MASTER

Investigation of the mechanical characteristics of a composite louver applicable to the integrated roof wind energy system

Stellingwerff, D.A.

Award date:
2014

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Master Thesis

Investigation of the mechanical characteristics of a composite louver applicable to the Integrated Roof Wind Energy System

D.A. Stellingwerff
17-11-2014
Master Thesis

Mechanical investigation of a composite louver applicable to the Integrated Roof Wind Energy System

Report

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0630230

November, 2014

A – 2014.72

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**Preface**

This thesis concludes my time at the Eindhoven University of Technology. It describes the final project which I have carried out at the faculty of Architecture, Building, and Engineering with a specialisation in Structural Design.

For my thesis subject I was searching for a problem with a practical application. An investigation which could give a solution to a practical problem. During the lectures of ‘Constructief Ontwerpen 8: Capita Selecta’ dr. ir. Alexander Suma presented a new and innovative way to harvest wind energy: the *Integrated Roof Wind Energy System* (IRWES). Triggered by this innovative product and the realisation of a solution for the upcoming energy shortage, I contacted Dr. ir. Alexander Suma about the possibilities to perform a graduation project related to IRWES.

The enthusiasm of dr. ir. Alexander Suma and a short chat quickly led to a component which needed further investigation: the louvers of the system. The desire to construct them out of composite materials and the investigation into the dynamic behaviour of the louvers, gave me the possibilities to learn more about two interesting topics. These topics have not been part of my curriculum so far. As a consequence a lot of effort was required to become familiar with these topics, which resulted into a large broadening of my knowledge.

During this process I have been supported by my graduation committee, for which I am truly grateful. I would like to thank prof. dr. ir. Akke Suiker with whom I spent many hours discussing the (aero)dynamic aspects of the louver, dr. ir. Alexander Suma providing the subject, encouragement, and new ideas, and ir. Rob van Oost who added practical knowledge about composites, and a different perspective at problems.

Furthermore, I would like to thank my friends and family for their support, interest in my work, and possibility express my frustrations. In particular I would like to thank Lianne for her support, motivation and advices on the approach of the project, my parents for believing in me and their encouragement, and my sister for her trust in me and providing different perspectives at times needed.

Douwe Stellingwerff

Eindhoven, November 2014
Abstract

As durable energy becomes more important, the industry of durable energy products is growing. In this industry the Integrated Roof Wind Energy System (IRWES) forms an innovative product. IRWES is a rooftop mounted wind energy system which integrates with the architecture of the building and uses funnelling of the wind to increase the amount of wind power generated. The system is surrounded by louvers, which ensure a laminar wind flow towards the wind turbine. These louvers form the main subject of this thesis.

It is desired that the louvers have a low mass. Therefore, it is chosen to construct the louvers out of a plastic reinforced material. This thesis focuses on the technical design of the louver. It investigates the stiffness and strength of the louver under certain wind conditions, for several cross-sections and boundary conditions. The research goal is specified as follows:

“Investigation into the mechanical characteristics and possibilities for a composite louver applicable to the Integrated Roof Wind Energy System.”

For this investigation the continuous problem of the louver is first discretised to a mass-spring system. With the use of equations of motion it is possible to model the displacement behaviour of the louvers, which are subjected to a wind loading. Besides stable responses, instability phenomena are investigated as well.

The modelled motions are subsequently used as input for a finite element program. Subjecting the numerical models of the louver to these motions, makes it possible to investigate the stress patterns in the louver.

A change of the cross-sectional design and/or the boundary conditions, influences the response of the louver to the same wind loading. Therefore, four different designs of the louver are made, with two types of cross-section and two types of boundary conditions. The influences of these changes on the displacement behaviour and mechanical response of the louvers are investigated.

For this thesis, it is assumed that the louvers are constructed out of a glass fibre reinforced composite. During the investigation of the displacement behaviour of the louver, it is shown that instability is not a problem. The louver becomes unstable at unrealistic wind speeds (U > 160 m/s). However, stiffness requirements are not unconditionally met. The influence of the boundary conditions on the deflection of the louver is tremendous. Assuming a hinged support instead of fully clamped support, results into a five times higher deflection. As a consequence, the stiffness requirement is failed for a hinged connected louver under high wind loading. Assuming fully clamped supports satisfies the stiffness requirement even for unrealistic high wind speeds.

The numerical analysis shows that the material has sufficient strength to withstand the occurring stresses. In case of an average wind signal the stress levels remain well within the fatigue limit (20% of the strength capacity of a glass fibre material). If the louver is subjected to an extreme wind loading the stresses in the louver exceed the fatigue limit locally, but do not reach the ultimate limit strength of the glass fibre material.
Samenvatting
De vraag naar duurzame energie wordt steeds groter, daardoor is de vraag naar producten die duurzaam energie opwekken ook steeds groter. Het *Integrated Roof Wind Energy System* (IRWES) is een innovatief product wat aan deze vraag voldoet. IRWES is een windenergiesysteem dat bovenop gebouwen wordt geplaatst en de architectuur van het gebouw ondersteunt. Door de wind op een slimme manier te versnellen, wordt er meer windenergie opgewekt. Om er voor te zorgen dat de windstroom de windturbine zo laminaire mogelijk bereikt, is het systeem omringd door lamellen. Deze lamellen vormen het onderwerp van dit onderzoek.

Er wordt gestreefd naar een laag eigengewicht van de lamellen, daarom zullen de lamellen worden gemaakt van een vezel versterkt composiet materiaal. Dit onderzoek richt zich op het technisch ontwerp van de lamellen. Daarbij worden de stijfheid en sterkte van de lamellen, ten gevolge van bepaalde windbelastingen onderzocht. Dit onderzoek is uitgevoerd voor verschillende ontwerpen van de lamellen. Het onderzoek doel is als volgt vastgesteld:

*“Onderzoek naar de mechanische eigenschappen en mogelijkheden van een lamel van composiet materiaal toepasbaar voor het Integrated Roof Wind Energy System.”*

Het continue probleem is voor dit onderzoek eerst gediscretiseerd tot een massa-veersysteem. Door de discretisatie is het mogelijk om het verplaatsingsgedrag van de lamellen, ten gevolge van de windbelasting, te beschrijven met bewegingsvergelijkingen. Naast onderzoek naar de stabiele responses van de lamellen, zijn instabiliteitsproblemen onderzocht.

De gemodelleerde verplaatsing is vervolgens als input gebruikt voor numerieke analyses van de lamel. Door in het numerieke model van de lamel deze verplaatsingen op te leggen, is het mogelijk om de spanningsverdeling in de lamel te onderzoeken.

Aanpassingen van de doorsnede van de lamel en/of de opleggingen, beïnvloeden het gedrag van de lamel ten gevolge van dezelfde windbelasting. Om deze invloeden te onderzoeken zijn vier verschillende ontwerpen van de lamel onderzocht. De vier ontwerpen zijn een combinatie van twee verschillende doorsneden en twee typen oplegging. Voor elk ontwerp zijn het verplaatsingsgedrag van de lamel en de spanningsverdeling in de lamel onderzocht.

In dit onderzoek is aangenomen dat de lamellen zijn gemaakt van een glasvezel composiet materiaal. De analyse van het verplaatsingsgedrag van de lamel heeft aangetoond dat instabiliteit niet zal optreden voor reële windsnelheden. Het eerste moment van instabiliteit treedt op bij een windsnelheid $U = 160$ m/s. Het belang van het toegepaste type oplegging is wel aangetoond. Als er wordt gekozen voor scharnierende opleggingen, zal in het geval van hoge windsnelheden niet worden voldaan aan de stijfheidseis van de lamel. Wanneer er voor wordt gekozen om de lamel volledig in te klemmen, zal de lamel altijd aan de stijfheidseis voldoen.

Met de numerieke analyses is aangetoond dat een glasvezel composiet materiaal voldoende sterk is om de optredende spanningen op te nemen. Voor een gemiddelde windbelasting blijven de spanningen onder de vermoeiingslimiet van het materiaal (20% van de sterkte van een glasvezel composiet). In het geval dat de lamel wordt belast door een extreme windbelasting, blijven de spanningen onder de maximale sterkte van het glasvezel composiet materiaal.
# Table of contents

Preface.................................................................................................................................4

Abstract..................................................................................................................................................6

Samenvatting ...........................................................................................................................................8

List of symbols ........................................................................................................................................14

1. Introduction.................................................................................................................................16

2. Composites........................................................................................................................................20
   2.1. Introduction of composites ...........................................................................................................20
   2.2. Different sorts of fibres ................................................................................................................21
       2.2.1. Glass fibres .............................................................................................................................21
       2.2.2. Carbon and graphite fibres ....................................................................................................21
       2.2.3. Other fibres............................................................................................................................21
   2.3. Matrix materials.........................................................................................................................22
   2.4. Review of composites ................................................................................................................23

3. Mechanical approximation and displacement behaviour of the louver .........................................26
   3.1. Schematisation and discretisation of the louver .........................................................................26
   3.2. Aerodynamic forces ....................................................................................................................28
   3.3. Steady aerodynamic model..........................................................................................................30
   3.4. Low-Frequency aerodynamic model............................................................................................34
   3.5. Low-frequency model including structural damping .................................................................35
   3.6. Instability of the louver .................................................................................................................37
       3.6.1. Torsional divergence...............................................................................................................38
       3.6.2. Flutter instability of the steady aerodynamic model.................................................................39
       3.6.3. Flutter instability low-frequency model .................................................................................41
       3.6.4. Flutter instability model including structural damping ...............................................................43
   3.7. External loading............................................................................................................................43
       3.7.1. Change in angle of attack .......................................................................................................48
       3.7.2. Validation of the response of the louver for U(t).....................................................................51

4. Design alternatives..........................................................................................................................54
   4.1. Alternative design of the louver, Louver B1................................................................................54
       4.1.1. Response Louver B1, steady aerodynamic model .................................................................55
       4.1.2. Response Louver B1, low-frequency model .........................................................................56
       4.1.3. Response Louver B1, low-frequency model including structural damping ............................58
4.1.4. Response Louver B1 subjected to an external loading ................................................. 59
4.1.5. Reflection on Louver B1 ................................................................................................... 61

4.2. Applying hinged connections, Louver A2 ........................................................................... 61
4.2.1. Response Louver A2, steady aerodynamic model ........................................................ 62
4.2.2. Response Louver A2, low-frequency model ................................................................. 64
4.2.3. Response Louver A2, low-frequency model including structural damping ............ 66
4.2.4. Response Louver A2 subjected to an external loading ................................................ 67
4.2.5. Reflection on Louver A2 ................................................................................................... 69

4.3. Combination of design alternatives, Louver B2 .......................................................................... 69
4.3.1. Steady aerodynamic model .............................................................................................. 69
4.3.2. Low-frequency model ....................................................................................................... 70
4.3.3. Low-frequency model including structural damping .................................................. 71
4.3.4. Louver B2 subjected to an external loading ................................................................... 72
4.3.5. Reflection on louver B2 ..................................................................................................... 74

4.4. Conclusions design alternatives .................................................................................................... 74

5. Numerical analysis ............................................................................................................................. 76
5.1. Modal analysis ................................................................................................................................. 77
5.1.1. Modal analysis Louver A1, analytical ........................................................................... 77
5.1.2. Modal analysis Louver A1, numerical ........................................................................... 79
5.1.3. Modal analysis Louver B1, analytical ............................................................................. 81
5.1.4. Modal analysis Louver B1, numerical ............................................................................ 82
5.1.5. Static validation numerical model .................................................................................. 84
5.2. Dynamic numerical analyses of the louver .................................................................................. 86
5.2.1. Dynamic behaviour Louver B2 under average wind loading ..................................... 87
5.2.2. Dynamic behaviour louver B2 under high wind loading............................................ 91
5.3. Conclusions numerical analysis .................................................................................................... 96

6. Conclusions and recommendations ................................................................................................. 98
6.1. Conclusions ...................................................................................................................................... 98
6.2. Recommendations ........................................................................................................................... 99

7. Bibliography ...................................................................................................................................... 100
Appendix A: Equations of motion ........................................................................................................... 102
A.1. Derivation of the coefficients of the equations of motion .................................................. 102
A.2. Input files MATLAB, no external loading .............................................................................. 103
## List of symbols

### Latin upper-case

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Unity</th>
<th>Explanation</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>[m^2]</td>
<td>Area</td>
</tr>
<tr>
<td>(A)</td>
<td>[-]</td>
<td>Aerodynamic damping matrix</td>
</tr>
<tr>
<td>(C)</td>
<td>[-]</td>
<td>Damping matrix</td>
</tr>
<tr>
<td>(C_i)</td>
<td>[Ns/m]</td>
<td>Aerodynamic damping coefficient</td>
</tr>
<tr>
<td>(C_{si})</td>
<td>[Ns/m]</td>
<td>Structural damping coefficient</td>
</tr>
<tr>
<td>(C_{L})</td>
<td>[-]</td>
<td>Lift coefficient</td>
</tr>
<tr>
<td>(E)</td>
<td>[N/m^2]</td>
<td>Young’s modulus</td>
</tr>
<tr>
<td>(G)</td>
<td>[N/m^2]</td>
<td>Shear modulus</td>
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<td>(I_t)</td>
<td>[m^4]</td>
<td>Torsional moment of inertia</td>
</tr>
<tr>
<td>(I_s)</td>
<td>[m^4]</td>
<td>Bending moment of inertia</td>
</tr>
<tr>
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<td>[kgm^2/m]</td>
<td>Mass moment of inertia</td>
</tr>
<tr>
<td>(K)</td>
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<td>Stiffness matrix</td>
</tr>
<tr>
<td>(K_h)</td>
<td>[N/m/m]</td>
<td>Translational spring stiffness</td>
</tr>
<tr>
<td>(K_{h;eq})</td>
<td>[N/m/m]</td>
<td>Equivalent translational spring stiffness</td>
</tr>
<tr>
<td>(K_\theta)</td>
<td>[Nm/rad/m]</td>
<td>Torsional spring stiffness</td>
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<tr>
<td>(L)</td>
<td>[m]</td>
<td>Length louver</td>
</tr>
<tr>
<td>(L_f)</td>
<td>[N/m]</td>
<td>Lift coefficient</td>
</tr>
<tr>
<td>(L_{Fex})</td>
<td>[N/m]</td>
<td>Lift coefficient due to (\theta_i)</td>
</tr>
<tr>
<td>(L_f(t))</td>
<td>[N]</td>
<td>Lift force</td>
</tr>
<tr>
<td>(M)</td>
<td>[-]</td>
<td>Mass matrix</td>
</tr>
<tr>
<td>(M)</td>
<td>[Nm]</td>
<td>Concentrated moment</td>
</tr>
<tr>
<td>(M_{EA})</td>
<td>[Nm/m]</td>
<td>Moment coefficient</td>
</tr>
<tr>
<td>(M_{EA;ex})</td>
<td>[Nm/m]</td>
<td>Aerodynamic coefficient due to (\theta_i)</td>
</tr>
<tr>
<td>(M_{EA}(t))</td>
<td>[Nm]</td>
<td>Aerodynamic moment about EA</td>
</tr>
<tr>
<td>(P_t)</td>
<td>[W]</td>
<td>Power produced by wind turbine</td>
</tr>
<tr>
<td>(Q_{L})</td>
<td>[N/m^1]</td>
<td>Aerodynamic lift coefficient</td>
</tr>
<tr>
<td>(Q_{L;ex})</td>
<td>[N/m^1]</td>
<td>External aerodynamic lift force coefficient</td>
</tr>
<tr>
<td>(Q_M)</td>
<td>[Nm/m^1]</td>
<td>Aerodynamic moment coefficient</td>
</tr>
<tr>
<td>(Q_{Mex})</td>
<td>[Nm/m^1]</td>
<td>External aerodynamic moment coefficient</td>
</tr>
<tr>
<td>(S)</td>
<td>[m^2]</td>
<td>Plan area louver</td>
</tr>
<tr>
<td>(S_0)</td>
<td>[kgm/m]</td>
<td>Static moment per unit length</td>
</tr>
<tr>
<td>(U)</td>
<td>[m/s]</td>
<td>Speed</td>
</tr>
<tr>
<td>(U_f)</td>
<td>[m/s]</td>
<td>Flutter wind speed</td>
</tr>
<tr>
<td>(U_{mean})</td>
<td>[m/s]</td>
<td>Mean wind speed</td>
</tr>
<tr>
<td>(U(t))</td>
<td>[m/s]</td>
<td>Dynamic wind signal</td>
</tr>
<tr>
<td>(V_F)</td>
<td>[%]</td>
<td>Fibre volume fraction</td>
</tr>
</tbody>
</table>

### Latin lower-case

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Unity</th>
<th>Explanation</th>
</tr>
</thead>
<tbody>
<tr>
<td>(c)</td>
<td>[m]</td>
<td>Chord length louver</td>
</tr>
<tr>
<td>(ec)</td>
<td>[m]</td>
<td>Eccentricity between AC and EA</td>
</tr>
<tr>
<td>(m_1)</td>
<td>[kg/m]</td>
<td>Mass louver per unit length</td>
</tr>
<tr>
<td>(h)</td>
<td>[m]</td>
<td>Continuous vertical translation louver</td>
</tr>
<tr>
<td>(h_{dis})</td>
<td>[m]</td>
<td>Discrete vertical translation of louver</td>
</tr>
</tbody>
</table>
\( h_0 \) [m] Thickness louver

\( h(t) \) [m] Translation louver in time

\( q \) [N/m²] Dynamic pressure

\( q_L \) [N/m²] Equally distributed lift force

\( q_M \) [Nm/m²] Equally distributed aerodynamic moment

\( t \) [sec] Time

\( u_i \) [-] Modal vector

\( x \) [m] Distance along length of the louver

**Greek lower-case**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Unity</th>
<th>Explanation</th>
</tr>
</thead>
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<tr>
<td>( \alpha )</td>
<td>[-]</td>
<td>Rayleigh coefficient related to ( M )</td>
</tr>
<tr>
<td>( \beta )</td>
<td>[-]</td>
<td>Rayleigh coefficient related to ( K )</td>
</tr>
<tr>
<td>( \zeta_i )</td>
<td>[-]</td>
<td>Damping ratio</td>
</tr>
<tr>
<td>( \theta )</td>
<td>[rad]</td>
<td>Continuous torsional rotation louver</td>
</tr>
<tr>
<td>( \theta_{dis} )</td>
<td>[rad]</td>
<td>Discrete torsional rotation louver</td>
</tr>
<tr>
<td>( \theta_0 )</td>
<td>[rad]</td>
<td>Initial rotation</td>
</tr>
<tr>
<td>( \theta_1 )</td>
<td>[rad]</td>
<td>Angle of non-horizontal wind stream</td>
</tr>
<tr>
<td>( \theta(t) )</td>
<td>[rad]</td>
<td>Rotation of louver in time</td>
</tr>
<tr>
<td>( \lambda )</td>
<td>[-]</td>
<td>Wind turbine efficiency</td>
</tr>
<tr>
<td>( \nu )</td>
<td>[-]</td>
<td>Poison factor</td>
</tr>
<tr>
<td>( \rho )</td>
<td>[kg/m³]</td>
<td>Density</td>
</tr>
<tr>
<td>( \sigma )</td>
<td>[N/mm²]</td>
<td>Stress</td>
</tr>
<tr>
<td>( \sigma_1 )</td>
<td>[N/mm²]</td>
<td>Maximal principal stress</td>
</tr>
<tr>
<td>( \sigma_3 )</td>
<td>[N/mm²]</td>
<td>Minimal principal stress</td>
</tr>
<tr>
<td>( \sigma_{fat} )</td>
<td>[N/mm²]</td>
<td>Fatigue stress limit</td>
</tr>
<tr>
<td>( \sigma_{max} )</td>
<td>[N/mm²]</td>
<td>Maximal material stress limit</td>
</tr>
<tr>
<td>( \tau_{max} )</td>
<td>[N/mm²]</td>
<td>Maximal shear stress</td>
</tr>
<tr>
<td>( \omega_{nci} )</td>
<td>[rad/s]</td>
<td>Natural frequency</td>
</tr>
<tr>
<td>( \omega_f )</td>
<td>[rad/s]</td>
<td>Fundamental frequency</td>
</tr>
</tbody>
</table>

**Abbreviations**

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Explanation</th>
</tr>
</thead>
<tbody>
<tr>
<td>AC</td>
<td>Aerodynamic centre</td>
</tr>
<tr>
<td>AMS</td>
<td>Automatic Multi-level Substructuring</td>
</tr>
<tr>
<td>CG</td>
<td>Centre of gravity</td>
</tr>
<tr>
<td>DEM</td>
<td>Discrete elementen methode</td>
</tr>
<tr>
<td>DEM</td>
<td>Discrete element method</td>
</tr>
<tr>
<td>EA</td>
<td>Elastic axis</td>
</tr>
<tr>
<td>EEM</td>
<td>Eindige elementen methode</td>
</tr>
<tr>
<td>FEM</td>
<td>Finite element analysis</td>
</tr>
</tbody>
</table>
1. Introduction

Life as we know today, with all the technological possibilities, has an enormous impact on the environment. Due to the increase in world population and the enhancement of technology, natural supplies will become scarce in time. One major issue within this global problem is the oncoming shortage of fossil fuels for our energy supply. Since a number of years, sustainable energy has become an important topic on the global agenda. A complete new industry has developed, and still is developing.

One of these sustainable energy sources is wind. A known example for harvesting energy out of wind are the wind mills as shown in Figure 1.1. Besides installing wind mills in the middle of the sea it is possible to apply them to a building. One example is the appliance of DonQi’s on the rooftop of a building, as shown in Figure 1.2. As can be seen the offered solution does not integrate with the architecture of the building. Figure 1.3 shows the World Trade Center in Bahrain, in the final design of the building the wind mills are more integrated with the architecture as is the case in Figure 1.2.

![Figure 1.1: Windmills at sea (Suma, 2012)](image1)
![Figure 1.2: DonQi’s mounted on top of a roof (Suma, 2012)](image2)
![Figure 1.3: World Trade Center, Bahrain (Suma, 2012)](image3)

Besides the aesthetic disadvantages, roof top mounted wind mills generate relatively small amounts of wind energy. The amount of wind power produced by a wind turbine, is determined by the basic equation of wind power generation (Suma, 2012):

\[ P_t = \frac{1}{2} \rho U^3 \lambda A \] (1.1)

With:
- \( P_t \) = power produced by the wind turbine [W]
- \( \rho \) = air density [g/m³]
- \( U \) = wind speed approaching the wind turbine [m/s]
- \( \lambda \) = wind turbine efficiency [-]
- \( A \) = projected area of the turbine perpendicular to the approaching wind [m²]

As can be seen in equation (1.1) the wind speed has an important role in the generation of wind power. Doubling the wind speed results into eight times more wind power!

*Ibis Power BV* has designed a new product for harvesting wind energy which integrates with the architecture of a building and utilises the beneficial contribution of the wind speed. The result is...
shown in Figure 1.4. By creating a dome inside the Integrated Roof Wind Energy System (IRWES) the wind speed is increased, thereby increasing the amount of wind power.

![Figure 1.4: Integrated Roof Wind Energy System (IRWES)](image)

However, IRWES can be applied on newly built buildings and existing buildings. Especially in case of the latter, it is desired to accomplish a low mass for the wind energy system. This thesis investigates the possibilities of a low weight louver, constructed out of a composite material. For this investigation, instability and strength characteristics are investigated as well as several cross-sectional designs and varying boundary conditions. The research goal is specified as follows:

“Investigation into the mechanical characteristics and possibilities for a composite louver applicable to the Integrated Roof Wind Energy System.”

To conduct this investigation the following research approach is used:

- Development of a discrete element model, simulating the fluid-structure interaction of a louver applied to IRWES;
- Investigation of the mechanical behaviour of the cross-section under dynamic wind loading, using a finite element model; and
- Studying the influences on the type of composite applied with regards to type of fibre reinforcement, matrix material and fibre orientation.

Firstly, the motion of the louver due to the wind is modelled by using the discrete element method (DEM). The motion in time of the discretised louver is described by the equations of motion. This leads to the input to be used for the finite element method (FEM) model.

The next step is to use the modelled motion of the louver as input for the FEM model. Using a dynamic analysis it becomes possible to investigate the stresses and strains of the louver in time for a certain wind loading.

Finally, the properties of the cross-section are modified, leading to different mass, damping, and stiffness parameters. This leads to a new response of the louver for the same wind loading. Therefore, the DEM is used again to model the input for the FEM model and the newly designed louver is investigated for its stresses and strains. Repeating this process results into an optimisation of the design of the louver.

This repetitive process is schematically shown in Figure 1.5.
The further outline of this report is as follows:

Chapter 2: Composites
A small introduction of available composite materials and their properties.

Chapter 3: Mechanical approximation and displacement behaviour of the louver
In this chapter both the mechanical approximation and the development of the discrete element model are explained. By the use of three aerodynamic models the motions of the louver in case of a free vibration and a forced vibration are modelled. Instability phenomena related to aerodynamically shaped structures are considered as well.

Chapter 4: Design alternatives
In this chapter, the influences of a modified cross-section and different boundary conditions are considered. The motions of the modified louver are modelled by all three aerodynamic models and the results are compared with the results of the other models.

