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Optimal Experiment Design
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Abstract
Investigating glucose kinetics under physiological conditions e.g. a meal or an oral glucose tolerance test (OGTT), is extremely valuable since diabetes mellitus is a pressing public health concern. In this study an expanded version of the glucose minimal model of Bergman has been developed and verified from a physiological and identification point of view using synthetic data produced from parameters found in literature.

In succession of the minimal model presented by [Bergman et al., Am. J. Phys. 236: E667-E677, 1979] which is widely used under non-physiological conditions, different minimal models are presented for use under physiological conditions during an OGGT e.g. [Natalucci et al., Int. Conf. IEEE EMB, Cancun, Mexico, 2003]. Natalucci’s et al. minimal model is expanded to include jointly the effects of capillary recruitment, Glut-4 translocation and the production and inhibition of hepatic glucose production. The expanded minimal model incorporates a physiological model of glucose appearance from gut into blood plasma and an interstitial glucose compartment which can be measured using the microdialysis technique.

The new model has been tested in all its identifiability aspects: on a priori global identifiability by using an algorithm based on differential algebra [Audoly et al., IEEE Trans Biomed., 48 1: 55-65, 2001] and a posteriori identifiability using model to model adjustment. Also, the sensitivity of the a priori identifiable parameters in relation with the estimated parameters has been examined showing large sensitivity for two of the six a priory parameters. Parameter estimation has been performed using Maximum likelihood estimation and the worst bias is calculated showing low bias for all estimated parameters. The oral glucose ingestion dosis has been optimised to reduce the bias for the estimated parameters and an algorithm has been designed to find the optimal sample schedule. Knowing the joint interaction between these mechanism in an novel, physiologically more detailed model and identifying the new oral glucose minimal model in all its aspects and performing optimal experiment design, is a great step forward to understand and quantify different potential causes of insulin resistance in type 2 diabetes.

Keywords: minimal compartmental model, glucose kinetics, OGTT, capillary recruitment, Glut-4 translocation, saturable receptor-mediated insulin transport, microdialysis, interstitial glucose, a priori global identifiability, a posteriori identifiability, maximum likelihood estimation, optimal experiment design

Introduction

1 Diabetes Mellitus

Diabetes mellitus is a metabolic disease characterized by varying or persistent elevated glucose levels in blood plasma. Diabetes mellitus is a pressing public health concern. In the Netherlands more than a half million people suffer from diabetes. Diabetes mellitus can be divided into two types: type 1 and type 2. Diabetes type 1 is caused by a lack of insulin
production and insulin secretion by the pancreas and type 2 diabetes is caused by decreased sensitivity of body tissues for insulin. This study focuses on type 2 diabetes.

The glucose regulatory system in humans consists of releasing insulin by the pancreatic β cells when the glucose concentration in blood rises above the normal range of 3.6 to 6.1 mM. Insulin stimulates glucose uptake by skeletal muscle (and adipose tissue) and suppress glucose production by the liver (hepatic glucose production). Insulin stimulates glucose uptake by skeletal muscle via two sequential mechanisms: recruitment of capillaries surrounding muscle fibers and glucose transporter-4 (Glut-4) translocation.

The second mechanism depends on the access of glucose and insulin to skeletal muscle. Glucose uptake by human skeletal muscle is regulated by insulin concentration in the interstitium [H. Herkner 2003]. Insulin is transported from blood plasma through the endothelium to reach the interstitium. After insulin has reached the interstitium, insulin is cleared due to internalization and degradation of insulin by muscle cells: Insulin binds to the insulin receptor in the interstitium which is the initiating step in insulin signalling pathways as discussed.

Two hypotheses have been proposed for the first mechanism: capillary recruitment and insulin-mediated increase of blood flow [Bergman 2003 and references in there]. In skeletal muscles two vascular flow routes exits: one route by nutritive capillaries that surround the skeletal muscles fibres responsible for the delivery of glucose and insulin and a second route by non-nutritive capillaries that has no contact with the skeletal muscles fibres. The first hypothesis suggests that insulin controls the blood flow to skeletal muscle at rest and during exercise by varying blood flow between the nutritive capillaries and the non-nutritive capillaries. It is believed that this process is very fast, 5 minutes after increase of insulin [Vincent 2004, Iwashita 2001, Porter 1997] and at insulin concentrations lower than required for the second hypothesis [Zhang 2004]. Changes in the distribution pattern can be caused by insulin resistance of the endothelial cells. Several studies [Clerk 2006, Gudbjörnsvöttir 2005, Gudbjörnsvöttir 2003] show an increase of capillary recruitment (increased permeability-surface area product (PS)) during enhanced physiological insulin concentration and a reduced capillary recruitment (lower PS) in type 2 diabetes humans than in healthy (obese) humans. The second hypothesis suggests that insulin acts as a vasodilator (alternating the blood flow at regular intervals through different capillary groups) controlling the access of glucose and insulin to skeletal muscle. Insulin binds to the insulin receptor on the luminal endothelial cells and this activates nitric oxide synthase (NOS). In response, endothelial cells release the vasodilator nitric oxide (NO) that prevents the smooth muscles cells of the precapillary arteriole to contract. This effect is more apparent at higher insulin concentration (pharmacologic concentrations) and after several hours. Impaired vasodilation can by due to insulin resistance. No increase in blood flow is seen during physiological insulin concentrations in obese healthy humans and type 2 diabetic humans [Bergman 2003 and reference in there, Gudbjörnsdóttir 2005, Gudbjörnsdóttir 2003]. However, [Clerk 2006] shows an increase during physiological insulin concentrations in lean healthy humans as well as in obese healthy humans. Insulin resistance can also have a more physical-chemical reason like a highly impermeable endothelium or reduced capillary density.

Several studies have addressed the mechanism of insulin transport from blood plasma through the endothelium to reach the interstitium by measuring glucose and insulin concentrations in blood plasma and interstitium. These studies result in conflicting conclusions either diffusion transport or a saturable receptor-mediated transport. These conflicting results have been analysed here (appendix A). It is concluded that a saturable
receptor-mediated insulin transport is considered when insulin concentrations are in a physiological range (up to 600 pM) whereas a non-saturable insulin transport is considered at pharmacologic insulin concentrations (++ 600 pM).

