MASTER

Analysis of infinite arrays of broadband antennas

Vreeken, I.A.

Award date:
1996

Link to publication
Analysis of infinite arrays of broadband antennas

door

I.A. Vreeken

EM-1-96
Analysis of infinite arrays of broadband antennas

I.A. Vreeken

February 14, 1996
Acknowledgements

This research has been performed in fulfilment of the requirements for the degree of Master of Science at the University of Technology (T.U.E.). It has been carried out at Holland Signaal Apparaten (H.S.A.) in Hengelo. The author wishes to thank both Dr.Ir. A.B.Smolders at H.S.A. and Dr. M.E.J.Jeuk at the T.U.E. for their many contributions and patience.
Abstract

Much research has already been done on suitable antenna-elements in phased-array antennas. Next to microstrips tapered-slot antennas seem to be very promising at this moment. A tapered-slot antenna consists of one or two pairs of protruding strips above a ground plane and possibly on a dielectric slab, where the tapering of the strips can be varied. Some of these types are the Constant-tapered Slot Antenna, Linearly-tapered Slot Antenna and the exponentially-tapered Antenna (Bunny-ear Antenna and Vivaldi Antenna).

In the early days of phased-array research behaviour of the array was modelled by just adding up the contributions of all elements. In todays array-theory mutual coupling is included, which is necessary for arrays with small element-distances. Mutual coupling causes elements within the array to behave differently and therefore has a strong influence on the overall performance. An important simplification is often made if the infinite-array concept is used. In this concept the array is assumed to be so large in terms of the wavelength that adding elements at the outside of the array will not influence the behaviour of the elements near the center. If the array is assumed to contain an infinite number of elements mathematics are significantly reduced.

The hybrid Greens function/Method of Moments is thought to be the best method to implement these concepts. Moreover it is a well-known method.

In the course of the research standard methods for arrays of plane elements such as microstrip antennas appeared to be less suitable for protruding antennas. The Equivalence Theorem was therefore used to obtain a structure in which calculations are made easier, on the cost of extra fictitious magnetic currents. After calculation of the Greens functions and application of the equivalence theorem boundary conditions are constructed and tested. The resulting matrix equation is then solved for the unknown current on the antenna. From this current input impedance, reflection coefficient and radiation pattern are then derived.

Initially the aim of this research was to develop and implement a suitable model to analyse protruding antennas in general and the bunny-ear in particular, all for the case of antennas in air.

A model has been developed and implemented. It has been validated with an infinite array of monopoles, strongly resembling results in literature. At this stage results for the bunny-ear cannot be given. The developed software is expected to be easily extended to other structures. This study must be considered as a starting study, as a basis for further research. Follow-up projects are necessary to generate results of practical use.
Contents

List of Figures

1 General Introduction

1.1 Phased-array antennas ........................................ 6
  1.1.1 The infinite-array concept ................................. 7
1.2 Numerical methods for the analysis of phased-array antennas .... 7
  1.2.1 Moment Method (MM) ........................................ 8
  1.2.2 Other methods .............................................. 8
  1.2.3 Choice of method .......................................... 9
1.3 Strategy ....................................................... 9

2 General Greens functions for infinite arrays ........................ 10

2.1 Introduction ................................................... 10
2.2 Single electric dipoles in air above a conducting ground-plane ... 11
  2.2.1 Maxwell's equations ......................................... 11
  2.2.2 Spectral Greens functions ............................ 13
2.3 An infinite array of electric dipoles in air above a conducting ground-plane .... 16
  2.3.1 Floquet modes in a rectangular grid .................. 17
2.4 An infinite array of magnetic dipoles in air above a conducting ground-plane .... 19

3 Greens functions for an infinite array of protruding antennas in air above a conducting ground-plane .... 23

3.1 Introduction .................................................. 23
3.2 The equivalence theorem .................................... 23
3.2.1 Why the equivalence theorem .................................................. 23
3.2.2 Step-wise application of the equivalence theorem ..................... 25
3.3 Greens functions in the equivalent structure ................................ 26
  3.3.1 Magnetic currents ............................................................... 27
  3.3.2 An y-directed electric dipole ................................................. 30
  3.3.3 A z-directed electric dipole .................................................. 34
  3.3.4 Check of continuity of the electric field in the aperture ............ 36

4 Formulation of the moment method .................................................. 38
  4.1 Introduction ............................................................................. 38
  4.2 Formulation of the remaining boundary-conditions ....................... 38
    4.2.1 Two-dimensional electric and magnetic field dyadic Greens functions .. 38
    4.2.2 Aperture and fin fields .................................................... 39
    4.2.3 The remaining boundary conditions ................................... 39
  4.3 Expanding and testing ............................................................. 40
    4.3.1 The matrix equation ......................................................... 40
  4.4 The impedance matrix ............................................................. 42
    4.4.1 The self-impedance submatrix ......................................... 43
    4.4.2 The aperture-fin coupling submatrix .................................. 44
    4.4.3 The fin-aperture coupling submatrix .................................. 44
    4.4.4 The self-admittance submatrix ......................................... 45
  4.5 The Feed model and excitation matrix ....................................... 46
  4.6 Expansion functions and testing functions .................................. 47
    4.6.1 Expansion functions ......................................................... 47
    4.6.2 Testing functions .............................................................. 51

5 Intermediate results ........................................................................... 54
  5.1 Convergence ............................................................................ 55
  5.2 Calculating efficiency .............................................................. 56
  5.3 Numerical stability .................................................................... 58
  5.4 Software users guide .................................................................. 59
CONTENTS

6 Conclusions and recommendations 64

bibliography 64

A Variation of constants applied to the inhomogeneous spectral domain Helmholtz equation 68

B Greens functions for airdielectric formulation 71

C Matrix elements for use in software 75

C.1 The impedance matrix ...................................................... 75
    C.1.1 The self-impedance submatrix ................................... 75
    C.1.2 The aperture-fin coupling submatrix ........................... 76
    C.1.3 The fin-aperture submatrix ...................................... 77
    C.1.4 The self-admittance submatrix ................................. 78

C.2 The source vector ......................................................... 78

D Integrals 80

E Literature research report 82
## List of Figures

1.1 Several types of tapers ..................................................  6
1.2 An array of bunny-ear antennas in air above a conducting ground-plane ....  7

2.1 A single electric dipole in y-direction in air above a conducting ground-plane . 11
2.2 Imaginary dipoles .......................................................... 16
2.3 An array of arbitrarily directed dipoles .................................. 17

3.1 A unit cell with two bunny-ears ........................................... 24
3.2 The equivalence theorem .................................................. 24
3.3 A general aperture problem ................................................. 25
3.4 Equivalent internal and external magnetic sources ......................... 25
3.5 The equivalence theorem applied to an array of protruding antennas ........ 26
3.6 A parallel-waveguide model ............................................... 31
3.7 The active boundary condition ............................................ 33

4.1 The delta-gap electric field feed model .................................. 47
4.2 Piecewise Constant function .............................................. 48
4.3 Piecewise Linear function ................................................ 48
4.4 Piecewise Sinusoidal function ............................................ 49
4.5 Rectangular modelling of electric fin current in bunny-ear antennas .......... 50
4.6 Tilted sinusoidals in modelling of currents in bunny-ear antennas .......... 50
4.7 Subdomain modes for the electric current ................................ 52
4.8 Entire-domain modes for the aperture current ............................ 52

5.1 Unit-cell with two monopoles ............................................. 54
<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.2</td>
<td>Magnitude of the reflection coefficient</td>
<td>55</td>
</tr>
<tr>
<td>5.3</td>
<td>Phase of the reflection coefficient</td>
<td>56</td>
</tr>
<tr>
<td>5.4</td>
<td>The Voltage Standing Wave Ratio</td>
<td>57</td>
</tr>
<tr>
<td>5.5</td>
<td>Convergence with the number of monopole expansion modes</td>
<td>58</td>
</tr>
<tr>
<td>5.6</td>
<td>Convergence with the number of monopole expansion modes</td>
<td>59</td>
</tr>
<tr>
<td>5.7</td>
<td>Convergence with the number of aperture modes in y-direction</td>
<td>60</td>
</tr>
<tr>
<td>5.8</td>
<td>Convergence with the number of aperture modes in z-direction</td>
<td>61</td>
</tr>
<tr>
<td>5.9</td>
<td>Convergence with the number of parallel-plate modes in y-direction</td>
<td>61</td>
</tr>
<tr>
<td>5.10</td>
<td>Convergence with the number of parallel-plate modes in y-direction</td>
<td>62</td>
</tr>
<tr>
<td>5.11</td>
<td>Convergence with the number of parallel-plate modes in z-direction</td>
<td>62</td>
</tr>
<tr>
<td>5.12</td>
<td>Convergence with the number of parallel-plate modes in z-direction</td>
<td>63</td>
</tr>
</tbody>
</table>
Chapter 1

General Introduction

1.1 Phased-array antennas

Phased-array antennas have been studied intensively in the last decades. In naval systems they offer many advantages in comparison with conventional scanning systems. The most important advantages are simultaneous scanning, multi-function operation and fewer moving parts.

In designing arrays one wishes to combine maximum scan-range with maximum bandwidth and a symmetrical beam. The bandwidth of micro-strip antennas has gone up from a few percent in the beginning to around 15-25%, using techniques such as stacked micro-strips (multi-layer). Tapered-slot antennas have proved to have an even larger bandwidth in a phased-array environment. They consist of a dielectricum with two strips printed on one side (single-sided) or two on every side (double-sided). These strips can have various shapes. Among these are the linearly tapered-slot antenna (LTA), constant width slot-antenna (CSWA) and Vivaldi-antenna (exponential tapering).

![Constant tapering, Linear tapering, Exponential tapering, Bunny ear](image)

Figure 1.1: Several types of tapers

In this report a start is made with the investigation of arrays of broadband elements in general and bunny-ear antennas in particular, with the help of a relatively novel method (equivalence). The bunny-ear antenna is a variation on the Vivaldi antenna concept and has a bandwidth of approximately 50% in an array environment. Both ends of the bunny-ear antenna diverge with a bunny-ear shaped bending.

Shown in 1.1 and 1.2 are some tapered elements and an array of tapered-slot antennas.

In elementary theory of antenna-arrays [3] mutual coupling between the elements was neglected. This is only a good approximation when the spacing between the elements is far more than half the wavelength. In most practical situations this is not the case. In arrays, currents and
fields differ from element to element in magnitude and phase and these differences vary as a function of frequency, element spacing, element size and scan-angle. The reason for this is the mutual interaction between elements, better known as mutual coupling. In large arrays it can even result in scan-blindness which corresponds to zeros in the radiation pattern. In that case all the power is reflected back from antenna to the feed and no power is radiated. Secondly mutual coupling can result in grating lobes appearing in the real (visible) part of the scan range. Transmitted/received power is then distributed over main lobe and grating lobes, less power is radiated/received in the main direction [14]. Grating lobes can appear in visible space when the quotient of spacing and wavelength is between 0.5 and 1.0. These problems can be minimized for one frequency by optimising the geometry but in large-bandwidth arrays the superposition of phases changes significantly through the bandwidth and problems can be expected.

1.1.1 The infinite-array concept

Mutual coupling in electrically large phased-array antennas can be analysed efficiently by using the infinite-array-model. In the infinite array model one assumes that adding more elements at the outer edge of the array will not have a significant influence on the elements near the center of the array. When the array is illuminated uniformly, every single element in the center is influenced by mutual coupling in the same way and edge-effects do not have to be accounted for. The array can be treated to have infinite dimensions with a periodic structure on which known analytic and numerical procedures can be applied. Only a unit cell consisting of one element needs to be examined with the help of appropriate boundary-conditions. The question is which of the following numerical methods best fits this concept of infinite arrays.

1.2 Numerical methods for the analysis of phased-array antennas

In simple field problems the classic differential equations are replaced by differential quotients. In most cases the functions are defined for the entire region of interest. For more complex electro-magnetic problems a large number of sophisticated solving methods is available. These methods have a lot in common or are derived from eachother. In [5] an overview is given.
1.2. NUMERICAL METHODS FOR THE ANALYSIS OF PHASED-ARRAY ANTENNAS

1.2.1 Moment Method (MM)

In [3] Amitay defines MM as general methods to reduce an operator equation to a matrix equation. In applications to arrays MM is used in combination with the Greens functions technique. In this hybrid Moment Method/Greens functions method first the Greens functions are determined. In electro-magnetism Greens functions are actually fields or potential functions arising from an elementary dipole source. The next step is to add all contributions of the array-elements, the Floquet-modes. Then MM is used: The element surface is divided in subdomains and all contributions to the vector potential are added. (For complicated structures as ours subdomains are preferred but simple geometries like for example wire antennas need only one domain). On each domain a current pattern with unknown magnitude is chosen. Next we impose boundary conditions for the total fields. They form sets of equations which have to be solved. Now the inner product of these boundary conditions and several weighting functions is evaluated, since the boundary conditions have to be satisfied on each subdomain. This is a variational method. For fast convergence the set of expansion functions can be used as weighting functions too (the Galerkin method). From the resulting matrix equation one can then extract the characteristic impedance and reflection coefficients. Because the reflection depends on scan angle, frequency etc. as mentioned before these coefficients are called active.

The disadvantage of using subdomain basis functions is the enormous amount of equations to be solved numerically. A disadvantage is the restricted modelling accuracy. MM/Greens functions combines exact analytic expressions for the entire domain with the ability to handle relatively complex structures. Structures in which the direction of the currents is perpendicular to the ground-plane have to be converted to 2 dimensions with the equivalence theorem. This is because the Greens function technique uses a 2D Fourier transform. MM/Greens functions technique is a very suitable method here because the infinite array model can easily be implemented.

A variation on MM is the Point-Matching Method (PMM) in which a set of dirac weighting functions is used to reduce the amount of unknowns and minimize the computational effort. Other combinations such as MM with the Geometrical Theory of Diffraction exist but are only used to calculate coupling between wires and large objects.

1.2.2 Other methods

1.2.2.1 The Finite Element Method (FEM)

In each region (possibly different of shape) a basis function is chosen (expansion) and the governing equation (for example Helmholtz) for the entire region is then converted into a matrix (a system of equations for each part which are called functionals) on which a variational principle can be applied. As a function of the expansion factors a minimum energy of the system is determined, i.e. the functional is minimized. As one usually has an idea of the function's shape the expansion functions are chosen in that way. FEM belong to the class of Ritz-Galerkin methods. FEM is very promising because complex structures can easily be implemented in software, although much computational effort is required. Implementation of the necessary boundary conditions of the infinite array model is difficult and little has been written on this subject.
1.3. STRATEGY

In the last years the combination of FEM/MM has scarcely been studied. It combines advantages of both methods but no actual research has been done in the field of arrays.

1.2.2.2 The Mode-Matching Method (MMM)

This method uses expansion functions in the entire domain. It doesn’t explicit uses Greens functions but source-fields. In our structure this method can not be used.

1.2.2.3 The Finite Difference Time Domain Method (FDTD)

It uses a grid in time and place. Since antenna design a frequency-domain problem this method is not efficient. Moreover, this method only suits relative simple geometries.

1.2.3 Choice of method

From the above analyses it was concluded that the most suitable method in this survey is the hybrid MM/Greens functions method.

1.3 Strategy

The structure of this report is as follows. Chapter 2 contains a derivation of the general vector potential Greens functions for elementary magnetic and electric sources above a conducting ground plane. In chapter 3 the vector potential Greens functions, which are of practical use for protruding structures are derived with the help of the equivalence, while in chapter 4 the Method of Moments is applied, thereby giving an overview of existing expansion and testing methods and making a choice. In chapter 5 some intermediate results with an array of simple monopole are given and remarks are made on numerical aspects such as numerical stability, calculating efficiency and convergence. A little software guide is included. The appendices contain the derivation of the Greens function particular solutions, the derivations of the Greens functions for the electric and magnetic fields, derivation of expressions for matrix elements and integrations which can be implemented in software.
Chapter 2

General Greens functions for infinite arrays

2.1 Introduction

The first thing that has to be done is the derivation of the dyadic Greens function for an infinite array of horizontal and vertical dipoles above a conducting ground plane in air. The dyadic Greens function is the vector potential caused by an arbitrary combination of elementary dipoles in x-, y-, and z-direction. Once we know the dyadic Greens function we can divide the surface in subdomains and integrate the product of the subdomain current and the Greens functions to obtain the total vector potential. Then the fields are also known because they can be expressed in this total potential. The electric vector potential created by an arbitrarily shaped antenna consisting of electric dipoles in the space domain can be written as,

\[ \vec{A}(\vec{r}, \vec{r'}) = \int_S \vec{G}(\vec{r}, \vec{r'}) \cdot \vec{J}(\vec{r'}) ds', \]  

(2.1)

where \( \vec{J}(\vec{r'}) \) is the surface current density on the antenna. \( ds' \) is the element surface \( dx dy dz \) and \( \vec{r} \) is the point of observation and \( \vec{r'} \) is a point on the source. The dyadic Greens function \( \vec{G} \) for a single arbitrarily directed electric dipole in space domain is written as,

\[ \vec{G} = \begin{bmatrix} G_{xx}^e & G_{xy}^e & G_{zz}^e \\ G_{yx}^e & G_{yy}^e & G_{yz}^e \\ G_{zx}^e & G_{zy}^e & G_{zz}^e \end{bmatrix}, \]  

(2.2)

where for example \( G_{xy}^e \) denotes the x-component of the vector potential caused by a dipole in y-direction.

Similar expressions can be written for elementary magnetic sources \( \vec{M} \). The total vector magnetic potential \( \vec{F} \) created by an arbitrarily directed antenna consisting of magnetic dipoles is then,
2.2. SINGLE ELECTRIC DIPOLES IN AIR ABOVE A CONDUCTING GROUND-PLANE

\[ \mathcal{F}(\vec{r}, r') = \int_S \mathcal{G}^m(\vec{r}, \vec{r}') \cdot \mathcal{M}(\vec{r'}) ds', \quad (2.3) \]

in which the dyadic Greens function for magnetic sources \( \mathcal{G}^m \) has the same form as (2.2).

In 2.2 the Greens functions will be given for single electric dipoles. In section 2.3 we will do the same for an array. Magnetic dipoles and arrays will be examined in section 2.4.

2.2 Single electric dipoles in air above a conducting ground-plane

In this section Greens functions of single dipoles above a conducting ground-plane and pointing in the x-, y- or z-direction are investigated. Shown is a picture of an x-directed dipole.

![Figure 2.1: A single electric dipole in y-direction in air above a conducting ground-plane](image)

2.2.1 Maxwell's equations

A \( e^{jut} \) dependence is assumed (steady state solutions). In the presence of an electric current density and charge density \( \rho_e \) in air the Maxwell equations in the space domain are,

\[
\begin{align*}
\nabla \times \vec{E} &= -j\omega \mu_0 \vec{H} \\
\nabla \times \vec{H} &= j\omega \varepsilon_0 \vec{E} + \vec{J} \\
\n\nabla \cdot \vec{E} &= \frac{\rho_e}{\varepsilon_0} \\
\n\nabla \cdot \vec{H} &= 0,
\end{align*}
\]

(2.4)

where \( \omega \) is the radial frequency, \( \varepsilon_0 \) and \( \mu_0 \) are the permittivity and permeability in air and \( \vec{J} \) is the electric current density. As described in [11] a magnetic vector potential \( \vec{A} \) can be chosen such that,
2.2. SINGLE ELECTRIC DIPOLES IN AIR ABOVE A CONDUCTING GROUND-PLANE

\[ \mathcal{H} = \frac{1}{\mu_0} \nabla \times \mathbf{A} \]

\[ \mathcal{E} = -\frac{j\omega}{k_0^2} \left( k_0^2 \mathbf{A} + \nabla \left( \nabla \cdot \mathbf{A} \right) \right), \]  

(2.5)

with \( k_0 = \omega \sqrt{\varepsilon_0 \mu_0} \) denoting the wavenumber in air. We can write (2.5) in rectangular components,

\[ \mathcal{H} = \frac{1}{\mu_0} \begin{bmatrix} \partial_y A_z - \partial_z A_y \\ \partial_x A_y - \partial_y A_x \\ \partial_z A_x - \partial_x A_z \end{bmatrix} \]

(2.6)

\[ \mathcal{E} = -\frac{j\omega}{k_0^2} \begin{bmatrix} (k_0^2 + \partial_x^2) A_x + \partial_x \partial_y A_y + \partial_x \partial_z A_z \\ \partial_y \partial_x A_x + (\partial_y^2 + \partial_z^2) A_y + \partial_y \partial_z A_z \\ \partial_z \partial_x A_x + \partial_z \partial_y A_y + (\partial_x^2 + \partial_y^2) A_z \end{bmatrix} \]

(2.7)

To facilitate the calculation of the Greens function Fourier transform will be used. It transforms expressions from the time domain to the spectral domain. In this report the Fourier transform and its inverse are respectively defined as,

\[ F(k_x, k_y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} F(x, y) e^{jk_x x} e^{jk_y y} dx dy \]

(2.8)

\[ F(x, y) = \frac{1}{4\pi^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} F(k_x, k_y) e^{-jk_x x} e^{-jk_y y} dk_x dk_y \]

(2.9)

We use a transform in the x and y direction because in these directions the geometry of the array is of infinite length. Some important rules by thumb in transforming are,

- The Fourier transform of \( \partial_x F \) is \( -jk_x F \) and the same goes for \( y \),
- The Fourier transform of \( \delta(x - x') \delta(y - y') \delta(z - z') \) is \( e^{jk_x x'} e^{jk_y y'} \delta(z - z') \).

Fourier transformation applied to equation (2.7) yields,

\[ \bar{H} = \frac{1}{\mu_0} \begin{bmatrix} -jk_y A_z - \partial_z A_y \\ \partial_x A_y + jk_x A_z \\ -jk_x A_y + jk_y A_x \end{bmatrix} \]

(2.10)

\[ \bar{E} = -\frac{j\omega}{k_0^2} \begin{bmatrix} (k_0^2 - \partial_x^2) A_x - k_x k_y A_y - jk_x \partial_z A_z \\ (k_0^2 - \partial_y^2) A_y - k_y k_x A_x - jk_y \partial_z A_z \\ (k_0^2 + \partial_x^2) A_x - jk_x \partial_z A_x - jk_y \partial_z A_y \end{bmatrix} \]

(2.11)
2.2. SINGLE ELECTRIC DIPOLES IN AIR ABOVE A CONDUCTING GROUND-PLANE

2.2.2 Spectral Greens functions

Substitution of equation (2.5) in equation (2.4) \[11\] results in the inhomogeneous Helmholtz equation in the space domain,

\[ \nabla^2 \vec{A} + k_0^2 \vec{A} = -\mu_0 \vec{J}, \]  
(2.12)

with \( \nabla^2 = \partial_x^2 + \partial_y^2 + \partial_z^2 \).

