MASTER

Endless polarization control using piezoelectric systems

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ENDLESS POLARIZATION CONTROL USING
PIEZOELECTRIC SQUEEZERS

W.H.J. Aarts

Report of the graduation work
performed from October 1987 till August 1988
Professor: prof.ir. G.D. Khoe

The department of electronical engineering of the Technical University Eindhoven is not responsible for the contents of graduation reports.
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Abstract

The first section of this report is a detailed introduction to polarization control.

On the topic of endless polarization control, the algorithm of Noé is described. In this scheme, three piezoelectric squeezers are used to control the polarization. Hereafter, a new approach to the problem is presented.

Experimental work has been done on the scheme proposed by Mahon. The scheme is a combination of polarization control and polarization diversity. The scheme can be simplified to a polarization diversity system without a ratio combiner but preceded by two piezoelectric squeezers.
Summary

In a coherent optical communication system, the polarization of the received signal light must be matched to the polarization of the local oscillator. The following solutions are proposed: polarization diversity and polarization control.

Since a state of polarization (SOP) has two degrees of freedom, two parameters must be controlled. In the polarization diversity scheme, the necessary parameters are controlled electronically by means of a phase control circuit and a ratio combiner. On the other hand, polarization control is performed by two devices which induce a particular form of birefringence.

In our case, piezoelectric squeezers having a limited retardation range, are implemented. To achieve endless polarization control, an extra squeezer and a special control algorithm i.e. reset algorithm are required to prevent signal losses during reset. Noé has proposed such an algorithm.

In general, the Poincaré sphere is used to illustrate the transformation from input SOP to output SOP. Another way of looking at the problem is to describe the transformation by the corresponding retardations of the three squeezers leading to the introduction of the S1,S2,S3-space. The S1,S2,S3-space is defined as a three-dimensional space having a coordinate system with the retardations scaled along the axes. A transformation is a point in this space, where the three coordinates stand for the retardations. The retardation range limits of the squeezers are represented by a box. The reset algorithm has to prevent situations having a combination of retardations corresponding to a point outside the box.

Moreover, the terms "Reset traces" and "Controllability" are introduced leading to the proposal of a new algorithm.

Mahon has proposed a scheme, which is a combination of polarization control and polarization diversity. One parameter of the SOP is controlled electronically using a phase control circuit and one parameter is controlled by means of a squeezer. An extra squeezer is added to achieve endless control.

An experimental set-up based on the scheme, was built. A critical part of the set-up is a beamsplitter plate on which the interference of two beams occurs in open air. Consequently, the set-up is very sensitive to air currents which causes instability. To improve this aspect, the beamsplitter plate is placed in a plastic box and polarization maintaining fibre is used for the optical pathes. Mahon's scheme can be simplified by omitting the beamsplitter, resulting in a polarization diversity system without a ratio combiner and preceded by two squeezers.
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1.0 Introduction

In the last decade, interest in optical communication systems has grown.

The first optical communication systems examined were based on the principle of direct detection. Early systems consisted of analogue schemes, later followed by digital transmission, where the laser is switched on-off depending on the information bit sent over (on-off keying). The receiver detects only the energy of the optical wave and no phase information is sent.

A more advanced scheme is the technique of coherent detection. Unlike direct detection, where the optical signal is converted directly into a demodulated electrical output, the coherent receiver first adds to the signal a locally generated optical wave (local oscillator) and then detects the sum. The resulting photocurrent is a replica of the original signal, translated down in frequency from the optical domain to the radio domain, where conventional electronic techniques can be used for further signal processing. Compared with direct detection, this method offers significant improvements in receiver sensitivity and wavelength selectivity.

On the other hand, the complexity of the receiver is increased; the linewidth of the laser must be much narrower and only single-mode fibre can be used.

A necessary condition for maximum receiver sensitivity is the matching between the states of polarization of the received signal and the local oscillator light. When no extra equipment is used for polarization matching, signal fading occurs due to polarization fluctuation of the signal light. The causes of polarization fluctuation are due to thermal and mechanical disturbances undergone by the fibre.

Different solutions are proposed for the problem of signal fading due to polarization mismatch. One of them is polarization diversity in which two orthogonal components of the received signal are detected separately and added later.

Another is polarization control at the receiver end. This consists of active matching of the signal and local oscillator states by fibre deformation. If the polarization control devices have an unlimited range, then two control devices are necessary. On the other hand, three devices are required, when they have a limited range. To overcome the problem of limited range, a control algorithm is needed to reset a device without introducing signal loss.

This report concerns my graduation work for the period October 1987 - August 1988.

In the group "Wideband Communication Systems" at the Philips Research laboratories, where I have performed my graduation work, coherent optical communication systems are being currently examined.

Mahon, BE, an ex-member of the group, has researched in polarization control and polarization diversity. He has proposed a scheme which is a combination of polarization control and polarization diversity. Much experimental work was also carried out by him.

My main task is to improve the existing set-up which was very susceptible to environmental perturbations, especially air currents. I have also performed some theoretical work on polarization control using polarization control devices of limited range.
2.0 Introduction to Polarization Control

2.1 Polarization

Before discussing polarization control in optical communication systems, it is necessary to have a basic knowledge of the characteristics and properties of polarized light. A detailed treatment of this topic is beyond the scope of this report, see references [1],[2].

Polarization is a property that is common to all types of vector waves. For all those types polarization refers to the behaviour with time of one of the field vectors observed at a fixed point in space. Light waves are electromagnetic in nature and require four basic field vectors for their complete description: $\hat{E}$, $\hat{H}$, $\hat{D}$ and $\hat{B}$. Of those four vectors the electric vector $\hat{E}$ is chosen to define the State Of Polarization (SOP) of light waves. The vector $\hat{E}$ can be split into two orthogonal components $\hat{E}_x$ and $\hat{E}_y$. For a monochromatic wave these components are described using the real parts of the complex harmonic function.

$$
\bar{E}_x = \text{Re}(|E_x| e^{i(\delta_x + \omega t - 2\pi n)}) \\
\bar{E}_y = \text{Re}(|E_y| e^{i(\delta_y + \omega t - 2\pi n)})
$$

with $\omega = 2\pi f$

$x,y,z$ is a right-handed Cartesian coordinate system with $z$ in the propagation direction of the wave.

The end-point of the electric vector traces an ellipse in space (see fig.2.1). Such an ellipse is periodically described at a repetition rate equal to the optical frequency. For complete specification of the elliptical polarization we need to know:

- The orientation in space of the plane of the ellipse of polarization.
- The orientation of the ellipse in its plane, its shape and the sense in which it is described.
- The size of the ellipse.
- The absolute temporal phase.

Only three of these parameters are important for the State Of Polarization (SOP). (refer to fig.2.1)

1) The azimuth $\theta$ is the angle between the major axis of the ellipse and the positive direction of the $x$-axis and defines the orientation of the ellipse in its plane.

2) The ellipticity $e$ is the ratio of the length of the semi-minor axis of the ellipse $'b'$ to the length of its semi-major axis $'a'$.

It is convenient to introduce an ellipticity angle $\epsilon$ such that
\[ e = \tan \varepsilon \]  

(2.2)

3) The handedness of the ellipse of polarization determines the sense in which the ellipse is described. It is a parameter that can assume only one of two discrete values. The polarization is right and left handed if the ellipse is traversed in a clockwise and a counter-clockwise sense, respectively, when looking against the propagation direction of the wave. It is mathematically convenient to incorporate the handedness in the definition of the ellipticity \( e \) by allowing the ellipticity to assume positive and negative values. So the SOP can be completely described by two parameters which assume positive and negative values.

![Fig. 2.1: The ellipse described by the end-point of the electric vector for an arbitrary state of polarization.](image)

When the common factor \( e^{i(\alpha - \phi)} \) is omitted in eqs.(2.1) and only the arguments of the 'Re' functions are considered, we get the Jones-vector.

\[
\vec{j} = \begin{pmatrix} |E_x|e^{i\phi_x} \\ |E_y|e^{i\phi_y} \end{pmatrix}
\]

(2.3)

The Jones-vector contains all the information about the SOP.

A linear optical device can be represented by the Jones-matrix \( J \), which is a 2x2-matrix of complex numbers. The relationship between the Jones-vector of the input polarization \( \vec{j}_{\text{in}} \) and the Jones-vector of the output polarization \( \vec{j}_{\text{out}} \) is given by:

\[
\vec{j}_{\text{out}} = J \vec{j}_{\text{in}}
\]

(2.4)

For a polarizer in the x-direction, we find.
For a linear retarder characterized by a retardation $\psi$ in the x-direction w.r.t. the y-direction, we get:

$$J_{\text{retarder}} = \begin{pmatrix} e^{-i\frac{\psi}{2}} & 0 \\ 0 & 1 \end{pmatrix}$$  \hspace{1cm} (2.6)

Using a rotational transformation, allows the determination of the Jones-matrix of a polarizer having an arbitrary direction. The polarizer is rotated through an angle $\phi$ w.r.t. the x-axis. $\phi$ is defined positive when the sense of rotation is clockwise, looking against the propagation direction of the wave. The Jones-matrix for the particular polarizer becomes:

$$J = \begin{pmatrix} \cos \phi & -\sin \phi \\ \sin \phi & \cos \phi \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} \cos^2 \phi & \sin \phi \cos \phi \\ \sin \phi \cos \phi & \sin^2 \phi \end{pmatrix}$$  \hspace{1cm} (2.7)

Accordingly, the Jones-vector of the polarization at the output of the polarizer is equal to:

$$\begin{pmatrix} F_{\text{out,}x} \\ F_{\text{out,y}} \end{pmatrix} = \begin{pmatrix} \cos^2 \phi |E_x|^2 e^{i\beta_x} + 1/2 \sin 2\phi |E_y|^2 e^{i\beta_y} \\ \sin^2 \phi |E_y|^2 e^{i\beta_y} + 1/2 \sin 2\phi |E_x|^2 e^{i\beta_x} \end{pmatrix}$$  \hspace{1cm} (2.8)

The output power is proportional to $|F_{\text{out}}|^2$:

$$|F_{\text{out}}|^2 = F_{\text{out,}x}^* F_{\text{out,}x} + F_{\text{out,}y}^* F_{\text{out,}y}$$  \hspace{1cm} (2.9)

$^*$ : complex conjugated.

Using the definition of the relative retardation $\delta = \delta_y - \delta_x$, we get,

$$|F_{\text{out}}|^2 = |E_x|^2 \cos^2 \phi + |E_y|^2 \sin^2 \phi + |E_x||E_y| \sin 2\phi \cos \delta$$  \hspace{1cm} (2.10)

### 2.2 Poincaré Sphere

As stated in the last paragraph, the SOP can be completely described using two parameters, therefore the polarization space is two-dimensional. In the present section we discuss the important polarization space in the form of a spherical surface whose points are in one-to-one correspondence with the different states of polarization of light. This spherical polarization space with unit radius is referred to as the Poincaré sphere. The following correspondence properties are obtained (see fig. 2.2).
- The 'south' and 'north' poles of the Poincaré sphere represent the left (L)- and right (R)-circular polarization states, respectively.
- Each point on the equator of the Poincaré sphere represents a distinct linear SOP.
- Excluding the south pole, the equator and the north pole all other points on the Poincaré sphere represent elliptical states of polarization. Below the equator (the southern hemisphere) all polarizations are left-handed. Above the equator (the northern hemisphere) all polarizations are right-handed.
- The equi-azimuth (θ) contours on the surface of the sphere constitute a family of semi-great circles drawn through the south and north poles. The longitude is double the azimuth angle of the ellipse of polarization.
- The equi-ellipticity contours on the surface of the sphere are represented by a family of coaxial circles whose common axis is the polar axis from the south to the north pole. The latitude is double the ellipticity angle of the ellipse of polarization.

The above properties of the Poincaré sphere can be summarized in a single statement: A state of polarization of azimuth \( \theta \) and ellipticity angle \( \varepsilon \) is represented on the surface of the Poincaré sphere by a point whose longitude is double the azimuth, \( 2\theta \), and whose latitude is double the ellipticity angle, \( 2\varepsilon \).

![Fig. 2.2: The Poincaré sphere representation of the polarization state space.](image)

The points X and Y on the sphere can be considered as the SOP's corresponding to the two
orthogonal modes. The normalized power difference \((|E_x|^2 - |E_0|^2)/(|E_x|^2 + |E_0|^2)\) between the two modes can be proven to be equal to the distance between the origin and the projection of the SOP at the axis through the points \(X\) and \(Y\).

In this approach the effect of a linear retarder is represented by a rotation of the Poincaré sphere around an axis in the equatorial plane (see fig.2.3). The angle of rotation is equal to the undergone retardation \(\psi\). The angle between the rotation axis and the X-Y-axis of the Poincaré sphere is double the angle \(\phi\), the angle between the retardation axis and the x-axis defined in physical space. The SOP's lying on the intersections of the rotation axis and the Poincaré sphere, are not effected by the linear retarder, and are therefore called the eigenstates of the retarder.

![Fig.2.3: The effect of a linear retarder with retardation \(\psi\) and orientation angle \(\phi\) at the state of polarization on the Poincaré sphere.](image-url)
2.3 Birefringence

Single mode fibres with nominal circular symmetry about the core axis allow the propagation of two degenerate modes with orthogonal polarizations. They are therefore bimodal supporting $HE_{11}$ and $HE_{11}'$ modes where the principal axes $x$ and $y$ are determined by the symmetry elements of the fibre cross section. Thus the fibre behaves as a birefringent medium due to the difference in the effective refractive indices and hence phase velocities for these two orthogonally polarized modes. The modes therefore have different propagation constants $\beta_x$ and $\beta_y$ which are dictated by the anisotropy of the fibre cross section. When the fibre cross section is independent of the fibre length $L$ in the $z$ direction then the modal birefringence $B_F$ for the fibre is given by,

$$B_F = \frac{\beta_x - \beta_y}{2\pi/\lambda} \quad (2.11)$$

The difference in phase velocities causes the fibre to exhibit a linear retardation $\psi$ which is proportional to the fibre length $L$ and inversely proportional to the wavelength in vacuum:

$$\psi = (\beta_x - \beta_y)L = \frac{2\pi}{\lambda_0} \left( \frac{1}{N_x} - \frac{1}{N_y} \right)L \quad (2.12)$$

The fibre length corresponding to a retardation of $2\pi$ (it is acting like a full wave plate) is called the beat length $L_B$:

$$L_B = \frac{2\pi}{\beta_x - \beta_y} \quad (2.13)$$

Typical single-mode fibres are found to have beat lengths of a few centimeters.

Modal birefringence can be separated into two components: a geometrical contribution and an effective material birefringence due to strain. For a small core-cladding index difference strain birefringence does make the dominant contribution to the birefringence [3].

During manufacture introduction of perturbations such as strain or variation in the fibre geometry and composition, is inevitable. These perturbations lead to coupling of energy from one mode to the other (cross-coupling). Different thermal expansion coefficients of the materials used for the core and cladding create an extra strain which also leads to undesired coupling.

For a normal SM-fibre the phase retardation and the cross-coupling are very sensitive to temperature changes and the presence of external stress on the fibre such as stress due to bending, twisting, mounting.

These effects are also the cause of polarization fluctuation at the end of a fibre used for long-distance communication.

To ensure repeatability of the measurements, it was necessary to take precautions concerning these effects. The particular fibres were held as straight as possible by gently taping them onto a flat surface at regular but not necessarily identical intervals to minimize inducing any stress in the fibre.
2.4 Polarization in Heterodyne Optical Systems

A heterodyne optical communication system uses the principle of coherent detection. Unlike direct detection, where the optical signal is converted into a demodulated electrical output, the coherent receiver first adds to the signal a locally generated optical wave and then detects the sum using a nonlinear detector, refer to fig.2.4. The resulting photocurrent is a replica of the original signal, translated down in frequency from the optical domain (about $10^5$ GHz) to the radio domain (a few GHz), where conventional electronic techniques can be used for further signal processing, such as filtering and demodulation. Compared with direct detection this method offers significant improvements in receiver sensitivity and wavelength selectivity.

Fig.2.4: Block diagram of a coherent fibre optic transmission system.

While combining the local oscillator (LO) beam and the signal beam, the following conditions should be satisfied, according to O.E. DeLange [4],

1) The two beams must have the same mode structure, which usually means limiting operation to the fundamental mode.

2) The two beams must be coincident and their diameters must be equal.

3) The beams must propagate in the same direction that is, their Poynting-vectors must be coincident.

4) The wave fronts must have the same curvature.

5) The beams must be identically polarized.
The last condition is explained as follows.

The $x$- and $y$-component of the signal are equal to,

$$E_{x,x} = |E_{x,x}| \cos (\omega + \omega_{IF})t$$

$$E_{y,y} = |E_{y,y}| \cos [(\omega + \omega_{IF})t + \delta]$$ (2.14)

For the LO we get,

$$E_{LO,x} = |E_{LO,x}| \cos \omega t$$

$$E_{LO,y} = |E_{LO,y}| \cos (\omega t + \Omega)$$ (2.15)

The photocurrent through the detector is proportional to the sum of the detected light intensities of the $x$- and $y$-component.

$$I_{detector} \propto (E_{x,x} + E_{LO,x})^2 + (E_{y,y} + E_{LO,y})^2$$ (2.16)

Neglecting the DC-terms which contain no information, the IF-terms which fall within the bandwidth of the receiver (all higher terms are neglected) are given below:

$$IF \propto |E_{x,x}| |E_{LO,x}| \cos (\omega_{IF}t) + |E_{y,y}| |E_{LO,y}| \cos (\omega_{IF}t + \delta - \Omega)$$ (2.17)

Maximizing the amplitude of the IF-signal, the following conditions can be derived,

$$\delta = \Omega$$ (2.18)

$$\frac{|E_{x,x}|}{|E_{LO,x}|} = \frac{|E_{y,y}|}{|E_{LO,y}|}$$ (2.19)

Or similarly, at the detector the state of polarization (SOP) of the LO must be equal to the signal SOP.

However, the signal SOP varies randomly due to mechanical and thermal effects (cf. paragraph 2.3), which results in fading. Generally, the polarization state in a single-mode fibre changes slowly; the variation of azimuth and ellipticity are about $1^\circ \text{min}^{-1}$ in a 10-km length fibre installed under the sea [5].

A number of solutions have been proposed to this problem:

1) Polarization maintaining fibres.

PMF fibres are at a premature stage of development, are costly and exhibit high attenuation in comparison with normal fibres [6].

2) Polarization diversity.

Polarization diversity increases receiver complexity, insofar as two receivers and a ratio combiner are required [7] (see paragraph 5.2).
3) Polarization control.
   Since this is the subject of this report, it will be elaborately described. In the next paragraph, polarization control devices are discussed.

2.5 *Polarization Control Devices*

At the receiver side the SOP of the LO must be controlled, so that it matches the signal SOP. Different effects can be used to change the SOP at the output of a fibre,

1) Photoelastic effect.
   External pressure on the fibre causes a change in birefringence of the fibre.

2) Faraday-effect.
   The Faraday-effect produces a rotation of the plane of polarization about a magnetic field vector. The angle of rotation is equal to the product of the Verdet constant, the magnitude of the field and the optical path length.

3) Electrooptic effect.
   The birefringence of an electrooptic crystal changes when a voltage is applied.

According to T. Okoshi [8] the following schemes are proposed based on one of the last mentioned effects.

1) Electromagnetic or piezoelectric fibre squeezers.
   The squeezers introduce a transversal strain into the fibre and so introduce birefringence. The birefringence axes lie along and perpendicular to the direction of the applied pressure [9].

2) Rotatable fibre coils.
   Three coils with a few windings of fibre can transform an arbitrary input polarization to an arbitrary output polarization as shown in fig.2.5 [10].
   This method is based on the photoelastic effect.

3) Rotatable fibre cranks.
   In this scheme, the polarization control device consists of a bended short fibre section in a crank form as shown in fig.2.6 [11]. This method uses also the photoelastic effect.

4) Electrooptic crystals.
   The birefringence is controlled by the voltage applied to the particular crystal [12].

5) Faraday rotator.
   The rotation of the plane of polarization is controlled by the current through the coil which generates the magnetic field [13].
6) Rotatable phase plates.
   In this scheme, rotatable quarter-wave and half-wave plates are used. The phase plates have the
great advantage in that the rotation is endless [5].

A polarization control device can be characterized by four features:

1) Endlessness in control
2) Insertion loss
3) Temporal response
4) Mechanical fatigue

These features are compared for the mentioned polarization control devices in table 1.

<table>
<thead>
<tr>
<th>Polarization-state control scheme</th>
<th>Insertion loss</th>
<th>Endlessness in control</th>
<th>Temporal response</th>
<th>Mechanical fatigue</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fibre squeezers</td>
<td>low</td>
<td>no</td>
<td>medium</td>
<td>present</td>
</tr>
<tr>
<td>Rotatable fibre coils</td>
<td>low</td>
<td>no</td>
<td>slow</td>
<td>present</td>
</tr>
<tr>
<td>Rotatable fibre cranks</td>
<td>low</td>
<td>yes</td>
<td>slow</td>
<td>present</td>
</tr>
<tr>
<td>Electrooptic crystals</td>
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<td>no</td>
<td>fast</td>
<td>absent</td>
</tr>
<tr>
<td>Faraday rotators</td>
<td>low</td>
<td>no</td>
<td>fast</td>
<td>absent</td>
</tr>
<tr>
<td>Phase plates</td>
<td>medium</td>
<td>yes</td>
<td>slow</td>
<td>small</td>
</tr>
</tbody>
</table>

Table 1: Features of six polarization state control schemes.