The second mechanism by which insulin stimulates glucose clearance is by increased uptake by myocytes. The transport of glucose through the muscle cell membrane is facilitated by a special carrier mediated protein called Glut-4. In the early morning fasting state only a small portion of the total amount of Glut-4 is located in the outer muscle cell membrane (insulin independent). In response to the binding of insulin to the insulin receptor in the interstitium, triggering a cascade of protein phosphorylation reactions that ultimately activate the translocation of Glut-4 stored in microsomal membranes are transported to the outer muscle cell membrane. (insulin dependent). [Lauritzen 2006] shows a time delay of 10 minutes between insulin leaving the interstitium and the first translocations of Glut-4 after intravenous insulin infusion. Maximum translocation of Glut-4 was reached after 30 minutes.

Three impairments exist in the above described mechanisms for type 2 diabetic humans: Impairment of Glut-4 translocation, impairment of the recruitment of muscle capillaries, and impairment of the inhibition of hepatic glucose production [Zijlstra 2004]. The contribution of each of these impairments is not clear, since only an overall insulin sensitivity ($S_I$), the efficacy of insulin to glucose uptake by body tissues [$1/(\text{pM} \cdot \text{min})$], is used in experiments to quantify the whole insulin resistance.

The goal of this study is to develop a method to understand and quantify different potential causes of insulin resistance in type 2 diabetes under normal physiological conditions. For this purpose a novel model is to be developed and to be verified. In general the model will incorporate the main glucose regulating mechanism to obtain descriptions of the variables involved and to enable estimation of physiologically relevant parameters using measured data of glucose profiles, insulin profiles or others profiles of processes in a human.

The model should meet all the conditions for valid use in widespread clinical use. The oral glucose minimal model must describe the mechanism by which insulin regulates glucose uptake by muscle cells or adipose tissues as best as possible. Simplification of the model leads to undermodelling of the glucose regulatory system giving erroneous estimates. Overmodelling leads to complex model which are difficult to identify. The data from clinical measurements used for the novel model must be obtained during normal life conditions, the possibilities for invasive measurements in humans are limited and the costs of the measurements must be kept under a reasonable financial budget. Furthermore, the model must provide reliable unique estimates for the model parameters which describe the effects of Glut-4 translocation, the recruitment of muscle capillaries, and impairment of the inhibition of hepatic glucose production with high precision using the measurements techniques available, in other words the model needs to be identifiable for those parameters.

The first part of this report consists of modelling glucose kinetics in humans. First the mechanisms of glucose uptake by skeletal muscle and its possible impairments were discussed in this section. Further a closer analyse was made of the mechanism of transport of insulin from blood plasma through the endothelium to reach the interstitium list in appendix D. Secondly, several minimal models modelling the rate of appearance of glucose from the gut in the blood plasma are discussed. Thirdly, the new oral glucose minimal model is presented with is modifications compared to the classical minimal model of [Bergman 1979]. Finally the new oral glucose minimal model is represented in state space and the plasma
glucose, interstitial glucose, interstitial insulin and the rate of appearance of glucose are simulated using measured insulin profiles. A different approach could have been to use a model to simulate insulin (insulin profiles) that is secreted by pancreatic $\beta$-cells via the liver into the portal vein. A study discussing the model is listed in appendix B.

The second part of this report consists of parameter estimation to identify the unknown parameters in the glucose metabolism model as optimal as possible. First, the glucose metabolism model is tested on a priori global identifiability and a posteriori identifiability. Secondly, the sensitivity of a priori parameters in relation with the estimated parameters is examined. Thirdly, the parameter estimation algorithm used is explained and also a technique is presented to calculate a lower bound of the precision that can be achieved for each parameter when estimating the parameters. Fourthly, an algorithm is designed to optimize the glucose input doses and an algorithm is designed to find the optimal sampling scheduling. Then the results are shown. Finally, a discussion is made and suggestions for future research are given.

2 Minimal Models

$S_I$ can be estimated in experiments with the Clamp technique, with the intravenous glucose tolerance test (IVGTT), with the meal oral glucose tolerance test (MGTI) or with the oral glucose tolerance test (OGTT). The Clamp technique consists of intravenous injecting a known insulin concentration together with an infusion of glucose to control the glucose concentration in blood plasma to a fixed value. The ratio of injected insulin and infused glucose provides a measure of insulin sensitivity. The IVGTT consists of injecting a known glucose concentration into the blood plasma. The glucose and insulin concentration in blood plasma are measured in time. The minimal model [R.N. Bergman 1979] is used to estimate parameters from which $S_I$ is calculated. The CLAMP technique as well as the IVGTT use non-normal life conditions. The MGTI and OGTI are tests using normal life conditions. The MGTI uses an ingested meal containing glucose and the OGTI uses an ingested liquid containing glucose. The glucose and insulin profiles in blood plasma are measured. Also less surgical intervention is needed (only arterial and venous blood samples) than during a CLAMP or IVGTT. The minimal model of Bergman needs to be adapted to incorporate the rate of appearance of glucose from the gut in the blood plasma / systemic circulation of nutrient taken by mouth ($Ra(t)$).

In the past two mathematical models without physiological meaning have successfully been used to estimate $Ra(t)$. The “integral equation” method is based on the assumption that $Ra(t)$ is an anticipated version of blood plasma glucose concentrations [A. Caumo 2000]. The method calculates the areas under the measured glucose and insulin blood plasma concentrations. To properly calculate the areas, the glucose and insulin concentration have to return to basal levels. This may cause problems if the “integral equation” method is applied to humans with either impaired glucose tolerance or impaired insulin secretion, in which the return of glucose and insulin to the pretest levels requires a rather long time. Compared to the Bergman minimal model for an IVGTT, only two additional parameters need to be estimated. The “differential equation” method uses a piecewise-linear parametric model of $Ra(t)$ [Dalla Man 5-2002]. An advantage of this method is that glucose and insulin concentrations do not have to return to basal value at the end of the test, hence, the experiment is dramatically shortened. Four additional parameters have to be estimated.
Two mathematical models with physiological meaning have been used successfully to estimate $R_a(t)$. [D.R.L. Worthington 1997] shows a physiological model which was tested on a type 1 diabetic humans. A one compartmental method was proposed, having the following physiological parameters: fraction content of glucose in the food normalized to glucose distribution volume, time constant of gastric emptying and the transport delay between ingestion of glucose and appearance of glucose in blood plasma. Advantages of this method are that only three additional parameters have to be estimated and the parameters are physiological.

