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Electromagnetic interaction of a circular coil and layered tissue

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Electromagnetic interaction of a circular coil and layered tissue

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<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>ABSTRACT</td>
<td>2</td>
</tr>
<tr>
<td>INTRODUCTION</td>
<td>3</td>
</tr>
<tr>
<td>Magnetic resonance imaging</td>
<td>3</td>
</tr>
<tr>
<td>The parameters of interest</td>
<td>3</td>
</tr>
<tr>
<td>Transmitting properties</td>
<td>4</td>
</tr>
<tr>
<td>The formulation of the problem</td>
<td>4</td>
</tr>
<tr>
<td>THE BOUNDARY VALUE PROBLEM</td>
<td>6</td>
</tr>
<tr>
<td>Introduction</td>
<td>6</td>
</tr>
<tr>
<td>Configuration and tissue properties</td>
<td>6</td>
</tr>
<tr>
<td>Maxwell’s equations and the boundary conditions</td>
<td>7</td>
</tr>
<tr>
<td>Definition of the excitation signal</td>
<td>9</td>
</tr>
<tr>
<td>Methods to be applied</td>
<td>9</td>
</tr>
<tr>
<td>SOLUTION TO THE BOUNDARY VALUE PROBLEM (BVP)</td>
<td>10</td>
</tr>
<tr>
<td>The BVP in circular cylinder coordinates</td>
<td>10</td>
</tr>
<tr>
<td>Hankel transform</td>
<td>11</td>
</tr>
<tr>
<td>Application of the Hankel transform</td>
<td>11</td>
</tr>
<tr>
<td>Determination of the coefficients $A_i$ and $B_i$</td>
<td>13</td>
</tr>
<tr>
<td>The vector notation</td>
<td>14</td>
</tr>
<tr>
<td>ANALYSIS OF THE IMPEDANCE</td>
<td>17</td>
</tr>
<tr>
<td>Solution procedure</td>
<td>18</td>
</tr>
<tr>
<td>NUMERICAL APPROXIMATION</td>
<td>19</td>
</tr>
<tr>
<td>Numerical integration</td>
<td>19</td>
</tr>
<tr>
<td>Estimation of $n$ and $h$</td>
<td>19</td>
</tr>
<tr>
<td>Approximation of infinite integrals</td>
<td>20</td>
</tr>
<tr>
<td>Preconditions for correct integral computations</td>
<td>20</td>
</tr>
<tr>
<td>The boundary conditions</td>
<td>21</td>
</tr>
<tr>
<td>The $\tau$-sensitivity</td>
<td>21</td>
</tr>
<tr>
<td>SIMULATIONS</td>
<td>22</td>
</tr>
<tr>
<td>Configurations</td>
<td>22</td>
</tr>
<tr>
<td>Permittivities in different media</td>
<td>23</td>
</tr>
<tr>
<td>Format of $E_o$, $H_z$, $H_z$ and $Z_c$</td>
<td>24</td>
</tr>
<tr>
<td>Results and conclusions with respect to $E_o$, $H_z$ and $H_z$</td>
<td>24</td>
</tr>
<tr>
<td>Results and conclusions with respect to coil impedance</td>
<td>32</td>
</tr>
<tr>
<td>Appendix 1</td>
<td>37</td>
</tr>
<tr>
<td>Manual of the available programs</td>
<td>37</td>
</tr>
<tr>
<td>Input data</td>
<td>37</td>
</tr>
<tr>
<td>Some precautions on input variables</td>
<td>39</td>
</tr>
<tr>
<td>Test programs</td>
<td>39</td>
</tr>
<tr>
<td>EMFIELD and IMPCOIL</td>
<td>41</td>
</tr>
<tr>
<td>The graphics interface programs</td>
<td>41</td>
</tr>
<tr>
<td>Appendix 2</td>
<td>43</td>
</tr>
<tr>
<td>Programs in meta language</td>
<td>43</td>
</tr>
<tr>
<td>EMFIELD metalanguage</td>
<td>43</td>
</tr>
<tr>
<td>IMPCOIL metalanguage</td>
<td>44</td>
</tr>
<tr>
<td>TSTLAMBD metalanguage</td>
<td>44</td>
</tr>
<tr>
<td>TSTBOND metalanguage</td>
<td>45</td>
</tr>
<tr>
<td>Appendix 3</td>
<td>46</td>
</tr>
<tr>
<td>Q-FACTOR</td>
<td>46</td>
</tr>
<tr>
<td>List of references</td>
<td>47</td>
</tr>
</tbody>
</table>
ABSTRACT

In development of surface coils, as used in Magnetic Resonance Imaging (MRI), the need arose of a theoretic means to determine sensitivity for electromagnetic fields of surface coils and impedance of surface coils.

In this report the transmitting properties of a circular coil are investigated theoretically. The position of the coil is parallel to a plane, stratified half-space. The media are supposed to be homogenous, linear, isotropic, dispersive and dissipative.

The theoretical analysis leads to integral expressions for the electric field, the magnetic field and the impedance of the coil. These integral expressions have been evaluated numerically. To do so, a software package has been developed.

Comparison of the spatial distribution of the electric field strengths close to the coil when surrounded only by air and in case the coil is positioned above matter, does not show remarkable differences. The same holds for the magnetic field strength. On the other hand, the spatial distribution of the phase of the electric- and magnetic-field and direction of the magnetic field all show a significant dependence on both frequency and permittivities of the media.

The complex coil impedance has been investigated as a function of coil thickness, width and radius, respectively. In addition the influence of the coil-to-object distance on this impedance has been examined. To obtain a maximal $Q$-factor, the coil must be flat and narrow.

Acknowledgements

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INTRODUCTION

Magnetic resonance imaging

Magnetic resonance imaging (MRI) is an imaging technique often used as a diagnostic means in medicine [1]. This technique is based on nuclear induction. Nuclei of many different atoms produce a small magnetic field. The nucleus of hydrogen, which is a single proton, shares this property. Since hydrogen is abundantly present in a human body and its magnetic moment is large, it is a suitable isotope to be imaged.

The direction of the fields generated by the individual nuclei can be aligned. For this purpose a homogenous magnetic field is transmitted into the body to be depicted. Next an alternating electromagnetic field is transmitted into that body. After turning off the alternating part of the electromagnetic field, also called the excitation field, a decaying net magnetization, caused by the nuclei, will remain. This response is measured either by the transmitting coil or by a separate receiving coil. If the excitation field is well coded, decoding the response will result in an image, showing the various water-containing tissues in the human body.

Often the separate receiving coil is positioned directly on the body and therefore it is called a surface coil. There are several parameters of the coil and its positioning that are to be optimized to obtain a better image quality. In order to determine these parameters, as far as the electromagnetic behaviour is concerned, it is necessary to acquire a more profound insight in the electromagnetic field distribution in the environment of the coil.

The performance of the coil with respect to electromagnetic field and impedance is investigated. Various dimensions will be based on values as they appear in MRI-systems. Calculations and simulations are executed employing parameters often occurring in MRI-development. This is done so because this research arose as a need in MRI-coil development. Knowing more about the electromagnetic interaction between tissue and surface coil may lead to improvement of design and performance of the surface coil and the system as a whole.

The parameters of interest

The following parameters are subject to change to get a better performance:

- Coil material. It is not allowed to use magnetic materials in the system. The surface coil is often made from copper.
- Material in which the coil is embedded.

- Dimensions of the coil.  
The dimension and shape of the surface coil is based on  
the region to be depicted.

- Position of the coil relative to the body.  
The plane of the coil is positioned parallel to the  
body.

- Frequency and magnitude of the excitation field.  
Changing frequency and/or magnitude of the excitation  
fields requires an extensive adjustment of the system,  
so these quantities are to be taken as parameters.

Next, there are also parameters which cannot be  
controlled such as:

- Permittivities and permeabilities of body tissue.

- Shape of the body.  
The model, dimension and electromagnetic properties of  
the object to be imaged are different for each case and  
they vary over a wide range.

- Impedance and current in the coil.  
In the receiving state of the system the current in the  
coil contains the image information and therefore this  
current is regarded as the output variable.

Transmitting properties

Calculating the current in the coil as a result of a  
supplied field distribution is a difficult problem to tackle.  
When the coil is used as transmitter the load of this coil is  
composed from the patient and its environment. This has  
effect on the electric impedance of the coil. There is a  
conjecture that the sensitivity of the coil, used as a  
receiver, is also determined by the electric impedance.

Therefore, it is expected to get a sufficient insight in  
the performance of the coil when the transmitting properties  
are determined.

The formulation of the problem

Now, the goal of the investigation can be defined in a  
more formal way:

Develop a theoretical means which can provide  
qualitative and quantitative information about the  
coil used as a transmitter with respect to  
- electromagnetic field distribution;  
- impedance.
The outcome of the theoretical analysis, will be integrated in a computer program allowing us to do some simulations. The results of these simulations are compared to the results of some well known cases.
THE BOUNDARY VALUE PROBLEM

Introduction

In this chapter the electromagnetic field problem will be written as a boundary value problem. The boundary value problem (BVP) is presented in this sequence:

- Configuration of the coil and the tissue(s);
- Tissue-properties;
- Imposed electric current distribution.

For an exact solution of the BVP we use Maxwell's equations and the boundary conditions.

Configuration and tissue properties

To solve the BVP using Maxwell's equations and the boundary conditions the following assumptions about the configuration are made:

- The coil is circular and its cross-section is rectangular (see Figure 2.1 and Figure 2.2);
- The body is layered;
- Each tissue-layer is assumed to be flat and of infinite extent;
- The orientation of all tissue-layers and the coil is parallel.

The positioning of the coil and tissues is depicted below.

Fig. 2.1a Top view of coil and body.
This illustration will serve as the model in the following chapters. It is obvious that this model shows an axial symmetry. For this reason the configuration is positioned in a circularly cylindrical coordinate system. \( (z,r,\theta) \)

Each layer is indexed by an integer \( i = [1,n+1] \). Also each boundary is indexed by an integer \( i = [1,n] \) so, that the \( i \)-th layer is on top of the \( i \)-th boundary. \( h_i \) \( i = [1,n] \) equals the \( z \)-value at which the \( i \)-th boundary is positioned.