In a homogeneous medium that extends to infinity the vector potential caused by a dipole has only one component, in the direction of the dipole. In the next paragraph it will be shown that in the presence of a superconducting ground-plane only one component is still sufficient to satisfy the boundary condition on the ground-plane.

2.2.2.1 An x-directed dipole

An infinitesimal horizontal electric dipole with unit strength in \( \vec{r} = \vec{r}' \) is investigated.

\[ \vec{J} = \delta(x - x')\delta(y - y')\delta(z - z')\vec{e}_x, \]

in which \( \vec{e}_x \) is the unit vector in x-direction. The Helmholtz equation (2.12) reads,

\[ \nabla^2 \vec{A} + k_0^2 \vec{A} = -\mu_0 \delta(x - x')\delta(y - y')\delta(z - z')\vec{e}_x \]  
(2.13)

Writing out the x-component and applying Fourier transform we get,

\[ \partial_x^2 A_x^e + k_y^2 A_x^e = -\mu_0 e^{jk_x z'} e^{jk_y y'} \delta(z - z'), \]  
(2.14)

with \( k^2 = k_0^2 - k_x^2 - k_y^2 \). Using equation (2.1),

\[ A_x = G_{xx}^e, \]  
(2.15)

so \( A_x = G_{xx} \). The vector potential \( \vec{A} \) caused by an elementary current source is the Greens function \( G \). We have,

\[ \partial_z^2 G_{xx}^e + k^2 G_{xx}^e = -\mu_0 e^{jk_x z'} e^{jk_y y'} \delta(z - z'), \]  
(2.16)

A general solution of equation (2.16) is,

\[ G_{xx}^e = \mu_0 \left[ Ce^{-jk_x z} + \frac{1}{2jk} e^{-jk|z - z'|} e^{jk_x z'} e^{jk_y y'} \right], \]  
(2.17)
2.2. SINGLE ELECTRIC DIPOLES IN AIR ABOVE A CONDUCTING GROUND-PLANE

in which the first part of the term between brackets is the homogeneous solution. The complement of this solution has been made zero because of the Sommerfeld radiation condition. The second term between brackets is the particular solution and can be obtained by variation of constants (see Appendix A).

The tangential electrical field on the ground plane is zero so the following boundary condition has to be satisfied,

\[ \vec{e}_z \times \vec{E} = 0 \mid_{z=0}, \quad (2.18) \]

with \( \vec{e}_z \) the unit vector in z-direction. Using (2.11) and the fact that \( A_x = G^e_{xx} \) we obtain,

\[ G^e_{xx} = 0 \mid_{z=0} \quad (2.19) \]

Bearing in mind that \( z = 0 > z' \) a solution for the constant \( C \) is,

\[ C = -\frac{1}{2jk} e^{-jkz'} \quad (2.20) \]

Substitution in equation (2.17) yields,

\[ G^e_{xx} = \frac{\mu_0}{2jk} [e^{-jk|z-z'|} - e^{-jk(z+z')}]|e^{jkz}e^{jkz'} \quad (2.21) \]

It is allowed to take the absolute value of the exponent in the second part of this formula, because \( z \geq -z' \); both observation points \( z \) and source-points \( z' \) are in the positive halfspace.

\[ G^e_{xx} = \frac{\mu_0}{2jk} [e^{-jk|z-z'|} - e^{-jk|z+z'|}] e^{jkz}e^{jkz'}, \quad (2.22) \]

2.2.2.2 y- and z-directed dipoles

The derivation for a horizontal dipole in y-direction with unit strength is identical to the one for an x-directed dipole. We get

\[ G^e_{yy} = \frac{\mu_0}{2jk} [e^{-jk|z-z'|} - e^{-jk(z+z')}]|e^{jkz}e^{jkz'} \quad (2.23) \]

For the z-directed dipole with unit strength the equation,

\[ \partial_z^2 G^e_{zz} + k^2 G^e_{zz} = -\mu_0 e^{jkz}e^{jkz'} \delta(z-z'), \quad (2.24) \]

has general solutions,

\[ G^e_{zz} = \mu_0 [Be^{-jkz} + \frac{1}{2jk} e^{-jk|z-z'|}] e^{jkz}e^{jkz'} \quad (2.25) \]
2.2. SINGLE ELECTRIC DIPOLES IN AIR ABOVE A CONDUCTING GROUND-PLANE

With boundary condition (2.18) and equations (2.11) and (2.25) we get,

$$\partial_z G^{e}_{zz} = 0 \mid_{z=0},$$  \hspace{1cm} (2.26)

and with regard to the relative position of $z'$ to $z$,

$$-jk\mu_0 [Be^{-jkz} - \frac{1}{2jk}e^{-jk(z'-z)}] = 0 \mid_{z=0}$$

$$B = \frac{1}{2jk}e^{-jkz'}$$ \hspace{1cm} (2.27)

The expression for $G^{e}_{zz}$ is,

$$G^{e}_{zz} = \frac{\mu_0}{2jk} [e^{-jk|z-z'|} + e^{-jk|z+z'|}] e^{jkz} e^{jk'y'}$$ \hspace{1cm} (2.28)

The dyadic Greens function in spectral domain becomes,

$$\overline{G} = \begin{bmatrix} G^{e}_{xx} & 0 & 0 \\ 0 & G^{e}_{yy} & 0 \\ 0 & 0 & G^{e}_{zz} \end{bmatrix} = \mu_0 e^{jkx'} e^{jk'y'} \overline{G}^{e}_{r},$$ \hspace{1cm} (2.29)

with,

$$\overline{G}^{e}_{r} = \begin{bmatrix} G^{e}_{x} & 0 & 0 \\ 0 & G^{e}_{y} & 0 \\ 0 & 0 & G^{e}_{z} \end{bmatrix},$$ \hspace{1cm} (2.30)

and,

$$G^{e}_{x} = \frac{1}{2jk} [e^{-jk|z-z'|} - e^{-jk|z+z'|}]$$

$$G^{e}_{y} = \frac{1}{2jk} [e^{-jk|z-z'|} - e^{-jk|z+z'|}]$$

$$G^{e}_{z} = \frac{1}{2jk} [e^{-jk|z-z'|} + e^{-jk|z+z'|}]$$

From these formulas it is observed that the vector potential caused by a dipole above a superconducting ground plane could be regarded as the vector potential created by two dipoles, a real dipole in $\vec{r} = \vec{r}'$ and an imaginary one in $\vec{r} = -\vec{r}'$.

The difference in sign between $G^{e}_{xx}$ and $G^{e}_{zz}$ can be explained as follows [4]: To satisfy the boundary condition for a horizontally-directed dipole an imaginary horizontal dipole must be
2.3. AN INFINITE ARRAY OF ELECTRIC DIPOLES IN AIR ABOVE A CONDUCTING GROUND-PLANE

Figure 2.2: Imaginary dipoles

directed 180 degrees out of phase with it (a reflection coefficient of -1), for a vertically directed dipole it must be placed in phase (reflection coefficient is +1). This is shown in figure 2.2.

Another way to calculate these Greens functions uses active boundary conditions for the region of the dipole in combination with a more general solution for $G_{xx}, G_{yy}$ and $G_{zz}$ [17].

For an arbitrarily shaped single antenna, the vector potential can be calculated now. Using inverse Fourier transformation in (2.1) gives,

\[
\tilde{A}(\mathbf{r}) = \frac{1}{4\pi^2} \int_{S} \left[ \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \overline{G}(k_x, k_y, z, x', y', z') e^{-j k_x x} e^{-j k_y y} dk_x dk_y \right] \\
\cdot \mathcal{F}(x', y', z') dx' dy' dz'
\]

Using equation (2.29) and changing the order of integration,

\[
\tilde{A}(\mathbf{r}) = \frac{\mu_0}{4\pi^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \overline{G}(k_x, k_y, z, z') \left[ \int_{S} \mathcal{F}(\mathbf{r}') e^{j k_x x'} e^{j k_y y'} dx' dy' dz' \right] \\
\cdot e^{-j k_x x} e^{-j k_y y} dk_x dk_y
\]

(2.31)

2.3 An infinite array of electric dipoles in air above a conducting ground-plane

An expression for $\tilde{A}$, the dyadic vector potential or dyadic Greens function of an infinite array of arbitrarily directed dipoles is now derived.In figure 2.3 the geometry of such an array is shown.

Because of the infinite-array concept we can apply several theorems, such as the Floquet theorem [3] and the Poisson summation formula [12]. As a result of this concept we can use the unit-cell approach later: The array forms a periodic rectangular lattice with rectangular coordinates. The behaviour of the entire array is examined using only one unit cell containing two elements. The $(m, n)^{th}$ element is located at,

\[
m a \vec{e}_x + n b \vec{e}_y,
\]

\[\text{SIGNAAL}^{\oplus}\]
2.3. AN INFINITE ARRAY OF ELECTRIC DIPOLES IN AIR ABOVE A CONDUCTING GROUND-PLANE

Figure 2.3: An array of arbitrarily directed dipoles

with $a$ and $b$ denoting the grid distance between the elements in respectively the x-direction and y-direction.

2.3.1 Floquet modes in a rectangular grid

To scan in a direction with spherical coordinates $\theta, \phi$, the current on the $m, n^{th}$ element must be phased as,

$$e^{-jk_0(mau + nbv)},$$

with $u = \sin \theta \cos \phi$ and $v = \sin \theta \sin \phi$. Using superposition of all elements (the Floquet theorem) in equation (2.31) the total vector potential for an array of arbitrarily shaped antennas consisting of electric dipoles can be expressed in $\vec{A}$,

$$\vec{A}(r, \theta, \phi) = \frac{\mu_0}{4\pi^2} \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} G_r(k_x, k_y, z, \zeta') e^{-jk_0(mau + nbv)} e^{jk_x(x' + ma)} e^{jk_y(y' + nb)} dx' dy' dz'$

$$e^{-jk_z z'} e^{-jk_y y'} dk_z dk_y$$

$$= \frac{\mu_0}{4\pi^2} \int_{S} \vec{J}(\vec{r'}) e^{-jk_0(mau+nbv)} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} G_r(k_x, k_y, z, \zeta') e^{-jk_z(x' + ma - z)} e^{jk_y(y' + nb - y)} dk_z dk_y dx' dy' dz'$$
2.3. AN INFINITE ARRAY OF ELECTRIC DIPOLES IN AIR ABOVE A CONDUCTING GROUND-PLANE

This can be written as,

\[ \mathbf{\tilde{A}}(\mathbf{r}, \mathbf{r}') = \int_S \mathbf{\tilde{F}}(\mathbf{r}') \cdot \mathbf{\tilde{A}}(\mathbf{r}, \mathbf{r}') d\mathbf{r}', \quad (2.32) \]

where,

\[
\mathbf{\tilde{A}}(\mathbf{r}, \mathbf{r}') = \frac{\mu_0}{4\pi^2} \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \left[ e^{-jk_0(xa+nb)} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \mathbf{G}(k, k_y, z, z') \right. \\
\left. \times e^{jk_x(x'+ma-x)} e^{jk_y(y'+nb-y)} \right] dk_x dk_y
\]  

(2.33)

We now use a special derivative of the Poisson summation formula to eliminate the integrations in formula (2.33),

\[
\int_{-\infty}^{\infty} \mathbf{F}(t) e^{j\omega t} dt' = T \sum_{l=-\infty}^{\infty} \mathbf{F}(t+\omega T) e^{-j\omega T},
\]  

(2.34)

Let in formula (2.33) be \( l = m, t = k_0 u, \omega_0 = a, k_x = t', dk_x = dt', \omega = x'-x \) and \( T = \frac{2\pi}{a} \) then formula (2.33) can be simplified. Doing the same for the second summation we get the final expression for the dyadic Greens function of an infinite array of infinite electric dipoles with components of unit strength,

\[
\mathbf{\tilde{A}} = \frac{\mu_0}{ab} \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \mathbf{G}(k_x, k_y, z, z') e^{-jk_x(x'-x)} e^{-jk_y(y'-y)}
\]  

(2.35)

\[
= \begin{bmatrix}
A_x & 0 & 0 \\
0 & A_y & 0 \\
0 & 0 & A_z
\end{bmatrix}
\]  

(2.36)

where \( \mathbf{G} \) is given by equation (2.30) and \( k_x^m \) and \( k_y^n \) now have discrete values,

\[
k_x^m = k_0 u_0 + \frac{2\pi m}{a}
\]

\[
k_y^n = k_0 v_0 + \frac{2\pi n}{b}
\]

and,

\[
u_0 = \sin \theta \cos \phi
\]

\[
v_0 = \sin \theta \sin \phi
\]
2.4. AN INFINITE ARRAY OF MAGNETIC DIPOLES IN AIR ABOVE A CONDUCTING GROUND-PLANE

We have,

\[ A_x = \frac{\mu_0}{ab} \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \frac{1}{2jk_{mn}} e^{jk_m^z(x'-x)} e^{jk_n^y(y'-y)} \left[ e^{-jk_{mn}|z'-z|} - e^{-jk_{mn}|z+z'|} \right] \]  

(2.37)

\[ A_y = \frac{\mu_0}{ab} \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \frac{1}{2jk_{mn}} e^{jk_m^z(x'-x)} e^{jk_n^y(y'-y)} \left[ e^{-jk_{mn}|z'-z|} - e^{-jk_{mn}|z+z'|} \right] \]  

(2.38)

\[ A_z = \frac{\mu_0}{ab} \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \frac{1}{2jk_{mn}} e^{jk_m^z(x'-x)} e^{jk_n^y(y'-y)} \left[ e^{-jk_{mn}|z'-z|} + e^{-jk_{mn}|z+z'|} \right] \]  

(2.39)

which is the same as,

\[ A_x = \frac{\mu_0}{ab} \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \frac{1}{k_{mn}} e^{jk_m^z(x'-x)} e^{jk_n^y(y'-y)} \left\{ \begin{array}{ll} e^{-jk_{mn}|z'|} \sin k_{mn}z' & z > z' \\ e^{-jk_{mn}z'} \sin k_{mn}z & z' > z \end{array} \right. \]  

(2.40)

\[ A_y = \frac{\mu_0}{ab} \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \frac{1}{k_{mn}} e^{jk_m^z(x'-x)} e^{jk_n^y(y'-y)} \left\{ \begin{array}{ll} e^{-jk_{mn}|z'|} \sin k_{mn}z' & z > z' \\ e^{-jk_{mn}z'} \sin k_{mn}z & z' > z \end{array} \right. \]  

(2.41)

\[ A_z = \frac{\mu_0}{ab} \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \frac{1}{k_{mn}} e^{jk_m^z(x'-x)} e^{jk_n^y(y'-y)} \left\{ \begin{array}{ll} e^{-jk_{mn}|z'|} \cos k_{mn}z' & z > z' \\ e^{-jk_{mn}z'} \cos k_{mn}z & z' > z \end{array} \right. \]  

(2.42)

In equation (2.36) the first column represents the vector potential caused by an infinite array of x-dipoles and so on (compare with (2.29) for a single dipole). The wavenumbers in x-direction, y-direction and z-direction, respectively \( k_x, k_y \) and \( k \) are now called \( k_m^z, k_n^z \) and \( k_{mn} \) because of their \( mn \)-dependance in an array.

2.4 An infinite array of magnetic dipoles in air above a conducting ground-plane

In the presence of a magnetic current density \( \vec{\mathcal{M}} \) and magnetic charge density \( \rho_m \) in air above an electrically perfectly conducting ground-plane the Maxwell equations are given by,

\[ \nabla \times \vec{\mathcal{E}} = -j\omega \mu_0 \vec{\mathcal{H}} - \vec{\mathcal{M}} \]

\[ \nabla \times \vec{\mathcal{H}} = j\omega \varepsilon_0 \vec{\mathcal{E}} \]

\[ \nabla \cdot \vec{\mathcal{E}} = 0 \]

\[ \nabla \cdot \vec{\mathcal{H}} = \frac{\rho_m}{\mu_0} \]

An electric vector potential \( \vec{\mathcal{F}} \) can be defined as,

\[ \vec{\mathcal{E}} = -\frac{1}{\varepsilon_0} \nabla \times \vec{\mathcal{F}} \]

\[ \vec{\mathcal{H}} = -\frac{j\omega}{k_0^2} (k_0^2 \vec{\mathcal{F}} + \nabla (\nabla \cdot \vec{\mathcal{F}})) \]
2.4. AN INFINITE ARRAY OF MAGNETIC DIPOLES IN AIR ABOVE A CONDUCTING GROUND-PLANE

or in space domain rectangular components,

\[
\vec{E} = -\frac{1}{\varepsilon_0} \begin{bmatrix} \partial_y F_z - \partial_z F_y \\ \partial_z F_x - \partial_x F_z \\ \partial_x F_y - \partial_y F_x \end{bmatrix} \tag{2.43}
\]

\[
\vec{H} = -\frac{\mu_0}{k_0^2} \begin{bmatrix} (\partial_z^2 + k_0^2) F_x + \partial_x \partial_y F_y + \partial_x \partial_z F_z \\ (\partial_y + k_0^2) F_y + \partial_x \partial_z F_z + \partial_y \partial_z F_y \\ (\partial_x^2 + k_0^2) F_z + \partial_x \partial_z F_z + \partial_y \partial_z F_y \end{bmatrix} \tag{2.44}
\]

Written in the spectral domain,

\[
\vec{\tilde{E}} = -\frac{1}{\varepsilon_0} \begin{bmatrix} -jk_y F_x - \partial_z F_y \\ \partial_z F_x + jk_y F_z \\ -jk_x F_y + jk_y F_z \end{bmatrix} \tag{2.45}
\]

\[
\vec{\tilde{H}} = -\frac{\mu_0}{k_0^2} \begin{bmatrix} (k_0^2 - k_x^2) F_x - k_x k_y F_y - jk_x \partial_z F_z \\ (k_0^2 - k_y^2) F_y - k_y k_x F_x - jk_y \partial_z F_z \\ (k_0^2 + \partial_z^2) F_z - jk_x \partial_z F_x - jk_y \partial_z F_y \end{bmatrix} \tag{2.46}
\]

The inhomogeneous Helmholtz equation in the space domain becomes,

\[
\nabla^2 \vec{\tilde{F}} + \kappa_0^2 \vec{\tilde{F}} = -\varepsilon_0 \vec{\tilde{M}}, \tag{2.47}
\]

For an elementary magnetic dipole with unit strength located in \( \vec{r} = \vec{r}' \) and pointing in the x-direction, the magnetic current density in the space domain is given by,

\[
\vec{\tilde{M}} = \delta(x - x') \delta(y - y') \delta(z - z') \vec{e}_x
\]

Equation (2.47) then reads,

\[
\nabla^2 \vec{\tilde{F}} + \kappa_0^2 \vec{\tilde{F}} = -\varepsilon_0 \delta(x - x') \delta(y - y') \delta(z - z') \vec{e}_x, \tag{2.48}
\]

or in the spectral domain,

\[
\partial_z^2 \vec{\tilde{F}} + k^2 \vec{\tilde{F}} = -\varepsilon_0 \delta(z - z') e^{jk_z (x-x')} e^{jk_y (y-y')}, \tag{2.49}
\]

with \( k^2 = k_0^2 - k_x^2 - k_y^2 \). With (2.3),

\[
\partial_z^2 G_{zz}^m + k^2 G_{zz}^m = -\varepsilon_0 \delta(z - z') e^{jk_z (x-x')} e^{jk_y (y-y')}, \tag{2.50}
\]

with general solutions,
2.4. AN INFINITE ARRAY OF MAGNETIC DIPOLES IN AIR ABOVE A CONDUCTING GROUND-PLANE

\[ G_{xx}^m = \mu_0 \left[ B e^{-jkz} + \frac{1}{2jk} e^{-jk|z-z'|} e^{jkz} e^{jk_y'y'} \right] \]  \hspace{1cm} (2.51)

By applying the boundary condition (2.18) with equations (2.43) we get,

\[ \partial_z G_{xx}^m = 0 \]  \hspace{1cm} (2.52)

A solution is,

\[ G_{xx}^m = \frac{1}{2jk} \left[ e^{-jk|z-z'|} + e^{-jk|z+z'|} \right] e^{jkz} e^{jk_y'y'} \]  \hspace{1cm} (2.53)

Doing the same for y and z-directed dipoles we get,

\[ G_{yy}^m = \frac{1}{2jk} \left[ e^{-jk|z-z'|} + e^{-jk|z+z'|} \right] e^{jkz} e^{jk_x'x'} \]

\[ G_{zz}^m = \frac{1}{2jk} \left[ e^{-jk|z-z'|} - e^{-jk|z+z'|} \right] e^{jkz} e^{jk_y'y'} \]

These results are in agreement with the image theory in [4]. To satisfy the boundary conditions on the ground-plane imaginary horizontal magnetic dipoles must be thought in phase with the real dipole and vertical magnetic dipoles 180 degrees out of phase. Comparing \( G_{xx}^m, G_{yy}^m, G_{zz}^m \) with \( G_x^m, G_y^m, G_z^m \) the difference in sign is caused by the difference between the boundary conditions for each component (2.19), (2.52).

A general formula for the total vector potential of an array of arbitrarily directed shaped magnetic antennas with surface S is,

\[ \vec{F}(\vec{r}, \vec{r'}) = \int_S \vec{M}(\vec{r'}) \cdot \vec{F} d\vec{r'}, \]  \hspace{1cm} (2.54)

with \( \vec{F} \) the dyadic Greens function for an array of elementary magnetic dipoles with components of unit strength,

\[ \vec{F} = \begin{bmatrix} \mathcal{F}_x & 0 & 0 \\ 0 & \mathcal{F}_y & 0 \\ 0 & 0 & \mathcal{F}_z \end{bmatrix} \]  \hspace{1cm} (2.55)

We have,

\[ \mathcal{F}_x = \frac{\varepsilon_0}{ab} \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \frac{1}{2jk_{mn}} e^{jk_{mn}(x'-z)} e^{jk_{mn}(y'-y')} \left[ e^{-jk_{mn}|z-z'|} + e^{-jk_{mn}|z+z'|} \right] \]  \hspace{1cm} (2.56)
2.4. AN INFINITE ARRAY OF MAGNETIC DIPOLES IN AIR ABOVE A CONDUCTING GROUND-PLANE

\[
\mathcal{F}_y = \varepsilon_0 \frac{ab}{J} \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \frac{1}{2j_{kmn}} e^{jk_{m}^y(x' - x)} e^{jk_{n}^y(y' - y)} [e^{-j_{kmn}|z - z'|} + e^{-j_{kmn}|z + z'|}] \tag{2.57}
\]

\[
\mathcal{F}_z = \varepsilon_0 \frac{ab}{J} \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \frac{1}{2j_{kmn}} e^{jk_{m}^z(x' - x)} e^{jk_{n}^z(y' - y)} [e^{-j_{kmn}|z - z'|} - e^{-j_{kmn}|z + z'|}] \tag{2.58}
\]

which is the same as,

\[
\mathcal{F}_x = \varepsilon_0 \frac{ab}{J} \sum_{n=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} \frac{1}{j_{kmn}} e^{jk_{m}^x(x' - x)} e^{jk_{n}^x(y' - y)} \left\{ \begin{array}{ll}
\cos k_{mn} z' & z' > z' \\
\cos k_{mn} z & z > z' \\
\end{array} \right. \tag{2.59}
\]

\[
\mathcal{F}_y = \varepsilon_0 \frac{ab}{J} \sum_{n=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} \frac{1}{j_{kmn}} e^{jk_{m}^y(x' - x)} e^{jk_{n}^y(y' - y)} \left\{ \begin{array}{ll}
\cos k_{mn} z & z > z' \\
\cos k_{mn} z' & z' > z \\
\end{array} \right. \tag{2.60}
\]

\[
\mathcal{F}_z = \varepsilon_0 \frac{ab}{J} \sum_{n=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} \frac{1}{k_{mn}} e^{jk_{m}^z(x' - x)} e^{jk_{n}^z(y' - y)} \left\{ \begin{array}{ll}
\sin k_{mn} z & z > z' \\
\sin k_{mn} z' & z' > z \\
\end{array} \right. \tag{2.61}
\]

Again the wavenumbers have been given indices to indicate their \( mn \)-dependance.
Chapter 3

Greens functions for an infinite array of protruding antennas in air above a conducting ground-plane

3.1 Introduction

In chapter 2 the Greens functions were derived for an infinite array of arbitrarily directed electric and magnetic dipoles in air above a conducting ground-plane. In this chapter these results will be used in the derivation of Greens functions for an infinite array of protruding antennas in air. As an example of a protruding antenna figure 1.2 shows an array of bunny-ear antennas.

