Characterization of a hydrogen capillary discharge as a waveguide for laser wakefield acceleration

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Characterization of a hydrogen capillary discharge as a waveguide for laser wakefield acceleration

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Summary

A capillary hydrogen plasma has been investigated at electron densities in the range $10^{23} - 10^{24}$ m$^{-3}$ as a waveguide for Laser Wakefield Acceleration. Low densities of $10^{23}$ m$^{-3}$ are attractive for this application due to a larger possible electron energy gain and an easier synchronization of injected electron bunches and the plasma wave. The experiments are performed with an advanced version of a so called slow capillary discharge, which includes a new capillary design. In this design the hydrogen leak rate is kept low by the usage of an automatic pressure control system and electromechanical shutters. A cw HeNe laser is used in combination with an ICCD camera to carry out time resolved measurements. Interferometry experiments are performed for different pressures, discharge currents and time delays measured from the beginning of the discharge current. A matched spot size of about 67 μm has been achieved for a density of $5 \cdot 10^{23}$ m$^{-3}$ and for a time period of 200 ns in a capillary of 250 μm inner radius. The optical guiding properties are checked at a density of $2 \cdot 10^{24}$ m$^{-3}$. High quality optical guiding has been demonstrated for time delays up to 500 ns after the triggering of the discharge for a capillary plasma with a matched spot size of 67 μm. An external axial magnetic field of 5 Tesla has been applied in the capillary discharge to improve the guiding properties. The matched spot size has been determined for pressures between 100 and 10000 Pa for both the cases with and without magnetic field. At the lower pressures a decrease of the matched spot size of up to 30% is obtained.
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1 Introduction

1.1 Particle accelerators

Particle accelerators [1] are devices, which produce beams of energetic particles such as ions and electrons, which are used for many different purposes. Particle accelerators are for example one of the most used tools for the exploration of subatomic particles, but are also used in many other applications, such as in material analysis and modification and in spectrometry.

About half of the world's 15,000 accelerators are used as ion implanters for surface modification and for sterilization and polymerization. The ionization arising, when charged particles are stopped in matter, is often utilized in radiation surgery and therapy of cancer. At hospitals about 5,000 electron accelerators are used for this purpose. Accelerators also produce radioactive elements that are used as tracers in medicine, biology and material science. A growing area in material science is the ion and electron accelerator, which produces neutrons and photons (light sources) over a wide range of energies.

Particle accelerators work with the principle that the particles, which are electrons in the experiments described in this report, are accelerated by means of an electric field. Usually this electric field is created in a gap between 2 plates with opposite charges. The electron is accelerated from a low voltage towards a high voltage.

The first high-voltage particle accelerator had a single gap with a potential drop in the order of 100 kilovolts and was invented by Cockcroft Walton in 1920. Nowadays the most common potential-drop accelerator in use is the Van de Graaff accelerator, named after its inventor Robert Jemison Van de Graaff.

In 1924 the idea for the nowadays well known linear accelerators came from Gustav Ising. He suggested that the maximum energy could be increased by replacing the single gap, holding a DC voltage, by placing along a straight line several hollow cylindrical electrodes, holding pulsed voltages. This idea was improved by Rolf Wideröe, who proposed that, if the phase of the alternating voltage changed by 180 degrees during a particle's trip between two gaps, the particle could gain energy in each gap. Based on this idea he built a three-stage accelerator for sodium ions.

This technique has a major drawback. When the potential difference is too large, breakdown between the electrodes will occur. The limit of energy, which an electron can gain per unit of length, is at maximum 100 MV/m. Thus in order to reach very large electron energies, the accelerator setup should be very large.

The first circular accelerator based on the principle of repetitive acceleration was the cyclotron, invented by Ernest Orlando Lawrence. In a cyclotron the charged particles circulate in a strong magnetic field due to the Lorentz force and they are accelerated by electric fields in several gaps. After having passed a gap, the particles move inside an electrode and are screened from the electric field. When the particles exit from the screened area and enter the next gap, the phase of the time-varying voltage has changed by 180 degrees so that the particles are again accelerated.

But also this technique has drawbacks. The Lorentz force has to be in equilibrium with the centrifugal force. Large kinetic energies of the electrons cause a large centrifugal force. Therefore the energy gain of the particles is limited by the magnetic field strength and again by the dimensions of the accelerator. However the major drawback is that, when the particles reach high energies, the loss factor due to synchrotron radiation becomes larger and larger.

So, even when the magnetic field strength and the dimensions are large enough, there is a limit in the maximum of energy that the particles can reach.
1.2 Laser Wakefield Acceleration

An alternative technique is Laser Wakefield Acceleration (LWFA). In 1979 Tajima and Dawson came up with the idea to use a plasma as a medium for accelerating electrons [2]. In plasmas much higher electric fields can be created than in linear and cyclotron accelerators. Electric fields up to TV/m are possible, which is 4 orders of magnitude larger than the linear and cyclotron accelerators.

LWFA requires high power density laser pulses, which are sent through a plasma. The electromagnetic field of the laser pulse pushes the electrons on its path away from their equilibrium position due to the ponderomotive force \( F_p \). This causes a so-called wakefield. The ions remain at their place, because of their large mass. The plasma aspires quasi-neutrality. Therefore the electrons, which are pushed away, will move back to their initial position. However, they have gained so much potential energy, that they will overshoot, resulting in an oscillation around their initial position. All these oscillations have a small different phase with respect to the direction of the laser pulse. So there will be a potential wave. When electrons are injected in this potential wave with an initial energy, which is large enough to keep up with the potential wave, they will accelerate from the low electric potential wave trough towards the high electric potential wave top.

In order to gain the maximum energy for the electrons, they have to be coupled out of the plasma at the moment that they are at the electric potential wave top. If the out coupling is too late, the electrons will start to decelerate and if the out coupling is too early, the electrons will not have gained the maximum amount of energy.

1.3 Plasma waveguides

In order to accelerate electrons to high energies in a plasma wakefield, laser pulses with high power density \( (10^{22} \text{ W/m}^2) \) are required. These laser pulses are focused to spots with a radius of several tens of μm. Laser pulses with such small dimensions diverge very fast, i.e. the Rayleigh length \( Z_R = \frac{\pi w_0^2}{2} \) (\( w_0 \) is the size of the laser beam waist) is very short. This means that the power of the laser is spread over a larger area, resulting in a smaller acceleration gradient in a plasma accelerator [2]. Therefore the laser pulse needs to be guided through the plasma, so that its spot size does not diverge.

A plasma waveguide is a plasma channel with a hollow parabolic radial electron density profile. Considering \( \Delta n_e \) as the difference between the electron density at radius \( r_e \) of the channel and the electron density at the axis, these values define the so-called matched spot size

\[
W_M = \sqrt{\frac{r_e^2}{\pi r_e \Delta n_e}}.
\]

If a Gaussian laser beam is focused to a waist equal to \( W_M \) at the entrance of the channel, it will propagate without diverging. So the energy will not be spread over a larger area.

The main parameters of plasma waveguides are the physical dimensions, the matched spot size and the electron density \( n_{e,0} \) on the axis.

A number of different approaches to create such plasma waveguides exists. Below a short overview is given together with some typical parameters of these waveguides.

The creation of a linear spark by means of focusing a laser beam with an axicon lens [3 - 6] gives an \( n_{e,0} \) in the range of \( 10^{25} \text{ m}^{-3} \) with a \( W_M = 10 - 15 \mu\text{m} \) and length values from several mm up to 1 cm.

The ablation of the capillary wall by a discharge [7 - 9] gives an \( n_{e,0} = 2 \cdot 4 \cdot 10^{24} \text{ m}^{-3} \) and a \( W_M = 28 - 35 \mu\text{m} \) with a length of several cm.
A third type of a waveguide has been produced by the so called fast capillary discharge \[10, 11\]. It can be created with a length in the order of several cm. The numerical estimations show an \(n_{\text{e0}} = 2 \cdot 10^{23} \text{ m}^{-3}\) and a \(W_M\) of about 20 - 25 \(\mu\m\).

Hooker et al \[12 -15\] have proposed and studied another type of plasma waveguides, obtained in a so called slow capillary discharge. This type of discharge produces the heating of hydrogen gas inside a capillary with an inner radius of 150 - 250 \(\mu\m\). A discharge voltage of 5 - 20 kV is used to get total ionization. The typical time of the discharge is about several hundreds of nanoseconds. The plasma is cooler near the wall, resulting in a temperature gradient in the capillary. Assuming a constant pressure, there will also be an electron density gradient, which can be used as a waveguide. Hooker et al reported guiding lengths up to 10 cm and a \(W_M\) of 37.5 \(\mu\m\) with an \(n_{\text{e0}}\) of \(3 \cdot 10^{24} \text{ m}^{-3}\).

### 1.4 Goal and overview

In 2001 the FOM research program 55 on Laser Wakefield Accelerators started under supervision of Prof.dr. M.J. van der Wiel. This is a collaboration project between the FOM Institute for Plasma Physics Rijnhuizen, Eindhoven University of Technology and the University of Twente. The program addresses the physics and realization of a compact plasma accelerator for electron energies of several hundred MeV, based on a laser-driven wakefield. The part of the research performed at Rijnhuizen contains the realization and characterization of a suitable plasma waveguide.

Within this research at Rijnhuizen, the goal of this graduation project is to characterize fully ionized slow capillary discharges by means of interferometry measurements at different pressures, discharge currents, capillary lengths etc. The final aim is the usage of this plasma as a waveguide for LWFA. The emphasis in this project is on small electron densities \(n_{\text{e0}}\) in the range of \(10^{23} \text{ m}^{-3}\), which are attractive from the point of view of potentially larger electron energy gain \[2\] and an easier synchronization of injected electron bunches and the plasma wave.

For this purpose the earlier mentioned parameters of plasma waveguides at low hydrogen pressures – down to 1000 Pa, resulting in an electron density on axis in the range \(10^{23} - 10^{24} \text{ m}^{-3}\) – are studied. A set of experiments is performed with an advanced version of the slow capillary discharge, which includes a pressure control system and electromechanical shutters to keep the hydrogen leak rate low. The advantages of this shutter system are that the pressure inside the capillary can be determined more accurately and that the background pressure can easier be kept lower. The main diagnostic tool used for obtaining experimental results about the discharge is an optical interferometer (Mach-Zehnder).