The coil, or source is placed between \( h_{i-1} \) and \( h_i \), \( r = r_c^1 \) and \( r = r_c^2 \) and its parameters are indexed by "s". The \( i \)-th \( i = [2,n] \) layer is defined to be positioned at \( z = [h_i,h_{i-1}] \).

The first layer is positioned at \( z = [h_1,h_1] \) and the \( n+1 \)-th layer is positioned at \( z = [h_n,h_n] \).

In every layer, the permeability is assumed to equal the permeability in air. As far as the dielectric constants are concerned we assume the tissues to be: isotropic
- homogenous
- linear
- dispersive
- dissipative

Maxwell's equations and the boundary conditions

The electric and magnetic fields satisfy Maxwell's equations and the boundary conditions. The boundary conditions express continuity of the tangential components of the electric and magnetic field and boundedness of the fields at infinite distance of the source.
In the frequency domain, Maxwell's equations are:

\[
\begin{align*}
\text{rot } E_i + j\omega \mu_0 \text{H}_i &= 0 & \text{for } i = [1,n+1] \\
\text{rot } \text{H}_i - j\omega \varepsilon_0 E_i &= 0 & \text{for } i = [1,n+1]
\end{align*}
\]  

(2.1)  

(2.2)

and the boundary conditions are:

\[
\begin{align*}
\text{E}_{n+1}^{z} &= \text{E}_{n+1}^{z+1} & \text{for } z = h_1 \text{ and } i = [1,n] \\
\text{H}_{n+1}^{z} &= \text{H}_{n+1}^{z+1} & \text{for } z = h_1 \text{ and } i = [1,n]
\end{align*}
\]  

(2.3)  

(2.4)

\[
\lim_{z \to \infty} \{E_i, H_i\} \text{ is bounded for } r = [0,\infty)
\]  

(2.5)

\[
\lim_{z \to -\infty} \{E_i, H_i\} \text{ is bounded for } r = [0,\infty)
\]  

(2.6)

\[
\lim_{z \to \pm \infty} \{E_i, H_i\} \text{ is bounded for } z = \pm \infty, i = [1,n+1]
\]  

(2.7)

\[
\lim_{z \to 0} \{E_i, H_i\} \text{ is bounded for } z = \infty, i = [1,n+1]
\]  

(2.8)

employing:

- \(E_i\) : The electric field in the \(i\)-th layer;
- \(H_i\) : The magnetic field in the \(i\)-th layer;
- \(J\) : The current-density in the source;
- \(\varepsilon_i\) : The permittivity in the \(i\)-th layer;
- \(\mu_0\) : The permeability in vacuum;
- \(\omega = 2\pi f\) : The angular frequency;
- \(E_{i,n+1}\) : The tangential component of \(E\) in the \(i\)-th layer i.e. \(E_{r,i}\) and \(E_{e,i}\);
- \(H_{i,n+1}\) : The tangential component of \(H\) in the \(i\)-th layer i.e. \(H_{r,i}\) and \(H_{e,i}\).

Since the tissues are dispersive and isotropic, the permittivity is a complex scalar. Because each tissue is homogenous and dispersive, the permittivity is a function of the frequency. This leads to:

\[
\varepsilon_i(\omega) = \varepsilon_0 (\varepsilon'_i - j\varepsilon''_i) & \text{ for } i = [1,n+1]
\]  

(2.9)

\(\varepsilon_0\) is the permittivity in vacuum and \(\varepsilon'_i\) and \(\varepsilon''_i\) are the real and negative imaginary part of the relative permittivity, respectively.
Definition of the excitation signal

Like the response i.e. the electric and magnetic fields (resp. \( \mathbf{E} \) and \( \mathbf{H} \)), the excitation signal \( \mathbf{J} \) is presented in the frequency domain. The current in the coil \( I \) is only a function of the frequency, \( I = I(f) \). \( I \) is related to the spectral current density \( \mathbf{J} \) as follows:

\[
\mathbf{J} = J_0 \mathbf{\varepsilon}_0 = \frac{I(U(z-h_\text{a}) - U(z-h_{\text{a}-1})(U(r-r_{\text{c}_{\text{1}}})-U(r-r_{\text{c}_{\text{2}}}))}{(h_{\text{a}-1}-h_\text{a}) (r_{\text{c}_{\text{2}}}-r_{\text{c}_{\text{1}}})} \mathbf{\varepsilon}_0 \tag{2.8}
\]

using the unity step function:

- \( U(z) = 1 \) when \( z > 0 \)
- \( U(z) = 0 \) when \( z < 0 \)
- \( U(z) = 0.5 \) when \( z=0 \)

\( \mathbf{J} \) is defined as a current density in air and therefore the material aspects of the coil are abandoned.

Methods to be applied

The BVP is formulated by equations 2.1 to 2.8, the given configuration and the assumptions made for the tissues. Initially, these equations are written in circularly cylindrical coordinates. Because of the axial symmetry of the model application of the Hankeltransform proves to be useful. Finally, the results are calculated numerically.
SOLUTION TO THE BOUNDARY VALUE PROBLEM (BVP)

The BVP in circular cylinder coordinates

Because of the axial symmetry of the configuration and because the imposed current is assumed to be independent on the \( \theta \)-coordinate, it can be concluded that the field shows no angular dependency. So \( \delta_\theta E = 0 \) and \( \delta_\theta H = 0 \).

Then the field equations read:

\[
\begin{align*}
\delta_z E_{\theta 1} - j \mu_0 H_{r 1} &= 0 \quad (3.1a) \\
\delta_z E_{r 1} - \delta_r E_{z 1} + j \mu_0 H_{\theta 1} &= 0 \quad (3.1b) \\
\delta_r E_{\theta 1} + (1/r) E_{\theta 1} + j \mu_0 H_{z 1} &= 0 \quad (3.1c) \\
\delta_z H_{\theta 1} + j \omega \xi_1 E_{r 1} &= 0 \quad (3.1d) \\
\delta_r H_{r 1} - \delta_r H_{z 1} - j \omega \xi_1 E_{\theta 1} &= J_e \quad (3.1e) \\
\delta_r H_{\theta 1} + (1/r) H_{\theta 1} - j \omega \xi_1 E_{z 1} &= 0 \quad (3.1f)
\end{align*}
\]

for \( i = [1,n+1] \)

and the boundary conditions are:

\[
\begin{align*}
E_{r 1} &= E_{r 1+1} \text{ for } z = h_i \text{ and for } i = [1,n] \quad (3.2a) \\
E_{\theta 1} &= E_{\theta 1+1} \text{ for } z = h_i \text{ and for } i = [1,n] \quad (3.2b) \\
H_{r 1} &= H_{r 1+1} \text{ for } z = h_i \text{ and for } i = [1,n] \quad (3.2c) \\
H_{\theta 1} &= H_{\theta 1+1} \text{ for } z = h_i \text{ and for } i = [1,n] \quad (3.2d) \\
\lim_{r \to \infty} \{ E_{r 1}, E_{z 1}, E_{\theta 1}, H_{r 1}, H_{z 1}, H_{\theta 1} \} &\text{ is bounded for } z = \leftarrow, \rightarrow \text{ and for } i = [1,n+1] \quad (3.2e) \\
\lim_{r \to 0} \{ E_{r 1}, E_{z 1}, E_{\theta 1}, H_{r 1}, H_{z 1}, H_{\theta 1} \} &\text{ is bounded for } z = \leftarrow, \rightarrow \text{ and for } i = [1,n+1] \quad (3.2f) \\
\lim_{z \to \infty} \{ E_{1}, H_{1} \} &\text{ is bounded for } r = [0, \rightarrow] \quad (3.2g) \\
\lim_{z \to -\infty} \{ E_{n+1}, H_{n+1} \} &\text{ is bounded for } r = [0, \rightarrow] \quad (3.2h)
\end{align*}
\]

From these equations it is seen that the current density \( J_e \) excites the field components \( H_{r 1}, H_{z 1} \) and \( E_{\theta 1} \) in each layer \( i = [1,n+1] \). The other components \( E_{r 1}, E_{z 1} \) and \( H_{\theta 1} \) are assumed to be 0 because they are not excited by the source, nor by the tissues. (see the boundary conditions)

Now, the field equations are:

\[
\begin{align*}
\delta_z E_{\theta 1} - j \mu_0 H_{r 1} &= 0 \quad (3.3a) \\
\delta_r E_{\theta 1} + (1/r) E_{\theta 1} + j \mu_0 H_{z 1} &= 0 \quad (3.3b) \\
\delta_z H_{r 1} - \delta_r H_{z 1} - j \omega \xi_1 E_{\theta 1} &= J_e \quad (3.3c)
\end{align*}
\]

for \( i = [1,n+1] \)

Equation 3.3b can be written as:

\[
(1/r) \delta_r (r E_{\theta 1}) + j \mu_0 H_{z 1} = 0 \quad (3.3d)
\]

for \( i = [1,n+1] \)
The boundary conditions are:

\[ E_{el} = E_{el+1} \text{ for } z = h_i \text{ and for } i = [1,n] \]  \hspace{1cm} (3.3e)
\[ H_{el} = H_{el+1} \text{ for } z = h_i \text{ and for } i = [1,n] \]  \hspace{1cm} (3.3f)
\[ \lim_{z \to -\infty} [E_{el}, H_{r1}, H_{z1}] \text{ is bounded for } z = (-, \rightarrow) \text{ and for } i = [1,n+1] \]  \hspace{1cm} (3.3g)
\[ \lim_{z \to \infty} [E_{el}, H_{r1}, H_{z1}] \text{ is bounded for } z = (-, \rightarrow) \text{ and for } i = [1,n+1] \]  \hspace{1cm} (3.3h)
\[ \lim_{z \to \infty} [E_{e1}, H_{r1}, H_{z1}] \text{ is bounded for } z = (0, \rightarrow) \]  \hspace{1cm} (3.3i)
\[ \lim_{z \to \infty} [E_{e1}, H_{r1}, H_{z1}] \text{ is bounded for } r = (0, \rightarrow) \]  \hspace{1cm} (3.3j)

Hankel transform

Introduction of the Hankel transform will lead to a useful simplification of this set of equations. The Hankel transform of order \( n \) is defined as follows:

\[
\hat{F}(\tau) = H[F(r)] = \int_{0}^{\infty} rF(r)J_n(\tau r)dr
\]  \hspace{1cm} (3.4a)

and is a transformation from the \( r \) domain to the \( \tau \) domain. The inverse Hankel transform of order \( n \) is obtained by interchanging \( \tau \) and \( r \) as well as \( F \) and \( \hat{F} \).

\[
F(r) = H^{-1}[\hat{F}(\tau)] = \int_{0}^{\infty} \tau \hat{F}(\tau)J_n(\tau r)d\tau
\]  \hspace{1cm} (3.4b)

using: \( J_n \): Bessel function of the first kind and integer order \( n \) \((n \geq 0)\).
\( \tau \): Transformation variable.

