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Investigation of the reliability of electronic products under a mechanical dynamic load failure behaviour of solderjoints on multilayer printed circuit boards

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INVESTIGATION ON THE RELIABILITY OF ELECTRONIC PRODUCTS UNDER A MECHANICAL DYNAMIC LOAD:

FAILURE BEHAVIOUR OF SOLDERJOINTS ON MULTILAYER PRINTED CIRCUIT BOARDS

Final report

by:

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List of symbols

\( a \) - acceleration (chap.4), height (App.E) \([\text{m/s}^2], [\text{mm}]\)

\( A \) - area \([\text{mm}^2]\)

\( \alpha \) - angular displacement \([-]\)

\( b \) - width \([\text{mm}]\)

\( c \) - constant \([\text{N/mm}]\)

\( d \) - diameter \([\text{mm}]\)

\( D \) - diameter (chap.2), cross force (App.E) \([\text{mm}],[\text{N}]\)

\( D_0 \) - footdiameter of solderjoint \([\text{mm}]\)

\( e \) - strain (chap.2), excentricity (chap.4) \([-],[\text{mm}]\)

\( E \) - ratio of moduli of elasticity \([-]\)

\( F \) - force \([\text{N}]\)

\( g \) - gravity \([\text{m/s}^2]\)

\( h \) - height of solderjoint (chap.2), height (App.E) \([\text{mm}]\)

\( I_{yy} \) - moment of inertia of area \([\text{mm}^4]\)

\( k \) - constant \([\text{N/mm}^2]\)

\( K \) - constant \([\text{kg/m}^2]\)

\( l \) - length \([\text{mm}]\)

\( L \) - shear force \([\text{N}]\)

\( m \) - mass (chap.4), reduced bending moment (chap.2) \([\text{kg}],[\text{Nmm}]\)

\( M \) - moment \([\text{Nmm}]\)

\( N \) - normal force \([\text{N}]\)

\( P \) - force \([\text{N}]\)

\( r \) - radius \([\text{mm}]\)

\( s \) - stress \([\text{N/mm}^2]\)

\( S \) - static moment of inertia of area \([\text{mm}^3]\)

\( t \) - time \([\text{s}]\)

\( \tau \) - shear stress \([\text{N/mm}^2]\)

\( u \) - displacement \([\text{mm}]\)

\( v \) - length \([\text{mm}]\)

\( \psi \) - angle between point of application of the load \( P \) and "stress point" A \([-]\)

\( x \) - displacement (chap.4) \([\text{mm}]\)

\( \omega \) - angular speed \([\text{1/s}]\)

lowerindices:

1 2 - interface between material 1 and 2

1 - wire

2 - solder

I - cross section I

II - cross section II

adm - admissible

A - point A

b - bending

bs - bending solder

bw - bending wire

c - crack

max - maximum

P - point P

s - solder

tot - total

w - wire
upperindices:

',  - dimensionless (chap.2), first derivative (chap.4)
'\' - second derivative

iii
Definitions:

Failure - an occurrence of an inability of an item to perform its function when called upon to do so [IBM].

Fault - a physical state or condition of an item which may cause a failure of the item [IBM,IEEE].

Flaw - a physical state or condition of an item with potential to become a fault [IBM].

Failure mechanism - the physical, chemical or mechanical process which results in a fault [IBM,IEC].

Stress - to subject an item to an environment which exceeds normal usage as defined by engineering specification, with the intention of precipitating existing faults in the item without creating new faults.

Stress parameters - environmental conditions which are controlled at levels which exceed normal usage as defined by engineering specifications.

Screen - a go or no-go test with the objective to separate failure suspectable items.

Stress testing - the investigative phase of a stress program, which results in identification of the most prevalent failure mechanism in a product and definition of the stress conditions which produce the most effective process stress screen oraccelerated reliability test.

Stress screening - a screening process which utilizes stress parameters.
Introduction

This report describes the subject, which was studied in accordance with the final project in mechanical engineering study at Eindhoven University of Technology, the Netherlands. The work was carried out at the IBM company in Amsterdam during the period of April 16, 1984 until December 31, 1985. From University the task was accompanied by prof.dr.ir. D.H. van Campen and dr.ir. A. de Kraker. Coaching at IBM was done by ir. F. van der Post.

Generally the commission was given to come to a product independent mathematical theory for vibrational stress screening. With help of this theory stress screens could be calculated and be more optimal than those used nowadays.

A theory as mentioned could not be derived during this project. However a basic strategy is formulated and the bottlenecks to a complete theory are indicated. It appeared that a lot of investigation has to be done on the subject of mathematical descriptions for the behaviour of failure mechanisms under a mechanical dynamical load. Therefore part of this study was focussed on a more profound investigation towards the failures of solderjoints. The study had to be restricted to only this failure mechanism because of the time available. Additional investigations thus can be made in possible future studies on the same subject.
Report summary

The report starts with a brief discussion on today's stress screening tests. A new strategy, based on a system approach is given, to come to a mathematical model for calculating optimal stress screens. The need for fundamental investigations on the behaviour of failure mechanisms under a mechanical vibrational load is highlighted (chapter 1). The report is continued with the study on the failure mechanism of solderjoints.

Chapter 2 describes an analytical model for the mechanical stresses in solderjoints on multilayer printed circuit boards. A parameter analysis, wherein several geometric properties are changed concludes the chapter. Additional numerical calculations, with aid of the finite element modeling technique, are presented in chapter 3. A comparison is made towards the analytical model.

Tests on the solder process of testspecimen and fatigue tests with those specimen are described in chapter 4. The fatigue tests were setup to study the fractures in solderjoints when the point of application of the fatigue load on the wire has different distances towards the solderjoint.

Finally conclusions and critical remarks are written in chapter 5 and 6.

The appendices contain additional graphs and plots of the stresses in the studied solderjoints. Technical descriptions of the test equipment are also found in the appendix.
1. A NEW LIGHT ON STRESS SCREENING

It is well known that products sometimes show failures after shipping and installation. Two major causes can be given for these failures:
- failures due to poor design.
- failures that are caused in the assembly process including packaging the product.

In this study it is assumed that the design will not lead to any failures. The only kind of failures left are the assembly failures, or assembly faults or flaws, which will eventually lead to failures. With so called stress screening tests it should be possible to accelerate these flaws and faults to become failures, which can be detected before the product is shipped to the customer. Generally stress screening methods can be divided into mechanical, electrical and thermal tests. This study is concentrated on the first method. From the mechanical tests we look at those, which make use of (random) vibration for screening a product. Terms with their definitions, which are related to the subject are given in the list of definitions (page iv).

Looking at the mechanical tests one can feel there must be a relationship between the environmental stimuli and the way flaws are growing into failures. There may be a dimensionless parameter (like the Reynolds number in flow problems) which describes the relationship between for instance frequency spectrum \(f\), time duration \(t\) and intensity \(g\) of the vibration in relation to characteristic product dimensions:

\[
\text{PAR}(f,t,g,...) = f(\text{product type and dimensions}) \quad (1.1)
\]

When such a parameter can be found, it will be possible to calculate an optimal stress screen within the limits of the available test equipment. So it becomes possible that someone calculates his productscreen for a specific equipment configuration. Another one, with e.g. less powerfull shakers, can also calculate an optimal screen for the same product but the testtime may be longer in comparison with the first one. However, both screens will stimulate the product with an equal amount of environmental influence, because the PAR value is the same in both cases.

The need for a proper description of stress screens is also read in literature (literature survey [2].[10]). Many investigators use different methods of stress screening, and they say they are succesful with it. But nobody can say that he is using the right screen. There is always an uncertainty about the power of the screen. And therefore it stays possible that products will be rejected because they show several failures, which could be caused by the screening process itself. In that case the power of the screen is to high. On the other hand a screen could be so wide that products with potential failures slip through it and cause problems in the field. A mathematical tool as proposed in (1.1) can bring clearness in these problems so
investigators can estimate the consequences of the screen they use better than before.

A thing that must be kept in mind when reading literature about stress screening is the approach from the author. There are people writing about stress screening what we would call environment simulation. The tests they describe are based on a true simulation of the environment, in which the product will be during lifetime. Primary goal of such tests is to check the product performance under several environmental influences. For example look at random vibration packaging tests. Here a field measured shipping spectrum is applied to packed products to see if they can withstand the shipping environment. Talking about stress screening however, is talking about tests which are set up to detect failures in a bare product, not looking at the environment the product eventually will be in. These tests thus can be seen as stronger ones than the environmental tests because the goal is to get products with zero defects, which are always able to withstand the future environmental conditions.

![Diagram of product system](image)

Fig.1.1. Example of the structure of the product system.

To give more substance to formula (1.1) it is necessary to take a closer look at the product and what is inside the product. Fig.1.1 gives an example how a product can be divided in several systems. All systempaths end in a box
called FAILURES, which represents all possible failure mechanisms and their vibrational mathematical description. The basic strategy to derive eq. (1.1) is studying the system paths with their transfer functions and combine them at the product level. For example look at the bold drawn path in fig.1.1. SUBASM3 in the product consists of a bolt joint, a PCB and a connector, which are fixed in a subframe. \( H_5 \) is the transfer function between the outside of the product and the spot where failures occur (on the PCB). It is assumed that the failure behaviour of the entire PCB is describable. With \( H_5 \) and this failure description a vibrational condition on product level which stimulates the failure mechanisms on the PCB can be derived. Eventually when doing this for all items in the system, an overall input condition, which stimulates all the failure mechanisms in the most optimal way is obtained. In other terms, knowing the permitted local stresses at the failure level a global stress on product level can be calculated. We note here, that the system approach, as presented here, is also applicable for other stimuli.

