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cold rolling process

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Abstract

In this report one advanced modern control method called Model Predictive Control (MPC) is presented for a cold rolling process in Corus. A linear model of the cold rolling process is derived and validated for one coil. A MPC algorithm is discussed, which is based on the infinite prediction horizon. It is proven that the closed-loop stability of the MPC is guaranteed for the nominal model.

The performance based on the simulation of the MPC is compared to the controller developed by Corus. It is proven from the simulation that the MPC provides a better control performance than its competitor. Besides, it has also been proven that the closed-loop stability of the MPC is guaranteed in the assumed worst case.

Key words: Model Predictive Control, MPC, Rolling Process, Cold Rolling Process, Infinite Prediction Horizon, Stability, Closed-loop Stability, Nominal Stability.
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\frac{\partial v}{\partial T} \quad \text{sensitivity of exit strip speed to interstand tension variation (ms}^2\text{(MN)}^{-1})

\textit{WECO} \quad \text{type material (-), developed by Corus}

\alpha_i \quad \text{roll force ratio between stand } i \text{ and stand } 4 (-)

\beta_i \quad \text{motor power ratio between stand } i \text{ and stand } 4 (-)

\mu \quad \text{friction coefficient (-)}

\tau_{tr,i} \quad \text{transport time lag from the thickness sensor located before stand } i \text{ to stand } i \text{ (s)}

\tau_{tr,i+o} \quad \text{transport time lag from stand } i \text{ to its thickness sensor (s)}

\tau_{RG} \quad \text{time constant of the hydraulic capsule (-)}

\tau_{m,j} \quad \text{time constant of the motor model of stand } i, \text{ second order term (-)}

\tau_{m,j} \quad \text{time constant of the motor model of stand } i, \text{ first order term (-)}

\omega \quad \text{motor angle speed (s}^{-1})

\text{CM11} \quad \text{Cold Mill 11}

\text{CSPIJ} \quad \text{Corus Strip Products IJmuiden}

\text{CRDT} \quad \text{Corus Research, Development & Technology}

\text{CV} \quad \text{Controlled Variable}

\text{GPC} \quad \text{Generalized Predictive Control}

\text{HYD} \quad \text{Hydraulic Capsule}

\text{IIO} \quad \text{Increment-Input-Output}

\text{IO} \quad \text{Input-Output}

\text{MOT} \quad \text{Motor}

\text{MPC} \quad \text{Model Predictive Control}

\text{MV} \quad \text{Manipulated Variable}

\text{RPM} \quad \text{Repetitive Preset Method}

\text{RPMA} \quad \text{Repetitive Preset Method Alternative}

\text{SIWA} \quad \text{Simulatie Walsen, a simulation program developed by Corus for rolling process}
Notation & Abbreviations

$\Delta$  symbol of variation, must be used with other symbols together (-)

$b$  strip width (m)

$D$  diameter of work roll (m)

$F$  roll force (MN)

$\frac{\partial F}{\partial H}$  sensitivity of roll force to entry thickness variation (MN/mm)

$\frac{\partial F}{\partial h}$  sensitivity of roll force to exit thickness variation (MN/mm)

$\frac{\partial F}{\partial T}$  sensitivity of roll force to interstand tension variation (-)

$H$  entry thickness (mm)

$h$  exit thickness (mm)

$i$  subscript, stand number (-)

$K$  controller gain (-)

$k$  strip hardness (-)

$M$  mill modulus (MN/mm)

$MT$  motor torque (MNm)

$\frac{\partial MT}{\partial H}$  sensitivity of motor torque to entry thickness variation (MNm/mm)

$\frac{\partial MT}{\partial h}$  sensitivity of motor torque to exit thickness variation (MNm/mm)

$\frac{\partial MT}{\partial T}$  sensitivity of motor torque to interstand tension variation (m)

$P$  motor power (MW)

$R$  radius of work roll (m)

$RG$  roll gap (mm)

$S$  forward slip (-)

$s$  Laplace operator (-)

$T$  interstand tension (MN)

$V$  work roll speed (m/s)

$\nu$  exit strip speed (m/s)
Chapter 1 Introduction

Corus is an international metal company, providing steel and aluminium products and services to customers worldwide. With an annual turnover of £9 billion, the company is comprised of four Divisions, Strip Products, Long Products, Distribution & Building Systems and Aluminium. One of the major sites in the Division Strip Products is Corus Strip Products IJmuiden (CSPIJ), in the Netherlands. CSPIJ manufactures hot rolled, cold rolled and metal-coated steels for many industries. They include the automotive and transport industries, building and construction, consumer appliances and electronics, and general engineering. The annual output at CSPIJ is about 6.5 million tons a year at the writing of this report.

One of the plants at CSPIJ is the Cold Mill 11 (CM11). In CM11, the input is hot rolled product with a strip thickness between 1.8 and 2.4 mm, and the output is cold rolled product with a strip thickness between 0.22 and 0.70 mm. The process in CM11 is in general called a cold rolling process. It is a process by which the sheet metal or strip stock is introduced between rollers and then compressed and squeezed, see figure 1.1 below. Here two work rolls are used to reduce the strip thickness; the two back-up rolls reduce deflection of the work rolls. Since the variation in the strip width during a cold rolling process is usually very small and can be neglected here, we may consider this process as a two dimensional system.

In figure 1.1, the strip thickness is reduced in one step. This reduction consumes motor power and introduces roll force on the work rolls. However, in the industry, both motor power and roll force are limited. Therefore, too many reductions in one step is physical impossible. To solve this problem, in CM11, four subsequent steps are applied. In every step a certain reduction is
carried out, so that in the last step the thickness can meet the requirements of the clients. Reduction in several steps in this way is called a continuous rolling process. The steps are called stands in industry, so CM11 is a four stand cold mill, whose structure is presented in figure 2.1 of chapter 2.

The hardness and other material properties of the finished cold rolled product are not only dependent on the thickness reductions, but also depend on the amount of strain. Because we are here talking about a continuous rolling process, the strain gets a name as interstand tension, i.e. the tension between the stands. In CM11, both the thickness reduction and interstand tension are online controlled by basic controllers; see the blue rectangle blocks in figure 2.1. At this moment, these basic controllers provide sufficient performance. However, the question is which reference values these basic controllers should have, namely the setup values of the thickness and interstand tension, etc. In figure 2.1, these setup values are indicated by red colour.

To determine the setup values, Corus has developed an offline model; they give it a name as preset model. The setup values of the basic controllers obtained from the preset model are currently implemented in the factory. Its calculation is based on certain criteria. One concrete mathematic form of the criteria is given by Eq.4.1. In this chapter we simply name these criteria as the roll force and motor power ratios between stands. Consider the roll force at stand 4 as the nominal one, the offline obtained setup values must satisfy that the roll force of other stands are in certain ratio to the roll force of stand 4, similarly for the motor power. The values of these ratios are obtained from the factory based on experience. In this assignment, we consider that these ratio values are given correctly.

If the preset model is good enough, i.e. setup values are correctly calculated, a constant desired ratio is guaranteed during online process. However, if the preset model makes a wrong calculation or when the incoming strip has variations in thickness and hardness, the distribution pattern of the roll forces and motor powers sometimes become inappropriate. Since the distribution pattern of the roll force and motor powers are not online controlled, any errors in the ratios remain during the whole online process. When a roll force varies to a great extent, strip shape defects may occur. When a motor power exceeds its maximum value, usually all roll speeds are reduced manually and the productivity decreases [19]. The factory employ an
operator to manual control the setup values of the basic controllers. How he is doing the manual control purely depend on his experience, here we consider what one operator does as an example of manual control, see figure 1.2 below.

![Figure 1.2 Block structure of rolling process](image)

To improve the controlling performance, Corus Research introduces a method called Repetitive Preset Method (RPM); more detail about RPM is presented in section 4.2. In this method, the preset model is repeatedly called during online processing, i.e. the setup values will be resent at certain moments during processing. Obviously, the biggest shortage in this method is lack of consideration of the dynamics of the system. Therefore, it is foreseeable that this solution is not optimal. To overcome this, i.e. to take the system's dynamic into account, since years ago, some people in the research department have already thought about the implementation of an advanced control method like Model Predictive Control (MPC). This provides the opportunity of setting up this graduate assignment.

We define the main question of this project: **What kind of performance can a MPC controller make against RPM?**

In chapter 2, we start with the modeling of CM11 including all its basic controllers. In chapter 3, based on the process model introduced in chapter 2, a MPC controller is designed. Both the advantages and the disadvantages of the designed MPC will be discussed in chapter 3. In chapter 4, we briefly illustrate the core structure of RPM. Besides, one alternative is introduced to approximate the RPM, because the researchers, who designed the RPM, do not provide the simulation result but planned to directly implement it in the near future. Both the simulation result of the MPC and the RPM alternative can be found in chapter 5. In the last chapter of this report, a conclusion of this graduate assignment is drawn. Besides, there will be some acknowledge words for the people, who give so much essential assistance to this project.
It is assumed that the reader of this report has basic knowledge of Control Engineering. For the one, who is not familiar with control theory, please refer to the literature [1].
Chapter 2 Modelling Of Cold Mill 11 (CM11)

In this chapter, the whole cold rolling process of CM11 in Corus is presented. It is described by a dynamic process model. In this process model, the Controlled Variables (CV) are written as a linear function of the Manipulated Variables (MV). Be aware that this linearization is only valid around the operation point, which is determined offline.

Figure 2.1 Current Mill line layout of CM11 in Corus

In figure 2.1, the current mill line layout of CM11 is sketched. The values of symbols in red colour are offline obtained, namely the set up values. Their physical meanings will be further discussed in section 2.1 and 2.2 in this chapter. The rectangle blocks in blue colour are currently implemented basic controllers. In Corus, these basic controllers are indicated as level 1 controllers, see figure 2.2 below. Here all the MV's and CV's of level 1 controller are denoted with a subscript Level 1, similarly for the later introduced MPC. The physical meanings of the
MV's and CV's of level 1 controller are explained later in this chapter, for MPC in chapter 3.

In section 2.1, we concentrate ourselves on the modelling of the Cold Rolling Process part. In section 2.2, the structure of the Level 1 Controller is presented. In section 2.3, there is a short description how the model parameters used in section 2.1 and 2.2 are obtained. The topic of section 2.4 is the validation of the Process Model; some modifications on the model presented in section 2.1 will also be discussed here. In section 2.5, the disturbances on the Process Model will be introduced. In section 2.6, we deal with the model uncertainty problem. Finally, in the last section of this chapter, a complete layout of the Process Model is presented.

**2.1 Process model**

Five paragraphs are included in this section, they are: the modelling of exit thickness deviations based on the deformation model in 2.1.1; based on the mass balance theory in 2.1.2; the modelling of the interstand tension variation in 2.1.3; the modelling of the rolling force and motor power in 2.1.4 and 2.1.5.

**2.1.1 Model of exit thickness deviation based on the deformation model**

In general [2], roll force is a function of entry thickness, exit thickness, entry tension, exit tension, friction between roll and strip, hardness and width of the strip, see Eq.(2.1) below.

\[ F_i = f(H_i, h_i, T_{i-1}, T_{i+1}, \mu_i, k_i, b_i) \]  

(2.1)

Where subscript \( i \) indicates the stand number, namely \( i \in \{1, 2, 3, 4\} \) for CM11;

- \( F \) - roll force (MN)
- \( H \) - entry thickness (mm)
- \( h \) - exit thickness (mm)
- \( T \) - interstand tension (MN)
\[ \mu \quad \text{friction coefficient (-)} \]
\[ k \quad \text{strip hardness (-)} \]
\[ b \quad \text{strip width (m)} \]

According to literature [3], the variation of the roll force at stand \( i \) can be described by the following linear equation:

\[
\Delta F_i = \frac{\partial F_i}{\partial H_i} \Delta H_i + \frac{\partial F_i}{\partial h_i} \Delta h_i + \frac{\partial F_i}{\partial T_{i-1,i}} \Delta T_{i-1,i} + \frac{\partial F_i}{\partial T_{i,i+1}} \Delta T_{i,i+1} \quad (2.2)
\]

Where

\[
\frac{\partial F_i}{\partial H_i} \quad \text{sensitivity of roll force of stand } i \text{ to entry thickness variation at stand } i \text{ (MN/mm)}
\]
\[
\frac{\partial F_i}{\partial h_i} \quad \text{sensitivity of roll force of stand } i \text{ to exit thickness variation at stand } i \text{ (MN/mm)}
\]
\[
\frac{\partial F_i}{\partial T_{i-1,i}} \quad \text{sensitivity of roll force of stand } i \text{ to tension variation between stand } i-1 \text{ and } i
\]
\[
\frac{\partial F_i}{\partial T_{i,i+1}} \quad \text{sensitivity of roll force of stand } i \text{ to tension variation between stand } i \text{ and } i+1
\]

\( \Delta F_i \) is defined as the difference between the actual value and the preset value. For example,

\[ \Delta F_i = F_{i,act} - F_{i,pre} \]

Obviously, in Eq.2.2, the variations on \( \mu, k, \) and \( b \) are neglected.

In general, exit thickness is the sum of roll gap and roll deformation, see sketch below for their physical explanations.

Therefore, the following equation is valid:
\[ h_i = RG_i + \frac{F_i}{M_i} \]  

(2.3)

Where

\( RG_i \) roll gap of stand \( i \) (mm)

\( M_i \) mill modulus of stand \( i \) (MN/mm)

In literature [3], a constant mill modulus is assumed. Therefore, linearization of Eq.2.3 leads to:

\[ \Delta h_i = \Delta RG_i + \frac{\Delta F_i}{M_i} \]  

(2.4)

Substitute Eq.2.2 into Eq.2.4, for stand 1, namely for \( i = 1 \), it results:

\[ \Delta h_1 = \Delta RG_1 + \frac{\frac{\partial F_1}{\partial H_1} \Delta H_1 + \frac{\partial F_1}{\partial h_1} \Delta h_1 + \frac{\partial F_1}{\partial T_{l1}} \Delta T_{l1} + \frac{\partial F_1}{\partial T_{l2}} \Delta T_{l2}}{M_1} \]  

(2.5)

\[ \Rightarrow \]

\[ \Delta h_1 = \Delta RG_1 + \frac{\frac{\partial F_1}{\partial H_1} \Delta H_1 + \frac{\partial F_1}{\partial T_{l1}} \Delta T_{l1} + \frac{\partial F_1}{\partial T_{l2}} \Delta T_{l2}}{M_1} \]  

(2.6)

In general, \( \frac{\partial F_1}{\partial T_{l,j-1}} \approx 5 \times \frac{\partial F_1}{\partial T_{l,j+1}} \), however, since the tension variation \( \Delta T_{l1} \) is usually very small, approx. one tenth of \( \Delta T_{l2} \), which results the third term in Eq.2.5 is the half of the last term. Therefore, we simplified Eq.2.5 to the equation below:

\[ \Delta h_1 = \frac{M_1}{M_1 - \frac{\partial F_1}{\partial h_1}} \Delta RG_1 + \frac{\frac{\partial F_1}{\partial H_1} \Delta H_1 + \frac{\partial F_1}{\partial T_{l2}} \Delta T_{l2}}{M_1 - \frac{\partial F_1}{\partial h_1}} \]  

(2.7)

In literature [3], the adjustment of the roll gap is approached by a first order transfer function:

\[ \Delta RG_i(s) = \frac{1}{\tau_{RG_i,s} + 1} \Delta RG_{i,ref}(s) \]  

(2.8)
Where subscript \( ref \) indicates a reference value, \( s \) denotes a Laplace transform.

\[ \tau_{RG_i} \] time constant of the hydraulic capsule of stand \( i \) (-)

In figure 2.1, it can be notified that there is a certain distance between the stand and its thickness sensor, which leads to the following two equations:

\[
\Delta H_i(s) = e^{-\tau_{m,t} \cdot \Delta H_{i,m}(s)} \quad (2.8)
\]

\[
\Delta h_{i,m}(s) = e^{-\tau_{m,t} \cdot \Delta h_i(s)} \quad (2.9)
\]

Where subscript \( m \) indicates a measurement value

\[ \tau_{in,j} \] transport time lag from the thickness sensor located before stand \( i \) to stand \( i \) (s)

\[ \tau_{t,o} \] transport time lag from stand \( i \) to its thickness sensor (s)

Remark:

The transport time lag in the stand is here neglected. This is allowed since the distance in the stand is in the order of millimetre, but between the stands is in the order of meter, see sketch on the right side.