Application of the Hankel transform

Hankel transformation of the field equations 3.3a and 3.3c with \( n=1 \) and 3.3d with \( n=0 \) results in:

\[ \delta_{el}E_{el} = j\Omega d_{el} \hat{H}_{r1} \]  \hspace{1cm} (3.5a)
\[ \tau\delta_{el}E_{el} = -j\Omega d_{el} \hat{H}_{z1} \]  \hspace{1cm} (3.5b)
\[ \delta_{el}H_{r1} + \tau\delta_{el}H_{z1} - j\Omega d_{el}E_{el} = \delta_{el} \]  \hspace{1cm} (3.5c)
for \( i = [1,n+1] \).
with:

\[ \hat{E}_{01}(\tau) = \int_0^{\infty} rE_{01}(r)J_1(\tau r)dr \]  
(3.6a)

\[ \hat{H}_{r1}(\tau) = \int_0^{\infty} rH_{r1}(r)J_1(\tau r)dr \]  
(3.6b)

\[ \hat{H}_{z1}(\tau) = \int_0^{\infty} rH_{z1}(r)J_1(\tau r)dr \]  
(3.6c)

\[ \hat{F}_0(\tau) = \int_0^{\infty} rJ_0(\tau r)dr \]  
(3.6d)

for \( i = [1,n+1] \)

under the condition that:

\[ \lim_{r \to 0} \sqrt{F(E,H)} = [0,0] \]

This condition will be satisfied in all cases as the field components are decaying exponentially towards infinity because of dissipative matter.

Now, the boundary conditions are:

\[ \hat{E}_{01} = \hat{E}_{01+1} \quad \text{for} \quad z = h_i, \quad i = [1,n] \]  
(3.5d)

\[ \hat{H}_{r1} = \hat{H}_{r1+1} \quad \text{for} \quad z = h_i, \quad i = [1,n] \]  
(3.5e)

\[ \lim_{z \to -\infty} [\hat{E}_{01},\hat{H}_{r1},\hat{H}_{z1}] \text{ is bounded} \]  
(3.5f)

\[ \lim_{z \to -\infty} [\hat{E}_{a+1},\hat{H}_{r+1},\hat{H}_{z+1}] \text{ is bounded} \]  
(3.5g)

Substitution of eq. 3.5a and 3.5b in eq. 3.5c leads to the scalar inhomogenous differential equation in the \( \tau,z \) domain:

\[ \delta^2_z \hat{E}_{01} + k_z^2 \hat{E}_{01} = S_i \quad \text{for} \quad i = [1,n+1] \]  
(3.7a)

where \( k_z^2 = k_i^2 - \tau^2 \) for \( i = [1,n+1] \) with \( k_i^2 = \Omega' \varepsilon_1 \mu_0 \).

Now:

\[ S_i = jQ\mu_0 I/((r_{c2}-r_{c1})(h_s - h_{s-1})) \int_{r_{c1}}^{r_{c2}} rJ_1(\tau r)dr \text{ when } i = s \]  
(3.7c)

\[ S_i = 0 \text{ when } i = 1,n+1]/s \]
using the axial wavenumber in the i-th layer, $k_{z,i}$ and the wavenumber in the i-th layer, $k_i$.

Since $k_i$ and $k_{z,i}$ are both square roots of a complex number, two solutions in the complex plane emerge. Only one solution for $k_i$ and $k_{z,i}$ is allowed. Because $\varepsilon_1$ lies in the fourth quadrant of the complex plane, we define $k_i$ and $k_{z,i}$ also to be in this quadrant. Since the tissues are always dissipative, $\varepsilon''_1$ is positive and the imaginary parts of $k_{z,i}$ and $k_i$ cannot equal 0, so:

$$\text{re}[k_{z,i},k_i] = [0,\rightarrow] \quad \text{for } i = [1,n+1]$$
$$\text{im}[k_{z,i},k_i] = [\leftarrow,0] \quad \text{for } i = [1,n+1]$$

A particular solution, $\hat{E}_{i,p}$, of the differential-equation reads:

$$\hat{E}_{i,p} = S_i/k_{z,i}^2 \quad \text{for } i = [1,n+1]$$

whereas the homogenous solution, $\hat{E}_{i,h}$, is given by:

$$\hat{E}_{i,h} = A_i \exp(-jk_{z,i}z) + B_i \exp(jk_{z,i}z)$$
for $z = [h_i,h_{i-1}]$ when $i = [2,n]$
for $z = [h_i,\rightarrow]$ when $i = 1$
for $z = [\leftarrow,h_a]$ when $i = n+1$

This leads to the general solution of the differential equation:

$$\hat{E}_i = A_i \exp(-jk_{z,i}z) + B_i \exp(jk_{z,i}z) + S_i/k_{z,i}^2 \quad \text{(3.8a)}$$
for $z = [h_i,h_{i-1}]$ when $i = [2,n]$
for $z = [h_i,\rightarrow]$ when $i = 1$
for $z = [\leftarrow,h_a]$ when $i = n+1$

An expression for $\hat{H}_z$ is found by combining eq.'s 3.5a, 3.5b and 3.8a:

$$\hat{H}_z = (kj_1/\mu_o)\left[-A_i \exp(-jk_{z,i}z) + B_i \exp(jk_{z,i}z)\right] \quad \text{(3.8b)}$$
for $z = [h_i,h_{i-1}]$ when $i = [2,n]$
for $z = [h_i,\rightarrow]$ when $i = 1$
for $z = [\leftarrow,h_a]$ when $i = n+1$

Determination of the coefficients $A_i$ and $B_i$ [6]

Substitution of the expression for $\hat{E}_{i,h}$ in the boundary conditions results in a set of equations from which the coefficients $A_i$ and $B_i$ are solved:
\[ A_i \exp(-jkz_1 h_i) + B_i \exp(jkz_1 h_i) + S_i / k_e^* =\]
\[ A_{i+1} \exp(-jkz_{1+1} h_{i+1}) + B_{i+1} \exp(jkz_{1+1} h_{i+1}) + S_{i+1} / k_e^* \]  
\text{(3.9a)}

for \( i = [1,n] \)

\[ k_{z_1} (A_i \exp(-jkz_1 h_i) - B_i \exp(jkz_1 h_i)) =\]
\[ k_{z_{1+1}} (A_{i+1} \exp(-jkz_{1+1} h_{i+1}) - B_{i+1} \exp(jkz_{1+1} h_{i+1})) \]  
\text{(3.9b)}

for \( i = [1,n] \)

\( B_i \exp(jkz_1 z) \) increases if \( z \) increases because \( \text{Im}[kz_1] \leq 0 \). Since \( \lim_{z \to \infty} B_i \) is bounded this implicates for \( B_i \) that:

\[ B_i = 0 \]  
\text{(3.9c)}

For the same reason, \( A_i \exp(-jkz_1 z) \) decreases when \( z \) decreases. Since \( \lim_{z \to -\infty} A_i \) is bounded this implicates for \( A_{i+1} \) that:

\[ A_{i+1} = 0 \]  
\text{(3.9d)}

The vector notation

Equation 3.9a, 3.9b, 3.9c and 3.9d are sufficient to calculate the coefficients \( A_i \) and \( B_i \). For this goal the vector notation is introduced. For typographical reasons a column vector will be noted in square brackets:

\[ \begin{pmatrix} x \\ y \\ z \end{pmatrix} := [x,y,z] \]

Introduce:

\[ P_i = [1,kz_1,0] \exp(-jkz_1 h_i) \quad \text{for} \quad i = [1,n+1] \]
\[ Q_i = [1,-kz_1,0] \exp(jkz_1 h_i) \quad \text{for} \quad i = [1,n+1] \]
\[ S_p = S_i / k_e^* \quad \text{for} \quad i = [1,n+1] \]
\[ S_p = [S_i / k_e^*, 0, 0] \quad \text{for} \quad i = [1,n+1] \]

with \( i \) denoting the layer index and \( j \) denoting the boundary index. Now, 3.9a and 3.9b are written as:

\[ A_i P_i + B_i Q_i = A_{i+1} P_{i+1} + B_{i+1} Q_{i+1} \]  
\text{(3.10a)}

\[ A_i P_i^s + B_i Q_i^s + S_p = A_{i+1} P_{i+1}^s + B_{i+1} Q_{i+1}^s \]  
\text{(3.10b)}

\[ A_i P_i^{s-1} + B_i Q_i^{s-1} + S_p = A_{i-1} P_{i-1}^{s-1} + B_{i-1} Q_{i-1}^{s-1} \]  
\text{(3.10c)}

The vector product (\( \times \)) of eq. 3.10b and \( Q_i^{s+1} \) and the vector product of \( P_i^{s+1} \) and eq. 3.10b yield:

\[ A_s (P_i^{s} \times Q_i^{s+1}) + B_s (Q_i^{s} \times Q_i^{s+1}) + S_p X Q_i^{s+1} = A_{s+1} W_{s+1} \]  
\text{(3.11b)}

\[ A_s (P_i^{s+1} \times X P_i^{s}) + B_s (P_i^{s+1} \times X Q_i^{s}) + P_i^{s+1} X S_p = B_{s+1} W_{s+1} \]  
\text{(3.11c)}
with $W_i = (P_i X Q_i) = [0,0,W_i]$ wherein $W_i = -2k_{z_i}$ is the Wronskian.

