Eindhoven University of Technology

MASTER

Ground buried resonators
analytical and numerical modelling of their noise reducing effect for sound propagating outdoors from traffic noise sources

van der Aa, B.A.

Award date:
2010

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Ground buried resonators:
Analytical and numerical modelling of their noise reducing effect for sound propagating outdoors from traffic noise sources

BART A. VAN DER AA

Department of Civil and Environmental Engineering
Division of Applied Acoustics, Vibroacoustics Group
CHALMERS TEKNISKA HÖGSKOLA
Gothenburg, Sweden 2010
Report S10:01
Master's Thesis

Ground buried resonators - Analytical and numerical modelling of their noise reducing effect for sound propagating outdoors from traffic noise sources

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Göteborg, Sweden, 2010

accord 2010-07-12
H.J. Marti
Ground buried resonators - Analytical and numerical modelling of their noise reducing effect for sound propagating outdoors from traffic noise sources

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Master's project (Master's thesis, Eindhoven University of Technology)
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Sweden
Telephone: +46(0)31-722 1000

Cover: Insertion loss at three different source locations (the source locations are marked with a black-cross) in 200 Hz 1/3 octave band with 30 energetically averaged logarithmically spaced frequency components). The model includes 25 (5x5) resonators for which the natural frequency increases further away from the source. For further information of this geometry please read chapter four.

Printed by
Chalmers Reproservice
Göteborg, Sweden, 2010
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Chalmers University of Technology

Abstract

Environmental noise pollution and strategies to reduce its impact on inhabitants of urban environments remain to be an important topic in the broad field of acoustics. Still, most ongoing research and practice, in which the propagation path in between source and receiver is of interest, aims to attenuate sound around 1000 Hz and above. However, for an overall optimization of outdoor sound quality, reduction in the frequency range 200-1000 Hz is an important supplement.

In general reduction of low- and mid-frequency sound with passive means is somewhat more troublesome than at higher frequencies due to larger wavelengths and resulting practical limitations due to size and thickness needed to, e.g. achieve a sufficiently high absorption using any kind of porous material. A different approach to reduce sound, specifically, at lower frequencies, is to create a field of Helmholtz resonators that are buried in the ground. By changing the values of physical parameters, such as volume and the opening diameter, a single resonator can be tuned to any desired resonance frequency. By means of combined effects, a large set of these devices can achieve broadband reduction in a specific octave band. However, important herein is to find the right configuration, e.g. mutual distance between orifices and variation of resonance frequencies, because most energy is steered rather then absorbed. In some cases bad design might even lead to an amplified sound field.

To predict the noise reducing effect of a field of ground buried resonators, a numerical three-dimensional model which is able to handle multiple coupled resonators, with arbitrary resonance frequencies, loss factors and opening positions, is developed. This model, based on the equivalent sources method (ESM), represents every resonator by an equivalent source and has theoretically no upper limit for the amount of resonators included. Its validity is shown for a single resonator geometry by a comparison with measurements giving good agreement.

Key words: Helmholtz resonators, coupled resonators, outdoor sound propagation, road traffic noise.
List of notations

Throughout this thesis the following abbreviations are used:

**Roman upper case letters**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A$</td>
<td>Body radius</td>
<td>[m]</td>
</tr>
<tr>
<td>$F$</td>
<td>Force</td>
<td>[N]</td>
</tr>
<tr>
<td>$G$</td>
<td>Green's function</td>
<td>[-]</td>
</tr>
<tr>
<td>$L_b$</td>
<td>Body length</td>
<td>[m]</td>
</tr>
<tr>
<td>$L_n$</td>
<td>Uncorrected neck length</td>
<td>[m]</td>
</tr>
<tr>
<td>$L_{inner}$</td>
<td>Inner-end correction</td>
<td>[m]</td>
</tr>
<tr>
<td>$L_{outer}$</td>
<td>Outer-end correction</td>
<td>[m]</td>
</tr>
<tr>
<td>$M_n$</td>
<td>Neck mass</td>
<td>[kg/m$^3$]</td>
</tr>
<tr>
<td>$M_b$</td>
<td>Body mass correction</td>
<td>[kg/m$^3$]</td>
</tr>
<tr>
<td>$R_n$</td>
<td>Neck resistance</td>
<td>[kg/s]</td>
</tr>
<tr>
<td>$S_b$</td>
<td>Body surface</td>
<td>[m$^2$]</td>
</tr>
<tr>
<td>$S_n$</td>
<td>Neck opening surface</td>
<td>[m$^2$]</td>
</tr>
<tr>
<td>$V$</td>
<td>Resonator volume</td>
<td>[m$^3$]</td>
</tr>
<tr>
<td>$Q$</td>
<td>Ground reflection coefficient</td>
<td>[-]</td>
</tr>
<tr>
<td>$Z_A$</td>
<td>Acoustical impedance</td>
<td>[Ns/m$^5$]</td>
</tr>
<tr>
<td>$Z_{hr}$</td>
<td>Mechanical impedance</td>
<td>[Ns/m]</td>
</tr>
</tbody>
</table>

**Roman lower case letters**
List of notations

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>Neck opening radius</td>
<td>[m]</td>
</tr>
<tr>
<td>c</td>
<td>Speed of sound</td>
<td>[m/s]</td>
</tr>
<tr>
<td>$c_p$</td>
<td>Heat capacity at constant pressure</td>
<td>[J/KgK]</td>
</tr>
<tr>
<td>d</td>
<td>Neck opening diameter</td>
<td>[m]</td>
</tr>
<tr>
<td>e</td>
<td>Natural logarithm base</td>
<td>[-]</td>
</tr>
<tr>
<td>$f_0$</td>
<td>Natural frequency</td>
<td>[Hz]</td>
</tr>
<tr>
<td>f</td>
<td>Frequency</td>
<td>[Hz]</td>
</tr>
<tr>
<td>$g_{nm}$</td>
<td>Coupling function between $n^{th}$ and $m^{th}$ opening</td>
<td>[-]</td>
</tr>
<tr>
<td>j</td>
<td>Imaginary unit $\sqrt{-1}$</td>
<td>[-]</td>
</tr>
<tr>
<td>k</td>
<td>Wavenumber</td>
<td>[rad/m]</td>
</tr>
<tr>
<td>l</td>
<td>Number</td>
<td>[-]</td>
</tr>
<tr>
<td>p</td>
<td>Pressure</td>
<td>[Pa]</td>
</tr>
<tr>
<td>q</td>
<td>Unknown source strength</td>
<td>[Pa]</td>
</tr>
<tr>
<td>$q_0$</td>
<td>Initial source strength</td>
<td>[Pa]</td>
</tr>
<tr>
<td>s</td>
<td>Spring stiffness</td>
<td>[N/m]</td>
</tr>
<tr>
<td>v</td>
<td>Velocity</td>
<td>[m/s]</td>
</tr>
<tr>
<td>x</td>
<td>Displacement</td>
<td>[m]</td>
</tr>
</tbody>
</table>

Various symbols

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda$</td>
<td>Wavelength</td>
<td>[m]</td>
</tr>
<tr>
<td>$\rho_0$</td>
<td>Density of air</td>
<td>[kg/m³]</td>
</tr>
<tr>
<td>$\omega$</td>
<td>Angular frequency</td>
<td>[rad/s]</td>
</tr>
<tr>
<td>$\omega_0$</td>
<td>Angular natural frequency</td>
<td>[rad/s]</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>ratio of specific heats</td>
<td>[-]</td>
</tr>
<tr>
<td>$\mu_0$</td>
<td>the dynamical viscosity of air</td>
<td>[Ns/m²]</td>
</tr>
<tr>
<td>$\nu$</td>
<td>Thermal conductivity</td>
<td>[W/mK]</td>
</tr>
<tr>
<td>$\Re$</td>
<td>Real part of argument</td>
<td>[-]</td>
</tr>
</tbody>
</table>
Acknowledgements

After a fruitful study period of five months in Göteborg, during the spring term of academic year 2008 – 2009, I felt honestly sad because I had to leave Sweden. Fortunately for me and probably a bit less for my dear friends, family and Reni, it was only the beginning of a whole new chapter in my life. This adventure was not continued without the generous help and advise of Maarten Hornikx. I still think that his advise has let me see possibilities which I, most likely, otherwise would not have been able to see. Therefore many thanks!

Eventually, I went back to Sweden and wrote this thesis which documents the work done at Applied Acoustics, at Chalmers University of Technology. The period where all the 'magic' happened, November 2009 until June 2010, contained both, nice and awful weather. Since I come from the Netherlands which is, beside Tulips, cheese and wooden shoes (we don’t wear them anymore) also a nation of skating, I am supposed to be familiar with wintry weather. Here I learned this is really not the case. Especially, when the street’s surface changes from 'nice' black carpets into something better known as a skating track. Hopefully mother nature is more moderate next winter. If not, I shall consider to organize a new annual event in Göteborg, something like the Dutch "Elf steden tocht".

It should be noted here that I participated in a bilateral Erasmus Agreement between Chalmers University of Technology and my home university Technische Universiteit Eindhoven (TU/e). However, this would not have been possible without the support of my thesis supervisor, Heiko Martin. Together we convinced the exam committee, of the Architecture, Building and Planning faculty in the Netherlands, that a continuation in Sweden for me personally would be a great opportunity.

Of course a special thank to all the people at Applied Acoustics is necessary, I appreciate the pleasant environment created by all of you very much! Last but definitely not least I hereby want to thank my supervisor in Sweden, Jens Forssén, for his concern in me and my work and the confidence he gave me. The best thing I can say is that I hope to continue working like we did the last period.
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1. Introduction

1.1. Background

During most of the history of human beings, we have lived a rural lifestyle. Not too long ago, around the seventeenth century, only 3 percent of the world's population lived in urbanized areas [1]. However, after the industrial revolution things started to change and by 1900 not less than 14 percent of the world's population lived in urban areas. From the 1950's over 30 percent were urbanites and 83 cities had a population of plus 1 million [1]. In 2008 a doubtful milestone in history was reached with an equal distribution of people living in urban and rural areas. Since then there are more than 400 cities having 1 million inhabitants or more and 19 mega cities having a minimum population of 10 million people. It is expected that in future this tendency will continue [1].

As a consequence of urbanization, millions of people in Europe and billions (thousand million) of people on a global scale are affected by traffic noise. Traffic noise can lead to a variety of complaints, where 'simply' annoyance is the most represented complaint. Besides, sleep disturbance, certain cardiovascular diseases and raised blood pressures are known effects of exposure to traffic noise [2]. Mainly due to possible adverse health effects, the World Health Organization (WHO) recognized environmental noise, where traffic noise is a part, as a serious threat to public health. Since in our modern society choices are often driven by economical reasons, the costs of environmental noise for the society is estimated. In a conservative estimation EUR 40 billion a year is 'spent' on effects of environmental noise in Europe and hence it is very costly for the society. The major contribution (circa 90 percent) of environmental noise is caused by person cars and lorries, however rail traffic noise is also of significance [2]. According to WHO, more than 40 percent of the population living in Europe is exposed to a day-time averaged equivalent A-weighted sound level exceeding \( L_d = 55 \text{ dB}(A) \), which is caused by road traffic, 20 percent is even exposed to levels exceeding \( L_d = 65 \text{ dB}(A) \). As a guideline the WHO published maximum noise levels for specific environments as well as different periods of the day [2]. For equivalent levels outdoor the threshold is set to \( L_d = 55 \text{ dB}(A) \), where the criteria is serious annoyance [3].
1. Introduction

During the past decades research to reduce road traffic noise has been conducted widely. Different approaches which can be summarized into three spearheads are therefore used, see e.g. Hornikx [3];

- Reducing the sound power produced by the road traffic vehicles and their interaction with the road;
- Reducing the transfer of the sound intensity from sound sources (the vehicles) to the receiver position, e.g. building façade or a position outside;
- Reducing the transfer of the sound intensity from the outside to inside by changes in the outer wall construction.

Plenty of examples for these three approaches can be summarized, yet the focus in this work is on sound propagating outdoors to an outdoor receiver. Whereas the indoor sound environment with closed windows can be protected by ordinary measures as façade insulation and optimized glazing, the outdoor environment is more difficult to realize as a good sound environment. The most straightforward and known measure in outdoor situations are clearly noise barriers. Nevertheless, noise barriers are mostly effective for receiver positions nearby a source and sometimes aesthetically unwanted or not applicable from a practical point of view, e.g. inside city centres. Besides noise barriers, speed limitations of traffic and porous asphalt are as well common measures.

A complementary approach in order to protect inhabitants from environmental noise pollution, proposed by, among others, Kilhman is to give citizens access to a place where the sound pressure levels are substantially lower; a 'quiet side' [4]. This is an approach which can be of particular interest in city centres – where backyards shielded from the street are common. In relation to this, among others, Hornikx modelled the sound propagation to closed urban court yards [5]. A comparable study was performed by Van Renterghem and Bottledooren. They developed a method able to predict sound pressure levels at the screened side of a parallel street canyon geometry [6]. Both methods can be used to evaluate effects of noise abatement schemes, while taking complex propagating factors as multiple reflections, screening, edge diffraction, scattering and atmospheric effects into account.

A series of prediction tools and noise abatement strategies for a better outdoor sound quality has been established, yet sustainability has rarely been paid attention to [7]. Within the SEVENTH FRAMEWORK PROGRAMME, THEME 7; "Sustainable Surface Transport" the HOSANNA project should supplement this lack. The full project
1. Introduction

name of HOSANNA reads: "Holistic and sustainable abatement of noise by optimized combinations of natural and artificial means". In this project several studies concerning the optimization of green areas, in terms of their acoustic impact on road and railway traffic noise will be covered [7]. Since the costs of having green areas and surfaces in urban and rural environments are well accepted and established without the noise issue being considered, exploiting these green areas and surfaces to at the same time minimize the noise impact on citizens of Europe leads to a better use of financial resources [7].