All the pure time delay elements are later replaced by second order Padé approximations.

End Remark

Assume that all the interstand tensions between the stands are uniformly distributed, which means the following equation is valid.

Assumption:

\[
\Delta T_{i-1,i,m}(s) = \Delta T_{i-1,i}(s)
\]

\[
\Delta T_{i,i+1,m}(s) = \Delta T_{i,i+1}(s)
\] \{A.1\}

Substitute Eq.2.7 – 2.9 and A.1 into Eq.2.6, For \( i = 1 \), it leads:
Remark:
The structure in figure 2.3 is also called as Eq.2.10. In this assignment, it is chosen to present the derivation result in a visual way instead of pure mathematic equations. Here the symbols in the left side are the source; the symbols in the right side are the sink; the rectangle block represents a multiplication.

**End Remark**

2.1.2 Model of exit thickness deviation based on the mass balance theory

In literature [3], the following mass flow balance equation is presented:

\[ v_{i-1}h_{i-1} = v_i h_i \]  

(2.11)

Where

- \( v_{i-1} \) exit strip speed of stand \( i-1 \) (m/s)
- \( v_i \) exit strip speed of stand \( i \) (m/s)

Remark:

In Eq.2.11, the strip width and strip density have already been considered as constant values.

**End Remark**

However, Eq.2.11 presented in literature [3] is not true when dynamics are taken into account. A more explicit form is given by:
In Eq. 2.12 it is assumed that the exit strip speed of stand \( i-1 \) is equal to the enter strip speed of stand \( i \).

Linearization of Eq. 2.12 leads to:

\[
v_{i-1} \cdot \Delta H_i + \Delta v_{i-1} \cdot H_i = v_i \cdot \Delta h_i + \Delta v_i \cdot h_i
\]  

(2.13)

Divide Eq. 2.13 by Eq. 2.12, it gives:

\[
\frac{\Delta H_i}{H_i} + \frac{\Delta v_{i-1}}{v_{i-1}} = \frac{\Delta h_i}{h_i} + \frac{\Delta v_i}{v_i}
\]  

(2.14)

By introducing forward slip, the following equation is obtained:

\[
v_i = V_i (1 + S_i)
\]  

(2.15)

Where

- \( S_i \): forward slip of stand \( i \) (\(-\))
- \( V_i \): work roll speed of stand \( i \) (m/s)

Therefore, linearization of Eq. 2.15 gives:

\[
\Delta v_i = \Delta V_i (1 + S_i) + V_i \cdot \Delta S_i \approx \Delta V_i (1 + S_i)
\]  

(2.16)

In literature [3], the forward slip variation (\( \Delta S_i \)) in Eq. 2.16 is neglected.

Substitute Eq. 2.15 & Eq. 2.16 into Eq. 2.14, it gives:

\[
\frac{\Delta H_i}{H_i} + \frac{\Delta V_{i-1}}{V_{i-1} (1 + S_{i-1})} = \frac{\Delta h_i}{h_i} + \frac{\Delta V_i}{V_i (1 + S_i)}
\]  

\[
\Delta h_i = h_i \left( \frac{\Delta H_i}{H_i} + \frac{\Delta V_{i-1}}{V_{i-1}} \right)
\]  

(2.17)

In literature [3], the adjustment of the motor speed is approached by a second order transfer function:

\[
\Delta V_i (s) = \frac{1}{\tau_{v,2}s^2 + \tau_{v,1}s + 1} \Delta V_{i, ref} (s)
\]  

(2.18)

Where

- \( \tau_{v,2} \): time constant of the motor model of stand \( i \), second order term (\(-\))
Substituting of Eq.2.8, 2.9 and 2.18 into Eq.2.17 results in:

\[ \Delta H_{i,m}(s) e^{-\tau_{n,i} s^2} \Delta H_i(s) \]

\[ \frac{1}{H_i} \]

\[ \Delta V_{i-1}(s) \]

\[ \frac{1}{V_{i-1}} \]

\[ \Delta V_i(s) \]

\[ \frac{1}{V_i} \]

\[ + \]

\[ h_i e^{-s_i \alpha} \Delta h_i(s) \]

\[ \Delta h_{i,m}(s) \]

Figure 2.4 Model of exit thickness deviation of stand \( i \)

The models of exit thickness deviations of stand 2, 3 and 4 are all obtained from Eq.2.19 for \( i = 2, 3, 4 \).

2.1.3 Model of interstand tension variation

According to literature [3] the linearized model of process \( \Delta T_{i2} \) is given by the following mathematical equations:

Reconsider Eq.2.2 and 2.4 again, substituting of Eq.2.4 into Eq.2.2 results in:

\[ \Delta h_i = \Delta R G_i + \frac{\partial F_i}{\partial H_i} \Delta H_i + \frac{\partial F_i}{\partial h_i} \Delta h_i + \frac{\partial F_i}{\partial T_{i-1,j}} \Delta T_{i-1,j} + \frac{\partial F_i}{\partial T_{i,j+1}} \Delta T_{i,j+1} \]

\[ \Leftrightarrow \]

\[ \frac{\partial F_i}{\partial T_{i-1,j}} \Delta T_{i-1,j} = -M_i \Delta R G_i \]

\[ + \frac{\partial F_i}{\partial H_i} \Delta H_i + \left( M_i - \frac{\partial F_i}{\partial h_i} \right) \Delta h_i - \frac{\partial F_i}{\partial T_{i,j+1}} \Delta T_{i,j+1} \]

\[ \Leftrightarrow \]

\[ \Delta T_{i-1,j} = -M_i \Delta R G_i + \frac{\partial F_i}{\partial H_i} \Delta H_i + \frac{\partial F_i}{\partial h_i} \Delta h_i + \frac{\partial F_i}{\partial T_{i-1,j}} \Delta T_{i-1,j} - \frac{\partial F_i}{\partial T_{i,j+1}} \Delta T_{i,j+1} \]

(2.20)

In general, \( \frac{\partial F_i}{\partial T_{i-1,j}} \approx 5 \times \frac{\partial F_i}{\partial T_{i,j+1}} \), which means the last term in Eq.2.20 may be roughly neglected. It results:
Substituting of Eq. 2.7, 2.8 and A.1 into Eq. 2.21 results in:

\[
\Delta T_{i-1,i} = -\frac{M_i}{\partial F_i} \Delta RG_i + \frac{\partial F_i}{\partial h_i} \Delta h_i + \frac{M_i}{\partial F_i} \frac{\partial F_i}{\partial T_{i-1,i}} \Delta h_i
\]

(2.21)

Substituting of Eq. 2.7, 2.8 and A.1 into Eq. 2.21 results in:

\[
\Delta H_{i,m}(s) = e^{-\tau_{RQ}s} \Delta H_i(s)
\]

\[
\Delta h_{i,m}(s) \approx \Delta h_i(s)
\]

\[
\Delta RG_{i,m}(s) = \frac{1}{\tau_{RQ} s + 1} \Delta RG_i(s)
\]

\[
\Delta T_{i-1,i,m}(s) = -\frac{M_i}{\partial F_i} \Delta RG_i + \frac{\partial F_i}{\partial h_i} \Delta h_i + \frac{M_i}{\partial F_i} \frac{\partial F_i}{\partial T_{i-1,i}} \Delta h_i
\]

(2.22)

Figure 2.5 Model of interstand tension deviation between stand \( i-1 \) and \( i \)

The models of interstand tension deviations, namely \( \Delta T_{12}, \Delta T_{23} \) and \( \Delta T_{34} \) are all obtained from Eq. 2.22 for \( i = 2, 3, 4 \).

**Remark:**

In Eq. 2.22, exit thickness deviation of stand \( i \) is approximated by its measurement value.

**End Remark**

2.1.4 Model of roll force

The modelling of roll force is directly derived from Eq. 2.4 under assumption that exit thickness deviation is removed by the level 1 controller:

\[
\Delta h_i = \Delta RG_i + \frac{\Delta F_i}{M_i} \Leftrightarrow
\]

\[
0 = \Delta RG_i + \frac{\Delta F_i}{M_i} \Leftrightarrow
\]

\[
\Delta F_i \approx -M_i \cdot \Delta RG_i
\]

(2.23)
2.1.5 Model of motor power

In general, motor power is a function of motor torque and motor angle speed:

\[ P_i = MT_i \cdot \omega_i \]  

(2.24)

\( P_i \)    motor power of stand \( i \) (MW)

\( MT_i \)   motor torque of stand \( i \) (MNm)

\( \omega_i \) motor angle speed of stand \( i \) (s\(^{-1}\))

The motor angle is related to speed according to:

\[ \omega_i = \frac{V_i}{R_i} \]  

(2.25)

Where

\( R_i \)    radius of work roll of stand \( i \) (m)

Substituting of Eq.2.25 into Eq.2.24 results in:

\[ P_i = MT_i \cdot \frac{V_i}{R_i} \]  

(2.26)

Assuming a constant radius of work roll, linearization of Eq.2.26 leads to:

\[ \Delta P_i = \frac{1}{R_i} \left( \Delta MT_i \cdot V_i + MT_i \cdot \Delta V_i \right) \]  

(2.27)

The second term in Eq.2.27, namely \( \Delta V_i \) is the MV of level 1 controller. The model of \( \Delta V_i \) will be presented in section 2.2. Here we only show the model of the first term in Eq.2.27, namely \( \Delta MT_i \).

According to the Corus deformation model [20], the following equation is directly obtained:

\[ \Delta MT_i = \frac{\partial MT_i}{\partial H_i} \Delta H_i + \frac{\partial MT_i}{\partial h_i} \Delta h_i + \frac{\partial MT_i}{\partial T_{i-1,j}} \Delta T_{i-1,j} + \frac{\partial MT_i}{\partial T_{i,j+1}} \Delta T_{i,j+1} \]  

(2.28)

Where

\( \frac{\partial MT_i}{\partial H_i} \) sensitivity of motor torque of stand \( i \) to entry thickness variation (MNm/mm)
\[ \frac{\partial MT_i}{\partial h_i} \] sensitivity of motor torque of stand \( i \) to exit thickness variation (MNm/mm)

\[ \frac{\partial MT_i}{\partial T_{i-1,i}} \] sensitivity of motor torque of stand \( i \) to tension variation between stand \( i-1 \) and \( i \) (m)

\[ \frac{\partial MT_i}{\partial T_{i,i+1}} \] sensitivity of motor torque of stand \( i \) to tension variation between stand \( i \) and \( i+1 \) (m)

For \( i=1 \), \( \Delta T_{01} \) is neglected in Eq.2.28, see Eq.2.6.

For \( i=4 \), \( \Delta T_{45} \) is neglected in Eq.2.28, because the tension variation \( \Delta T_{45} \) is also very small, approx. one tenth of \( \Delta T_{34} \).

**2.2 Controller model**

This section contains seven paragraphs. It presents the models of all level 1 controllers used in CM11, see figure 2.1 for the position of these level 1 controllers in cold rolling process.

**2.2.1 Model of FF1**

The CV of FF1 is the enter thickness of stand 1, namely FF1 is used to remove the enter thickness variation at stand 1.

The MV of FF1 is the roll gap at stand 1. Because there are other level 1 controllers, which also manipulate the roll gap position at stand 1, we denote the MV sent by FF1 as: \( MV_{FF1} \), similar for other level 1 controllers.

The design of FF1 is based on Eq.2.6 with assumptions that exit thickness variation at stand 1 and interstand tension variation between stand 1 and 2 are removed by other level 1 controllers, namely \( \Delta h_1 \approx 0 \) and \( \Delta T_{12} \approx 0 \). With these assumptions, Eq.2.6 can be rewritten as:
\[
\Delta h_i = \frac{M_i}{M_i - \frac{\partial F_i}{\partial h_i}} \Delta RG_i + \frac{\partial F_i}{\partial H_i} \Delta H_i + \frac{\partial F_i}{\partial T_{12}} \Delta T_{12} \quad \Leftrightarrow \\
0 = \frac{M_i}{M_i - \frac{\partial F_i}{\partial h_i}} \Delta RG_i + \frac{\partial F_i}{\partial H_i} \Delta H_i + \frac{\partial F_i}{\partial T_{12}} \Delta T_{12} \quad \Leftrightarrow \\
-M_i \Delta RG_i = \frac{\partial F_i}{\partial H_i} \Delta H_i \quad \Leftrightarrow \\
\Delta RG_i = \frac{-\frac{\partial F_i}{\partial H_i}}{M_i} \Delta H_i
\] (2.29)

Remark
Eq. 2.30 is different from Eq. 2.7. For the modelling of cold rolling process we use Eq. 2.7, for the modelling of FF1 controller we use Eq. 2.30.

End Remark

For \( i = 1 \), Substitute Eq. 2.8 and 2.29 into Eq. 2.30, it results:

\[
\Delta H_{1m}(s) \quad e^{-(\tau_{RG1} - \tau_{in})s} \cdot \frac{\partial F_i}{\partial H_i} \cdot \frac{K_{FF1}}{M_i} \Delta RG_{1,FF1}(s)
\]

(2.31)

Figure 2.6 Model of FF1

Where
\[
K_{FF1} \quad \text{FF1 controller gain (\cdot)}
\]
\[
\tau_{RG1} \leq \tau_{in}
\]

Remark:
FF1 is a feedforward controller. This is why it is denoted as FF.

In Eq.2.31, the nominal value of $K_{FF1}$ is 1, but the exact value is online tuned, similarly for all other level 1 controller gains.

**End Remark**

### 2.2.2 Model of FB1

The CV of FB1 is the exit thickness of stand 1; The MV of FB1 is the roll gap at stand 1.

The design of FB1 is again based on Eq.2.6. With assumptions $\Delta H_1 \approx 0$ and $\Delta T_{12} \approx 0$, it results:

$$\Delta h = \frac{M_1}{M_1} \frac{\partial F_1}{\partial h_1} \Delta R G_i + \frac{\partial F_1}{\partial h_1} \cdot 0 + \frac{\partial F_1}{\partial T_{12}} \cdot 0 \Leftrightarrow$$

$$\Delta R G_i = \frac{M_1}{M_1} \frac{\partial F_1}{\partial h_1} \cdot \Delta h$$

(2.32)

Since the thickness sensor is located after stand 1, which introduces a transport lag from the stand to the thickness sensor, a Smith Predictor is added to FB1. Besides, there is also one integrator added to FB1. Therefore, the following structure of FB1 is obtained:

(2.32A)

![Figure 2.7 Model of FB1](image)

Where
\( K_{FB1} \) FB1 controller gain (-) 

\( \tau_{FB1} \) time constant of the Smith Predictor (-) 

**Remark:**

For people who are not familiar with control theory like the benefits of Smith Predictor or why one integrator is added here, please refer to literature [1].

FB1 is a feedback controller. This is why it is denoted as FB.

FB1 is a pure Integral controller; it is not a Proportional-Integral controller. Besides, the Smith Predictor used by FB1 in figure 2.7 is different from a standard Smith Predictor, which is shown in figure 2.8 below [1]. However, since the improvement of level 1 controller is not the concern of this assignment, these two remarks are left to the readers.

The CV of FF2 is the enter thickness of stand 2, namely FF2 is used to remove the enter thickness variation at stand 2.

The MV of FF2 is the roll speed at stand 1.