Accordingly, the vector product $(X)$ of eq. 3.10c and $Q_i^{-1}$ and the vector product of $P_i^{-1}$ and eq.3.10c yield:

$$A_{i-1}(P_i^{-1}XQ_i^{-1}) + B_{i-1}(Q_i^{-1}XQ_i^{-1}) - S_p XQ_i^{-1} = A_0W_0$$ (3.11d)

$$A_{i-1}(P_i^{-1}XP_i^{-1}) + B_{i-1}(P_i^{-1}XQ_i^{-1}) - P_i^{-1}XS_p = B_0W_0$$ (3.11e)

Each vector product results in a vector with the following shape: $[0,0,factor]$. In general "factor" does not equal 0. Eq.'s 3.11b,c,d and e can now be written as:

$$\Delta_{i+1} = C_i \Delta_i + \mathcal{S} \Delta_i$$ for $i = [1,n]$ (3.12a)

using $A_i = [A_i,B_i]$ and:

$$C_i = 1/W_{i+1} \begin{pmatrix} C_{i11} & C_{i12} \\ C_{i21} & C_{i22} \end{pmatrix}$$ for $i = [1,n]$ (3.12b)

with:

$$C_{i11} = (-k_{z_{i+1}} + k_{z_i}) \exp (k_{z_{i+1}} - k_{z_i})h_i$$
$$C_{i12} = (k_{z_{i+1}} - k_{z_i}) \exp (k_{z_{i+1}} + k_{z_i})h_i$$
$$C_{i21} = (-k_{z_{i+1}} - k_{z_i}) \exp (-k_{z_{i+1}} - k_{z_i})h_i$$
$$C_{i22} = (-k_{z_{i+1}} + k_{z_i}) \exp (-k_{z_{i+1}} + k_{z_i})h_i$$

for $i = [1,n]$

$$\mathcal{S} \Delta_i = \frac{\mathcal{S}_p}{s} [\exp (k_{s+1}h_s), \exp -j(k_{s+1}h_s)]$$ $i = s$ (3.12c)
$$= -\frac{\mathcal{S}_p}{s} [\exp (k_{s+1}h_s), \exp -j(k_{s+1}h_s)]$$ $i = s-1$
$$= 0$$ $i = [1,n+1]/s$

The inverse matrix $C_i^{-1}$ is:

$$C_i^{-1} = 1/W_i \begin{pmatrix} C_{i11} & C_{i12} \\ C_{i21} & C_{i22} \end{pmatrix}$$ for $i = [1,n]$ (3.13b)

with:

$$C_{i11} = (-k_{z_{i+1}} - k_{z_i}) \exp (-k_{z_{i+1}} + k_{z_i})h_i$$
$$C_{i12} = (k_{z_{i+1}} - k_{z_i}) \exp (k_{z_{i+1}} + k_{z_i})h_i$$
$$C_{i21} = (k_{z_{i+1}} + k_{z_i}) \exp (-k_{z_{i+1}} - k_{z_i})h_i$$
$$C_{i22} = (-k_{z_{i+1}} + k_{z_i}) \exp (k_{z_{i+1}} - k_{z_i})h_i$$

for $i = [1,n]$

and can be used to calculate the amplitudes in reverse direction:
\[ A_i = C_i^{-1} A_{i+1} - S P_{i}^{1_{\alpha \nu}} \quad \text{for } i = [1,n] \quad (3.13a) \]

\[ S P_{i}^{1_{\alpha \nu}} = \frac{1}{2} S_p [\exp(j k_1 h_x), \exp(-j k_1 h_x)] \quad i = s \quad (3.13c) \]

\[ = -\frac{1}{2} S_p [\exp(j k_s h_x), \exp(-j k_s h_x)] \quad i = s-1 \]

\[ = 0 \quad i = [1,n+1]/s \]

Since \( B_1 = 0 \) and \( A_{n+1} = 0 \) we write:

\[ A_1 = [1,0] \alpha \quad (3.14a) \]
\[ A_{n+1} = [0,1] \beta \quad (3.14b) \]

\( A_n \) is calculated in a top-down direction \( (A_n = F(A_{i+1}) \) and \( A_i \) is calculated in a bottom-up direction \( (A_i = G(A_{n+1}) \). This leads to:

\[ C_{s-1} \ldots C_{i} [1,0] \alpha - S P_{s-1} = A_i = C_{s-1} \ldots C_{i} [0,1] \beta - S P_{i}^{1_{\alpha \nu}} \quad (3.15) \]

Calculation of \( \alpha \) happens by taking the scalar product of \( (1,0) \) and:

\[ C_s \ldots C_{s-1} \ldots C_1 (1,0) \alpha = [0,1] \beta + C_s \ldots C_1 (S_P_{s-1} - S P_{i}^{1_{\alpha \nu}}) \]

which leads to:

\[ \alpha = \frac{(1,0) C_s \ldots C_i (S_P_{s-1} - S P_{i}^{1_{\alpha \nu}})}{(1,0) C_s \ldots C_1 [1,0]} \quad (3.16a) \]

\( \beta \) is determined in a similar way now taking the scalar product with \( (0,1) \), so:

\[ \beta = \frac{(0,1) C_{i}^{-1} \ldots C_{s}^{-1} (-S P_{s-1} + S P_{i}^{1_{\alpha \nu}})}{(0,1) C_{s}^{-1} \ldots C_{i}^{-1} [0,1]} \quad (3.16b) \]

Equation 3.16a and eq. 3.12. are used to determine \( A_i \) for \( i = [1.s] \) while eq. 3.16b and eq. 3.13. serve to determine \( A_i \) for \( i = [s,n+1] \). In the coil layer these amplitudes have to match.

Now a means is provided to compute the amplitudes \( A_i \) and \( B_i \) of \( E_{i+1} \) in each layer "\( i \)". By substitution of \( A_i \) in eq. 3.8a the differential equation in \( E_{i+1} \) is analytically solved for this model \( (i = [1,n+1]) \). Employing eq's 3.8b and 3.8c the expressions for \( H_r \) and \( H_z \) can be calculated.
ANALYSIS OF THE IMPEDANCE

The impedance of the coil consists of two components i.e. the internal and the external impedance. The impedance presented at the input of the coil is the sum of these components and reads written as [3]:

\[ Z_c = (-1/|I|^2) \int_V (E \cdot J^*) \, d^3x \]  

(4.1a)

using:

- \( J^* \): Complex conjugate of the current;
- \( Z_c \): Internal and external coil impedance;
- \( I \): Short-circuit current of the coil;
- \( \cdot \): Scalar product of two vectors.

In circular cylinder coordinates the coil impedance is written as:

\[ Z_c = (-1/|I|^2) \int_0^{2\pi} \int_0^\infty \int_0^h (E \cdot J^*) \, r \, dz \, dr \]  

(4.1b)

In our model all components of \( E \) and \( H \) and the excitation \( J \) show no \( \theta \)-dependence. Furthermore, the only non-zero components of \( E \) and \( J \) are \( E_0 \) and \( J_0 \). So it is written that:

\[ Z_c = (-2\pi/|I|^2) \int_0^{h_0} (E_0 \cdot J_0^*) \, r \, dr \]  

(4.1c)

Parseval's theorem [2] allows the following substitution:

\[ \int_0^\infty (E_0 \cdot \hat{J}_0) \, r \, dr = \int_0^\infty \hat{E}_0 \cdot \hat{J}_0 \, r \, d\tau \]

This results in:

\[ Z_c = (-2\pi/|I|^2) \int_0^{h_0} (\hat{E}_0 \cdot \hat{J}_0) \, r \, dz \, d\tau \]  

(4.2a)

Where \( \hat{E}_0 \) is the Hankel transform of \( E_0 \) in the coil layer and \( \hat{J}_0 \) is the Hankel transform of \( J_0 \) in the coil layer.
Executing the integration over \( z \) yields:

\[
Z_c = (-2\pi/|I|^2) \int_0^\infty F(\tau) d\tau \tag{4.3}
\]

The integrand reads:

\[
F(\tau) = \left( J^*_s / j(k_s) \right) \left( -A_s \exp(-jk_z s h_s - jh - jh) + B_s \left( \exp(jk_z s h_s - jh) - \exp(jk_z s h_s) \right) - S_s [h_s - jh]/(jk_s) \right)
\]

Solution procedure

All equations necessary to calculate the required field distribution and coil impedance are derived. Solution of the set of equations is to be continued numerically because of the complexity of the expressions of the amplitudes \( A_s \) and the inverse Hankel transform of \( E_z \), \( H_r \), \( H_z \) and the Hankel transform of \( J_s \).
### NUMERICAL APPROXIMATION

**Numerical integration**

To compute an integral \( I \) the compound Simpson rule is used. This rule reads:

\[
I = \int_{a}^{b} f(x) \, dx = T_a + R_a
\]  

\( T_a = \frac{1}{3h} [f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + \ldots + 4f(x_{n-1}) + f(x_n)] \)  

\( R_a = -\frac{1}{90} h^5 f^{(4)}(\bar{c}) \)

The integrand \( f(x) \) is calculated at \( n \) integration intervals and thus \( f(x) \) is calculated at \( n+1 \) discrete values \( x_i, \ i = [0,n] \) of the integration variable. The \( x_i \)'s are equidistant so \( h = x_i - x_{i-1} = \) constant, \( i = [1,n] \) and \( x_0 = a \) and \( x_n = b \). The calculated integrands \( f(x_i), \ i = [0,n] \) are weighed and added. The outcome of this sum is multiplied by \( h \) which results in an approximated value of the integral \( T_a \) [4]. The error made by this approximation is \( R_a \).