Because of the infinite-array concept which was mentioned before, only one unit cell with two elements in the array is enough to calculate properties of the array, i.e. we use Greens functions for an entire array but apply them to one cell. Figure 3.1 shows a unit cell with two bunny-ear elements in it. Both bunny-ears shall be involved in the next calculations (boundary conditions).

In section 3.2 it is explained why the equivalence theorem is used and applied to the bunny-ear array. In section 3.3 the Greens functions in the equivalent structure are given.

3.2 The equivalence theorem

3.2.1 Why the equivalence theorem

In chapter 2 Greens functions for arrays with xy-periodicity were derived ( formulas (2.40) through (2.42) and (2.59) through (2.61). Love's Field equivalence theorem is used to convert the geometry of figure 3.1 to a structure in which these formulas can be used too.

The main reason for using this theorem is that the mentioned vector potentials were derived for
3.2. THE EQUIVALENCE THEOREM

a super-conducting ground-plane with homogeneity in x and y-direction, i.e. with all boundaries in planes defined by \( z = \text{constant} \). In a situation with protruding antennas in the integral equations (2.32) two different versions must then be used for the formulas for the magnetic vector potential (formulas 2.40) through (2.42), depending on the relative position of \( z' \) to \( z \).

Love's Field equivalence-theorem [2]:

There are many different sources which produce the same field. These sources are called equivalent within a region because they produce the same field in that region. Let for example be the source internal to \( S \) and free space external to \( S \). This is equivalent to zero-field within \( S \) and equivalent electric and magnetic currents on \( S \) which are equal to the tangential field on \( S \),

\[
\vec{J}_s = \vec{n} \times \vec{H} \quad \quad \quad \vec{M}_s = \vec{E} \times \vec{n}
\]

Using the uniqueness theorem (which states that fields in the lossy region external to \( S \) are uniquely specified by the tangential component of the electric or magnetic field on \( S \)) it has now been proved that the fields external to \( S \) are the same as in the original situation.
3.2. THE EQUIVALENCE THEOREM

3.2.2 Step-wise application of the equivalence theorem

One wants to translate the internal $z$-directed current distribution into equivalent aperture currents (see figure (3.3)). In aperture problems equivalence can be applied in the following way. We assume only electric currents $\mathcal{J}_{\text{fin}}$ in region A.

![Figure 3.3: A general aperture problem](image)

**Step 1** Region B is closed at infinity and the aperture $S_a$ is open. The electric fin current $\mathcal{J}_{\text{fin}}$ causes fields $\vec{E}, \vec{H}$ in both regions.

**Step 2** Closing the aperture with an infinitely thin electric plate above fields are zero in region B and non-zero in region A. In this region fields $\vec{E}_1, \vec{H}_1$ do exist and satisfy the inhomogenous Maxwell-equations.

The existing fields $\vec{E}_1, \vec{H}_1$ at the place of the aperture can be thought of as sources and therefore as imaginary magnetic surface currents on the aperture.

**Step 3** With the aperture opened and removing the fin-currents, in both regions fields, $\vec{E}_2, \vec{H}_2$ are generated by the afore mentioned equivalent magnetic current sources on top of the conducting plate. See fig 3.4. These fields stisfy the homogenous Maxwell-equations.

![Figure 3.4: Equivalent internal and external magnetic sources](image)

The aperture is imaginary so there is no reason for discontinuities in the total fields.

$\vec{n} \times \vec{E}_1$ is zero at the aperture. $\vec{n} \times \vec{E}_2$, which is stisfied by making $\vec{M}^e = -\vec{M}^i$. This shall now be proved.
3.3. GREENS FUNCTIONS IN THE EQUIVALENT STRUCTURE

Assume that \( \vec{n} \) is always pointing into region A. The electric field boundary condition in \( z = 0 \) is then expressed [23] as,

\[
\lim_{z \to 0^-} E \times \vec{n} - \lim_{z \to 0^+} E \times \vec{n} = \vec{M}_i + \vec{M}_e,
\]

which shows continuity of the electric field only when interior and exterior magnetic currents are equal.

\( \vec{H}_1 \) is discontinuous in \( S_e \) so \( \vec{H}_2 \) must have an opposite discontinuity too.

The formulas to be derived in the following text should support these characteristics.

Applying the equivalence theorem to the unit cell of (3.1) at \( z = 0 \) we get a structure which consists of two parallel electrically perfectly conducting plates in \( z = 0 \) and \( z = -d \). See figure 3.5. On the cost of an extra surface (\( z = 0 \) with fictitious currents more convenient formulas for the magnetic vector potential can be derived. The magnetic vector potential now has the same form as in a parallel-plate waveguide system, in which it is independant of the relative position of \( z \) to \( z' \).

![Figure 3.5: The equivalence theorem applied to an array of protruding antennas](image)

As can be seen in figure 3.5 in the equivalent structure electric currents exist in the yz-plane and magnetic currents flow in the xy-plane. The former currents generate a magnetic vector potential and the later currents cause an electric vector potential.

The structure is now a superposition of two regions, an external region for \( z > 0 \) and an internal region for \( -d \leq z < 0 \). Internal magnetic currents \( \vec{M}_i \) in \( z = 0 \) create a field within \( S \) and external sources \( \vec{M}_e \) create fields external to \( S \).

The form of the fins is explicitly accounted for in the application of the moment method.

3.3 Greens functions in the equivalent structure

In the case of the magnetic currents, we have a source above one ground plane, which can be described with the theory of chapter 2. The electric currents perpendicular to the equivalent parallel plates will be treated after that. The formulas in chapter two for electric currents can not be used then.
3.3. GREENS FUNCTIONS IN THE EQUIVALENT STRUCTURE

We begin with the derivation of the Greens functions of the magnetic sources in \( z = 0 \). After that the electric currents are examined. The exact functions are determined from the results in chapter 2 by enforcing active boundary conditions [8].

3.3.1 Magnetic currents

3.3.1.1 Internal magnetic dipoles

The fields within the unit cell are created by equivalent internal magnetic sources and electric sources. For the electric vector potential associated with a \( x \)-directed internal magnetic current of unit strength, we use the formula,

\[
\vec{M}^t = \lim_{\epsilon \to 0} \delta(x - x')\delta(y - y')\delta(z - \epsilon)\vec{e}_x,
\]

we use the formula,

\[
\mathcal{F}_x^i = \frac{\varepsilon_0}{ab} \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \frac{L_{mn}}{jk_{mn}} e^{jk_{x_n}(x' - x)} e^{jk_{y_n}(y' - y)} \cos k_{mn}(z + d),
\]

which is just a shifted version of formula (2.59) because we now have a ground plane at \( z = -d \) instead of \( z = 0 \) and \( z' = 0 > z \). The \( mn \)-dependent constant \( L_{mn} \) has been inserted to satisfy the space-domain active boundary condition on the \( x \)-directed source,

\[
\lim_{x \to 0} \vec{E} \times \vec{n} = \lim_{\epsilon \to 0} \delta(x - x')\delta(y - y')\delta(z - \epsilon)\vec{e}_x
\]

\[
\lim_{z \to 0} \vec{E}_y = -\delta(x - x')\delta(y - y'),
\]

in which \( \vec{n} \) is the normal vector \( \vec{n} = -\vec{e}_z \). It says that the total tangential field on the source due to the entire array is zero. Actually by means of this active boundary condition now the field \( \vec{E}_2 \) due to the \( \vec{E}_1 \) field on the aperture or the equivalent magnetic current is calculated. With formula (2.43),

\[
\lim_{x \to 0} \frac{1}{\varepsilon_0} \frac{\partial}{\partial x} \mathcal{F}_x^i = \delta(x - x')\delta(y - y')
\]

The point \((x', y', 0)\) is the only point on the aperture in which the tangential electric field is not zero. So with (3.1) we get,

\[
\frac{j}{ab} \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} L_{mn} e^{jk_{x_n}(x' - x)} e^{jk_{y_n}(y' - y)} \sin k_{mn}d = \delta(x - x')\delta(y - y'),
\]

On the interval \( x \in [0, a] \) the functions \( f_m(x) = e^{jk_{x_n}x} \) with \( k_{x_n} = k_0u_0 + \frac{2\pi n}{a} \) form an orthogonal set. This means that,
3.3. GREENS FUNCTIONS IN THE EQUIVALENT STRUCTURE

\[ \int_{x=0}^{a} f_m(x) \cdot f_p'(x) \, dx = \int_{0}^{a} e^{j2\pi(m-p)x} \, dx = \frac{a e^{j2\pi(m-p)} - 1}{j2\pi(m-p)} = a\delta(m - p), \]

because,

\[ \lim_{\alpha \to 0} \frac{e^{j\alpha} - 1}{j\alpha} = \lim_{\alpha \to 0} \frac{1 + j\alpha + \frac{(j\alpha)^2}{2} + \cdots - 1}{j\alpha} = 1. \]

In the same way on the interval \( y \in [0, b] \) the functions \( f_n(y) = e^{jky} \) with \( k_y = k_0 v_0 + \frac{2\pi n}{a} \) form an orthogonal set. This characteristic is now used in equation (3.2). Multiplying both sides of (3.2) with \( e^{jky}e^{jky} \) and then integrating we get,

\[ \frac{1}{ab} \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} e^{jkmz} e^{jkn'y} \cdot L_{mn} \int_{x=0}^{a} e^{j2\pi(m-p)x} \, dx \int_{y=0}^{b} e^{j2\pi(n-q)y} \, dy \cdot \sin(k_{mn}d) \]

\[ = \int_{x=0}^{a} \delta(x - x') e^{jkmz} \, dx \int_{y=0}^{b} \delta(y - y') e^{jkn'y} \, dy \]

\[ = e^{jk_{x'}z'} e^{jk_{y'}y'} \cdot \int_{x=0}^{a} \delta(x - x') \, dx \int_{y=0}^{b} \delta(y - y') \, dy \]

\[ = e^{jk_{x'}z'} e^{jk_{y'}y'} \cdot U(x - x')|_{x=0}^{a} \cdot U(y - y')|_{y=0}^{b}, \]

in which \( U \) denotes a step function and the following rule is used: \( f(x) \cdot \delta(x - x') = f(x') \delta(x - x') \). With \( x' \in [0, a], y' \in [0, b], \)

\[ \frac{1}{ab} \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} e^{jkmz} e^{jkn'y} \cdot L_{mn} a \delta(m - p) b \delta(n - q) \sin(k_{mn}d) = e^{jk_{x'}z'} e^{jk_{y'}y'}, \]  \hspace{1cm} (3.3)

or,

\[ je^{jk_{x'}z'} e^{jk_{y'}y'} L_{pq} \sin(k_{pq}d) = e^{jk_{x'}z'} e^{jk_{y'}y'} \]  \hspace{1cm} (3.4)

As it is now allowed to substitute for \( m \) for \( p' \) and \( n \) for \( q \),

\[ L_{mn} = \frac{1}{j \sin k_{mn}d} \]

Indeed the element constant is dependent on \( mn \) through \( k_{mn} \).
3.3. GREENS FUNCTIONS IN THE EQUIVALENT STRUCTURE

3.3.1.2 An y-directed internal dipole

The same can be done for an y-directed internal source.

\[
\mathcal{M}_y^i = \lim_{\epsilon \to 0} \delta(x - x') \delta(y - y') \delta(z - \epsilon) \cdot \vec{e}_y
\]  

(3.5)

Applying the boundary condition,

\[
\lim_{z \to 0} \vec{E} \times \vec{n} = \lim_{\epsilon \to 0} \delta(x - x') \delta(y - y') \delta(z - \epsilon) \vec{e}_y
\]

(3.6)

with \( \vec{n} = -\vec{e}_z \) yields,

\[
\lim_{z \to 0} \frac{1}{\epsilon_0} \partial_z \vec{F}_y = \delta(x - x') \delta(y - y')
\]

(3.7)

The constant in,

\[
\mathcal{F}_y^i = \frac{\epsilon_0}{ab} \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} M_{mn} e^{jk_x (x - x')} e^{jk_y (y - y')} \cos k_m (z + d),
\]

(3.8)

is the determined as

\[
M_{mn} = \frac{1}{j \sin k_m d}
\]

With formula (2.44) it can be seen that for \( \mathcal{F}_x^i \), the resulting electric field is a \( TE_x \) field, i.e. the x-component is zero. Analog \( \mathcal{F}_y^i \) creates a \( TE_y \) field.

3.3.1.3 External magnetic dipoles

The external magnetic currents are,

\[
\mathcal{M}_x^e = -\lim_{\epsilon \to 0} \delta(x - x') \delta(y - y') \delta(z + \epsilon) \vec{e}_x
\]

\[
\mathcal{M}_y^e = -\lim_{\epsilon \to 0} \delta(x - x') \delta(y - y') \delta(z + \epsilon) \vec{e}_y
\]

Fields outside the unit-cell are caused by the external magnetic sources. As this half-space is similar to the geometry in chapter 2 with the addition that \( z > z' = 0 \) we can write (2.59) and (2.60) as,

\[
\mathcal{F}_x^e = \frac{\epsilon_0}{ab} \sum_{n=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} N_{mn} e^{jk_x (x' - x')} e^{jk_y (y' - y')} e^{-jk_m z}
\]
3.3. GREENS FUNCTIONS IN THE EQUIVALENT STRUCTURE

\[ \mathcal{J}_y^e = \frac{\varepsilon_0}{ab} \sum_{n=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} \frac{O_{mn}}{jk_{mn}} e^{jk_{mn}(x'-x)}e^{jk_{mn}(y'-y)}e^{-jk_{mn}z} \]

We now have dipoles pointing in the negative direction, while in chapter 2 formulas were derived for positive directed dipoles. Analog to the internal currents the constant in this formulas is obtained. It will account for the opposite orientation. Using the boundary condition for the x-directed magnetic dipole,

\[ \lim_{z \to 0} E \times \mathbf{n} = -\lim_{z \to 0} \delta(x-x')\delta(y-y')\delta(z+\epsilon)\mathbf{\hat{e}_x} \quad (3.9) \]

With \( \mathbf{n} = -\mathbf{\hat{e}_z} \) and formula (2.43) this is,

\[ \lim_{z \to 0} \frac{1}{\varepsilon_0} \partial_z \mathcal{J}_x = -\delta(x-x')\delta(y-y'), \quad (3.10) \]

or,

\[ N_{mn} = 1 \]

Doing the same for \( \mathcal{M}_{y}^e \) we get,

\[ \lim_{z \to 0} \frac{1}{\varepsilon_0} \partial_z \mathcal{J}_y = -\delta(x-x')\delta(y-y'), \quad (3.11) \]

and,

\[ O_{mn} = 1 \]

As said in section 3.2.2 the tangential magnetic field \( \mathbf{n} \times \mathbf{\hat{H}}_1 \) due to the fin current is discontinuous on the aperture because it is zero for \( z \geq 0 \). The total tangential field is continuous on the aperture. Hence \( \mathbf{\hat{H}}_2 \) must have an equally big but opposite discontinuity too. In chapter 4 one of the two integral equations is constructed out of these conditions.

3.3.2 An y-directed electric dipole

Expressions for the electric current Greens functions can be found by solving for the magnetic vector potential in a parallel-plate waveguide with plates at \( z = 0, -d \), see

\[ \mathcal{J}_y = \delta(x-x')\delta(y-y')\delta(z-z')\mathbf{\hat{e}_y} \quad (3.12) \]

In the x and y-direction the configuration is unbounded and solutions will have an exponential form. Looking at figure 3.1 one sees that the configuration is periodic in x and y. Moreover
in the $y$-direction there is homogenity (the same situation as in chapter 2) while in the $x$-direction boundaries exist between air and antenna surface. This is why the $y$-periodicity of the vector potential can be described with a Floquet-mode term. For the $x$-component a general nonperiodic form is chosen, in which the constants will be determined through the conditions of periodicity and the source jump condition. In the bounded $z$-direction solutions contain a sine or cosine. The total solution is,

$$A_y = \mu_0 \sum_{n=-\infty}^{\infty} \left[ A e^{j\beta x} + B e^{-j\beta x} \right] e^{k_0^x (y'-y)} [E \sin(k_z z) + F \cos(k_z z)],$$

with $\beta^2 = k_0^2 - k_z^2 - (k_y^0)^2$. Now the boundary conditions are enforced,

### 3.3.2.1 Perfect electric conductors in $z=0,-d$

On the equivalent plates in $z=0,-d$ the tangential electric field disappears. Actually this is the $\mathcal{E}_1$ field due to the fin currents from section 3.2.2. Formulated in an active boundary condition in the space domain,

$$\vec{n} \times \vec{E} = \vec{0} \quad z=0,-d,$$

in which $\vec{n} = -\vec{e}_z$ in $z=0$ and $\vec{n} = \vec{e}_z$ in $z=-d$. We get,

$$\mathcal{E}_x = \mathcal{E}_y = 0 \quad z=0,-d,$$

or with formula (2.6),
3.3. GREENS FUNCTIONS IN THE EQUIVALENT STRUCTURE

Substituting formula (3.13) yields \( F = 0 \) and \( k_z = \frac{E_z}{d} \) with \( p = 1, 2, 3 \cdots \). It follows that \( \beta_{np} = \sqrt{k_0^2 - \left(\frac{E_z}{d}\right)^2 - (k_y)^2} \). We get,

\[
A_y(x, y, z) = \mu_0 \sum_{n=-\infty}^{\infty} \sum_{p=1}^{\infty} [A_{np} e^{j\beta_{np} x} + B_{np} e^{-j\beta_{np} x}] e^{jk_z(y-y')} \sin\left(\frac{p\pi z}{d}\right) = 0 \quad z = 0, -d
\]

In these series \( p = 0 \) is excluded because for \( p = 0 \) the fields \( \vec{E}, \vec{H} \) become zero due to the sine in formula (3.15) (non-existence of any waves). To obtain the expressions for the constants \( A \) and \( B \) we need two independent equations. The first one expresses the periodicity in the \( x \)-direction (compare \( \beta_{np} \) to \( k_z = k_0 u_0 + \frac{2\pi m}{a} \)) and the second is the source condition.