An interesting physiological model is studied by [J.D. Elashoff 1982]. This model is used in this study. The model uses the exponential curve which follows a set of physiological and mathematical characteristics. The most important mathematical characteristic is that at time zero the fraction of glucose ingested ($DS$) [g] is 1.0 and at infinite time the ingested fraction is 0.0. Further, the power exponential curve ($i$) also allows to have a slow initial emptying followed by an accelerated emptying ($\beta>1$). If $\beta<1$ the opposite occurs. This phenomenon is often seen in humans who have undergone an operation. If $\beta=1$ the exponential curve is obtained. Physiologically, the model uses the inverse of the time constant of gastric emptying ($k$) [1/min] and $\beta$.

$$f(t) = e^{-(k+1)^{\beta}}$$

(1)

Data of older studies obtained by measuring the contents of different ingested meals (e.g. glucose contents) and taken samples of the contents of stomach and using by different healthy humans as well as by healthy humans with operated stomach were fit to (1) using nonlinear least squares regression. This model showed high goodness of fit with the measured data.

Another study [J. Schirra 1996] performed an experiment with eight healthy non-obese males using this power exponential curve (1). The experiment used 50 or 100 grams glucose dissolved in 400 ml water and drank in 2 minutes showing $k$'s and $\beta$'s with a very high precision.

Natalucci coupled the mono compartmental model [S. Natalucci 2003] using the power exponential curve (1) with the adapted minimal model [Bergman 1979] for an OGTT. Figure 1 shows this model were $G_{gut}(t)$ [1/kg] is the mass of glucose in the gut and $K_{abs}$ [1/min] is the rate constant of glucose absorption from the gut into blood plasma. The model was tested on nine non diabetic humans using a doses of 75 grams of glucose $(DS)$ dissolved in water. However, the parameters of the power exponential curve were not estimated, but the values $\beta=1.23$ and $k=0.014$ [1/min] were taken from [J. Schirra 1996]. The experiment showed $S_i$ and $K_{abs}$ with low variance.

\[ DS \xrightarrow{f(t)} G_{gut}(t) \xrightarrow{K_{abs}} R_a(t) \]

Figure 1: Mono compartmental model of the gut
3 New Oral Glucose Minimal Model

Since the contribution of the different potential impairments in the glucose regulatory system in type 2 diabetes, as stated in the introduction, is not clear yet, we propose to split up $S_i$ in the individual components $S_{i-cap}$, $S_{i-glut-4}$ and $S_{i-liver}$. $S_{i-cap}$ is the efficacy of insulin to stimulate insulin and glucose transport from blood plasma into interstitium by capillary recruitment. $S_{i-glut-4}$ is the efficacy of insulin to stimulate glucose clearance in muscle and adipose tissue by GLUT-4 translocation. $S_{i-liver}$ is the efficacy of insulin to inhibit hepatic production and stimulate glucose uptake by liver. The aim of this study was to modify the method developed by Bergman [R.N. Bergman 1979] to make this splitting possible.

[Zijlstra 2004] suggested a modification of the minimal model of Bergman. This modification consists of adding a (measured) interstitial glucose compartment to the classic minimal model, to model the control of glucose transport into the interstitium by insulin as discussed in the introduction. Adding this compartment could make the identification of the rate constant of transport of plasma glucose into the interstitial compartment possible. This extra compartment is not new, a remote (non-accessible, only exchange with the glucose blood plasma compartment) glucose compartment was already used during an IVGTT in [Cobelli 1999], during an OGTT [Dalla Man 10-2002] and references in there to avoid overestimation of $S_G$ (the insulin independent glucose uptake) and underestimation of $S_i$ seen in one Compartmental Minimal Models (1CMM) like the classic Bergman model. This extra compartment was also used in [Caumo 1993] and references in there to derive a time profile of endogenous hepatic glucose production during a labeled IVGTT. However, this extra compartment was not measured in all earlier models.

Another difference in the new oral glucose minimal model with the classic minimal model of Bergman is that inhibition of hepatic production and stimulation of glucose uptake by liver is regulated by plasma insulin instead of interstitial insulin as access to hepatocytes is immediate, due to a highly fenestrated endothelium [Bergman 2003]. Also, plasma insulin regulates the transport of insulin and glucose from plasma to interstitium as discussed in the introduction.

In the classic minimal model of Bergman $S_G$ consists of the net rate of glucose produced by the liver and insulin independent glucose uptake by tissues like the brain. Unfortunately, those effects cannot be estimated independently. By using ingested labelled glucose, a distinction is made between ingested (labelled) glucose and the net rate of (non-labelled) glucose produced by the liver. By measuring the concentration of labelled glucose in blood plasma $S_{i-liver}$ can be obtained as e.g. performed in [Dalla Man 10-2002].

The suggestions made by Zijlstra [Zijlstra 2004] together with the new suggestions made in this section, the previously discussed physiologic description of glucose appearance from the gut into blood plasma and with the saturable receptor-mediated insulin transport hypothesis (appendix D) resulted in the new oral glucose minimal model shown in figure 2.
Figure 2: New oral glucose minimal model for glucose metabolism, derived from the classic minimal model of Bergman including an interstitial glucose compartment \( G'(t) \), inhibition of hepatic production and stimulation of glucose uptake by liver regulated by plasma insulin \( I(t) \) instead of by interstitial insulin \( I'(t) \) and a physiologic description of glucose appearance from the gut into blood plasma \( G(t) \) and using a saturable receptor-mediated insulin transport from blood plasma to interstitium for insulin concentrations in physiological range.

This new model consists of four compartments \( G(t) \) [mM], glucose plasma, \( G'(t) \) [mM], interstitial Glucose, \( I(t) \) [pM] insulin plasma and \( I'(t) \) [pM] interstitial insulin. \( Rge \) [g/min] is the rate of gastric emptying, \( DS \) [g] is the amount of glucose ingested, \( G_{gut} \) (t) [I/g] is the mass of glucose in the gut and \( K_{abs} \) [I/min] is the rate constant of glucose absorption from the gut into blood plasma.