**Estimation of \( n \) and \( h \)**

It is easily shown that \( R_a \approx \frac{1}{15}(T_{2a} - T_a) \) which can serve as a criterium to calculate \( R_a \) in relation to \( T_{2a} \). Using this criterium it is proven that if \( f(x) = \text{Acos}(2\pi f_x x) \), \( a = 0 \) and \( b = \pi + 1/f_x \), then for a sufficient reduction of \( R_a \), \( n \geq 6 \). Consider the fourier transform of \( f(x) \), \( F(f_x) \) which is a function of the frequency \( f_x \). Let \( h \) fit minimally eight times in the lowest period (= 1/highest frequency) of \( F(f_x) \).

In general \( F(f_x) \) will be non-zero at all frequencies \( f_x \). Therefore \( f(x) \) has to be investigated to determine \( f_x \) for calculation of \( n \) and \( h \).

This can be applied on the integration of the source integral (see eq. 3.7c). Now \( a = r_c \), \( b = r_{c2} \), \( x = \tau \) and \( f(x) = rJ_{1}(\tau r) \). The minimal distance between two zero-crossings of the integrand is determined by \( J_{1}(\tau r) \) for large arguments \( \tau r \) and equals \( n \). \( J_{1}(\tau r) \) behaves as a cosine for large values of \( \tau r \). So we regard \( J_{1}(\tau r) \) as a cosine with respect to the assessment of \( h \). \( \tau r \cdot \delta r = 2\pi \) and \( n \geq 8 \) in one period. \( \delta r \approx n/4\tau \) and \( n = (b-a)/\delta r \geq 4\tau(b-a)/n \) in the complete integration interval.

Sometimes the high frequencies reside in a relatively small part of the integration interval \([a,b]\). In that case there is no need to let \( n \) and \( h \) depend on this small part for the complete integration interval. The integration interval
is divided in sub-intervals. Each sub-interval has its own \(n\) and \(h\). This is to prevent unnecessary many integrand computations at intervals where the integrand is relatively smooth.

For a correct computation of the Simpson rule, \(n\) has to be odd in each interval. This will be tested and when necessary, \(n\) will be incremented by 1.

**Approximation of infinite integrals**

If \(b (b > a)\) approaches infinity then the integration interval has to be reduced to \([a, b]\) since computers only handle finite numbers. In respect of our application \(b \rightarrow \infty\) when an inverse Hankel transform has to be computed. The criterium for allowing this interval reduction is determined after investigation of the integrand \(g(\tau)\) of an inverse Hankel transform of the function \(h(\tau) = \{E_0, R_r, R_z\}\). Now \(g(\tau)\) looks like:

\[
g(\tau) = h(\tau) + J_1(\tau)\]

or

\[
g(\tau) = h(\tau) + J_0(\tau)\]

Then this criterium sounds:

\[
|g(\tau)| < 0.01 \text{ MAX}(g) \text{ for } \tau = [b, \infty)
\]

where \(\text{MAX}(g)\) is the maximal value of \(g(\tau)\) for \(\tau = [a, b]\). Now \(I\) reads:

\[
I = \int_a^b g(\tau) \, d\tau = \int_a^a g(\tau) \, d\tau + \int_a^b g(\tau) \, d\tau \approx \int_a^b g(\tau) \, d\tau
\]

As most integrations are inverse Hankel transforms their integration intervals range from \([0, \infty)\). The integration is taken over \([0, b]\). The integration over \([b, \infty)\) is neglected. For every inverse Hankel transform the behaviour of the integrand has to be investigated in the complete interval \([0, \infty)\).

**Preconditions for correct integral computations**

The accuracy of a computer is always limited. If a number is presented as a fractional part and an exponential part we will operate at an accuracy of 16 digits in the fractional part. Occasionally, some rejected digits are important for further computations. For this reason two tests have to be performed to increase the reliability of the algorithm:

- boundary conditions;
- \(\tau\)-sensitivity.
The boundary conditions

These are tested in the \( \tau \)-domain because the inverse Hankel transform is not expected to raise leaps in \( E_i \) or \( H_i \) when \( z = h_i \), \( i = [1,n] \) and omitting the inverse Hankel transform saves a considerable amount of computation time. The algorithm for computation of \( A_i \) \( [i = 1,n+1] \) might be the cause of some problems in this respect.

The \( z \)-dependence of \( \hat{E}_{i+1} \), \( \hat{H}_{i+1} \) and \( \hat{H}_{i+1} \) shows up in the common term \( \exp(-jk_{i+1}z) \) and the computed results for \( \hat{E}_{i+1} \), \( \hat{H}_{i+1} \) and \( \hat{H}_{i+1} \) are bounded when \( z \rightarrow \infty \). The \( z \)-dependence of \( \hat{E}_{i+1} \), \( \hat{H}_{i+1} \) and \( \hat{H}_{i+1} \) shows up in the common term \( \exp(jk_{i+1}z) \) and the computed results \( \hat{E}_{i+1} \), \( \hat{H}_{i+1} \) and \( \hat{H}_{i+1} \) are bounded when \( z \rightarrow -\infty \). The boundary conditions to be tested are:

\[
\begin{align*}
\hat{E}_{i+1} &= \hat{E}_{i+1} \quad \text{for } z = h_i, \quad i = [1,n] \\
\hat{H}_{i+1} &= \hat{H}_{i+1} \quad \text{for } z = h_i, \quad i = [1,n]
\end{align*}
\]

Since \( \hat{E}_{i+1} \), \( \hat{H}_{i+1} \) and \( \hat{H}_{i+1} \) are calculated at discrete points and layer \( i \) is defined to be positioned in \([h_i, h_{i-1})\), the boundary conditions are checked according to:

\[
\begin{align*}
\text{lim } \frac{\hat{E}(z=h_i) - \hat{E}(z=h_i-\delta z)}{\delta z} &\leq 10 \quad \text{for } i = [1,n] \quad (5.3a) \\
\text{lim } \frac{\hat{H}(z=h_i) - \hat{H}(z=h_i-\delta z)}{\delta z} &\leq 10 \quad \text{for } i = [1,n] \quad (5.3b)
\end{align*}
\]

These equations represent the relative variation of \( \hat{E}_{i+1} \) and \( \hat{H}_{i+1} \) at the \( i \)-th boundary as a function of \( z \), respectively.

The \( \tau \)-sensitivity

The second test is performed by executing two times the numerical computation. For the second computation each indexed \( \tau \), \( \tau_j \), \( j = [0,n] \) will be multiplied by \( 1+\delta \). For a stable algorithm, the relative variation in the final result has to be of the same order as \( |\delta| \); the absolute value of the relative variation in \( \tau_j \), \( j = [0,n] \). In other terms:

\[
\frac{2 \left| \left[ \{E_0, H_0, H_z \} (\tau_j) - \{E_0, H_0, H_z \} ((1+\delta)\tau_j) \right] \right|}{\left| \{E_0, H_0, H_z \} (\tau_j) + \{E_0, H_0, H_z \} ((1+\delta)\tau_j) \right|} < 10 |\delta| \quad (5.4)
\]

The algorithms for computation of \( J_0 \) and \( J_1 \) and the inverse Hankel transform are already tested thoroughly and proven to remain stable on input disturbances. The computation of \( A_i \) and \( B_i \), \( i = [1,n+1] \) might introduce instability.

Therefore, some worst case situations are simulated to investigate the stability of the algorithm for computation of \( A_i \) and \( B_i \) by substituting large leaps in the permittivities in the contiguous layers of the configuration. Until now, no problems are encountered.
SIMULATIONS

Configurations

$E_o$, $H_r$, $H_z$ and $Z_c$ are computed in the following configurations:

1. Coil in air. This is depicted in Figure 6.1

![Fig. 6.1 Cross-section of the coil in air.]

2. Coil above phantom. See Figure 6.2

![Fig. 6.2 Cross-section of the coil above phantom.]
3. The third configuration is a simulation of a body surface. In this simulation it is assumed that the surface consists of three, plane layered tissues of infinite extent. The tissues are skin, fat and muscle. Nearest to the coil is a layer of skin, then a layer of fat which on the other side borders on a halfspace of muscle. This is illustrated in Figure 6.3.

![Figure 6.3 Cross-section of the coil above skin, fat and muscle.]

For each of the configurations, two frequencies, often used in MRI, are substituted. These frequencies are 21 MHz and 64 MHz.

**Permittivities in different media**

The permittivity is defined as \( \varepsilon_{r,i} = \varepsilon_r'^i - j\varepsilon_r'' \). In air \( \varepsilon_{r,i} = \varepsilon_r^i \) for \( i, j \in [1, 3] \). The conductivity \( \sigma \) in air is assumed to be \( 10\text{E-6} \, (\Omega\text{m})^{-1} \) and relates to \( \varepsilon_r'' \) as \( \sigma = \Omega \varepsilon_r \varepsilon_r'' \). This leads to \( \varepsilon_{r,1} = 1 - j87.8\text{E-5} \) when \( f = 21 \text{ MHz} \) and \( \varepsilon_{r,1} = 1 - j28.8\text{E-5} \) when \( f = 64 \text{ MHz} \).

The relative permittivities used in the computations are listed in Table 7.1.