In our case is chosen for the piezoelectric fibre squeezer which is not endless in control.
Fig. 2.5: Rotatable fibre coils.
   a) Sketch of device.
   b) Configuration of fibre.
Fig. 2.6: A rotatable fibre crank.

a) Shape and dimensions.

b) Driving mechanism for 'translational' movement.
3.0 Polarization Control Using Two Squeezers

3.1 Polarization Control System

Two squeezers are required to control the two independent variables or degrees of freedom of the output SOP (ellipticity and orientation) (c.f. fig.3.1). The squeezers introduce a transversal strain into the fibre and so induce birefringence. The birefringence axes lie along and perpendicular to the direction of the applied pressure. Accordingly, a squeezer can be represented by a linear retarder having the retardation axis along the direction of the applied pressure.

Considering the Poincaré sphere, the effect of a squeezer is equivalent to rotation of the sphere (c.f. paragraph 2.2) around an axis in the equatorial plane. The angle of rotation is equal to the undergone retardation which is proportional to the applied voltage of the squeezer. The angle between the rotation axis and the X-Y-axis of the Poincaré sphere is double the angle $\phi$, where $\phi$ is the angle between the squeezer direction and the x-axis of the physical space. Consequently, if the azimuths of the two squeezers are rotated at an angle of 45 degrees with respect to each other around the axis of the optical fibre, the corresponding rotation axes of the Poincaré sphere are perpendicular to each other (fig.3.2).
The aim of the polarization control system is the transformation of an arbitrary input SOP to the desired output SOP. For a coherent system the desired output SOP equals the SOP of the local oscillator which does not change with time. Assuming the SOP of the LO is linearly polarized along the x-axis (desired output SOP is point 'X' in fig. 3.2), the IF-term is proportional to $|E_x|$ of the received optical signal (see eq. 2.17). In a coherent system an electrical signal proportional to the IF-term is available at the polarization control system. By means of feedback, this signal is maximized to get minimal signal fading.

A complete coherent system is not needed to test the polarization control system. The power of the electrical signal, available in the coherent system, $(|E_x|^2)$, can be retrieved from a photodetector preceded by a polarizer in the x-direction (fig.3.1). Maximizing $|E_x|^2$ equals maximizing $|E_x|$, so the control algorithm of the test set-up is equal to the algorithm used in the coherent system.

By controlling the squeezer voltages, an arbitrary input SOP can be transformed to the point 'X' on the Poincaré sphere (linearly polarized light in the x-direction). If point 'X' is taken as the north pole of the sphere, then the rotation of the Poincaré sphere caused by squeezer 1 (S1) and squeezer 2 (S2) is equal to the latitude and longitude of the arbitrary input SOP, respectively.

During normal control, the polarization control system has to track the input SOP. Since a squeezer is not endless (paragraph 2.5), a reset of a squeezer is necessary, when the particular squeezer reaches a range limit. The reset provokes momentary signal losses.
3.2 Angular Misalignment

In the previous discussion it is assumed that the angle between the squeezers is precisely 45 degrees. In practice, there is of course a deviation from this optimal value. Designating the variable \( \theta \) to the deviation, the angle between the squeezers becomes \( 45^\circ + \theta \). Now it is impossible to transform the SOP's in the lens-shaped regions (indicated in fig.3.3) to point ‘X’ (S1 must always set before S2). Consequently, the output SOP cannot be always equal to the desired SOP (point ‘X’) resulting in signal fading after the polarizer.

![Diagram of orientation of the rotation axes of the squeezers in the non-ideal situation](image)

Fig.3.3: Orientation of the rotation axes of the squeezers in the non-ideal situation, i.e. rotation at an angle of \( 45^\circ + \theta \) of the squeezers with respect to each other.

Indicated lens-shaped regions (region 2) consist of input SOP's which cannot be transformed to the point ‘X’.

In a digital communication system, the effect of the misalignment is a degradation of the bit error rate. The following expression concerning the probability of error is valid for a heterodyne communication system where polarization control is optimal [14],

\[
P_e \sim \frac{1}{2} \exp \left[ -\frac{E_s}{2N_0} \right]
\]

(3.1)

where \( E_s \) is the average signal power.

\( N_s \) is the total average noise power.
Calculation of the probability of error for non-optimal polarization control is now carried out.

We must differentiate between two cases. One case considers the input SOP's outside the lens-shaped regions of the Poincaré sphere and the other case deals with the input SOP's inside the lens-shaped regions. For both cases the probability of error can be calculated. The total probability of error is equal to the weighted average of these probabilities.

\[ P_{\text{total}} = \Pr[\text{SOP in region 1}]P_1 + \Pr[\text{SOP in region 2}]P_2 \]  

(3.2)

where

- region 1: the part of the Poincaré sphere outside the lens-shaped regions.
- region 2: the part of the Poincaré sphere consisting of two lens-shaped regions.

\[ P_1 \]  
probability of error for input SOP's in region 1.

\[ P_2 \]  
probability of error for input SOP's in region 2.

\section*{CASE 1}

The probability of the input SOP being in a particular region of the Poincaré sphere equals the ratio of the area of the region to the total area of the sphere (= 4\pi). It is convenient to consider an infinitesimally thin slice of the Poincaré sphere perpendicular to the rotation axis of squeezer S1 and at a distance L to the origin (as shown in fig. 3.4).

![Fig.3.4: Definition of position and dimensions of the slice described in the text.](image)

The probability of the input SOP lying on the slice (situated at a distance L from the origin) is given by,
\[ \Pr(L) = \frac{2\pi RA}{4\pi} = \frac{1}{2} RA \] 

where \( R \) is the radius of the slice. 
\( \Lambda \) is the width of the slice.

Considering fig.3.5, the following equation can be easily derived,

\[ \cos \phi = R = \frac{dL}{\Lambda} \] 

where \( dL \) is the thickness of the slice.

![Diagram of a slice](image)

Fig.3.5: Looking against the slice of fig.3.4.

Substituting eq.(3.4) into eq.(3.3) gives,

\[ \Pr(L) = \frac{1}{2} \, dl \] 

For the probability density function, we find,

\[ p(L) = \frac{1}{2} \quad \text{when} \quad -1 < L < 1 \] 

The part of the Poincaré sphere outside the lens-shaped regions is characterized by \(|L| < \cos 2\theta\) (see fig.3.7). Accordingly, the probability of the presence of an input SOP in this region is given by,

\[ \Pr[\text{SOP in region 1}] = \int_{-\cos 2\theta}^{\cos 2\theta} \frac{1}{2} \, dl = \cos 2\theta \]
In this case every input SOP can be transformed to the desired output SOP. Consequently, there is no signal fading and we find for the probability of error,

\[ P_e = \frac{1}{2} \exp \left( -\frac{E_t}{2N_o} \right) \]  

(3.8)

- CASE 2

The situation is more intricate than in case 1.

An arbitrary SOP \( t \) in one of the lens-shaped regions is transformed by squeezer S1 to SOP \( t' \) along the surface of the sphere (see fig.3.6-3.7) thus having maximal throughput at the output polarizer. Squeezer S2 cannot improve the throughput and therefore gives no retardation. If SOP \( t \) now lies in the opposite lens-shaped region (to the one shown in fig.3.6), then squeezer S2 has to produce a retardation \( \pi \).

**Fig.3.6:** Looking along the retardation axis of squeezer S1 to the Poincaré sphere. SOP \( t \) cannot be transformed to point ‘X’, the nearest attainable point to ‘X’ is SOP \( t' \).
Fig. 3.7: Top view of the Poincaré sphere shown in fig. 3.6. SOP t cannot be transformed to point 'X', the nearest attainable point to 'X' is SOP t'.

For the normalized power difference between the two modes of SOP t', we find (see fig. 3.7 and paragraph 2.2),

\[
\frac{|E_x|^2 - |E_y|^2}{|E_x|^2 + |E_y|^2} = L \cos 2\theta + \sqrt{1 - L^2} \sin 2\theta = \sin (\arcsin(L) + 2\theta)
\]

(3.9)

For the passed-through intensity of the polarizer, we obtain,

\[
\frac{1}{2} |E_x|^2 = \frac{1}{2} E_x (1 + \sin (\arcsin(L) + 2\theta))
\]

(3.10)

The probability of bit error due to output SOP t' is then given by,

\[
\Pr[\text{output SOP t' and bit error}] = \frac{1}{2} \exp \left(- \frac{|E_x|^2}{4N_o}\right)
\]

(3.11)

All input SOP's on an infinitesimally thin slice containing SOP t can be transformed to output SOP t'. The probability of error due to such an input SOP is given by eq. (3.11).
The mean error probability over complete lens-shaped regions, is found using,

$$Pr[SOP \text{ in region 2}]P_e^2 = \frac{1}{2} \int_{\cos \theta}^{1} \exp \left( -\frac{|E_x|^2}{4N_o} \right) \rho(L) dL + \frac{1}{2} \int_{-1}^{\cos \theta} \exp \left( -\frac{|E_x|^2}{4N_o} \right) \rho(L) dL \quad (3.12)$$

Substitution of

$$L = \cos \psi$$
$$dL = - \sin \psi d\psi$$

$$\rho(L) = \frac{1}{2}$$

$$|E_x|^2 = E_s(1 + \sin(arcsin(L) + 2\theta))$$

into eq.(3.12), gives,

$$Pr[SOP \text{ in region 2}]P_e^2 = \frac{1}{2} \exp \left( -\frac{E_s}{4N_o} \right) \int_0^{2\theta} \exp \left[ -\frac{\cos(\psi - 2\theta)E_s}{4N_o} \right] \sin \psi d\psi \quad (3.13)$$

Using eq.(3.2) gives,

$$P_e^{tot} = \frac{1}{2} \cos 2\theta \exp \left( -\frac{E_s}{2N_o} \right) + \frac{1}{2} \exp \left( -\frac{E_s}{4N_o} \right) \int_0^{2\theta} \exp \left[ -\frac{\cos(\psi - 2\theta)E_s}{4N_o} \right] \sin \psi d\psi \quad (3.14)$$

Numerical evaluation of the last expression leads to the plot shown in fig.3.8. Here, the probability of error versus the signal-to-noise ratio is plotted for different values of $\theta$ .

Conclusively, the probability of error is not much degraded by a deviation of the angle between the squeezers from the optimal value of 45 degrees.
Fig. 3.8: Probability of error on a logarithmic scale versus signal-to-noise ratio ($E_s/N_0$) for different values $\theta$ (angular misalignment between the two squeezers).
4.0 Endless Polarization Control

4.1 Introduction

In a polarization control system having two squeezers, the reset provokes momentary signal losses. To prevent these losses different schemes are proposed [15]-[20]. In general, an extra squeezer is required. Also a special controlling algorithm i.e. reset algorithm must be developed. First, I shall discuss the algorithm proposed by Noé [19]. Then I discuss a new reset algorithm.

4.2 Algorithm developed by Noé

In this scheme described in [19] three squeezers are used. Squeezers S1 and S3 have the same orientation with respect to the x-axis. Squeezer S2 is rotated at an angle of \(45^\circ\) with respect to these two squeezers as shown in fig.4.1. The retardation ranges needed are \([0, \pi]\) for S1, \([-\pi, \pi]\) for S2 and \([0, \pi]\) for S3.

![Fig.4.1: Set-up of the endless polarization control system having three squeezers.](image)

The transformation from input SOP to output SOP using three squeezers is shown in fig.4.2.
Fig. 4.2: The transformation from input SOP to output SOP using the set-up of fig. 4.1.

The control algorithm consists of two main stages. The first stage, normal operation, refers to operation when a reset is not required. The output SOP is then controlled only by squeezers S2 and S3. This is achieved by letting S2 rotate the input SOP to the semi-circle XRY. Squeezer S3 then rotates the SOP to point 'X' (fig. 4.2).

The second stage, reset operation, uses all three squeezers. The algorithm assumes that the input SOP remains fairly constant throughout the reset period. Separate algorithm sections are used, depending upon which squeezer (S2 or S3) reach a range limit.
Consider the case when S2 reaches a range limit. Then S1 and S3 must ensure that the input SOP to S2 becomes an eigenstate of S2. Consequently, S2 is free to reset. This can be explained using the Poincaré sphere representation in fig. 4.3. When the input SOP reaches the position shown, a reset of S2 must take place, since this is the range limit of S2. The squeezer S1 must transform the input SOP to an eigenstate of S2. In order to preserve the system output SOP, S3 must also be active.

Note that when the eigenstate condition occurs, squeezer S3 gives zero retardation and S1 determines the output SOP. Squeezer S2 is now free to reset by $2\pi$. After this step, S1 and S3 return to their original positions.
Consider now the case when S3 reaches a range limit (see fig.4.4). The input SOP is either point 'X' or 'Y'. Then S2 is in an eigenstate condition and is switched over by $\pi$ in a direction chosen so as not to exceed the S2 range. The direction of S3 movement reverses and overflows are prevented.
4.3 New Algorithm

In the previous discussion the Poincaré sphere was used to visualize the effect of the squeezers. Another way of looking at the problem is to describe the transformation from input SOP to output SOP by the corresponding retardations of the three squeezers.

The S1,S2,S3-space is defined as the Euclidean three-dimensional space having a Cartesian coordinate system with the retardations scaled along the axes. A transformation can be represented by a point in this space, where the three coordinates represent the retardations.

In this approach, a retardation limit of a squeezer is given by a plane, orthogonal to the axis of the squeezer in the S1,S2,S3-space. Consideration of all six range limits in this space, gives six planes, which form a box. Inside the box are the possible combinations of retardations. The reset algorithm has to prevent situations having a combination of retardations corresponding to a point outside the box.

Before discussing the algorithm, the terms "Reset traces" and "Controllability" are explicitly described.

4.3.1 Reset traces

Ideal situation

Using three squeezers, three degrees of freedom are available to control the output SOP; two parameters of the output SOP have to be controlled so one degree of freedom is over. This extra degree of freedom is used to arrange the reset procedure.

In the S1,S2,S3-space the reset procedure is represented by a trace having the property that all the points of the trace give retardations which carry out a similar transformation from input SOP to output SOP.

In the next discussion, the superscripts $x'$, $y'$, $z'$ refer to a Cartesian coordinate system in the space of the Poincaré sphere having the $x'$-axis along the XY-axis of the sphere, the $y'$-axis along the PQ-axis and the $z'$-axis along the RL-axis. The origin coincides to the centre point of the Poincaré sphere (see fig.4.2).

A state of polarization (SOP) is represented by a vector with respect to the coordinate system.

The transformation from input SOP to output SOP is given by a succession of three rotations (fig.4.2):

1) Rotation around the PQ-axis eq. $y'$-axis at an angle $S_1$.
2) Rotation around the XY-axis eq. $x'$-axis at an angle $S_2$.
3) Rotation around the PQ-axis eq. $y'$-axis at an angle $S_3$.

Or in matrix representation,

$$\text{SOP}_{\text{out}} = S[\text{SOP}_{\text{in}}]$$  \hspace{1cm} (4.1)

where
\[
S = \begin{pmatrix}
\cos S_3 & 0 & \sin S_3 \\
0 & 1 & 0 \\
-\sin S_3 & 0 & \cos S_3
\end{pmatrix}
\begin{pmatrix}
1 & 0 & 0 \\
0 & \cos S_2 & -\sin S_2 \\
0 & \sin S_2 & \cos S_2
\end{pmatrix}
\begin{pmatrix}
\cos S_1 & 0 & \sin S_1 \\
0 & 1 & 0 \\
-\sin S_1 & 0 & \cos S_1
\end{pmatrix}
\] (4.2)

\[
\overline{\text{SOP}}_{\text{out}} = \begin{pmatrix}
\text{SOP}^{x'}_{\text{out}} \\
\text{SOP}^{y'}_{\text{out}} \\
\text{SOP}^{z'}_{\text{out}}
\end{pmatrix}
\] and \[
\overline{\text{SOP}}_{\text{in}} = \begin{pmatrix}
\text{SOP}^{x'}_{\text{in}} \\
\text{SOP}^{y'}_{\text{in}} \\
\text{SOP}^{z'}_{\text{in}}
\end{pmatrix}
\] (4.3)

Substituting \(\overline{\text{SOP}}_{\text{out}} = \text{point 'X' }=(1,0,0)\) and inverting the transformation gives,

\[
\overline{\text{SOP}}_{\text{in}} = S^{-1}[\overline{\text{SOP}}_{\text{out}}] = S^{-1}\begin{pmatrix}
1 \\
0 \\
0
\end{pmatrix} = \begin{pmatrix}
\cos S_1 \cos S_3 - \sin S_1 \cos S_2 \sin S_3 \\
\sin S_2 \sin S_3 \\
\sin S_1 \cos S_3 + \cos S_1 \cos S_2 \sin S_3
\end{pmatrix}
\] (4.4)

Differentiating eq.(4.4) with respect to \(S_1, S_2, S_3\) and taking the output SOP variation \(d\overline{\text{SOP}}_{\text{out}} = 0\), gives,

\[
d\overline{\text{SOP}}_{\text{in}} = A \begin{pmatrix}
dS_1 \\
dS_2 \\
dS_3
\end{pmatrix}
\] (4.5)

where

\[
A = \begin{pmatrix}
-\sin S_1 \cos S_3 - \cos S_1 \cos S_2 \sin S_3 & \sin S_1 \sin S_2 \sin S_3 & -\cos S_1 \sin S_3 - \sin S_1 \cos S_2 \cos S_3 \\
0 & \cos S_2 \sin S_3 & \sin S_2 \cos S_3 \\
\cos S_1 \cos S_3 - \sin S_1 \cos S_2 \sin S_3 & \cos S_1 \sin S_2 \sin S_3 & -\sin S_1 \sin S_3 + \cos S_1 \cos S_2 \cos S_3
\end{pmatrix}
\] (4.6)

Matrix \(A\) can be described as a product of matrix \(B\), which represents a rotation at an angle \(S_1\) around the \(y'\)-axis, and matrix \(C\), whose components are a function of \(S_2,S_3\) only.
\[ A = B \times C \] \hspace{1cm} (4.7)

where

\[
B = \begin{pmatrix}
\cos S1 & 0 & -\sin S1 \\
0 & 1 & 0 \\
\sin S1 & 0 & \cos S1
\end{pmatrix}
\] \hspace{1cm} (4.8)

and

\[
C = \begin{pmatrix}
-\cos S2 \sin S3 & 0 & -\sin S3 \\
0 & \cos S2 \sin S3 & \sin S2 \cos S3 \\
\cos S3 & -\sin S2 \sin S3 & \cos S2 \cos S3
\end{pmatrix}
\] \hspace{1cm} (4.9)

The differential equations of the reset traces (no input and output SOP variations) are given by,

\[
A \begin{pmatrix}
dS1 \\
dS2 \\
dS3
\end{pmatrix} = \begin{pmatrix} 0 \\
0 \\
0
\end{pmatrix}
\] \hspace{1cm} (4.10)

Using \( A = B \times C \), we get,

\[
C \begin{pmatrix}
dS1 \\
dS2 \\
dS3
\end{pmatrix} = \begin{pmatrix} 0 \\
0 \\
0
\end{pmatrix}
\] \hspace{1cm} (4.11)

The solution of eq.(4.11) is given by,

\[
\begin{pmatrix}
dS1_{\text{res}} \\
dS2_{\text{res}} \\
dS3_{\text{res}}
\end{pmatrix} = \begin{pmatrix}
tan S3 \\
- \sin S2 \\
- \cos S2 \tan S3
\end{pmatrix} d\lambda
\] \hspace{1cm} (4.12)

where \( \lambda \) is a parameter.

Note the independence on \( S1 \).
Numerical integration of the above equation gives the reset curves in \( S1,S2,S3 \)-space. Three of them are shown in fig.4.5 for different start conditions.
Fig. 4.5: Reset curves in the $S_1, S_2, S_3$-space, starting at the points,

- $S_1 = 0$, $S_2 = \frac{\pi}{32}$, $S_3 = \frac{\pi}{2}$
- $S_1 = 0$, $S_2 = \frac{\pi}{4}$, $S_3 = \frac{\pi}{2}$
- $S_1 = 0$, $S_2 = \frac{\pi}{2} - \frac{\pi}{32}$, $S_3 = \frac{\pi}{2}$

Other reset curves can be obtained by horizontally shifting and mirroring the reset curves of fig. 4.5. The projection onto the $S_2$-$S_3$ plane of these reset curves is shown in fig. 4.6.

Conclusion: For every combination of retardations $(S_1, S_2, S_3)$, squeezer $S_1$ can be reset.
Fig. 4.6: Reset curves projected onto the S2,S3-plane

**Misalignment**

The reset curves of fig. 4.5 are valid for the ideal situation; perfect alignment of the squeezers. In practice, misalignment can play an important role. To incorporate the misalignment, the orientation angles of the retardation axes of squeezers S1 and S2 are given by $45^\circ + \frac{1}{2} \theta_1$ and $\frac{1}{2} \theta_2$, respectively. The misalignment between squeezer S3 (orientation angle: $45^\circ$) and the polarizer (orientation angle: $0^\circ$) (fig. 4.1) is not incorporated, since in practice the misalignment can be easily compensated by rotation of the polarizer.