Glucose \( G(t) \) is transported at a rate proportional to \( k_7 \) [I/min] to the interstitium and cleared to muscle and adipose tissue at a rate proportional to \( k_1 \) [I/min]. Glucose plasma \( G(t) \) is also cleared to tissue in a non-insulin dependent way at a rate proportional to \( p_6 \) [I/min].

Insulin \( I(t) \) is transported at a rate proportional to \( k_2 \) [I/min] to the interstitium. Insulin \( I(t) \) controls the inhibition and stimulation of hepatic glucose uptake by liver \( k_6 \) [I/(pM·min)]. Also, insulin controls the transport of plasma glucose \( k_8 \) [I/(pM·min)] and plasma insulin \( k_9 \) [I/pM] to the remote compartments. Interstitial insulin controls glucose uptake by muscles \( k_4 \) [I/(pM·min)] and is degraded at a rate proportional to \( k_3 \) [I/min].

Figure 3 shows the reparameterized model of figure 2 (model I). In model 1, \( p_2 \) equals \( k_3 \) [I/min], \( S_{cap} \) equals \( k_2 \cdot k_9 \) [I/(pM·min)], \( p_4 \) equals \( k_7 \cdot k_8 \) [I/(pM·min)], \( p_5 \) equals \( k_5 \) [I/min], \( p_6 \) equals \( S_c \) [I/min], \( S_{gut} \) equals \( k_4 \cdot k_1 \) [I/(pM·min)] and \( S_{liver} \) equals \( k_6 \) [I/(pM·min)].
The reparameterized new oral glucose minimal model 1 of figure 2 can be modified further by including ingested labelled glucose $GL(t)$ resulting in the new oral glucose minimal model 2 as shown in figure 4 (index $L$ denotes the labelled glucose version of a compartment). Remark: Another method could be to enrich blood plasma with labelled glucose by injecting intravenous labelled glucose resulting in the new oral glucose minimal model 3. The new oral glucose minimal model 2 can easily be modified to describe this experimental setup.
The new oral glucose minimal model describes jointly the effects of Glut-4 translocation, the recruitment of muscle capillaries, and impairment of the inhibition of hepatic glucose production using existing minimal models in literature and modifying them using recent insights in the mechanism by which insulin regulates glucose uptake by muscle cells or adipose tissue. Knowing the joint interaction between these mechanisms is a great step forward to understand and quantify different potential causes of insulin resistance in type 2 diabetes. Furthermore, analyzing and verifying the new oral glucose minimal model in all its aspects (a priori and a posterior identifiability, a priori parameter sensitivity) and performing optimal experiment design (optimal input and optimal sample schedule) is a great step forward in modelling glucose kinetics since to the authors' knowledge it has never been applied all in one to a minimal model of glucose kinetics before.

Methods

4.1 Measurements Techniques

The data will be obtained in the near future at the School of Sport and Exercise Sciences at the University of Birmingham. Sophisticated techniques are used to measure the concentrations of plasma glucose and insulin and interstitial glucose during an OGTT. Glucose and insulin in blood plasma are measured by taking samples by inserting an intravenous cannula (BD Venflon™) into one of the antecubital veins. The subject's arm is placed in a so-called 'hot box' in which the high temperature will arterialise the venous blood. The blood samples will be centrifuged at 3000g at 4°C for 10 min and stored at -70°C for later analysis of glucose and insulin.

Plasma glucose concentrations will be determined by the Cobas Mira Plus automated analyser (ABX Diagnostics, UK) with a coefficient of variation (CV) of 2% if measured in 10-fold. Plasma Insulin concentrations will be determined by a commercially available kit (DRG instruments GmbH, Germany) with a CV of 10%. Further, the interstitial glucose concentrations are measured by the microdialysis technique using custom made CMA107 microdialysis probe with a length of 3 cm, a MW cut-off of 20000 and a coefficient of variation (CV) of 4%. Details of this technique are discussed in the next section.

4.1.1 Microdialysis

The microdialysis technique [Lönroth 1987, Lönroth 1997] consists of the diffusion of a water-soluble substance over a dialysis membrane into a perfusion medium. It enables measurement of the interstitial fluid. The inlet of the catheter is perfused by a pump with a perfusion fluid (water containing isotonic saline) at a rate of 0.667 μl/min. Exchange occurs between the perfusion fluid and the interstitial fluid. After equilibrium is reached, subsequent dialyses samples (20 μl) are collected at the outlet of the catheter. The collected sample is a mean measurement over the time needed to reach equilibrium. The samples are analyzed for interstitial glucose. Due to the permeability of the dialysis membrane, maximal 85-90% of interstitial glucose is recovered in the dialysis. The microdialysis technique is not suitable for interstitial insulin measurements since very low recovery (+/- 3%) is achieved yet [Sjöstrand 2005, Sjöstrand 1999]. Another drawback of this method is that data samples
suffer from slow dynamics: interstitial mean glucose concentrations are only available after reaching equilibrium in about 30 minutes.

4.2.1 Model Structure in State Space

The oral glucose minimal model can be represented in state space. The state space model structure and its state space measurement model of a dynamic system is shown in equations (2, 3) and (4), respectively, where $x$ is the $n$-state vector, $u$ the multi input vector, $f$ are linear or nonlinear functions which describe the structure of the system parameterized by vector $\theta_0$, $g$ are nonlinear or linear functions which describe the output configuration parameterized by vector $\theta_o$, $y_i$ are the measurements of the $v$th output sampled at $N_v$ discrete times $t_{iv}$, $y_m(t, \theta_j)$ is the modeled $v$th output at time $t$ or sampled at $N_v$ discrete times $t_{iv}$, $e_i$ is the measurement error, assumed to be additive zero mean white noise with known variance $\sigma^2_{v}(t_i)$ and $M$ the number of measured outputs.