In chapter 2, it is shown how the transverse profile of a Gaussian beam develops and how the matched spot size is defined and calculated. Further is described how from the curvature of the fringes in an interference pattern the electron density profile can be calculated and what the effect is of an external axial magnetic field on the electron density distribution.

In chapter 3 the experimental setup is introduced, including the interferometer, the capillary unit and the external magnetic field. Also is described how a time resolved measurement is performed for characterizing the time development of the plasma.

Chapter 4 shows how the electron density on the axis of the capillary and the electron density profile are determined from the interferograms by means of the least square method. The experimental results are discussed in chapter 5.

Chapter 6 contains the conclusions, followed by an outlook on future work in chapter 7. In appendix A an article, submitted to Phys. Rev. E, is inserted, which is also about the subject discussed in this report.
2 Theory

2.1 Gaussian optics

The experiments described in this report are performed with a helium neon laser with a wavelength $\lambda$ of 632.8 nm. A helium neon laser beam profile can be approximated very well with a theoretical Gaussian intensity profile. Therefore Gaussian beam optics is very useful to describe the beam's transverse profile, which is needed for the analysis of the interferometry measurements described later in this report. The derivation can be found in more detail in reference [16].

Starting from the general wave equation for a monochromatic field $u$:

$$\nabla^2 u + k^2 u = 0$$  \hfill (1)

where $k = 2\pi / \lambda$ is the propagation constant in the medium. The beam's field is concentrated near the $z$-axis and the transverse change of $u$ is much slower than the $z$-axis $k$-oscillations. Then $u$ can be described as:

$$u = \psi(x, y, z) \exp(-ikz)$$  \hfill (2)

in which $\psi$ is a slowly varying complex function, which describes the differences between a laser beam and a plane wave.

Substituting this into formula 1 and assuming $\frac{\partial^2 \psi}{\partial z^2} = 0$:

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} - 2ik \frac{\partial \psi}{\partial z} = 0$$  \hfill (3)

Converting this to cylindrical coordinates, using that the beam is azimuthally symmetric:

$$\frac{1}{r} \frac{\partial}{\partial r} r \frac{\partial \psi}{\partial r} - 2ik \frac{\partial \psi}{\partial z} = 0$$  \hfill (4)

This has the exact solution:

$$\psi = \exp \left( -i \left[ P(z) + \frac{k}{2q(z)} r^2 \right] \right)$$  \hfill (5)

$P(z)$ is a complex phase shift and $q(z)$ is a complex beam parameter that describes the beam variation in intensity and curvature of the phasefront. $q$ is related to 2 real beam parameters, the curvature $R$ of the phasefront and the beam width $w$, which is radius of the $1/e^2$ contour of the intensity of the beam, by:

$$\frac{1}{q} = \frac{1}{R} - i \frac{\lambda}{\pi w^2}$$  \hfill (6)
The beam waist \( w_0 \) is the radius of the \( 1/e^2 \) intensity contour of the beam, where the wavefront is flat. This is the minimum radius of the beam. In the beam waist \( R \to \infty \) and therefore:

\[
q_0 = i \frac{\pi w_0^2}{\lambda}
\]  

(7)

in which the index 0 denotes the waist.

Consider a cylindrical capillary plasma with a parabolic radial electron density profile. The refractive index \( N \) has then also a parabolic radial profile, given by:

\[
N = N_0 - \frac{1}{2} N_2 r^2
\]

(8)

Here \( N_0 \) is the refractive index at the axis and \( N_2 \) is a parameter for the radial dependency. The capillary is placed in such a way that the waist of the beam is at the beginning of the capillary plasma, so there \( q = q_0 \). Using standard beam transformation matrices and taking \( N_0 \approx 1 \) for low pressures, the value of the parameter \( q \) at the exit of the capillary with length \( b \) will be:

\[
q_{\text{probe}} = \frac{(q_0 \cos(b \sqrt{N_2}) + \sin(b \sqrt{N_2})/\sqrt{N_2})}{(-q_0 \sqrt{N_2} \sin(b \sqrt{N_2}) + \cos(b \sqrt{N_2}))}
\]

(9)

For a beam in free space the corresponding value of \( q \) after a distance \( b \) is:

\[
q_{\text{ref}} = q_0 + b
\]

(10)

Index 0 represents the input plane at the entrance of the capillary and index 1 represents the output plane at the exit of the capillary.

In free space the wave’s field in formula 2 can be rewritten as:

\[
u(r, z) = \frac{w_0}{w} \exp\left( -ikz + i\Phi - i \frac{k}{2R} r^2 - \frac{r^2}{w^2} \right)
\]

(11)

with:

\[
\Phi = \arctan\left( \frac{k z}{\pi w_0^2} \right)
\]

(12)

### 2.2 Matched spot size

Laser beams with small dimensions diverge very fast. This means that the power of the laser is spread over a larger area, resulting in a smaller acceleration gradient in a plasma accelerator [2]. Therefore the laser needs to be guided through the plasma, so that its spot size does not diverge and the power is not spread over a larger area. The laser spot size, which propagates
in a waveguide without diverging, is called the matched spot size \( W_M \). Its dependence on the radial electron density profile can be calculated as follows.

From formula 11 the \( z \)-component of the propagation vector is calculated by taking the spatial derivative of the total phase \( \varphi \).

\[
\frac{\partial \varphi}{\partial z} = k_z = k \left( 1 + \frac{r^2}{2} \frac{z_0^2 - z^2}{(z^2 + z_0^2)^2} \right)
\]  

(13)

with \( z_0 = \pi \omega_0^2 / \lambda \). The spatial derivative of \( \Phi \) is negligible small in comparison with \( k \) and has no radial dependency.

In the waist of the beam, where \( z = 0 \), the phase velocity in the \( z \)-direction is:

\[
v_{\text{phase}} = \frac{\omega}{k_z} = \frac{\omega}{k} \left( 1 - \frac{r^2}{2z_0^2} \right)
\]  

(14)

This radial dependency of the phase velocity on the radius represents the divergence of the beam. To maintain the beam spot size in the capillary, this divergence should be compensated by the radial dependency of the refractive index in the plasma. Then the phase velocity of the light in the capillary is the same at every radial position and the beam is guided through the plasma without diverging. Thus for guiding the following has to be satisfied, using formulae 8 and 14:

\[
\frac{\omega}{k} \left( \frac{r^2}{2z_0^2} \right) = \frac{\omega}{k} \left( \frac{N_z r^2}{2} \right) \Rightarrow \frac{1}{z_0^2} = N_z = w_0 = \left( \frac{\lambda^2}{\pi^2 N_z} \right)^{\frac{1}{4}}
\]  

(15)

Therefore only a beam with this particular spot size is guided in a plasma with a certain parabolic refractive index profile. This spot size is called the matched spot size and usually is denoted as \( W_M \). The refractive index is indirectly dependent on the electron density according to:

\[
N = \sqrt{1 - \frac{\omega_{\text{plasma}}^2}{\omega_{\text{laser}}^2}}
\]  

(16)

\( \omega_{\text{plasma}} \) is the plasma frequency and \( \omega_{\text{laser}} \) is the laser frequency, which are defined as:

\[
\omega_{\text{plasma}} = \sqrt{\frac{n_e e^2}{\varepsilon_0 m_e}},
\]  

(17)

\[
\omega_{\text{laser}} = \frac{2 \pi c}{\lambda}.
\]  

(18)

Here \( n_e \) is the electron density, \( e \) is the elementary charge, \( \varepsilon_0 \) is the permittivity of free space, \( m_e \) is the electron mass and \( c \) is the velocity of light.

Combining formula 16, 17 and 18, this dependency of \( N \) on \( n_e \) is approximated by:
When assuming a parabolic electron density profile, \( n_e \) can be written as [13]:

\[
 n_e = n_{e,0} + n_{e,2} \frac{r^2}{r_m^2}
\]  

(20)

in which \( n_{e,0} \) is the density on the axis of the capillary and \( n_{e,2} \) is the increase in the electron density at radius \( r = r_m \).

Combining formula 8, 19 and 20 results in:

\[
 N_0 = 1 - \frac{\frac{e^2 \lambda^2}{2 \cdot 4 \pi^2 c^2 \varepsilon_0 m_e}}{n_{e,0}}
\]  

(21)

and:

\[
 N_2 = \frac{\frac{e^2 \lambda^2}{4 \pi^2 c^2 \varepsilon_0 m_e r_m^2}}{r_m^2}
\]  

(22)

Substituting formula 22 in formula 15 gives an expression for the matched spot size depending on the parabolic electron profile parameter \( n_{e,2} \):

\[
 W_M = \left( \frac{4 c^2 \varepsilon_0 m_e r_m^2}{e^2 n_{e,2}} \right)^{\frac{1}{4}}
\]  

(23)

Using the formula for the classical electron radius \( r_e \):

\[
 r_e = \frac{1}{4 \pi \varepsilon_0 m_e c^2}
\]  

(24)

results in:

\[
 W_M = \left( \frac{r_m^2}{\pi r_m n_{e,2}} \right)^{\frac{1}{4}}
\]  

(25)

\( n_{e,2} \) and therefore also \( W_M \) is dependent on the radius of the capillary, the initial gas pressure and the capillary discharge current.

### 2.3 Beam sizes

Using formulae 6, 7 and 10, the laser beam radius in free space can be described with:
\[ w(z) = w_0 \sqrt{1 + \left( \frac{\lambda z}{\pi w_0^2} \right)^2} \].

(26)

\( w(z) \) is radius of the \( 1/e^2 \) contour of the intensity of the beam after a distance \( z \), measured from the beam waist \( w_0 \).

The Rayleigh length \( z_R \) is defined as the distance over which the beam radius spreads by a factor of \( \sqrt{2} \):

\[ z_R = \frac{\pi w_0^2}{\lambda} . \]

(27)

For Gaussian beams the lens equation changes with respect to geometrical optics. The waist of the input beam corresponds to the object and the waist of the output beam to the image.