The squeezers' rotation axes on the Poincaré sphere are indicated in fig. 4.7.
The transformation matrix $S$ becomes,

$$S = M_{S_3} M_{S_2} M_{S_1} \quad (4.13)$$

where

$$M_{S_1} = \begin{pmatrix} \cos \varepsilon_1 & \sin \varepsilon_1 & 0 \\ -\sin \varepsilon_1 & \cos \varepsilon_1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \cos S_1 & 0 & \sin S_1 \\ 0 & 1 & 0 \\ -\sin S_1 & 0 & \cos S_1 \end{pmatrix} \begin{pmatrix} \cos \varepsilon_1 & -\sin \varepsilon_1 & 0 \\ \sin \varepsilon_1 & \cos \varepsilon_1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$M_{S_2} = \begin{pmatrix} \cos \varepsilon_2 & \sin \varepsilon_2 & 0 \\ -\sin \varepsilon_2 & \cos \varepsilon_2 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos S_2 & -\sin S_2 \\ 0 \sin S_2 & \cos S_2 & 0 \end{pmatrix} \begin{pmatrix} \cos \varepsilon_2 & -\sin \varepsilon_2 & 0 \\ \sin \varepsilon_2 & \cos \varepsilon_2 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$M_{S_3} = \begin{pmatrix} \cos S_3 & 0 & \sin S_3 \\ 0 & 1 & 0 \\ -\sin S_3 & 0 & \cos S_3 \end{pmatrix}$$

In the same way as above, the reset curves can be obtained. For different misalignment angles $\varepsilon_1$ and $\varepsilon_2$, the reset curves are plotted in fig.4.8-4.9.

Conclusion: In the presence of angular misalignment of the squeezers, there are points where reset of $S_1$ is impossible. For the points on the reset curves having a diameter less than a particular boundary value, reset of $S_1$ still remains possible. This boundary value depends on the magnitude of the misalignment angles.
Fig. 4.8: Reset curves in the situation of misalignment.
Dotted line indicates projection onto the $S_2S_3$-plane.
Fig. 4.9: Reset curves in the situation of misalignment.
Dotted line indicates projection onto the S2,S3-plane.
4.3.2 Controllability

$S1,S2$-control

The effect of a squeezer upon a SOP depends on the position of the SOP on the Poincaré sphere. The two linear polarization states along the retardation axes are the eigenstates of the squeezer; these SOP’s cannot be changed by the squeezer. In the Poincaré space, these eigenstates correspond to the two intersections of the rotation axis and the Poincaré sphere. The greater the distance between an intersection and a SOP, the greater the effect of the squeezer.

During normal operation, the squeezers have to prevent variations of the output SOP. Due to polarization fluctuation, the output SOP can move over the surface of the Poincaré sphere in an arbitrary direction. By controlling the retardations of the squeezers, the variation of the output SOP is compensated for.

The $dS1,dS2,dS3$-space is a three-dimensional space containing the retardation change vectors $(dS1,dS2,dS3)$. The control space is defined as the space containing all vectors $(dS1,dS2,dS3)$ which are available to control the output SOP. Since two parameters of the output SOP (orientation and ellipticity) must be controlled, the control space is a two-dimensional surface in the $dS1,dS2,dS3$-space.

In the next discussion, squeezers $S1$ and $S2$ are used to control the output SOP. Hence, the control space equals the plane perpendicular to the vector $(0,0,1)$ containing the vectors $(dS1,dS2,0)$.

The response time of a squeezer is proportional to the required retardation change. In the polarization control system, the squeezers are adjusted independently. Consequently, the response time of the polarization control system is determined by the squeezer having the largest retardation change. Hence, restriction of the response time leads to restriction of the largest retardation change. If the retardation change is limited to $\varepsilon$, then the achievable vectors $(dS1,dS2,dS3)$ lie inside a square having corner points $(-\varepsilon,-\varepsilon,0),(-\varepsilon,\varepsilon,0),(\varepsilon,\varepsilon,0)$ and $(\varepsilon,-\varepsilon,0)$ in the $dS1,dS2,dS3$-space. In the following discussion, the square is normalized with respect to $\varepsilon$.

The vectors $(dS1,dS2,0)$ are transformed using matrix $A$ (eq.(4.5)) to the vectors $d\mathbf{SOP}_{\text{out}}$. The vectors $d\mathbf{SOP}_{\text{out}}$ form a plane in the Poincaré space through point X and perpendicular to the X-Y axis.

The vector $d\mathbf{SOP}_{dS1}$ is defined as the $d\mathbf{SOP}_{\text{out}}$ due to vector $(dS1,dS2,dS3)=(1,0,0)$. Similarly, the vector $d\mathbf{SOP}_{dS2}$ is defined as the $d\mathbf{SOP}_{\text{out}}$ due to vector $(dS1,dS2,dS3)=(0,1,0)$. Since the transformation from the $dS1,dS2,dS3$-space to the $d\mathbf{SOP}_{\text{out}}$-space is linear, the square in the $dS1,dS2,dS3$-space having corner points $(-1,-1,0),(-1,1,0),(1,1,0)$ and $(1,-1,0)$ is transformed to the parallelogram in the $d\mathbf{SOP}_{\text{out}}$-space (fig.4.10) having angular points,\[ -d\mathbf{SOP}_{dS1} - d\mathbf{SOP}_{dS2}, -d\mathbf{SOP}_{dS1} + d\mathbf{SOP}_{dS2}, d\mathbf{SOP}_{dS1} + d\mathbf{SOP}_{dS2}, d\mathbf{SOP}_{dS1} - d\mathbf{SOP}_{dS2}.\]
Fig. 4.10: Transformation (matrix A) of the square in the dS1,dS2 plane to the parallelogram in the $\text{dSOP}_{\text{out}}$-space.

For maximal control $\text{dSOP}_{\text{out}}$ is on the perimeter of the parallelogram. The controllability is defined as the minimal normalized value of $|\text{dSOP}_{\text{out}}|$ while using maximal control. In other words, the controllability is the minimum distance between a point on the outline of the parallelogram and the origin.

In the next calculation, the distances between the sides of the parallelogram and the origin are determined. Since the parallelogram is symmetrical with respect to the origin, only two sides have to be considered. The controllability is given by the minimum of these values. The two distances are called $|\text{dSOP}_{\text{min1}}|$ and $|\text{dSOP}_{\text{min2}}|$.

The distance between a straight line $K$ described by equation $K = \lambda \vec{V}_1 + \vec{V}_2$, and the origin is given by (see fig. 4.11),

$$d(\text{origin, line } K) = \sqrt{|\vec{V}_2|^2 - \frac{(\vec{V}_1 \cdot \vec{V}_2)^2}{|\vec{V}_1|^2}}$$

(4.14)

Fig. 4.11: Geometrical deduction of an expression for the distance between a line $K$ ($= \lambda \vec{V}_1 + \vec{V}_2$) and the origin $O$. 
Eq. (4.14) applied to our case, we obtain,

\[
|d \text{SOP}_{\text{min1}}|^2 = |d \text{SOP}_{\text{ds1}}|^2 - \frac{(d \text{SOP}_{\text{ds1}}, d \text{SOP}_{\text{ds2}})^2}{|d \text{SOP}_{\text{ds1}}|^2} \tag{4.15}
\]

\[
|d \text{SOP}_{\text{min2}}|^2 = |d \text{SOP}_{\text{ds1}}|^2 - \frac{(d \text{SOP}_{\text{ds1}}, d \text{SOP}_{\text{ds2}})^2}{|d \text{SOP}_{\text{ds2}}|^2} \tag{4.16}
\]

The vectors \(d \text{SOP}_{\text{ds1}}\) and \(d \text{SOP}_{\text{ds2}}\) are equal to the vectors \(d \text{SOP}_{\text{ext}}\) due to the vectors \((dS_1,dS_2,dS_3) = (1,0,0)\) and \((dS_1,dS_2,dS_3) = (0,1,0)\), respectively. According to eq. (4.5), we find,

\[
d \text{SOP}_{\text{ds1}} = A \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} -\sin S1 \cos S3 - \cos S1 \cos S2 \sin S3 \\ \cos S1 \cos S3 - \sin S1 \cos S2 \sin S3 \\ \end{pmatrix} \tag{4.17}
\]

\[
d \text{SOP}_{\text{ds2}} = A \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} \sin S1 \sin S2 \sin S3 \\ \cos S2 \sin S3 \\ \cos S1 \sin S2 \sin S3 \end{pmatrix} \tag{4.18}
\]

Using these expressions, we obtain,

\[
|d \text{SOP}_{\text{ds1}}|^2 = \cos^2 S3 + \cos^2 S2 \sin^2 S3 \tag{4.19}
\]

\[
|d \text{SOP}_{\text{ds2}}|^2 = \sin^2 S3 \tag{4.20}
\]

\[
(d \text{SOP}_{\text{ds1}}, d \text{SOP}_{\text{ds2}}) = -\sin S2 \sin S3 \cos S3 \tag{4.21}
\]

Substituting the last quantities into eqs. (4.15) and (4.16) gives,

\[
|d \text{SOP}_{\text{min1}}| = |\cos S2| \tag{4.22}
\]

\[
|d \text{SOP}_{\text{min2}}| = \sqrt{\frac{\cos^2 S2 \sin^2 S3}{\cos^2 S3 + \cos^2 S2 \sin^2 S3}} \tag{4.23}
\]

The minimum of \(|d \text{SOP}_{\text{min1}}|\) and \(|d \text{SOP}_{\text{min2}}|\) is the controllability. It is evident that the controllability is independent of \(S1\). The controllability versus \(S2, S3\) is plotted in fig.4.12.
Fig 4.12: Controllability versus S2, S3 in the case of S1,S2-control.
The situation is analogous to the case of S1, S2-control. The control space is the plane perpendicular to (0, 1, 0) in the dS1, dS2, dS3-space.

In the same way as above, we get,

\[ |\overline{dSOP}_{\text{min1}}|^2 = |dSOP_{dS1}|^2 - \frac{(dSOP_{dS1} dSOP_{dS3})^2}{|dSOP_{dS3}|^2} \quad (4.24) \]

\[ |\overline{dSOP}_{\text{min2}}|^2 = |dSOP_{dS3}|^2 - \frac{(dSOP_{dS1} dSOP_{dS3})^2}{|dSOP_{dS1}|^2} \quad (4.25) \]

For dSOP_{dS1} and dSOP_{dS3} we find,

\[
\overline{dSOP}_{dS1} = A \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} -\sin S1 \cos S3 - \cos S1 \cos S2 \sin S3 \\ \cos S1 \cos S3 - \sin S1 \cos S2 \sin S3 \\ 0 \end{pmatrix} \quad (4.17)
\]

\[
\overline{dSOP}_{dS3} = A \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} -\cos S1 \sin S3 - \sin S1 \cos S2 \cos S3 \\ \sin S2 \cos S3 \\ -\sin S1 \sin S3 + \cos S1 \cos S2 \cos S3 \end{pmatrix} \quad (4.26)
\]

Using these expressions, we obtain,

\[ |\overline{dSOP}_{dS1}|^2 = \cos^2 S3 + \cos^2 S2 \sin^2 S3 \quad (4.19) \]

\[ |\overline{dSOP}_{dS3}|^2 = 1 \quad (4.27) \]

\[ (dSOP_{dS1} \overline{dSOP}_{dS3}) = \cos S2 \quad (4.28) \]

Substituting the last quantities into eqs. (4.24) and (4.25) gives,

\[ |dSOP_{\text{min1}}| = |\cos S3 \sin S2| \quad (4.29) \]

\[ |dSOP_{\text{min2}}| = |\cos S3 \tan S2| \quad (4.30) \]

The minimum of |dSOP_{\text{min1}}| and |dSOP_{\text{min2}}| is the controllability, we get,

\[ \text{CONTROLLABILITY} = |\cos S3 \sin S2| \quad (4.31) \]

The controllability versus S2, S3 is plotted in fig. 4.13.
Fig. 4.13: Controllability versus $S_2$, $S_3$ in the case of $S_1,S_3$-control.
**S2,S3-control**

The situation is analogous to the case of S1,S2-control. The control space is the plane perpendicular to \((1,0,0)\) in the dS1,dS2,dS3-space.

In the same way as previously treated, we get,

\[
|\bar{d}\text{SOP}_{\text{min}}|^2 = |\bar{d}\text{SOP}_{\text{dS2}}|^2 - \left( \frac{(\bar{d}\text{SOP}_{\text{dS2}} \cdot \bar{d}\text{SOP}_{\text{dS3}})^2}{|\bar{d}\text{SOP}_{\text{dS2}}|^2} \right) \tag{4.32}
\]

\[
|\bar{d}\text{SOP}_{\text{min2}}|^2 = |\bar{d}\text{SOP}_{\text{dS3}}|^2 - \left( \frac{(\bar{d}\text{SOP}_{\text{dS2}} \cdot \bar{d}\text{SOP}_{\text{dS3}})^2}{|\bar{d}\text{SOP}_{\text{dS2}}|^2} \right) \tag{4.33}
\]

For \(\bar{d}\text{SOP}_{\text{dS2}}\) and \(\bar{d}\text{SOP}_{\text{dS3}}\) we find,

\[
\bar{d}\text{SOP}_{\text{dS2}} = A \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} \sin S1 \sin S2 \sin S3 \\ \cos S2 \sin S3 \\ \cos S1 \sin S2 \sin S3 \end{pmatrix} \tag{4.18}
\]

\[
\bar{d}\text{SOP}_{\text{dS3}} = A \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} -\cos S1 \sin S3 - \sin S1 \cos S2 \cos S3 \\ \sin S2 \cos S3 \\ -\sin S1 \sin S3 + \cos S1 \cos S2 \cos S3 \end{pmatrix} \tag{4.26}
\]

Using these expressions, we obtain,

\[
|\bar{d}\text{SOP}_{\text{dS2}}|^2 = \sin^2 S3 \tag{4.20}
\]

\[
|\bar{d}\text{SOP}_{\text{dS3}}|^2 = 1 \tag{4.27}
\]

\[
(\bar{d}\text{SOP}_{\text{dS2}} \cdot \bar{d}\text{SOP}_{\text{dS3}}) = 0 \tag{4.34}
\]

Substituting the last quantities into eqs.(4.32) and (4.33) gives,

\[
|\bar{d}\text{SOP}_{\text{min}}| = |\sin S3| \tag{4.35}
\]

\[
|\bar{d}\text{SOP}_{\text{min2}}| = 1 \tag{4.36}
\]

The minimum of \(|\bar{d}\text{SOP}_{\text{min}}|\) and \(|\bar{d}\text{SOP}_{\text{min2}}|\) is the controllability, we get,

\[
\text{CONTROLLABILITY} = |\sin S3| \tag{4.37}
\]

The controllability versus S2, S3 is plotted in fig.4.14.
Fig. 4.14: Controllability versus S2, S3 in the case of S2,S3-control.
Optimal control

Up to now, only control schemes using two squeezers have been described. An optimal control method can be achieved using all three squeezers.

When two squeezers are used, the retardation change vector \((dS_1, dS_2, dS_3)\) has a component in the direction of the reset-vector \((dS_{1_{\text{res}}}, dS_{2_{\text{res}}}, dS_{3_{\text{res}}})\) which does not contribute to the change \(d\text{SOP}_{\text{out}}\) of the output SOP.

Consequently, the aim of the three-squeezer control scheme is to prevent a component in the direction of the reset-vector. Hence, the control space is the plane perpendicular to the reset-vector. The equation of the plane is given by,

\[
(dS_{1_{\text{res}}})dS_1 + (dS_{2_{\text{res}}})dS_2 + (dS_{3_{\text{res}}})dS_3 = 0
\]  \hspace{1cm} (4.38)

The condition that the retardation change is limited to \(\varepsilon\) must now be considered for three squeezers instead of two. In the \(dS_1, dS_2, dS_3\)-space, the restriction is represented by a cube of side \(2\varepsilon\) and a centre-point equal to the origin. The cube normalized with respect to \(\varepsilon\) is shown in fig.4.15. The attainable vectors \((dS_1, dS_2, dS_3)\) lie inside the intersection of the cube and the control space (eq.4.38). In general, the intersection is a mathematical figure having six sides as shown in fig.4.15.
Fig. 4.15: Intersection of the control space (plane perpendicular to the reset-vector \(\bar{d}S_{\text{res}}\)) and the cube (see text).

Since the figure is symmetrical with respect to the origin, only three sides have to be considered. First, the side determined by the intersection of the following two planes is considered,

\[
dS_1 = 1
\]

\[
(dS_{1}\text{res})dS_1 + (dS_{2}\text{res})dS_2 + (dS_{3}\text{res})dS_3 = 0
\]

Substituting the first equation into the last equation gives,

\[
dS_2 = -\frac{dS_{3}\text{res}}{dS_{2}\text{res}}dS_3 - \frac{dS_{1}\text{res}}{dS_{2}\text{res}}
\]

Substitution of \(dS_3 = \lambda\) results in,
\[
\begin{pmatrix}
\frac{dS_1}{dS_3} \\
\frac{dS_2}{dS_3}
\end{pmatrix} = \lambda \begin{pmatrix}
0 \\
-\frac{dS_{3\text{res}}}{dS_{2\text{res}}}
\end{pmatrix} + \begin{pmatrix}
1 \\
-\frac{dS_{1\text{res}}}{dS_{2\text{res}}} \\
0
\end{pmatrix} = \lambda \vec{p} + \vec{q}
\tag{4.42}
\]

The straight line \( \overline{dS} = \lambda \vec{p} + \vec{q} \) is transformed by matrix \( A \) to the straight line \( \overline{dSOP_{\text{out}}} = \lambda A[\vec{p}] + A[\vec{q}] \).

Our interest is the distance of the last line to the origin. For the distance, we find (c.f. eq.(4.14)),

\[
|dSOP_{\text{min}}| = \sqrt{|A[\vec{q}]|^2 - \frac{(A[\vec{p}]A[\vec{q}])^2}{|A[\vec{p}]|^2}}
\tag{4.43}
\]

where

\[
\vec{p} = \begin{pmatrix}
0 \\
-\frac{dS_{3\text{res}}}{dS_{2\text{res}}}
\end{pmatrix}
\]

\[
\vec{q} = \begin{pmatrix}
1 \\
-\frac{dS_{1\text{res}}}{dS_{2\text{res}}} \\
0
\end{pmatrix}
\]

\[
\begin{pmatrix}
\frac{dS_{1\text{res}}}{dS_{3\text{res}}} \\
\frac{dS_{2\text{res}}}{dS_{3\text{res}}}
\end{pmatrix} = \begin{pmatrix}
\tan S3 \\
-\sin S2 \\
-\cos S2 \tan S3
\end{pmatrix}
\tag{4.14}
\]
In the same way as above, we get,

\[ |d\text{SOP}_{\text{min}2}| = \sqrt{|A[\tilde{q}]|^2 - \frac{(A[\tilde{\rho}]A[\tilde{q}])^2}{|A[\tilde{\rho}]|^2}} \]  

(4.44)

where

\[
\tilde{\rho} = \begin{pmatrix}
-dS_{3\text{res}} \\
0 \\
dS_{1\text{res}}
\end{pmatrix}
\]

\[
\tilde{q} = \begin{pmatrix}
-dS_{2\text{res}}/dS_{1\text{res}} \\
1 \\
0
\end{pmatrix}
\]

and

\[ |d\text{SOP}_{\text{min}3}| = \sqrt{|A[\tilde{q}]|^2 - \frac{(A[\tilde{\rho}]A[\tilde{q}])^2}{|A[\tilde{\rho}]|^2}} \]  

(4.45)

where

\[
\tilde{\rho} = \begin{pmatrix}
-dS_{2\text{res}} \\
dS_{1\text{res}} \\
0
\end{pmatrix}
\]

\[
\tilde{q} = \begin{pmatrix}
-dS_{3\text{res}}/dS_{1\text{res}} \\
0 \\
1
\end{pmatrix}
\]

The controllability is the minimum of \(|d\text{SOP}_{\text{min}1}|, |d\text{SOP}_{\text{min}2}|\) and \(|d\text{SOP}_{\text{min}3}|\).

The controllability versus S2, S3 is plotted in fig.4.16. Comparing the plot with the plots of fig.4.12-4.14, then we see for every squeezer setting (S2,S3), the controllability of the optimal control scheme is better than the controllability of the other three control schemes as should be expected.
Fig. 4.16: Controllability versus $S_2$, $S_3$ in the case of optimal control.
4.3.3 Algorithm

The aim of the reset algorithm is to overcome the problem of signal loss due to the reset of a squeezer. As already mentioned in the introduction, a retardation limit of a squeezer is given by a plane, orthogonal to the axis of the squeezer in the $S_1,S_2,S_3$-space. Consideration of all six range limits in this space, gives six planes which form a box. The reset algorithm has to prevent situations having a combination of retardations corresponding to a point outside the box.

The algorithm proposed here, prevents combinations of $S_2,S_3$-retardations outside the shaded region shown in fig.4.17. The edges of regions 1 and 3 correspond to the projection of reset curves onto the $S_2,S_3$-plane (c.f. fig.4.6). The regions 1 and 3 are chosen so that for the points inside the regions a reset is possible, whereas for the points outside the regions, a reset leads to winding up of $S_2$ and/or $S_3$ due to a misalignment of the squeezers (figs.4.8-4.9).

The retardation of $S_1$ is limited by two planes orthogonal to the $S_1$-axis. In other words, the allowable combinations of retardations lie inside a closed 3-dimensional volume as shown in fig.4.18. The box (dashed lines in fig.4.18) which is obtained from the physical range limits, has to enclose the 3-dimensional volume.

Fig.4.17: Allowed combinations of $S_2,S_3$-retardations.
Fig. 4.18: Allowable combinations of $S_1, S_2, S_3$-retardations.

