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Periodic persistent currents in nano-volcanoes

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Periodic persistent currents in nano-volcanoes

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Summary

Recently it has been shown that by a specific growth procedure InAs/GaAs quantum dots can be transformed into quantum ring structures. Cross-sectional STM data showed that these structures are characterized by a depression rather than an opening in the center and that they can be strongly asymmetric. Theoretical calculations have shown that such "nano-volcanoes" can still manifest Aharonov-Bohm oscillations, which are specific to ideal rings. A state of the art sample has been designed and grown consisting of 30 layers of nano-volcanoes. The periodic Aharonov-Bohm oscillations in the persistent current are observed in low temperature magnetization measurements using a torque magnetometer in magnetic fields up to 15T. The results are in excellent agreement with theoretical predictions for strained nano-volcanoes. Photoluminescence measurements in magnetic fields support the magnetization experiments.
4.6.2 PL measurements in magnetic field on sample 2B

5 Conclusions and Suggestions
   5.1 Conclusions
   5.2 Suggestions

6 Acknowledgments

7 Appendix A

8 Appendix B
Chapter 1

Introduction

The last decades have shown an enormous progress in the control of the growth of semiconductor materials. Low-dimensional semiconductor structures comprise an important area of research nowadays. Low-dimensional semiconductor structures confine electrons to less than three dimensions. In this thesis two of such structures are discussed, namely the InAs self-assembled quantum rings and quantum dots, which confine electrons to zero dimensions. The confinement of electrons to zero dimensions combined with the small sizes of the quantum rings and dots, typically in the order of several nanometers, lead to quantum systems, which show striking similarities with atoms. For this reason these structures are often called 'artificial atoms'.

The application of these structures can be legion. For example these structures can be used in a new generation of lasers with a wavelength closer to the wavelength used in optical communication than conventional lasers. Furthermore we are able to influence the states of these artificial atoms by for instance applying magnetic fields. This tunability of such zero dimensional systems is most interesting in quantum computing. The computers no longer have to work with binary bits but with so-called quantum bits, also called qubits. The elements representing the data are no longer only given by a serie of 0's and 1's. The states of the system of a nanostructure are a combination or superposition\(^1\) of state '0' and state '1'. This enables a number of possibilities to be computed simultaneously, vastly speeding up the processing.

One of the most intriguing quantum mechanical phenomena is the existence of the Aharonov-Bohm effect. This effect has no classical analogue at all and is therefore counter intuitive. Historically, this effect was first mentioned in a paper by W. Ehrenburg and R.E. Siday in 1949 [2], but their remarks were completely overlooked. It was Y. Aharonov and D. Bohm who pointed out to this previously overlooked aspect and the importance of this effect [3]. In short the Aharonov-Bohm effects describes the behavior of a charged particle in the vicinity of an electromagnetic field. Even though this charged particle is in a field free region, there is still a vector potential not equal to zero due to the near-by presence of the electromagnetic field. Since the Hamiltonian in quan-

\(^1\)For example see [1]. Due to the exchange interaction the state of a two electron system is given by a combination of spin up (the 1's) and spin down (the 0's) states.
Quantum mechanics uses potentials instead of the actual fields, the charged particle is feeling this vector potential and consequently its motion is influenced by it. In the article Aharonov and Bohm propose the following experiment. A coherent beam of electrons is split into two parts, each passing a solenoid on opposite sides. The beams are combined to interfere after passing around the solenoid. Both parts of the beam have picked up an additional phase caused by the vector potential. Depending on the vector potential the beams will have constructive or destructive interference. The Aharonov-Bohm effect was first experimentally shown by Tonomura et al. [4] in 1986, almost thirty years after the effect was predicted.

InAs self-assembled quantum rings form a special class of topologically non-simple connected quantum nanostructures, which are expected to show Aharonov-Bohm like quantum interference effects. Up to now mainly voltage-capacitance and optical techniques have been used to explore the specific quantum effects in self-assembled quantum rings [5]. Unfortunately there is still a strong sceptic opinion whether quantum interference really plays a role in InAs self-assembled quantum rings. In order to measure unambiguously the Aharonov-Bohm effect in these structures we use a very sensitive torque magnetometer implemented in the High Field Magnetic Laboratory in Nijmegen. This master thesis will discuss the Aharonov-Bohm effect, the used semiconductor structures and the used setup into detail. Above all, we will show remarkable experimental results which are supported by theoretical calculations based on X-STM, AFM and PL measurements.
Chapter 2

Theory

2.1 General properties of semiconductors

Most low dimensional semiconductor structures that are studied nowadays consist of alloys based on combinations of group III and V elements. In particular alloys based on the elements Al, Ga, In and As are of major interest, where group II and IV elements are added to these semiconductor compounds as dopants. Figure 2.1 shows the most common semiconductor elements.

In figure 2.2 various semiconductor compounds are shown. Their lattice constants \( a_0 \) are plotted along the horizontal axis against the bandgap energy \( E_g \) and corresponding wavelength \( \lambda \) along the vertical axis. The values are for room temperature. Heterostructures consist of different layers of semiconductor compounds. As shown in figure 2.2 different semiconductor compounds have different lattice constants. Therefore heterostructures can consist of strained layers. These strained layers change the conduction and valence bands and consequently bandgap energy \( E_g \).
Photoluminescence (PL) is a way to examine and analyse the band structure of semiconductor structures. First the sample is illuminated. An electron absorbs a photon when the energy of the photon is larger than the bandgap energy $E_g$. The electron is excited to the conduction band and leaves a hole behind in the valence band. This absorption process can be used for bandgap analysis. PL is the emission of a photon as a result of the recombination of an electron and a hole. The resulting PL spectrum can give a lot of information about nano structures such as its structural properties.

In figure 2.2 a distinction is made between direct and indirect semiconductor compounds. When electrons in the minimum of the conduction band and holes in the maximum of the valence band have the same momentum, it is called a direct gap. Because the momentum of the hole and electron are matched, recombination via a photon, which has a small momentum, is a likely process. The well-known semiconductor compound GaAs has a direct gap of $E_g=1.42$eV, which corresponds with a wavelength of $\lambda=871$nm. InAs has a direct gap of $E_g=0.36$eV and a corresponding wavelength of $\lambda=3444$nm. These values are valid at room temperature. For lower temperatures the gap energies are slightly higher. For example, the direct bandgap of GaAs at a temperature of zero Kelvin is $E_g=1.52$eV [7].
An indirect gap semiconductor compound has a difference in the momentum of the electron at the minimum of the conduction band and the hole at the maximum of the valence band. Optical transitions occur only if other contributions of momentum are provided to satisfy conservation of total momentum. Phonons and impurities can provide these contributions.

### 2.2 From quantum dots to quantum rings and quantum volcanoes

Low dimensional semiconductors confine electrons to less than three dimensions. The most well known low dimensional semiconductor is the quantum well. The electrons in a quantum well are confined in one direction and are free to move in the plane perpendicular to this direction. Quantum dots (QDs) and quantum rings (QRs) are of particular interest, since they confine electrons to all three dimensions. This confinement leads to discrete energy spectra, which are comparable to the spectra we obtain from atoms. That is why these semiconductor structures are often called 'artificial atoms'. One of the major technical advantages of structures such as the QDs and QRs is that they can be grown by self-assembly. Figure 2.3 illustrates the Stranski-Krastanow (SK) growth mode of self assembled QDs. The Stranski-Krastanow growth mode is characterized by the strain-induced transition from two-dimensional layer-by-layer growth towards three-dimensional island growth. This transition occurs after growing 1.5 mono layer (ML) of InAs on GaAs. Droplets of InAs, the quantum dots, are formed on top of a thin layer of InAs, called the wetting layer (WL). After deposition of more than 2.5 ML dislocations in the QDs are formed. These dislocations lead to considerable quenching of the PL of the QDs and are therefore not preferable. The self-assembled dots can for instance be grown by MBE (Molecular Beam Epitaxy) technique, see [8].

The formation of the quantum rings occurs when the dots are capped with a thin layer of GaAs (typically in the order of 2nm) with an subsequential annealing process. The formation of self-assembled InAs on GaAs QRs was first

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<1.5ML ~1.5ML ~2ML >2.5ML

Figure 2.3: Stranski-Krastanow growth mode of self assembled quantum dots. For more than 1.5 ML of InAs on GaAs the QDs are formed. This formation occurs on top of a thin layer of InAs, which is called the wetting layer (WL).

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\[1\] This process shows striking similarities with the behavior of a thin sheet of water on a metal plate. By increasing the amount of water on this metal plate, a strain driven transition occurs towards the formation of droplets.
observed by García et al. [9]. The process is schematically shown in figure 2.4.

The annealing process causes a migration of indium away from the center of the dot giving rise to a second wetting layer. The remaining parts of the quantum dot now form a quantum ring. For a detailed description of the growth method of QRs see [10], [11] and [12].

Figure 2.5: X-STM image of a pyramid shaped QD. The black arrow indicates the WL. The bright area corresponds with the indium rich area.

Scanning Probe Microscopy (SPM) is used to analyse the structural properties such as shape, size and composition of dots and rings. From AFM (Atomic Force Microscopy) measurements it was shown that the QRs are having a larger diameter and smaller height as compared to the dimensions of QDs. The QDs and QRs can have an asymmetry both in diameter and height subtly depending
Figure 2.6: Filled state X-STM image of a self-assembled InAs quantum rings cleaved in the (110)-plane (a) and in the (110) plane (b) showing the asymmetry of the rings on this sample. The inset shows the 3D model of the rings observed by X-STM. Image from [13].

on the growing conditions. Typical sizes for dots as observed by AFM are 60nm in diameter and 10nm in height for dots and 80nm in diameter and 1nm in height for QRs.

Only recently it has been shown by P. Offermans et al. that the actual sizes found by X-STM (Cross-sectional Scanning Tunnelling Microscopy) for the dots and rings capped with a thick layer of GaAs, are considerably smaller compared to the sizes found by AFM for surface dots and rings [13]. Moreover X-STM measurements show that the rings have a depression in the center of the ring instead of a hole; therefore the rings are sometimes called craters or volcanoes. The X-STM images of the dots and the rings are shown in figures 2.5 and 2.6 respectively. The rings are cleaved in two different directions showing the asymmetry of the ring as was observed by AFM measurements (see paragraph 4.2.1). The inset of figure 2.6 shows a 3D model of the InAs rings as observed by X-STM.

The difference in size found by AFM and X-STM measurements has two origins. The first origin is the convolution of the shape of the tip and the shape of the observed structure in AFM measurements, schematically shown in figure
2.7. This convolution results into a larger diameter measured by AFM compared with the actual value of the diameter measured by X-STM. Deconvolution of the AFM data is only possible if the exact shape of the AFM tip is known, which in general is not the case. The convolution of tip shape and structure does not occur during X-STM measurements since the X-STM tip is scanning a nearly flat cleaved surface\(^2\). Of course on the atomic scale a convolution of the wave function of the last atom at the apex of the tip with the surface wave function still occurs, but this happens at subnanometer scale.

The second origin of the difference in size of the structures is that the AFM measurements show the rings underneath the 2nm GaAs capping after annealing (step (3) in figure 2.4). Therefore the AFM measurements do not show the actual (buried) ring structure. As observed by X-STM measurements the diameter of the rings is comparable with the diameter of the dots in contrast to the observation by AFM measurements.

The decrease in height of the dots and the rings when observed with X-STM has a different origin. During the burying of the dots and rings with GaAs, there is a reordering of material. Indium from the top of the dots and rings is migrating to the wetting layer, resulting in a reduced height of the buried dots and rings. From the X-STM measurements we find a typical diameter of 25nm and a height of 6nm for the dots and a diameter of 25nm and a height of 4nm for the rings. These exact sizes are strongly dependent on the exact growth conditions. Moreover, the sizes differ from nanostructure to nanostructure (typical a size distribution of 10% is observed), giving rise to some inhomogeneous broadening of the PL.

The sizes of the QDs and QRs are such that the behavior of electrons confined in these structures cannot be explained with the use of classic mechanics. Quantum mechanica comes into play and leads to some intriguing phenomena, such as the Aharonov-Bohm effect.

\(^2\)The relaxation of strain on the cleaved surface results in a surface relaxation. The error due to the convolution of the X-STM tip with the structure is small, typically in the order of hundreds of picometer.

Figure 2.7: The dotted line indicates the image obtained with AFM from a QD due to the convolution of the shape of the tip with the shape of the dot. This convolution results in diameter increased by \(2\Delta r\).
2.3 Quantum rings in a magnetic field: the Aharonov-Bohm effect

The Aharonov-Bohm (AB) effect is a well-known quantum mechanical effect, which has no classical analogue. The AB effect occurs when an electron travels through or is confined in a structure with ring geometry threaded by a magnetic field. The quantum mechanical approach in the ideal case of a one dimensional ring will be discussed in the first subparagraph. The semi-classical approach is discussed in the next subparagraph in order to gain more intuition on the AB effect. The AB effect in self-assembled InAs quantum rings on GaAs is discussed thoroughly in the last subparagraph. This discussion is based on theoretical calculations performed by V. Fomin, V.N. Gladilin and J.T. Devreese of the TFVS group in the Department of Physics at Antwerp University. These calculations are a result of a close collaboration with the HGF group of Eindhoven University of Technology and are performed using the data obtained from the X-STM measurements on quantum rings performed by P. Offermans. Part of the results were already presented at different conferences [14] and [15] and in an APL paper by P. Offermans et al. [13].

2.3.1 Quantum mechanics of an electron in an one dimensional ring

In figure 2.8 the ideal case of an one dimensional ring of radius R is shown. A magnetic field is concentrated only in the opening of the ring.

Although the electrons confined in the ring do not feel the magnetic field itself, they are affected by the vector potential of the magnetic field. This is a consequence of the Hamiltonian in quantum mechanics, which contains potentials instead of the fields themselves. The single electron Hamiltonian in the presence of a magnetic field $\mathbf{B}$ is given by:

$$\mathcal{H} = \frac{1}{2m_e^*} \left( \mathbf{p} - e \mathbf{A} \right)^2 + V,$$

(2.1)
where $V$ contains details of the electronic confinement potential. $A$ is the vectorpotential, which is related to $\vec{B}$ via:

$$\vec{B} = \nabla \times \vec{A} \quad (2.2)$$

The Hamiltonian is connected to the energy $E$ and the wave function $\psi$ of the electron via the Schrödinger equation, given by:

$$\mathcal{H} \psi = E \psi \quad (2.3)$$

We are now going to follow the same approach as in [16]³. For practical reasons we set the electronic confinement potential to zero. In absence of a magnetic field, $\vec{B}=0$, we calculate the wave function $\psi$ of an electron in an one dimensional ring. This wave function only depends on the azimuthal angle $\phi$, so $\psi=\psi(\phi)$. Using spherical coordinates the Schrödinger equation leads to⁴:

$$-\frac{\hbar^2}{2m_e^*} \nabla^2 \psi(\phi) = -\frac{\hbar^2}{2m_e^*} \frac{1}{R^2} \frac{\partial^2}{\partial \phi^2} \psi(\phi) = E \psi(\phi) \quad (2.4)$$

The wave function is single valued, so $\psi(\phi)=\psi(\phi+2\pi)$. Normalization demands $\int_0^{2\pi} |\psi(\phi)|^2 d\phi=1$. For these conditions the solution to the Schrödinger equation is given by:

$$\psi(\phi) = \frac{1}{\sqrt{2\pi}} \left( \frac{R}{\hbar} \right)^{1/2} \exp \left( \pm \frac{im}{\hbar} \sqrt{2m_e^*} E \phi \right) \quad (2.5)$$

The energy values $E$ are quantized because of the periodic boundary conditions⁵. This leads to discrete energy values:

$$E_m = \frac{m^2 \hbar^2}{2m_e^* R^2}, \quad m = 0, \pm 1, \pm 2, \pm 3, \ldots \quad (2.6)$$

The wave functions belonging to $E_m$ are $\psi(\phi)=\frac{1}{\sqrt{2\pi}} \exp (im\phi)$. Actually, there are two solutions: $\frac{1}{\sqrt{2\pi}} \exp (im\phi)$ and $\frac{1}{\sqrt{2\pi}} \exp (-im\phi)$. We cover the latter by allowing $m$ to run negative. We choose to label the energies and wave functions with quantum number $m$, because of its analogue with the angular momentum quantum number $m$ [1], [18], sometimes denoted as magnetic quantum number⁶.