So far the strategy. Is it really possible to walk down the way mentioned above and solve the problem? The answer is no. A lot of steps in the procedure can be solved but coming to the point of mathematical descriptions of the failure mechanisms no answer can be given yet. The solvable steps are for instance the definition of transfer functions \( H_i \). They can be derived either by calculation with aid of the systems theory, or they can be measured with model analysis tools. The FAILURES blackbox is the trouble and therefore this study is looking closer to that point. To get a first insight in the problem, a brief overview of failure mechanisms commonly occurring in today's electronics is given:

- **Printed circuit boards (PCB's):** broken PCB, cracks
  - loosen or broken solderjoints
  - broken components
  - failing components
  - broken wires
  - broken printtracks
  - corrosion

- **Connectors:** broken pins
  - detached
  - corrosion

- **Cables:** poor solderjoints
  - broken cables (internal)

- **Electrical parts:** defective switches

- **Mechanical parts:** loose screws, nuts, bolts, springs, buttons, etc.
  - broken parts
  - cracks
Failing components can be subdivided into several failure types. However that will enter an area where mechanical tests through vibration are less efficient, while thermal tests are more significant in failure acceleration.

As already underlined in the previous list the attention will go out to the failure mechanism of the broken or loose solderjoints. There are two reasons for doing so:

- Bad solderjoints is a very common failure and from an economical point of view an important failure to do something about.

The last reason is very important because this lead to a tool, to install printed circuit boards containing zero defects. When also knowing that PCB's are more and more the main part of a product, such a test can add a great amount of reliability to it.

The rest of this report is focussed on the failure mechanism of solderjoints. Though this is a very small area in the entire world of stress screening, this choice is made because a better insight in solderjoint breakdown, together with the already known theories, can lead to a mathematical description as mentioned in (1.1) for at least printed circuit boards. Translating the existing theories to the area of electronic products could be another approach for this study, but eventually will not give an answer to the question how to arrange a stress screen for optimal testing.
2. ANALYTICAL MODEL AND PARAMETER STUDY ON THE STRESSES IN SOLDERJOINTS

2.1 APPROXIMATION BY STEINBERG

Looking at a solderjoint on a printed circuit board with plated through holes, Steinberg [11] gives a description of the crack mechanism. He assumed that a solderjoint, with a shape as drawn in fig. 2.1, will show a fatigue crack on approximately $\frac{2}{3}$ of the height ($h$) of the joint, after a certain amount of load changes in the bending moment $M$. The stresses due to the bending moment, on the spot of the assumed crackplane, are given by Steinberg as follows:

For the maximum tensile stress on the outside of the wire:

$$s_w = \frac{Md_w / 2}{nd_w^4 / 64} = \frac{32M}{nd_w^3} \quad (2.1)$$

For the maximum stress on the solder surface:

$$s_s = \frac{Md_s / 2}{nd_s^4 / 64} = \frac{32M}{nd_s^3} \quad (2.2)$$

These stresses, together with the fatigue data of solder and the wire material, are used to predict the duration of life of the solderjoint.

A closer look at the theory of Steinberg shows that his approach is too simple, because of the great difference in moduli of elasticity between the solder and the wire material. This is not taken into account in the above formulas. In both formulas Steinberg assumed linear stress distributions over the entire sections. Formula (2.1) reflects a linear stress distribution in the wire only, and no stress in the solder. Formula (2.2) shows a linear stress distribution over the entire section as all material would be solder. It is better to derive stress formulas in which the difference of elasticity is included. This results in a stress distribution which consists of two linear fields.

2.2 REFINED CALCULATIONS

Considering the laminated section as given in fig. 2.2 a stress distribution which is in equilibrium with the external bending moment $M$ can be calculated.
Under the assumption of Bernoulli's hypothesis for pure bending the displacement field in a cross section is given by:

\[ u = czx \]

Therefore the axial strain will be:

\[ e_{xx} = \frac{\partial u}{\partial x} = cz \]

And stress in the direction of x follows by Hooke's law:

\[ s_{xx} = E e_{xx} \]

Due to the differences in moduli of elasticity the above formula will be different for both materials so that:

\[ s_{xx1} = E_1 e_{xx} = E_1 cz \quad \text{for the wire} \]
\[ s_{xx2} = E_2 e_{xx} = E_2 cz \quad \text{for the solder} \]

The internal moment, caused by the stress distribution, must be in equilibrium with the external moment so:

\[ dM = s_{xx1} z dA_1 + s_{xx2} z dA_2 \]
\[ = c(E_1 z^2 dA_1 + E_2 z^2 dA_2) \]

Integration for the whole section leads to:

\[ M = c(E_1 \int z^2 dA_1 + E_2 \int z^2 dA_2) \]
\[ = c(E_1 I_{yy1} + E_2 I_{yy2}) \]

So the two linear stress distributions can be written as:

\[ s_{xx1} = E_1 cz = \frac{E_1}{E_1 I_{yy1} + E_2 I_{yy2}} Mz \quad \text{for the wire} \quad (2.3) \]
\[ s_{xx2} = E_2 cz = \frac{E_2}{E_1 I_{yy1} + E_2 I_{yy2}} Mz \quad \text{for the solder} \quad (2.4) \]
When the stress distributions are sketched the two linear paths of both stresses in solder and wire are seen. Note the discontinuity which occur on the border of the two materials if $E_1 \neq E_2$.

Example:

To show the differences between Steinberg and the new theory his example on pag.288-289 [11] is recalculated. In a solderjoint with dimensions given in fig.2.5 Steinberg supposes a fatiguecrack in the solderjoint on a distance of 0.015 inch from the PCB. There he calculates a bending moment of 0.0463 lbin, which in his theory results in two stresses:

$$d_s = 0.025 \text{ in.} = 0.635 \text{ mm}$$

$$d_w = 0.010 \text{ in.} = 0.254 \text{ mm}$$

$$M = 0.0463 \text{ lbin} = 5.23 \text{ Nmm}$$

$$h_c = 0.015 \text{ in.} = 0.38 \text{ mm}$$

Recalculation of the problem, when taking into account the E-moduli differences, leads to:

A solder stress: $S_{bs} = 30200 \text{ lb/in}^2 = 208.3 \text{ N/mm}^2$

A wire stress: $S_{bw} = 59000 \text{ lb/in}^2 = 406.9 \text{ N/mm}^2$
A solder stress with formula (2.4) with:

\[ E_1 = 120000 \, \text{N/mm}^2 \]
\[ E_2 = 30000 \, \text{N/mm}^2 \]
\[ I_{yy1} = \left(\frac{n}{64}\right)0.508^4 = 0.00327 \, \text{mm}^4 \]
\[ I_{yy2} = \left(\frac{n}{64}\right)(0.635^4 - 0.508^4) = 0.00471 \, \text{mm}^4 \]
\[ M = 5.23 \, \text{Nmm} \]
\[ z = 0.32 \, \text{mm} \]

results in:

\[ 30000 \times 5.23 \times 0.32 \]
\[ 120000 \times 0.00327 + 30000 \times 0.00471 \]

\[ = 94.1 \, \text{N/mm}^2 \]

A wire stress with formula (2.3) with:

\[ z = 0.25 \, \text{mm} \]

results in:

\[ 120000 \times 5.23 \times 0.25 \]
\[ 120000 \times 0.00327 + 30000 \times 0.00471 \]

\[ = 294.0 \, \text{N/mm}^2 \]

It is clear that stresses which are calculated with the new theory are quite different from those Steinberg gives. Steinberg's theory leads to higher stresses which result in a lower amount of allowable load cycles (s\text{bs}: 208.3 vs 94.1 and s\text{bw}: 409.6 vs 294.0 N/mm\(^2\)). Translated to stress screening tests this means the product is understressed and thus more failures are left.

Back to the solderjoint, the above theory can be applied to derive a formula for the stresses on the surface of the solder, and those on the wire, as a function of the height. Only these stresses are taken into account because they are important when studying the fatigue behaviour of the structure. Yet, fatigue crack initiation mostly starts at the surface of a material when it is exposed to an alternating load [16]. The stress along the wire is calculated because it may be possible that this stress initiates a crack in the wire on a spot, which is inside the solderjoint. Later on such a crack could grow into both solder and wire.

The new model is given in fig.2.6. The PCB is assumed to be fixed. The diameter D(x) of the joint on position x is taken variable so that the formulas can be used for every geometry of the solderjoint. Furthermore formulas for two load cases are calculated. A constant bending moment at the end of the wire, and a load in the cross section plane. In practice the load induced by a component on a PCB will be a combination of these two loads. It must be noticed that the refined theory still stays an approximation because of the fact that stresses are calculated close to the fixture and therefore some common theories like Bernoulli's hypothesis
lose part of there validity.

\[ \frac{32M(x)d}{\pi d^4(E_1-E_2) + D^4(x)E_2} \]

\[ S_{xx1} = \frac{E_1}{E_1(\pi/4)(d/2)^4 + E_2(\pi/4)((D(x)/2)^4 - (d/2)^4)} \]

\[ S_{xx2} = \frac{E_2}{E_1(\pi/4)(d/2)^4 + E_2(\pi/4)((D(x)/2)^4 - (d/2)^4)} \]

For 0<=x<=h:

Wire surface stresses as shown in (2.3) and with:

\[ I_{yy1} = (\pi/4)(d/2)^4 \]
\[ I_{yy2} = (\pi/4)((D(x)/2)^4 - (d/2)^4) \]
\[ z = d/2 \]

lead to a general expression for the wire stress \( s_{xx1} \):

Solder surface stresses as shown in (2.4) with:

\[ z = D(x)/2 \]

lead to the general expression for \( s_{xx2} \):

Fig.2.6. New model with different E-moduli.
For \( h \leq x \leq 1 \):

Wire surface stress is:

\[
S_{xx1} = \frac{M(x)z}{I_{yy1}} = \frac{M(x)d/2}{(\pi/4)(d/2)^4} = \frac{32M(x)}{\pi d^3}
\]

(2.7)

For both load cases the stress formulas are found by entering the right terms for \( M(x) \) in the above formulas:

For the constant bending moment:

\( M(x) = M_b \)  
(2.8)

For the cross load:

\( M(x) = P(l-x) \)  
(2.9)

2.3 PARAMETER STUDY ON THE REFINED MODEL

To get more insight in the progress of surface stresses calculated with the model, a simple case is examined in which the diameter \( D(x) \) is approximated by a linear function of \( x \):

\[
D(x) = \frac{D_0 - d}{h} \cdot x + D_0 \quad (0 \leq x \leq h)
\]

(2.10)

Fig. 2.7. Simplified model used in the parameter study.

