frame shown in Figure 94. In this paragraph the results obtained during these analyses will be compared and a DLF will be derived for the considered load case. For this comparison the time history of the deflection at the position of the removed column is considered. The results of the three analyses where the column was assumed to be notionally removed are shown in Figure 106. From this graph it might be concluded that it is worthwhile to perform a full dynamic analysis rather than just assuming a dynamic load factor of two.
Based on the results obtained with the previously performed analyses a dynamic load factor (DLF) was derived for the current system. This DLF can be determined by two procedures [48]. The first procedure is that the DLF is defined as the ratio of the maximum dynamic displacement to the static displacement. However, this procedure is only applicable to linear systems and would for the current research still result in a quite large DLF. This will be shown next. The static displacement as previously determined was 20.5 mm and the dynamic displacement was 39.75 mm. According to [6.1] this would result in a DLF of 1.94. However, this is only slightly lower than the dynamic load factor of 2.

\[
DLF = \frac{y_{\text{dynamic}}}{y_{\text{static}}} = \frac{39.75}{20.5} = 1.94
\]

[6.1]

The second way to determine the DLF is more appropriate for nonlinear systems as considered in the current study. The DLF is now defined as the static force required to deflect the structure by the same amount as the maximum dynamic displacement, divided by the maximum dynamic force [11]. As a starting point for this procedure the maximum bending moment found with the static and dynamic analyses are used as shown in [6.2].

\[
DLF = \frac{M_{\text{dynamic}}}{M_{\text{static}}} = \frac{419252000}{275713000} = 1.52
\]

[6.2]

Again a static analysis was performed with the previously determined DLF of 1.52. The results are shown in Figure 107. The static deflection found with this procedure is 37.96 mm. As can be seen in Figure 107 the deflection found with a DLF determined with the second procedure provides a better approximation of the dynamic response than the first procedure. The difference in the deflection found between the static analysis with a DLF of 1.52 and the dynamic analysis is now only 4.5%. Referring to the DLF of 2 which is often used for design purposes would result in a very conservative design. The dynamic load that might be used for the design with a DLF of 2 would provide an overestimation of the load with 24% for this specific situation. However, due to the limited number of analyses performed in this study it was not possible to draw valuable conclusions about the establishment of a dynamic load factor. Further research is needed to establish useful recommendations on this topic.
Figure 107: analysis with DLF=1.52 versus dynamic analysis
7 Concluding remarks and recommendations

New requirements with respect to progressive collapse came into force with the introduction of the Eurocode in 2012 in the Netherlands. One of the main design strategies to prevent progressive collapse recommended in the Eurocode is the application of horizontal and vertical ties to prevent collapse by catenary action. However, different researchers expressed their concerns about the proper functioning of catenary action especially with respect to the sudden loss of a corner support in reinforced concrete frame structures. A literature study to the state of the art with respect to alternate load paths and catenary action was performed first during this project to investigate if these concerns are justified.

During the literature study it was found that in case of the sudden failure of an intermediate column catenary action can develop as long as the beams are able to deform sufficiently. Debonding the rebar’s can improve this deformation capacity of the beams. However, for the collapse of a corner support no catenary action was observed which might indicate that the cable mechanism cannot develop for this situation. Some researchers included the influence of the floors and found that this highly improved the progressive collapse resistance in case of failure of a corner column. Further some researchers performed experimental tests on existing buildings that would be demolished to study their vulnerability to progressive collapse. It was observed that none of the existing structures collapsed after the sudden removal of a column. The deflections measured were really small. For further research it is recommended to study some existing structures that will be demolished anyway. Studies to these structures can really increase our knowledge with respect to progressive collapse.

As mentioned previously there are doubts about the functioning of the cable mechanism as included in the Eurocode in case of failure of a corner column. Therefore an alternate load path by means of the flexural capacity of the beams was considered in this study as alternative for catenary action. For such an alternate load it is important to include the dynamic nature of the load caused by the sudden failure of a column. Therefore a second goal was specified for the current thesis.

The second goal was the development of a dynamic load factor which is less conservative than the often used load factor of 2. Therefore a finite element model was developed which was able to analyze the behavior of a reinforced concrete frame structure after the sudden loss of a column. Because no experimental data was available simple analytical models were developed to check if the results obtained with the finite element model build in Abaqus provides explicable results. It was found that it is very important to verify the results obtained with a finite element model since errors are easily made. Especially when considering reinforced concrete it is important to benchmark the results. The behavior of reinforced concrete is highly nonlinear due to effects like strain softening, strain hardening and yielding of the reinforcement and therefore more sensitive for errors in the input.

For this research thesis it can be concluded that the specification of the correct material behavior of the reinforced concrete was the most challenging subject. By performing a lot of benchmark tests confidence was obtained in the proper functioning of the finite element model with respect to the simulation of progressive collapse. It was observed that the model is able to describe the full nonlinear response of the reinforced concrete. Further it was found that the model was able to describe the maximum response due to dynamically applied loads. However, the behavior after the first peak displacement was reached is not yet been clarified. Therefore, more research is needed to the cyclic behavior of the reinforced concrete and how this behavior can be modeled within the finite element software Abaqus. Especially the stiffness degradation due to cyclic loading and the effect of strain rates on the progressive collapse resistance are important properties to consider for further research. If the stiffness degradation can be modeled correctly a more reliable
estimation of the resistance against progressive collapse can be obtained that includes the full strain softening behavior of the concrete in compression. According to various researchers it is further useful to take the effects of strain rates into account since this effect increase the strength properties of both the concrete and reinforcement.

This study also showed that a dynamic load factor of 2 which is often used to describe the dynamic nature of the progressive collapse load will result in an over conservative design. It was found that this factor of two is only valid for structures which behave perfectly elastic. However, as discussed in this thesis, reinforced concrete does not behave linear elastic and therefore a load factor smaller than 2 would be more appropriate. The dynamic analysis on the reference frame showed that for that particular case a dynamic load factor of 1.52 would be more appropriate.

With respect to the procedure to simulate the sudden column loss as proposed in this thesis it was found that it is able to simulate the event of sudden column loss correctly. With this procedure column removal scenarios on various locations can be considered. For further research it is recommended to perform a parameter study to the progressive collapse resistance of the considered reinforced concrete frame structure. Parameters like bay size, reinforcement ratio, concrete properties, beam dimension and a variation of the number of story’s and positions of column removal could be considered.

In this thesis only the flexural capacity of the beams is considered without any check of the resistance of the columns against among other buckling. For future research it is recommended to investigate the behavior of the columns after the instantaneous loss of adjacent columns. Due to failure of, for instance, a corner column the load on the adjacent column will increase. This increase of load on the adjacent column might result in failure caused by among others stability problems. Further, it is recommended to investigate the influence of the flexural capacity of the columns since this will improve the progressive collapse resistance of the frame (for instance by Vierendeelaction).

Among other the UFC guidelines allow 3D analysis only for the design of an alternate load path. Since this thesis considered a 2D structure only it might be useful to consider a 3D structure for further research. Here the effect of floors could be included and also effects like torsion of the beams might be important to analyze. When referring to a perimeter beam in a precast structure where, for instance, a hollow core floor causes an eccentrically applied load on the beam effects like torsion of the beam may be normative. In such cases a 3D representation of the structure would be very useful.
Bibliography


[38] G. Lagidogna, F. Accornero, M. Corrado and A. Carpinteri, "Crushing and fracture energies in concrete specimens monitored by accoustic emission," in International Conference on Fracture Mechanics of Concrete and Concrete Structures.


A Appendix: progressive collapse properties

Types of progressive collapse

There are various different mechanisms of collapse possible. In which way a structure collapses depends on various features like the type of construction, the detailing and the nature of the first local collapsing element. Starossek [4] describes six different types of collapse which will be treated in the subsequent paragraphs. For each type of progressive collapse the characteristic features will be described.

Pancake-type collapse

The term pancake collapse originates from floor slabs of a building which pile on top of each other, resembling a stack of pancakes. This type of collapse occurred at the World Trade Center towers after a terrorist attack. The mechanism of a pancake-type collapse exhibits the following features:

- Initial failure of vertical load bearing elements
- Separation of structural components and their fall in vertical ridged body motion
- Transformation of gravitational potential energy into kinetic energy
- Impact of separated and falling components on remaining structure
- Failure of other vertical load bearing elements due to the axial compression forces that result from the impact loading
- Failure progression in the vertical direction

Zipper-type collapse

A zipper collapse occurs by the initial failure of a load bearing element and progressing due to the resulting overloading and failure of adjacent elements. An example of a zipper collapse can be observed in the footage of the collapse of the original Tacoma Narrows Bridge in 1940. The mechanism of a zipper-type collapse exhibits the following features:

- Initial failure of one or a few load bearing elements
- Redistribution of the forces carried by these element into the remaining structure
- Impulsive dynamic loading due to the suddenness of the initial failure and redistribution of forces
- Dynamic response of the remaining structure to that impulsive dynamic loading
- Concentration of forces in load bearing elements that are similar in type and function to and adjacent to or in the vicinity of the initially failing elements due to the combined static and dynamic structural response to that failure
- Overloading and failure of those elements
- Failure progression in a direction transverse to the principal forces in the failing elements

Domino-type collapse

A row of dominoes falls by pushing over one of the pieces which causing a chain reaction that will make the entire row of stones falling. This phenomenon can also occur in a structure. The mechanism behind a domino collapse is as follows:

- Initial overturning of one element
- Fall of that element in an angular rigid body motion around a bottom edge
- Transformation of gravitational potential energy into kinetic energy
- Lateral impact of the upper edge of the overturning element on the side face of an adjacent similar element
- The horizontal pushing force transmitted by that impact is of both static and dynamic origin because it results from both the tilting and the motion of the impacting element
- Overturning of the adjacent element due to the horizontal pushing force from the impacting element
- Failure progression in the overturning direction

The occurrence and the importance of impact forces suggest a relationship with the pancake-type collapse. The principal forces in the failing elements, however, are now
orthogonal to the direction of failure progression, and the elements constitute a parallel load transfer system. Any group of individual structures that are susceptible to overturning and are placed in a repetitive horizontal arrangement, similar to a row of dominoes, could collapse in such a manner. An example of a domino collapse can be found by the collapse of overhead transmission line towers triggered by ice accretion in 2005 in Germany.

**Section-type collapse**
When a cross-section is loaded by a bending or tensile force the stresses will be distributed over the whole cross-section. However, when a part of the cross-section is cut, this will result in a potential increase of stresses in the remaining cross-section. This increase of stresses can cause, in certain parts of the cross section, a rupture. This kind of failure is usually not called progressive collapse, but brittle fracture or fast fracture. A section-type collapse contains the same list of features as a zipper-type collapse, provided the terms “part of cross-section” and “remaining cross-section” are substituted for the term “element” and “remaining structure”. The main difference are that the cross-section of a beam or tie rod is unstructured, continuously contiguous, and comparatively homogeneous, whereas a structure, for instance a cable-stayed bridge, is generally structured consisting of discrete elements that are interconnected in various ways and have different properties.