The design of FF2 is based on Eq.2.17 for \( i = 2 \) with the assumption that exit thickness variations at stand 2 are removed by another level 1 controller. Besides, the roll speed variation at stand 2 is neglected, namely \( \Delta h_2 \approx 0 \) and \( \Delta V_2 \approx 0 \). Therefore, Eq.2.17 can be rewritten as:
\[ \Delta h_2 = h_2 \left( \frac{\Delta H_{2,1}}{H_2} + \frac{\Delta V_1}{V_1} - \frac{\Delta V_2}{V_2} \right) \]
\[ 0 = h_2 \left( \frac{\Delta H_{2,1}}{H_2} + \frac{\Delta V_1}{V_1} - \frac{0}{V_2} \right) \]
\[ \Delta V_i = -\frac{V_i}{H_2} \Delta H_2 \] (2.33)

For stand 1, the driving motor dynamic is described by Eq.2.18 with \( i = 1 \):
\[ \Delta V_i(s) = \frac{1}{\tau_{V1,2}s^2 + \tau_{V1,1}s + 1} \Delta V_{i,\text{ref}}(s) \]
\[ \Delta V_{i,\text{ref}}(s) = (\tau_{V1,2}s^2 + \tau_{V1,1}s + 1) \Delta V_i(s) \] (2.34)

Substituting of Eq.2.33 into Eq.2.34 leads to:
\[ \Delta V_{i,\text{ref}}(s) = -\frac{V_i}{H_2} \left( \tau_{V1,2}s^2 + \tau_{V1,1}s + 1 \right) \Delta H_2(s) \] (2.35)

However in a physical world, \((\tau_{V1,2}s^2 + \tau_{V1,1}s + 1)\) in Eq.2.35 is not realizable, that is why Eq.2.35 is approximated by:
\[ \Delta V_{i,\text{ref}}(s) = -\frac{V_i}{H_2} \frac{\tau_{V1,2}s^2 + \tau_{V1,1}s + 1}{(P_d s + 1)(T_d s + 1)} \Delta H_2(s) \] (2.36)

Where
\( P_d \) Phase advance (-)
\( T_d \) Time constant (-)

Substituting of Eq.2.8 with \( i = 2 \) into Eq.2.36 results in:
\[ (2.37) \]

\[ h_{i,\text{ref}}(s), h_{i,m}(s), \Delta H_{2,\text{m}}(s), e^{-\tau_{\text{int}} s}, K_{FF2}, \frac{-V_{1,1} s^2 + \tau_{V1,1}s + 1}{H_2 (P_d s + 1)(T_d s + 1)} \]

\[ \Delta V_{i,FF2}(s) \]

Figure 2.9 Model of FF2

Where
\( K_{FF2} \) FF2 controller gain (-)
2.2.4 Model of FB2

The CV of FB2 is the exit thickness of stand 2; The MV of FB2 is the roll speed at stand 2. The structure of the controller is directly presented in figure 2.10 below:

\[ h_{2,\text{ref}}(s) + K_{FB2} \Delta V_{2FB2}(s) \rightarrow h_{2,m}(s) \]

Figure 2.10 Model of FB2

Where

\[ K_{FB2} \] FB2 controller gain (-)

Remark:

However, from figure 2.11 below it seems that the use of FB2 has a big disadvantage. Obviously the exit thickness variation of stand 3 (blue curve in the upper subfigure) is mainly caused by \[ \Delta V_{2FB2} \] (green curve). It leads that the FBT controller at stand 4 (blue curve in the lower subfigure) has to take action to remove the thickness variation of stand 3 and keep the thickness of stand 4 to its reference value. Since the FBT controller is a feedback controller and the thickness variation caused by FB2 is predictable, introducing one extra feedforward controller like FF4 to actively compensate this effect is probably a better way.

End Remark
2.2.5 Model of FF3

The CV of FF3 is the enter thickness of stand 3, namely FF3 is used to remove the enter thickness variation at stand 3.

The MV's of FF3 are the roll speed at stand 2, the roll speed at stand 1 and the roll gap of stand 3. The main MV of FF3 is the roll speed at stand 2. The other two MV's are used to compensate the influence of the main MV on CV's of other level 1 controllers.

The design of FF3 is through the same procedure as applied for FF2. Therefore, we may directly draw the following conclusion according to Eq.2.36:

\[
\Delta V_{2,\text{ref}}(s) = -\frac{V_2 \tau_{2,2}s^2 + \tau_{2,1}s + 1}{H_3 \left( P_d s + 1 \right) \left( T_d s + 1 \right)} \Delta H_3(s) \tag{2.38}
\]

The calculation of the roll speed correction at stand 1 is based on Eq.2.17 for \( i = 2 \) with assumptions \( \Delta h_2 = 0 \) and \( \Delta H_2 = 0 \):

\[
\Delta h_2 = h_2 \left( \frac{\Delta H_2}{H_2} + \frac{\Delta V_1}{V_1} - \frac{\Delta V_2}{V_2} \right) \quad \Leftrightarrow \quad 0 = h_2 \left( \frac{0}{H_2} + \frac{\Delta V_1}{V_1} - \frac{\Delta V_2}{V_2} \right) \quad \Leftrightarrow \\
\Delta V_1 = \frac{V_1}{V_2} \cdot \Delta V_2 \tag{2.39}
\]

Approximate \( \Delta V_1 \) by \( \Delta V_{1,\text{ref}} \) for \( i = 1, 2 \), Eq.2.39 is approximated by:

\[
\Delta V_{1,\text{ref}}(s) = \frac{V_1}{V_2} \cdot \Delta V_{2,\text{ref}}(s) \tag{2.40}
\]

The calculation of the roll gap correction at stand 3 is based on Eq.2.2, Eq.2.17 and Eq.2.4 for \( i = 3 \) with assumptions \( \Delta h_3 \approx \Delta V_3 \approx \Delta T_{23} \approx \Delta T_{34} \approx 0 \):

\[
\begin{align*}
\Delta F_3 &= \frac{\partial F}{\partial H_3} \Delta H_3 \\
0 &= \frac{\Delta H_3}{H_3} + \frac{\Delta V_2}{V_2} \\
0 &= \Delta R G_3 + \frac{\Delta F_3}{M_3} \tag{2.41}
\end{align*}
\]

Solving Eq.2.41 leads to:
\[
\Delta R G_3 = \frac{\partial F_3}{V_2} \cdot \frac{\partial H_3}{M_3} \cdot \Delta V_2
\]  
(2.42)

Approximating \( \Delta R G_3 \) by \( \Delta R G_{3,\text{ref}} \) and \( \Delta V_2 \) by \( \Delta V_{2,\text{ref}} \), it results Eq.2.42 to be simplified to:

\[
\Delta R G_{3,\text{ref}} (s) = \frac{H_3}{V_2} \cdot \frac{\partial H_3}{M_3} \cdot \Delta V_{2,\text{ref}} (s)
\]  
(2.43)

Based on Eq.2.40, 2.43 and Eq.2.38 with substitution of Eq.2.8 for \( i = 3 \), the structure of FF3 is obtained:

\[
\begin{align*}
\Delta H_{3m} (s) & = e^{-\tau_{2} s} K_{FF3} \frac{-V_2 \tau_{2} s^2 + \tau_{2} s + 1}{V_2 \tau_{2} s^2} \\
\Delta V_{2,FF3} (s) & = \frac{V_1}{V_2} \\
\Delta V_{FF3} (s) & = \frac{\partial F_3}{H_3} \frac{\partial H_3}{M_3}
\end{align*}
\]

(2.43A)

Figure 2.12 Controller model of FF3

Where

\( K_{FF3} \) FF3 controller gain 1 (-)
\( K_{FF3,\text{no3}} \) FF3 controller gain 2 (-)

2.2.6 Controller model of FBT

The CV of FBT is the exit thickness of stand 4; The MV's of FBT are the roll speed and the roll gap at stand 4. The main MV of FBT is the roll speed at stand 4. The other MV is used to
compensate the tension variation between stand 3 and 4, which is caused by the main MV.

The design of FBT is based on Eq.2.17 for $i=4$ with assumptions $\Delta H_4 \approx 0$ and $\Delta V_4 \approx 0$.

Therefore, Eq.2.17 can be rewritten as:

$$
\Delta h_4 = h_i \left( \frac{\Delta H_4}{H_4} + \frac{\Delta V_3 - \Delta V_4}{V_3 - V_4} \right) \quad \Leftrightarrow \\
\Delta h_4 = h_i \left( \frac{0}{H_4} + \frac{0 - \Delta V_4}{V_3 - V_4} \right) \quad \Leftrightarrow \\
\Delta V_4 = \frac{V_4}{h_i} \Delta h_4
$$

(2.44)

The calculation of the roll gap correction at stand 4 is based on Eq.2.2, Eq.2.4 and Eq.2.44 for $i=4$ with assumptions $\Delta H_4 \approx \Delta T_{34} \approx \Delta T_{45} \approx 0$:

$$
\begin{align*}
\Delta F_4 &= \frac{\partial F_4}{\partial h_4} \Delta h_4 \\
\Delta h_4 &= \Delta RG_4 + \frac{\Delta F_4}{M_4} \\
\Delta V_4 &= \frac{V_4}{h_i} \Delta h_4
\end{align*}
$$

(2.45)

Solving Eq.2.45 leads to:

$$
\Delta RG_4 = -\frac{h_i}{V_4} \cdot M_4 \cdot \frac{\partial F_4}{\partial h_4} \cdot \Delta V_4
$$

(2.46)

Approximating $\Delta RG_4$ by $\Delta RG_{4,ref}$ and $\Delta V_4$ by $\Delta V_{4,ref}$, it results Eq.2.46 to be simplified to:

$$
\Delta RG_{4,ref} = -\frac{h_i}{V_4} \cdot M_4 \cdot \frac{\partial F_4}{\partial h_4} \cdot \Delta V_{4,ref}
$$

(2.47)

Since the thickness sensor is located after stand 4, which introduces a transport lag from stand to the sensor, a Smith Predictor is added to FBT. Based on Eq.2.44, 2.47 and added Smith Predictor, we obtain:

(2.47A)
Figure 2.13 Model of FBT

Where

\( K_{FBT} \)  
FBT controller gain (-)

\( K_{RG4} \)  
FBT controller gain (-)

\( \tau_{FBT} \)  
time constant of the Smith Predictor (-)

2.2.7 Model of tension controller

Define that \( T_{j} \) is the name of the tension controller where \( j \in \{1, 2, 3\} \).

The CV of \( T_{j} \) is the interstand tension between stand \( i-1 \) and \( i, \ j = i-1 \); The MV of \( T_{j} \) is the roll gap at stand \( i \).

Under assumption that \( \Delta H_{i} \approx \Delta h_{i} \approx \Delta T_{i,j+1} \approx 0 \), Eq.2.2 and 2.4 can be approximated by:

\[
\begin{align*}
\Delta F_{i} &= \frac{\partial F_{i}}{\partial T_{i-1,j}} \Delta T_{i-1,j} \\
0 &= \Delta RG_{i} + \frac{\Delta F_{i}}{M_{i}}
\end{align*}
\]  
(2.48)

Solving Eq.2.48 leads to:

\[
\Delta RG_{i} = \frac{-\frac{\partial F_{i}}{\partial T_{i-1,j}}}{M_{i}} \Delta T_{i-1,j}
\]  
(2.49)

Substituting of Eq.2.7 into Eq.2.49 results in:
When an integrator is added to Eq. 2.50, the following result is obtained:

\[
\Delta G_{i,\text{ref}}(s) = (\tau_{RG_i}s + 1) \frac{\frac{\partial F_i}{\partial T_{i-1,j}}}{M_i} \cdot \Delta T_{i-1,j} (2.50)
\]

Where

\[K_{\text{TT}}\quad \text{Tension controllers gain (\text{-})}\]

\[i = 2, 3, 4 \quad j = i - 1\]

### 2.3 Model parameters calculation

The presented model in section 2.1 and 2.2 will be validated in section 2.4. This section describes briefly how the model parameters are obtained.

The model parameters can roughly be classified into two categories. Physical parameters like distance between stands, distance between stand and its thickness sensors, mill modulus of stands, internal motor controller gains and internal hydraulic actuator controller gains are assumed to be well-known and time-invariant values. The second category contains all calculated process model parameters, which are considered as unknown and could be time-variant values.

**Remark**

The model uncertainty discussed in section 2.6 is mainly caused by the second category parameters since they could be time-variant values.

**End Remark**

All the model parameters in the second category are denoted as:
\[ \text{sen} = \left\{ \frac{\partial F_i}{\partial H_i}, \frac{\partial F_i}{\partial h_i}, \frac{\partial F_i}{\partial T_{i-1,j}}, \frac{\partial F_i}{\partial T_{i+1,j}}, \frac{\partial MT_i}{\partial H_i}, \frac{\partial MT_i}{\partial h_i}, \frac{\partial MT_i}{\partial T_{i-1,j}}, \frac{\partial MT_i}{\partial T_{i+1,j}}, \frac{\partial v_i}{\partial T_{i+1}} \right\} \quad i = 1, 2, 3, 4 \]

To calculate the value of \text{sen}, a program named SIWA (Simulatie Walsen) is used. This program has been developed by the Corus Research Department [21]. It is the core of the Preset Model, which is currently implemented in CM11. How this program is realized is not the topic of this report. Here we only focus ourselves on the use of this program.

To use the SIWA program, the following parameters are essential. All of them are denoted as:

\[ \text{SIW}_\text{input} = \{ \text{WECO}, H_i, h_i, T_{i-1,j}, T_{i+1,j}, v_i, D_i, \mu_i \} \quad i = 1, 2, 3, 4 \]

Where

\begin{itemize}
  \item \text{WECO} \quad \text{type material (-), developed by Corus, see page 56 of [18] for more detail}
  \item \text{D}_i \quad \text{diameter of work roll of stand } i \text{ (m)}
\end{itemize}

Obviously, if the value of \text{SIW}_\text{input} is well-known and correct, assuming that the SIWA program is perfect, the value of \text{sen} can be obtained correctly during offline procedure. In other words, any faults in the value of \text{SIW}_\text{input} will produce a wrong calculation of the \text{sen} value. This kind of model uncertainty problem will be further discussed in section 2.6. In the next section, we are going to simulate and validate the model presented in section 2.1 and 2.2.

\section*{2.4 Simulation & Validation}

Based on the model described in section 2.1 and 2.2, a complete process model of CM11 is build up. In section 2.7, its general block diagram is presented. In this section, we concentrate ourselves on the validation of this complete process model. The Coil with ID number 36264 has been chosen to validate the process model. In figure 2.15 below, all the online values of the red symbols in figure 2.1 are presented, except \( R G_{i,pre} \) and \( h_{i,ref} \). Because the process model is only valid around one operating point, the input values are given by their relative values instead of their absolute values.
The corresponding output data are plotted in figure 2.16 below. Here the grey curve is the measurement data, the red curve is the simulation of the process model and the green curve is the simulation of the process model but using measurement data of the exit thickness of stand 2 instead of simulated $\Delta h_2$. 

Figure 2.15 Input data
Figure 2.16 Validation of process model

In figure 2.16, the green curve matches the grey one much better than the red curve. Therefore, we directly draw the conclusion that the model of exit thickness at stand 2 needs to be improved. This will be done through the following procedure.

Reconsider Eq. 2.14 again, for \( i = 2 \), we have:

\[
\frac{\Delta H_2}{H_2} + \frac{\Delta v_1}{v_1} = \frac{\Delta h_2}{h_2} + \frac{\Delta v_2}{v_2}
\]  

(2.51)

According to the Corus deformation model (part of SIWA), the exit strip speed \( (v_i) \) is described
as:

\[ v_i = f\left(V_1, H_1, h_i, T_{0i}, T_{i2}\right) \]  

(2.52)

Neglecting the influence of \( H_1, h_i \) and \( T_{0i} \), by linearization of Eq.2.52 the following result is obtained:

\[ \Delta v_i = \Delta V_1 \left(1 + S_i\right) + \frac{\partial v_i}{\partial T_{i2}} \Delta T_{i2} \]  

(2.53)

Where

\[ \frac{\partial v_i}{\partial T_{i2}} \]  

sensitivity of exit strip speed of stand 1 to interstand tension variation between stand 1 and 2 (ms\(^{-1}\)(MN\(^{-1}\))

**Remark**

The difference between Eq.2.16 and Eq.2.53 is that the slip variation at stand 1 is not neglected.

In Eq.2.53, the slip variation is given as a function of the tension variation between stand 1 and 2.