Using the ray transfer matrices mentioned in reference [16], the lens equation is denoted in terms of the Rayleigh length as:

\[ \frac{1}{s + z_R^2 / (s - f)} + \frac{1}{s''} = \frac{1}{f} . \]

(28)

Here is \( s \) the distance from the lens to the waist of the input beam, \( s'' \) the distance from the lens to the waist of the output beam and \( f \) the focal length of the lens.

The magnification of a lens is then given by:

\[ m = \frac{w''}{w_0} = \frac{1}{\sqrt{1 - (s / f)^2 + (z_R / f)^2}} , \]

(29)

in which \( w'' \) is the waist of the output beam and \( w_0 \) is the waist of the input beam.

For the lens equation and the magnification equation are 2 cases of special interest.

For the case that \( s = 0 \):

From formula 28:

\[ s'' = \frac{f}{1 + \left( \frac{\lambda f}{\pi w_0^2} \right)^2} \]

(30)

and from formula 29:

\[ w''_0 = \frac{\lambda f / \pi w_0}{\sqrt{1 + \left( \frac{\lambda f}{\pi w_0^2} \right)^2}} . \]

(31)

For the case that \( s = f \):

From formula 28:

\[ s'' = f \]

(32)

and from formula 29:

\[ w''_0 = \frac{\lambda f}{\pi w_0} . \]

(33)
2.4 Interferometry

A Mach-Zehnder interferometer works according to the following principle. See figure 2.1.

![Figure 2.1: Schematic overview of a Mach-Zehnder interferometer.](image)

The light from the laser is separated in 2 arms with equal optical paths. One of the arms goes through a medium, which has to be investigated, like a capillary discharge, and is called the probe beam. The other arm goes usually through vacuum and is called the reference beam. After that, both arms are brought together and interfere. When the arms are brought together with an angle, an interference pattern with light fringes will appear in the plane of the detector. The fringe separation will depend on the angle between the 2 beams. The fringe separation is large for small angles. For a better visualization the angle in this schematic overview is drawn larger than it is in the experimental setup.

The beams have the same phase at the second beam splitter. After the plasma channel the probe beam gets an additional phase shift, because of the difference in refractive index between the plasma and vacuum. This causes a shift of the light fringes in the interference pattern. The magnitude of this phase shift of the fringes, which corresponds to a particular change of the refractive index in the probe beam, is calculated as:

$$\Delta \Phi_{p-v} = -L \frac{\omega}{c} (N_{\text{plasma}} - N_{\text{vacuum}}).$$  \hspace{1cm} (34)

Here $L$ is the length of the plasma channel, $\omega$ the laser frequency, $c$ the speed of light, $N_{\text{vacuum}}$ is the refractive index of the vacuum and $N_{\text{plasma}}$ is the refractive index of the plasma. The refractive index of the plasma is smaller than the refractive index of vacuum, which is 1. Thus the phase of the probe beam decreases with respect to the reference beam, which means that the optical path of the probe beam is smaller than the optical path of the reference beam. This is shown in figure 2.2, where both beams are represented by their wavefronts.
Figure 2.2: Above the wavefronts of the reference beam and below of the probe beam. A difference in phase arises due to the difference in refractive index in both paths. The optical path of the probe beam is smaller than the optical path of the reference beam.

The effect of this phase shift of the fringes in the interference pattern at the camera is shown in figure 2.3 and 2.4:

In figure 2.3 the point where the 2 beams interfere is shown. When the plasma is created, the probe beam, which is in red, has a smaller optical path with respect to the reference beam, which is in blue. So the wavefronts of the probe beam shift forward to the place of the green dashed wavefronts. In figure 2.4 the effect on the interference pattern is shown in a top-view 1-dimensional projection. When the detector is placed at a certain fixed position, where the beams interfere (see figure 2.1), the wavefronts of both beams cross for example at the positions at the camera as imaged in figure 2.4a, where the blue dots indicate the fringe maxima. At that position of the detector, these cross sections of the wavefronts shift due to the phase shift of the probe beam. This is imaged in figure 2.4b. In this 1-dimensional projection the fringes move upwards due to the plasma. In the recorded 2-dimensional interference pattern this is to the left.

From figure 2.3 it also becomes clear that a phase shift of $2\pi$ radians of the probe beam results in a phase shift of $2\pi$ radians of the fringes in the interference pattern. If the wavefronts shift forward like in figure 2.3, the fringes shift to the left in the plane of the detector. When taking this into account in formula 34 and also that the refractive index $N_{\text{vacuum}}$ of vacuum is 1, the fringe shift $\Delta \varphi$ is given by formula 35:
\[ \Delta \varphi_f = -L \frac{\omega}{c} (N_{\text{plasma}} - 1), \]  

The fringe shift is related to the refractive index \( N_{\text{plasma}} \) of the plasma and therefore also related to the electron density according to formula 16. When formulae 16, 17, 18 and 35 are combined, a relation between the phase shift of the fringes and the electron density is obtained:

\[ \Delta \varphi_f = -L \frac{2\pi}{\lambda} \left( \sqrt{1 - \frac{n_e e^2 \lambda^2}{4\pi^2 e^2 \varepsilon_0 m_e}} - 1 \right). \]  

The electron density in a capillary discharge plasma is usually not distributed homogeneously. However, due to the symmetrical geometry of the capillary, it is assumed that there is no azimuthal or axial dependency on the electron density, but only a radial dependency. Therefore the fringe shift due to the plasma is different for different positions in the plane of the interference pattern. This radial dependency of the electron density is explained as follows.

The electron-ion collision time \( \tau_{ei} \) is:

\[ \tau_{ei} = \frac{2.8 \cdot 10^5 T_e^{3/2}}{n_e \ln \Lambda} \quad [17] \]

where \( T_e \) is the electron temperature in K and \( \ln A \) is the Coulomb logarithm. For the capillary plasma in the experiments described in this report the following approximating values can be taken: \( T_e \approx 9 \cdot 10^4 \) K, \( n_e \approx 10^{24} \) m\(^{-3} \) and \( \ln A \approx 6 \). Thus \( \tau_{ei} \) can be estimated at \( 1 \cdot 10^{-12} \) s. This is much shorter than the lifetime of the plasma, which is about \( 1 \cdot 10^{-6} \) s.

The mean free path length of the electrons \( \lambda_{ei} \) is:

\[ \lambda_{ei} = \frac{1.5 \cdot 10^9 T_e^{3/2}}{n_e \ln \Lambda} \quad [17]. \]

When taking \( T_e \approx 9 \cdot 10^4 \) K, \( n_e \approx 10^{24} \) m\(^{-3} \) and \( \ln A \approx 6 \), \( \lambda_{ei} \) is \( 7.2 \cdot 10^{-9} \) m. This is much smaller than the radius of a capillary plasma, which is often about \( 10^{-4} \) m. So, when the plasma in the capillary is created and the equilibrium is reached, the ion- and electron pressure in the plasma are constant, because of these small values of \( \tau_{ei} \) and \( \lambda_{ei} \).

However the wall of the capillary is cooler than the hot plasma at the axis. Therefore a temperature gradient is induced in the plasma. Approximating this state of the plasma by a thermodynamic equilibrium the pressure \( p_e \) of the electrons is given by:

\[ p_e = n_e k T_e, \]  

in which \( k \) is the Boltzmann constant and \( T_e \) is the electron temperature in K. This means, with \( p_e \) constant, that when a gradient in \( T_e \) is present, also a gradient in \( n_e \) is present. Thus in the case of a lower temperature at the wall than in the centre of the capillary, there is a hollow density profile. This is an electron density profile with a smaller density at the axis and a larger density near the wall of the capillary. The refractive index near the wall is then smaller than 1 and smaller than in the centre.
For a better understanding of the effect of this radial electron density gradient on the interference pattern, the gradient is separated in a horizontal and a vertical component. Because a medium is created in the probe beam, the fringes will shift. However, because of the vertical component of the electron density gradient, there will be a larger fringe shift in the upper and lower part of the interference pattern than in the mid part. Therefore the fringes will be curved. See figure 2.5:

![Figure 2.5: Schematic change of fringes due to a vertical density gradient.](image)

When concentrating on the horizontal component of the gradient, there will be a larger fringe shift in the left and the right part of the interference pattern than in the mid part. This causes the fringe spacing to change across the fringe pattern. See figure 2.6:

![Figure 2.6: Schematic change of fringes due to a horizontal density gradient.](image)

When these 2 effects are combined, there will be curved fringes with a changing fringe spacing, because of the radial gradient of the electron density. See figure 2.7:

![Figure 2.7: Schematic change of fringes due to both a vertical and horizontal density gradient.](image)

In first approximation the electron density profile is assumed to be parabolic. Using this assumption and formula 36, the electron density and its radial dependency can be determined from the curvature and the spacing of the fringes.
2.5 Coherence length

For an interferometer, the coherence time, or coherence length, is important, because the relative delay in the two arms of the interferometer must be less than the coherence time of the source to see interference fringes. When the optical path difference of the arms is equal to half the coherence length of the laser, the fringe contrast will decrease to approximately half its value.

The coherence length is defined by the spectral width of the laser spectral line. Its value is about the inverse value of the spectral line width times the light velocity. If two points on the axis of the laser beam are separated further from each other than the coherence length, the relative phases will fluctuate in a random way.

If the interferometer has a difference in arm length, which is larger than the coherence length, then the interference pattern will fluctuate very fast. It will jitter on a distance larger than the spacing between the fringes in a timescale, which is about the inverse value of the laser spectral line width. So, when a detector with a finite time response is used, there will be no fringes visible at all, but a homogeneous section of the beam.

Thus, if a stable interference pattern is visible, the difference in arm length is within the coherence length.

The difference in optical path length of the arms in the interferometer, which is described in this report, due to the angle between the arms is less than half a cm. The coherence length of a HeNe laser is about 10^{-30} cm, which is more than 20 times larger than the difference in optical path length. Coherence length is therefore not an issue in the measurements reported here.

2.6 Magnetic fields in a capillary discharge plasma

When an externally applied axial magnetic field \(B_0\) is applied in a capillary discharge, the modulation of the electron density is improved [18]. This can be explained as follows.