State equation:

$$x(t, \theta_j) = f(x(t, \theta_j); u(t), \theta_j) \quad x(t_0, \theta_j) = x_0$$  \hspace{1cm} (2)

Output equation:

$$y_v(t, \theta_j) = g_v(x(t, \theta_j); \theta_j)$$  \hspace{1cm} (3)

Measurement model:

$$y_{iv} = y_{iv} + e_{iv} \quad i=1, 2, 3, ..., N_v, \quad v=1, 2, 3, ..., M$$  \hspace{1cm} (4)

This framework includes non-equidistant time samples, that not all outputs (blood plasma glucose and interstitial glucose) are sampled at the same time points and outputs measured with different accuracy. It should be noted that for the glucose minimal model two different types of input variables have to be discriminated. The orally ingested glucose can be manipulated by the experimenter and, therefore, is an independent input, which, in system theory, is commonly denoted by $u(t)$. In our approach the measured insulin profile is also applied as an input to the minimal model. However, since this input is a variable which results from the response of the system to the independent input $u(t)$, it is not independent.

The concept of 'dependent inputs' allows the opening of some of the feedback loops that connect the system of interest to the rest of the closed loop real system [van Riel 2006]. The model can focus on a smaller part of the system, as long as this subsystem is still integrated in its real system through the measured input signal. If the system contains a subsystem (state variables) which output can be measured, this subsystem (state variables) can be substituted by using the measured signal as input to the system (dependent input $v(t)$) as shown in (5). The number of states of the system is reduced. The drawback is a loss of predictive power, because the 'open-loop' model can only be used to simulate situations for which the dependent inputs have been measured in the real system. To solve the differential equations for the used integration step, the measured dependent input signal $v(t)$ is linearly interpolated. If the dependent input(s) $v(t)$ can be (accurately) measured in time, these signal can be used as forcing functions for these input(s).

$$x(t, \theta_j) = f(x(t, \theta_j); u(t), v(t), \theta_j) \quad x(t_0, \theta_j) = x_0$$  \hspace{1cm} (5)
4.2.2 Simulation

The differential equations have been solved in Matlab by the differential equation solver Ode45. The relative error tolerances (RelTol) of the solutions were set four orders lower than the amplitudes of the outputs to $1 \cdot 10^{-4}$ and the absolute error tolerances (AbsTol) were set two orders smaller than RelTol to $1 \cdot 10^{-6}$.

4.3 Identifiability

The parameters of the new oral glucose minimal model 1, 2 and 3 are tested on a priori global identifiability and a posteriori identifiability to analyse which parameters can be estimated using the measured techniques discussed in section 4.1. A parameter can be estimated if its variance has an acceptable low value.

A priori global identifiability consists of analyzing the structure of the model to verify which parameters can be estimated using the measured techniques discussed in section 4.1 and using noise-free data without having any a priori information about the parameters. In contrast to a priori global identifiability, a posteriori identifiability verifies which parameters can be estimated by analysing the effects of the dimensions of the parameters, the effects of different input profiles and the effects of different sample schedules.

4.3.1 A Priori Global Identifiability

A priori global identifiability under noise-free conditions has been examined by using an algorithm based on differential algebra [Audoly 2001]. Briefly, the algorithm consists of first deriving one equation which is algebraic observable, in other words may only contain the model input and output variables and their derivatives and not contain any states or its derivatives. The input-output differential equation representation of a dynamic model has this property. Hence, the differential equation has to be derived from the state space representation of the model. In [Audoly 2001] this is done by eliminating all states in each equation of the state space representation by substituting state equations or their derivatives in each other, the so called characteristic set of the model, resulting in one equation without states, the so-called input-output relation of the model.

The differential equation of the model is identical to the input-output relation of the model. The differential equation is used to find the set of identifiable parameters by extracting the coefficients of the differential equation of the model. These coefficients are subtracted by the same coefficients which are substituted by a random numerical point in the parameter space for the unknown parameters. An example is shown below:

Example A:

Differential equation:

\[ c_1 \cdot c_2 \cdot x(t) + c_3 / c_2 \cdot x(t) = 0 \]  \hspace{1cm} (a1)

Coefficients of the differential equation:

\[ c_1 \cdot c_2, \frac{c_3}{c_2} \]  \hspace{1cm} (a2)
A random numerical point in the parameter space for the unknown parameters:

\[ c_1 = 12 \]  
\[ c_2 = 4 \]  
\[ c_3 = 36 \]

Coefficients of the differential equation subtracted by the same coefficients substituted by random numerical point in the parameter space for the unknown parameter:

\[ c_1 \cdot c_2 - 48 = 0 \]  
\[ c_3 / c_2 - 9 = 0 \]

The obtained set of coefficients (example 1: \( a_4, a_5 \)) of the differential equation, which are called exhaustive summary in [Audoly 2001], are solved by the Buchberger algorithm [Buchberger 2001] obtaining the Gröbner basis as shown in (a6). If this basis has a finite number of solutions, all parameter can be estimated properly. Else, non-identifiable parameters have to be estimated jointly, to obtain a finite number of solutions of the Gröbner basis. In example 1 \( C_1 \) and \( C_2 / C_3 \) are identifiable.

\[ 9c_2 - c_3 \]  
\[ -9 + c_3 / c_2 \]  
\[ c_1 - 48 / c_2 \]

The algorithm described in [Audoly 2001] assumes generic initials conditions for the state space model. Generic initials conditions are initial equations which are not restricted to a particular value. If non-generic initials conditions are used, parameters could not be observable in the differential equation leading to parameters which cannot be tested on identifiability using this algorithm. Hence, if non-generic initials conditions are used a manual check needs to be done to verify if unknown parameters are not observable when deriving the input-output relation of the model. If parameters vanish, the set of state equations of the state space representation needs to be enlarged by adding the polynomials evaluated at these initial conditions. In other words, the initial condition is filled in the state space representation of the model and this polynomial is added as extra equation in the state space representation of the model. The input-output relation must be recalculated.