**Instability-type collapse**
If a particular element is sensitive to instability, additional structural components can be used that brace or stiffen the structure. However, failure of a bracing element due to some small triggering event can make a structure unstable and result in collapse. This type of collapse is called instability collapse. The mechanism of an instability-type collapse exhibits the following features:

- Initial failure of bracing or stiffening elements that have been stabilizing load-bearing elements in compression
- Instability of the elements in compression that cease to be stabilized
- Failure – caused by the stability failure of these destabilized compressed elements – failure of bracing or stiffening elements of other load-bearing elements in compression
- Failure progression in the same repetitive manner

**Mixed-type collapse**
Not every collapse can be classified in one of the above mentioned categories. In certain kinds of structures, particularly certain kinds of buildings, it even seems possible that features of the four basic categories interact and jointly contribute to failure progression. This is called a mixed type collapse.

**Important structural properties**

For regular structural design maximum resistance and stiffness are the most important properties. In design with regard to accidental action and prevention of progressive collapse also parameters such as the maximum deformation, total strain energy and ductility are important parameters to consider [2] [4].

**Brittle material behavior**
Brittle material behavior is not desirable for progressive collapse due to a lack of plastic deformation capacity. Ductile material, on the other hand, is able to absorb and dissipate kinetic energy due to the significant plastic deformation capacity. Due to this a redistribution of forces is permitted which will result in a reduction of force concentrations. Ductile material behavior increases the robustness of a structure through the provision of alternative load paths.
Over strength and ductile material behavior
Over strength and ductile material behavior may in some cases be detrimental. Starossek [4] gives an example with a transmission line where the negative effect of over strength becomes clear. In his example he describes a domino type collapse in which the propagating action is transmitted by the transmission line. This action results in a force in the mediating elements larger than that occurring under normal conditions. To limit that force, the strength of the mediating elements or their connections should in some cases be limited, and thus over strength should be avoided. This also applies for ductile material behavior.

By restricting the strength and ductility of a well defined element, it is possible to create segment boarders. A segment boarder is an expedient of a design method aimed at restricting the extent of collapse through isolating the collapsing part by means of segmentation.

Continuity or discontinuity
Also for the features continuity and discontinuity applies that they can be implemented only in connection with the type of collapse. Continuity can be favorable when the design aims at increasing the robustness of a structure by providing alternative load paths. But continuity can also be harmful when an element fails and therefore the other elements will be loaded more heavily and fail because of the extra load. In this situation discontinuity would be a better solution.

Series or parallel load transfer
When the structural system consists of series load transfer, the initial failure of any load bearing element can be expected to cause collapse. A serial load transfer means that there is mainly a vertical load transfer. In this case the initial failing element is the whole of the vertical load bearing members of a story. Series load transfer is in principle undesirable, however, it is inevitable in some structures such as slender high-rise buildings.

In the case of parallel load transfer, the forces carried by failing elements can be redistributed into the remaining structure. A parallel load transfer means that there is mainly a horizontal load transfer.

Spatial orientation, size, and slenderness
Structures are often aligned in a primary direction, high rise buildings for example are mainly vertical aligned while bridges mainly aligned along a horizontal axis. This has a significant influence on the type of collapse. A pancake-type collapse is more likely in a vertical aligned structure.

Also the size of a building has influence on the type of collapse. A large building is more susceptible to progressive collapse than a small building. Especially the height is an important feature. The reason for this height dependency lies in the ratio of the gravitational potential energy to the elastic potential energy, which increases with the height.

A high rise building often is a slender structure. However, slenderness is a structural feature that promotes pancake type collapse. This is because the ratio between the extent of accidental circumstances and the susceptibility to failure progression decreases with an increase in slenderness. Because slenderness facilitates failure from overturning and instability, it is a property that also favors domino type and instability type collapse.
Structuredness
Structuredness is the degree to which a structure possesses a definite pattern of load bearing elements, which is also a collapse promoting feature. For example the structure of a high-rise building with its pattern of beams, slabs and columns is highly structured while an industrial chimney of the same height designed as a compact reinforced concrete tube is comparatively unstructured. Structuredness is mainly a condition which favors pancake-type collapse. This follows from the release and subsequent reintroduction of a large amount of gravitational energy associated with the fall and impact of components which is difficult to develop in a structure lacking structuredness. This also applies for a domino-type of collapse.
Appendix: determination ties EC

Here the procedure to determine the dimensions of the ties according to the Eurocode is discussed. First the procedure as included in Eurocode 1 is described and thereafter the procedure as included in Eurocode 2.

Eurocode 1: General actions

Eurocode 1 first gives a method for the determination of horizontal ties in case of a structure with columns. These ties should be placed at each floor and roof level in two mutually perpendicular directions to connect the columns and walls with the structure. The ties should be applied continuous and placed as close as possible to the edge of floors and lines of columns and walls. At least 30% of the ties should be placed in the vicinity of grid lines from columns and walls. The horizontal ties may consist of rolled steel sections, reinforcement bars or reinforcement meshes in concrete floor slabs and profiled steel plates in steel-concrete floors, also a combination of these ties is allowed. Internal ties should be able to resist a tensile force equal to equation [B.1] and peripheral ties a tensile force equal to equation [B.2].

\[ T_i = 0.8(G_k + \Psi Q_k) \cdot s \cdot L \geq 75kN \]
\[ T_p = 0.4(G_k + \Psi Q_k) \cdot s \cdot L \geq 75kN \]

Here, \( s \) is the distance between the ties and \( L \) the length of the tie.

For structures with load bearing walls a deviation is made for structures in consequence class 2a and 2b. For consequence class 2a there are no additional requirements given, only that the building should have sufficient robustness and that collaboration between all elements must be realized. For buildings in consequence class 2b horizontal ties should be applied. These ties should be internal ties which are spread over the floor in both orthogonal directions, and peripheral ties which are placed along the periphery of the floor slabs within a width of 1,2 m. The tensile force for internal ties must be determined by the largest value of \( F_t \) in [B.4] or equation [B.3] and for peripheral ties the smallest value of \( F_t \) as shown in equation [B.4].

\[
F_t = \frac{G_k + \Psi Q_k}{7,5 \cdot 5} \cdot z
\]
\[
F_t = 20 + 4 \cdot n_s \text{ or } 60 \text{ kN/m} \text{ (smallest value should be chosen)}
\]

Here, \( n_s \) indicate the number of storeys and \( z \) is the smallest value of:
- Five times the storey height, or;
- The largest distance in the direction of the tie, between the center lines of the columns or other load bearing elements

The last section contains a method for the determination of the vertical ties. Every column and wall should provide a continuous tie from the foundation to roof level. In case of a building with a framework, the columns and walls should be able to resist a design value of an exceptional tensile force equal to the maximum design value of the vertical reaction by permanent and live load in the column by each storey. For structures with load bearing walls, the vertical ties can be regarded as effective if:

- The minimal thickness of masonry walls is 150mm and if they possess at least a compressive strength of 5 N/mm².
- The free height of the wall is not more than 20 times the thickness of the wall
- They are designed and calculated to resist a tensile force equal to equation [B.5].
Here \( A \) is the cross section of the wall.

- The vertical ties are grouped on distances of maximal 5m along the wall and are located on no more than 2,5 m from a unsupported end of the wall.

**Eurocode 2: Design of concrete structures**

Also Eurocode 2 [8] contains rules for the determination of the tie reinforcement. First a calculation method is given for horizontal peripheral ties. These ties should be placed at each floor or roof level around the total floor, within a distance of 1,2m form the edge of the floor. The peripheral ties should be able to take a tensile force equal to the force resulting from equation [B.6].

\[ F_{tie;per} = l_i \cdot q_1 \leq q_2 \]  \[ \text{[B.6]} \]

Here \( F_{tie;per} \) is the tensile force, \( l_i \) is the length of the end span and \( q_1 \) and \( q_2 \) are forces which are dependent of the consequence class of the building.

The next section contains a calculation method for internal ties. These ties should be placed at each floor and roof level in two directions, parallel and perpendicular to the span of the floor units. These ties should be applied over the entire floor length and should be duly anchored at the peripheral ties, unless they are applied as continuous ties through columns or walls. The internal ties may, in whole or in part, be spread evenly in the floor units or may be grouped at or in the beams, walls or other appropriate positions.

In each direction, the internal ties should have a design value that equals the tension force \( F_{tie;int} \) as given below:
- \( F_{tie;int}=0 \) KN/m for consequence class 1 and 2a
- \( F_{tie;int}=20 \) KN/m for consequence class 2b and 3

In floors without topping where ties cannot be distributed across the span direction, the transverse ties may be grouped along the beam lines. In this case, the minimum force at an internal beam is equal to equation [B.7].

\[ F_{tie} = \frac{l_1 + l_2}{2} \cdot q_3 \leq q_4 \]  \[ \text{[B.7]} \]

Here, \( l_1 \) and \( l_2 \) are the floor spans on both sides of the beam and \( q_3 \) and \( q_4 \) are forces which are dependent on the consequence class of the building. Further, the internal ties should be connected with the peripheral ties to ensure a transfer of forces.

The last section for horizontal ties contains the determination of ties on columns or walls. Here, it is required that the corner columns and walls are connected with the structure on each floor- or roof level. These ties should be able to absorb a tensile force \( F_{tie;fac} \) per meter façade. For columns it should be able to absorb a force equal to \( F_{tie;col} \). Both \( F_{tie;fac} \) and \( F_{tie;col} \) are dependent on the consequence class of the building. The corner columns should be connected in two directions.

The last section contains requirements for vertical ties. Here, it is required that buildings with five or more storey’s should be provided with vertical ties in columns and walls. These vertical ties should limit the extent of damage due to failure of the lower parts of the structure by suspension of the excessive loads to the remaining structure. These ties should be applied continuously from the lowest to the highest level of the building, and should be able to carry the load on the floor above the failed column or wall.
C Appendix: catenary action

Here the basics of a cable mechanism will be discussed. When a cable is completely straight it can bear no load other than a tensile force parallel to its axis since a cable does not have any resistance against bending or compression. Because of this, there is in no equilibrium possible in the undeformed state when a load is applied perpendicular to the axis of the cable. Only when the cable deforms equilibrium becomes possible. Consider a cable which is supported by two hinges and loaded by a point load in the middle as shown in Figure 108.

![Figure 108; point load on cable](image)

Due to the load the cable will deform and an equilibrium state can be reached. Now the horizontal force $H$ can be determined by considering the bending moment equilibrium with \[\text{[C.1]}\]. The horizontal reaction force $H$ is an important force since this force must be resisted by the remaining structure after the sudden loss of a load bearing element.

$$\sum M = \frac{F}{2} \cdot \frac{L}{2} - \delta \cdot H = 0 \rightarrow H = \frac{F \cdot L}{4 \cdot \delta}$$

[C.1]

Another important force is the tensile force $T$ in the cable. This cable force can be determined by the square root of the vertical and the horizontal component as shown with \[\text{[C.2]}\].

$$T = \sqrt{H^2 + V^2} = \sqrt{\left(\frac{F \cdot L}{4 \cdot \delta}\right)^2 + \left(\frac{F}{2}\right)^2} = \frac{F}{2} \cdot \sqrt{\left(\frac{L^2}{28}\right)^2 + 1}$$

[C.2]

In \[\text{[C.8]}\] and \[\text{[C.9]}\] it can be seen that the deformation of the cable has a large influence on both the horizontal and cable force. When the deformation $\delta$ increases the horizontal force $H$ and cable force $T$ decreases. The cable must extend to be able to deform. How much the cable extends depends on the strain capacity of the cable and the possibility to slip at the support. The required cable strain can be described with equation \[\text{[C.3]}\] [49].

$$\varepsilon_{\text{tie}} = \frac{\Delta L}{L_0} = \varepsilon_{\text{slip}} = \frac{\left(\frac{L^2}{2} + \delta^2 - \frac{L}{2}\right)}{\frac{L}{2}} = \frac{\text{slip}}{\frac{L}{2}}$$

[C.3]

The procedure as discussed above is also valid for a cable that is loaded by a uniformly distributed load as shown in Figure 109.