We now calculate the wave functions and corresponding energies in the presence of a magnetic field. We assume the magnetic field shown in figure 2.8 to be homogeneous and time independent and directed perpendicular to the plane.

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³[17] gives also a clear alternative representation of the AB effect in rings.

⁴Spherical coordinates: $\nabla^2 = \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 \frac{\partial}{\partial r}) + \frac{1}{r^2 \sin(\theta)} \frac{\partial}{\partial \theta} (\sin(\theta) \frac{\partial}{\partial \theta}) + \frac{1}{r^2 \sin^2(\theta)} \frac{\partial^2}{\partial \phi^2}$. For the one dimensional ring the wave function only depends on the azimuthal angle $\phi$. The polar angle $\theta=\pi$ and $r$ is equal to the radius of the ring $R$. This simplifies to $\nabla^2 = \frac{1}{r^2} \frac{\partial^2}{\partial \phi^2}$.

⁵The wave function is single valued and leads to the periodic boundary conditions $\psi(\phi)=\psi(\phi+2\pi)$. Using equation 2.5: $\exp (\pm i\frac{m}{\hbar} \sqrt{2m_e^*} E \phi) = \exp (\pm i\frac{m}{\hbar} \sqrt{2m_e^*} E (\phi + 2\pi))$. This is valid only if $\exp (\pm i2\pi \frac{m}{\hbar} \sqrt{2m_e^*} E) = 1 = \exp (i2\pi m)$ with $m$ an integer.

⁶The solution of the azimuthal part of the Schrödinger equation in spherical coordinates leads to $\exp (im\phi)$, with $m$ the angular momentum quantum number. This solution is identical to the solution of the Schrödinger equation of an one dimensional ring.
of the ring. At the rim of the ring no magnetic field is present, since it is only
penetrating the opening of the ring. This leads to $\mathbf{B} = 0$ at the rim of the ring
and using $2.2^7$ leads to $\mathbf{A} = \nabla \chi$. The magnetic flux $\Phi$ penetrating the ring is,
using Stokes’s theorem, related to $\chi$ by:

$$\Delta \chi = \oint \mathbf{A} \cdot d\mathbf{l} = \int \int \mathbf{B} \cdot d\mathbf{S} = \Phi$$  \hspace{1cm} (2.7)$$

We now define a new wave function $\psi'$:

$$\psi' = \psi \exp \left( \frac{ie}{\hbar} \chi \right)$$ \hspace{1cm} (2.8)$$

The advantage of this gauge transformation is the elimination of the vectorpo-
tential $\mathbf{A}$ in the Schrödinger equation, which is proven in Appendix A. Using
definition 2.8 together with the periodic boundary condition leads to a relative
change in the phase of the wave function given by:

$$\frac{\psi'(\phi)}{\psi'(\phi + 2\pi)} = \frac{\psi(\phi) \exp \left( \frac{ie}{\hbar} \chi(\phi) \right)}{\psi(\phi + 2\pi) \exp \left( \frac{ie}{\hbar} \chi(\phi + 2\pi) \right)}$$ \hspace{1cm} (2.9)$$

Wave function $\psi$ is single valued$^8$ and applying equation 2.7 we get the relation
between $\psi'(\phi)$ and $\psi'(\phi + 2\pi)$ given by:

$$\psi'(\phi) = \psi'(\phi + 2\pi) \exp \left( i2\pi \frac{\Phi}{\Phi_0} \right),$$ \hspace{1cm} (2.10)$$

where we introduced the elementary flux quantum $\Phi_0$, which is given by $\Phi_0 = \frac{\hbar}{e}$. The consequence of the gauge transformation is thus a change in the phase of the wave function $\psi'$ when the azimuthal angle $\phi$ changes by $2\pi$. Moreover from
2.10 we get a periodic dependence of $\psi'$ on $\Phi$. The exponent is equal to unity if $\Phi$ takes on integral values of $\Phi_0$ leading to no change in the phase of the wave
function when $\phi$ changes by $2\pi$. The wave function in the more general case, not only at $\phi = 0$ and $2\pi$, is given in equation 2.11.

$$\psi'(\phi) = \frac{1}{2\pi} \exp \left( im \phi \right) \exp \left( i2\pi \frac{\Phi}{\Phi_0} \right)$$ \hspace{1cm} (2.11)$$

The energy levels belonging to the wave function $\psi'$, and thus $\psi'$, are obtained
applying the same procedure as used in the absence of the magnetic field. In
equation 2.12 the energy levels are given as function of the angular momentum
quantum number $m$:

$$\mathcal{E}_m = \frac{\hbar^2}{2m_e R^2} \left( m + \frac{\Phi}{\Phi_0} \right)^2, \quad m = 0, \pm 1, \pm 2, \ldots$$ \hspace{1cm} (2.12)$$

The energy levels as function of the magnetic flux penetrating the center of the
ring are given in figure 2.9. For increasing magnetic field, the ground energy

$^7$From vector calculus: $\nabla \times \nabla \chi = 0$

$^8$\psi is single valued, $\psi'$ not. This is the price you pay for eliminating the vector potential from the Hamiltonian.
state will change from angular momentum quantum number \( m = 0 \) to a state with higher value of \( m \). In the case that the magnetic field is oriented along the positive \( z \)-axis, as in figure 2.9, the value of \( m \) will become higher negative. The transitions in \( m \) occur for rational values of the magnetic flux given by \( \Phi = (p + \frac{1}{2}) \Phi_0 \), with \( p = 0, \pm 1, \pm 2, \ldots \).

The transitions in angular momentum quantum number are intimately related to jumps in the magnetization. The relation between the energy and the magnetization \( M \) of a many particle system is given by one of the thermodynamical Maxwell equations:

\[
M = -\frac{\partial F}{\partial B} \mid_{N,T},
\]  

(2.13)

where \( F \) is the total free energy, \( N \) the total number of electrons and \( T \) the temperature. In the case of an electron in an one dimensional ring and at zero temperature the free energy is equal to the energy given in equation 2.12. In figure 2.10 the magnetization of an electron in the one dimensional ring, \( \mu^0 \), is plotted against the magnetic flux. The linear relation between magnetic field and magnetization is expected; the energy depends quadratically on the magnetic flux and the magnetization is the derivative of the energy. The jumps in the magnetization appear for rational values of the magnetic quantum flux.

\(^9\)One electron has a magnetic moment of \( \mu \). The total magnetization \( M \) of \( N \) electrons is \( M = N \mu \).
Figure 2.10: The magnetization $\mu$ against the magnetic flux penetrating the center of the ring. The magnetization abruptly changes sign at the values of the magnetic flux $\Phi=(p+\frac{1}{2})\Phi_0$ and is zero at $\Phi=p\Phi_0$, where $p$ is an integer.

$\Phi_0$. The magnetization is zero for integer values of $\Phi_0$. In the next paragraph a semi-classical approach is used in order to gain more intuition of this behavior.

2.3.2 Semi-classical approach of an electron in an one dimensional ring

This subparagraph treats the electron as if circulating in the ring due to the presence of a vector potential. This is quite similar to the case of the cyclotron motion of an electron in a magnetic field. However, the cyclotron resonance is an effect due to the Lorentz force and a direct consequence of carriers in a magnetic field, whereas the AB effect is a consequence of the presence of a vector potential. Equation 2.14 gives the current $I$ of an electron with velocity $v$ in a one dimensional ring with radius $R$.

$$I = \frac{e}{T} = \frac{ev}{2\pi R} = \frac{ehk}{2\pi R m_e^*},$$

(2.14)

here $T$ is the period time needed for the electron to travel the ring once. The wave vector $k$ of the electron is introduced by comparing the quantum mechanical and classical relation $^{10}$ of the momentum $p$, $p=\hbar k=m_e^*v$.

In absence of a magnetic field the wave vector $k$ of the electron in the ring has to be an integer value $m$ times $\frac{2\pi}{2\pi R}$ when the azimuthal angle changes by $2\pi$. $^{11}$ The integer $m$ is equal to the angular momentum quantum number. From the previous paragraph we obtained that the presence of the magnetic field will change the boundary conditions and therefore the wave vector and energy of the electron. The wave vector $k$ picks up an additional phase proportional to $\lambda$ the wavelength of the electron which must fit the circumference an integer times to prevent destructive interference.

$^{10}$This is the reason why it is called the semi-classical approach. This problem is not solvable dealing it in a purely classical approach.

$^{11}$k=$\frac{\lambda}{2\pi}$, with $\lambda$ the wavelength of the electron which must fit the circumference an integer times to prevent destructive interference.
the enclosed magnetic flux $\Phi$. Equation 2.15 gives the $k$ vector in the presence of a magnetic field\textsuperscript{12}.

$$ k = \frac{1}{R} (m + \frac{\Phi}{\Phi_0}) $$

(2.15)

Substituting $k$ in the equation of $I$ and rewriting $\frac{\Phi}{\Phi_0}$ as $\frac{\Phi R^2}{\hbar/e}$ leads to:

$$ I = \frac{e^2 B}{4\pi m^*} + \frac{ehm}{2\pi R^2 m^*} $$

(2.16)

We assume a realistic value of the radius of the ring of $R=10\text{nm}$ and a magnetic field of $B=10\text{T}$. For these conditions an electron in the ground state has angular momentum quantum number $m=0$. A very rough approximation of an InAs self-assembled ring is obtained when the effective mass of the electron is taken to be the effective mass of an electron in bulk InAs\textsuperscript{13}, so $m^*=0.023m_0$. The value for the current using these approximations is $I\approx 9.7\times10^{-7}\text{A}$. The magnetization of an electron traversing a ring with area $A$ is given by:

$$ \mu = IA = I\pi R^2 $$

(2.17)

So in our example the value of the single electron magnetization $\mu \approx 3\times10^{-22}\text{J/T}$. We now introduce the Bohr magneton:

$$ \mu_B = \frac{eh}{2m_e} = 9.274\times10^{-24}\text{J/T} $$

(2.18)

In terms of the Bohr magneton, the magnetization is in the order of $\mu \approx 33\mu_B$.

For a certain magnetic field the magnetic flux through the ring is exactly $\frac{\Phi_0}{2}$. The ground state instantaneously changes from $m=0$ to $m=-1$, as shown in figure 2.9. In equation 2.16 the second term becomes larger than the first term resulting in a negative current. This corresponds to an instantaneous change of the direction of the current. For a magnetic flux equal to the magnetic flux quantum, $\Phi=\Phi_0$, the current is exactly zero since both terms in equation 2.16 cancel. When the current changes instantaneously, the magnetization also changes direction. The magnetization is zero when there is no current, so at integral values of the magnetic flux quantum. This is exactly the behavior shown in figure 2.10.

To complete the semi-classical picture of a current to describe the magnetization we have to take into account the coherence time of an electron traversing the ring. This has to be at least the time it takes for an electron to traverse the ring. Equation 2.19 gives the coherence time $\tau$ for an electron in a bulk material.

$$ \tau = \frac{m^*\mu}{e} $$

(2.19)

Note that $\mu$ in equation 2.19 is the mobility. It is known that under the right growth circumstances QDs have no dislocations. Assuming the same low level

\textsuperscript{12}This can quickly be seen by comparing the free electron energy $E=\frac{\hbar^2k^2}{2m_e}$ with the energy of an electron in a one dimensional ring in the presence of a magnetic field given by equation 2.12.

\textsuperscript{13}The effective mass in GaAs is $m^*=0.063m_0$. In reality the QDs consist of InGaAs, which has an effective mass in between the ones of InAs and GaAs. This would lead to a weaker current and thus to a weaker magnetization signal as calculated in this model.
of dislocations in QRs, we estimate the mobility of the electron in a ring to be in the order of \( \mu = 100 \text{m}^2/\text{Vs} \), corresponding to high mobility InAs bulk. This leads to a coherence time in the order of \( \tau \sim \text{ps} \). The velocity of the traversing electron in an one dimensional ring is approximately \( v = 3.8 \times 10^5 \text{m/s} \). From this the period \( T \) is calculated to be in the order of \( T \sim 10\text{fs} \), which is a factor of 100 less than the coherence time. This estimation makes sure that the AB oscillation will not be destroyed due to incoherence effects.

2.3.3 Magnetization of the quantum ring

As discussed in paragraph 2.2, the quantum ring does not have an opening in its center and depending on the growth condition it can have a certain asymmetric shape. The crucial question has become whether electrons in these asymmetric quantum craters still are able to perform AB oscillations. In contrast to the previous paragraph these quantum craters have finite thickness and width of the rim through which in the experiments magnetic field is penetrating. This results in a combination of pure quantum mechanical AB behavior and the classical Lorentz motion.

First of all, we look at the influence of the finite thickness of the ring. The additional magnetic field in the rim produces an additional Lorentz force on electrons revolving around the ring. The electrons are now not only performing a circular motion around the ring (due to the AB effect), but also a second circular motion (the cyclotron motion) due to this Lorentz force. The resulting total motion will be a superposition of these two motions. The cyclotron radius belonging to the Lorentz force is given by:

\[
\tau = \frac{m_e v}{eB}
\]

The velocity will be estimated by using the equations of an 1D-ring in a magnetic field. The wave function of an one dimensional ring in a magnetic field is given by equation 2.11. Using the momentum operator and integrating over \( \phi \) we find for the velocity:

\[
v_\phi = \langle \psi^\dagger | \frac{\vec{p}_\phi}{m_e} | \psi \rangle = \frac{1}{2\pi} \int_0^{2\pi} \exp(-im\phi) \exp(-i\frac{\Phi}{\Phi_0}) \cdot -i\hbar \frac{1}{m_e \tau} \frac{\partial}{\partial \phi} (\exp(im\phi) \exp(i\frac{\Phi}{\Phi_0})) d\phi
\]

\[
= \frac{\hbar}{m_e \tau} \left( m + \frac{\Phi}{\Phi_0} \right)
\]

We substitute this result in 2.20 and take a ring with a radius of \( R = 10 \text{nm} \). At a magnetic field of 10T the electron in this ring is in the ground state characterized by \( m = 0 \). Again we estimate the effective mass to be the bulk effective mass of InAs. For these estimations a cyclotron radius of \( r \approx 5 \text{nm} \) is obtained.

The influence of the Lorentz force on the electrons traversing the ring changes the trajectory. Figure 2.11 shows the skipping orbit trajectories of electrons due

\[\footnote{Calculated for the case where the ring radius \( R = 10\text{nm} \) and \( B = 10\text{T} \).} \]
to the Lorentz force only in a ring with a large rim width. In the real ring there is no infinite high barrier present. The electrons reflect on the increasing potential towards the center of the ring. This reflection ends the cyclotron orbit of the electron. The electron starts a new cyclotron orbit until it is reflected again. However, the rim width of the quantum rings are typically 3nm and such behavior as shown in figure 2.11 cannot occur. Its seems realistic that the trajectory in a self-assembled InAs quantum ring is mainly determined by the AB effect.