Chapter 5: Numerical analyses
This chapter explains the numerical analyses conducted for this thesis. Both static and dynamic analyses have been performed providing insight in the mechanical behaviour of the louver under wind loading.

Chapter 6: Conclusions and recommendations
In the final chapter the conclusion of this thesis are drawn and recommendations are given regarding future research and development of the louver.
2. Composites

This chapter gives an introduction of composite materials. A material is considered to be composite when it exists out of two or more distinct materials. In this thesis, a composite material is regarded to be a material which is constructed out of reinforcing fibres embedded in a matrix material (fibre reinforced plastics).

2.1. Introduction of composites

Composite materials are (most often) designed to improve the mechanical properties of a structural element. A composite material is characterised by a discontinuous phase (i.e. the reinforcing material) and a continuous phase (i.e. a matrix material). Therefore, the desired properties can be obtained by applying the reinforcing material and the matrix material in a product specific way. As the properties of composite materials are mostly determined by the applied reinforcement, Figure 2.1 shows a classification of composite materials according to the type of reinforcement.

![Classification of composite materials](image)

The first distinction to be made is, whether it is a particle type of reinforcement (i.e. a relatively high cross-section to length ratio) or a fibre reinforced composite (i.e. a relatively low cross-section to length ratio).

Secondly, the material can be classified by the way the reinforcement is applied. In case of particle-reinforced composites, the orientation of the fibres can be either random or guided. For the fibre reinforced composites, the material can be classified by the number of layers applied. A single layer composite is strong in the direction of the fibre and weak perpendicular to the fibre’s direction. While a multilayer composite can be equally strong in different directions, as its fibres are directed in multiple orientations. However, the ultimate strength of a multilayer composite is (generally) about half of the ultimate strength of a single layer composite.
Finally, a distinction can be made between continuous fibres and discontinuous fibres. The continuous fibres can be unidirectional or bidirectional, while the discontinuous fibres can be either random orientated or guided.

### 2.2. Different sorts of fibres

There are numerous fibres available for the construction of composite materials. The ones which are best known are probably glass fibres, and carbon/graphite fibres. This paragraph will shortly discuss both fibres and some others.

#### 2.2.1. Glass fibres

Glass fibres are widely used for the fabrication of composite materials. This is mostly due to the combination of low costs and high strength. Besides these advantages, glass fibres have disadvantages like abrasion, with the result that glass fibres cannot be loaded to their full capacity. Also, fibres made out of Kevlar or carbon have higher stiffness properties than glass fibres. However, depending on the problem to be solved, glass fibres may have sufficient stiffness after all.

Figure 2.2 shows different forms of glass fibres available.

![Figure 2.2: Fibreglass roving (left), woven roving (middle), and chopped-strand matt (right) (Nautic Expo, 2013)](image)

#### 2.2.2. Carbon and graphite fibres

During the last decades, the appliance of carbon fibres has increased tremendously as a result of the increase in availability and a reduction in costs. Nowadays, carbon and graphite fibres are the most used high-strength, and high-modulus reinforcement fibres in the fabrication of high-performance polymer-matrix composites.

The application of carbon fibre composites has expanded. Besides appliance in e.g. aerospace engineering, carbon fibres are nowadays used in sports goods, automotive engineering, and civil engineering as well. However, carbon fibres are mostly used if there is a genuine desire for high-performance products, as the costs of carbon fibres are significantly higher than the costs of glass fibres.

#### 2.2.3. Other fibres

Besides these well-known fibres, other types of fibres are available, like aramid fibres, or better known as Kevlar. Kevlar is an organic fibre with a substantially higher strength and elastic modulus compared to other types of (organic) fibres. Another advantage of Kevlar is its low density. However, the cost price of this fibre is very high, which makes it unfavourable for simple applications.
Another sort of fibre which may not be as well-known as the previous sorts, are boron fibres. Boron fibres are produced out of boron trichloride with hydrogen on a tungsten or carbon monofilament substrate. These types of fibres can be as strong as Kevlar fibres. However, their density is substantially higher and the fabrication process is extensive.

It might be clear by now that composite materials gain their advantageous behaviour through the appliance of fibres. These fibres gain their properties as a consequence of limited discrepancies in the small cross-section of the fibres. Therefore, the fibre strength is almost as high as the theoretical strength of the material. Table 2.1 gives an overview of different fibre materials and conventional materials, regarding their stiffness, strength, and density.

<table>
<thead>
<tr>
<th>Material</th>
<th>Tensile Modulus (E) [GPa]</th>
<th>Tensile Strength (σₜ) [GPa]</th>
<th>Density (ρ) [g/cm³]</th>
<th>Specific modulus (E/ρ) [-]</th>
<th>Specific strength (σₜ/ρ) [-]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fibres</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>E-glass</td>
<td>72,4</td>
<td>3,5⁺</td>
<td>2,54</td>
<td>28,5</td>
<td>1,38</td>
</tr>
<tr>
<td>S-glass</td>
<td>85,5</td>
<td>4,6⁺</td>
<td>2,48</td>
<td>34,5</td>
<td>1,85</td>
</tr>
<tr>
<td>Graphite (high modulus)</td>
<td>390,0</td>
<td>2,1</td>
<td>1,90</td>
<td>205,0</td>
<td>1,1</td>
</tr>
<tr>
<td>Graphite (high tensile strength)</td>
<td>240,0</td>
<td>2,5</td>
<td>1,90</td>
<td>126,0</td>
<td>1,3</td>
</tr>
<tr>
<td>Boron</td>
<td>385,0</td>
<td>2,8</td>
<td>2,63</td>
<td>146,0</td>
<td>1,1</td>
</tr>
<tr>
<td>Silica</td>
<td>72,4</td>
<td>5,8</td>
<td>2,19</td>
<td>33,0</td>
<td>2,65</td>
</tr>
<tr>
<td>Tungsten</td>
<td>414,0</td>
<td>4,2</td>
<td>19,30</td>
<td>21,0</td>
<td>0,22</td>
</tr>
<tr>
<td>Beryllium</td>
<td>240,0</td>
<td>1,3</td>
<td>1,83</td>
<td>131,0</td>
<td>0,71</td>
</tr>
<tr>
<td>Kevlar 49 (aramid polymer)</td>
<td>130,0</td>
<td>2,8</td>
<td>1,50</td>
<td>87,0</td>
<td>1,87</td>
</tr>
</tbody>
</table>

| Conventional Materials          |                           |                             |                     |                           |                              |
| Steel                           | 210,0                     | 0,34-2,1                    | 7,80                | 26,9                      | 0,043-0,27                   |
| Aluminium alloys                | 70,0                      | 0,14-0,62                   | 2,70                | 25,9                      | 0,052-0,23                   |
| Glass                           | 70,0                      | 0,7-2,1                     | 2,50                | 28,0                      | 0,28-0,84                    |
| Tungsten                        | 350,0                     | 1,1-4,1                     | 19,30               | 18,1                      | 0,057-0,21                   |
| Beryllium                       | 300,0                     | 0,7                         | 1,83                | 164,0                     | 0,38                         |

⁺ Virgin strength values. Actual strength values prior to incorporation are approximately 2,1 GPa

Table 2.1: Properties of fibres and conventional bulk materials (Bhagwand D. Agarwal, 2006)

### 2.3. Matrix materials

An important part of a composite material is the matrix material which bonds the fibres and assures structural applicability. The choice of matrix material has a great influence on the mechanical properties of the composite material, such as the shear modulus, shear strength, and compression properties.

When the composite material is applied in an environment where the temperature remains below 260 °C, a polymer matrix can be applied. Several polymeric matrix materials are known, where we distinguish between thermoplastic polymers and thermosetting polymers.

The most commonly known thermosetting polymers are polyester and epoxy resins. Their main advantages are the easy processability and good chemical resistance. A polyester resin is generally a polyester solid dissolved in a monomer which can be polymerised. By varying the
processing techniques and raw materials, it is possible to obtain the desired properties for the polyester.

The epoxy resin is a viscous liquid of which the viscosity depends on the degree of polymerisation. It is possible for this polymer to harden at room temperature. However, a higher temperature, will achieve a higher degree of hardening.

Thermoplastic polymers are most often used in case of discontinuous composites and in large volume products. In general, the production costs are lower because these composites can be manufactured by mass production methods. Composites which are constructed in this way are most often regarded as higher strength and higher stiffness replacements for plastics, instead of being considered high-performance load-bearing materials. However, it is also possible to fabricate thermoplastic polymers with higher service temperature and maximum use temperatures than some epoxies.

2.4. Review of composites

Composite materials are most often applied for their low-weight and high-strength and -stiffness properties compared to other conventional materials. This is the case for the louvers of IRWES as well.

The desired low-weight material would argue for the application of carbon or Kevlar fibres. However, since IRWES is a commercial product, costs form an important aspect. This forms a disadvantage for these materials. Besides, the loads on the louver may not be of a magnitude that favours the use of these high-stiffness and high-strength fibres.

Considering the wind load on the louvers, it is expected that continuous fibres in multiple directions are the favourable choice. This also justifies the application of an epoxy resin. Comparing the properties of cast thermosetting polymers to cast epoxy resin, the properties of the latter are more favourable over the properties of the first.

Table 2.2 gives an overview of several cross-ply composites and their properties. The E-glass-epoxy composite forms the basis for further elaboration in this thesis. This study will show if this composite material satisfies the strength and stiffness requirements of the louvers for IRWES.
<table>
<thead>
<tr>
<th>Material</th>
<th>Fibre Volume Fraction, $V_F$ (%)</th>
<th>Tensile Modulus, $E$ (GPa)</th>
<th>Tensile Strength, $\sigma_u$ (GPa)</th>
<th>Density, $\rho$ (g/cm$^3$)</th>
<th>Specific modulus $(E/\rho)$</th>
<th>Specific strength $(\sigma_u/\rho)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mild Steel</td>
<td>-</td>
<td>210</td>
<td>0,45-0,83</td>
<td>7,8</td>
<td>26,9</td>
<td>0,058-0,106</td>
</tr>
<tr>
<td>Aluminium 2024-T4</td>
<td>-</td>
<td>73</td>
<td>0,41</td>
<td>2,7</td>
<td>27</td>
<td>0,152</td>
</tr>
<tr>
<td>Aluminium 6061-T6</td>
<td>-</td>
<td>69</td>
<td>0,26</td>
<td>2,7</td>
<td>25,5</td>
<td>0,096</td>
</tr>
<tr>
<td>E-glass-epoxy</td>
<td>57</td>
<td>21,5</td>
<td>0,57</td>
<td>1,97</td>
<td>10,9</td>
<td>0,26</td>
</tr>
<tr>
<td>Kevlar 49-epoxy</td>
<td>60</td>
<td>40</td>
<td>0,65</td>
<td>1,4</td>
<td>29</td>
<td>0,46</td>
</tr>
<tr>
<td>Carbon fibre-epoxy</td>
<td>58</td>
<td>83</td>
<td>0,38</td>
<td>1,54</td>
<td>53,5</td>
<td>0,24</td>
</tr>
<tr>
<td>Boron-epoxy</td>
<td>60</td>
<td>106</td>
<td>0,38</td>
<td>2</td>
<td>53</td>
<td>0,19</td>
</tr>
</tbody>
</table>

Table 2.2: Properties of conventional structural materials and bidirectional (cross-ply) fibre composites (Bhagwand D. Agarwal, 2006)
3. Mechanical approximation and displacement behaviour of the louver

This chapter discusses the mechanical approximation of the louver as well as its displacement behaviour. First the mechanical approach to model the louver is discussed. Thereafter, the influences of three different aerodynamic models on the displacement behaviour of the louver are investigated.

3.1. Schematisation and discretisation of the louver

As a consequence of the wind loading on the louver, the louver is subjected to a vertical equally distributed load \( q_L \) [N/m] and a torsional equally distributed moment \( q_M \) [Nm/m]. These forces occur as a result of lift forces and an eccentricity between the aerodynamic centre and the elastic axis of the louver, respectively. Firstly, it is assumed that the louver is fully clamped at both sides. The complete schematisation of the louver is given in Figure 3.1 (note: the direction of the forces are strictly arbitrary and for illustrative purposes).

Due to the loadings on the louver, the motion of the louver includes a vertical translation and a torsional rotation about its longitudinal axis. In order to be able to describe these motions, the continuous problem is discretised as a two-degree-of-freedom (2-DOF) system.

![Figure 3.1: Schematisation of fully clamped louver including aerodynamic loadings](image)

During this thesis the motion of the louver is modelled per unit length. Therefore, the coefficients of the equations of motion are determined per unit length of the louver.

To determine the spring stiffnesses of the louver per unit length, a comparison is made between the displacements of the continuous system and the discretised system. First the translational stiffness of the louver is considered. The translation of a continuous beam subjected to an equally distributed load is given by the following general differential equation (Bouma, 1989):

\[
E I_x \frac{d^4 h}{dx^4} - q(x) = 0, \text{ with } q(x) = q_L
\]  

(3.1)

With:  
- \( h \) = vertical displacement [m]  
- \( x \) = position along the x-axis [m]  
- \( q_L \) = equally distributed load [N/m]  
- \( L \) = length louver [m]  
- \( E \) = Young's modulus [N/m²]  
- \( I_x \) = bending moment of inertia [m⁴]
Solving equation (3.1) for the boundary conditions \( h'(0) = h'(L) = h(0) = h(L) = 0 \) results into an expression describing the translation along the length of the louver:

\[
h(x) = \frac{1}{24} \frac{q}{E I_x} x^4 - \frac{1}{12} \frac{q L}{E I_x} x^3 + \frac{1}{24} \frac{q L^2}{E I_x} x^2 \tag{3.2}
\]

The translation \( h(x) \) represented by equation (3.2) has to be equal to the discretised translation \( h_{\text{dis}} \) as shown in Figure 3.2. As can be seen in Figure 3.2 the translational spring stiffness \( K_h \) is given per unit length. It can be derived that the discrete translation equals:

\[
h_{\text{dis}} = \frac{q L}{K_h} \tag{3.3}
\]

With: 
- \( h_{\text{dis}} \) = discretised translation [m]
- \( q \) = equally distributed load [N/m²]
- \( K_h \) = translational spring stiffness [N/m/m]

The point of interest is located at the louver’s midspan (i.e. \( x = \frac{L}{2} \)), the location with the largest deformation. Therefore, the continuous translation \( h(x) \) has to equal the discretised translation \( h_{\text{dis}} \) at this point. Solving equation (3.2) for \( x = \frac{L}{2} \) and equating this solution to the discretised displacement in equation (3.3), results into an expression for \( K_h \) at the louver’s midspan:

\[
\frac{1}{384} \frac{q L^4}{E I_x} = \frac{q L}{K_h} \Rightarrow K_h = 384 \frac{E I_x}{L^4} \text{ N/m/m} \tag{3.4}
\]

In order to derive the rotational spring stiffness, \( K_\theta \), a similar approach is used. The torsion of a continuous beam subjected to an equally distributed moment is given by (Bouma, 1989):

\[
-G I_t \frac{d^2 \theta}{dx^2} = q M \Rightarrow \frac{d^2 \theta}{dx^2} = -\frac{q M}{G I_t} \tag{3.5}
\]

With: 
- \( \theta \) = rotation [rad]
- \( x \) = position along the x-axis [m]
- \( q M \) = equally distributed moment [Nm/m]
- \( L \) = length of the louver [m]
- \( G \) = shear modulus [N/m²]
- \( I_t \) = torsional moment of inertia [m⁴]
To describe the rotation of the fully clamped louver, equation (3.5) is solved for the following boundary conditions: $\theta(0) = \theta(L) = 0$. This results into an expression describing the rotation of the continuous louver along its length:

$$\theta(x) = -\frac{1}{2} \frac{q_M}{Gl_t} x^2 + \frac{1}{2} \frac{q_M L}{Gl_t} x$$

(3.6)

Figure 3.3 schematically shows the discretised louver regarding the torsional stiffness. It is assumed that the rotation $\theta$ is constant over the length of 1.0 meter. Similar as in the case of $K_h$ in Figure 3.2), $K_\theta$ is per unit length. It is derived that the discretised rotation, $\theta_{dis}$, is represented by:

$$\theta_{dis} = \frac{q_M}{K_\theta}$$

(3.7)

With:

- $\theta_{dis}$ = discretised rotation [rad]
- $q_M$ = equally distributed moment [Nm/m$^1$]
- $K_\theta$ = torsional spring stiffness [Nm/rad/m$^1$]

![Figure 3.3: Discretised system regarding rotation per unit length](image)

The point of interest is located at $x = \frac{1}{2}L$ again. Solving equation (3.6) for $x = \frac{1}{2}L$ and equating the solution to the discretised rotation expressed in equation (3.7), results into an expression for $K_\theta$:

$$\frac{1}{8} \frac{q_M L^2}{Gl_t} = q_M \frac{K_\theta}{K_\theta} \rightarrow K_\theta = \frac{8}{L^2} \frac{Gl_t}{L^2} \text{Nm/rad/m}^1$$

(3.8)

Both derived expressions for $K_h$ and $K_\theta$, are used later on to describe the motion of the louver.

### 3.2. Aerodynamic forces

The louver is aerodynamically shaped and therefore the forces acting upon the louver is of aerodynamic origin. Figure 3.4 schematically shows the typical section of the louver, including the aerodynamic lift force, $L_F$, and the stiffness parameters.

The aerodynamic force $L_F$ is a function of the rotation of the louver in time $\theta(t)$. Thereby, it is characterised by the dynamic pressures, which depends on the air density ($\rho$) and the free stream wind speed ($U$). The lift force is expressed by (Hulshoff, 2003):

$$L_F(t) = q S c_L \alpha \theta(t)$$

(3.9)

With:

- $L_F$ = lift force [N]
- $q$ = dynamic pressure ($= \frac{1}{2} \rho U^2$) [N/m$^2$]
\[ S = \text{plan area louver [m}^2\text{]} \]
\[ C_{L\alpha} = \text{lift coefficient [-]} \]
\[ \theta = \text{rotation louver relative to zero lift line [rad]} \]

It may be clear that \( L_F \) causes a moment \( M_{EA} \) about the elastic axis. This moment depends on the eccentricity \( ec \) between the aerodynamic centre and the elastic axis (Hulshoff, 2003):

\[ M_{EA}(t) = (ec)L_F(t) \quad (3.10) \]

With:
- \( M_{EA} \) = moment about elastic axis EA [Nm]
- \( ec \) = eccentricity between aerodynamic centre AC and elastic axis EA [m]

With:
- \( U \) = freestream wind speed [m/s]
- \( \theta \) = rotation louver [rad]
- \( L_F \) = aerodynamic lift [N]
- \( AC \) = aerodynamic centre [-]
- \( EA \) = elastic axis [-]
- \( ec \) = eccentricity between AC and EA [m]
- \( c \) = chord length [m]
- \( K_\theta \) = torsional spring stiffness [Nm/rad/m]
- \( K_h \) = translational spring stiffness [N/m/m]

**Figure 3.4: Schematic representation of the louver as a 2-DOF system**

In this thesis a symmetric louver is investigated and is approximated by thin aerofoil theory. As a result of the symmetric profile, the louver has no camber. The lift coefficient according to thin aerofoil theory for an aerofoil without camber is given by (Caughey, 2006):

\[ C_{L\alpha} = 2\pi \quad (3.11) \]

As the spring stiffnesses of the louver are determined per unit length (equations (3.4) and (3.8)), the aerodynamic forces are constructed to be dependent of the chord length \( c \) rather than the plan area \( S \). Rewriting the expressions of the aerodynamic forces in equations (3.9) and (3.10) for the lift coefficient \( C_{L\alpha} \) given by equation (3.11) and substituting \( S \) by \( c \) results into:
\[ L_F(t) = 2qc\pi\theta(t) \quad (3.12) \]
\[ M_{EA}(t) = 2q(ec)\pi\theta(t) \quad (3.13) \]

With:  
- \( L_F \) = lift force \([N/m^1]\)  
- \( M_{EA} \) = moment about elastic axis \([Nm/m^1]\)

For convenience equations (3.12) and (3.13) are rewritten as:

\[ L_F(t) = 2qc\pi\theta(t) = Q_L\theta(t) \quad (3.14) \]
\[ M_{EA}(t) = 2q(ec)\pi\theta(t) = Q_M\theta(t) \quad (3.15) \]

With:  
- \( Q_L \) = aerodynamic lift coefficient \( (= 2qc\pi \) \([N/m^1]\)  
- \( Q_M \) = aerodynamic moment coefficient \( (= 2q(ec)\pi \) \([Nm/m^1]\)

The response of the louver loaded by a certain wind loading is modelled by means of three different aerodynamic models. The first model is the steady aerodynamic model, for which the forces are given by equations (3.14) and (3.15). The second model is the low frequency model, which includes the vertical translation speed \( \dot{h}(\theta) \) in the angle of attack. Finally, a model will be considered for which structural damping has been taken into account.

### 3.3. Steady aerodynamic model

To be able to model the motion of the louver the equations of motion of the mass-spring-system of Figure 3.4 are formulated (Inman, 1994). The equations of motion relative to the elastic axis become:

\[
\begin{bmatrix}
m_1 & S_0 \\
S_0 & I_0
\end{bmatrix}
\begin{bmatrix}
\dot{h} \\
\dot{\theta}
\end{bmatrix}
+ \begin{bmatrix}
K_h & 0 \\
0 & K_\theta
\end{bmatrix}
\begin{bmatrix}
h \\
\theta
\end{bmatrix}
= \begin{bmatrix}
0 \\
0
\end{bmatrix}
\quad (3.16)
\]

With:  
- \( m_1 \) = mass of the louver \([kg/m^1]\)  
- \( S_0 \) = static moment about EA \([kgm/m^1]\)  
- \( K_h \) = translational spring stiffness \([Nm/m^1]\)  
- \( I_0 \) = mass moment of inertia \([kgm^2/m^1]\)  
- \( K_\theta \) = torsional spring stiffness \([Nm/rad/m^1]\)  
- \( h \) = vertical displacement \([m]\)  
- \( \theta \) = rotation \([rad]\)

The system of equations in equation (3.16) does not yet contain the aerodynamic contributions. When a mass-spring-system is subjected to an externally applied force, this force is usually written on the right hand side of the equations of motion. However, since the aerodynamic forces linearly depend on one of the degrees of freedom (rotation \( \theta \)) the forces are written on the left hand side of the equations of motion as well. Thereby, it has to be noted that the lift force is by definition negative when acting upwardly and the moment about the elastic axis is positive in case of nose up motion. Writing these forces on the left-hand side of the equation yields the opposite i.e. a positive lift contribution and a negative moment contribution.
Including the aerodynamic stiffness contributions, as given by equations (3.14) and (3.15), into the equations of motion results into:

\[
\begin{bmatrix}
  m_1 & S_0 \\
  S_0 & I_0
\end{bmatrix}
\begin{bmatrix} h \\ \theta \end{bmatrix} + \begin{bmatrix} K_h & 0 \\
  0 & K_\theta \end{bmatrix}
\begin{bmatrix} h \\ \theta \end{bmatrix} + \begin{bmatrix} 0 & Q_L \\
  0 & -Q_M \end{bmatrix}
\begin{bmatrix} h \\ \theta \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}
\]

(3.17)

As the equations of motion for the typical section are known, the response for certain conditions can be determined. However, a reference model of the actual cross-section of the louver has to be introduced to determine the mass and stiffness coefficients in equation (3.17).

![Figure 3.5: Louver A1, four-digit NACA profile design of the louver (dimensions in mm)](image)

Figure 3.5 shows Louver A1, the preliminary design of the cross-section of the louver. This design is used to derive the mass, and stiffness parameters for the equations of motion. The cross-sectional properties of the louver are given in Table 3.1. It is assumed that the louver is constructed out of a bi-directional glass fibre composite material. Its material properties are given in Table 3.2.