![Figure 109; distributed load on cable](image)
The horizontal force can now be determined with \([C.4]\).

\[
\sum M = q \cdot \frac{L}{2} \cdot \frac{L}{2} - q \cdot \frac{L}{2} \cdot \frac{L}{4} - H \cdot \delta = 0 \rightarrow H = \frac{q \cdot L^2}{8 \cdot \delta}
\]  \([C.4]\)

The corresponding cable force can be derived with \([C.5]\).

\[
T = \sqrt{H^2 + V^2} = \sqrt{\left(\frac{q \cdot L^2}{8 \cdot \delta}\right)^2 + \left(\frac{q \cdot L}{2}\right)^2} = \frac{q \cdot L}{8} \sqrt{\frac{L^2}{\delta} + 4}
\]  \([C.5]\)

Besides catenary action in one direction also catenary action in two directions can develop as shown in Figure 110. This has a favorable effect on both the cable force and the horizontal reaction force.

Figure 110; point load 3D cable

To show the afford of a three dimensional cable structure as shown in Figure 110 the horizontal reaction force and cable force will be determined again. It is assumed that the spans and the diameter of the ties are equal in both directions and that the cable is loaded by a point load in the middle. As can be observed in \([C.6]\) and \([C.7]\) is that both the reaction force and cable force are half of the force as for the one dimensional cable.

\[
\sum M = \frac{F}{4} \cdot \frac{L}{2} - H \cdot \delta = 0 \rightarrow H = \frac{F \cdot L}{8 \cdot \delta}
\]  \([C.6]\)

\[
T = \sqrt{H^2 + V^2} = \sqrt{\left(\frac{F}{8 \cdot \delta}\right)^2 + \left(\frac{F}{4}\right)^2} = \frac{F}{4} \sqrt{\frac{L^2}{2 \cdot \delta} + 1}
\]  \([C.7]\)
**D  Appendix: alternate path method**

This appendix provides a brief description on how the various factors needed to perform an alternate load path analysis according the UFC guidelines can be determined. First the linear static procedure will be treated and thereafter the nonlinear static procedure.

**Linear static procedure**

According to the UFC guidelines only for regular and irregular structures that meet certain requirements (section 3-2.11.1 of UFC 4-023-03) a linear static procedure can be used. Further two dimensional analyses are not permitted, only a three dimension model may be used for linear static analyses. The loads that need to be applied on the structure for collapse analysis are slightly different compared to the Eurocode. For the determination of the factors always the most restrictive structural component is decisive. This component must be selected in the bays around the location where the loss of structural components is assumed.

Now the procedure for determining the load factor will be treated first. As mentioned before, the UFC guidelines use a so called m-factor to determine the Load Increase Factor for deformation controlled actions. In Table 10 the load increase factors for different materials are shown.

<table>
<thead>
<tr>
<th>Material</th>
<th>Structure type</th>
<th>LIF deformation controlled</th>
<th>LIF force controlled</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steel</td>
<td>Framed</td>
<td>0,9 $m_{UF}$ + 1,1</td>
<td>2,0</td>
</tr>
<tr>
<td>Reinforced concrete</td>
<td>Framed</td>
<td>1,2 $m_{UF}$ + 0,8</td>
<td>2,0</td>
</tr>
<tr>
<td></td>
<td>Load bearing wall</td>
<td>2,0 $m_{UF}$</td>
<td>2,0</td>
</tr>
<tr>
<td>Masonry</td>
<td>Load bearing wall</td>
<td>2,0 $m_{UF}$</td>
<td>2,0</td>
</tr>
<tr>
<td>Wood</td>
<td>Load bearing wall</td>
<td>2,0 $m_{UF}$</td>
<td>2,0</td>
</tr>
<tr>
<td>Cold-formed steel</td>
<td>Load bearing wall</td>
<td>2,0 $m_{UF}$</td>
<td>2,0</td>
</tr>
</tbody>
</table>

The factor $m_{lf}$ can be determined with, among others, table 4.2 of the UFC 4-023-03. A preview of this table is presented in Table 11 where the m-factor for reinforced concrete can be determined.

<table>
<thead>
<tr>
<th>Conditions</th>
<th>m-factor type</th>
<th>m-factor type</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Beam controlled by flexure</td>
<td>Primary</td>
<td>Secondary</td>
</tr>
<tr>
<td>$\rho - \rho'$ $\rho_{bal}$ Trans. Reinfr.</td>
<td>$V$ $b_{w} \cdot d \cdot \sqrt{f_{c}}$</td>
<td></td>
</tr>
<tr>
<td>≤0,0</td>
<td>C</td>
<td>≤3</td>
</tr>
<tr>
<td>≤0,0</td>
<td>C</td>
<td>≥6</td>
</tr>
<tr>
<td>≥0,5</td>
<td>C</td>
<td>≤3</td>
</tr>
<tr>
<td>≥0,5</td>
<td>C</td>
<td>≥6</td>
</tr>
</tbody>
</table>

Before the m-factor can be derived from Table 11 some parameters must be determined. The factor $\rho$ and $\rho'$ represents the reinforcement ratio of the compression and tension reinforcement respectively. The factor $\rho_{bal}$ represents the reinforcement ratio that produces balanced strain conditions, $\rho_{bal}$ can be determined with [D.1].

$$\rho = \frac{A_{s}}{b \cdot d}$$  \hspace{1cm} \text{[D.1]}$

$$\rho_{bal} = 0,85 \cdot \beta_{i} \cdot \frac{f_{c}}{f_{y}} \cdot \frac{\varepsilon_{u}}{\varepsilon_{u} + \varepsilon_{y}}$$  \hspace{1cm} \text{[D.2]}$
The factor $\beta_1$ depends on the concrete compressive strength. According to ACI 318-08, $\beta_1 = 0.85$ for $f'_c$ between 2500 psi and 4000 psi (17.2 N/mm² and 27.6 N/mm² respectively). For $f'_c$ values above 4000 psi, $\beta_1$ will be reduced linearly with a rate of 0.05 for each 1000 psi (6.9 N/mm²) of strength in excess of 4000 psi with a minimum of 0.65.

The term “C” beneath the column with “transverse reinforcement” is an abbreviation for conforming. A component can be classified as conforming if the stirrups are spaced at $\leq d/3$ within the flexural plastic hinge region. The second requirement for conforming for components of moderate and high ductility demand is that the strength provided by the hoops is at least three fourths of the design shear. If not, the component is considered nonconforming. This parameter is quite important because the shear reinforcement has a large effect on the compressive strength and ductility of the concrete [41]. In Figure 111 the effect of different shear reinforcement ratios on the compressive strength and strain is shown.

![Figure 111; effect of lateral reinforcement ratio on the behavior of steel confined concrete [41]](image)

The level of confinement is determined by the amount and constitutive behavior of the confining steel and can result in an increase of compressive strength and/or ultimate compressive strain as shown in Figure 112.

![Figure 112; compressive stress-strain relation for confined concrete [41]](image)

The third row in Table 11 includes a factor based on the shear force. Here, it should be noted that it is important to use the right unit’s since the UFC express dimensions and force in inches and pounds respectively. When using millimeters and Newton’s it will affect the outcome and a wrong m-factor will be obtained from the table. Therefore, Table 12 includes a conversion from the units used in the UFC guidelines to the units used in Europe.

<table>
<thead>
<tr>
<th>Table 12: conversion units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Conversion units used in UFC guidelines to European units</td>
</tr>
<tr>
<td>1 kip                  = 4448.22 N = 1000 psi</td>
</tr>
<tr>
<td>1 ksi                  = 6894.8 N/mm²</td>
</tr>
<tr>
<td>1 in                   = 25.4 mm</td>
</tr>
</tbody>
</table>
After all parameters are known the m-factor can be obtained from the table and subsequently the LIF can be determined. According to Mckay [50] an effective multiplier can be applied to the load equal to LIF/m, this effective multiplier must be determined for all primary structural components adjacent to the removed column. The largest effective multiplier is then decisive and must be applied on all loads directly above the removed column.

**Nonlinear static procedure**

The main steps to determine the dynamic increase factor for the nonlinear static procedure will be shown here. The UFC guidelines distinguish different dynamic increase factors for different materials. Table 13 shows how these dynamic increase factors can be determined. In this table \( \Omega_n \) stands for the dynamic increase factor that will be used to magnify the loads with. Also here applies that the most decisive component must be used just as for the linear static procedure.

**Table 13; dynamic increase factors for nonlinear static analysis [10]**

<table>
<thead>
<tr>
<th>Material</th>
<th>Structure type</th>
<th>( \Omega_n )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steel Framed</td>
<td>Framed</td>
<td>( 1,08+0,76/(\theta_{pra}/\theta_y+0,83) )</td>
</tr>
<tr>
<td>Reinforced concrete</td>
<td>Framed</td>
<td>( 1,04+0,45/(\theta_{pra}/\theta_y+0,48) )</td>
</tr>
<tr>
<td>Masonry Load bearing wall</td>
<td>Load bearing wall</td>
<td>2</td>
</tr>
<tr>
<td>Wood Load bearing wall</td>
<td>Load bearing wall</td>
<td>2</td>
</tr>
<tr>
<td>Cold-formed steel</td>
<td>Load bearing wall</td>
<td>2</td>
</tr>
</tbody>
</table>

Further Table 13 contains two factors \( \theta_{pra} \) and \( \theta_y \) which stands for the plastic rotation angle and yield rotation respectively. It can be observed that the determination of the dynamic increase factors for RC depends on the ratio of the allowable plastic rotation to the yield rotation specified in the acceptance criteria. The maximum allowable rotation angle \( \theta_{pra} \) can be determined with Table 14 for a reinforced concrete beam. The parameters to determine the plastic rotation angle are equal to the parameters as specified for the linear static procedure. The yield rotation can be determined with [D.3]. To determine the yield rotation the effective stiffness of the reinforced concrete must be used. The term \( l_p \) in [D.3] indicates the plastic hinge length.

\[
\theta_y = \left( \frac{M_y}{F_{ce}} \right) l_p
\]

**Table 14; nonlinear acceptance criteria for reinforced concrete beams [10]**

<table>
<thead>
<tr>
<th>Conditions</th>
<th>Acceptance criteria Plastic rotations angle, radians</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Primary</td>
</tr>
<tr>
<td>i. Beam controlled by flexure</td>
<td>( \rho - \rho' )</td>
</tr>
<tr>
<td>( \rho - \rho' ) ( \rho_{bat} )</td>
<td>( V )</td>
</tr>
<tr>
<td>( \leq 0,0 )</td>
<td>C</td>
</tr>
<tr>
<td>( \leq 0,0 )</td>
<td>C</td>
</tr>
<tr>
<td>( \geq 0,5 )</td>
<td>C</td>
</tr>
<tr>
<td>( \geq 0,5 )</td>
<td>C</td>
</tr>
</tbody>
</table>
Appendix: ULS design

Loads

In Table 15 an overview is given of the dead and live loads which act on the structure. The floor is assumed to be a wide slab floor. Due to the fact that the wide slab floor is considered as a beam on 4 supports in the direction perpendicular to the span direction of the beam only 40% of the floor load acts on the perimeter beam. Further it is not necessary to consider live loads like wind for the assessment of the progressive collapse resistance [7].