In the theoretical model of the Antwerp team both the Lorentz force and AB effect are taken into account. The effect of the Lorentz force results in to the shift of the AB-peak in the magnetization to higher fields and to a broadening and amplitude decrease of the magnetization peak as will be shown in figure 2.21.

Based on the structural information of self-assembled InAs QRs on GaAs extracted from the X-STM measurements performed by P.Offermans [13], the Antwerp team\textsuperscript{15} modelled the ring structure with a varying-thickness InGaAs layer embedded in a matrix of GaAs. The bottom of the InGaAs layer is assumed to be perfectly flat and parallel to the x,y-plane. The analytical equations given by 2.22 and 2.23:

\begin{equation}
\frac{h(\rho, \phi)}{\gamma_0} = h_0 + \frac{(h_M(1+\xi\cos(2\phi)) - h_0)\gamma_0^2}{R^2} \times \frac{R^2 - (\rho - R)^2}{(\rho - R)^2 + \gamma_0^2}, \quad \rho \leq R \quad (2.22)
\end{equation}

\begin{equation}
\frac{h(\rho, \phi)}{\gamma_\infty} = h_\infty + \frac{(h_M(1+\xi\cos(2\phi)) - h_\infty)\gamma_\infty^2}{(\rho - R)^2 + \gamma_\infty^2}, \quad \rho > R \quad (2.23)
\end{equation}

are used to describe the height $h$ of the ring as function of the radial coordinate $\rho$ and the azimuthal $\phi$. The radius of the ring is given by $R$, $h_0$ is the height of

\begin{itemize}
\item V.Fomin, V.N. Gladilin and J.T. Devreese, TFVS, Department of Physics of Antwerp University.
\end{itemize}
Figure 2.12: The shape of the ring as modelled by equations 2.22 and 2.23, with $R=11.5\text{nm}$, $h_0=1.6\text{nm}$, $h_M=3.6\text{nm}$, $h_\infty=0.4\text{nm}$, $\gamma_\circ=3\text{nm}$ and $\gamma_\infty=5\text{nm}$. The anisotropy parameter $\xi$ is set on 0.2.
the ring in the center, $h_M$ is the ring height and $h_\infty$ is the height of the InGaAs layer far away from the ring. The parameters $\gamma_0$ and $\gamma_\infty$ define the inner and outer slopes of the rim and $\xi$ defines the asymmetry in the rim height. In figure 2.12 the modelled ring is given for realistic values of the parameters.

The fraction $x$ of In$_x$Ga$_{1-x}$As is determined using the surface relaxation and lattice distortion of the cleaved surface observed in X-STM. This method is described intensively in [19]. Figure 2.13 shows the fraction $x$ in In$_x$Ga$_{1-x}$As for a cross-section of the modelled ring.

The shape of the ring and the concentration of In in the ring determines band parameters such as the effective mass and the band gaps. These parameters are used to calculate the potential of the ring. The Schrödinger equation is solved using the adiabatic approximation [20]. In this approximation the motion of the electron is separated in a fast motion along the z-direction and a slow motion in the x,y-plane\textsuperscript{16}. In equation 2.24 the Schrödinger equation is shown.

$$\left[ T_z + T_{in} + V_f(r, \phi) + V_I(r, \phi, z) \right] \Psi = E \Psi$$  \hspace{1cm} (2.24)

The kinetic operator is separated in the kinetic operator belonging to the z-motion, $T_z$, and the in-plane motion, $T_{in}$. The potential is separated into a potential that depends on the polar coordinates only, $V_f(r,\phi)$, and a

\textsuperscript{16}The confinement energy in the z-direction is higher compared with the confinement energy in the lateral direction due to the smaller height of the ring compared with the lateral dimensions of the ring.
Fig. 2.14: The adiabatic potential of a quantum ring with the same parameters as in figure 2.12. The potential has valleys where the rims are higher and hills where the rims are smaller. The potential reflects the asymmetry of the ring. In the x,y-plane the top view of the ring of figure 2.12 is shown.

potential that depends on both the polar coordinates and the height z of the ring, \( V_{11}(\rho, z) \). A certain wave function \( \psi(\rho, z) \) is defined such that:

\[
\left[ \hat{T}_z + V_{11}(\rho, z) \right] \psi(\rho, z) = \varepsilon(\rho)\psi(\rho, z)
\]  

The fast motion equation leads to the adiabatic potential \( \varepsilon(\rho) \), which will later on be used as a potential in the slow motion equation. This can be done, because effectively the slow motion is governed by the adiabatic potential giving the average potential of the fast motion. The calculated adiabatic potential \( \varepsilon(r, \phi) \) for a uniform \(^{17}\) In\(_{0.6}\)Ga\(_{0.4}\)As is shown in figure 2.14.

The wave function \( \Psi \) can be written as \( \Psi = \psi(\rho, z)\xi(\rho) \). Substituting this and equation 2.25 in the Schrödinger equation we get:

\[
\left[ \hat{T}_m + V_1(\rho) + \varepsilon(\rho) \right] \psi(\rho, z)\xi(\rho) = \mathcal{E}\psi(\rho, z)\xi(\rho)
\]  

Multiplication with the conjugate of \( \psi \) and integration over \( z \) gives:

\[
\int \psi^*(\rho, z) \left[ \hat{T}_m + V_1(\rho) + \varepsilon(\rho) \right] \psi(\rho, z)\xi(\rho)dz = \mathcal{E}\int |\psi(\rho, z)|^2 dz,
\]  

which simplifies to \(^{18}\):

\[
\left[ \hat{T}_m + V_1(\rho) + \varepsilon(\rho) \right] \xi(\rho) = \mathcal{E}\xi(\rho).
\]

\(^{17}\)The adiabatic potential belonging to an indium distribution as shown in figure 2.13 is more complicated.

\(^{18}\)We neglect in this step the dependence of \( \psi(\rho) \) on \( \rho \). This may be done for smooth dependence of \( \psi \) on \( \rho \).
The potential $V_1$ consists mainly of terms dictated by the strain in the QRs. The strain effects in the rings are considerable. The strain is calculated with the use of a fortran program called ABACUS, where the X-STM measurements are used as input. A small additional effect on the potential, the piezo-effect, is finally added. This effect is described in detail in [21]. The piezo-effects are important in large QDs, but negligible in small rings.

Knowing the potentials, we calculate the wave function $\xi(\rho)$ from equation 2.28. From the wave function we calculate the current and finally the magnetization. The magnetization of an unstrained quantum ring for different radii using the above method of calculation is given in figure 2.15.

The rings in our experiments are expected to have a diameter of approximately 10nm. According to figure 2.15 this would correspond to a jump in the magnetization around 12T or higher. For rings with large radii, the jump in the magnetization occurs at lower magnetic field compared to rings with small radii. Obviously a large ring, with a large inner area, needs a smaller magnetic field to enclose one flux quantum. The overall trend to negative magnetization values is due to the diamagnetic properties of InAs. The decrease in amplitude is due to the asymmetry of the ring. This asymmetry of the ring leads to a localization of the electrons in the parts where the rim is highest (see figure 2.14). A magnetic field strengthens this localization, which effectively reduces the current traversing the ring. This is shown in figure 2.16. In this figure the iso-density profiles of the wave function are shown for $B=0T$ and $B=45T$ for rings with increasing asymmetry parameter $\xi$. For a symmetric ring, $\xi = 0$, the probability to find an electron is constant along the ring, whereas the symmetry breaking results in a localization.
Figure 2.16: Iso-density profile of the probability of finding the electron at a certain position in the ring $\Psi^2$. Increasing the magnetic field and increasing the asymmetry of the ring effectively localizes the electron in places where the rim is highest. Note that the picture shows iso-density profiles for a certain value of $|\Psi|^2$. Positions in the ring where the profile is not shown do not mean that $|\Psi|^2=0$. 
The energy levels as function of magnetic field are given in figure 2.17. The energy levels are no longer indicated with the use of the angular momentum quantum number $m$, since in the asymmetric quantum craters this is no longer a quantum number. The lowest energy level consist of a series of angular momentum quantum number with $m=0, -2, -4, \ldots$. As is observed from figure 2.17, the intermixing of states is mainly due to the asymmetry of the system. Although the lowest state consist of a series of even angular momentum quantum numbers, the $m=0$ nature is still strongest present. The second highest energy level consist of a serie of odd angular momentum quantum numbers where the $m=-1$ behavior dominates. In figure B1 in Appendix B the intermixing of different angular momentum quantum numbers is visualized by plotting the real part of the wave function as function of magnetic field and symmetry. The intermixing between odd and even states becomes possible when the $C_2$-symmetry of the quantum ring is broken. In our model we can still apply a rotation over $\pi$ resulting in the same quantum ring shape. In reality this will not be the case and the ground state will consist of a mixture of all angular momentum quantum numbers, but still with a dominating $m=0$ behavior at low magnetic fields. When this is the case, there will no longer be any crossings of the energy levels; so called anti-crossings occur. Instead of a instantaneous transition of a state with even angular momentum quantum number to a state with odd angular
momentum momenta, there will be a more gradual change of a state which is dominated by \( m=0 \) to a state which is dominated by \( m=-1 \). This results to a gradual shift in magnetization instead of an instantaneous jump.

In figure 2.18 the effect of strain on the energy levels is taken into account for an asymmetric quantum ring. The main difference with the unstrained energy levels of figure 2.17 is the increase in energy. The energy levels allow us to calculate PL energies of the rings as well as the magnetization of rings (using equation 2.13). The effect of the strain on the magnetic moment \( \mu \) of an electron is given in figure 2.19. The magnetization both with and without taking into account strain are shown. Due to a reduced potential barrier at the center of a strained ring, the effective electron radius decreases when taking into account strain. Correspondingly, the transition magnetic fields, where the electron ground state changes its \( m \)-like behavior, are higher in a strained ring than in an unstrained one. From figure 2.19 we expect per quantum ring a jump in magnetization of approximately \( 10\mu_B \) per ring, assuming a population of one electron per ring. This is an enormous challenge to detect. Moreover, the real jump has a decreased amplitude and is smoothed, due to inhomogeneous size broadening of the rings. The rings have a size distribution of approximately 10\%, measured by X-STM. A way to simulate the effect of size distribution can be done by increasing the temperature. Figure 2.20 shows the magnetization of an electron calculated for an inhomogeneous size distribution of the rings and figure 2.21 shows the magnetization of an homogeneous size distribution of
Figure 2.19: Magnetic moment $\mu$ of an electron, calculated with (solid curve) and without (dashed curve) taking into account effects due to strain.

Figure 2.20: Average magnetic moment as a function of the applied magnetic field for rings with $h_0=1.6$ nm, $h_M=3.6$ nm, and a Gaussian distribution of radii, which is characterized by the average radius $<R> = 12$ nm and the standard deviation $\sigma_R$. 
Figure 2.21: Magnetic moment of an electron, calculated for different temperatures with (heavy curves) and without (thin curves) taking into account effects due to strain. At least up to 10K the AB oscillations are clearly recognizable.

The important conclusion is that in strained, asymmetric quantum craters with an inhomogeneous size distribution the Aharonov-Bohm oscillation still survive.

2.4 Free electrons in a magnetic field: De Haas-van Alphen oscillations

The Aharonov-Bohm oscillations are not the only quantum mechanical effect of electrons in semiconductor structures in a magnetic field. De Haas-van Alphen oscillations are a well known phenomenon in bulk and 2DEG structures. Assume a magnetic field in the z-direction, \( \mathbf{B} = (0, 0, B) \). The electrons in the plane normal to \( \mathbf{B} \), so in the x,y-plane, execute circular trajectories due to the Lorentz force, where the cyclotron radius is given in equation 2.20 and the cyclotron frequency is given by:

\[
\omega_c = \left| \frac{eB}{m_e} \right|. \tag{2.29}
\]
Figure 2.22: The constant density of states for a 2D system collapses into a series of δ peaks, the Landau levels. The separation between two of these peaks is equal to $h\omega_c$. Each Landau level contains many degenerate states with different $k_x$. Due to scattering the peaks have a Gaussian profile of width $\Gamma$. For small magnetic fields (b) the levels overlap significantly. By increasing the magnetic field, the levels become more distinct (c).

Using the Landau gauge, which defines the vector potential as $\vec{A}=(0,Bx,0)$, the 3D-Schrödinger equation results in 2.30:\(^{10}\):

$$-\frac{\hbar^2}{2m_e} \nabla^2 \psi - \frac{i e \hbar B x}{m_e} \frac{\partial \psi}{\partial y} + \frac{(e B x)^2}{2m_e} + V(z) = \mathcal{E} \psi(x,y,z), \quad (2.30)$$

with $V(z)$ the electronic confinement potential in the z-direction. In the absence of a magnetic field, the wave vectors $k_x$, $k_y$ and $k_z$ are good quantum numbers and define properties of the system such as the energy. However, for $B \neq 0$ the system is not longer described using $k_x$ and $k_y$, since they are not quantum numbers anymore. The total energy $\mathcal{E}$ is given by:

$$\mathcal{E} = \mathcal{E}_i + \mathcal{E}_n, \quad (2.31)$$

where $\mathcal{E}_i$ is the confinement energy in the 2D potential and $\mathcal{E}_n$ is the energy of the harmonic oscillator given by:

$$\mathcal{E}_n = (n - \frac{1}{2})h\omega_c. \quad (2.32)$$

The energy is thus described by quantum number $n$, the Landau quantum number. This has as consequence that the density of states collapses from a constant density of states of a 2D system to a series of δ-functions, see figure 2.22. These degenerate energy levels are called the Landau levels. In practice these Landau levels will not be sharp δ-functions, but the Landau levels will have a Gaussian or Lorentzian distribution due to the scattering of electrons. The energy spread is denoted with $\Gamma$, depicted in figure 2.22b. When the energy separation between the different Landau levels increases, due to an increase of magnetic field, the different levels become more distinct.

\(^{10}\)For more details, see [8]
The occupation of Landau levels and the filling factor $\nu$ as function of increasing field. The Fermi level is moved to maintain a constant density of electrons.

The degeneracy per Landau level $n_B$ is extracted from the area of figure 2.22a. The relation between the area of the constant density of states which collapses into one Landau level and the degeneracy of a Landau level $n_B$ is given by:

$$2n_B = \frac{m^* e^2}{\pi \hbar^3} \omega_c = \frac{2eB}{h},$$

(2.33)

where the factor of 2 is included to account for the spin. The number of occupied Landau levels is given by the filling factor $\nu$, defined by the ratio between the density of the electrons in 2D and the Landau degeneracy:

$$\nu = \frac{n_{2D}}{n_B},$$

(2.34)

where the definition counts the two spins as separate levels. In figure 2.23 the dependence of the filling factor on the field is illustrated.

In order to keep the total electron density constant, the Fermi level shifts up and down in energy. The conductance is determined by the density of states of the electrons at the Fermi level. The density of states at the Fermi level drops to zero when $\nu=n$, where $n$ is an integer, assuming there are clear gaps between the Landau levels, which is the case when $\Gamma$ is small. The electrons providing the conductance have a low scattering possibility for these integral values of $\nu$, since electron-electron scattering is small for a low density of states. Consequently it reduces the longitudinal resistance. If the Fermi level is positioned in the middle of the Landau level the possibility of scattering has increased due to the increase of the electron density of states. This increases the longitudinal resistance. So there is an oscillating behavior between the longitudinal resistance as function of $B$. These oscillations are called Shubnikov-de Haas oscillations and are periodic with the inverse of the magnetic field. In figure 2.24 these typical oscillations are shown. These oscillations can also be observed in the magnetization. This can be seen in equation 2.13. The energy of the system fluctuates as function of magnetic field and thus the magnetization fluctuates. These are the so called De Haas-van Alphen oscillations and they are schematically shown in figure 2.25 for a 2DEG and in 2.26 for a 3DEG.
Figure 2.24: The oscillatory behavior of the longitudinal magneto resistance $R_{xx}$ of a 2DEG as function of the magnetic field, also known as Shubnikov-de Haas oscillations. They are a consequence of the shifting of the Fermi level with respect to the Landau levels. The oscillations are periodic with the inverse magnetic field. Image from [22].

