MASTER

Three-dimensional magnetostatic field of a video head

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J.F.G.M. Savelkoul* and
S.B. Luitjens

THREE-DIMENSIONAL MAGNETOSTATIC
FIELD OF A VIDEO HEAD

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THREE DIMENSIONAL MAGNETOSTATIC FIELD
OF A VIDEO HEAD

Notebooks: none

* This study has been made in fulfilment of the requirements for the Degree of Master of Science (Ir.) at the Eindhoven University of Technology, Department of Electrical Engineering, under the supervision of Prof. Dr. M.P.H. Weenink, Eindhoven University of Technology and Dr. Ir. S.B. Luitjens, Philips Research Laboratories, Eindhoven.
SUMMARY

This report treats the three-dimensional calculation of the magnetic field strength of a video head, excited by a permanent magnet, which is a model for the head's coil. With the help of the magnetic field strength at the position of the tape, the side reading of the low frequency track following signal by the constriction of the head can be studied.

The calculations are done with the program package PADDY, that uses a finite element method for solving magnetostatic problems. With the PADDY results also the efficiency and the self inductance of the head are calculated.

The numerical Fourier Transform of these results is complicated by the relatively small number of "field points" in a finite part of 3D-space. Therefore an analytical model is proposed, that consists of magnetically charged plates. With this model the effect of the constriction can be described properly. The surface charge on these plates is chosen considering the results of PADDY. This model is a first step towards a proposed method, which allows calculation of the magnetic field in every point in space (in principal). This method uses the surface charge density of the head, that can be obtained from the PADDY results.

ACKNOWLEDGEMENT

I wish to thank those at the Philips Research Laboratories and at the Eindhoven University of Technology who made this work possible. In particular I would like to thank Prof. Dr. M.P.H. Weenink and Dr.ir. S.B. Luitjens for their stimulating guidance.

Furthermore I am grateful to Drs. J. van Welij (ISA PHILIPS) for his advices on the use of the program package PADDY. I also wish to thank Drs. J. van Welij and Ing. A. Rijckaert for reading parts of the manuscript for this report.

J. Savelkoul
Eindhoven,
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THREE-DIMENSIONAL MAGNETOSTATIC FIELD OF A VIDEO HEAD

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CHAPTER ONE

GENERAL INTRODUCTION AND BACKGROUND OF THE PROBLEM

1.0. INTRODUCTION

1.1. WHY THREE DIMENSIONAL?

1.2. THE APPLICATION OF THE CONSTRICTION IN A VIDEO HEAD

1.3. THE EFFECT OF THE CONSTRICTION IN A VIDEO HEAD

1.4. SOME RESULTS OF MAGNETIC RECORDING THEORY
1.0. INTRODUCTION

In this chapter a brief introduction into magnetic recording is given. The question why the calculation of the magnetic field should be three dimensional is treated. The major problem in increasing track density: crosstalk, is discussed and an application of it in the V2000 systems is given. Finally some results of magnetic recording theory are presented and their importance is elucidated.
1.1. WHY THREE DIMENSIONAL?

To answer this question we have to obtain some insight into the magnetic recording process.

The magnetic recording process transforms a time varying electrical signal, representing the information to be stored, into a pattern of spatially varying magnetization in a magnetizable medium (write process, Figure 1.1a). This is achieved by moving the magnetizable medium, with constant velocity, past a write head. The write head transforms the electrical signal into a time-varying magnetic field, which in its turn, magnetizes the medium: the information is stored.

When the stored information has to be reproduced, the medium is moved past the read head. The read head transforms the spatially varying pattern of magnetization into an electrical signal (read process, Figure 1.1b), which after being properly processed, can be made to resemble the original write current (van Herk, 1980).

Figure 1.1

The magnetic medium is usually a tape or a disk, on which the information is stored in straight or circular tracks. These tracks are lying side by side in a thin magnetic layer on a non-magnetic carrier material. When the electrical signal is represented by the variation of the magnetic flux,
tization in the direction of transportation of the medium, we have the so-called longitudinal recording mode (Figure 1.2). The other directions of magnetization mentioned in Figure 1.2 are related to transverse and perpendicular recording.

Up to now most recording systems operated in the longitudinal mode. However since the paper of Iwasaki and Nakamura (1977) much effort has been put into developing systems that use the perpendicular mode. In this report we will concentrate on the longitudinal process.

![Diagram of magnetic recording](image)

**Figure 1.2**

An account of the fact that magnetic recording techniques are of widespread use for storing a tremendously growing amount of information, there is a constant demand for higher storage capacity. One of the important parameters governing this storage capacity, is the width of the tracks that are recorded on the magnetic medium.

At the beginning of magnetic recording the first recorders that utilized a flat tape, needed the entire tape width for recording a single track in the longitudinal mode. To increase storage capacity the track width was reduced and more tracks were laid side by side on the tape. It was soon discovered that a drastic reduction of the track width and of the distance between adjacent tracks leads to difficulties.
The source of these difficulties is the fringing field of the head illustrated in Figure 1.3.

![Figure 1.3](image)

Its influence on the write and read properties of the head can be distinguished into several adverse effects (Van Herk 1980). Two of them are: the side writing and the crosstalk of adjacent tracks. They put a primary limitation on the track density available (Figure 1.4).

![Figure 1.4](image)
With the help of the reciprocity theorem in magnetic recording theory it can be made clear, that the structure of the magnetic field external to a magnetic recording head determines, through its introduction with the recording medium, the recording and reproduction properties of the configuration (De Hoop, 1977). In the fifties Westmijze (1953) and Karlqvist (1954) already published a theoretical analysis of this field. These calculations, however, were based on a two-dimensional approximation of the problem. This means that the resulting field has only an x and an y component which are independent of the z coordinate.

The field of the structure in Figure 1.5 is properly described in the symmetry plane z=0 by means of a two dimensional approach. For \( L \gg d_1 \) and \( L \gg d_2 \) this approximation is valid for \( |z| < 1/2L \). Near \( |z| = 1/2L \) the field is not properly described anymore.

![Figure 1.5](image-url)

For a one track recording system a two dimensional description of the field is sufficient, since there are no neighbouring tracks in the z direction.

The relative dimensions of today's magnetic heads in smalltrack-recording systems make, that certain characteristic effects cannot be described with only an x and an y dependence of the field components. One of these effects is crosstalk. Since it is our goal to investigate this phenome-
non, the dependence of the field components of the z coordinate should be taken in account. So the calculation should be three dimensional.

The consequence is that the calculation cannot be done analytically for an arbitrary magnetic head. Only for special head analytical three-dimensional expressions for the field exist, for instance: "the infinite-wedge modes (Van Herk, 1977)". So for practical heads a numerical calculation is needed.
1.2. THE APPLICATION OF CROSSTALLK IN THE V2000 SYSTEMS

In the previous paragraph we described crosstalk as a troublesome phenomenon, that restricts the density of tracks. Yet crosstalk can be made useful, as will be shown below.

Available consumer video-recorders record the tracks with the so-called "helical-scan" process. With this technique the tracks are written inclined on the tape. The video-2000 systems use a track width of 22 micrometer. To position the heads on the right track, Dynamic Track Following (D.T.F.) is used (Bahr 1981). In the explanation of D.T.F. and throughout the rest of this report we shall denote the track, the head should be positioned on with: "the actual track".

Figure 1.6 shows the read signal caused by a track of width 22 micrometer and read with a head of the same width. The recorded signal in Figure 1.6 has a wavelength of 50 μm. This is a low frequency signal, since the velocity with which head and tape move relative to each other is 5 m/sec (V2000).

![Figure 1.6](image-url)
For such a signal it can be seen, that when the head is positioned exactly next to the actual track, the amplitude of the read signal decreases 10 dB. From this figure it is clear, that the signal loss is an indication for the position of the head relative to the actual track. If it would be possible to distinguish between the amplitudes of the low-frequency signals of the two neighbouring tracks, seen by the video head as crosstalk, one could develop a control loop: The head should be moved in the direction of the crosstalk signal with the lower amplitude until both measured amplitudes are equal. This is illustrated in Figure 1.7.

A way to distinguish between the crosstalk signals is to see to it, that they have different frequencies. In order to create these frequencies, during recording on each four successive tracks low frequency signals are written (Figure 1.8a). The order is given by Figure 1.8b.
The frequencies \( f_1, \ldots, f_4 \) are chosen in such a way, that mixing the low frequency signal of the actual track with the low frequency crosstalk of the adjacent tracks, always gives two signals of 15 kHz and 47 kHz. By comparing the amplitudes of these signals a decision can be made in which direction the head should be moved, in order to position it exactly on the actual track. So the head should be movable in the upward and downward direction (Figure 1.9).
In order to make the head movable, it is placed on a so-called actuator (Figure 1.9). It consists of two polarized piezo-electrical plates, which are partly placed over each other, as shown in Figure 1.10. Dependent on the polarity of the applied control voltage the head can be moved upwards and downwards.

Figure 1.10

So the crosstalk phenomenon has been used to solve the problem of the positioning of the head. The consequence for the shape of the head to achieve optimum tracking will be discussed in the next section.
1.3. THE EFFECT OF THE CONSTRICTION IN A VIDEO HEAD

For the track following system as described in the former section a head is needed, which reads the low frequency information and suppresses the high frequency components. A magnetic head that displays such a function in the ring head with constriction, that is applied in the V2000 systems. To understand the operation of such a head, we shall elucidate the ring head itself first.

The head consists of an annular core of magnetic material. This "ring" is interrupted by an air gap (Figure 1.11), where a fringing field is present during the write process. This field exceeds the coercive force of the medium over slightly more than the entire width of the head: the information is recorded on a track.