**End Remark**

Substituting of Eq.2.53, 2.15 (\( i = 1, 2 \)) and 2.16 (\( i = 2 \)) into Eq.2.51 gives:

\[ \Delta h_2 = h_2 \left( \frac{\Delta H_2}{H_2} + \frac{\Delta V_2}{V_2} + \frac{V_1}{1 + S_i} \Delta T_{i2} - \frac{\Delta V_1}{V_1} \right) \]  

(2.54)

The blue curve in figure 2.17 shows the simulation result of the process model, in which the model of \( \Delta h_2 \) is remodelled according to Eq.2.54. Obviously the blue curve matches the gray one much better than the red curve, especially on the outputs: exit thickness of stand 2 and 3 (\( \Delta h_2, \Delta h_3 \)); roll speed at stand 2 and 4 (\( \Delta V_2, \Delta V_4 \)); roll gap of stand 3 (\( \Delta RG_3 \)). The difference between the grey curve and the blue curve is classified as the unknown disturbance and will be removed by feedback controllers.
Define:

\[ \alpha_i = \frac{F_{i,\text{pre}}}{F_{i,\text{pre}}} \quad \beta_i = \frac{P_{i,\text{pre}}}{P_{i,\text{pre}}} \quad i \in \{1, 2, 3, 4\} \quad \alpha_4 = \beta_4 = 1 \]

The simulation result of roll force and motor power are given in figure 2.18 below. Similarly as in figure 2.17, the grey curve is the measurement and the blue one is remodeled process model. \( \Delta F_i - \alpha_i \Delta F_4 \) and \( \Delta P_i - \beta_i \Delta P_4 \) are the CV's of the MPC. This will be further discussed in chapter 3. The difference between the simulation and the measurement is considered as the external disturbance, which will be further discussed in section 2.5.
Figure 2.18 Validation of process model
2.5 External disturbance

This section describes the external disturbance on the process model applied for creating figure 2.17 & 2.18.

To determine the disturbance signal, a free run test is executed. In this test, all the red symbols in figure 2.1 are kept at their preset values, except the entrance thickness at stand 1, which is not controllable. In other words, the variations of these red symbols except \( \Delta H_{1m} \) are set to zeros during online processing, see figure 2.19 below.

![Figure 2.19 Input data of free run test](image)

The corresponding output data are presented in figure 2.20 below. Roughly, the difference between the blue curve and the gray one (measurement) is considered as the sum of one low frequent component plus one high frequent component. The high frequent disturbance is probably as a result of the eccentricity problem in the work roll, which will not be further discussed in this assignment. Here we only concentrate ourselves on the low frequent disturbance.
The above discussed low frequent disturbance is modeled as an integrated white noise signal.
However, the design of the MPC controller later in chapter 3 is based on a stable process model. Introducing an integrator will make the controller design unrealizable. Therefore, we use \( \frac{1}{s + \varepsilon} \) instead of a pure integrator \( \frac{1}{s} \). Here \( \varepsilon \) is chosen as a small but positive real value.

Suppose that the disturbance only takes place on the outputs \( F_i \) and \( P_i \). Add the disturbance to the process model, it results:

\[
\text{Process Model} \quad \frac{W_{Ki}}{s + \varepsilon} \quad F_i \\
\text{Modified Process Model} \quad \frac{W_{Pi}}{s + \varepsilon} \\
\text{White Noise}
\]

![Figure 2.21 Modified Process Model](image)

In figure 2.21, \( w_{Ki} \) and \( w_{Pi} \) are dimensionless constants that will be determined offline. This modified process model will be used in chapter 3 for the design of the MPC controller. The next section will discuss uncertainty in the process model.

### 2.6 Model uncertainty

The process model presented in sections before can be different from the actual situation. The difference between the actual process and the process model is characterised by uncertainty in the process model parameters and/or errors in the process model. We neglect errors in the process model and quantify only the uncertainty in this section.

In section 2.3, it was already known that the uncertainty in the \( SIWA_{input} \) value can lead to the uncertainty in the process model parameters. In this section, we first locate which element
in the $SIWA_{input}$ is most unpredictable or most time-variant. After it, we focus on determining the maximum variations of the process model parameters, which can occur in reality.

In $SIWA_{input}$, $D_i$ are physical parameters, which can be assumed as time-invariant values. $H_i$, $h_i$, $T_{i-1,j}$, $T_{j,i+1}$, $v_i$ were considered as state variables in the process model. Therefore, their influence on the process model parameters will be neglected to avoid creating a time-variant process model, which can not be controlled by a default MPC controller. This is allowed because their influences on the model parameters are much less than the frictions coefficients. $WECO$ is used to number the type of material, it will be assumed to be constant. The remaining element in $SIWA_{input}$ are the friction coefficients $\mu_i$. This one is the toughest one to be predicted, namely the nominal friction coefficient ($\mu_{i,\text{nom}}$) is usually estimated wrong during the offline procedure. Besides, in the process model, it is considered that $\mu_i$ are time-invariant values, i.e. $\Delta \mu_i$ are set to zeros. However, figure 2.22 shows that the actual $\mu_i$ are time-variant values. The $\mu_i$ here are calculated at a sample period of about 50 seconds by a Corus program, which uses the measured feedback data like rolling forces, strip thickness, rolling speed, motor power and etc. For more detail about this program see [4]. Here we just show that $\mu_i$ can be considered as the only parameters, who cause the variations in the process model parameters. In fact, we may regard the $\mu_i$ as a kind of black box and put all uncertain factors in the process model into it.

Therefore, the following assumption is allowed:

The uncertainties in the process parameters are only caused by variations in the friction coefficients $\{\Delta \mu_i\}$

Define:

![Figure 2.22 Path of actual friction coefficients](image-url)
\( \mu_{i,\text{min}} \) the minimum value of friction coefficient at stand \( i \) (-)

\( \mu_{i,\text{max}} \) the maximum value of friction coefficient at stand \( i \) (-)

\( \text{sen}_{\text{min}} \) the minimum \( \text{sen} \) value, i.e. the absolute value of each element in \( \text{sen}_{\text{min}} \) must be its minimum value.

\( \text{sen}_{\text{max}} \) the maximum \( \text{sen} \) value, i.e. the absolute value of each element in \( \text{sen}_{\text{max}} \) must be its maximum value.

We make another assumption:

For any \( \mu_i \subset [\mu_{i,\text{min}}, \mu_{i,\text{max}}] \), it is valid that \( \text{sen} \subset [\text{sen}_{\text{min}}, \text{sen}_{\text{max}}] \). \{A.3\}

A.3 is illustrated in the following example:

Suppose only the friction coefficient at stand 1 is wrong estimated, but it is limited to \( \mu_{1,\text{min}} \) and \( \mu_{1,\text{max}} \). For \( \mu_1 = \mu_{1,\text{min}}, \mu_2 = \mu_{2,\text{nom}}, \mu_3 = \mu_{3,\text{nom}}, \mu_4 = \mu_{4,\text{nom}} \), we obtain \( [\text{sen}_{\text{min}}, \text{sen}_{\text{max}}] \).

According to A.3, for any \( \mu_i \subset [\mu_{i,\text{min}}, \mu_{i,\text{max}}] \), it is valid that \( \text{sen} \subset [\text{sen}_{\text{min}}, \text{sen}_{\text{max}}] \).

Remark

A.3 is not proven. It is introduced in this assignment to obtain a rough estimate for the value of \( \text{sen}_{\text{min}} \) and \( \text{sen}_{\text{max}} \). Further research on obtaining the correct \( \text{sen}_{\text{min}} \) and \( \text{sen}_{\text{max}} \) is required. \[ \text{End Remark} \]

According to A.2 and A.3, for a 4 stand cold rolling process, there are totally \( 2^4 = 16 \) possible candidates of the worst case. For example, \( \mu_1 = \mu_{1,\text{min}}, \mu_2 = \mu_{2,\text{max}}, \mu_3 = \mu_{3,\text{max}}, \mu_4 = \mu_{4,\text{max}} \) is one candidate, but \( \mu_1 = \mu_{1,\text{min}}, \mu_2 = \mu_{2,\text{min}}, \mu_3 = \mu_{3,\text{max}}, \mu_4 = \mu_{4,\text{max}} \) is also possible the worst case, etc. In chapter 3, the designed MPC must guarantee closed-loop stability for each of the 16 worst case candidates. However, the question now is how to obtain the \( \mu_{i,\text{min}} \) and \( \mu_{i,\text{max}} \) value. We continue with it in this section.

In figure 2.22, the path of the friction coefficients is shown for one coil. From the measurement data of hundreds of other coils, it can be notified that a maximum variation above 30% of
\( \mu_{thrust} \) almost never occurs. Therefore, we may introduce the following criteria:

\[
\mu_i \in [0.7 \mu_{thrust}, 1.3 \mu_{thrust}]
\]

\{C.1\}

However, not every \( \mu_i \), which meets the criteria C.1 will give a solution from the SIWA program. Therefore, the second criteria below is used to determine the \( \mu_{i,min} \) and \( \mu_{i,max} \), namely:

\[
\mu_i \in [\mu_{i,SIWA_{min}}, \mu_{i,SIWA_{max}}]
\]

\{C.2\}

Where

- \( \mu_{i,SIWA_{min}} \) the minimum value of the friction coefficient at stand \( i \) that gives a solution in the SIWA program (-)
- \( \mu_{i,SIWA_{max}} \) the maximum value of the friction coefficient at stand \( i \) that gives a solution in the SIWA program (-)

Here \( \mu_{i,min} \) and \( \mu_{i,max} \) must meet both criteria C.1 and C.2. The obtained values are later used in chapter 3 for the stability test of the designed MPC.

Next section is the last section of this chapter. Here a complete block diagram of the process model will be presented. It may be considered as a summary of the results of this chapter.

### 2.7 Block diagram of the process model

Before presenting the block diagram of the process model, which will be used for the controller design in chapter 3, we briefly discuss here which MV's could be available for the MPC.

![Figure 2.23 Paired \( \Delta h_{ref} \) and \( \Delta V_{l,pre} \)](image)

From the measurement data in figure 2.23, we conclude that \( \Delta h_{ref} \) and \( \Delta V_{l,pre} \) are paired
inputs, similarly for \( \Delta h_{2,ref} \) and \( \Delta V_{2,pre} \). Their relationships are approximated by the following two equations:

\[
\Delta h_{1,ref} = -\frac{h_{ref}}{V_{1,pre}} \Delta V_{1,pre} \tag{2.55}
\]

\[
\Delta h_{2,ref} = -\frac{h_{ref}}{V_{2,pre}} \Delta V_{2,pre} \tag{2.56}
\]

According to literature [5], the operator has bad experience with online implementation of \( RG_{1,pre} \). Therefore, for security reasons, we will not consider \( RG_{1,pre} \) as MV's of the MPC, further research is required to investigate the correct implementation of \( RG_{1,pre} \) during online operation. Besides, the value of \( h_{ref} \) can not be changed, otherwise it will disturb the thickness of the end product. This is not desired by clients.

Based on the above discussion, the following block diagram of the process model is presented in figure 2.24. The symbols in red except \( \Delta H_{1,m} \) are the MV's of the MPC; \( \Delta H_{1,m} \) is considered as well-known disturbance; the green symbols represent the CV's of the MPC. For more detail about chosen CV's of MPC see chapter 3. The dashed line here indicates a line not connected to its crossover lines or crossover blocks.
Figure 2.24 Block diagram of the Process Model
Chapter 3 Model Predictive Control

In this chapter, we will concentrate ourselves on the controller design based on the process model described in chapter 2.

At first, in section 3.1, the objective function of the controller will be defined. After it, in section 3.2, we will present the algorithm of the applied controller, namely MPC algorithm. The topic in section 3.3 and 3.4 are nominal closed-loop stability and robustness of the controller, respectively. In the last section of this chapter, the MPC presented in section 3.2 will be compared with a Generalized Predictive Control (GPC) from literature [6]. Here advantages and shortages of the two controllers will be discussed. The corresponding simulation results of the MPC can be found in chapter 5.

We now start with the problem definition.

3.1 Definition of controller objective function

According to literature [7], an automatic controller is desired by the CM11, which can realize the roll force and the motor power distribution between the stands. It is also desired for a minimum error of the thickness of the strip at the last stand. Besides, from the economic perspective, people want to 'roll' as quickly as possible. Consider stand 4 as the reference stand, the above discussed requirements can be translated to the following objective function, namely:

\[ J = \sum_{i=1}^{3} (F_i - \alpha_i \cdot F_4)^2 + \sum_{i=1}^{3} (P_i - \beta_i \cdot P_4)^2 + (h_4 - h_{4,\text{ref}})^2 + (V_4 - V_{4,\text{pre}})^2 \]  

(3.1)

Where

- \( h_{4,\text{ref}} \) The desired thickness of the clients (mm)
- \( V_{4,\text{pre}} \) By offline calculation obtained maximum speed of stand 4 (m/s)

Remark

According to most Corus researchers of the Cold Rolling Process department, an uniform distribution of the roll force and the motor power between the stands makes a good quality in the form of the strip. That is why this is desired. However, there are no documents found in Corus, which describe the exact relationship between a uniform distribution and the
corresponding strip quality. The idea that a uniform distribution makes a good quality strip purely comes from practical experience. Besides, for different materials, also different distribution patterns are used, i.e. specific $\alpha_i$'s and $\beta_i$'s are used for the corresponding material. In this assignment, we assume that those $\alpha_i$'s and $\beta_i$'s obtained from practical experience are perfect. Therefore, their values will not be changed during online operation.

End Remark

The designed automatic controller must also satisfy the following inequality constraints:

\[
\begin{align*}
F_i &< F_{i,\text{max}} & i = 1, 2, 3, 4 \\
P_i &< P_{i,\text{max}} & i = 1, 2, 3, 4 \\
T_{i+1,\text{min}} &< T_{i+1} < T_{i+1,\text{max}} & i = 1, 2, 3 \\
h_{i,\text{min}} &< h_i < h_{i,\text{max}} & i = 1, 2, 3 
\end{align*}
\] (3.2)

Putting Eq.3.1 and Eq.3.2 together and assuming of constant $\alpha_i$'s and $\beta_i$'s the objective function can be rewritten as:

\[
J = \sum_{i=1}^{3} (\Delta F_i - \alpha_i \cdot \Delta F_4)^2 + \sum_{i=1}^{3} (\Delta P_i - \beta_i \cdot \Delta P_4)^2 + (\Delta h_i)^2 + (\Delta V_4)^2
\]

\[
\begin{align*}
\Delta F_i &< \Delta F_{i,\text{max}} & i = 1, 2, 3, 4 \\
\Delta P_i &< \Delta P_{i,\text{max}} & i = 1, 2, 3, 4 \\
\Delta T_{i+1,\text{min}} &< \Delta T_{i+1} < \Delta T_{i+1,\text{max}} & i = 1, 2, 3 \\
\Delta h_{i,\text{min}} &< \Delta h_i < \Delta h_{i,\text{max}} & i = 1, 2, 3 
\end{align*}
\] (3.3)

Define:

\[
U = \begin{bmatrix} 
\Delta V_{1,\text{pre}} & \Delta V_{2,\text{pre}} & \Delta V_{3,\text{pre}} & \Delta T_{12,\text{ref}} & \Delta T_{23,\text{ref}} & \Delta T_{34,\text{ref}} 
\end{bmatrix}_{1 \times 6}^T
\]

Where subscript $T$ denotes the transposed matrix.

In section 2.7 it has already been shown that $U$ includes all available MV's. Obviously here $U$ contains only six elements, but the $J$ in Eq.3.3 exists of totally eight elements. Therefore, it is not guaranteed that all elements in $J$ can be minimized as a subject to $U$. Therefore, we redefine the objective function in Eq.3.3 as:
In Eq.3.4, we neglect the following two terms from Eq.3.3: $\Delta h_4$ and $\Delta V_4$

The reason that $\Delta h_4$ is removed is that in reality there are certain tolerances allowed for the thickness deviations in the end product. Their tolerances can be translated as the inequality in Eq.3.4, namely $\Delta h_{4,min} < \Delta h_4 < \Delta h_{4,max}$. Sacrificing the correctness in the thickness within the tolerances to win back the quality in the form of the strip is certainly not a bad idea.