<table>
<thead>
<tr>
<th></th>
<th>( f = 21 \text{ MHz} )</th>
<th>( f = 64 \text{ MHz} )</th>
</tr>
</thead>
<tbody>
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<td>Air</td>
<td>( \varepsilon_r'^1 )</td>
<td>( \varepsilon_r'^1 )</td>
</tr>
<tr>
<td></td>
<td>( 1.0 )</td>
<td>( 1.0 )</td>
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<tr>
<td></td>
<td>( 87.8\text{E-5} )</td>
<td>( 28.8\text{E-5} )</td>
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<td>90</td>
</tr>
<tr>
<td></td>
<td>750</td>
<td>250</td>
</tr>
</tbody>
</table>

**Table 7.1 Relative complex permittivities of media involved.**
Format of $E\,\gamma$, $H_\rho$, $H_z$ and $Z_c$

The quantities to be derived from the complex values of $E\,\gamma$, $H_\rho$, and $H_z$ are:

- $\text{mod}(E) = \sqrt{(E_\gamma E_\gamma^*)}$: electric field strength;
- $\text{arg}(E) = \arctan(\text{im}(E)/\text{re}(E))$: phase of the electric field;
- $\text{mod}(H) = \sqrt{(H_\rho H_\rho^* + H_z H_z^*)}$: magnetic field strength;
- $\text{arg}(H_\rho) - \text{arg}(H_z) = \arctan(\text{im}(H_\rho)/\text{re}(H_\rho)) - \arctan(\text{im}(H_z)/\text{re}(H_z))$: phase difference of $H_\rho$ and $H_z$;
- $\text{dir}(H) = \arctan(H_z/|H_\rho|)$: direction of the magnetic field.

Finally, $Z_c$ is investigated as a function of:

- $h_{s-1} - h_s$: coil thickness;
- $r_{c2} - r_{c1}$: coil width;
- $[r_{c2} + r_{c1}]/2$: average coil radius;
- $h_s - h_{s+1}$: coil-to-matter distance.

Quantities, derived from $Z_c$ are:

- $R_c$: Coil resistivity;
- $L_c$: Coil inductivity;
- $Q = \Omega L_c/R_c$: $Q$-factor of the coil. (see app. 3)

Results and conclusions with respect to $E\,\gamma$, $H_\rho$ and $H_z$

The spatial electric field strengths in air, the phantom case and the tissue case are shaped similarly for a frequency of either 21 MHz or 64 MHz. This is seen in Figure 6.4 to Figure 6.7 which are graphical illustrations of the electric field strengths in air and in the phantom case when the frequency is 21 MHz and when the frequency is 64 MHz. The greatest difference of the electric field strength is found at $f = 64$ MHz and in $r = 7.5E-2$ m, $z = -6.0E-2$ m. At this position $\text{mod}(E) = 0.016 \, \text{v/m}$ in air and $\text{mod}(E) = 0.0098 \, \text{v/m}$ in the phantom.

It is seen in Figure 6.4 to Figure 6.7 that $\text{mod}(E(f=64\text{MHz})) \approx 3\text{mod}(E(f=21\text{MHz}))$. 
Fig. 6.4 Electric field strength of coil in air for a frequency of 21 MHz.

Air
$f = 21.0E+6$ Hz
Thickn. coil = 2.0E-3 m
$RC_{1,2} = 38E-3 42E-3$ (m)
$\varepsilon_{\text{rl, Air}} = 1 - j86.0E-5$

Fig. 6.5 Electric field strength of coil in air for a frequency of 64 MHz.

Air
$f = 64.0E+6$ Hz
Thickn. coil = 2.0E-3 m
$RC_{1,2} = 38E-3 42E-3$ (m)
$\varepsilon_{\text{rl, Air}} = 1 - j28.0E-5$
Fig. 6.6 Electric field strength of coil above phantom for a frequency of 21 MHz.

Phantom
\( f = 21.0 \text{E}+6 \text{Hz} \)
Thickn. coil = 2.0E-3
RC1,2 = 38E-3 42E-3
Epsr\((\text{Air})\) = 1 - j86.0E-
Epsr\((\text{phm})\) = 100 - j170E-
Dist. to phm = 8.0E-3

Fig. 6.7 Electric field strength of coil above phantom for a frequency of 64 MHz.

Phantom
\( f = 64.0 \text{E}+6 \text{Hz} \)
Thickn. coil = 2.0E-3
RC1,2 = 38E-3 42E-3
Epsr\((\text{Air})\) = 1 - j28.0E-
Epsr\((\text{phm})\) = 100 - j57E-
Dist. to phm = 8.0E-3
In air, the phase of $E_0$ is $\pi/2$ rad. For a frequency of 64 MHz, the spatial phase variation of $E_0$ in the phantom case has a range of approximately 1.3 rad, whereas the corresponding variation in the tissue case is limited to appr. 0.6 rad. (See Figure 6.8 and Figure 6.9) From Figure 6.10 it is seen that the spatial phase variation of $E_0$ in the phantom case has a range of appr. 0.5 rad for a frequency of 21 MHz. Therefore, it is concluded that the spatial phase variation of $E_0$ depends strongly on both configuration and frequency.

The magnetic field strength in air and the tissue case for a frequency of 64 MHz are depicted in Figure 6.11 and Figure 6.12, respectively. The biggest differences in these field strengths are found at $r = 7.5 \times 10^{-2}$ m and $z = -6.0 \times 10^{-2}$ m. At this position $\text{mod}(\mathbf{H}) = 0.687 \times 10^{-3}$ A/m in air and $\text{mod}(\mathbf{H}) = 0.753 \times 10^{-3}$ A/m in the muscle. In general $\text{mod}(\mathbf{H}(f=64\text{MHz})) \approx \text{mod}(\mathbf{H}(f=21\text{MHz}))$.

In air the phase difference of $H_r$ and $H_z$ is 0. Just as the phase of $E_0$, the phase difference of $H_r$ and $H_z$ in matter varies considerably as a function of $r$ and $z$. Graphic illustrations are troubled by noise at the positions where $\text{mod}(H_z) \approx 0$ since the value of $\arctan(|\text{im}(H_z)|/|\text{re}(H_z)|)$ is dependent on noise on $\text{im}(H_z)$ and $\text{re}(H_z)$. Therefore, the phase difference of $H_r$ and $H_z$ can better be evaluated from the input data file of the graphics generator.

![Fig. 6.8 Phase of $E_0$ of coil above phantom for a frequency of 21 MHz.](image)
Tissue
\[ f = 64.0E+6 \text{Hz} \]
Thickn. coil = 2.0E-3 m
\[ RC_{1,2} = 38E-3 42E-3 \text{ (m)} \]
\[ \text{Epsr(Air)} = 1 - j28.0E-5 \]

Phantom
\[ f = 64.0E+6 \text{Hz} \]
Thickn. coil = 2.0E-3 m
\[ RC_{1,2} = 38E-3 42E-3 \text{ (m)} \]
\[ \text{Epsr(Air)} = 1 - j28.0E-5 \]
\[ \text{Epsr(phantom)} = 100 - j570 \]
Dist. to phm = 0.0E-3 m

Fig. 6.9 Phase of \( E_\phi \) of coil above tissues for a frequency of 64 MHz.

Fig. 6.10 Phase of \( E_\phi \) of coil above phantom for a frequency of 64 MHz.
Fig. 6.11 The magnetic field strength in air for a frequency of 64 MHz.

Air

f = 64.0E+6Hz
Thickn. coil = 2.0E-3 m
RC1,2 = 38E-3 42E-3 (m)
Epsr(Air)=1 - j28.0E-5

Fig. 6.12 The magnetic field strength of coil above tissue for a frequency of 64 MHz.

Tissue

f = 64.0E+6Hz
Thickn. coil = 2.0E-3 m
RC1,2 = 38E-3 42E-3 (m)
Epsr(Air)=1 - j28.0E-5
From Figure 6.14 and Figure 6.15 it is seen that there is a significant difference in direction of the magnetic fields when the coil is in air and when the coil is positioned above phantom. The rectangles in these illustrations are the cross-sections of the coil. The solid line in Figure 6.14 coincides with the boundary of the phantom.

Now, a second coil can be placed on top of the first one. The middle of the upper coil is positioned at a different r-value as the bottom coil. This is depicted in Figure 6.13. If the current in the upper coil $I_2$ has the correct amplitude and phase in relation to the current in the bottom coil, a circularly polarized magnetic field $H_{circ}$ is generated at at least one position $(r_{circ}, z_{circ})$. The direction of the magnetic fields, generated either coil and thus $(r_{circ}, z_{circ})$ are dependent on the configuration. In other words if a circularly polarized magnetic field has to be generated at $(r_{circ}, z_{circ})$, the positioning of the second coil depends on the configuration.

Fig. 6.13 Cross-section of two coils.
Fig. 6.14 Direction of the magnetic field of the coil in air.

Air dir(H)
f = 64.0E+6Hz
Thickn. coil = 2.0E-3 m
RC1,2 = 38E-3 42E-3 (m)
Epsr(Air)=1 - j28.0E-5

Fig. 6.15 Direction of the magnetic field of the coil above phantom.

Phantom dir(H)
f = 64.0E+6Hz
Thickn. coil = 2.0E-3 m
RC1,2 = 38E-3 42E-3 (m)
Epsr(Air)=1 - j28.0E-5
Epsr(phm)=100 - j570
Dist to phm = 6.0E-3 m
Results and conclusions with respect to coil impedance

In MRI a high Q-factor is desired. Next, if the coil is used as a receiver, its inductivity is proportional to the induced voltage. Therefore, we aim at a low coil resistivity, a high coil inductivity and thus a high Q-factor.

When \( 0.1E-3m < h_s - h_a < 2.0E-3m \) there is a small change in coil resistivity and Q-factor. \( R_c \approx 3.3 \) ohm and \( Q \approx 22 \). In Figure 6.16 it is seen that coil inductivity decreases at an increasing coil thickness.

```
Phantom
f = 64.8E+6 Hz
RC1,2 = 38E-3 42E-3 (m)
Epsr(Air) = 1 - j28.0E-5
Epsr(phm) = 100 - j578
Dist. to phm = 8.0E-3 m
```

Fig. 6.16 Coil inductivity as function of coil thickness.