[Saccomani 2002, Saccomani 2001] presented a modified algorithm based on [Audoly 2001] for dealing with those non-generic initial conditions. The algorithm is based on the lack of reachability / accessibility from the initial state. If the system is accessible from all initial states, the characteristic set of the model is valid. Else the characteristic set needs to be enlarged adding the polynomials evaluated at the initial conditions which lacks of accessibility. The accessibility property of a model is checked by using the accessibility Lie Algebra (6) where \( \frac{\partial}{\partial x} \) denotes the Jacobian matrix, \( A \) are the polynomials of the state space model which are multiplied by the states \( x \) and \( B \) are the polynomials of the state space model which are multiplied by the input \( u \) as defined in (7). A model is accessible from its initial conditions if the so-called accessibility rank condition holds for each initial condition [Sontag 1998]. The accessibility rank condition is satisfied when the linear span of all vectors functions that can be obtained starting with the \( A \)'s and taking any numbers of Lie brackets of them as shown in (8) has the same dimension as the state space model.
\[ [A, B] = \left( \frac{\partial}{\partial x} B \right) A - \left( \frac{\partial}{\partial x} A \right) B \]  
(6)

\[ x = A \cdot x + B \cdot u \]  
(7)

(8)

### 4.3.2 A posteriori Identifiability

The model to model adjustment [Damen 2005] approach has been used to analyze the a posteriori identifiability using different input profiles under noise and noise-free conditions and under. The effect using when using different samples schedules can also be studied. Model to model adjustment consists of analyzing the error cost function (9) for combinations of parameters \( \hat{\theta} \) with different values in the parameter space where \( y_v(t_{iv}) \) are the measurements of the \( v \text{th} \) output sampled at \( N_v \) discrete times \( t_{iv} \), \( y_{vm}(t_{iv}, \hat{\theta}) \) are the modeled \( v \text{th} \) output sampled at \( N_v \) discrete times \( t_{iv} \) using the parameters \( \theta_v \), \( z_v \) is the \( v \text{th} \) output weighting factor, \( \theta_v \) are the true parameters, \( N_v \) the number of samples taken for each output \( v \) and \( M \) the number of outputs.

\[
J(\hat{\theta}) = \frac{1}{N_v} \sum_{i=1}^{N_v} \sum_{v=1}^{M} z_v \left( y_v(t_{iv}, \theta_v) - y_{vm}(t_{iv}, \hat{\theta}) \right)^2 
\]  
(9)

The 2 dimensional contour plot, plots the cost function (the model error) for two parameters for a range of values. Other parameters in the parameter space are kept fixed at the true parameter \( \theta_v \). If the combination of parameter is identifiable, the contour plot shows concentric ellipses centered at the supposed true values of the two parameters (global minima). The more narrow and steeper the concentric ellipsis are, how more accurate the estimation of both parameters will be.

### 4.3.3 A Priori Parameters Sensitivity

Parameters which are not identifiable by the used model setup in combination with the used experimental setup have been set a priori. Unfortunately, parameters can vary among population. The parameter sensitivity shows if a non-identifiable parameter can be set a priori without losing significant precision of the parameters when the non-identifiable parameter varies significantly among population. Therefore the parameters sensitivities have been calculated by varying the a priori parameters one by one over a fixed range with respect to the true known parameter, setting the value of the other a priori parameters fixed, estimating the corresponding unknown identifiable parameters and finally calculating the corresponding parameter covariance matrix \( \Delta(\theta) \) which will be explained in section 4.4. If the identifiable parameters are unbiased, the determinants of the obtained covariance matrices are an indication for the parameter sensitivity for variation of the a priori parameter. Else, if estimates were not unbiased, the worstcase bias technique, as stated in the last paragraph of section 4.4, has been used.
4.4 Parameter Estimation

Several algorithms exist to estimate parameters of a model, such as: Weight Least Squares estimation (WLS), Maximum Likelihood estimation (ML), Generalized Least Squares estimation and Bayesian estimation (MAP). In this study Maximum Likelihood estimation has been used since the noise introduced by the measurements techniques used (section 4.1) can be assumed to be white, Gaussian, zero mean and with variance $\sigma^2_v$.

The ML criterion assumes that the measured vth output $y_v$, sampled at $N_v$ discrete times $t_{v,i}$ consists of the modelled vth output $y_{vm}$ depending on true parameters $\theta$, and noise $e$ as shown in (10), which is different from (4) since the parameter vector has been split in a priori unidentifiable parameters which were kept fixed $\theta_{fix}$ and the identifiable parameters which have been estimated $\theta_{est}$.

$$y_v(t_{v,i}) = y_{vm}(t_{v,i}, \theta_{fix}, \theta_{est}) + e_v(t_{v,i}) \quad i = 1, 2, 3, ..., N_v, \quad v = 1, 2, 3, ..., M \quad (10)$$

The probability density function of Gaussian noise is shown in (10).

$$p_e(e_v(i)) = \frac{1}{\sqrt{2\pi\sigma_v^2}} e^{-\frac{e_v^2(i)}{2\sigma_v^2}} \quad (11)$$

Combining equations (10) and (11) and assuming that the consecutive noise samples are independent, results in the likelihood function for the residue of the vth output with variance $\sigma^2_v$ (12).

$$L(y_v | \theta_{est}) = \left( \frac{1}{\sqrt{2\pi\sigma_v^2}} \right)^{N_v} e^{-\frac{1}{2\sigma_v^2} \sum_{i=1}^{N_v} (y_v(t_{v,i}) - y_{vm}(t_{v,i}, \theta_{fix}, \theta_{est}))^2} \quad (12)$$

The noise is assumed to be white, Gaussian, zero mean and with variance $\sigma^2_v$. This assumption has been checked, as will be discussed in this section, after having estimated the parameters.