The algorithm can be split into two different phases: control phase and reset phase.

Control phase

Optimal control achieves the largest controllability for an arbitrary combination $S_2, S_3$ (c.f. previous paragraph). During optimal control, three squeezers must be controlled simultaneously. In practice, it is much easier to control only two squeezers at a time. Using two squeezers, there are three control possibilities: $S_1, S_2$-control, $S_1, S_3$-control and $S_2, S_3$-control. In figs. 4.12-4.14, controllability versus $S_2, S_3$ is plotted for these control methods. The aim is largest controllability. Consequently, in regions 1 and 3 (fig.4.17) squeezers $S_2$ and $S_3$ are controlled and in region 2 squeezers $S_1$ and $S_3$ are controlled.
Reset phase

Fig. 4.19: Paths of the algorithm in the $S_1, S_2, S_3$-space.

The reason for a reset is the winding up of squeezer $S_1$. In region 1 or region 3 (fig.4.17), $S_1$ does not wind up, since squeezer $S_1$ is not used for control ($S_2, S_3$-control) (path 1 in fig.4.19). Accordingly, no reset is required in these regions.

At the edge of region 1 (or region 3), $S_1$ must be reset until the combination $S_2, S_3$ lies at the border between region 1 and region 2 (path 2 in fig.4.19). The direction of winding is chosen so as not to exceed the retardation range limits of $S_1$. In practice, $S_1$ moves slowly in the desired direction and during this movement, $S_2$ and $S_3$ control the output power at a maximum.

In region 2, $S_1$ can wind up, since it is used for control ($S_1, S_3$-control). Resetting $S_1$ in region 2 can result in winding up $S_2$ and/or $S_3$ due to misalignment of the squeezers (c.f. figs.4.8 and 4.9). Therefore, no reset is allowed in region 2. Moreover, the possibility of winding up of $S_1$ is present. Consequently, signal fading can occur due to the fact that squeezer $S_1$ may become stuck at one of its physical range limits (path 3 in fig.4.20). The probability of such a situation is proportional to the length of region 2, which depends on the size of regions 1 and 3. The sizes of regions 1 and 3 are determined by the magnitude of the misalignment angles of the squeezers (see fig.4.7).
The retardation ranges needed are $[0, \pi]$ and $[-\pi, \pi]$ for $S_2$ and $S_3$, respectively. The physical retardation ranges must contain these ranges, eventually shifted over a multiple of $\pi$. The length of the retardation range of $S_1$ must be greater than or equal to $2\pi$.

The reset conditions require knowledge of the retardations $S_2$ and $S_3$. In practice, the relation between the control voltage and the retardation of a squeezer is not be exactly known due to drift and hysteresis. In this case, the values of $S_2$ and $S_3$ are not known, but they can be retrieved by determination of the reset-vector. The retardation of $S_1$ is increased with $dS_1$. To compensate, squeezers $S_2$ and $S_3$ must be changed with $dS_2$ and $dS_3$, respectively. From eq.(4.12), the values for $S_2$ and $S_3$ can be determined. Linearity of the relation between control voltage and retardation is required.

To overcome the problems of angular misalignment and the uncertain relation between control voltage and retardation, an extra squeezer may be used. Therefore, research on polarization control systems using four squeezers is necessary.
5.0 Endless Control Using Two Squeezers

5.1 Introduction

In the previous chapter, an endless polarization control system having three squeezers was discussed. The amount of squeezers may be reduced by one using the scheme proposed by Mahon [21][22]. In this scheme, a squeezer is omitted and passive optical components are added. The scheme is a combination of polarization diversity and polarization control. Therefore the polarization diversity scheme is explained first and then the scheme of Mahon is discussed.

5.2 Polarization Diversity Scheme

The polarization diversity scheme is discussed with reference to fig. 5.1.

![Fig. 5.1: Polarization diversity receiver.](image)

The electric field of the incoming signal is given by,

\[ E_{x1} = |E_{x1}| \cos \omega t \]  
\[ E_{y1} = |E_{y1}| \cos (\omega t + \delta_p) \]

Subscript 1 corresponds to point 1 in fig. 5.1.

The electric field is split into a x-component and a y-component using a polarization splitter (PS).

\[ E_{x2} = |E_{x2}| \cos \omega t \]  
\[ E_{y3} = |E_{y3}| \cos (\omega t + \delta_p) \]
These components are heterodyned separately using half of the intensity of the local oscillator (LO) for each.

\[ E_{x4} = \frac{1}{\sqrt{2}} |E_{LO}| \cos \omega_{1f} t \] (5.5)

\[ E_{y5} = \frac{1}{\sqrt{2}} |E_{LO}| \cos (\omega_{1f} t + \delta_{LO}) \] (5.6)

3 dB-couplers are used for the heterodyning. The effect of a 3 dB-coupler is shown in fig.5.2 using complex notation.

\[ \bar{E}_{in1} = \frac{1}{\sqrt{2}} \bar{E}_{in1} + \frac{1}{\sqrt{2}} \exp(-j \frac{\pi}{2}) \bar{E}_{in2} \]

\[ \bar{E}_{in2} = \frac{1}{\sqrt{2}} \bar{E}_{in2} + \frac{1}{\sqrt{2}} \exp(-j \frac{\pi}{2}) \bar{E}_{in1} \]

Fig. 5.2: Effect of a 3 dB-coupler. Complex notation is used.

Application to our case, gives,

\[ E_{x6} = \frac{1}{\sqrt{2}} |E_{x,x}| \cos \omega t + \frac{1}{2} |E_{LO}| \cos (\omega_{1f} t - \frac{\pi}{2}) \] (5.7)

\[ E_{y7} = \frac{1}{\sqrt{2}} |E_{x,y}| \cos (\omega t - \frac{\pi}{2}) + \frac{1}{2} |E_{LO}| \cos (\omega_{1f} t) \] (5.8)

\[ E_{y8} = \frac{1}{\sqrt{2}} |E_{x,y}| \cos (\omega t + \delta_x) + \frac{1}{2} |E_{LO}| \cos (\omega_{1f} t + \delta_{LO} - \frac{\pi}{2}) \] (5.9)

\[ E_{y9} = \frac{1}{\sqrt{2}} |E_{x,y}| \cos (\omega t + \delta_x - \frac{\pi}{2}) + \frac{1}{2} |E_{LO}| \cos (\omega_{1f} t + \delta_{LO}) \] (5.10)

The balanced detectors are placed after the couplers. A balanced detector consists of two photodiodes, whose photocurrents are subtracted and the result is the output signal [23].

We find,
IF₁ = \frac{1}{2\sqrt{2}} |E_{sx}| |E_{LO}| \cos\left((\omega - \omega IF)t + \frac{\pi}{2}\right) \tag{5.11}

IF₂ = \frac{1}{2\sqrt{2}} |E_{sy}| |E_{LO}| \cos\left((\omega - \omega IF)t + \delta₁ - \delta₁LO + \frac{\pi}{2}\right) \tag{5.12}

The IF-signals are added together to form the output of the polarization diversity receiver. Before addition, the phase difference between the IF-signals is compensated electronically using a phase control mechanism. Accordingly, the phase difference reduces to zero and the output becomes maximum.

The IF-signals become after phase control,

IF₁ = \frac{1}{2\sqrt{2}} |E_{sx}| |E_{LO}| \cos\left((\omega - \omega IF)t + \frac{\pi}{2}\right) \tag{5.13}

IF₂ = \frac{1}{2\sqrt{2}} |E_{sy}| |E_{LO}| \cos\left((\omega - \omega IF)t + \frac{\pi}{2}\right) \tag{5.14}

In this manner, we have improved the signal-to-noise ratio of the output. Another way of doing this, is the implementation of a ratio combiner. The operation of the ratio combiner is explained as follows.

Generation of the photocurrent at the photodiode introduces inevitably shot noise. The energy of the shot noise is proportional to the total light energy detected by the photodiode. Since the energy of the local oscillator (LO) is much larger than the received signal energy, the LO-energy determines the shot noise energy.

Each of the IF-signals of eqs.(5.11) and (5.12) are obtained using half the LO-energy \(= \frac{1}{4} |E_{LO}|^2\). Hence, a shot noise term caused by half the LO-energy must be added to the r.h.s. of eqs.(5.13) and (5.14),

IF₁ = \frac{1}{2\sqrt{2}} |E_{sx}| |E_{LO}| \cos\left((\omega - \omega IF)t + \frac{\pi}{2}\right) + n₁ \left(\frac{1}{4} C|E_{LO}|^2\right) \tag{5.15}

IF₂ = \frac{1}{2\sqrt{2}} |E_{sy}| |E_{LO}| \cos\left((\omega - \omega IF)t + \frac{\pi}{2}\right) + n₂ \left(\frac{1}{4} C|E_{LO}|^2\right) \tag{5.16}

where C is the proportional constant of the relation between the detected light power and the shot noise energy.

\(n₁(x)\) and \(n₂(x)\) are independent Gaussian noise processes having energy \(x\).

Addition of the IF-signals without a ratio combiner gives for the signal energy \(S\), noise energy \(N\) and the ratio \((S/N)\) of the output,

\[ S = \frac{1}{16} (|E_{sx}| |E_{sy}|)^2 |E_{LO}|^2 \tag{5.17} \]

\[ N = 2 \times \frac{1}{4} C|E_{LO}|^2 \tag{5.18} \]
\[(S/N)_{\text{ratio combiner}} = \frac{1}{8C} \left( |E_{x,x}| + |E_{y,y}| \right)^2 \] (5.19)

The ratio can be improved using a ratio combiner. The ratio combiner weights the IF-terms before addition. The IF1-term is multiplied by \(|E_{x,x}|\) and the IF2-term is multiplied by \(|E_{y,y}|\). Since \(|E_{x,x}|\) and \(|E_{x,y}|\) vary with time, the multiplication factors have to be adapted continuously.

The weighted IF-signals are added, and expressions for the quantities \(S\), \(N\), \((S/N)\) of the output are derived,

\[ S = \frac{1}{16} (|E_{x,x}|^2 + |E_{x,y}|^2) E_{\text{sub LO}}^2 \] (5.20)

\[ N = (|E_{x,x}|^2 + |E_{y,y}|^2) \frac{1}{4} C E_{\text{LO}}^2 \] (5.21)

\[ (S/N)_{\text{ratio combiner}} = \frac{1}{8C} \left( |E_{x,x}|^2 + |E_{y,y}|^2 \right) \] (5.22)

Comparing the \((S/N)\) ratios of eqs. (5.19) and (5.22), gives,

\[ (S/N)_{\text{no ratio combiner}} \leq (S/N)_{\text{ratio combiner}} \] (5.23)

which may be derived from the inequality,

\[ (|E_{x,x}| - |E_{y,y}|)^2 \geq 0 \] (5.24)

The equality sign is valid for the situation,

\[ |E_{x,x}| = |E_{x,y}| \] (5.25)

Eq.(5.23) proves the use of the ratio combiner.

In summary, two mechanisms are involved in improving the \((S/N)\) ratio of the output. The phase control compensates for the phase difference between the IF-signals and the ratio combiner takes care of the weighting of the IF-terms. Hence, two parameters must be controlled electronically, which agrees with the amount of parameters, which describe the output SOP.

### 5.3 Scheme Proposed by Mahon

#### 5.3.1 Theory

In both the polarization control and polarization diversity scheme, two parameters must be controlled to achieve maximum receiver sensitivity. In the polarization diversity scheme, both the parameters are controlled electronically, whereas during polarization control, the two parameters are controlled optically using two squeezers.

Mahon’s scheme is a combination of polarization diversity and polarization control. The system configuration is shown in fig.5.3.
The optical signal energy is divided equally over the two heterodyning branches by the system configuration of Fig.5.4. Accordingly, there is no need for a ratio combiner (c.f. paragraph 5.2). One parameter, the phase difference between the IF-terms, is controlled electronically. The other parameter is controlled optically using squeezer S2 (Fig.5.4). An extra squeezer (S1) is required to reset squeezer S2.

Considering the configuration of Fig.5.4, the light intensities of the two outputs must be equalized using squeezer S2.

The electric field at point 1 is given by,

\[ E_{x1} = |E_{x1}| \cos \omega t \]  
\[ E_{y1} = |E_{y1}| \cos (\omega t + \delta) \]  

Subscript 1 corresponds to point 1 in Fig.5.4.

The electric field is split into an x-component and a y-component using a polarization splitter (PS).
Linear polarization in the x-direction at point 3 is transformed into a linear polarization in the y-direction using mode conversion.

\[ E_{3\text{y}} = |E_{3\text{x}}| \cos \omega t \]  
(5.28)

\[ E_{4\text{y}} = |E_{3\text{x}}| \cos (\omega t + \delta) \]  
(5.29)

The path between points 3 and 5 represents a delay, equivalent to the optical path length difference \( \Delta L_{\text{opt}} \), between paths 2-4 and 3-5.

\[ E_{5\text{y}} = |E_{4\text{y}}| \cos (\omega t + \delta - \Delta L_{\text{opt}}) \]  
(5.30)

The two SOP’s of points 4 and 6 are combined in the 3 dB-coupler. The effect of a 3 dB-coupler is shown in fig. 5.2.

\[ E_{6\text{y}} = |E_{3\text{x}}| \cos \omega t + |E_{3\text{y}}| \cos (\omega t + \delta - \Delta L_{\text{opt}} - \frac{\pi}{2}) \]  
(5.31)

\[ E_{7\text{y}} = |E_{3\text{x}}| \cos (\omega t - \frac{\pi}{2}) + |E_{3\text{y}}| \cos (\omega t + \delta - \Delta L_{\text{opt}}) \]  
(5.32)

The amplitudes of \( E_{6\text{y}} \) and \( E_{7\text{y}} \) are given by,

\[ |E_{6\text{y}}|^2 = (|E_{3\text{x}}| + |E_{3\text{y}}| \sin (\delta - \Delta L_{\text{opt}}))^2 + (|E_{3\text{y}}| \cos (\delta - \Delta L_{\text{opt}}))^2 \]  
(5.33)

\[ |E_{7\text{y}}|^2 = (|E_{3\text{x}}| - |E_{3\text{y}}| \sin (\delta - \Delta L_{\text{opt}}))^2 + (|E_{3\text{y}}| \cos (\delta - \Delta L_{\text{opt}}))^2 \]  
(5.34)

The amplitudes are equal when,

\[ \delta - \Delta L_{\text{opt}} = 0 \text{ or } \delta - \Delta L_{\text{opt}} = \pi \]  
(5.35)

For the SOP at point 1, the retardation \( \delta \) between the two modes must be equal to \( \Delta L_{\text{opt}} \) or \( \Delta L_{\text{opt}} + \pi \) to get equally distributed energies at the outputs. The ratio of the energies of the two modes \( ||E_{\text{xx}}||^2 ||E_{\text{yx}}||^2 \) at point 1 has no influence on the distribution.

The SOP’s which have the same phase relation between the two modes (but different relative intensities) can be represented by a circle on the Poincaré sphere as shown in fig. 5.5. The angle between the plane of the circle and the equatorial plane equals \( \Delta L_{\text{opt}} \).
Squeezer $S_2$ transforms the input SOP onto this circle. A reset is performed when the input SOP crosses the circle $XRYL$. Then squeezer $S_1$ orientated at an angle of $45^\circ$, transforms the input SOP to point 'X' or point 'Y', the eigenstates of squeezer $S_2$. Consequently, $S_2$ can be reset. After the reset, squeezer $S_1$ is brought back to the original state.
5.3.2 Experimental set-up

*Alternatives*

The experimental set-up used by Mahon is shown in fig. 5.6. Comparing the set-up with the system configuration of fig. 5.4, the various components can be recognized easily. For example, the \(\lambda/2\)-plate orientated at an angle of 45° performs the mode conversion, the 50/50 beamsplitter replaces the 3 dB-coupler.

Since the whole set-up is in air, air currents have a significant effect upon the optical path length difference \(\Delta L_{\text{opt}}\).

![Diagram of experimental set-up](image)

*Fig. 5.6: Experimental set-up used by Mahon.*

The susceptibility to air currents can be reduced considerably by using optical fibre between the components as shown in fig. 5.7.
For correct operation, the SOP must be maintained between the polarization beam splitter and the 50/50-beamsplitter. Therefore, polarization maintaining fibre (PMF) is used to connect these components. Mode conversion can be realized by simply rotating the particular PMF-fibre at a 90° angle about its axis. After the beamsplitter, the SOP is not important. Important is only the loss incurred between the beamsplitter and the photodiode. The loss is reduced by using graded-index (GI) fibre having a large core diameter and therefore a large numerical aperture.

Beamsplitters and lensconnectors are separated by air, thus the influence of air currents is still present. The air gaps may be avoided by using fibre couplers as shown in fig.5.8.

Fig.5.7: Experimental set-up using fibre between the components.

Fig.5.8: Experimental set-up using fibre couplers.

The attainability of polarization splitting and polarization maintaining fibre couplers is rather restricted, since manufacturing technology is still not perfected.

Notwithstanding, a polarization maintaining coupler was available for measuring. The measurement results (see appendix A) deviated a lot from the specifications and thus the coupler couldn’t be used.

A polarization splitting coupler was not available.

Consequently, the set-up using the fibre couplers couldn’t be realized.
The set-up shown in fig.5.7 was realized and is described in the next paragraph.

**Chosen set-up**

The set-up which is built and tested is shown fully in fig.5.9. The set-up may be split into an optical part and an electrical part.

![Diagram of realized and tested set-up](image)

**OPTICAL PART**

The operation of the optical part has already been described elaborately. Hence, only the differences between the original set-up of fig.5.7 and the built set-up are discussed. Also, the optical alignment procedure is described.

The measurement results and the specifications of the separate electrical and optical components can be found in appendix B.

The first difference between the original set-up and the built set-up is the amount of photodiodes used. In the built set-up, one photodiode is used instead of two. The reason for this is to save on components. First, the aim of the control algorithm was equalization of the detected light energies of the two photodiodes. Now, the aim is to keep the detected energy of only one photodiode at a reference level, which equals half the total light energy on the beamsplitter plate.

Another difference is the presence of wheels 1 and 2. Each set of wheels consists of three wheels having two, four, two windings of single-mode fibre, respectively [10]. The wheels induce
birefringence. By means of adjusting the wheels, an arbitrary input SOP may be transformed to an arbitrary output SOP.

Wheels 1 is not a fundamental part of the set-up and is used for alignment and measurement purposes.

To reduce the influence of air currents, the polarization beamsplitter and the 50/50-beamsplitter are placed in a plastic box. The effect of the plastic box on the detected light power is shown in fig.5.10.

![Graph showing the effect of the plastic box on light power]

**Fig.5.10: Effect of the plastic box.**

The single-mode fibre, leading from the squeezers to the polarization beamsplitter, is passed through a gap in the box. This cannot be done without introducing birefringence in the particular piece of single-mode fibre. The birefringence is not desired and must be compensated by wheels 2.

**Optical alignment**

The various optical components must be aligned with respect to each other. In particular, the lensconnectors. Lensconnector 1 is in a fixed position w.r.t. the polarization beamsplitter. Accordingly, the extinction ratio of the deviated beam cannot be improved by changing the angle of incidence (c.f. paragraph 3.4.2 of appendix B).

Lensconnector 1 is fixed in a V-groove. If lensconnector 3 is placed in a V-groove opposite and aligned with the V-groove of lensconnector 1, the coupling between the connectors separated by the polarization beamsplitter (PBS) is reduced due to the beam deviation of the PBS. Hence, lensconnector 3 must be aligned using an alignment apparatus (see chapter 10 of appendix B).

Lensconnector 2 is also aligned using a similar alignment apparatus.

The polarization maintaining fibre (PMF) is not rotation symmetric. Only two SOP's at the input of the PMF fibre are preserved, corresponding to linear SOP's along the principal axes of the PMF-fibre (c.f. chapter 5 of appendix B). Therefore lensconnectors 2 and 3 must be rotated so that one of the principal axes of the PMF-fibre is perpendicular to the plane of drawing of fig.5.9.
The optimal orientation gives the largest extinction ratio (ratio of energies of the two modes) at the output of the PMF-fibre. The extinction ratio is measured using a polarizer and a photodiode after lensconnectors 4 and 5, respectively.

The mounting of the 50/50-beamsplitter plate is shown in fig.B.31 of appendix B. The orientation of the plate may be adjusted using screws. Lensconnectors 4 and 5 are fixed in a V-groove. By means of rotating the connectors, the linear SOP's coming out the two connectors are aligned. A polarizer and a photodiode at one of the outputs of the beamsplitter are used to recognize optimal alignment.

At the beamsplitter plate, maximal interference is desired. An IR-camera placed at one of the outputs of the beamsplitter, displays the interference pattern. The orientation of the beamsplitter plate is adjusted until the spots of the beams from lensconnectors 4 and 5 coincide and maximal interference occurs. In the realized set-up, about 10 % of the light does not contribute to the interference due to misadaptation of the interfering beam spots.

The next step is to place lensconnector 6 in the beamsplitter mounting. Optimal operation requires the same losses for the optical pathes A and B. Using wheels 1, the light power through path A is maximized. The maximum power is measured by the photodiode of fig.5.9. In the same way, the maximum throughput of path B is determined. The two maxima must be the same. If not, the largest of the two values is equalized to the other by introducing a misalignment of a lensconnector in the particular branch (lensconnector 2 for path A and lensconnector 3 for path B).