For an overall optimization of the outdoor sound environment low- and midrange frequency noise reduction is an important supplement to the more classical noise abatement schemes. Analysing traffic noise spectra composed of a mix of vehicles passing-by and considering different driving speeds, a peak between 800-1250 Hz in 1/3 octave bands is clearly displayed [8]. Most often the peak is somewhere around 1000 Hz and after A-weighting this peak will be even more prominent. In the following pages, speaking about midrange frequencies, means within the range of 200-2000 Hz.

Measures to reduce traffic noise are limited, especially in the region where traffic noise peaks. For example the use of porous asphalt shows a clear difference compared to ordinary dense asphalt – a dense road surface mainly reflects the sound energy, whereas a porous road surface introduce some absorption [9]. However, porous asphalt is mostly effective above 1250 Hz. Promising alternatives to porous asphalt, having better acoustic properties in relation to traffic noise spectra are hybrid systems – e.g. a porous material combined with Helmholtz resonators. The Helmholtz resonators are then tuned to the frequency range of interest, whereas the porous asphalt is mostly effective in the higher frequency regions. Different kind of construction designs for these hybrid systems, based on basically the sample principle, are patented, mostly in the United States of America. The use of it, however, is not that convenient - probably due to the unfamiliarity at acoustic consultancies as well as the complexity of predicting the insertion loss of such systems. In addition the costs might be high and the road industry can be rather conservative and therefore it often takes time to establish innovative road constructions. A different approach using the same principle in order to attenuate low- to midfrequency noise produced by road traffic is to create a field of resonators in the ground. The acoustic effect of a field of Helmholtz resonators will be studied in this thesis.

Numerous studies on the use and prediction of Helmholtz resonators (HR:s) have been performed during the past decades. On one hand, they are meant for use as sound amplifying devices. On the other hand, as passive noise control devices – which is the
1. Introduction

focus in this work. (It could be noted that recent work also has considered actively controlled resonators.)

The application to sound reduction has been investigated by many, yet most extensively by Ingard and Selamet. Absorption and scattering from single HR:s, in free-field conditions or mounted in a wall, was a topic investigated by Ingard. Furthermore, he developed a basic theory to calculate the interaction between two circular orifices using an interaction impedance approach [10]. As a continuation Selamet et al. investigated several formations where HR:s are coupled to a duct - an application which is of special interest in the building service industry as well as the car industry. Specifically they investigated the effect of absorbing material inside the resonator body [11], and circular asymmetric resonators attached to a circular duct [12].

To make use of Helmholtz resonators in an outdoor environment, many resonators are necessary to gain a reasonable insertion loss. A possible solution is then to install an array of resonators neighbouring road- or railways. Even though typical traffic noise spectra show a sharp peak around 800-1250 Hz, the effectiveness of identical resonators having a perfectly smooth surface is still far too narrow to cover this region. In order to broaden the damping peak, additional absorbing material inside the resonator or varying the resonator's natural frequency are possible measures – e.g. decreasing the resonators volume will shift the natural frequency upward. At first glance, adding absorbent material is not very suitable in an outdoor environment, since it will be exposed to rain, snow and cold. However, nor is the use of smooth surfaced resonators, if water can leak inside the resonators volume and change the resonators properties. For both of these practical issues solutions can probably be found, e.g. by replacing a smooth resonator's bottom by a porous bottom enabling water to leak in the ground and creating increased losses by air leakage. Hence, to get started, these practical limitations are neglected throughout this work.

1.2. Purpose and problem definition

The purpose of this thesis is to investigate the interaction between multiple Helmholtz resonators and model their noise reducing effect for sound propagating outdoors from traffic noise sources. To predict potential noise reduction, in the frequency range 200–1000 Hz, from a field of resonators, a numerical model able to handle multiple resonators having arbitrary resonator natural frequencies, loss factors and opening positions is developed.
1. Introduction

1.3. Limitations

The use of a homogeneous non-moving atmosphere should be named as the most important limitation of this work. As a result, people using this model should be aware of the fact that presented results may change significantly if a down- or upward refracting atmosphere is modelled. Furthermore, only the case of a single cylindrically shaped resonator, mounted in a baffle, is modelled and validated against measurements. For this reason predicting the noise reduction of differently shaped resonators is not possible.

1.4. Thesis structure

This thesis is structured as following:

- **Chapter two**: Contains the theoretical description of both developed analytical models, e.g. single and multiple Helmholtz resonators mounted in a baffle.

- **Chapter three**: Covers the validation of a single Helmholtz resonator mounted in a baffle by comparing measurements against predictions.

- **Chapter four**: Presents several parameter studies using the developed multiple resonator model.

- **Chapter five**: Finally, results and findings are summarized. In addition research topics for continuation of this work are discussed.
2. Theory

This chapter starts with a brief introduction to resonating absorbers and their applications. Thereafter, a Helmholtz resonator and its sound reducing mechanisms is described in more detail. The derivations for a single Helmholtz resonator in a baffle and multiple resonators in a baffle are then given. As a remark the author would like to point out that both complete derivations are omitted from the main text and moved to the appendices.

2.1. Resonant absorbers

The use of resonant absorbers goes back to the Greeks, who, we believe, used them to modify the reverberation time of open-air theatres. The Danish and Swedish followed by using resonant absorbers in churches as early as the thirteenth century, whereas others start to make use of this principle in later centuries [13].

Nowadays resonant absorbers, most often of the Helmholtz type, are used in an abundance of devices like rockets, exhaust systems and ducts, e.g. for air supply. Beside, resonant absorbers are an indispensable measure in room acoustics - especially to gain absorption in the low- to midfrequency range. The use of resonant absorbers in room acoustics becomes mainly interesting when porous absorbers are inefficient due to a low particle velocity at room boundaries and corners or by practical limitations due to the size and thickness needed to achieve a sufficiently high absorption at lower frequencies [14]. Examples for the use of resonant absorbers in an outdoor environment nowadays are at least not common, even though they could be a good supplement to the more ordinary measures to reduce traffic noise.

Talking about resonating absorbers one can mainly distinguish two different types - membrane and Helmholtz absorbers. Both are, however, based on the same physical principle, a mass which vibrates against a spring. A brief introduction to both types will be discussed.
2. Theory

2.1.1. Membrane absorbers

Membrane absorbers are often used for base traps in listening rooms or other rooms where damping of very low frequencies is of interest. A membrane absorber or differently called a panel absorber basically consists of a rigidly-backed cavity, closed by a membrane, with or without additional absorption material. The membrane, for instance made of gypsum board or wood, forms the mass of the resonating system. The membrane, exited by an incoming sound wave starts, depending on the frequency content of the impinging wave, to vibrate. As a consequence the motion of the membrane leads to a compression of the air on the back of the membrane - the membrane converts the pressure fluctuation into air motion so to speak. Gas-compression naturally increases its temperature and hence sound attenuation take place. Even though membrane absorbers can be interesting to study, the focus in this thesis will be on Helmholtz resonators.

2.1.2. Helmholtz resonators

In order to eventually model a coupled system of interacting resonators the theoretical background of a single resonator is introduced first. One of the first who studied individual resonators in detail, though for a completely different purpose, was Herman von Helmholtz (1821-1894). He developed a series of hollow spherically shaped resonators made of glass and metal having two openings; one rather great opening in the bottom, adapted for insertion in the ear, and an opening on top through which sound enters the resonator body. The air enclosed in the resonator, the resonator neck, together with that in the aural passage and the drumskin, forms an elastic system which is capable of vibrating. Especially the prime tone of the resonator, which is much deeper than other present proper tones, can be set into a very powerful vibration. The ear which is in immediate connection with the air inside the resonator can perceive the amplified prime tone directly. In the past such devices were especially helpful for unpractised ears to detect individual faint tones even if they were accompanied by other tones [15]. Helmholtz used his resonators to detect individual tones, however, they can also be used as a damping device. Taking Helmholtz design as a starting point, the ear-opening in the bottom through which the prime tone is detected should then be covered so that the resonator is completely closed, except for the narrow opening on top. A resonator where the enclosed air inside the volume is set into a powerful vibration may affect the sound field outside the resonator, e.g. direct and reflected sound waves, by re-radiating energy with opposite phase. Obviously this effect is strongest near to the resonator and will decrease further away from the resonator.
2. Theory

2.2. Physical description of a single Helmholtz resonator

A single Helmholtz resonator basically consists of a closed volume, \( V_b \) of arbitrary shape, which is connected to the neck having an opening-area \( S_n \), see Fig. (2.1). The neck-opening functions as a port through which the resonator communicates with the external medium, here assumed to be a homogeneous non-moving atmosphere. A plug of air enclosed in the neck of the resonator will move downwards after a pressure pulse is impinging on the resonators opening and will increase the pressure inside the resonators volume. It is, however, important to realize that the resonators neck is small with respect to the wavelength of interest, both in length and width. Hence the neck particle velocity throughout the neck can be assumed as equally distributed. Differently said an acoustical lumped element approach may be used to characterize the behaviour of the resonator. Using this approach one assumes the physical parameter of interest to be concentrated, or lumped, at a specific point [16]. On the other hand one could use a distributed elements method, which implies a field that varies over space. The assumption of an equal neck velocity for a Helmholtz resonator (HR) has been proven to agree well in previous studies, see among others [17].

Figure 2.1.: Representation of a single cylindrical HR. Here, \( L_n = \) uncorrected length of the neck, \( L_b = \) length of the body, \( A = \) radius of the body, \( a = \) radius of the neck opening, \( S_n = \) opening surface and \( S_b = \) body surface.

Back to the HR, a plug of air forms a concentrated mass which moves due to a pressure difference between outside and inside. The mass of the air plug is, mainly, located in the resonators neck and is given by,
2. Theory

\[ M_n = S_n \rho_0 L_n, \]  
(2.1)

with \( S_n \) the opening-surface, \( \rho_0 \) the density of air and \( L_n \) the neck length. Because the air directly in- and outside the end of the resonator neck oscillates together with the air enclosed in the neck, the length of the air plug is somewhat larger than its physical length. To correct for this difference an inner end-correction \( L_{\text{inner}} \) and an outer end-correction \( L_{\text{outer}} \) should be added to its physical length. Several studies have focussed on deriving a formulation for the inner end-correction. The most convenient in this work may be the one derived by Ingard, see [10]. He modelled a circular piston radiating into a cylinder,

\[ L_{\text{inner}} = 0.48 \sqrt{S_n} (1 - 1.25 \cdot \frac{a}{\lambda}) \]  
(2.2)

The outer-end correction is taken from Rayleigh, who modelled the well known case of a circular piston in a baffle, see e.g. [18]

\[ L_{\text{outer}} = \frac{8}{3\pi} a. \]  
(2.3)

Classical analysis of Helmholtz resonators stated that the resonance frequency is independent on the shape of the cavity. Later studies, e.g. Panton and Miller, showed this is, in case of a cylindrical HR, only valid for length dimensions less than 1/16 of the wavelength (\( \lambda \)), which is significantly shorter than the often quoted 1/4 \( \lambda \) [19]. They derived therefore an additional mass correction factor for the resonator body,

\[ M_b = \frac{1}{3} \frac{\rho_0 L_{\text{r}}}{S_0} S_n^2. \]  
(2.4)

which is an addition to the earlier given neck mass, \( M_n \).

If the plug of air is, due to an incoming pressure pulse, driven into the resonator's volume, the air inside the volume will be compressed and as a consequence the pressure inside the volume will rise from static pressure. The plug moves then back to its original position if the restoring force in the volume becomes larger than the incoming pressure. Due to the springiness of the air trapped in the body this results, if no new force is applied to the system, in a back and forth movement of the air plug. However, due to several resistive components the oscillation amplitude of the plug will decrease after each cycle, and eventually die out completely. For this reason a HR can be simplified into a harmonic oscillator having a single degree of freedom (SDOF). This an analogy
well known from mechanical engineering, and in the early days of acoustics first used by Rayleigh to describe the behaviour of a single resonator. Without damping included a HR behaves as an undamped simple mass-spring system - where the mass is formed by the air plug in the neck and the spring is formed by the air inside the resonator volume.

By applying Hooke's law, \( F = -sx \) an expression for the elastic stiffness of the air enclosed inside the resonator volume can be derived [20],

\[
s = \frac{\rho_0 c^2 S_n^2}{V},
\]

with, \( V \) = the resonator's volume, \( S_n \) = the orifice surface and \( c \) = the speed of sound.

### 2.3. Losses in the resonator

Even though no additional damping is included, a Helmholtz resonator dissipates energy at and around the natural frequency due to losses in the resonator's neck. The acoustic dissipation is mainly due to viscosity and heat conduction losses on the surfaces of the resonator - both have a linear character. Non-linear effects, however, will become important if a turbulent flow is modelled or high sound intensities are of interest [10]. Though interesting, non-linear effects are out of the scope for this work and hence not included. The total linear acoustic dissipation in the resonator neck can be formulated as,

\[
R_n = \frac{L_n}{a} \frac{\sqrt{2\mu_{eff} \rho_0 \omega}}{\pi a^2} + 2 \frac{\sqrt{2\mu_0 \rho_0 \omega}}{\pi a^2},
\]

where

\[
\mu_{eff} = \mu_0 \left(1 + (\gamma - 1) \sqrt{\frac{\nu}{\mu_0 c_p}}\right)^2.
\]

Here, \( \omega = 2\pi f \), \( f \) is the frequency, \( \mu_0 \) the dynamical viscosity of air, \( \gamma \) the ratio of specific heats, \( c_p \) is the heat capacity at constant pressure. The first right-hand term in Eq. (2.6) represents the viscous losses in the neck wall [10]. Viscous losses are in particular of interest for openings having a small radius - e.g. a resonator neck. This can be clarified more by showing an exaggerated velocity profile which takes this form due to viscous friction, see Fig. (2.2).