### 3.3.2.2 Periodicity in \( x \)

The electric and magnetic fields in \( x = 0 \) must be the same as the fields in \( x = a \), apart from a phase factor in \( x = a \) \( (e^{j k_0 u_0 a}) \) as shown in the periodicity condition,

\[
(\vec{E}, \vec{H})_{x=0} = e^{j k_0 u_0 a} (\vec{E}, \vec{H})_{x=a}
\]

With formulas (2.6), (2.7) and formula (3.15) we get the same equation for each field component,

\[
A_{np} + B_{np} = e^{j k_0 u_0 a} (A_{np} e^{j\beta_{np} a} + B_{np} e^{-j\beta_{np} a})
\]

which can be written as,

\[
A_{np} = -B_{np} \frac{1 - e^{j a(k_0 u_0 - \beta_{np})}}{1 - e^{j a(k_0 u_0 + \beta_{np})} (3.17)}
\]

### 3.3.2.3 The source jump condition

As in [9] on the y-directed dipole an active condition is used with which the constants \( A_{np} \) and \( B_{np} \) can be determined. In \( x = 0 \) the simple spatial domain boundary condition,

\[
\vec{n} \times [\vec{H}_{x=0} - \vec{H}_{x=10}] = \vec{J}_{z=0}
\]

is equal to,

\[
\vec{n} \times [e^{j k_0 u_0 a} \cdot \vec{H}_{x=1} - \vec{H}_{x=10}] = \delta(y - y')\delta(z - z') \vec{e}_y
\]
3.3. GREENS FUNCTIONS IN THE EQUIVALENT STRUCTURE

Figure 3.7: The active boundary condition

or,

\[ e^{j k_0 u a} \cdot H_{x=x_0} - H_{x=x_0} = \delta(y-y')\delta(z-z') \]  

(3.19)

With formulas (2.6) and (3.15) we get,

\[
\sum_{n=-\infty}^{\infty} \sum_{p=1}^{\infty} e^{j k_n^p (y - y')} \sin \frac{p \pi z}{d} j \beta_n p [A_{n p} e^{j \beta_n p a} - B_{n p} e^{-j \beta_n p a}] e^{j k_0 u a} \\
- \sum_{n=-\infty}^{\infty} \sum_{p=1}^{\infty} e^{j k_n^p (y - y')} \sin \frac{p \pi z}{d} j \beta_n p [A_{n p} - B_{n p}] = \delta(y-y')\delta(z-z')
\]

This is,

\[
\sum_{n=-\infty}^{\infty} \sum_{p=1}^{\infty} e^{j k_n^p (y - y')} \sin \frac{p \pi z}{d} j \beta_n p [e^{j k_0 u a} (A_{n p} e^{j \beta_n p a} - B_{n p} e^{-j \beta_n p a}) - A_{n p} + B_{n p}] = \delta(y-y')\delta(z-z'),
\]

(3.20)

Substituting (3.17) in (3.20) gives,

\[
\sum_{n=-\infty}^{\infty} \sum_{p=1}^{\infty} e^{j k_n^p (y - y')} \sin \frac{p \pi z}{d} j \beta_{q p} B_{q p} (1 - e^{j a (k_0 u a - \beta_{q p})}) = \frac{1}{2} \delta(y-y')\delta(z-z')
\]

(3.21)

Multiplying both sides with \(e^{j k_2^q y}\) and integrating \(y\) over \([0, b]\) as we did with formula (3.2) we get,

\[
\sum_{p=1}^{\infty} \sin \frac{p \pi z}{d} j \beta_{q p} B_{q p} (1 - e^{j a (k_0 u a - \beta_{q p})}) = \frac{1}{2b} \delta(z-z')
\]

(3.22)

Multiplying both sides with \(\sin(\frac{p \pi z}{d})\) and integrating \(z\) over \([0, d]\),

\[
\int_{z=0}^{d} \sum_{p=1}^{\infty} \sin \frac{p \pi z}{d} \sin \frac{p' \pi z}{d} j \beta_{q p} (1 - e^{j a (k_0 u a - \beta_{q p})}) dz = \int_{z=0}^{d} \frac{1}{2b} \sin \frac{p' \pi z}{d} \delta(z-z') dz
\]

(3.23)
3.3. GREENS FUNCTIONS IN THE EQUIVALENT STRUCTURE

Now,

\[ \int_{z=0}^{d} \sum_{p=1}^{\infty} \sin \frac{p\pi z}{d} \sin \frac{p'\pi z}{d} \, dz = \frac{1}{2} \sum_{p=1}^{\infty} \int_{z=0}^{d} \cos \left( \frac{p-p'}{d} \pi z \right) dz - \frac{1}{2} \sum_{p=1}^{\infty} \int_{z=0}^{d} \cos \left( \frac{p+p'}{d} \pi z \right) dz \]

\[ = \frac{d}{2} \sum_{p=1}^{\infty} \frac{\sin(\pi(p-p'))}{\pi(p-p')} - \frac{d}{2} \sum_{p=1}^{\infty} \frac{\sin(\pi(p+p'))}{\pi(p+p')} , \]

of which the second term is always zero because \( p, p' \geq 0 \). The first term is zero too except for \( p = p' \). So formula (3.23) becomes,

\[ \sum_{p=1}^{\infty} j\beta_{qp}B_{qp} \frac{d}{2} \delta(p-p')(1 - e^{i\alpha(k_{0}u_{0} - \beta_{np})}) = \frac{1}{2b} \sin \left( \frac{p'\pi z'}{d} \right) \cdot U(z - z') \bigg|_{z=0} \]

(3.24)

and then,

\[ j\beta_{qp}B_{qp} \frac{d}{2} (1 - e^{i\alpha(k_{0}u_{0} - \beta_{qp'})}) = \frac{1}{2b} \sin \left( \frac{p'\pi z'}{d} \right) \]

(3.25)

Now \( p' \) is changed to \( p \) and \( q \) to \( n \). Combining equation 3.25 with expression (3.17) finally provides us with expressions for the constants \( A_{np} \) and \( B_{np} \) in formula (3.15),

\[ A_{np} = -\frac{\sin \frac{p\pi z'}{d}}{jbd\beta_{np}(1 - e^{i\alpha(\beta_{np} + k_{0}u_{0})})} \]

(3.26)

\[ B_{np} = +\frac{\sin \frac{p\pi z'}{d}}{jbd\beta_{np}(1 - e^{-i\alpha(\beta_{np} - k_{0}u_{0})})} \]

(3.27)

3.3.3 A z-directed electric dipole

For a z-directed electric dipole,

\[ \vec{J} = \delta(x - x')\delta(y - y')\delta(z - z')\vec{e}_{z}, \]

a general solution is,

\[ A_{z} = \mu_{0} \sum_{n=-\infty}^{\infty} \left[ C e^{i\beta z} + D e^{-i\beta z} \right] e^{ik_{n}y(y' - y)} [K \sin(k_{z}z) + L \cos(k_{z}z)] \]

(3.28)
3.3. **GREENS FUNCTIONS IN THE EQUIVALENT STRUCTURE**

3.3.3.1 Perfect conductors in \( z = 0, -d \)

The boundary condition \( \mathbf{n} \times \mathbf{E} = 0 \) with \( \mathbf{n} = -\mathbf{e}_z \) in \( z = 0 \) and \( \mathbf{n} = \mathbf{e}_z \) in \( z = -d \) yields \( \mathcal{E}_x = \mathcal{E}_y = 0 \) in \( z = 0, -d \) and with equation (2.6) we get \( \partial_z \mathcal{A}_z = 0 \) in \( z = 0, -d \). Substituting expression (3.28) it follows that \( L = 0 \) and \( k_z = \frac{\beta \eta}{d} \). Formula (3.28) becomes,

\[
\mathcal{A}_z = \mu_0 \sum_{n = -\infty}^{\infty} \sum_{p = 0}^{\infty} [C e^{\beta x} + D e^{-\beta x}] e^{j \eta(y'-y)} \cos\left(\frac{p \pi z}{d}\right)
\]  

(3.29)

3.3.3.2 Periodicity in \( x \)

Combining condition (3.16) with formulas (3.29) and (2.6),(2.7), for each field component the same formula is obtained,

\[
C_{np} = -D_{np} \frac{1 - e^{ja(k_0 u_0 - \beta np)}}{1 - e^{ja(k_0 u_0 + \beta np)}}
\]  

(3.30)

3.3.3.3 The source-jump condition

The active boundary condition (2.31) is now,

\[
e^{j k_0 u_0 a} \cdot \mathcal{H}_{y,x=a} - \mathcal{H}_{y,x=0} = \delta(y - y')\delta(z - z')
\]  

(3.31)

Using formulas (2.7) and (3.30), multiplying both sides with \( e^{j \eta y} \) and integrating in \( y \) over \([0, b]\) gives us,

\[
\sum_{p=0}^{\infty} \cos \frac{p \pi z}{d} j \beta_{np} C_{qp} (1 - e^{\eta (k_0 u_0 - \beta np)}) = \frac{1}{2b} \delta(z - z')
\]  

(3.32)

Both sides are multiplied with \( \cos\left(\frac{p \pi z}{d}\right) \) and integrating in \( z \) over \([0, d]\) then yields,

\[
\int_{z=0}^{d} \sum_{p=0}^{\infty} \cos \frac{p \pi z}{d} \cos \frac{p' \pi z}{d} j \beta D_{qp} (1 - e^{\eta (k_0 u_0 - \beta np)}) dz = \int_{z=0}^{d} \frac{1}{2b} \cos \frac{p' \pi z}{d} \delta(z - z') dz
\]  

(3.33)

Now,

\[
\int_{z=0}^{d} \sum_{p=0}^{\infty} \cos \frac{p \pi z}{d} \cos \frac{p' \pi z}{d} dz = \frac{d}{2} \sum_{p=0}^{\infty} \frac{\sin(\pi(p - p'))}{\pi(p - p')} + \frac{d}{2} \sum_{p=0}^{\infty} \frac{\sin(\pi(p + p'))}{\pi(p + p')}
\]  

(3.34)

Both terms in 3.34 are equal to zero except for \( p = p' = 0 \) (both terms) and \( p = p' \) (only the first term) in which cases they are equal to \( \frac{d}{2} \). Using this in (3.33) and (3.30) we get,
3.3. GREENS FUNCTIONS IN THE EQUIVALENT STRUCTURE

\[ A_z = \mu_0 \sum_{n=-\infty}^{\infty} \sum_{p=0}^{\infty} \left[ C_{np} e^{j\beta_{np} x} + D_{np} e^{-j\beta_{np} x} \right] e^{jk_y (y' - y)} \cos \frac{p\pi z}{d} \]  

(3.35)

with

\[ C_{np} = -\frac{\cos \frac{p\pi z'}{d}}{jbd\beta_{np}(1 - e^{j\alpha(\beta_{np} + k_0 u_0)})} \cdot \frac{1}{\varepsilon_p} \]  

(3.36)

\[ D_{np} = +\frac{\cos \frac{p\pi z'}{d}}{jbd\beta_{np}(1 - e^{-j\alpha(\beta_{np} - k_0 u_0)})} \cdot \frac{1}{\varepsilon_p} \]  

(3.37)

and,

\[ \varepsilon_p = \begin{cases} 2 : & p = 0 \\ 1 : & p > 0 \end{cases} \]  

(3.38)

In (3.35) we have a cosine because of the first boundary condition on the plates in \( z = 0, -d \) (3.14). For \( p = 0 \) the wave doesn’t vanish as in \( A_y \), so \( p = 0 \) can be included in the series. As before it is clear that \( A_y \) generates a \( TM_y \) field, i.e. zero magnetic field in \( y \)-direction and \( A_z \) creates a \( TM_z \) pattern. The superposition of these TE and TM waves covers every angle in positive halfspace, as could of course be expected.

3.3.4 Check of continuity of the electric field in the aperture

As it is the basis for the rest of the calculations the obtained constants and formulas must be checked by means of the continuity of the electric field in \( z = 0 \). Physically this is expressed as,

\[ \lim_{z \downarrow 0} \vec{E} \times \vec{n} - \lim_{z \uparrow 0} \vec{E} \times \vec{n} = \vec{0} \]  

(3.39)

with \( \vec{n} = -\vec{e}_z \) the \( x \)-component and \( y \)-component of this boundary condition are written as,

\[ -\lim_{z \downarrow 0} \vec{E}_y + \lim_{z \uparrow 0} \vec{E}_y = 0 \]

\[ \lim_{z \downarrow 0} \vec{E}_x - \lim_{z \uparrow 0} \vec{E}_x = 0 \]

Both electric and magnetic currents must be taken into account. Since \( \partial_y \partial_z A_z = 0 \) and \( A_y = 0 \)

for \( z = 0 \) it follows with equation (2.6) and (2.43) that,

\[ \lim_{z \downarrow 0} \partial_z \mathcal{I}_x - \lim_{z \uparrow 0} \partial_z \mathcal{I}_x = 0 \]

\[ -\lim_{z \downarrow 0} \partial_z \mathcal{I}_y + \lim_{z \uparrow 0} \partial_z \mathcal{I}_y = 0 \]
3.3. GREENS FUNCTIONS IN THE EQUIVALENT STRUCTURE

Substituting the denoted formulas leads to the conclusion that the boundary conditions are indeed satisfied.
Chapter 4

Formulation of the moment method

4.1 Introduction

In general, with Greens functions both transmit- and scattering problems can be tackled. In scattering problems the reflecting properties of a body for an incident field are examined (for example a plane that has to be invisible for radar). In transmitting problems the reflection coefficient from feed to antenna is analysed (i.e. for example the input impedance) as a function of the scan angle. This is a typical antenna problem. Because of the reciprocity of antennas receiving properties are identical to the transmitting properties.

The vector potential Greens functions for a general infinite array of protruding antennas on a conducting ground-plane have been derived in chapter 3. As in [8] in Appendix B the Greens functions for the electric and magnetic fields are calculated with these results.

In the first section of this chapter the remaining boundary conditions shall be given and worked out to yield integral equations. In the succeeding sections the unknown electric and magnetic currents are expanded and the resulting integral equations are tested to obtain an impedance matrix. Expressions will be given for all matrix elements.

4.2 Formulation of the remaining boundary-conditions

4.2.1 Two-dimensional electric and magnetic field dyadic Greens functions

As we want expressions for the fields generated by the entire surfaces of fins and aperture (the plate in $z = 0$) we must use formulas as (2.1) and (2.3). Therefore field Greens functions in dyadic form are needed. Since we need to know only the tangential fields these dyadic functions have two dimensions. We have,

$$\vec{E}^{ff} = \begin{bmatrix} E_{yy}^{ff} & E_{yz}^{ff} \\ E_{zy}^{ff} & E_{zz}^{ff} \end{bmatrix}$$
4.2. FORMULATION OF THE REMAINING BOUNDARY-CONDITIONS

\[
\begin{align*}
\vec{\mathcal{E}} f_a &= \begin{bmatrix} \mathcal{E}_{y'z'} \mathcal{E}_{y'z'} \\ \mathcal{E}_{y'z'} \mathcal{E}_{y'z'} \end{bmatrix} \\
\vec{\mathcal{H}}^a f &= \begin{bmatrix} \mathcal{H}_{x'y'} \mathcal{H}_{x'y'} \\ \mathcal{H}_{x'y'} \mathcal{H}_{x'y'} \end{bmatrix} \\
\vec{\mathcal{H}} a,ai &= \begin{bmatrix} \mathcal{H}_{x'i} \mathcal{H}_{x'i} \\ \mathcal{H}_{x'i} \mathcal{H}_{x'i} \end{bmatrix} \\
\vec{\mathcal{H}} a,ae &= \begin{bmatrix} \mathcal{H}_{x'i} \mathcal{H}_{x'i} \\ \mathcal{H}_{x'i} \mathcal{H}_{x'i} \end{bmatrix}
\end{align*}
\]

In these dyadic Greens functions for the fields the indices a,f,e,i stand for aperture, fins, external, internal. The first character in the indices refers to the field point and the second to the source point. So \( \mathcal{H}^{a,zx} \) is the z-component of the magnetic field on the fin surface created by a x-directed magnetic unit source on the aperture surface. All fields and their components have a \((r,\tau, k_{mn})\) dependence, so this indication is left out.

4.2.2 Aperture and fin fields

Similar to (2.1) and (2.3) we can write,

\[
\begin{align*}
\vec{\mathcal{E}} f &= \int_{S_f} \vec{\mathcal{E}} f \cdot \mathcal{f}(y', z') \, dS_f \\
\vec{\mathcal{E}} f,ai &= \int_{S_a} \vec{\mathcal{E}} f,ai \cdot \mathcal{M}(x', y') \, dS_a \\
\vec{\mathcal{H}} f &= \int_{S_f} \vec{\mathcal{H}} f \cdot \mathcal{f}(y', z') \, dS_f \\
\vec{\mathcal{H}} a,ai &= \int_{S_a} \vec{\mathcal{H}} a,ai \cdot \mathcal{M}(x', y') \, dS_a \\
\vec{\mathcal{H}} a,ae &= \int_{S_a} \vec{\mathcal{H}} a,ae \cdot \mathcal{M}(x', y') \, dS_a
\end{align*}
\]

For example \( \vec{\mathcal{E}} f,ai \) is the tangential electric field on the fin surface due to a internal magnetic current distribution \( \mathcal{M}^i \) on the aperture. \( S_a \) and \( S_f \) denote fin surface and aperture, \( dS_a' = dx' dy' \) and \( dS_f' = dy' dz' \).

4.2.3 The remaining boundary conditions

The remaining boundary conditions are the zero tangential electric field on the fins and the continuity of the tangential magnetic field on the aperture,

\[
\begin{align*}
\vec{\mathcal{E}} \times \vec{n}_2 &= 0 \quad \tau \text{ on fins} \\
(\vec{n}_2 \times \vec{\mathcal{H}})_{x10} - (\vec{n}_2 \times \vec{\mathcal{H}})_{x10} &= 0 \quad \tau \text{ on aperture}
\end{align*}
\]
4.3. EXPANDING AND TESTING

in which \( \vec{n}_1 = (1,0,0) \) and \( \vec{n}_2 = (0,0,1) \). From (4.6) and (4.7),

\[
\begin{align*}
\vec{E}^{lf} + \vec{E}^{l,ai} &= -\vec{E}^g \\
\vec{H}^{a,l} + \vec{H}^{a,ai} - \vec{H}^{a,ae} &= \vec{0},
\end{align*}
\]

As said before we consider only the transmit case so there is no exciting tangential magnetic field on the aperture, the right-hand side of equation (4.9) is zero. On the antenna-feed a tangential electric field is impressed (subscript g means gap). Writing equations (4.8) and (4.9) with formulas (4.1) through (4.5) we obtain the following integral equations,

\[
\begin{align*}
\int_{S_f} \vec{E}^{lf} \cdot \vec{J}(y', z') \, dS_f' + \int_{S_a} \vec{E}^{l,ai} \cdot \vec{M}(x', y') \, dS_a' &= -\vec{E}^g \\
\int_{S_f} \vec{H}^{a,l} \cdot \vec{J}(y', z') \, dS_f' + \int_{S_a} (\vec{H}^{a,ai} - \vec{H}^{a,ae}) \cdot \vec{M}(x', y') \, dS_a' &= \vec{0}
\end{align*}
\]

Note that the difference in sign between interior and exterior currents has already been accounted for in the Greens functions.

4.3 Expanding and testing

4.3.1 The matrix equation

Eventually the currents in equations (4.10) and (4.11) have to be obtained. Therefore they are written as a sum of expansion functions,

\[
\begin{align*}
\vec{J} &= \sum_{q=1}^{Q} j_{qq} \vec{J}_{qq} + \sum_{q=Q+1}^{QQ} j_{zq} \vec{J}_{zq} \\
\vec{M} &= \sum_{s=1}^{S} m_{xs} \vec{M}_{xs} + \sum_{s=1}^{S} m_{ys} \vec{M}_{ys}
\end{align*}
\]

The electric currents \( \vec{J} \) are modelled with y-directed expansion modes (1 tot Q) and z-directed expansion modes (Q+1 tot QQ). The centers for the y-modes do not coincide with the z-mode centers. Hence they are not necessarily equal to each other in number. The magnetic currents \( \vec{M} \) on the aperture are written with summations over x-modes and y-modes which have the same centers.

In the expansion notation small characters denote the unknown expansion coefficients. Formulas (4.10) and (4.11) become,
4.3. EXPANDING AND TESTING

\[
\sum_{q=1}^{Q} \int_{S_f} \tilde{\mathbf{e}}^{ff} \cdot j_{yy} \tilde{J}_{yy} \, dS_f' + \sum_{q=Q+1}^{QQ} \int_{S_f} \tilde{\mathbf{e}}^{ff} \cdot j_{zq} \tilde{J}_{zq} \, dS_f' \\
+ \sum_{s=1}^{S} \int_{S_a} \tilde{\mathbf{e}}^{fa} \cdot m_{xs} \tilde{M}_{xs} \, dS_a' + \sum_{s=1}^{S} \int_{S_a} \tilde{\mathbf{e}}^{fa} \cdot m_{ys} \tilde{M}_{ys} \, dS_a' = -\tilde{\mathbf{e}}^g
\]  

(4.14)

\[
\sum_{q=1}^{Q} \int_{S_f} \tilde{\mathbf{h}}^{af} \cdot j_{yy} \tilde{J}_{yy} \, dS_f' + \sum_{q=Q+1}^{QQ} \int_{S_f} \tilde{\mathbf{h}}^{af} \cdot j_{zq} \tilde{J}_{zq} \, dS_f' \\
+ \sum_{s=1}^{S} \int_{S_a} (\tilde{\mathbf{h}}^{a,ai} + \tilde{\mathbf{h}}^{a,ae}) \cdot m_{xs} \tilde{M}_{xs} \, dS_a' \\
+ \sum_{s=1}^{S} \int_{S_a} (\tilde{\mathbf{h}}^{a,ai} + \tilde{\mathbf{h}}^{a,ae}) \cdot m_{ys} \tilde{M}_{ys} \, dS_a' = 0
\]  

(4.15)

The objective is to satisfy these boundary conditions in an average way over the fin surface and aperture. Therefore equations (4.14) and (4.15) are weighted with testing functions. An inner product is taken of the equation and these testing functions. For functions \( f \) (the equations) and \( g \) (the testing function) defined on a surface \( S \) this inner product is not-uniquely defined by,

\[
\langle f, g \rangle = \int_{S} f \, \bar{g} \, dS
\]  

(4.16)

Now equation (4.14) will be tested with \( \tilde{J}^{w}(y,z) \) over the fin surface and \( t^{th} \) aperture. Equation (4.15) will be tested over the aperture with \( \tilde{M}^{w}(x,y) \). The sums of testing functions have the same form as the expansion functions.

\[
\tilde{J}^{w} = \sum_{r=1}^{R} \tilde{J}^{w}_{yr} + \sum_{r=R+1}^{RR} \tilde{J}^{w}_{zr}
\]  

(4.17)

\[
\tilde{M}^{w} = \sum_{t=1}^{T} \tilde{M}^{w}_{xt} + \sum_{t=1}^{T} \tilde{M}^{w}_{yt},
\]  

(4.18)

The number of testing modes is equal to the number of expansion modes so summation limits \( R,RR,T,T \) are equal to the limits \( Q,QQ,S \). As said before testing with these functions actually means that boundary equations (4.14) and (4.15) are made valid for each \( r^{th} \) subdomain on fin surface and each \( t^{th} \) aperture. We get two sets of respectively \( r \) and \( t \) equations. As an example testing function 1 \( (r = 1) \) will be used on equation (4.14) to form the very first of all equations in the matrix equation.
4.4. THE IMPEDANCE MATRIX

\[
\sum_{q=1}^{Q} j_{yq} \int_{S_f} \int_{S_f} \vec{\mathcal{E}}_{f} \cdot \vec{\mathcal{J}}_{yq} \ dS_f \ dS_f + \sum_{q=Q+1}^{Q} j_{zq} \int_{S_f} \int_{S_f} \vec{\mathcal{E}}_{f} \cdot \vec{\mathcal{J}}_{zq} \ dS_f \ dS_f \\
+ \sum_{s=1}^{S} m_{xs} \int_{S_t} \int_{S_a} \vec{\mathcal{E}}_{f} \cdot \vec{\mathcal{J}}_{xs} \ dS_a \ dS_f + \sum_{s=1}^{S} m_{ys} \int_{S_t} \int_{S_a} \vec{\mathcal{E}}_{f} \cdot \vec{\mathcal{J}}_{ys} \ dS_a \ dS_f \\
= - \int_{S_f} \vec{\mathcal{E}} \cdot \vec{\mathcal{J}}_{y} \ dS_f
\]

Note that the weighting functions (superscript w) solely depend on unprimed coordinates, while the current distributions/expansion functions depend on primed coordinates. More will be said about the testing functions in subsection (4.6.2). Conveniently written as a matrix the two sets of equations look like,

\[
\begin{bmatrix}
Z_{yy, rq}^f & Z_{yz, rq}^f & T_{yf, rs}^f & T_{yf, rs}^f \\
Z_{yz, rq}^f & Z_{zz, rq}^f & T_{zf, rs}^f & T_{zf, rs}^f \\
\ldots & \ldots & \ldots & \ldots \\
T_{xf, ts}^a & T_{xf, ts}^a & Y_{aa, xt, ts}^a & Y_{aa, xt, ts}^a \\
T_{yx, ts}^a & T_{yx, ts}^a & Y_{aa, xt, ts}^a & Y_{aa, xt, ts}^a \\
\end{bmatrix}
\begin{bmatrix}
j_{yq} \\
j_{zq} \\
\ldots \\
m_{xs} \\
m_{ys} \\
\end{bmatrix}
= \begin{bmatrix}
V_r \\
V_l \\
\end{bmatrix} (4.19)
\]

The left matrix in the left-hand side of equation (4.19) is the impedance matrix, the right-hand side vector is the excitation matrix. The vector in the left-hand side is the current matrix, which has to be solved for.