The energy losses in the hydrogen plasma are mainly caused by electron thermal conductivity, from which the coefficient is given by [18]:

\[
\chi_e = \chi_0 T_e^{3/2} \frac{1}{1 + (\omega_L \tau_{ei})^2 (7 \omega_L \tau_{ei})} ,
\]

where \(\chi_0 = 3.2 n_e k_B T_e / m_e\), with \(T_e\) the electron temperature, \(\omega_L\) the Larmor frequency, \(\tau_{ei}\) the electron-ion collision time and \(\omega_L \tau_{ei}\) the dimensionless Hall parameter \(H_{ei}\) given by [18]:

\[
H_{ei} = \frac{5 \cdot 10^{16} T_e^{3/2}}{n_e \ln \Lambda} \left( B_0^2 + B_\theta^2 \right)^{1/2},
\]

where \(B_\theta\) is the azimuthal magnetic field component induced by the discharge current. The Hall parameter due to \(B_0\) varies between 1 and 100 and the Hall parameter due to \(B_\theta\) varies between 0.3 and 30, for densities of respectively \(10^{23}\) and \(10^{24}\) m\(^{-3}\), taking \(T_e \approx 9 \cdot 10^4\) K, \(B_0 \approx 5\) T, \(B_\theta \approx 1.5\) T and \(\ln \Lambda \approx 6\). When the Hall parameter is larger than 1, the electrons are called magnetized. Then an electron completes at least 1 cyclotron movement around a magnetic field line, before a collision occurs, which causes a radial displacement of the electron. Thus the electrons are only radially magnetized for the lower pressures.
In radial direction there is no magnetic field, so in that direction the Hall parameter is 0. Without external axial magnetic field, the ratio between the thermal conductivity in radial direction \( \chi_{r,r} \) and in azimuthal direction \( \chi_{e,a} \) is \( \chi_{r,r}/\chi_{e,a} = 0.9 - 0.005 \) for densities of respectively \( 10^{24} \) and \( 10^{22} \) m\(^{-3}\).

Thus for lower pressures the heat transport is smaller, which causes the electron density profile to become steeper. However, for lower pressures, a stronger effect is that a much smaller electron density modulation is possible. Besides, the electron-ion collision time increases for smaller pressures. Therefore the electrons transfer less energy to the ions near the axis of the capillary and more energy to the ions near the wall. This adds up in a much more flat radial temperature profile, resulting in a flatter radial density profile for lower pressures.

When an external axial magnetic field is applied, the ratio between the thermal conductivity in radial direction \( \chi_{r,r} \) and in both the azimuthal and axial direction \( \chi_{e,a,a} \) is \( \chi_{r,r}/\chi_{e,a,a} = 0.4 - 0.001 \) for densities of respectively \( 10^{24} \) and \( 10^{22} \) m\(^{-3}\). Thus the thermal conductivity in radial direction is for low pressures up to a factor 5 less and for higher pressures up to a factor 2 less than in azimuthal and axial direction, when an external axial magnetic field is applied.

In the case of no external axial magnetic field, \( H_{ci} = 0 \) and \( \chi_{e} \sim T^{3/2} \). For a large value of \( B_0 \), and using formula 39, the electron thermal conductivity coefficient \( \chi_e \sim T^{5/2} / H_{ci} \sim T n_e / B_0 \sim 1 / B_0 \). Thus a strong axial magnetic field leads to a radial plasma thermal conductivity with a weaker dependence on the plasma temperature.

For a large \( B_0 \) the thermal conductivity coefficient behaves like \( 1 / B_0 \), resulting in a decreasing overall heat flux to the capillary wall. This leads to an increase of the electron temperature, which corresponds to a smaller electron density, on the axis. Thus the electron density profile will become more hollow.

If the total number of particles is the same, the diminishing of the particle density near the axis will result in some increase near the wall. Therefore the electron density profile will be steeper as well at larger values of the axial magnetic field. In reference [18] it is shown in more detail that a smaller temperature dependence of \( \chi_e \) results in steeper temperature and density profiles.

Also is shown that applying an external axial magnetic field causes the electron density gradient near the axis to become less flat. This results in an electron density profile, which is closer to a parabolic shape, allowing higher quality guiding.

Because the thermal conductivity coefficient decreases with the magnetic field, the overall energy flux on the capillary wall decreases, with a corresponding increase of the electron temperature on axis. This lowers the plasma column resistivity and the value of the energy input. The smaller energy load on the capillary wall leads to a longer capillary lifetime.

It is concluded that, when an external axial magnetic field is applied, for the same required matched spot size larger capillaries can be used, which suffer less from erosion and can withstand a larger power input. Also the deeper, nearly parabolic, density profile is expected to be more suitable as a waveguide for laser pulses.
3 Experimental setup

3.1 Interferometer

A schematic overview of the experimental setup is shown in figure 3.1.

![Schematic overview of the interferometer setup](image)

A helium-neon cw-laser of 15 mW provides a Gaussian beam at a wavelength of 0.6328 μm with a waist of 540 μm. This beam is focused with 3 lenses at the entrance of a capillary. For the interferometry experiments the focussed spot radius is set to a waist of 180 μm and for the transmission experiments this is about 60 μm.

Using formulae 30 – 33, a proper combination of lenses is chosen using some restrictions. The distance from the last lens to the capillary should be at least 50 cm, because of the vacuum chamber and the beam splitter. The beam should also fit the lens sizes and therefore it can not be expanded too much. Besides, due to aberrations in the lenses and the finite thickness of the lenses, the size of the beam and the position of the waists will be somewhat larger than calculated.

After these lenses the beam goes to a beam splitter, which is set to an angle of 45°. The light is split in 2 arms. One arm goes via a 45° mirror first through a capillary and then through a second beam splitter towards an ICCD camera. The second arm goes via another mirror at an angle larger than 45° also to the second beam splitter, which is also at an angle larger than 45°, and then to the ICCD camera. So the beams are brought together at an angle. After the second beam splitter a positive lens is used, which images and magnifies the exit of the capillary on the ICCD camera. An interference filter with a typical 2.4 nm bandwidth centred around 0.6328 μm is placed in front of the camera. This filter blocks the light from the plasma.

It should be mentioned that the beams at the second beam splitter do not overlap in the experimental setup. The schematic overview gives a general idea of the setup and the propagation of the beams. See figure 3.1. Another point is that for the interferometry experiments the whole setup is used. However for the transmission experiments only the probe beam is used and the reference beam is blocked.

When a plasma is created in the capillary and when the reference beam and the probe beam come together as in figure 3.1, the fringes shift towards the left, according to the theory of the interferometer. This is checked by the following procedure. The capillary is filled with an increasing amount of hydrogen. Then the interference fringes shift more and more to a certain direction, because the refractive index in the capillary changes. Hydrogen has a refractive index larger than 1, while a hydrogen plasma has a refractive index smaller than 1. Thus the
fringe shift caused by the increasing amount of hydrogen is towards the other side than the fringe shift caused by the plasma. The shift due to hydrogen indeed appeared to be to the right in this case.

3.2 Capillary unit and plasma characteristics

![Diagram of capillary unit and discharge circuit]

Figure 3.2: Capillary, holder and discharge circuit.

The capillary is a hollow tube made of alumina, which is placed and sealed in a holder, as shown in figure 3.2. The capillary holder is placed in a vacuum chamber with a background pressure of $10^{-6}$ mbar. The inner radius of the capillary used in the experiments, described in this report, is 250 µm and the outer radius is 1.5 mm. The length can be chosen between 1 and 10 cm.

The capillary is filled with hydrogen gas with a pressure in the range of a few thousands Pascal. By applying a high voltage pulse of about 10 to 30 kV a plasma is created with a plasma current of 500 – 1000 A. Too high currents ($I > 2000$ A) might cause a pinch effect [18]. This would enhance the electron density at the axis, resulting in worse quality guiding or even no guiding at all. Besides, it might cause a kink instability, resulting in a non-constant matched spot size in time and position.

The discharge starts along the wall and then spreads to the middle of the capillary. This plasma has a lifetime of about 1 µs. The discharge circuit contains the capacitor $C$ (8 nF) and the semiconductor switch $S$. The discharge current is monitored by a magnetic probe (Pierson coil). A typical current curve is shown in Figure 3.3.
Instead of using a constant hydrogen flow through the holes of the capillary, electromechanical shutters close both ends of the capillary together with small ballast volumes $V_1$ and $V_2$, each of about 1 cm$^3$. When the shutters open, the gas from the ballast volumes will first flow out, because they are larger and have larger entrances, causing less flow resistance. The result is a buffer, which is emptied before the gas of the capillary flows out. The advantages of this shutter system are that the pressure inside the capillary can be determined more accurately and the background pressure can easier be kept lower. Also the modeling of the light’s trip through the plasma is more accurate than in reference [13], because the end effects due to out gassing are much less. The shutters open, when a TTL trigger pulse is applied to a special pulse generator, which produces a pulsed voltage for 2 electromagnets $E_1$ and $E_2$ that pull the shutters open. The discharge is produced, when the shutters at both ends of the capillary are totally open.

The time delay of the geometrical opening of the shutters is measured as follows. A photodiode is placed after the capillary and is connected to an oscilloscope. A TTL pulse is sent to the pulse generator for the electromagnets and also to the oscilloscope. The photodiode signal is acquired at the moment of the opening of the shutters. In the oscillogram in figure 3.4 the triggering TTL pulse and the signal from the photodiode are shown together.
It appears that the total mechanical delay of the shutter is about 5 ms with an accuracy of about 0.05 ms, but this delay depends upon the adjustment of the mechanical parts of the shutter construction. The real time of opening is about 0.2 ms. Simple estimations show that during this time the leak will be less than 0.1 cm$^3$ on each side of the capillary. Thus, the effective pressure inside the capillary will be approximately equal to the steady state pressure. The steady leak is about 3 orders of magnitude less than it is in the capillary system of Hooker et al [13]. The background pressure remains about $10^{-6}$ mbar, measured with a hydrogen pressure of 100 mbar inside the capillary. This pressure in the capillary is larger than the pressures used in the experiments.