Analytical examination of the stress functions was tried. In a constant bending moment load, only under several conditions, maxima in the stresses were found. The other load case however appears to be not diagnoseable by analytical methods. Because the study of this load case is important for the interpretation of the test results, where a cross load is applied, another method is used to get insight into it. A graphic parameter study was performed to
see the influences on the stress in consequence of parameter variations. To do so the expressions (2.5), (2.6) and (2.7) were made dimensionless. The variables were normed on the wire diameter. This is a reasonable assumption because all other geometric quantities are dependent on the diameter in general. The following dimensionless parameters are defined:

\[(x/d)\] dimensionless variable which indicates the distance to the fixed end;

\[(h/d)\] dimensionless height of the solder joint;

\[(D_0/d)\] diameter ratio of the bottom and the top of the solder cone;

\[(l/d)\] dimensionless length of the wire from fixed end to cross load;

\[(E_1/E_2)=\xi\] ratio of the moduli of elasticity;

With these, the dimensionless expressions for stresses along wire and solder surfaces are derived in appendix F.

The parameter analysis was made in relation to a default situation. Relative to the parameter settings in this case, the parameters were varied up and down between practical limits. Taking notice of the geometries which were seen at the fatigue tests, the following default values were assumed:

\[l/d = 5.0\]
\[E_1/E_2 = 4.0\]
\[h/d = 1.5\]
\[D_0/d = 2.5\]

Stresses along the surfaces of solder and wire due to the default values are sketched in fig.2.8. Stresses were calculated with a cross load applied at the end of the wire.

![Graph](attachment:image.png)

**Fig.2.8. Stress values for the default situation.**
This is done to link up with the fatiguetests where a cross load is used too. Further analysis is focussed on stresses along the wire inside the solderjoint. Outside the solderjoint wire stresses always show a linear decreasing value in case of the cross load, and a constant value in case of the constant bending moment. Therefore these are of no interest for investigation. Hereafter the influence of the several parameters is discussed and the range in which they are changed is indicated. Each time only one parameter is varied while the others are fixed at their default values.

1/d ratio:

The 1/d range is estimated by looking at the height of the connection wires of a component and the wirediameter. Assuming a general component diameter of 4 to 12 mm, gives a wirelength which varies between 2 and 6 mm. With the mostly used diameter of 0.8 mm the 1/d range will be:

1/d = 2.5 - 7.5

The default setting is determined as the average of the values and will be 1/d = 5.0. The other 1/d ratios used in the parameter analysis are:

1/d = 2, 4, 6, 8, 10

A somewhat larger 1/d ratio than the estimation is taken because, when components are not directly mounted against the PCB, and thus are hanging a bit above it, the connection wire is longer than half the component diameter and the 1/d ratio is greater.

The results of the parametervariation are given in appendix A1. Two plots are drawn. The upper one shows the dimensionless stresses along the solder surface, while the other one contains the dimensionless stresses on the wire surface. Notice the x-scale which does not go further then 1.5, as only the stresses in the solderjoint, which has the default h/d ratio of 1.5 are considered.

The first notable things are the stress levels in both plots. Stress levels on the wire surface are higher than those on the soldersurface. This phenomena could be expected because of the differences in the moduli of elasticity (see fig.2.4).

Secondly stress maxima are seen. The value of the maximum stress, for the solder surface, decreases with decreasing 1/d ratio and is more sensitive for lower 1/d ratios. Together with the decreasing value the maximum shifts inwards the solder joint. Wire surface stresses express the same progress though the maxima lie more to the top of the solderfillet.

Another thing is the stress progress towards the fixture. From the maximum it is always lowering, due to the fact that
the moment of inertia of area is growing with a power of four and local bending moment is only growing linear.

**$E_1/E_2$ ratio:**

A copper wire is assumed for the investigated solderjoints. In several works E-moduli for electrical copper, which vary from 105000 to 130000 N/mm$^2$ are found. The solder used can be one of the ASTM solders in the range from 30A to 60A (tin percentage) [12]. Therefore $E_2$ will vary from approximately 20500 to 31000 N/mm$^2$ These values result in a $E_1/E_2$ range of:

$$E_1/E_2 = 3.39 - 6.34$$

Default setting will be 4.0 because the most common used solder is 60/40 with an E of 30000 N/mm$^2$ while the average E-modulus of copper is 120000 N/mm$^2$. To cover the other values in the possible $E_1/E_2$ range five other values will be taken into account:

$$E_1/E_2 = 3.5, 4.5, 5.0, 5.5, 6.0$$

The outcome of the analysis for this parameter is given in appendix A2.

For greater $E_1/E_2$ ratios solderstresses lower but wire stresses increases. This is right because the wire can take up higher stresses at larger E-moduli. And therefore equilibrium with the same local bending moment is reached with lower solderstresses.

**$D_0/d$ ratio:**

The default ratio is figured out by taking a copperland diameter of 2 mm on the PCB, and a wire diameter of 0.8 mm, which makes $D_0/d = 2.5$. Also considering diameters of the copperland from 1 to 4 mm a $D_0/d$ range is made up from:

$$D_0/d = 1.25 - 5.00$$

The tool used to study the parameter changes is able to handle 6 values, therefore:

$$D_0/d = 1.5, 2.0, 2.5, 3.0, 3.5, 4.0$$

These values do not cover the entire $D_0/d$-range but they give an indication of the stress progress which is sufficient. Furthermore it is less probable that copperlands bigger than 4 times the wirediameter will occur in modern electronics.

In relation to the foregoing parameters, it is seen that stresses are quite sensitive for parameter changes in $D_0/d$ (appendix A3). A notable thing is the shift of the stress maximum on the soldersurface. Between a $D_0/d$ ratio of 1.5 and 2.0 it moves from 0.7d to 1.3d. This can be explained with the same theory about the progress of the moment of
inertia of area as mentioned in the discussion on the l/d ratio.

**h/d ratio:**

Based on the geometries of solderjoints used in the fatigue tests a default h/d ratio of 1.5 is chosen. The ratio is varied as follows:

\[ h/d = 0.75, 1.00, 1.25, 1.50, 1.75, 2.00 \]

Results of this variation are drawn in appendix A4. The x-axis is expanded to a x/d value of 2.0 because of the higher value of h/d. Lower solderjoints relatively show a higher stress together with a sharper maximum. The stress values are higher for lower joints because of the load case. The local bending moment due to the cross load increases towards the fixed end so stresses get a higher level into this direction.

In this chapter it was seen that the failure model for solderjoints as given by Steinberg is too simple. A new analytical model was presented, wherein all relevant geometric and material properties were taken into account. Through variation of these, the behaviour of the axial stresses at the surfaces was graphically made visible. Generally it was seen that stresses were smaller than Steinberg suggested. Furthermore maxima mostly occur near the neck of the solderjoint and not on \( 2/3 \) of the height of the solderjoint.
3. NUMERICAL MODEL AND PARAMETER STUDY ON THE STRESSES IN SOLDERJOINTS.

The analytical model as described in the previous chapter is a first approximation on the behaviour of stresses in the solderjoint. To check the model, some additional numerical calculations were made.

With the aid of numerical techniques such as the finite element method it is possible to evaluate the correctness of the assumptions made for the analytical model. The confrontation of both models will be used to determine the level of accuracy of the simplified analytical model. The numerical model will be investigated for several parameter changes.

To perform the numerical calculations the program IDEAS from SDRC (the same as CAEDS from IBM) was used. This program basically consists of a 3D solid modeling module and a finite element module, which contains the powerful enhanced mesh generator. Both modules were used to model and calculate the solderjoints.

3.1. CALCULATION SETUP.

As numerical calculations cost a lot of time and money for the modeling process, only a limited number were made. Before making the calculations two test cases were calculated, to become familiar with the programs and look for the best setup of the finite element model.

![Solderjoint test model](image)

Fig.3.1. Solderjoint test model. The shaded parts are left away in further analysis.

A half modelled solderjoint was calculated first. This could be done because the load case is symmetric in the centerplane of the solderjoint (fig.3.1). The joint was
fixed by adding boundary conditions on the planes, which lie along the plated through hole in the printed circuit board. The results of this calculation show that stresses in the joint, on the spot of the hole, were negligible. Therefore the model was reduced by taking only the upper part of the solderjoint on the component side. The cut plane, located in the middle of the hole, was modelled as a plane of symmetry.

In addition a test calculation with a piece of printed circuit board added to the joint was made. The board was fixed along the edges. The reason to this calculation was to find out weather the PCB influences the stress distribution in the joint. The results however differ less from the first calculations so it was decided to keep away the PCB in future calculations. The common outlook of the solderjoint models then look like that given in fig.3.2.

![Fig.3.2. The used solderjoint model seen in different views.](image)

To investigate the stress patterns in the solderjoint when several parameters are varied, seven numerical models were set up and calculated. The models were named as JOINT1 through JOINT7 and represent the following solderjoint parameters (see also fig.3.3):

- **JOINT1**: \( \frac{L}{d} = 5 \), \( \frac{h}{d} = 1.50 \), \( \frac{D_0}{d} = 2.5 \);
  shows the influence of a longer wire.
- **JOINT2**: \( \frac{L}{d} = 2 \), \( \frac{h}{d} = 1.50 \), \( \frac{D_0}{d} = 2.5 \);
  shows the influence of a shorter wire.
JOINT3:  l/d=3, h/d=1.50, D₀/d=2.5
default solderjoint.