<table>
<thead>
<tr>
<th>Line load on peripheral beam</th>
<th>Floor</th>
<th>Roof</th>
</tr>
</thead>
<tbody>
<tr>
<td>$G_{k;floor}$</td>
<td>$7,2 \times 0,4 \times 7,75 , \text{kN/m}^2$</td>
<td>$7,2 \times 0,4 \times 6,20 , \text{kN/m}^2$</td>
</tr>
<tr>
<td>$G_{k;beam}$</td>
<td>$1 \times 5,76 , \text{kN/m}$</td>
<td>$1 \times 5,76 , \text{kN/m}$</td>
</tr>
<tr>
<td>$G_{k;façade}$</td>
<td>$3,6 \times 3,0 , \text{kN/m}^2$</td>
<td>$3,6 \times 2,5 , \text{kN/m}^2 \times 0,5$</td>
</tr>
<tr>
<td>$G_{k;total}$</td>
<td>$38,9 , \text{kN/m}$</td>
<td>$23,7 , \text{kN/m}$</td>
</tr>
<tr>
<td>$Q_{k}$</td>
<td>$7,2 \times 0,4 \times 2,5 , \text{kN/m}^2$</td>
<td>$7,2 \times 0,4 \times 0,56 , \text{kN/m}^2$</td>
</tr>
<tr>
<td>$Q_{k;mom}$</td>
<td>$3,6 , \text{kN/m}$</td>
<td>$0 , \text{kN/m}$</td>
</tr>
</tbody>
</table>

In order to perform calculations for the ultimate limit state (ULS) some load combinations need to be considered. These load combinations are based on NEN-EN 1990-1-1 [51]. The required load combinations are shown in equation [E.1] and [E.2].

$$
\begin{align*}
\sum_{j=1}^{j=n} \gamma_{g,j} \cdot G_{k,j} + \gamma_{Q,1} \cdot Q_{k,1} + \sum_{j=1}^{j=n} \gamma_{Q,1} \cdot \psi_{0,j} \cdot Q_{k,j} & \quad \text{[E.1]} \\
\sum_{j=1}^{j=n} \gamma_{g,j} \cdot G_{k,j} + \gamma_{Q,1} \cdot Q_{k,1} + \sum_{j=1}^{j=n} \gamma_{Q,1} \cdot \psi_{0,j} \cdot Q_{k,j} & \quad \text{[E.2]}
\end{align*}
$$

Since the function of the building is assumed to be an office building the following load factors are required:

$$
\begin{align*}
\gamma_{j} & = 1,35 \\
\gamma_{d} & = 1,2 \\
\psi_{0,j} & = 0,9 \\
\gamma_{0,j} & = 1,5
\end{align*}
$$

With equation [E.1] and [E.2] and the earlier mentioned load factors the load combinations as included in Table 16 can be obtained.

<table>
<thead>
<tr>
<th>Design loads</th>
<th>Floor</th>
<th>Roof</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_{D;unfavorable;floor}$</td>
<td>$1,35 \times 38,9 , \text{kN/m} + 1,5 \times 3,6 , \text{kN/m}$</td>
<td>$1,35 \times 23,7 , \text{kN/m} + 1,5 \times 0 , \text{kN/m}$</td>
</tr>
<tr>
<td>$P_{D;unfavorable;floor}$</td>
<td>$1,2 \times 38,9 , \text{kN/m} + 1,5 \times 7,2 , \text{kN/m}$</td>
<td>$1,2 \times 23,7 , \text{kN/m} + 1,5 \times 0 , \text{kN/m}$</td>
</tr>
<tr>
<td>$P_{D;favorable;floor}$</td>
<td>$0,9 \times 38,9 , \text{kN/m}$</td>
<td>$0,9 \times 23,7 , \text{kN/m}$</td>
</tr>
</tbody>
</table>
When designing against progressive collapse the load combinations differ from the previously derived load combinations. The general formulation for this load combination is shown in [E.3].

\[
\sum_{j=1} g_{kj} + (\psi_{1,1} \text{ or } \psi_{2,1}) \cdot Q_{k,j} + \sum_{i=1} \psi_{2,i} \cdot Q_{i,j}
\]

For office buildings the combination factor $\psi_{1,1}$ is 0,5 and $\psi_{2,1}$ is 0,3 (Table NB.2, A1.1 NEN-EN 1990-1-1 [51]). When designing for progressive collapse the factor $\psi_{2,1}$ can be used (Table NB.7-A1.3 NEN-EN 1990-1-1). The applied loads are equal to the loads as presented in Table 15, the design loads on the peripheral beams with respect to progressive collapse are presented in Table 17.

<table>
<thead>
<tr>
<th>Design loads</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_{d,\text{unfavorable;floor}} =$ 38,9 kN/m + 7,2 kN/m0,3 = 41,1 kN/m</td>
</tr>
<tr>
<td>$P_{d,\text{unfavorable;roof}} =$ 23,7 kN/m = 23,7 kN/m</td>
</tr>
</tbody>
</table>

The aforementioned loads are applied on the structure and the internal forces are calculated with MatrixFrame. An overview of some input parameters is presented in Table 18.
Reinforcement ULS

Based on the results obtained with the MatrixFrame analysis the reinforcement was calculated for the main structural elements. The reinforcement was calculated according to the regular design rules as included in Eurocode 2. For this design concrete class C30/37 is used and steel grade B500. The environmental class for the concrete elements is XC1. The beam dimensions are assumed as bxh=450x600 mm² and the column dimensions bxh=450x450 mm².

First the beam bending and shear reinforcement was determined. The bending reinforcement was determined with [E.4] for both the top as bottom reinforcement. With the static analysis discussed previously the decisive bending moments at the support and the field were \( M_{Ed;support}=275\text{kNm} \) and \( M_{Ed;field}=167\text{kNm} \) respectively.

\[
M_{Rd} = A_s \cdot f_{yd} \cdot z \tag{E.4}
\]

The reinforcement at the support was determined first. An estimation of the reinforcement \( A_s \) was made which resulted in an amount of reinforcement steel of 1885 mm². In order to determine the internal lever arm \( z \) with [E.7], the compressive zone of the concrete and the effective height \( d \) must be known. The compressive zone can be calculated with [E.5] and the effective height with [E.6].

\[
x_u = \frac{4 \cdot A_s \cdot f_{yd} - 4 \cdot 1885 \cdot 435}{3 \cdot f_{ed} \cdot b} = \frac{3 \cdot 20 \cdot 450}{121 \text{mm}}
\]

\[
d = h - c - \phi_{stirrup} = \frac{\phi}{2} = 600 - 30 - 8 - 10 = 552\text{mm}
\]

\[
z = d - 0,39x_u = 552 - 0,39 \cdot 121 = 505\text{mm}
\]

The last step was the determination of the bending capacity of the beam \( M_{Rd} \) by filling in [E.4]:

\[
M_{Rd} = 1885 \cdot 435 \cdot 505 \cdot 10^{-6} = 413 \text{kNm} > M_{Ed;support}
\]

It turned out that the top reinforcement is sufficient large to resist the bending moment at the support. Now also the field reinforcement must determined. This was carried out in accordance with the previously described procedure only now the reinforcement is set to 1257mm². This calculation is not elaborated here.

The next step in the design process of the beam reinforcement was the determination of the shear capacity. This was done first for the cross section without any shear reinforcement. The occurring shear force \( V_{Ed} \) is 282 KN. The general equation to determine the shear capacity of the beam without shear reinforcement is presented in [E.8].

\[
V_{Rd,c} = \left[ C_{Rd,c} \cdot k(100\rho_1 \cdot f_{ck})^{0.5} + k_1 \cdot \sigma_{cp} \right] \cdot b_w \cdot d \tag{E.8}
\]

For the beam used for this project the next factors are valid: \( k_1 = 0.15 \) (art. 6.2.2 NEN-EN 1990-1-1 NB), \( b_w = 400\text{mm} \), \( d = 547\text{mm} \), \( f_{ck} = 30 \text{N/mm}^2 \). A justification of the other factors is presented in [E.9].

\[
C_{Rd,c} = \frac{0.10}{\gamma_c} = \frac{1.8}{1.5} = 0.12 \tag{E.9}
\]
In order to determine $k$ the effective height is needed. For this project the decisive shear force is situated near a support and thus the effective height for this cross section is needed ($d=547\text{mm}$). Because the same bar diameter was used for both the top and bottom reinforcement the effective height $d$ will be equal. If the effective height $d$ is known the factor $k$ can be determined with [E.10]

$$k = 1 + \frac{200}{d} = 1 + \frac{200}{552} = 1,6 \leq 2,0$$ \hspace{1cm} [E.10]

As mentioned previously, the decisive shear force occurred at a support. In order to determine the tensile reinforcement density in the cross-section the top reinforcement is used in [E.11].

$$\rho_t = \frac{A_{td}}{b_w \cdot d} = \frac{1885}{450 \cdot 552} = 0,007$$ \hspace{1cm} [E.11]

The last factor is the compressive stress in the cross section due a normal force in the beam as presented in [E.12]. However, for this project there is no normal force present in the beam and therefore $\sigma_{cp}=0$.

$$\sigma_{cp} = \frac{N_{ed}}{A_c} = 0 \leq 0,2 \cdot f_{cd}$$ \hspace{1cm} [E.12]

Now all factors are known they can be entered in [E.8].

$$V_{rd,x} = 0,12 \cdot 1,6(100 \cdot 0,007 \cdot 30)^{\frac{1}{3}} + 0 \cdot 450 \cdot 552 \cdot 10^{-3} = 125\text{kN} < V_{Ed}$$

The shear capacity of the beam without shear reinforcement is not sufficient to resist the occurring shear force, shear reinforcement is therefore necessary. The capacity of the shear reinforcement can be obtained with [E.13].

$$V_{rd,x} = \frac{A_{sw}}{s} \cdot z \cdot f_{ywd} \cdot \cot\theta$$ \hspace{1cm} [E.13]

Also here an estimation was made of the required reinforcement $A_{sw}$. The amount of reinforcement is set to $101\text{ mm}^2$ in each cross section. This corresponds to stirrups $\varnothing 8\text{-}200$. Next the slope of the concrete compression diagonal was determined with [E.15]. In order to determine the compression diagonal a strength reduction factor for concrete cracked by shear must be derived with [E.14].

$$v_1 = 0,6 \cdot \left(1 - \frac{f_{ck}}{250}\right) = 0,6 \cdot \left(1 - \frac{30}{250}\right) = 0,53$$ \hspace{1cm} [E.14]

$$\sin^2\theta = \frac{A_{sw} \cdot f_{ywd}}{b_w \cdot s \cdot v_1 \cdot f_{cd}} = \frac{101 \cdot 435}{450 \cdot 200 \cdot 0,53 \cdot 20} = 0,05 \rightarrow \theta = 0,22$$ \hspace{1cm} [E.15]

$$\cot\theta = \frac{\cos\theta}{\sin\theta} = \frac{\cos 0,22}{\sin 0,22} = 4,55 \rightarrow 1 \leq \cot\theta \leq 2,5 \rightarrow \cot\theta = 2,5$$ \hspace{1cm} [E.16]