The reason to remove the $\Delta V_4$ term is that in reality $V_{4,pre}$ in Eq.3.1 is not perfectly calculated. It is not advisable to approach a possibly wrong calculated preset value by sacrificing one freedom. Besides, the quality in the form of the strip is not dependent on the minimization of $\Delta V_4$, it only influences the speed of the cold mill. Similarly, we sacrifice here the speed to win back the quality. Besides, letting the automatic controller decide the roll speed of stand 4 does not always mean losing production speed. In fact, the chances for getting faster or getting slower are equal.

Define:

\[
Y = \begin{bmatrix} 
\Delta F_1 - \alpha_1 \Delta F_4 & \Delta F_2 - \alpha_2 \Delta F_4 & \Delta F_3 - \alpha_3 \Delta F_4 & \Delta P_1 - \beta_1 \Delta P_4 & \Delta P_2 - \beta_2 \Delta P_4 & \Delta P_3 - \beta_3 \Delta P_4 
\end{bmatrix}^{T}
\]

\[
Z_1 = \begin{bmatrix} 
\Delta F_1 & \cdots & \Delta F_4 & \Delta P_1 & \cdots & \Delta P_4 
\end{bmatrix}^{T}
\]

\[
Z_2 = \begin{bmatrix} 
\Delta h_1 & \cdots & \Delta h_4 & \Delta T_{12} & \cdots & \Delta T_{23} 
\end{bmatrix}^{T}
\]

\[
Z_{1,\text{max}} = \begin{bmatrix} 
\Delta F_{1,\text{max}} & \cdots & \Delta F_{4,\text{max}} & \Delta P_{1,\text{max}} & \cdots & \Delta P_{4,\text{max}} 
\end{bmatrix}^{T}
\]

\[
Z_{2,\text{min}} = \begin{bmatrix} 
\Delta h_{1,\text{min}} & \cdots & \Delta h_{4,\text{min}} & \Delta T_{12,\text{min}} & \cdots & \Delta T_{34,\text{min}} 
\end{bmatrix}^{T}
\]

\[
Z_{2,\text{max}} = \begin{bmatrix} 
\Delta h_{1,\text{max}} & \cdots & \Delta h_{4,\text{max}} & \Delta T_{12,\text{max}} & \cdots & \Delta T_{34,\text{max}} 
\end{bmatrix}^{T}
\]

Eq.3.4 can be written as:
\[ J = Y^T Y \]
\[ Z_1 \leq Z_{1\text{max}} \]
\[ Z_{2\text{min}} \leq Z_2 \leq Z_{2\text{max}} \]  

Y, \( Z_1 \), and \( Z_2 \) can be expressed as a function of \( U \) according to the process model presented in chapter 2. This will be done in the next section. Besides, a technique called "receding horizon" will be introduced, which is used to minimize the objective function in Eq.3.5 subject to \( U \) from current time to a receding finite future time instant [6]. The controller design based on this technique is in general called as Model Predictive Control (MPC). Because of the inequality in Eq.3.5, a Constrained MPC will be presented. That is also the biggest advantage to use the MPC, simply because it can handle constraints.

### 3.2 MPC control algorithm

The control algorithm of MPC introduced in this section is based on literature [8], [9], [10] and [11]. However, the errors in the literature references are illustrated and taken away here. Besides, a complete mathematical deduction in the general form is also presented here.

In general, one \( m \) input and \( p \) output stable system can be expressed in the following State Space equations:

\[ X(k+1) = A \cdot X(k) + B \cdot U(k) \]  
\[ Y(k) = C \cdot X(k) \]  

Where \( U(k) = \begin{bmatrix} u_1(k) \\ u_2(k) \\ \vdots \\ u_m(k) \end{bmatrix} \) \( Y(k) = \begin{bmatrix} y_1(k) \\ y_2(k) \\ \vdots \\ y_p(k) \end{bmatrix} \)

Remark

For the special case in Eq.3.5, it is known that \( m = p = 6 \), but we keep here the control algorithm in its general form.

End Remark

Define:
\[ J = \sum_{j=0}^{\infty} \left[ Y^T (k+j+1) \cdot Q \cdot Y(k+j+1) + U^T (k+j) \cdot R \cdot U(k+j) \right] \]  \hspace{1cm} (3.8)

Where \( Q \) is a symmetric positive semidefinite penalty matrix, \( R \) is a symmetric positive definite penalty matrix, subscript \( T \) denotes the transposed matrix.

**Remark**

The difference between Eq.3.8 and Eq.3.5 is the introduction of matrices \( Q \) and \( R \). These two matrices will be the controller turning parameters of the later designed MPC.

End Remark

Substituting of Eq.3.7 into Eq.3.8 results in:

\[ J = \sum_{j=0}^{\infty} \left[ X^T (k+j+1) \cdot C^T QC \cdot X(k+j+1) + U^T (k+j) \cdot R \cdot U(k+j) \right] \]  \hspace{1cm} (3.9)

**Remark**

Literature [8], [9], [10] and [11] all used the following objective function:

\[ J = \sum_{j=0}^{\infty} \left[ X^T (k+j) \cdot C^T QC \cdot X(k+j) + U^T (k+j) \cdot R \cdot U(k+j) \right] \]

At time \( k \), \( X(k) \) is not manipulable, it does not make any sense to take \( X(k) \) in the objective function.

End Remark

Define:

\[ U(k) = K \cdot X(k) + U_{mpc}(k) \]  \hspace{1cm} (3.10)

Where \( K \) is selected as the unconstrained LQR feedback gain.

The receding horizon regulator is based on minimization of the following infinite horizon open-loop quadratic objective function at time \( k \).

\[ \min_{U_{mpc}(k)} J = \sum_{j=0}^{\infty} \left[ X^T (k+j+1) \cdot C^T QC \cdot X(k+j+1) + U^T (k+j) \cdot R \cdot U(k+j) \right] \]  \hspace{1cm} (3.11)

Where \( U_{mpc}(k) = \left[ U_{mpc}(k) \ U_{mpc}(k+1) \ldots \ U_{mpc}(k+M-1) \right]^T \), \( M \) denotes the control horizon.

At time \( k+M \), the input vector \( U_{mpc}(k+j) \) is set to zero and kept at this value for all
In the open-loop objective function value calculation.

Assumption:

\[ U(k + j) = K \cdot X(k + j) \quad \text{for} \quad j \geq M \quad \{A.4\} \]

Substituting of A.4 into Eq.3.6 results in:

\[ X(k + j + 1) = (A + BK) \cdot X(k + j) \quad \text{for} \quad j \geq M \quad \{A.5\} \]

The infinite horizon open-loop objective function in Eq.3.11 with assumption A.4 can be expressed as:

\[
\begin{align*}
\min_{u_{n+k}(t)} J &= \sum_{j=0}^{M-1} \left[ X^T(k + j + 1) \cdot C^TQC \cdot X(k + j + 1) + U^T(k + j) \cdot R \cdot U(k + j) \right] \\
& \quad + X^T(k + M) \cdot K^T \cdot X(k + M) \\
& \quad + X^T(k + M + 1) \cdot \hat{Q} \cdot X(k + M + 1)
\end{align*}
\]

Where

\[ \hat{Q} = \sum_{i=0}^{\infty} (A + BK)^{iT} \cdot (C^TQC + K^T \cdot RK) \cdot (A + BK)^{i} \]

(3.13)

Proof of Eq.3.12

Eq.3.11 can be rewritten as:

\[
\begin{align*}
\min_{u_{n+k}(t)} J &= \sum_{j=0}^{M-1} \left[ X^T(k + j + 1) \cdot C^TQC \cdot X(k + j + 1) + U^T(k + j) \cdot R \cdot U(k + j) \right] \\
& \quad + U^T(k + M) \cdot R \cdot U(k + M) \\
& \quad + \sum_{i=M}^{\infty} 
\end{align*}
\]

(3.14)

Substituting of assumption \{A.4\} into Eq.3.14 results in:

\[
\begin{align*}
\min_{u_{n+k}(t)} J &= \sum_{j=0}^{M-1} \left[ X^T(k + j + 1) \cdot C^TQC \cdot X(k + j + 1) + U^T(k + j) \cdot R \cdot U(k + j) \right] \\
& \quad + X^T(k + M) \cdot K^T \cdot X(k + M) \\
& \quad + \sum_{j=M}^{\infty} X^T(k + j + 1) \cdot (C^TQC + K^T \cdot RK) \cdot X(k + j + 1)
\end{align*}
\]

(3.15)

Compare Eq.3.12 with Eq.3.15, Eq.3.12 is valid if:

\[ \sum_{j=M}^{\infty} X^T(k + j + 1) \cdot (C^TQC + K^T \cdot RK) \cdot X(k + j + 1) = X^T(k + M + 1) \cdot \hat{Q} \cdot X(k + M + 1) \]

\[ \Leftrightarrow \]

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\[
\sum_{j=M}^{\infty} X^T(k+j+1) \cdot (C^TQC + K^T RK) \cdot X(k+j+1)
\]

(3.16)

\[
\sum_{j=0}^{\infty} (A + BK)^j \cdot (C^TQC + K^T RK) \cdot (A + BK)^j \cdot X(k+M+1)
\]

When \( j = M \) and \( i = 0 \)

\[
X^T(k+M+1) \cdot (C^TQC + K^T RK) \cdot X(k+M+1)
\]

\[
= X^T(k+M+1)(A + BK)^{MT} \cdot (C^T QC + K^T RK) \cdot (A + BK)^{M} X(k+M+1)
\]

When \( j = M + N \) and \( i = N \), with assumption A.5, we obtain:

\[
X^T(k+M+N+1) \cdot (C^T QC + K^T RK) \cdot X(k+M+N+1)
\]

\[
= \left[ (A + BK)^N X(k+M+N+1) \right]^T \cdot (C^T QC + K^T RK) \cdot (A + BK)^N X(k+M+N+1)
\]

\[
= \left[ (A + BK)^{M} X(k+M+N+1) \right]^T \cdot (C^T QC + K^T RK) \cdot (A + BK)^{M} X(k+M+N+1)
\]

\[
= \ldots
\]

\[
= \left[ (A + BK)^N X(k+M+N+1) \right]^T \cdot (C^T QC + K^T RK) \cdot (A + BK)^N X(k+M+N+1)
\]

\[
= X^T(k+M+N+1)(A + BK)^{NT} \cdot (C^T QC + K^T RK) \cdot (A + BK)^N X(k+M+N+1)
\]

Obviously, for any \( N \subset [0, \infty) \), \( N \in \mathbb{Z} \), Eq.3.16 is valid.

End Proof

Define:

\[
X_M(k) = \begin{bmatrix}
X(k) \\
X(k+1) \\
X(k+2) \\
\vdots \\
X(k+M)
\end{bmatrix},
U_M(k) = \begin{bmatrix}
U(k) \\
U(k+1) \\
\vdots \\
U(k+M-1)
\end{bmatrix}
\]

\[
L_{4,M} = \begin{bmatrix}
0 & 1 & 0 & \cdots & 0 \\
0 & 0 & 1 & \cdots & 0 \\
\vdots & \vdots & \ddots & \ddots & \vdots \\
0 & 0 & \cdots & 0 & 1
\end{bmatrix}, \ (M \text{ times } I_s \text{ in second column diagonal block, subscript })
\]

\( s \) is the dimension of \( X \)

\[
Q_M = \text{diag} \left[ \begin{bmatrix} C^T QC & \cdots & C^T QC & C^T QC + K^T RK \end{bmatrix} \right], \ (M \text{ times } C^T QC \text{ in the block })
\]

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diagonal, where \( K^T R K \) is added to the last \( C^T Q C \)

\[
R_M = \text{diag} \left( \begin{bmatrix} R & R & \cdots & R \end{bmatrix} \right), \quad (M \text{ times } R \text{ in diagonal block})
\]

Eq. 3.12 can be rewritten as:

\[
\min_{u_{\text{mpc} \cdot M} (k)} J = X_M^T (k) L_{4 \cdot M}^T \cdot Q_M \cdot L_{4 \cdot M} X_M (k) + U_M^T (k) \cdot R_M \cdot U_M (k)
\]

\[
+ X^T (k + M + 1) \cdot \hat{Q} \cdot X(k + M + 1)
\]

\[ (3.17) \]

A linear quadratic problem has the following form:

\[
\min_{u_{\text{mpc} \cdot M} (k)} J = \frac{1}{2} U_{\text{mpc} \cdot M}^T \cdot H \cdot U_{\text{mpc} \cdot M} + F_{\text{mpc} \cdot M} U_{\text{mpc} \cdot M} \quad G \cdot U_{\text{mpc} \cdot M} \leq G_{\text{max}}
\]

\[ (3.18) \]

Where \( H, F \), G and \( G_{\text{max}} \) are well-known values at time \( k \) and \( U_{\text{mpc} \cdot M} \) is the unknown vector, which needs to be solved.

Assume at time \( k \) that all state variables \( X(k) \) in Eq. 3.6 are measurable. This assumption is usually not true, but it is a subject to build one observer to estimate all state variables.

In Eq. 3.17, the unknown variables are \( X_M (k), \quad X(k + M + 1), \quad U_M (k) \) and \( \hat{Q} \). The infinite sum in Eq. 3.13 can be determined from the solution of the following discrete Lyapunov equation.

\[
\hat{Q} = C^T Q C + K^T R K + (A + B K)^T \hat{Q} (A + B K)
\]

\[ (3.19) \]

There are standard methods available for the solution of this equation.

**Remark**

Eq. 3.19 is only valid if \( A + B K \) is stable, i.e. the pair of \( A \) and \( B \) is stabilizable, otherwise Eq. 3.13 can not be expressed by Eq. 3.19.

**End Remark**

The remaining unknown variables \( X_M (k), \quad X(k + M + 1) \) and \( U_M (k) \) can be expressed as a function of \( U_{\text{mpc} \cdot M} (k) \), see the procedure below.