In the read process the magnetic medium moves relatively to the head and causes a change in the magnetic flux \( \Phi \) (Wb) that is coupled into the head at the gap. The induced read voltage \( \mathcal{E}_R(t) \) at the terminals of the coil (n turns) is given by Faraday's law:

![Figure 1.11](image-url)
When a sinusoidal magnetization pattern is present on the medium, we can observe an interesting feature in the read voltage. At a wavelength near the gap length \((G)\) \(\mathcal{E}_R(t)\) is zero. This phenomenon is called the gap-loss effect. The reason for its occurrence is that at this wavelength, no flux is coupled into the head. The same behaviour can be noticed at \(G/l = 1, 2, 3, \ldots\). So we can define a gap-loss function (Westwijze, 1953):

\[
\frac{\sin\left(\frac{\pi G \lambda}{\lambda}ight)}{\left(\frac{\pi G \lambda}{\lambda}\right)}
\]

Therefore ring heads are used for frequencies lower than the frequency corresponding to the gap length.

In order to suppress the high frequency signals of the adjacent tracks, these tracks should be read with a head with a relatively large gap. Meanwhile the actual track should be read with a very small gap. To satisfy these seemingly contradictory demands, a constriction is made in the ring head. Figure 1.12a shows the pole-face after the constriction is made.

![Figure 1.12a](image_url)
Near the gap \(|x| < 20 \mu m\) we see that the width \((W_1)\) of the head is 23 \(\mu m\). For increasing \(|x|\) the width of the gap increases and for \(|x| > 60 \mu m\) even more strongly. The reason for this is illustrated in Figure 1.12b. Supposing that \(T_4\) is the actual track and \(T_3\) and \(T_5\) are its neighbours, it can be seen that the low frequencies of \(T_1\) and \(T_7\) can disturb the tracking process, unless they are read with a gap length that suppresses even these low frequencies.

Heads of the kind of the one illustrated in Figure 1.12a are used in the Philips V2000 recorder. In fact two heads are used, that write a track in turn. They have different azimuth as shown in Figure 1.12b. This results in an extra attenuation of the high frequency crosstalk of adjacent tracks due to mistracking, because it is read with a head that is 30° out of line. For long wavelengths the crosstalk is of far less importance. That is why we neglect it and take the gap perpendicular to the x-axis.

The head we used in our calculations (Figure 1.13 and 1.14) has two planes of symmetry \((x=0\) and \(z=0)\) instead of one \((x=0)\). Furthermore it has a flat pole-face instead of a
curved one. In this way the input for the program package PADDY is considerably simplified and we still have the real situation of a head pressed against the tape. The sizes of the head are slightly different as those in Figure 1.12a, but a comparison with former results (Van de Berg, 1981) is possible now.

Figure 1.13

<table>
<thead>
<tr>
<th>VIDEO HEAD</th>
</tr>
</thead>
<tbody>
<tr>
<td>DIMENSIONS IN MICROMETERS</td>
</tr>
<tr>
<td>POLE-FACE</td>
</tr>
<tr>
<td>217</td>
</tr>
</tbody>
</table>

| FRONT VIEW |
| 800 |
| 820 |

| CROSS-SECTIONAL VIEW |
| 128.5 |
| 3000 |
| -200 |

Figure 1.13
1.4. SOME RESULTS OF MAGNETIC RECORDING THEORY

In this section we give some results of magnetic recording theory and define most of the quantities we use. Throughout this report SI units are used.

In paragraph 1.1. we mentioned the constant velocity which the medium should be moved with past the head. This velocity \( v \) (m/sec) is related to the recorded wavelength \( \lambda \) (m) and the frequency \( f \) (1/sec) of the signal as given by:

\[
\nu = \lambda \cdot f
\]

The wave number \( k \) is defined by:

\[
k = \frac{2\pi}{\lambda}
\]

In the following we will use two Fourier transforms:

a) time \( (t) \) frequency \( (\omega) \)
b) space \( (x) \) wavelength \( (\lambda) \)

The transform pairs related to a) and b) are:

\[
\tilde{f}(\omega) = \int_{-\infty}^{\infty} f(t) e^{j\omega t} \, dt ; \quad f(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \tilde{f}(\omega) e^{-j\omega t} \, d\omega
\]

\[
\tilde{f}(k) = \int_{-\infty}^{\infty} f(x) e^{j-kx} \, dx ; \quad f(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \tilde{f}(k) e^{j-kx} \, dk
\]

\( \tilde{\varphi}(\omega) \), the Fourier Transform of the read voltage \( \varphi(t) \), is called the harmonic response of a magnetic head. Since
1.1. is an expression for the read voltage, we can write for the harmonic response

\[ \tilde{v}_R(\omega) = j\omega \tilde{\Phi}(\omega) \quad (\text{n} = 0, 1, 2, \ldots) \tag{1.6} \]

An expression for \( \tilde{\Phi}(t) \) is given by the reciprocity theorem of magnetic recording theory (De Hoop, 1977):

\[ \Phi(t) = m_0 \iint \mathbf{H}(x) \cdot \mathbf{M}(x,t) \, dV \tag{1.7} \]

where:

- \( V \) (m³) domain occupied by the recording head
- \( H(x) \) (A/m) head field obtained by a current of 1A through the head's coil in the absence of the tape
- \( M(x,t) \) (A/m) magnetization of the medium defined by \( M = (B/\mu_0) - \mathbf{H} \), where \( B \) is the magnetic induction in Tesla (T).

It is possible to derive an expression for \( \tilde{\Phi}(\omega) \) (Van Herk, 1980):

\[ \tilde{\Phi}(\omega) = m_0 \iint \tilde{M}(-k,y,z) \cdot \tilde{H}(k,y,z) \, dA \tag{1.8} \]

where

- \( A \) (m²) the cross section of the medium perpendicular to the direction of transportation
- \( \tilde{M}(k,y,z) \) and \( \tilde{H}(k,y,z) \) are the Fourier transforms of \( M(r,t) \) and \( H(r) \) respectively.

\( H(k;y,z) \) is called the harmonic flux sensitivity function of
the head. The flux sensitivities for the longitudinal and perpendicular components of \( M \) are then given by \( \overline{H}_x(k;y,z) \) and \( \overline{H}_y(k;y,z) \) respectively.

On the basis of the behaviour of the static magnetic field at the edges of a head and of the sensitivity functions, statements can be made about the crosstalk (Luitjens, Van Herk; 1982). Throughout this report cross talk has our major interest, therefore much attention is payed at the interpretation of the magnetic field and its Fourier transform.

Both the calculation of the field and the sensitivity function have to be done numerically. In chapter two we propose a model for the video head, to study the cross talk.
CHAPTER TWO

A THREE-DIMENSIONAL ANALYTICAL MODEL FOR THE EFFECT OF THE CONSTRICTION

2.0. INTRODUCTION

2.1. A THREE-DIMENSIONAL ANALYTICAL MODEL FOR THE GAP-FRINGING FIELD OF A RING HEAD

2.2. A MODEL FOR THE EFFECT OF THE CONSTRICTION
2.0. INTRODUCTION

In this chapter we start with an analytical model for a ring head. The necessary theory is treated and the results are compared with the ones found in literature. The second paragraph deals with the analytical model for the effect of the constriction. Results, related to a real situation, are presented and discussed.
2.1. A THREE-DIMENSIONAL ANALYTICAL MODEL FOR THE GAP-FRINGING FIELD OF A RING HEAD

The model described is an analytical one and therefore has the advantage, that the results can be computed easily. The fact that it is three dimensional, offers us the possibility to study the crosstalk effect. To explain the model, we have to treat some theory first.

We suppose that the ring head consists of a soft-magnetic body excited by a current carrying coil (write process) and positioned in free space. The near gap field is considered to be caused by the magnetization of the head only. This is justified by the fact that, near the gap, the contribution of the field of the current carrying coil is neglectable. In this way the problem is restricted to a magnetostatic one without currents. Problems of this kind can be described with the following equations of Maxwell.

\[ \nabla \times \mathbf{H} = 0 \]  
\[ \nabla \cdot \mathbf{B} = 0 \]

Furthermore we have:

\[ \mathbf{B} = \mu_0 (\mathbf{H} + \mathbf{M}) \]

The vector \( \mathbf{B} \) is called the magnetic induction (Tesla). The quantity \( \mu_0 \) is the magnetic pressurability of the vacuum. \( V \) is the bounded domain in space occupied by the head, the boundary surface of the head is called \( S \). On \( S \) 2.1. and 2.2. should be supplemented with the boundary conditions: \( \mathbf{n} \times \mathbf{H} \) and \( \mathbf{n} \cdot \mathbf{B} \) continuous when crossing \( S \). \( \mathbf{n} \) is the unit vector along the normal on \( S \) in the outward direction.

However we are interested in the field outside the head at the position of the tape. For this we take the right-handed system of Figure 1.14. The position vector of the observer is called \( \mathbf{r} \) and the position vector of a point on the
surface of the head: \( \xi \).

From 2.1. and 2.2. it follows that we may write:

\[
H(\xi) = -\nabla \cdot U(\xi) \tag{2.4}
\]

with

\[
U(\xi) = \int \left( -\nabla \cdot M(\xi^2) g(\xi, \xi') dU(\xi') \right) + \int \left( \nabla \cdot M(\xi^2) g(\xi, \xi') dS(\xi) \right) \tag{2.5}
\]

Furthermore we define the free-space Green function:

\[
g(\xi, \xi') = \frac{1}{\sqrt{\pi |\xi - \xi'|}} \tag{2.6}
\]

For the case of a magnetic material with a high \( \mu_r \) the magnetic potential \( U(\xi) \) changes significantly only on crossing the gap. Now an approximation for the field near the gap can be given by the field of two homogeneously charged quadrangles of opposite sign at the inner faces of the gap of the ring head. These two sheets represent the potential jump on crossing the gap in the direction of transportation (positive x-direction, Figure 2.1).
As mentioned earlier the model is three-dimensional. This is an advantage with respect to the two-dimensional models of Westwijze (1953) and Karlqvist (1954). A disadvantage however is, that the field does not satisfy the real boundary conditions on the pole-face near the gap. Despite this disadvantage the model has been used successfully by McCary (1971) and Chi (1974) for a ring head. Luitjens and Van Herk (1982) used the model to approximate a single pole head (perpendicular recording).