The likelihood function for the residues of all outputs $M$ is shown in (13).

$$L(y | \theta_{est}) = \prod_{v=1}^{M} \left( \frac{1}{\sqrt{2\pi\sigma_v^2}} \right)^{N_v} e^{-\frac{1}{2\sigma_v^2} \sum_{i=1}^{N_v} \sum_{v=1}^{M} (y_v(t_{v,i}) - y_{vm}(t_{v,i}, \theta_{fix}, \theta_{est}))^2} \quad (13)$$

Taking the log of (13) results in the cost function for the ML (14) [Damen 2005 p. 47-62].

$$J(\theta_{est}) = \sum_{i=1}^{N_v} \sum_{v=1}^{M} \frac{(y_v(t_{v,i}) - y_{vm}(t_{v,i}, \theta_{fix}, \theta_{est}))^2}{2\sigma_v^2} \quad (14)$$
The model parameters $\theta_{est}$ are estimated by minimizing (14) as shown in (18).

$$\theta_{est} = \arg \min_{\theta_{est}} J(\theta_{est})$$

(15)

Function minimization of (15) has been accomplished in Matlab using the non linear least square estimation algorithm lsqnonlin which uses the Medium-Scale Gauss-Newton Algorithm with Line search cubicpoly. The minimization of (15) stopped when the variation in cost function, between two subsequent iterations was less than TolFun or the absolute parameters value changed less than TolX. TolFun was set much lower than the amplitudes of the weighted outputs $z_{v,y}$ to $1 \cdot 10^{-4}$ and TolX, a factor 1000 lower than the lowest true value of the estimated parameter set to $1 \cdot 10^{-10}$. The minimization algorithm also stops after a number of iterations is reached. The maximal number of iterations was set to 800.

After having estimated the parameters, the residues of the criterium $J_v(\theta_{est})$ must be analyzed to ensure that the assumption made earlier that the noise is Gaussian, zero mean with variance $\sigma_v^2$ is true. For white noise it holds that consecutive noise samples are independent. Its mean is zero and its autocorrelation function $\phi_v(t_{shift})$ (16) takes the form of a pulse at zero time shift and is zero elsewhere (Dirac pulse).

$$\phi_v(t_{shift}) = \frac{1}{N_v} \sum_{i=1}^{N_v} J_v(t_{iv}, \theta_{est}) \cdot J_v(t_{iv} + t_{shift}, \theta_{est})$$

(16)

The lower bound of the parameter covariance matrix $\Delta(\theta_{est})$ of the estimated model parameters $\theta_{est}$ is obtained (using the Cramer-Rao theorem) from the inverse of the Fisher information matrix $F(\theta_{est})$ [Walter 1997 p. 245-250] as shown in (17). If MLE qualifies as an unbiased and minimum variance estimator, the lower bound in (17) becomes an equality.

An estimator is unbiased if (18) holds where $E$ denotes the expectation. In addition, if the variance $\sigma_v^2$ is set to the inverse of the data covariance of the disturbance and $N_v \gg n_0$ (number of parameters) holds, MLE qualifies as an asymptotically unbiased and minimum variance estimator [Damen 2005], respectively.

$$\lim_{N_v \to \infty} \Delta(\theta_{est}) = F^{-1}(\theta_{est}) = \sum_{v=1}^{M} E_{y_{vm} \theta_{est} \theta_{est} \theta_{est}} \left\{ \frac{\partial^2 \left[ - \ln \left( L(y_{vm} | \theta_{fix}, \theta_{est}) \right) \right]}{\partial \theta_{est} \partial \theta_{est}^T} \right\}^{-1}$$

$$\alpha = \sum_{v=1}^{M} \frac{1}{N_v} \sum_{i=1}^{N_v} \frac{\partial y_{vm}(t_{iv}, \theta_{fix}, \theta_{est})}{\partial \theta_{est}} \cdot \frac{\partial y_{vm}(t_{iv}, \theta_{fix}, \theta_{est})}{\partial \theta_{est}}^T$$

(17)

$$E(\theta_{est} - \theta_r) = 0 \text{ for } N_v \gg n_0$$

(18)
with $E$ the expected value and $n_0$ the number of estimated parameters. The variance $\sigma_n^2$ in (17) has been determined a posteriori. After having estimated the parameters, information is available concerning the variance $\sigma_n^2$. It is set to the squared 2-norm of the residues of $J(\theta_{est})$ (14).

If the parameters are biased ($E(\theta_{est} - \theta_t) \neq 0$) the parameter covariance matrix cannot be calculated using (17). In this case, the relative worst case bias is used to provide a measure of the (in)accuracy of the parameter estimations. The relative worst case bias as shown in (19) has been calculated for each identifiable parameter $\theta_{est}$ in the parameter space by varying the initial values of each identifiable parameter one $\theta_{est}$ by one between -50 to +50% of $\theta$ in steps of 10%, setting all other identifiable parameter fixed and estimating its value using the maximum likelihood estimation technique and adding for each estimation procedure $q$ a different sequence of noise to the synthetic data with a $\frac{1}{SNR}$ of 0%, 5% or 10%. The relative worst case bias is then the maximal difference found, among all $Q$ estimation procedures, between estimate $\theta_{est,q}$ and true value $\theta_{t,q}$ as shown in (20).

$$Bias_{rel}(\%)_{est,q} = \frac{|\theta_{est,q} - \theta_t|}{\theta_t} \cdot 100\% \quad q = 1, 2, 3 ..., Q$$

$$Bias_{rel}(\%)_{est} = \max_{\theta}(Bias_{rel}(\%)_{est,q})$$

(19)

(20)

4.5 Optimal Experiment Design

Designing an optimal experiment for parameter identification consists primarily of selecting the inputs and outputs to make all parameters of interest in the model identifiable (section 4.3). Secondary, it consists of choosing a suitable parameter estimation algorithm, optimizing the shape of the input signal, choosing the optimal sampling schedule, choosing the type of sensors etc. to get the maximal information from the obtained data.

Since the choices of inputs and outputs were restricted due to physical accessibility, optimal experiment design has been focused on optimizing the glucose input doses and choosing the optimal sampling schedule discussed in section 4.5.1 and 4.5.2 respectively.

4.5.1 Optimal Input Shaping

An optimal input can be determined by the D-optimality criterion [Walter 1997 p. 287-291] as applied in [Cobelli 1988] if the identifiable parameters are unbiased. D-optimality criterion consists of minimizing the determinant of the parameter covariance matrix, which is equivalent to maximizing the determinant of the Fisher information matrix (17) (unbiased and minimum variance estimator) for the input signal $u(t)$ using a conventional sample schedule that is intuitively used by experts. The optimal input criterion for each $v$th output is shown in (21).
If the identifiable parameters are biased, (17) cannot be used to estimate the variance of the parameters. In this case, the relative worst case bias technique has been used as measure of parameter accuracy, as discussed in the last paragraph of section 4.4.