At the wall boundaries the velocity of moving air particles is zero, whereas velocity reaches a maximum in the middle of the opening - the so-called free stream velocity.
2. Theory

![Diagram of a resonator](image)

**Figure 2.2.** Illustration of a resonator's neck and an exaggerated velocity profile inside the neck.

Considering a wider pipe, the area of free stream velocity is bigger and hence losses due to viscous friction relatively smaller. The second right-hand term in Eq. (2.6) represents the viscous losses of the neck ends [10]. This expression holds for neck ends, ending in an open space as well as for neck ends in an infinite baffle. The latter is comparable to a resonator's opening, mounted in an infinite plate. The inner neck end does, however, not fulfill any of the statements. Here, the finite radius of the resonator's volume should be taken into account and as a consequence the use of it is an approximation for the application studied here. However, since previous work found good agreement between calculations and measurements and no better description was found in the literature the described expressions are used here [17].

### 2.4. Impedance of a single Helmholtz resonator

All individual elements necessary to physically describe a Helmholtz resonator are discussed in previous paragraphs. How they relate to each other may be formulated as an impedance, $Z_{hr}$. An impedance approach, as proposed by Rayleigh, seems here a logical choice since an incoming pressure eventually results in a velocity of the piston, i.e. the air of the neck opening. The piston velocity, which is dependent on the resonator characteristics, can be seen as the speed at which the opening surface $S_n$ moves due to an incoming pressure. The impedance of the resonating absorber is constituted of its mass, spring-stiffness and resistance. Using a lumped element model makes it possible to visualize the working system by a few simple elements, see Fig. (2.3).

In Fig.(2.3) the displacement in z-direction is positive and the force, $F$, is chosen to be
2. Theory

Figure 2.3.: Left: Lumped element model of a Helmholtz resonator excited by a force, $F$. Furthermore, $M$ represent the total mass ($M_n + M_b$), $s$ the springs stiffness of the air enclosed in the resonator's volume and $R_n$ the resistance in the neck. Right: free body diagram of a single Helmholtz resonator.

in the opposite direction, whereby it has negative sign in Eq. (2.8). In order to more easily derive the impedance, $Z_{hr}$, a free body diagram is drawn. Such a diagram is set up by removing all elements acting on the mass and interchange them with a force. A force balance results in:

\[-F = Ma + R_nv + sz.\]  (2.8)

With;

$F=$ the external force (incoming pressure) which point downwards;
$Ma=$ the mass inertia, from Newton’s second law of motion;
$sz=$ the spring stiffness, from Hooke’s law;
$R_nv=$ damping, proportional to the velocity in the neck.

The impedance, relating pressure and velocity at a communal point, reads

\[Z = \frac{p}{v}.\]  (2.9)

Using this expression plus the definition of the pressure, $p = \frac{F}{S_n}$, with $S_n$ as the surface of the opening gives the final expression for the mechanical impedance, $Z_{hr}$;

\[Z_{hr} = -\frac{F}{S_v} = j\omega M + R_n - \frac{sz}{S_n}.\]  (2.10)

Where, $j = \sqrt{-1}$. A complete derivation of Eq. (2.10) is covered in Appendix A.
2.5. Single Helmholtz resonator mounted in an infinite baffle

In this section the derivation of a strongly idealized three-dimensional geometry including a single HR is presented. Like the derivation of a HR impedance, only the most important steps are reported in the main text and a complete derivation is given in Appendix B.

A single HR in a 3D semi-infinite space is of interest. Using the term semi-infinite suggest the space is bounded in a certain direction, however, is unbounded in other directions. Here a ground surface of infinite size is modelled, having a perfectly reflecting surface, i.e. reflection factor, $Q$, which equals 1. Somewhere in this perfectly reflecting ground surface a single cylindrically shaped HR, which has its circular opening at $z=0$, is implemented, see Fig. (2.4). Consequently, all other parts of the resonator (neck and volume) are buried and not 'visible' so to speak.

![Diagram of a Helmholtz resonator in a semi-infinite space, buried in an infinite ground surface.](image)

At an arbitrary position in the semi-infinite space a source generates an input signal and can activate the resonator. However, the amount of activity will depend on the strength of the pressure produced by the source and how near it is to the natural frequency of the resonator. The driving force of the system is chosen to be a point source. Since a traffic noise problem forms the basis of this work one can argue about the choice of a single point source description. However, for deriving the expression it makes life easier and later a set of uncorrelated point sources can simulate a traffic source. A receiver, positioned at an arbitrary position in the same spatial domain as the source, picks-up the total pressure which reaches that point. The total pressure at the receiver can be defined as the superposition of pressures from the direct and reflected sound path as well.
2. Theory

as the path from source to HR to receiver,

\[ p(x, y, z) = q_0 G(r_d) + q_0 G(r_r)Q + qG(r_{hr}). \]  

(2.11)

Where, \( q_0 \) is the source strength of the driving force, \( G(r) \) the Green's function for a point source in a three-dimensional unbounded space, \( Q \) the reflection factor of the ground and \( q \) an unknown source strength of the HR. The used Green's function represents an outgoing wave and is formulated as \([21]\),

\[ G(r) = \frac{e^{-jkr}}{r}. \]  

(2.12)

An outgoing wave means that the wave front created by the monopole source goes away from the centre point. Multiplying the Green's function by a source strength, \( q_0 \) (or \( q \)), describes the spherical sound field, where the pressure amplitude is \( \frac{Q}{r} \). It seems clear that the amplitude is thus decreasing with increasing distance \( r \). Since all other variables are known, the crux of finding a solution for Eq. (2.11) is to find the unknown source strength \( q \). The source strength \( q \), is related to the velocity at the resonator's opening by,

\[ v = \frac{2\pi q}{j\omega \rho S_n}, \]  

(2.13)

with, \( \omega = 2\pi f \), \( \rho \) the density of air and \( S_n \) the surface of the HR opening. This general solution for a small and flat source in a baffle holds however only for source-receiver distances which are large \([22]\). Furthermore, the source has a constant velocity over the area of the opening - an assumption which is true for a small opening (small compared to the wavelength of the incoming wave). Whereas the velocity for far-field receiver positions is relatively easy to obtain by using Eq. (2.13), the velocity for \( r_{hr} \rightarrow 0 \) is more complex to calculate. In order to find \( v \) and eventually \( q \) for \( r_{hr} \rightarrow 0 \) the receiver is placed exactly above the HR opening, precisely in the centre of it, see Fig. (2.5).

As a consequence the pressure at the receiver is equal to,

\[ p(x, y, z) = q_0(2G(r'_d)) + qG(r_{hr}). \]  

(2.14)

Here, the direct contribution is simply doubled, since \( r'_d = r_r \) and \( Q = 1 \). Yet the velocity is still unknown, but can be eliminated by applying the Huygens-Rayleigh integral \([22]\),
2. Theory

Figure 2.5.: Visualisation of a Helmholtz resonator in a semi-infinite space, buried in an infinite ground surface. Here, \( q_0 \) = source strength, \( p \) = received SPL and \( r_d \) = direct contribution.

\[
p(x, y, z) = \frac{j\omega \rho}{2\pi} v \int G(a) dS. \quad (2.15)
\]

The integral evaluates the pressure amplitude in point \((x, y, z)\) – here the centre point of the resonator’s opening. In fact the pressure due to its own velocity is what will be calculated by placing the receiver there. Solving the integral for a circular opening of radius \( a \) results in the following expression for the pressure,

\[
p(x, y, z) = p c(1 - e^{-jka})v. \quad (2.16)
\]

With, \( k = \frac{\omega}{c} \) and \( v \) the velocity in the centre point of the HR. The pressure at the HR-opening can now be formulated as the incoming pressure from the source plus an additional pressure from the HR,

\[
p(0, 0, 0) = q_0(2G(r'_d)) + p c(1 - e^{-jka})v. \quad (2.17)
\]

Still an important part in the expression is missing since the pressure at the opening should relate to the impedance of the resonator, \( Z_{hr} \). It should be denoted here that \( Z_{hr} \) already includes a reactive mass component above the resonator opening which is defined as outer end correction, \( \frac{8}{3\pi} a \). However, so does the derived expression \( p c(1 - e^{-jka})v \). Keeping both terms in the equation will give an additional shift in frequency and thus gives wrong results at every possible receiver point in the semi-infinite space. To overcome this problem two tracks can be followed, either remove the outer end correction in \( Z_{hr} \) or keep the real resistive part of \( p c(1 - e^{-jka})v \) and omit the complex reactive term. The latter approach is used throughout this work, because in a later section \( Z_{hr} \)
2. Theory

is validated against measurements and there the outer end correction is necessary again. Substituting Eq. (2.17) into Eq. (2.09) eventually results in the final expression for the velocity,

$$v = \frac{q_0(2G(r'_d))}{Z_{hr} - \Re(\rho c(1 - e^{-jka}))}.$$  \hspace{1cm} (2.18)

One additional substitution, namely Eq. (2.18) into Eq. (2.13), gives the unknown source strength of a single HR,

$$q = \frac{-j\omega \rho S}{2\pi} \left( \frac{q_0(2G(r'_d))}{Z_{hr} - \Re(\rho c(1 - e^{-jka}))} \right).$$  \hspace{1cm} (2.19)

If $q$ for a certain geometry is calculated, the total pressure including the influence of the resonator is obtained by applying Eq.(2.11), where, $h_{hr}$ represents the distance from HR to receiver. For the new source (the resonator's opening) monopole radiation is assumed. In practice this means it affects the sound field around a resonator equally much for receivers having the same distance $r$ from the resonator's centre.

2.6. Acoustic coupling between two resonators

The case of a single resonator in a baffle is rather extreme and rarely seen in 'real' life. In order to create a prediction tool which is more useful in practical situations, the effect of multiple resonators is investigated analytically. At first glance this can be obtained simply by calculating the effect of all individual resonators, applying the above derived expressions, and make a complex summation at the receiver. However, this is only true for some extreme cases where the distances between the openings are large. In case radiators are positioned in close proximity with other resonators it is proven that the total performance is affected by their acoustic interaction, see among others [23]. In order to explain this idea, two resonators and their acoustic interaction will be evaluated, see Fig. (2.6).

Except for one additional resonator the geometry is exactly the same as the previously used geometry for a single resonator mounted in an infinite baffle. As a consequence parts of the derived equations can be used here. Until the definition of its piston velocity no additional steps are implemented - except for the fact that there are now two piston velocities. Having only one resonator included, the contribution at the receiver is simply the source strength, Eq. (2.19), of that resonator multiplied by the Green's function, Eq. (2.11). Including two or more resonators there is one intermediate step missing. Because
2. Theory

Figure 2.6.: Visualisation of two Helmholtz resonator in a semi-infinite space.

The uncoupled velocity of HR,1 gives rise to the pressure at HR,2. The force acting on the second opening is defined by the incoming pressure of the original source plus an additional pressure from the first resonator, see Fig. (2.8).

Figure 2.7.: Visualisation of coupling between two resonator openings in a semi-infinite space.

On the other hand, the uncoupled velocity of HR,2 influences the pressures at the opening of HR,1. This can be written as follows,

\[ p_1 = q_0(2G(r_{o,1})) + \rho c(1 - e^{-j\alpha t})v_1 + \langle p_{21} \rangle, \quad (2.20) \]

with,

\[ p_{21} = \frac{j\omega \rho}{2\pi} v_2 \int G(r_{21})dS_1, \quad (2.21) \]

and "\( \langle p_{21} \rangle " declairing that the spatial average pressure of opening two has been used. Since \( p_{21} \) includes a propagation factor in the form of the Green's function, the coupling pressure may then, compared to the initially radiated pressure, \( p_1 \), of the resonator opening, be shifted in phase. This can be interpreted as the time it takes for a sound wave to travel from opening one to opening two, or vice versa. By relating the pressure...
2. Theory

at the opening of HR, one obtains,

\[ q_0(2G(r_{0,1})) + \langle p_{21} \rangle = (Z_{hr,1} - \Re(\rho c(1 - e^{-jka})))(p_{21}) \]  \tag{2.22}

The spatial averaging \( \langle p_{21} \rangle \) is approximated by taking the value in the centre point of the opening multiplied by its surface. For further analysis we also divide \( \langle p_{21} \rangle \) by \( v_2 \) to obtain the coupling function \( g_{21} \):

\[ g_{21} = \frac{\langle p_{21} \rangle}{v_2} \]  \tag{2.23}

which is approximated as

\[ g_{21} = \frac{j\omega \rho}{2\pi} G(r_{21}) S_1. \]  \tag{2.24}

The crux is now to find \( v_1 \) and \( v_2 \), both representing the coupled velocities of resonator one and two, respectively. This can be done in two ways, either by using the substitution method or by defining a two dimensional matrix. Because it is hard to keep track of all variables a substitution method is only manageable for a certain amount of resonators, bounded to a rather low upper limit. Since a numerical model for \( n \times m \) resonators is eventually desired, a matrix solution seems therefore the best choice. A coupled system of two resonators can be written as \( A \ast b = c \),

\[
\begin{bmatrix}
Z_{hr,1} - \Re(\rho c(1 - e^{-jka})) & -g_{21} \\
-g_{12} & Z_{hr,2} - \Re(\rho c(1 - e^{-jka}))
\end{bmatrix}
\ast \begin{bmatrix}
v_1 \\
v_2
\end{bmatrix}
= \begin{bmatrix}
q_0(2G(r_{0,1})) \\
q_0(2G(r_{0,2}))
\end{bmatrix},
\tag{2.25}
\end{equation}

with, \( A \) the matrix for coupling pressures with diagonal terms \( Z_{hr} - \Re(\rho c(1 - e^{-jka})) \), \( b \) the vector for coupled velocities and \( c \) the vector for source-to-resonator pressures. Since \( A \) and \( c \) are known, \( b \) can be calculated by applying a backslash-division operation, \( A \backslash c \), e.g. Gaussian elimination. The coupled source strength in general form is then given by,

\[ q'_n = \frac{j\omega \rho S_n}{2\pi} v_n. \]  \tag{2.26}

with, \( n \) denoting the \( n \)th resonator in order to make this expression applicable for multiple resonators too. Back to our coupled system; the pressure at the receiver, including both resonators as well as direct and reflected waves is determined as follows,

\[ p = q_0 G(r_d) + Qq_0 G(r_r) + q'_1 G(r_{hr,1}) + q'_2 G(r_{hr,2}), \]  \tag{2.27}

with, \( G(r) \) the Green's function as formulated in Eq.(2.12). Note that all terms in Eq.
(2.25) are summed complex at the receiver. The use of complex summation is necessary, because the pressures contain their phase information in the imaginary term.