In 3D the total energy $\mathcal{E}$ is given by:

$$\mathcal{E} = \frac{\hbar^2 k_x^2}{2m^*_e} + \mathcal{E}_n$$  \hspace{1cm} (2.35)

This effects the density of states of the LL in figure 2.22 and results in a different amplitude dependence. A 2D system has a saw-tooth like behavior on magnetic field with a constant amplitude, whereas the 3D system has smoother features and an increasing amplitude with magnetic field. For detailed derivations of the De Haas-Van Alphen effect we refer to papers by Vagner et al. or [23] and [24].

From [25] the periodic behavior of the magnetization with $1/B$ is given by:

$$\Delta(1/B) = \frac{2\pi e}{\hbar S},$$  \hspace{1cm} (2.36)

with $S$ the Fermi surface in k-space. The Fermi surface is related to the Fermi wave vector $k_F$ in 2D by:

$$S = \pi k_F^2.$$  \hspace{1cm} (2.37)

In n dimensions the volume element in k-space is approximated by $(\frac{2\pi}{L})^n$, where $L$ is the typical size of the system. So in a three dimensional system, the total electron density in a sphere of radius $k_F$ is given by

$$N = 2 \frac{4/3\pi k_F^3}{(\frac{2\pi}{L})^3} = \frac{V k_F^3}{3\pi^2},$$  \hspace{1cm} (2.38)

with $V$ the volume of the system. The electron density of the 3D system $n_{3D} = N/V$ can thus directly be extracted from the period of the oscillation by
Figure 2.25: The magnetization $M$ as function of the magnetic field $B$. The saw-tooth like De Haas-Van Alphen oscillations are periodic with the inverse magnetic field. The amplitude of the oscillation stays constant.

Figure 2.26: The magnetization $M$ as function of the inverse magnetic field $B^{-1}$. The 3D De Haas-Van Alphen oscillations are periodic with the inverse magnetic field. Furthermore the amplitude of the oscillations increases with increasing magnetic field.

Combining equations 2.36, 2.37 and 2.38. For a 2D system we find:

$$n_{2D} = \frac{k_F^2}{2\pi}.$$  \hspace{1cm} (2.39)

Further analysis on the De Haas-Van Alphen oscillations are performed in [26], where an analytical relation for the oscillations is derived.
Chapter 3

Experimental Setup

In this chapter the setups for the photoluminescence (PL) measurements both with and without magnetic field will be presented. The setup for doing the magnetization measurements will be treated extensively.

3.1 PL setup

3.1.1 PL setup in absence of magnetic field

In figure 3.1 the schematic representation of the PL setup is given. The PL measurements in absence of magnetic field were performed using a Nd:YAG laser. This is a green laser, working at a wavelength of 532nm and having a power of 25mW. The laser light passes a low pass filter F1, which filters out higher harmonics of the excitation laser. Lens L1 is used to collimate the laser beam. The collimated beam is reflected by a mirror on the sample. The spot size on the sample is approximately $4.2\, \text{mm}^2$, resulting in an excitation density of approximately $500\, \text{mW/cm}^2$. In the sample the laser excites electrons to the conduction band, leaving holes in the valence band. The majority of these excited carriers relax via the wetting layer into the dots/rings. Here recombination takes place and PL is emitted from the sample. Part of this PL (together with some reflected laser light) is collected and collimated by lens L2 and reflected by a gold mirror onto a filter F2. Here the remaining laser light is filtered out. The PL is focussed on the grating of the detector by focus lens L3. Via the grating the PL is projected on an InGaAs photodiode array. The array contains 512 pixels, each with a size of $50 \times 500\, \mu\text{m}$. The detector has a quantum efficiency (QE) larger than 80% in the near infrared (NIR) region for wavelengths between 0.8-1.6µm. The InGaAs detector is connected with a computer. Winspec software and hardware is used to analyse the data.

At various temperatures, down to approximately 5K, PL can be collected. The sample is placed in the inner chamber of the setup (see figure 3.1). In the outer chamber there is a vacuum ($\sim 10^{-7}\, \text{mbar}$) to isolate the system. In the inner chamber there is He gas functioning as contact gas. Liquid helium is transported from a Helium dewar, through the inner chamber where it cools the sample, to a heat exchanger. This heat exchanger is used to adjust the flux of liquid helium. Furthermore this heat exchanger is connect to a resistance in the
Figure 3.1: Schematic representation of the PL setup. A laser (green beam) excited carriers in a sample. The resulting PL (red beam) is measured with the use of an InGaAs detector.
inner chamber. By sending a current through this resistance the inner chamber is heated. Finally, the liquid Helium is converted into Helium gas and collected by an collection system.

### 3.1.2 PL in a magnetic field

The PL-setup in the presence of a magnetic field resembles the one without a magnetic field and is schematically shown in figure 3.2. A HeNe-laser, $\lambda=632.8\text{nm}$, is used as excitation laser. The laser is reflected via a beam splitter to objective lens L1. Here the beam is focussed on to the sample. The resulting PL is collected and collimated by the same lens L1 and send to an aluminum mirror. The PL reflects off this mirror through lens L2, which focusses the beam on the grid of the monochromator, which has 300 grooves/mm. The slit width of the monochromator was held constant on $50\mu\text{m}$. A band-pass filter is placed in front of the monochromator.
of the monochromator to filter out the remaining laser light. The total length of the monochromator is 30cm. The PL is detected with a silicon CCD detector. This CCD detector has 512x512 pixels with a pixel size of 24μm x 24μm. The resolution of the setup is 0.24nm at a wavelength of 1000nm, which corresponds to a resolution of 0.3meV in energy. The typical size of the excitation spot is ~1μm in diameter.

The sample is placed in the center of a bitter magnet. This magnet can apply fields up to 29T. The sample can be displaced in the horizontal plane using two screws implemented on top of the insert tube, which allows position dependent excitation experiments. Furthermore, the excitation density can be reduced by placing a HC-filter in front of the laser. This way the dependence of the photoluminescence on the excitation density is measured.

3.2 Torque Magnetometry setup

The magnetization measurements are performed at the High Field Magnetic Laboratory¹, which is part of the Radboud University Nijmegen. The HFML is committed to research in the highest available magnetic fields; up to 33T continuous field and 60T pulsed field.

A torque magnetometer is used to measure the magnetization. The principle is based on a torque induced by a magnetic moment in a magnetic field. This torque is translated into a displacement in the form of a rotation, which is detected. The torque \( \vec{\tau} \) experienced by a magnetic moment \( \vec{M} \) in a magnetic field \( \vec{B} \) is given by 3.1:

\[
\vec{\tau} = \vec{M} \times \vec{B} + \vec{r} \times (\vec{M} \cdot \vec{\nabla})\vec{B},
\]

(3.1)

with \( \vec{r} \) the distance from the rotation axis to a place on the sample. When the sample is mounted on the rotation axis and placed in a homogeneous magnetic field, the second term in equation 3.1 vanishes². A torque will only be detected if the sample is anisotropically magnetized at an angle with the magnetic field. The larger the angle of the sample normal with respect to the magnetic field, the higher the torque. However, implementing the sample under an angle lead to an in plane magnetic field, whereas in most experiments a magnetic field perpendicular to the sample is preferred. For the used setup, an optimized situation is obtained for an angle of 10° of the sample normal with respect to the magnetic field. Figure 3.3 gives a schematic representation of this angle and the setup.

The sample is mounted on a 20-mm-long phosphorbronze wire, which is stretched between two epoxy posts. The torque is translated into rotation using equation 3.2:

\[
\vec{\tau} = \left( \frac{\pi R^4 G}{2l} \right),
\]

(3.2)

where \( R \) is the radius of the wire, \( G \) is the torsional spring constant of the material and \( l \) is the length of the wire. The radius of the wire is approximately

¹web address:www.hfml.ru.nl

²An homogeneous magnetic field leads to \( \vec{\nabla} \vec{B} = 0 \). In addition \( \vec{\tau} = 0 \) if the sample is mounted on the rotation axis.
Figure 3.3: Schematic representation of the Torque magnetometer setup. A laser is reflected from the sample onto a quadrant detector. The torque induced by a magnetization in a magnetic field displaces the sample and thus the spot on the detector. The feedback coil is indicated with the dashed circle underneath the sample.
12.5µm, which leads to a rotation of $1.4 \times 10^{-4}$ degrees/pNm. The torque wire is glued into a removable part of the magnetometer to allow easy access for sample mounting.

As shown in figure 3.3 the rotation of the sample is measured using a laser. A laserdiode with a wavelength of 790nm and variable intensity is coupled to a fiber which enters the magnetometer. Here the light leaves the fibre and is focussed on a quadrant detector using a mobile lens. The system can be aligned using two adjustable mirrors. The first mirror reflects the laser on to the sample and the second mirror reflects the beam coming from the backside of the sample onto the quadrant detector. In order to make the backside of the sample reflective, an aluminum coated glass plate is mounted underneath the sample. The quadrant detector consists of four fibres. Each of these fibres is connected to four identical silicon diodes, which are used to determine the intensity distribution of the laser among the fibres. From this intensity distribution a normalized coordinate of the laser spot is deduced, which is independent of fluctuations in the total laser intensity. Furthermore the separation between the detection fibres is 1mm. The quadrant detector can resolve rotations as small as $3 \times 10^{-6}$ degrees.

A feedback based on a counter magnetization is build in the system. A small coil is mounted underneath the sample. This feedback coil is connected with a current source. Letting a current flow through the feedback coil induces a magnetization given by 3.3:

$$|M| = NI_A,$$

(3.3)

where $N$ is the number of windings, $I$ the current through the coil and $A$ the area of the coil. This feedback current is regulated such that the normalized coordinate is kept constant. Operation in this feedback mode has thus the important advantage that by keeping the normalized coordinate constant, it immediately gives the magnitude of the magnetization from the dimensions and number of windings of the coil. We used a coil of diameter 6mm made from 18µm Cu wire. A photo of the actual magnetometer is given in figure 3.4. Here the separate parts of the setup are indicated.

Unfortunately the movement of the sample is not only caused by the magnetization. Mechanical noise is also influencing the motion limiting the sensitivity of this setup. Generally, it is necessary to use an active feedback to damp the unwanted additional motion of the magnetometer and it is not sufficient to just filter out the mechanical noise. Touching the cryostat typically induces two types of movement. A slow movement of approximately 7Hz belonging to the rotational eigenfrequency of the system and a fast movement of approximately 117Hz belonging to the vertical vibration eigenfrequency of the system. The vibrational motions usually damp in seconds and are not always present whereas the rotational motions are never completely absent and take minutes to dampen. The active feedback should therefore be aimed at damping this motion. The feedback current is generated by a proportional-integral-differential controller (PID). The input of this PID controller is the normalized position $X$, consisting of a DC component we wish to measure and two AC components with the

3 Gold has a higher reflectivity, but is relatively easy magnetized leading to an interference with the magnetization signal.
frequencies of the two different eigenfrequencies of the system. More details of the feedback circuit can be found in [24].

A superconducting coil is used as magnet to reach fields up to 15.5T. This magnet is located in the cryostat. In the middle of the cryostat another cryostat is placed, the anti-cryostat. A tube which contains the magnetometer and contact gas is placed in the center of this anti-cryostat. Figure 3.5 shows a schematic representation of the complete setup.

The cryostat is pre-cooled with liquid nitrogen. The liquid nitrogen is subsequently blown out with nitrogen gas after which the cryostat if filled with liquid helium. At this temperature the superconducting magnet functions. Moreover the anti-cryostat is already pre-cooled by the cryostat and thus can be filled directly with liquid helium. The contact gas makes sure the sample is also cooled to this temperature. By decreasing the pressure in the anti-cryostat temperatures can be reached as low as 1.2K.

The sensitivity of this setup is $7 \cdot 10^{11} \mu_B$, obtained from the experiments discussed in the next chapter. More details of the setup are given in [24].
Figure 3.5: A schematic representation of the total magnetometer setup. The actual torque magnetometer is located in the center of the magnet. The laser light is detected by a quadrant detector. This quadrant detector calculates the normalized position X, which is used as input for the active feedback controllers. The output of the active feedback controllers is the feedback current.
Chapter 4

Results and Discussion

In this chapter the results obtained on different structures are discussed. A scala of measurements are performed, consisting of optical, magnetization and scanning probe microscopy measurements. These measurements are supplementary and provide a profound analysis of the structures examined. In the first section, different layer structures will be discussed and characteristics of the band structures will be derived using a one dimensional self-consistent Poisson equation and Schrödinger equation solver. These structures will be analyzed by AFM measurements in the second section. To obtain additional information on the size distribution of the nanostructures in the samples, optical experiments are performed. These optical experiments consist of photoluminescence measurements in the absence and presence of a magnetic field, respectively in section 3 and 6. In section 4 the torque magnetometer setup magnetization measurements on in total four different samples are presented and discussed. On two of the samples Shubnikov-De Haas measurements are performed as discussed in section 5.

4.1 Sample design and analysis using an one-dimensional self-consistent Schrödinger-Poisson solver

4.1.1 Quantum ring sample 1

The first sample used in the experiments consisted of 20 layers of QRs grown on a doped substrate. It was grown by Granados et al.\(^1\) with the intention to perform Cross-sectional Scanning Tunnelling Microscopy measurements on it. The layer structure is given in figure 4.1. The spacing between the different QR layers is 18nm. According to previous experiments [27], this spacing is large enough to decouple the different QR layers. The substrate is N++ doped in order to perform X-STM measurements in which the conductivity of the sample plays an important role. The doping in between the QR layers provides the rings with electrons. On the surface another layer of QRs is grown which enables the characterization of the sample using AFM.

\(^1\)D.Granados, J.M. García et al. from Instituto de Microelectrónica de Madrid.
With the use of a self-consistent Schrödinger-Poisson solver we analyse the band structure of this sample, see [28], [29] and [30]. The QRs are simulated using two different models. The first model approaches the QRs as a quantum well (QW) layer of a 2nm thick $\text{In}_{0.6}\text{Ga}_{0.4}\text{As}$ layer. The used concentration of Indium is obtained from X-STM measurements [13]. The band structure is shown in figure 4.2. The Fermi level in the structure is constant since the situation is shown for the equilibrium situation. The Fermi level crosses the QWs nearest to the substrate, but is lower than the QWs near the surface. This indicates that not all QRs will be occupied by the electrons provided by the doped layers. Figure 4.3 shows the free electron concentration in the QWs, which confirms this prediction. Only ten of the twenty QW layers contain electrons.

One clear disadvantage of the model is that QWs can confine large amounts of electrons. On the other hand, QRs have a limited amount of electrons they can confine before the electrons will begin to occupy the wetting layer.