JOINT4:  l/d=3, h/d=1.50, D₀/d=3.5
shows the influence of a bigger solderfillet
diameter.

JOINT5:  l/d=3, h/d=1.50, D₀/d=2.0
shows the influence of a smaller solderfillet
diameter.

JOINT6:  l/d=3, h/d=0.75, D₀/d=2.5
shows the influence of a lower solderfillet.

JOINT7:  l/d=3, h/d=2.25, D₀/d=2.5
shows the influence of a higher solderfillet.

The used parameters can be compared with those from the parameter analysis in the previous chapter. The default length was changed to a value of three times the wire
diameter. The value of 5d (used in chapter 2), will occur
less. Most components have dimensions which are small and a
resulting wire length of 3d is more common. All calculations
were made using a wire diameter of 0.8 mm and moduli of
elasticity of 30,000 N/mm² for the solder and 120,000 N/mm²
for the copper wire. The applied load was a vertical force
of 0.5 N on the upper end node of the wire. Because only
half the solderjoint was modelled this load equals a load of
1 N on a full solderjoint model. Only elastic calculations
were made. This will be sufficient when looking at stress
concentrations, which might initiate fatigue cracks.

3.2. CALCULATION RESULTS

The results of the seven numerical computations can be find
in appendix B. For each model four stress graphs and two
stress pattern plots are given. The graphs show the
calculation results in comparison with the analytical
values, for both wire and solder surfaces at the top and
bottom of the model. The coloured stress plots display the
axial and von Mises stress patterns which are found in the
plane of symmetry of the model (fig.3.4).

Looking along the surface, stress concentrations are seen in
all calculations. Mostly they occur just above the neck of
the solderjoint in the wire material. Situations wherein the
force is near to the solderfillet (JOINT2 and JOINT7) show
no relevant stress concentration on the top surface of the joint. The influence of the introduction of the force in the material is too strong here. A good insight of the force introduction can be taken from the von Mises plots. When the solderfillet is long and tall (JOINT5 and JOINT7), a second stress concentration is seen on the surface of the solderfillet. Fatigue cracks mostly start at the surface of a material, on a spot where stress concentrations are found. From that it can be concluded, that solderjoint fatigue cracks will be initiated in the neck of the joint or at the spot where the stress concentrates on the solderfillet surface. However the last mentioned stress concentration is lower and blunter than the one in the neck. Hence a crack beginning in this part of the joint is less probable. Of course this also depends on the fatigue curves of both materials.

Compared to our analytical model it is seen that the wire surface stresses outside the solderjoint, especially those on the bottom surface of the model, commonly agree with those found in the numerical calculations. On the upper surfaces, stresses diverge when coming to the point of application of the load. Inside the solderjoint differences are bigger. Taking a closer look at the stress plots it appears that the assumed linear stress distribution along a perpendicular cross section through the joint is no longer linear. A maximum is found in the stress distribution across the wire. Also there is not a sharp change-over between the wire and
solder stresses. Numerical wire surface stresses will be lower due to these phenomena's. Solder surface stresses will mostly be lower too (fig.3.5).

Not the way stresses are distributed inside the joint, but the value on the spot where a fatigue crack will occur is important for calculating the fatigue behaviour. At this point the analytical model shows much better results. The analytical stress value in the neck of the joint is almost equal to the top value which appears in the finite element analysis. This situation also applies to the stress concentration on the solder surface. On the assumption that the numerical calculations are a better approximation of reality, the position of the stress concentration is not correctly given by the analytical model. In those cases where a solder surface stress maximum does not occur, the maximum value of the analytical model is less valuable for the fatigue behaviour. From the results so far it seems that stress concentrations on the solder surface occur only when the ratio \((h/d):(D_0/d)\) exceeds 3:4.

Remarkable things occur in the numerical results for solderjoints where the load is applied near to the solderfillet (JOINT2 and JOINT7). The highest stress values in these configurations are found as long stretched areas inside the wire. They are positioned near the material interface between copper and solder.

The calculated model only indicates stress patterns before any crack starts growing. To say more about the crack growing process in relation to the stress concentrations, it is neccesary to calculate several geometric non-linear models wherein the growing crack is taken into account.
Also, seven numerical models of the solderjoint were calculated to evaluate the analytical model. Except in the area where the force is introduced, the agreement of both models is good for the wire surface stresses outside the solderjoint. Stresses at the joint itself differ. The numerical model shows a more decreasing stress profile in comparison with the analytical model. However, stress maxima appear to have the same value in both models. The positions of these maxima are more towards the point of application of the load in the numerical model.
4. SOLDERJOINT FATIGUE TESTS

To get more practical insight in the fatigue behaviour of the solderjoint, especially the crack growing process, a test rig was build to conduct fatigue tests. Also a solder dip bath was made to prepare the test specimens. To make sure that right solderjoints come out of the dip process also some metalurgical work has to be done. These practical considerations are described in the following paragraphs.

4.1. TEST RIG DESIGN

The entire test rig is build on a baseplate. Basically it consists of an electromotor with a fixture for the solderjoint specimen on its shaft (fig.4.1). Besides the electromotor some fixtures to support sensors and other electrical equipment are also mounted on the baseplate. The solderjoint specimen is mounted as good as possible in a direct line with the shaft centerline. The specimen itself consists of a little piece of PCB with plated through holes and a short piece of wire soldered at the center of it (see fig.4.6). When mounted in the fixture a small bearing of synthetic material is pushed over the end of the wire. The bearing is press fitted on the wire. A groove is made on the outside surface of the bearing. This groove must guide a spring which is put on the bearing with one end while on the other end a mass is hooked on (fig.4.2). The mass is radially guided in a sort of bowl, otherwise it turns out to swing
because of the friction moment which is caused by the friction of the spring on the bearing. With a setup like this it becomes possible to supply a rotating bending moment in the solderjoint. The load case in the test rig will not fully agree with the practical situation. In practice only a few vibrational modes are thinkable for components which are mounted on a PCB. Therefore the possible load cases are restricted to two perpendicular plane bending loads in the solderjoint [11]. So the fatigue test deals with a worst case situation because bending loads are induced in infinite bending planes due to the rotational movement of the solderjoint specimen.

A special remark is made towards the use of the little coupling spring in the test (fig.4.2). Looking at the rotating wire it is practically impossible to avoid a certain amount of excentricity on the point of application of the load. This excentricity will lead to an acceleration of the point of application of the force. The total applied force then consists of a static and a dynamic part, and therefore stresses in the wire and the solderjoint will no longer be perfectly alternating. The coupling spring has to eliminate the dynamic effects.

Looking at fig.4.3, we see point A on the wire surface and point P as the point of application of the force. It is assumed here, the mass is mounted on the wire without the before mentioned coupling spring. An angle $\Phi$ exists between both points, while the distance between A and P is given by length $l$. The acceleration of P due to the excentricity and the rotation of the wire, in the direction of gravity, can be derived as follows:

$$
\begin{align*}
  x_P &= -e\cos \alpha \\
  x_P' &= e\alpha' \sin \alpha \\
  x_P'' &= e\alpha'' \cos \alpha \\
  \alpha_p &= e\omega^2 \cos \alpha \\
\end{align*}
$$

where $\alpha = \omega t$

$\alpha' = \omega$
Total acceleration of mass $m$:

$$a_{tot} = g - a_p = g - e\omega^2 \cos \alpha$$  \hspace{1cm} (4.2)

Resulting inertia force at $P$:

$$F = a_{tot}m = m(g - e\omega^2 \cos \alpha)$$  \hspace{1cm} (4.3)

![Diagram showing the acceleration and force at point P.]

**Fig.4.3.**

The axial stress at point $A$ caused by force $F$ now will be:

$$s_A(\varphi) = \frac{F l (d/2)}{I_{yy}} \cos \varphi = K a_{tot} \cos \varphi$$  \hspace{1cm} (4.4)

at standstill, where:

$$K = \frac{m l (d/2)}{I_{yy}}$$  \hspace{1cm} (4.5)

$$s_A(\varphi, \alpha) = K a_{tot} \cos(\varphi + \alpha)$$  \hspace{1cm} (4.6)

when the wire has an angular displacement $\alpha$.

Rewriting of formula (4.6) leads to:

$$s_A(\varphi, \alpha) = K(g - e\omega^2 \cos \alpha) \cos(\varphi + \alpha)$$

$$= Kg\cos(\varphi + \alpha) - K\omega^2 \cos \alpha \cos(\varphi + \alpha)$$  \hspace{1cm} (4.7)
It is seen that the stress in point A consists of the pure alternating part A which is disturbed by part B, as a result of the excentricity of the point of application of the load. Part A is caused by the static part of the load while B is caused by the not desired dynamic part. A graphical representation of the stress shows the different stress patterns that occur at A for several values of $\epsilon^2$ (fig.4.4). It is clear that the fatigue load can vary from a purely alternating load in case of no excentricity to a repeated load. The stress pattern also depends on the position of A in relation to P. For instance when $\phi=0^\circ$ and $\epsilon^2$ is bigger than g, the pattern will look like a repeated compression stress, but for $\phi=180^\circ$ it will look like a repeated tensile stress. When the value of $\epsilon^2$ exceeds gravity, there will be a moment (when P is in its highest position) at which the mass looses contact with the wire. The greater $\epsilon^2$ the greater this contactless period will be. Therefore the maximum value of $\epsilon^2$ limits the application of the mass without a spring. In the beginning a number of specimens were tested without a spring. There often was a drop of the mass because the excentricity of the bearing was too large. To solve the problem the already mentioned spring was added to the test. Background of this application was to uncouple the variations in $a_p$ from the mass. Therefore a weak spring is used which follows the rapid vibrations of

![Graph showing stress patterns for different values of $\epsilon^2$](image-url)
the bearing. With this overcritical running was ensured. The load on the wire now can be assumed to be constant, because the only very little variations in load which will occur, are due to the stretching of the spring caused by the excentricity of the bearing (the mass can be assumed to hang still).