**Importance of acoustic coupling**

The importance of acoustic coupling is examined by comparing two acoustically coupled resonators against two resonator openings which are not coupled. The mutual spacing between orifices is then varied, a method proposed by Ingard [10]. Intuitively, coupling will increase for openings closer to each other and decrease when they are moved from each other. From a certain point of increasing the mutual distance they can even be treated as uncoupled resonators. Of course the amplitude and phase of re-radiated sound from both resonators is important herein. It is known that coupling is strongest for resonators having the same frequency response and hence identical resonator properties are chosen to for the example made here. The geometry shown in Fig. (2.8) has been investigated for various distances \( l_d \), where, \( d \) is the resonator diameter and \( l \) a number. Thus, \( l = 1 \) corresponds thus to a situation where both openings touch each other at the neck-border.

![Diagram](image)

Figure 2.8.: Visualisation of coupling between two resonator openings in a semi-infinite space. With, \( q_0 = \) source position, \( p_1 = \) receiver position one and \( p_2 = \) receiver position two.

The source height is here set to 0.01 m - a typical value when car tyres are the noise source of interest. The receivers are lifted to 1.60 m and 0.01 m where the former correspond roughly to ear height of a human being. To exhibit the effect of the implemented resonators, sound pressure levels relative to free field conditions \( (SPL_{re,free,active}) \) have been calculated,

\[
SPL_{re,free} = 20 \log_{10} \left( \frac{P_{total}}{P_{direct}} \right). \tag{2.28}
\]
2. Theory

With, $p_{total}$ the complex summation of direct and reflected pressures, plus both pressures from the coupled resonators and $p_{direct}$ the direct pressure at the receiver. To get rid of the contribution of reflected waves in the analysis of the results, Eq. (2.26) is also calculated for a situation where no resonators are included, $SPL_{re,free,inactive}$. Practically, $SPL_{re,free,inactive}$ implies that no influence of resonators is included in $p_{total}$. By calculating the difference between $SPL_{re,free,inactive}$ and $SPL_{re,free,active}$, better known as insertion loss, the influence of the resonators at any receiver is what remains,

$$IL = SPL_{re,free,inactive} - SPL_{re,free,active}.$$  \hfill (2.29)

Defining the insertion loss as in Eq. (2.27) means that positive values correspond to a reduction due to presence of resonators. Two arbitrary chosen resonators tuned at 1425 Hz having an opening diameter of 4 mm are implemented in the described geometry. In order to investigate the influence of mutual distance on acoustic coupling, $l$ is varied with values 2, 4, 8 and 16 see Fig. (2.9). To compare, one uncoupled variant for $l = 4$ is also shown in Fig. (2.9). Notice that only one graph of the uncoupled variants is included because the difference among the uncoupled variants was extremely small for the investigated opening distances. Hence, for this example, separations of $l = 4$ or larger seems to be well approximated without coupling the resonators.

Studying the results shown in Fig. (2.9) one can see directly that the effect of two resonators at the proposed receiver positions is rather weak. Of course this can be improved by implementing more resonators in the geometry and finding the optimum configuration in terms of resonator spacing and source-to-resonator distance. Furthermore, the use of lower pitch resonators naturally gives a higher reduction since more potential energy can be stored in bigger volumes. As a consequence the restoring force of the spring, when it decompresses, will be greater. The higher amplitude of the re-radiated energy results then, together with the direct pressure, in deeper or higher interference patterns - dependent on the receiver position. Another remarkable fact is that the highest calculated insertion loss is somewhat shifted from the natural frequency of the implemented resonators. This can be explained by the introduced phase shift of the resonator. However, this will be explained in more detail in a later section.

As can be concluded from Fig. (2.9) it is of importance to use a coupled system of resonators. Especially, when resonators are placed in close vicinity to other resonators $\Delta SPL_{re,free,active}$ is enhanced significantly. This is caused by matching phase angles between the uncoupled re-radiated pressure of a resonator and the coupling pressure
2. Theory

Figure 2.9.: Insertion loss of two coupled and uncoupled resonators in a semi-infinite space. Upper: IL at receiver position $p_1$ and lower: IL at receiver position $p_2$.

from the other resonator. Theoretically coupling enhances IL if the mutual distance divided by the wavelength is in the range \( \{2\pi + n \rightarrow \frac{\pi + n}{2}\} \). Furthermore, IL can as well be affected by destructive interference if the mutual distance divided by the wavelength is in the range \( \{ \frac{\pi + n}{2} \rightarrow \pi + n\} \).

2.7. Matrix of $n \cdot m$ coupled resonators

In this work it is of interest to gain a substantial difference in the insertion loss by applying ground buried Helmholtz resonators in an outdoor environment. One, rather obvious, conclusion drawn in the previous section was that a coupled system of two resonators only gains a small loss. To improve, a field of $n \cdot m$ resonators in between the propagation path from source to receiver can be installed. Predicting the insertion loss of such a geometry is possible by extending the earlier presented matrix solution for two coupled resonators to an arbitrary amount of resonators,
2. Theory

\[
A \times \begin{bmatrix}
    v_1 \\
    v_2 \\
    \vdots \\
    v_n
\end{bmatrix} = \begin{bmatrix}
    q_0(2G(r_{o,1})) \\
    q_0(2G(r_{o,2})) \\
    \vdots \\
    q_0(2G(r_{o,n}))
\end{bmatrix},
\]

with,

\[
A = \begin{bmatrix}
    Z_{hr,1} - \Re(\rho c(1 - e^{-jka_1})) & -g_{11} & \cdots & -g_{1n} \\
    -g_{12} & Z_{hr,2} - \Re(\rho c(1 - e^{-jka_2})) & \cdots & -g_{2n} \\
    \vdots & \vdots & \ddots & \vdots \\
    -g_{in} & -g_{2n} & \cdots & Z_{hr,n} - \Re(\rho c(1 - e^{-jka_n}))
\end{bmatrix}.
\]

Theoretically the amount of resonators which can be included in the model is infinite. However, computational power will put an upper limit to the highest amount of resonators which can be included. Considering a large field of resonators, e.g. the size of a football field, the question may arise if a resonator on one side of the field affects the sound field on the other side. The answer is obvious; no. However the method proposed here does not include this fact and simply adds the coupling pressures of all other resonators in the field, wherever they may be located. From a computational point of view this seems not too clever and could indeed be improved. Analysing from a physical viewpoint it seems, according to Pritchard, no additional assumptions are hereby introduced [23]. Since it is assumed that the resonators are mounted in an infinite baffle and the opening diameter is small compared to the wavelength of interest scattering is negligible. As a consequence, the coupling function, \( g_{nm} \) is only a function of the \( n^{th} \) and \( m^{th} \) resonator and independent of all others [23].
3. Simulations and experimental validation

In this chapter the acoustic response of a single HR mounted in a baffle is validated with measurements. In addition, the analytical expressions of the neck resistance and impedance of the resonator are validated separately. Due to time restrictions an array of resonators is not validated experimentally.

3.1. Measurement Setup

In order to measure in semi free-field conditions, the measurements are performed in the anechoic chamber of Applied Acoustics. The anechoic chamber was originally designed for the frequency range 75 Hz to 10 kHz and has effective dimensions of 8 m x 8 m x 6 m. Since the resonator used for validation is tuned at 1425 Hz, the influence of the room on the measured results is expected to be little.

As ground plane, a 1.12*1.12 m film faced plywood plate has been used because it has a smooth surface and hence good reflecting properties. A drawback of the plate is that it can not be treated as infinite around 1425 Hz. Consequently, edge diffraction shall influence the measured results, especially closer to the plate boundaries. Even though edge diffraction is not included in the numerical model, it is expected that the pressure at receiver positions in close vicinity to the resonator can still be predicted fairly exact.

In the middle of the plate a hole of 25 mm is drilled and a cylinder of the same diameter can be fitted in, see Fig. (3.1: Left). Since it is hard by nature and can be machined fairly exact steel has been used. It is of importance to use a rather hard material considering that differences in the execution can cause already a significant shift in resonance frequency. The cylinder which does look as an ordinary cylinder from the outside is machined by a milling machine in order to create a volume and neck inside the cylinder. The created neck diameter and neck length both measure 4 mm, whereas the body diameter and body height are 7 and 16 mm respectively. The resonator, made in two separate parts, contains a permanent microphone, CH3 (Panasonic coil microphone, not calibrated), in the bottom which enables to validate the neck resistance and resonator
3. Simulations and experimental validation

impedance, see Fig. (3.1: Right). After the source is activated the sound pressure on the plate is measured by CH1 (Larsson and Davies microphone, calibrated) and CH2 (Larsson and Davies microphone, calibrated). Here, 'CH1' functions as reference microphone, e.g. close to the source opening and 'CH2' as variable microphone, e.g. more close to the resonator opening. The source is a combination of a powerful sound insulated driver and a 25 mm diameter attached tube of circa 5 m. The end of the tube may be seen as a small loudspeaker having monopole properties up to a frequency where $\lambda/2$ approaches the tube diameter. Going higher in frequency there is a possibility that oblique waves are formed in the tube and the assumption of plane wave propagation and point source radiation is not longer valid [24]. A benefit of using this source is found in the fact that it can be moved from the receiver and hence the influence of sound radiation from the driver is negligible. Furthermore, the tube can be attached separate from the plate so that the source is hanging freely above the plate – avoiding influences of plate transmitted waves.

![Diagram of measurement setup](image)

Figure 3.1.: Left: Measurement set-up in anechoic chamber, with SO = source opening, CH1 = microphone channel one, CH2 = microphone channel two and HR = position of the Helmholtz resonator. Right: close-up of the cylindrical resonator, formed by an upper and lower part which can be combined to one closed volume. Here, CH3 = microphone channel three.

The measurement set-up shown in Fig. (3.2) has been used to compare against theory. All microphones are placed directly on the plate, only the source is slightly elevated to 40 mm (measured from the centre of the tube). The position of CH1 is here fixed, whereas CH2 is moved to different positions. Therefore, two receiver positions with the resonator centre as reference have been used; CH2a: 4 mm from the resonator centre
3. Simulations and experimental validation

Figure 3.2.: Top view of the measurement set-up used to validate against theory. With, SO = source, CH1 = reference microphone and CH2 = microphone with variable position and HR = resonator opening

and CH2:b-10 mm from the resonator centre.

The measurement system VXI was chosen to use as data acquisition system. Using this system it is possible to measure up to 16 input channels at once and specify an adjustable Matlab based recording script. Besides, an arbitrary source signal can be selected - a useful option which allows to design any kind of input signal in Matlab. Throughout this work impulse responses were measured in order to identify conflicting noise factors, like reflections at plate edges, reflections in the tube or from any other obstacle in the room. Using this technique most noise factors could be excluded completely. For the source function, \( S(t) \), has been used, as e.g. applied by Hornikx et. al.

\[
S(t) = A \cos(2\pi f_c(t - t_c)) e^{-a(t-t_c)^2},
\]

with amplitude \( A \), \( f_c \) and \( t_c \) the central frequency and time of the wavelet and \( a \) a constant determining the bandwidth [5]. It is advantageous to use a source function as described by Eq. (3.1), because most energy of the transmitted pulse is then centralized around \( f_c \). Fig. (3.3) shows a calculated time signal in free-field conditions determined according to Eq. (3.1) and transformed into frequency components by using the Fast Fourier Transform (FFT) technique, with the sampling frequency, \( f_s = 5.12 \) kHz , \( f_c = 1435 \) Hz, \( t_c = 0.005 \) s , \( a = 8f_c^2 \) s\(^{-2}\) and \( A \) = an arbitrary amplitude \( Nm/s \). Both plots are normalized to their maximum values.
3. Simulations and experimental validation

Figure 3.3.: Source function in free field, calculated with Eq. (3.1). Left: time signal; Right: frequency content of the time signal

3.2. Neck resistance

In order to compare the analytical expression of the neck resistance \( R_{\text{calc}} \) to measured results, the half-power bandwidth method has been used. This method needs as input the frequency bandwidth, \( \Delta f \), of the peak after the SPL dropped 3 dB from its highest level at the resonance frequency. To obtain this value, a relation between the quality-factor of the resonator \( Q \), resonance frequency, mass and neck resistance can be used,

\[
Q = \frac{\omega_0 M_n}{R},
\]

with, \( Q = \frac{\omega_0}{\Delta f} \) and \( \omega_0 = \) angular resonance frequency. Focussing on the expression of \( Q \) it seems clear that a large factor evidently implies a narrow damping peak. In other words, large absorption will only occur in a small region around the resonance frequency. The bandwidth of the peak can be broadened, e.g. by introducing a larger resistance inside the resonator, as can be concluded from Eq. (3.2). Substituting \( Q = \frac{\omega_0}{\Delta f} \) into Eq. (3.2) results in,

\[
R_{\text{meas}} = M_n \Delta \omega,
\]

The drawback of this method is that it is only able to relate \( R \), \( M_n \) and \( \Delta \omega \) at the natural frequency of the resonator. It is evident that a complete picture of the measured resistance would be the optimal case. On the other hand the resonator abate most energy
3. Simulations and experimental validation

at and around the natural frequency. If the neck resistance for the whole frequency spectra would be changed to a single value, namely the measured value at resonance, only a small error is expected. Hence this experiment should be seen as a check, more than a validation and is mainly of importance to keep track of the units and resulting magnitude shifts. To avoid confusion it should be reported that \( M_n \) is not measurable in a 'normal' acoustic laboratory and as a consequence the analytical expression of \( M_n \) is used to obtain \( R_{\text{meas}} \). The measured transfer function, defined as the ratio between the frequency content of CH3 and CH1, is shown in Fig. (3.4).