An expression for the field of a plane **triangularly shaped** distribution of magnetic charge with linearly varying density can be found in Van Herk (1980). In our case we need an expression for a **quadrangularly shaped** distribution \( A(x') \) of magnetic charge with a constant density \( \mathcal{M}(n \cdot M(x')) \).
where

\[ \mathbf{n}_m = \frac{\mathbf{a}_m \cdot \mathbf{a}_{m+1} + \mathbf{a}_m \mathbf{a}_{m+1}}{\mathbf{a}_m \cdot \mathbf{a}_m + \mathbf{a}_m \mathbf{a}_m} \]

The other symbols used, are defined in Table 2.1:

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \mathbf{r}_m )</td>
<td>position vectors of the vertices</td>
</tr>
<tr>
<td>( \mathbf{z}_m )</td>
<td>vectorial sides of the quadrangle ( \mathbf{z}_m = \mathbf{r}_m + 1 - \mathbf{r}_m )</td>
</tr>
<tr>
<td>( \mathbf{a}_m )</td>
<td>vectors from the point of observation ( \mathbf{r} ) to the vertices ( \mathbf{a}_m = \mathbf{r}_m - \mathbf{r} )</td>
</tr>
<tr>
<td>( \mathbf{n}_m )</td>
<td>unit vectors along the tangents to the sides of the quadrangle</td>
</tr>
<tr>
<td>( \mathbf{n}_\phi )</td>
<td>unit vector along the normal on the quadrangle, the direction of circulation along the boundary contour ( \mathbf{C} ) of the quadrangle forms a right-handed system with ( \mathbf{n} ) (Figure 2.3)</td>
</tr>
<tr>
<td>(</td>
<td>\mathbf{a}_m</td>
</tr>
</tbody>
</table>
$\Omega$ is the signed solid angle at which the quadrangle is observed from the point of observation. $\Omega$ is negative when the point of observation $r$ is at the same side of the quadrangle as to where $n_Q$ points and is positive in the other case.
The derivation of 2.8 is given in appendix A.

We now have an analytical expression for the magnetic field of a plane homogeneously charged quadrangle, that can be calculated numerically. Our model of the ring head consists of two quadrangles of opposite charge. So, the total field can be obtained by superposition of the fields of each of the quadrangles separately. This model has been implemented on the computer and the results shown in Figure 2.4. Since we are interested in the near edge field at the position of the tape, we made 3D-plots of the field components in the plane: y=0.1 μm above the head.
FIELD COMPONENTS OF THE RING HEAD MODEL \( Y = 0.1 \mu M \)

dimensions in \( \mu M \)

Figure 2.4
Looking at these results, the following points strike us:

- In contrast to Hx, Hy and Hz show an asymmetrical behaviour near the gap.

- Hx has a maximum at the position of the gap (x=0), it decreases for increasing y (not shown here).

- Hy is zero at the position of the gap (x=0) and reaches extrema on both sides of the gap. These extrema decrease and withdraw from the gap for increasing y (not shown here).

- In this configuration and at y=0.1 μm Hz differs significantly from zero only near the edge and even there it is much smaller than Hx and Hy.

- Although the maximum of |Hy| is smaller than the maximum of Hx, it can be clearly seen, that the maximum of |Hy| decreases faster than the maximum of Hx for constant y and increasing z. As a consequence in the perpendicular mode there will be less side writing than in the longitudinal recording mode (Figure 1.4).

To study the crosstalk phenomenon we calculated the Fourier transform (1.5b) of the magnetic field to obtain the sensitivity function $|\tilde{H}_x(k; y, z)|$ and $|\tilde{H}_y(k; y, z)|$. This calculation has been carried out in the plane $y=0.1$ μm at $z=0.0$, 12.0 and 20.0 μm respectively. The results are given in Figure 2.5.
Sensitivity functions - Ring head model

- Figure 2.5

- $z = 0.0 \mu m$ in the center of the head
- $z = 12.0 \mu m$ at the edge of the head
- $z = 20.0 \mu m$ way at the edge of the head
For very large wavelengths and for very small wavelengths the numerical calculation of the Fourier Transform gives unreliable solutions, but these wavelengths are of no importance for video recording, so they are left out in this report.

In Figure 2.5 we notice that in the region above the head: $|z| \leq \left(\frac{1}{2}\right) W$ (Fig. 2.1):

- first dip at $\lambda = 0.4 \, \mu m$;

- in the plane of symmetry $z=0.0$ and for $\lambda < 32 \, \mu m$ $|\tilde{H}_x| = |\tilde{H}_y|$; for $z$-values near the edges the equality disappears for smaller wavelengths;

- for increasing $y$ the equality disappears also for smaller wavelengths (not shown here).

In the region beside the head $|z| > 1/2 \, W$:

- The values of $|\tilde{H}_x|$ and $|\tilde{H}_y|$ are different for all wavelengths;

- for small wavelengths the longitudinal sensitivity function has larger values than the perpendicular one.

So in spite of the fact that this two plate model does not take into account the effect of the correct boundary conditions at the head, we find the first dip at exactly the gap length. In the symmetry plane $x=0$ the two-dimensional approach is valid for small wavelengths because $|H_x|$ and $|H_y|$ are equal and this is a property of two-dimensional head configurations (Appendix B). For large $\lambda$-values the three-dimensional character of the head becomes evident. The same is noticed for increasing $y$. As far as crosstalk is concerned, we find that the longitudinal recording mode is more sensible for crosstalk than the perpendicular mode (for small wavelengths). Analogous results are found by Luitjens and Van Herk (1982) with their "cubical head".
Stratton (1941) states that at larger distances the field of a magnetic source approached the field of a corresponding dipole. Similar to mechanics a dipole moment \( m_d \) and a geometrical point of gravity: \( \vec{r}_d \) can be defined

\[
m_d = \iiint \mathcal{M}(\vec{x}) \vec{r}' \, dV(\vec{x}') \]

\[
\vec{r}_d = \frac{\iiint \vec{r}' \, dV(\vec{x}')}{\iiint dV(\vec{x}')}
\]

where \( V(\vec{x}) \) is the volume of the magnetic source involved and \( \mathcal{S}(\vec{x}') \) the volume charge density. In our case we have a longitudinal dipole \( m_d = -\vec{M} A_1 \) and \( \vec{r}_d = (0, -1/2H, 0) \) pointing in the negative \( x \)-direction (Figure 2.1). For the sensitivity functions \( \tilde{H}_{dx} \) and \( \tilde{H}_{dy} \) analytical expressions exist (Van Herk, 1980). In our problem they reduce to:

\[
\tilde{H}_{dx}(k; y, x) = \left(\pi a^2\right)^{-1} \int_{m_x} \left[ m_x z a \frac{d}{dz} K_2(az) - (a z)^2 K_1(az) \right] dz
\]

\[
\tilde{H}_{dy}(k; y, x) = \left(\pi a^2\right)^{-1} \int_{m_x} \left[ j m_x (a z)^2 \frac{d}{dz} K_2(az) \right] dz
\]

where \( K_1 \) and \( K_2 \) are modified Bessel functions of the second kind. Furthermore the following expression holds (Abramowitz, 1964)
It is interesting now to compare the behaviour of the F.T. of the dipole field with that of the field of the ring head model for large wavelengths. The sensitivity functions for the dipole are plotted in Figure 2.6. Examining these results we see:

- for all z-values the longitudinal sensitivity functions approximate the ones of the ring model for the large wavelengths;
- the perpendicular sensitivity function of the dipole has larger values than the corresponding one of the ring head model, this difference decreases for increasing z-values.

The results above indicate that a ring head in three dimensional space, at least the perpendicular sensitivity function cannot always be approximated by the one of a corresponding dipole for large wavelengths. Apparently the behaviour of the two fields for smaller distances of the head, causes the sensitivity functions to differ for larger wavelengths (Luitjens, Van Herk 1982).
Figure 2.6
2.2. A MODEL FOR THE EFFECT OF THE CONSTRICTION

In section 2.1, we used two homogeneously charged quadrangles (of opposite sign) to approximate the near gap field of a ring head. The dimensions of these quadrangle and their distance (gap length) were chosen equal to those of the inner faces of the gap of the video head (Figures 1.14 and 2.1). To approximate the effect of the constriction in a video head, the number of quadrangles is increased. The "video head model" consists of the four sloping sides of the constriction (\(-h \leq y \leq 0\)) and the two plates of the ring head model (Figure 2.7).

Figure 2.7

It is obvious that in this model, just as in the ring head model, the boundary conditions at the surface of the head are not satisfied. Furthermore the charge density \(\nabla M_i\) (i=3, ..., 6) will change as a function of the position on the corresponding quadrangles. In our model, however, we only use quadrangle with a constant charge density, so choosing a value for \(\nabla M_i\) implies a compromise.

In attaching the \(\nabla M_i\)'s we anticipate on the results obtained with the program package PADDY (Chapter 3). To approximate the almost homogeneous gap field calculated with PADDY (\(H \approx H_{ix}\) and \(H_x \approx 1.4 \times 10^5\) A/m, page 62 we take \(|\nabla M_{1,2}|=1.4 \times 10^5\) A/m. In order to define the \(|\nabla M_i|\), the
factor \( a \) is chosen such, that the extrema of \( H_x \) along the line \( z = 40.0 \ \mu m \) in our model match with the extrema of \( H_x \) of the field calculated with PADDY along the same line \( (a = 0.03062, \text{ page } 62) \).

The resulting \( H_x \) and \( H_y \) component are shown as a function of \( x \) along lines of constant \( z \) value in Figures 2.9 and 2.10 in the plane \( y = 0.25 \ \mu m \). The \( z \) values are:

- \( z = 0.0 \): symmetry plane
- \( z = 12.0 \ \mu m \): near the edge of the gap
- \( z = 20.0 \ \mu m \): way at the edge of the gap
- \( z = 40.0 \ \mu m \): way at the edge of the gap

It should be noted, that in these figures \( -100 \ \mu m \leq x \leq 100 \ \mu m \), while in Section 2.1. Figure 2.4: \( -1.0 \ \mu m \leq x \leq 1.0 \ \mu m \).