**Composing** \( X_M (k) \) & \( X(k + M + 1) \)

At time \( k \), according to Eq.3.6 we have
\[
\begin{bmatrix}
X(k+1) \\
X(k+2) \\
\vdots \\
X(k+M)
\end{bmatrix} =
\begin{bmatrix}
A \cdot X(k) \\
A \cdot X(k+1) \\
\vdots \\
A \cdot X(k+M-1)
\end{bmatrix}
+ \begin{bmatrix}
B \cdot U(k) \\
B \cdot U(k+1) \\
\vdots \\
B \cdot U(k+M-1)
\end{bmatrix}
\] (3.20)

Substituting of Eq.3.10 into Eq.3.20 results in:

\[
\begin{bmatrix}
X(k+1) \\
X(k+2) \\
\vdots \\
X(k+M)
\end{bmatrix}
= \begin{bmatrix}
A \cdot X(k) \\
A \cdot X(k+1) \\
\vdots \\
A \cdot X(k+M-1)
\end{bmatrix}
+ \begin{bmatrix}
B \cdot (K \cdot X(k)+U_{npe}(k)) \\
B \cdot (K \cdot X(k+1)+U_{npe}(k+1)) \\
\vdots \\
B \cdot (K \cdot X(k+M-1)+U_{npe}(k+M-1))
\end{bmatrix}
\Rightarrow
\begin{bmatrix}
X(k+1) \\
X(k+2) \\
\vdots \\
X(k+M)
\end{bmatrix}
= \begin{bmatrix}
A_c \cdot X(k) \\
A_c \cdot X(k+1) \\
\vdots \\
A_c \cdot X(k+M-1)
\end{bmatrix}
+ \begin{bmatrix}
B \cdot U_{npe}(k) \\
B \cdot U_{npe}(k+1) \\
\vdots \\
B \cdot U_{npe}(k+M-1)
\end{bmatrix}
\] (3.21)

Where \( A_c = A + BK \)

In Eq.3.21, we have:

\[
X(k+1) = A_c \cdot X(k) + B \cdot U_{npe}(k)
\]

\[
X(k+2) = A_c \cdot X(k+1) + B \cdot U_{npe}(k+1)
\]

\[
= A_c \cdot (A_c \cdot X(k) + B \cdot U_{npe}(k)) + B \cdot U_{npe}(k+1)
\]

\[
= A_c^2 \cdot X(k) + A_c B \cdot U_{npe}(k) + B \cdot U_{npe}(k+1)
\]

\vdots

\[
X(k+M) = A_c \cdot X(k+M-1) + B \cdot U_{npe}(k+M-1)
\]

\[
= A_c \cdot (A_c \cdot X(k+M-2) + B \cdot U_{npe}(k+M-2)) + B \cdot U_{npe}(k+M-1)
\]

\vdots

\[
= A_c^M \cdot X(k) + A_c^{M-1} B \cdot U_{npe}(k) + \cdots + B \cdot U_{npe}(k+M-1)
\]

Therefore, Eq.3.21 may be rewritten as:

\[
\begin{bmatrix}
X(k+1) \\
X(k+2) \\
\vdots \\
X(k+M)
\end{bmatrix}
= \begin{bmatrix}
A_c \cdot X(k) \\
A_c \cdot X(k) \\
\vdots \\
A_c \cdot X(k)
\end{bmatrix}
+ \begin{bmatrix}
B \cdot 0 \cdots 0 \\
A_c B \cdot B \cdots \cdots 0 \\
\vdots \\
A_c^{M-1} B \cdot A_c^{M-2} B \cdots \cdots B
\end{bmatrix}
\begin{bmatrix}
U_{npe}(k) \\
U_{npe}(k+1) \\
\vdots \\
U_{npe}(k+M-1)
\end{bmatrix}
\] (3.22)

Adding \( X(k) = I_p \cdot X(k) \) to Eq.3.22 leads to:
\[
\begin{bmatrix}
X(k) \\
X(k+1) \\
X(k+2) \\
\vdots \\
X(k+M)
\end{bmatrix} = 
\begin{bmatrix}
I_c \\
A_c \\
A_c^2 \\
\vdots \\
A_c^M
\end{bmatrix}
\begin{bmatrix}
X(k) \\
X(k) \\
X(k) \\
\vdots \\
X(k)
\end{bmatrix} +
\begin{bmatrix}
0 & 0 & \cdots & 0 \\
B & 0 & \cdots & 0 \\
A_c B & B & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
A_c^{M-1}B & A_c^{M-2}B & \cdots & B
\end{bmatrix}
\begin{bmatrix}
U_{mpc}(k) \\
U_{mpc}(k+1) \\
\vdots \\
U_{mpc}(k+M-1)
\end{bmatrix}
\]

\text{(3.23)}

Define:
\[
A_{c,M} = 
\begin{bmatrix}
I_c \\
A_c \\
A_c^2 \\
\vdots \\
A_c^M
\end{bmatrix}
\quad 
B_M = 
\begin{bmatrix}
0 & 0 & \cdots & 0 \\
B & 0 & \cdots & 0 \\
A_c B & B & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
A_c^{M-1}B & A_c^{M-2}B & \cdots & B
\end{bmatrix}
\]

Eq.3.23 may be rewritten as:
\[
X_{c,M}(k)_n = A_{c,M} \cdot X(k) + B_{c,M} \cdot U_{mpc,M}(k)
\]

\text{(3.24)}

According to Eq.3.6 and assumption A.4, we have
\[
X(k+M+1) = A_c X(k+M) \iff
X(k+M+1) = A_{c}^{M+1} \cdot X(k) + \begin{bmatrix} A_c M B & A_c^{M-1} B & \cdots & A_c B \end{bmatrix} \cdot U_{mpc,M}(k) \iff
X(k+M +1) = A_{c}^{M+1} \cdot X(k) + B_1 \cdot U_{mpc,M}(k)
\]

\text{(3.25)}

Where
\[
B_1 = \begin{bmatrix} A_c M B & A_c^{M-1} B & \cdots & A_c B \end{bmatrix}
\]

End Composing \(X_{c,M}(k) \& X(k+M+1)\)

\text{Composing} \(U_{M}(k)\)

Similarly, at time \(k\), according to Eq.3.10 we have
\[
\begin{bmatrix}
\begin{bmatrix}
U(k) \\
U(k+1) \\
\vdots \\
U(k+M-1)
\end{bmatrix} = 
\begin{bmatrix}
K \cdot X(k) \\
K \cdot X(k+1) \\
\vdots \\
K \cdot X(k+M -1)
\end{bmatrix} + 
\begin{bmatrix}
U_{mpc}(k) \\
U_{mpc}(k+1) \\
\vdots \\
U_{mpc}(k+M-1)
\end{bmatrix}
\end{bmatrix}
\]

\text{(3.26)}

Define:
\[
K_M = \text{diag}([K \ K \ \cdots \ K]), \ (M \ \text{times} \ K \ \text{in diagonal block})
\]

\[51\]
\[
L_{3,M} = \begin{bmatrix}
I_s & 0 & \cdots & 0 & 0 \\
0 & I_s & \cdots & \vdots & \vdots \\
\vdots & \vdots & \ddots & \vdots & \vdots \\
0 & 0 & \cdots & I_s & 0 \\
\end{bmatrix}, \quad (M \text{ times } I_s \text{ in diagonal block})
\]

Eq. 3.26 can be rewritten in a matrix form:

\[
U_M(k) = K_M \cdot L_{3,M} X_M(k) + U_{\text{mpc},M}(k)
\]  
\[ (3.27) \]

Substituting of Eq.3.24 into Eq.3.27 results in:

\[
U_M(k) = K_M \cdot L_{3,M} \left[ A_{c,M} \cdot X(k) + B_M \cdot U_{\text{mpc},M}(k) \right] + U_{\text{mpc},M}(k) \\
= K_M L_{3,M} A_{c,M} \cdot X(k) + K_M L_{3,M} B_M \cdot U_{\text{mpc},M}(k) + U_{\text{mpc},M}(k) \\
= K_M L_{3,M} A_{c,M} \cdot X(k) + (K_M L_{3,M} B_M + I_{nM}) \cdot U_{\text{mpc},M}(k)
\]

\[
\Rightarrow
U_M(k) = K_{LA} \cdot X(k) + K_{LB} \cdot U_{\text{mpc},M}(k)
\]  
\[ (3.28) \]

Where \( K_{LA} = K_M L_{3,M} A_{c,M} \) and \( K_{LB} = K_M L_{3,M} B_M + I_{nM} \)

End Composing \( U_M(k) \)

Substituting of Eq.3.24, Eq.3.25 and Eq.3.28 into Eq.3.17 results in:

\[
\min_{U_{\text{mpc},M}(k)} J = \sum_{t=1}^{M} X_t^T(k) L_{4,M}^T \cdot Q_M \cdot L_{4,M} X(k) + U_M^T(k) \cdot R_M \cdot U_M(k) \\
+ X^T(k + M + 1) \cdot \hat{Q} \cdot X(k + M + 1) \\
= \left[ A_{c,M} \cdot X(k) + B_M \cdot U_{\text{mpc},M}(k) \right]^T L_{4,M}^T \cdot Q_M \cdot L_{4,M} \left[ A_{c,M} \cdot X(k) + B_M \cdot U_{\text{mpc},M}(k) \right] \\
+ \left[ K_{LA} \cdot X(k) + K_{LB} \cdot U_{\text{mpc},M}(k) \right]^T \cdot R_M \left[ K_{LA} \cdot X(k) + K_{LB} \cdot U_{\text{mpc},M}(k) \right] \\
+ \left[ A_{c}^{M+1} \cdot X(k) + B_{1} \cdot U_{\text{mpc},M}(k) \right]^T \cdot \hat{Q} \left[ A_{c}^{M+1} \cdot X(k) + B_{1} \cdot U_{\text{mpc},M}(k) \right] \\
= U_{\text{mpc},M}(k)^T \cdot \left[ L_{4,M}^T \cdot Q_M \cdot L_{4,M} B_M + K_{LA}^T R_M K_{LB} + B_{1}^T \hat{Q} B_{1} \right] \cdot U_{\text{mpc},M}(k) \\
+ 2X^T(k) \left[ A_{c,M} \cdot L_{4,M}^T \cdot Q_M \cdot L_{4,M} B_M + K_{LA}^T R_M K_{LB} + (A_{c}^{M+1})^T \cdot \hat{Q} B_{1} \right] \cdot U_{\text{mpc},M}(k) \\
+ R_{\text{est}}(k)
\]

Where the \( R_{\text{est}}(k) \) term contains no \( U_{\text{mpc},M}(k) \) element.

\[
\Rightarrow
\min_{U_{\text{mpc},M}(k)} J = \frac{1}{2} U_{\text{mpc},M}(k) \cdot H \cdot U_{\text{mpc},M}(k) + X^T(k) F \cdot U_{\text{mpc},M}(k)
\]  
\[ (3.29) \]

Where
\[ H = 2 \left[ B_M L_{4,M}^T Q_M L_{4,M} B_M + K_{LB}^T R_M K_{LB} + B_M^T \hat{Q} B_M \right] \]

\[ F = 2 \left[ A_{c,M}^T L_{4,M}^T Q_M L_{4,M} B_M + K_{LB}^T R_M K_{LB} + \left( A_{c,M+1} \right)^T \hat{Q} B_M \right] \]

In an unconstrained case, the minimization problem in Eq.3.29 can be directly solved by:

\[ U_{\text{mpc, } M}(k) = -H^{-1}F^T X(k) \quad (3.30) \]

Where \( H \) and \( F \) can be offline calculated.

For the inequalities in Eq.3.5, we continue ourselves with composing of \( G \) and \( G_{\text{max}} \) in Eq.3.18.

Define:

\[ Z(k) = E \cdot X(k) \quad (3.31) \]

\[ Z_{\text{min}} \leq Z(k) \leq Z_{\text{max}} \quad (3.32) \]

\( ne \) is the number of rows of the \( E \) matrix.

Where \( Z_{\text{min}} = \begin{bmatrix} z_{1,\text{min}} \\ \vdots \\ z_{ne,\text{min}} \end{bmatrix} \) and \( Z_{\text{max}} = \begin{bmatrix} z_{1,\text{max}} \\ \vdots \\ z_{ne,\text{max}} \end{bmatrix} \)

Define:

\[ U_{\text{min}} \leq U(k) \leq U_{\text{max}} \quad (3.33) \]

Where \( U_{\text{min}} = \begin{bmatrix} u_{1,\text{min}} \\ \vdots \\ u_{m,\text{min}} \end{bmatrix} \) and \( U_{\text{max}} = \begin{bmatrix} u_{1,\text{max}} \\ \vdots \\ u_{m,\text{max}} \end{bmatrix} \)

Define:

\[ \Delta U(k) = U(k) - U(k-1) \quad (3.34) \]

\[ \Delta U_{\text{min}} \leq \Delta U(k) \leq \Delta U_{\text{max}} \quad (3.35) \]
Where \( \Delta U_{\text{min}} = \begin{bmatrix} \Delta u_{1,\text{min}} \\ \Delta u_{2,\text{min}} \\ \vdots \\ \Delta u_{m,\text{min}} \end{bmatrix} \) and \( \Delta U_{\text{max}} = \begin{bmatrix} \Delta u_{1,\text{max}} \\ \Delta u_{2,\text{max}} \\ \vdots \\ \Delta u_{m,\text{max}} \end{bmatrix} \)

Combining Eq.3.32, Eq.3.33 and Eq.3.35 together results in:

\[
\begin{cases}
Z_{\text{min}} \leq Z(k) \leq Z_{\text{max}} \\
U_{\text{min}} \leq U(k) \leq U_{\text{max}} \\
\Delta U_{\text{min}} \leq \Delta U(k) \leq \Delta U_{\text{max}}
\end{cases}
\]

(3.36)

Eq.3.36 can be rewritten in the following form:

\[
G \cdot U_{\text{spec,le}}(k) \leq G_{\text{max}}
\]

(3.37)

For the definition of \( G \) and \( G_{\text{max}} \) see "Composing Constraints" below.

**Composing Constraints**

Rewrite Eq.3.32 as:

\[
\begin{cases}
Z_{\text{min}} \leq Z(k) \leq Z_{\text{max}} \\
Z_{\text{min}} \leq Z(k) \\
Z(k) \leq Z_{\text{max}} \\
-Z(k) \leq -Z_{\text{min}} \\
Z(k) \leq Z_{\text{max}}
\end{cases}
\]

(3.38)

Where \( I_{ne2} = \begin{bmatrix} -1_{ne} \\ I_{ne} \end{bmatrix} \) and \( Z_{\text{new, max}} = \begin{bmatrix} -Z_{\text{min}} \\ Z_{\text{max}} \end{bmatrix} \)

Substitute Eq.3.31 into Eq.3.38, for \( M \) samples, it results:

\[
I_{ne2} \cdot X(k + j) \leq Z_{\text{new, max}}, \quad 1 < j \leq M
\]

\[
\begin{bmatrix}
I_{ne2} \cdot X(k + 1) \\
I_{ne2} \cdot X(k + 2) \\
\vdots \\
I_{ne2} \cdot X(k + M)
\end{bmatrix} \leq
\begin{bmatrix}
Z_{\text{new, max}} \\
Z_{\text{new, max}} \\
\vdots \\
Z_{\text{new, max}}
\end{bmatrix}
\]

\[
\iff
\begin{bmatrix}
I_{ne2} \cdot X(k + 1) \\
I_{ne2} \cdot X(k + 2) \\
\vdots \\
I_{ne2} \cdot X(k + M)
\end{bmatrix}
\]

54
\[ E_M L_{4,M} X_M(k) \leq Z_{M,\text{max}} \] (3.39)

Where

\[ E_M = \text{diag} \left( \begin{bmatrix} I_{ne2,E} & I_{ne2,E} & \cdots & I_{ne2,E} \end{bmatrix} \right), \quad (M \times \text{times} \ I_{ne2,E} \text{ in diagonal block}) \]

\[ Z_{M,\text{max}} = \begin{bmatrix} Z_{\text{new, max}} & Z_{\text{new, max}} & \cdots & Z_{\text{new, max}} \end{bmatrix}^T, \quad (M \times \text{times} \ Z_{\text{new, max}}) \]

**Remark**

In literature [8] and [9], the following inequality is taken:

\[ I_{ne2,E} \cdot X(k) \leq Z_{\text{new, max}} \]

This is a mistake, since \( X(k) \) is not manipulable, it should not be taken in the inequality as an extra requirement.

Literature [11] presents:

\[ I_{ne2,E} \cdot X(k + j) \leq Z_{\text{new, max}}, \quad 0 < j \leq M - 1 \]

Because control horizon of the MPC is defined as \( M \), it is very confusing here to use \(< \) instead of \( \leq \). People may easily interpret \( 0 < j \leq M - 1 \) as \( 0 \leq j \leq M - 1 \), it will be better to replace \( 0 < j \leq M - 1 \) by \( 1 \leq j \leq M \) like in Eq.3.39.