Further details on the electrical design of the test rig can be found in appendix C.

4.2. SOLDER POT

The first series of test specimen were made by handsoldering with a solder iron. Because it was difficult to get a constant solderjoint in this way, a solderpot was build.

Fig. 4.5. Overview solderpot. On the left side the temperature controller. On the right side the solderpot as set up for the calibration procedure. Instead of the specimen fixture a thermometer is mounted.
With such a tool more regularity in the material and geometric properties of the joint was pursued. The joint formation is then caused by the gravity and the capillary properties of the wire-PCB combination and no longer depends on how and how long a solder iron is placed on the joint to form it. When also controlling bath temperature and dip-in time it is reasonable to assume that repetitive solderjoints will occur. As no solder pot was in-plant available, some improvisation leads to the solution as described in appendix D.

4.3 SOLDER TESTS

Before the specimens used in the tests could be made, several dip tests were required to get the right properties for solder bath temperature and dip-in time. Also some experiments were carried out in finding the flux for optimal wetting.

To begin with the last, three fluxes were applied to investigate their wetting capabilities:

MULTICORE COLLOPHONY FLUX
MULTICORE ACTIEC 5 FLUX
ORGANOFLUX

The first two are collophony fluxes with ascending activities. Organoflux is a chemical flux with very active wetting characteristics. As expected the wetting increased with the application of more active fluxes. However for a geometry of the solderjoint which shows good wetting the most active flux was required. The differences in solderjoint geometry caused by the different fluxes can be seen in fig.4.6.

Fig.4.6. Specimen as they come out of the dip process with different fluxes used. From the left to the right the used fluxes are MULTICORE COLLOPHONY, MULTICORE ACTIEC 5 and ORGANOFLUX.
Secondly test specimens were made to determine the bath temperature and dip-in time for an optimal joint. Such a joint is defined as one which shows a fine eutectic structure through the entire solder area. This requirement is made because joints which show inclusions and/or a rough structure probably lead to stress concentrations and non homogenous stress distributions in the mechanical loaded joint. Therefore residues of the wire tincoating and gasholes may not occur either in the joint. Several dip-in times were used at two presetted temperatures of the controller:

- Controller temperature 300°C, which makes a bath temperature of 260°C:
  3 samples at every dip-in time of 5, 7 and 10 seconds (date: 1985-7-1)

- Controller temperature 315°C, makes a bath temperature of 270°C:
  3 samples at every dip-in time of 1, 2, 3 and 5 seconds (date: 1985-6-27)
  3 samples at dip-in times of 5, 7 and 10 seconds (date: 1985-7-12)

In all these tests the applied flux was ORGANOFLUX. Tests were started at a solder bath temperature of 270°C. This was done because a higher temperature assures that the solderbath never gets a temperature below the recommended one (fig.4.9) after 5 specimens are dipped. In practice it was seen that temperature decreases 10°C after 5 specimens were dipped, so it remains in the recommended area. Knowing this, additional tests were carried out at 260°C. With those the development of gas holes in the joint, which were caused by the aggressive flux, had to be avoid as much as possible. It was thought that a lower temperature lead to fewer gas holes in reaction on less sputtering of the flux. However, examination of both structures made at 270°C and 260°C shows no significant differences in the formation of gas holes. Gas holes will be formed at both temperatures, and it seems impossible to avoid this formation in the improvised dip soldering process used. So in the tests some limitations on the perfect solderjoint were made because of the probability of the occurrence of gas holes. The requirements about the metalurgical structure in the joint could all be met.

To see how the structure reacts on several parameters the solderjoints made at 260°C bath-temperature and dip-in times of 5, 7 and 10 seconds are discussed first. In comparison with the first made series at 270°C, longer dip-in times were used to get a comparable amount of heat in the joint at this lower temperature. In the upper part of all joints it is seen that tin of the pretinned wire is not solved in the solder (fig.4.7). An explanation of this effect can be given when looking at the heat transfer in the dip process. Due to the relatively great mass of cold aluminum of the wire fixture which is brought into contact with the wire during
the process (see fig.D.2), wire temperature will decrease towards that fixture. Therefore a lower temperature than bath temperature in the upper part of the joint will be found. This lower temperature leads to the visible effect of the non-solved tin coating. To eliminate this effect it is necessary to elongate dip-in time so the aluminum wire fixture get a higher temperature and the molten solder in the upper part of the joint is capable of solving the entire tin coating on the wire.

**Fig. 4.7.** Detailed structure of upper part of the solderjoint, \( t_{\text{dip}} = 5\text{s}, T_{\text{path}} = 260^\circ\text{C} \). The white area is the unsolved tin coating of the wire. Clearly seen are the separated drops of tin from the coating. (M=113x, 85/7/4, s.n. 5pII lu).

**Fig. 4.8.** Detailed structure for the same type of solderjoint as shown in fig.4.7. Inside the eutectic alloy structure the coagulated tin drops are seen. (M=113x, 85/7/4, s.n. 5pIII ru).

An other effect seen only in the 5-sec-joint is a tin-rich eutecticum look alike in the upper part of the joint. This can be caused by little pieces of tin which were original part of the tin coating of the wire. In the upper part of the joint solder temperature is lower than bath-temperature. Because bath-temperature is already near to the melting point of tin (see fig.4.9), the molten solder at the top of the joint gets a temperature which is too low to properly solve the wire tin coating. Drops of tin are than separated in the tin concentrated area along the wire. Obviously the surface tension of these drops is too high, at this
temperature and dip-in time, to reach solvation. Fig.4.7 and 4.8 show the effect. In fig.4.7 the drop separation from the tin coating is seen and fig.4.8 shows these drops when they are mixed into the molten solder. The effect dissapears at longer dip-in times because solder at the top of the joint reaches higher temperatures in those cases. Also there is more time for diffusion of the tin drops into the molten solder.

Both foregoing effects lead to a joint which is not desired in the tests. Although the last imperfection dissapears at longer dip-in times there still is a poor solvability of the last piece of tin coating on the wire.

Setting a bath-temperature of 270°C, six dip-in times were applied for the specimens. The first good looking joint is reached at a dip-in time of 5 seconds. For dip-in times less than 5 seconds a poor wetted joint or a joint which contains tin-coating residues on the wire occurs. The last phenomena can be explained in the same way as done for the 260°C series. Poor wetting is caused by the very short dip-in times, which give the molten solder no occasion to flow along the wire. Further examination of the joints brings out a difference in structure between top and bottom of the joint. In the lower part of the joint a structure with lead-rich particles is seen. These structures become finer as dip-in time increases. The effect is made visable in fig.4.10-4.15, where structures in the upper and lower parts of the solderjoints, with dip-in times of 5, 7 and 10 seconds, are recorded. Finally at a dip-in time of 10 seconds a joint with a fine structure over the entire solder area is seen. Because this is the required joint for the tests all testspecimen were dipped at this time and temperature with ORGANOFLUX as the applied flux.
Fig. 4.10. Structure in the upper part of the solder joint. $t_{dir}=5s$, $T_{bath}=270^\circ C$. A fine eutectic structure is seen. Because of the fast cooling of the solder primer $a$-lead particles do not have the occasion to grow. (M=283x, 85/7/4, s.n. 5aII lu).

Fig. 4.11. Structure in the lower part of a joint like that in fig. 4.10. Note the crystallization of lead-rich particles due to slower cooling of the solder. (M=283x, 85/7/4, s.n. 5aIII rl).

Fig. 4.12. Fig. 4.13.
Fig. 4.12. Solderjoint for $t_{\text{dip}} = 7\text{s}$, $T_{\text{bath}} = 270^\circ \text{C}$. The structure in the upper part of the joint is a fine eutectic one. 
(M=113x, 85/9/11, s.n. 7a ru).

Fig. 4.13. Same solderjoint as in fig. 4.12, which shows a lead-rich particled structure in the lower part. In comparison with fig. 4.11 the structure is finer because the cooling is slower and more crystallization nodes can be formed. 
(M=113x, 85/9/11, s.n. 7a rl).

Fig. 4.14. Upper part of a solderjoint for $t_{\text{dip}} = 10\text{s}$ and $T_{\text{bath}} = 270^\circ \text{C}$. The same fine structure like those in joints at dip-in times of 5 and 7 seconds is seen. 
(M=113x, 85/9/11, s.n. 10b ru).

Fig. 4.15. Lower part of the joint from fig. 4.14. At this longer dip-in time so much crystallization nodes can be formed, that a fine structure results. 
(m=283x, 85/9/11, s.n. 10b rl).

4.4. FATIGUE TESTS

Two fatigue test runs, RUN1 and RUN2, were performed with the test rig as described in 4.1. The goal of the fatigue tests was to derive fatigue graphs for solderjoints and get insight in the crack mechanism which occur during fatigue. RUN1 was carried out with 14 hand soldered specimen. The applied force had a point of application which lies on a distance of approximately 12.4d from the PCB. To examine the effects when the force is applied closer to the PCB, at a distance of approximately 2.8d, RUN2 was carried out. The 17
specimen used during this run were made with the developed solder process (4.2-4.3).

For every tested specimen a record was made which was stored in a case together with some possible remark sheets. The test record contains data about the shape of the solderjoint, the precise point of application of the load, the applied mass and the duration of the test.

The results of both test runs are given in the fatigue graphs, fig.4.16, 4.18. The stresses along the y-axis are the absolute tensile stresses which occur on the wire surface in the neck of the solderjoint. The position of the neck was derived by taking the average solderjoint height of the specimen. Stresses were calculated at this point because it was seen that crack growing starts from there. Data points, which are indicated with an arrow, represent specimen, which did not fail after the applied number of cycles. Besides the data points a fatigue curve of 99.9% copper cold drawn [11] is plotted in the graphs as a reference to the test results.