In Figure 2.8 the Fourier Transform for corresponding \( z \) values at \( y = 0.25 \ \mu m \) of the ring head model are given. The homogeneous gap field corresponds to the one of the video-head model.
Sensitivity Functions

Ring Head Model in Y=0.25μM

Figure 2.8
$H_x$ COMPONENT OF VIDEO HEAD MODEL

IN $Y=0.25 \mu M$

Figure 2.9
HY COMPONENT OF VIDEO HEAD MODEL

IN Y=0.25 μM

Figure 2.10
In Figures 2.9 and 2.10 we notice:

- In the symmetry plane $z=0$ the results conform with those of Figure 2.8 at $|x|=20$ $\mu m$ the influence of the four sloping plates can be seen.

- $z=12.0$ $\mu m$: the gap fringing field still dominates.

- $z=20.0$ $\mu m$: the gap fringing is hardly perceptible and the field of sloping quadrangles dominates.

- $z=40.0$ $\mu m$: there is only the field due to the sloping plates.

- Due to the absence of magnetic material the $H_x$ component crosses zero at the position of the plates and the $H_y$ component reaches its extrema there.

In Figure 2.10 we used the distance between the extrema of $H_y$ in the $x$ direction to denote the: "secondary gap", due to the constriction along a line of constant $z$ value. We will use these in the discussion of the results of the Fourier Transform of the field components (Figure 2.11).
Figure 2.11

-38a- 5895
SENSITIVITY FUNCTIONS VIDEO HEAD MODEL IN Y=0.25µM
Figure 2.11 shows the sensitivity functions \( |\tilde{H}_x(k; y, z)| \) and \( |\tilde{H}_y(k; y, z)| \) for the same \( z \) values as in Figures 2.9 and 2.10 and for the same plane \( 0.25 \mu m \).

The following remarks can be made:

- In the symmetry plane \( z=0 \) \( |H_x| \) and \( |H_y| \) are equal for small \( \lambda \) again (\( \lambda < 32 \mu m \)) and the influence of the sloping quadrangles results in a small dip (\( \lambda \approx 45 \mu m \)). The values for larger wavelengths \( \lambda > 100 \mu m \) are larger than in the ring head model.

- \( z=12.0 \mu m \): the first dips in \( |H_x| \) and \( |H_y| \) are at \( \lambda \approx 39 \mu m \) as expected (Figure 2.10). The other dips do not coincide. This is merely due to the fact, that the gap fringing field is dominating. Furthermore the fact that we are near the edge of plates nr. 3 and nr. 5 contributed.

- \( z=20.0 \mu m \): the influence of the gap has vanished almost completely. The dips coincide at the expected wavelengths.

- \( z=40.0 \mu m \) the same accounts as for 20 \( \mu m \).

Compared to the ring head model the sensitivity functions of the video head model have larger values for \( |z| > 11.5 \mu m \) (above the constriction). Within the limitations of this model we conclude that the video frequencies of the neighbouring tracks are suppressed due to the secondary gap effect of the constriction. The suppression starts at about 125 kHz, so some of the track following frequencies would be suppressed too (Figure 1.8a). This is due to the dimensions of the constriction as chosen for this particular model (Figure 1.13).
CHAPTER THREE

PADDY A PROGRAM PACKAGE FOR SOLVING THREE-DIMENSIONAL MAGNETOSTATIC PROBLEMS.

3.1. FACILITIES OF THE PROGRAM PACKAGE

3.2. THE ALGORITHM BLOCK

3.3. THE PADDY INPUT FOR THE VIDEO HEAD
INTRODUCTION

Chapter three is dedicated to the description of the program package PADDY. Firstly we describe some facilities. Then we will treat the specification of the mesh statements. In the last paragraph the input for the video head problem is treated.

In PADDY the solution is calculated in terms of the magnetic potential, using a finite element method. A brief description of the solution method of our own PADDY problem will be given in Appendix C.
3.1 FACILITIES OF THE PROGRAM PACKAGE

Paddy offers facilities for the approximate solution of three dimensional static problems. It is primarily meant for magnetic fields but can equally well be used for electrostatics or with some restrictions for heat distribution (De Beer, 1981).

A magnetic problem may contain soft isotropic materials or hard anisotropic materials, static currents and air. The sources may be permanent magnets, currents, boundary potentials or imposed fluxes.

The basic quantities involved are:

- \( H \) magnetic field strength (A/M)
- \( B \) magnetic induction (Tesla: T)
- \( U \) magnetic potential (scalar) (A)
- \( j \) static current density (A/m²).

Maxwell's equation for magnetostatic problems are:

\[
\nabla \times H = j 
\]

3.1

\[
\nabla \cdot B = 0 
\]

3.2

The vectors \( B \) and \( H \) are related by

\[
B = \mu_0 \mu_r (H) H + B_r 
\]

3.3

where \( \mu_0 = 4\pi \times 10^{-7} \text{ Vsec/Am} \) and \( B_r \) is the remanent induction.

We will restrict ourselves to problems with materials that have a constant \( \mu_r \).

The basic quantity in PADDY is the scalar potential \( U \).

The solution \( U \) of the problem is approximated using the finite element method. In our case this method leads to a system of linear equations, which can be solved numerically.
The way this system is achieved, will be described in Appendix C: The mathematical formulation. The numerical solution of the system is not treated.

The general structure of the PADDY package is shown in Figure 3.1.

![Figure 3.1](image)

The input can be given in the problem-oriented language PADLAN (De Beer, 1981). Input for PADDY is always screened for errors by the interpreter part and a transformed form is put on data files. If user-chosen functions are found or if the program needs extra workspace to do the calculations, a few lines of FORTRAN are generated and put on the FORTRAN file (PADDY is completely written in FORTRAN).

When this file has not been generated, processing continues immediately, otherwise the file is compiled first and linked to the existing program, after which processing is resumed. The user has access to a B-H curve library and to the computed values of PADDY for the calculation of secondary results.

As stated in Appendix C, the program package computes the magnetic potential $U$ in certain points in a finite domain a space, specified by the user. Even in a finite part of space, the number of points can take large values. In our
problem this number is about $2 \times 10^4$. So in PADDY much attention is paid to simplifying the designation of these so-called nodes.

By connecting all points one gets a mesh and this in fact is the way, the definition of the nodes takes place. In this mesh broken surfaces can be noticed, which are called: "mesh planes". In the input of the problem the user defines these meshplanes by means of mesh statements (section 3.2).

A problem-oriented language PADLAN has been developed to define the problem. PADLAN offers the possibility to distinguish the input into different sorts of information. We can distinguish in the input:

- information about the mesh;
- information about the structure and the kind of materials and sources used;
- information about the desired output.

In PADDY these parts are called: "blocks" each block has its own identifier. The most important are:

- **ALGORITHM**: the mesh of the finite part of free space is defined.

- **REGION**: the structure and the dimensions of materials are distinguished. For each material a specific REGION block should be given, the same accounts for the magnetic sources.

- **PRINT/PLOT**: information on the results, that should be printed and/or plotted can be given in this block.

- **FORTRAN**: user-chosen function names can be defined. Even user written subroutines to enable post-processing of the results may be declared.
3.2. THE ALGORITHM BLOCK

A magnetostatic problem consists of magnetic materials, currents and air. The air actually extends to infinity. However as described before, the problem is restricted to a finite domain. The magnitude of this domain is adapted to the accuracy requirements for the far field (De Beer 1981).

In order to specify the nodes in which the potential should be calculated, the mesh planes are defined in the ALGORITHM block of the input. The basic element which the mesh is built from, is the hexahedron, which can be looked upon as a distorted cube (Figure 3.2)

![Figure 3.2](image)

On such an element the solution is approximated in a so-called isoparametric way; specifically, the function is trilinear on the cube, and taken through the same distortion as the element shape itself (Appendix C).

Along an edge this results in a linear function of the magnetic potential. Consequently the field component along that edge cannot vary, since $\mathbf{H} = -\nabla U$ (Appendix C). The size of the elements thus determines the accuracy with which the true solution can be approximated. Experience will have to guide the user in his choice.
As stated before, in the assembly of elements one can distinguish what we will call mesh planes. If one connects the topsides of all right, left, front and back neighbour elements, a broken surface with all the element tops as facets is obtained. In a similar way the mesh planes of the other two directions can be defined. The outer boundaries of the problem should be defined by such mesh planes. Obviously there must be six of these. The consequence is that each mesh plane in one direction contains the same number of element sides. The mesh must be laid out such that all boundaries of materials and sources consist of sides of elements.

In PADDY one can use cartesian, cylindrical or spherical coordinates. We will only treat the mesh specification in cartesian coordinates. So the above mentioned three directions are the x, y and z direction in this frame.

To define a mesh plane one has to use mesh statements. The simplest case is when a mesh plane is also a coordinate plane. For the case of the z-coordinate the mesh statement would be

\[ Z(A) = B \]

where A is the number of the plane and B is the value of the Z-coordinate.

A more general form of the mesh statement is necessary if the mesh plane is not a coordinate plane. Now the mesh plane is defined by giving the mesh lines in its. A mesh line is the interaction of two mesh planes. When we start in a coordinate plane, e.g.:

\[ X(C) = D \]
The specification of a mesh line for the z-coordinate is given by:

\[ Z(c, \text{*, } F) = E \]  

This statement means: the mesh line is number \( F \) out of the set of mesh lines for coordinate \( z \). This line runs in the \( y \) direction, indicated by the place of the star, and lies in the \( x \)-plane number \( c \) (Figure 3.3).

![Figure 3.3](image)

Now all \( z \) mesh lines have to run in the \( y \)-direction. Finally \( E \) is the value of the \( x \) coordinate in plane \( c \).