The second method to describe the QRs is to describe them as deep donors. The typical binding energy of an electron in a QR is $200\text{meV}^2$. In this representation it is not possible to have more than one electron per quantum ring. As a result, the real QR system is probably in between the QW and deep donor approximation. The QR density of this sample is estimated from AFM measurements, which lead to a density of $n_{QR}=7\cdot10^9\text{cm}^{-2}$. The average number of electrons per ring is estimated to be two, which combined with $n_{QR}$ leads$^3$ to a

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{figure4.png}
\caption{Layer structure of quantum ring sample 1. It consists of 20 layers of QRs which are covered with GaAs and one additional layer of QRs in order to perform AFM. In between are doping layers to provide the rings with electrons.}
\end{figure}

\begin{itemize}
\item QR = QD + 2 nm GaAs
\item $x20$
\item QR = QD + 2 nm GaAs
\item $11 \text{nm} \text{GaAs}$
\item $9 \text{nm} \text{GaAs}$
\item $2 \text{nm} \text{GaAs:Si (1x10^{17})}$
\item $7 \text{nm} \text{GaAs}$
\item $9 \text{nm} \text{GaAs}$
\item $2 \text{nm} \text{GaAs:Si (1x10^{17})}$
\item $9 \text{nm} \text{GaAs}$
\item $85 \text{nm} \text{GaAs}$
\item $170 \text{nm} \text{GaAs:Si (3x10^{19})}$
\item \text{GaAs:Si Wafer N$^{++}$}
\end{itemize}

$^2$Derived from calculation shown in paragraph 2.3.

$^3$Thickness of the deep donor layer is 2nm.
Figure 4.2: The QR layers are modelled as In$_{0.6}$Ga$_{0.4}$As QWs. The conduction and valence bands, $E_c$ and $E_v$ respectively, as function of position are shown. The surface of the sample corresponds to a thickness of 0 and the substrate corresponds to large thicknesses. The calculated Fermi level $E_F$ indicated.

Figure 4.3: The electron density as function of position in the sample. Only ten of the twenty QW layers contain electrons.
Figure 4.4: Ionized deep donor concentration $n_D$ as function of position in the sample. This model shows that 12 layers of donors, which represent the QRs, are not or only partially filled with electrons.

The ionized deep donor concentration of $n_{dd} = 7 \cdot 10^{16} \text{cm}^{-3}$. In figure 4.4 the ionization profile of the deep donors is shown as function of position in the sample. Depletion near the surface causes 11 of the 20 deep donor layers to be fully ionized and one additional layer to be partially ionized, meaning that most of the modelled QR layers do not confine electrons. An additional effect is the presence of a depletion region in between the undoped GaAs layer and the n-doped GaAs layer.

Both methods, although extremes, conclude that at least half of the layers containing QRs are not filled with electrons. This is mainly due to the depletion near the surface. For doing the magnetization experiments, it is crucial to fill all the layers of rings, since the signal caused by the electrons in the rings will be critical to detect. In the next subparagraph a new sample is designed in order to achieve the filling of all the layers of rings with electrons.

4.1.2 Quantum ring sample 2

Using the self-consistent Schrödinger-Poisson solver a new sample is designed. The layer structure is shown in figure 4.5. The sample is optimized for AB measurements. The doped substrate is replaced by a normal GaAs substrate. This way the electrons in the substrate will not interfere with AB oscillations. The substrate contains as less chromium (Cr) as possible, since Cr is a magnetic impurity. To account for the depletion of the surface, the top layer region at the surface is made larger, 1µm, compared with the QR sample 1, 11nm. The donor concentration of the doping layers in between the QR layers are determined from the QR density of previous grown samples. We assume a QR density of $n_{QR} = 7 \cdot 10^9 \text{cm}^{-2}$. We are aiming for 2 electrons per ring, which
Figure 4.5: The structure consists of 29 capped QR layers and 1 surface layer of QRs. In between the capped QR layers, doping layers are grown to provide the rings with approximately 2 electrons. Beneath and above the serie of QR layers additional doping layers with a higher concentration are grown, the so-called boarder layers.

leads to a donor concentration of \( n_D = 7 \times 10^{16} \text{cm}^{-3} \) for a 2nm thick doping layer. Moreover the number of layers containing QRs is increased from 20 to 29, which will increase the possible magnetization signal by a factor of 1.5 compared to sample 1. We also choose for a larger spacing layer of 50nm between the ring layers, to exclude any coupling between the QR layers in this heavily strained sample. Additional, there is a doped layer above and underneath the QR layers, the outer most doping layers, with a higher concentration as the ones in between the QR layers. These concentrations are to be carefully determined. If these donor concentration are too low, the conduction and valence band will bend upwards in the region where the QR layers are present. This results in the same effect as shown in figure 4.2 of the previous subparagraph. The Fermi level will not be crossing all the potential wells caused by the QRs, resulting in an insufficient occupation of these QR layers. However, doping the two outer most doping layers too heavily will increase the number of electrons per QR. It can be estimated that a population of more than 2 electrons per ring will lead to a decrease in the magnetization signal.

The exact donor concentration of the outer most doping layers is determined from the two models of the previous paragraph. In figure 4.6 the band diagram is shown for the layer structure of figure 4.5 using the model in which the QRs are simulated using QWs. The Fermi level is crossing all the QWs, resulting in an occupation of every QW. For a donor concentration of the outer most doping layers of \( n_{outer} = 1.6 \times 10^{17} \text{cm}^{-3} \) there are approximately 2 electrons in each ring, corresponding with a concentration of \( n_D = 7 \times 10^{16} \text{cm}^{-3} \), see figure 4.7.
Figure 4.6: The QWs are used to simulate the QR layers. The Fermi level lies in the QWs, resulting into a filling of the QW layer with electrons from the doped layers in between the stacked QW layers. This is shown in figure 4.7.

Figure 4.7: The QWs are filled with electrons from the doping layers. An electron density of the outer most doping layers of $n_{outer} = 1.6 \cdot 10^{17} \text{cm}^{-3}$ results in a electron density in the QWs of approximately $n_D = 7 \cdot 10^{16} \text{cm}^{-3}$, corresponding with 2 electrons per ring.
The simulation of QRs with deep donors lead to a similar figure as figure 4.4. The concentration \( n_{outer} \) is chosen such that all donors confine their electrons, leading to the absence of an ionized deep donor concentration. This simulation also leads to a value of \( n_{outer} = 1.6 \times 10^{17} \text{cm}^{-3} \), which therefore is used in the sample design.

The sample is grown by García et al. Unfortunately the wafer was not rotated during growth resulting into a gradient in the distribution of several elements from the different cells in the MBE. Keeping this in mind we will use two samples from different part of the wafer and show the similarities and differences between the two samples throughout the subsequent part of this chapter.

4.2 Atomic Force Microscopy measurements

Scanning Probe Microscopy (SPM) Techniques such as AFM and X-STM are indispensable in determining the structural information of nanostructures. AFM is usually used as a first tool to investigate the global properties of the sample structures. The AFM is often used as tool to verify that the growth process went as planned and the samples are of good quality. The presented AFM measurements were performed using the AFM Digital Nanoscope III part of the Dimension™ 3100 Series from Digital Instruments. The setup is vibration isolated from the surroundings using a damping table which is decoupled from the building by a damping platform.

4.2.1 AFM on quantum ring sample 1

Figure 4.8 shows an AFM image of QRs on sample 1. In the image in the background the atomic steps of different plateaus are resolved. The AFM shows QRs that are elongated in the [110] direction. The preferable immigration of In in the [110] direction during the formation of the rings leads to the elongation in this direction. Furthermore there is a variation of the height of the rim of the ring. The model used by Fomin et al. in section 2.3.3 is based on the structural analysis, performed with X-STM, of these rings and reflect these asymmetries. The density of the QRs is determined by counting the QRs in different AFM images and dividing them by the scanned area. The density of this sample is determined on \( n_{QR} = 7 \times 10^9 \text{cm}^{-2} \). The outer size of the QRs is about 100 by 70 nm and an average height of 1 nm. The holes in the center of the islands are asymmetric as well with a size of 30 by 20 nm and a depth of about 0.5-1.5 nm.

Remark that the asymmetry of the depression in the center is opposite to the asymmetry of the ring. No explanation for this phenomenon has been given yet in literature. One way to explain this is to neglect migration of In in the [110] direction during the QR formation progress. In the center of the dot the indium concentration is highest \([19]\), leading to a locally high strain. Here the migration happens on a faster rate than away from the center of the QR, where is less indium. Assuming such a migration rate profile leaves a depression in the center with opposite asymmetry of the ring. AFM measurements on this sample are also published \([13]\).
Figure 4.8: The left figure shows an AFM image of QRs on sample 1. The scanned area is 2.17µm x 2.17µm and the z-range is 3.2nm. The rings are elongated along the [110] direction. The image on the right shows a zoom of the area indicated in the left figure. The zoom has a length and width of 0.35µm x 0.32µm.

4.2.2 AFM on quantum ring sample 2

The measurements on QR sample 2 are performed on two samples from different parts of the wafer. These two samples contain different QR structures due to the growth without rotation. From now on the first sample will be labelled as QR sample 2A and the second sample as QR sample 2B.

AFM on quantum ring sample 2A

Figure 4.9 shows an AFM image of QRs on sample 2A. From one of the rings the cross-section is shown, which is a typical cross-section of the rings in this area. The rings have a symmetric round shape in contrast with the rings showed in figure 4.8. The rings do have an undulation of height across the rim. The typical sizes, obtained by taking height profiles of several rings, are 75-95nm in outer diameter. The inner diameter is 30-40nm and the depth of the depression in the center of the ring is typically 2nm. The QR density of this sample is determined on \( n_{QR} = 6 \cdot 10^9 \text{cm}^{-2} \), comparable with the QR density of sample 1.

AFM on quantum ring sample 2B

An AFM image of the QRs on sample 2B are shown in figure 4.9. Typical sizes are a 0.8nm for the depression in the center. The outer diameter of the QV is 55-75nm and the inner diameter is 20-25nm. The QR density of this sample is determined on \( n_{QR} = 9 \cdot 10^9 \text{cm}^{-2} \). The depression in the center of the QRs of sample 2B is less pronounced as the depression in the center of the QRs of
Figure 4.9: The upper part of the figure shows the AFM image of sample 2A. The z-range is 6.4nm. The scanned area is 1.07µm×1.07µm. As can be seen, the height varies across the rim of the ring, introducing an asymmetric shape. The height profile of one of the rings is shown in the right figure. A clear depression in the center of the ring is shown. The lower part of the figure shows AFM image of QRs on sample 2B. The z-range is 3.5nm and the scanned area is 1µm×1µm. The right figure shows the height profile of the ring indicated in the left figure. The rings show a more shallow depression in the center of the ring compared with the rings of sample 2A.
Figure 4.10: PL spectrum of quantum ring sample 1. The peak consists of several peaks. The exposure time is 0.3s and the excitation density is 0.5W/cm² and the slit width is 250µm. The spectrum is taken at T=5K using a grating of 200 grooves/mm with a blaze of 1.7µm.

sample 2A. This might due to the growth without rotation of the wafer. The growth process is slightly different as function of position on the sample, which could have lead to a more pronounced eruption of material from the dots on sample 2A compared with the dots on sample 2B. However, further analysis with X-STM would be needed to be conclusive, as AFM does not reveal the actual sizes of the buried rings underneath the eruption material, see paragraph 2.2.

4.3 Photoluminescence measurements in the absence of a magnetic field

The result presented in this section are based on experiments performed with both PL setups discussed in chapter 3 in the absence of magnetic fields.

4.3.1 PL measurements on quantum ring sample 1

The spectra of this sample are obtained using the setup explained in paragraph 3.1.1. Figure 4.10 shows the PL spectrum as function of wavelength and energy. The PL-profile consists of a strong peak at 1.36eV and a shoulder that comprises weaker peaks on the low energy side. All peaks have a Gaussian like distribution. This Gaussian distribution is the direct consequence of the inhomogeneous size distribution of the QRs; this broadening of the peak is therefore due to inhomogeneous broadening. The peak with highest intensity is on the
Figure 4.11: PL spectrum of quantum ring sample 1 as function of excitation density. The shape of the peak is independent of the excitation density. The different peaks are indicated by the arrows. They probably belong to QRs with different height. The wetting layer is also indicated. The exposure time is 10s and the excitation density for ND=0 is 0.5W/cm². The slit width is minimized and the spectra are taken at T=5K using a grating of 200 grooves/mm with a blaze of 1.7µm.

The high energy side of the spectrum at an energy of 1.36eV corresponding with a wavelength of 911nm. These wavelengths correspond to the typical wavelength found for self-assembled InAs QRs [31],[27]. In order to determine the origin of the peaks, we measured the excitation density dependence of the spectra, shown in figure 4.11. The excitation density is changed by ND filters. The ND-factor of the filters corresponds to a reduced excitation density with a factor of $10^{ND}$. The slit width is minimized\(^4\) in order to obtain maximal resolution.

The shape of the spectra do not depend on the excitation density. This means that the additional peaks are not due to excited states. Moreover excited states should appear on the higher energy side of the main peak and we obtain a shoulder on the lower energy side. The peaks can most probably be described to QRs with different heights\(^5\). This multimodal size distribution in the height is reported recently for InAs/GaAs self-assembled QDs [32], [33] and QRs [34].

\(^4\)The exact slit width could not be determined. The screw determining the slit width was set on zero, but light is coming through anyway.

\(^5\)The height mainly determines the confinement energy as the height has smaller dimensions compared to the lateral sizes of the QRs.
Figure 4.12: PL of QR sample 1 as function of temperature. The exposure time is 10s and the excitation density is 0.5W/cm² and the slit width is minimized. A grating of 200 grooves/mm with a blaze of 1.7µm is used. The spectrum taken at T=264K is taken with a slit width of 250µm.

The rings with smallest height have the highest confinement energies and correspond to the peak at 1.36eV. With the increase of the height of the ring, the confinement energy decreases and the second peak at 1.29eV is obtained. Further increase of height leads to peaks at 1.26eV and 1.23eV.

Figure 4.12 shows the temperature dependence of the PL. The peak shifts to lower energies. This red-shift is a result of the decrease of the bandgap energy with temperature. The decrease of the PL intensity with the increase of temperature is due to the interaction of electrons and holes with phonons. At low temperatures the phonons states are not populated, but their at higher temperatures is pronounced. At room temperature phonon assisted recombination is most important and the PL originating from recombination of electrons and holes in the rings is strongly suppressed.

The change of the shape of the PL peak is remarkable. This is explained by a redistribution over the electrons over the different rings. Whereas the rings with a smaller height have highest intensity at lower temperature, the larger rings are mainly contributing to the PL in the higher temperature regime (compare T=85K and T=120K in figure 4.12); a redistribution takes place of electrons from the low QRs to the high QRs. At low temperatures the electrons are confined in the low rings which comprise the majority of the rings. The electrons need additional energy to overcome the confinement barrier of the low rings in order to hop to the higher rings where they can further reduce their energy. This hopping is thermally assisted and is a well-known process for inhomogeneously broadened QD distributions [35].
4.3.2 PL measurements on quantum ring sample 2A

In figure 4.13 the PL spectrum is shown at full excitation intensity using the setup described in paragraph 3.1.2. The PL-profile consists of two separate peaks. As is shown in figure 4.33 in paragraph 5.5.1, the peak at higher energy disappears with decreasing excitation density, characteristic for PL from excited states. We therefore conclude that the peak corresponding to the excited states of the exciton is centered at 1.350eV. The peak centered at 1.303eV (950nm) corresponds to the recombination of excitons in the ground state. This PL-peak does not reflect a multimodal size distribution as in sample 1. The recombination energies of the rings in this sample are lower compared to the QRs in sample 1. These smaller energies are a consequence of the smaller confinement of the electrons in these types of QRs due to size differences.

An energy level can be populated by only two electrons (holes) with opposite spin as a consequence of the Pauli principle. There already are 2 electrons in the ring due to the doping. To have a transition between the excited states we need a hole and an electron in the excited level. The excited state consists thus at least of 3 holes and 5 electrons. Complicated PL spectra result from such many particle systems [36], [37] and [38]. The energy splitting between the hole levels is smaller compared to the energy splitting of the electron levels because of the larger effective mass of the holes compared to the effective mass of the electrons. The energy splitting between the ground state and the excited state, 48meV, can thus be assigned due to energy splitting of the electron levels. In
The splitting between the ground state PL (fitted with Lorentzian 1) and the excited state PL (fitted with Lorentzian 2) is 38meV.