![Figure 3.4.: Measured transfer function between CH3 and CH1 (normalized to the input pressure at the resonator's orifice) and two marked intersection points at -3 dB from the maximum value.](image)

Here, CH1 is normalized to the input pressure at the resonator's orifice. This is realized by assuming that the tube-end is a perfect monopole source. That means, the source strength can be obtained by applying,

\[
q_0 = \frac{p_1 \tau_1}{e^{-jkr_1}},
\]

with, \( p_1 = \) measured pressure at CH1 and \( \tau_1 = \) receiver-to-source distance. The normalized input pressure is then calculated by multiplying \( q_0 \) with the Green's function, Eq.(2.12). Two dashed lines, drawn in Fig. (3.4), define the maximum level of the transfer function as 0 dB and the half power level as -3 dB. Both resulting intersection points between transfer function and half power level define the width of \( \Delta \omega \). Numerical
3. Simulations and experimental validation

<table>
<thead>
<tr>
<th>Table 3.1.: Physical properties of air, used to calculate $R_{calc}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Property</td>
</tr>
<tr>
<td>Density $\rho_0$</td>
</tr>
<tr>
<td>Speed of sound $c$</td>
</tr>
<tr>
<td>Dynamical viscosity $\mu$</td>
</tr>
<tr>
<td>Ratio of specific heats $\gamma$</td>
</tr>
<tr>
<td>Thermal conductivity $\nu$</td>
</tr>
<tr>
<td>Heat capacity at constant pressure $c_p$</td>
</tr>
</tbody>
</table>

Figure 3.5.: Calculated resistance, $R_{calc}$ obtained by applying Eq. (2.6) and the measured resistance $R_{meas}$.

values used to predict the analytical neck resistance ($R_{calc}$) are given in Tab. (3.1).

Finally, the proposed analytical expression for viscous losses in the neck, $R_{calc}$ obtained from Eq. (2.6) and measured resistance, $R_{meas}$ show good agreement at the natural frequency of the resonator, as can be seen in Fig. (3.5). Throughout the entire work this equation is therefore used to estimate viscous losses in the neck.

3.3. Resonator Impedance

Validating the analytical expression of the resonator impedance is of interest in this section. Its value, depending on the used description for mass, spring stiffness and resistance, will be examined by comparing the measured transfer function to the calculated transfer function. The measured transfer function is defined as the ratio between the pressure inside the resonator, CH3, to the pressure in front of the opening. The pressure
in front of the opening is again indirectly obtained by normalizing CH1 to the pressure at the resonator opening. The geometry under consideration can be visualized as shown in Fig. (3.6).

![Figure 3.6: Cross-section of Helmholtz resonator, with \( p_3 \) = microphone channel 3 and \( p_{1,\text{norm}} \) = normalized input pressure from microphone channel 1](image)

A useful relation between the volume velocity of the neck and body is given by,

\[
S_n v_n = S_b v_b. \tag{3.5}
\]

It reveals that the velocity inside the body is smaller than the neck velocity, obviously only if the body diameter is larger than the neck diameter. If pressure and velocity at a common point are known, an impedance description can be used. Because a homogeneous pressure and velocity distribution is assumed, the acoustical impedance \( Z_A \) inside the resonator can be written as [24],

\[
Z_A = \frac{p_3}{v_b S_b}. \tag{3.6}
\]

It should be reminded that there is a difference between acoustical impedance and mechanical impedance in terms of units. The unit of acoustical impedance is the acoustical ohm (i.e. \( Ns/m^5 \)), whereas the unit of mechanical impedance is the mechanical ohm (i.e. \( Ns/m \)) [24]. The difference actually represents a factor \( S_n^2 \) which can be applied in both ways. The acoustical impedance used here focuses on the body part of the resonator which can be seen as a resonant tube. If the tube length and diameter are small compared to the wavelength of interest, \( Z_A \) may be written approximately as [24],
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\[ Z_A(\omega) \approx \frac{\rho_0 c^2}{j \omega V}. \] (3.7)

In subsection 2.3.3, but more extensive in Appendix A, the impedance of a single resonator is derived. Here it relates the pressure above the opening with the neck velocity. Since the pressure in the centre of the resonator is hard to measure one can write,

\[ Z_{hr} = \frac{p_{1,norm}}{v_n}. \] (3.8)

where, \( p_{1,norm} \) is the normalized input pressure at the centre of the resonator, obtained by applying Eq. (3.4) and Eq. (2.12). In order to obtain the analytical transfer function \( (TF_{calc}) \) Eq. (3.6) and Eq. (3.8) are then combined,

\[ TF_{calc} = \frac{p_3}{p_{1,norm}} = \frac{Z_A S_n v_n}{Z_{hr} v_n} = \frac{Z_A S_n}{Z_{hr}}. \] (3.9)

The analytical expression of \( Z_{hr} \) can now be evaluated by comparing \( TF_{meas} \) with \( TF_{calc} \), see Fig. (3.6). Even though there is a small pitch shift of the resonance frequency and a small difference in peak amplitudes, both transfer functions show a good agreement as can be concluded from Tab. (3.2). The table shows relative discrepancies between measured and predicted values.

<table>
<thead>
<tr>
<th>Difference in:</th>
<th>Measured</th>
<th>Calculated</th>
<th>Discrepancy</th>
</tr>
</thead>
<tbody>
<tr>
<td>Q-value [-]</td>
<td>22.70</td>
<td>22.94</td>
<td>-1.05 %</td>
</tr>
<tr>
<td>( f_0 [Hz] )</td>
<td>1425</td>
<td>1451</td>
<td>-1.82 %</td>
</tr>
<tr>
<td>( A [dB] )</td>
<td>25.75</td>
<td>27.20</td>
<td>1.45 dB</td>
</tr>
</tbody>
</table>

Possible causes of deviations between the measured and predicted transfer function can be found in the use of lumped elements and approximate expressions for viscous losses, spring stiffness and mass terms. It is important to observe that a -1.82 % difference between both resonance frequencies is mainly caused by the underestimated mass term in the analytical solution. An even more exact analytical expression for the mass, which shall be found in both end correction terms, shifts the resonance frequency somewhat and will result in a slightly better fit. Focussing on the phase, see Fig. (3.7:lower), a change from \( \pi \) to zero is obtained in both transfer functions. As shown in Fig. (3.7:lower), the biggest shift is concentrated around the resonance frequency after which the phase
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![Graphs showing comparison between measured and calculated transfer functions and phase angles.](image)

Figure 3.7.: Comparison between $TF_{\text{meas}}$ and $TF_{\text{calc}}$. Upper figure: transfer function. Lower figure: unwrapped phase angles.

approaches zero. If damping in the resonator neck is eliminated completely a sudden jump from $\pi$ to zero will result. On the other hand the transfer function will increase in amplitude and narrow in frequency. As a consequence the resonator is then more effective, however, in a smaller frequency region.

3.4. Single resonator mounted in a baffle

An independent approach to validate the neck resistance and resonator impedance is presented and discussed in previous sections. Yet, a geometry where the resonator influences the sound field on the plate is still missing and will therefore be discussed here. To be able to do so, the measurement set-up described in section 3.1 has been used. Since both expressions for resistance and impedance showed a good agreement, the main interest is thus to validate the expressions affecting the transfer between resonator opening and receiver. The reader should, however, keep in mind that all deviations showed in previous sections will also affect the results presented here. Effectively, ”the transfer between resonator opening and receiver” comprises the approximations of a
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Spatially averaged velocity and the propagation from resonator centre to an arbitrary receiver in the computational half-space. It is expected that this approximation is least valid for receiver positions very close to the resonator and will approach the exact solution several diameters from the resonator opening. To be complete; the exact solution involves an integral from each point of the resonator opening to the receiver. Since multiple points can be included, the possibility of a variable velocity distribution is then introduced. The amount of points naturally depends on the level of discretization and is hence arbitrary. One of the great disadvantages of the proposed 'exact' solution is definitely the computational effort it will involve. Especially if interaction of multiple resonators is of interest the computation time will increase rapidly.

The measured transfer function, $TF_{meas}$ showed in Fig. (3.8), reveals the ratio between CH1 and CH2 at 10 mm from the resonator centre. To obtain $TF_{calc}$ the earlier discussed geometry was evaluated numerically. One can immediately see that the deepest peak, if the region beneath 100 Hz is omitted from evaluation, is at 1475 Hz and somewhat shifted from the resonance frequency of the resonator, i.e. 1425 Hz. This can be explained by means of the resonator which is shifted in phase compared with the sum of the direct and the ground reflected wave. Under controlled circumstances at a perfect spot, e.g. near to the centre of the resonator, an exact shift of $\pi$ will lead to maximum cancellation of direct and reflected sound. Besides, a level increase due to the presence of the resonator is shown in the $TF_{calc}$ around 1400 Hz. The same argumentation used to explain the shifted 'damping' peak can as well be applied to the level increase. One could validate this by introducing an 'artificial' phase shift of $\pi$ at the receiver instead of including the re-radiated pressure of the resonator. The resulting function shape will, roughly, correspond to the calculated transfer function as shown in Fig. (3.8). The only difference is that a resonator introduce a gradual phase shift, e.g. phase angles change over frequency, resulting in a much smoother function.

As can be concluded from Fig. (3.8) the dip due the to presence of the resonator is predicted correctly in frequency and amplitude. However, at a relatively poor signal to noise ratio. Overall one can conclude that the measured transfer function corresponds well with predictions. However, a distinct discrepancy between both is obtained around 1400 Hz; the predicted increase of $\Delta SPL_{re,free}$ is not followed by measurements. In order to prove that the predicted function shape is likely to be correct; but in measurements lost due to weakness of the re-radiated pressure of the resonator, the transfer function is measured even closer to the opening, see. Fig. (3.9). The geometry under investigation is identical to the previous investigated variant, except for the alteration that microphone
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Figure 3.8.: Left: comparison between $TF_{meas}$ and $TF_{calc}$. Right: illustration of the investigated geometry.

position CH2:a (4 mm from the resonator centre) has been used instead. A significant frequency shift between measured and predicted transfer function is still shown, but since it follows the earlier obtained discrepancy of the analytical impedance expression it becomes unimportant. In other words, a more detailed description of the mass term would improve the accuracy of both predicted transfer functions simultaneously, i.e. inside the resonator and on top of the plate. It is thus proven that the proposed analytical model agrees well with measurements.

Figure 3.9.: Left: comparison between $TF_{meas}$ and $TF_{calc}$. Right: illustration of the investigated geometry.

The proposed model can, if the earlier discussed restrictions are taken into account, be used to predict the effect of a single resonator of arbitrary tuning in semi-free field conditions having a perfectly reflecting ground surface. It is shown that, even in controlled laboratory conditions, the effect of one single resonator tuned to 1425 Hz is rather weak. This might be the case for high pitched devices, but resonators in a comparable setting tuned to lower frequencies will be more effective due to higher amplitudes of the re-radiated pressure. As a consequence they will also have a bigger impact on distant
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receiver positions, e.g. more pronounced destructive interference patterns. After all, applying resonators in an outdoor environment will only be worthy if the strength of combined powers is used. Due to time restrictions the multiple resonator model could not be validated against measurements. However, taking a minimum centre-centre-centre distance of three times the opening diameter into account, will, according to the published results be on the safe side. Moving the openings closer to each other conflicts with the point source assumption and should thus be avoided. It is even better to take a large distance, up to 20 times the diameter, cause every resonator may then be treated as a single operating device. An important drawback is that the power of coupling, e.g. the synergy that two coupled resonators perform better then two single devices, will then be lost.
4. Analysis & Results

In this chapter results computed with the earlier discussed numerical model for multiple resonators are presented. Having such a model allows to investigate endless many geometrical variations. The analysis is therefore restricted to some variants which are, from the viewpoint of general understanding, important to discuss. One could see the outcomes then as rough design rules which can serve as a handle for later studies and applications of large(r) resonator fields.

4.1. General understanding

In the following paragraphs most of the analysis will be done by comparing either octave- or third-octave band results. For this reason it might be useful to emphasize first on how these averaged levels are calculated, but moreover what they cover. To be able to do so, a geometry shown in Fig. (4.1) has been investigated. It includes a fixed source position, \( q_0 \) and \( n \) identical resonators tuned at circa 200 Hz. The frequency dependent loss factor is obtained with the physical properties of air shown in Tab. (3.1) and an uncorrected neck length of 40 mm as well as a neck radius of 10 mm. The number of active resonators is varied as 5x1, 5x3 and 5x5, where in each case 5 resonators are placed along a line perpendicular to the source-receiver line. Here, the left row of resonators will be called R1 whereas R5 corresponds to the right row.

To be clear; 5x3 thus means that all five resonators in the first three rows, i.e. R1, R2 and R3, are activated. Of course any resonator design could have been implemented, yet 200 Hz corresponds to third-octave band centre frequency and additionally is fairly low in terms of the frequency range of interest in this work. Since the reduction through presence of multiple resonators is very much dependent on the location, three receiver positions, varied over height, are analysed, see Fig. 4.1. In a later stage more receiver positions will be added so that the reduction over space can be visualized. For now this is however not of great importance. The insertion loss at defined receiver positions is showed in Fig. (4.2).
4. Analysis & Results

Figure 4.1.: Investigated geometry to understand the origin of averaged results. With, dotted-line = top view of resonator field, \( q_0 \) = source position and \( p_1 \) t/m \( p_3 \) = receiver positions.

Figure 4.2.: Upper: Insertion loss (dB) of variant 5x1, Middle: IL (dB) of variant 5x3 and Lower: IL (dB) of variant 5x5.

A beneficial recurring trend, reduction on third-octave band basis, is obtained for all studied variants. Due to a broad range of magnitudes in narrowband frequency range.
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their significance is however impossible to study and therefore an overview is given in Tab. (4.1).