On the basis of an example we can show how a broken mesh plane can be achieved. Consider the case that we have 5 \( x \)-planes with the numbers 1, 2, 3, 4 and 5 (Figure 3.4); they are coordinate planes, so they can easily be defined with statements \( X(1) = \eta \), \( X(2) = \varphi \), \( X(3) = r \) etc. Now the
z lines can be specified with

\[ Z(1, *, A) = F_1 \]
\[ Z(2, *, A) = F_1 \]
\[ Z(3, *, A) = F_2 \]
\[ Z(4, *, A) = F_2 \]
\[ Z(5, *, A) = F_2 \]

\[ \text{Figure 3.4} \]

Or shorter

\[ Z(1:2, *, A) = F_1 \]
\[ Z(3:5, *, A) = F_2 \]

In figure 3.4 it can be seen that out of a set of z-lines with the same number (A) a flat plane \((F_1=F_2)\) or a broken one can arise \((F_1 \neq F_2)\).

All the mesh statements together result in a set of broken planes and/or coordinate planes. If the mesh number of
succeeding planes in the mesh definition is raised with more than one, PADDY specifies the intermediate planes by means of the interpolation (Figure 3.5).

The regular pattern of the mesh system implies, that, if in some area a rather fine mesh is needed to obtain a reliable field (near the gap of the video head). The same number of elements must be used throughout, giving use to very many unknown values, that have to be calculated. In order to avoid this effect, it is allowed to have elements without volume. By making their sides coincide the interior disappears. It will be obvious, that at the point where voluminous and collapsed elements meet, other than hexahedral elements are found.

Despite the efforts to simplify the mesh statements, the specification of the mesh can be a time consuming occupation.

Coordinates in PADLAN must be given in meters or radians. However, this may not be convenient. Therefore the units can be adapted by the unit statements. Per coordinate a unit statement may be given, specifying a factor by which the given coordinates must be multiplied.
For example, the statement:

$$\text{UNIT}(x) = 1 \times 10^{-6}$$

implies that the x-coordinate can be given in micrometers.

The last statement we want to describe is the symmetry statement. If the structure that is considered has planes of symmetry, the user can impose a symmetrical behaviour of the field at these planes. Symmetry with respect to a straight boundary surface implies that the B vectors are perpendicular to this surface. Antisymmetry implies that the B vectors be in the surface.

According to the expected behaviour of the field, the user can apply the symmetry statement. At surfaces where it is not given the program assumes antisymmetry. In this way it is possible to reduce the domain which the problem should be solved in.
3.3. THE PADDY INPUT FOR THE VIDEO HEAD

Before we specified our own mesh, we used an existing one (v.d. Berg, 1981) to gain some experience. With the help of this experience we could formulate some requirements for our own mesh. They are:

- The mesh definition should be simple and the statements conveniently arranged, so that they can easily be altered and understood by others. As a consequence more statements are used than strictly necessary.

- At the position of the tape the element effect (the normal component of \( B \) is not continuous when crossing the element's boundary surface, Appendix C) should be restricted to the direction of transportation of the tape (x direction as much as possible).

The second requirement is a consequence of the Fourier Transform, that is carried out on the field along a line of constant \( z \) in the x direction at the position of the tape. So \( z \) mesh planes should be coordinate planes as much as possible. We have tried to satisfy these demands in the definition of the mesh.

In chapter one we specified the structure of our video head (Figure 1.13). Its characteristics are:

- a flat pole-face
- two planes of symmetry \((x=0, z=0)\).

These two assumptions are made to reduce the extent and the complexity of the input of the problem.

Since we are merely interested in the field at the position of the tape and consider the tape in contact with the pole-face everywhere, the assumption of a flat pole-face is allowed (Figure 3.6).
Therefore most of the y-mesh planes coincide with the y coordinate planes, which simplifies the input considerably.

The two planes of symmetry offer us the possibility to use the symmetry statement described in the previous section. We expect the flux lines to cross the plane x=0 perpendicularly (x=0 is an equipotential plane). At the other plane of symmetry, the flux lines lay in the plane (Figure 3.7).

Thus by defining a symmetry statement on x=0 (if no symmetry is defined on z=0, the program assumes antisymmetry...
we only have to specify a quarter of the videohead and the surrounding air as shown in (Figure 3.7).

For some of the results, the size of the finite part of 3D-space used is important. The dimensions of the finite area are given in fig. 3.8.

The easiest way to define a mesh occurs when all the mesh planes are coordinate planes. All the mesh statements would be of the kind shown in 3.4. This is a situation, which is desirable but cannot be achieved, since at the coil chamber and at the constriction we have sloping sides. Despite these set-backs we will try to describe the mesh on the basis of mesh planes that coincide with coordinate planes as much as possible.

Considering the intersections of the head's material with the x coordinate planes, (Figure 3.8) we see that these intersections contain two quadrangles at most.

Figure 3.8

Due to the constriction and the structure of the coil changer the upper one of the two quadrangler changes for
0.2μm < x < 400μm. We can distinguish between a change of the upper quadrangle in the positive z direction and a change in the negative y-direction. The former lasts for 20μm < x < 108.5μm, the latter for 0.2μm < x < 400μm. With the help of Figure 3.9 and 3.10 show, how these planes decay as a function of x in the plane of the pole-face and the symmetry plane z=0 respectively.

If we want to observe our requirement that the y and z mesh planes should be coordinate planes as much as possible, then Figure 3.9 and 3.10 show, how these planes decay as a function of x in the plane of the pole-face and the symmetry plane z=0 respectively.

It should be noticed, that these planes have to follow the surface of the material for x u < x < x u (in these figures.)
By using the possibility of coinciding mesh planes/lines we can obtain the desired behaviour as illustrated in Figure 3.11 for the y mesh plane in Figure 3.11.

For \( x \leq x_l \), all x-planes have two coinciding mesh lines A and B with y value \( y_A \). At plane \( x_{cr} \) mesh line A still has y value \( y_A \) but meshline B suddenly has \( y_B \). For \( x > x_{cr} \), this remains the same. PADDY interpolates between the mesh planes and in this way y mesh lines are generated that have the desired behaviour. This can be done in every z-plane and now we have defined a y mesh plane that coincides with the sloping surface of the coil chamber for \( x_l < x < x_{cr} \). The same procedure can be followed at the constriction.

To describe this procedure in mesh statement we have chosen for:

- y as the running variable in the x statements
- z " " " " " " y " "
- y " " " " " z " "

In order to arrange the mesh statements in a convenient way, we described the y and z planes in the region \( 0.2 \mu m < \kappa < 400 \mu m \) by giving these intersections with the x mesh planes. The statements for the y mesh lines in all these
planes have to result in y mesh planes that, one after another, follow the sloping surface of the coil chamber for a certain distance in the x direction as shown in Figure 3.12.

![Figure 3.12](image)

The same accounts for the z lines. To make things even worse, for $20\mu m < x < 108.5\mu m$ the procedure illustrated in Figure 3.11 should be explored simultaneously for the y and the z lines. In our case we needed about 700 mesh statements to describe the mesh of the video head. However, per x mesh plane only minor changes have to be made in the statements describing the intersecting mesh lines, so most of the work could be done by the editor of the computer system we used. Out of the area $20\mu m < x < 400\mu m$ all planes coincide with coordinate planes, so we don't pay special attention to the statements needed.
To reduce the number of unknown at places where the mesh is too refined, compared to the behaviour of the expected field, we can let several mesh planes coincide in all three directions. In our case we restricted the number of unknown to $2 \times 10^4$.

In the PAOLAN file for our video head we started with the mesh statements. Firstly we specified the y and z lines that are the same for every x plane, because only one statement (as described in section 3.2 statement 3.8) is needed to define such a line in every x plane. After this in each x plane for $0.2 \mu m \leq \zeta \leq 400 \mu m$ all the remaining intersecting y and z lines are specified. For $\zeta > 400 \mu m$ the distribution of the intersecting y and z lines does not change anymore. So the statements for these planes are like 3.4 and 3.8 again.

Then the unit statement defines that all the values given in the mesh specification are in micrometers. The symmetry statement is used as discussed previously. For this case Figures 3.13 up and until 3.16 show plots of mesh plane and also the configuration of the head's materials.

The source was chosen identical to the one v.d. Berg (1981) used: a permanent magnet. The strength of this source (the value of the remanent induction: $B_r$) corresponds to the case of an exitation by a coil with 16 turns and a current of 5.325 mA. The $\mu_z$ outside the permanent magnet was chosen 500, this is an estimated value (Peters, 1979) for the magnetic permeability of applied magnetic head materials at the video frequencies (Figure 1.8a). In the permanent magnet of length $L=400 \mu m$ (Figure 3.13), a hard magnetic material, $\mu$ is a tensor. The tensor approximately has the form (Zijlstra, 1976):

$$\mu_z(H) = \begin{pmatrix} \mu_{zz}(H) & 0 \\ 0 & \mu_{yy} \end{pmatrix}$$
\( H_\parallel \) is the component in the direction of \( B_r \). In our case \( B_r = -B_r \parallel \) and \( \mu_0 \mu_r = 500 \) for \( \mu_r \) also the value 500 is chosen. This is done to simulate the situation in the soft magnetic material as much as possible.

The problem can only be solved in a restricted domain, shown in Figures 3.13 and 3.14. The boundaries of the mesh may not be too close to the boundaries of the head, to avoid influence on the field near the head. On the other hand, they should not be too far away, in order to restrict the number of unknowns. This, again implies a compromise.

By means of a TEST statement in the input file, plots of mesh planes can be made like those in Figures 3.13 up and including 3.16. This offers a possibility to check if the mesh statements represent the desired pattern.