<table>
<thead>
<tr>
<th>Cross-Sectional Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chord length (c) [m]</td>
<td>0,800</td>
</tr>
<tr>
<td>Thickness louver (hL)[m]</td>
<td>0,100</td>
</tr>
<tr>
<td>Cross-sectional area (A) [m²]</td>
<td>0,0088</td>
</tr>
<tr>
<td>Position aerodynamic centre (AC) [m]</td>
<td>0,200</td>
</tr>
<tr>
<td>Position elastic axis (EA) [m]</td>
<td>0,297</td>
</tr>
<tr>
<td>Eccentricity between AC and EA (ec) [m]</td>
<td>0,097</td>
</tr>
<tr>
<td>Position centre of gravity (CG) [m]</td>
<td>0,386</td>
</tr>
<tr>
<td>Bending moment of inertia (Iₚ) [m⁴]</td>
<td>1,02·10⁻⁵</td>
</tr>
<tr>
<td>Torsional moment of inertia (Iₜ) [m⁴]</td>
<td>3,26·10⁻⁵</td>
</tr>
<tr>
<td>Total length louver (L) [m]</td>
<td>6,00</td>
</tr>
</tbody>
</table>

Table 3.1: Cross-sectional properties louver

<table>
<thead>
<tr>
<th>Material Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Density (ρ) [kg/m³]</td>
<td>1 970</td>
</tr>
<tr>
<td>Young’s modulus (E) [N/m²]</td>
<td>2,15·10¹⁰</td>
</tr>
<tr>
<td>Poison factor (ν) [-]</td>
<td>0,23</td>
</tr>
<tr>
<td>Shear modulus (G) [N/m²]</td>
<td>8,74·10⁹</td>
</tr>
</tbody>
</table>

Table 3.2: Material properties applied composite material (Bhagwand D. Agarwal, 2006)

As all the relevant parameters and constants are known, the coefficients of the mass matrix are determined first. It has to be noted that the louver is considered per unit span. Therefore the coefficients of the mass matrix are per unit length of the louver. The exact calculation of the coefficients is given in Appendix A.1.
The first value of the mass matrix, $m_1$, is determined as:

$$m_1 = 17.33 \text{ kg/m}^1$$  \hspace{1cm} (3.18)

Next the static moment $S_0$ can be determined as:

$$S_0 = 1.54 \text{ kglm/m}^1$$  \hspace{1cm} (3.19)

Leaving the mass moment of inertia $I_0$ to be determined for the mass matrix (Dimitriadis, 2007):

$$I_0 = 1.11 \text{ kgm}^2/\text{m}^1$$  \hspace{1cm} (3.20)

Next step is to determine the spring stiffnesses of the louver. The translational spring stiffness ($K_h$) and the rotational spring stiffness ($K_\theta$) are calculated in agreement with equations (3.4) and (3.8) respectively:

$$K_h = 6.48 \cdot 10^4 \text{ N/m/m}$$  \hspace{1cm} (3.21)

$$K_\theta = 6.34 \cdot 10^4 \text{ Nm/rad/m}$$  \hspace{1cm} (3.22)

As given by equations (3.14) and (3.15) the aerodynamic stiffness contributions ($Q_L$ and $Q_M$) depend upon the dynamic pressure $q$. In equation (3.9) it is explained that $q$ depends on the density of air ($\rho$) and the wind speed ($U$). For now, it is assumed that the dynamic pressure is constant and determined according to the values given by Table 3.3. The wind speed $U = 32.6$ m/s represents a Beaufort force 12. The aerodynamic coefficients are determined in agreement with (3.14) and (3.15) and the geometrical parameter $c$ and $ec$ as given by Table 3.1:

$$Q_L = 3285.34 \text{ N/m}$$  \hspace{1cm} (3.23)

$$Q_M = 318.68 \text{ Nm/m}$$  \hspace{1cm} (3.24)

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Air density ($\rho$) [kg/m$^3$]</td>
<td>1.23</td>
</tr>
<tr>
<td>Wind velocity ($U$) [m/s]</td>
<td>32.6</td>
</tr>
</tbody>
</table>

Table 3.3: Air density and wind velocity (Cavcar) (KNMI, 2011)

Now all the coefficients are known the equations of motion become (equation (3.17)):

$$
\begin{bmatrix}
17.33 & 1.54 \\
1.54 & 1.11
\end{bmatrix}
\begin{bmatrix}
h \\
\dot{h}
\end{bmatrix}
+\left(\begin{bmatrix}
6.48 \cdot 10^4 & 0 \\
0 & 6.34 \cdot 10^4
\end{bmatrix} + \begin{bmatrix}
0 & 3285.34 \\
0 & -318.68
\end{bmatrix}\right)
\begin{bmatrix}
h \\
\dot{\theta}
\end{bmatrix}
= \begin{bmatrix}
0 \\
0
\end{bmatrix}
$$  \hspace{1cm} (3.25)

The system of equations given by equation (3.25) is evaluated by using MATLAB R2013A (MATLAB). However, MATLAB is only capable to solve first order differential equations. As the system of equations contains second order derivatives, the equations have to be rewritten into state space formulation. This formulation is based upon the following principle (The Mathworks, Inc, 2013):
\[
\begin{align*}
    y_1 &= h \\
    \dot{y}_1 &= \dot{h} = y_2 \\
    y_2 &= \dot{h} \\
    \dot{y}_2 &= \dot{h} = y_3 \\
    y_3 &= \theta \\
    \dot{y}_3 &= \dot{\theta} = y_4 \\
    y_4 &= \dot{\theta}
\end{align*}
\] (3.26)

A full derivation of the state space script is attached in Appendix A.2.1., it is noted that the input vector \([\dot{y}_1 \, \dot{y}_2 \, \dot{y}_3 \, \dot{y}_4]^T\) is replaced by a vector expressed in terms that can be recognised by MATLAB: \([x(1) \, x(2) \, x(3) \, x(4)]^T\).

Since the system of equations in equation (3.25) is not subjected to an external loading (i.e. right-hand side of the equation equals zero) the system requires a set of initial conditions to respond to. The set of initial conditions is given in Table 3.4.

<table>
<thead>
<tr>
<th>Initial conditions</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial translation ((h_0)) [m]</td>
<td>0,01</td>
</tr>
<tr>
<td>Initial translational velocity ((\dot{h}_0)) [m/s]</td>
<td>0</td>
</tr>
<tr>
<td>Initial rotation ((\theta_0)) [rad]</td>
<td>0,0015</td>
</tr>
<tr>
<td>Initial rotational velocity ((\dot{\theta}_0)) [rad/s]</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 3.4: Initial conditions

Solving the system of equations given by equation (3.25) with MATLAB, results into the response of the louver as shown in Figure 3.6, for \(0 \leq t \leq 1\).

![Figure 3.6: Response Louver A1 according to the steady aerodynamic model, for \(U = 32.6\ m/s\)](image)

The graphs of Figure 3.6 show an expected outcome. No damping is included so a harmonic response is expected. Secondly, as the system contains two degrees of freedom, two different frequencies can be distinguished. The rotational response (Figure 3.6B) seems to follow the frequency of the translations, but is strongly influenced by the frequency of the rotational degree of freedom. Another aspect is that for the given dynamic pressure the system remains stable, as no divergence occurs.
3.4. Low-Frequency aerodynamic model

As previously stated, the low-frequency model takes the vertical translation speed, $\dot{h}(t)$, of the louver into account. This is shown in Figure 3.7. Since speed is the first derivative of displacement, the equations of motion is expanded by a damping matrix.

Assuming a small angle approximation, the total rotation of the louver, including the vertical translation speed, becomes (Hulshoff, 2003):

$$\theta_{tot}(t) = \theta(t) + \frac{\dot{h}(t)}{U} \quad (3.27)$$

With:
- $\theta$ = rotation louver relative to zero lift line [rad]
- $\dot{h}$ = translation speed [m/s]
- $U$ = free stream wind speed [m/s]

Inserting the total rotation of the louver, as given by equation (3.27), into the aerodynamic forces given by equations (3.12) and (3.13) results into:

$$L_F(t) = 2qc\pi \left( \theta(t) + \frac{\dot{h}(t)}{U} \right) \quad (3.28)$$

$$M_{EA}(t) = 2q(ec)\pi \left( \theta(t) + \frac{\dot{h}(t)}{U} \right) \quad (3.29)$$

The introduction of $\dot{h}(t)$ in equations (3.28) and (3.29) results into an aerodynamic damping matrix $A$, defined by:

$$A = \begin{bmatrix} 2q\frac{\pi}{U} & 0 \\ -2q\frac{\pi}{U}(ec) & 0 \end{bmatrix} = \begin{bmatrix} c_1 & 0 \\ -c_3 & 0 \end{bmatrix} \quad (3.30)$$

Inserting $A$ as given by equation (3.30) into equation (3.17) results into the new equations of motion, describing a damped motion of the louver:
\( \begin{bmatrix} m_1 & S_0 \\ S_0 & I_0 \end{bmatrix} \begin{bmatrix} \dot{h} \\ \dot{\theta} \end{bmatrix} + \begin{bmatrix} C_1 & 0 \\ -C_3 & 0 \end{bmatrix} \begin{bmatrix} h \\ \theta \end{bmatrix} + \left( \begin{bmatrix} K_h & 0 \\ 0 & K_\theta \end{bmatrix} + \begin{bmatrix} 0 & Q_L \\ 0 & -Q_M \end{bmatrix} \right) \begin{bmatrix} \dot{h} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \) \tag{3.31}

\( C_1 \) and \( C_3 \) are determined in agreement with equation (3.30). The dynamic pressure \( q \) is given in equation (3.9), with \( \rho = 1.23 \text{ kg/m}^3 \) and \( U = 32.6 \text{ m/s} \). Geometrical parameters \( c \) and \( ec \) are given in Table 3.1, \( c = 0.8 \text{ m} \) and \( ec = 0.097 \text{ m} \). The aerodynamic damping coefficients are:

\[
C_1 = 100.78 \text{ Ns/m/m}^1 \tag{3.32}
\]

\[
C_3 = 9.78 \text{ Ns/m}^1 \tag{3.33}
\]

Now all the coefficients of the equations of motion are known, the equations of motion for the low-frequency model are represented by (see equation (3.25)):

\[
\begin{bmatrix} 17.33 & 1.54 \\ 1.54 & 1.11 \end{bmatrix} \begin{bmatrix} \dot{h} \\ \dot{\theta} \end{bmatrix} + \begin{bmatrix} 100.78 & 0 \\ -9.78 & 0 \end{bmatrix} \begin{bmatrix} h \\ \theta \end{bmatrix} + \begin{bmatrix} 6.48 \times 10^4 & 0 \\ 0 & 6.34 \times 10^4 \end{bmatrix} \begin{bmatrix} h \\ \theta \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \tag{3.34}
\]

In order to solve equation (3.34) with MATLAB, equation (3.34) is rewritten into state space format. The state space-file is attached in Appendix A.2.2. Solving the system of equations in agreement with the initial conditions given in Table 3.4, results into the response of the louver as shown in Figure 3.8, for \( 0 \leq t \leq 1 \).

![Figure 3.8: Response Louver A1 according to the low-frequency model, for U = 32.6 m/s](image)

Comparing Figure 3.8 to Figure 3.6, shows the influence of the aerodynamic damping matrix \( A \) on the response of the louver. The distinction between two frequencies is still clear, but the amplitude of the motion decreases as time \( t \) increases.

### 3.5. Low-frequency model including structural damping

The final step to complete the model of the louver is to include structural damping. Therefore an additional damping matrix, \( C_s \), is constructed. \( C_s \) is constructed by applying Rayleigh damping to the system. This method is based on the following principle (Inman, 1994):
\[ C = \alpha M + \beta K \]  

Where

\[
\begin{bmatrix}
\alpha \\
\beta
\end{bmatrix} = 2 \begin{bmatrix} \omega_{n,1} \omega_{n,2} \\
\omega_{n,2}^2 - \omega_{n,1}^2
\end{bmatrix} \begin{bmatrix}
\frac{-\omega_{n,1}}{\omega_{n,2}} & 1 \\
1 & \frac{-\omega_{n,1}}{\omega_{n,2}}
\end{bmatrix} \begin{bmatrix}
\zeta_1 \\
\zeta_2
\end{bmatrix}
\]

With:
- \( \alpha \) = Rayleigh coefficient related to \( M \) [-]
- \( M \) = mass matrix [-]
- \( \beta \) = Rayleigh coefficient related to \( K \) [-]
- \( K \) = stiffness matrix [-]
- \( \omega_{n,i} \) = natural frequencies (\( i = 1, 2 \)) [rad/s]
- \( \zeta_i \) = damping ratio (\( i = 1, 2 \)) [-]

To be able to derive both Rayleigh coefficients, the natural frequencies (\( \omega_{n,1} \) and \( \omega_{n,2} \)) of the system have to be known. To obtain these frequencies, a general solution has to be found for equation (3.16), where the undamped equations of motion are without any contribution of the aerodynamic forces. The general solution is of the form (Inman, 1994):

\[ x = \tilde{x} e^{i \omega t} \]  

with

\[ x = [h \ \theta]^T \] and \( \tilde{x} = [\tilde{h} \ \tilde{\theta}]^T \) and \( i = \sqrt{-1} \)

Substituting equation (3.36) into equation (3.16) results into:

\[
\begin{bmatrix}
-\omega^2 m_1 + K_h \\
-\omega^2 S_\theta
\end{bmatrix} \tilde{x} e^{i \omega t} = 0
\]

Solving equation (3.37) for a non-trivial solution, leads to two absolute values for \( \omega_{n,1} \) and \( \omega_{n,2} \):

\[ \omega_{n,1} = 60.89 \text{ rad/s} \quad \omega_{n,2} = 256.33 \text{ rad/s} \]  

(3.38)

As the natural frequencies are known, the Rayleigh coefficients \( \alpha \) and \( \beta \) can be determined. It has to be noted that it is assumed that the damping ratio \( \zeta \) does not differ for both modes i.e. \( \zeta_1 = \zeta_2 = \zeta \). It is assumed that \( \zeta = 0.05 \). Elaborating equation (3.35) by first determining the Rayleigh coefficients followed by \( C \) results into:

\[
\begin{bmatrix}
\alpha \\
\beta
\end{bmatrix} = \begin{bmatrix} 4.92 \\
3.15 \cdot 10^{-4}
\end{bmatrix}
\]

(3.39)

Now the damping matrix \( C \), representing the structural damping, is determined, the complete equations of motion can be constructed. Inserting \( C \) into equation (3.31) results into:

\[
\begin{bmatrix}
m_1 & S_\theta \\
S_\theta & I_\theta
\end{bmatrix} \begin{bmatrix}
\dot{h} \\
\dot{\theta}
\end{bmatrix} + \begin{bmatrix}
C_{s,1} & C_{s,2} \\
C_{s,3} & C_{s,4}
\end{bmatrix} \begin{bmatrix}
h \\
\theta
\end{bmatrix} + \begin{bmatrix}
C_{1} - C_3 \\
C_3
\end{bmatrix} \begin{bmatrix}
0 \\
0
\end{bmatrix} \begin{bmatrix}
h \\
\theta
\end{bmatrix} + \begin{bmatrix}
K_h & 0 \\
0 & K_\theta
\end{bmatrix} \begin{bmatrix}
h \\
\theta
\end{bmatrix} + \begin{bmatrix}
0 & L \\
0 & -M_EA
\end{bmatrix} \begin{bmatrix}
\dot{h} \\
\dot{\theta}
\end{bmatrix} = \begin{bmatrix} 0 \\
0
\end{bmatrix}
\]

(3.40)

As all the coefficients in equation (3.40) are known the response of the louver is determined. Solving equation (3.40) with MATLAB in accordance with the initial conditions given in Table 3.4,
a dynamic pressure corresponding to a wind velocity \( U = 32.6 \text{ m/s} \) and air density \( \rho = 1.23 \text{ kg/m}^3 \), the response is as shown in Figure 3.9. The full state space script is attached in Appendix A.2.3.

Comparing Figure 3.9 to the response of the louver in Figure 3.8, the influence of the structural damping can clearly be distinguished. Due to structural damping, the louver returns faster to its neutral position and the influence of a second frequency is less dominant in Figure 3.9B as it is in Figure 3.8B.

### 3.6. Instability of the louver

Until now, only stable responses of the louver have been evaluated. However, the point at which the louver becomes unstable is of importance as well. The instability behaviour is characterised by flutter, which is a self-sustaining oscillation due to fluid-structure interactions between the airflow over the louver and the louver itself. A famous example of flutter instability is the collapse of the Tacoma Narrows Bridge, see Figure 3.10.

Figure 3.10: Tacoma Narrows bridge minutes before collapse (Image World, 2012)

Figure 3.11 shows Collar’s triangle and illustrates several aeroelastic problem types in relation to the forces considered within the model of the louver.
Although multiple aerospace problems are shown in Figure 3.11, only the divergence and the flutter problem are investigated in this thesis. To illustrate both, a simplified model will be discussed to illustrate the divergence problem first. Next, the flutter behaviour of the three aerodynamic models is discussed.

### 3.6.1. Torsional divergence

Divergence is a phenomenon which is related to aerodynamic and structural forces, while flutter is related to inertial forces as well. It can be imagined that an increase in dynamic pressure $q$ results into an increase in the aerodynamic forces, $L_F(t)$ and $M_{EA}(t)$. However, as long as the rotational spring stiffness $K_θ$ is able to withstand these forces, the louver remains stable. If $q$ causes aerodynamic forces which cannot be withstand by the torsional spring stiffness $K_θ$, then $θ$ grows unboundedly, since $K_θ$ remains constant.

The lowest wind speed, $U$, at which $θ$ grows unboundedly, is called the torsional divergence speed. This speed can be determined by formulating the sum of the moments about $EA$ for a 1-DOF system (see Figure 3.12). In case of a symmetric profile as designed for the louver and assuming small angle approximation, it is derived that the moment equation about $EA$ is:

$$\Sigma M_{EA} = 0 \rightarrow 2q(ec)\pi θ - K_θθ = 0 \quad (3.41)$$

![Figure 3.12: 1-DOF system for the divergence problem](image)
When considering the stability of the system, an upper limit for the dynamic pressure is found by considering the stability of the system for small perturbations, for which yields (Hulshoff, 2003):

$$\frac{dM_{EA}}{d\theta} \leq 0$$  \hspace{1cm} (3.42)

The upper limit of the dynamic pressure can be found by computing the derivative of (3.41) in agreement with (3.42) and rewriting it into:

$$q \leq \frac{K_\theta}{2(ec)c\pi}$$  \hspace{1cm} (3.43)

Computing this upper limit of the dynamic pressure leads to an upper limit of the wind speed for which the louver remains stable. Deriving the upper limit for the wind velocity ($U_{max}$), of the current cross-sectional design of the louver (using the parameters given by Table 3.1, Table 3.3, $q$ as given by (3.9) and $K_\theta$ as given by (3.22)) results into:

$$\frac{1}{2}\rho U^2 \leq \frac{K_\theta}{2(ec)c\pi} \rightarrow U = U_{div} \leq 459.8 \text{ m/s}$$  \hspace{1cm} (3.44)

As long as the wind speed remains below $U_{div}$, the louver remains stable for the 1-DOF system considered. This is shown in Figure 3.13, for $\theta_0 = 0.05 \text{ rad}$. Increasing the wind speed to $U = 459.9 \text{ m/s}$, results into an unlimited rotation of the louver, see Figure 3.14. In the following paragraphs, the influences of inertial effects on a 2-DOF system (i.e. rotation and translation) are discussed.

**3.6.2. Flutter instability of the steady aerodynamic model**

To investigate flutter instability of the steady aerodynamic model a solution has to be found for the equations of motion, for convenience equation (3.17) is recapitulated:

$$\begin{bmatrix} m_1 & S_0 \\ S_0 & I_0 \end{bmatrix} \begin{bmatrix} \dot{h} \\ \dot{\theta} \end{bmatrix} + \begin{bmatrix} K_h & 0 \\ 0 & K_\theta \end{bmatrix} \begin{bmatrix} h \\ \theta \end{bmatrix} + \begin{bmatrix} 0 & Q_L \\ 0 & -Q_M \end{bmatrix} \begin{bmatrix} h \\ \theta \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$  \hspace{1cm} (3.45)

To find a solution for (3.45) it is assumed that the response will be of the form (Hulshoff, 2003):

$$x = \hat{x}e^{pt}$$  \hspace{1cm} (3.46)

with $x = [h \quad \theta]^T$ and $\hat{x} = [\dot{h} \quad \dot{\theta}]^T$ and $p = (\sigma + i\omega)$
Rewriting the equations of motion given by equation (3.45) in agreement with equation (3.46) results into:

\[
\begin{bmatrix}
  m_1 p^2 + K_h & S_0 p^2 + Q_L \\
  S_0 p^2 & I_0 p^2 + K_0 - Q_M
\end{bmatrix}
\begin{bmatrix} \dot{x} \\ \dot{e} \end{bmatrix} = 0
\]  
(3.47)

Recall equations (3.14) and (3.15):

\[ Q_L = 2 q c \pi \quad \text{and} \quad Q_M = 2 q (ec) \pi \]  
(3.48)

In order to find a non-trivial solution for \( \dot{x} \), the determinant of the coefficient matrix in equation (3.47) needs to be zero. This leads to the characteristic equation of the system (Hulshoff, 2003):

\[ a_4 p^4 + a_2 p^2 + a_0 = 0 \]  
(3.49)

With:

\[ a_4 = m_1 I_0 - S_0^2 \]
\[ a_2 = m_1 K_0 + I_0 K_h - 2 q c \pi (m_1 (ec) + S_0) \]
\[ a_0 = K_h (K_0 - 2 q (ec) \pi) \]

The flutter point, at which the cross-section transfers from a stable to an unstable configuration, can be determined by equating the discriminant of the characteristic equations to zero (Hulshoff, 2003). The discriminant of equation (3.49) is given by:

\[ a_2^2 - 4 a_4 a_0 = 0 \]  
(3.50)

Solving equation (3.50) according to the values of the parameters for the steady aerodynamic model, recapitulated by Table 3.5, results into two values for the dynamic pressure \( q \).

<table>
<thead>
<tr>
<th>Parameters matrix coefficients typical section</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>( m_1 ) [kg/m(^1)]</td>
<td>17.33</td>
</tr>
<tr>
<td>( S_0 ) [kgm/m(^1)]</td>
<td>1.54</td>
</tr>
<tr>
<td>( I_0 ) [kgm(^2)/m(^1)]</td>
<td>1.11</td>
</tr>
<tr>
<td>( K_0 ) [Nm/rad/m(^1)]</td>
<td>6.34 \times 10^4</td>
</tr>
<tr>
<td>( K_h ) [N/m/m(^1)]</td>
<td>6.48 \times 10^4</td>
</tr>
</tbody>
</table>

Table 3.5: Values parameters steady aerodynamic model

\[ q_1 = 46196.2 \text{ N/m}^2 \quad q_2 = 90282.0 \text{ N/m}^2 \]  
(3.51)

The lowest value of the dynamic pressure, \( q_i \) in equation (3.51), represents the flutter point of the louver, dependent on the flutter wind speed \( U_f \). The value of \( U_f \) can be determined by solving \( q_i \) for \( U_f \) (see equation (3.9)), whereby it is still assumed that \( \rho = 1.23 \text{ kg/m}^3 \):

\[ q_1 = \frac{1}{2} \rho U^2 \rightarrow U = U_f = 274.1 \text{ m/s} \]  
(3.52)

For the new system of equations, the aerodynamic stiffness contributions (equations (3.14) and (3.15)) are derived according to the wind speed \( U_f \) as given by equation (3.52). Similar to the procedure in paragraph 3.3, the new system of equations for the steady aerodynamic model is constructed. Using MATLAB to solve this system of equations, according to the initial conditions given by Table 3.4, results into the response as given by Figure 3.15. The self-excited vibration can clearly be seen. The amplitude keeps increasing over time which leads to failure of the louver.
\[ Q_L = 232 \ 207.4 \, \text{N/m} \quad (3.53) \]
\[ Q_M = 22 \ 524.1 \, \text{Nm/m} \quad (3.54) \]
\[
\begin{bmatrix}
17.33 & 1.54 \\
1.54 & 1.11
\end{bmatrix}
\begin{bmatrix}
\dot{h} \\
\dot{\theta}
\end{bmatrix} + \begin{bmatrix}
0 & 6.48 \times 10^4 \\
6.34 \times 10^4 & 0
\end{bmatrix}
\begin{bmatrix}
\dot{h} \\
\dot{\theta}
\end{bmatrix} + \begin{bmatrix}
0 & 232 \ 207.4 \\
22 \ 524.1 & 0
\end{bmatrix}
\begin{bmatrix}
h \\
\theta
\end{bmatrix} = \begin{bmatrix} 0 \\
0 \end{bmatrix} 
\quad (3.55)
\]

The peculiarity of the flutter point is illustrated by decreasing the wind velocity with 1 m/s to \( U = 273 \, \text{m/s} \). Calculating the aerodynamic forces, and solving the equations of motion for these new aerodynamic contributions and the same initial conditions as previously, results in Figure 3.16. It can be seen that the motion of the louver does not grow unboundedly, although the amplitude of the response fluctuates in time instead of being constant (as shown in Figure 3.6).

Comparing the instability phenomena of Figure 3.15 and Figure 3.14 shows the typical difference. The 1-DOF model diverges rapidly after reaching the upper limit of \( q \), while the 2-DOF shows a self-sustaining amplitude. Another difference is the strong influence of the inertia effects on the upper limit of \( q \), which is reduced by more than 40% in the last situation.

\[ \begin{align*}
\text{Figure 3.15: Flutter instability of the steady aerodynamic model, for } U = 274.1 \, \text{m/s} \\
\text{Figure 3.16: Response steady aerodynamic model, for } U = 273 \, \text{m/s}
\end{align*} \]

\[ \text{3.6.3. Flutter instability low-frequency model} \]

The second aerodynamic model takes a damping term into account, including a negative damping coefficient affecting the maximal dynamic pressure. It is expected that the maximum dynamic pressure decreases due to this negative contribution.