<table>
<thead>
<tr>
<th>Variant</th>
<th>Position</th>
<th>p1</th>
<th>p2</th>
<th>p3</th>
</tr>
</thead>
<tbody>
<tr>
<td>5x1</td>
<td></td>
<td>0.08</td>
<td>0.09</td>
<td>0.11</td>
</tr>
<tr>
<td>5x3</td>
<td></td>
<td>0.14</td>
<td>0.18</td>
<td>0.24</td>
</tr>
<tr>
<td>5x5</td>
<td></td>
<td>0.05</td>
<td>0.15</td>
<td>0.27</td>
</tr>
</tbody>
</table>

Table 4.1.: Insertion loss (dB) in 200 Hz 1/3 Octave band.

As can be seen overall the averaged reduction is weak, but more interesting; a larger field did not automatically result in a higher reduction at all investigated receiver positions. Without detailed looking into the presented graphs one would expect a higher reduction on third-octave band basis. However, the actual dip, which mostly appeared at and around the resonance frequency, is narrow and smeared out over frequencies for which the insertion loss approaches zero decibels.

Before sticking to the insertion loss exclusively one exemplifying intermediate step will be discussed first. As presented in Eq. (2.27) the insertion loss is here defined as the difference in SPL relative to free field conditions, between a geometry with and without resonators. Figure 4.3 shows these intermediate results of variant 5x5 at receiver position p3.

Figure 4.3.: Intermediate results towards IL (dB) of variant 5x5 at receiver position p3.
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4.2. Array configuration

At first glance there is no obvious relation between ultrasonic phased transducer arrays and a field of Helmholtz resonators applied in an outdoor environment. To give some further information, phased transducer arrays are for instance known from the antenna industry, but also from medicinal applications which couple many single transducer to focus and/or steer waves by controlling phase and amplitude information of each individual element. Considering a field of resonators and treating each opening in the field as a transducer element, some parallels can however be drawn. Each resonator can, for example, be moved apart from each other or organized more close together in order to modify the phase difference. Obviously, organizing resonators, by means of varying the distance between openings, can hardly be as powerful as arrays controlled by an electric circuit whereby time delay controlling is possible. Still, it might be interesting to study different resonator configurations and their reduction, because some specific settings possibly gives a better performance than others, e.g. through stronger coupling effects.

In order to examine different resonator configurations the geometry shown in Fig. (4.4) was used. As can be seen, the (monopole) source is slightly elevated to 0.01 m and is, horizontally measured, located 2 meters from the marked (crossed in Fig. 4.4) area of the base. Inside the boundaries of this area the resonators will be installed so that the prior conditions, e.g. source position, are equal for the investigated configurations.

![Figure 4.4. Investigated geometry to compare different resonator configurations. With, dashed boxes = horizontal and vertical calculation planes and crossed box = location of the resonators.](image)

The whole base is modelled as perfectly reflecting, in other words the velocity in normal direction is equal to zero, except for the location of each single resonator. Observe that
each resonator is represented by an equivalent source and actually corresponds to a single point. To give insight in the three-dimensional sound field caused by a field of resonators the Insertion Loss has been calculated for a horizontal and vertical plane; both are, in Fig. (4.4), indicated with dashed lines. The horizontal plane is elevated to 1.6 m and measures 10.0 x 3.25 m whereas the vertical plane is 10.0 x 2.0 m and intersects the field of resonators through its centre. Here the evaluation height, 1.6 m, is chosen because it corresponds to an average height of the aural passage for human beings. Naturally, there is a large variation of ear heights among people, but, as will be showed later on, it only affects the results to a very small extent and is therefore of minor importance.

Resonators can be organized in different patterns, the most practical however are founded on two basic configurations shown in Fig. (4.5). The left hand configuration can be seen as a "square pattern" whereas the right one calls up associations with number five on a dice and is therefore called "dice pattern". The mutual distance between neighbouring resonators is, in case of the square pattern resonators, set to 0.5 m, and corresponds roughly to a quarter wavelength of 200 Hz. The right hand configuration is organized in such a way that the outer circumference of both fields is equal. Since both configurations then cover exactly the same amount of ground surface which fits the marked area of the base, and include 25 identical resonators tuned at 200 Hz, the obtained reduction due to the presence of these fields can be compared. To be able, an energetic average over 30 logarithmically spaced frequency samples in the 200 Hz third-octave band is computed so that possible peaks and dips on single frequency basis are averaged out.

![Figure 4.5.](image)

Figure 4.5.: Two resonator configurations; Left: resonators organized in a "square pattern" and Right: resonators organized in a "dice pattern".

The reduction of 25 identical resonators organized in a dice pattern is plotted in Fig. (4.6). It is shown that some areas, both in the horizontal and vertical plane, are affected
by constructive interference, i.e. increased sound level compared to a situation without resonators. However, for most receiver positions a level decrease due to destructive interference has been calculated. It seems interesting that regions of higher amplitudes arise in particular on the source side and above the resonator array while reduction mainly focusses on the right side of the resonator field. One can interpret this as a time shift of re-radiated pressure between the first row of resonators and the last row of resonators which steer the main lobe (highest pressures) away from the receiver positions on the right side. Obviously spacing between resonators is herein important. Furthermore, the source location plays a part since all resonators are, in this model, excited from a source located on the left side of the resonator array. Considering the source height to be 0.01 m, for which the angle of incidence quickly approaches 90°, an repetitive phase differential between neighbouring orifices will be obtained.

Figure 4.6.: Insertion loss due to 25 identical resonators organized in dice pattern. Calculated as 200 Hz 1/3-octave band with 30 energetically averaged logarithmic spaced frequencies. Upper: vertical calculation plane, 10.0 x 2.0 m. Lower: horizontal calculation plane 10.0 x 3.25 m, height 1.6 m.

An identical geometry, except for the fact that a dice pattern organization of resonators has been replaced by a square arrangement, is investigated and shown in Fig. (4.7).
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Figure 4.7: Insertion loss due to 25 identical resonators organized in square pattern. Calculated as 200 Hz 1/3-octave band with 30 energetically averaged logarithmic spaced frequencies. Upper: vertical calculation plane, 10.0 x 2.0 m. Lower: horizontal calculation plane 10.0 x 3.25 m, height 1.6 m.

On basis of these plots the difference seems marginal and can hardly be judged on its significance. In order to visualize the difference between both arrangement better, the insertion loss at 1.6 m height is taken from both vertical calculation planes and is shown in Fig. (4.8). It follows that there is only a small difference between both arrangements at the plotted height. Though minor, a slightly better reduction in benefit of the resonators organized in square pattern was obtained. One should keep in mind that the influence of resonator arrangement on the insertion loss might increase for extended geometries. Since computation and evaluation of both arrangement would be time-consuming a square pattern resonator organization is used throughout the remaining studies.
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Figure 4.8.: Comparison of 25 identical resonators organized in square and dice pattern. IL is 200 Hz 1/3-octave band with 30 energetically averaged logarithmically spaced frequencies. Evaluated points = junction between horizontal and vertical plane shown in Fig. (4.4).

Up till now the evaluation has only focussed on 200 Hz third-octave band results because the resonance peak of the applied resonators is covered herein. However, by recalling the results presented in the validation measurements section, one could see that the frequency spectrum over which a resonator is active was fairly broad. Consequently it is not unlikely that parts of the resonator's frequency response are not covered inside the 200 Hz third-octave band. An evaluation using the wider full octave bands (1/1 OB) is carried out to determine in which regions of the frequency spectrum, and to what extent, a geometry of 25 identical resonators organized in a square pattern affects the sound field in the external space, see Fig. (4.9). Again the junction between the horizontal and vertical evaluation planes, shown in Fig. (4.4), was here of interest. Each single 1/1 OB is computed with 90 logarithmically spaced frequencies.

As can be concluded from Fig. (4.9) the sound field is mainly affected through applied resonators within the 125 and 250 Hz 1/1 OB boundaries. Besides, small ripples are calculated inside the 500 and 1000 Hz 1/1 OB, but these can be seen as tail parts for which $IL \to 0\text{dB}$ and hence of minor importance. The shape of the 250 Hz 1/1 OB is, compared to the 200 Hz 1/3 OB, somewhat depressed in amplitude. In practice it means certainly that the resonance peak is smeared out over a larger frequency spectrum which basically depress the total obtained averaged values. For further studies a choice between continuing the analysis on 1/3 or 1/1 OB had to be made. Both have their pros and cons, but since the applied resonators will either be identical or just slightly shifted in pitch analysis on third-octave band basis seems a natural choice.
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![Graph showing IL vs horizontal distance](image)

Figure 4.9.: Identical resonators, 25, organized in square pattern. IL is 1/1-octave band results with 90 energetically averaged logarithmically spaced frequency components. Evaluated points are given by the junction between the horizontal and vertical plane shown in Fig. (4.1).

4.3. Expanding the geometry

A general sense which remained from previous sections is that an array of 25 coupled identical resonators achieves a rather weak reduction. A way to improve the reduction in a certain third-octave band might be through extending the amount of resonators in the geometry. There are then two possible tracks to follow, either condensing or expanding the treated area with additional resonators. From a practical point of view the latter is definitely most interesting and is therefore discussed in this section.

Considering a situation with one row of infinite many resonators parallel to an infinitely straight road where one car is driving, the question may arise at which point the influence of distant resonators seen from the car can be treated as negligible. In order to study this, the geometry shown in Fig. (4.10:right) has been used. It consists at maximum of $N \times M = 1 \times 17$ identical (pitch 200 Hz) resonators with a fixed mutual spacing of 0.5 m. They are, horizontally measured, located 2 meters away from the source and the number of active resonators is varied with values: 3, 5, 9 and 17. Observe that the line of receivers, at 1.6 m height, will always cross the centre of the field.

It is shown that on the source side and directly above the resonators, i.e. between 0 and 4 meters, an increased number of resonators has no effect on the insertion loss. Here resonators nearest to the receiver mainly contribute to the presented reduction. Taking further receiver positions into consideration, more resonators are contributing, but still
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the difference among variants N5 x M1 - N17 x M1 is little. Obviously, these findings may change for different source positions and hence cannot be applied on different geometries.

![Graph showing IL vs horizontal distance](image)

Figure 4.10.: Left: $IL = 200 \text{ Hz } 1/3$-octave band results with 30 energetically averaged logarithmically spaced frequency components. Right: illustration of the geometry under investigation. With, dashed-dotted-line = evaluation points and dotted-line = break line.

Initially the expectation was that expanding the geometry with additional resonators automatically improves the insertion loss. However, recalling the results from the first paragraph of this chapter it seems to be less straightforward. Since paragraph 4.1 only showed the insertion loss at specific points it might be worthwhile to study expanded geometries over a series of receiver positions. As starting point variant N9 x M1 is taken. It means that all variables as; resonance frequency, loss factor, source position, mutual spacing and receiver positions are identical to the previous study of this section. What however will be varied is the number of rows included in the geometry: 1, 3, 5 and 9. The results shown in Fig. (4.11: left) do not bring up completely new facts. Nevertheless, some interesting details might be worth to discuss here. A better reduction, compared to N9 x M1, is for instance obtained for variants N9 x M3 and N9 x M5 whereas N9 x M9 resulted in an additional region of decreased insertion losses. Because it goes against common sense the latter variant is by far the most interesting one to discuss. However, its complexity makes it difficult to explain. Roughly it is tried by means of the source location and resonance frequency of the resonators in combination with the adopted mutual distance. Here two resonators in line with the source are taken to exemplify this phenomenon. As discussed earlier an initial phase differential of $\delta \approx \lambda/4$ between neighbouring orifices is obtained through the chosen source location. Consequently the
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Figure 4.11: Left: $IL = 200$ Hz 1/3-octave band results with 30 energetically averaged logarithmically spaced frequencies. Right: Illustration of the geometry under investigation. With, dashed-dotted-line = evaluation points and dotted-line = break line.

2nd resonator, observed from the source side, is then excited approximately a of the oscillation period later later than the first resonator. In order to obtain the pressure above orifice two, the initial pulse is added by the re-radiated coupling pressure from orifice one. Since the resonator introduces a $180^\circ$ phase shift (half a wavelength) and additionally needs to travel to the second opening (quarter-wavelength), the net phase difference is then half a wavelength. From this point the oscillation amplitude of both resonators decrease. However, every oscillation each resonator re-radiate pressure back in space and thereby excite the other resonator again. In case of variant N9 x M9 the complex interactive system of all resonators introduced a new lobe around 9 meters from the source, at height 1.6 m.

4.4. Broadband reduction using variable resonator volumes

A drawback of the proposed method thus far might be the narrow frequency region in which a field of identical resonators is active. Moreover, expanding the geometry with additional rows of resonators did not automatically improve the reduction to a large extent. In order to gain a higher reduction on 1/3 octave band basis a set of different natural frequencies has been considered, using the fact that a decreased resonator volume results in a higher natural frequency. Again considering a field of resonators, however this time gradually varying the resonator's volume, covers then a broader frequency spectrum. A possible disadvantage might be that fields of increased SPLs due to constructive
interference are pushed outside the 1/3 octave band of interest and thereby outside the scope of analysis. To avoid this, results of both neighbouring 1/3 octave bands will therefore be given in addition. Another possible threat can be found in the loss of coupling between resonators if their individual frequency responses are too different. Of course this is purely a design aspect and hence basically dependent on the choices made by aims and objectives in view.

Considering broadband reduction using volume variation the mayor 'design' task to face, should be, whether there exists an optimum balance between frequency spreading of resonators and reduction on 1/3 octave band basis. Due to time restrictions the optimum setting could not be studied. However, another closely related variable; positioning of different types of resonators relative to the source, could be studied. To be able to do so, five types of resonators; 199.2 Hz, 194.5 Hz, 190.2 Hz, 186.2 Hz and 182.3 Hz are implemented in the geometry which is shown in Fig. (4.12). Four variants illustrated in Tab. (4.2) are then selected for evaluation. The variation in resonance frequency is obtained by increasing the body length of the highest pitch, 199.2 Hz, resonator. It should be noted that the choice of varying the resonance frequency is rather uncontrolled and could be improved, but here serves as a starting point. On the other hand the neck geometries remain identical which means there is no extra damping is introduced, and additionally it allows to exemplify the strength of volume variation.