At the gap the mesh is more refined, since we expect a strongly varying field there. In Figures 3.15 and 3.16 the areas of the coil chamber and the constriction are enlarged. It can be seen now, how the \( y \) and the \( x \) planes decay as a function of \( x \) at the sloping sides of the heads material. Also the coinciding planes to reduce the number of unknown can be noticed.
Figure 3.13

\[ z = 0 \, \text{\(\mu\text{m}\)} \]
Figure 3.14
Figure 3.15

$z = 0 \text{ um}$
CHAPTER FOUR

RESULTS OBTAINED WITH PADDY

4.0 INTRODUCTION

4.1 RESULTS OBTAINED WITH THE VIDEO MESH

4.2 RESULTS OBTAINED WITH A REFINED MESH

4.3 COMPARISON OF RESULTS
4.0 INTRODUCTION

In chapter 4 the results obtained with PADDY are presented and discussed. Firstly the results of the video mesh as described in Chapter 3 are given. Then a refined part of this mesh is defined and a potential is imposed on its boundaries, the results of this calculation are shown in Section 4.2. Finally a comparison is made between the PADDY results and the results of the video head model of section 2.2.
4.1. RESULTS OBTAINED WITH THE VIDEO MESH

After the problem is defined (Section 3.3) the PADDY job can be started. The results are presented in plots and/or prints of the different quantities. It can be checked now, if the output satisfies the following physical expectations.

- The decay of the potential can give information about the quality of the mesh. A carefully chosen mesh will show no non physical changes in the sign and the value of the field components.
- The value and the homogeneity of the field in the gap.
- The value of the z component in the symmetry plane (z=0).
- The boundary conditions at the surface of the medium.

In 4.3. a comparison with the results of our video head model will be made.

The gap field is almost homogeneous:

\[ H = H_x \neq x \] and \[ H_z \approx 1.4 \times 10^5 A/m. \] With this value the so-called efficiency \( \eta \) of the head can be calculated.

\[
\eta = \frac{\int_{\text{gap}} H_z \, dl}{\int_{\text{head}} H_z \, dl} = \frac{\int_{\text{gap}} H_z \, dl}{NIT} \quad 4.1
\]

The efficiency is in fact the relation between the potential jump over the gap divided by the M.M.F. of the source. It was found to be: 66%, in agreement with Vd Berg (1981).

The values of the \( H_z \) component of the magnetic field in \( z=0 \), were almost zero as expected. So in \( z=p \) the magnetic field lines can be imagined as the orthogonal trajectories of the equipotential lines. In Figure 4.1 a plot of these lines is given in \( z=0 \).

The lines denote a constant increment of the value of the potential. We see that this value increases linear in the permanent magnet. The larger distance in the soft
magnetic material compared to the air, is due to the difference in $\mu$. Outside the head and in the coil changer the flux leakage can be seen.

Due to the relatively high value of $\mu_r$ in the soft magnetic material, the field lines are almost perpendicular to the surface. In Figure 4.2 the equipotential lines near the gap are shown.
After verification of the results, another characteristic of the head can be calculated: the self inductance $L$. Together with the resistance and the stray capacity of the coil, the self inductance of the head forms a resonance circuit.

In a recording system (e.g. videorecorder) one has to avoid a strong resonance in the frequency range of interest. On the other hand it is advisable to use as many turns as possible, to enhance the read signal. This implies a compromise. For the calculation of the self inductance:

$$L = \frac{(N \int B \cdot dA)}{I} \quad (\text{HENRY}) \quad 4.2$$

we need the flux in the head, cause by the source:

$$\Phi = \int B \cdot dA \quad (\text{WEBER}) \quad 4.3$$

$\Phi$ is calculated in the permanent magnet at $y=700\mu m$. In this calculation a factor 2 should be included accounting for the fact that only a quarter of the head was implemented (Figure 3.8). Furthermore one should bare in mind that the total head is excited by two permanent magnets of equal strength and opposite sign. Figure 4.3.

---

Figure 4.3
The self induction is found to be

\[ L \approx 0.85 \mu H \]

Measurements on practical video heads give a self induction of 1.5\( \mu H \). The difference can be due to the fact that:
- a permanent magnet is used instead of a coil
- \( H \) is assumed to be a constant
- the structure of our video head is an approximation of a real video head.
- It appears that a large fraction of the flux does not reach the gap. It leaks away outside the soft magnetic circuit. This leakage flux does contribute to the inductance. The amount of leakage flux is probably rather low in this problem because of the rather small 3D-space, that is taken into account in the z-direction (and positive y-direction). As a consequence the self inductance may be underestimated.

We now proceed with the magnetic field strength along lines of constant z value in a plane at the position of the tape. A post processing routine (Section 3.1) was used to obtain the values of the field components. The \( H_x \) and the \( H_y \) component are shown in Figure 4.4 and 4.5. PADDY calculates only in a quarter part of the total area (Figure 3.3). Therefore the results are extended to the total area (with the proper sign) by using the already symmetry properties of the head (page 14).

The results are normalised on the deep gap field. The pole-face of the video head is shown too.
We notice that the $H_x$ component reaches its maximum exactly above the gap. The value of the maximum decreases quickly for increasing $|z|$. $H_y$ shows the expected asymmetrical behaviour. The $H_z$ component differs significantly from zero only at the edge of the gap and is omitted here.

For $z$ values near the edge of the gap we see the influence of both the gap and the constriction. In Section 4.2, with the help of a more refined mesh, more detailed plots of the magnetic field components at the constriction are given.
4.2. RESULTS OBTAINED WITH A REFINED MESH

Despite the fact that there are almost $2 \cdot 10^4$ nodes, the number of nodes along a line of constant $z$ value for (Figure 3.8) is only 38. In order to describe the effect of the construction properly this number should be increased.

In PADDY there is a possibility to refine the mesh in a certain area and impose a potential on the new boundaries. The boundary potential should be calculated from the results of the original problem of the video head. This is done for the area shown in Figure 4.6 and 4.7.
The potential at the boundaries is calculated by means of the following procedure.

1. Define a new PADDY problem (in a restricted domain of the original problem) with a mesh, material specification etc. and a FORTRAN function (section 3.1): FPOT. This function gives the potential in the nodes at the boundaries of the new mesh. PADDY calculates all the nodes in the mesh and asks for the potential in the nodes at the boundary. Since this potential is unknown yet, FPOT is made zero, meanwhile the coordinates of these nodes are stored in a file.

2. The file with the coordinates serves as an input file for a post processing job (Section 3.1) on the results of the original problem. In these boundary nodes the post-processing job asks for the potential. The new mesh is more refined than the original one, so interpolation of the original results is necessary. PADDY takes care of that (Appendix C). In this way a new file can be made, with the coordinates and the potential at the boundary nodes.

3. The new PADDY job is runned with this file as an input file.
The new mesh is generated in the same way as the original one (Section 3.3), with the difference that there is only one sloping side now: the one of the constriction. The new mesh is illustrated in Figure 4.8 and 4.9.

Figure 4.8
Figure 4.9
The results obtained with this mesh are shown in Figures 4.10 and 4.11.

In these Figures we notice:

- The decay of the $H_x$ and $H_y$ as already seen in Figures 4.4 and 4.5. The maximum of the $H_x$ is twice the value of the extrema of $H_y$. The same phenomenon can be found in the Karlquist model (De Hoop, 1976).

- At $z = 12.0 \mu m$ the influence of the constriction is hardly perceptible, while the gap fringing has decreased only with a factor 2.

- At $z = 20.0 \mu m$ there is still the influence of the gap, but the contribution of the constriction has increased.

- At $z = 40.0 \mu m$ there is only the effect of the constriction left. Compared to the results at $z = 20.0 \mu m$ the extreme have smaller values.

- Due to the boundary condition at the pole face, the component is almost zero above the material. The extreme in $H_y$ are reached after crossing the sloping sides of the construction.

- Even in this refined mesh a lack of "field points" (nodes) is displayed at the constriction.
$H_x$ COMPONENT OF VIDEO HEAD

IN Y=0.25\mu M

Figure 4.10
Hy COMPONENT OF VIDEO HEAD

IN Y=0.25 μM

Figure 4.11
If the results of the original and the refined problem are combined, the number of field points is raised from 38 to 58 for $\times \gg \Omega$ (The Fourier Transform in Chapter was calculated with about 400 field points in the range $\times \gg \Omega$). Furthermore since,

$$\int_{-\infty}^{\infty} H_{x,y} e^{ikx} dx = \int_{-\infty}^{\infty} H_{x,y} dx = 0$$

the sensitivity functions should decrease for large wavelengths ($\lambda \to \infty$, $k \to 0$). In PADDY the potential is calculated in a finite domain. Therefore the Fourier Transform of the PADDY results for wavelengths of about the length of the domain (in the $\alpha$ direction, and larger, will be unreliable.

The numerical F.T. of the PADDY results is further complicated by the lack of field points. In Figure 4.12 only the results of $\left| H_{x}(k; y, z) \right|$ are shown for $z=0.0$ and $z=12.0 \mu m$. 
In Figure 4.12 we see that the first dip is at the right wavelength for \( z = 0 \). At \( z = 12.0 \mu m \) the sensitivity function has lower values, but the same form as the one at \( z = 0 \). This shows that the effect of the construction is only minor compared to the effect of the gap at this line. For higher \( z \) values the first dips are at the expected wavelengths (not shown here).

In conclusion we can remark that the effect of the constriction is shown along \( z \) lines not too close to the edge of the gap. So suppression of video frequencies due to the constriction works well, except for the adjacent tracks.
Figure 4.12
4.3. COMPARISON OF RESULTS

In this section we compare the results obtained with the video head model (briefly called: "model" here) of Section 2.2 and the PADDY results (called the results of: "the video head").

The following remarks can be made:

- Near the gap the field of the video head is larger than the field of the model, despite the fact that both of them have the same homogeneous gap field.

- The extrema in $H_x$ and $H_y$ of the video head along lines of constant $z$ value above the constriction, decrease for increasing $|z|$ values. This is due to the increasing distance of the sloping sides. Since the surface density of the sloping plates in the model is a constant, this decrease is not shown in the video head model.