**End Remark**

Substituting of Eq.3.24 into Eq.3.39 results in:

\[ E_M L_{4,M} \left[ A_{c,M} \cdot X(k) + B_{M} \cdot U_{\text{mpc,ld}}(k) \right] \leq Z_{M,\text{max}} \quad \Leftrightarrow \]

\[ E_M L_{4,M} B_{M} \cdot U_{\text{mpc,ld}}(k) \leq Z_{M,\text{max}} - E_M L_{4,M} A_{c,M} \cdot X(k) \quad \Leftrightarrow \]

\[ G_1 \cdot U_{\text{mpc,ld}}(k) \leq G_{1,\text{max}} \] (3.40)

Where

\[ G_1 = E_M L_{4,M} B_{M} \quad G_{1,\text{max}} = Z_{M,\text{max}} - E_M L_{4,M} A_{c,M} \cdot X(k) \]

Similarly, Eq.3.33 can be rewritten as:
\[ U_{\text{min}} \leq U(k) \leq U_{\text{max}} \iff \]
\[ \begin{cases} U_{\text{min}} \leq U(k) \\ U(k) \leq U_{\text{max}} \end{cases} \iff \]
\[ -U(k) \leq -U_{\text{min}} \iff \]
\[ U(k) \leq U_{\text{max}} \]
\[ I_{m2} U(k) \leq U_{\text{new,max}} \quad (3.41) \]

Where \( I_{m2} = \begin{bmatrix} -I_m \\ I_m \end{bmatrix} \quad U_{\text{new,max}} = \begin{bmatrix} -U_{\text{min}} \\ U_{\text{max}} \end{bmatrix} \)

For \( M \) samples, according to Eq.3.41, we directly get:
\[ I_{m2,M} U_M(k) \leq U_{M,\text{max}} \quad (3.42) \]

Where
\[ I_{m2,M} = \text{diag} \left( \begin{bmatrix} I_{m2} \\ I_{m2} \\ \vdots \\ I_{m2} \end{bmatrix} \right), \quad (M \text{ times } I_{m2} \text{ in diagonal block}) \]
\[ U_{M,\text{max}} = \begin{bmatrix} U_{\text{new,max}} \\ U_{\text{new,max}} \\ \vdots \\ U_{\text{new,max}} \end{bmatrix}^T, \quad (M \text{ times } U_{\text{new,max}}) \]

Substituting of Eq.3.28 into Eq.3.42 gives:
\[ I_{m2,M} \left[ K_{LA} \cdot X(k) + K_{LB} \cdot U_{\text{mpc,M}}(k) \right] \leq U_{M,\text{max}} \iff \]
\[ I_{m2,M} K_{LB} \cdot U_{\text{mpc,M}}(k) \leq U_{M,\text{max}} - I_{m2,M} K_{LA} \cdot X(k) \iff \]
\[ G_2 \cdot U_{\text{mpc,M}}(k) \leq G_{2,\text{max}} \quad (3.43) \]

Where
\[ G_2 = I_{m2,M} K_{LB} \quad G_{2,\text{max}} = U_{M,\text{max}} - I_{m2,M} K_{LA} \cdot X(k) \]

Similarly, Eq.3.35 can be rewritten as:
\[ \Delta U_{\text{min}} \leq \Delta U(k) \leq \Delta U_{\text{max}} \iff \]
\[ \begin{cases} \Delta U_{\text{min}} \leq \Delta U(k) \\ \Delta U(k) \leq \Delta U_{\text{max}} \end{cases} \iff \]
\[ -\Delta U(k) \leq -\Delta U_{\text{min}} \iff \]
\[ \Delta U(k) \leq \Delta U_{\text{max}} \]
\[ I_{m2} \Delta U(k) \leq \Delta U_{\text{new,max}} \quad (3.44) \]
For \( M \) samples, according to Eq. 3.44, we have:

\[
I_{m2,M} \Delta U_M(k) \leq \Delta U_{M,\text{max}} \tag{3.45}
\]

Where

\[
\Delta U_M(k) = \begin{bmatrix}
\Delta U(k) & \Delta U(k+1) & \cdots & \Delta U(k+M-1)
\end{bmatrix}^T
\]

\[
\Delta U_{M,\text{max}} = \begin{bmatrix}
\Delta U_{\text{new, max}} & \Delta U_{\text{new, max}} & \cdots & \Delta U_{\text{new, max}}
\end{bmatrix}^T, \text{ (} M \text{ times } \Delta U_{\text{new, max}} \text{)}
\]

According to the definition of \( \Delta U(k) \) in Eq. 3.34, \( \Delta U_M(k) \) can be expressed as:

\[
\Delta U_M(k) =
\begin{bmatrix}
\Delta U(k) = U(k) - U(k-1) \\
\Delta U(k+1) = U(k+1) - U(k) \\
\vdots \\
\Delta U(k+M-1) = U(k+M-1) - U(k+M-2)
\end{bmatrix}
\begin{bmatrix}
1 & 0 & 0 & \cdots & 0 \\
-1 & 1 & 0 & \cdots & 0 \\
\vdots & \vdots & \ddots & \ddots & \vdots \\
0 & \cdots & 0 & -1 & 1
\end{bmatrix}
\cdot U_{M}(k)
\begin{bmatrix}
-I_m \\
0 \\
\vdots \\
1
\end{bmatrix}

= L_{1,M} \cdot U_{M}(k) + L_{2,M} \cdot U(k-1) \tag{3.46}
\]

Where

\[
L_{1,M} =
\begin{bmatrix}
1 & 0 & 0 & \cdots & 0 \\
-1 & 1 & 0 & \cdots & 0 \\
\vdots & \vdots & \ddots & \ddots & \vdots \\
0 & \cdots & 0 & -1 & 1
\end{bmatrix}
\]

\[
L_{2,M} =
\begin{bmatrix}
-I_m \\
0 \\
\vdots \\
1
\end{bmatrix}
\]

Substituting of Eq. 3.46 into Eq. 3.45 gives:

\[
I_{m2,M} [L_{1,M} \cdot U_{M}(k) + L_{2,M} \cdot U(k-1)] \leq \Delta U_{M,\text{max}}
\]

\[
I_{m2,M} L_{1,M} \cdot U_{M}(k) \leq \Delta U_{M,\text{max}} - I_{m2,M} L_{2,M} \cdot U(k-1) \tag{3.47}
\]

Substituting of Eq. 3.28 into Eq. 3.47 leads to:

\[
I_{m2,M} L_{1,M} \left[ K_{LA} \cdot X(k) + K_{LB} \cdot U_{\text{upc,}M}(k) \right] \leq \Delta U_{M,\text{max}} - I_{m2,M} L_{2,M} \cdot U(k-1)
\]

\[
I_{m2,M} L_{1,M} K_{LB} \cdot U_{\text{upc,}M}(k) \leq \Delta U_{M,\text{max}} - I_{m2,M} L_{2,M} \cdot U(k-1) - I_{m2,M} L_{1,M} K_{LA} \cdot X(k)
\]
Finally, combined Eq.3.40, Eq.3.43 and Eq.3.48 together, it leads:

\[ G \cdot U_{mpc,M}(k) \leq G_{max} \]  

(3.49)

Where

\[ G = \begin{bmatrix} G_1 \\ G_2 \\ G_3 \end{bmatrix}, \quad G_{max} = \begin{bmatrix} G_{1,\text{max}} \\ G_{2,\text{max}} \\ G_{3,\text{max}} \end{bmatrix} \]

End Composing Constraints

Eq.3.29 and Eq.3.49 lead to the following quadratic problem to be solved repetitive at every moment. This technique is called as receding-horizon implementation

\[
\begin{cases}
\min_{U_{mpc,M}(k)} J = \frac{1}{2} U_{mpc,M}^T(k) \cdot H \cdot U_{mpc,M}(k) + X^T(k) \cdot F \cdot U_{mpc,M}(k) \\
G \cdot U_{mpc,M}(k) \leq G_{max}
\end{cases}
\]

(3.50)

There are standard methods to solve this problem.

### 3.3 Nominal stability & online implementation suggestion

When there are no constraints, the value of \( U_{mpc}(k) \) is negligible, which means assumption A.4 is valid for all future time. Therefore, Eq.3.6 can be rewritten as:

\[
X(k+1) = A \cdot X(k) + B \cdot U(k) = A \cdot X(k) + B \cdot K \cdot X(k) = (A + BK) \cdot X(k)
\]

Since \( A \) is convergent and \( K \) is the optimal LQ gain, the convergence of \( A + BK \) follows immediately, see literature [11]. Therefore, the nominal closed-loop stability is guaranteed for any tuning parameters of the MPC.

**Remark**

The guarantee of the nominal closed-loop stability results from the introduction of the infinite prediction horizon in Eq.3.8. The word 'nominal' here emphasizes that there is no model uncertainty and no external unknown disturbance. However, this is not true in reality. In section
3.4 we will discuss the robustness of the designed MPC.

End Remark

However, the MATLAB programming shows possible numerical problems. For example, the $H$ matrix in Eq.3.50 must be symmetric according to the theory described in the last section, while this is not true in the MATLAB programming. Therefore, MATLAB recalculates a new symmetric $H$ according to $H_{\text{new}} = \left( H + H^T \right) / 2$. To avoid the instability of the system caused by this modification, it is recommended to carry out the following security check in a practical implementation.

Security Check

Define:

$$L_{5,M} = \begin{bmatrix} I_m & 0 & \cdots & 0 \end{bmatrix} \quad (M-1 \text{ times } 0)$$

From the definition of $U_{\text{mpc},M}(k)$, we have:

$$U_{\text{mpc}}(k) = L_{5,M} U_{\text{mpc}},M(k)$$  \hspace{2cm} (3.51)

Substituting of Eq.3.30 into Eq.3.51 results in:

$$U_{\text{mpc}}(k) = -L_{5,M} H^{-1} F^T X(k)$$  \hspace{2cm} (3.52)

Substituting of Eq.3.52 into Eq.3.10 results in:

$$U(k) = (K - L_{5,M} H^{-1} F^T) \cdot X(k)$$  \hspace{2cm} (3.53)

Substituting of Eq.3.53 into Eq.3.6 gives:

$$X(k+1) = \left[ A + B \left( K - L_{5,M} H^{-1} F^T \right) \right] \cdot X(k) = S_c \cdot X(k)$$  \hspace{2cm} (3.54)

Where $S_c = A + B \left( K - L_{5,M} H^{-1} F^T \right)$  \hspace{2cm} (3.55)

$S_c$ is convergent if all the eigenvalues of $S_c$ are inside the unit circle. This security check can be executed as the final step in the offline procedure to avoid closed-loop instability of the system due to numerical problems in the programming.

End Security Check

When the constraints are taken into account, Eq.3.50 is not guaranteed to be converged with a solution $U_{\text{mpc},M}(k)$. If this happens, the easiest way is turning off the MPC controller, where a
LQ controller remains in the system, namely $U(k)$ is obtained from $U(k) = K \cdot X(k)$.

However, a LQ controller does not take any constraints into account, which can cause unwanted system failure. Therefore, it is maybe good first to try the following algorithm before turning off the MPC controller.

**Algorithm 1**

STEP 0: Choose a finite control horizon $M$

STEP 1: Solve Eq.3.50

STEP 2: If Eq.3.50 is solvable, exit the algorithm 1 with finding $U_{mpc,M}(k)$

STEP 3: Increase $M$

STEP 4: If $M > M_{\text{max}}$, $\Rightarrow U_{mpc,M}(k) = 0$, exit the algorithm 1

STEP 5: If $M \leq M_{\text{max}}$, go to STEP 1

End Algorithm 1

The reason that $M$ is capped here with $M_{\text{max}}$ is that increasing $M$ makes the computational time longer. It is not allowed that the computational time is larger than the sample time of the system, so the value of $M_{\text{max}}$ is dependent on the CPU of the applied computer system.

An alternative is directly choosing the maximum control horizon $M_{\text{max}}$ in STEP 0 of Algorithm 1. If Eq.3.50 is unsolvable, then turn off the MPC controller immediately for security reason. Further research is required for the implementation of MPC.

**3.4 Robustness of the MPC**

For the robustness of the MPC controller we only focus ourselves on the closed-loop stability of the system. In section 3.2, it is already known that the nominal stability of the MPC is guaranteed. However, this is not true when there are model uncertainties or external unknown disturbances.
In Figure 3.1, $P_{nom}$ is the nominal process model derived from chapter 2, $X$ represents all state variables, $Y$ represents all measurable state variables, $C_p$ represents the relationship between $X$ and $Y$, $d$ represents all unknown variables described in section 2.5, subscript $real$ denotes a true value, subscript $exp$ denotes an expected value and subscript $cor$ denotes an expected value with correction based on the measurement data. In the block Observer, the Kalman gain is chosen as the observer gain, which is derived from the nominal process model. For more detail about Kalman gain see literature [13]. In the block MPC, The control algorithm described in section 3.2 is used. Here $\Delta P$ represents the model uncertainty, which is described in section 2.6.

Remark

The level 1 controller gains are also updated during online processing. For the robustness analysis of the MPC, we will consider them as constant values. This consideration is valid if the "updating" is not in a wrong direction.

End Remark

The aim of the MPC controller is that the closed-loop stability of the system is guaranteed in the worst case. In section 2.6, it is assumed that there are 16 possible worst case candidates. Therefore, all of them will be checked for the designed MPC controller to guarantee closed-loop stability. Besides, the disturbance $d$ is added at a certain moment during the simulation. The result is plotted in figure 5.5 of chapter 5.

3.5 Advantages and disadvantages of MPC

This section describes briefly advantages and disadvantages of the MPC presented in section...
3.2 compared to a Generalized Predictive Control (GPC) from literature [6].

The biggest advantage of the MPC is the guarantee of the nominal stability, which is described in section 3.3. Although in reality that model uncertainty or external disturbance makes this advantage less attractive, we still consider it as a good reason to use this MPC.

However, the presented MPC is based on an Input-Output (IO) model, not on an Increment Input-Output (IIo) model. In literature [13], it is proven that the steady-state error is inevitable by the controller design based on IO model.

Using the transformation procedure presented on page 37 of literature [13], the process model described in chapter 2 can be transformed in its IIo form. However, this transformation will introduce one integrated characteristic in the process model, which makes the system unstable.

For an unstable system, i.e. if the pair of $A$ and $B$ is not stabilizable, Eq. 3.13 can no more be expressed as the discrete Lyapunov equation in Eq. 3.19. Therefore, the infinite prediction horizon technique can no longer be used and the prediction horizon must be bounded. GPC, which is based on finite prediction horizon and IIo model, has the advantage that there is no steady-state error, but its shortage is no more guarantee of the nominal closed-loop stability.

We must make a choice between one of them: guarantee of nominal stability or no steady-state error!

Obviously, in this assignment it is considered that the closed-loop stability is the definitive factor, which means the steady-state behavior here is sacrificed. To make the steady-state error as small as possible, we let the penalty matrix $R$ on the absolute $U$ in Eq. 3.8 be very small. The unwanted result here is that the designed MPC becomes very aggressive, see figure 5.3 in chapter 5. To overcome this, a big measurement noise signal is added to the calculation of the observer gain. This makes the MPC be very careful with the measurement data and results in a suppression of the aggressive characteristic of the MPC (see figure 5.3).

The next chapter will open a new topic, namely the concurrent of MPC, Repetitive Preset Model (RPM), which has been developed by Corus since years ago.

In chapter 5, different simulation results of MPC and RPM will be presented.
Chapter 4 Repetitive Preset Method

This chapter describes briefly how to set up a simulation of RPM for CM11. In section 4.1, the original plan is presented. However, for certain reasons, this original plan can not be realized. Therefore, at first in section 4.2, we deeply analyse the original plan. After it, in section 4.3, an alternative solution will be found to replace the original plan presented in section 4.1. In chapter 5, the simulation result of this alternative will be plotted.

4.1 Original plan

RPM has been developed by Corus Research Department since years ago. From the original plan in this graduate assignment, it is expected that the RPM is available in an executable file, which can be directly implemented in the Matlab simulink model. According to this original plan, there is one RPM.exe file, which recalculates the controller outputs at a sample period of 50 seconds, see figure 4.1 below. Here U and Y are defined as in section 3.2. W is defined as $[\Delta V_1 \cdots \Delta V_3 \ \Delta T_{12} \ \Delta T_{23} \ \Delta T_{34}]^T$, F is defined as $[\Delta F_1 \cdots \Delta F_4]^T$, P is defined as $[\Delta P_1 \cdots \Delta P_4]^T$. Subscript abs indicates an absolute value, subscript pre indicates the offline obtained setup value, a indicates an average value from the measurement data, The "Process Model" is the one obtained from chapter 2, in which external disturbance is added.