The calculation are performed using a single electron model. It is known that the energy levels can change drastically, when coulomb interaction are taken into account [39], [40]. The effect of the repulsive coulomb interaction between electrons increases the separation of the energy levels. This effect might account for difference in the energy splitting of ground state and excited state calculated in theory and measured by experiment.

4.3.3 PL measurements on quantum ring sample 2B

The PL-peak at the high energy in figure 4.14 disappears when using lower excitation densities, as shown in figure 4.35 in paragraph 4.5.2, indicating this peak is a result of the recombination of excited state excitons. The peak at 1.22eV corresponds to the recombination of the ground state excitons. This PL energy is lower compared to the PL energy of sample 2A. This might be related to the difference in size and shape observed by AFM measurements. The AFM measurements showed a more shallow depression of the rings in sample 2B as compared to sample 2A. Therefore the rings on sample 2B are more dot-like\(^6\) corresponding to lower energy PL. We will show later that this difference is mainly related to a difference in height of the rim between QR sample 2A and 2B.

\(^6\)InAs on GaAs self-assembled dots show luminescence at lower energy (PL energy typically at \(E=1.1eV\)) due to lower confinement in dots compared to rings (PL energy typically at \(E=1.35eV\)[31].
Figure 4.15: The PL spectrum of sample 2B for different temperatures. The PL intensity is on logarithmic scale. A red-shift of the PL peak is observed. The shape of the peak does not change indicating an unimodal distribution of the height of the rings. This has also been observed for sample 2A.

Since the rings on sample 2B have less confinement as compared to the rings on sample 2A, this should result in a reduction of the energy level splittings. This is the case since the energy splitting between the ground state and the first excited state is 38meV, which is 10meV less as compared to sample 2A.

Figure 4.15 shows the temperature dependence of the PL-peak. Similar temperature dependence is observed for sample 2A. In contrast to sample 1 the shape of the PL-profile does not change. Apparently this sample contains an unimodal distribution in the height of QRs since no redistribution of electrons to higher rings is observed as was the case for sample 1.

4.4 Torque magnetometer measurements

In this section the actual magnetization measurements are discussed. We will show that sample 1 is not suitable due to an incorrect design. In sample 2A and 2B we used a properly designed structure that indeed showed magnetization related to the rings in the sample.

\[ \text{This can be derived using the QW analogue. When increasing the width of the QW, the splitting between the energy levels is reduced.} \]
4.4.1 Magnetization measurements on sample 1

Sample 1 is polished to a thickness of 200-250μm. This is done in order to reduce the magnetization caused by the substrate. Moreover, the decrease of sample thickness reduces the weight of the sample. This way the torsion wire will not break due to the weight of the sample. The sample has a length and width of 7.5mm×7.5mm. The total number of rings on this sample is: (20 layers)*(7·10^9 QRs/cm²)*(0.75cm×0.75cm)≈ 8·10^{11} QRs.

In paragraph 4.1.1 it is shown that at most half of the 20 layers contain 2 electrons per ring or less. Assuming 2 electrons per ring we estimate from figure 2.21 that the magnitude of the change in the magnetization signal for one ring approximately 10μB is\(^8\). This leads to a maximal expected jump in the magnetization of 8·10^{12} μB, which is equal to 7.4·10^{-11}J/T using 2.18. The sensitivity reported in [24] is 2·10^{-13}J/T, which should be enough to detect the signal.

Figure 4.16 shows the measured magnetization signal of three different runs. The magnetization is determined by measuring the feedback current\(^9\) (typical signal in the order of several μA). The behavior of the magnetization signal begins unstable followed by a decaying background. At low magnetic fields (B<1T), the system has difficulties regulating the feedback\(^10\). In figure 4.17 the position of the laser spot is shown as function of magnetic field. This figure reflects the instable behavior up to fields of 1T. The normalized position of the laser spot serves as feedback input. Up to 1T the feedback system is not able to regulate the feedback current and thus the sample position is unstable. For fields larger than 1T the feedback is working properly and the obtained data is reliable.

The decaying background has several origins. It is known from other experiments that the background depends for instance on the type of wafer used. A wafer of similar material, for instance GaAs, but from a different manufacturer leads to a different background behavior. Part of the background caused by the substrate is due to the diamagnetic behavior of GaAs as well as magnetic impurities in the GaAs substrate. Furthermore the sample is mounted under an angle in the magnetic field. This leads to a magnetization oriented in the plane of the sample, which is leading to an additional torque. An additional origin of the background is caused by the gradient in the magnetic field in the superconducting magnet. Although the sample is centered in the magnet, every place on the sample experiences a slightly different magnetic field. We expect that this has only a small contribution to the background as our sample has small sizes compared to the lateral gradient in the magnetic field. Another contribution to the background comes from the glue used to attach the sample and the feedback coil on to the torsion wire. The total scala of contributions leads to a smooth background. Unfortunately the background differs from measurement to measurement, which makes it impossible to use a standard procedure in subtracting the background. These

\(^8\)We deal with two electrons per ring. Electron-electron interaction is not taken into account in this approximation. The estimation is done using the curve belonging to a temperature of 4K.
\(^9\)The feedback coil consists of 5 windings.
\(^10\)The feedback current is infinite high at zero fields since the counter torque produced by the feedback current is proportional with the magnetic field (equation 3.1).
Figure 4.16: The magnetization as function of magnetic field for three different runs; one run gives the magnetization by sweeping the magnetic field upward, one run gives the magnetization by sweeping the magnetic field downward and one run gives the magnetization for an upward sweep where the sample is displaced 2.5cm from the center of the magnet. At low field the signal shows some unstable behavior. For magnetic fields larger than 1T there is a decaying background and on top of this background an oscillations appears. These oscillations are the De Haas-Van Alphen oscillations of free carriers in the doped substrate.
Figure 4.17: Normalized position of the laser spot as function of magnetic field. Up to 1T the feedback system is not able to regulate the feedback current leading to an unstable behavior of the normalized position.

types of backgrounds in magnetization experiments are well-known and reported in literature by other groups, for example see [41], [42] and [43].

From figure 4.16 it is observed that for fields larger than 4.5T an oscillation starts. The oscillations have an increasing period and are growing in magnitude. These properties are characteristic for De Haas-Van Alphen oscillations. The upward and downward sweep curves show a good reproducibility. The black curve shows the magnetization after the sample was displaced 2.5cm in the magnet away from the maximum field, which leads to an effectively smaller field. This explains the displacement of the oscillations to higher fields. Figure 4.18 shows the periodicity of the curves by plotting the magnetization as function of the reciprocal magnetic field. Our measurement has striking similarities to the De Haas-Van Alphen oscillations shown in figure 2.26. From figure 4.18 we determine the period of the De Haas-Van Alphen oscillations. The measurement taken during the upward and downward sweeps give a period of $\Delta(1/B)=(0.031\pm0.001)T^{-1}$ and the measurement with the displaced sample gives a period of $\Delta(1/B)=(0.030\pm0.001)T^{-1}$. These periods are equal to one and another to within the uncertainty error. To conclude unambiguously that the De Haas-Van Alphen are due to a 3DEG, we use the equations of paragraph 2.4 and calculate the electron densities, both in the case of a 2DEG or a 3DEG. The results give a 2D electron density of $n_{2D}=6.2\cdot10^{12}\text{cm}^{-2}$ and a 3D electron density of $n_{3D}=1.0\cdot10^{18}\text{cm}^{-3}$.

We now compare the measured electron densities with the sample design (see figure 4.1). The 2DEG electron concentration is a factor of 300 larger as
Figure 4.18: Magnetization as function of the reciprocal magnetic field. There is a clear periodicity of the oscillations with \(1/B\), a characteristic feature of De Haas-Van Alphen oscillations.

expected from the design\(^\text{11}\), which is not likely. The bulk density is in the typical range of doping for a N++ substrate. Both the increase in amplitude of the De Haas-Van Alphen oscillations and the electron concentration lead to the conclusion that the De Haas-Van Alphen oscillation are due to a 3DEG with \(n_{3D}=1.0\cdot10^{18}\text{cm}^{-3}\), most probably caused by the doped substrate.

We conclude that this sample is not suitable for magnetization measurements on the ring structures. The sample is not optimal by design to confine 2 electrons in each ring (see paragraph 5.1.1) and the De Haas-Van Alphen oscillations caused by the doped substrate swamp the possible signal resulting from AB oscillations in the rings.

### 4.4.2 Magnetization measurements on sample 2

Sample 2 is grown on an undoped substrate to prevent the pronounced De Haas-Van Alphen oscillations from free carriers in the substrate. The drawback of sample 2 is that it is probably not suitable for X-STM measurements. In this paragraph an additional sample is introduced. In this sample the epilayers containing the rings have been removed and thus we can directly measure the magnetization of the substrate only.

\(^\text{11}\)The design gives a density of the doping layers of \(1\cdot10^{17}\text{cm}^{-3}\). The thickness of this doping layer is 2nm, resulting in a 2DEG concentration of \(n_{2D}=2\cdot10^{-7}\text{cm}^{-1}\cdot1.10^{17}\text{cm}^{-3}=2\cdot10^{10}\text{cm}^{-2}\).
Figure 4.19: Normalized position X as function of magnetic field. No feedback current is applied. The oscillatory behavior is due to mechanical vibrations of the setup. The overall behavior of the curve is the behavior of the background.

Magnetization measurements on sample 2A

The magnetization measurement is performed on a sample of size 7mm x 8mm and a thickness of approximately 200μm. First a magnetic field is applied up to 5T without regulating the feedback. The normalized position as function of magnetic field is shown in figure 4.19. The normalized position is increasing with increasing magnetic field. The feedback current applied to the feedback coil underneath the sample (see figure 3.3) must be regulated such that the normalized position is kept constant. The feedback current will also suppress the oscillations due to mechanical vibrations of the system, therefore the feedback current is sensitive to external vibrations. Figure 4.20 shows the steps in normalized position X as function of a DC current applied directly to the feedback coil. From this figure it is clear that an increase in the feedback current corresponds with an increase of the normalized position X. The feedback will thus apply a decreasing feedback current to counteract the increasing normalized position shown in figure 4.19.

The feedback current as function of magnetic field is shown in figure 4.21. As we see in the inset, the normalized position is kept constant for fields larger as 1T. For fields lower as 1T the X is unstable and therefore the data is not reliable, which is reflected in a strange behavior of the feedback current. For a magnetic field of approximately 13T an oscillation is observed.

To convert the feedback current into a magnetization one has to look care-
Figure 4.20: Normalized position X as function of a DC current sent through the feedback coil at B=5T and T=4.2K. A step in the feedback current corresponds to a step in the normalized position.

Figure 4.21: Feedback current as function of the magnetic field. An oscillation on top of a background is observed at approximately 13T. The inset shows the normalized position as function of magnetic field. The feedback is regulated such that at higher fields as 1T the normalized position is kept constant.
fully into the geometry of the system. The magnetometer geometry states that an increase in $X$ reflects a decrease in the tilt angle, the angle between the normal of the sample and the magnetic field. When the total magnetization of the system, consisting of a background and the magnetization of the sample, increases, the increased torque makes the sample rotate to a smaller tilt angle and thus to an increase in $X$. The feedback system counteracts this rotation by sending a smaller feedback current to increase the tilt angle back to its original value and keeping $X$ constant, see figure 4.20. Therefore an increase in sample magnetization is characterized by a decrease of the feedback current as observed in 4.21. The magnetization can thus be obtained by multiplying the feedback current with $-1$. This is in correspondence with figure 4.19. Figure 4.22 shows the magnetization in units of Bohr magneton as function of the magnetic field.

In order to study the magnetization in more detail we subtract a phenomenological background. Figure 4.23 shows the magnetization of different sessions after an exponential background subtraction. This exponential background is obtained by fitting the magnetization signal in the range of 3-11T. The fitting parameters obtained by this procedure are used to extrapolate this exponential background to the whole range. It may be clear that this type of background correction has to be used carefully as it can induce artefacts in the final curves. The figure shows magnetization curves measured at two different temperatures, 4.2K and 1.2K. In two measurements the intensity of the laser is doubled to see

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\[ \text{In the measurements on sample 2 the feedback coil consists of 2 windings. Via equation 3.3 we can calculate the magnetization.} \]
Figure 4.23: Magnetization after exponential background subtraction. A small dip is followed by a large peak at approximately 14.5T. The curves are reproducible and independent on temperature and laser intensity.

A distinct peak at 14.2T is observed in all measurements. The value of the magnetic field halfway the rise and the peak corresponds with the critical field where the ground state changes its angular momentum quantum number $m$ (see figure 2.21). It directly reflects the occurrence of the AB-effect in the persistent current. The measured critical or transition magnetic field $B_{trans}$ is $B_{trans}=13.4T$. The average magnitude of the peak is $(1.3\pm0.7)\times10^{13}\mu_B$, measured from the value at $B=8T$ to the top of the peak.\(^{15}\)

In order to give an analytical description of the background, a physical understanding of this background is needed. As discussed in the previous paragraph the background has many origins. Therefore it is impossible to justify any approach which subtracts an analytical background based on a physical understanding from the raw data. A straightforward method of avoiding any influence on the magnetization caused by photo induced carriers\(^{14}\).

\(^{14}\)The laser does not directly excite carriers in the sample since it is reflected by an aluminium mirror beneath the sample. However, scattered light can reach the sample.

\(^{15}\)This average is calculated without taking into account the largest and the smallest peak.
Figure 4.24: Panel (a) shows the theoretical differential magnetization curves for different temperatures. The theoretical calculations are based on figure 2.21. A distinct peak is obtained at a magnetic field corresponding to the transition field $B_{\text{trans}}$. Panel (b) shows the experimental differential magnetization curves for the magnetization measurements at $T=1.2K$. Panel (c) shows the experimental differential magnetization curves for the magnetization measurements with doubled laser intensity (also at $T=1.2K$). The differential magnetization for the measurements at 4.2K is shown in panel (d).
background subtraction uses the differential magnetization, where we calculate
the derivative of the magnetization with respect to the magnetic field. Panel
(a) of figure 4.24 shows the theoretical differential magnetization at different
temperatures. The theoretical curve shows a clear feature of the first AB os­
cillation in the magnetization. A peak in the differential magnetization occurs
at the position of the first AB oscillation. The peak smoothes with increasing
temperature and finally disappears for \( T=20 \text{K} \).

In order to show the differential magnetization of the experimental magne­
tization curves a smoothing procedure is used\(^{16}\). Figure 4.24 shows the differ­
ential magnetization curves for \( T=1.2 \text{K} \) (panel (b)), \( T=1.2 \text{K} \) measured with
doubled laser intensity (panel (c)) and the differential magnetization measured
at \( T=4.2 \text{K} \) (panel (d)).

**Magnetization measurements on sample 2B**

Figure 4.25 shows the normalized position \( X \) as function of magnetic field with­
out the regulation of the feedback current. The background has an opposite
color compared to the sample 2A. We have no explanation for this opposite
behavior. It is well-known the background can deviate strongly from sample to
sample.

The contacts of the feedback coil are connected the other way round for
sample 2B. The same feedback current as in figure 4.20 leads to an opposite
rotation of the sample, as shown in figure 4.25. The magnetization of sample 2B
has the same sign as the feedback current and can therefore directly be obtained.
Figure 4.27 shows the feedback current, which is related to the magnetization,
as function of magnetic field. Two maxima are just visible at high fields.