The equations of motion of the low-frequency model are recapitulated (equation (3.31)):
\[
\begin{bmatrix} m & S & 0 \\
S & I_0 & 0 \\
0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
\dot{h} \\
\dot{\theta}
\end{bmatrix} + \begin{bmatrix}
C_1 \\
-C_3 \\
0
\end{bmatrix}
\begin{bmatrix}
\dot{h} \\
\dot{\theta}
\end{bmatrix} + \begin{bmatrix}
K_h & 0 \\
0 & K_\theta \\
0 & 0
\end{bmatrix}
\begin{bmatrix}
h \\
\theta
\end{bmatrix} + \begin{bmatrix}
0 \\
0 & -Q_M
\end{bmatrix}
\begin{bmatrix}
h \\
\theta
\end{bmatrix} = \begin{bmatrix} 0 \\
0 \end{bmatrix} 
\quad (3.56)
\]

In order to obtain a solution for the system of equations given by equation (3.56), it is assumed that the solution is of the form as given by equation (3.46). Rewriting equation (3.56) in agreement with (3.46), results into:
Recall equations (3.14), (3.15), and (3.30):

\[ Q_L = 2q\pi \quad Q_M = 2q(\text{ec})\pi \quad C_1 = 2 \frac{q}{U} \pi \quad \text{and} \quad C_3 = 2 \frac{q}{U} (\text{ec})\pi \]  

(3.58)

A non-trivial solution of equation (3.57) is obtained by a similar procedure as in paragraph 3.6.2. The characteristic equation of the low-frequency model is given by:

\[ a_4 p^4 + a_3 p^3 + a_2 p^2 + a_1 p + a_0 = 0 \]  

(3.59)

With:

\[
\begin{align*}
  a_4 &= m_1 I_0 - S_0^2 \\
  a_3 &= 2 \frac{q}{U} \pi (I_0 + S_0 (\text{ec})) \\
  a_2 &= m_1 K_0 + I_0 K_h - 2q \pi (m_1 (\text{ec}) + S_0) \\
  a_1 &= 2K_0 \frac{q}{U} \pi \\
  a_0 &= K_h (K_0 - 2q (\text{ec})\pi)
\end{align*}
\]

The roots of the characteristic equations define the types of motion of the louver. However, as the characteristic equation has become a full fourth order polynomial it is much harder to derive the flutter point analytically. Nevertheless, it is numerically shown that the flutter point for the low-frequency model is reached for \( U_f = 160 \, \text{m/s} \). The translation and rotation graphs are given by Figure 3.17. Note that for this case \( 0 \leq t \leq 5 \).

![Figure 3.17: Response Louver A1 according to the low frequency model, for \( U = 160 \, \text{m/s} \)]

As expected the flutter point has decreased to \( U_f = 160 \, \text{m/s} \). However, it can be seen in Figure 3.17, that the amplitudes do not grow as rapidly as in the case of the steady aerodynamic model (see Figure 3.15).
3.6.4. Flutter instability model including structural damping

As has already been explained, the third model takes structural damping into account. It is assumed that the structural damping will have a positive influence on \( U_f \) i.e. \( U_f \) has a higher value when compared to the low-frequency model.

The full equations of motion for the model including structural damping are recapitulated (equation (3.40)):

\[
\begin{align*}
\begin{bmatrix}
m_1 & S_0 & 0 \\
S_0 & I_0 & 0 \\
0 & 0 & 0 \\
\end{bmatrix}
\begin{bmatrix}
h \\
\dot{h} \\
\theta \\
\dot{\theta} \\
\end{bmatrix}
+
\begin{bmatrix}
C_{s,1} & C_{s,2} \\
C_{s,3} & C_{s,4} \\
-C_3 & 0 \\
0 & 0 \\
\end{bmatrix}
\begin{bmatrix}
h \\
\dot{h} \\
\theta \\
\dot{\theta} \\
\end{bmatrix}
+
\begin{bmatrix}
K_h & 0 & 0 \\
0 & K_\theta & 0 \\
0 & -Q_L & 0 \\
\end{bmatrix}
\begin{bmatrix}
\theta \\
\dot{\theta} \\
\phi \\
\dot{\phi} \\
\end{bmatrix}
&=
\begin{bmatrix}
0 \\
0 \\
0 \\
0 \\
\end{bmatrix}
\end{align*}
\]

(3.60)

Applying the same method to derive the characteristic equation for this model as for the low-frequency model (paragraph 3.6.3) leads to the following characteristic equation:

\[
a_4 p^4 + a_3 p^3 + a_2 p^2 + a_1 p + a_0 = 0
\]

(3.61)

With:

\[
\begin{align*}
a_4 &= m_1 I_0 - S_0^2 \\
a_3 &= m_1 C_{s,4} + I_0 C_{s,1} - S_0 (C_{s,2} + C_{s,3}) + 2 \frac{q}{U} \pi (I_0 + S_0 (ec)) \\
a_2 &= m_1 K_\theta + I_0 K_h + C_{s,1} C_{s,4} - C_{s,2} C_{s,3} + 2 \frac{q}{U} \pi C_{s,2} (ec) + C_{s,4}) - 2 q \pi (m_1 (ec) + S_0) \\
a_1 &= C_{s,1} K_\theta + C_{s,4} K_h - 2 q \pi C_{s,1} (ec) + C_{s,3}) + 2 K_h \frac{q}{U} \pi ec \\
a_0 &= K_h (K_\theta - 2 q (ec) \pi)
\end{align*}
\]

The characteristic equation is a full fourth order polynomial again. Similar to the previous model, the value for \( U_f \) is determined numerically. It can be shown that for \( U_f = 229 \) m/s the louver becomes unstable. Figure 3.18 shows the response of the louver in case of aerodynamic forces for \( U_f = 229 \) m/s.

![Figure 3.18: Response Louver A1 including structural damping, for U = 229 m/s](image)

3.7. External loading

Until now it has been assumed that the wind flow is in line with the horizontal position of the louver. As can be imagined, this will not always be the case in the environment in which the louver is applied. The effect of a non-horizontal wind direction can be taken into account by...
introducing the relative angle of attack $\theta_1$, see Figure 3.19. A positive value for $\theta_1$ has a positive influence on the lift force i.e. an increase in $\theta_1$ leads to a decrease in lift force $L_F$.

![Figure 3.19: Cross-section louver including direction U ($\theta_1$)](image)

In the previous paragraphs the aerodynamic forces $L_F(t)$ and $M_{EA}(t)$ were fully dependent on the rotation of the louver. As a consequence of the introduction of $\theta_1$ these forces have become partially dependent of a constant, which results into a right-hand side of the equations of motion.

In addition it is assumed that $U$ is not constant anymore. Therefore, an external fluctuating wind signal $U(t)$ is introduced. The definition of $U(t)$ is given later on.

Recalling the equations for $L_F(t)$ and $M_{EA}(t)$, (3.12) and (3.13) respectively, and rewriting these equations incorporating $\theta_1$ and $U(t)$ results into:

$$L_F(t) = \frac{1}{2} \rho U(t)^2 \times 2c\pi(\theta(t) - \theta_1)$$

$$M_{EA}(t) = \frac{1}{2} \rho U(t)^2 \times 2(ec)c\pi(\theta(t) - \theta_1)$$

(3.62) (3.63)

With:  
$U(t)$ = external fluctuating wind signal [m/s]  
$\theta_1$ = angle of attack relative to zero lift line [rad]

Note that the dynamic pressure $q$, as given by equation (3.9), is fully elaborated to make clear that it is not a constant anymore.

Inserting equations (3.62) and (3.63) into the equations of motion for the steady aerodynamic model as given by equation (3.17), results into:

$$\begin{bmatrix} m_1 & S_\theta & h \end{bmatrix} + \begin{bmatrix} K_h & 0 & 0 \\ 0 & 0 & -\frac{1}{2} \rho U(t)^2 \times 2(\text{ec})c\pi \\ 0 & -\frac{1}{2} \rho U(t)^2 \times 2(\text{ec})c\pi & 0 \end{bmatrix} \begin{bmatrix} h \\ \theta_1 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} \rho U(t)^2 \times 2c\pi\theta_1 \\ -\frac{1}{2} \rho U(t)^2 \times 2(\text{ec})c\pi\theta_1 \end{bmatrix}$$

(3.64)

Recall that the lift acting upwardly is by definition negative. For convenience equation (3.64) is rewritten into:
With:  
\[ Q_{\text{L,fl}} = \text{fluctuating lift force coefficient} = \rho U(t)^2 \tan(\theta) \text{ [N/m]} \] 
\[ Q_{\text{L,ex}} = \text{external lift force coefficient} = \rho U(t)^2 \tan(\theta) \text{ [N/m]} \] 
\[ Q_{\text{M,fl}} = \text{fluctuating moment coefficient} = \rho U(t)^2 \tan(\theta) \text{ [Nm/m]} \] 
\[ Q_{\text{M,ex}} = \text{external moment coefficient} = \rho U(t)^2 \tan(\theta) \text{ [Nm/m]} \] 

\[ U(t) = \text{fluctuating wind signal [m/s]} \]

The external loading is represented by a wind signal measured by the IRWES team during a storm on a roof. This signal is shown in Figure 3.20. It can be clearly seen how the speed of the wind fluctuates in time. The wind signal will be approximated using a Fourier Series, see equation (3.66) (Meirovitch, 2001) & (The Mathworks, Inc, 2013). When an eight-term Fourier Series is applied to the wind signal as shown in Figure 3.20, the Fourier coefficients are as given by Table 3.6. Figure 3.21 shows the approximation of the Fourier Series to the measure points of the wind signal in Figure 3.20.

\[ U(t) = a_0 + \sum_{n=1}^{\infty} (a_n \cos(n\omega t) + b_n \sin(n\omega t)) \quad (3.66) \]

<table>
<thead>
<tr>
<th>Fourier coefficients for n = 8</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>( a_0 )</td>
<td>13,51</td>
</tr>
<tr>
<td>( a_1 )</td>
<td>-0,7446</td>
</tr>
<tr>
<td>( a_2 )</td>
<td>-1,620</td>
</tr>
<tr>
<td>( a_3 )</td>
<td>2,613</td>
</tr>
<tr>
<td>( a_4 )</td>
<td>0,7562</td>
</tr>
<tr>
<td>( a_5 )</td>
<td>-0,2481</td>
</tr>
<tr>
<td>( a_6 )</td>
<td>0,2751</td>
</tr>
<tr>
<td>( a_7 )</td>
<td>0,6015</td>
</tr>
<tr>
<td>( a_8 )</td>
<td>0,05051</td>
</tr>
</tbody>
</table>

Table 3.6: Fourier coefficients eight-term approximation
Before considering the response of the louver under the fluctuating wind speed, the influence of the initial conditions on the response under a static wind load is investigated. Therefore, a wind load corresponding to the mean wind speed ($U_{\text{mean}}$) of Figure 3.21 is used. The Fourier analysis shows that $U_{\text{mean}} = 13.51 \text{ m/s}$ (equal to variable $a_0$ in Table 3.6). This investigation is done according to the low-frequency model.

Determining the aerodynamic contributions to the equations of motion of the low-frequency model (equation (3.31)), according to equations (3.14), (3.15), and (3.30) and for $U = 13.51 \text{ m/s}$, results into:

\[
\begin{align*}
Q_L &= 564.23 \text{ N} \\
Q_M &= 54.73 \text{ Nm} \\
C_1 &= 41.76 \text{ Ns/m}^2 \\
C_3 &= 4.05 \text{ Ns/m}
\end{align*}
\]

Considering the same cross-sectional design of the louver as before (see Figure 3.5), the full equations of motion for the given wind load are:

\[
\begin{bmatrix}
17.33 & 1.54 \\
1.54 & 1.11
\end{bmatrix} \ddot{h} + \begin{bmatrix}
-41.76 & 0 \\
0 & 0
\end{bmatrix} \dot{h} + \begin{bmatrix}
6.48 \cdot 10^4 & 0 \\
0 & 6.34 \cdot 10^4
\end{bmatrix} \begin{bmatrix}
0 \\
0
\end{bmatrix} + \begin{bmatrix}
564.23 \\
0
\end{bmatrix} \begin{bmatrix}
\theta_1 \\
\theta_1
\end{bmatrix} = \begin{bmatrix}
0 \\
0
\end{bmatrix}
\]

Figure 3.22 shows the first response of the louver for the new aerodynamic loading. This response is in accordance with the initial conditions given in Table 3.7. Different sets of initial conditions are given in Table 3.8 to Table 3.11 and the corresponding response graphs in Figure 3.23 to Figure 3.26 respectively.
<table>
<thead>
<tr>
<th>Initial conditions</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial displacement, $h_0$ [m]</td>
<td>0.01</td>
</tr>
<tr>
<td>Initial translational velocity, $h_0$ [m/s]</td>
<td>0</td>
</tr>
<tr>
<td>Initial rotation, $\theta_0$ [rad]</td>
<td>0.0015</td>
</tr>
<tr>
<td>Initial rotational velocity, $\theta_0$ [rad/s]</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 3.7: First set of initial conditions

<table>
<thead>
<tr>
<th>Initial conditions</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial displacement, $h_0$ [m]</td>
<td>0.0015</td>
</tr>
<tr>
<td>Initial translational velocity, $h_0$ [m/s]</td>
<td>0</td>
</tr>
<tr>
<td>Initial rotation, $\theta_0$ [rad]</td>
<td>0.01</td>
</tr>
<tr>
<td>Initial rotational velocity, $\theta_0$ [rad/s]</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 3.8: Second set of initial conditions

<table>
<thead>
<tr>
<th>Initial conditions</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial displacement, $h_0$ [m]</td>
<td>0.1</td>
</tr>
<tr>
<td>Initial translational velocity, $h_0$ [m/s]</td>
<td>0.05</td>
</tr>
<tr>
<td>Initial rotation, $\theta_0$ [rad]</td>
<td>0.1</td>
</tr>
<tr>
<td>Initial rotational velocity, $\theta_0$ [rad/s]</td>
<td>0.05</td>
</tr>
</tbody>
</table>

Table 3.9: Third set of initial conditions

Figure 3.22: Response Louver A1 in accordance with the initial conditions given by Table 3.7

Figure 3.23: Response Louver A1 in accordance with the initial conditions given by Table 3.8

Figure 3.24: Response Louver A1 in accordance with the initial conditions given by Table 3.9
### Changing the initial conditions

Changing the initial conditions leads to different responses and situations. For example if one of the displacement degrees of freedom is given a significantly higher initial value than the other, the degree of freedom with the high initial value is dominant in the response graphs of the louver. A high initial velocity leads to an increase in displacement (immediately after $t = 0$) instead of a decrease is shown in the other cases.

### 3.7.1. Change in angle of attack

Next step is to change the angle of attack i.e. introduce $\theta_1 \neq 0$. The influence of $\theta_1$ is investigated for all three aerodynamic models. In the following, it is assumed that $\theta_1 = 10^\circ (\approx 0.1745 \text{ rad})$. Since a positive value for $\theta_1$ is assumed, it is expected that the louver is subjected to a negative rotation and a positive translation. See Figure 3.19 for the positive definition of $\theta$ and $h$. The MATLAB scripts for the louver subjected to the external wind signal, are attached in Appendix A.3. for all three aerodynamic models.
Recalling equation (3.65) gives the equations of motion for the steady aerodynamic model under influence of $\theta_1$ and $U(t)$:

$$
\begin{bmatrix}
m & S \\
S & I_0
\end{bmatrix}
\dot{\begin{bmatrix} h \\ \theta \end{bmatrix}}
+
\begin{bmatrix}
K_h & 0 \\
0 & K_\theta
\end{bmatrix}
\begin{bmatrix} h \\ \theta \end{bmatrix}
+
\begin{bmatrix}
0 & Q_{L,U} \\
0 & -Q_{M,U}
\end{bmatrix}
\begin{bmatrix} h \\ \theta \end{bmatrix}
= \begin{bmatrix} Q_{L,ex} \\ -Q_{M,ex}\end{bmatrix}
$$

(3.72)

Due to the introduction of an external wind loading, all initial conditions are set to zero. Figure 3.27 shows the response of the steady aerodynamic model if the wind signal $U(t)$ is introduced, for $0 \leq t \leq 60$. Although the louver vibrates strongly, the input of $U(t)$ can be distinguished. It can clearly be seen that the translation of the louver (Figure 3.27A) is positive and the rotation of the louver (Figure 3.27B) is negative.

To eliminate the strong vibrations as shown in Figure 3.27, the low-frequency model is considered. This model takes the damping effects of the fluid-structure interaction into account. The aerodynamic damping matrix $A$, is influenced by $U(t)$ as well. Similar to the procedure in paragraph 3.4, it is shown that $A$ is now represented by:

$$
A = \begin{bmatrix}
\rho U(t) c_p & 0 \\
-p U(t) c_p & 0
\end{bmatrix}
= \begin{bmatrix}
C_{1,U} & 0 \\
-C_{3,U} & 0
\end{bmatrix}
$$

(3.73)

Inserting equation (3.73) into the equations of motion as given by equation (3.72), including $\theta_1$ and $U(t)$, results into:

$$
\begin{bmatrix}
m & S \\
S & I_0
\end{bmatrix}
\dot{\begin{bmatrix} h \\ \theta \end{bmatrix}}
+
\begin{bmatrix}
C_{1,U} & 0 \\
-C_{3,U} & 0
\end{bmatrix}
\begin{bmatrix} h \\ \theta \end{bmatrix}
+
\begin{bmatrix}
K_h & 0 \\
0 & K_\theta
\end{bmatrix}
\begin{bmatrix} h \\ \theta \end{bmatrix}
+
\begin{bmatrix}
0 & Q_{L,U} \\
0 & -Q_{M,U}
\end{bmatrix}
\begin{bmatrix} h \\ \theta \end{bmatrix}
= \begin{bmatrix} Q_{L,ex} \\ -Q_{M,ex}\end{bmatrix}
$$

(3.74)

Assuming no initial conditions and a relative angle of attack $\theta_1 = 10^\circ$, the low-frequency model gives a response of the louver as shown in Figure 3.28, for $0 \leq t \leq 60$. The strong vibrations as shown in Figure 3.27, have been limited to only adjustment vibrations, which are damped out by the damping terms of the fluid-structure interaction.
In the third model structural damping is taken into account. Therefore, the structural damping matrix \( C \) is inserted into the equations of motion given by equation (3.74). It has already been shown that for the case \( \zeta = 0.05 \), the structural damping matrix becomes (see equation (3.39)):

\[
C = \begin{bmatrix}
105.69 & 7.59 \\
7.59 & 25.44
\end{bmatrix}
\]  

(3.75)

Including the damping matrix \( C \) into the equations of motion and solving the system of equations assuming no initial conditions and \( \theta_1 = 10^\circ \), results into the response of the louver as shown in Figure 3.29.

Due to the structural damping the adjustment vibrations are limited. Furthermore, the response is much similar to the response given by Figure 3.28.
The graph of Figure 3.29A shows that the maximal deflection of the louver is just less than 5 mm. This value is well within the limits of the stiffness requirements of the building code, which states that the limit value is $0.004 \times 6\,000\,\text{mm} = 24\,\text{mm}$ (Eurocodes, 2011).

### 3.7.2. Validation of the response of the louver for $U(t)$

It can be shown that a relation between the value of $\theta_1$ and the initial conditions $[x_0]$ exists. Different values for the relative angle of attack $\theta_1$ and initial conditions $[x_0]$ lead to different responses of the louver. This can be summarised as follows:

- If $\theta_1 = 0$ and $[x_0] = 0$ → No motion of the louver
- If $\theta_1 = 0$ and $[x_0] \neq 0$ → Motion of the louver
- If $\theta_1 \neq 0$ and $[x_0] = 0$ → Motion of the louver
- If $\theta_1 \neq 0$ and $[x_0] \neq 0$ → Motion of the louver

Regarding the first condition, as a consequence of the initial conditions ($[x_0]$) being equal to zero, the louver rests in its neutral position. Besides, since the aerodynamic forces depend upon the angle of attack, it is obvious that no forces are exerted on the louver for $\theta_1 = 0$. Therefore, the louver does not show any motion under these conditions.

The other three conditions are illustrated by Figure 3.30 to Figure 3.33. To illustrate these conditions the low-frequency model, including structural damping, is used.

Figure 3.30 shows the response in case of $\theta_1 = 0$ and the initial conditions $h_0 = 0.01\,\text{m}$ and $\theta_0 = 0.0015\,\text{rad}$. Figure 3.31 shows the first second of the response as shown in Figure 3.30 and shows the influence of the initial conditions. Since there is no external loading (since $\theta_1 = 0$), the louver shows a damped vibration.

![Figure 3.30: Response Louver A1 according to the model including structural damping, for $\theta_1 = 0$ and $[x_0] \neq 0$](image-url)
For the third condition, a relative angle of attack $\theta_1 = 10^\circ (\approx 0.1745 \text{ rad})$ is assumed and all initial conditions are set to zero. The response, shown in Figure 3.32, shows a clear similarity to the wind signal in Figure 3.21. Although the louver is subjected to some disturbance in the beginning, as is shown in Figure 3.29 as well.

The result of the final condition, a relative angle of attack and a set of non-zero initial conditions, is shown in Figure 3.33. For this condition, an initial translation $h_0 = 0.01 \text{ m}$, an initial rotation $\theta_0 = 0.015 \text{ rad}$, and a relative angle of attack $\theta_1 = 10^\circ (\approx 0.1745 \text{ rad})$ are applied.

Figure 3.33 shows the influence of the initial conditions on the beginning of the response graphs of the louver. However, where the vibration in Figure 3.30 becomes fully damped, the contribution of $\theta_1$ causes an ongoing vibration.
Figure 3.33: Response louver according to the model including structural damping, for $\theta_1 \neq 0$ and $[x_0] \neq 0$
4. Design alternatives

This chapter discusses the influences of design changes on the response of the louver. Two types of design alternatives are investigated. Firstly, the design of the cross-section is adjusted. Secondly, the boundary conditions of the louver regarding vertical displacement are changed. This will only influence the translational stiffness of the louver, not the rotational stiffness. Finally, the consequences of changing both the cross-section and the boundary conditions of the louver at the same time are investigated.

4.1. Alternative design of the louver, Louver B1

In paragraph 3.7.1 it has been shown that the deflection of the louver for the given wind load is well within the limits of the building codes. Therefore, a different design of the cross-section is investigated. The vertical beams in the cross-section of Figure 3.5, which act as stiffeners, are removed from the design. The proposed change has the advantage of limiting the amount of material and the louvers can easier be fabricated. One possible disadvantage is local instability of the top or bottom side of the louver. The newly designed cross-section of the louver, *Louver B1*, is shown in Figure 4.1.

![Figure 4.1: Alternative design cross-section, Louver B1](image)

The cross-sectional properties of *Louver B1* are given in Table 4.1. The material properties are recapitulated by Table 4.2.

<table>
<thead>
<tr>
<th>Cross-Sectional Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chord length (c) [m]</td>
<td>0,800</td>
</tr>
<tr>
<td>Thickness louver (hL) [m]</td>
<td>0,100</td>
</tr>
<tr>
<td>Cross-sectional area (A) [m²]</td>
<td>0,0080</td>
</tr>
<tr>
<td>Position aerodynamic centre (AC) [m]</td>
<td>0,200</td>
</tr>
<tr>
<td>Position elastic axis (EA) [m]</td>
<td>0,218</td>
</tr>
<tr>
<td>Eccentricity between AC and EA (ec) [m]</td>
<td>0,018</td>
</tr>
<tr>
<td>Position centre of gravity (CG) [m]</td>
<td>0,390</td>
</tr>
<tr>
<td>Bending moment of inertia (Iₓ) [m⁴]</td>
<td>9,74·10⁻⁶</td>
</tr>
<tr>
<td>Torsional moment of inertia (Iₜ) [m⁴]</td>
<td>3,04·10⁻⁵</td>
</tr>
<tr>
<td>Total length louver (L) [m]</td>
<td>6,00</td>
</tr>
</tbody>
</table>

Table 4.1: Cross-sectional properties louver

<table>
<thead>
<tr>
<th>Material Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Density (ρ) [kg/m³]</td>
<td>1 970</td>
</tr>
<tr>
<td>Young's modulus (E) [N/m²]</td>
<td>2,15·10¹⁰</td>
</tr>
<tr>
<td>Poison factor (ν) [-]</td>
<td>0,23</td>
</tr>
<tr>
<td>Shear modulus (G) [N/m²]</td>
<td>8,74·10⁹</td>
</tr>
</tbody>
</table>

Table 4.2: Material properties applied composite material
In the following the responses of Louver B1 for all three aerodynamic models are constructed. The exact derivations and calculations of matrices have been omitted. Both the stable configuration (for \( U = 32.6 \) m/s) and the unstable configuration (for \( U_f \)) are considered. Finally the redesigned cross-section is subjected to an external wind signal and its responses are compared to the responses found for Louver A1.

### 4.1.1. Response Louver B1, steady aerodynamic model

Constructing the equations of motion corresponding to the steady aerodynamic model for Louver B1 (similar to the procedure in paragraph 3.3) results into:

\[
\begin{bmatrix}
15.80 & 2.72 \\
2.72 & 1.37 \\
\end{bmatrix}
\begin{bmatrix}
\frac{\partial h}{\partial t} \\
\frac{\partial \theta}{\partial t} \\
\end{bmatrix}
+ \begin{bmatrix}
6.20 \cdot 10^4 & 0 \\
0 & 5.90 \cdot 10^4 \\
\end{bmatrix}
+ \begin{bmatrix}
0 & 3.285,34 \\
0 & -59.14 \\
\end{bmatrix}
\begin{bmatrix}
\frac{\partial h}{\partial t} \\
\frac{\partial \theta}{\partial t} \\
\end{bmatrix}
= \begin{bmatrix}
0 \\
0 \\
\end{bmatrix}
\quad (4.1)
\]

The response as determined by MATLAB is shown in Figure 4.2, computed according to the initial conditions given by Table 4.3.