By filling in the previously determined values in [E.13] the shear capacity of the beam by shear reinforcement can be determined.

$$V_{rd,x} = \frac{101}{200} \cdot 0,9 \cdot 552 \cdot 435 \cdot 2,5 \cdot 10^{-3} = 272\text{kN} > V_{Ed}$$
To determine the column reinforcement a decisive column was considered. First the effective length of the column was determined which largely depends on the boundary condition at the ends of the column. To determine the effective length of the column in the "strong" direction first the rotational stiffness at the column ends must be derived with \[E.22\] and \[E.23\]. An explanation of the parameters is shown in Figure 114. In these equations the actual stiffness of the beams and columns must be used to take the influence of cracked concrete into account. The actual stiffness was determined first for the beam and thereafter for the column. For the calculation of the column it was assumed that the cross section contain 4020 bars (\(A_s=2\times628\text{mm}^2\)) and the beams the reinforcement as determined previously. Eurocode 2 contains a table that can be used to determine the actual stiffness of the beam. The actual stiffness for cross sections loaded by bending can be determined with \[E.18\]. Here the reinforcement ratio \(\rho\) over the average reinforcement must be specified with \[E.17\].

\[
\rho = \frac{A_{st} + A_{sc}}{A_c} = \frac{1571 + 1257}{450 \cdot 600} = 0.010 \quad [E.17]
\]

\[
E_{\text{effective}} = (2.85 + 620\rho)10^3 = (2.85 + 620 \cdot 0.010)10^3 = 9343 \text{N/mm}^2 \geq 4450 \quad [E.18]
\]

Now the effective young's modulus for the column can be determined with \[B.21\].

\[
\rho = \frac{A_{st} + A_{sc}}{A_c} = \frac{628 + 628}{450 \cdot 450} = 0.006 \quad [E.19]
\]

\[
\alpha_n = \frac{N_{ed}}{f_{ct} \cdot A_c + (A_{st} + A_{sc}) \cdot f_{yd}} = \frac{889 \cdot 10^3}{20 \cdot 450^2 + 1256 \cdot 435} = 0.19 \quad [E.20]
\]

\[
E_{\text{eff}} = 1960 + 43200\rho + (20000 - 196000\rho) \cdot \alpha_n = 1960 + 43200 \cdot 0.006 + (20000 - 196000 \cdot 0.006) \cdot 0.19 = 8129 \text{N/mm}^2 \quad [E.21]
\]

The length of the beams that support the column equals 7200mm. The column under consideration is supported by one beam on the right side at the top and one beam at the bottom as shown in Figure 113. The column length is 3300 mm.

Now the relative flexibility at the column ends can be determined with \[E.22\] and \[E.23\].

\[
k_1 = 2 \frac{E_{l1} f_{l1} + E_{l2} f_{l2}}{E_{l1} f_{l1} + 2 E_{l2} f_{l2}} = \frac{1/12 \cdot 450 \cdot 450^3}{3300 \cdot 2} \cdot \frac{8129}{3300} = 0.82 \quad [E.22]
\]

\[
k_2 = 2 \frac{E_{l1} f_{l1} + E_{l2} f_{l2}}{E_{l1} f_{l1} + 2 E_{l2} f_{l2}} = \frac{2 \cdot 1/12 \cdot 450^4}{7200} \cdot \frac{8129}{3300} = 0.82 \quad [E.23]
\]

The next step is the determination of the effective length of the column in the y-direction with \[E.24\].
Graduation thesis progressive collapse

\[ l_0 = 0.5 \cdot l \cdot \sqrt{\left(1 + \frac{k_1}{0.45 + k_1}\right) \cdot \left(1 + \frac{k_2}{0.45 + k_2}\right)} \]

\[ = 0.5 \cdot 3300 \cdot \sqrt{\left(1 + \frac{0.82}{0.45 + 0.82}\right) \cdot \left(1 + \frac{0.82}{0.45 + 0.82}\right)} \]

\[ = 2713 \text{mm} \] \[ \text{[E.24]} \]

The effective length of the column in the z-direction is equal to the storey height due to the horizontal support provided by the floors. The effective length in the z-direction is therefore 3300mm. Hereafter, the slenderness of the column in the y-direction was determined with [E.26].

\[ i_y = \frac{l}{\sqrt{A_c}} = \sqrt{\frac{1/12 \cdot 450^4}{450^2}} = 129.9 \]

\[ \lambda_y = \frac{l_0}{i_y} = \frac{2713}{129.9} = 20.9 \] \[ \text{[E.26]} \]

Now the first order bending moments were determined for the y-direction. From the ULS analysis it turned out the moment at the top equals 121 kNm and at the bottom 130 kNm. The axial force in the column equals 889 kN.

\[ e_t = \max\left(\frac{l_0}{400} \cdot \frac{h}{30}\right) \geq 20\text{mm} \rightarrow e_t = 20\text{mm} \] \[ \text{[E.27]} \]

\[ M_{01} = \min\{M_{\text{bottom}}, M_{\text{top}}\} + e_t \cdot N_{\text{Ed}} = 121 + 0.02 \cdot 889 = 139\text{ kNm} \] \[ \text{[E.28]} \]

\[ M_{02} = \max\{M_{\text{bottom}}, M_{\text{top}}\} + e_t \cdot N_{\text{Ed}} = 130 + 0.02 \cdot 889 = 149\text{ kNm} \] \[ \text{[E.29]} \]

In the z-direction there only acts a moment caused by eccentricity as shown in [E.30].

\[ M_{01} = M_{02} = e_t \cdot N_{\text{Ed}} = 0.02 \cdot 889 = 18\text{ kNm} \] \[ \text{[E.30]} \]

A check was performed to see if it is necessary to perform a second order analysis for the y-direction with [E.31].

\[ \lambda_{\text{lim}} = \frac{20 \cdot A \cdot B \cdot C}{\sqrt{n}} = \frac{20 \cdot 0.83 \cdot 1.13 \cdot 2.6}{\sqrt{0.22}} = 106 > \lambda \rightarrow \text{no second order moment} \] \[ \text{[E.31]} \]

Equations [E.32] to [E.35] contain the factors that are required for [E.31].

\[ A = \frac{1}{1 + 0.2 \phi_{\text{ef}}} = \frac{1}{1 + 0.2 \cdot 1} = 0.83 \] \[ \text{[E.32]} \]

\[ B = \sqrt{1 + 2 \phi} = \sqrt{1 + 2 \cdot \frac{A_s \cdot f_{yd}}{A_c \cdot f_{cd}}} = \sqrt{1 + 2 \cdot 0.13} = 1.26 \] \[ \text{[E.33]} \]

\[ C = 1.7 - r_m = 1.7 - \frac{m_{01}}{m_{02}} = 1.7 - \frac{139}{149} = 0.77 \] \[ \text{[E.34]} \]

\[ n = \frac{N_{\text{Ed}}}{A_c \cdot f_{cd}} = \frac{889 \cdot 10^3}{450^2 \cdot 20} = 0.22 \] \[ \text{[E.35]} \]
It turned out that also for the y-direction no second order moment needed to be considered. Further, it was shown that double bending will not occur. The final step was the determination of the amount of reinforcement. This is done by using a column chart as shown in Figure 115. The factors needed for this graph are shown in [E.36] till [E.38]

\[
\frac{d_i}{h} = \frac{48}{450} = 0.11 \quad \text{[E.36]}
\]

\[
\frac{N_{Ed}}{b \cdot h \cdot f_{cd}} = \frac{889 \cdot 10^3}{450^2 \cdot 20} = 0.22 \quad \text{[E.37]}
\]

\[
\frac{M_{Ed}}{b \cdot h \cdot f_{cd}} = \frac{149 \cdot 10^6}{450^3 \cdot 20} = 0.08 \quad \text{[E.38]}
\]

\[
\frac{A_s \cdot f_y}{h \cdot b \cdot f_{cd}} = 0.05 \quad (\text{from graph}) \rightarrow A_s = 466 \text{mm}^2 \quad \text{[E.39]}
\]

The applied amount of reinforcement equals 1257 mm\(^2\) and is sufficient to resist the occurring loads.

Figure 115; column chart for rectangular column with symmetrical reinforcement

Figure 116 and Figure 117 contain an overview of the reinforcement of the beam and column respectively according to the current ULS analysis. For simplicity, this reinforcement will be added on the entire structure.

Figure 116; beam reinforcement

Figure 117; column reinforcement
Appendix: transformation factors SDOF system

Here the derivation of the load and mass transformation factors for a beam on two supports with a uniform distributed load is shown. The transformation factors are based on the static deflection shape corresponding to a particular load distribution. For a beam on two supports with a uniform distributed load these deflection shapes are indicated in Figure 118 and Figure 119 for the elastic and plastic response respectively [47].

The assumed shapes are expressed in [F.1] and [F.2] for the elastic and plastic shape function respectively.

\[ \phi_{el}(x) = \frac{16}{5L^4}(L^3x - 2L^3x^2 + x^3) \]  
\[ \phi_{pl}(x) = \frac{2x}{L} \text{ for } x < \frac{L}{2} \]  

The transformation factor for the mass indicates the ratio of the equivalent mass \( m_e \) to the actual mass \( m_t (=L \cdot m) \) of the structure and can be written as [F.3]. The equivalent mass of the SDOF system can be written as [F.4] [47].

\[ \kappa_m = \frac{m_e}{m_t} = \frac{m_e}{L \cdot m} \]  
\[ m_e = \int_0^L m \cdot \phi(x)dx \]  

For the elastic stage [F.1] can be substituted in [F.4] and thereafter [F.4] can be substituted in [F.3]. Now [F.3] can be written as [F.5] which also includes the solution of \( \kappa_m \) which is 0.504 for linear elastic response.

\[ \kappa_{M,el} = \frac{1}{L} \int_0^L \left[ \frac{16}{5L^4}(L^3x - 2L^3x^2 + x^3) \right]^2 = \frac{3968}{7875} \approx 0.504 \]  

This procedure is also valid for the plastic response and can be expressed with [F.6].

\[ \kappa_{M,pl} = 2 \cdot \frac{1}{L} \int_0^{L/2} \left[ \frac{2x}{L} \right]^2 = \frac{2}{6} \approx 0.333 \]  

The transformation factor for the load can be obtained in a similar way. The transformation factor for the load can be written as [F.7].

\[ \kappa_L = \frac{F_e}{F_t} = \frac{F_e}{q \cdot L} \]  

Here \( F_e \) denotes the equivalent force on the SDOF system and \( F_t (=q \cdot L) \) the real applied load. The equivalent force on the SDOF system can be expressed with [F.8].
The shape function as shown with [F.1] is also used for the derivation of the load factor. Substituting [F.1] in [F.8] and thereafter in [F.7] results in [F.9]. Solving the integral results in a value of 0.64 for the linear elastic response.

\[
F_x = \int_0^L q(x) \cdot \phi(x) \, dx \tag{F.8}
\]

The shape function as shown with [F.1] is also used for the derivation of the load factor. Substituting [F.1] in [F.8] and thereafter in [F.7] results in [F.9]. Solving the integral results in a value of 0.64 for the linear elastic response.

\[
k_{L,el} = \frac{1}{L} \int_0^L \frac{16}{5E} (L^3x - 2Lx^3 + x^4) = \frac{16}{25} = 0.64 \tag{F.9}
\]

The same procedure is also valid for the plastic response as shown with [F.10]. The load factor for the plastic response is 0.5.

\[
k_{L,pl} = 2 \cdot \frac{1}{L} \int_0^{L/2} \frac{2x}{L} = \frac{2}{4} = 0.5 \tag{F.10}
\]

The last step is the derivation of the load-mass factors used in this thesis. For the elastic response it is shown in [F.11] and for the plastic response it is shown in [F.12].

\[
k_{LM,el} = \frac{k_{M,el}}{k_{L,el}} = \frac{0.504}{0.64} \approx 0.787 \tag{F.11}
\]

\[
k_{LM,pl} = \frac{k_{M,pl}}{k_{L,pl}} = \frac{0.333}{0.5} = 0.666 \tag{F.12}
\]

To approve the correctness of these values a table provided by [47] is added (see Table 19). The values for the transformation factors as found with the previously discussed procedure are in accordance with Table 19.