In the measurements on the sample 2B the background showed a linear be­
havior between 6-8T. In this range the background is fitted and extrapolated to
higher fields. Subtracting this background from the raw data gives the sample
magnetization, shown in figure 4.28 for different runs. The main features are in
agreement with the sample 2B, however there is now a more pronounced
structure before the high peak. The origin of this structure is still unknown.
The transition field at which the AB effect is in resonance (at the crossing of
the \( m=0 \) and \( m=-1 \) state) is determined on \( B_{\text{trans}}=12.8 \text{T} \). The magnitude of
the jump of the magnetization is \( 1.8 \cdot 10^{13} \mu_B \), measured from the value of the
magnetization at \( B=8 \text{T} \) to the top of the peak at 14T.

Figure 4.29 shows the differential magnetization of sample 2B. The char­
acteristic peak predicted by theory is observed in the experimental curve. However,
there is more structure present just before the characteristic peak. This structure,
both observed in magnetization and the differential magnetization is also
present in the differential magnetization of sample 2A at \( T=4.2 \text{K} \).

\(^{16}\)This is necessary since small noise signal on the raw magnetization signal lead to large
features in the derivative. The used smoothing procedure first smoothes the magnetization
data by using an adjacent averaging procedure over 200 points. Then we calculate the differ­
ential magnetization. This curve is than smoothed with an adjacent averaging procedure over
5 points.
Figure 4.25: Normalized position X as function of magnetic field. No feedback current is applied. The overall behavior of the curve is the behavior of the background, which has an opposite character as compared to the QR sample.

Figure 4.26: Normalized position X as function of a DC current sent through the feedback coil at B=5T and T=4.2K. A step in the feedback current corresponds to a step in the normalized position opposite to the QR sample. Remark that the feedback is not regulated.
Figure 4.27: Feedback current as function of the magnetic field. An oscillation on top of a background is observed at approximately 13T. The inset shows the normalized position as function of magnetic field.

Figure 4.28: Magnetization as function of magnetic field of sample 2B obtained after a linear background subtraction. A distinct peak at 14T is observed. The origin of the structure before the high peak is still unknown.
Substrate sample

An additional sample from a different part of the wafer of samples 2A and 2B has been prepared by polishing the rings away. This way we are able to measure the magnetization of the substrate. Figure 4.30 shows the magnetization of the substrate as function of magnetic field for different runs. The inset shows a close up of the range between 8-16T. No structure is observed on the background in contrast to the case of samples 2A and 2B. The noise on the background differs for the different runs. During the measurement the noise on the background decreased, most probably due to decreasing external noise (for instance working activities in the environment). The overall behavior of the background on the magnetic field is comparable. Note that the magnetization caused by the background behaves similar to the background found in the magnetization of sample 2B.

Comparison with theory

The transition field is in excellent agreement with the theoretical model. Sample 2A has $B_{\text{trans}}=13.4T$ and sample 2B has $B_{\text{trans}}=12.8T$, whereas the theoretical model gives $B_{\text{trans}}=14T$ (see figure 2.21). The theory is based on X-STM data obtained on sample 1. However, the rings on sample 2A and 2B are slightly different. Therefore we expect a slightly different transition field. Furthermore the shape of the magnetization curves of samples 2A and 2B, respectively in figures 4.23 and 4.28, is broadened mostly due to the inhomogeneous size dis-
Figure 4.30: Feedback current in µA and magnetization in µB. The inset zooms in on the range where the sample 2A and 2B showed oscillating structure. The substrate only shows a background with no features on top of it.

The height of the magnetization peak of sample 2B is higher in comparison with the jump in magnetization of sample 2A due to the higher density of rings in sample 2B as compared to sample 2A. The ratio of densities obtained from AFM measurements is \( \frac{A}{B} = 0.67 \). The ratio of the magnitude of the magnetization

\[ \text{Sample } 2B \text{ has a higher QR density of } 9 \cdot 10^9 \text{ cm}^{-2}, \text{ leading to a stronger jump in the magnetization of } 1.5 \cdot 10^{12} \mu_B. \]

\[ \text{Sample } 2A \text{ determined by AFM measurements is } 6 \cdot 10^9 \text{ cm}^{-2}. \text{ The sample is designed to have 2 electrons per ring. The sample has a surface of } 0.56 \text{ cm}^2 \text{ and contains 29 layers of rings. The jump in magnetization for an ring containing 2 electrons is approximately } 10 \mu_B \text{ as predicted by theory. The total expected jump in the magnetization is therefore } 29 \cdot 10 \mu_B \cdot 0.56 \text{ cm}^{-2} = 9.7 \cdot 10^{12} \mu_B. \text{ Sample } 2B \text{ has a higher QR density of } 9 \cdot 10^9 \text{ cm}^{-2}, \text{ leading to a stronger jump in the magnetization of } 1.5 \cdot 10^{12} \mu_B. \]

\[ \text{The QR density of sample } 2A \text{ determined by AFM measurements is } 6 \cdot 10^9 \text{ cm}^{-2}. \text{ The sample is designed to have 2 electrons per ring. The sample has a surface of } 0.56 \text{ cm}^2 \text{ and contains 29 layers of rings. The jump in magnetization for an ring containing 2 electrons is approximately } 10 \mu_B \text{ as predicted by theory. The total expected jump in the magnetization is therefore } 29 \cdot 10 \mu_B \cdot 0.56 \text{ cm}^{-2} = 9.7 \cdot 10^{12} \mu_B. \text{ Sample } 2B \text{ has a higher QR density of } 9 \cdot 10^9 \text{ cm}^{-2}, \text{ leading to a stronger jump in the magnetization of } 1.5 \cdot 10^{12} \mu_B. \]

\[ \text{The} \text{ theo} \text{ ry predict} \text{s an} \text{ magnitude of the} \text{ jump in magnetization of } 9.7 \cdot 10^{12} \mu_B \text{ at most for sample } 2A \text{ and } 1.5 \cdot 10^{13} \mu_B \text{ at most for sample } 2B, \text{ where the experimenta} \text{l values are } (1.3 \pm 0.7) \cdot 10^{13} \mu_B \text{ for sample } 2A \text{ and } (1.8 \pm 0.7) \cdot 10^{13} \mu_B \text{ for sample } 2B. \text{ These values are thus in excellent agreement with theoretical calculations of the amplitude of the magnetization.} \]

The theory predicts an magnitude of the jump in magnetization of \( 9.7 \cdot 10^{12} \mu_B \) at most for sample 2A and \( 1.5 \cdot 10^{13} \mu_B \) at most for sample 2B, where the experimental values are \( (1.3 \pm 0.7) \cdot 10^{13} \mu_B \) for sample 2A and \( (1.8 \pm 0.7) \cdot 10^{13} \mu_B \) for sample 2B. These values are thus in excellent agreement with theoretical calculations of the amplitude of the magnetization.
tion signal $\frac{2A}{2B}=0.72$. This ratios are in thus in good agreement and again prove the magnetization signal is due to the magnetization of the QRs on both samples.

To exclude any artefacts in the data analysis that may have occurred in figure 4.23 we use a background independent analysis by calculating the differential magnetization. The differential magnetization data of samples 2A and 2B both show a characteristic peak in good agreement with the theoretical curve for a value of the temperature between 4K and 10K.

We will now compare AFM, magnetization and PL measurements of samples 2A and 2B with each other, in order to qualitatively determine the difference between the size of the QRs on both samples. AFM shows a larger diameter for the rings on sample 2A, approximately 85nm, compared to the rings of sample 2B, approximately 65nm. The magnetization measurements show that the transition magnetic field of sample 2A is 0.6T larger compared to sample 2B. This indicates the electrons in sample 2A are captured in rings with a slightly smaller diameter compared to the electrons captured in the rings in sample 2B\textsuperscript{18}, which on first sight contradicting to the AFM measurements. However, the actual sizes of the buried rings are unknown. What we see in AFM measurements is the erupted material. By PL measurements it is observed that the confinement energy of the rings in sample 2A is higher, 1.303eV, compared to the rings in sample 2B, 1.22eV. The confinement is mainly determined by the confinement in the z-direction. Therefore the difference in PL energy indicate a lower height of the rings in sample 2A compared to the rings in sample 2B. Moreover the energy difference between excited state energy $E_1$ and ground state energy $E_0$, $\Delta(E_1 - E_0)$, is larger for sample 2A compared to sample 2B, respectively $\Delta(E_1 - E_0)=48\text{meV}$ and $\Delta(E_1 - E_0)=38\text{meV}$. This also indicates a higher confinement in the z-direction due to a smaller height of the rings in sample 2A compared to sample 2B.

We conclude that the difference in the shape of the ring is thus mainly expressed in the lower height of the rings on sample 2A compared with the rings on sample 2B. A lower height of the ring means in principal a more pronounced eruption process. A more pronounced eruption of material of the rings on sample 2A might lead to larger lateral sizes found by AFM measurements.

Possible other causes for the magnetization signal

Although we give convincing evidence of the observation of the AB effect in the magnetization, we cannot explain the structure occurring just before the high peak in the magnetization and differential magnetization, which is especially pronounced in the measurements on sample 2B. We will therefore exclude other possible causes for the magnetization signal.

First of all the magnetization signal is not influenced by the increase in laser intensity, as shown in figure 4.23, indicating that the signal we obtain is not caused by photo induced carriers in the sample. As shown in figure 4.23 the decrease in temperature from 4.2K to 1.2K does not influence the shape and\textsuperscript{18}A smaller diameter corresponds with larger magnetic field required to enclose a magnetic flux quant ($\frac{\phi}{\epsilon}$).
magnitude of the magnetization signal. Therefore it can be excluded that the magnetization is due to a De Haas-Van Alphen effect of thermally excited carriers.

We will now consider the possibility that the two peaks observed in figure 4.28 are caused by De Haas-Van Alphen oscillations. In order to prove that this assumption is incorrect, we calculate the electron density that might be obtained from the period of the two peaks and compare it with the electron density expected from the sample design. The increase in amplitude would suggest a 3D De Haas-Van Alphen effect (see paragraph 2.4). Nevertheless we calculate both the 2D and 3D concentration. From figure 4.28 and paragraph 2.4 we obtain $n_{2D}=6.2\times10^{12}\text{cm}^{-2}$ and $n_{3D}=1.10^{18}\text{cm}^{-3}$. The sample design shows a concentration of the 2D doped layers in the order of $3\times10^{10}\text{cm}^{-2}$, if the carriers are not captured by the QRs. Therefore it is highly unlikely that the magnetization signal is caused by the 2D De Haas-Van Alphen oscillations.

If for any reason there is a a transfer of all the carriers from the doping layers to the 3D epilayer, which has a thickness of 1µm, a 3D De Haas-Van Alphen concentration in the order of $10^{15}\text{cm}^{-3}$ is expected from the design, which in fact is very close to the normally expected p-type concentration in the epilayer due to background impurities. This is three orders of magnitude less than found by treating the peaks in the magnetization as De Haas-Van Alphen oscillations. Therefore 3D De Haas-Van Alphen can be excluded.

The magnetization of the substrate did not show any structure, excluding the De Haas-Van Alphen oscillation are due to free carries in the substrate. Furthermore, the independence of the magnetization on temperature indicates that the magnetization is not a result of the De Haas-Van Alphen effect, because De Haas-Van Alphen oscillations are strongly dependent on temperature[26].

In order to even further exclude that the magnetization signal is due to electrons in the doped layers we will perform Shubnikov-De Haas measurements on the original samples used in the magnetization measurements, discussed in the next paragraph 4.5.

### 4.5 Shubnikov-De Haas measurements

To exclude that the magnetization signal is produced by carriers originating from the doping layers in between the rings, we perform Shubnikov-De Haas measurements on both samples 2A and 2B. In order to do this, small contacts are implemented on the sample by diffusion through annealing. The annealing time is chosen such that the contacts reach well below the doping layers. The experiments are performed at 4.2K in a bitter magnet running up to 20T in the HFML in Nijmegen. This bitter magnet is used up to 20T. The measurements are performed at $T=4.2K$.

Shubnikov-De Haas oscillations in the magneto resistance will only occur when the different Landau levels are well separated. The Landau levels are split enough to give rise to Shubnikov-De Haas if the condition $\mu B > 1$ holds, where $\mu$ is the mobility of the carriers. If this condition is satisfied we can estimate the electron density from the oscillation period of the Shubnikov-De Haas oscillations.
The voltage in V of a two-point measurement when applying a current of 10µA for three series; up to 20T and down from 20T to 0T and another seris measured fields down to -20T after illumination. The voltage shows no sign of Shubnikov-De Haas oscillations. The uncertainty error at zero magnetic field is 0.0025V. From the slope we determine the carrier concentration corresponding to the magneto resistance.

The measurements were performed on a sample with a Van der Pauw geometry. However, not all contacts showed good conductivity. This may be due to bad contacts or to internal cracks19 in the sample through which electrons are poorly conducted. It is also likely that the sample has poor conductivity due to a very low carrier density. A current of 10µA is send through two of the contacts and the voltage is measured between these same two contacts; such a measurement is called a two-point measurement. Figure 4.31 shows the voltage as function of magnetic field for three series; an up sweep to positive magnetic field of 20T, a down sweep from 20T to 0T and an up sweep to negative magnetic fields up to -20T. The measurement at negative magnetic fields is performed after illumination of the sample with a LED. Since this measurement has the same behavior as the measurement without illuminating the sample, we exclude any effect on the conductivity of photo-excited carriers. The three measurements do not show the Shubnikov-De Haas oscillations.

19These internal cracks can be due to the cooling down and warming up of these heavily strained samples.
Figure 4.32: The voltage for a current of 100nA to prevent possible heating of the sample. Again the voltage shows no sign of any Shubnikov-De Haas oscillations. From the slope we determine the carrier concentration corresponding to the magneto resistance.

The applied current of 10µA may cause heating of the sample. A temperature increase results in the broadening of the Fermi-Dirac distribution. This results in an averaging of the density of states over the different Landau levels and consequently lead to absence of quantum oscillations. To exclude this argument, we decrease the current to 100nA. The result of this low-current measurement is shown in figure 4.32. The signal is more noisy as the one shown in figure 4.31 since we are measuring smaller signals. Again we observe an increase in the voltage without any Shubnikov-De Haas oscillations.

At room temperature sample 2B showed a resistance in the order of MΩ's between different contacts. After cooling down the sample showed no conductivity anymore, implicating a freeze out of all carriers.

Both figure 4.31 and 4.32 show no Shubnikov-De Haas oscillations, which are expected in the case that the magnetization signal is caused by De Haas-Van Alphen oscillations. Both samples 2A and 2B have poor conductivity, most probably caused by a low concentration of carriers in the samples.

The linear increase in voltage with magnetic field observed in the measurements shown in figures 4.31 and 4.32 is a result of the Hall effect. The magneto resistance $R(B)$ of a 2D sample is given by 4.1:

$$R(B) = \frac{l}{b n_{2D} e \mu} + \alpha R_H(B),$$  

(4.1)
with \( l \) the length of the sample and \( b \) the width of the sample. \( R_H(B) \) is the Hall resistance and \( \alpha=1 \) for a two-points measurement. In this calculation \( n_{2D} \) is the carrier concentration, since both electrons or holes can provide the conductance. The Hall resistance is given by (4.2):

\[
R_H(B) = \frac{B}{n_{2D}e},
\]

(4.2)

Therefore the differential magneto resistance is therefore proportional to \( \frac{1}{n_{2D}e} \).