Comparing the response of Louver B1 in Figure 4.2 to the response of Louver A1 (Figure 3.6), it can be seen that the rotational response is less influenced by a second frequency and the rotation increases immediately after \( t = 0 \). The former of the two observations is a result of the decreased eccentricity between AC and EA. As a result, the moment acting about EA has decreased and therefore reduced the rotation of the louver.

<table>
<thead>
<tr>
<th>Initial conditions</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial displacement (( h_0 )) [m]</td>
<td>0,01</td>
</tr>
<tr>
<td>Initial translational velocity (( v_0 )) [m/s]</td>
<td>0</td>
</tr>
<tr>
<td>Initial rotation (( \theta_0 )) [rad]</td>
<td>0,0015</td>
</tr>
<tr>
<td>Initial rotational velocity (( \theta_0 )) [rad/s]</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 4.3: Initial conditions

![Figure 4.2: Response Louver B1 steady aerodynamic model, for \( U = 32.6 \) m/s](image)

To derive the flutter point of the newly designed cross-section, the characteristic equation of equation (4.1) is constructed. Similar to the procedure in equation (3.49), it is found that:
\[a_4p^4 + a_2p^2 + a_0 = 0\]  \hspace{1cm} (4.2)

With:
\[
\begin{align*}
a_4 &= m_1l_0 - S_0^2 \\
a_2 &= m_1K_0 + l_0K_b - q2\pi(m_1(ec) + S_0) \\
a_0 &= K_b(K_0 - 2q(ec)\pi) 
\end{align*}
\]

Determining the dynamic pressure \( q \) for which the discriminant of equation (4.2) becomes negative, results into two values for \( q \) (see equation (3.50)):
\[
q_1 = 38\,012.9 \text{ N/m}^2 \quad q_2 = 95\,290.1 \text{ N/m}^2 \]  \hspace{1cm} (4.3)

The lowest value for \( q \) is governing. Therefore, converting the given dynamic pressure \( q_1 \) to a wind speed results into the flutter speed \( U_f \) (see equation (3.52)), assuming \( \rho = 1.23 \text{ kg/m}^3 \):
\[
\frac{1}{2} \rho U^2 \rightarrow U = U_f = 248.62 \text{ m/s} \]  \hspace{1cm} (4.4)

The upper limit of the wind speed has decreased compared to the flutter point of \( Louver A1 \) (for which \( U_f = 274.1 \text{ m/s} \)). However, since the stiffness of the louver is reduced, this result is expected. The response of the louver corresponding to the aerodynamic forces belonging to \( U_f = 248.62 \text{ m/s} \) is given in Figure 4.3.

Figure 4.3: Flutter instability Louver B1 steady aerodynamic model, for \( U = 248.62 \text{ m/s} \)

4.1.2. Response Louver B1, low-frequency model

Deriving the equations of motion for the low-frequency model, using a similar procedure as in paragraph 3.4, including the aerodynamic contributions for \( U = 32.6 \text{ m/s} \), results into:
\[
\begin{bmatrix}
15.80 & 2.72 \\
2.72 & 1.37
\end{bmatrix}
\begin{bmatrix}
h \\
\dot{h}
\end{bmatrix}
+ \begin{bmatrix}
100.78 & 0 \\
-1.81 & 0
\end{bmatrix}
\begin{bmatrix}
\dot{h} \\
\ddot{h}
\end{bmatrix}
+ \begin{bmatrix}
6.20 \cdot 10^4 & 0 \\
0 & 5.90 \cdot 10^4
\end{bmatrix}
\begin{bmatrix}
h \\
\dot{h}
\end{bmatrix}
+ \begin{bmatrix}
0 \\
3.285,34
\end{bmatrix}
\begin{bmatrix}
\dot{\theta} \\
\ddot{\theta}
\end{bmatrix}
= \begin{bmatrix}
0 \\
0
\end{bmatrix}  \hspace{1cm} (4.5)
\]

Solving this system of equations by using \textit{MATLAB} for the initial conditions, given by Table 4.3, leads to the response as shown in Figure 4.4. The same behaviour as shown in Figure 4.2 can be
distinguished. However, due to the damping matrix, the amplitude decreases and the louver will eventually return to its neutral position.

![Figure 4.4: Response Louver B1 low-frequency model, for U = 32.6 m/s](image1)

As has already been mentioned during the investigations of Louver A1, deriving the flutter point for the low-frequency model is fairly complicated (paragraph 3.6.3). It can be found that the given cross-section becomes unstable for $U_f = 192$ m/s. The response of Louvre B1 for this wind speed is given in Figure 4.5.

Comparing the flutter point of Louver A1 to the flutter point of Louver B1, it is shown that for the latter the flutter point has decreased from $U_f = 274.1$ m/s to $U_f = 248.6$ m/s for the steady aerodynamic model, while in case of the low-frequency model this point is increased from $U_f = 160$ m/s to $U_f = 192$ m/s. This is due to the influence of the aerodynamic damping matrix $A$ and the reduced eccentricity between AC and EA.

![Figure 4.5: Flutter instability Louver B1 low-frequency model, for U = 192 m/s](image2)
4.1.3. Response Louver B1, low-frequency model including structural damping

Finally the response of Louver B1 is determined by the use of the aerodynamic model which includes structural damping. Since the mass and stiffness parameters have changed, the structural damping matrix \( C \) has changed. Similar to the procedure for Louver A1 (paragraph 3.5) it is shown that for this configuration, \( C \) is represented by:

\[
C = \begin{bmatrix}
97.97 & 13.55 \\
13.55 & 25.17
\end{bmatrix}
\] (4.6)

Constructing the complete equations of motion, for \( U = 32.6 \) m/s, similar to the procedure in paragraph 3.5 results into:

\[
\begin{bmatrix} 15.80 & 2.72 \\ 2.72 & 1.37 \end{bmatrix} \begin{bmatrix} \dot{h} \\ \dot{\theta} \end{bmatrix} + \begin{bmatrix} 97.97 & 13.55 \\ 13.55 & 25.17 \end{bmatrix} \begin{bmatrix} h \\ \theta \end{bmatrix} + \begin{bmatrix} 100.78 & 0 \\ -1.81 & 0 \end{bmatrix} \begin{bmatrix} h \\ \theta \end{bmatrix} + \begin{bmatrix} 6.2 \cdot 10^4 & 0 \\ 0 & 5.9 \cdot 10^4 \end{bmatrix} \begin{bmatrix} h \\ \theta \end{bmatrix} + 3 \begin{bmatrix} 285.34 \\ -59.14 \end{bmatrix} \begin{bmatrix} h \\ \theta \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}
\] (4.7)

Using MATLAB to solve this system of equations, with the initial conditions as given in Table 4.3, results into the response as shown in Figure 4.6. The influence of a second frequency in Figure 4.6B can hardly be seen. Besides that, the motion is damped stronger as for the situation without structural damping in Figure 4.4.

![Figure 4.6: Response Louver B1 including structural damping, for U = 32,6 m/s](image)

The flutter point is derived similar to the procedure in paragraph 3.6.4. Figure 4.7 shows the response of the louver for \( U = 226 \) m/s. For this wind speed, the louver has just transferred from a stable configuration to an unstable configuration. Although the upper limit of the wind speed in the previous two aerodynamic models differed from the values found for Louver A1, the upper limit in this model is hardly influenced (for Louver A1 \( U_f = 229 \) m/s).
4.1.4. Response Louver B1 subjected to an external loading

Similar to the procedure in paragraph 3.7 the second design of the cross-section is subjected to an external wind signal \( U(t) \). The external wind signal \( U(t) \) is shown in Figure 3.21. In order to conduct a fair comparison it is again assumed that \( \theta_1 = 10^\circ \) (\( \cong 0.1745 \text{ rad} \)). The procedure to incorporate \( U(t) \) into the equations of motion is explained in paragraph 3.7.

For the steady aerodynamic model loaded by \( U(t) \), the response is given in Figure 4.8. It has to be noted that for this response the initial conditions of the louver are set to zero. The translation of the louver (Figure 4.8B) shows great similarity with the graph of Figure 3.27A. However, Figure 4.8B shows a more constant amplitude for the rotation of the louver, when compared to Figure 3.27B. Since the eccentricity between AC and EA has become smaller, the moment acting about EA has decreased, which explains the smaller fluctuation in rotation.
Figure 4.9 shows the response of *Louver B1* under an external loading according to the low-frequency model. Again all initial conditions are assumed to be zero.

The strong vibrations in the response of the louver in Figure 4.8 are eliminated. The graphs show a clear behaviour of the louver. It has also become clear how small the fluctuations in the rotational response of the louver are.

The response of *Louver B1* loaded by $U(t)$ is finally investigated for the low-frequency model including structural damping. The response of this model is given in Figure 4.10, again it is assumed that the initial conditions are zero.

As expected, the addition of structural damping has hardly any influence on the course of the graphs when compared to Figure 4.9. However, the effects of the vibrations in the beginning of the response graphs are limited.

Figure 4.9: Response Louver B1 low-frequency model, for $U(t)$

Figure 4.10: Response Louver B1 low-frequency model including structural damping, for $U(t)$
4.1.5. Reflection on Louver B1

As this paragraph has already shown, the change in cross-sectional design has consequences for the response of the louver under the same circumstances as for Louver A1.

Considering the responses of the louver when no external loading is part of the system, it can be seen that the rotational response is less influenced by another frequency. For the same initial conditions, the rotation is less dominated by the translations of the louver. Besides, due to the reduction in rotational stiffness, the rotation starts with an increase to $2.0 \times 10^{-3}$ rad after $t = 0$, while for Louver A1 the rotation stays limited to its initial value $1.5 \times 10^{-3}$ rad (Figure 4.2 and Figure 3.6 respectively).

Regarding the responses of the louver loaded by $U(t)$, it can be seen that the translational responses hardly differ for both designs of the louver. However, the rotation of Louver B1 is strongly reduced compared to the rotation of Louver A1. This is the result of the small eccentricity between EA and AC.

4.2. Applying hinged connections, Louver A2

The boundary conditions for Louver A1 are adjusted. Instead of a fully clamped beam the translational displacements are governed by hinges. Furthermore, the same cross-sectional design and properties as for Louver A1 are assumed (Figure 3.5). The schematisation regarding the translation of the new louver, Louver A2, is given by Figure 4.11.

The translational stiffness of Louver A2 is derived similar to the procedure in paragraph 3.1. The differential equation for this system is given by equation (4.8) and the boundary conditions for the situation as schematised in Figure 4.11 are: $h(0) = h(L) = 0$ and $M(0) = M(L) = 0$.

![Figure 4.11: New schematisation vertical loading (Louver A2)](image)

\[
\frac{d^4 h}{dx^4} - q(x) = 0, \text{ with } q(x) = q
\]  

\[
h(x) = \frac{1}{24 \cdot EI} \cdot q \cdot x^4 - \frac{1}{12 \cdot EI} \cdot q \cdot L \cdot x^3 + \frac{1}{24 \cdot EI} \cdot q \cdot L^3 \cdot x
\]  

With:
- $h$ = vertical displacement [m]
- $x$ = position along the $x$-axis [m]
- $q$ = equally distributed load [N/m]
- $L$ = length louver [m]
- $E$ = Young’s modulus [N/m$^2$]
- $I$ = bending moment of inertia [m$^4$]
The discrete translational stiffness remains as derived in equation (3.3) and is recapitulated for convenience:

\[ h_{\text{dis}} = \frac{q L}{K_h} \]  

(4.10)

With:

- \( h_{\text{dis}} \) = discretised translation [m]
- \( q \) = equally distributed load [N/m]
- \( K_h \) = translational spring stiffness [N/m/m²]

Deriving the translational spring stiffness at \( x = \frac{1}{2}L \) by equating equations (4.9) and (4.10), results into:

\[
\frac{5}{384} \frac{q L^4}{E I} = \frac{q}{K_h} \Rightarrow K_h = \frac{384 E I}{5} \frac{N}{m/m^4}
\]  

(4.11)

It appears that by assuming hinged connections for the beam, the translational stiffness becomes five times smaller when compared with a fully clamped beam (see equation (3.4)). Deriving the new value for \( K_h \) in agreement with the material and cross-section properties given by Table 3.1 and Table 3.2 results into:

\[ K_h = 1.30 \times 10^4 \frac{N}{m/m^4} \]  

(4.12)

The influence of this reduced stiffness is considered for all three aerodynamic models. Once for the case \( U = 32.6 \text{ m/s} \) and once for the flutter point of the louver. Thereafter, the louver is subjected to an external wind loading.

### 4.2.1. Response Louver A2, steady aerodynamic model

Constructing the equations of motion for Louver A2 in case of the steady aerodynamic model (similar to the procedure in paragraph 3.3) and aerodynamic contributions corresponding to \( U = 32.6 \text{ m/s} \) results into:

\[
\begin{bmatrix}
17.33 & 1.54 \\
1.54 & 1.11
\end{bmatrix}
\begin{bmatrix}
h \\
\theta
\end{bmatrix}
+ \begin{bmatrix}
1.30 \times 10^4 & 0 \\
0 & 6.34 \times 10^4
\end{bmatrix}
\begin{bmatrix}
h \\
\theta
\end{bmatrix}
+ \begin{bmatrix}
0 & 3.285.34 \\
0 & -318.68
\end{bmatrix}
\begin{bmatrix}
h \\
\theta
\end{bmatrix}
= \begin{bmatrix}
0 \\
0
\end{bmatrix}
\]  

(4.13)

Using MATLAB to solve equation (4.13) in agreement with the initial conditions given by Table 4.4 results into the response as given in Figure 4.12. The influence of the reduced translational stiffness can clearly be seen, as the period of the translational vibration has increased. Figure 4.12B shows that the response regarding the rotation of the louver is still influenced by the frequency of the translational displacement. However, this influence is much smaller as in the case of Louver A1.

<table>
<thead>
<tr>
<th>Initial conditions</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial displacement ((h_0) [m])</td>
<td>0,01</td>
</tr>
<tr>
<td>Initial translational velocity ((h_0) [m/s])</td>
<td>0</td>
</tr>
<tr>
<td>Initial rotation ((\theta_0) [rad])</td>
<td>0,0015</td>
</tr>
<tr>
<td>Initial rotational velocity ((\theta_0) [rad/s])</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 4.4: Initial conditions
Figure 4.12: Response Louver A2 steady aerodynamic model, for $U = 32.6 \text{ m/s}$

The flutter point of this cross-section is determined similar to the steady aerodynamic model explained in paragraph 3.6.2. It is found that the louver becomes unstable for a dynamic pressure $q$ equal to:

$$ q = 57,906,6 \text{ N/m}^2 $$

(4.14)

This dynamic pressure corresponds to the following wind speed, assuming that $\rho = 1.23 \text{ kg/m}^3$, (equation (3.52)):

$$ U_f = 306.85 \text{ m/s} $$

(4.15)

Figure 4.13: Flutter instability Louver A2 steady aerodynamic model, for $U = 306.85 \text{ m/s}$

Comparing the value of $U_f$ in case of Louver A1 ($U_f = 274.1 \text{ m/s}$) and the value found by equation (4.15), it appears that a decrease in translational stiffness leads to an increase of the flutter point, i.e. a higher dynamic pressure. This is not expected at first thought.
Table 4.5 shows a comparison between increases and decreases of both the translational and rotational spring stiffness, while the mass parameters remain constant. The first row shows the spring stiffnesses and the flutter point of Louver A1 and forms the reference point for the comparison.

Out of the results provided by Table 4.5 it can be concluded that a louver which has a higher stiffness in the translational sense, is easier affected by flutter instability than a louver which has a lower stiffness in this direction. While a reduction in rotational stiffness results into a lower flutter point and an increase in rotational stiffness into a higher flutter point. Besides, a change in both stiffness parameters has approximately the same effect as the sum of both separate.

Summarised it can be concluded that the flutter point is mainly governed by the rotational stiffness and that a more flexible louver (in translational sense) has a positive influence on the flutter point.

<table>
<thead>
<tr>
<th>Situation</th>
<th>Translational spring stiffness (K_h) [N/m/m]</th>
<th>Rotational spring stiffness (K_θ) [Nm/rad/m]</th>
<th>Flutter point (q) [N/m²]</th>
<th>Difference in flutter point [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reference point</td>
<td>6.48×10⁴</td>
<td>6.34×10⁴</td>
<td>46 196.2</td>
<td>0</td>
</tr>
<tr>
<td>K_h -20%</td>
<td>5.18×10⁴</td>
<td>6.34×10⁴</td>
<td>48 368.5</td>
<td>+4.7</td>
</tr>
<tr>
<td>K_θ -20%</td>
<td>7.78×10⁴</td>
<td>6.34×10⁴</td>
<td>44 265.3</td>
<td>-4.2</td>
</tr>
<tr>
<td>K_h +20%</td>
<td>6.48×10⁴</td>
<td>5.07×10⁴</td>
<td>35 050.6</td>
<td>-24.1</td>
</tr>
<tr>
<td>K_θ +20%</td>
<td>6.48×10⁴</td>
<td>7.61×10⁴</td>
<td>57 583.7</td>
<td>+24.7</td>
</tr>
<tr>
<td>K_h -20% &amp; K_θ -20%</td>
<td>5.18×10⁴</td>
<td>5.07×10⁴</td>
<td>36 956.9</td>
<td>-20.0</td>
</tr>
<tr>
<td>K_h +20% &amp; K_θ -20%</td>
<td>7.78×10⁴</td>
<td>5.07×10⁴</td>
<td>33 363.4</td>
<td>-27.8</td>
</tr>
<tr>
<td>K_h -20% &amp; K_θ +20%</td>
<td>5.18×10⁴</td>
<td>7.61×10⁴</td>
<td>59 993.8</td>
<td>+29.9</td>
</tr>
<tr>
<td>K_h +20% &amp; K_θ +20%</td>
<td>7.78×10⁴</td>
<td>7.61×10⁴</td>
<td>55 435.4</td>
<td>+20.0</td>
</tr>
</tbody>
</table>

Table 4.5: Comparison spring stiffnesses and flutter point

4.2.2. Response Louver A2, low-frequency model

To investigate the responses of Louver A2 to the low-frequency model, the equations of motion are adjusted to (see paragraph 3.4):

\[
\begin{bmatrix}
17.33 & 1.54 \\
1.54 & 1.11
\end{bmatrix} \begin{bmatrix}
\dot{\mathbf{h}} \\
\dot{\mathbf{\theta}}
\end{bmatrix} + \begin{bmatrix}
100.78 & 0 \\
-9.78 & 0
\end{bmatrix} \begin{bmatrix}
\mathbf{h} \\
\mathbf{\theta}
\end{bmatrix}
+ \left(\begin{bmatrix}
1.30 \cdot 10^4 & 0 \\
0 & 6.34 \cdot 10^4
\end{bmatrix} + \begin{bmatrix}
0 & 3.285.34 \\
0 & 318.68
\end{bmatrix}\right) \begin{bmatrix}
\dot{\mathbf{h}} \\
\dot{\mathbf{\theta}}
\end{bmatrix} = \begin{bmatrix}
0 \\
0
\end{bmatrix}
\]

(S4.16)

Solving this system of equations, according to the initial conditions given in Table 4.4, results into the response of the louver as shown in Figure 4.14, for U = 32.6 m/s. As expected after considering the steady aerodynamic model, the response has become a damped vibration. Similar to the response in Figure 4.12, the translational stiffness has a small influence on the rotation of the louver.
When considering the instability of Louver A2 (similar to the procedure in paragraph 3.6.3), according to the low-frequency model, it is found that the flutter point is reached at $U_f = 160$ m/s, as shown in Figure 4.15. For this case, the dynamic pressure at which the louver becomes unstable is hardly influenced by the reduced translational stiffness when compared to the results found for Louver A1 ($U_f = 160$ m/s).

However, some differences can be distinguished when the response of Figure 4.15 is compared to the response of Louver A1 in Figure 3.17. Besides the influences due to the lower translational stiffness, the rotational excitation in Figure 4.15 at $t = 5$ is twice as large as in Figure 3.17. Therefore, it can be concluded that the flutter point is slightly influenced by the reconsidered schematisation of the louver, but hardly noticeable in reality as this can be caused by a very small change in wind speed.

Figure 4.14: Response Louver A2 low-frequency model, for $U = 32.6$ m/s

Figure 4.15: Flutter instability Louver A2 low-frequency model, for $U = 160$ m/s
4.2.3. Response Louver A2, low-frequency model including structural damping

The third aerodynamic model is considered with a reduced translational stiffness. Both the stable response and the unstable response are presented for this case. However, since the stiffness matrix has changed, the structural damping matrix has changed. Similar to the procedure used in paragraph 3.5, it is shown that for this system the damping $C$ is:

$$
C = \begin{bmatrix}
47,41 & 3,80 \\
3,80 & 25,19
\end{bmatrix}
$$

(4.17)

Similar to the procedure in paragraph 3.5, the equations of motion for $U = 32,6$ m/s are expressed by:

$$
\begin{bmatrix}
17,33 & 1,54 \\
1,54 & 1,11
\end{bmatrix} \begin{bmatrix}
\dot{h} \\
\dot{\theta}
\end{bmatrix} + \begin{bmatrix}
47,41 & 3,80 \\
3,80 & 25,19
\end{bmatrix} \begin{bmatrix}
\ddot{h} \\
\ddot{\theta}
\end{bmatrix} + \begin{bmatrix}
100,78 & 0 \\
-9,78 & 0
\end{bmatrix} \begin{bmatrix}
\dot{h} \\
\dot{\theta}
\end{bmatrix} + \begin{bmatrix}
1,30 \cdot 10^4 \\
6,34 \cdot 10^4
\end{bmatrix} \begin{bmatrix}
\dot{h} \\
\dot{\theta}
\end{bmatrix} + \begin{bmatrix}
0 & 3 \cdot 285,34 \\
0 & -318,68
\end{bmatrix} \begin{bmatrix}
\dot{h} \\
\dot{\theta}
\end{bmatrix} = \begin{bmatrix}
0 \\
0
\end{bmatrix}
$$

(4.18)

Figure 4.16 shows the response of the louver for $U = 32,6$ m/s. Comparing Figure 4.16 and Figure 4.14 learns that the structural damping merely effects the damping period of the louver.

The flutter point is derived similar to the procedure in paragraph 3.6.4. Figure 4.17 shows that for the louver to remain stable $U < 233$ m/s. Comparing this result with the result found for Louver A1 in Figure 3.18, it can be seen that flutter occurs at a slightly higher wind speed. In case of Louver A1 it has been found that $U_f = 229$ m/s.
4.2.4. Response Louver A2 subjected to an external loading

In order to complete the investigation of Louver A2, the responses are considered for all three aerodynamic models loaded by wind signal U(t) (Figure 3.21). The relative angle of attack \( \theta_1 = 10^\circ \) (= 0.1745 rad) and all initial conditions are zero for all three models. See paragraph 3.7 for the procedure of incorporating the external loading into the equation of motion.

Figure 4.18 shows the response of Louver A2 loaded U(t) for the steady aerodynamic. Comparing Figure 4.18 with Figure 3.27, it can be noted that the graphs are a little less dense as a result of the reduced translational stiffness. However, looking at the scale of the vertical axis of the graphs it can be seen that the translation of the louver in Figure 4.18A is five times as large as in Figure 3.27A. The rotation in Figure 4.18B hardly differs from the rotation in Figure 3.27B, since the rotational stiffness has not changed.
To eliminate the strong vibrations in the response of the louver shown in Figure 4.18, the low-frequency model is considered. Its response is given in Figure 4.19. Similar observations can be done as have been done for the responses of the steady aerodynamic model in Figure 4.18. The translation is five times as large and the rotation of the louver is hardly influenced, when compared with the responses for the same situation in Figure 3.28.

![Figure 4.19: Response Louver A2 low-frequency model, for U(t)](image)

The response of Louver A2 for the low-frequency model including structural damping is shown in Figure 4.20. The response shown is very similar to the response given in Figure 4.19, though the strong vibrations at the begin of the graph are limited to the very first seconds.

![Figure 4.20: Response Louver A2 including structural damping, for U(t)](image)

It has to be noted that the deflection has increased with a factor five due to the reduction in translational stiffness. For the given configuration, the louver just satisfies the deflection requirement \( h_{\text{max}} = 24 \text{ mm} \) as determined in paragraph 3.7.1.
4.2.5. Reflection on Louver A2
Reducing the translational stiffness of the louver has several consequences, both favourable and unfavourable.

When considering the aerodynamic models without the external wind loading \( U(t) \), the frequency of the translational displacements is strongly reduced. The frequency of the rotation is maintained since \( K_\theta \) has not changed. The flutter point for the steady aerodynamic model is increased by almost 12%, when compared to the result of Louver A1, while for the other two aerodynamic models, the flutter point is minimally affected.