The RUN1 fatigue test (fig.4.16) shows good agreement with the reference curve of copper. Examination of the broken joints points out fractures starting at the neck of the solderjoint (fig.4.17). For these the fatigue crack has only grown through the copper wire, and the correspondence of test results and fatigue curve is explicable.

![Fatigue test data from RUN1.](image-url)
Looking at the RUN2 results (fig.4.18), a difference is seen between the test data and the fatigue curve of copper. In this run higher cross loads were applied to get the same bending moment in the joint. Fig.4.19 shows that the nature of the fracture which occur in these tests differ from that from RUN1. The wire fatigue crack is found on a spot which lies inside the solderjoint. The place where the crack is growing into the wire could not be taken from the test specimen. The dimensions of the solderjoint were too small to measure easily. Although it was seen that the final fracture of the wire mostly took place beneath half the height of the joint.

Fig.4.17. Typical fracture for RUN1. (M=15x, 85/12/16, s.n.2).

Fig.4.18. Fatigue test data for RUN2.
solderfillet. Taking into account this different fracture mechanism, stresses better can be calculated on the spot the crack is growing into the wire, but not at the upperside of the solderfillet as done in RUN1. Because this spot is nearer to the fixed end of the solderjoint, stresses shall be higher, and therefore the data points will move upwards in the fatigue graph. Eventually they might be in the same area as the fatigue curve of copper and the fatigue bahaviour can be compared with the one that came out of RUN1. The effect is shown in fig.4.18 where the stresses are recalculated. Assumed in this recalculation is a crack growing into the wire at a height of half the height of the solderfillet.

The crack mechanism which is found in the RUN2 tests can be explained by looking at the shear stress in planes parallel to the wire axis close to the interface between copper and solder. To get the idea, this stress is derived for a simple model of a rectangular sandwich beam (appendix E). The progress of the shear stress across a cut section is seen in fig.4.20. Maximum stress occur at the center of the beam, while a nod is seen at the interface. Here the stress value is called $\tau_{12}$. Along a beam with increasing height $h$ a maximum of $\tau_{12}$ is seen (fig.4.20). This geometry can be compared with the solderjoint.

Supposing an admissible shear strength $\tau_{adm}$, for the upper layer of the sandwich beam (the solder), a "shear crack" appears in the area where $\tau_{12}$ is greater than $\tau_{adm}$. Once such a crack is formed, the state of stress will change. Therefore it is possible that shear stress $\tau_{12}$ becomes higher than $\tau_{adm}$ on spots where it did not reach that value before any crack has grown. Than further crack growing results. However this mechanism is not studied further. After "shear cracking" the little triangle shaped piece of layer material which rests (at the left) is to weak to resist the residual axial stress and will break. Such a piece of material is also seen in fig.4.19 near the neck of the solderjoint.
The explanation for the observed solderjoint fractures follows directly out of the foregoing theory. Because of the higher cross loads in the RUN2 tests, shear stress in the solder exceeds the admissible shear strength and a crack occur as discussed. Once this crack exists the final fatigue crack into the wire is initiated on a spot at the end of the "shear crack".

As the recalculated figures from RUN2 and the calculated test data from RUN1 are both lying in the area of the fatigue curve of the wire material, it can be concluded that the fatigue behaviour of the solderjoint is commonly equal to that of the wire material. The fatigue characteristics of solder are of no importance.
The fatigue stress that must be calculated depends on the crack mechanism which occur. When the maximum shear stress in the solder stays beneath the admissible stress, the fatigue stress can be calculated at the top of the solderfillet. In cases of higher shear stress a fatigue crack is found inside the solderjoint and therefore fatigue stresses must be calculated at that specific spot. Besides the height of the joint, the diameter progress of the solderfillet is also an important property in this calculations.

Summarized this chapter discussed the tools which has been used to perform fatigue tests on solderjoints. A considerable amount of work has been made to get the solderjoint test specimen as perfect as possible. For this, some metalurgical investigation has to be done. Eventually it has been seen that a not suspected crack mechanism occur from the fatigue tests. Further examination of this phenomena resulted in an explanation which shows that the shear stress in the solder of the joint is very important to the crack mechanism that appear.
5. CONCLUSIONS

To define mathematically an effective vibrational stress screen, a system approach towards the product is needed. Missing parts in such a theory are the descriptions of the behaviour of possible failure mechanisms under a mechanical dynamical load. Literature survey showed no knowledge on this area. However the additional knowledge to accomplish a mathematical model is already there. This mainly consists of vibration analysis techniques applied to electronic equipment [11].

The failure model of solderjoints as presented by Steinberg [11], is disproved by the results of this study. A refined analytical model of the solderjoint demonstrates that the position and value of stress concentrations, on the outside surface (solder or wire) of the joint, depend on the geometry of the joint and the point of application of the load. Therefore a fatigue crack will not commonly occur at 2/3 of the height of the solderfillet as Steinberg suggested. Numerical calculations showed the same stress profile along the surfaces, although the stress concentrations on the solderfillet are somewhat shifted towards the top of it in comparison with the analytical model.

Practical tests showed a different way of cracking when the point of application of a cross load is varied. Short distances from this point to the printed circuit board cause a fracture of the joint which will be in the wire inside the solderfillet. For that the failed solderjoint will look cratershaped. As the distance from the printed circuit board to the applied force increase, the joint will crack across the wire above the solderfillet. In both cases the fatigue behaviour of the solderjoint can be described by that of the wire material. The crack mechanism which appears, depends on the value of the shear stress in the solderfillet on a spot just next to the interface with the wire.
6. EPILOGUE

With this report a more fundamental approach to the reliability of electronic products under a mechanical dynamical load has been presented. However it does not contain a complete new theory how to calculate an optimal stress screen. The report indicates a strategy to derive a mathematical description for stress screens. Also it is made clear that a lot of investigation has to be done to get formulations for the individual failure mechanisms which can occur during stress screening. The report is finished with some remarks on the work that was carried out for the failure mechanism of solderjoints.

An investigation had been made to get the most proper solderjoint for the test specimen (chapter 4). Nevertheless it was not possible to avoid gas holes in the joint. Another thing that had to be taken for granted was a constant solder bath temperature. By dipping the specimen the bath temperature dropped 10°C. Yet constant temperature is important in making a good joint, and it is therefore suggested to use a professional wave solder bath whenever conducting the fatigue tests again. This will also have a positive effect on the formation of gas holes. To reduce heat losses by the aluminum wire fixture another solution has to be developed to prevent the wire from falling down the printed circuit board. When this is done, differences in metallurgical structures in the solder will be smaller. The model described in [14] gives more insight in the process of heat transfer during soldering. It can be helpfull in finding an optimal fixture.

The fatigue tests were run to determine the effect of the distance of the load towards the printed circuit board. No other solderjoint geometries (e.g. different D₀/d ratio) were investigated. An investigation of the fatigue and fracture behaviour for these solderjoints can be a subject in a follow-up study.

No practical investigations are made yet on the crack growing process. To get more insight in this phenomena some fatigue tests, with similar load cases, should be interrupted after different amounts of cycles. The progress in crack growing then can be made visible by making cross sections of the joints.

A final remark is made towards the numerical calculations. One of the basic things that must be kept in mind is the fact that these calculations are only an approximation for the real stress patterns. For example the dimensions of the elements used appoint the accurateness of the results. Recalculation of a numerical model with smaller elements probably lead to a sharper change over in stresses at the interface of copper and solder. Additional recalculations also can be made for the load case of a purely bending moment at the end of the wire. This calculations were left out of consideration in this study, because the fatigue test load was a cross load and could be compared with the
numerical models. The extended numerical analysis can be a subject for further study too.
Acknowledgements

During my project I was supported by many people. They made it possible to successfully finish the investigations and write the report. For their help I like to give my acknowledgements.

I appreciate the fact that the IBM company gave me the opportunity to do the teachable commission in their plant at Amsterdam. I owe thanks to the employees of the Technical Services Lab (TSL), for the metalurgical examinations and those from the Technical Automation Department (TAD), for being my colleagues for the period I was with them. Special thanks to mr. ir. F. van der Post and mr. ir. L.J. Kniesenburg, both Automation Engineer, for their coaching and constructive criticism on the report and the performed tests. Also special thanks to mr. J.F.A. Wallart, Plant Education Manager, for the arrangements he took to give me a suitable task and the social accompaniment during the time it was carried out. Because most of the photos in this report are made by mr. B.H. Veenstra, I will express my gratitude towards him for the work he did. Finally I give my acknowledgements to mr. prof.dr.ir. D.H. van Campen and mr. dr.ir. A. de Kraker, from the department of Mechanical Engineering of Eindhoven University of Technology, for the support and the occasion they gave to me to do a final project outside University.