Figure 4.31 gives a 2D carrier concentration of \( n_{2D}=2.69\cdot10^{11} \text{cm}^{-2} \). Figure 4.32 gives a carrier concentration of \( n_{2D}=9.16\cdot10^{10} \). The difference between these two measurement proves that a two-point measurement for such low concentration samples is not an accurate method to determine the electron density. The best method would involve a four-point measurement preferably on a Hall-bar structure.

We will now compare the carrier concentrations to typical carrier concentrations that can be expected in the grown structure. The density of the electrons in the doping layers between the rings was \( 1.4\cdot10^{10} \text{cm}^{-2} \), an order of magnitude lower compared to \( n_{2D} \) obtained from the two-point measurement. The growers indicated a considerable error in the doping concentration due to difficulties in growing low doped layers. However a factor of 10 seems too much.

The Hall effect can also be caused by the extra doped layer above or beneath the rings, which were grown to take into account the depletion of the sample. These sheet densities were \( 3.2\cdot10^{10} \text{cm}^{-2} \). Even if all these electrons will contribute to a conduction layer instead of being bound to impurities, the carrier concentration found by the Hall-measurements are factor larger in magnitude.

Finally, the epilayer of 1\( \mu \text{m} \) of GaAs could cause the Hall effect. The 3D concentration of carriers is related to \( n_{2D} \) by \( n_{3D}=t \cdot n_{2D} \), with \( t \) the thickness of the epilayer. This leads to a 3D carrier concentration in the order of \( n_{3D}=1\cdot10^{15} \text{cm}^{-3} \). It is known [44] that the MBE growth of GaAs typically results in a p-type background with a hole concentration of \( p_{3D}=10^{14}-10^{15} \text{cm}^{-3} \). This is in good agreement with the value of the carrier density we find from the measurements. Therefore the Hall-effect is most likely caused by background p-type impurities in the grown epilayer.

4.6 Magnetic field dependent photoluminescence measurements

In order to support the magnetization measurements, we perform \( \mu \)-PL in high magnetic fields on samples 2A and 2B. The measurements are performed at 4.2K. Details of the setup are discussed in paragraph 3.2. A spot size of \( 1\mu \text{m} \) in diameter leads to an excitation of at least 1500 rings. The PL spectra reflect the energies of the excitons in the rings. As will be shown, the shift of the energy peak corresponding to the excitons in the QRs is mainly due to the shift of the energy of the electrons. Figure 2.18 shows the electron energy levels as function of magnetic field. According to this figure the shift of the energy peak must therefore show an increase in energy with a kink at the field where the ground
state character changes from \( m=0 \) to \( m=-1 \) (from the magnetization experiment this we expect this around 13T) and an additional kink at 24T for the ground state transition from \( m=-1 \) to \( m=-2 \) (see figure 2.21).

4.6.1 PL measurements in magnetic field on sample 2A

We measure PL spectra as function of magnetic field. A serie consists of spectra taken at intervals of 0.5T up to a magnetic field of 29T. The characteristics of the different series are:

- **Serie 1**: Unfocused spot (excitation density \( I_{exc} = \text{medium} \)) at position 1 on sample.
- **Serie 3**: Unfocused spot (\( I_{exc} = \text{medium} \)) at position 2 on sample.
- **Serie 4**: Focused spot (\( I_{exc} = \text{high} \)) at position 3 on sample.
- **Serie 5**: Focused spot (\( I_{exc} = \text{high} \)) at position 4 on sample.
- **Serie 6**: HC09-filter\(^\text{20}\) in front of laser, resulting in \( I_{exc} = \text{low} \) at position 5 on sample.
- **Serie 7**: HC09-filter in front of laser, resulting in \( I_{exc} = \text{low} \) at position 6 on sample.

Figure 4.33 shows the PL spectra of the different series at zero magnetic field. As discussed in paragraph 4.3.2 at high \( I_{exc} \) the excited states show up at the high energy side of the spectrum. As we already have two electrons in our

\(^{20}\)A HC09-filter decreases the excitation density with to only 5-10% of its original value.
Figure 4.34: The left panel show the average peak position \( \langle E \rangle \) as function of magnetic field for the series 1, 3 and 6. All three show clear features around 13T (indicated with thick red arrow) and 24T (indicated with thin black arrow). Series 1 and 3 are also analysed using a Lorentzian fitting procedure. Both methods show similar features at the transition fields.

rings, the main peak corresponds to exciton complexes formed by 3 electrons and 1 hole or 4 electrons and 2 holes\(^{21}\). The excited states correspond with at least 3 holes and 5 electrons as discussed in paragraph 4.3.2.

Figure 4.34 shows the peak position shift as function of magnetic field for the series 1 and 3 using two different methods to determine the peak position. The panels on the right of the figure show peak positions determined from fitting the spectra with a Lorentzian distribution. The panels on the left show the weighted average energy as function of magnetic field. The weighted average

\(^{21}\)We assume an equal population of the ring with electrons and holes as a consequence of the excitation. The main peak corresponds to the recombination of holes and electrons in the ground state.
energy is determined by 4.3:

\[
< E > = \frac{\sum_i E_i I_i}{\sum_i I_i}
\]

(4.3)

with \( I_i \) the PL intensity at energy \( E_i \). Figure 4.34 also shows the result for serie 6 using the statistical method. This serie also shows distinct features at the transition fields.

Although the exact shape of the curve is different each time, the behavior shows a reproducible trend in the different series. We expect to see a feature in the shift of the PL-peak energy around \( 13T \) from the magnetization measurements. Indeed, as is shown in figure 4.34, there is a clear feature at this field corresponding with the transition of the angular momentum quantum number \( m \). The change in the position of the sample for different series might relate to slightly different transition fields\(^{22}\).

The exact shape of the curves in figure 4.34 is not explained yet. Moreover the detailed structure of the PL-profile is not reliable because the excitation density differs slightly with magnetic field. This change in excitation density is caused by a defocussing of the spot, which is caused by the movement of a tube used to protect and connect the sample holder and the objective lens.

The PL measurements are a result of the recombination of electron-hole pairs, whereas the magnetization is only due to electrons. To couple both measurements to one another we examine the contribution of the holes to the PL transitions. In different papers \([45\), [46\] and [47\] calculations have been reported on the so called light-dark transitions. Since the electron and hole have a different effective mass they perform different circular orbits in the InAs quantum rings and enclose a different area. Therefore the electron and the hole require different magnetic field strengths to enclose the same amount of magnetic flux. When for instance the orbit of the electrons is larger compared to the holes, they will change angular momentum \( m \) at lower field than the holes. In the region of magnetic field where the hole angular momentum is not equal to the electron angular momentum the recombination between electron and holes is forbidden. These 'dark' regions might be observed in time resolved PL. Calculations by the Antwerp team show however that in our model of the InAs self-assembled quantum ring, the holes are strongly bound in the potential pockets of an undulated ring\(^{23}\). The hole procession around the ring is therefore strongly suppressed. Calculations for a ring occupied by a single hole show a shift of the energy of the hole of \( 1\text{meV} \) at \( 50T \), which is negligible with the calculated energy shift of the electron of \( 20\text{meV} \) at \( 50T^{24}\). Therefore the results of the magnetization measurements and the PL measurements can be compared.

\(^{22}\) Different position on the sample means slightly different structures.
\(^{23}\) Since the effective mass of the holes is quite large, the holes are stronger bound to these potential minima.
\(^{24}\) Theoretical calculations based on the rings of sample 1. We have different rings and therefore different strain fields. The exciton energy shift we observe is in the order of \( 6\text{meV} \), a factor of 3 lower compared with the calculations. As can be seen from figure 2.18 strain influences the whole picture considerable and therefore it is hard to compare the theoretical and experimental values.
So we conclude that the features we see in figure 4.34 are mainly caused by transitions of the electron energy ground state to higher angular momentum quantum number. These transition fields are in good agreement with the magnetization measurements and theoretical calculations. Both in PL and magnetization measurements we see a clear fingerprint of the AB oscillations in the InAs self-assembled ring structures. A detailed description of the energy shift has still to be given.

4.6.2 PL measurements in magnetic field on sample 2B

We measured different PL series. The characteristics of the different PL series are:

- **Serie 1**: HC08-filter\(^{25}\) in front of the laser, position 1 on sample.
- **Serie 2**: HC08-filter in front of the laser, position 2 on sample.
- **Serie 4**: HC08-filter in front of the laser, position 3 on sample.
- **Serie 5**: No filter in front of the laser, position 4 on sample.
- **Serie 6**: HC08-filter in front of the laser, position 4 on sample.
- **Serie 7**: HC09-filter in front of the laser, position 4 on sample.

Series 1, 2, 3 and 6 use an integration time of 5s. Serie 5 uses an integration time of 0.5s and serie 7 an integration time of 10s. The PL spectra of sample 2B of the different series at B=0T is shown in figure 4.35.

The statistical approach is used to analyse the different spectra of the different series as function of magnetic field. Figure 4.36 shows the average energy as function of magnetic field for the two series 2 and 6. The shift to higher energy again shows changes in trends at the orbital momentum transition field, however these features are not that pronounced as with sample 2A.

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\(^{25}\) A HC08-filter decreases the initial \(I_{exc}\) to 10-15%.
Figure 4.35: PL spectra of the different series at B=0T. The spectrum of serie 5 has highest $I_{exc}$ and shows both PL from ground state and excited state excitons. The spectra of series 1, 2, 3 and 6 are taken with a HC08-filter to decrease $I_{exc}$. Spectrum 7 is taken using a HC09-filter to even further decrease $I_{exc}$. The corresponding integration times of the different series are indicated.

Figure 4.36: The average energy $<E>$ as function of magnetic field for sample 2B. The change in trends are not that pronounced as with sample 2A, but they are present.
Chapter 5

Conclusions and Suggestions

5.1 Conclusions

We performed magnetization measurements on different samples containing self assembled InAs quantum rings. X-STM on a sample containing 20 layers of QRs has shown that quantum rings do not have a open hole in the center, but rather show a depression in the center \[13\]. Furthermore the self-assembled InAs quantum rings are strained and asymmetric. It was initially not clear if these asymmetric and strained quantum volcanoes can perform AB oscillations in the persistent currents. These AB oscillations are typical for ideal ring-like geometries and a clear fingerprint of the quantum mechanical behavior of the electrons in such rings. Calculations performed by the Antwerp team showed that the AB oscillations should nevertheless be observable.

Sample 1 on which X-STM is performed did not show AB oscillations, due to the presence of pronounced De Haas-Van Alphen oscillations caused by the heavily doped substrate. Moreover the sample design is not optimized for double electron occupancy in each ring. A new sample was designed using a 1D self-consistent Poisson-Schrödinger solver, which is suitable for magnetization measurements. The sample was grown without rotation of the wafer during growth. This leads to slight variation in the quantum ring parameters and the doping concentrations as function of the position on the wafer. This slight variation in quantum ring parameters has been observed in AFM measurements and other measurements.

Magnetization measurement were performed by using a torque magnetometer setup. Two samples showed a peak in magnetization around 13T having a magnitude of approximately \(1.5 \times 10^{13} \mu_B\), in good agreement concerning position and amplitude with theoretical predictions. Therefore these peaks are positively identified as the first AB oscillation in the persistent current of these nano-volcanoes.

To further support the magnetization measurements we performed \(\mu\)-PL
measurements in high magnetic fields on two samples. The observed features in
the PL are not reproduced in detail in each run. However they clearly show a
feature at magnetic fields corresponding to the peaks in the magnetization. A
second feature is observed at higher fields (24T), which indicates the second AB
oscillation.

To conclude, we are the first to observe oscillations in the persistent current
due to the AB-effect occurring in self-assembled InAs quantum rings. The ob­
servations are in perfect agreement with calculations based on a sophisticated
theoretical model. The µ-PL in high magnetic fields support the magnetization
measurements.

5.2 Suggestions

In order to see more than one AB oscillation, we need to go to higher fields,
preferably up to 33T. This will be possible at the HMFL in the near future.
For these measurements it is best to use a different sample. A new stacked QR
sample has recently been grown with rotation, to minimize variations in the
shape and structure of the rings.

To check that the ring topology is responsible for the observed effects, it
would be nice to do magnetization measurements on another sample containing
multiple layers of quantum dots. Calculations of the Antwerp group have shown
that we expect no AB oscillations in the magnetization for such disk-like shaped
structures.

The PL measurements can be improved drastically by doing these measure­
ments on a sample containing a single layer of QRs. This way we can perform
single ring spectroscopy. This allows us to follow sharp PL lines as function
of magnetic field, which allows us to determine the energy shift as function of
magnetic field far more accurately. Moreover, single ring spectroscopy can be
used to investigate the electronic structure and the AB effect of the rings into
detail.
Chapter 6

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I want to conclude by saying I had a great time during my master project. I had some very nice experiences and met nice and interesting people. I feel lucky to start my promotion in this group.
Bibliography


Chapter 7

Appendix A

The system of an electron in a time-independent magnetic field is described by the Hamiltonian 7.1:
\[ \mathcal{H} = \frac{1}{2m_e}(-i\hbar \nabla + e A)^2 \] (7.1)
and the time independent Schrödinger equation is given by 7.2:
\[ \mathcal{H}\psi = E\psi \] (7.2)

In absence of a vector potential the Hamiltonian reads 7.3:
\[ \mathcal{H}_0 = \frac{1}{2m_e}(-i\hbar \nabla)^2 \] (7.3)
and the Schrödinger equation 7.4 leads to:
\[ \mathcal{H}_0\psi_0 = E_0\psi_0 \] (7.4)

Equation 7.1 and 7.3 can be transformed into each other by introducing a phase factor in the wave function; this transformation is a type of gauge transformation. We have the general expression 7.5:
\[ \psi = \exp(i\epsilon`\Gamma)\psi_0 \] (7.5)

Now \(-i\hbar \nabla + e A\) working on 7.5 and using the chain rule, leads to 7.6:
\[ (-i\hbar \nabla + e A)\psi = e\hbar \nabla \Gamma \exp(i\epsilon`\Gamma)\psi_0 + \exp(i\epsilon`\Gamma)(-i\hbar \nabla + e A)\psi_0 \] (7.6)

Now choose \(\Gamma\) so that it obeys 7.7:
\[ e\hbar \nabla \Gamma + e A = 0 \] (7.7)
then from 7.6 follows:
\[ (-i\hbar \nabla + e A)\psi = \exp(i\epsilon`\Gamma)(-i\hbar \nabla)\psi_0 \] (7.8)
So from this we finally obtain 7.9:
\[ \mathcal{H}\psi = \exp(i\epsilon`\Gamma)\mathcal{H}_0\psi_0 = \exp(i\epsilon`\Gamma)E_0\psi_0 = E\psi \] (7.9)
We have just shown that a Schrödinger equation with a vectorpotential $\vec{A}$ can be rewritten by a gauge transformation to a Schrödinger equation without a vectorpotential.

The condition of the function $r$ given in equation 7.7 can be solved if $\vec{B}=0$, which does not mean that $\vec{A}=0$. It is solved by a line integral, resulting in $r=-\frac{i}{\hbar}\Phi$. 

88
Chapter 8

Appendix B

Figure 8.1 shows the real part of the wave function as function of magnetic field and the anisotropy parameter $\xi_h$. 
Figure 8.1: The real part of the wave function as function of magnetic field and the anisotropy parameter $\xi$. The ground state wave function increases orbital momentum from $m=0$ to $m=1$ and so on for increasing magnetic field in the case of $\xi=0$. The introduction of asymmetries in the quantum ring structure lead to the intermixing of the different $m$ states. This can be seen for example in the case of $B=0$ and $\xi=0.2$. Instead of the ideal wave function belonging to an $m=0$ state, it consists of components of $m=2$ and other even even $m$. Breaking another symmetry in the topological modelling of the ring, will cause intermixing of both even and odd orbital momentum states.