However, when the louver is subjected to the external wind loading \( U(t) \) the influence of the reduction in translational spring stiffness becomes very clear. As \( K_h \) has become a factor five smaller, the deflections are five times as large for all three models. Therefore, it can be concluded that instead of stability, the stiffness of the louver may be governing.

4.3. Combination of design alternatives, Louver B2
In the previous paragraphs, the influences of two separate adjustments have been investigated. Since both adjustments are favourable regarding fabrication and construction of the wind energy system, the consequences of adjusting both the cross-section and boundary conditions of the louver are investigated. Therefore, the cross-section of Louver B2 is designed as shown in Figure 4.1 and it deflection is governed by hinges. The investigation for Louver B2 is again conducted with a constant wind speed \( U = 32.6 \) m/s and for the situation where the louver is loaded by an external wind signal \( U(t) \), shown in Figure 3.21.

4.3.1. Steady aerodynamic model
First, the steady aerodynamic model is considered. Changing the equations of motion, similar to the procedure in paragraph 3.3, and using MATLAB to solve these equations for the initial conditions given in Table 4.6, results into Figure 4.21. As the rotational stiffness is reduced, the translational frequency influences the rotational response a bit more than was the case for Louver A2 (see Figure 4.12B). The translational response of Louver B2 is hardly influenced when compared to the translational response of Louver A2 (see Figure 4.12A).

<table>
<thead>
<tr>
<th>Initial conditions</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial displacement ( (h_0) ) [m]</td>
<td>0,01</td>
</tr>
<tr>
<td>Initial translational velocity ( (\dot{h}_0) ) [m/s]</td>
<td>0</td>
</tr>
<tr>
<td>Initial rotation ( (\theta_0) ) [rad]</td>
<td>0,0015</td>
</tr>
<tr>
<td>Initial rotational velocity ( (\dot{\theta}_0) ) [rad/s]</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 4.6: Initial conditions
Figure 4.21: Response Louver B2 steady aerodynamic model, for $U = 32.6$ m/s

With a similar procedure as explained in paragraph 3.6.2, it can be shown that the flutter point of the louver is reached at $U = 283.99$ m/s. This is an increase in wind speed compared to the values found for Louver A1 and Louver B1 ($U_f = 274$ m/s and $U_f = 248$ m/s respectively) and a decrease compared to the situation for Louver A2 ($U_f = 306.85$ m/s).

Figure 4.22: Flutter point of Louver B2, for $U = 283.99$ m/s

4.3.2. Low-frequency model

If the damping matrix due to fluid-structure interactions is implemented into the equations of motion (see paragraph 3.4) and then solved for the initial conditions given in Table 4.6. The response of Louver B2 is as shown in Figure 4.23. As expected Figure 4.23 shows a damped motion with similar frequencies and influences as seen in Figure 4.21.
Similar to the procedure in paragraph 3.6.3, it is derived that Louver B2 becomes unstable at a wind speed $U = 192$ m/s. Which is the equal to the flutter point of Louver B1 and higher than the values of the flutter points in case of Louver A1 and Louver A2 (both $U_f = 160$ m/s).

4.3.3. Low-frequency model including structural damping

Constructing the equations of motion for Louver B2, similar to the procedure in paragraph 3.5, results into the response shown in Figure 4.25. Where the response in Figure 4.23 still strongly vibrates, is the amplitude of the response in Figure 4.25 stronger damped.
Similar to the procedure in paragraph 3.6.4, it is derived that the flutter point for this configuration of the louver lies at about $U = 234 \text{ m/s}$. This does not differ much from the results found for Louver A1, Louver A2, and Louver B1 ($U_f = 229 \text{ m/s}$, $U_f = 233 \text{ m/s}$ and $U_f = 226 \text{ m/s}$ respectively).

4.3.4. Louver B2 subjected to an external loading

To complete the comparison, the external wind signal $U(t)$ of Figure 3.21 is introduced to Louver B2 (see paragraph 3.7 for the procedure). The responses for all three aerodynamic models are given in Figure 4.27 to Figure 4.29. The introduction of $U(t)$ to this configuration of the louver shows the influences of the changes in cross-section and boundary conditions. The consequences of these changes, which were separately found previously, are now combined. The reduction of $K_h$ results into an increase in vertical deflection and the reduced eccentricity between AC and EA has led to only small rotations.
Figure 4.27: Response Louver B2 according to the steady aerodynamic model, for $U(t)$

Figure 4.28: Response Louver B2 according to the low-frequency model, for $U(t)$

Figure 4.29: Response Louver B2 according to the low-frequency model including structural damping, for $U(t)$
4.3.5. Reflection on louver B2

The combination of a differently designed cross-section and assuming that the louver is hinged in the vertical direction, has both favourable and unfavourable consequences.

Firstly, the reduction in translational stiffness causes the vertical translations to increase by a factor five. Although the stiffness requirements are met for this particular case, an increase in wind speed could lead to the conclusion that the louver has insufficient stiffness. Secondly, as has already been seen in the case of Louver B1, the rotation of the louver is limited due to the decrease of the eccentricity between AC and EA.

4.4. Conclusions design alternatives

Now four different designs of the louver have been discussed regarding the displacement behaviour, the results are summarised by Table 4.7.

<table>
<thead>
<tr>
<th></th>
<th>Louver A1</th>
<th>Louver A2</th>
<th>Louver B1</th>
<th>Louver B2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$U_f$ Steady aerodynamic model [m/s]</td>
<td>274,1</td>
<td>306,9</td>
<td>248,6</td>
<td>284,0</td>
</tr>
<tr>
<td>$U_f$ Low frequency model [m/s]</td>
<td>160,0</td>
<td>160,0</td>
<td>192,0</td>
<td>192,0</td>
</tr>
<tr>
<td>$U_{inc.}$ structural damping [m/s]</td>
<td>229,0</td>
<td>233,0</td>
<td>226,0</td>
<td>234,0</td>
</tr>
<tr>
<td>Maximal translation h for U(t) [m]</td>
<td>0,0048</td>
<td>0,024</td>
<td>0,0050</td>
<td>0,025</td>
</tr>
<tr>
<td>Maximal rotation $\theta$ for U(t) [rad]</td>
<td>4,79-10^-4</td>
<td>4,79-10^-4</td>
<td>9,58-10^-5</td>
<td>9,56-10^-5</td>
</tr>
</tbody>
</table>

Table 4.7: Results for all four louver designs

The influence of the considered aerodynamic model on the flutter point of the louver is very clear. It is shown that the lowest wind speed for which the louver becomes unstable, is at $U_f = 160$ m/s. Which is an unrealistic wind speed.

Furthermore, the influence of the support of the louver is clearly shown. Assuming hinged supports results into a translation which is five times as large compared to fully clamped supports. The decrease in translational stiffness of Louver B1 and Louver B2, relative to Louver A1 and Louver A2, has no significant influence on the vertical translation. However, the rotation of the louver is strongly reduced.
5. Numerical analysis

This chapter discusses the numerical analysis of the louver conducted with the FEM program Abaqus 6.14-1 (Abaqus). First, a validation of the FEM model is done by a comparison between analytically and numerically determined results. This is done by means of a modal analysis and a static deflection test of the FEM model. Next, a dynamic model is used to investigate the occurring stresses in the material of the louver for a median wind speed signal. Finally, the louver is subjected to an extreme wind signal and analysed for the occurring stresses.

Before conducting the analysis, a mesh density investigation is performed. Since the FEM analysis approximates the real solution, a reasonable approximation is desired. The FEM analysis has to be reasonable in terms of accuracy and computation time. A coarse meshed model has a short computation time, but the analysis is not accurate. Increasing the mesh density increases the computations time, but causes a convergence of the accuracy as well. The mesh density test is done for Louver B1. It is assumed that these results apply to the other models as well.

The mesh density test is performed by dividing the top edge (highlighted in Figure 5.1) of the louver into multiple elements. Figure 5.2 shows that applying more than 50 elements, does not result into a significant increase in accuracy of the solution. Therefore, the top side of the louver is divided into 50 elements, to limit the computational effort. The same number of elements are applied to the bottom side of the louver.

It has to be noted that for numerical modelling purposes, the units of all variables are converted to values in agreement with millimeters.

![Figure 5.1: Highlighted top side of the louver](image)

![Figure 5.2: Mesh density test](image)
5.1. Modal analysis

A modal analysis is conducted to determine the natural frequencies and the mode shapes of the louver, partially characterised by the eigenvalues of the louver. The analytical derivation of the modal vectors is done for both Louver A1 and Louver B1. These results are compared with the numerical modal analyses conducted with FEM program Abaqus.

Before continuing with the modal analyses it is investigated whether or not the system is classically damped. For a system to be classically damped it must yield that (Peres-Da-Silva, Cronin, & Randolph, 1994):

$$\mathbf{CM}^{-1}\mathbf{K} = \mathbf{KM}^{-1}\mathbf{C}$$ (5.1)

With:  
\[
\begin{align*}
\mathbf{C} &= \text{structural damping matrix [-]} \\
\mathbf{M} &= \text{mass matrix [-]} \\
\mathbf{K} &= \text{stiffness matrix [-]}
\end{align*}
\]

This is also valid for a system which is proportionally damped, e.g. when Rayleigh-damping is applied. The modal vectors of a classically damped system depend solely on \(\mathbf{M}\) and \(\mathbf{K}\), and are independent of \(\mathbf{C}\), regardless of how heavily damped the system is (Gavin, 2012).

Since damping matrix \(\mathbf{C}\) in the equations of motion of the louver is determined by Rayleigh-damping (as shown in paragraph 3.5), the modal analyses are conducted by assuming an undamped system. This makes the procedure much more straightforward.

5.1.1. Modal analysis Louver A1, analytical

To derive the modal vectors of Louver A1, the undamped equations of motion, without aerodynamic contributions, are recapitulated (equation (3.16)). The values of the different coefficients, as determined in paragraph 3.3, are recapitulated as well:

\[
\begin{bmatrix}
\mathbf{m}_1 & \mathbf{S}_0 \\
\mathbf{S}_0 & \mathbf{I}_0
\end{bmatrix}
\begin{bmatrix}
\mathbf{h} \\
\mathbf{\theta}
\end{bmatrix} +
\begin{bmatrix}
\mathbf{K}_h & 0 \\
0 & \mathbf{K}_\theta
\end{bmatrix}
\begin{bmatrix}
\mathbf{h} \\
\mathbf{\theta}
\end{bmatrix} =
\begin{bmatrix}
0 \\
0
\end{bmatrix}
\] (5.2)

With:  
\[
\begin{align*}
\mathbf{m}_1 &= \text{mass louver per unit length} (= 17,33 \text{ kg/m}) \\
\mathbf{S}_0 &= \text{static moment louver} (= 1,54 \text{ kgm/m}) \\
\mathbf{I}_0 &= \text{mass moment of inertia louver} (= 1,11 \text{ kgm}^2/\text{m}) \\
\mathbf{K}_h &= \text{translational stiffness louver} (= 6,48\times10^4 \text{ N/m/m}) \\
\mathbf{K}_\theta &= \text{rotational stiffness louver} (= 6,34\times10^4 \text{ Nm/rad/m}) \\
\mathbf{h} &= \text{translation [m]} \\
\mathbf{\theta} &= \text{rotation [rad]}
\end{align*}
\]

In order to derive the modal vectors of the system of equations in equation (5.2), the eigenvalue problem has to be solved first. The eigenvalue problem is represented by the following equations (Meirovitch, 2001):

\[
\begin{align*}
(\mathbf{K}_h - \omega^2 \times \mathbf{m}_1)\mathbf{h} + (-\omega^2 \times \mathbf{S}_0)\mathbf{\theta} &= 0 \\
(-\omega^2 \times \mathbf{S}_0)\mathbf{h} + (\mathbf{K}_\theta - \omega^2 \times \mathbf{I}_0)\mathbf{\theta} &= 0
\end{align*}
\] (5.3)
Equation (5.3) possesses a non-trivial solution if the determinant of its coefficients matrix is zero, this is represented by (Meirovitch, 2001):

\[
\det \begin{bmatrix}
K_h - \omega^2 \times m_1 & -\omega^2 \times S_\theta \\
-\omega^2 \times S_\theta & K_\theta - \omega^2 \times I_\theta
\end{bmatrix} = 0
\]  
(5.4)

Solving equation (5.4) for \( \omega^2 \), in agreement with the coefficients as presented by equations (5.2), results into the eigenvalues of the system:

\[
\omega_1^2 = 3\,707.46 \text{ rad/s} \quad \omega_2^2 = 65\,706.61 \text{ rad/s}
\]  
(5.5)

Now the eigenvalues of the system are known, it is possible to determine the mode shapes of the louver. Therefore, the eigenvalue problem of equation (5.3) is rewritten into (Meirovitch, 2001):

\[
\begin{align*}
(K_h - \omega_i^2 m_1)h_i + (-\omega_i^2 S_\theta)\theta_i &= 0 \\
(-\omega_i^2 S_\theta)h_i + (K_\theta - \omega_i^2 I_\theta)\theta_i &= 0
\end{align*}
\]  
(5.6)

The expressions in equation (5.6) represent two sets of two homogenous algebraic equations, one for \( i = 1 \) and one for \( i = 2 \). Since the equations are homogeneous, it is only possible to solve for the ratios \( \theta_i / h_i \) (\( i = 1, 2 \)). These ratios represent the shape of the displacement profile of the system while it oscillates at the eigenfrequency \( \omega_i \). To determine the modal vectors, the values of the variables as given by equations (5.2) and (5.5) are implied into equation (5.6). Solving either the first or the second expression of equation (5.6) for the ratio \( \theta_i / h_i \) (\( i = 1, 2 \)), results into the first (for \( i = 1 \)) or the second modal vector (for \( i = 2 \)). For Louver A1 the modal vectors are given by:

\[
\begin{align*}
u_1 &= \begin{bmatrix} h_1 \\ \theta_1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0,10 \end{bmatrix} \\
u_2 &= \begin{bmatrix} h_2 \\ \theta_2 \end{bmatrix} = \begin{bmatrix} 1 \\ -10,61 \end{bmatrix}
\end{align*}
\]  
(5.7)

Figure 5.3 and Figure 5.4 show schematically the mode shapes for \( \mathbf{u}_1 \) and \( \mathbf{u}_2 \) respectively.

---

**Figure 5.3:** Schematic representation of the first mode shape Louver A1

**Figure 5.4:** Schematic representation of the second mode shape Louver A1
5.1.2. Modal analysis Louver A1, numerical

The numerical modal analysis is conducted with FEM program *Abaqus* by means of a *Frequency step*. To conduct a proper modal analysis, the louver is modelled over its full length (6.0 meter). The boundary conditions of the louver are fully clamped, for translation and rotation, at both ends (Figure 5.5).

*Abaqus* has a number of eigensolvers to compute the eigenvalues of the louver. The method of *Automatic Multi-level Substructuring (AMS)* is applied for this case as it generates the eigenvectors at every node of the model (Simulia, 2014).

Table 5.1 gives an overview of the 20 first eigenvalues and natural frequencies for the louver as shown in Figure 5.5.

![Figure 5.5: Abaqus model Louver A1](image)

<table>
<thead>
<tr>
<th># Mode</th>
<th>Eigenvalue</th>
<th>Frequency (rad/s)</th>
<th>Frequency (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4580</td>
<td>67.67</td>
<td>10.77</td>
</tr>
<tr>
<td>2</td>
<td>31855</td>
<td>178.48</td>
<td>28.41</td>
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<td>3</td>
<td>78542</td>
<td>280.25</td>
<td>44.60</td>
</tr>
<tr>
<td>4</td>
<td>109317</td>
<td>330.63</td>
<td>52.62</td>
</tr>
<tr>
<td>5</td>
<td>173304</td>
<td>416.30</td>
<td>66.26</td>
</tr>
<tr>
<td>6</td>
<td>206204</td>
<td>454.10</td>
<td>72.27</td>
</tr>
<tr>
<td>7</td>
<td>239477</td>
<td>489.36</td>
<td>77.89</td>
</tr>
<tr>
<td>8</td>
<td>326077</td>
<td>571.03</td>
<td>90.88</td>
</tr>
<tr>
<td>9</td>
<td>390646</td>
<td>625.02</td>
<td>99.48</td>
</tr>
<tr>
<td>10</td>
<td>401766</td>
<td>633.85</td>
<td>100.88</td>
</tr>
<tr>
<td>11</td>
<td>483391</td>
<td>695.26</td>
<td>110.65</td>
</tr>
<tr>
<td>12</td>
<td>518126</td>
<td>719.81</td>
<td>114.56</td>
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<td>13</td>
<td>548228</td>
<td>740.42</td>
<td>117.84</td>
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<td>563092</td>
<td>750.39</td>
<td>119.43</td>
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<td>601552</td>
<td>775.60</td>
<td>123.44</td>
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<td>603906</td>
<td>777.11</td>
<td>123.68</td>
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<td>634943</td>
<td>796.83</td>
<td>126.82</td>
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<td>18</td>
<td>710850</td>
<td>843.12</td>
<td>134.19</td>
</tr>
<tr>
<td>19</td>
<td>751285</td>
<td>866.77</td>
<td>137.95</td>
</tr>
<tr>
<td>20</td>
<td>802481</td>
<td>895.81</td>
<td>142.57</td>
</tr>
</tbody>
</table>

*Table 5.1: First 20 eigenvalues and natural frequencies Louver A1*
The first eigenmode is shown in Figure 5.6, as can be seen this is a translational eigenmode. According to Table 5.1 the first eigenmode occurs at a frequency of $\omega = 67.67$ rad/s. This is close to the first eigenfrequency of the DEM model, which is represented by the square root of the first eigenvalue in equation (5.5), $\omega_1 = 60.89$ rad/s.

Figure 5.7 shows the mode shape corresponding to the first eigenmode of the louver, as determined by *Abaqus*. Investigating the ratio between the translation and the rotation of the louver’s cross-section as shown in Figure 5.7, learns that the modal vector $u_1$ is represented by:

$$u_1 = \begin{bmatrix} h_1 \\ \theta_1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0.094 \end{bmatrix}$$

Comparing $u_1$ of equation (5.8) to $u_1$ of equation (5.7), shows that the modal vectors are similar.

The modal vector $u_2$ of the DEM model, represents a torsional mode at a natural frequency $\omega_2 = 256.33$ rad/s. This natural frequency is determined by the square root of $\omega^2$ in equation (5.5). Looking at the eigenmodes determined by *Abaqus*, this should match with eigenmode 3 from Table 5.1. Figure 5.8 shows this eigenmode and clearly shows a torsional mode shape. The ratio between the translation and rotation for this eigenmode is represented by:

$$u_2 = \begin{bmatrix} h_2 \\ \theta_2 \end{bmatrix} = \begin{bmatrix} 1 \\ -2.7 \end{bmatrix}$$

Although, the natural frequencies corresponding to the torsional mode of the DEM and FEM model are off by about 10% (256.33 rad/s and 280.25 rad/s, respectively), the rotation is strongly reduced compared to the analytical result of $u_2$ in equation (5.7). However, it is known that analytical analyses for higher frequency mode shapes can be off by 20-30% or more.
5.1.3. Modal analysis Louver B1, analytical

In order to compute the modal vectors of Louver B1, the undamped equations of motion are recapitulated first (equation (3.16)). The values of the different coefficients, as determined in paragraph 4.1, are recapitulated as well:

\[
\begin{bmatrix} m_1 & S_\theta \\ S_\theta & I_\theta \end{bmatrix} \begin{bmatrix} \ddot{h} \\ \ddot{\theta} \end{bmatrix} + \begin{bmatrix} K_h & 0 \\ 0 & K_\theta \end{bmatrix} \begin{bmatrix} h \\ \theta \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}
\] (5.10)

With: \( m_1 \) = mass louver per unit length (= 15.80 kg/m)
\( S_\theta \) = static moment louver (= 2.72 kgm/m)
\( I_\theta \) = mass moment of inertia louver (= 1.37 kgm²/m)
\( K_h \) = translational stiffness louver (= 6.20 \times 10^4 N/m/m²)
\( K_\theta \) = rotational stiffness louver (= 5.90 \times 10^4 Nm/rad/m²)

\( h \) = translation [m]
\( \theta \) = rotation [rad]

Similar to the procedure in the case of the modal analysis for Louver A1 the eigenvalue problem has to be solved first (paragraph 5.1.1). Applying equation (5.4) to the system of equations in equation (5.10) results into the following eigenvalues:

\[ \omega_1^2 = 3\ 798.47 \quad \omega_2^2 = 67\ 591.68 \] (5.11)

Comparing the eigenvalues of equation (5.11) to the eigenvalues found for Louver A1 (equation (5.5)) shows that they do not differ much. However, as the change in cross-section of the louver has not strongly influenced the equations of motion of the louver, the outcome is as expected.
Applying the same principle used to derive the modal vectors of \textit{Louver A1} (see equation (5.7)), to derive the modal vectors of \textit{Louver B1} results into:

\[
\mathbf{u}_1 = \begin{bmatrix} h_1 \\ \theta_1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0.19 \end{bmatrix} \quad \mathbf{u}_2 = \begin{bmatrix} h_2 \\ \theta_2 \end{bmatrix} = \begin{bmatrix} 1 \\ -5.47 \end{bmatrix}
\] (5.12)

Figure 5.10 and Figure 5.11 schematically show the mode shapes of \textit{Louver B1}, corresponding to the modal vectors of equation (5.12).

\begin{figure}[h]
\centering
\includegraphics[width=0.8\textwidth]{figure10.png}
\caption{Analytical representation of first mode shape of Louver B1}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=0.8\textwidth]{figure11.png}
\caption{Analytical representation of the second mode shape of Louver B1}
\end{figure}

\textbf{5.1.4. Modal analysis Louver B1, numerical}

The FEM analysis of \textit{Louver B1} is conducted similar to the case of \textit{Louver A1} (paragraph 5.1.2). The first 20 eigenmodes of \textit{Louver B1} are given by Table 5.2.

The lowest natural frequency determined by the DEM model is \( \omega_1 = 61.63 \text{ rad/s} \), the square root of \( \omega_1^2 \) in equation (5.11). This frequency represents a translational eigenmode of the louver. The first eigenmode according to the FEM model is reached at \( \omega = 66.66 \text{ rad/s} \) (Table 5.2). The mode shape is as shown in Figure 5.12. A cross-section of the louver at half the span and in its first eigenmode, is given in Figure 5.13. Investigation of this cross-section learns that the modal vector of the first eigenmode equals:

\[
\mathbf{u}_1 = \begin{bmatrix} h_1 \\ \theta_1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0.15 \end{bmatrix}
\] (5.13)

Comparing \( \mathbf{u}_1 \) from equation (5.12) to \( \mathbf{u}_1 \) from equation (5.13), shows a difference of about 25%. However, this might be due to the rigid body approximation of the analytical analysis and leaving out the vertical beams in the numerical modelled louver. The absence of the vertical beams influences the local stiffness of the numerical model.
<table>
<thead>
<tr>
<th>#</th>
<th>Mode</th>
<th>Eigenvalue</th>
<th>Frequency [rad/s]</th>
<th>Frequency [Hz]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4</td>
<td>443</td>
<td>66,66</td>
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<td>310</td>
<td>455,31</td>
<td>72,47</td>
</tr>
</tbody>
</table>

Table 5.2: First 20 eigenvalues and natural frequencies Louver B1

The second natural frequency corresponding to the second eigenvalue of the louver represents a torsional eigenmode. However, this torsional eigenmode is not determined by the FEM analysis. Besides other high frequency translational eigenmodes, types of ‘breathing’ modes are computed.

An explanation for the absence of a torsional mode in the numerical analysis, may be given by the two different approaches of the DEM analysis and FEM analysis. In the DEM modal analysis, the cross-section is assumed to behave as a rigid-body. This is not the case for the FEM analysis.
Furthermore, comparing the cross-section of *Louver A1* to the cross-section of *Louver B1* (see Figure 5.14 and Figure 5.15 respectively), shows that the eccentricity between the aerodynamic centre (AC) and the elastic axis (EA) has decreased. As a result the moment exerted on the louver has decreased, therefore the torsional mode is not excited.

![Figure 5.14: Cross-section Louver A1](image1)

![Figure 5.15: Cross-section Louver B1](image2)

### 5.1.5. Static validation numerical model

In this paragraph the spring stiffnesses of the DEM models in chapter 3 and 4 are validated. Therefore, the FEM model of the louver is loaded by an equally distributed ‘unity load’ of 1 000 N/m\(^1\). Using the deflection of the louver due to this ‘unity load’, an equivalent spring stiffness (\(K_{\text{h,eq}}\)) is derived. In turn this equivalent spring stiffness is compared to the spring stiffness (\(K_h\)) of the DEM models, which is derived corresponding to the procedure as described in paragraph 3.1.

Before continuing with this validation, the application of forces and displacements on the FEM model of the louver is explained. In the FEM model the elastic axis is specified as a reference point (RP). Subsequently the reference point is given a rigid body constraint with tie nodes. Due to this procedure the louver behaves as a rigid body at the vertical cross-section through the reference point (Simulia, 2014). This is illustrated in Figure 5.16.