Willem Knoop
References:

[1] Glossary of terms applying to hardware quality and reliability, IBM Corporate Practice, C-P 0-2011-002&003, IBM, November 1984 (updated)


[5] Stress screening: its role in electronics reliability, Tustin, W., Quality Progress, June 1982, pp. 18-22


[8] What is random vibration, Tustin, W., Conference Papers

[9] Stress screening using multiaxial vibration, Hobbs, G.K., Holmes, J.L., Mercado, R. Conference Papers


PARAMETER ANALYSIS $DO/d$ RATIO

A: 1.5  B: 2.0  C: 2.5  D: 3.0  E: 3.5  F: 4.0

APPENDIX A3
PARAMETER ANALYSIS h/d RATIO

DIMENSIONLESS STRESS

RELATIVE LENGTH (x/d)

A: 0.75  B: 1  C: 1.25  D: 1.5  E: 1.75  F: 2

DIMENSIONLESS WIRESTRESSES

RELATIVE LENGTH (x/d)

A: 0.75  B: 1  C: 1.25  D: 1.5  E: 1.75  F: 2

APPENDIX A4
JOINT1 (upper wire surface)
Analysis data versus theoretical values

\[
\begin{align*}
1/d &= 5.00 \\
E_1/E_2 &= 4.00 \\
D_0/d &= 2.50 \\
h/d &= 1.50
\end{align*}
\]

JOINT1 (upper solder surface)
Analysis data versus theoretical values

---
APENDIX B1
**JOINT1 (lower wire surface)**

Analysis data versus theoretical values

- $l/d = 5.00$
- $E_1/E_2 = 4.00$
- $D_0/d = 2.50$
- $h/d = 1.50$

**JOINT1 (lower solder surface)**

Analysis data versus theoretical values

**APPENDIX B2**
JOINT2 (upper wire surface)

Analysis data versus theoretical values

1/d = 2.00
E1/E2 = 4.00
D0/d = 2.50
h/d = 1.50

JOINT2 (upper solder surface)

Analysis data versus theoretical values

THEORY + NUMERICAL ANALYSIS

APPENDIX B4
JOINT3 (upper wire surface)

Analysis data versus theoretical values

\[ \frac{1}{d} = 3.00 \]
\[ \frac{E_1}{E_2} = 4.00 \]
\[ \frac{D_0}{d} = 2.50 \]
\[ \frac{h}{d} = 1.50 \]

--- THEORY --- + NUMERICAL ANALYSIS

--- THEORY --- + NUMERICAL ANALYSIS

JOINT3 (upper solder surface)

Analysis data versus theoretical values

--- THEORY --- + NUMERICAL ANALYSIS

--- THEORY --- + NUMERICAL ANALYSIS

APPENDIX B7
JOINT3 (lower wire surface)
Analysis data versus theoretical values

--- THEORY + NUMERICAL ANALYSIS

1/d = 3.00
E1/E2 = 4.00
D0/d = 2.50
h/d = 1.50

JOINT3 (lower solder surface)
Analysis data versus theoretical values

--- THEORY + NUMERICAL ANALYSIS

APPENDIX B8
JOINT4 (upper wire surface)

Analysis data versus theoretical values

\[
\begin{align*}
1/d &= 3.00 \\
E_1/E_2 &= 4.00 \\
D_0/d &= 3.50 \\
h/d &= 1.50
\end{align*}
\]

---

JOINT4 (upper solder surface)

Analysis data versus theoretical values

---

APPENDIX B10
JOINT4 (lower wire surface)
Analysis data versus theoretical values

\[ l/d = 3.00 \]
\[ E_1/E_2 = 4.00 \]
\[ D_0/d = 3.50 \]
\[ h/d = 1.50 \]

**Theory** + **Numerical Analysis**

---

JOINT4 (lower solder surface)
Analysis data versus theoretical values

**Theory** + **Numerical Analysis**

APPENDIX B11
JOINT5 (upper wire surface)
Analysis data versus theoretical values

$1/d = 3.00$
$E_1/E_2 = 4.00$
$D_0/d = 2.00$

$\text{SRESS YY (N/mm^2)}$

0 0.4 0.8 1.2 1.6 2 2.4 2.8

--- THEORY + NUMERICAL ANALYSIS

JOINT5 (upper solder surface)
Analysis data versus theoretical values

$1/d = 1.50$

$\text{SRESS YY (N/mm^2)}$

0 0.4 0.8 1.2 1.6 2 2.4 2.8

--- THEORY + NUMERICAL ANALYSIS

APPENDIX B13
**JOINT5 (lower wire surface)**

Analysis data versus theoretical values

\[
\frac{1}{d} = 3.00 \\
\frac{E_1}{E_2} = 4.00 \\
\frac{D_0}{d} = 2.00 \\
h/d = 1.50
\]

---

**JOINT5 (lower solder surface)**

Analysis data versus theoretical values

---

*APPENDIX B14*
**JOINT6 (upper wire surface)**

Analysis data versus theoretical values

---

**JOINT6 (upper solder surface)**

Analysis data versus theoretical values

---

**APPENDIX B16**
JOINT6 (lower wire surface)

Analysis data versus theoretical values

\[
\begin{align*}
1/d &= 3.00 \\
E_1/E_2 &= 4.00 \\
D_0/d &= 2.50 \\
h/d &= 0.75
\end{align*}
\]

--- THEOREY + NUMERICAL ANALYSIS

--- THEOREY + NUMERICAL ANALYSIS

JOINT6 (lower solder surface)

Analysis data versus theoretical values
JOINT7 (upper wire surface)
Analysis data versus theoretical values

STRESS YY (N/mm²)

1/d = 3.00
E₁/E₂ = 4.00
D₀/d = 2.50
h/d = 2.25

THEORY + NUMERICAL ANALYSIS

JOINT7 (upper solder surface)
Analysis data versus theoretical values

STRESS YY (N/mm²)

--- THEORY + NUMERICAL ANALYSIS
JOINT7 (lower wire surface)

Analysis data versus theoretical values

\[ \text{STRESS YY (N/mm}^2\text{)} \]

- \( l/d = 3.00 \)
- \( E_1/E_2 = 4.00 \)
- \( D_0/d = 2.50 \)
- \( h/d = 2.25 \)

---

JOINT7 (lower solder surface)

Analysis data versus theoretical values

\[ \text{STRESS YY (N/mm}^2\text{)} \]

---

APPENDIX B20
The electrical design of the test rig is setup to get a standalone device. The test rig switches off by itself when the test specimen fails and also registrates the total amount of fatigue cycles applied. To do the job, two sensors are installed. One registrates the rotations of the motor shaft (S1), the other (S2) senses the location of the mass which hangs on the specimen. Look at fig.C.1 to see an electrical diagram of the test rig. To start a run the specimen and

---

Fig.C.1. Electrical wiring diagram of the test rig.
mass are installed. In this condition relay 1 (Re1) is set and stays in this position until the reset pushbutton is activated. After resetting the test rig the motor goes on running because relay 2 (Re2) is activated in response of the reset of relay 1. At the same time the connection between the shaft sensor and the counter is made. Now when the specimen fails and the mass drops, the mass sensor switches off relay 1. This results in a reset of relay 2 which switches off the motor and breaks the counter circuit. So the test run is stopped automatically, and the number of rotations can be read on the counter. The reset switch keeps relay 1 activated until a new specimen is mounted and the mass sensor uncouples point 10 on relay 1 from ground. Again pressing the reset button starts the new run. An hourscounter is added to the test rig for test runs which will run so long, that counter overflow occur. The counter only registrates up to 1 million cycles, which is equivalent of a test duration of 5.56 h. After that amount of cycles it starts again and indicates an overflow condition. So when test runs exceeded 1 million cycles, the total amount of test cycles can be obtained by dividing the total test time by 5.56. For more refinement the last 6 numbers of this quantity can be read on the counter.

Fig.C.2. Rear view on the test rig. On top of the plastic cover the reset button and hourscounter. Under it the relays.

Power for the control section (24V) is delivered by a simple power supply while power for the electromotor comes out of the three-phase current plant network. The optocouplers were
installed to buffer the sensors. Optocoupler 1 (OC1) was put out of action because the applied sensor appear to be capable to directly drive the counter.
D. SOLDER POT DESIGN

The solder pot itself consists of a little ceramic bowl, which is fixed in an aluminum support by heat transmitting paste. The combination of these two was put into a furnace to burn out the pollution from the bowl. Furthermore a regular 150W solder iron was put upside down into a consisting stand, and the aluminum support was fixed into it. Bath-temperature is thermostatically controlled. The temperature is measured with a probe which holds a thermo-couple sensor. An overview of the solderpot with the temperature controller is given in fig. 4.5.

Fig.D.1. The two-parted special fixture used to clench the solderjoint specimen.

First the temperature probe was installed in the molten solder. However the great thermal delay between the heating element of the solder iron and the solder itself, caused a considerable oscillation in solder temperature. Next the probe was directly contacted with the heating element of the solder iron which gave much better results. The only thing that must be taken into account is the cooling of the solderbath by the dipprocess. Temperature measurements before and after several dips show a little temperature reduction. But it is possible to dip up to 5 specimens after eachother without great losses of temperature. Then waiting for 5 to 10 minutes is neccesary until the solder bath temperature is stabilized. To reduce the temperature losses and also avoid direct danger of touching the heat source, an isolation of glass wool is added to the tool.
To dip the specimen in the molten solder, a special fixture was made which hangs above the solder pot (fig.D.1). One part of this fixture consists of a movable hook which springs up automatically. On the hook the second part of the fixture can be hinged. This is a modified electrical clip that holds the specimen. To avoid the wire falling through the PCB, it is fixed in a little bush which rests on the PCB during the dip process (fig.D.2).

When the solder tool was mounted, a calibration procedure was carried out to get the relation between bath temperature and presetted temperature value on the control device. The calibration procedure was carried out by measuring the solder bath temperature with a regular mercurial thermometer. Those measurements were carried out at several settings of the controller, see fig.D.3. It is seen that a straight line can be drawn through the measured points. In the calibration graph an area containing recommended solder temperatures for the used 60/40 tin/lead solder is indicated, see fig.4.9 [13].
### Solderpot Calibration Measurements and Resulting Graph (dd.1985-6-25/26)

<table>
<thead>
<tr>
<th>$T_{\text{control}}$ (°C)</th>
<th>$T_{\text{bath}}$ (°C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>245</td>
<td>219</td>
</tr>
<tr>
<td>265</td>
<td>234</td>
</tr>
<tr>
<td>270</td>
<td>238</td>
</tr>
<tr>
<td>280</td>
<td>245</td>
</tr>
<tr>
<td>290</td>
<td>253</td>
</tr>
<tr>
<td>300</td>
<td>259</td>
</tr>
<tr>
<td>310</td>
<td>267</td>
</tr>
<tr>
<td>315</td>
<td>270</td>
</tr>
<tr>
<td>320</td>
<td>274</td>
</tr>
<tr>
<td>330</td>
<td>280</td>
</tr>
</tbody>
</table>

**Recommended Solder Temperature for Sn/Pb 60/40**

![Graph showing solderpot calibration measurements and resulting graph with recommended solder temperature.]
E. SHEAR STRESS IN A RECTANGULAR SANDWICH BEAM

In this problem formulas are derived for shear stress \( \tau \) in planes parallel to the center axis of a rectangular sandwich beam. Fig.E.1 shows the problem. The beam consists of core material 1 with a layer material 2. The beam is fixed at one end. At the other end a cross force \( P \) is applied.