During the adjustment of wheels 2, path A is blocked. Consequently, the optical circuit acts as a simple polarizer. A varying voltage is applied to squeezer S2. The fluctuating voltage causes a variation of the detected light. Wheels 2 is adjusted so that the variation in detected light power is minimized.

**ELECTRICAL PART**

The central part of the electrical circuit is the microcomputer. The microcomputer is an Applecomputer (model Euro) and is provided with two DAC-cards (Digital to Analog Convertor) and one ADC-card (Analog to Digital Convertor). The ADC-card translates linearly an input voltage in the range 0..10,28 Volt to a digital value in the range 0..1028. The DAC-card translates linearly a digital value in the range 0..1028 to an output voltage in the range 0..10,28 Volt. A high-voltage operational amplifier (HiVo Opamp) amplifies the output of the DAC with a factor 13. Consequently, the output voltage range of the amplifier is then 0..130 Volt, the necessary voltage range to control the squeezer.

The piezoelectric squeezers (appendix B) are mounted in metal clamps as shown in fig.5.11. Adjustment of the clamp gives a preset retardation.
Apart from the force applied, the retardation is also directly proportional to the length of fibre pressed by the squeezer. To increase the induced retardation, the pressing region is enlarged to about 10 mm using a metal brace between the fibre and the squeezer (fig. 5.12).

The rounded corners are to prevent damage to the fibre. Another precautionary measure is the use of aluminium foil between the fibre and the metal brace. Damage to the fibre can lead to extra loss in the fibre depending on the squeezer pressure.

In the ideal case, the retardation is directly proportional to the applied control voltage. The retardation as a function of control voltage is measured using the set-up in fig. 5.13. To get minimum throughput, the retardation induced by the squeezer must be compensated for by using a Soleil-Babinet Compensator (SBC). Then the retardation of the SBC is the reverse of the retardation caused by the squeezer. Since the retardation of the SBC is shown on a digital read-out, the retardation of the squeezer is known.
Fig. 5.13: Set-up to measure the retardation of a squeezer.

Fig. 5.14: Squeezer retardation versus control voltage.
The results are plotted in fig. 5.14. If we observe the plot it is apparent that non-linearity and hysteresis due to the piezoelectric material must be considered. The plot does not show the retardation drift due to the clamps.

## 5.3.3 Algorithm

The algorithm has one input parameter and two output parameters. The aim of the algorithm is to keep the detected light at a fixed reference value. Therefore, the two voltages applied to the squeezers, are controlled. Due to electrical capacities of the squeezers, there is a delay between the change of the squeezers' voltages and the resulting change in detected light. Accordingly, the algorithm has to wait after a change in output voltage until the detected light is settled. The corresponding waiting time is called settle time. The settle time is determined empirically. The smaller the settle time, the faster is the control system. If the settle time is chosen too small, the system is not stable anymore and tends to oscillate.

The algorithm can be split into three phases:

1) acquisition
2) control
3) reset

### Acquisition

During acquisition, a search routine is performed to find a value for the voltage of squeezer S2 (VS2) giving a detected light power (PD) equal to the reference value (REF). In other words, a zero of PD-REF as a function of VS2 must be found. The search process is illustrated in fig. 5.15.

The algorithm increases or decreases VS2 using steps of variable size. In the first stage, the step size is taken MAXSTP. VS2 is increased until the sign of PD-REF changes. Then the search routine reverses (VS2 decreases) and the step size is divided by DIVSTP. In this second stage, the expected amount of steps to reach the zero, is less than DIVSTP. The amount of steps needed may be larger due to hysteresis of the squeezer. If the amount of steps exceeds 2*DIVSTP an error is reported and the acquisition procedure is started over.

The process of reversing the search and dividing the step size is repeated until the step size is less than MINSTP.
CONTROL

Squeezer S2 is used to control the detected light (PD) at the reference value.

The control algorithm is a gradient algorithm, where the control step size $\Delta V_{S2}$ is directly proportional to $PD - \text{REF}$,

$$\Delta V_{S2} = \frac{\lambda (PD - \text{REF})}{SLOPE}$$

(5.37)

where $\lambda$ is the gradient multiplicator which is determined by observation.

SLOPE is the derivative of PD with respect to VS2 and is determined by the computer program before the control phase.

The response time of the system is decreased by increments of $\lambda$. If $\lambda$ is chosen too large the system becomes instable and tends to oscillate as shown in fig.5.16 for $\lambda = 2$. 

Fig.5.15: Illustration of the search process.
Fig. 5.16: Oscillations of the detected light due to a large gradient multiplicator $\lambda$ ($\lambda = 2$).

**RESET**

When the input SOP crosses the circle XRYL of the Poincaré sphere, a reset of squeezer $S_2$ is possible (see fig. 5.5). The situation is characterized by $S_2 = \Delta l_{opt} + k\pi$ ($k$ is an integer value). The voltage of $S_2$ corresponding to this situation is input to the computer (VS2RES). Due to air currents, $\Delta l_{opt}$ changes. Therefore the computer program must be stopped and VS2RES must be adapted. This makes the reset procedure semi-automatic.

The reset procedure consists of three stages:

1) **stretch:** $\quad$ VS2 is increased by a step of size MINSTP and the corresponding change in detected light power (DIF) is measured. This action is repeated until DIF reaches a fixed value (MAXDIF). The amount of steps (T) taken depends on the value of VS1. VS1 is controlled until T is at a maximum. Then squeezer $S_2$ is in an eigenstate and can be reset without signal losses.

2) **reset of VS2:** the retardation of squeezer $S_2$ is changed about $2\pi$ away from the closest physical range limit by changing VS2. The magnitude of the change of VS2 equals VS2PERIOD.

3) **restabilization:** VS1 is brought back to equilibrium while controlling squeezer $S_2$.

The algorithm is translated to a computer program. The program language used is PASCAL. A listing of the program is shown in appendix C.
5.3.4 System Performance

Three performance criteria are discussed:
1) system control speed
2) insertion loss
3) intensity loss due to non-optimal control

SYSTEM CONTROL SPEED

The response time of the system is determined by the computer calculation time needed and the time constant of the squeezers. The time constant equals \( RC = 0.8 \text{ ms} \), where \( R \) is the output resistance of the Hi-Vo-amplifier (= 1k) and \( C \) is the capacitance of the squeezer (= 0.8 \( \mu \text{F} \)). The computer calculation time between two iterations is about 75 ms. Consequently, the response time is completely determined by the computer system.

INSERTION LOSS

The power loss between the single-mode ball lens connectors at the polarization beamsplitter gives the largest contribution to the insertion loss. The coupling is dependent on the distance between the connectors (0.14 dB/cm). In the set-up, the distance between the ball-lens connectors is about 4 cm, corresponding to a loss of about \( 1 + 4 \times 0.14 = 1.6 \text{ dB} \) (see chapter 4 of appendix B). Hence, the insertion loss is about 1.5 dB.

INTENSITY LOSS DUE TO NON-OPTIMAL CONTROL

Adjusting wheels 1 (fig. 5.9), the input SOP changes. The detected output power is measured as a function of time.
During normal control, the intensity variation is a few percent. During reset, the maximum intensity deviation from the reference value is about 10 \%.
5.4 Simplified Scheme

Mahon's scheme can be simplified by changing the orientation of the squeezers and omitting the mode conversion section and the 3 dB-coupler (c.f. fig.5.3). The system configuration obtained in this way, is shown in fig.5.17. In principle, it is a polarization diversity scheme preceded by two squeezers. A polarization diversity scheme preceded by one endless polarization control device was already proposed by T. Okoshi in ref.[7].

In the new scheme, a ratio combiner is not necessary, since the squeezers divide the received light equally over the two heterodyning branches. In other words, the energies of the two modes must be equal at the input of the polarization splitter. Accordingly, the desired SOP at the input of the polarization splitter is on the circle PRQL of the Poincaré sphere (see fig.5.18).
Fig. 5.18: Circle of the desired SOP's on the Poincaré sphere (see text).

Squeezer $S_2$ transforms the input SOP to this circle. A reset is performed when the input SOP crosses the circle PRQL. Then squeezer $S_1$ orientated at an angle of $0^\circ$, transforms the input SOP to point 'P' or point 'Q', the eigenstates of squeezer $S_2$. Consequently, $S_2$ can be reset. After the reset squeezer $S_1$ is brought back to the original state.

The reset procedure is analogous to the reset procedure proposed by Mahon. Therefore, the previous discussed algorithm can be used to control the squeezers.
Conclusions

A new approach to the problem of endless polarization control using three squeezers has been developed, leading to the proposal of a new algorithm.

The realized experimental set-up based on Mahon's scheme, is less sensitive to air currents than the original set-up.

The system is not completely automatic: the conditions for a reset must be input to the computer system.

The response time of the system is large due to the computer system used.

Mahon's scheme can be simplified to a polarization diversity system preceded by two squeezers.
Recommendations

The misalignments present in placing the squeezers and the unclear relation between control voltage and squeezer retardation (caused by hysteresis and drift) led to problems which may be overcome by using an extra squeezer. Therefore, it may be interesting to investigate a polarization control system using four squeezers.

Mahon's scheme need not be examined further, since a simplified scheme is available.

To decrease the response time of the polarization control system, the computer system must be replaced by a faster computer system.

To automize the polarization control system, the conditions for a reset must be determined by the algorithm.
References


List of Symbols

ARABIC SYMBOLS

a  length of the semi-major axis of the polarization ellipse
b  length of the semi-minor axis of the polarization ellipse
\( B_f \)  modal birefringence
\( c_o \)  light velocity
e  ellipticity of the polarization ellipse
\( \vec{E} \)  electric field vector
\( f \)  frequency
\( \vec{H} \)  magnetic field vector
\( \vec{j} \)  Jones-vector
\( J \)  Jones-matrix
k  wavenumber (\( = 2\pi/\lambda \))
L  left-handed circular polarization
\( L_a \)  beat length
\( N_o \)  noise energy
\( N_x \)  refractive index of the x-direction
\( N_y \)  refractive index of the y-direction
\( P \)  linear polarization having an angle of 45\(^\circ\) with the x-axis
\( Q \)  linear polarization having an angle of 135\(^\circ\) with the x-axis
\( R \)  right-handed circular polarization
S1,S2,S3  squeezers or their retardations
\( X \)  horizontal linear polarization
\( Y \)  vertical linear polarization
\( x,y,z \)  right-handed Cartesian coordinate system where z is the propagation direction of the wave and x,y correspond to the directions of the two orthogonal modes

GREEK SYMBOLS

\( \beta_x, \beta_y \)  propagation constants of the two modes
\( \delta \)  phase delay between the two modes
\( \epsilon \)  ellipticity angle of the polarization ellipse
\( \epsilon_o \)  permittivity of free space
\( \theta \)  angle between the major axis of the polarization ellipse and the x-axis
\( \lambda \)  wavelength of the light
\( \lambda_o \)  wavelength of the light in free space
\( \mu_o \)  magnetic permeability in free space
\( \psi \)  retardation of one mode with respect to the other mode
\( \omega \)  angular frequency (\( = 2\pi f \))
MATHEMATICAL SYMBOLS

$Pr[]$ probability function

$|.|$ (complex) amplitude

$\dagger$ complex conjugation

$\Sigma$ summation

$\vec{a}$ vector

$\vec{a} \times \vec{b}$ outer vector product of the vectors $\vec{a}$ and $\vec{b}$

$(\vec{a},\vec{b})$ inner vector product of the vectors $\vec{a}$ and $\vec{b}$

ABBREVIATIONS

ADC Analog-to-Digital Convertor

BS BeamSplitter

DAC Digital-to-Analog Convertor

DFB Distributed Feedback

GI Graded-Index

IF Intermediate Frequency

LO Local Oscillator

OPAMP Operational Amplifier

PBS Polarization BeamSplitter

PMF Polarization Maintaining Fibre

SMF Single-Mode Fibre

SOP State Of Polarization
APPENDIX A

PMF-COPLER
Polarization maintaining coupler

Fibre couplers can be manufactured using different techniques such as polishing method, fibre biconical taper method (FBT) [A.1]. The last mentioned method is used for the coupler examined here. The coupler consists of YORK-Polarization Maintaining Fibre (see appendix B).

In fabrication of the polarization maintaining coupler the principal axes of the YORK-PMF must be parallel and precisely aligned. The effect of a misalignment is an increase in cross-coupling or mode-coupling.

If the input polarization is along one of the principal axes, the power ratio (ratio of the energies of the two modes) observed at the output of the splitted branch equals [A.1],

\[
\text{POWER RATIO} = 10 \log(\tan^2 \phi) \tag{A.1}
\]

where \( \phi \) is the angle of misalignment.

The power ratio's of the straight-through branch (A-X) and the splitted branch (A-Y) are measured as a function of input polarization angle. The set-up is shown in fig.A.1. The light source is a solid-state Fabry-Perot laser at \( \lambda_s = 1.551 \, \mu m \).

---

Fig. A.2: Experimental results of the coupler.

The experimental results are shown in the plot of fig. A.2. Maximal power ratio corresponds to an input polarization along one of the principal axes. The cross-coupling or maximal power ratio of the splitted branch equals about 6.5 dB, which does not agree with the value of more than 20 dB mentioned in the specifications.

Using eq. (A.1), we get for the angular misalignment,

$$\phi = 25^\circ$$  \hspace{1cm} (A.2)

Possibly, there are other causes of the increase in cross-coupling such as cracks in the stress-applying elements of the PMF-fibre.

Conclusively, the coupler does not satisfy the specifications and therefore cannot be used in the set-up.
APPENDIX B

COMPONENTS OF THE
EXPERIMENTAL SET-UP
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1.0 Laser

The laser is based on the Distributed FeedBack (DFB) principle. The laser is mounted in a TO5-package and is provided with a pigtail. The fibre output versus the laser current is plotted in fig. B.1.

Fig. B.1: Fibre output versus laser current.
2.0 Piezoelectric squeezer

The piezoelectric squeezer consists of several piezoelectric ceramic plates which are arranged on top of one another and connected in parallel electrically (c.f. fig.B.2). When a voltage is applied, the electrical field points in the direction of polarization, which leads to enlargement of the plate's thickness and therefore to an expansion of the stack.

Fig.B.2: Piezoelectric squeezer.

In comparison with other piezoelectric squeezers, the squeezer used here (Physik Instrumente, P-820.20) needs a low control voltage (0..120 V). A disadvantage is the high capacitance (0,8 μF) of the squeezer (table B.1).

<table>
<thead>
<tr>
<th>P.820.20</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Enlargement at 120 V (μm)</td>
<td>30</td>
</tr>
<tr>
<td>Maximal load (N)</td>
<td>100</td>
</tr>
<tr>
<td>Electrical capacitance (μF)</td>
<td>0,8</td>
</tr>
<tr>
<td>Length (mm)</td>
<td>44</td>
</tr>
</tbody>
</table>

Table B.1: Specifications of the squeezer.
3.0 Polarization Beam Splitter

3.1 Introduction

The polarization beam splitter which has been examined (model 03 PBS 082, Melles Griot), has the shape of a cube and is from now on called the cube. The cube splits the polarization of the input beam into two orthogonal components. The transmitted beam is almost entirely plane-polarized in the p-direction (parallel to the plane of incidence), while the reflected beam is nearly all plane-polarized in the s-direction (perpendicular to the plane of incidence). (see fig.B.3)

![Fig.B.3: The polarization of the input beam splitted into two orthogonal components by the cube.](image)

Inside the cube is a multilayer structure consisting of alternating high- and low-index dielectric layers which are deposited onto the hypotenuses of two right-angle prisms. These prisms are cemented together to form a cube.

The principle of operation of the cube is the condition that the angle of incidence inside the multilayer equals the Brewster angle. Brewster angle incidence implies that at a boundary of two layers the reflected beam is purely s-polarized, but the transmitted beam is not entirely p-polarized, there is still a s-component. By making a pile or a stack of layers the s-component in the transmitted beam can be reduced. In practice the optical thickness of the layers is chosen $\lambda/4$. Advantage is taken of the appearing interference inside the multilayer. This implies that the cube is designed for one wavelength and can be used at that particular wavelength only.
3.2 Theory

3.2.1 General

The following condition must be true for the cube to be a polarization beam splitter (PBS):

\[ N_G^2 = \frac{2N_L^2N_H^2}{N_L^2 + N_H^2} \] (B.1)

This formula could be deduced by use of:

Snell’s law:

\[ N_G \sin \theta_G = N_L \sin \theta_L = N_H \sin \theta_H \] (B.2)

Brewster angle condition:
Eq. B.3 ensures Brewster incidence within the multilayer, but at the two boundaries of the stack and the glass there is no Brewster incidence, so there is reflection of p-polarized light.

By ensuring that the phase thickness is an odd multiple of \( \pi/2 \) at the design wavelength, extinction of the boundary p-component reflection is ensured. The \((LH)^n\) structure satisfies above condition. This structure consists of alternating layers with a high and a low refractive index.

The problem is to find the reflectivity and transmissivity for p-polarized and s-polarized light. There are different methods to solve this problem (eg. admittance method, characteristic matrix method). Because of my electronic background the admittance method is chosen. The calculations have already been done by B. Salzberg [B.1]. These calculations are repeated and focused to the \( \lambda/4 \) -stack and the \((LH)^n\) structure. Interference effects are automatically considered in this model. It's assumed that the materials used are lossless (no absorption).

The calculus used here is the usual complex calculus.

All components of the electric and magnetic vectors in each layer can be regarded as consisting of a transmitted wave propagated along the positive \( z \)-axis, and a reflected wave propagated in the opposite direction. Since only tangential components of the electric and magnetic vector are directly involved in energy flow across the layer boundaries, no further explicit consideration need to be
given to the normal components.

Wave equations for the tangential components, applicable to each layer are:

\[ E(z) = E_k^i \exp (jk_z(z - z_0)) + E_k^r \exp (jk_z(z - z_0)) \]  \hspace{1cm} (B.5)

\[ H(z) = H_k^i \exp (jk_z(z - z_0)) - H_k^r \exp (jk_z(z - z_0)) \]  \hspace{1cm} (B.6)

with

\[ k_z = k \cos \theta = \frac{\omega N}{c_0} \cos \theta \]

\( E_t \) and \( E_r \) are the transmitted and reflected amplitudes of \( E \). (see fig.B.5)

\( H_t \) and \( H_r \) are the transmitted and reflected amplitudes of \( H \).

Each of these quantities contains the common factor \( \exp (j\omega t) \) and the amplitudes on the right side of both expressions contain the factor \( \exp (-jk_z x) \).
It's well known that in a non-absorbing dielectric material the following equation is valid for one plane wave:

\[
\frac{\vec{H}}{\vec{E}} = N \sqrt{\frac{\varepsilon_0}{\mu_r}} = A
\]  

(B.7)

When \( \vec{H} \) is perpendicular to the plane of incidence (p-polarized light) it follows:

\[
\left( \frac{H_k'}{E'_k} \right) = \left( \frac{H_k'}{E_k'} \right) = \frac{A}{\cos \theta} = A'
\]  

(B.8)

When \( \vec{E} \) is perpendicular to the plane of incidence (s-polarized light) it follows:

\[
\left( \frac{H_k'}{E_k'} \right) = \left( \frac{H_k'}{E_k} \right) = A \cos \theta = A'
\]  

(B.9)

After some calculations, the use of eqs. (B.7) to (B.9) and the conditions of continuity (the tangential components of the electric and magnetic vector are continuous at a boundary), we obtain:

\[
E_k' = E_{k+1}' \cos (k_d d_k) + j(H_{k+1}'/A') \sin (k_d d_k)
\]  

(B.10)

\[
H_k' = jE_{k+1}' A' \sin (k_d d_k) + H_{k+1}' \cos (k_d d_k)
\]  

(B.11)

where \( d \) is the layer thickness.

The input admittance \( Y_k = H_k'/E_k' \) of the \( k \)-th layer is obtained by taking the ratio of eq. (B.11) to eq. (B.10).

\[
Y_k = \frac{A' \left[ Y_{k+1} + jA' \tan (k_d d_k) \right]}{A' + jY_{k+1} \tan (k_d d_k)}
\]  

(B.12)

In our case ( \( \lambda/4 \)-stack) the optical thickness \( k_d \) is equal to \( \pi/2 \). Eqs. (B.10) to (B.12) become:

\[
E_k' = \frac{jH_{k+1}'}{A'_k}
\]  

(B.13)

\[
H_k' = jA'_k E_{k+1}^{in}
\]  

(B.14)

\[
Y_k = \frac{(A'_k)^2}{{Y}_{k+1}^{in}}
\]  

(B.15)
Fig. B.6: The choice of the numbering for the layers (2m is total amount of layers).