Variation of coil width happens by variation of only \( r_c \). This way, the flux is constant. \( R_c \approx 3.4 \) ohm when \( 0.1E-3m < r_c < 1.0E-3m \). In this case \( h_s - h_a = 1.0E-4 \) m. \( L_c(r_c - r_{c1}) \) and \( Q(r_c - r_{c1}) \) are depicted in Figure 6.17 and Figure 6.18, respectively. Increasing coil width leads to a small diminishment of both coil inductivity and Q-factor.
Phantom
\[ f = 64.0 \times 10^6 \text{ Hz} \]
\[ \text{hs-1 - hs} = 1.0 \times 10^{-4} \text{ m} \]
\[ \text{RC1} = 40.0 \times 10^{-3} \text{ m} \]
\[ \text{Epsr}(\text{Air}) = 1 - j28.0 \times 10^{-5} \]
\[ \text{Epsr}(\text{phm}) = 100 - j570 \]
\[ \text{Dist. to phm} = 8.0 \times 10^{-3} \text{ m} \]

Fig. 6.17 Coil inductivity as function of coil width.

Phantom
\[ f = 64.0 \times 10^6 \text{ Hz} \]
\[ \text{hs-1 - hs} = 1.0 \times 10^{-4} \text{ m} \]
\[ \text{RC1} = 40.0 \times 10^{-3} \text{ m} \]
\[ \text{Epsr}(\text{Air}) = 1 - j28.0 \times 10^{-5} \]
\[ \text{Epsr}(\text{phm}) = 100 - j570 \]
\[ \text{Dist. to phm} = 8.0 \times 10^{-3} \text{ m} \]

Fig. 6.18 Q-factor as function of coil width.
In MRI, large coils are necessary if a larger region has to be depicted. $R_c([r_{c2} + r_{c1}]/2)$, $L_c([r_{c2} + r_{c1}]/2)$ and $Q([r_{c2} + r_{c1}]/2)$ are illustrated in Figure 6.19, 6.20 and 6.21, respectively. Where $2.0E-2m < [r_{c2} + r_{c1}]/2 < 8.0E-2m$.

At increasing coil radius, the Q-factor decreases considerably but coil inductivity increases.

**Fig. 6.19** Coil resistivity as function of coil radius.

**Fig. 6.20** Coil inductivity as function of coil radius.
Since the phantom is a load for the coil it is expected that if the distance from coil to matter increases, coil resistivity decreases and Q-factor increases. This is seen in Figure 6.22 and 6.23 for the phantom case. This does not mean that the coil-to-matter distance must be maximized because the sensitivity of the coil for a magnetic field distribution in matter decreases significantly if the distance to that matter increases.

$L_c \approx 1.7 \times 10^{-7}$ when $0.1E-2m < h_n - h_{n+1} < 2.0E-2m$. 

**Phantom**

- $f = 64.0E+6$ Hz
- $h_0 - h_5 = 1.0E-4$ m
- $RC2-RC1 = 4.0E-3$ m
- $\varepsilon_{r(\text{Air})} = 1 - j20.0E-5$
- $\varepsilon_{r(\text{pm})} = 100 - j570$
- Dist. to $\text{pm} = 8.0E-3$ m
Fig. 6.22 Coil resistivity as function of coil-to-phantom distance.

Phantom
- $f = 64.0 \times 10^6 \text{ Hz}$
- $h_s - 1 - h_s = 2.0 \times 10^{-3} \text{ m}$
- $R_{C1,2} = 38 \times 10^{-3} - 42 \times 10^{-3} \text{ (m)}$
- $\varepsilon_{r_{\text{Air}}} = 1 - j28.0 \times 10^{-5}$
- $\varepsilon_{r_{\text{Phm}}} = 100 - j570$

Fig. 6.23 Q-factor as function of coil-to-phantom distance.

Phantom
- $f = 64.0 \times 10^6 \text{ Hz}$
- $h_s - 1 - h_s = 2.0 \times 10^{-3} \text{ m}$
- $R_{C1,2} = 38 \times 10^{-3} - 42 \times 10^{-3} \text{ (m)}$
- $\varepsilon_{r_{\text{Air}}} = 1 - j28.0 \times 10^{-5}$
- $\varepsilon_{r_{\text{Phm}}} = 100 - j570$
Appendix 1

Manual of the available programs

As explained in the previous chapter, the input data of the simulation programs has to pass several inspections before we can conclude that the simulated field distribution or coil impedance is reliable. For this purpose several test programs are written. The output of these test programs and the simulation programs can better be evaluated when it is presented in a graphical way.

Several graphics generators are available. However, every generator uses its own format for the input data. Therefore interface programs are necessary to put the results of our computations in the correct format for the graphics generators used.

Input data

In general the input data for the simulation programs EMFIELD and IMPCOIL, and the test programs TSTLAMBDA and TSTBOND contains the same parameters. These parameters are read from the input files "PAR.DAT" and "VAR.DAT".

"PAR.DAT" only consists of integers. After reading the contents of this file the following parameters are specified:

* s: Index of the sourcelayer
* n+1: Number of layers to be included in the system
* noz: Number of coordinates in the z direction
* nor: Number of coordinates in the r direction
* notint: Number of \( \tau \)-integration intervals (maximal 5)

Second line
* not(\( \tau \)int): Number of integration steps in each interval: \( \tau \)int = [1,5]

PAR.DAT looks like:

\[
\begin{array}{cccccc}
s & n+1 & noz & nor & notint \\
\text{not(1)} & \text{not(2)} & \text{not(3)} & \text{not(4)} & \text{not(5)}
\end{array}
\]

Fig. 6.1 configuration of PAR.DAT

On both lines, the integers are read on position 5, 10, 15, 20 and 25, respectively.
"VAR.DAT" contains real and complex variables, preceded by character expressions. These character expressions are not to be changed as their existence is tested by the program before the data is read. The expressions are:

- IFEPSMUO: Data, read in the following line is:
  - I in Ampere
  - f in Hertz
  - $\varepsilon'_i u_0$ in m$^4$/m$^2$
  - $\mu'_i$ in mkgC$^{-2}$

- EPSILONS: $\varepsilon'_i$ and $-\varepsilon''_i$, $i = [1,11]$ The four lines following this command consist of a maximum of eleven pairs, three pairs in either of the first three lines and two pairs in the fourth line;

- HVALUES: The next two lines contain $h_i$, $i = [1,11]$. First line: $h_i$, $i = [1,6]$ Second line: $h_i$, $i = [7,11]$;

- LAMBDainter: This is followed by the $\tau$-values for each interval. $\tau_{i\text{int}}(i\text{int})$, $i\text{int} = [1,\text{not}\text{int}+1]$ The upper boundary of one interval serves simultaneously as the lower boundary of the subsequent interval;

- RC_Z_R_RNG: The next line contains:
  - $r_{c1}$ in m
  - $r_{c2}$ in m
  - $z_{\text{min}}$ and $z_{\text{max}}$ in m: the z-range where $E_z$, $H_r$ and $H_z$ are calculated
  - $r_{\text{min}}$ and $r_{\text{max}}$ in m: the r-range where $E_z$, $H_r$ and $H_z$ are calculated;

- END_OF_VAR: No further information will be read.
VAR.DAT has the following configuration:

**IFEPSMUO**

<table>
<thead>
<tr>
<th>I</th>
<th>f</th>
<th>$\varepsilon_0^*\mu_0$</th>
<th>$\mu_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$\varepsilon_r$ (1)</td>
<td>$\varepsilon_r$ (2)</td>
</tr>
</tbody>
</table>

**EPSILONS**

- $\varepsilon_r$ (1) - $\varepsilon_r$ (2) - $\varepsilon_r$ (3) - $\varepsilon_r$ (4) - $\varepsilon_r$ (5) - $\varepsilon_r$ (6) - $\varepsilon_r$ (7) - $\varepsilon_r$ (8) - $\varepsilon_r$ (9) - $\varepsilon_r$ (10) - $\varepsilon_r$ (11)

**HVALUES**

- $h_1$ - $h_2$ - $h_3$ - $h_4$ - $h_5$ - $h_6$

**LAMBDAINT**

- tint(1) - tint(2) - tint(3) - tint(4) - tint(5) - tint(6)

**RC_Z_R_RNG**

- $r_{c_1}^*$ - $r_{c_2}^*$ - zmin - zmax - rmin - rmax

**END_OF_VAR**

---

Fig. 6.2 Configuration of the inputfile VAR.DAT

Now the position where the subsequent values are to be read is a multiple of 10 in each line, except for $\varepsilon_0^*\mu_0$ and $\mu_0$ which are to be read at position 20 and 45, respectively.

Some precautions on input variables

A small value for $\varepsilon_r^*$ has to be substituted in order to prevent singularities in the integrand of the inverse Hankel transform. Singularities will also emerge when the argument ($\tau r$) substituted in the algorithm for the Bessel function equals 0. Therefore $\tau$ and $r$ are not allowed to be 0. As a consequence, the integrand must also be inspected on the 40dB criterion at the lowest $\tau$-value.

Test programs

The test programs employing this input data are:

- **TSTLAMBD**
  - **purpose:** Determination of the $\tau$-interval for a correct inverse Hankel transform.
  - **method:** The integrand of each inverse Hankel transform is computed as a function of $\tau$.
  - **output:**
    - $E_0^*(\tau, z)$ in EF.DAT
    - $E_r^*(\tau, z)$ in HR.DAT
    - $\bar{H}_r^*(\tau, z)$ in HZ.DAT
    - $\tau$-values in ULVA.DAT
    - $z$-values in UZVA.DAT
Appendix 1 40

- TSTBOND in co-operation with PREBOND

purpose: Check if the boundary conditions are met.

method: Computation of \( \hat{E}_0, \hat{H}_r, \) and \( \hat{H}_z \) when \( z = h_i \) and \( z = h_i - \delta z, i = [1,n] \).

output: \( \hat{E}_0 (\tau, h_i) \) and \( \hat{E}_0 (\tau, h_i - \delta z) \) for \( i \in [1,n] \) in EF.DAT

\( \hat{H}_r (\tau, h_i) \) and \( \hat{H}_r (\tau, h_i - \delta z) \) for \( i \in [1,n] \) in HR.DAT

\( \hat{H}_z (\tau, h_i) \) and \( \hat{H}_z (\tau, h_i - \delta z) \) for \( i \in [1,n] \) in HZ.DAT

\( \tau \)-values in ULVA.DAT

z-values in UZVA.DAT

Selection of boundary (i) and \( \hat{E}_0, \hat{H}_r, \) or \( \hat{H}_z \) and the quantity to be compared (absolute value, real or imaginary part) happens in PREBOND. The relative difference is to be read in BOND.CMD. (See eq. 5.3) The maximal relative difference returns on screen.