The impedance matrix consists of four submatrices. The self-impedance submatrix \(Z_{f}^{I}\) expresses the coupling from the \(q^{th}\) y-directed electric fin current to the \(r^{th}\) y-directed fin element. \(Y_{rs}^{a}\) is the self-admittance matrix for the aperture. The matrix \(T_{f}^{a}\) indicates the influence from the \(s^{th}\) x-directed magnetic aperture current on the \(r^{th}\) y-directed fin element (\(T_{q}^{f}\) vice versa). The size of the matrix is determined by the number of expansion modes and test modes that are used \((q,r,s,t)\).

Every submatrix consists of four subsubmatrices because both expansion and testing functions have two coordinates.

The impedance matrix must be inverted to obtain a solution for the unknowns in the current matrix.

4.4 The impedance matrix

From equations (4.14) and (4.15) in this section the four submatrices in the impedance matrix will be written derived. Note that the final integral expressions are actually Fourier transforms of the current distributions. Expressions which are ready for implementation in software can be found in appendices C and D.
4.4. THE IMPEDANCE MATRIX

4.4.1 The self-impedance submatrix

Taking the y-component of both electric expansion and testing functions we get,

\[
Z_{yy, rq}^{ff} = \int_{S_f} \mathcal{J}_y^{\omega} \left[ \int_{S_f} \mathcal{E}_y^{ff} \cdot \mathcal{J}_y \, dS_f \right] \, dS_f \\
= \int_{S_f} \mathcal{J}_y^{\omega} \left[ \int_{S_f} \mathcal{E}_y^{ff} \cdot \mathcal{J}_y \, dS_f \right] \, dS_f
\]

Using the Greens functions from Appendix B, splitting the integrals in unprimed and primed coordinates and substituting \( x' = x = 0 \) in the integrals we get,

\[
Z_{yy, rq}^{ff} = \frac{1}{j\omega \varepsilon_0 b d} \sum_{n=-\infty}^{\infty} \sum_{p=1}^{\infty} \frac{k_0^2 - (k_y^n)^2}{\beta_{np}} \frac{\sin(\beta_{np}a)}{\cos(\beta_{np}a) - \cos(k_0 u_0 a)} \\
\cdot \int_{S_f} \mathcal{J}_y e^{jk_y^n y'} \sin\left(\frac{p \pi y'}{d}\right) \, dy' \, dz' \cdot \int_{S_f} \mathcal{J}_y e^{-jk_y^n y} \sin\left(\frac{p \pi y}{d}\right) \, dy \, dz
\]

Doing the same for the other submatrices \( Z \) we get,

\[
Z_{yy, rq}^{ff} = \frac{1}{j\omega \varepsilon_0 b d} \sum_{n=-\infty}^{\infty} \sum_{p=1}^{\infty} \frac{\pi k_y^n}{\beta_{np} d} \frac{\sin(\beta_{np}a)}{\cos(\beta_{np}a) - \cos(k_0 u_0 a)} \\
\cdot \int_{S_f} \mathcal{J}_y e^{jk_y^n y'} \cos\left(\frac{p \pi y'}{d}\right) \, dy' \, dz' \cdot \int_{S_f} \mathcal{J}_y e^{-jk_y^n y} \cos\left(\frac{p \pi y}{d}\right) \, dy \, dz
\]

\[
Z_{zz, rq}^{ff} = \frac{1}{j\omega \varepsilon_0 b d} \sum_{n=-\infty}^{\infty} \sum_{p=1}^{\infty} \frac{\pi k_y^n}{\beta_{np} d} \frac{\sin(\beta_{np}a)}{\cos(\beta_{np}a) - \cos(k_0 u_0 a)} \\
\cdot \int_{S_f} \mathcal{J}_y e^{jk_y^n y'} \cos\left(\frac{p \pi y'}{d}\right) \, dy' \, dz' \cdot \int_{S_f} \mathcal{J}_y e^{-jk_y^n y} \cos\left(\frac{p \pi y}{d}\right) \, dy \, dz
\]

with \( \beta_{np}^2 = k_0^2 - (k_y^n)^2 - (\frac{\pi}{d})^2 \)
4.4. THE IMPEDANCE MATRIX

4.4.2 The aperture-fin coupling submatrix

Written out the subsubmatrix $T_{y^a,rs}$ in the aperture-fin coupling matrix is,

$$T_{y^a,rs} = \int_{S_f} J_{y^r}^w \left[ \int_{S_a} \alpha_{y^a,rs} dS_a \right] dS_f$$

$$= \int_{S_f} J_{y^r}^w \left[ \int_{S_a} \alpha_{y^a,rs} dS_a \right] dS_f$$

Using Appendix B, splitting the integrals and substituting $x=0$ we have,

$$T_{y^a,rs} = -\frac{1}{ab} \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \frac{1}{\sin k_{mn} d} \cdot \int_{S_a} \mathcal{M}_{xz} e^{j k_x^m x'} e^{j k_y^m y'} dx' dy' \cdot \int_{S_f} J_{y^r}^w e^{-j k_y^m y} \sin k_{mn} (z + d) dydz$$

Performing similar derivations for the other submatrix elements,

$$T_{y^v,rs} = 0$$

$$T_{y^z,rs} = \int_{S_f} J_{y^r}^z \left[ \int_{S_a} \alpha_{y^z,rs} dS_a \right] dS_f$$

$$= \frac{1}{ab} \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \frac{j k_y^n}{k_{mn} \sin k_{mn} d} \cdot \int_{S_a} \mathcal{M}_{xz} e^{j k_x^m x'} e^{j k_y^n y'} dx' dy' \cdot \int_{S_f} J_{y^r}^z e^{-j k_y^n y} \cos k_{mn} (z + d) dydz$$

$$T_{y^l,rs} = \int_{S_f} J_{y^l}^l \left[ \int_{S_a} \alpha_{y^l,rs} dS_a \right] dS_f$$

$$= -\frac{1}{ab} \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \frac{j k_x^m}{k_{mn} \sin k_{mn} d} \cdot \int_{S_a} \mathcal{M}_{yl} e^{j k_x^m x'} e^{j k_y^n y'} dx' dy'$$

$$\cdot \int_{S_f} J_{y^l}^z e^{-j k_y^n y} \cos k_{mn} (z + d) dydz$$

with $k_{mn}^2 = k_0^2 - (k_x^m)^2 - (k_y^n)^2$.

4.4.3 The fin-aperture coupling submatrix

The submatrix element $T_{y^z,tq}$ can be written as,
4.4. THE IMPEDANCE MATRIX

\[ T_{xz,tq}^{af} = \int_{S_a} \mathcal{M}_{zt}^w \left[ \int_{S_f} \frac{\mathcal{H}_{yz}^a \mathcal{J}_{yz} dS_f}{\mathcal{H}_{zz}^a} \right] dS_a \]

\[ = \int_{S_a} \mathcal{M}_{zt}^w \left[ \int_{S_f} \mathcal{H}_{xy}^a \mathcal{J}_{yq} dS_f \right] dS_a \]

Using Appendix B, splitting the integrals and substituting \( z = 0 \) yields,

\[ T_{xz,tq}^{af} = \frac{1}{bd} \sum_{n=-\infty}^{\infty} \sum_{p=1}^{\infty} \frac{1}{p} \frac{1}{\beta_{np} \cos(\beta_{np}a) - \cos(k_0 u_0a)} \]

\[ \cdot \int_{S_f} \frac{\mathcal{J}_{yq} e^{ik_y y'} \sin \left( \frac{p \pi z'}{d} \right) dy' dz'}{dS_a} \cdot \int_{S_a} \mathcal{M}_{zt}^w \left[ e^{-j k_0 u_0a \sin \beta_{np} x} - \sin \beta_{np}(x - a) \right] e^{-j k_y y} dxdy \]

The other submatrices are,

\[ T_{xx,tq}^{af} = \int_{S_a} \mathcal{M}_{zt}^w \left[ \int_{S_f} \mathcal{H}_{zz}^a \mathcal{J}_{yz} dS_f \right] dS_a \]

\[ = \frac{1}{d} \sum_{n=-\infty}^{\infty} \sum_{p=0}^{\infty} \frac{1}{\beta_{np} \cos(\beta_{np}a) - \cos(k_0 u_0a)} \frac{1}{\beta_{np} a} \]

\[ \cdot \int_{S_f} \frac{\mathcal{J}_{yz} e^{ik_y y'} \cos \left( \frac{p \pi z'}{d} \right) dy' dz'}{dS_a} \cdot \int_{S_a} \mathcal{M}_{yt}^w \left[ e^{-j k_0 u_0a \cos \beta_{np} x} - \cos \beta_{np}(x - a) \right] e^{-j k_y y} dxdy \]

\[ T_{yy,tq}^{af} = 0 \]

\[ T_{yz,tq}^{af} = \int_{S_a} \mathcal{M}_{yt}^w \left[ \int_{S_f} \mathcal{H}_{yz}^a \mathcal{J}_{yz} dS_f \right] dS_a \]

\[ = \frac{1}{d} \sum_{n=-\infty}^{\infty} \sum_{p=0}^{\infty} \frac{1}{\beta_{np} \cos(\beta_{np}a) - \cos(k_0 u_0a)} \frac{1}{\beta_{np} a} \]

\[ \cdot \int_{S_f} \frac{\mathcal{J}_{yz} e^{ik_y y'} \cos \left( \frac{p \pi z'}{d} \right) dy' dz'}{dS_a} \cdot \int_{S_a} \mathcal{M}_{yt}^w \left[ e^{-j k_0 u_0a \cos \beta_{np} x} - \cos \beta_{np}(x - a) \right] e^{-j k_y y} dxdy \]

with \( \beta_{np}^2 = k_0^2 - (k_y^2 - p \pi)^2 / d^2 \).

4.4.4 The self-admittance submatrix

\[ Y_{xx,tq}^{ab} = \int_{S_a} \mathcal{M}_{zt}^w \left[ \int_{S_f} (\mathcal{H}_{zz}^a - \mathcal{H}_{zz}^a) \mathcal{J}_{yz} dS_f \right] dS_a \]

\[ = \int_{S_a} \mathcal{M}_{zt}^w \left[ \int_{S_f} (\mathcal{H}_{zz}^a - \mathcal{H}_{zz}^a) \mathcal{J}_{yz} dS_f \right] dS_a \]
4.5. THE FEED MODEL AND EXCITATION MATRIX

Again using Appendix B, splitting the integrals in primed and unprimed coordinates and
substituting $z = 0$ we have,

$$Y_{xx,ts}^{aa} = \frac{1}{\omega \mu_0 ab} \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \frac{k_m^2 - (k_n^m)^2}{k_{mn}} (\cot k_{mn}d + j)$$

$$\int_{S_a} M_{xt} e^{ik_m^m x'} e^{ik_n^m y'} dx'dy' \int_{S_a} M_{xt} e^{-ik_m^m x} e^{-ik_n^m y} dx dy$$

The other elements are,

$$Y_{xy,ts}^{aa} = \int_{S_a} M_{yt} \left[ \int_{S_a} (h_{xy}^{ai} - h_{xy}^{ae}) M_{ys} dS_a \right] dS_a$$

$$= \frac{1}{\omega \mu_0 ab} \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \frac{k_m^x k_n^y}{k_{mn}} (\cot k_{mn}d + j)$$

$$\int_{S_a} M_{ys} e^{ik_m^m x'} e^{ik_n^m y'} dx'dy' \int_{S_a} M_{yt} e^{-ik_m^m x} e^{-ik_n^m y} dx dy$$

$$Y_{yx,ts}^{aa} = \int_{S_a} M_{yt} \left[ \int_{S_a} (h_{yx}^{ai} - h_{yx}^{ae}) M_{zs} dS_a \right] dS_a$$

$$= \frac{1}{\omega \mu_0 ab} \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \frac{(k_m^2 - (k_n^m)^2)}{k_{mn}} (\cot k_{mn}d + j)$$

$$\int_{S_a} M_{ys} e^{ik_m^m x'} e^{ik_n^m y'} dx'dy' \int_{S_a} M_{yt} e^{-ik_m^m x} e^{-ik_n^m y} dx dy$$

4.5 The Feed model and excitation matrix

The elements in the bunny-ear array will be fed through protruding slotlines. This is shown in
figure (1.1) and (1.2). In the same way as coaxially feeds and other simple feed-types this type
of feed can be modelled with a delta-gap voltage source [7], as shown in figure (4.1). In [8] the
agreement of this type of feed (and other types) with reality is qualified for several types of
antennas. From [8] it was concluded that a delta-gap source is good enough for our application.
To validate this model the assumption has to be made that the gap is very small compared
to the other antenna-dimensions [4]. The location and orientation of this source coincide with
the physical structure and will be expressed in $E_g$ in the integral equation (4.14) and further.
It represents the electric coupling from feed to fin.
4.6. EXPANSION FUNCTIONS AND TESTING FUNCTIONS

Other delta-gap sources are the electric gap current or magnetic filament current. They are more accurate. In contrast to the voltage source they account for both electric and magnetic coupling which does exist in reality.

Using the delta-gap voltage source and with the help of the delta function rule, the \( r \)-th element of the excitation matrix can be written as,

\[
\mathcal{E}_g = \lim_{\Delta y \to 0} \left( -\frac{V_g}{\Delta y} \right) \cdot \mathcal{E}_y
= -\partial_y V_g \cdot \mathcal{E}_y
= -V_g \delta(\vec{r} - \vec{r}_g) \cdot \mathcal{E}_y
\]  

Since there is no exciting tangential field on the aperture, the right-hand side of equation 4.18 is zero and thus the elements of \( V_i \) are zero too.

4.6 Expansion functions and testing functions

4.6.1 Expansion functions

In this subsection a choice is made for the expansion functions for the \( y/z \) components of the electric fin current and the \( x/y \) components of the magnetic aperture currents. This is a crucial step in the process. It is important that the smoothness of the expected current pattern is in balance with the smoothness of the chosen expansion and testing patterns. Not smoothly enough chosen patterns yield inaccurate solutions, when the patterns are too smooth calculation becomes inefficient.

In general expansion functions can be divided in sub-domain and entire-domain functions. Entire-domain functions are of great use in configurations with a-priori knowledge of the sought
current pattern, for example a simple wire-antenna. To obtain complex current densities subdomain functions are often used. The spatial area under view is then divided in subdomains, each with a locally defined function. By choosing appropriate functions and subdomains which are small enough one can control accuracy. Subdomain expansion functions can have any form. Some types that are often used [4] are,

- A Piece-wise Constant function (PWS) in x-direction has the following shape,

\[ f_m(x) = \begin{cases} 
1 & x_{m-1} \leq x \leq x_m \\
0 & \text{elsewhere} 
\end{cases} \quad (4.22) \]

- A Piecewise Linear function (PWL) or rooftop-function in x-direction looks like,

\[ f_m(x) = \begin{cases} 
\frac{x-x_{m-1}}{c} & x_{m-1} \leq x \leq x_m \\
\frac{x_{m+1}-x}{c} & x_m \leq x < x_{m+1} \\
0 & \text{elsewhere} 
\end{cases} \quad (4.23) \]

In which c is a constant that determines the amplitude of the rooftop-function. A combination of PWL functions in two directions is called the tetrahedral function. By taking small overlapping areas with these functions any total distribution can be modelled without discontinuities. Disadvantage is the large amount of subdomains needed for accurate modelling, because of the lack of smoothness in the PWL and PWC functions.

- The Piecewise Sinusoidal function is a smoother function and is generally defined as,

\[ f_m(x) = g_m(x) \cdot h_m(y) \quad x_{m-1} \leq x \leq x_{m+1} \quad y_{m-1} \leq y \leq y_{m+1} \quad (4.24) \]

with,
4.6. EXPANSION FUNCTIONS AND TESTING FUNCTIONS

The function \( h_m(y) \) can have various forms but here only the simplest is mentioned.

\[
h_m(y) = \frac{1}{W_m} \quad y_{m-1} \leq y \leq y_{m+1} \quad (4.26)
\]

The symbols \( x_m, y_m \) denote respectively the borders of the \( m \)th subdomain while \( k_e \) is the PWS wavenumber and \( W_m = y_{m+1} - y_{m-1} \). This wavenumber can be chosen arbitrarily but it has been found that setting \( k_e \) equal to the effective wavenumber of the used substrate is a good choice [20]. The effective wavenumber of the substrate is defined as \( k_{eff} = k_0 \sqrt{\mu \varepsilon_{eff}} \). An approximation for the effective dielectric constant \( \varepsilon_{eff} \) is \( \varepsilon_{eff} \approx \frac{\varepsilon_{r+1}}{2} \) with \( \varepsilon_r \) the relative dielectric constant of the substrate. At this point a model without substrate is used, in the software \( k_e = k_0 \) is used.

- Other expansion functions are for example the truncated cosine [4] and the spline function [19].

Looking at the domain \( f_m(x) \) is defined on it is clear that each of the given \( m \)th expansion functions overlap neighbouring functions.

4.6.1.1 Expansion functions used in similar problems

- In a study on wideband tapered element phased-array antennas without a substrate, Chu divided the fins in equally sized rectangular patches [10]. See figure 4.5 (it must be said that only the patches on the fins are actually used.)

Using this grid PWS electric current distributions are chosen for the \( y \) direction and \( z \) direction. In [10] the equivalence principle is not used, so no magnetic currents are involved.

- In [7] Schaubert treats an infinite stripline-fed slot-antenna array with a ground-plane in a different way. For both magnetic and electric equivalent currents non-uniform PWS
4.6. EXPANSION FUNCTIONS AND TESTING FUNCTIONS

functions are used in tilted directions, i.e. the negative and positive half of the sinusoidal are not directed in the same way. See figure 4.6.

Curves in the fin-surface can more accurately be modelled in this way, on the expense of calculating time and complexity.

Bayard uses overlapping non-uniform tilted PWS functions in the modelling of equivalent electric currents in a protruding dipole [9], where in [18] he uses only simple non-tilted uniform functions.

- In [8] Cooley examines an infinite array of tapered-slot antennas with equivalence. Electric currents are modelled with non-uniform PWS functions an extra type of tilting while the magnetic current is expanded with Floquet-mode terms, i.e. a summation over neighbouring array elements.

4.6.1.2 Choice of expansion functions

It is important to note that the electric currents are expanded and tested using overlapping subdomain functions (summations over subdomains, formulas (4.12) and (4.17)), but for the magnetic expansion and testing entire domain functions will be used (summations over neighbouring array-elements, formulas (4.13) and (4.18)).

Figure 4.5: Rectangular modelling of electric fin current in bunny-ear antennas

Figure 4.6: Tilted sinusoidals in modelling of currents in bunny-ear antennas
4.6. EXPANSION FUNCTIONS AND TESTING FUNCTIONS

For the electric fin-currents uniform subdomains with overlapping PWS functions for $y$ and $z$ are used. Length and width are respectively $2h$ and $W$. Accuracy can reasonably well be controlled by choosing the grid-size to the smoothness of the geometry.

As done in [8] by Cooley, the magnetic current will be expanded with Floquet-mode terms. In fact the single summations in formulas (4.13) and (4.18) are double summations over $m$ and $n$. All these Floquet-modes added up model the magnetic currents which flow on the aperture. Note that these aperture modes are not the same as the Floquet-summations over $n$ in the vector potentials. In the next paragraph it will become clear that this choice will minimize calculating effort. In (4.12) and (4.13) the following expansion functions are chosen,

$$\mathcal{J}_{yy} = \mathcal{J}_{yz}$$

$$\mathcal{J}_{yz} = \sin k_e (h - |y' - y_q|)$$

on

$$y_q - h \leq y' \leq y_q + h, y_q = y_{q-1} + 2h$$

$$z_q - \frac{W}{2} \leq z' \leq z_q + \frac{W}{2}, z_q = z_{q-1} + W$$

$$\mathcal{J}_{zz} = \mathcal{J}_{zx}$$

$$\mathcal{J}_{zx} = \sin k_e (h - |z' - z_q|)$$

on

$$y_q - \frac{W}{2} \leq y' \leq y_q + \frac{W}{2}, y_q = y_{q-1} + W$$

$$z_q - h \leq z' \leq z_q + h, z_q = z_{q-1} + 2h$$

$$\tilde{\mathcal{M}}_{xs} = \tilde{\mathcal{M}}_{ys}$$

$$\tilde{\mathcal{M}}_{ys} = \tilde{\mathcal{M}}_{xs}$$

$$\mathcal{M}_{xs} = e^{-j (k_x^m x' + k_y^n y')}$$

$$\mathcal{M}_{ys} = e^{-j (k_x^m x' + k_y^n y')}$$

on

$$0 \leq x' \leq a$$

$$0 \leq y' \leq b$$

with $k_x^m$ and $k_y^n$ as in section 2.3.1. In these formulas $(y_q, z_q)$ indicate centers of a subdomain/mode. The constants $2h, W$ denote the distance between mode centers in the direction of the current and the direction perpendicular to the current, respectively. It is taken constant through the antenna surface. See figure (4.7).

4.6.2 Testing functions

Looking at the integral expressions for the matrix elements section 4.3 it is clear that it is convenient to make the electric current testing functions equal to the expansion functions (Galerkin testing) and the magnetic current testing functions equal to the conjugate of the expansion functions. This choice causes orthogonality in the summations and in some cases symmetry in matrices, which in turn speeds up convergence and reduces calculation effort. Hence expressions (4.17) and (4.18) can be specified,
4.6. EXPANSION FUNCTIONS AND TESTING FUNCTIONS

Figure 4.7: Subdomain modes for the electric current

Figure 4.8: Entire-domain modes for the aperture current

$$\tilde{J}_y^w = \tilde{e}_y J_y^w$$

$$J_y^w = \frac{\sin k_e (h - |y - y_r|)}{W \sin (k_e h)}$$  on

$$y_r - h \leq y \leq y_r + h, y_r = y_{r-1} + 2h$$

$$z_r - \frac{W}{2} \leq z \leq z_r + \frac{W}{2}, z_r = z_{r-1} + W$$

$$\tilde{J}_z^w = \tilde{e}_z J_z^w$$

$$J_z^w = \frac{\sin k_e (h - |z - z_r|)}{W \sin (k_e h)}$$  on

$$y_r - \frac{W}{2} \leq y \leq y_r + \frac{W}{2}, y_r = y_{r-1} + W$$

$$z_r - h \leq z \leq z_r + h, z_r = z_{r-1} + 2h$$

$$M_x^w = \tilde{e}_x M_x^w$$

$$M_y^w = \tilde{e}_y M_y^w$$

$$M_z^w = e^{-j(k_x z + k_y y)}$$
4.6. EXPANSION FUNCTIONS AND TESTING FUNCTIONS

\[ M_{yt} = e^{-j(k_{x}x' + k_{y}y')} \quad \text{on} \]
\[ 0 \leq x \leq a \]
\[ 0 \leq y \leq b \]
Chapter 5

Intermediate results

To test the developed model and software an array of monopoles has been examined. A unit-cell with two monopoles is shown in figure 5.1.

Magnitude and phase of the reflection coefficient $\Gamma$ have been investigated as a function of scan-angle $\theta$ and for $\phi = 0$ and $\phi = 45$, as shown in pictures 5.2 and 5.3.