**Table 19; transformation factors SDOF system [47]**

<table>
<thead>
<tr>
<th>Loading diagram</th>
<th>Shown range</th>
<th>Load factor $k_L$</th>
<th>Mass factor $k_m$</th>
<th>Load-mass factor $k_{LM}$</th>
<th>Maximum resistance $R_m$</th>
<th>Spring constant $k$</th>
<th>Dynamic reaction $V$</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1" alt="Elastic" /></td>
<td>Elastic</td>
<td>0.64</td>
<td>0.60</td>
<td>0.78</td>
<td>$\frac{93R_e}{L}$</td>
<td>0.39R + 0.11F</td>
<td></td>
</tr>
<tr>
<td><img src="image2" alt="Plastic" /></td>
<td>Plastic</td>
<td>0.50</td>
<td>0.33</td>
<td>0.66</td>
<td>$\frac{93R_e}{L}$</td>
<td>0</td>
<td>0.38R + 0.12F</td>
</tr>
<tr>
<td><img src="image3" alt="Elastic" /></td>
<td>Elastic</td>
<td>1.0</td>
<td>0.49</td>
<td>1.0</td>
<td>$\frac{93R_e}{L}$</td>
<td>0.78R - 0.29F</td>
<td></td>
</tr>
<tr>
<td><img src="image4" alt="Plastic" /></td>
<td>Plastic</td>
<td>1.0</td>
<td>0.33</td>
<td>1.0</td>
<td>$\frac{93R_e}{L}$</td>
<td>0</td>
<td>0.75R - 0.25F</td>
</tr>
<tr>
<td><img src="image5" alt="Elastic" /></td>
<td>Elastic</td>
<td>0.97</td>
<td>0.70</td>
<td>0.85</td>
<td>$\frac{93R_e}{L}$</td>
<td>0.52R + 0.22F</td>
<td></td>
</tr>
<tr>
<td><img src="image6" alt="Plastic" /></td>
<td>Plastic</td>
<td>1.0</td>
<td>0.56</td>
<td>1.0</td>
<td>$\frac{93R_e}{L}$</td>
<td>0</td>
<td>0.53R - 0.33F</td>
</tr>
</tbody>
</table>
Appendix: linear elastic SDOF system

For a linear elastic undamped single degree of freedom mass spring system a simple mathematical solution can be obtained for a forced system. The equation of motion can be written as:

\[ m\ddot{y} + c\dot{y} + ky = F(t) \]  \[\text{[G.1]}\]

If it is assumed that the load is suddenly applied and has a constant magnitude and that damping is neglected then equation [G.1] can be written as [G.2].

\[ m\ddot{y} + ky = F \]  \[\text{[G.2]}\]

The general solution of equation [G.2] can be written as [G.3].

\[ y(t) = C_1 \cdot \sin(\omega t) + C_2 \cdot \cos(\omega t) + \frac{F}{k} \]  \[\text{[G.3]}\]

Here \( \omega = \sqrt{\frac{k}{m}} \). Assume the initial boundary condition \( y(t=0) = 0 \) and \( \dot{y}(t=0) = 0 \). With these boundary conditions the unknowns \( C_1 \) and \( C_2 \) can be determined.

\[ y(t=0) = C_1 \cdot \sin(0) + C_2 \cdot \cos(0) = 0 \rightarrow C_2 = -\frac{F}{k} \]  \[\text{[G.4]}\]

\[ \dot{y}(t=0) = C_1 \cdot \omega \cdot \cos(0) + C_2 \cdot \omega \cdot \sin(0) = 0 \rightarrow C_1 = 0 \]  \[\text{[G.5]}\]

Substituting the values found for \( C_1 \) and \( C_2 \) in equation [G.3] results in:

\[ y(t) = -\frac{F}{k} \cdot \cos(\omega t) + \frac{F}{k} = \frac{F}{k} (1 - \cos(\omega t)) \]  \[\text{[G.6]}\]

The maximum deflection of a linear elastic system subjected to an instantaneous applied uniform distributed load can be obtained by deriving the maximum of [G.6]. The result of this maximum is shown in [G.7].

\[ y_{\text{max}} = \frac{2F}{k} = 2y_{\text{static}} \]  \[\text{[G.7]}\]

In [G.7] it can be observed that the deflection of a dynamically applied load is twice that of the same load applied statically for a linear elastic system.
Appendix: central difference method

Here the solution procedure to solve the SDOF system as mentioned in section 5.2 will be treated. As discussed in 5.2 the central difference method is used to solve the equation of motion for the SDOF system. Small time steps $\Delta t$ are assumed. For each time step the displacement $y$ is solved by means of the two previously determined displacements.

The general form of the equation of motion of the single degree of freedom system can be written as:

$$m \ddot{y} + c \dot{y} + ky = F(t) \quad [H.1]$$

Here $m$ is the mass of the system, $c$ is the damping and $k$ is the stiffness. In this thesis the damping is left out of consideration. Further, $\ddot{y}$, $\dot{y}$ and $y$ are the acceleration, velocity and displacement respectively.

Consider Figure 120, the velocity at time $t - \Delta t/2$ and $t + \Delta t/2$ can be expressed with equation [H.2] and equation [H.3] respectively. Consequently the acceleration at time $t$ can be determined with equation [H.4].

$$y'(t + \Delta t/2) = \frac{y(t + \Delta t) - y(t)}{\Delta t} \quad [H.2]$$
$$y'(t - \Delta t/2) = \frac{y(t) - y(t - \Delta t)}{\Delta t} \quad [H.3]$$
$$y''(t) = \frac{y'(t + \Delta t/2) - y'(t - \Delta t/2)}{\Delta t} \quad [H.4]$$

The general difference equation for the velocity is described in equation [H.5]. When equation [H.2] and [H.3] are substituted into equation [H.4] this results in the general equation for the acceleration as shown with equation [H.6].

$$y'(t) = \frac{y(t + \Delta t) - y(t - \Delta t)}{2\Delta t} \quad [H.5]$$
$$y''(t) = \frac{y(t + \Delta t) - 2y(t) + y(t - \Delta t)}{\Delta t^2} \quad [H.6]$$
By substituting equation [H.5] and [H.6] in equation [H.1] and neglecting the damping this results in [H.7]. For each time step equation [H.7] is solved.

\[ m \cdot y(t + \Delta t) = \Delta t^2 \cdot F(t) + [2m - \Delta t^2k]y(t) - m \cdot y(t - \Delta t) \]  

[H.7]

To start the central difference procedure the deflection, acceleration and force at t=0 must be known in advance. For this procedure it is assumed that the structure is at rest before the load is applied and therefore the initial deflection and velocity at t=0 are zero. The initial acceleration at t=0 can be determined with [H.8].

\[ \ddot{y}(0) = \frac{F(0) - k \cdot y(0)}{m} \]  

[H.8]

For the nonlinear materials the equation of motion as shown in [H.1] is slightly modified. Since the stiffness of the spring will also behave inelastic when the material becomes inelastic the stiffness used in the equation of motion will also vary. Therefore for nonlinear material the equation of motion is written as [H.9]. Here the internal force in the spring \( k \cdot y \) is replaced by a function which describes the internal force as a function of the deformation \( f(y) \).

\[ m \ddot{y} + c \dot{y} + f(y) = F(t) \]  

[H.9]

For example, when the material reaches its full plastic capacity \( R_m \) the term \( f(y) \) becomes equal to this maximum capacity \( R_m \) as shown in [H.9]. This system can be solved with the same procedure as discussed previously.

\[ m \ddot{y} + c \dot{y} + R_m = F(t) \]  

[H.9]
Appendix: input files Abaqus

Statically loaded beam on two supports

*HEADING
Reinforced concrete beam on two supports

*PARAMETER
nodes = 11
element = 10

*NODE ,NSET=ENDS
1.0.
<nodes>,3600

*NGEN, NSET=ALL
1,<nodes>

*NSET, NSET=load
1,

*NSET, NSET=deflection
<nodes>

*ELEMENT,TYPE=B23,ELSET=BEAM
1,1,2

*ELGEN, ELSET=BEAM
1,<element>

*ELSET, ELSET=STRESSES
<element>

*ELSET, ELSET=MK
<element>,

*ELSET, ELSET=mk

*BEAM SECTION,SECTION=RECT,ELSET=BEAM,MATERIAL=A1
450,600
0.,0.,-1
61,

*************************************************

*MATERIAL,NAME=A1

*DENSITY
2400E-9,

*ELASTIC
32837.,19

*CONCRETE DAMAGED PLASTICITY
38,0.1,1.16,0.6666,0.0004

*CONCRETE COMPRESSION HARDENING
5.638 , 0
15.525 , 0.0000622595711458398
20.773 , 0.000124585879243309
25.312 , 0.000208043898527687
30.488 , 0.000360040285522889
35.144 , 0.00061500812397357
37.998 , 0.00118620376679221
35.954 , 0.00177212209387281
32.496 , 0.0022251723238899
28.117 , 0.0026619497825446

*MATERIAL,NAME=A1

*DENSITY
2400E-9,

*ELASTIC
32837.,19

*CONCRETE DAMAGED PLASTICITY
38,0.1,1.16,0.6666,0.0004

*CONCRETE COMPRESSION HARDENING
5.638 , 0
15.525 , 0.0000622595711458398
20.773 , 0.000124585879243309
25.312 , 0.000208043898527687
30.488 , 0.000360040285522889
35.144 , 0.00061500812397357
37.998 , 0.00118620376679221
35.954 , 0.00177212209387281
32.496 , 0.0022251723238899
28.117 , 0.0026619497825446

*MATERIAL,NAME=A1

*DENSITY
2400E-9,

*ELASTIC
32837.,19

*CONCRETE DAMAGED PLASTICITY
38,0.1,1.16,0.6666,0.0004

*CONCRETE COMPRESSION HARDENING
5.638 , 0
15.525 , 0.0000622595711458398
20.773 , 0.000124585879243309
25.312 , 0.000208043898527687
30.488 , 0.000360040285522889
35.144 , 0.00061500812397357
37.998 , 0.00118620376679221
35.954 , 0.00177212209387281
32.496 , 0.0022251723238899
28.117 , 0.0026619497825446