![Figure 4.12.: Cross-section of investigated geometry linked to Tab. (4.2). With: $q_0 =$ source position, dashed-line = brake line and dashed-dotted-line = evaluation line.](image)

Briefly the four variants imply; A: all resonators have the same resonance frequency, B: resonators decrease in volume, seen from the source side, C: resonators increase in
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Table 4.2: A: 45 identical resonators, B: decreasing resonance frequency, C: increasing resonance frequency and D: random resonance frequency. With: dashed-line = brake line and dashed-dotted-line = evaluation line.

![Diagram of resonators]

Concerning B, C and D it is important to see that the same collection of resonators is used for all variants and only their formations differ. From a viewpoint of reduction variant D shall, most likely, perform weakest since the strength of correlation between resonators is expected to be lower. On the other hand it is interesting to prove that the reduction is, to a certain extent, manipulatable taking certain design rules into account.

In Fig. (4.13) the insertion loss of variants A, B, C and D is compared on basis of 160, 200 and 250 Hz 1/3 octave bands. Each 1/3 octave band is, again, computed with 30 energetically averaged, logarithmically spaced, frequencies. These plots demonstrate that the discrepancy between investigated variants is mostly concentrated inside the third-octave band wherein the resonator's natural frequencies occur. Still, a minor influence, i.e. +/- 0.1 dB, due to the presence of the resonators on the averaged relative sound pressure levels is calculated for both other third-octave bands. Inside the 160 Hz third-octave band sound is mostly enhanced through constructive interference whereas it is affected by destructive interference within the 250 Hz third-octave band. Yet, it can be
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Figure 4.13.: Comparison of variant A, B, C and D on the insertion loss (dB) in 1/3-octave bands using 30 energetically averaged logarithmic spaced frequencies. Upper: 160 Hz 1/3 octave band, middle: 200 Hz 1/3 octave band and lower: 250 Hz 1/3 octave band.

Concluded that both neighbouring third-octave bands are to some extent affected by the resonators though not so much by the differences in configuration. One exception is the increased insertion loss for randomly distributed resonators inside the 250 Hz 1/3-OB, but this will be omitted from further analysis. Consequently, the evaluation becomes less complicated because the focus may go the 200 Hz third-octave band, exclusively. Herein it is clear that variant D, i.e. randomly distributed resonators, has a significantly lower reduction compared to variants B and C at distant receiver positions, i.e. plus 7 meters. The explanation was already given earlier; resonators which are randomly distributed are expected to have weaker mutual coupling and consequently show a lower reduction overall. Finally, it is shown by means of variant B and C that broadband reduction can be improved by changing resonator properties and their positioning. The improved reduction may, partly, be addressed to the presence of identical resonators in the same row.
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so that they are enhanced by mutual coupling. Besides, a broader frequency spectrum is covered through differently tuned resonators, see Fig. (4.14). The insertion loss over frequency is here presented between the boundaries of the 200 Hz 1/3 octave band at receiver location \((x,z) = (10,1.6)\) of the geometry shown in Fig. (4.12).

![Graphs showing frequency response of different variants](image)

Figure 4.14.: Frequency response of the four studied variants. Up-left: variant A, Up-right: variant B, Down-left: variant C and Down-right: variant D.

4.5. Source position

Up till now a fixed source position has been used throughout the studied variants. However, intuitively the reduction may be improved either through moving the source closer to the resonators or by moving our field of resonators towards the source, because the point of specular reflection moves then closer to the field of resonators. Theoretically, both describe the same measure if a rigid ground surface is assumed, but practically
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the latter makes more sense. Here the influence of the horizontal source-to-resonator distance is examined using a fixed source height of 0.01 m and resonators which increase in pitch further away from the source. To make life easier variant C known from the last section is reused. Briefly it implies a geometry shown in Fig. (4.12) which contains 45 equally resonators with a spacing of 0.50 m. The horizontal source-to-resonator distance is varied with values; 2.0, 1.0, 0.5 and 0.25 m, and its response over distance at 1.6 m height is shown in Fig. (4.15). Observe that the first resonator in the field is located at 3 meters and hence the source coordinates (x,z) are; (1.0, 0.01), (2.0, 0.01), (2.5, 0.01) and (2.75, 0.01) respectively. As expected the gained reduction increased when the distance with respect to the resonators decreased. More interesting however is the trend among the four investigated variants. It seems that a distance doubling each time resulted in an additional reduction of about 0.1 dB. Though promising one should keep in mind that a specific geometry is studied and outcomes will differ if any transformation concerning the geometry is carried out.

Figure 4.15.: Study on variable source locations using a fixed geometry shown in Fig. (4.12). The implemented resonator configuration is sketched in Tab. (4.2:C). Insertion loss (dB) in 200 Hz 1/3 octave band (30 energetically averaged logarithmic spaced frequencies).

So, in a way the reduction can, theoretically, be modified simply by reducing the horizontal distance between source and resonators. However, taking our departure from the idea that these resonators will be applied in fields and parks adjoining a road surface, there do exist physical limitations. Resonators being installed directly along the road side are from a viewpoint of reduction the optimum scenario physically possible. Taking this into account a source-to-resonator distance of 0.25 m seems then to be unrealistic.
whereas 0.5 m is on the edge, but an acceptable lower limit.

Moving the source closer to the field of resonators led to a higher reduction and is in that sense beneficial. On the other hand elevating the source, e.g. when the engine is the main noise source, may as well affect the reduction. Identifying the main noise sources of interest in traffic noise problems, i.e. person cars and lorries, source heights up to 0.75 m are common and for some exceptions, e.g. heavy vehicles with high exhaust, a source at approximately 3.5 m is even imaginable. In Nord2000 Road, the Nordic method for predicting road traffic noise, source heights are generally characterized at 0.01, 0.30 and 0.75 m [25]. This can be seen by a brief source identification of noise coming from tyres and noise produced by the engine. Noise coming from tyres can hardly be seen separately from the road surface and so one can speak of tyre/road interaction. Hereby, several phenomena contribute to the total source description and not all are as important in the 200-1000 Hz region which is of interest in this work. In general however most focus at and around the contact area between tire and road surface and hence 0.01 m is often taken as source height for car tyres. Considering radiation from the engine most energy will escape along the bottom part and only a minor part is radiated from the steel body. According to the developers of Nord2000 prediction method a proper generalized source height for cars is therefore 0.30 m whereas 0.75 m can be taken into account for trucks.

Simulations for which the source height has been varied at three horizontally measured source-to-resonator distances are performed according to the geometry illustrated in Fig. (4.16). At each distance three source heights, adopted from Nord2000 road, are examined and thereby nine individual source positions were defined. The implemented resonator formation, variant C, is again borrowed from the previous section.
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Figure 4.16.: Cross-section of investigated geometry for variable source locations. With: $q_1$ to $q_9$ inclusive = source positions, dashed-line = brake line and dashed-dotted-line = evaluation height.

In Fig. (4.17) the insertion loss for different source locations is shown on 200 Hz third-octave band basis. Since all plots basically present the same trend a detailed discussion of how each variant differs among each other is omitted here. One can generally state that elevating the source to 0.3 m did not affect the existing results of previous sections to a very large extent. This however in contrary to the studied source height equal to 0.75 m which introduced a not earlier appeared local dip.

Figure 4.17.: Insertion loss (dB) of source locations $q_1$ to $q_9$ inclusive. Geometry of interest is showed in Fig. (4.16).

Since a picture often says more than a thousand words the appearance of the dip is explained by showing both graphs which constitute the insertion loss, i.e. $SPL_{re, free, active}$ and $SPL_{re, free, inactive}$. Due to similarities there is no need of showing all source positions
and hence only $q_3$, $q_6$ and $q_9$ are selected to plot in Fig. (4.18). Now it seems clear that the ground interference dip is somewhat depressed by the resonators and consequently it has a negative side effect on the insertion loss.

![Graphs](image)

**Figure 4.18:** Constitution of insertion loss of source locations $q_3$, $q_6$ and $q_9$. Geometry of interest is showed in Fig. (4.16).

Finally, to complete the picture the insertion loss for three source locations, i.e. $q_3$, $q_6$ and $q_9$, is showed in a two-dimensional vertical plane, see Fig. (4.19). This plane measures 10.0 x 2.0 m and intersect the field of resonators through its centre.
Figure 4.19: Insertion loss (dB) of source locations $q_0$, $q_5$ and $q_9$. e.g. source locations are marked with a black-cross.
5. Conclusions & Future work

The aim of this work was to investigate the interaction between multiple Helmholtz resonators and model their noise reduction for sound propagating outdoors from traffic noise sources. Conclusions regarding the complete work; but mainly ongoing the validation measurements and variant studies using the developed numerical prediction tool for multiple resonators, are briefly summarized in the first paragraph. The 2nd paragraph covers research proposals which might be interesting to study in the future.

5.1. Conclusions

An important, however far from new, conclusion which can be drawn from this work, but mostly measurements, is that the phase inverts at and around the resonance frequency of the resonator. Nevertheless, it might still be interesting to discuss, because the way resonators 'work' in an outdoor environment is often misunderstood. The reason is probably that Helmholtz resonators are, in general, quickly linked to the use in room acoustics. One of the differences between Helmholtz resonators applied in room acoustics and outdoor acoustics can be found in the absence of room boundaries in the latter case. It means outdoors, oblique sound incidence is mostly of interest whereas normal sound incidence, e.g. due to room specific standing waves, is of importance indoors. The presence of standing waves in rooms can, for instance, be treated by a panel of Helmholtz resonators having an acoustic absorptive material inside the cavity or the neck. From a physical point of view the neck would be the best choice since the velocity is there highest. However, considering practical limitations, material inside the cavity is easier to realize and therefore generally applied. It is often, correctly, quoted that absorption material inside resonator's neck or cavity broadens the 'damping peak'; but it also introduces real absorption in addition to viscous losses in the neck. Consequently, during every oscillation, e.g. spring is compressed and elongated, more energy is dissipated into heat than compared to a case where no additional absorption material is included. In outdoor applications adding ordinary damping material will be problematic due to weather influences and hence a more robust design is probably necessary. If a
5. Conclusions & Future work

practical design without additional damping is desired, only a small part of the total reduction will be gained by absorption due to viscous losses in the resonator neck and the rest is obtained by destructive interference due to the inverted phase of re-radiated energy.

**Validation measurements: resonator impedance**

The analytical expression of the resonator impedance plus introduced correction terms is validated against measurements by using a transfer function method, i.e. one microphone inside the resonator's volume and one outside the resonator. The relative discrepancy between measured and calculated resonance frequency and Q-value, for the investigated resonator tuned at 1425 Hz, was 1.82% and 1.05%, respectively. The relative difference in peak amplitude was found to be 1.45 dB, taking the measurements as reference.

**Validation measurements: single resonator mounted in a baffle**

The predicted insertion loss due to presence of a single resonator mounted in an infinite rigid baffle has been validated against measurements in the anechoic chamber of Applied Acoustics, Göteborg, Sweden. These experiments has proven that both shape and amplitude can be predicted fairly exact by using the proposed model. Assumptions like an equal velocity distribution in the resonator neck and representation of a resonator by an equivalent source seems thus to hold quite well. A drawback during the measurements however was the short distance between resonator opening and microphone necessary to measure the re-radiated sound from the resonator. In other words more distant microphone positions resulted in a poorer signal-to-noise ratio (SNR). Note that the signal here is not defined as the initial excited pulse, but the strength of the new source (resonator opening), whereas the noise shall be a combination of 'regular' background noise and late room reflections excited by the initial pulse. After all it is shown that, even in well controlled laboratory conditions, the effect of one single resonator is rather weak and applying resonators in an outdoor environment will only be worthy if the strength of combined powers is used. Due to time restrictions the multiple resonator model could not be validated against measurements. However, taking a minimum centre-centre-centre distance of three times the opening diameter into account, will, according to the published results be on the safe side. Moving the openings closer to each other conflicts with the assumption of an equivalent source and should thus be avoided. It is even better to take a large distance, up to 20 times the diameter, because every resonator may then be treated as a single operating device. An important drawback is that the
5. Conclusions & Future work

power of coupling, i.e. the synergy that two coupled resonators perform better than twice that of a single device, will then be lost.

Multiple resonator model: array configuration

The insertion loss of 25 identical resonators organized in square and dice pattern were evaluated and compared. It was shown that the reduction of both arrangements on third-octave band basis was nearly identical for distant receiver positions. Nearer to the resonator array the differences are more distinct, but still marginal. As expected some receiver positions showed increased sound pressure levels compared to a situation without resonators. However, for most receiver positions a level decrease between 0.1-0.5 dB due to destructive interference has been calculated. Interesting is that regions of higher amplitudes arise in particular on the source side (left from resonators) and above the resonator array while reduction mainly focuses on the right side of the resonator field. One can interpret this as a time shift of re-radiated pressure between the first row of resonators and the last row of resonators which steer the main lobe (highest pressures) away from the receiver positions on the right side. Source location and mutual spacing between resonators are herein important. Since the angle of incidence quickly approaches 90° for low height sources, an repetitive phase differential between neighbouring orifices is then obtained. This may lead to strong coupling effects if the mutual distance between orifices matches a quarter-wavelength or multiples of the re-radiated pulse from the resonator.

Multiple resonator model: expanding the geometry

Improving the reduction in a certain third-octave band by means of expanding the amount of applied resonators has been studied for identical resonators. Intuitively one would expect the reduction to increase for larger resonator arrays. It is however shown that in case identical resonators are applied and their mutual distance approaches a quarter wavelength the complex interactive coupling between resonators may introduce unwanted level increases.