### 3.3 Time resolved interference measurements

In order to perform a time-resolved experiment, an intensified CCD camera, Andor DH 734-18F-03 (1024×1024, 13µ×13µ), is used as a detector. When all experimental parameters are fixed, every time a plasma with the same density characteristics is produced, within an uncertainty of about $2 \cdot 10^{22}$ m$^{-3}$. During the existence of the plasma, an image of the interference pattern is obtained at a certain time delay with a gate width of 10 ns. This is performed at different time delays, each time using a new plasma. By doing so, a time resolved characterization of the plasma density is obtained.

The procedure per measurement is as follows. A schematic overview is given in figure 3.5.

![Timing of the measurement](image)

**Figure 3.5: Timing of the measurement.**

A TTL trigger pulse from a trigger box is applied to a special pulse generator. This pulse generator produces a pulsed voltage for the 2 electromagnets that pull the shutters open. As it is shown in the previous paragraph, it takes about 5 ms before the shutters are totally open. With the photodiode this time is determined and, because of the uncertainty in this measured value, the delay is chosen 0.05 ms larger. After these 5 ms the discharge is triggered. It takes about 400 ns before the discharge takes place. The ICCD camera has an internal delay of 40 ns. Therefore the ICCD camera is triggered at least 360 ns after the discharge trigger. By increasing this last delay, every measurement is performed at a different time delay during the existence of the plasma.

The ICCD camera and the semiconductor switch have a 50 Ω input impedance, which prevents noise signals from triggering the camera or the semiconductor switch.
3.4 External axial magnetic field setup

In order to perform experiments with an externally applied magnetic field, the setup shown in figure 3.6 is used.

![Diagram of the capillary unit with the addition of a coil for applying an external axial magnetic field.]

**Figure 3.6**: Capillary unit with the addition of a coil for applying an external axial magnetic field.

It differs from the setup for the experiments without magnetic field at the following points. The semiconductor switch for the discharge plasma is replaced by a thyratron T. This has the advantage of less jitter in the moment of the starting of the plasma current. For the semiconductor switch this is about 20 ns, while for the thyratron this is 2 ns. Also other capacitors are used. A characteristic current profile for a discharge voltage of 19 kV is shown in figure 3.7 in red. It can be seen that the discharge current pulse is shorter than the current pulse from the electric circuit with the semiconductor switch.

![Graph showing plasma current at 19 kV with thyratron and current through the coil around the capillary at 10 kV.]

**Figure 3.7**: In red the plasma current at a voltage of 19 kV with the thyratron and in green the current through the coil around the capillary at a voltage of 10 kV.
A second point of difference is the coil around the capillary, which provides the axial magnetic field. The coil is connected to another capacitor (C = 0.5 \mu F) and a semiconductor switch. A characteristic current profile of the current through the coil with a capacitor voltage of 10 kV is shown in figure 3.7 in green. The current through the coil is triggered so, that its maximum is at the same time as the discharge current maximum. During the discharge the current through the coil is assumed to be constant. Therefore the magnetic field is assumed to be constant in time as well during the discharge and it is estimated at about 5 T.

It should be mentioned that a capillary with a length of 1.9 cm is used with in the middle a coil of 4 mm. So, when the external magnetic field is applied, only the mid part of the capillary is magnetized, while the electron density measurement is line integrated. Therefore the calculated matched spot size is a mean value and can differ locally.
4 Data analysis

In order to calculate the density parameters from the interferograms, the following procedure is used. A Fortran program is written, which creates a model for the interferograms. This model contains all parameters, which characterize the position, width and amplitude of the interference pattern, the spatial frequency and the phase of the fringes and the density parameters $n_{e,0}$ and $n_{e,2}$ (see formula 20).

The parameter values are initialized with best-guess values for the capillary dimensions, density parameters, beam parameters and fringe parameters. Then the sum of the amplitudes of the reference beam and the probe beam with the corresponding relative phases is calculated, taking into account the phase difference due to the angle between the beams. The phase modulation due to the electron density distribution in the plasma and the deflection of the probe beam, also mentioned in [13], which manifests itself as some focusing, are also taken into account using formula 11. Taking this focusing into account makes it possible to use longer capillaries and stronger electron density gradients.

The square of the sum of the amplitudes is compared with the experimentally measured intensity of the interference pattern at the ICCD camera. This is performed by applying the least square method. In this procedure the squared difference between the calculated and the measured intensity is determined for every pixel. All these squared differences are summated. By variation of the mentioned model parameters the minimum of the summated squared differences is determined. In this minimum the values of the parameters fit the experimental data best.

The least square method is first applied to a few reference interferograms, taken without plasma. From this the parameters for the position, width and amplitude of the interference pattern and the spatial frequency and phase of the fringes are determined. Using these parameters, the density parameters $n_{e,0}$ and $n_{e,2}$ are determined from shift and the curvature of the curved fringes.

Then from the value of the $n_{e,2}$ parameter the matched spot size is calculated, using formula 25. The $n_{e,0}$ parameter is not fully determined in the least square method, because to its corresponding phase shift of the fringes a whole number times $2\pi$ can be added or subtracted. This corresponds to one total fringe shift. The $n_{e,0}$ parameter is only determined in a whole series of measurements during the plasma. It is obtained by first taking the last interference pattern in time, where the plasma is extinguished totally. Then the next measurement, one step back in time delay, is taken. A criterion for the decision whether to add or to subtract a fringe is that the density at the axis of the capillary gradually changes in time. Another criterion is that the plasma establishes in about 50 ns. This is done for the whole series.

According to formula 39 for the hydrogen gas, it appears that for 1000 Pa, 2000 Pa and 4000 Pa the mean electron density is respectively 0.5, 1.0 and $2.0\cdot10^{24}$ m$^{-3}$, when the plasma is fully ionized. When the electron density appears to be larger, there can be desorption from the wall.
5 Results

5.1 Interferometry

Several series of the interferograms are obtained at the following initial hydrogen pressures: 1000 Pa, 2000 Pa and 4000 Pa. The plasma is created with a high voltage pulse 18 kV. The maximal current which has been measured during the experiments is about 650 A (see figure 5.1). A capillary with 250 μm inner radius and 15 mm length is used. Each series contains a set of frames, corresponding to different time delays with respect to the beginning of the discharge current with intervals between 50 and 100 ns. A part of such a set is shown in figure 5.1 for an initial pressure of 4000 Pa together with the current curve. It roughly shows how the interference pattern changes in time during the discharge current.

![Figure 5.1: Interference pattern changes in time during the plasma current.](image)

The pattern evolution of the interferograms consists of several stages. It can be seen that before the high voltage pulse is applied, the fringes have no curvature. The interferograms, obtained within the first 50 ns after the breakdown, have a very low contrast of fringes. The reason is the scattering of the probe beam. This stage of the discharge corresponds to the decrease of the capillary waveguide transmission, mentioned in reference [3]. The probe beam scattering can be connected with azimuthal inhomogeneity of the capillary breakdown [19].

The next stage corresponds to the time delay range from 50 ns up to 800 ns, which is the end of the discharge current. During this stage curved fringes are visible in the interference pattern due to the radial dependence of the refractive index of the discharge plasma. This curvature of the fringes corresponds to a ‘hollow’ electron density radial profile of the plasma, which can serve as a waveguide. The maximum curvature is located at the peak of the plasma current, at about 200 ns.
After the total damping of the discharge current the fringe curvature changes its sign. This corresponds to a bulb-shaped electron density profile. The curvature is less strong than the curvature from the hollow density profile. After 1200 ns the plasma extinguishes and the fringes become straight again.

The density parameters $n_{e,0}$ and $n_{e,2}$ are determined from each interference pattern. These parameters are the electron density on the axis of the capillary and the second order parameter, which describes the parabalic electron density profile, as mentioned in formula 20. In figure 5.2 time evolution of the electron density $n_{e,0}$ at the axis of the capillary is shown.

![Figure 5.2: Electron density $n_{e,0}$ at the axis of the capillary for different time delays and for different initial hydrogen pressures with in red 4000 Pa, in green 2000 Pa and in blue 1000 Pa.](image)

The plasma reaches a maximum electron density around 200 - 400 ns delay. It recombines gradually due to the decrease of the discharge current, caused by the characteristics of the electrical circuit.

As mentioned in chapter 4, the mean electron density of the 1000 Pa measurements is about $5 \cdot 10^{23} \text{ m}^{-3}$ in the case of full ionization. However between 200 and 400 ns the density on axis is more than 60% larger. It is $8 \cdot 10^{23} \text{ m}^{-3}$ instead of $5 \cdot 10^{23} \text{ m}^{-3}$. In these values the additional increase of the electron density near the wall due to the parabolic electron density profile is not taken into account. This result has the consequence that a considerable desorption of the substance from the walls of the capillary is detected. Most likely, it is desorption of adsorbed hydrogen.

Due to vibrations, the data analysis and the fact that the plasma is every time a little different, the accuracy of the interferograms is about $1 \cdot 10^{23} \text{ m}^{-3}$. The points before 50 ns have a larger uncertainty in the axial density of about $2 \cdot 10^{23} \text{ m}^{-3}$, because of the turbulence in this stage. During the first 800 ns of the 2000 Pa and the 4000 Pa series there is an extra uncertainty in the addition or subtraction of a fringe. This corresponds to $2 \cdot 10^{23} \text{ m}^{-3}$. This extra uncertainty can be avoided by performing much more measurements, for example every 5 ns instead of 10 ns. Another possibility is to count the amount of shifted fringes with a photodiode.
In figure 5.3 the time evolution of the second order parameter $n_{e,2}$ is shown.

![Graph](image)

**Figure 5.3:** Second order parameter $n_{e,2}$ for different time delays and for different initial hydrogen pressures with in red 4000 Pa, in green 2000 Pa and in blue 1000 Pa.

The second order density parameter $n_{e,2}$ is determined from the curvature of the fringes. From figure 5.3 it appears that the maximum second order parameter is around 200 ns for each pressure and its value is about $0.35 \cdot 10^{24}$ m$^{-3}$ for all pressures. However, the larger the pressure, the slower the second order parameter decreases after its peak value.