Equilibrium of the local bending moment \( M \) with the axial stress distribution across the beam results in:

\[
M = E_1 \int_{-a}^{b} \int_{-b}^{h} z^2 \, dA_1 + E_2 \int_{-a}^{b} \int_{-b}^{h} z^2 \, dA_2 \quad \text{(see also chapter 2)}
\]

\[
= E_1 \left[ \int_{-a}^{b} \int_{-b}^{h} z^2 \, dy \, dz \right] + E_2 \left[ \int_{-a}^{b} \int_{-b}^{h} z^2 \, dy \, dz \right]
\]

\[
= E_1 cy \left[ (1/3)z^3 \right] + 2E_2 cy \left[ (1/3)z^3 \right]
\]

\[
= E_1 c22b(1/3)2a^3 + 2E_2 c22b(1/3)(h^3-a^3)
\]

Therefore:

\[
c = \frac{3}{4b a^3(E_1-E_2)+h^3E_2} \cdot \frac{1}{M}
\]

with \( k = \frac{3}{4b a^3(E_1-E_2)+h^3E_2} \).

Now assuming an \( E_1/E_2 \)-ratio of 4, which can be compared with a layer material of solder and a core of copper, \( k \) will be:

\[
E_1 = 4E_2
\]

\[
k = \frac{3}{4b a^3(4E_2-E_2)+h^3E_2} \cdot \frac{1}{M}
\]
To derive the shear stress, a piece of the beam is isolated [15], see fig. E.2.

Fig. E.2. Isolated piece of beam.

At the end surfaces of the hatched volume two normal forces can be calculated. When \( M_{II} > M_{I} \) it follows that \( N_{II} > N_{I} \). To meet equilibrium a third shear force \( L \) can be thought at the bottom surface of the volume. For the sandwich beam this works out as follows:

- for \( a < z \leq h \) or \( -h \leq z < -a \) (material 2):

\[
\begin{align*}
    s_{I2} &= E_{2}cz = E_{2}M_{I}kz
    \\
    s_{II2} &= E_{2}cz = E_{2}M_{II}kz
\end{align*}
\]

\[
\begin{align*}
    dN_{I} &= s_{I2}dA_{2}
    \\
    N_{I} &= \int s_{I2}dA_{2}
    \\
    &= E_{2}M_{I}k\int zdA_{2}
    \\
    &= E_{2}M_{I}kS_{2}(z)
\end{align*}
\]

Where \( S_{2}(z) \) is the static moment of inertia of area of the end surface from the hatched volume:

\[
S_{2}(z) = 2b(b-z)(z+(1/2)(h-z))
= b(h^{2}-z^{2})
\]

In the same way \( N_{II} \) is derived as:

\[
N_{II} = E_{2}M_{II}kS_{2}(z)
\]
This lead to a shear force $L$ of:

$$L_2(z) = N_{II} - N_I = (M_{II} - M_I) E_2 k S_2(z) \quad (E.2)$$

- when $-a \leq z \leq a$ (material 1):

$$s_{II} = E_1 c z = E_1 M_I k z$$
$$s_{I1} = E_1 c z = E_1 M_{II} k z$$

$$dN_I = s_{I2} d A_2 + s_{I1} d A_1$$
$$N_I = \int s_{I2} d A_2 + \int s_{I1} d A_1$$
$$A_2 = A_1(z)$$
$$= E_2 M_{II} k \int z d A_2 + E_1 M_I k \int z d A_1$$
$$= E_2 M_{II} k S_2(a) + E_1 M_I k S_1(z)$$

Where $S_1(z)$ is the static moment of inertia of area of the cross hatched area:

$$S_1(z) = 2b(\frac{a}{2} - z)(z + (1/2)(a - z)) = b(2z^2 - az^2)$$

The normal force at end II will be:

$$N_{II} = E_2 M_{II} k S_2(a) + E_1 M_{II} k S_1(z)$$

So the resulting shear force for material 1 becomes:

$$L_1(z) = N_{II} - N_I = (M_{II} - M_I) k (E_2 S_2(a) + E_1 S_1(z)) \quad (E.3)$$

Because no cross loads are applied on the isolated piece of beam, $D_I$ equals $D_{II}$. From the equilibrium of bending moments results:

$$M_I + Dv - M_{II} = 0$$
$$M_{II} - M_I = Dv = P_v$$

Which is used to rewrite formulas E.2 and E.3. When also assuming an equal distribution of the shear force in its working plane, the final formulas for the shear stress will be:

$$\tau_1 = \frac{L_1}{2b v}$$

material 2: $\tau_2(z) = \frac{P_v k E_2 S_2(z)}{2b v}$

$$= \frac{P_k E_2 S_2(z)}{2b}$$

$$= \frac{P_k E_2(2b)(h^2 - z^2)}{2b}$$

$$= \frac{P_k E_2}{2}(h^2 - z^2) \quad (a < |z| < h) \quad (E.4)$$

APPENDIX E3
material 1: $\tau_1(z) = \frac{Pvk(E_2S_2(a)+E_1S_1(z))}{2bv} = \frac{2bP(E_2S_2(a)+E_1S_1(z))}{Pk(E_2b(h^2-a^2)+4E_2b(a^2-z^2))} = \frac{2b}{2b} \frac{P(E_2b(h^2-a^2)+4E_2b(a^2-z^2))}{Pk}\left(\frac{h^2-a^2}{4}\right) + \frac{4(a^2-z^2)}{4} (|z|\leq a) \quad (E.5)

To study the stress profiles which appear when the thickness of layer 2 changes, the shear stress can be thought a function $\tau(z,h)$. Applied to a beam with $2b = (6/8)P$, a stress progress as shown in fig. 4.20 occur. Maximum shear stress at the centerline ($\tau_{max}$) of the beam and that which is find at the interface ($\tau_{12}$) are given by:

$$\tau_{12} = \frac{\tau_1(a,h) = \tau_2(a,h)}{PKE_2} = \frac{(h^2-a^2)}{2} = \frac{4bE_2}{3a^3+h^3} \frac{1}{PE_2 8.3} = \frac{(h^2-a^2)}{4} \frac{6PE_2}{3a^3+h^3} \frac{1}{1} = \frac{(h^2-a^2)}{3a^3+h^3} \quad (E.6)$$

$$\tau_{max} = \frac{\tau_1(0,h)}{PKE_2} = \frac{(h^2-a^2)+4a^2)}{2} = \frac{4bE_2}{3a^3+h^3} \frac{1}{PE_2 8.3} = \frac{(h^2+3a^2)}{4} \frac{6PE_2}{3a^3+h^3} \frac{1}{1} = \frac{(h^2+3a^2)}{3a^3+h^3} \quad (E.7)$$

A remark must be made towards formulas E.6 and E.7. Because the expressions for the shear stress are derived in case of a prismatic beam, they do not give an exact description for the beam with changing cross section. The equations of equilibrium of stress are not obeyed in that case. However the stress progresses are derived to indicate the rough patterns which will occur in a solderjoint and therefore the approximation is sufficient.
F. DIMENSIONLESS STRESSES USED IN THE PARAMETER ANALYSIS

With the dimensionless parameters as defined in paragraph 2.3 it follows:

\[ D(x) = - \frac{D_0 - d}{h} x + D_0 \]
\[ = - \frac{D_0 - d}{(h/d)} (x/d) + D_0 \]

\[ D'(x) = \frac{((D_0/d) - 1)}{d} = - \frac{1}{(h/d)} \frac{D_0 - d}{(h/d)} \]

\[ D'(x) = \frac{1}{(h/d)} \frac{D_0 - d}{(h/d)} \quad (0 \leq x \leq h) \] (F.1)

\[ M(x) = M_b \]
\[ M'(x) = 1 \quad \text{for the constant bending moment} \quad (F.2) \]

\[ M(x) = P(1-x) = P_l(1-(x/l)) = P_l(d/l)((l/d)-(x/d)) \]
\[ M'(x) = P_l \]

\[ M'(x) = \frac{1}{(d/l)((l/d)-(x/d))} \quad \text{for the cross load} \quad (F.3) \]

This transforms (2.5) into:

\[ s_{xx1}(x) = \frac{32mM'(x)d}{\eta d^4(\varepsilon-1)E_2 + d^4D^4(x)E_2} \]

with \( m = \frac{M(x)}{M'(x)} \), which is equal to \( M_b \) for the constant bending moment load, and \( P_l \) for the cross load.

\[ s_{xx1}(x) = \frac{32m}{\eta d^3(\varepsilon-1) + D^4(x)} \]
\[ s'_{xx1}(x) = \frac{s_{xx1}(x)\eta d^3}{32m} = \frac{M'(x)}{\varepsilon-1 + D^4(x)} \quad (0 \leq x \leq h) \] (F.4)

(2.6) transforms into:

\[ s'_{xx2}(x) = \frac{s_{xx2}(x)\eta d^3}{32m} = \frac{M'(x)D'(x)}{\varepsilon-1 + D^4(x)} \quad (0 \leq x \leq h) \] (F.5)

and (2.7) will be:

\[ s'_{xx1}(x) = \frac{s_{xx1}(x)\eta d^3}{32m} = M'(x) \quad (h < x < l) \] (F.6)