- TSTCOEF

purpose: Inquiry of the stability of the algorithm for the computation of the coefficients \( A_i \) and \( B_i \).

method: Computation and comparison of \( A_i (\exp(\pm jkh_i)) \) and \( A_i ((1+\delta)\exp(\pm jkh_i)) \) for \( i \in [1,n+1] \).

output: The maximal relative change is monitored.
EMFIELD and IMPCOIL

- EMFIELD
  purpose: $E_0, H_r$ and $H_z$ are computed.
  output: $E_0(r,z)$ in EF.DAT
           $H_r(r,z)$ in HR.DAT
           $H_z(r,z)$ in HZ.DAT
           $\tau$-values in ULVA.DAT
           $r$-values in URVA.DAT
           $z$-values in UZVA.DAT

  It will be asked if the computation has to take place with disturbed $\tau$-values. If the answer is "y", $\delta$ has to be inserted. (See equation 5.4) If the answer is "n", $\delta = 0$. If $E_0, H_r$ and $H_z$ are computed two times, i.e. $\{E_0, H_r, H_z\}(\delta>0)$ and $\{E_0, H_r, H_z\}(\delta=0)$ then these results can be compared using PRELSTAB.

- PRELSTAB
  purpose: Computation of the relative difference of the contents of two files.
  method: According to equation 5.4.
  input: * names of the files to be compared.
         * $\delta$ (See eq.5.4)
  output: The relative difference of the selected field values at matching spatial positions in LSTAB.CMD and the relative difference(s) exceeding $\delta$ on screen.

- IMPCOIL
  purpose: Computation of $Z_c$.
  method: As described in chapter 4.
  output: $Z_c$ on screen.
          IMPL.CMD wherein the behaviour of $F(\tau)$ can be studied. $F(\tau)$ has to satisfy the 40dB requirement in the neglected interval before the monitored impedance has any meaning.

The graphics interface programs

The data in EF.DAT, HR.DAT, HZ.DAT, ULVA.DAT, URVA.DAT and UZVA.DAT is unformatted. Interface programs are developed to rearrange the contents of these files so that the output files can be used as input for the graphics generator. The graphics generator is developed by BUTS ELECTRONIC SYSTEMS. The output data is always preceded by some plot directives. A brief explanation of the interface programs is given below:

- PREPLOT
  purpose: Either the magnitude, phase, real or imaginary part of either $E_0$, $H_r$ or $H_z$ is stored in 3DFILE.CMD to serve as input for 3DPLOT which generates a 3-dimensional image on the screen.
  input: EF.DAT, HR.DAT or HZ.DAT and both URVA.DAT and UZVA.DAT
  output: 3DFILE.CMD
- PREFUNC
purpose: Storage in 2DFILE.CMD of a selected integrand of one of the inverse Hankel transforms. (See eq. 5.2a) This program succeeds TSTLAMBD. 2DFILE.CMD is input for the 2-dimensional image generator: PLOTUTIL.
input: The z-value for $g(\tau,z)$ has to be inserted from the console.
output: 2DFILE.CMD

- PREARROW
purpose: Storage of the magnetic field direction in ARROW.CMD (See chapter 6) An arrow picture is a 2-dimensional illustration of the magnetic field direction at the selected, discrete positions where the magnetic field is computed: $\text{Dir}(H(r_i,z_j))$, $i = [1,\text{nor}]$, $j = [1,\text{noz}]$.

Magnetic field direction and phase difference are computed and stored in MAGN.CMD and PHASEDIF.CMD, respectively. (See chapter 6)

input: It will be asked if the user wants to select a part of $(r_i,z_j)$, $i = [1,\text{nor}]$, $j = [1,\text{noz}]$. If the answer is "y" then selection takes place by inserting the:
* minimal r-value;
* maximal r-value;
* minimal z-value;
* maximal z-value;
* maximal number of r- and z-coordinates in the grid to be imaged.

If the answer is "n", the direction of the magnetic field is computed at all positions where the magnetic field is computed i.e. $(r_i,z_j)$, $i = [1,\text{nor}]$, $j = [1,\text{noz}]$.

output: ARROW.CMD
MAGN.CMD
PHASEDIF.CMD
Appendix 2

Programs in meta language

In this appendix, the structure of EMFIELD, IMPCOIL, TSTLAMBD and TSTBOND is presented in meta language. This meta language can be considered as a high level programming language wherein the name of each statement clarifies the result of its action.

TSTLAMBD, TSTBOND do have the same structure as EMFIELD. They compute intermediate results of EMFIELD.

EMFIELD metalanguage

Program EMFIELD

read parameters from par.dat
read variables from var.dat

fill $E_e (r_1, z_1), H_r (r_1, z_1), H_z (r_1, z_1)$ with $(0,0)$

do 100 $\tau$ _index = 1 to max_$\tau$ _index
  compute $\tau$
  compute $kz(i), i=[1,n+1]$
  compute $S_p$
  compute $A_1, i=[1,n+1]$
  do 200 $\tau$ _index = 1 to max_$\tau$ _index
    compute $E_e (\tau, z_1), A_r (\tau, z_1), A_z (\tau, z_1)$
    do 300 $r$ _index = 1 to max_$r$ _index
      integrate $E_e * J_1 (\tau * r_1) * \tau$ over $\tau$
      integrate $A_r * J_0 (\tau * r_1) * \tau$ over $\tau$
      integrate $A_z * J_1 (\tau * r_1) * \tau$ over $\tau$
    300 continue
  200 continue
100 continue

store $E_e (r_1, z_1), H_r (r_1, z_1), H_z (r_1, z_1)$
End
Appendix 2

IMPCOIL metalanguage

Program IMPCOIL

read parameters from par.dat
read variables from var.dat
define \( r_{e1}, r_{e2}, h_{e-1}, h_e \)

fill \( E_0(r_1,z_1) \) with \((0,0)\)

do 100 \( \tau\_index = 1 \) to \( \text{max}\_\tau\_index \)
   compute \( \tau \)
   compute \( kz(i), i=[1,n+1] \)
   compute \( S_p \)
   compute \( A_i, i=[1,n+1] \)
   compute \( F(\tau) \) (see eq. 4.3)
   integrate \( F(\tau) \) over \( \tau \)
100 continue

display \( Z_c \)
End

TSTLAMBD metalanguage

Program TSTLAMBD

read parameters from par.dat
read variables from var.dat

fill \( E_0(\tau,z_1), H_r(\tau,z_1), H_z(\tau,z_1) \) with \((0,0)\)

do 100 \( \tau\_index = 1 \) to \( \text{max}\_\tau\_index \)
   compute \( \tau \)
   compute \( kz(i), i=[1,n+1] \)
   compute \( S_p \)
   compute \( A_i, i=[1,n+1] \)
   do 200 \( z\_index = 1 \) to \( \text{max}\_z\_index \)
   compute \( E_0(\tau,z_1), H_r(\tau,z_1), H_z(\tau,z_1) \)
200 continue
100 continue

store \( E_0(\tau,z_1), H_r(\tau,z_1), H_z(\tau,z_1) \)
End
TSTBOND metalanguage

Program TSTBOND

read parameters from par.dat
read variables from var.dat
\[ z_{11} := h_1, \quad z_{11-1} := h_1 - \delta z \quad \text{for}\ i = [1,n+1] \]

fill \( \hat{E}_0(\tau,z_1), \hat{R}_r(\tau,z_1), \hat{R}_z(\tau,z_1) \) with \( (0.0) \)

do 100 \( \tau_{\_index} = 1 \) to max_\( \tau_{\_index} \)
compute \( \tau \)
compute \( k_z(i), \ i=[1,n+1] \)
compute \( S_0 \)
compute \( \hat{\Delta}_1, \ i=[1,n+1] \)
do 200 \( z_{\_index} = 1 \) to max_\( z_{\_index} \)
compute \( \hat{E}_0(\tau,z_1), \hat{R}_r(\tau,z_1), \hat{R}_z(\tau,z_1) \)
200 continue
100 continue

store \( \hat{E}_0(\tau,z_1), \hat{R}_r(\tau,z_1), \hat{R}_z(\tau,z_1) \)
End
Appendix 3

Q-FACTOR

The impedance of the coil \( Z_c \) is seen as a series connection of an inductor \( L_c \) and a resistor \( R_c \) so:

\[
Z_c = R_c + j\omega L_c
\]

\( R_c \) does not include copper losses. \( Z_c \) connected parallel to a capacitor \( C_p \) results in a resonant circuit. (See Figure A.1)

![Resonant circuit 1](image1)

**Fig. A.1** Resonant circuit 1.

![Resonant circuit 2](image2)

**Fig. A.2** Resonant circuit 2.

The \( Q \)-factor of the circuit (\( Q_r \)) depicted in Figure A.2 is defined as \( Q_r = R_p \sqrt{C_p / L_p} \) [5] Substitution of \( R_p \) and \( L_p \) by \( Z_c \) is allowed if:

\[
R_c + j\omega L_c = 1/(1/R_p + 1/(j\omega L_p))
\]

So \( R_c = Q^2 L_p R_p / (R_p^2 + Q^2 L_p^2) \) and \( L_c = L_p R_p^2 / (R_p^2 + Q^2 L_p^2) \). If \( Q \approx 1/\sqrt{L_p C_p} \) then:

\[
Q_r \approx R_p / Q L_p = Q L_c / R_c
\]

Assumed that \( C_p \) is an ideal capacitor and thus \( R_p \) evolves from the coil, the \( Q \)-factor of the coil (\( Q \)) is defined as:

\[
Q = Q L_c / R_c
\]

\( Q \) is an important factor in MRI coil development.
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