An uncertainty in the value of the second order parameter is introduced by the fact that the plasma differs every time and by the data analysis. The turbulent stage introduces again an extra uncertainty. The uncertainty is estimated at $0.5 \cdot 10^{23}$ m$^{-3}$. 

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The second order parameter \( n_{e,2} \) defines the matched spot size \( W_M \) according to formula 25. The time evolution of the matched spot size is shown in figure 5.4.

![Graph showing the time evolution of the matched spot size for different initial hydrogen pressures](image)

**Figure 5.4: Matched spot size for different time delays and for different initial hydrogen pressures with in red 4000 Pa, in green 2000 Pa and in blue 1000 Pa.**

In the range of our capillary discharge parameters we get the minimal value for \( W_M \) of about 67 \( \mu \)m at time delays between 150 – 250 ns, depending upon the initial pressure. The larger the pressure the larger the time delays in which a minimal match spot size is reached. This can be addressed to a decrease of the average electron temperature due to lower energy deposition per electron. A lower temperature will result in a decrease of the electron thermal conductivity (formula 40) and therefore it will increase the time of reaching equilibrium. The minimal \( W_M \) only slightly depends upon the initial pressure. However for larger pressures, the matched spot size is for a longer period close to the minimal value and after 400 ns it remains smaller and more stable than for lower pressures. The uncertainty in the value of the matched spot size is estimated at 2 \( \mu \)m.

The electron density in the capillary is determined by the electron density at the axis and the second order parameter, which adds a radial dependent component. For the measurement at 4000 Pa with a high voltage pulse of 18 kV the electron density profile is shown for some time delays in figure 5.5 a-c. The corresponding matched spot size for each time delay is shown in the legend between brackets.
Figure 5.5 a-c: Electron densities at different time delays at a pressure of 4000 Pa and a high voltage pulse of 18 kV. The corresponding value of the matched spot size is shown in the legend for each time delay.
5.2 Transmission

A series of experiments are performed to study the guiding properties of the capillary discharge in the linear regime. A HeNe laser is used for these experiments. The reference beam arm in the setup (see figure 3.1) is blocked. The laser beam is focused at the entrance of the capillary to a size of about the matched spot size of the plasma as determined in the previous paragraph at about 60 μm. A set of measurements of the laser spot at the exit of the capillary is performed at different time delays. The initial pressure in the capillary is 4000 Pa and the maximal discharge current is about 650 A with a high voltage pulse of 18 kV. For these measurements a capillary with an inner radius of 250 μm and length of 50 mm is used. A global overview of the obtained laser spots is shown in figure 5.6.

Before the high voltage pulse is applied, there is no guiding and the beam diverges in the capillary. According to formula 27 a beam with a waist of 60 μm has a Rayleigh length of about 1.8 cm. So according to formula 26 the radius of the beam would be 180 μm. Due to the Gaussian intensity distribution there will be a small amount of light outside the radius of 250 μm and diffract at the wall. In figure 5.6 this is visible in a very weak ring around the spot. The spot has indeed a radius of about 180 μm. The radius of the outer ring in the diffracted pattern is about 250 μm, which corresponds to the inner radius of the capillary.

In the chaotic starting stage of the discharge this diffraction pattern is totally disturbed. As expected, the time delays in the range of 50 - 800 ns correspond to the waveguiding regime with good quality of the guided beam. The measured spot size at the exit of the capillary is about 60 μm. In the last stage, when the discharge current is totally damped, the beam gradually defocuses.

Figure 5.6: The transmitted laser spot changes in time during the high voltage pulse. An initial pressure of 4000 Pa is used.
The waist of the beam should be aligned very accurately at the entrance of the capillary. Otherwise it will diverge before entering the capillary and it will not be guided properly. Without plasma in the capillary the diverging beam will diffract at the wall and a diffraction pattern will be visible as shown in figure 5.7.

![Figure 5.7: Misalignment of the waist causes diffraction at the wall, resulting in a diffraction pattern.](image)

In order to investigate how much light is guided through the capillary, the intensity of non-guided and the guided spot are both integrated over the area within their 1/e² radius. It is obtained that 92±5% of the light is guided through the capillary without diverging.

### 5.3 External axial magnetic field

Interferometry measurements are performed with and without external axial magnetic field for several pressures. The time delay is chosen around 300 ns from the beginning of the plasma current. This is the stage of the plasma where the fringes have a maximal curvature, resulting in a minimal matched spot size, which is needed for LWFA. This delay corresponding to a maximal curvature is different from the delay value in the interferometry experiments, because of the different electronic circuit. The magnetic field is 5 Tesla and the peak discharge current is about 800 A. The pressures at which the experiments are performed are: 100 Pa, 500 Pa, 1000 Pa, 1500 Pa, 2000 Pa, 3000 Pa, 4000 Pa, 5000 Pa, 7000 Pa and 10000 Pa. The time delay is 320 ns. In figure 5.8 the matched spot size for both cases with and without magnetic field is plotted against the pressure for 288 ns time delay.
Figure 5.8: Matched spot size for different pressures with and without external axial magnetic field, which are in green and red respectively.

The values of the matched spot size are much larger than the ones in the interferometry experiments. This is due to the different electronic circuit, which provides less power to the discharge, because the current pulse is shorter. Therefore the plasma is heated less, resulting in less electron density modulation.

When an external axial magnetic field is applied, the matched spot size is significantly smaller for all pressures, varying from 15 μm up to 23 μm. Depending on the pressure the matched spot size is about 20 – 30 % smaller. The difference is the largest for small pressures. This corresponds to the theory, because for smaller densities the Hall parameter is larger and the thermal conductivity coefficient is smaller.

However, in the case of an initial pressure of 100 Pa and 500 Pa, there is not enough hydrogen in the capillary to produce an electron density modulation, which corresponds to a matched spot size of about 62 μm. This small value of the matched spot size is therefore an indication for desorption of hydrogen from the wall, which causes the pressure during the discharge to be larger than the measured value.

For an initial pressure of 1000 Pa a time delay scan is performed for both the cases of with and without an external axial magnetic field. The matched spot size is plotted against the time delay measured from the beginning of the discharge current in figure 5.9.
Figure 5.9: The matched spot size for different time delays for both with and without external axial magnetic field at an initial pressure of 1000 Pa.

When an axial magnetic field is applied, the matched spot size is up to about 20 μm smaller in the stage of the beginning of the discharge current at the position of the maximal curvature around 400 ns. After this stage the difference becomes smaller down to about 5 μm. The maximal difference corresponds to a 4 times larger curvature of the fringes. Again it should be mentioned that this is the effective value of the matched spot size, because of non-homogeneity of the magnetic field. In the middle of the coil the local value of the matched spot size can be less and outside the coil it can be larger.
6 Conclusions

Capillary discharges as optical waveguides at electron densities in the range $10^{23} - 10^{24}$ m$^{-3}$ are investigated. A new design of a capillary discharge set up has been presented. It has been used to create plasmas as optical waveguides. The most important new feature of the setup is that it does not use a steady gas flow through the capillary, but instead of that an electromechanical shutter system is implemented with a very small leak rate (almost static pressure inside the capillary). This makes it possible to measure and control the initial hydrogen pressure inside the capillary in a straightforward way and to decrease the effective leak rate with 3 orders of magnitude in comparison with the experimental setup in reference [11]. Also the background pressure can easier be kept lower.

The discharge plasma is investigated by means of the interferometer setup, which uses a Gaussian "needle-like" beam of a HeNe laser as a probe. The following stages during the plasma evolution are found:

1) The initial stage up to 50 ns after the beginning of the discharge. It is characterized by strong scattering of the probe beam due to optical inhomogeneities.

2) The optical guiding stage at delays between 50 ns and 800 ns. The time evolution of the matched spot size $W_M$ is studied at different initial pressures. It is found, that an increase of the initial hydrogen pressure results in an increase of the time delay of getting the minimal $W_M$, which is about 67 µm.

3) After the total damping of the discharge current (time delay is more than 800 ns) the radial electron density profile changes from a "hollow" profile to a "bulb" profile, resulting in non waveguiding optical properties.

4) The plasma totally recombiners after 1200 ns measured from the beginning of the discharge.

The results of the axial electron density showed that for an initial pressure of 1000 Pa there is desorption of hydrogen from the wall of the capillary.

The optical properties of the created plasma channel are checked via transmission measurements of a focused Gaussian laser beam. The results show high quality optical guiding of the beam with an energy transmission of 92%. The beam, which is focused at a size of about 60 µm, propagates without diverging.

Applying an external axial magnetic field has a large effect on the guiding properties of the plasma. The matched spot size decreases with 23 – 15 µm for different pressures between respectively 100 and 10000 Pa at a time delay, which corresponds to a maximal curvature. This is a decrease of about 20-30 %.

For an initial pressure of 1000 Pa a time delay scan is performed for both the cases of with and without external axial magnetic field. In the case of an external magnetic field the matched spot size is up to 20 µm less up to a delay of 400 ns.

So it is confirmed that when an external axial magnetic field is applied, smaller pressures can be used to achieve the same required matched spot size.
7 Outlook

7.1 Integration experimental setups FOM – TU/E

After some additional external magnetic field measurements the capillary setup will be moved to Eindhoven. There it will be integrated in their setup for LWFA and the electron acceleration experiments will be performed. A schematic overview of the setup is shown in figure 7.1 [20].

\[ \text{Figure 7.1: Capillary unit integrated in LWFA setup in Eindhoven} \]

A 2 TW Ti:Sapphire laser pulse is split into two pulses. One of them is focused at the entrance of the capillary and creates the wakefield. The other one is used for the generation of short intense electron bunches in a RF photo-gun. This is shown in figure 7.2. A short (30 fs) pulse of UV laser light frees electrons from a copper cathode, which are accelerated by a Radio Frequent electric field to an energy of about 6.5 MeV [20].

\[ \text{Figure 7.2: RF photo-gun} \]

The bunches from the photo-gun are focused within the plasma channel using a solenoid and then combined with the TW laser pulse, generating the wakefield in the plasma. The bunch will enter the plasma just as the accelerating fields are being generated, allowing for an acceleration to energies (~100 MeV) over a length of about 5-10 cm. Note that using classical acceleration technology, this would require an accelerator of almost 10 m [20].
7.2 A new way of creating a waveguide: electron beam

In 2005 V.V. Ivanov came with the idea to create a plasma optical waveguide in a new way [21]. A schematic overview of the setup is shown in figure 7.3. The idea is the creation of a sharply focused pulsed electron beam with an energy of about 10 keV and a current of 0.1 A, which is focused by means of a magnetic field inside a hydrogen filled cell. The electron beam ionizes the hydrogen gas. The degree of ionization is around $10^2$ at an initial molecular density of about $10^{23} \text{ m}^{-3}$. In the dissociative recombination processes and elastic collisions the energy is transferred to the heavy components (hydrogen atoms, ions and molecules) producing a relatively slow expansion of the gas column. The result is a hollow neutral radial density profile.