*MATERIAL,NAME=A1

*DENSITY
2400E-9,

*ELASTIC
32837.,19

*CONCRETE DAMAGED PLASTICITY
38,0.1,1.16,0.6666,0.0004

*CONCRETE COMPRESSION HARDENING
5.638 , 0
15.525 , 0.0000622595711458398
20.773 , 0.000124585879243309
25.312 , 0.000208043898527687
30.488 , 0.000360040285522889
35.144 , 0.00061500812397357
37.998 , 0.00118620376679221
35.954 , 0.00177212209387281
32.496 , 0.0022251723238899
28.117 , 0.0026619497825446

*MATERIAL,NAME=A1

*DENSITY
2400E-9,

*ELASTIC
32837.,19

*CONCRETE DAMAGED PLASTICITY
38,0.1,1.16,0.6666,0.0004

*CONCRETE COMPRESSION HARDENING
5.638 , 0
15.525 , 0.0000622595711458398
20.773 , 0.000124585879243309
25.312 , 0.000208043898527687
30.488 , 0.000360040285522889
35.144 , 0.00061500812397357
37.998 , 0.00118620376679221
35.954 , 0.00177212209387281
32.496 , 0.0022251723238899
28.117 , 0.0026619497825446

*MATERIAL,NAME=A1

*DENSITY
2400E-9,

*ELASTIC
32837.,19

*CONCRETE DAMAGED PLASTICITY
38,0.1,1.16,0.6666,0.0004

*CONCRETE COMPRESSION HARDENING
5.638 , 0
15.525 , 0.0000622595711458398
20.773 , 0.000124585879243309
25.312 , 0.000208043898527687
30.488 , 0.000360040285522889
35.144 , 0.00061500812397357
37.998 , 0.00118620376679221
35.954 , 0.00177212209387281
32.496 , 0.0022251723238899
28.117 , 0.0026619497825446

*MATERIAL,NAME=A1

*DENSITY
2400E-9,
*CONCRETE TENSION DAMAGE
0   ,  0
0.0890343721040365 ,  0.0000683657702674782
0.193484514165973 ,  0.000132912240558842
0.307042308380837 ,  0.00019443366301245
0.421812530884085 ,  0.000253559119064377
0.627001924144929 ,  0.000474637018701815
0.871072638420282 ,  0.000579734378188659
0.95376038567569 ,  0.000916473139381494
0.991499003821554 ,  0.0010673998344066
*************************************************
*REBAR,ELEMENT=BEAM,MATERIAL=A2,NAME=REBBOT
BEAM,1257,0.,-250
*REBAR,ELEMENT=BEAM,MATERIAL=A2,NAME=REBTOP
BEAM,1885,0.,250
*MATERIAL,NAME=A2
*DENSITY
7850E-9,
*ELASTIC
2.0E5 ,  0.3
*PLASTIC
500 ,  0.00000
560 ,  0.04720
*BOUNDARY
1,2
<nodes>,XSYM
*****************************************************************************
*STEP, INC=2000, NLGEOM=YES
*STATIC, RIKS
**0.011,1.0,1.0E-16,,,<nodes>,2,-75
*BOUNDARY
<nodes>,2,2,-75
*****************************************************************************
*CONTROLS, ANALYSIS=DISCONTINUOUS
*CONTROLS, PARAMETERS=LINE SEARCH
10,
*NODE PRINT
U,
CF,
RF,
*EL PRINT, ELSET=BEAM
E,
S,
SM1,
SK1,
*EL PRINT, REBAR=REBBOT
Appendix: input files Abaqus

E,S
*NODE FILE,NSET=ALL
U,
CF,
RF,
*OUTPUT,FIELD
*NODE OUTPUT,NSET=ALL
U,
CF,
RF,
*OUTPUT,HISTORY,FREQUENCY=1
*ELEMENT OUTPUT, ELSET=STRESSES
E, S
*ELEMENT OUTPUT, ELSET=MK
SM1, SK1
*NODE OUTPUT, NSET=load
RF2,
*NODE OUTPUT, NSET=deflection
U2,
*ELEMENT OUTPUT, ELSET=mk
SK1, SM1
*END STEP
**Statically loaded beam on three supports**

*HEADING
Beam on three supports

*PARAMETER
nodes = 41
element = 40
midnode = 21
length=14400

*NODE ,NSET=ENDS
1,0.
<nodes>,<length>

*NGEN, NSET=ALL
1,<nodes>

*NSET, NSET=load
1,

*NSET, NSET=deflection
<nodes>

*ELEMENT,TYPE=B23,ELSET=BEAM
1,1,2

*ELGEN, ELSET=BEAM
1,<element>

*ELSET, ELSET=STRESSES
7,8,9,10,18,19,20

*ELSET, ELSET=mk

*BEAM SECTION,SECTION=RECT,ELSET=BEAM,MATERIAL=A1
450,600
0.,0.,-1
61,

*************************************************
*MATERIAL,NAME=A1
*DENSITY
2400E-9,
*ELASTIC
32837,.19

*CONCRETE DAMAGED PLASTICITY
38,0.1,1.16,0.6666,0.0004

*CONCRETE COMPRESSION HARDENING
5.638 , 0
15.525 , 0.0000622595711458398
20.773 , 0.000124585879243309
25.312 , 0.000208043898527687
30.488 , 0.00036651155210399
35.144 , 0.00061500812397221
37.998 , 0.00118620376679221
35.954 , 0.00177212209387281
32.496 , 0.0022251723238899
28.117 , 0.0026619497825446

*CONCRETE TENSION STIFFENING,TYPE=STRAIN
2.9 , 0
2.296923202 , 0.0000683657702674782
1.819260757 , 0.000132912240558842
1.440931808 , 0.00019443366301245
1.141279207 , 0.000253559119064377
0.715960164 , 0.00036651155210399
0.449144217, 0.000474637018701815
0.281762223, 0.000579734378188659
0.121724299, 0.000764608085422304
0.060481522, 0.000916473139381494
0.030051638, 0.0010673998344066
*CONCRETE TENSION DAMAGE
0, 0
0.0890343721040365, 0.0000683657702674782
0.193484514165973, 0.000132912240558842
0.307042308380837, 0.00019443366301245
0.421812530884085, 0.000253559119064377
0.627001924144929, 0.000366511552103999
0.776290269658308, 0.000474637018701815
0.871072763842028, 0.000579734378188659
0.95376038567569, 0.00064608085422304
0.980298564762287, 0.000916473139381494
0.991499003821554, 0.0010673998344066
*****************************************************************
*REBAR,ELEMENT=BEAM,MATERIAL=A2,NAME=REBBOT
BEAM,1257,0.,-250
*REBAR,ELEMENT=BEAM,MATERIAL=A2,NAME=REBTOP
BEAM,1885,0.,250
*MATERIAL,NAME=A2
*DENSITY
7850E-9,
*ELASTIC
2.0E5, 0.3
*PLASTIC
500, 0.00000
560, 0.04720
*BOUNDARY
1,2
1,1
21,2
41,2
*****************************************************************
*STEP, INC=1500, NLGEOM=YES
*STATIC, RIKS
0.011,1.0,1.0E-16,,1.0
*DLOAD
*BEAM,PY,-50
*****************************************************************
*SOLVER CONTROLS
10E-12,
*CONTROLS, ANALYSIS=DISCONTINUOUS
*CONTROLS, PARAMETERS=LINE SEARCH
15,
*NODE PRINT
U,
CF,
RF,
*EL PRINT, ELSET=BEAM
E,
S,
SM1,
SK1,
*EL PRINT, REBAR=REBTOP
E,S
*EL PRINT, REBAR=REBBOT
E,S
*NODE FILE,NSET=ALL
U,
CF,
RF,
*OUTPUT,FIELD
*NODE OUTPUT,NSET=ALL
U,
CF,
RF,
*ELEMENT OUTPUT, ELSET=BEAM
SF,
*OUTPUT, HISTORY, FREQUENCY=1
*ELEMENT OUTPUT, ELSET=STRESSES
E, S
*NODE OUTPUT, NSET=load
RF2,
*NODE OUTPUT, NSET=deflection
U2,
*END STEP
### Complete frame

*HEADING
FRAME
*NODE, NSET=ENDS

<table>
<thead>
<tr>
<th>Node</th>
<th>X</th>
<th>Y</th>
<th>Z</th>
</tr>
</thead>
<tbody>
<tr>
<td>21</td>
<td>7200</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>61</td>
<td>21600</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>221</td>
<td>7200</td>
<td>3600</td>
<td></td>
</tr>
<tr>
<td>261</td>
<td>21600</td>
<td>3600</td>
<td></td>
</tr>
<tr>
<td>401</td>
<td>0</td>
<td>3600</td>
<td></td>
</tr>
<tr>
<td>471</td>
<td>25200</td>
<td>3600</td>
<td></td>
</tr>
<tr>
<td>601</td>
<td>0</td>
<td>3600</td>
<td></td>
</tr>
<tr>
<td>661</td>
<td>21600</td>
<td>3600</td>
<td></td>
</tr>
<tr>
<td>801</td>
<td>0</td>
<td>7200</td>
<td></td>
</tr>
<tr>
<td>861</td>
<td>21600</td>
<td>7200</td>
<td></td>
</tr>
<tr>
<td>1001</td>
<td>0</td>
<td>7200</td>
<td></td>
</tr>
<tr>
<td>1071</td>
<td>25200</td>
<td>7200</td>
<td></td>
</tr>
<tr>
<td>1201</td>
<td>0</td>
<td>7200</td>
<td></td>
</tr>
<tr>
<td>1261</td>
<td>21600</td>
<td>7200</td>
<td></td>
</tr>
<tr>
<td>1401</td>
<td>0</td>
<td>10800</td>
<td></td>
</tr>
<tr>
<td>1461</td>
<td>21600</td>
<td>10800</td>
<td></td>
</tr>
<tr>
<td>1601</td>
<td>0</td>
<td>10800</td>
<td></td>
</tr>
<tr>
<td>1671</td>
<td>25200</td>
<td>10800</td>
<td></td>
</tr>
<tr>
<td>1801</td>
<td>0</td>
<td>10800</td>
<td></td>
</tr>
<tr>
<td>1861</td>
<td>21600</td>
<td>10800</td>
<td></td>
</tr>
<tr>
<td>2001</td>
<td>0</td>
<td>14400</td>
<td></td>
</tr>
<tr>
<td>2061</td>
<td>21600</td>
<td>14400</td>
<td></td>
</tr>
<tr>
<td>2201</td>
<td>0</td>
<td>14400</td>
<td></td>
</tr>
<tr>
<td>2271</td>
<td>25200</td>
<td>14400</td>
<td></td>
</tr>
<tr>
<td>2401</td>
<td>0</td>
<td>14400</td>
<td></td>
</tr>
<tr>
<td>2461</td>
<td>21600</td>
<td>14400</td>
<td></td>
</tr>
<tr>
<td>2601</td>
<td>0</td>
<td>18000</td>
<td></td>
</tr>
<tr>
<td>2661</td>
<td>21600</td>
<td>18000</td>
<td></td>
</tr>
<tr>
<td>2801</td>
<td>0</td>
<td>18000</td>
<td></td>
</tr>
<tr>
<td>2871</td>
<td>25200</td>
<td>18000</td>
<td></td>
</tr>
</tbody>
</table>