![Diagram](image_url)

**Figure 4.1 Original simulation structure of RPM**

**Remark**

When in a variable a subscript abs or pre is used, it means that the sign of the variable

63
is removed. For example:

\[ U = \begin{bmatrix} \Delta V_{1\text{pre}} & \Delta V_{2\text{pre}} & \Delta V_{3\text{pre}} & \Delta T_{12\text{ref}} & \Delta T_{23\text{ref}} & \Delta T_{34\text{ref}} \end{bmatrix}_n \]

\[ U_{\text{abs}} = \begin{bmatrix} V_{1\text{pre}} & V_{2\text{pre}} & V_{3\text{pre}} & T_{12\text{ref}} & T_{23\text{ref}} & T_{34\text{ref}} \end{bmatrix}_n, \text{recalculated during the online process.} \]

\[ U_{\text{pre}} = \begin{bmatrix} V_{1\text{pre}} & V_{2\text{pre}} & V_{3\text{pre}} & T_{12\text{ref}} & T_{23\text{ref}} & T_{34\text{ref}} \end{bmatrix}_n, \text{offline obtained.} \]

End Remark

However, this "RPM.exe" file is not available. To still realize the simulation of the RPM, we need to understand the "RPM.exe" basics. This is discussed in next section.

### 4.2 Mathematical structure of RPM

From literature [4], [14], [15] and [16], we directly present the core algorithm of RPM.exe in the following flowcharts in figure 4.2.

![Figure 4.2 Flowcharts of RPM.exe](image)

Where

\[ \eta = [\eta_1 \ldots \eta_4]_4 \]

\[ \beta = [\beta_1 \beta_2 \beta_3]_3 \]
\[
J = \sum_{i=1}^{3} \left( F_i(U_{obs}, \mu_{new}) - \alpha_i \cdot F_i(U_{obs}, \mu_{new}) \right)^2 \\
+ \sum_{i=1}^{3} \left( P_i(U_{obs}, \mu_{new}) - \beta_{i,new} \cdot P_i(U_{obs}, \mu_{new}) \right)^2 \\
+ \left( h_4 - h_{4,ref} \right)^2 + \left( V_4 - V_{4,prev} \right)^2 \\
F_j(U_{obs}, \mu_{new}) \leq F_{j,max} \\
P_j(U_{obs}, \mu_{new}) \leq P_{j,max} \\
T_{i,i+1,min} \leq T_{i,i+1}(U_{obs}, \mu_{new}) \leq T_{i,i+1,max} \\
h_{i,min} \leq h_i(U_{obs}, \mu_{new}) \leq h_{i,max} \\
\]

\text{(4.1)}

Remark

Eq.4.1 is not obtained from the literature, because there are no documents found in Corus, which describe the currently used objective function in RPM. Eq.4.1 is obtained from the conversation with the Corus researcher Matijs Toose.

Eq.4.1 is based on a static model, it consists of non-linear equations.

End Remark

Define:

\[
Z_{max} = [Z_{1,\text{max}} \quad Z_{2,\text{min}} \quad Z_{2,\text{max}}]^T \\
\]

Where

\[ Z_{1,\text{max}}, \ Z_{2,\text{min}} \ \text{and} \ \ Z_{2,\text{max}} \ \text{are defined as in section 3.1.} \]

With given \( \mu_{\text{new}} \) and \( Z_{\text{max}} \), Eq.4.1 can be expressed as the following:

\[
\begin{bmatrix}
J = f(U_{obs}) \\
g(U_{obs}) \geq 0
\end{bmatrix}
\]

\text{(4.2)}

To minimize the Eq.4.2 as a subject to \( U_{obs} \), it results the following nonlinear programming problem (NLP):

\[
f(U_{obs}) = \min_{U_{obs} \in \Omega} \\
\Omega = \{ U_{obs} \in \mathbb{R} : g(U_{obs}) \geq 0 \}
\]

\text{(4.3)}

According to Matijs Toose, Eq.4.3 is solved by a Sequential Quadratic Programming (SQP) presented in literature [17]:
\[ \nabla f(U_{\text{guess}})^T U_{\text{cor}} + \frac{1}{2} U_{\text{cor}}^T B_{\text{guess}} U_{\text{cor}} = \min, \]  
(4.4) \[ \nabla g_{\lambda_{\text{guess}}} (U_{\text{guess}})^T U_{\text{cor}} + g_{\lambda_{\text{guess}}} (U_{\text{guess}}) = 0 \]

Where

\( U_{\text{guess}} \) is the guess value of \( U_{\text{obs}} \) in Eq.4.3

\( U_{\text{cor}} \) is the correction factor

\( B_{\text{guess}} \) is one symmetric positive semi definite matrix

\( \lambda_{\text{guess}} \supseteq \{ j : j \in \{1, \ldots, m \} \text{ and } g_j (U_{\text{guess}}) \leq 0 \} \) and the definition of \( m \) here is not given by [17].

Solving Eq.4.4 is a very complicated process and it is far outside the concern of this assignment. That is why an alternative is used to approach the algorithm described in figure 4.2. This will be done in next section.

### 4.3 Approximated RPM algorithm

In this section, an alternative is presented, which will be used for the simulation in chapter 5.

For convenience, we call this alternative "Repetitive Preset Method Alternative" (RPMA). Figure 4.3 below shows the general structure of RPMA.

![Diagram of RPMA](image)

Figure 4.3 Simulation structure of RPMA

Obviously, for the core algorithm of RPMA we use the MPC algorithm described in chapter 3. The difference compared to the MPC is that the RPMA outputs are filtered through a low pass filter with transfer function \( \frac{1}{20s + 1} \). Besides, the measurement data are given to the RPMA per 50 seconds instead of 0.2 second. In other words, here we keep the same control algorithm...
and introduce the different methods to adapt the measurement data, one is for the MPC and the other is according to the original RPM in figure 4.1.

In next chapter, the simulation result of the RPMA compared with the MPC described in chapter 3 will be presented.
Chapter 5 Simulations of MPC & RPMA

In this chapter we present the simulation results of both MPC & RPMA controllers. The corresponding files of these simulations can be found in the delivered CD. For using the simulation files there is a brief user manual included in this report, see Appendix B. The default values of the tuning parameters of used MPC and RPMA in this chapter can be found in Appendix A. The values other than their default values will be specified in each section.

Figure 5.1 below shows the structure of the model used for the plant in the simulation. It is based on the process model described in chapter 2. However, there are external disturbances added to the process model. $w_1$ and $w_2$ here have the same values as the one in figure 2.21.

![Diagram of model for the plant in the simulation]

In figure 5.2 below the step disturbance is introduced at a certain time. For the well-known disturbance, namely the entry thickness variation of the strip, we simply take the measurement data of one coil before.

For the controller design, both MPC and RPMA are based on the modified process model presented in figure 2.21. The white noise signal is kept as zeros. The sampling time of the MPC is 0.2 second. In this chapter three sections are included. In section 5.1, we show the control performance of the MPC described in chapter 3 by adjusting its tuning parameters. In section
5.2, the controller performance of the MPC, including its robustness analysis, is presented. Finally, in last section of this chapter, we will see the advantages of the MPC compared to its concurrent, i.e. RPMA described in section 4.3.

![Figure 5.2 Step disturbance and incoming thickness variations](image)

### 5.1 MPC turning parameters

From chapter 3, we know that $R$ is defined as the penalty matrix on MV's. For convenience, we consider the penalty on all MV's equal. With this consideration it is valid that $R = R \cdot I_p$. In figure 5.2, we present the simulation results for two different $R$'s. The red curve is obtained when $R = 1$ and the green curve is obtained by $R = 0.001$. Obviously the red one suffers from the steady state error and the green one not. Therefore, we may draw the following conclusion:

The designed MPC causes steady-state error in a closed-loop system. However, this error can be minimized by keep decreasing $R$. 

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Remark

\( R = 0.001 \) is used as the default value of the MPC.

In figure 5.2 there are measurement noises added to all measurable state variables.

End Remark

However, when \( R \) is small, the MPC becomes very aggressive. To suppress this undesired feature of the MPC, there are measurement noises added to all measurable state variables.

Define \( \nu_1 (k) \) is the measurement noise at time \( k \) on one measurable state variable and \( R_{\nu} = E \{ \nu_1 \cdot \nu_1^T \} \) is defined as the covariance of \( \nu_1 \).

For a number of \( mm \) measurable state variables with \( mm \) uncorrelated measurement noise, we have \( R_{\nu} = \text{diag} \left( \begin{bmatrix} R_{\nu_1} & R_{\nu_2} & \cdots & R_{\nu_{mm}} \end{bmatrix} \right) \). For convenience, we consider the measurement noise on every measurable state variable is equal and rewritten \( R_{\nu} \) as \( R_{\nu} = R_{\nu} \cdot I_{mm} \).

When \( R_{\nu} = 0.001 \), i.e. the measurement noise is almost neglected, the red curve in figure 5.3
below shows a very aggressive MPC. When $R_v = 1$, the green curve here shows that the aggressive property of the MPC is suppressed by sacrificing the total performance. The total performance here means that at every moment, the sum of all the six outputs in the red curve is always less than the sum in the green one. This is difficult to see in the figure, but it is surely true according to the presented MPC algorithm in chapter 3. However, the problem with such an aggressive MPC is when there are any uncertainties in the process model, the closed-loop performance drops quickly. In worst case, it can cause the system to become unstable. To avoid this, namely to improve the robustness of the MPC, we choose $R_v = 1$ as the default parameter of the MPC.

![Figure 5.3 Influence of measurement noise on the aggressive feature of MPC](image)

The prediction horizon of the presented MPC is infinite, the only one remain in the tuning parameters is the control horizon of the MPC. This value is always set up as five in this report. Increasing $M$ makes simulation time longer but decreasing $M$ makes the possibility that the quadratic problem in Eq.3.50 being unsolvable higher. It has been found for the given disturbances in figure 5.1 and the given constraints in figure 5.4 that for $M = 5$, Eq.3.50 is
always solvable and simulation time is acceptable for the programmer.

5.2 MPC performance

This section shows the control performance of MPC against no controller. In figure 5.4, the green curve is for the MPC and the blue one is without any controllers. The red curve in the figure represents the constraints. Obviously from the top six subfigures in figure 5.4, when there is no controller present in the system, any error in the outputs will remain there. However, with the MPC the output error is removed at a settling time of around 5 to 25 seconds. Besides, the MPC takes the constraints into account, see specially the upper-right subfigure in the second figure and the third figure of figure 5.4.

Here a careful reader may find that the outputs $\Delta P_1 - \beta_1 \Delta P_4$ and $\Delta P_3 - \beta_3 \Delta P_4$ are suffering from a steady state error from time 560 to 580. The reason here is the restriction of $\Delta T_{23}$ during this time interval, see the upper-right subfigure of the third figure. This means further research is required to investigate the correctness in calculation of the constraints. Besides, the constraints here are faked; actual constraints are required from the factory.

![Figure 5.4](image-url)
The robustness of the MPC is presented in figure 5.5 below. Here the MPC algorithm is always
based on the nominal process model. In the red curve, the nominal process model is used for the plant model. The green curves represent the plant model, in which the process model parameters are obtained by the 16 worst case candidates described in section 2.7.

![Diagram](image)

**Figure 5.5** Robustness of the MPC

From figure 5.5 we see that the closed-loop stability of the system is guaranteed. Besides, the control performance here is also not poor, except on the outputs \( \Delta F_1 - \alpha \Delta F_4 \) and \( \Delta P_3 - \beta \Delta P_4 \). These two outputs are more sensitive to the variations in the model parameters.

### 5.3 MPC VS RPMA

The described RPM in section 4.2 has been developed by Corus Research department since years ago. In Corus, it is planned to directly implement RPM in the near future without any simulations to predict how the RPM will work. Since the "RPM.exe" in figure 4.1 is not available, one alternative called RPMA is used to approach the function of "RPM.exe". In this section, we will compare the control performance of MPC against RPMA.
In Figure 5.6, the dark green curve shows the control performance of the MPC, the blue curve is the one of the RPMA. Obviously for the disturbance rejection here, MPC is an absolute winner. Besides, RPMA suffers from a big steady-state error!

Next chapter is the last chapter of this report, a summary of this graduate assignment with acknowledgements will be presented.
Chapter 6 Conclusions & Acknowledgements

The research goal of this project was to design a MPC controller which is able to keep a constant roll force and motor power ratio between the stands of CM11 during online process. One linear dynamic process model was presented for CM11. This model is validated around one operation point, i.e. when the cold mill is at its run speed. Further research is required to provide a model, which is also valid around the start and end point of cold rolling process.

The mathematic algorithm of the designed MPC controller was discussed; it gives guarantee of the nominal stability. In this assignment, it is assumed that the worst case of process model is related to the worst case in the value of friction coefficients. For the assumed worst case, the closed-loop stability has been proven to be guaranteed. However, if the assumption is not valid, further research is required to estimate the worst case in the model uncertainty more accurately.

The structure of the original RPM, which is not simulated but will be directly implemented in the near future, was presented. Here one alternative called RPMA is discussed, which is used to approximate the function of the original RPM.

Both the simulation results of MPC and RPMA were presented. It is illustrated how to influence the control performance by tuning the controller parameters of MPC, i.e. MPC is a very user-friendly controller. Besides, the simulation result indicates that the designed MPC provides a better control performance than the RPM. Therefore, it is expected that the strip quality by using MPC will be better than the RPM.

There are some other options, which people can acquire by implementing MPC in a cold rolling process. The first one here I like to mention is that MPC is also a good reference tracker. Imagine that in the future the strip thickness of one coil is varied during online process; see the sketch below.
This can be simply done by expanding the MPC objective function. The suggested extra input for the MPC is the motor speed at stand 4. In contrast to it, RPM is not able to do it directly, since it can not guarantee the quality in the strip form when the end thickness is varying.

The second extra option of the MPC is 'helping' the accuracy of the level 1 controller. In an extreme case, the whole level 1 controller system can be replaced by the MPC. This idea is not meaningless since in chapter 2 it is clearly seen that the design of level 1 controllers are all based on lots of simplifications. In other words, their design is originally based on several SISO systems, while in fact the cold rolling process is clearly a complex MIMO system. However, the replacement of level 1 controller by MPC is really a big step and requires deeply further research on it.

I end this assignment with some acknowledgements. First of all, I really appreciate the valuable comments from my direct supervisor at Corus, Jan Schuurmans. Secondly, I would like to thank my professor at TUE, Ton Backx, for the good comments on the area of MPC. Besides, I can not deny saying acknowledgements to my roommate at Corus during the whole project, Jacob Anema. He helped me solve lots of little problems and the daily talks with him inspired me in many ways.

The last people I would like to thank are my friends and family for their support during this project. Especially my sister for her help and inspiration and my girlfriend Jiayue Du, who helped me through times of doubt and uncertainty with all the patience and love one could hope for.

This thesis could not have been written without the above-mentioned people.

Hua Zhang
Reference


Mill-balance Control Technique for Tandem Cold Mill, ISIJ International, 
Vol.42, No.6, pp. 624-628, 2002


## Appendix A Process Model Parameters

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<td>1.6929</td>
<td>Meter per second per Mega Newton</td>
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<td>Value</td>
<td>Unit</td>
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<td>MPC sampling time</td>
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<tr>
<td>$\epsilon$</td>
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</table>
Appendix B Simulation programmer manual

To make the simulations presented in this report, take the following steps:

1. Copy all the contents of attached CD to your hard drive
2. Start the MATLAB environment
3. Go to the folder, in which the contents of the CD are included
4. Run the file 'firstrun.m'

Functions descriptions

<table>
<thead>
<tr>
<th>Function</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>init</td>
<td>Initialisation the operation point and get the values of all offline parameters including model parameters, quadratic parameters, etc.</td>
</tr>
<tr>
<td>sen</td>
<td>To calculate the values of the model parameters</td>
</tr>
<tr>
<td>rdata</td>
<td>Read the data source obtained from the factory</td>
</tr>
<tr>
<td>mpcdia.m</td>
<td>Make a diagonal matrix</td>
</tr>
<tr>
<td>Kmmpc.m</td>
<td>S-function, used to solve quadratic problem online</td>
</tr>
<tr>
<td>RPMaverage.m</td>
<td>Average measurement data every 5 seconds</td>
</tr>
<tr>
<td>Plotdata.m</td>
<td>Used to plot the simulation in figures</td>
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</table>

For more detail it is recommended to read the comments in the mentioned files.