- As a consequence of the first remark, the influence of the constriction near the edge of the gap is hardly perceptible in the magnetic field of the video head, in contrast to the model.

- The Fourier Transform of the magnetic field along lines of constant $z$ at the plane of the tape shows the secondary gap effect clearly. The F.T. of the corresponding PADDY results is reliable only for small wavelengths and small $|z|$ values.

Considering these remarks it can be noticed, that there is quite a difference between the video head and the model. This is due to the absence of the proper boundary conditions, caused by the magnetic material of the head. However as already stated in section 2.2, the video head model is only used to describe the effect of the constriction. It was not our purpose to approximate the field of a video head
with this model.

However, it should be noted here that in principle the field of the video head can be approximated analytically by taking into account all the charged planes at the surface of the video mesh and by deriving the surface charge from PADDY.
FINAl CONCLUSIONS AND FOLLOW UP

At the end of this report we give some final conclusions and some possibilities for a continuation.

Although the PADDY calculation results in a reliable field in the head and in the area next to it, it is difficult to study the crosstalk of neighbouring tracks by means of the Fourier Transform of the magnetic field along lines of constant z value at the place of the tape.

To improve the results of the numerical Fourier Transform, smoothing of the PADDY results is necessary. Furthermore the field at larger distances should be approximated by an analytical function.

A first step towards another way of smoothing the PADDY results is given in Chapter 2. In this Chapter an expression is derived for the magnetic field strength of a plane homogeneously charged quadrangle. The surface of the video head as specified in the mesh, consists of quadrangles and triangles, that can be thought to have a constant surface charge density:

\[ \mathbf{\nabla} \mathbf{H}(\mathbf{r}) = \left( \mathbf{M}(\mathbf{r}) \right) \mathbf{n} \]  

It is possible now, to calculate the magnetic field in every point in principal, by summation of all the contributions of the charged polygons on the surface of the head. The implementation of this calculation on a computer is a time consuming affair, for which there was no time left in this graduation period. Instead, the simple models of Chapter 2 were implemented. The results of these models, look very promising.

Eindhoven, November 1983
Philips Research Laboratories
Nederlandse Philips Bedrijven B.V.
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APPENDIX A

THE MAGNETIC-FIELD STRENGTH OF A PLANE HOMOGENEOUSLY CHARGED QUADRANGLE

The expression for this magnetic field is given by 2.12

\[ H_A(x) = \frac{\mu_0}{4\pi} \int \nabla' \cdot \left( \frac{Q}{r^3} \right) \, dA(x') \]  \hspace{1cm} 2.12

First we designate

\[ R = |x' - \ell| = \left[ (x' - \ell) \cdot (x - \ell) \right]^{1/2} \geq 0 \quad \text{A.1} \]

and rewrite:

\[ H_A(x) = \frac{\mu_0}{4\pi} \int \nabla' R'' \cdot dA(x') \quad \text{A.2} \]

\[ \nabla' R'' = (\nabla \cdot \nabla') \nabla' R'' \]

\[ \nabla \times (\nabla' R'') \times \nabla' \phi + (\nabla' \phi \cdot \nabla' R'') \nabla' \phi = \]

\[ (\nabla \cdot \nabla' R'') \times \nabla' \phi + (\nabla' \phi \cdot \nabla' R'') \nabla' \phi = \]

\[ (\nabla \times \nabla' R'') \times \nabla' \phi R'' + (\nabla' \phi \cdot \nabla' R'') \nabla' \phi \quad \text{A.3} \]

A.3 is a decomposition of \( \nabla' R'' \) in a component perpendicular to \( \nabla' \phi \) and one parallel to it. \( \nabla' \phi \) is a constant vector on A.
On account of Stokes' theorem we have (Figure A.1):

$$H_A(x) = \frac{\nabla \cdot \mathbf{F}}{4\pi} \int \mathbf{F} \times \mathbf{n}_Q \cdot \mathbf{R}^{-1} + (\mathbf{F} \cdot \nabla \mathbf{R}^{-1}) \mathbf{n}_Q \, dA(x)$$  \hspace{1cm} \text{A.4}

\[ \text{Figure A.1} \]

$$\int (\mathbf{n} \times \mathbf{v}) \times \mathbf{n}_Q \mathbf{R}^{-1} \, dA(x) = \int (\mathbf{v} \times \mathbf{n}_Q) \mathbf{R}^{-1} \, dA(x) \]

$$= -\mathbf{n}_Q \times \mathbf{v} \mathbf{R}^{-1} \, dS(\mathbf{x})$$  \hspace{1cm} \text{A.5}

C denotes the boundary contour of the quadrangle A and \( \mathbf{v} \) is the unit vector along the tangent to C in the direction of circulation along C, the latter forming a right-handed system with \( \mathbf{n}_Q \).

Now we can write for A.4

$$H_A(x) = \frac{\nabla \cdot \mathbf{F}}{4\pi} \left[ -\mathbf{n}_Q \times \mathbf{v} \mathbf{R}^{-1} \, dA(x) + \mathbf{n}_Q \int (\mathbf{F} \cdot \nabla \mathbf{R}^{-1}) \mathbf{n}_Q \, dA(x) \right]$$  \hspace{1cm} \text{A.6}
The second integral in A.5 is the signed solid angle at which \( \mathbf{A} \) is observed from the point of observation

\[
\iiint_{A(x')} n_\varphi \cdot \nabla \mathbf{R}'' n_\varphi \, dA(x') = \frac{\mathbf{n}_\varphi \cdot (\mathbf{x'} - \mathbf{x})}{|\mathbf{n}_\varphi| (\mathbf{x'} - \mathbf{x})} \iint_{A(x')} \frac{\mathbf{n}_\varphi \cdot (\mathbf{x'} - \mathbf{x})}{|\mathbf{x'} - \mathbf{x}|} \, dA(x')
\]

\[
= -\text{sgn} (\mathbf{n}_\varphi \cdot (\mathbf{x'} - \mathbf{x})) \Omega = \Omega
\]

\[\text{A.7}\]

So \( \Omega \) is negative when the point of observation is at the same side of the quadrangle as to where \( n_\varphi \) points (a in Figure A.2) and is positive in the other case (b).

For the solid angle \( \Omega \) an expression in terms of geometrical quantities can be derived. To obtain this expression we...
Todhunter and Leatham (1911) give an expression for the area of the projection of a polygon on a sphere with radius $r_a$. In our situation, we have a quadrangle and a sphere of radius $r_a=1$, the expression is as follows:

$$\omega = (\alpha_1 + \alpha_2 + \alpha_3 + \alpha_4) - 2\pi$$ \hspace{1cm} \text{A.9}$$

The angles $0 \leq \alpha_m \leq \pi$ are illustrated in Figure A.4. They can be found by calculating the unit vectors along the normals of the including planes $n_1$ and $n_2$ (Figure A.5).
\[ \alpha = \arccos(-\mathbf{n}_1 \cdot \mathbf{n}_2) \]
The contour of a quadrangle consists of four sides. Along such a side, $\xi = \xi_m$ is a constant vector (see table 2.1). Further we introduce (Figure A.6):

$$\xi' - \xi = \beta_m + \lambda a_m \quad \text{with} \quad 0 \leq \lambda \leq 1 \quad A.9$$

$$ds = a_m \, d\lambda \quad a_m = |a_m| \quad A.10$$

Now we can write for the contour integral along the $m$-th side of the quadrangle

$$\int_{\xi} R^{-1} \, ds(\xi) = \int_{0}^{1} a_m \, R^{-1} \, d\lambda \quad A.11$$

in which

$$R = \left[ (a_m \cdot a_m) \lambda^2 + 2 (a_m \cdot \beta_m) \lambda + \beta_m \cdot \beta_m \right]^{1/2} \geq 0 \quad A.12$$

A.11 now becomes
\[ \int_{a_m} R^{-d} \alpha = a_m' \ln L_m \quad A.13 \]

and

\[ L_m = \ln (\frac{a_m \cdot \beta_m + a_m \cdot \omega_m}{a_m \cdot \beta_m + a_m \cdot \omega_m}) \quad A.14 \]

By rewriting A.10 we can see in which cases computing of \( L_m \) needs special care. With the help of Figure A.6 we find

\[ L_m = \ln (\frac{\beta_m \cdot (i + \cos(\alpha_m))}{\beta_m \cdot (i + \cos(\beta_m))}) \quad A.15 \]

Two interesting cases arise:

1) \( \alpha_m = 0 \) and \( \beta_m = \pi \)

\( L_m \) is singular, so an observation point on a side of the quadrangle is not allowed.

2) \( \alpha_m = \pi \) and \( \beta_m = \pi \)

The point of observation then lies on the extension of the \( m \)-th side along \( a_m \)

![Figure A.7](image)

The distance \( d \) can be expressed in terms of \( \phi_m \) and \( \bar{\phi}_{m+1} \)

and the angles \( \alpha_m \) and \( \beta_m \) (Figure A.7)
So finally we have come to the following equation, the magnetic-field strength of a plane homogeneously charged quadrangle:

\[
\frac{H(x)}{A} = \frac{\sqrt{\pi}}{m} \left\{ \left( \sum_{m=1}^{\infty} \left( \frac{2\pi}{x-m} \right)^2 \right) + 2\pi \right\} 
\]
APPENDIX B

DERIVATION OF A CHARACTERISTIC OF TWO-DIMENSIONAL HEAD CONFIGURATIONS

We define D as a part of two-dimensional space (vacuum $\mu=\mu_0$):

$$D: \quad -\infty < x < \infty \quad \text{and} \quad y > 0$$

![Figure B.1](image)

The equations of Maxwell for two-dimensional magnetostatic problems without currents in D:

$$\nabla \times \mathbf{H} = \mathbf{0} \quad \text{(B.1)}$$

$$\nabla \cdot \mathbf{H} = \mathbf{0}$$

so:

$$\nabla \times \nabla \times \mathbf{H} = \mathbf{0} \quad \text{(B.2)}$$

$$\nabla (\nabla \cdot \mathbf{H}) = \mathbf{0}$$

Now we can use a vector identity:

$$\nabla (\nabla \cdot \mathbf{H}) = \nabla^2 \mathbf{H} + \nabla \times \nabla \times \mathbf{H} \quad \text{(B.3)}$$

and

$$\nabla^2 \mathbf{H} = \mathbf{0} \quad \text{(B.4)}$$

In components:
The Fourier Transform of $\nabla \cdot H = 0$ is:

$$\frac{\partial^2 H_x}{\partial x^2} + \frac{\partial^2 H_y}{\partial y^2} = 0$$

$$\frac{\partial^2 H_y}{\partial x^2} + \frac{\partial^2 H_x}{\partial y^2} = 0$$

Taking the Fourier Transform of B.5 with respect to $x$ (1.5b) we obtain:

$$\frac{\partial^2 \tilde{H}_x}{\partial y^2} + k_x^2 \tilde{H}_x = 0$$

$$\frac{\partial^2 \tilde{H}_y}{\partial y^2} + k_y^2 \tilde{H}_y = 0$$

The solution of B.6 is:

$$\tilde{H}_x(k; y) = \tilde{H}_x(k; 0) e^{-ik_y y}$$

$$\tilde{H}_y(k; y) = \tilde{H}_y(k; 0) e^{-ik_x x}$$

where we assume that $H_x(k; 0)$ and $H_y(k; 0)$ are known.