Formulas (B.13) and (B.14) give:

\[ E_{k}^{in} = - E_{k+2}^{in} \left( \frac{A'_{k+1}}{A'_{k}} \right) \]  
\[ E_{1}^{in} = (-1)^{m} \left( \frac{\bar{N}_{H}}{\bar{N}_{L}} \right)^{m} E_{2m+1}^{in} \]  

For p-polarized light:

\[ \bar{N}_{L} = \frac{N_{L}}{\cos \theta} \]
\[ \bar{N}_{n} = \frac{N_{H}}{\cos \theta} \]

For s-polarized light:

\[ \bar{N}_{L} = N_{L} \cos \theta \]
\[ \bar{N}_{n} = N_{H} \cos \theta \]
There is no reflected wave in the region $2m + 1$ using this:

$$Y_{2m+1} = A'_{2m+1} = A'_0 = \frac{\bar{N}_L}{\bar{N}_H} \frac{\sqrt{\frac{\varepsilon_0}{\mu_0}}}{\cos \theta_G} (B.18)$$

With eq. (B.15):

$$Y_i = \left( \frac{\bar{N}_L}{\bar{N}_H} \right)^{2m} Y_{2m+1} = \left( \frac{\bar{N}_L}{\bar{N}_H} \right)^{2m} A'_0 (B.19)$$

The following formula for the electric transmission coefficient $t$ of one layer can be obtained by use of eqs. (B.5) to (B.9).

$$t_k = \frac{E_{k+1}^i}{E_k^i} = \frac{2A'_k}{A'_k + Y_{k+1}} (B.20)$$

Now we can calculate the over-all electric transmission coefficient $t_{overall}$ with eqs. (B.12), (B.14) and (B.15).

$$t_{overall} = \frac{E_{2m+1}^i}{E_0^i} = \frac{E_{2m+1}^i}{E_1^i} \frac{E_1^i}{E_0^i} = (-1)^m \left( \frac{\bar{N}_L}{\bar{N}_H} \right)^m \frac{2A'_o}{A'_o + Y_1} = \frac{2(-1)^m \left( \frac{\bar{N}_L}{\bar{N}_H} \right)^m}{1 + \left( \frac{\bar{N}_L}{\bar{N}_H} \right)^{2m}} (B.21)$$

For the over-all magnetic transmission coefficient $t'_{overall}$ we find:

$$t'_{overall} = \frac{H_{2m+1}^i}{H_0^i} = \left( \frac{A'_{2m+1}}{A'_0} \right) t_{overall} (B.22)$$

Transmissivity $T$ is the ratio of the transmitted to the incident intensity. ($\ast$ means complex conjugated)

$$T = \frac{Re\left\{E_{2m+1}^i H_{2m+1}^{i\ast}\right\}}{Re\left\{E_0^i H_0^{i\ast}\right\}} = \left| t_{overall} \right|^2 \frac{Re\left\{A'_{2m+1}\right\}}{A'_o} = \left| t_{overall} \right|^2 (B.23)$$

$$T = \frac{4 \left( \frac{\bar{N}_L}{\bar{N}_H} \right)^{2m}}{(1 + \left( \frac{\bar{N}_L}{\bar{N}_H} \right)^{2m})^2} (B.24)$$

And for the reflectivity we find:
This last result is in agreement with the mentioned formula in a publication of P.B. Clapham et al [B.2].

### 3.2.2 Extinction Ratio of the Straight-Through Beam

If the extinction ratio of the straight-through beam is analyzed, only the transmissivity of s-polarized light has to be considered.

\[
R = 1 - T = \left( \frac{\frac{\bar{N}_L}{\bar{N}_H}}{\frac{\bar{N}_L^{2m}}{\bar{N}_L^{2m} + \bar{N}_H^{2m}}} \right)^2
\]  

\[(B.25)\]

With:

\[
\bar{N}_L = \frac{N_L}{\cos \theta} \quad \bar{N}_H = \frac{N_H}{\cos \theta}
\]

The relation between \(\theta_i, \theta_L, \) and \(\theta_H\) is Snell's law (see fig.B.4):

\[
\sin \theta_i = N_G \sin(\theta_G - \frac{\pi}{4}) \quad (B.26)
\]

\[
N_G \sin \theta_G = N_L \sin \theta_L = N_H \sin \theta_H
\]  

\[(B.2)\]

For given \(N_G, N_H, N_L\) we can plot now the extinction ratio versus the angle of incidence (see fig.B.11).

### 3.2.3 Extinction Ratio of the Deviated Beam

If the extinction ratio of the deviated beam is analyzed, only the reflectivity of p-polarized light has to be considered.

\[
R = \left( \frac{\frac{\bar{N}_L}{\bar{N}_H}}{\frac{\bar{N}_L^{2m}}{\bar{N}_L^{2m} + \bar{N}_H^{2m}}} \right)^2
\]  

\[(B.25)\]

With:

\[
\bar{N}_L = \frac{N_L}{\cos \theta} \quad \bar{N}_H = \frac{N_H}{\cos \theta}
\]
The relation between $\theta_s$, $\theta_l$, and $\theta_H$, mentioned above, is also valid here.

For given $N_s$, $N_H$, $N_L$, we can plot now the extinction ratio versus the angle of incidence (see fig.B.12).

### 3.2.4 Light-Throughput as Function of Linear Input Polarization

We assume at the input of the cube a linear polarization under an angle $\phi$ with the plane of incidence. This linear polarization can be splitted into a p-polarized component with an amplitude $|\vec{E}| \sin \phi$ and a s-polarized component with an amplitude $|\vec{E}| \cos \phi$. The p-polarized component is almost entirely transmitted and the s-polarized component is almost entirely reflected by the cube.

The throughput $H_{\text{deviated}}$ measured in the deviated branch is equal to:

$$H_{\text{deviated}} = \alpha_{\text{deviated}} \sin^2 \phi = \alpha_{\text{deviated}} \left( \frac{1}{2} - \frac{1}{2} \cos 2\phi \right)$$  \hfill (B.27)

with $\alpha_{\text{deviated}}$ is the loss in the deviated branch.

The throughput $H_{\text{straight}}$, measured in the straight-through branch is equal to:

$$H_{\text{straight}} = \alpha_{\text{straight}} \cos^2 \phi = \alpha_{\text{straight}} \left( \frac{1}{2} + \frac{1}{2} \cos 2\phi \right)$$  \hfill (B.28)

with $\alpha_{\text{straight}}$ is the loss in the straight-through branch.

These functions are plotted for certain $\alpha_{\text{straight}}$ and $\alpha_{\text{deviated}}$ (see fig.B.14). The horizontal shift of the plot will be explained in the paragraph 3.4.3.

### 3.2.5 Beam Deviation

The straight-through beam is in practice not precisely parallel to the incident beam, because the front- and endface of the cube are not exactly plane parallel.

The relation between $\theta$ (the angle between the front- and endface) and $\phi$ (the beam deviation) is (see fig.B.7):

For $\theta \ll 1$:

$$\phi = (N-1) \theta$$  \hfill (B.29)

where $N$ is the refractive index of the material and $\theta$ the particular angle in radian.

In our case $N = 1,5$ (BK7-glass), so:

$$\phi = 0,5 \times \theta$$  \hfill (B.30)
Fig. B.7: Beam deviation through a cube with two unparallel planes.

3.2.6 Insertion Loss

The insertion loss is the loss caused by reflection at the faces of the cube, absorption and wavefront distortion. The last two contributions are not considered here.

The reflectivity for normal incidence at a boundary is equal to:

$$R = \left( \frac{N_1 - N_2}{N_1 + N_2} \right)^2$$

(B.31)

where $N_1$, $N_2$ are the refractive indices of the particular materials.

For a glass-air or air-glass boundary $N_1 = 1.5$ and $N_2 = 1$ or vice versa, we obtain:

$R = 4\%$

This reflectance is reduced by use of an Anti-Reflection (AR) coating on the faces of the cube.
3.3 Experiments

3.3.1 Measurement Set-up

Introduction

On account of the high extinction ratio (up to 40 dB) of the cube very low intensity of light has to be detected. At this level of intensity the noise (e.g. amplifier noise) disturbed the measurements. A synchronous detection scheme has been used to reduce this disturbance.

Fig. B.8: Block diagram of a basic lock-in system which illustrates the lock-in technique.

A lock-in amplifier performs a synchronous detection operation to eliminate asynchronous signals and achieves narrow bandwidth (see fig.B.8). Functionally a lock-in amplifier is a narrowband filter followed by a rectifier. The center frequency is controlled by an external reference signal, and the output is a DC level proportional to the amplitude of an input signal that is coherent with this external reference. In this case the reference signal comes from a light beam chopper.

Extinction Ratio Straight-Through Beam
Fig. B.9: Experimental set-up for the measurement of the extinction ratio of the straight-through beam. The letters x, y, z, θ, φ, ψ stand for the possibility to adjust the corresponding device in that direction. The definition of the directions is indicated by the four figures below.

- Definition of X, Y, Z with respect to the plane of drawing.
- Definition of θ with respect to X, Y.
- Definition of ψ with respect to X, Z.
- Definition of φ with respect to Y, Z.

In fig. B.9 you can see the total set-up for the measurement of the extinction ratio of the straight beam. Now some remarks will be made on the different parts of this set-up. Doing this from the left to the right, starting with the laser. The laser is a distributed feedback (DFB). Its wavelength (\( \lambda = 1.503 \, \mu m \)) is in the IR-region. The pigtail of the laser is ended with a ferrule which has an endface at an angle of 10° with the optic axis, the reason for this is to prevent reflections back in the laser; DFB-lasers are very sensitive to reflections.

In the pigtail, a polarization control is used which consists of three wheels with resp. one, two, one windings of single-mode fiber [B.3]. The so induced birefringence can change an arbitrary input polarization to an arbitrary output polarization.

The next block is the chopper with two blades. The chopper generates an electronical signal with the same frequency and phase as the mechanical turning of the blades (reference signal for the lock-in amplifier). This frequency can be tuned (during the measurements it was tuned to 460 tpm.)

Behind the chopper is the polarizer (extinction ratio better than 50 dB). This polarizer (model PF-6, Optics for Research) has been built in a circular holder which can be rotated. The amount of rotation is shown by means of a scale printed on the outside of the holder. So the direction of the polarization axis of the polarizer can be adjusted at an arbitrary amount of degrees.
The cube has been discussed before and the photodetector is treated. The detector must be sensitive for IR-light, and therefore a Germanium-photodiode has been choosen (model J16-P1-ϕ13, Judson).

The lock-in amplifier (Dynatrac lock-in amplifier, model 397EO, Ithaco) used in the set-up, has a built-in current preamplifier, so the photodetector has been directly connected to the lock-in amplifier.

The fiber used in the set-up is matched cladding single-mode fiber from Corning.

A general remark: the letters x, y, z, θ, φ, ψ in fig.B.9 stand for the possibility to adjust the corresponding device in that direction. For a definition of the directions see also fig.B.9.

**Extinction Ratio Deviated Beam**

The set-up for the measurement of the extinction ratio of the deviated beam is the same as above, save the position of ball-lens connector 4. This connector is placed in the deviated branch.

**Light Throughput as Function of Linear Input Polarization**

For the measurement of the light throughput in both output beams as function of the input polarization at the cube, we use a simpler set-up. The values of light power to be measured are much greater so the lock-in amplifier and the chopper can be replaced by an ordinary light power meter.
(model 88XL, Photodyne) (see fig.B.10).

Fig. B.10: Experimental set-up for the measurement of the light-throughput in both branches. The letters x, y, z, \( \theta \), \( \phi \), \( \psi \) stand for the possibility to adjust the corresponding device in that direction. The definition of the directions is indicated by the four figures below. From the left to the right:

Definition of \( X \), \( Y \), \( Z \) with respect to the plane of drawing.
Definition of \( \theta \) with respect to \( X \), \( Y \).
Definition of \( \psi \) with respect to \( X \), \( Z \).
Definition of \( \phi \) with respect to \( Y \), \( Z \).

**Beam Deviation**

The set-up is the same set-up as used for the measurement of the extinction ratio of the straight-through beam (see fig.B.9), with in addition an extra fiber and photodiode placed in the deviated branch.

**Insertion Loss**

The set-up is the same set-up as used for the measurement of the extinction ratio of the straight-through beam (see fig.B.9), with in addition an extra fiber and photodiode placed in the deviated branch.

### 3.3.2 Measurement Procedure

**Extinction Ratio Straight-Through Beam**

The alignment procedure will be first described. A serious problem is the alignment of the ball-lens connectors 3 and 4, because the coupling between the two connectors is rapidly degraded by an
angular misalignment (see paragraph 4.3). During the alignment procedure the polarizer is set at $\phi = 0^\circ$ (maximal throughput of the total system).

The first stage of the alignment procedure is to use visible HeNe-light. Ball-lens connector 3 is replaced by a ball-lens connector which passes HeNe-light and the other ball-lens connector is replaced by a diaphragm. The beam of the first connector is aimed through the diaphragm. This aiming is made precise by using diaphragms with smaller gaps. When this has been done the ball-lens connector with HeNe-light and the diaphragm are changed from position and the same actions are repeated.

The second stage is to use a graded-index fiber (GI) between ball-lens connectors 4 and 5; this on account of a higher numerical aperture of a GI-fiber in comparison with a single-mode fiber (SM). The output power is controlled to a maximum by adjustment of the orientation of ball-lens connector 4.

The last stage is to use SM-fiber between connectors 4 and 5 and to control the output power again to a maximum.

Now the polarizer is rotated until the minimal throughput of the total system. During this rotation the alignment of ball-lens connectors 3 and 4 is maintained. With the polarization control wheels the maximal throughput of the polarizer is maintained.

In the same way the maximal throughput of the total system has been measured. The ratio of the minimal to the maximal throughput gives the extinction ratio.

**Extinction Ratio Deviated Beam**

The procedure is analogous as above, except for one difference, during the alignment procedure the polarizer is set at $\phi = 90^\circ$ (maximal throughput for the deviated beam).

**Light-Throughput as Function of Linear Input Polarization**

The alignment procedure is analogous as above.

The adjustment of the input polarization at the cube operates as follows. This input polarization is controlled by means of the wheels and the polarizer. The polarizer is rotated to the desired direction of polarization and the wheels are controlled at maximal throughput of light at the polarizer. When this has been done the input polarization of the cube is linear in the direction of the polarizer. Only a linear polarization can be established at the input of the cube.

**Beam Deviation**

First the measurements were carried out with the cube between ball-lens connectors 3 and 4 (see fig.B.9). The angle of incidence was set to obtain the lowest extinction ratio in the deviated beam (see fig.B.12). After this, the polarization at the input of the cube was set parallel to the plane of incidence (p-polarized) by means of the polarizer and the wheels. This state of polarization can be recognized by detection of minimum signal in the deviated branch.

The set-up remains aligned throughout these adjustments.
The insertion loss of the cube was then measured.

The cube was then taken away, and the alignment was not recovered. As a consequence is an angular misalignment caused by the beam deviation of the cube occurred. The angular misalignment of course degrades the coupling between ball-lens connectors 3 and 4. The measurement of this loss was needed in the determination of the beam deviation (see paragraph 3.4.4).

**Insertion Loss**

The procedure for the measurement of the insertion loss of the straight-through beam is the same as the procedure, previously described except for one single modification; the alignment is recovered after the cube is taken away.

For the measurement of the insertion loss of the deviated beam, the input polarization of the cube is set perpendicular to the plane of incidence (s-polarized light). First the intensity of the straight-through beam is measured without cube. Then the intensity of the deviated beam is measured with the cube. The ratio in dB's is the particular insertion loss.
3.4 Results

3.4.1 Extinction Ratio Straight-Through Beam

This extinction ratio is rather independent of the angle of incidence (see fig.B.11). Since the angle of incidence is measured relatively, it's impossible to give the absolute angle of incidence with respect to the frontface of the cube. It's reasonable to assume that the angle of incidence for the next measurement is in the range $-1^\circ$ to $1^\circ$.

Extinction ratio $= 0.75 \times 10^{-3} = -31.2$ dB.

This is in good agreement with the theory (see fig.B.11).

---

Fig.B.11: The extinction ratio of the straight-through beam versus the angle of incidence.
3.4.2 Extinction Ratio Deviated Beam

In contrary with the ratio discussed above, the extinction ratio of the deviated beam is strongly dependent of the angle of incidence (see fig.B.12).

The angle of incidence has been measured relatively. The angle of incidence with the lowest extinction ratio has been taken zero, this corresponds with a horizontal shift of the experimental curve until it matches the theoretical curve.

Another way to match the two curves was the choice of the amount of layers (2m) and the greatest refractive index (\( N_H \)). These two quantities are not specified in the specification of the cube, so 'free' to choose.

The cube has been made of BK7 glass (see specification).

For BK7 glass: \( N_0 = 1.50 \) at \( \lambda = 1.50 \mu m \).

\( N_L \) is determined by formula (B.1).

The best choice of \( N_H \) and 2m is (also considering the influence at the extinction ratio of the straight-through beam):

\[
N_H = 1.598
\]

\[
2m = 36
\]

According to Melles Griot (the manufacturer of the cube) the total amount of layers is somewhere between 30 and 45. So the determined value of 36 is very reasonable.

Fig.B.12 shows that the two curves are matching quite well.

If we demand an extinction ratio of maximal \( 2\% \), it can be seen in fig.B.12 that the useful incident angular diameter is about \( 3^\circ \). This result is in contrary with the specifications (see paragraph 3.5), where a value of at least \( 10^\circ \) is ensured. This difference could be explained by the difference between
the measuring wavelength (\( \lambda = 1.503 \, \mu m \)) and the specified wavelength (\( \lambda = 1.550 \, \mu m \)).

Fig. B.12: The extinction ratio of the deviated beam versus the angle of incidence.

In fig. B.13 the extinction ratio is plotted once more as a function of the angle of incidence. The difference with fig. B.12 is the logarithmic scale for the extinction ratio. Now it can be seen that there are some deviations from the theory in the neighbourhood of \( \theta_i = 0^\circ \). This can be explained by the absorption of the cube which has been neglected in the theory. The best achievable extinction ratio seems to be about 40 dB.
Fig. B.13: The extinction ratio of the deviated beam in dB's versus the angle of incidence.
3.4.3 Light Throughput as Function of Input Polarization

Fig. B.14: The light-throughput of both branches as function of the angle of the linear input polarization ($= \phi$ for the definition of $\phi$ see fig. B.10).

If we look at fig. B.14 it strikes that the amplitudes of the sinusoidal functions representing the throughput of the deviated and the straight-through beam, are unequal. This is explained by an extra loss in the deviated branch, caused by a greater optical distance in the deviated branch. The difference between these two distances is about 4 cm. This represents an loss of
4 x 0,14 dB = 0,56 dB (see fig.B.20). In paragraph 4.2 is the relation between the coupling and the distance between two ball-lens connectors examined.

A deviation from the theory is also the horizontal shift (−5°) of the whole plot. This is probably a consequence of the fact that the direction of polarization is not correctly calibrated indicated on the holder of the polarizer. A reference polarizer was not available, so the last mentioned statement could not be verified.

### 3.4.4 Beam Deviation

Three measurements have been carried out for reproducibility. An average value is used for the calculation of the loss caused by the beam deviation. We obtain:

\[
\text{Loss} = 0,94 \text{ dB}
\]

With the help of fig.B.23 the beam deviation is determined:

\[
\text{Beam deviation} = 3,5 \text{ arcminutes}
\]

The result is not in the manufacturers specified range but lies very close (specifications (paragraph 3.5): beam deviation < 3 arcminutes).

The beam deviation was also directly measured. The spot of a visible HeNe-laser (λ = 633 μm) was projected onto a screen and the position marked. Then the displacement of the spot was measured with the cube placed in the beam. The measured displacement was 6,5 mm and the distance between the screen and the cube was 6,3 m. Therefore the beam deviation:

\[
\text{Beam deviation at } \lambda = 633 \text{ nm} = \frac{6,5 \times 10^{-3}}{6,3} (\text{rad}) = 1,0 \times 10^{-3} (\text{rad}) = 3,54 \text{ arcminutes}
\]

Our interest is the beam deviation at the wavelength λ = 1500 nm. The beam deviation calculated above, must be multiplied with the ratio of the refractive index of BK7-glass at λ = 1500 nm minus one to the index at λ = 633 nm minus one (see formula (B.29)).

\[
\text{Beam deviation at } \lambda = 1500 \text{ nm} = \frac{N_{1500} - 1}{N_{633} - 1} \times 3,54 \text{ arcminutes} = 3,52 \text{ arcminutes}
\]

### 3.4.5 Insertion Loss

Three measurements were made; the average of which were used in the calculation of the insertion loss of the straight-through beam. We find:

\[
\text{Insertion loss straight-through branch} = 1,8 \, \% = 0,08 \text{ dB}
\]

The loss of the deviated branch is determined by one measurement:
Loss = 4,9 % = 0,22 dB

The difference between the optical distance of the deviated and the straight-through branch is about 1 cm in air, giving an extra loss of about 0,14 dB (see ball-lens connector fig. B.20). The insertion loss of the deviated branch is found by subtracting the measured loss and the last mentioned extra loss, giving:

Insertion loss deviated branch = 0,22 dB - 0,14 dB = 0,08 dB

This result is in good agreement with the measured insertion loss in the straight-through branch. Theoretically the values should be equal to each other; since both branches contain two cube-air boundaries.

The reflection at the four faces of the cube is specified as less than 0,25 %, implying that the insertion loss is less than 0,50 % (two cube-air boundaries). There seems to be a contradiction between the measurements (1,8 %) and the specifications (paragraph 3.5). The difference may be wholly or partially attributable to the two other causes of loss, absorption and wavefront distortion. Another factor may be the difference between the measuring wavelength ( $\lambda = 1,503 \mu m$ ) and the specification wavelength ( $\lambda = 1,550 \mu m$ ), as the effectiveness of the AR-coating is strongly dependent on wavelength.
3.5 Specifications

SPECIFICATIONS: POLARIZING BEAM splitters
Dimensions: ±0.2mm

Principal Transmittance: >98% T for p-polarization;
<2% T for s-polarization
Principal Reflectance: >98% R for s-polarization;
<2% R for p-polarization.