In figure 5.4 the Voltage Standing Wave Ratio is shown as a function of the scan-angle $\theta$ and for $\phi = 0$ and $\phi = 45$ [24]. From these diagrams it is concluded that the monopole-array has maximum reflectivity in the direction $\theta = 0$. In this direction no power is transmitted ($\Gamma = 1$, $VSWR = +\infty$), while in the direction $\theta = 60$ maximum power is transmitted ($\Gamma = 0$, $VSWR = \pm 0$).

The calculations in figures 5.2 to 5.4 are done with $L = \lambda/4$, $d = \lambda/2$, $a = b = \lambda/2$, $NRMC = 7$, $EMAX = 9$ and $NN = PP = 30$. The array-elements are assumed to be connected to a 50-ohm load. The VSWR and reflection coefficient are given by,

$$VSWR = 20 \log \frac{1 + |\Gamma|}{1 - |\Gamma|}$$

$$\Gamma = \frac{Z_{in} - 50}{Z_{in} + 50}$$
5.1. CONVERGENCE

A vertical monopole above a groundplane

Figure 5.2: Magnitude of the reflection coefficient

The input-impedance of the monopole-array has been examined too. Convergence has been tested at the same time so results for the input-impedance are given in the next section.

5.1 Convergence

It is important to look at the convergence-behaviour because of accuracy-considerations and computation-effort. For this investigation we used \( W = 226.\text{E-5m} \), \( L = 147.\text{E-4m} \), \( a=b=606.\text{E10-4} \) and \( \phi = 45 \). Scan-angle \( \theta \) was determined by the used frequency by the formula,

\[
\theta = \arcsin \frac{\lambda}{a\sqrt{2}}
\]

Convergence of real and imaginary part of the input impedance with the number of subdomains (expansion-functions) NRMC on the monopole is shown in pictures 5.5 and 5.6. EMAX, NN and PP were chosen respectively 49, 50 and 50. It is seen that both real and imaginary part of the input impedance reach accurate values for NRMC=7. Comparing these pictures with [13], figure 3.1a/b, only a small deviation is noticed, probably caused by the difference used monopole shape and source model. In [13] a cylindrical monopole with diameter \( R \) is used in combination with a magnetic-frill generator as a source, while in this report a flat strip with equivalent width \( W = 2.\text{R} \) and a voltage-gap generator is used.
Convergence of the input impedance with the number of aperture expansion modes $EMAX$ is pictured in figures 5.7 and 5.8 (NRMC=5,NN=50,PP=50). This diagrams show even faster convergence, for $EMAX = 9$ sufficiently accurate results are obtained. For $EMAX$ the remarks made for NRMC must be beared in mind too. As the distance $d$ between equivalence plane and ground plane is chosen larger fewer aperture expansion modes are needed for well-converged results because the aperture to fin coupling is getting less with larger $d$ (more evanescent modes).

Convergence of the input impedance with the number of parallel-plate waveguide modes $NN,PP$ is shown in figure 5.9 to 5.12, whereby one of them is chosen 50 each time. Comparing to figure 3.2a/b in [13] there is fairly good agreement again. Moreover it shows that $NN = PP = 30$ yields accurate results.

Further it became clear that for the case of monopoles convergence exists with the monopole thickness $W$. For $W = 1.E - 5$ input impedance was significant with four figures. The place of the voltage gap source was of little influence on output, as long as the offset with the ground plane was not bigger than one millimeter.

5.2 Calculating efficiency

Special attention is paid to calculating efficiency, for else calculation time would be prohibitive.
5.2. CALCULATING EFFICIENCY

As it was said in section 5.1 choosing a relatively large distance $d$ between the equivalence plane and ground plane reduces the number of aperture expansion modes that is needed for well-converged results and thereby the calculation time.

Apart from that several things can be concluded from the formulas in appendix C.

The number of summations has been substantially reduced by choosing appropriate expansion functions and testing functions. Orthogonality in $m,n$ causes some summations to disappear in the aperture-fin and fin-aperture coupling matrix. In the selfadmittance matrix they totally disappear yielding a diagonal matrix. Nevertheless much summations have to carried out, especially in $Z_{ff}$. A practical example is given.

Suppose a structure with 50 expansion modes for the electric current and 121 for the magnetic current. With $NN=PP=50$, $Z_{ff}$ then contains $4*50*(2*50+1)*50*50=505.E+7$ summations, $T_{ff}$ contains $3*50*50*121=9075.E+2$ summations and $T_{fa}$ contains $3*50*50=7500$ summations. In improving efficiency the submatrix $Z_{ff}$ must be emphasized since it is obviously very much bigger than the rest. Storing integration results and constants in matrices beforehand and then calculating the elements is applied and reduced calculation time significantly.

Further reduction can be achieved by using the fact that some susubmatrices are equal to eachother, for example in $Y_{aa}$. Examining $Z_{ff}$ it is discovered that in the H-plane $\phi = 90$ reciprocity symmetry exists in this subarray. This means that element "r" has exactly the same influence on "q" as vice versa, $Z_{yy,rq} = Z_{yy,qr}$ and $Z_{yz,rq} = Z_{yz,qr}$. For $\phi = 90$, $v = \sin(\theta)\sin(\phi)$ becomes zero so $k_n^y = \frac{2\pi n}{d}$. In this case in the integral formulas for $Z_{yy}, Z_{yz}$ "r" and "q" may be interchanged since,
5.3. NUMERICAL STABILITY

The wavenumbers $\beta_{np}, k_{mn}$ in the x-direction and z-direction must be handled with care since they can take imaginary values which, in combination with exponential functions possibly leads to exploding solutions. To avoid results behaving as noise a unique sign must be chosen consistently for the square-root wavenumbers solutions, for example,

$$k_{mn} = \begin{cases} \sqrt{k_0^2 - (k_x^m)^2 - (k_y^m)^2} & k_0^2 > (k_x^m)^2 + (k_y^m)^2 \\ -j\sqrt{(k_x^m)^2 + (k_y^m)^2 - k_0^2} & k_0^2 < (k_x^m)^2 + (k_y^m)^2 \end{cases}$$

$$\beta_{np} = \begin{cases} \sqrt{k_0^2 - (k_y^m)^2 - \left(\frac{pT}{d}\right)^2} & k_0^2 > (k_y^m)^2 + \left(\frac{pT}{d}\right)^2 \\ -j\sqrt{(k_y^m)^2 + \left(\frac{pT}{d}\right)^2 - k_0^2} & k_0^2 < (k_y^m)^2 + \left(\frac{pT}{d}\right)^2 \end{cases}$$

Exponential expressions with these wavenumbers $(C_{np1,2})$ must consequently be rewritten in
5.4. SOFTWARE USERS GUIDE

an asymptotic form, based on the choice for $k_{mn}, \beta_{np}$. If this is not done problems might occur nevertheless,

In some cases underflow can not be avoided. Some terms in the $n,p$-summations do become very small, physically meaning that far-away array-elements have little influence on each other and that higher parallel-waveguide modes (evanescent modes) can be neglected.

5.4 Software users guide

Basically two programs have been written using Fortran-77 for Suns. They are called genarray.for, monoarr1.for. The first one is a complete implementation of the derived theory to handle protruding antennas with electric currents in both $y$ and $z$-direction such as Tapered-Slot antennas. It is still in need for an update. The second one is an undressed version which was made to test the theory on an array of monopoles (from the second one a Matlab version is made for flexibility).

The Fortran programs have a relatively simple structure. They use only one external subroutine, named zgeco.for. This subroutine inverts the impedance matrix, then solves the matrix equation and puts the solution in the right-hand side vector. In both programs comments have been inserted which speak for themselves. Input for monoarr.for is done from a separate file, in which several parameters can easily be input, depending on the precise choice of the programmer. Default the following parameters must be input in this separate file (put in the right order).

- f:frequency
Figure 5.7: Convergence with the number of aperture modes in y-direction

- $\text{fff}$: number of theta-values for which calculations are done
- $\phi$: scan angle
- $\text{nrmc}$: number of expansion modes on monopole
- $\text{MMAX/NMAX}$: number of aperture expansion modes in x/y direction
- $\text{NN/PP}$: number of parallel-waveguide modes in y/z direction
- $\text{dd}$: offset of source location from ground plane

Coordinates and dimensions of expansion modes are automatically calculated with the monopole length and number of expansion modes. Length and width of the monopole as well as the dimensions $a, b, d$ of the unit-cell in turn are automatically calculated as a function of the wavelength.
A vertical monopole above a groundplane

Figure 5.8: Convergence with the number of aperture modes in z-direction

A vertical monopole above a groundplane

Figure 5.9: Convergence with the number of parallel-plate modes in y-direction
5.4. SOFTWARE USERS GUIDE

Figure 5.10: Convergence with the number of parallel-plate modes in y-direction

Figure 5.11: Convergence with the number of parallel-plate modes in z-direction
Figure 5.12: Convergence with the number of parallel-plate modes in z-direction
Chapter 6

Conclusions and recommendations

A model has been developed and implemented to analyse infinite arrays of protruding antenna-structures. It has for a great deal been validated with an array of monopoles.

To reach the final goal of this project, a follow-up is necessary for which at this moment the following recommendations can be formulated.

- Finishing the validation of the model with monopoles
- Verification of the model with literature on bunny-ears [10]
- Extension of the model to antennas on dielectric sheets [7]
- Extension of the model to a double-printed dielectricum
- Improvement of accuracy by using tilted sinusoidals in fin-current model [8]
- Modelling and analysis of physical coupling from antenna to ground plane.

Apart from these objectives it could be interesting to use this report for analysis of the dipole-array which is used in SMART-L. SMART-L is a volume-radar developed by HSA. This array uses little walls between the dipoles to avoid hinder from mutual-coupling. In the radiation pattern of this array grating-lobes exist. The question is which influence the little walls have on this behaviour. The model described in this report could eventually help in modelling different situations and comparing them.
Bibliography

[1] ibson,P.J.
   The Vivaldi Aerial.
   Ninth European Conference on Microwaves,p101-105
   Brighton,UK,1979

   Time-Harmonic Electromagnetic Fields

   Theory and analysis of phased-array antennas.
   Wiley-Interscience,1972

   Antenna theory.
   Wiley and sons
   1982

   Analysis methods for electro-magnetic wave problems.
   Artech house,1990

   Moment methods in antenna and scattering.
   Artech House,1990

   Moment Method Analysis of Infinite Stripline-Fed Tapered Slot Antenna Arrays with a
   Groundplane.

   Radiation and scattering Analysis of infinite arrays of endfire slot antennas with a
   ground-plane.
Analysis of infinite arrays of printed dipoles on dielectric sheets perpendicular to a ground-plane.

[10] Chu, R.S. et al.
Analysis of wideband tapered element phased-array antennas.

Electromagnetic Antennas 1.
Eindhoven University of Technique, Faculty of Electrical Engineering, lecture notes, in Dutch
Eindhoven 1988

Analysis of an infinite micro-strip array.
Eindhoven University of Technique, Faculty of Electrical Engineering, Master of Science thesis, nr. EM-02-92
April 1992

Analysis of an infinite array of micro-strip antennas embedded in a thick substrate.
Eindhoven University of Technique, Faculty of Electrical Engineering, Master of Science thesis, nr. EM-7-94
August 1994

Phased-array antenna handbook.
Artech House
Boston/London 1994

[15] B. Smolders
Analysis of micro-strip antennas in the spectral domain using a moment method.
Eindhoven University of Technique, Faculty of Electrical Engineering, Master of Science thesis, nr. ET-15-1989
Eindhoven October 1989

[16] B. Smolders
Microstrip Phased-array antennas, a finite-array approach.
Eindhoven University of Technique, Faculty of Electrical Engineering, Ph.D. thesis
Eindhoven October 1994

[17] G. Dolmans
Stacked antennas embedded in a two-layer substrate.
Eindhoven University of Technique, Faculty of Electrical Engineering, Master of Science thesis, nr. EM-11-1992
Eindhoven October 1992
[18] J.P.R.Bayard  
Analysis of infinite arrays of micro-strip-fed dipoles printed protruding dielectric substrates and covered with a dielectric radome.  
IEEE Transactions on Antennas and Propagation, vol42 1994,p82-89

Computational electromagnetics- frequency domain method of moments.  
IEEE proceedings  
New-York,1992

[20] D.M.Pozar  
Input Impedance and mutual coupling of Rectangular Microstrip Antennas.  
IEEE transactions on Antennas and Propagation,vol AP30,no.6,1982

[21] Cooley,M.E.  
Analysis of infinite arrays of endfire slot antennas.  
Ph.D.thesis, University of Massachusets, Department of Electronic and Computational Engineering  
February 1992

[22] Harrington,R.F. and Mautz  
A general Network Formulation for aperture problems.  
IEEE Transactions on Antennas and Propagation,pp870-873  
November 1976

[23] Scharten,T.  
Electromagnetisme 2.  
Eindhoven University of Technique, Faculty of Electrical Engineering,lecture notes, in Dutch  
Eindhoven 1992

[24] Fenn,A.J.  
Theoretical and Experimental Study of Monopole Phased Array Antennas.  
IEEE Transactions on antennas and propagation, vol.AP-33 1985,No.10

[25] Janaswamy,R.  
An accurate Moment Method Model for the Tapered Slot Antenna.  

[26] Johansson,J.F.  
A moment method analysis linearly tapered slot antennas.  
IEEE Antennas and Propagation Symposium, Dig. vol.1 1989,p383-386

[27] Koksal,A. et.al.  
Moment Method Analysis of Linearly Tapered Slot Antennas.  
IEEE Antennas and Propagation Symposium, Dig. vol.1 1991,p314-317

[28] Lee,J.J. et.al.  
Wide Band Bunny-ear radiating element.  
IEEE Antennas and Propagation Symposium, Dig. vol.5 1993,p1604-1607
Appendix A

Variation of constants applied to the inhomogeneous spectral domain Helmholtz equation

With variation of constants we want to obtain particular solutions for the equation,

$$\partial^2_{zz}G_{zz} + k^2G_{zz} = -\mu_0 \delta(z - z')e^{jkz'}e^{jkz'},$$  \hspace{1cm} (A.1)

The found solution will then be tested to comply with the given physical geometry. A general solution of the homogeneous equation is,

$$G_{zz} = Ce^{jkz} + De^{-jkz}.$$  \hspace{1cm} (A.2)

Now suppose C and D are functions of z. We calculate the first and second derivative of $G_{zz}$ and then state that in this derivative the part with a derivative of C and/or D is zero, except for the highest (second) derivative of $G_{zz}$,

$$\partial_z G_{zz} = jkC(z)e^{jkz} - jkD(z)e^{-jkz} + [\partial_z C(z)]e^{jkz} + [\partial_z D(z)]e^{-jkz},$$  \hspace{1cm} (A.3)

with,

$$[\partial_z C(z)]e^{jkz} + [\partial_z D(z)]e^{-jkz} = 0$$  \hspace{1cm} (A.4)

$$\partial^2_{zz}G_{zz} = (jk)^2C(z)e^{jkz} + (-jk)^2D(z)e^{-jkz} + jk[\partial_z C(z)]e^{jkz} - jk[\partial_z D(z)]e^{-jkz}$$

$$\partial^2_{zz}G_{zz} = -k^2C(z)e^{jkz} - k^2D(z)e^{-jkz} + jk[\partial_z C(z)]e^{jkz} - jk[\partial_z D(z)]e^{-jkz},$$  \hspace{1cm} (A.5)

Substituting equations (A.2) and (A.5) in equation (A.1) yields,
A Variation of constants applied to the inhomogeneous spectral domain Helmholtz equation

\[-[\partial_z D(z)]e^{-j kz} + [\partial_z C(z)]e^{jkz} = -\frac{\mu_0}{jk} \delta(z - z')e^{jkz'}e^{jkz}\]  \hspace{1cm} (A.6)

From equation (A.4) we have,

\[\partial_z C(z) = -\partial_z D(z)e^{-2jkz}\]  \hspace{1cm} (A.7)

Using this in equation (A.6) we get,

\[\partial_z D(z) = \frac{\mu_0}{2jk} \delta(z - z')e^{j kz'}e^{jkz'}e^{jkz}\]  \hspace{1cm} (A.8)

In general for a continuous function \(f(z)\) may be written,

\[f(z) \delta(z - z') = f(z') \delta(z - z')\]  \hspace{1cm} (A.9)

It follows that,

\[D(z) = \frac{\mu_0}{2jk} e^{j kz'}e^{jkz}e^{jkz'}\int \delta(z - z')dz\]

\[D(z) = \frac{\mu_0}{2jk} e^{j kz'}e^{jkz}e^{jkz'}(U(z - z') + A),\]

in which \(U(z - z')\) is a delayed step-function and \(A\) is an integration constant. With equation (A.7) we obtain,

\[\partial_z C(z) = -\frac{\mu_0}{2jk} \delta(z - z')e^{j kz'}e^{jkz'}e^{-jkz'}\int \delta(z - z')dz\]

\[C(z) = -\frac{\mu_0}{2jk} e^{j kz'}e^{jkz}e^{-jkz'}(U(z - z') + B),\]

in which \(B\) again is an integration constant. Now using equation (A.2) the expression for this particular solution \(G_{xx}\) becomes,

\[G_{xx} = -\frac{\mu_0}{2jk} e^{j kz'}e^{jkz'}[e^{-jk(z' - z)}(U(z - z') + B) - e^{-jk(z - z')}(U(z - z') + A)]\]  \hspace{1cm} (A.10)

Writing \(G_{xx}\) for the two subdomains in \(z\) we get,

\[z > z' \hspace{1cm} G_{xx} = -\frac{\mu_0}{2jk} e^{j kz'}e^{jkz'}[e^{-jk(z' - z)}(1 + B) - e^{-jk(z - z')}(1 + A)]\]  \hspace{1cm} (A.11)

\[z < z' \hspace{1cm} G_{xx} = -\frac{\mu_0}{2jk} e^{j kz'}e^{jkz'}[e^{-jk(z' - z)}B - e^{-jk(z - z')}A]\]  \hspace{1cm} (A.12)
A Variation of constants applied to the inhomogeneous spectral domain Helmholtz equation

\( G_{xx} \) must satisfy Sommerfeld's radiation condition; waves only travel away from the source and not towards the source, i.e., for \( z < z' \) all waves travel to the left, for \( z > z' \) all waves travel to the right. From (A.11) it is then observed that \( B = -1 \) and from (A.12) that \( A = 0 \). An expression for the particular solution of differential equation (A.1), valid for the entire \( z \)-domain is then,

\[
G_{xz} = \frac{\mu_0}{2jk} e^{jkz} e^{jkz'} e^{-jk|z-z'|} \quad \forall z
\]  

(A.13)
Appendix B

Greens functions for airdielectric formulation

In this appendix the electric and magnetic field greens functions are given for electric and magnetic dipoles in respectively \((0, y', z')\) and \((x', y', 0)\).

From chapter 3 we have the Greens functions for vector potentials associated with respectively internal and external magnetic and electric unit sources,

\[
\mathcal{F}_x^i = -\frac{\varepsilon_0}{ab} \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \frac{1}{k_{mn} \sin k_{mn} d} e^{ik_m^T(x'-x)} e^{ik_n^T(y'-y)} \cos k_{mn}(x + d) \tag{B.1}
\]

\[
\mathcal{F}_y^i = \mathcal{F}_x^i \tag{B.2}
\]

\[
\mathcal{F}_x^e = \frac{\varepsilon_0}{ab} \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \frac{1}{j k_{mn}} e^{ik_m^T(x'-x)} e^{ik_n^T(y'-y)} e^{-j k_{mn} z} \tag{B.3}
\]

\[
\mathcal{F}_y^e = \mathcal{F}_x^e \tag{B.4}
\]

with \(k_{mn}^2 = k_0^2 - (k_x^m)^2 - (k_y^n)^2\).

\[
\mathcal{A}_y = \mu_0 \sum_{n=-\infty}^{\infty} \sum_{p=1}^{\infty} [A_{np} e^{j\beta_{np} x} + B_{np} e^{-j\beta_{np} x}] e^{ik_n^T(y'-y)} \sin \frac{p\pi x}{d} \tag{B.5}
\]

\[
\mathcal{A}_z = \mu_0 \sum_{n=-\infty}^{\infty} \sum_{p=0}^{\infty} [C_{np} e^{j\beta_{np} x} + D_{np} e^{-j\beta_{np} x}] e^{ik_n^T(y'-y)} \cos \frac{p\pi x}{d} \tag{B.6}
\]

with,

\[
A_{np} = -\frac{\sin \frac{p\pi x'}{d}}{jbd\beta_{np}(1 - e^{j(\beta_{np}k_{0u})})}
\]
B Greens functions for airdielectric formulation

\[
B_{np} = \frac{\sin \frac{\pi x}{d}}{\sqrt{b d \beta_{np}(1 - e^{-j\alpha(\beta_{np} - k_{0}u_{0})})}}
\]

\[
C_{np} = -\frac{\cos \frac{\pi x'}{d}}{\sqrt{b d \beta_{np}(1 - e^{j\alpha(\beta_{np} + k_{0}u_{0})})}} \frac{1}{\epsilon_{p}}
\]

\[
D_{np} = \frac{\cos \frac{\pi x'}{d}}{\sqrt{b d \beta_{np}(1 - e^{-j\alpha(\beta_{np} - k_{0}u_{0})})}} \frac{1}{\epsilon_{p}}
\]

\[
(\beta_{np})^{2} = k_{0}^{2} - \left(\frac{\omega}{\beta_{np}}\right)^{2} - (k_{y}^{n})^{2}
\]

\[
\epsilon_{p} = \begin{cases} 
2: & p = 0 \\
1: & p > 0
\end{cases}
\]

in which for example \( \mathcal{F}^{i}_{x} \) is the (only not-zero) \( x \)-component of the electric vector potential due to an \( x \)-directed magnetic unit source in \( \mathbf{r} = r' \). With the following rewriting,

\[
e^{j\beta_{np}x} - e^{-j\beta_{np}x} = j\frac{e^{-j\alpha(x - a)}}{\cos(\beta_{np}x) - \cos(k_{0}u_{0}a)}
\]

and formulas (2.6),(2.7),

\[
\mathcal{E}_{yy}^{ff} = \frac{-3\omega}{k_{0}^{2}}(\partial_{y}^{2} + k_{y}^{2})A_{y}
\]

\[
= \frac{-3\omega \mu_{0}}{k_{0}^{2}} \sum_{n=-\infty}^{\infty} \sum_{p=1}^{\infty} \frac{k_{0}^{2} - (k_{y}^{n})^{2}}{\beta_{np}} e^{j\kappa_{y}(y' - y)} \sin \frac{\pi x}{d} \sin \frac{\pi x'}{d} T_{1}(x)
\]

\[
\mathcal{E}_{yz}^{ff} = \frac{-3\omega}{k_{0}^{2}} \partial_{y} \partial_{z} A_{z}
\]

\[
= \frac{3\omega \mu_{0}}{k_{0}^{2}} \sum_{n=-\infty}^{\infty} \sum_{p=1}^{\infty} \frac{-j\kappa_{n} p x}{b} e^{j\kappa_{y}(y' - y)} \sin \frac{\pi x}{d} \cos \frac{\pi x'}{d} \epsilon_{p} T_{1}(x)
\]

\[
\mathcal{E}_{zy}^{ff} = \frac{3\omega}{k_{0}^{2}} \partial_{y} \partial_{z} A_{y}
\]

\[
= \frac{3\omega \mu_{0}}{k_{0}^{2}} \sum_{n=-\infty}^{\infty} \sum_{p=1}^{\infty} \frac{j\kappa_{n} p x}{b} e^{j\kappa_{y}(y' - y)} \cos \frac{\pi x}{d} \sin \frac{\pi x'}{d} T_{1}(x)
\]

\[
\mathcal{E}_{yy}^{ff} = \frac{3\omega}{k_{0}^{2}} \partial_{y} \partial_{z} A_{z}
\]

\[
= \frac{3\omega \mu_{0}}{k_{0}^{2}} \sum_{n=-\infty}^{\infty} \sum_{p=1}^{\infty} \frac{j\kappa_{n} p x}{b} e^{j\kappa_{y}(y' - y)} \cos \frac{\pi x}{d} \sin \frac{\pi x'}{d} T_{1}(x)
\]
B Greens functions for airdielectric formulation

\[ \mathcal{E}_{zz}^{ff} = -\frac{j\omega}{k_0^2} (\partial_z^2 + k_0^2) A_z \]

\[ = \frac{1}{j\omega \varepsilon_0 bd} \sum_{n=-\infty}^{\infty} \sum_{p=0}^{\infty} \left( \frac{k_0^2 - \left( \frac{p\pi}{d} \right)^2}{\beta_{np}} \right) e^{jk_n^p(y'-y)} \cos \frac{p\pi x}{d} \cos \frac{p\pi z'}{d} T_1(x) \]

with \( \beta_{np}^2 = k_0^2 - \left( \frac{p\pi}{d} \right)^2 - (k_y^p)^2 \) and,

\[ T_1(x) = \frac{e^{-jko_{qa} \sin \beta_{np} x - \sin \beta_{np}(x-a)}}{\cos \beta_{np} a - \cos ko_{qa}} \quad (B.7) \]

In the derivation of \( \mathcal{E}_{zz}^{ff} \) for \( p = 0 \) the expression is zero because of the sine and thus \( \varepsilon_p \) is not needed anymore.