The plasma waveguide can be created by a high power femtosecond laser pulse, which is focused on the entrance of the hydrogen channel. This step is partly analogous to the idea of guiding in glass capillaries, filled with a gas [22]. The difference is that in the electron beam case the necessary spatial radial distribution of hydrogen is prepared in advance. An injected laser pulse of 0.5-1 J will produce a practically immediate optical field induced ionization, resulting in a proper (hollow) electron density profile and will also excite a wakefield. The energy, needed to ionize 1 cm of such a hollow hydrogen channel is about $10^{-4}$ J, which is negligible in comparison with energies of femtosecond pulses in the range of 0.5-1 J. This means that a large fraction of the energy in the pulse can be used for exciting strong wakefields.

An important advantage of this method is that a small matched spot size of about 30 μm is feasible for a low density of $10^{23} \text{ m}^{-3}$. As opposed to the slow capillary discharge there is no erosion from the wall. Another advantage is the easy construction of waveguides in the length scale of about 1 m at a time scale of about 1 μs, circumventing synchronization problems.

![Figure 7.3: Schematic overview of the electron beam setup](image-url)
Acknowledgements

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References


Appendix A

Article submitted to Phys. Rev. E about the interferometry results discussed in this report:

Optical waveguiding characteristics of a slow capillary discharge at low electron densities

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Abstract

A hydrogen plasma waveguide has been investigated at electron densities in the range $10^{17} - 10^{18}$ cm$^{-3}$ as a waveguide for Laser Wakefield Acceleration. Low densities are attractive for this application due to a larger possible electron energy gain and an easier synchronization of injected electron bunches and the plasma wave. Interferometry experiments are performed in an advanced version of the slow capillary discharge, including a new capillary arrangement. A significant influence of desorption is found at an initial discharge pressure of 1000 Pa. A matched spot size of about 60 µm has been achieved at a density of $8 \times 10^{17}$ cm$^{-3}$ and for a time period of 200 ns in a capillary of 500 µm inner diameter. The optical guiding properties have been checked at an initial pressure of 4000 Pa. High quality optical guiding has been demonstrated for time delays up to 500 ns after the triggering of a discharge with a matched spot size, which is defined from the interferometry measurements.

Introduction.

The plasma optical waveguide is an object, which has been studied for quite some time. In the research area of Laser Wakefield Acceleration (LWFA) the main goal for using these optical waveguides is to achieve interactions of high power density laser pulses with plasmas at length scales much larger than the Rayleigh length $Z_R = \pi W_0^2 / \lambda$ ($W_0$ is the size of the laser beam waist). A plasma waveguide is a channel in a plasma with a parabolic radial profile of the electron density, increasing towards the wall of the channel. Considering $\Delta n_e$ as the difference between the electron density at the radius $r_c$ of the channel and the electron
density at the axis, these values define the so called matched spot size \( W_m \). If a laser beam is focused to a waist equal to \( W_m \) at the entrance of the channel, it will propagate without changing its transverse profile.

The main parameters of plasma waveguides are the physical dimensions, the matched spot size and the electron density \( n_{e0} \) on the axis.

A number of different approaches to create plasma waveguides exists. Below a short overview is given together with the typical parameters of these waveguides.

The creation of a linear spark by means of focusing a laser beam with an axicon lens [1-4] gives an \( n_{e0} \) in the range of \( 10^{19} \) cm\(^{-3} \) with a \( W_m = 10-15 \) \( \mu \)m and length values from several mm up to 1 cm. The ablation of the capillary wall by a discharge [5-7] gives an \( n_{e0} = 2-4 \times 10^{18} \) cm\(^{-3} \) and a \( W_m = 28-35 \) \( \mu \)m with a length of several cm. A third type of a waveguide has been produced by the so called fast capillary discharge [8,9]. It can be created with a length in the order of several cm. The numerical estimations show an \( n_{e0} = 2 \times 10^{17} \) cm\(^{-3} \) and a \( W_m \) of about 20-25 \( \mu \)m. Hooker et al [10-13] have proposed and studied another type of plasma waveguides, obtained in a so called slow capillary discharge. This type of discharge produces the heating of hydrogen gas inside a capillary with an inner diameter 300-500 \( \mu \)m. A discharge current of 300-600 A is used to get total ionization. The typical time of the discharge is about several hundreds of nanoseconds. Due to the constant pressure in the plasma the cooling near the wall results in an increase of the electron density. Hooker et al reported guiding lengths up to 10 cm and a \( W_m \) of 37.5 \( \mu \)m at \( n_{e0} \) of \( 3 \times 10^{18} \) cm\(^{-3} \).

In this paper we report about the study of the earlier mentioned parameters of plasma waveguides at low hydrogen pressures – down to 1000 Pa, resulting in an axial electron density of \( 10^{17} \) cm\(^{-3} \). The final aim is the usage of this plasma waveguide for LWFA [14]. Smaller electron densities \( n_{e0} \) in the range of \( 10^{17} \) cm\(^{-3} \) are attractive from the point of view of larger electron energy gain [15] and an easier synchronization of injected electron bunches and the plasma wave. A set of experiments is performed with an advanced version of the slow capillary discharge, which includes a pressure control system and electromechanical shutters to keep the hydrogen leak rate low. The principle of the shutter system makes the capillary edge effects considerably less than in constant flow systems [10-13]. The main diagnostic tool used for obtaining experimental results about the discharge is an optical interferometer (Mach-Zehnder).
The capillary unit

Our capillary holder design contains new features compared with the capillary unit which has been used by Hooker et al [11]. Instead of using a constant hydrogen flow through the capillary, two electromechanical shutters are used, which close the capillary ends together with small ballast volumes $V_1$ and $V_2$, each about $1 \text{ cm}^3$ (see Fig.1). When a TTL trigger pulse is applied to a special pulse generator, which produces a voltage pulse for the electromagnets $E_1$ and $E_2$ (also see Fig.1), the shutters are pulled open. After this voltage pulse the shutters are closed by springs, which are attached at the other side of the shutters.

The capillary is made of alumina and is placed in a holder. Capillaries with varying inner diameters can be placed in the capillary holder. The outer diameter is constrained by the hole in the flange and is $3 \text{ mm}$. The length of the capillary can be chosen between $10 \text{ mm}$ and $100 \text{ mm}$.

The discharge is produced, when the apertures at both ends of the capillary are totally open. A measurement is carried out to determine the time scale of the geometrical opening of the shutters. To this end, the capillary and a photodiode are placed in the beam of the HeNe laser (see Fig.1). The result is demonstrated in the oscillogram in Fig.2, where the triggering TTL pulse and the signal from the photodiode are shown together. It appears that the total mechanical delay of the shutter is about $5 \text{ ms}$. However, this depends on the adjustment of the mechanical parts of the construction. The time of opening is about $0.2 \text{ ms}$.

Simple calculations show that during this time of opening the leak will be less than $0.1 \text{ cm}^3$ on each side of the capillary. Therefore the effective pressure inside the capillary will be approximately equal to the steady state pressure in the gas system. The major advantage of the capillary shutter system is that the steady gas leak is about 3 orders of magnitude less than it is in for instance [11]. The pumping rate is $500 \text{ l/s}$ and, with a pressure of $10000 \text{ Pa}$ inside the capillary, there is a residual background pressure of about $10^{-4} - 10^{-3} \text{ Pa}$.

The discharge circuit contains the capacitor $C$ (8 nF) and the semiconductor switch $S$. The charging voltage can be set up to $25 \text{ kV}$. The discharge current is monitored by a magnetic probe (Pierson coil). A current curve is shown in the experimental results.

The interferometer setup

The general scheme of the interferometer setup is represented in Fig.3. It is a Mach-Zehnder interferometer, in which a $15 \text{ mW CW HeNe}$ laser is used as a light source. In order to perform a time-resolved experiment, an intensified CCD camera, Andor DH 734-18F-03
(1024×1024 pixels, 13μm×13μm pixel size), is used as a detector. It has a minimum time gate of 2 ns and a time delay at any value beginning from 40 ns after the triggering pulse.

In contrast to the experimental setup of Hooker et al [11], a “needle”-like Gaussian beam is used as a probe for the capillary. The laser beam is first expanded and then focused on the entrance of the capillary as shown in figure 3. The waist of the probe beam can be tuned by changing the magnification of the beam expander. The exit of the capillary is imaged on the entrance of the ICCD through the narrow-band multilayer interference filter. The probe beam and the reference beam are superimposed at the detector plane at a certain angle. This gives rise to an interference pattern of straight fringes in absence of perturbations of the refraction index in the capillary.

The quantitative analysis of the capillary plasma is based upon the assumption that it has a parabolic electron density profile, as is convincingly proven in [11,16]. This assumption seems reasonable for the analysis at time delays corresponding to good channeling properties of the capillary. The electron density profile is then represented as a best fit to a parabolic profile, using the least square method.

The capillary plasma will act as a quadratic phase modulator. Thus the Gaussian beam at the entrance of the capillary will be transformed into another Gaussian beam [17]. The propagation and the transformation of the Gaussian beam can be described in terms of changes of its parameter $q$ [17]. These changes need to be determined.

The waist of the probe beam is focused on the entrance of the capillary tube, so the initial value of $q$ is

$$q_0 = i \pi w_0^2 / \lambda$$

in which $w_0$ is the waist radius at the level 1/e² of the maximum intensity.