Beam Polarization Purity (with unpolarized input):
98% pure
p-polarization in straight-through beam, 98% pure
s-polarization in 90° deviated beam

Transmission (ratio of straight-through output to total
unpolarized input) for single polarizer acting alone
\( \frac{1}{2}(k_1 + k_2) = 49\% \)

Open Straight-Through Transmission of pair
assuming unpolarized input: \( H_o = \frac{1}{2}k_1^2 + k_2^2 = 48\% \)

Extinction Ratio or closed straight-through transmission of
pair, assuming unpolarized input: \( H_o = k_1k_2 = 0.02 \)

Beam Deviation: < 3 arc minutes

Useful Angular Diameter: 10°

Entrance/Exit Surface Flatness: <λ/10 at 633nm
Wavefront Distortion: <λ/2 at 633nm

Cosmetic Surface Quality: 40-20 scratch and dig

Material: BK 7 Grade A Fine Annealed

AR Coating: ±0.25% R — all four faces

Unused faces are fine ground, and all edges are lightly beveled.

Fig. B.15: Specifications of the polarization splitter cube.
4.0 Ball-Lens Connector

4.1 Introduction

The ball-lens connector consists optically of a fibre endface placed at the focal point of a lens (see fig.B.16) made of BK7-glass with a diameter of 3 mm. The lens is spherically shaped so orientation problems are avoided. A disadvantage however is spherical aberration since a ball is not the ideal lens shape. A remaining justification problem is the distance between the ball-lens and the fibre endface. In fig.B.17 the solution to this problem is shown. The fibre endface is set against the edge of the metal holder (as shown) and the ball-lens is held in the right position w.r.t. the fibre endface by the spacer. In our case the distance between the lens and the fibre endface is equal to 727 μm with a tolerancy of 3 μm. A high degree of accuracy is needed because the coupling between two ball-lens connectors rapidly decreases with defocus.

![Diagram of Ball-Lens Connector](image)

A part of the holder is turned in a precision lathe, so that its mechanical axis coincides with the optical axis of the complete connector. The turned section is shown in fig.B.17 as the small outside diameter.

The formulas used to describe the coupling efficiency are taken from Tholen [B.4], where the ball-lens connector is elaborately described. The formulas are an approximation using the Gaussian
beam method. The method characterizes a beam by two parameters, the waist and the radius. The waist is taken as half the beam halfwidth at the narrowest point along the Gaussian beam, where the beamwidth in turn is defined at the 1/e-intensity points. The radius is related to the divergence. Introduction of a lens into a beam, causes the radius and waist to change. In the ball-lens configuration the radius and waist are both larger for the parallel beam (beam between the lenses) than for the highly divergent beam propagating from the fibre.

![Figure B.18: The Gaussian beams between two connectors.](image)

### 4.2 Distance between two connectors

#### 4.2.1 Theory

The beam between the lenses is characterized by a large radius and therefore is not very divergent, consequently the degradation by variation in the distance is not so critical. The next formula gives the quantitative relation between coupling and distance:

\[
T_{\text{distance}} \approx T_{\text{opt}} \frac{1}{1 + \frac{k^2 W_1^2 X^2}{16f^4}} \tag{B.32}
\]

Where

\[ f = 2239 \, \mu m = \text{focusing distance} \]

\[ k = \frac{2\pi N}{\lambda} = \frac{2\pi \times 1.5}{1.5 \times 10^{-6}} \]

\[ W_1 = \text{the waist of the light beam between lens and fibre endface.} \]

\[ X = \text{the distance between the connectors} \]

\[ T_{\text{opt}} = \text{the coupling efficiency at zero distance.} \]
The relation between coupling efficiency and loss in dB's is as follows:

\[ \text{Loss} = 10 \times \log_{10}(T) \]  

(B.33)

The loss in dB's is plotted against the distance in fig.B.20.

So far we have not considered the Fresnel reflection. The Fresnel reflection for an air-glass boundary is equal to 4 % of the incident light. In a connector there are a total of 6 air-glass boundaries (see fig.B.17). This corresponds to a loss of 6 x 4 % = 24 % = 1,08 dB. For the AR-coated lenses there are only Fresnel reflections at the fibre endfaces (the reflections at the lenses are neglected) this corresponds with a loss of 2 x 4 % = 8 % = 0,36 dB. The loss caused by spherical aberration is theoretically equal to 0,32 dB (see Tholen [B.4]). On account of the mechanical tolerances a value of 0,5 dB is practically achievable.

4.2.2 Experiments

![Diagram](image)

Fig. B.19: Experimental set-up for the measurement of the coupling efficiency. The letters x, y, z, θ, φ, ψ stand for the possibility to adjust the corresponding device in that direction. The definition of the directions is indicated by the four figures below. From the left to the right:

- Definition of X, Y, Z with respect to the plane of drawing.
- Definition of θ with respect to X, Y.
- Definition of ψ with respect to X, Z.
- Definition of φ with respect to Y, Z.

The set-up is shown in fig.B.19.
First the intensity of light coming out of ball-lens connector 1 is measured. After this ball-lens connectors 1 and 2 are aligned and the intensity of light coming out of ball-lens connector 3 is measured. The ratio of these two intensities is the coupling efficiency.

In order to examine the effect of AR-coating on the coupling efficiency, several lenses (both AR-coated and uncoated) were used.

In the first measurement ball-lens connectors 2 and 3 (see fig.B.19) were both uncoated.

In the second measurement, a new section of fibre was introduced, so that connector 2 was AR-coated and connector 3 was replaced by an uncoated normal ferrule (see curve A.R.1 in fig.B.20).

In order to examine the reproducibility a similar section of fibre was introduced into the system and the same measurement was repeated (see curve A.R.2 in fig.B.20).

### 4.2.3 Results

![Graph](image-url)

**Fig.B.20:** Loss versus distance between two connectors.
The slope of the theoretical curve was matched to that of the experimental curve NO A.R., allowing $W_1$ to be determined (see fig.B.20). In this manner $W_1$ was found to be equal to $4.65 \mu m$ which is in agreement with the value given in the technical note by Tholen ($W_1 = 4.5 \mu m$).

Fig.B.21: Plot of equal intensity contours from the light-spot at a single-mode fibre endface. The centre-point corresponds to 100 %. The inner circle corresponds to 80 %, the next circle to 60 % and so on.

Fig.B.22: Profile of the light-spot from the single-mode fibre endface across the lateral distance.
The waist $W_1$ was also measured directly. Using a IR-sensitive camera and a microscope objective an image of the fibre endface was formed which was then digitized. This digitized image was passed to a computer system. The contours of equal intensity are plotted in fig.B.21. From the digitized light-spot, a plot of intensity versus distance (intensity profile) is also obtained (see fig.B.22). From this profile the waist $W_1$ may be directly determined, at the 1/e-points (points at 37 % of maximum intensity). The diameter of the spot is about $8,2 \mu m$ so the waist $W_1$ is equal to $4,1 \mu m$. The difference between the theoretical value of $4,65 \mu m$ and the experimental value of $4,1 \mu m$ can be explained by the fact that the measurement resolution is about one wavelength (1.55 \mu m).

The slope of the theoretical curve (see fig.B.20) corresponds to a loss of 0.14 dB/cm. The vertical shift between the theoretical and experimental curves is explained by the extra loss caused by Fresnel reflection, absorption and spherical aberration of the lens. For uncoated lenses we get theoretically, 1.08 dB (reflections) + 0.50 dB (spherical aberration) = 1.58 dB. This is in agreement with the vertical shift observed in the plot (about 1.65 dB). For AR-coated lenses (the reflections at the surfaces are neglected; only reflections at the fibre endfaces are considered) we get 0.36 dB (reflections) + 0.50 dB (spherical aberration) = 0.86 dB which is in agreement with the loss measured at zero distance (see fig.B.20). The rather large difference between the two curves for connectors with AR-coated lenses is maybe caused by not equal thickness of the AR-coating on the sphere.

### 4.3 Angle between the two connectors

#### 4.3.1 Theory

The coupling between two connectors is very rapidly degraded by an angular misalignment which is shown in the plot of loss versus angle $\alpha$ (see fig.B.23). The coupling efficiency as a function of the angle can be calculated using the following formula:

$$T_{\text{angle}} \approx T_{\text{opt}} \exp \left(-k^2 W_2^2 \left(\frac{\alpha}{2}\right)^2\right) \quad (B.34)$$

where

$W_2$ = the waist of the lightbeam between the two lenses

$$k = \frac{2\pi N}{\lambda} = \frac{2\pi \times 1.5}{1.5 \times 10^{-6}}$$

$\alpha$ = the angle between the connectors.

$T_{\text{opt}}$ = the coupling efficiency without angular misalignment ($\alpha = 0^\circ$).
4.3.2 Experiments

The set-up is shown in fig.B.19. First the distance between the connectors was set to about 5 cm and the connectors were aligned. The greater the distance, the more precise the angular alignment. The connectors were then brought together while keeping the alignment. After the procedure the set-up was ready for the measurement.

4.3.3 Results

![Graph showing loss versus angle between connectors.]

The experimental curve is matched to the theoretical curve by the parameter $W_2$. These two curves are plotted in fig.B.23. We find $W_2 = 140 \mu m$. This value is verified by direct measurement of $W_2$. 

in a similar manner as before.

Fig.B.24: Plot of equal intensity contours from the light beam from a lens connector. The inner-most contour represents maximum intensity, the next contour 80% and so on.
Fig. B.25: Profile of the light beam from a lens connector across the lateral distance.

Using fig. B.24 and fig. B.25 the diameter of the spot at $1/e = 37\%$ of the maximal intensity equals about $300\,\mu m$ corresponding to a waist $W_2$ of $150\,\mu m$. This is in agreement with the value of $W_2'(140\,\mu m)$ obtained by curve matching.
5.0 Polarization Maintaining Fibre (PMF)

5.1 Introduction

An optical fibre exhibits a linear retardation between the two modes $\psi$ which is proportional to the fibre length $L$ and inversely proportional to the wavelength in vacuum:

$$\psi = (\beta_x - \beta_y)L = \frac{2\pi}{\lambda_0} \left( \frac{1}{N_x} - \frac{1}{N_y} \right)L$$  \hspace{1cm}  (B.35)

The fibre length corresponding to a retardation of $2\pi$ (acting like a full wave plate) is called the beat length $L_B$:

$$L_B = \frac{2\pi}{\beta_x - \beta_y}$$  \hspace{1cm}  (B.36)

Typical single-mode fibres are found to have beat lengths of a few centimeters. The YORK-PMF has a beat length of typically 1.3 mm, max. 2.0 mm (see specifications).

In a non-perfect fibre various perturbations along the fibre length such as strain or variation in fibre geometry and composition lead to coupling from one polarization to the other. This process is called cross-coupling.

For isolated perturbations, the cross-coupled power is reduced by increasing the fibre birefringence. If the individual perturbations are distributed along the fibre with a period $\Omega$, the energy transfer is at a maximum when this period is equal to the beat length. An arbitrarily distributed perturbation along the fibre can be described by some perturbation function which can be transformed into a perturbation spectrum (the spatial frequency dependence of the strength of the perturbations). This spectrum decreases rapidly over the period range $1 \text{ mm} < \Omega < 5 \text{ mm}$ [B.5]. Thus, a high-birefringence fibre with $L_B < 1 \text{ mm}$ could be expected to preserve the state of light polarized along one of its eigenmodes, even in the presence of normal perturbations. This is the basis of the polarization maintaining fibre.

The term 'polarization maintaining' does not mean that the two parameters (the power ratio of the two modes and the phase between the two modes) which completely describe the state of polarization, do not change with the fibre length. Only one parameter, the power ratio, is preserved. The other parameter, the phase, is strongly dependent on change in temperature and mechanical stress.

High-birefringence is induced in the fibres by exploiting the thermal stress produced by high-expansion regions incorporated within the cladding. The shape of these regions is for the YORK-PMF like a bow-tie as shown in fig.B.26. The YORK-PMF has an elliptical core to increase the material birefringence.

The polarization preserving capability is measured in terms of the cross-coupling.
5.2 Measurement of Cross-Coupling and Orientation

5.2.1 Procedure and Setup

The procedure of the measurement is performed as follows. First, linearly polarized light with an arbitrary angular direction is coupled into the PMF. Accordingly, the ellipticity of the output polarization is measured using an analyser (polarizer). With the knowledge of the phase difference between the two modes, the ellipticity can be directly related to the power ratio of the two modes. If the phase difference is equal to $\pi/2$ the minor axis and the major axis of the ellipse coincide to the principal axes of the fibre, the ellipticity corresponds to the power ratio of the two modes.

From the measurement of the power ratio as a function of the input polarization angle, the orientation of the principal axes at the input and the cross-coupling of the YORK-PMF can be determined. The input polarization angle giving maximum power ratio corresponds to the fibre orientation angle; the cross-coupling is equal to maximal power ratio.

The phase difference at the output is dependent upon the retardation of the birefringent PMF which varies with temperature and mechanical stress.

The retardation so the phase difference at the output is inversely proportional to the wavelength (see formula (B.35)). Hence, with a large wavelength spread as in the case of a broadband source, the phase difference can assume every value in the interval $[0,2\pi]$.

When the analyser (output polarizer) is placed at an angle $\phi$ with respect to the x-axis (one of the principal axes of the fibre), the passed-through intensity is equal to (see formula (2.10)):

$$ |E_\phi|^2 = |E_x|^2 \cos^2 \phi + |E_y|^2 \sin^2 \phi + |E_x||E_y| \sin 2\phi \cos \delta $$  \hspace{1cm} (B.37)

where $\delta$ is the phase difference between the two modes.
In the case of a broadband source, $\delta$ is a stochastic variable uniformly distributed over the interval $[0,2\pi]$. The following integral has to be calculated:

$$|E_{\phi}|^2 = \frac{1}{2\pi} \int_0^{2\pi} (|E_x|^2 \cos^2 \phi + |E_y|^2 \sin^2 \phi + |E_x| E_y \sin 2\phi \cos \delta) d\delta$$  

$$= |E_x|^2 \cos^2 \phi + |E_y|^2 \sin^2 \phi$$  

(B.38)

The situation is analogous to the situation where $\delta$ is a deterministic variable with a value $\pi/2$. Hence the ratio of the minor axis to the major axis corresponds to the cross-coupling. The question is: what is the minimal bandwidth of the source, needed for the measurements? It is rather easy to give a rough estimate of the minimal bandwidth needed. The knowledge of the beat length (about 1.3 mm) and the length of the measurement fibre (1.5 m) allows us to calculate the particular quantity.

The absolute phase retardation $\psi_{abs}$ is equal to $(1500/1,3)\times 2\pi = 2308\pi$. The retardation is inversely proportional to the wavelength (eq (B.35)), so a relative wavelength change $\delta \lambda/\lambda_0$ causes the same relative change of the phase retardation in the opposite direction. A retardation of $2\pi$ is necessary, so the relative wavelength change equals:

$$\frac{\delta \lambda}{\lambda_0} = \frac{2\pi}{2308\pi} = 8,7 \times 10^{-4}$$

corresponding to an absolute wavelength change of:

$$\delta \lambda = 1,3 \text{ nm}$$

Conclusion: a source (LED, FP-laser) with a bandwidth of a few nm's satisfies the broadband criterium.

When a narrow bandwidth source is used in the measurement set-up, the cross-coupling is over-optimistic, because the maximum and minimum of the ellipse determined with an analyser do not correspond in general to the fibre axes.

There is another condition which must be considered, the combined source and detector spectral response must be in the single-mode region of the fibre (for the YORK-PMF $> 1500$ nm, see specifications).
Fig. B.27: Set-up for the measurement of the cross-coupling.

The source used in the set-up is a LED with a central wavelength at 1550 nm and a bandwidth defined at half maximum equal to 100 nm.

5.2.2 Results

For every angle of input polarization the ellipticity of the output polarization is determined. The power ratio of the ellipse versus the angle of input polarization is shown in fig. B.28.

Assuming no cross-coupling the ratio can easily be obtained theoretically.

The ratio of the amplitudes of the electric vectors corresponding to the principal axes (at the input):

\[
\frac{|E_y|}{|E_x|} = \tan \theta_i
\]  \hspace{1cm} (B.39)

where \( \theta_i \) is the input polarization angle.

The power ratio of the axes:

\[
\frac{|E_y|^2}{|E_x|^2} = \tan^2 \theta_i
\]  \hspace{1cm} (B.40)

The power ratio in dB's:

\[
\text{power ratio} = 10 \log_{10}(\tan^2 \theta_i)
\]  \hspace{1cm} (B.41)

Since there is no cross-coupling assumed, the power ratio at the output is the same as the power ratio at the input.

Eq. (B.41) represents the theoretical curve plotted in fig. B.28.

The difference between the theory and the experimental results can be explained by the occurrence of cross-coupling.
The sections of the plot with the highest power ratio are the regions of most interest, hence one of the sections is measured and plotted with a higher resolution of polarization input angle (see fig. B.29).

The cross-coupling ratio is defined as the ratio of the transversed power to the remaining power in the original mode on the account that the input polarization is linear along one of the principal axes. Hence, the cross-coupling ratio is equal to the maximal power ratio. The cross-coupling ratio of the YORK-PMF with a length of 1.5 m is found to be about 16 dB (see fig. B.29). The specifications (paragraph 5.3) mention a value of 20 dB over 1 km fibre for the cross-coupling ratio. A possible cause of the difference between the specifications and the experimental results is the appearance of cracks in the stress-applying elements of the PMF-fibre as consequence of polishing and glueing the fibre in the ferrule.

The orientation of the fibre for achieving the maximal cross-coupling ratio is not so critical, 5 degrees deviation from the optimum results in a degradation of the cross-coupling ratio of 1 dB. The orientation of the fibre axes can be better found by considering the sharp minimum instead of the flat maximum (see fig. B.28).

![Figure B.28](image-url)  
Fig. B.28: The power ratio at the output versus the input polarization angle for the YORK-PMF of 1.5 m.
Fig. B.29: The power ratio at the output versus the input polarization angle for the YORK-PMF of 1.5 m. The resolution of the angle is smaller than in fig. B.28.
### 5.3 Specifications

<table>
<thead>
<tr>
<th>Fiber type:</th>
<th>HB 450</th>
<th>HB 600</th>
<th>HB 750</th>
<th>HB 800</th>
<th>HB 1250</th>
<th>HB 1500</th>
</tr>
</thead>
<tbody>
<tr>
<td>Operating wavelengths $\lambda_{op}$ (nm):</td>
<td>488, 514</td>
<td>633</td>
<td>780</td>
<td>830</td>
<td>1300</td>
<td>1550</td>
</tr>
<tr>
<td>Cut-off wavelengths (nm):</td>
<td>&lt;488</td>
<td>&lt;600</td>
<td>&lt;750</td>
<td>&lt;800</td>
<td>&lt;1250</td>
<td>&lt;1500</td>
</tr>
<tr>
<td>Attenuation at $\lambda_{op}$ (dB/km):</td>
<td>&lt;100</td>
<td>&lt;12</td>
<td>&lt;8</td>
<td>&lt;5</td>
<td>&lt;2</td>
<td>&lt;2</td>
</tr>
</tbody>
</table>

Other specifications:

- Polarization cross-coupling** (or extinction ratio): -20dB over 1km (typical)
- Beatlength at 633 nm: typical 1.3mm, max. 2mm
- Fiber diameter:
  - all types: 125 µm ± 3 µm (r.m.s. variation < 1 µm)
  - HB 800 also available with: 80 µm ± 3 µm
- Coating diameter: 220 µm (nominal)
- Coating type: mode stripping acrylate
- Core diameter: 2-8 µm depending on wavelength
- Core refractive index difference: 0.01 nominal

* Typical operating wavelength shown. As with any singlemode fiber, a broad operating window of about 200nm is available provided care is taken to avoid dual mode effects near the cut-off wavelength and bend losses at long wavelengths.

Fig.B.30: The specifications of the PMF-fibre.

Note: the PMF-fibre HB1500 is used.
6.0 50/50 beamsplitter plate

The beamsplitter plate is used in the mounting of fig.B.31. The angle of incidence is 15°.

![Fig.B.31: Mounting of the beamsplitter plate.](image)

A: V-groove  
B: lensconnector  
C: beamsplitter plate

The measured values for transmittance and reflectance of the p- and s-component are given by table B.2.

<table>
<thead>
<tr>
<th>Component</th>
<th>Transmitted</th>
<th>Reflected</th>
</tr>
</thead>
<tbody>
<tr>
<td>s-component</td>
<td>53 %</td>
<td>47 %</td>
</tr>
<tr>
<td>p-component</td>
<td>53 %</td>
<td>47 %</td>
</tr>
</tbody>
</table>

Table B.2: Transmittance and reflectance of the beamsplitter plate.
7.0 Photodiode and I-V amplifier

The photodiode (J16-P1) is manufactured by Judson.
The specifications and dimensions are depicted in fig.B.32.

---

<table>
<thead>
<tr>
<th>Standard Sizes</th>
<th>10mm</th>
<th>13mm</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Peak Response</td>
<td>1.5</td>
<td></td>
<td>µ</td>
</tr>
<tr>
<td>Responsivity</td>
<td>.65</td>
<td></td>
<td>A/W</td>
</tr>
<tr>
<td>Dark Current (-1V)</td>
<td>0.5</td>
<td>1</td>
<td>mA</td>
</tr>
<tr>
<td>Breakdown Voltage</td>
<td>2</td>
<td>2</td>
<td>V</td>
</tr>
<tr>
<td>NEP x 10^-12</td>
<td>12</td>
<td>16</td>
<td>W Hz 1/2</td>
</tr>
<tr>
<td>Cutoff Frequency</td>
<td>500</td>
<td>200</td>
<td>kHz</td>
</tr>
<tr>
<td>Capacitance²</td>
<td>30</td>
<td>50</td>
<td>nF</td>
</tr>
<tr>
<td>Impedance RD min.</td>
<td>400</td>
<td>250</td>
<td>Ω</td>
</tr>
</tbody>
</table>

---

Fig.B.32: Specifications and dimensions of the photodiode.