Multiple resonator model: broadband reduction

A drawback of applying identical resonators is the narrow frequency region wherein they are active. Furthermore, simply expanding the geometry did not necessarily improve the reduction. It is therefore tried to improve the insertion loss on third-octave band basis through applying gradual varying resonator volumes, and thereby the natural frequencies.
5. Conclusions & Future work

Three new variants were defined and compared against a geometry including identical resonators only. Briefly the four variants are; A: all resonators have the same resonance frequency, B: resonators decrease in volume, seen from the source side, C: resonators increase in volume, seen from the source side and D: resonator's natural frequencies. It is shown that variant D has a significantly lower reduction compared to variants A, B and C at distant receiver positions inside the third-octave band wherein the resonators are applied. This can be explained by means of weaker mutual coupling between resonators and consequently overall a lower obtained reduction. On the other hand variants B and C showed an improved reduction compared to variant A even though the insertion loss of both neighbouring third-octave bands remained the same. It is thus possible to manipulate the insertion loss to some extent by means of a gradual change of resonator volumes. Still, one should group them so that identical resonators are kept together in order to take advantage of coupling.

5.2. Multiple resonator model: source position

All presented conclusions thus far are based on simulations using a fixed source-to-resonator orientation. However, identifying the main noise sources of interest in traffic noise problems, i.e. person cars and lorries, variable source locations are more natural. Nine source positions varied in height and distance toward the resonators were therefore defined and tested with an invariable resonator geometry. It is shown that moving the source closer to the field of resonators generally led to a higher reduction and is in that sense beneficial. On the other hand elevating the source could introduce special conditions if the ground interference dip matches the resonators profile. In other words, the ground interference dip might then be weakened through presence of resonators and results in a negative insertion loss (increased sound pressure levels).

5.3. Future work

Once finished this thesis, it seemed several questions could not be answered, mainly due to time restrictions of the project. The following aspects are, according to the author, most relevant to discuss:

Validation measurements of multiple resonators

The main drawback of this work might be the lack of experimental data with regard to the proposed model for multiple Helmholtz resonators and their interaction. However,
it is already shown, theoretically, that resonators in close proximity to other resonators can not be treated as uncorrelated devices, on the contrary, they are affected by cross-travelling energy between orifices. An effect here called; coupling pressure. In spite of a large number of theoretical extensions to the coupling theory could be bring up, the amount of experimental data is limited and is important to test the legitimacy of the coupling theory for resonators. Creating a geometry whereby two resonators are mounted in a baffle and the distance between the orifices can be varied from far-to-close, could already validate the coupling theory to a very large extent. It is advisable to investigate identical resonators through the fact that more prominent coupling is expected than for resonators having slightly different resonance frequencies. Extending the geometry from two to an indefinite number of resonators may give possibilities to examine the validity of the theory for a resonator field in terms of the total gained reduction.

**Optimum Helmholtz resonator design for an outdoor environment**

As discussed previously it might be troublesome to add ordinary absorption material in resonator cavities which will be applied in an outdoor environment. On the other hand a completely closed, hard surfaced, volume where water has no possibility to leak out might as well not be desirable. An optimum case would be to combine these shortcomings into a new resonator design - one where water is drained into the ground through an acoustically absorptive material. Despite that this is a rather vague description which can be modified in many ways it seems clear that such a material should have special characteristics, namely absorbent for acoustic purposes and ricocheting for water particles. Of course it might be challenging, and may be utopian, to find a material fitting this profile; but if, resonators in an outdoor environment can be optimized to have better absorbing properties.

**Power of averaging**

The published results were merely shown in 1/3 and 1/1-octave bands to average peaks and dips on single frequency basis out. A valuable supplement to frequency averaging would however be spatial averaging of receiver positions and also multiple source locations might be worthy to study. In case of spatially averaged receiver positions one could think about defining an arbitrarily sized volume somewhere in space and average the obtained insertion losses covered, so that height, depth and width variations are equalized. The geometry of interest can, for instance, be split up in three volumes, e.g. source-region, resonator-region and distant receiver region. Each volume then represents a
single, geometry dependent, reduction value. These values can be collected, for example table-wise, to make results more accessible and applicable for engineering toolboxes and so forth. Besides the reduction through multiple source positions, e.g. an uncorrelated line source, is wished. Using the proposed model and Matlab script allows to implement one source position during every calculation cycle. Yet, varying the source location, e.g. by means of a for-loop, and average the insertion loss at a specific location, is equivalent to modelling a uncorrelated line source straight ahead. A possible drawback is then definitely the time required to average over frequency, space and source positions. It is however not clear if implementing a line source directly will improve the execution time to a large extent because the sizes of the matrices will approach the same dimensions.

**Optimum configuration**

A rather limited parameter study concerning broadband reduction using variable resonator volumes (and hence natural frequencies) looked promising, but needs to be developed further. One remaining question is still whether there exists an optimum balance between frequency spreading of resonators, loss factors, mutual distance between openings and the obtained reduction on 1/3 octave band basis. It is most likely that there exists an optimal solution and a possible way to track it might be grid-like scanning of a simple configuration. One can think about a 3x3 sized matrix of resonators for which the resonators are, per row, step-wise changed in resonance frequency. Repeating this process, parallel to step-wise changing of mutual distances, would then, hopefully, result in an optimum configuration to obtain the highest possible reduction.

**Finite impedance**

The proposed model is somewhat idealized since a perfectly reflecting ground surface was assumed. For some surfaces like asphalt or stony materials this might be a good approximation if lower frequency regions are of interest. However, grassy or in some way comparable surfaces typically introduce a phase change to the reflected sound wave and may affect the presented results. An improved model should thus include the possibility to define finite impedance ground types.
Bibliography


Bibliography


A. Derivation of a Helmholtz resonator impedance

In the following an expression for the resonator's impedance, $Z_{hr}$ is derived. The impedance of the resonating absorber is constituted of its mass, spring-stiffness and the resistance in the neck. Using a Lumped element model makes it possible to visualize the working system by a few simple elements, Fig. (A.1).

Figure A.1.: Left: Lumped element model of a Helmholtz resonator excited by a force, $F$. Furthermore, $M$ represent the total mass $(M_n + M_b)$, $s$ the spring stiffness of the air enclosed in the resonator’s volume and $R_n$ the resistance in the neck. Right: free body diagram of a single Helmholtz resonator.

In Fig.(A.1) the displacement in $z$-direction is positive and the force, $F$, is chosen to be in the opposite direction, whereby it has negative sign. In order to derive the impedance, $Z_{hr}$, more easily a free body diagram is drawn. Such a diagram is set up by removing all elements acting on the mass and interchange them with a force. A force balance results in;

\[-F = Ma + R_n v_n + sz. \tag{A.1}\]

With;
A. Derivation of a Helmholtz resonator impedance

$-F=$ the external force (incoming pressure) which point downwards;
$Ma =$ the mass inertia, from Newton's second law of motion;
$sz =$ the spring stiffness, from Hooke's law;
$R_nv_n =$ damping, proportional to the velocity in the neck.

To keep the notation as simple as possible complex notation is used instead of real notation. The following conversions, based on $z = |z|e^{j\omega t}$ are therefore used:

$$v = \frac{dz}{dt} = j\omega z, \quad (A.2)$$
$$z = \frac{v}{j\omega}, \quad (A.3)$$
$$a = \frac{d^2 z}{dt^2} = j\omega \frac{dz}{dt} = j\omega v. \quad (A.4)$$

Using these conversions result in,

$$-F = MJ\omega v + R_n v + \frac{1}{j\omega} sv, \quad (A.5)$$
$$-F = v \left( MJ\omega + R_n + \frac{1}{j\omega} s \right), \quad (A.6)$$
$$-F = v \left( MJ\omega + R_n - \frac{j}{\omega} s \right). \quad (A.7)$$

Using the definition of the pressure, $p = \frac{F}{S}$, with $S_n$ as the surface of the opening gives the final expression for the mechanical impedance, $Z_{hr}$;

$$Z_{hr} = \frac{-F}{Sv} = \frac{j\omega M + R_n - \frac{j}{\omega} s}{S_n}. \quad (A.8)$$
B. 3D analytical model for a single-ground buried - Helmholtz resonator

This appendix is devoted to the derivation of a three-dimensional analytical model for a single-ground buried - Helmholtz resonator. Assumptions which are made will be briefly introduced at first and are followed by the derivation itself.

To start with, a Helmholtz resonator (HR) in an semi-infinite space is modelled. Using the term semi-infinite suggest the space is bounded in a certain direction, however, is unbounded in other directions. Here a ground surface of infinite size, having an perfectly reflecting surface, e.g. reflection factor (Q-factor) which equals 1, is modelled. Somewhere in this perfectly reflecting ground surface a single Helmholtz resonator, which has its circular opening at \( z=0 \), is implemented. Consequently, all other parts of the resonator (neck and volume) are buried and not 'visible' so to speak, Fig. (B.1).

![Figure B.1: Visualisation of a Helmholtz resonator in a semi-infinite space, buried in an infinite ground surface. Here, \( q_0 = \) source strength, \( p = \) received SPL, \( r_d = \) direct contribution, \( r_r = \) reflected wave, \( r_{hr} = \) contribution from the HR.](image)

The driving force of the system is here chosen as a point source. Since a traffic noise problem forms the basis of this work one can argue the choice of a single point source description. However, for setting up the actual model it fits and later on a set of uncorrelated point sources can simulate a line source.

The pressure at the receiver position can be formulated as the summation of the direct
path, the reflected path and the path from source-to-HR-to-receiver. Mathematically this can be formulated as,

\[ p(x, y, z) = q_0 G(r_d) + q_0 G(r_r)Q + qG(r_{hr}). \] 

(B.1)

Here, \( q_0 \) source strength of the driving force, \( G(r) = \text{Green's function for a point source in a three-dimensional unbounded space, } Q = \text{reflection coefficient of the ground and } q = \text{unknown source strength of the HR.} \) Where the Green's function is formulated as;

\[ G(r) = \frac{e^{-jkr}}{r}. \] 

(B.2)

Since all other variables are known the crux of finding a solution for Eq. (B.1) is to find the unknown source strength of the HR. The source strength, \( q \), is related to the velocity at the opening by,

\[ qG(r) = \frac{j\omega \rho}{2\pi} \int_S G(r) dS, \] 

as a consequence the velocity is expressed by,

\[ v = \frac{2\pi q}{j\omega \rho S}. \] 

(B.3)

With, \( \omega = 2\pi f, \rho = \text{density of air and } S = \text{surface of the opening.} \) This general solution for a small, flat source in a baffle holds, but only for source-receiver distances which are large [22]. Furthermore, the source has a constant velocity over the area of the opening - an assumption which is true for a small opening (small compared to the wavelength of the incoming wave). Where the velocity for far-field receiver positions is relatively easy to obtain, the velocity for \( r_{hr} \to 0 \) is not.

In order to find \( v \) and eventually \( q \) for \( r_{hr} \to 0 \) the receiver is placed exact above the HR opening, precisely in the centre of it, see Fig. (B.2).

As a consequence the pressure at the receiver is equal to,

\[ p(x, y, z) = q_0(2G(r_d)) + qG(r_{hr}). \] 

(B.5)

Here, the direct contribution is simply doubled, since \( r_d = r_r \). Yet the velocity is still unknown, but can be eliminated by applying the Huygens-Rayleigh integral [22],
B. 3D analytical model for a single-ground buried-Helmholtz resonator

Figure B.2.: Visualisation of a Helmholtz resonator in a semi-infinite space, buried in an infinite ground surface. Here, $q_0 =$ source strength, $p =$ received SPL and $r_d =$ direct contribution.

$$p(x, y, z) = \frac{j \omega \rho}{2 \pi} v \int_{S} G(r_0) dS. \tag{B.6}$$

The integral evaluates the pressure amplitude in point $(x,y,z)$, here chosen as $(0,0,0)$, see Fig. (B.3). In fact the pressure due to it’s own velocity is what will be calculated by placing the receiver there - an important intermediate step towards the pressure at the receiver.

Figure B.3.: Top-view of a circular shaped Helmholtz resonator opening, having a receiver position at $(0,0,0)$.

Since a circular opening has to be evaluated one can skirt the more difficult integration over $x$- and $y$-coordinates. Instead, $r_0$ is used from zero to $a$, which results in,

$$p(x, y, z) = \frac{j \omega \rho}{2 \pi} v \int_{0}^{a} \frac{e^{-jkr_0}}{r_0} 2 \pi r_0 dr_0. \tag{B.7}$$
B. 3D analytical model for a single - ground buried - Helmholtz resonator

This can be simplified into,

\[
p(x, y, z) = \frac{j\omega \rho}{2\pi} v \int_0^a e^{-jkr_0} 2\pi dr_0. \tag{B.8}
\]

Finally one can solve the integral,

\[
p(x, y, z) = j\omega \rho v \left[ \left( \frac{1}{-jk} e^{-jka} \right) - \left( \frac{1}{-jk} e^{-jko} \right) \right]. \tag{B.9}
\]

Using the substitution, \( k = \frac{\omega}{c} \) gives,

\[
p(x, y, z) = \rho c (1 - e^{-jka}) v. \tag{B.10}
\]

The pressure at the HR-opening can now be formulated as the incoming pressure from the source plus an additional pressure from the HR,

\[
p(0, 0, 0) = q_0(2G(r_d)) + \rho c (1 - e^{-jka}) v. \tag{B.11}
\]

Yet, an important part in Eq. (10) is missing, because the pressure at the opening should relate to the impedance of the HR. A generally known relation between pressure and velocity can therefore be used [20],

\[
Z = \frac{p}{v}. \tag{B.12}
\]

Substituting Eq. (11) into Eq. (10) gives,

\[
Z v - \rho c (1 - e^{-jka}) v = q_0(2G(r_d)). \tag{B.13}
\]

Or differently written,

\[
v(Z - \rho c (1 - e^{-jka})) = q_0(2G(r_d)). \tag{B.14}
\]

This results in the final expression for the velocity,

\[
v = \frac{q_0(2G(r_d))}{Z - \rho c (1 - e^{-jka})}. \tag{B.15}
\]

Substituting Eq. (B.14) into Eq. (B.3) gives the unknown source strength of a single HR,

\[
q = \frac{-j\omega \rho S}{2\pi} \left( \frac{q_0(2G(r_d))}{Z - \rho c (1 - e^{-jka})} \right). \tag{B.16}
\]