If the capillary plasma has a length $b$ and the radial refractive index profile is:

$$n(r) = 1 - \frac{n_2}{2} r^2,$$

where the small deviation of $n(0)$ from 1 manifests itself in the total phase shift of the probe beam, then the value of the parameter $q$ at the exit of the capillary will be [17]:

$$q_{\text{probe}} = \frac{q_0 \cos(b \sqrt{n_2}) + \sin(b \sqrt{n_2})}{(-q_0) \sqrt{n_2} \sin(b \sqrt{n_2}) + \cos(b \sqrt{n_2})}$$

(2)

The corresponding value of the reference beam parameter will be:

$$q_{\text{ref}} = q_0 + b$$

(3)
Now the values of the radii $R$ of the phase surface curvature and the values of the width parameter $w$ for both beams can be obtained from the relation \([17]\):

$$\frac{1}{q} = \frac{1}{R} - i \frac{\lambda}{\pi w^2}$$  \hspace{1cm} (4)

A numerical model of the interference pattern is constructed in the following way: the square of the sum of the amplitudes of the reference beam and the probe beam with the corresponding relative phases is calculated, including the homogeneous phase shift because of the deviation of $n(0)$ from 1 and taking into account the phase difference due to the angle between the beams. The phase modulation due to the electron density distribution in the plasma and also the deflection of the probe beam, mentioned in \([11]\), which manifests itself as some focusing, are taken into account in Eq. (2). The latter makes it possible to use longer capillaries and stronger electron density gradients.

The modeled interference pattern is compared with the experimentally measured interference pattern. This is performed by applying the least square method. In this method several parameters, which characterize the position, width and amplitude of the interference pattern, the spatial frequency of the fringes and the parameter $n_2$, are used as variables. This set of parameters totally defines the modeled interference pattern. The difference between the experimental intensities and the modeled interference pattern is calculated for every pixel. These differences are squared and their sum $L$ is calculated. The set of parameters, which gives the minimal value of $L$, is considered as the best fit for the experimental data.

The value of the parameter $n_2$ from the best fit is directly related to the value of the matched spot size $w_m$:

$$w_m^4 = \frac{\lambda^2}{\pi^2 n_2}$$  \hspace{1cm} (5)

The electron density on the axis of the capillary is obtained from the analysis of the total fringe shift, which is also one of the variable parameters in the least square method.

It should be stressed that the evolution of the phase surfaces of the probe and reference beams is defined by the diffraction. The phase difference between the reference and the perturbed probe beam changes during their propagation. Even its sign can change. But, as it can be shown by simple matrix transformations of $q$, the precise geometrical imaging of the exit of the capillary allows the total recovery of the spatial pattern of the phase difference of the beams, corresponding to the exit of the capillary, taking into account the value of the image magnification.
Results interferometry

Several series of interferograms have been obtained at the following initial hydrogen pressures: 1000 Pa, 2000 Pa and 4000 Pa. The maximal current which has been used during the experiments is about 650 A (see Fig.4). A capillary with 500 μm inner diameter and 15 mm length is used.

Each series of interferograms contains a set of frames, corresponding to different time delays with respect to the beginning of the discharge current. A part of such a series is shown in Fig.4 for an initial pressure of 4000 Pa together with the current curve.

One can observe several stages during the pattern evolution of the interferograms. It can be seen that before the high voltage pulse is applied, the fringes have no curvature. The interferograms, obtained within the first 50 ns after the breakdown, have a very low contrast of fringes. The reason is the scattering of the probe beam. This stage of the discharge corresponds to the decrease of the capillary waveguide transmission, mentioned in reference [12]. The probe beam scattering can be connected with azimuthal inhomogeneity of the capillary breakdown [18]. The next stage corresponds to the time delay range between 50 ns and 800 ns (the end of the discharge current). Here it can be observed that the fringes are curved, which is due to the radial profile of the refractive index of the discharge plasma. This curvature of the fringes corresponds to a ‘hollow’ electron density radial profile of the plasma, which can thus serve as a waveguide. After the total damping of the discharge current the fringe curvature changes its sign. This corresponds to a bulb-shaped electron density profile. After 1200 ns the plasma extinguishes and the fringes become straight again.

The density parameters $n_{e_0}$ and $n_{e_2}$ are determined from each interference pattern. These parameters describe the radial profile of the electron density as $n_e(r) = n_{e_0} + n_{e_2} \cdot r^2$. Fig.5 shows the time evolution of the electron density $n_{e_0}$ at the axis of the capillary for an initial hydrogen pressure of 1000 Pa. The plasma establishes in about 50 -100 ns after the breakdown. It reaches a maximum electron density around 200-400 ns delay. The plasma recombines gradually due to the decrease of the discharge current, caused by the characteristics of the electrical circuit. The maximal axial electron density is more than 60% higher than can be expected from the initial hydrogen pressure. It is $8 \times 10^{17}$ cm$^{-3}$ instead of $5 \times 10^{17}$ cm$^{-3}$. In this latter value the additional increase of the density near the wall due to the parabolic electron density profile is not taken into account. This result has an important consequence, namely that we detect considerable desorption of the substance from the wall of the capillary. Most likely, it is desorption of adsorbed hydrogen.
The second order parameter $n_{c,2}$ defines the matched spot size $W_m = (1/\pi r_e n_{c,2})^{1/4}$ in which $r_e$ is the classical electron radius. The time evolution of the matched spot size is shown in Fig. 6 and Fig. 7. In the range of our capillary discharge parameters we get the minimal value of $W_m$ at time delays between $150 \text{ ns} - 250 \text{ ns}$ depending upon the initial pressure. The minimal $W_m$ depends only slightly upon the initial pressure. This can also be attributed to desorption from the wall of the capillary, which effectively changes the value of the mean electron density. The larger the pressure, the larger the time delays in which a minimal matched spot size is reached. The increase of the time delay of the minimal matched spot size can be ascribed to a decrease of the average electron temperature due to a lower energy deposition per electron by the increase of the total amount of hydrogen inside a capillary. A lower temperature will result in a decrease of the electron thermal conductivity and thus in an increase of the time of reaching equilibrium.

The optical guiding properties of the plasma channel.

We have studied the optical properties of the capillary discharge waveguide in the linear regime. A HeNe laser is used for these experiments. The reference beam arm in the set up (see Fig. 2) is blocked. The laser beam is focused at the entrance of the capillary at a radius of $60 \mu m$, which is about the matched spot size of the plasma as determined in the previous paragraph. A set of measurements of the laser spot at the exit of the capillary is performed at different time delays. In Fig. 8 the result is shown for a time delay before the discharge and in Fig. 9 for a time delay in the stage, which corresponds to a hollow electron density profile. For these measurements the initial pressure in the capillary is taken $4000 \text{ Pa}$ and the maximal discharge current is about $650 \text{ A}$. A capillary with an inner diameter of $500 \mu m$ and length of $50 \text{ mm}$ is used. Before the high voltage pulse is applied, the beam diverges in the capillary and diffracts at the wall. In the chaotic starting stage of the discharge this diffraction pattern is totally disturbed. As expected, the time delays in the range of $50-800 \text{ ns}$ correspond to the waveguiding regime with high quality of the guided beam. During this stage the relative amount of transmitted energy is estimated at between $90 - 100\%$, as concluded from time-resolved transmission measurements using the CW HeNe probe laser and the gated ICCD camera. In the last stage of the discharge, when the discharge current is totally damped, the beam is defocused. Transmission measurements in the non-linear range, i.e. using fs laser pulses at TW power levels, are yet foreseen.
Conclusions.

Capillary discharges as optical waveguides at electron densities in the range $10^{17} - 10^{18}$ cm$^{-3}$ are investigated in the linear regime. A new design of the capillary discharge set up is presented. The most important feature of the set up is that it does not use a steady gas flow through the capillary, but an electromechanical shutter system, which allows a near static pressure inside the capillary. This makes it possible to measure and control the initial hydrogen pressure inside the capillary in a straightforward way and to decrease the effective leak rate with 3 orders of magnitude in comparison with the experimental setup in reference [11]. This is one of the demands for integrating the capillary unit in the LWFA setup.

The discharge plasma is investigated by means of the interferometer setup, which uses a Gaussian “needle-like” beam of a HeNe laser as a probe. The following stages during the plasma evolution are found:

a) The initial stage up to 50 ns after the beginning of the discharge. It is characterized by strong scattering of the probe beam due to optical inhomogeneities.

b) The optical guiding stage at delays between 50 ns and 800 ns. The time evolution of the matched spot size $W_m$ is studied at different initial pressures. It is found, that an increase of the initial hydrogen pressure results in an increase of the time delay of obtaining the minimal $W_m$.

c) After the total damping of the discharge current (time delay is more than 800 ns) the radial electron density profile changes from a “hollow” profile to a “bulb” profile, resulting in unsuitable waveguiding optical properties.

d) The plasma totally recombines after 1200 ns measured from the beginning of the discharge.

It is found that the axial electron density at an initial pressure of 1000 Pa is considerably larger than can be expected from the total amount of hydrogen atoms. This fact can be explained by the desorption of the substance from the wall of the capillary (adsorbed hydrogen molecules or the material of the wall). So it can be a problem to achieve an axial electron density down to $10^{17}$ cm$^{-3}$ in the slow capillary discharges.

The optical properties of the created plasma channel are checked via transmission measurements of a focused Gaussian laser beam. The results show high quality optical guiding of the beam.
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References.

Figure 0.1. Schematic drawing of the capillary unit.

Figure 0.2. Transient time between triggering pulse and the opening of the shutters.

Figure 0.3. Schematic drawing of the experimental setup.
Figure 0.4. Time evolution of the interference pattern for an initial hydrogen pressure of 4000 Pa.

Figure 0.5. The time evolution of the axial electron density in the capillary discharge at an initial hydrogen pressure.
Figure 0.6. The time evolution of the matched spot size $W_m$ at different initial hydrogen pressures.

Figure 0.7. Zoom of the time evolution of the matched spot size $W_m$ at different initial hydrogen pressures.

Figure 0.8 and 0.9. The transmitted laser spot without guiding and the guided laser spot.