The electrical circuit of fig.B.33 is a transconductance amplifier, which amplifies the photocurrent and converts it to a voltage. The amplification factor can be adjusted using the potentiometer of 1MΩ.
An offset voltage at the input of the opamp causes an erroneous current through the impedance $R_D$ of the photodiode ($= 250 \, \Omega$). The current equals,

$$I_{\text{erroneous}} = \frac{V_{\text{offset}}}{R_D} \quad (B.42)$$

The minimal detectable power is about $1 \, \mu W$, the corresponding photocurrent equals,

$$I_{PD} = \text{Responsivity} \times \text{Lightpower} = 0.65 \, (\Lambda/\mu W) \times 1(\mu W) = 0.65 \, \mu A \quad (B.43)$$

To neglect the erroneous current, it must be an magnitude smaller than the photocurrent. Therefore the input offset voltage of the opamp must be smaller than about $10 \, \mu V$. Considering the specifications of the opamp OP-07A (fig.B.34), manufactured by PMI, the opamp has a typical input offset voltage of $10 \, \mu V (25 \, \mu V \text{ max.})$. The input offset voltage may be nulled by a variable resistor between pins 1 and 8 of the opamp. Consequently, this opamp satisfies the requirement.

The input offset current of the opamp equals $0.3 \, \text{nA} (2.0 \, \text{nA max.})$ and can be neglected with respect to the minimal photocurrent of $0.65 \, \mu A$. 

Fig.B.33: Transconductance amplifier.
**ELECTRICAL CHARACTERISTICS** at $V_0 = \pm 15V$, $T_A = 25^\circ C$, unless otherwise noted.

<table>
<thead>
<tr>
<th>PARAMETER</th>
<th>SYMBOL</th>
<th>CONDITIONS</th>
<th>MIN</th>
<th>TYP</th>
<th>MAX</th>
<th>UNITS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Input Offset Voltage</td>
<td>$V_{os}$</td>
<td>(Note 1)</td>
<td>$-10$</td>
<td>$0$</td>
<td>$10$</td>
<td>nV</td>
</tr>
<tr>
<td>Long-Term Input Offset</td>
<td>$\Delta V_{os}/\text{Time}$</td>
<td>(Note 2)</td>
<td>$0.2$</td>
<td>$1.0$</td>
<td></td>
<td>µV/Ms</td>
</tr>
<tr>
<td>Input Offset Current</td>
<td>$I_{os}$</td>
<td></td>
<td>$0.3$</td>
<td>$2.0$</td>
<td></td>
<td>nA</td>
</tr>
<tr>
<td>Input Bias Current</td>
<td>$I_B$</td>
<td></td>
<td>$0.7$</td>
<td>$2.0$</td>
<td></td>
<td>nA</td>
</tr>
<tr>
<td>Input Noise Voltage</td>
<td>$R_{n}$</td>
<td>$0.1$ to $10$ Hz</td>
<td>$3$</td>
<td>$0.36$</td>
<td>$0.8$</td>
<td>µV/s</td>
</tr>
<tr>
<td>Input Noise Voltage Density</td>
<td></td>
<td>$100$ Hz</td>
<td>$3$</td>
<td>$10.0$</td>
<td>$13.0$</td>
<td>µV/√Hz</td>
</tr>
<tr>
<td>Power Supply Rejection Ratio</td>
<td>$PSRR$</td>
<td></td>
<td>$4$</td>
<td>$10$</td>
<td></td>
<td>µV/V</td>
</tr>
<tr>
<td>Output Voltage Swing</td>
<td>$V_O$</td>
<td>$R_1 \geq 1k\Omega$, $V_O = \pm 12V$</td>
<td>$300$</td>
<td>$500$</td>
<td></td>
<td>V/MV</td>
</tr>
<tr>
<td>Power Consumption</td>
<td>$P_O$</td>
<td>$V_0 = \pm 15V$, No Load</td>
<td>$75$</td>
<td>$120$</td>
<td></td>
<td>mW</td>
</tr>
<tr>
<td>Offset Adjustment Range</td>
<td>$R_O = 20k\Omega$</td>
<td></td>
<td>$54$</td>
<td></td>
<td></td>
<td>mV</td>
</tr>
</tbody>
</table>

**NOTES:**

1. OP-07A grade $V_{os}$ is measured approximately one minute after application of power. For all other grades $V_{os}$ is measured approximately 30 seconds after application of power.

2. Long-Term Input Offset Voltage Stability refers to the averaged input line of $V_{os}$ in Time over extended periods after the first 30 days of operation.

3. Sample tested

4. Guaranteed by design

Fig. B.34: Specifications of the opamp OP07A.
8.0 ADC- and DAC-cards

ADC

---

Dual Analog to Digital Convertor

-------------------------------

Based on:
- AD571 (Analog Devices)

IC description:
- Analog to Digital Convertor
- Conversion time: 30 usec
- Resolution: 10 bits, 10 mV/step
- Input voltage: Unipolar: 0 V .. 10.23 V
  Bipolar: -5.12 V .. 5.11 V

Card description:
This card consists of two converting circuits. Each circuit converts an analog input signal between 0 V and 10.23 V (unipolar) or between -5.12 V and +5.11 V (bipolar) into a digital 10 bits output signal which can be read by the Apple.

DAC

---

Dual Digital to Analog Convertor

-------------------------------

Based on:
- AD561 (Analog Devices)

IC description:
- Digital to Analog convertor
- Settling time (to 0.5 LSB): 250 nsec
- Resolution: 10 bits, 10 mV/step
- Output voltage: Unipolar: 0 V .. 10.23 V

Card description:
This card consists of two converting circuits. Each circuit converts a 10 bits digital input signal, produced by the Apple, into an analog output signal between 0 V and 10.23 V. The settling time is 1 msec.
Also situated on the DAC card is a reed relay which can be switched on or off.

Fig.B.35: Specifications of the ADC- and DAC-cards.
9.0 Hi-Vo amplifier

Fig.B.36: High-voltage amplifier.

The central part of the high-voltage amplifier is the opamp BB3584 manufactured by Burr-Brown (fig.B.36). The opamp is used as a non-inverting amplifier and the amplification factor equals 13, which is determined by the values of the resistances of the feedback loop.

The output of the amplifier is loaded with a piezoelectric squeezer which can be represented by a capacitor of about 0.8 µF (see specifications squeezer). To limit the output current of the amplifier, an output resistor of 1 kΩ is used. Accordingly, the time constant τ of the amplifier can be calculated,

$$\tau = RC = 1 \times 10^3 \times 0.8 \times 10^{-6} = 0.8 \text{ ms} \quad (B.44)$$

The slew rate SR of the amplifier is determined by the maximal output current (= 25 mA) and the load (= 0.8 µF).

$$SR = \frac{25 \times 10^{-3}}{0.8 \times 10^{-6}} \frac{V/s}{0.3 \mu s} = 0.3 \frac{V}{\mu s} \quad (B.45)$$
## SPECIFICATIONS

### ELECTRICAL

Typical at 25°C and ±5V, max unless otherwise noted

<table>
<thead>
<tr>
<th>MODELS</th>
<th>3584JM</th>
</tr>
</thead>
</table>

#### POWER SUPPLY
- Voltage, 2V:
  - ±70 to ±150 VDC
- Quiescent Current, max: 25.5 mA

#### RATED OUTPUT
- Voltage, ±1 V ±5VDC, min: ±85 to ±145 VDC
- Current, max: 3.15 mA
- Current, Short Circuit: ±253 mA
- Load Capacitance, max: 10 nF

#### OPEN LOOP GAIN
- No Load, DC: 120 dB
- Rated Load, DC, min: 100 dB

#### FREQUENCY RESPONSE
- Unity Gain Bandwidth: ±0.1 Hz
- Frequency Bandwidth, G = 100: 1.35 kHz
- Gain Bandwidth: ±0.1 Hz
- Bandwidth: ±5 MHz
- Phase Margin: ±10°
- Power Output: ±0 dB
- Phase Margin: ±10°
- Bandwidth: ±5 MHz
- Phase Margin: ±10°
- Bandwidth: ±5 MHz

#### INPUT OFFSET VOLTAGE
- Initial @ 25°C, max: ±3 mV
- Drift vs Temp, max: ±25 μV/°C
- Drift vs Supply Voltage: ±20 μV/V
- Drift vs Time: ±50 μV/mo

#### INPUT BIAS CURRENT
- Initial @ 25°C, max: ±20 μA
- Drift vs Temp: doubles every 10°C
- Drift vs Supply Voltage: 0.2 μA/V
- Drift vs Supply Voltage: 0.2 μA/V

#### INPUT OFFSET CURRENT
- Initial @ 25°C, max: ±30 pA
- Drift vs Temp: doubles every 10°C
- Drift vs Supply Voltage: 0.2 pA/V
- Drift vs Supply Voltage: 0.2 pA/V

#### INPUT IMPEDANCE
- Differential: 10/Ω ± 10 pF
- Common Mode: 10/Ω

#### INPUT NOISE
- Voltage: 0.01 Hz to 10 Hz p-p: ±3 μV
- Current: 0.01 Hz to 10 Hz p-p: ±0.3 μA

#### INPUT VOLTAGE RANGE
- Max Safe Differential Voltage:
  - ±(Vcc + 1.2 Vcc)
- Max Safe Common Mode Voltage: ±4 Vcc
- Common Mode Voltage, Linear Operation: ±(Vcc + 10 VCC)
- Common Mode Rejection: ±110 dB

#### TEMPERATURE RANGE (Case)
- Operating: 0°C to 70°C
- Storage: -55°C to +125°C

---

### MECHANICAL

#### TO-3 CONNECTOR
- 2.54 mm (0.1")
- 39.62 mm (1.56")
- 0.10 mm (0.004")
- 0.63 mm (0.025")
- 0.85 mm (0.033")

---

**Fig. B.37: Specifications of the opamp BB3584.**
10.0 Alignment Apparatus

The angular alignment in one direction can be realized with two screws and a spring, which pushes the ball-lens connector against the screws. The principle of operation is illustrated in fig.B.38.

![Diagram](image)

Fig.B.38: The operation of the alignment apparatus.

By turning one of the screws the angle of the connector can be changed. Another effect of turning the screw is to introduce a lateral displacement of the connector. The angular and lateral alignment therefore cannot be split into two independent actions using this scheme. In practice, the angular alignment is achieved quite easily, since the coupling between two connectors is much more sensitive to angular misalignment than to lateral misalignment.

By turning both the screws at the same rate a lateral translation is performed. In this way lateral alignment can be achieved.

The angular and lateral alignment perpendicular to the plane of drawing (fig.B.38) is realized in the same way by two other screws.

The alignment apparatus uses the section of the connector (larger diameter) for which the optical and the mechanical axes don't coincide. A consequence is that the connector cannot removed from the alignment apparatus and replaced without loss of alignment.

A detailed drawing of the alignment apparatus is given in fig.B.39.
Fig. B.39: Detailed drawing of the alignment apparatus.
11.0 References


APPENDIX C

LISTING COMPUTER PROGRAM
PROGRAM POLCON;
USES LABI01;
VAR  REF, VS1, VS2, VS1STP, VS2STP, VS2PER, VS2RES : INTEGER;
     PD, PD1, SLOPE, DIFF, MAXDIF : INTEGER;
     COUNT, COUNT2, WAIT, LAMBDA, SETTLE : INTEGER;
     DIR, T, TP, TM, TMAX, STEPMI, LOSS : INTEGER;
     MINSTP, MAXSTP, DIVSTP : INTEGER;
     ERROR : BOOLEAN;
PROCEDURE INITIATE;
BEGIN
  WRITELN('REFERENCE VALUE (REF)');
  READLN(REF);
  WRITELN('GRADIENT MULTIPLICATOR (LAMBDA)');
  READLN(LAMBDA);
  WRITELN('SETTLE TIME=');
  READLN(SETTLE);
  WRITELN('VS2 PERIOD');
  READLN(VS2PER);
  WRITELN('VS2 RESET VOLTAGE');
  READLN(VS2RES);
  WRITELN('VS2 STEP DURING RESET');
  READLN(VS2STP);
  WRITELN('VS1STP=');
  READLN(VS1STP);
  WRITELN('MAXIMUM LOSS');
  READLN(LOSS);
  WRITELN('MAXDIF=');
  READLN(MAXDIF);

  MINSTP:=1;
  MAXSTP:=64;
  DIVSTP:=4;
END;
PROCEDURE ACQUIS(VAR ERROR:BOOLEAN);
VAR STEP, DIR, I, SIGN :INTEGER;

BEGIN
STEP:=MAXSTP;
VS2:=512;
WRTDAC(VS2,1,3);
FOR WAIT:=1 TO 1000 DO;
PD:=RDADC(0,4);
DIR:=1;
ERROR:=FALSE;
WHILE (STEP) MINSTP) AND (NOT ERROR) DO
BEGIN
I:=0;
SIGN:=-1;
IF PD>REF THEN SIGN:=1;
WHILE (I<8) AND (PD*SIGN) REF*SIGN) DO
BEGIN
VS2:=VS2+DIR*STEP;
WRTDAC(VS2,1,3);
FOR WAIT:=1 TO 10*STEP DO;
PD:=RDADC(0,4);
WRITELN('VS2=',VS2,' PD=',PD);
I:=I+1;
END;
DIR:=-DIR;
STEP:=STEP DIV DIVSTP;
WRITELN(STEP);
IF I=8 THEN ERROR:=TRUE;
END;
END;

PROCEDURE PSLOPE(DELTA:INTEGER);
BEGIN
WRTDAC(VS2+DELTA,1,3);
FOR WAIT:=1 TO 50 DO;
PD1:=RDADC(0,4);
WRTDAC(VS2,1,3);
FOR WAIT:=1 TO 500 DO;
PD:=RDADC(0,4);
SLOPE:=PD1-PD;
END;

PROCEDURE CONTROL;
VAR STEP :INTEGER;
BEGIN
WRTDAC(VS2,1,3);
FOR WAIT:=1 TO SETTLE DO;
PD:=RDADC(0,4);
DIFF:=PD-REF;
STEP:=-LAMBDA*DIFF DIV SLOPE;
VS2:=VS2+STEP;
END;
PROCEDURE STRETCH;
BEGIN
PD1:=RDADC(0, 4);
DIFF:=0;
T:=0;
WHILE (DIFF<LOSS) DO
BEGIN
WRTDAC(VS2-MINSTP*T, 1, 3);
FOR WAIT:=1 TO SETTLE DO;
PD:=RDADC(0, 4);
DIFF:=ABS(PD-PD1);
T:=T+1;
END;
WRTDAC(VS2, 1, 3);
FOR WAIT:=1 TO SETTLE DO;
END;

PROCEDURE RESET;
BEGIN
REPEAT
VS2:=VS2-VS2STP;
WRTDAC(VS2, 1, 3);
FOR WAIT:=1 TO SETTLE DO;
UNTIL (VS2<(VS2RES-VS2PER));
END;

PROCEDURE RESTABIL;
BEGIN
IF VS1>512 THEN
BEGIN
REPEAT
REPEAT
CONTROL;
UNTIL (ABS(DIFF)<MAXDIF);
VS1:=VS1-VS1STP;
UNTIL (VS1<512);
END
ELSE
BEGIN
REPEAT
REPEAT
CONTROL;
UNTIL (ABS(DIFF)<MAXDIF);
VS1:=VS1+VS1STP;
UNTIL (VS1>512);
END;
END;
BEGIN
INITIATE:

VS1:=512;
WRDAD(VS1,1,2);
FOR WAIT:=1 TO SETTLE DO;

ACQUIS(ERROR);
IF ERROR THEN WRITELN('ERROR');

PSLOPE(5);
IF SLOPE=0 THEN SLOPE:=1;

DIFF:=0;
REPEAT
   CONTROL;
UNTIL (VS2>VS2RES);

WRITELN('RESET');

TM:=0;
DIR:=1;
REPEAT
   VS1:=VS1+DIR*VS1STP;
   WRDAD(VS1,1,2);
   FOR WAIT:=1 TO SETTLE DO;
   REPEAT
      CONTROL;
   UNTIL (ABS(DIFF)<MAXDIF);
   SHIFT;
   IF (T<=TM) THEN DIR:=-DIR;
   TM:=T;
UNTIL (T>TMAX);
RESET;
RESTART;
END.
APPENDIX D

SOLEIL-BABINET COMPENSATOR
Soleil-Babinet Compensator

The Soleil-Babinet compensator can be thought of as a continuously adjustable retardation plate. It is possible to achieve any desired relative retardation by simply turning a micrometer screw. The Soleil-Babinet Compensator (SB-10, Optics for Research) has a digital read-out which is proportional to the retardation exerted by the compensator. The proportional constant has to be determined. In other words, the retardation per unit digital read-out is measured, this quantity is dependent on wavelength. The retardation $\psi$ of a birefringent material is inversely proportional to the wavelength in vacuum $\lambda_e$ as shown by the next equation,

$$\psi = \frac{2\pi}{\lambda_e} \left( \frac{1}{N_x} - \frac{1}{N_y} \right) L \quad (D.1)$$

where $N_x$ is the refractive index in the x-direction.
$N_y$ is the refractive index in the y-direction.
$L$ is the length of the lightpath through the birefringent material.

For the Soleil-Babinet compensator (SBC) we get:

$$\psi = \frac{C_{SBC}}{\lambda_e} R + \psi_0 \quad (D.2)$$

where $C_{SBC}$ is the proportional constant.
$R$ is the digital read-out of the SBC.
$\psi_0$ is the retardation by the SBC when the digital read-out indicates zero.

The relationship between wavelength and retardation is investigated using two laser sources operating at different wavelengths (1,301 $\mu m$ and 1,503 $\mu m$). The set-up used for the measurements is shown in fig.D.1.
Fig.D.1: The set-up for the calibration of the SBC.

The angle between the polarizer and the SBC is equal to 45°.

In general, the output power of a polarizer oriented at an angle \( \phi \) w.r.t. the x-axis, is equal to (see eq. (2.10)):

\[
|E_{\text{out}}|^2 = |E_x|^2 \cos^2 \phi + |E_y|^2 \sin^2 \phi + |E_x||E_y| \sin 2\phi \cos \delta
\]  
(\text{D.3})

where \( \delta \) is the phase difference between the two modes before the polarizer.

The directions of the principal axes are chosen so that they coincide with the retardation axes of the SBC. Hence, the effect of the SBC is only a retardation of the modes. Accordingly, the angle of the polarizer w.r.t. the x-axis, one of the principal axes, is equal to 45°.

Using \( \phi = 45^\circ \), eq. (D.3) becomes:

\[
|E_{\text{out}}|^2 = \frac{1}{2} |E_x|^2 + \frac{1}{2} |E_y|^2 + |E_x||E_y| \cos \delta
\]  
(\text{D.4})

At the beginning of the measurements there is established minimal throughput of the system (see fig.D.1) using the polarization controle wheels, so that the polarization at the input of the polarizer is linear characterized by a direction orthogonal to the polarization axis of the polarizer (45° w.r.t. the x-axis). The total of retardation introduced by the wheels and the SBC is consequently equal to 180°. The amplitudes \( |E_x| \) and \( |E_y| \) are equal to each other.

With \( |E_x| = |E_y| = \frac{1}{\sqrt{2}} |E_{\text{in}}| \), we get:

\[
|E_{\text{out}}|^2 = \frac{1}{2} |E_{\text{in}}|^2 (1 + \cos \delta)
\]  
(\text{D.5})

Using eq. (D.2), eq. (D.5) becomes:
\[
|E_{\text{out}}|^2 = \frac{1}{2} |E_{\text{in}}|^2 \left( 1 + \cos \left( \frac{C_{\text{SBC}} R}{\lambda_o} + \pi \right) \right)
\]  

(D.6)

The experimental determined relationship between the output power \(|E_{\text{out}}|^2\) and the digital read-out \(R\) is plotted in fig.D.2.

![Graph](image)

\(\lambda_o = 1.301 \mu m\)

\(\lambda_o = 1.503 \mu m\)

Fig.D.2: Output power versus the setting of the SBC.

Dividing the output power by the input power \(|E_{\text{in}}|^2\), subtracting one, and taking the arcsine, gives:

\[
\arcsin \left( \cos \left( \frac{C_{\text{SBC}}}{\lambda_o} R + \pi \right) \right) = \frac{C_{\text{SBC}}}{\lambda_o} R - \frac{\pi}{2}
\]

(D.7)

The straight lines corresponding to eq. (D.7) are plotted in fig.D.3.
Fig. D.3: Arcsine of the normalized output power versus the setting of the SBC.

The following values for $C_{SBC}$ can be deduced from the plot:

$C_{SBC} = 2.54 \times 10^{-8}$ for $\lambda_0 = 1.503 \mu m$

$C_{SBC} = 2.56 \times 10^{-8}$ for $\lambda_0 = 1.301 \mu m$

The difference between those values can be explained by the difference between the refractive index at $\lambda_0 = 1.301 \mu m$ and the refractive index at $\lambda_0 = 1.503 \mu m$.

In this appendix only the relative scale of the SBC is determined, for the knowledge of the absolute retardation caused by the SBC, other measurements should be carried out.