Using formulas (2.44),(2.43),

\[ \mathcal{E}_{yx}^{fa} = -\frac{1}{\varepsilon_0} \partial_x \mathcal{F}_x^f = -\frac{1}{ab} \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} e^{jk_x^m(x'-x)} e^{jk_n^p(y'-y)} \sin k_{mn}(z + d) \]

\[ \mathcal{E}_{zz}^{fa} = \frac{1}{\varepsilon_0} \partial_y \mathcal{F}_x^f = \frac{1}{ab} \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} jk_y^p e^{jk_n^p(x'-x)} e^{jk_n^p(y'-y)} \cos k_{mn}(z + d) \]

\[ \mathcal{E}_{zy}^{fa} = -\frac{1}{\varepsilon_0} \partial_z \mathcal{F}_x^f = -\frac{1}{ab} \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} jk_x^m e^{jk_x^m(x'-x)} e^{jk_y^p(y'-y)} \cos k_{mn}(z + d) \]

\[ \mathcal{E}_{yy}^{fa} = 0 \]

with \( k_{mn}^2 = k_0^2 - (k_{xx}^m)^2 - (k_{yy}^p)^2 \). Using formulas (2.6),(2.7),

\[ \mathcal{H}_{xy}^{af} = -\frac{1}{\mu_0} \partial_x \mathcal{A}_y = -\frac{1}{bd} \sum_{n=-\infty}^{\infty} \sum_{p=1}^{\infty} \frac{p\pi}{\beta_{np} d} e^{jk_n^p(y'-y)} \cos \frac{p\pi x}{d} \sin \frac{p\pi z'}{d} T_1(x) \]

\[ \mathcal{H}_{xx}^{af} = \frac{1}{\mu_0} \partial_y \mathcal{A}_x = \frac{1}{bd} \sum_{n=-\infty}^{\infty} \sum_{p=0}^{\infty} jk_y^p e^{jk_n^p(y'-y)} \cos \frac{p\pi x}{d} \cos \frac{p\pi z'}{d} T_1(x) \]

\[ \mathcal{H}_{yx}^{af} = -\frac{1}{\mu_0} \partial_z \mathcal{A}_x = -\frac{1}{bd} \sum_{n=-\infty}^{\infty} \sum_{p=0}^{\infty} e^{jk_n^p(y'-y)} \cos \frac{p\pi x}{d} \cos \frac{p\pi z'}{d} T_2(x) \]

\[ \mathcal{H}_{yy}^{af} = 0 \]

with \( \beta_{np}^2 = k_0^2 - (\frac{p\pi}{d})^2 - (k_y^p)^2 \).

\[ \mathcal{H}_{xx}^{aa} = -\frac{j\omega}{k_0^3} (\partial_x^2 + k_0^2) \mathcal{F}_x^f = -\frac{1}{j\omega \mu_0 ab} \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} (k_0^2 - (k_{xx}^m)^2) e^{jk_x^m(x'-x)} e^{jk_n^p(y'-y)} \cos \frac{k_{mn}(z + d)}{k_{mn} \sin k_{mn} d} \]

\[ \mathcal{H}_{yx}^{aa} = -\frac{j\omega}{k_0^3} \partial_x \partial_y \mathcal{F}_x^f = \frac{1}{j\omega \mu_0 ab} \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} k_x^m k_y^p e^{jk_x^m(x'-x)} e^{jk_n^p(y'-y)} \cos \frac{k_{mn}(z + d)}{k_{mn} \sin k_{mn} d} \]
B Greens functions for airdielectric formulation

\[ H_{y_y}^{a,ai} = -\frac{j\omega}{k_0^2} (\partial_y^2 + k_0^2)F_y^i = -\frac{1}{j\omega\mu_0ab} \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} (k_0^2 - (k_y^n)^2) e^{jk_x^n(x'-x)} e^{jk_y^n(y'-y)} \frac{\cos k_m(z + d)}{k_m \sin k_m d} \]

\[ H_{y_z}^{a,ai} = -\frac{j\omega}{k_0^2} \partial_z \partial_y F_y^i = H_{yz}^{a,ai} \]

\[ H_{x_x}^{a,ae} = -\frac{j\omega}{k_0^2} (\partial_x^2 + k_0^2)F_x^e = \frac{1}{j\omega\mu_0ab} \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} (k_0^2 - (k_x^m)^2) e^{jk_x^m(x'-x)} e^{jk_y^n(y'-y)} \frac{e^{-jk_m n z}}{jk_m} \]

\[ H_{y_z}^{a,ae} = -\frac{j\omega}{k_0^2} \partial_x \partial_y F_x^e = H_{xz}^{a,ae} \]

\[ H_{y_y}^{a,ae} = -\frac{j\omega}{k_0^2} (\partial_y^2 + k_0^2)F_y^e = \frac{1}{j\omega\mu_0ab} \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} (k_0^2 - (k_y^n)^2) e^{jk_x^m(x'-x)} e^{jk_y^n(y'-y)} \frac{e^{-jk_m n z}}{jk_m} \]

\[ H_{x_y}^{a,ae} = -\frac{j\omega}{k_0^2} \partial_x \partial_y F_y^e = H_{yz}^{a,ae} \]

with \( k_m^2 = k_0^2 - (k_x^m)^2 - (k_y^n)^2 \) and,

\[ T_2(x) = \frac{e^{-jk_0 x a} \cos \beta_{np}(x - a) - \cos \beta_{np}(x - a)}{\cos \beta_{np} a - \cos k_0 x a} \] (B.8)
Appendix C

Matrix elements for use in software

The expressions for the subsubmatrix elements which were derived in chapter (4.4) are not suitable for direct implementation in software. Departing from the formulas in (4.4) in this appendix first the expansion and testing functions are substituted. Summation limits are given finite values, integrations are further splitted in x,y,z and integration domains are specified. In the case of the aperture-fin submatrix, fin-aperture submatrix and self admittance matrix then use is made of orthogonality to rewrite the integrals. All integrals in these expressions are written in abbreviated form to enhance overview. In appendix (D) closed form solutions for these integrals are given.

C.1 The impedance matrix

C.1.1 The self-impedance submatrix

\[
Z_{yy rq}^{ff} = \frac{1}{j\omega_0 bd} \sum_{n=-N}^{N} \sum_{p=1}^{P} \frac{k_0^2 - (k_{np}^n)^2}{\beta_{np}} \sin(\beta_{np}a) \cos(\beta_{np}d) - \cos(k_0 u_0 d) \\
\cdot \int_{z_q-W/2}^{z_q+W/2} \int_{y_q-h}^{y_q+h} \frac{\sin k_e(h - |y' - y_q|)}{W \sin(k_e h)} e^{jk_{np}^n y'} \sin \left(\frac{p \pi z'}{d} \right) dy' dz' \\
\cdot \int_{z_r-W/2}^{z_r+W/2} \int_{y_r-h}^{y_r+h} \frac{\sin k_e(h - |y' - y_r|)}{W \sin(k_e h)} \sin \left(\frac{p \pi z}{d} \right) dy dz \\
= \frac{1}{j\omega_0 bd} \sum_{n=-N}^{N} \sum_{p=1}^{P} \frac{k_0^2 - (k_{np}^n)^2}{\beta_{np}} \sin(\beta_{np}a) \cos(\beta_{np}d) - \cos(k_0 u_0 d) \\
\cdot \int_{y_q-h}^{y_q+h} e^{jk_{np}^n y} \sin k_e(h - |y' - y_q|) dy' \int_{z_q-W/2}^{z_q+W/2} \sin \left(\frac{p \pi z'}{d} \right) dz' \\
\cdot \int_{y_r-h}^{y_r+h} e^{-jk_{np}^n y} \sin k_e(h - |y' - y_r|) dy' \int_{z_r-W/2}^{z_r+W/2} \sin \left(\frac{p \pi z}{d} \right) dz \\
= \frac{1}{j\omega_0 bd} \sum_{n=-N}^{N} \sum_{p=1}^{P} (k_0^2 - (k_{np}^n)^2) C_{np1} I_0(y_q) I_1(z_q) I_0(y_r) I_1(z_r)
C.1. THE IMPEDANCE MATRIX

\[ Z_{yz, rq}^{ff} = \frac{1}{j \omega \varepsilon_0 \beta} \sum_{n=1}^{NN} \sum_{p=1}^{PP} \frac{j \pi k_n}{d} C_{np} I_1^*(y_q) I_4(z_q) I_6^*(y_r) I_3^*(z_r) \]

\[ Z_{zy, rq}^{ff} = -\frac{1}{j \omega \varepsilon_0 \beta} \sum_{n=1}^{NN} \sum_{p=1}^{PP} \frac{j \pi k_n}{d} C_{np} I_6(y_q) I_3^*(z_q) I_4(z_r) I_1^*(y_r) \]

\[ Z_{zz, rq}^{ff} = \frac{1}{j \omega \varepsilon_0 \beta} \sum_{n=-NN}^{NN} \sum_{p=0}^{PP} (k_0^2 - \left(\frac{\pi}{d}\right)^2) \frac{1}{\varepsilon_p} C_{np} I_1^*(y_q) I_4(z_q) I_6^*(y_r) I_3(z_r) \]

\[ \beta_{np}^2 = k_0^2 - \left(\frac{\pi}{d}\right)^2 \]

\[ C_{np} = \frac{\sin(\beta_{np} \alpha)}{\beta_{np}(\cos(\beta_{np} \alpha) - \cos(k_0 \alpha))} \]

C.1.2 The aperture-fin coupling submatrix

\[ T_{yx, rs}^{fa} = -\frac{1}{ab} \sum_{m=-MM}^{MM} \sum_{n=-NN}^{NN} \frac{1}{\sin(k_{mn} d)} \int_{y_{-h}}^{y_{h}} \int_{r_{-W/2}}^{r_{+W/2}} e^{-j k_0 x'} e^{j k_n y' \sin(k_{mn} d)} dx' dy' \]

\[ \int_{r_{-W/2}}^{r_{+W/2}} \sin(k_{mn} d) \int_{y_{-h}}^{y_{h}} e^{j k_n y' \sin(k_{mn} d)} dy' \]

\[ \int_{y_{-h}}^{y_{h}} \int_{r_{-W/2}}^{r_{+W/2}} e^{j k_n y' \sin(k_{mn} d)} dy' \]

The integration over \( y' \) can be reduced,

\[ \int_{0}^{b} e^{-j(k_{0} x' + \frac{2\pi}{b} x')} e^{j(k_{0} x' + \frac{2\pi}{b} x')} dy' = \int_{0}^{b} e^{j \frac{2\pi}{b} (n_n s)} dy' \]

\[ = \frac{b}{2\pi (n_n s)} \left[e^{j \frac{2\pi}{b} (n_n s)} - 1\right] \]

\[ = b \delta(n_n s) \]

For \( x' \) the same can be done. Then,

\[ T_{yx, rs}^{fa} = -\frac{1}{ab} \sum_{m=-MM}^{MM} \sum_{n=-NN}^{NN} \frac{1}{\sin(k_{mn} d)} a b \delta(m_m s) \delta(n_n s) I_6^*(y_r) I_3^*(z_r) \]

\[ = -\frac{1}{\sin(k_{mn} d)} I_6^*(y_r) I_3^*(z_r) \]
in which \( mn_s \) indicate that \( m, n \) take the value of \( s \), i.e. the \( mn \)-value of the \( s^{th} \) aperture under consideration. This must also be taken in account in the integrations. In a similar way,

\[
\begin{align*}
T_{yy,rs}^{fa} &= 0 \\
T_{zz,rs}^{fa} &= \frac{jk_{yn}^{m,s}}{k_{mn,s} \sin(k_{mn,s}d)} T_{1}^{*b}(y_r) T_{3}^{b}(z_r) \\
T_{yz,rs}^{fa} &= -\frac{jk_{yn}^{m,s}}{\sin(k_{mn,s}d)} T_{1}^{*b}(y_r) T_{3}^{b}(z_r) \\

\end{align*}
\]

\[
\beta_{np}^{2} = k_{yn}^{2} - (k_{yn})^{2} - \left( \frac{p\pi}{d} \right)^{2} \quad n = n_s, m = m_s
\]

C.1.3 The fin-aperture submatrix

Applying the same technique as in above paragraph we get,

\[
\begin{align*}
T_{zy,tq}^{af} &= \frac{1}{bd} \sum_{n=-NN}^{NN} \sum_{p=1}^{PP} \frac{p\pi}{\beta_{np}d \cos(\beta_{np}a) - \cos(k_{0}u_{0}a)} \\
&\cdot \int_{y_{q}+W/2}^{y_{q}+h} \int_{y_{h}+h}^{y_{h}-h} \frac{\sin k_{e}(h-|y'-y_{q}|)}{W \sin(k_{e}h)} e^{jk_{e}y} \sin\left(\frac{p\pi z'}{d}\right) dy' dz' \\
&\cdot \int_{0}^{a} \int_{0}^{a} e^{-jk_{e}x} \left[ e^{-jk_{0}u_{0}a} \sin \beta_{np} x - \sin \beta_{np}(x-a) \right] e^{-jk_{e}y} dx dy \\
&= \frac{1}{bd} \sum_{n=-NN}^{NN} \sum_{p=1}^{PP} \frac{p\pi}{\beta_{np}d \cos(\beta_{np}a) - \cos(k_{0}u_{0}a)} \\
&\cdot \int_{y_{q}+h}^{y_{q}+h} e^{jk_{e}y'} \sin k_{e}(h-|y'-y_{q}|) dy' \cdot \int_{z_{q}+W/2}^{z_{q}-W/2} \sin\left(\frac{p\pi z'}{d}\right) dz' \\
&\cdot \int_{0}^{a} \int_{0}^{a} e^{-jk_{e}x} \left[ e^{-jk_{0}u_{0}a} \sin \beta_{np} - \sin \beta_{np}(x-a) \right] dx \cdot \int_{0}^{b} e^{-jk_{e}y} e^{-jk_{e}y} dy \\
&= -\frac{1}{d} \sum_{p=1}^{PP} \frac{p\pi}{d\beta_{np}} \cdot I_{6}(y_{q}) \cdot I_{3}^{b}(z_{q}) \cdot I_{10}(x)
\end{align*}
\]

and,

\[
\begin{align*}
T_{yy,tq}^{af} &= 0 \\
T_{zz,tq}^{af} &= -\frac{1}{d} \sum_{p=0}^{PP} C_{np2} \frac{jk_{yn}^{m}}{\varepsilon_{p}\beta_{np}} \cdot I_{1}^{b}(y_{q}) \cdot I_{4}(z_{q}) \cdot I_{10}(x) \\
T_{yz,tq}^{af} &= -\frac{1}{d} \sum_{p=0}^{PP} C_{np2} \frac{1}{\varepsilon_{p}} \cdot I_{1}(y_{q}) \cdot I_{4}(z_{q}) \cdot I_{11}(x)
\end{align*}
\]
C.1.4 The self-admittance submatrix

In this submatrix orthogonality is easily applied in both expansion and test integrals to get,

\[
\begin{align*}
Y_{xx,ts}^{aa} &= -\frac{1}{j\omega\mu_0 ab} \sum_{m=-MM}^{M} \sum_{n=-NN}^{N} \frac{k_0^2 - (k_{m}^n)^2}{k_{mn}} (\cot k_{mn}d - j)\delta(m - m_s)\delta(n - n_s)\delta(m - m_t)\delta(n - n_t) \\
Y_{xy,ts}^{aa} &= \frac{ab}{j\omega\mu_0 k_{mn,ts}^2} (\cot k_{mn,ts}d + j) \\
Y_{yx,ts}^{aa} &= \frac{ab}{j\omega\mu_0 k_{mn,ts}^2} (\cot k_{mn,ts}d + j) \\
Y_{yy,ts}^{aa} &= -\frac{ab}{j\omega\mu_0 k_{mn,ts}^2} (\cot k_{mn,ts}d + j)
\end{align*}
\]

with,

\[
\beta_{np}^2 = k_0^2 - (k_{m}^n)^2 - (k_{n}^m)^2 \quad mn = t, s
\]  

(C.1)

These \(Y_{aa}\) subsubmatrices are diagonal because of the double orthogonality. The elements are zero except for the case that \(t = s\).

C.2 The source vector

From section (4.5) we continue with the calculation of the elements of the right-hand side vector of the matrix equation (4.19).

\[
V_r = \mathcal{J}_r^w(\gamma_r, \gamma_r) V_g \int_{S_f} \delta(\bar{r} - \bar{r}_g) dS_f
\]

\[
= V_g \left\{ \begin{array}{ll}
\frac{1}{W} & y_r = y_g, z_r = z_g \\
0 & \text{else}
\end{array} \right.
\]

\[
\int_{z_r - W/2}^{z_r + W/2} \int_{y_r - h}^{y_r + h} \delta(x - x_g) \delta(y - y_g) \delta(z - z_g) dy dz
\]

\[
\frac{\sin k_e(h - |y_r - y_q|)}{W \sin(k_e h)}
\]

\[
\int_{z_r - W/2}^{z_r + W/2} \int_{y_r - h}^{y_r + h} \delta(x - x_g) \delta(y - y_g) \delta(z - z_g) dy dz
\]

\[
\int_{z_r - W/2}^{z_r + W/2} \int_{y_r - h}^{y_r + h} \delta(x - x_g) \delta(y - y_g) \delta(z - z_g) dy dz
\]

\[
\frac{\sin k_e(h - |y_r - y_q|)}{W \sin(k_e h)}
\]

\[
\int_{z_r - W/2}^{z_r + W/2} \int_{y_r - h}^{y_r + h} \delta(x - x_g) \delta(y - y_g) \delta(z - z_g) dy dz
\]

\[
\frac{\sin k_e(h - |y_r - y_q|)}{W \sin(k_e h)}
\]

\[
\int_{z_r - W/2}^{z_r + W/2} \int_{y_r - h}^{y_r + h} \delta(x - x_g) \delta(y - y_g) \delta(z - z_g) dy dz
\]

\[
\frac{\sin k_e(h - |y_r - y_q|)}{W \sin(k_e h)}
\]

\[
\int_{z_r - W/2}^{z_r + W/2} \int_{y_r - h}^{y_r + h} \delta(x - x_g) \delta(y - y_g) \delta(z - z_g) dy dz
\]

\[
\frac{\sin k_e(h - |y_r - y_q|)}{W \sin(k_e h)}
\]

\[
\int_{z_r - W/2}^{z_r + W/2} \int_{y_r - h}^{y_r + h} \delta(x - x_g) \delta(y - y_g) \delta(z - z_g) dy dz
\]

\[
\frac{\sin k_e(h - |y_r - y_q|)}{W \sin(k_e h)}
\]

\[
\int_{z_r - W/2}^{z_r + W/2} \int_{y_r - h}^{y_r + h} \delta(x - x_g) \delta(y - y_g) \delta(z - z_g) dy dz
\]
The last step originates in the fact that in general an expansion/test mode is put right on top of the source. All elements of the source vector are zero except for one element.
Appendix D

Integrals

Because of their amount integrations are carried out in this separate appendix. Several remarks must be made.