![Figure 5.16: Constraint of reference point to louver](image3)

The validation of the spring stiffness is conducted for two models of the louver, first for *Louver A1* and thereafter for *Louver B2*. For ease of numerical modelling this equally distributed load is replaced by a concentrated load \(Q_L\). In case of *Louver A1* it is shown that the concentrated load is equal to (see Appendix B):

\[
Q_L = 1 \,500 \, N
\]  
(5.14)
As can be seen in Figure 5.17 the maximal deflection of the louver is equal to 13,98 mm (= 0,01398 m). It has to be kept in mind that in chapter 3 and 4 the spring stiffnesses are determined per unit length (i.e. $K_h = N/m/m$). Therefore, the equivalent spring stiffness is computed by dividing the unit load by the given displacement. This results into the following equivalent spring stiffness:

$$K_{h,eq} = \frac{1000}{0,01398} = 7,15 \cdot 10^4 \text{ N/m/m}^1$$ \hspace{1cm} (5.15)

Comparing the value of $K_{h,eq}$ as determined by equation (5.15) to the value of the spring stiffness as determined in paragraph 3.3 ($K_h = 6,48 \cdot 10^4 \text{ N/m/m}^1$), shows that for this case the stiffness is off by 9,4%.

Next the translational stiffness of Louver B2 is validated. Transforming the equally distributed ‘unity load’ to a concentrated force for Louver B2, results into the following concentrated force (see Appendix B):

$$Q_L = 1875 \text{ N}$$ \hspace{1cm} (5.16)

Applying the concentrated load as given by equation (5.16) to the elastic axis of Louver B2, results into the deformation of the louver as shown in Figure 5.18. It is shown that the maximal deflection of the louver under this load is equal to 76,17 mm.

Similar to the procedure in equation (5.15) the equivalent spring stiffness of Louver B2 is derived:

$$K_{h,eq} = \frac{1000}{0,07617} = 1,31 \cdot 10^4 \text{ N/m/m}^1$$ \hspace{1cm} (5.17)
Comparing the value of $K_{\text{eq}}$ as determined by equation (5.17) to the value of the spring stiffness as determined in paragraph 4.3 ($K_b = 1.24 \times 10^4 \text{ N/m/m}$), shows that in case of Louver B2 values differ about 5.6% from each other.

Summarised, it has been shown that the eigenmodes of the DEM model are similar to the eigenmodes of the FEM model. Subsequently it has been proven that the stiffness applied in the DEM models corresponds to the stiffness of the FEM models. Therefore, the numerical model is validated and determined to be quite accurate.

### 5.2. Dynamic numerical analyses of the louver

As the FEM model of the louver is validated, two dynamical simulations are conducted. As the computation time of such an analysis is very long, only Louver B2 is considered. Louver B2 is the governing design regarding stress levels, due to large deformations and reduced material usage.

Two dynamic analyses are conducted. The first analysis investigates whether or not a fatigue analysis is required. Therefore, Louver B2 is subjected to the average wind signal $U(t)$ as shown in Figure 3.20, with $U = 24,05 \text{ m/s}$ as the maximum wind speed approximated by the Fourier Series.

The second dynamical analysis investigates the stress levels in the louver due to extreme wind loadings. The worst storm of the last decades in the Netherlands took place on January 25 1990. During this storm gusts have been measured with speeds up to 158 km/h ($= 43.9 \text{ m/s}$) (KNMI, 2012). The extreme wind signal, which includes the value of $U = 43.9 \text{ m/s}$, is determined later on. The analysis will show if the louver is sufficiently strong for these extreme conditions.

For both cases the louver is analysed by use of the low frequency aerodynamic model including structural damping. This aerodynamic model is explained in paragraph 3.5.

The results of the stress investigations have to be matched to the strength properties of the material applied. As has been stated in chapter 2, it is for this thesis assumed that the louvers are fabricated out of a glass fibre reinforced composite material. Table 5.3 gives an overview of the maximal strength limit and fatigue limit of a glass fibre composite material. It is a general rule of thumb that for glass fibre reinforced composites, the fatigue limit is characterised by 20% of the material’s ultimate strength limit. The strength properties are provided by ‘Performance Composites Ltd.,’ which produces a similar glass fibre reinforced composite. A complete overview of the glass fibre composite produced by ‘Performance Composites Ltd.,’ is attached in Appendix C.

For the stress analysis in which the louver is loaded by the average wind signal $U(t)$, the stress levels should not exceed the values of $\sigma_{\text{fat}}$. The stresses due to the extreme wind loading may exceed the fatigue limit, but must not exceed the strength limits of the material.

<table>
<thead>
<tr>
<th>Property</th>
<th>Strength limit ($\sigma_{\text{max}}$) [N/mm$^2$]</th>
<th>Fatigue limit ($\sigma_{\text{fat}}$) [N/mm$^2$]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tensile strength</td>
<td>120</td>
<td>24</td>
</tr>
<tr>
<td>Compressive strength</td>
<td>120</td>
<td>24</td>
</tr>
<tr>
<td>In plane shear strength</td>
<td>150</td>
<td>30</td>
</tr>
</tbody>
</table>

Table 5.3: Strength properties of bidirectional glass fibre composite
5.2.1. Dynamic behaviour Louver B2 under average wind loading

The dynamic behaviour of *Louver B2* is first considered for the average wind signal. The response of *Louver B2* due to the wind loading $U(t)$, as determined in paragraph 4.3.4, is recapitulated in Figure 5.19.

![Figure 5.19: Response Louver B2 under average wind loading $U(t)$](image)

For the dynamical stress analysis the louver’s stresses are first investigated at six points, which are schematically indicated by the crosses in Figure 5.20. The stresses at these six points are plotted as a function of the time $t$. The location of these points is approximately at $x = 2900$ mm to prevent influences from the symmetry boundary condition at $x = 3000$ mm.

Subsequently, the stresses along the louver’s length are investigated at $t = 17.5$ sec, at which the louver is subjected to its largest displacements (see Figure 5.19). The stresses in the louver at this time are investigated for six sections across the louver, indicated by the red lines in Figure 5.20. The stresses are given over a section from the support of the louver (at $x = 0$) to the middle of the louver’s span (at $x = 3000$).

![Figure 5.20: Schematic indication of investigated nodes](image)

Rather than the stresses related to the coordinate system of the FEM model, the principal stresses are given as the results of this investigation. The principal stresses show the maximal occurring normal stresses, therefore the stress analysis has more accuracy.
Figure 5.21 and Figure 5.22 show the maximal in plane principal stress ($\sigma_1$) and minimal in plane principal stress ($\sigma_3$), respectively. Figure 5.21 clearly shows that the highest stress level occur at the bottom side of the louver and as it is a positive value it represents a tensional stress. However, the value of $\sigma_1$ at the top side of the louver remains negative, representing a compressional stress. Since the louver is mainly subjected to bending this is as expected.

![Figure 5.21: Maximal in-plane principal stresses Louver B2, for average wind signal U(t)](image1)

![Figure 5.22: Minimal in-plane principal stresses Louver B2, for average wind signal U(t)](image2)

Similar observations as has been done for $\sigma_1$ in Figure 5.21 can be done for $\sigma_3$ in Figure 5.22. The lowest value of $\sigma_3$ occurs at the top side of the louver and represents a compressional stress. While the lowest stress level at the bottom side of the louver remains positive, indicating that the bottom side remains under tensile stress. These observations indicate bending as well.

The maximal shear stresses are computed with the use of Mohr’s circle. It can be shown that the maximal shear stress is derived by (Ferdinand P. Beer, 2004):
\[
\tau_{\text{max}} = \frac{1}{2}(\sigma_1 - \sigma_3)
\]

With:  
\(\tau_{\text{max}}\) = maximal shear stress [N/mm\(^2\)]  
\(\sigma_1\) = maximal in plane principal stress [N/mm\(^2\)]  
\(\sigma_3\) = minimal in plane principal stress [N/mm\(^2\)]

Applying equation (5.18) to the principal stresses as shown in Figure 5.21 and Figure 5.22, results into the shear stresses for Louver B2 loaded by the average wind signal, see Figure 5.23. As is shown in Figure 5.23 the shear stresses reach the highest stress levels at the top and bottom side of the louver. These parts are most influenced by the torsion of the louver as a result of the wind loading.

As has already been stated the stresses are investigated along the length of the louver as well. This investigation is conducted along the in Figure 5.20 indicated sections at \(t = 17.5\) sec, for which the louver is subjected to its highest deflection. The maximal and minimal in plane principal stresses are given Figure 5.24 and Figure 5.25, respectively. Figure 5.26 shows the maximal shear stresses along the louver’s length, the shear stresses are derived using the principle of equation (5.18).

As expected the top and bottom side of the louver show the highest stress levels. Besides the sections at the top and bottom side of the louver, it is shown that the stresses are relatively constant. Furthermore, some influences of the boundary conditions can be distinguished at the beginning and end of the graphs. These influence result in high stresses at the beginning of the graphs and some disturbance at the end of the graphs.
Figure 5.24: Maximal in-plane principal stresses Louver B2, for U(t) at t = 17.5 sec

Figure 5.25: Minimal in-plane principal stresses Louver B2, for U(t) at t = 17.5 sec

Figure 5.26: Maximal in-plane shear stresses Louver B2, for U(t) at t = 17.5 sec
As the occurring stresses under an average wind signal have become clear, it has to be checked whether or not the material limits are exceeded. It has been shown in Figure 5.21 to Figure 5.26 that the highest absolute stress level is the maximal in plane principal stress at the bottom side of the louver. The high stress levels due to the influences of the boundary conditions are neglected. However, this absolute maximal stress level does not exceed 9 N/mm², which is well within the fatigue limit of the glass fibre reinforced composite material. Therefore, no further fatigue analysis is required.

5.2.2. Dynamic behaviour louver B2 under high wind loading

The second analysis concerns the dynamic behaviour of the louver under an extreme wind load, with \( U = 43.9 \text{ m/s} \) as maximal gust speed. As the IRWES team has not been able to measure a wind signal which contains such high gust speeds, another signal is linearly scaled to a signal which contains \( U = 43.9 \text{ m/s} \). The new wind signal \( U(t) \) is given in Figure 5.27. The wind signal is again approximated by a Fourier Series, which is expressed by equation (5.19) (Meirovitch, 2001) & (The Mathworks, Inc, 2013) and the coefficients as given by Table 5.4. The Fourier approximation of the new wind signal is shown in Figure 5.28.

\[
U(t) = a_0 + \sum_{n=1}^{\infty} (a_n \cos(\omega_n t) + b_n \sin(\omega_n t))
\]  

(5.19)

| Fourier coefficients for \( n = 8 \) |
|-----------------|-----------------|-----------------|
| \( a_0 \)       | 36.45           | \( \omega_T \)  | 0.09835         |
| \( a_1 \)       | 0.06375         | \( b_1 \)       | -0.3713         |
| \( a_2 \)       | -1.388          | \( b_2 \)       | -0.9378         |
| \( a_3 \)       | -1.191          | \( b_3 \)       | -0.5236         |
| \( a_4 \)       | 2.085           | \( b_4 \)       | 1.254           |
| \( a_5 \)       | -0.8245         | \( b_5 \)       | 0.7601          |
| \( a_6 \)       | 0.5358          | \( b_6 \)       | -1.957          |
| \( a_7 \)       | -0.5622         | \( b_7 \)       | 0.4216          |
| \( a_8 \)       | 0.8035          | \( b_8 \)       | -0.7878         |

Table 5.4: Fourier coefficients of the wind signal in Figure 5.27
Before continuing to the FEM analysis of the louver, the response of Louver B2 to the wind signal of Figure 5.28 is determined. Using a similar procedure as in paragraph 4.3.4, it is shown that the response of the louver is as given by Figure 5.29.

Comparing the response given in Figure 5.29 to the response of Louver B2 for the average wind signal (Figure 5.19), a clear increase in vertical translation can be seen. For the average wind signal, the deflection of the louver just exceeds the stiffness limitations ($h = 25$ mm and $h_{\text{max}} \leq 24$ mm). In case of the extremr wind signal in Figure 5.29, the maximal deflection $h = 75$ mm. This is more than three times $h_{\text{max}}$. The maximal rotation of the louver in Figure 5.29 is $\theta = -3.16 \times 10^{-4}$ rad, while the rotation in Figure 5.19 is limited to $\theta = -9.5 \times 10^{-5}$ rad.

Although, the maximal wind speed has almost doubled (form $U = 24.05$ m/s to $U = 43.9$ m/s), the translation and the rotation of the louver are both more than three times as high as for the
average wind loading. However, a linear relation between the increase of the wind speed and the motion of the louver was not expected, since the aerodynamic force contain the wind speed squared. Nevertheless, the strong influence is impressive.

Similar to the analysis in paragraph 5.2.1 the stresses are first investigated at the six points as indicated by Figure 5.20. The maximal shear stresses are again determined with the use of Mohr’s circle, as explained by equation (5.18). The maximal in-plane stresses are given by Figure 5.30 and Figure 5.31 and the maximal shear stresses are given in Figure 5.32.

**Figure 5.30: Maximal in-plane principal stresses Louver B2, for extreme wind signal U(t)**

**Figure 5.31: Minimal in-plane principal stresses Louver B2, for extreme wind signal U(t)**
The influence of the extreme wind loading on the stress levels in the louver is clear. Comparing the stress responses of Figure 5.30 – Figure 5.32 with the stress responses as shown in Figure 5.21 – Figure 5.23, it can be seen that the stress levels have increased. However, as the loading on the louver has increased, the increase in stress levels is expected.

Similar to the observations of the nodes in case of the average wind signal in paragraph 5.2.1, the highest stress levels occur at the top and bottom side of the louver. The highest value of the maximal in plane principals stress $\sigma_{3} = 32,01\,\text{N/mm}^2$. Although this value exceeds the fatigue limit of the glass fibre material, it is well within its ultimate strength (see Table 5.3). Regarding the minimal in plane principal stress and maximal shear stress, both highest values do not even exceed the fatigue limit of the glass fibre material.

Similar to the analysis in case of an average wind loading, the stresses in the louver along the six sections as indicated in Figure 5.20 are investigated as well. As can be seen in Figure 5.29 the largest deformations occur at $t = 31,0\,\text{sec}$. Figure 5.30 to Figure 5.32 show the principal stresses across the sections for $t = 31,0$. 

![Maximal Shear Stresses Louver B2 at Nodes in Time](image)

**Figure 5.32: Maximal shear stresses Louver B2, for extreme wind signal U(t)**
Figure 5.33: Maximal in-plane principal stresses Louver B2, for $U(t)$ at $t = 31.0$ sec

Figure 5.34: Minimal in-plane principal stresses Louver B2, for $U(t)$ at $t = 31.0$ sec

Figure 5.35: Maximal shear stresses Louver B2, for $U(t)$ at $t = 31.0$ sec
Similar to the procedure in case of the average wind signal in paragraph 5.2.1, Figure 5.33 to Figure 5.35 show the stresses in the louver due to the extreme wind loading along the in Figure 5.20 indicated sections at $t = 31.0$ sec. At this point in time the louver is subjected to its largest deformation, see Figure 5.29.

Comparing graphs Figure 5.33 to Figure 5.35 with the graphs in Figure 5.24 to Figure 5.26, shows that both the compressive and tensile stress levels have increased due to the extreme wind load. Besides, the stress graphs of Figure 5.33 to Figure 5.35 show a stronger fluctuation in the course of the stress graphs. These fluctuations can be the result of three dimensional effects.

Furthermore, it can be noticed that in case of the minimal in plane principal stresses in Figure 5.34, the stress level are more evenly distributed than in the case of the average wind signal (see Figure 5.25). A similar observation can be done in case of the maximal shear stresses, the graphs of the different sections are close to each other, indicating an evenly distributed stress over.

### 5.3. Conclusions numerical analysis
By using a modal analysis and comparing the translational stiffness of the DEM model to the translational stiffness of the FEM model, it has been shown that both models correspond with each other.

The first dynamic analysis for which the louver is loaded by an average wind loading, has shown that the maximal stress levels under these circumstances do not exceed the fatigue limit. Thereby, showing that fatigue is not an issue for the louver. In the second dynamic analysis the louver is loaded by an extreme wind signal, including a maximal wind gust of 43.9 m/s. Although the displacements of the louver are very large, the strength of the glass fibre material is sufficient.
6. Conclusions and recommendations

This chapter discusses the conclusions and recommendations as a result of the investigation of the mechanical behaviour of a louver applicable to IRWES. First the conclusions of this investigation are drawn and subsequently several recommendation are given.

6.1. Conclusions

First of all, it is verified that the simplified aerodynamic model, as described in this thesis, is suited for first estimations of the aerodynamic behaviour of the louvers. It has to be kept in mind that the results are obtained in agreement with thin aerofoil theory.

During this thesis it has been assumed that the louvers are fabricated out of a glass fibre reinforced composite material.

The displacement behaviour of the louver has been investigated for four designs. The four designs are a combination of two different cross-sectional designs and two different boundary conditions, see Figure 6.1.

![Figure 6.1: The four investigated louver designs](image)

It has been shown that instability phenomena are not a problem. In the worst case scenario the louver becomes instable for a wind speed $U = 160 \text{ m/s}$, this the case for Louver A1 and Louver A2. This is an unrealistic wind speed.

Furthermore, it is shown that the deflections of Louver A2 and Louver B2 are five times as large, compared to the deflections of Louver A1 and Louver B1, respectively. The largest deflections occur for Louver B2. However, Louver B2 satisfies the stiffness requirement according to the building codes, for wind speeds up to 24 m/s. This corresponds to a Beaufort force 9.

The numerical analyses have shown that the stress levels do not exceed the material’s strength capacity under any circumstances. Although the louver is subjected to a combination of translation and rotation, the stresses at the top and bottom side of the louver are governing.

During the dynamic numerical analysis it has been shown that the stress levels remain below 20% of the material’s strength capacity. It is a general rule of thumb that for glass fibre reinforced composites, the fatigue limit is at 20% of the material capacity. Therefore, fatigue is not an issue if the louver is constructed out of a glass fibre reinforced composite.

Summarised, it is concluded that the desired design is Louver B2. As a result of the absence of the longitudinal beams, the production process of Louver B2 is simpler and saves material. Furthermore, the construction of a hinged support is simpler, then the construction of a fully clamped support. Finally, regarding the stiffness requirement, it can be argued that in case of very high wind speeds (i.e. Beaufort force over 9), it is accepted that the stiffness limit is exceeded.
6.2. Recommendations

This thesis provided a template for the design of the louvers. However, there are still aspects left that require further investigation.

Regarding the mechanical approximation of the louver, the results in this thesis are based on thin aerofoil theory. Therefore, it is recommended to validate this assumption and investigate the influences of it on the results.

Furthermore, it is supposed that the rotations of the louver are limited, therefore, drag has not been a part of the investigation. It is recommended to conduct experimental research on the louvers. With the experimental investigation it is possible to verify the theoretical model established in this thesis and determine the influences of drag on the louver. This can be done by measuring the deflection and rotation of the louver, and the wind speed at the same time.

During this thesis only one type of material has been investigated. However, many more materials can be applied, whether or not a composite material. It has been shown that stiffness is governing over the strength of the material. Therefore, applying a material with a high Young’s modulus and a higher density could be beneficial in the end, if the amount of material can be reduced. Aluminium is one material suited for further investigation.

In this thesis it has been shown that the supports of the louver are important regarding the displacement behaviour of the louver. Therefore, it is advised to design the supports carefully.

Finally, it is recommended to investigate the behaviour of the louvers on the opposite side of the system, where the wind exits the system. As the flow of the wind over the louver changes, the forces on the louver are changed as well.
7. Bibliography


Appendix A: Equations of motion

This appendix explains the construction of the equations of motion regarding the calculations of the different coefficients part of the equations of motion. The MATLAB input files, in order to determine the responses of the louver for the different situations considered, are given as well.

A.1. Derivation of the coefficients of the equations of motion

The general derivations of all the coefficients which are part of the equations of motion are elaborated to give more insight into the construction of these equations.

The mass matrices, \[
\begin{bmatrix}
m_1 & S_0 \\
S_0 & I_0
\end{bmatrix}
\]
are constructed conform the following expressions:

\[
m_1 = A \times \rho \quad (A.1)
\]

With \(m_1\) = mass louver per unit length \([\text{kg/m}]\)
\(A\) = cross-sectional area louver \([\text{m}^2]\)
\(\rho\) = density of the applied material \([\text{kg/m}^3]\)

\[
S_0 = (d_{CG} - d_{EA}) \times m_1 \quad (A.2)
\]

With \(S_0\) = static moment about the elastic axis EA per unit length \([\text{kgm/m}]\)
\(d_{CG}\) = distance from leading edge to centre of gravity CG \([\text{m}]\)
\(d_{EA}\) = distance from leading edge to elastic axis EA \([\text{m}]\)
\(m_1\) = mass louver per unit length (see equation (A.1)) \([\text{kg/m}]\)

\[
I_0 = \frac{1}{3} \times m_1 (c^2 - 3 \times c \times d_{EA} + 3 \times d_{EA}^2) \quad (A.3)
\]

With \(I_0\) = mass moment of inertia per unit length \([\text{kgm}^2/\text{m}]\)
\(m_1\) = mass louver per unit length \([\text{kg/m}]\)
\(c\) = chord length louver \([\text{m}]\)
\(d_{EA}\) = distance from leading edge to elastic axis EA \([\text{m}]\)

The structural damping matrices are in this thesis constructed by means of Rayleigh damping. First step is to determine the Rayleigh coefficients conform:

\[
\begin{bmatrix}
[\alpha] \\
[\beta]
\end{bmatrix}
= 2 \times \frac{\omega_{n,1} \omega_{n,2}}{\omega_{n,2}^2 - \omega_{n,1}^2} \begin{bmatrix}
\omega_{n,2} & -\omega_{n,1} \\
1 & 1
\end{bmatrix}
\begin{bmatrix}
\zeta_1 \\
\zeta_2
\end{bmatrix} \quad (A.4)
\]

With: \(\alpha\) = Rayleigh coefficient related to the mass matrix 
\(\beta\) = Rayleigh coefficient related to the stiffness matrix 
\(\omega_{n,i}\) = natural frequencies \((i = 1, 2)\) \([\text{rad/s}]\)
\(\zeta_i\) = damping ratio \((i = 1, 2)\) 

The second step is to multiply \(\alpha\) with the mass matrix and \(\beta\) with the stiffness matrix and summing up both results:

\[
\begin{bmatrix}
C_{S:1} & C_{S:2} \\
C_{S:3} & C_{S:4}
\end{bmatrix}
= \alpha \begin{bmatrix}
m_1 & S_0 \\
S_0 & I_0
\end{bmatrix} + \beta \begin{bmatrix}
K_h & 0 \\
0 & K_b
\end{bmatrix} \quad (A.5)
\]
With: $C_{si} =$ structural damping coefficient ($i = 1, ..., 4$) [Ns/m/m$^3$]

$m_i =$ mass louver per unit length [kg/m$^2$]

$S_\theta =$ static moment about the elastic axis EA per unit length [kgm/m$^3$]

$I_0 =$ mass moment of inertia per unit length [kgm$^3$/m$^2$]

$K_h =$ translational spring stiffness per unit length [N/m/m]

$K_\theta =$ rotational spring stiffness per unit length [Nm/m]

As the equations of motion are described about the elastic axis of the cross-section, the stiffness matrix $K$ contains only two coefficients on the diagonal, which are determined by:

For Louver A1 and Louver B1

$$K_h = \frac{384EI_x}{L^4}$$

For Louver A2 and Louver B2

$$K_h = \frac{384EI_x}{5L^4}$$  \tag{A.6}

With: $K_h =$ translational spring stiffness per unit length (N/m/m)

$E =$ Young’s modulus (N/m$^2$)

$I_x =$ area moment of inertia (m$^4$)

$L =$ total length of the louver (m)

For all four louvers

$$K_\theta = 8\frac{GI_t}{L^2}$$  \tag{A.7}

With: $K_\theta =$ rotational spring stiffness per unit length (Nm/m)

$G =$ shear modulus (N/m$^2$)

$I_t =$ torsional moment of inertia (m$^4$)

$L =$ total length of the louver (m)

The derivation of the aerodynamic contributions in the equations of motion are extensively elaborated in the text and are therefore no part of this appendix.

### A.2. Input files MATLAB, no external loading

Not all different input files are given in this appendix, but only for the case of Louver A1. All input files can constructed by adjusting the coefficients of the equations of motion conform the values valid for the design of the louver considered.