The Fourier Transform of $\nabla \cdot H = 0$ is:

$$-k_x \tilde{H}_x + \frac{\partial}{\partial y} \tilde{H}_y = 0$$

because

$$\int_{-\infty}^{\infty} \frac{\partial H_y}{\partial y} e^{ikx} dx = \frac{\partial}{\partial y} \int_{-\infty}^{\infty} H_y e^{ikx} dx = \frac{\partial}{\partial y} \tilde{H}_y$$
Substitution of B.7 in B.8 brings us to

\[-j k \tilde{H}_x(k,0) - k \tilde{H}_y(k,0) = 0\]

\[\tilde{H}_x(k,0) = j \tilde{H}_y(k,0)\]

With B.8 we find:

\[j \tilde{H}_x(k,y) = \tilde{H}_x(k,y)\]

\[|\tilde{H}_y(k,y)| = |\tilde{H}_x(k,y)|\]

So for two-dimensional head configurations the moduli of the sensitivity functions are equal.

In order to make an inverse Fourier Transform possible one should derive an expression for the case \((k < 0)\) too. In this case also B.13 is found, the derivation is omitted here.
APPENDIX C

THE MATHEMATICAL FORMULATION OF PADDY

In this appendix we describe how, in our case, a system of linear equations is achieved. The solution of this system is not treated.

The equations of Maxwell that describe a magnetostatic problem with currents and permanent magnets as sources are:

\[ \nabla \times \mathbf{H} = \mathbf{J} \quad (C.1) \]
\[ \nabla \cdot \mathbf{B} = 0 \quad (C.2) \]

With \( B_r \) as the remanent induction (Tesla:T) we have:

\[ \mathbf{B} = \mu \mathbf{H} + \mathbf{B}_r \quad (C.3) \]

Firstly we define \( \mathbf{H} \) as the field from the currents when all the magnetizable materials are omitted from the problem. So

\[ \nabla \times \mathbf{H} = \mathbf{J} \quad (C.4) \]
\[ \nabla \cdot \mathbf{H} = 0 \quad (C.5) \]

This field \( \mathbf{H} \), also called primary field, can be calculated explicitly via Biot-Savart's law (Stratton, 1941). If we restrict space to a part where \( \mathbf{J} = 0 \) and, if necessary, insert cuts to make it simply connected, we can define a potential \( U_j \) by:

\[ \mathbf{H} = -\nabla U_j \quad (\text{where } \mathbf{J} = 0) \quad (C.6) \]

* to reduce writing most arguments are omitted
Returning to the original problem, we define

\[ H = H_m + H_j \]  \hspace{2cm} \text{(C.8)}

By taking the curl on both sides, we see that \( \nabla \times H_m = 0 \) everywhere, so there exists a potential \( U_m \) such that:

\[ \nabla U_m = - \nabla U_m \]  \hspace{2cm} \text{(C.9)}

This is called a reduced potential. If space is restricted again, one can define a total (or full) potential \( U_f = U_m + U_j \) such that:

\[ H = - \nabla U_f \quad (\text{where} \ J = 0) \]  \hspace{2cm} \text{(C.9)}

In materials with high \( \mu \), \( H_m \) is almost equal to \( -H_j \), giving rise to numerical cancellation problems. Therefore it is advantageous to use the total potential in these areas. So we split space into two parts, one called \( T_f \), containing the region with high \( \mu \), the other \( T_j \), containing the region where the current floor. Other regions (e.g. the air) may be included in either of these. We will solve the following equations.

\[ \nabla \cdot B = \nabla \cdot (\mu (\nabla U_m + H_j) + Br) = 0 \text{ in } T_j \]  \hspace{2cm} \text{(C.10)}

\[ \nabla \cdot B = \nabla \cdot (\mu (\nabla U_f + Br) = 0 \text{ in } T_f \]  \hspace{2cm} \text{(C.11)}
At the interface between $T_j$ and $T_f$ the potential $U_f$ is discontinuous. The jump is equal to $U_j$. This quantity is calculated in advance using the normal component of $H_j$ as a boundary condition in the equation that follows after substitution of C.6 in C.5:

$$\nabla \cdot (\nabla U_j) = 0 \quad \text{(C.12)}$$

If no currents are present in the problem we have an expression like C.11 that applied for the whole region $T = T_j + T_f$, bounded by the surface $S$. Since nothing essential is left out, we will continue with an expression like C.11:

$$\nabla \cdot B = \nabla \cdot (\mu \nabla U + B_k) = 0 \quad \text{(C.13)}$$

For computational reasons $T$ has to be restricted to a finite domain in space bounded by $S$. Furthermore we will only consider problems with materials for which: $\mu = \text{constant}$.

A weak solution of C.13 can be found with the distribution (Strang and Fix, 1973)

$$\int_T \nabla \cdot B \, dT = 0 \quad \forall \omega \in K \quad \text{(C.14)}$$

$K$ is a set of so-called test functions.

We can rewrite C.14 into:

$$-\int_T B \cdot \nabla \omega \, dV + \int_T \nabla \cdot (\omega B) \, dV = 0 \quad \forall \omega \in K \quad \text{(C.15)}$$

$$-\int_T B \cdot \nabla \omega \, dV + \int_S \omega \omega \, ds = 0 \quad \forall \omega \in K \quad \text{(C.16)}$$
Boundary conditions imposed by the user determine the value of the surface integral in C.16.

From now on we approximate:

- the space $\bar{K}$ by a finite set $K$ (N test functions)
- the solution $U(\xi)$ by

$$U(\xi) = \sum_{i=1}^{N} u_i(x_i) \omega_i(\xi) \quad \omega_i \in K \quad \text{C.14}$$

In C.17 we suppose that it is sufficient to determine the potential in a number of $N$ nodes $\xi_i$ in $T$. The functions $\omega_i(\xi)$ are again chosen out of set $K$.

Substitution of C.16 and C.17 in C.13 brings us to:

$$\sum_{i} \int_{T} \nabla u_i \cdot \nabla \omega_i \, d\sigma = \sum_{i} \int_{T} B_i \cdot \nabla \omega_i \, d\sigma$$

$$+ \int_{\partial T} B_i \, dS = 0 \quad \omega_i \in K \quad \text{C.18}$$

C.18 can be rewritten into:

$$\sum_{i} \int_{T} \nabla u_i \cdot \nabla \omega_i \, d\sigma = \int_{T} B_i \cdot \nabla \omega_i \, d\sigma$$

$$- \int_{\partial T} B_i \, dS \quad \omega_i \in K \quad \text{C.19}$$

The right-hand side of C.19 is known.
We now have achieved a set of linear equations:

$$M \mathbf{u} = \mathbf{x}$$  \hspace{1cm} (C.20)

The square matrix $M$ consists of element

$$M_{ij} = \int \int \int \nabla u_i \cdot \nabla u_j \, \, dx \, dy \, dz$$  \hspace{1cm} (C.21)

$u$ represents the unknown $u_1, \ldots, u_N$ and $x$ the right-hand sides of C.19 for all the test functions $\omega_j$ out of $K$. It can be shown that $M$ is a positive definite matrix.

To specify the test functions we first introduce the notion: "mesh". The mesh arises when all the nodes are connected with each other. The basic element from which this mesh is built, is the hexahedron, which is most easily imagined as a distorted cube (Figure C.1). Furthermore we define a function $\varphi$, mapping the unit cube on the distorted element:

$$\varphi = f(p_1, p_2, p_3)$$  \hspace{1cm} (C.22)

The test function for point $i$ of the element can now be specified:
Assuming that $f$ is invertable. The function $b$ for point $i$ in Figure C.1 is:

$$b_i(x, y, z) = (1 - x_i)(1 - y_i)(1 - z_i) \quad C.24$$

outside the cube $b_i$ is zero.

The test functions for the other points of the element can be specified in a similar way. We see in C.17 already that the magnetic potential undergoes the same distortion as the element itself. This is called an iso-parametric way of approximating the solution.

The test functions $\omega_i$ are chosen unequal to zero only in the neighbourhood of the nodes. They are called finite elements. Now the contribution of $U_i$ to the potential near node $x_i$ is unequal to zero only in the eight mesh elements surrounding $x_i$. This results in a sparse matrix $M$ (C.21), which is solved using the "Incomplete Choleski Conjugated Gradient" method (I.C.C.G.). We will not treat this method here. An analysis of the finite element method is given by Strang and Fix (1973).

In conclusion we remark, that, since the potential is continuous at the surface of a mesh element, the tangential component of $B$ is continuous. The fact that the normal component of $B$ is not continuous is called: "the element effect".