- Two types of integration domains are used. When integrating in the direction of the current $h$ is used in the integration path, when integrating in the direction perpendicular to the current $W$ is used. The latter type has index $b$ (broad)

- The complex conjugate of a function is denoted with the superscript $^*$

- In $z = -d$ half-modes are used for $z$-directed expansion modes, i.e. integration is carried out over half of the domain (index $h$, half)

\[
I_{1b}(y_r) = \int_{y_r-W/2}^{y_r+W/2} e^{jky_y} \, dy = \frac{2}{k_y} e^{jky_r} \sin\left(\frac{k_y W}{2}\right)
\]

\[
I_{3b}(z_r) = \int_{z_r-W/2}^{z_r+W/2} \sin\left(\frac{p\pi z}{d}\right) \, dz = \frac{2d}{p\pi} \sin\left(\frac{p\pi z_r}{d}\right) \sin\left(\frac{p\pi W}{2d}\right)
\]

\[
I_{4}(z_r) = \int_{z_r-h}^{z_r+h} \mathcal{I}_z \cos\left(\frac{p\pi z}{d}\right) \, dz = \frac{2k_e \cos\left(\frac{p\pi z}{d}\right)}{W \sin(k_e h)(k_e^2 - (\frac{p\pi}{d})^2)} \left[\cos\left(\frac{p\pi h}{d}\right) - \cos(k_e h)\right]
\]

\[
I_{6}(y_r) = \int_{y_r-h}^{y_r+h} \mathcal{I}_y e^{jky_y} \, dy
\]

\[
= \frac{2k_e e^{jky_r}}{W \sin(k_e h)(k_e^2 - k_y^2)} \left[\cos(k_y h) - \cos(k_e h)\right] = I_{6}(y_r)
\]

\[
I_{7}(y_r) = \int_{y_r-h}^{y_r+h} \mathcal{I}_y e^{-jky_y} \, dy
\]

\[
= \frac{2k_e e^{-jky_r}}{W \sin(k_e h)(k_e^2 - k_y^2)} \left[\cos(k_y h) - \cos(k_e h)\right] = I_{7}(y_r)
\]

\[
I_{8}(z_r) = \int_{z_r-h}^{z_r+h} \sin(k_{mn}(z + d)) \, dz = -\frac{2}{k_{mn}} \sin(k_{mn}(z_r + d)) \sin\left(\frac{k_{mn} W}{2}\right)
\]
D Integrals

\[ I_9(z_r) = \int_{z_r-h}^{z_r+h} J_z \cos(k_{mn}(z_r + d)) \, dz = \frac{2k_e \cos(k_{mn}(z_r + d))}{W \sin(k_e h)(k_e^2 - k_{mn}^2)} \left[ \cos k_{mn} - \cos(k_e h) \right] \]

\[ I_{10}(x) = e^{-j \beta_{np} a} \int_0^a e^{j k_z x} \sin(\beta_{np} x) \, dx - \int_0^a e^{j k_z x} \sin(\beta_{np}(x - a)) \, dx = \frac{e^{-j \beta_{np} a} e^{j k_z a}(-\beta_{np} \cos(\beta_{np} a) + jk_z \sin(\beta_{np} a) + \beta_{np}) - \beta_{np} \cos(\beta_{np} a) - e^{j k_z a} - jk_z \sin(\beta_{np} a)}{\beta_{np}^2 - (k_z^2)^2} \]

\[ I_{11}(x) = e^{-j \beta_{np} a} \int_0^a e^{j k_z x} \cos(\beta_{np} x) \, dx - \int_0^a e^{j k_z x} \cos(\beta_{np}(x - a)) \, dx = \frac{e^{-j \beta_{np} a} \beta_{np} e^{j k_z a} \sin(\beta_{np} a) + jk_z (e^{j k_z a} \cos(\beta_{np} a) - 1) - \beta_{np} \sin(\beta_{np} a) - jk_z (e^{j k_z a} - \cos(\beta_{np} a)}{\beta_{np}^2 - (k_z^2)^2} \]

Half-modes are,

\[ I_i^h(z_r = -d) = \int_{-d}^{-d+h} J_z \cos\left(\frac{p \pi z}{d}\right) \, dz = \frac{1}{2W \sin(k_e h)} \left[ -\cos(p \pi + k_e h) + \cos(p \pi - \frac{p \pi h}{d}) + \cos(\frac{p \pi h}{d} - p \pi) - \cos(k_e h - p \pi) \right] \]

\[ I_i^h(z_r = -d) = \int_{-d}^{-d-h} J_z \cos(k_{mn}(z + d)) \, dz = \frac{k_e}{W \sin(k_e h)} \left[ \frac{\cos(k_{mn} h) - \cos(k_e h)}{k_e^2 - k_{mn}^2} \right] \]
Appendix E

Literature research report

Application of literature training to the M.Sc. Thesis:

ANALYSIS OF INFINITE ARRAYS OF BROADBAND ANTENNAS

Author: I.A. Vreeken
id.nr: 363597
Company: Hollandse Signaal Apparaten (HSA)
Division: Radar and Sensors, Functional Design, Mathematical Support (RS-FD-MMS)
University: University of Eindhoven, Fac. of Elec. Eng, Electromagnetism group (TUE)
Supervisor (HSA): dr. ir. A.B. Smolders
Supervisor (TUE): dr. M.E.J. Jeuken
E.0.0.1 SUMMARY OF THE GRADUATION PROJECT ASSIGNMENT

Introduction: Bunny-ear antennas are ultra-widebanded, because of the specific form of the metal structure placed on the dielectricum. From the original design antenna-types as the "bunny-ear" antenna and the "tapered slot" have been derived. In most applications an array of antennas is more useful than a single element. The problem that then arises is the mutual coupling, which means that the electromagnetic behaviour of the elements will change. Mutual coupling thus has a big influence on the radiation properties and adaption (i.e. bandwidth) of each element. In order to simplify the analysis the array will be considered to be of infinite size.

Aim of the graduation project:

The goal is to develop a theoretical model and accompanying software for the analysis of infinite arrays of protruding antennas in general and bunny-ears in particular. To examine the validity of the results some experiments on monopole-arrays will be performed.

A phased approach to the problem:

- Literature research on bunny-ear-alike antennas and arrays.
- Using the equivalence theorem to obtain a convenient antenna-structure.
- Derivation of the Greens functions for horizontal and vertical dipoles in air above a metal ground-plane in a unit-cell of the infinite array.
- Application of the Moment Method on an array with arbitrarily-shaped antennas.
- Validation with a monopole-array.
- Extension to bunny-ear antennas in air.

E.0.0.2 LITERATURE RESEARCH ASSIGNMENT

Find English literature on single Vivaldi-alike antennas such as bunny-ear antennas and tapered-slot antennas, as well on arrays of these elements. Only papers dated later than 1978/1979 are expected to contain relevant information, so the research is restricted to the period 1978/1979-1995. A complete collection is not necessary; a choice of key-articles with detailed information and derivations is made. Especially literature with theory on infinite arrays of protruding antennas in which use is made of the equivalence theorem is made, are interesting. Articles and Ph.D. thesis are preferred to conference papers and intermediate research reports for they generally contain more detailed information.
E.0.0.3 CONCEPT TABLE OF CONTENTS OF GRADUATION REPORT

List of Figures

1. GENERAL INTRODUCTION
   1.1 Phased-array antennas
      1.1.1. The infinite-array concept
   1.2 Numerical methods for the analysis of phased-array antennas
      1.2.1. Moment Method (MM)
   1.2.2. Other Methods
   1.2.3. Choice of method
   1.3. Strategy

2. GENERAL GREENS FUNCTIONS FOR INFINITE ARRAYS
   2.1. Introduction
   2.2. Single electric dipoles in air above a conducting ground-plane
      2.2.1. Maxwell's equations
   2.3. An infinite array of electric dipoles in air above a conducting ground-plane
      2.3.1. Floquet-modes in a rectangular grid
   2.4. An infinite array of magnetic dipoles in air above a conducting ground-plane

3. GREENS FUNCTIONS FOR AN INFINITE ARRAY OF PROTRUDING ANTENNAS IN AIR ABOVE A CONDUCTING GROUNDPLANE
   3.1. Introduction
   3.2. The equivalence theorem
      3.2.1. Why the equivalence theorem?
   3.3. Greens functions in the equivalent structure
      3.3.1. Magnetic currents
   3.3.2. An y-directed electric dipole
   3.3.3. A z-directed dipole
   3.3.4. Check of continuity of the electric field in the aperture

4. FORMULATION OF MOMENT METHOD
   4.1. Introduction
   4.2. Formulation of the remaining boundary-conditions
      4.2.1. Two-dimensional electric and magnetic field dyadic Greens functions
   4.2.2. Aperture and fin fields
   4.2.3. The remaining boundary conditions
   4.3. Expanding and testing
      4.3.1. The matrix equation
   4.4. The impedance matrix
      4.4.1. The self-impedance matrix
   4.4.2. The aperture-fin coupling matrix
   4.4.3. The fin-aperture coupling matrix
   4.4.4. The self-admittance matrix
4.5. The feed model and excitation matrix
4.6. Expansion functions and testing functions
4.6.1. Expansion functions
4.6.2. Testing functions

5. Intermediate results
5.1. Convergence
5.2. Calculating efficiency
5.3. Numerical Stability
5.4. Software users guide

6. Conclusions and recommendations

Bibliography
A Variation of constants applied to the inhomogenous spectral domain Helmholtz equation
B Green's functions for airdielectric formulation
C Matrix elements for use in software
D Integrals

E.0.0.4 LIST OF CATCHWORDS

The catchwords that are used in the literature research are:

- slot(-)antenna(s)
- tapered-slot antenna(s)
- bunny-ear antenna(s)
- bunny-ear element
- Vivaldi(-)antenna(s)
- Vivaldi aerial

E.0.0.5 LIST OF USED SOURCES

The following source were consulted:

- Vubis:
- Science Citation Index (printed version: period 1979-1995)
  The SCI was used in the following ways:
First the catch-words were used as a title-word in the subject-index (Permuterm) to and subsequently relevant information was obtained in the Source-index with the found author’s name. With the citation-index publications were found that referred to the publications found with the subject-index (see the diagrams further on in this appendix.)

- INSPEC (CD-ROM): mainly conference papers
- Conference papers donated by colleagues.

In the resulting list references in found publications are included.

E.0.0.6 SELECTED NUMBER OF REFERENCES IN FIRST INSTANCE

- Vubis: 25
- SCI: 10 (subject-index/citation-index)
- INSPEC: 22
- Donations: 2

E.0.0.7 SELECTION CRITERIA FOR INSERTION IN FINAL LITERATURE LIST

The selection criteria were:

- Contents: the contents has to agree with the graduation project assignment. This means that aforement antenna-types and/or arrays and their theoretical and practical behaviour are the central subject. Practical applications as well as flared-slot antennas and radial-slot antennas are of no interest. Some short research reports with practical results are included as a reference for experiments. Further the used antenna-modelling in the publications must be the Moment Method, because this method was used in the research too.
- Date of publication: after 1978/1979
- Language: only dutch, english, french and german literature can be handled, but in practice only english publications were found.
- Delivery time, ease of obtaining: enough publications were found in first instance, so literature which is relatively hard to get was not obtained.
- Reliability: conference papers and intermediate company research reports are less reliable then for example IEEE-Transactions.
E.0.0.8 LIST OF LITERATURE FROM SNOWBALL- AND CITATION METHOD

Arranged in order of year and in alphabetical order within a year hereunder a list of literature is given which was found. In the next paragraphs they are presented in diagrams. Literature from INSPEC is not included in these diagrams.

1. Gibson, P.J.
   The Vivaldi Aerial.
   Ninth European Conference on Microwaves, p.101-105
   Brighton, UK, 1979

2. Gibson, P.J.
   The Vivaldi Aerial.
   Ninth European Conference on Microwaves, p.120-124
   Brighton, 1979

3. Kollberg, E.L.
   Vivaldi antenna/finline circuit for SIS mixers.
   Sixth International Conference on Infrared Millimeter Waves, 1981

4. Thungren, T. et al.
   Vivaldi antennas for single beam integrated receivers.
   Twelfth European Conference on Microwaves, p.361-366
   Helsinki, Finland, 1982

5. Kollberg, E.L.
   New results on tapered slot endfire antennas on a dielectric substrate.
   IEEE Eighth International Conference on Infrared and Millimeter Waves, p.F3.6/1-2
   Miami Beach, Florida, USA, 1983

6. Pozar, D.M.
   Imaging system at 94Ghz using tapered slot antenna elements.
   Eighth International Conference on Infrared and Millimeter Waves
   1983

   Endfire Tapered Slot Antennas on Dielectric Substrates.

8. Janaswamy, R.S.
   Radiation pattern analysis of the tapered slot antenna.
   Ph.D. thesis, University of Massachusetts, Amherst, 1986
   Analysis of the transverse electromagnetic mode in linearly tapered slot antennas
   Radio Science, vol. 21, 1986, no. 5, p797-804

10. Janaswamy, R.S. et al.
    Analysis of the Tapered Slot Antenna.

11. Gazit, E.
    Improved design of the Vivaldi antenna

12. Johansson, J.F.
    Tapered slot antennas and focal plane imaging systems.
    Ph.D. thesis, School of Electrical and Computational Engineering
    Chalmers University of Technology
    Goteburg, Sweden, Augustus 1988

13. Catedra, M.F. et al.
    Analysis of microstrip and Vivaldi antennas using a CGFFT scheme that allows the study
    of finite dielectric sheets with arbitrary metallization on both sides.
    IEEE Antennas and Propagation Symposium, Dig., p1332-1335
    San Jose, 1989

14. Janaswamy, R.
    An accurate Moment Method Model for the Tapered Slot Antenna.

15. Johansson, J.F.
    A moment method analysis linearly tapered slot antennas.

    The Tapered Slot Antenna-A New Integrated Element for Millimeter-Wave Applications.

17. Kim, Y.S.M. et al.
    Characterization of Tapered Slot Antenna Feeds and Feed Arrays.

18. Ndagijimana, F. et al.
    Tapered slot antenna analysis with 3-D TLM method.
19. Cooley, M.E.
Radiation and scattering Analysis of infinite arrays of endfire slot antennas with a ground-plane.

Moment Method Analysis of Linearly Tapered Slot Antennas.
IEEE Antennas and Propagation Symposium, Dig. vol.1 1991, p314-317

21. Scharstein, R.W.
Receive and transmit mode aperture field of the parallel-plate fed slot antenna
IEEE el.magn. vol.33 1991, p59-61

22. Wunsch, A.D.
The vector effective length of slot antennas.

23. Cooley, M.E.
Analysis of infinite arrays of endfire slot antennas.

24. Feld, R.N.
Diffractional slot antennas—the theory and computations (An overview).
Radiotek. EL vol.36 1992, no.12, p2257-2280

Analysis of printed linear slot antenna using lossy transmission line model.
Electronic Letters, vol.28 1992, no.6, p598-601

Waveguide-Fed Parallel Plate Slot Array Antenna.

27. Simons, R.N. et al.
New techniques for exciting linearly tapered slot antennas with coplanar waveguide.
28. German, F. et.al.
   Analysis of flared slot antennas for phased array applications.
   IEEE Antenna and Propagation Symposium, Dig. p1600-1603
   Ann Harbor 1993

29. Koksal, A. et.al.
   Analysis of linearly tapered slot antennas on a dielectric substrate.
   IEEE Antennas and Propagation URSI Symposium Dig. p338-341
   July 1993

30. Langley, J.D.S. et.al.
    Novel ultrawide-bandwidth Vivaldi antenna with low crosspolarisation.

    Wide Band Bunny-ear radiating element.
    IEEE Antennas and Propagation Symposium, Dig. vol.5 1993, Ann Harbor, p1604-1607

32. Akhavan, H.G. et.al.
    Approximate model for microstrip fed slot antennas.

33. Biffl Gentili, G. et.al.
    FDTD analysis of broad-band tapered slot antennas.
    International Conference on progress in electromagnetic research.
    European Space Research and Technology Centre, Piers
    Noordwijk, 11-15 July 1994

34. Chu, R.S. et.al.
    Analysis of wideband tapered element phased-array antennas.

35. Koksal, A. et.al.
    Parametric study of linearly tapered slot antennas in air.
    International Conference on progress in electromagnetic research
    European Space Research and Technology Centre, Piers
    Noordwijk, 11-15 July 1994

36. Kormanyos, B.K. et.al.
    CPW-Fed Active Slot Antennas.
    IEEE Transactions on Microwave Technique, vol.42 1994, no.4, p541-545
37. Schaubert, D.H. et al.
Moment Method Analysis of Infinite Stripline-Fed Tapered Slot Antenna Arrays with a Groundplane
E.0.0.9 DIAGRAM OF THE SNOWBALL METHOD

1994
1993
1992
1991
1990
1989
1988
1987
1986
1985
1984
1983
1982
1981
1980
1979

1
2

3
4
5
6
7
8
9
10
11
12
13
14
15
16
20
23
29
31
33
34
35
37

1981
E.0.0.10 DIAGRAM OF THE CITATION METHOD
E.0.0.11 LIST OF LITERATURE FOUND WITH INSPEC ON CD-ROM

1. Macedo Filho, A.D. et. al.
   Cross polarization performance of vivaldi antennas in DC transverse magnetic fields.
   Conference Proceedings. 21st European Microwave Conference, Microwave '91, p. 1084-8
   Microwave Exhibitions Publishers, Tunbridge Wells, UK, 1991

2. Yngvesson, K.S.
   Review of integrated millimeter wave tapered slot antennas and arrays.
   1990 International Symposium Digest. Antennas and Propagation. Institute of Electrical and Electronics Engineers. Merging Technologies for the 90's (Cat. No. 90CH2776-3) p. 1406-8 vol.4

   32 GHz power-combining TSA array with limited sector scanning.
   Merging Technologies for the 90's (Cat. No.90CH2776-3),p. 1150-3 vol.3

   Parametric studies of the linearly tapered slot antenna (LTSA).
   Microwave and Optical Technology Letters
   Vol: 4 Iss: 5 p. 200-7, April 1991, USA

5. Safavi-Naini, S.
   Dispersion characteristics of wide slotlines on thin dielectric substrates in a two dimensional phased array environment.

6. Yngvesson, K.S. et. al.
   Characterization of tapered slot antenna feeds and feed arrays.

7. Dias de Macedo Filho, A. et. al.
   The Ortho-Mode Vivaldi antenna.
8. Sierra, M. et.al.
   Multibeam antenna array.
   IEEE, New York, NY, USA, 1989 3 vol. x+1754 pp., USA

9. Catedra, M.F. et. al.
   Analysis of arrays of Vivaldi and LTSA antennas.
   IEEE, New York, NY, USA, 1989 3 vol

10. Schaubert, D.H.
    Radiation characteristics of linearly tapered slot antennas.
    IEEE, New York, NY, USA, 1989 3 vol

11. Johansson, J.F.
    A moment method analysis of linearly tapered slot antennas.
    IEEE, New York, NY, USA

    Analysis of radiation from the tapered slot antenna.

    Characterization of Vivaldi antennas utilizing a microstrip-to-slotline transition
    International Symposium Digest Antennas and Propagation (Cat. No.93CH3289-6), p. 1212-15 vol.3
    IEEE, New York, NY, USA, 1993 3 vol.

14. Taylor, R.M.
    A broadband omnidirectional antenna
    IEEE. 1994 International Symposium Digest Antennas and Propagation
15. Chinglung Chen et. al.
Spectral domain analysis of microstripline fed arbitrarily-shaped aperture antennas.
IEEE. 1994 International Symposium Digest Antennas and Propagation
(Cat. No.94CH3466-0),p. 158-61 vol.1
IEEE,New York, NY, USA,1994 3 vol.

An explanation of some E-plane scan blindnesses in
single-polarized tapered slot arrays.
1612-15 vol.3
IEEE,New York, NY, USA,1993 3 vol.

17. Simons, R.N. et. al.
Linearly tapered slot antenna with dielectric superstrate.
IEEE Antennas and Propagation Society International Symposium 1993
International Symposium Digest Antennas and Propagation
(Cat. No.93CH3289-6),p. 1482-5 vol.3
IEEE,New York, NY, USA,1993 3 vol.

Characteristics of linearly tapered slot antennas with CPW feed on high resistivity silico
International Symposium Digest Antennas and Propagation
(Cat. No. 93CH3289-6), p588-91 vol.2

Non-planar linearly tapered slot antennas with balanced microstrip feed.
IEEE Antennas and Propagation Society International Symposium 1992
Digest. Held in Conjunction with:
URSI Radio Science Meeting and Nuclear EMP Meeting
(Cat.No.92CH3178-1),p. 2109-12 vol.4

20. Simons, R.N. et. al.
Effect of a dielectric overlay on a linearly tapered.
slot antenna excited by a coplanar waveguide
Microwave and Optical Technology Letters
Vol: 6 Iss: 4 p. 225-8, 20 March 1993, USA

21. Frayne, P.G. et. al.
Wideband measurements on Vivaldi travelling wave antennas.
IEE Colloquium on 'Multi-Octave Microwave Circuits', (Digest No.166) p. 5/1-6

22. Ndagijimana, F. et. al.
Introduction to the calculation of currents and radiation of tapered slot antennas.
Journees Internationales de Nice sur la Antennes
Conferences (Proceedings of Nice International Conference on Antennas), p. 301-4

E.0.0.12 RELATION BETWEEN THE FOUND REFERENCES AND THE CONCEPT TABLE OF CONTENTS

In the next table the relation is given between the references in the reference-list above and the concept table of contents above. A * means that the reference was used in the chapter. Publications which have no obvious relation to the chapters are publications which are expected to be of importance in the last phase of the research.

<table>
<thead>
<tr>
<th>Chapt.</th>
<th>Lit.</th>
<th>1</th>
<th>14</th>
<th>15</th>
<th>19</th>
<th>20</th>
<th>23</th>
<th>31</th>
<th>34</th>
<th>37</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>*</td>
<td></td>
<td></td>
<td>*</td>
<td>*</td>
<td>*</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>*</td>
<td>*</td>
<td></td>
<td>*</td>
<td>*</td>
<td>*</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

E.0.0.13 CONCLUSIONS and RECOMMENDATIONS

Looking at the (number of) publications from the first selection that according to the table above are used in the M.Sc. thesis a couple of conclusions can be drawn:

- The literature research assignment was very useful but many of the problems encountered during the research were solved using other literature.
- References [19],[23] were the most vital references, as they were used throughout the report. Authors who were regularly encountered during the project were D.H. Schaubert,
M.E. Cooley and J.P.R. Bayard. They provided in useful and often more detailed theoretical information on the subject of modelling protruding antennas.

INSPEC has been used only on CD-ROM. For further research it is recommended that the printed version is consulted. It contains more recent publications for it more often updated (till now).

E.0.0.14 THE FINAL LITERATURE LIST

From the reference list produced with this the literature research assignment above only a part was used, as a result of the Selection Criteria above. Other reasons for this were both the sufficient amount of articles obtained in an earlier stage and the completeness of certain publications. From the INSPEC-listing no literature is used at all. Nevertheless it possibly contains useful information for follow-up studies. They are given in the final literature list below:

1. Gibson, P.J.
   The Vivaldi Aerial.
   Ninth European Conference on Microwaves, p101-105
   Brighton, 1979

2. Janaswamy, R.
   An accurate Moment Method Model for the Tapered Slot Antenna.

3. Johansson, J.F.
   A moment method analysis linearly tapered slot antennas.
   IEEE Antennas and Propagation Symposium, Dig. vol.1 1989, p383-386

4. Cooley, M.E.
   Radiation and scattering Analysis of infinite arrays of endfire slot antennas with a ground-plane.

5. Koksal, A. et.al.
   Moment Method Analysis of Linearly Tapered Slot Antennas.
   IEEE Antennas and Propagation Symposium, Dig. vol.1 1991, p314-317

6. Cooley, M.E.
   Analysis of infinite arrays of endfire slot antennas.
   Ph.D. thesis, University of Massachusetts, Amherst, 1992
   Wide Band Bunny-car radiating element.
   IEEE Antennas and Propagation Symposium, Dig. vol.5, p1604-1607
   Ann Arbor, 1993

   Moment Method Analysis of Infinite Stripline-Fed Tapered Slot Antenna Arrays with a Groundplane.

9. Chu, R.S. et al.
   Analysis of wideband tapered element phased-array antennas.