**ADDITIONAL COLUMNS

<table>
<thead>
<tr>
<th>Node</th>
<th>X</th>
<th>Y</th>
<th>Z</th>
</tr>
</thead>
<tbody>
<tr>
<td>1001</td>
<td>3600</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>1021</td>
<td>3600</td>
<td>3600</td>
<td></td>
</tr>
<tr>
<td>1061</td>
<td>3600</td>
<td>3600</td>
<td></td>
</tr>
<tr>
<td>1081</td>
<td>3600</td>
<td>7200</td>
<td></td>
</tr>
<tr>
<td>1121</td>
<td>3600</td>
<td>7200</td>
<td></td>
</tr>
<tr>
<td>1141</td>
<td>3600</td>
<td>10800</td>
<td></td>
</tr>
<tr>
<td>1181</td>
<td>3600</td>
<td>10800</td>
<td></td>
</tr>
<tr>
<td>1201</td>
<td>3600</td>
<td>14400</td>
<td></td>
</tr>
<tr>
<td>1241</td>
<td>3600</td>
<td>14400</td>
<td></td>
</tr>
<tr>
<td>1261</td>
<td>3600</td>
<td>18000</td>
<td></td>
</tr>
</tbody>
</table>

*NGEN, NSET=FRAME

<table>
<thead>
<tr>
<th>Node</th>
<th>X</th>
<th>Y</th>
<th>Z</th>
</tr>
</thead>
<tbody>
<tr>
<td>21</td>
<td>61</td>
<td>20</td>
<td></td>
</tr>
<tr>
<td>221</td>
<td>261</td>
<td>20</td>
<td></td>
</tr>
<tr>
<td>401</td>
<td>471</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>601</td>
<td>661</td>
<td>20</td>
<td></td>
</tr>
<tr>
<td>801</td>
<td>861</td>
<td>20</td>
<td></td>
</tr>
<tr>
<td>1001</td>
<td>1071</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>1201</td>
<td>1261</td>
<td>20</td>
<td></td>
</tr>
<tr>
<td>1401</td>
<td>1461</td>
<td>20</td>
<td></td>
</tr>
</tbody>
</table>
1601, 1671, 1
1801, 1861, 20
2001, 2061, 20
2201, 2271, 1
2401, 2461, 20
2601, 2661, 20
2801, 2871, 1
*NSET, NSET=ADDCOL
10011, 10211, 10611, 10811, 11211, 11411, 11811,
12011, 12411, 12611,
*NSET, NSET=RFSUPP
601
*****************PN SETS************************
*NSET, NSET=PIN1_1, GENERATE
221, 261, 20
*NSET, NSET=PIN1A, GENERATE
421, 461, 20
*NSET, NSET=PIN1, GENERATE
401, 461, 20
*NSET, NSET=PIN1_2, GENERATE
601, 661, 20
*NSET, NSET=PIN2_1, GENERATE
801, 861, 20
*NSET, NSET=PIN2, GENERATE
1001, 1061, 20
*NSET, NSET=PIN2_2, GENERATE
1201, 1261, 20
*NSET, NSET=PIN3_1, GENERATE
1401, 1461, 20
*NSET, NSET=PIN3, GENERATE
1601, 1661, 20
*NSET, NSET=PIN3_2, GENERATE
1801, 1861, 20
*NSET, NSET=PIN4_1, GENERATE
2001, 2061, 20
*NSET, NSET=PIN4, GENERATE
2201, 2261, 20
*NSET, NSET=PIN4_2, GENERATE
2401, 2461, 20
*NSET, NSET=PIN5_1, GENERATE
2601, 2661, 20
*NSET, NSET=PIN5, GENERATE
2801, 2861, 20
***********************************************************************
*ELEMENT, TYPE=B23, ELSET=BEAM1
1, 401, 402
71, 1001, 1002
141, 1601, 1602
211, 2201, 2202
281, 2801, 2802
*ELGEN, ELSET=BEAMS
1, 70
71, 70
141, 70
211, 70
Appendix: input files Abaqus

281 , 70
*BEAM SECTION, SECTION=RECT, ELSET=BEAMS, MATERIAL=A1
450 , 600
0 , 0 , -1
61 ,
*ELEMENT, TYPE=B23, ELSET=COLUMN1
1001 , 21 , 221
1005 , 601 , 801
1009 , 1201 , 1401
1013 , 1801 , 2001
1017 , 2401 , 2601
*ELGEN, ELSET=COLUMNS
1001 , 3 , 20 , 1
1005 , 4 , 20 , 1
1009 , 4 , 20 , 1
1013 , 4 , 20 , 1
1017 , 4 , 20 , 1
*BEAM SECTION, SECTION=RECT, ELSET=COLUMNS, MATERIAL=A1
450 , 450
0 , 0 , -1
9 ,
*ELEMENT, TYPE=B23, ELSET=ADDCOLUMN
101001 , 10011 , 10211
101005 , 10611 , 10811
101009 , 11211 , 11411
101013 , 11811 , 12011
101017 , 12411 , 12611
*BEAM SECTION, SECTION=RECT, ELSET=ADDCOLUMN, MATERIAL=A1
450 , 450
0 , 0 , -1
9 ,
*MPC
PIN , PIN1_1 , PIN1A
PIN , PIN1 , PIN1_2
PIN , PIN2_1 , PIN2
PIN , PIN2 , PIN2_2
PIN , PIN3_1 , PIN3
PIN , PIN3 , PIN3_2
PIN , PIN4_1 , PIN4
PIN , PIN4 , PIN4_2
PIN , PIN5_1 , PIN5
**ADDITIONAL COLUMNS
PIN , 10211 , 411
PIN , 411 , 10611
PIN , 10811 , 1011
PIN , 1011 , 11211
PIN , 11411 , 1611
PIN , 1611 , 11811
PIN , 12011 , 2211
PIN , 2211 , 12411
PIN , 12611 , 2811
***************OUTPUT SETS*********************
*NSET, NSET=SUPPORTS
21, 41, 61
*ELSET, GENERATE, ELSET=BEAMMONITOR
281, 351, 1
*ELSET, ELSET=BEAMSTRESS
288, 289, 299, 300, 301
*ELSET, ELSET=MC
10, 80, 150, 220, 290
*************************************************
*MATERIAL, NAME=A1
*DENSITY
2400E-9,
*DAMPING, ALPHA=0.05
*ELASTIC
32837, 19
*CONCRETE DAMAGED PLASTICITY
38, 0.1, 1.16, 0.6666, 0.0004
*CONCRETE COMPRESSION HARDENING
5.638, 0
15.525, 0.000622595711458398
20.773, 0.000124585879243309
25.312, 0.000208043898527687
30.488, 0.000360040285522889
35.144, 0.00061500812397357
37.998, 0.00118620376679221
35.954, 0.00177212209387281
32.496, 0.0022251723238899
28.117, 0.0026619497825446
*CONCRETE TENSION STIFFENING, TYPE=STRAIN
2.9, 0
2.296923202, 0.000683657702674782
1.819260757, 0.000132912240558842
1.440931808, 0.00019443366301245
1.141279207, 0.000253559119064377
0.715960164, 0.000366511552103999
0.449144217, 0.000474637018701815
0.281762223, 0.00057973437818659
0.121724299, 0.000764608085422304
0.060481522, 0.000916473139381494
0.030051638, 0.0010673998344066
*CONCRETE TENSION DAMAGE
0, 0
0.0890343721040363, 0.000683657702674782
0.193484514165973, 0.000132912240558842
0.307042308380837, 0.00019443366301245
0.421825308840885, 0.000253559119064377
0.627001924144929, 0.000366511552103999
0.776290269658308, 0.000474637018701815
0.871072763842028, 0.00057973437818659
0.95376038567569, 0.000764608085422304
0.980298564762287, 0.000916473139381494
0.991499003821554, 0.0010673998344066
*REBAR, ELEMENT=BEAM, MATERIAL=A2, NAME=REBBOT
BEAMS, 1257, 0., -250
*REBAR, ELEMENT=BEAM, MATERIAL=A2, NAME=REBTOP
BEAMS, 1885, 0., 250
*REBAR, ELEMENT=BEAM, MATERIAL=A2, NAME=REBCOL1
COLUMNS, 628, 0., -200
*REBAR, ELEMENT=BEAM, MATERIAL=A2, NAME=REBCOL2
COLUMNS, 628, 0., 200
*MATERIAL, NAME=A2
*DENSITY
7850E-9,
*ELASTIC
2.0E5, 0.3
*PLASTIC
500, 0.00000
560, 0.04720
*************************************************
*BOUNDARY
10011, 1
10011, 2
**601, 2
21, 2
41, 2
61, 2
21, 1
41, 1
61, 1
471, XSYMM
1071, XSYMM
1671, XSYMM
2271, XSYMM
2871, XSYMM
*************************************************
*AMPLITUDE, NAME=RAMP1
0, 0, 300, 1.0
*AMPLITUDE, NAME=RAMP2
0, 1, 0.005, 0
*************************************************
*STEP, NLGEOM=YES, INC=5000
step 1 - static loading
*DYNAMIC
0.2, 350.0, 1.0E-10
*DLOAD, AMPLITUDE=RAMP1
BEAMS, PY, -41.1
*CLOAD, AMPLITUDE=RAMP1
601, 2, 321336
*SOLVER CONTROLS
10E-8,
*CONTROLS, ANALYSIS=DISCONTINUOUS
*CONTROLS, PARAMETERS=LINE SEARCH
15,
*************************************************
*NODE PRINT, NSET=SUPPORTS
RF,
*EL PRINT, ELSET=BEAMMONITOR
SM1,
*OUTPUT, FIELD, FREQUENCY=1
*NODE OUTPUT, NSET=FRAME
U,
CF,
RF,
**Graduation thesis progressive collapse**

*NODE OUTPUT, NSET=ADDCOL
U, CF, RF
*ELEMENT OUTPUT, ELSET=BEAMS
SF, SE,
*OUTPUT, HISTORY, FREQUENCY=1
*ELEMENT OUTPUT, ELSET=MC
SM1, SK1,
*NODE OUTPUT, NSET=RFSUPP
RF, u,
*END STEP

*************************************************

*STEP, NLGEOM=YES, INC=5000
step 2 – unloading column force
*DYNAMIC
0.01, 100.0, 1.0E-10
*CLOAD, AMPLITUDE=RAMP2
601,2,321336
*SOLVER CONTROLS
10E-8,
*CONTROLS, ANALYSIS=DISCONTINUOUS
*CONTROLS, PARAMETERS=LINE SEARCH
15,
*************************************************

*NODE PRINT, NSET=SUPPORTS
RF,
*EL PRINT, ELSET=BEAMMONITOR
SM1,
*OUTPUT, FIELD, FREQUENCY=1
*NODE OUTPUT, NSET=FRAME
U, CF, RF,
*NODE OUTPUT, NSET=ADDCOL
U, CF, RF
*ELEMENT OUTPUT, ELSET=BEAMS
SF, SE,
*OUTPUT, HISTORY, FREQUENCY=1
*ELEMENT OUTPUT, ELSET=MC
SM1, SK1,
*NODE OUTPUT, NSET=RFSUPP
RF, u,
*END STEP