#### A.2.1. MATLAB files steady aerodynamic

The state space file:

```matlab
function x = SteadyAerodynamicModel(~,y)

m = 17.33; % mass
S = 1.54;  % static moment
I = 1.11;  % mass moment of inertia
kh = 6.48*10^4; % translational spring stiffness
kr = 6.34*10^4; % rotational spring stiffness
L = 3285.34; % lift coefficient
MEA = 318.68; % moment about EA coefficient
x(1) = y(2); % h(dot)
```

Eindhoven University of Technology
\[ x(2) = \frac{1}{m} - \frac{S}{1/(m*I-S^2)} \left( y(1)^2 * S * kh - y(2) * (S * c1 + c3 * m) + y(1) * S * kh - y(3) * (m * kr - S * L - m * MEA) \right) - \frac{y(1)}{L} * y(3); \quad \% \text{h(double dot)} \]

\[ x(3) = y(4); \quad \% \text{theta(dot)} \]

\[ x(4) = \frac{1}{m * S^2} \left( y(1) * S * kh - y(3) * (m * kr - S * L - m * MEA) \right); \quad \% \text{theta(double dot)} \]

\[ x = [x(1); x(2); x(3); x(4)]; \]

The response file:

\[ tspan = [0: 0.0001: 1]; \quad \% \text{specification of time} \]

\[ y0 = [0.01; 0; 0.0015; 0]; \quad \% \text{initial condition} \]

\[ [t, y] = \text{ode45}('\text{SteadyAerodynamicModel}', tspan, y0); \quad \% \text{type of solver} \]

```
function x = LowFrequencyModel(t, y)
m = 17.33; \quad \% \text{mass}
S = 1.54; \quad \% \text{static moment}
I = 1.11; \quad \% \text{mass moment of inertia}
c1 = 100.78; \quad \% \text{first damping coefficient related to}
\quad \% \text{translational speed}
c3 = 9.78; \quad \% \text{second damping coefficient related to}
\quad \% \text{translational speed}
kh = 6.48*10^4; \quad \% \text{translational spring stiffness}
kr = 6.34*10^4; \quad \% \text{rotational spring stiffness}
L = 3285.34; \quad \% \text{lift coefficient}
MEA = 318.68; \quad \% \text{moment about EA coefficient}
x(1) = y(2); \quad \% \text{h(dot)}
x(2) = \frac{1}{m} - \frac{S}{1/(m*I-S^2)} \left( y(2)^2 * (S * c1 + c3 * m) + y(1) * S * kh - y(3) * (m * kr - S * L - m * MEA) \right) - \frac{y(2)}{L} * y(3); \quad \% \text{h(double dot)}
x(3) = y(4); \quad \% \text{theta(dot)}
x(4) = \frac{1}{m * S^2} \left( y(2) * (S * c1 + c3 * m) + y(1) * S * kh - y(3) * (m * kr - S * L - m * MEA) \right); \]
```

A.2.2. MATLAB files low frequency model

The state space file:
% theta(double dot)

\[ x = [x(1);x(2);x(3);x(4)]; \]

The response file:

\[
t = [0: 0.0001: 1]; \quad \text{% specification of time}
\]
\[
y0=[0.01;0;0.0015;0]; \quad \text{% initial condition}
\]
\[
[t,y]=ode45('LowFrequencyModel',tspan,y0); \quad \text{% type of solver}
\]
\[
subplot(2,1,1) \quad \text{% plotting properties}
plot(t,y(:,1))
grid
xlabel('Time t (sec)')
ylabel('Translation h (m)')
title('Translation h in time (t)')
\]
\[
subplot(2,1,2)
plot(t,y(:,3))
grid
xlabel('Time t (sec)')
ylabel('Rotation \theta (rad)')
title('Rotation \theta in time (t)')
\]

A.2.3. MATLAB files low frequency model including structural damping

The state space file:

\[
\text{function } x = \text{LowFrequencyModelIncDamping}(\cdot,y)
\]
\[
m = 17.33; \quad \text{% mass}
\]
\[
S = 1.54; \quad \text{% static moment}
\]
\[
I = 1.11; \quad \text{% mass moment of inertia}
\]
\[
cs1 = 105.69; \quad \text{% first structural damping coefficient}
\]
\[
cs2 = 7.59; \quad \text{% second structural damping coefficient}
\]
\[
cs3 = 7.59; \quad \text{% third structural damping coefficient}
\]
\[
cs4 = 25.44; \quad \text{% fourth structural damping coefficient}
\]
\[
c1 = 100.78; \quad \text{% first damping coefficient related to translational speed}
\]
\[
c3 = 9.78; \quad \text{% second damping coefficient related to translational speed}
\]
\[
kh = 6.48*10^4; \quad \text{% translational spring stiffness}
\]
\[
kr = 6.34*10^4; \quad \text{% rotational spring stiffness}
\]
\[
L = 3285.34; \quad \text{% lift coefficient}
\]
\[
MEA = 318.68; \quad \text{% moment about EA coefficient}
\]
\[
x(1) = y(2); \quad \text{% h(dot)}
\]
\[
x(2) = 1/m*(-S*(1/(I*m-S^2)*(-y(2)*(m*cs3-m*c3-S*cs1-S*c1)-y(4)*(m*cs4-S*cs2)+y(1)*k1-y(3)*(m*k4-m*MEA-S*L)))-cs1*y(2)-cs2*y(4)-c1*y(2)-k1*y(1)-L*y(3)); \quad \text{% h(double dot)}
\]
\[
x(3) = y(4); \quad \text{% theta(dot)}
\]
\[
x(4) = 1/(I*m-S^2)*(-y(2)*(m*cs3-m*c3-S*cs1-S*c1)-y(4)*(m*cs4-
% theta(double dot)

\[ x = [x(1); x(2); x(3); x(4)]; \]

The response file:

\[
\begin{align*}
\text{tspan} &= [0: 0.001: 1]; \quad \% \text{specification of time} \\
y0 &= [0.01; 0; 0.0015; 0]; \quad \% \text{initial condition} \\
[t, y] &= \text{ode45('LowFrequencyModelIncDamping', tspan, y0); } \quad \% \text{type of solver}
\end{align*}
\]

% plotting properties

\[
\begin{align*}
\text{subplot}(2,1,1) \\
\text{plot}(t, y(:,1)) \\
\text{grid} \\
\text{xlabel}('Time t (sec)') \\
\text{ylabel}('Translation h (m)') \\
\text{title}('Translation h in time (t)') \\
\end{align*}
\]

\[
\begin{align*}
\text{subplot}(2,1,2) \\
\text{plot}(t, y(:,3)) \\
\text{grid} \\
\text{xlabel}('Time t (sec)') \\
\text{ylabel}('Rotation \theta (rad)') \\
\text{title}('Rotation \theta in time (t)') \\
\end{align*}
\]

A.3. Input files MATLAB, with external loading

To model the response of the louver under a dynamic wind loading the MATLAB files have to be adjusted. In the case of the situations without an external wind loading the aerodynamic forces where characterised by constant coefficient, however, for this case the aerodynamic contributions exist out of two parts: a constant part and the fluctuating wind speed. Both parts are brought together again in the actually state space formulation.

A.3.1. MATLAB files steady aerodynamic model

The state space file:

\[
\begin{align*}
\text{function} \quad x &= \text{SteadyAerodynamicModelIncU}(t, y) \\
m &= 17.33; \quad \% \text{mass} \\
S &= 1.54; \quad \% \text{static moment} \\
I &= 1.11; \quad \% \text{mass moment of inertia} \\
kh &= 6.48 \times 10^4; \quad \% \text{translational spring stiffness} \\
kR &= 6.34 \times 10^4; \quad \% \text{rotational spring stiffness} \\
L &= 3.09; \quad \% \text{constant part lift force (without } U(t)\text{)} \\
MEA &= 0.30; \quad \% \text{constant part moment (without } U(t)\text{)} \\
w &= 0.1015; \quad \% \text{fundamental frequency of signal} \\
Ut &= 13.51 - 0.7446 \cos(t \times w) + 1.5 \sin(t \times w) - 1.62 \cos(t \times w^2) - 1.503 \sin(t \times w^2) + 2.613 \cos(t \times w^3) - 0.5211 \sin(t \times w^3) + 0.7562 \cos(t \times w^4) + 0.7715 \sin(t \times w^4) - 0.2481 \cos(t \times w^5) + 0.6971 \sin(t \times w^5) + 0.2751 \cos(t \times w^6) - 0.8395 \sin(t \times w^6) + 0.6015 \cos(t \times w^7) - 0.7389 \sin(t \times w^7) + 0.05051 \cos(t \times w^8) + 1.85 \sin(t \times w^8); \quad \% \text{Fourier}
\end{align*}
\]
T0 = 0.1745;  \quad \% \text{angle of attack in rad}

x(1) = y(2);  \quad \% h(\dot{\text{dot}})

x(2) = \frac{1}{m^*}(-S*(1/(m*I-S^2))*(y(1)*S*kh-y(3)*(m*kr-S*L*Ut^2-m*MEA*Ut^2)-T0*(S*L*Ut^2+m*MEA*Ut^2)))-kh*y(1)-y(3)*L*Ut^2+L*Ut^2*T0);  \quad \% h(\ddot{\text{dot}})

x(3) = y(4);  \quad \% \theta(\dot{\text{dot}})

x(4) = \frac{1}{(m*I-S^2)}(y(1)*S*kh-y(3)*(m*kr-S*L*Ut^2-m*MEA*Ut^2)-T0*(S*L*Ut^2+m*MEA*Ut^2));  \quad \% \theta(\ddot{\text{dot}})

x = [x(1);x(2);x(3);x(4)];

The response file:

tspan = [0: 0.0001: 60];  \quad \% \text{specification of time}

y0=[0;0;0;0];  \quad \% \text{initial condition}

[t,y]=ode45(’SteadyAerodynamicModelIncU’,tspan,y0);  \quad \% \text{type of solver}

subplot(2,1,1)  \quad \% \text{plotting properties}
plot(t,y(:,1))
grid
xlabel(’Time t (sec)’)
ylabel(’Translation h (m)’)
title(’Translation h in time (t)’)

subplot(2,1,2)
plot(t,y(:,3))
grid
xlabel(’Time t (sec)’)
ylabel(’Rotation \theta (rad)’)
title(’Rotation \theta in time (t)’)

A.3.2. MATLAB files low frequency model

The state space files:

function x = LowFrequencyModelIncU(t,y)

m = 17.33;  \quad \% \text{mass}
S = 1.54;  \quad \% \text{static moment}
I = 1.11;  \quad \% \text{mass moment of inertia}

c1 = 3.09;  \quad \% \text{first constant part damping coefficient related to translational speed}
c3 = 0.30;  \quad \% \text{second constant part damping coefficient related to translational speed}

kh = 6.48*10^4;  \quad \% \text{translational spring stiffness}
kr = 6.34*10^4;  \quad \% \text{rotational spring stiffness}

L = 3.09;  \quad \% \text{constant part lift force (without U(t))}
MEA = 0.30;  \quad \% \text{constant part moment (without U(t))}
w = 0.1015;  \quad \% \text{fundamental frequency of signal}
The response file:

tspan = [0: 0.0001: 60]; % specification of time
y0=[0;0;0;0]; % initial condition
[t,y]=ode45('LowFrequencyModelIncU',tspan,y0); % type of solver
subplot(2,1,1)
plot(t,y(:,1))
grid
xlabel('Time t (sec)')
ylabel('Translation h (m)')
title('Translation h in time (t)')

subplot(2,1,2)
plot(t,y(:,3))
grid
xlabel('Time t (sec)')
ylabel('Rotation \theta (rad)')
title('Rotation \theta in time (t)')

A.3.3. MATLAB files low frequency model including structural damping

The state space file:

```
function x = LowFrequencyModelIncDampingIncU(t,y)
m = 17.33; % mass
S = 1.54; % static moment
I = 1.11; % mass moment of inertia
cs1 = 105.69; % first structural damping coefficient
cs2 = 7.59; % second structural damping coefficient
cs3 = 7.59; % third structural damping coefficient
cs4 = 25.44; % fourth structural damping coefficient
```

c1 = 3.09; % first constant part damping coefficient related to translational speed
c3 = 0.30; % second constant part damping coefficient related to translational speed
kh = 6.48*10^4; % translational spring stiffness
kr = 6.34*10^4; % rotational spring stiffness
L = 3.09; % constant part lift force (without U(t))
MEA = 0.30; % constant part moment (without U(t))
w = 0.1015; % fundamental frequency of signal
Ut = 13.51-0.7446*cos(t*w)+1.5*sin(t*w)-1.62*cos(t*w*2)- 1.503*sin(t*w*2)+2.613*cos(t*w*3)- 0.5211*sin(t*w*3)+0.7562*cos(t*w*4)+0.7715*sin(t*w*4)- 0.2481*cos(t*w*5)+0.6971*sin(t*w*5)+0.2751*cos(t*w*6)- 0.8395*sin(t*w*6)+0.6015*cos(t*w*7)- 0.7389*sin(t*w*7)+0.05051*cos(t*w*8)+1.85*sin(t*w*8); % Fourier approximation of U(t)
T0 = 0.1745; % angle of attack

x(1) = y(2); % h(dot)
x(2) = 1/m*(-S*(1/(I*m-s^2)*(-y(2)*((m*cs3-m*c3*Ut-S*cs1-S*c1*Ut)- y(4)*((m*cs4-S*cs2)+y(1)*S*kh-y(3)*((m*kr-m*MEA*Ut^2-S*L*Ut^2)- T0*(S*L*Ut^2+m*MEA*Ut^2))) -cs1*y(2)-cs2*y(4)-c1*Ut*y(2)-kh*y(1)- L*Ut^2*y(3)+L*Ut^2*T0)); % h(double dot)
x(3) = y(4); % theta(dot)
x(4) = 1/(I*m-s^2)*(-y(2)*((m*cs3-m*c3*Ut-S*cs1-S*c1*Ut)- y(4)*((m*cs4-S*cs2)+y(1)*S*kh-y(3)*((m*kr-m*MEA*Ut^2-S*L*Ut^2)- T0*(S*L*Ut^2+m*MEA*Ut^2))); % theta(double dot)
x = [x(1);x(2);x(3);x(4)];

The response file:

tspan = [0: 0.001: 60]; % specification of time
y0=[0;0;0;0]; % initial condition
[t,y]=ode45('LowFrequencyModelIncDampingIncU',tspan,y0); % type of solver
subplot(2,1,1) % plotting properties
plot(t,y(:,1))
grid
xlabel('Time t (sec)')
ylabel('Translation h (m)')
title('Translation h in time (t)')

subplot(2,1,2)
plot(t,y(:,3))
grid
xlabel('Time t (sec)')
ylabel('Rotation \theta (rad)')
title('Rotation \theta in time (t)')
Appendix B: Translation of an equally distributed force to a concentrated load

For the ease of numerical modelling the equally distributed load is translated to a concentrated force at the elastic axis of the louver, half way along the louver’s span. Therefore, the concentrated force has to result into the same deflection as is the case for a louver subjected to an equally distributed load. Requiring both deflections to be equal results into a concentrated force represented by the equally distributed load.

B.1. General derivation of the concentrated forces

Figure B.1 and Figure B.2 show the schematisations of a fully clamped beam subjected to an equally distributed load \( q_L \) and an concentrated force \( Q_L \) respectively. The expressions of the maximal deflections \( h \) are given by the ‘vergeet-me-nietjes’ (MCB Nederland). Equating both deflections results into a concentrated force represented by the equally distributed load.

\[
\frac{1}{384} \frac{q_L L^4}{E I_x} = \frac{1}{192} \frac{Q_L L^3}{E I_x} \quad \rightarrow \quad Q_L = \frac{1}{2} q_L L \quad (B.1)
\]

With:
- \( q_L \) = equally distributed load \([N/m^1]\)
- \( Q_L \) = concentrated force \([N]\)
- \( L \) = length louver \([m]\)
- \( E \) = Young’s modulus \([N/m^2]\)
- \( I_x \) = bending moment of inertia \([m^4]\)

The concentrated load in case of a fully clamped beam loaded by \( q_L = 1000 \text{ N/m}^1 \), results into:

\[
Q_L = \frac{1}{2} \left( \frac{1}{2} \times 1000 \times 6,0 \right) = 1500 \text{ N} \quad (B.2)
\]

The concentrated lift force determined by (B.2) is applied at the elastic axes of Louver A1.

To determine the concentrated forces on Louver B2 a similar approach is applied. Figure B.3 and Figure B.4 show both schematisations for the situations considered. The standard expressions for the deflection of the illustrated beams are given as well (MCB Nederland).
Maximal deflection $h = \frac{5}{384} qL^4 \frac{qL^4}{EI}$

Figure B.3: Vertically hinged louver with equally distributed load

Maximal deflection $h = \frac{1}{48} \frac{FL^3}{EI}$

Figure B.4: Vertically hinged louver with concentrated force

In case of a beam supported by hinges and loaded by $qL = 1000$ N/m, the equivalent concentrated load is:

$$Q_L = \frac{1}{2} \left( \frac{5}{8} \times 1000 \times 6.0 \right) = 1875 \text{ N} \quad (B.4)$$
Appendix C: Material strength properties
Reference strength properties of glass fibre reinforced composite material, provided by *Performance Composites Ltd.*

<table>
<thead>
<tr>
<th>Symbol</th>
<th>GPa</th>
<th>Units</th>
</tr>
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<tr>
<td>$E_{1}$</td>
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<td>Std. CF</td>
</tr>
<tr>
<td>$E_{2}$</td>
<td>17</td>
<td>Std. CF</td>
</tr>
<tr>
<td>$G_{12}$</td>
<td>17</td>
<td>Std. CF</td>
</tr>
<tr>
<td>$G_{23}$</td>
<td>17</td>
<td>Std. CF</td>
</tr>
<tr>
<td>$G_{31}$</td>
<td>17</td>
<td>Std. CF</td>
</tr>
<tr>
<td>$v_{12}$</td>
<td>17</td>
<td>Std. CF</td>
</tr>
<tr>
<td>$v_{23}$</td>
<td>17</td>
<td>Std. CF</td>
</tr>
<tr>
<td>$v_{31}$</td>
<td>17</td>
<td>Std. CF</td>
</tr>
<tr>
<td>$v_{13}$</td>
<td>17</td>
<td>Std. CF</td>
</tr>
<tr>
<td>$v_{21}$</td>
<td>17</td>
<td>Std. CF</td>
</tr>
<tr>
<td>$v_{32}$</td>
<td>17</td>
<td>Std. CF</td>
</tr>
<tr>
<td>$v_{31}$</td>
<td>17</td>
<td>Std. CF</td>
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</table>

<table>
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<th>Material</th>
<th>In Plane Shear Strength</th>
<th>Compressive Strength</th>
<th>Tensile Strength</th>
<th>Poisson's Ratio</th>
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<td>110</td>
<td>0.34</td>
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<td>100</td>
<td>120</td>
<td>0.34</td>
</tr>
<tr>
<td>Steel</td>
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<td>120</td>
<td>120</td>
<td>0.34</td>
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<tr>
<td>Al</td>
<td>150</td>
<td>120</td>
<td>120</td>
<td>0.34</td>
</tr>
</tbody>
</table>

Note: These tables are for reference only and are NOT a guarantee of performance.
Appendix D: Input file abaqus

Step 1: Import sketch
Import the cross-section of the louver as a sketch. The cross-section is drawn in AutoCAD and saved as a .iges file. It is noted that the cross-section is drawn in millimeters.

Step 2: Create a part
Name the part Louver and create a 3D deformable shell and chose for the type extrusion. The approximate size is set to 1 000.

Click continue and add the sketch imported in ‘Step 1’. After closing the sketch editor edit the depth of the part to 3 000, as only half the louver is modelled due to symmetry.

Step 3: Edit part
A reference point is used to model the elastic axis of the louver. The reference point is introduced by ‘Tools’ -> ‘Reference point’ in the menu bar. Enter the coordinates of the reference point relative to the datum point. In case of Louver B2 the (x, y, z)-coordinates are: 218, 0, 3 000.

Next three sets are defined. Double click set in the model tree, name the set RP, select ‘Geometry’ and click ‘Continue’. Select the reference point and click ‘Done’ in the prompt area. Next, again double click set in the model tree, name the second set Left, select ‘Geometry’ and click ‘Continue’. Select the top and bottom edge of the louver near the reference point and click ‘Done’ in the prompt area.

Step 4: Creating the material
In this thesis it is assumed that it is sufficient to create an elastic isotropic material definition. To create a material double click ‘Materials’ in the model tree. Name the material GFRC. For dynamical analysis the density of the material has to be known, select ‘General’ -> ‘Density’. Accept the default setting and enter 1,97e-9 as the materials density, this agrees to tonnes/mm$^3$ as the sketch was in millimeters as well.

Next select ‘Mechanical’ -> ‘Elasticity’ - > ‘Elastic’, accept the default setting and enter the value of the Young’s modulus (21 500 N/mm$^2$) and the poison ratio (0,23).

Step 5: Creating and assigning a section
To assign section properties to the louver a section is created. Double click ‘Sections’ in the model tree, in the dialog box that appears name the section LouverSection and chose for a homogeneous shell section, click ‘Continue’. Chose ‘During analysis’ as section integration. On the first tab of the dialog box enter 5 mm as the shell thickness value and select GFRC in the material’s drop down list. Click ‘Ok’ to close the dialog box.

Next double click ‘Section Assignments’ in the model tree. Select whole the louver in the viewport and click ‘Done’ in the prompt area. Chose LouwerSection and accept the default setting, click ‘Ok’.

The part changes color after the part has been assigned section properties.
Step 6: Create mesh
To create a mesh for the model select in the menu bar ‘Seed’ -> ‘Edges’. Select the edges of the top and bottom side of the louver and click ‘Done’ in the prompt area. Select ‘By number’ as method to seed the edges and enter 50 for the number of elements, click ‘Ok’.

In the menu bar select ‘Mesh’ -> ‘Controls’, select the whole louver in the viewport and click ‘Done’ in the prompt area. Chose ‘Quad’ as the element shape and ‘Structured’ as the technique. Click ‘Ok’.

Next select ‘Mesh’ -> ‘Element type’ and select the whole louver, click ‘Done’ in the prompt area. Select S4 as the element type by toggling of ‘Reduced integration’. Click ‘Ok’.

As final step mesh the model by selecting ‘Mesh’ -> ‘Part’ in the menu bar and click ‘Yes’ in the prompt area.

Step 7: Creating sets for nodal stress investigation
To be able to investigate the stresses in the nodes near the middle of the louver’s span, six sets are created. To create the first set double click ‘Sets’ in the model tree, name the set BottomLouver and select ‘Node’ as the type of set, click ‘Continue’. Select a node at the bottom side of the louver, situated some distance from the edge.


Step 8: Assembling the model
Make an assembly of the model by expanding ‘Assembly’ in the model tree, double click ‘Instances’. Select the part Louver and click ‘Ok’. For this analysis the instance has been rotated in such a way that the origin is located at the right side of the leading edge. Make sure that the z-axis in pointing downwards and the y-axis points in the longitudinal direction of the louver.

Step 9: Define step
Double click ‘Steps’ in the model tree, name the step DynDisp (dynamic displacement). Select a ‘General’, ‘Dynamic, Implicit’ procedure type and click ‘Continue’. Enter 60 as the time period and toggle on ‘Non Linear Geometry’. Continue to the ‘Incrementation’ and specify the maximum number of increments to be 10 000. Set the initial increment size to 0,0125, the minimum increment size to be 1∙10\(^{-5}\) and the maximum increments size to 0,025. Click ‘Ok’.

It is chosen to use the ‘Dynamic, Implicit’ procedure because of the quasi-static nature of the loading and the duration of the dynamic behaviour (Simulia, 2014), (MSC Nastran Novice, 2013).

Step 10: Create output
Accept the default field output and delete the default history output. Create a new history output for every set created in ‘Step 7’. Double click ‘History Output Requests’ and name the output after the set for which it is formed (e.g. BottomLouver). Select for the domain ‘Set’, chose the same set as the name of the history output request. Select the desired components to be part of the output.
Step 11: Create constraint
To prevent a free floating reference point, the reference point is constrained to the top and bottom side of the louver.

Double click ‘Constraints’ in the model tree, name the constraint and select ‘Rigid body’. In the region type select tie nodes and select the set Left as the region. For the reference point, select the set RP. Click ‘Ok’.

Step 12: Import amplitudes
Double click ‘Amplitude’ in the model tree, name the first amplitude Translation and select ‘Tabular’. Select ‘Step Time’ as time span of the amplitude and import translational response of the louver in the columns. Make sure that the response starts with no initial excitation and increase with steps of 0,1.

Repeat the procedure for the rotational response of the louver.

Step 13: The boundary conditions
Four types of boundary conditions have to be created. The first forms the support of the louver. Double click ‘BCs’ in the model tree and create a ‘Symmetry/Antisymmetry/Encastre’ boundary condition in the initial step. Select the nodes in the leading and trailing edge near the origin of the louver, click ‘Done’ and select ‘Pinned’ and click ‘Ok’.

The second boundary condition is the symmetry boundary condition at the reference point RP. Repeat the previous procedure but select RP to be allocated a boundary condition, next select YSYM as the type of boundary condition.

The third and fourth boundary conditions are the forced displacements of the reference point. Create a displacement boundary condition in the DynDisp step, select RP, toggle on ‘U3’ and assign this parameter the value 1. From the amplitude’s drop down menu select translation. Repeat this procedure to enter the forced rotation of the louver.

Step 14: Create and submit the analysis
Double click ‘Jobs’ in the model tree, name the job, select the model in which the louver is modelled and click ‘Continue’. If desired the usage of the memory and processors can be adjusted. Create the job by clicking ‘Ok’.

The job is submitted by right-clicking on the created job and selecting ‘Submit’.

Step 15: Analyse the results
The results can be analysed by selecting results after completion of the numerical analysis.