4.5.2 Optimal Sample Schedule

The parameter covariance matrix $\Delta(\theta_{est})$ also depends on the used sample schedule according to the inverse of the Fisher information matrix (17).

The optimal sample schedule for each output is determined by maximizing the determinant of the Fisher matrix (17) (D-optimality) (unbiased and minimum variance estimator) for each sample taken ($t_v$) if the identifiable parameters are unbiased. The optimal sample schedule criterion for each $v$th output is shown in (22).

\[
\begin{align*}
\text{MIN}_{t_v}\{\text{det}[\Delta(\theta_{est},u(t))]} & \\
= \text{MAX}_{t_v}\{\text{det}[F(\theta_{est},u(t))]\} & = \text{MAX}_{t_v}\left\{\text{det}\left(-\sum_{m=1}^{M} E_{y_m} \frac{\partial^2 \ln(L(y_m|\theta_{fix},\theta_{est},u(t)))}{\partial \theta_{est} \partial \theta_{est}^T}\right)\right\} \\
= \text{MAX}_{t_v}\left\{\text{det}\left(\frac{1}{N_v} \sum_{i=1}^{N_v} \frac{1}{2\sigma^2} \frac{\partial (y_{m}(t_{iv},\theta_{fix},\theta_{est},u(t)))^2}{\partial \theta_{est}} \frac{\partial (y_{m}(t_{iv},\theta_{fix},\theta_{est},u(t)))^2}{\partial \theta_{est}^T}\right)\right\} \\
\end{align*}
\]

If the identifiable parameters are biased the Fisher information matrix cannot be used. In this case the relative worst case bias, as stated in the last paragraph of section 4.4, has been used to provide the variance of the parameter estimations.

Several algorithms are based on starting with a conventional sample schedule (the sample schedule which is used normally) where each individual sample is optimized by comparing it with its neighbouring samples in the conventional sample schedule. If by taking a neighbouring sample, the determinant of the Fisher matrix is increased, this new sample is exchanged with the old one [Mori 1979], [Landaw 1982], [Xianjin 2000].
The algorithm developed in this study starts at the first sample of the conventional sample schedule searching for the first sample within a fixed horizon of samples that increases the precision. This sample is exchanged with the old one. This process is repeated until the maximal duration of the experiment is achieved. If after finishing, the optimal sample schedule contains too many samples than ethical/financial possible, the fixed horizon of samples is increased. Changing the horizon of samples is stopped when the optimal sample schedule contains the desired number of samples.

The search horizon was taken arbitrary to reduce the number of samples taken: twice the time needed to measure interstitial glucose. The optimal samples have been selected in the range (domain) of the conventional samples

The outputs do not have to be sampled at the same time points. The samples are separated by at least the time needed to take a measurement of the output. Measurement of $G(t)$ takes much less than one minute and measurement of $G'(t)$ takes thirty minutes.

Results

5.1.1 Model Structure in State Space

The new oral glucose minimal model 1, valid for insulin concentrations in a physiological range (up to 600 pM), is a multi input multi output (MIMO) nonlinear dynamic system consisting of a four state-vector (26-29) having one independent input, one dependent input and two outputs. The independent input is $DS [g]$, the dependent input $I(t) [pM]$ and the outputs (30-31) are $G(t, \theta_i) [mM]$ and $G'(t, \theta_i) [mM]$ a mean over each 30 minutes as stated in section 4.1.2. The dependent input could become an independent input if simulated using the $\beta$-cell insulin secretion models which are list in Appendix B. The outputs are sampled at discrete time $t_i$ with $N_G$, $N_C$ samples, respectively, as shown in (32, 33) were $G_m[t_j, \theta_{jst}, \theta_{est}]$ is the modelled quantized glucose output and $G'_m[t_j, \theta_{jst}, \theta_{est}]$ is the modelled quantized mean interstitial glucose output over each 30 minutes. Due to measurement restrictions, the minimum time intervals of samples of $G(t, \theta_i)$ and $G'(t, \theta_i)$ were taken as 1 minute and 30 minutes, respectively. $e_v$ is the measurement error, assumed to be additive zero mean white noise with known variance $\sigma_v^2$ for output $v$. The remaining parameters are $G_b [mM]$ which is the basal glucose concentration, $I_b [pM]$ the basal insulin concentration, $\sigma_i^2$ the variance of the measurement noise and $\sigma_2^2$ the variance of the measurement noise.

\[
\begin{align*}
\dot{G}(t) &= -[S_{I_{-\text{tireg}}} + p4] \cdot [G(t) - G_b] \cdot [I(t) - I_b] + p5 + S_{G} \cdot G(t) \cdot \frac{Kabs \cdot G_{\text{glu}}(t)}{V} \\
G(0) &= G_b \\
\dot{I}'(t) &= -p2 \cdot I'(t) + S_{I_{-\text{cap}}} \cdot [I(t) - I_b] \\
I'(0) &= 0 \\
\dot{G}'(t) &= -S_{I_{-\text{glu}}} \cdot I'(t) \cdot G'(t) + p4 \cdot [G(t) - G_b] \cdot [I(t) - I_b] \\
G'(0) &= G'_b
\end{align*}
\]
\[ G_{gw}(t) = DS \cdot k \cdot \beta \cdot e^{(-k \cdot t)} - Kabs \cdot G_{gw}(t) \]
\[ y_i(t, P_i) = G(t, \Theta) \]
\[ y_2(t, P_i) = G'(t, \Theta) \]
\[ y_{1i}[t] = G_m[t_i, \theta_{fit}, \theta_{ext}] + e_{i}[t_i] \]
\[ y_{2j}[t_j] = G'_m[t_j, \theta_{fit}, \theta_{ext}] + e_{j}[t_j] \]

\[ G_{gw}(0) = 0 \] (29)
\[ (30) \]
\[ (31) \]
\[ (32) \]
\[ (33) \]

5.1.2 Simulation

Synthetic datasets are produced by simulating the model states space representation in Matlab using parameters found in literature for healthy non-obese humans and measured insulin input profile from [Wagenmakers 2006]. Sixth datasets were produced. Details of the datasets and plots used are shown in appendix C. Plots of the simulated glucose and insulin plasma and interstitial glucose data are also found in appendix C.

The parameters \( k, Kabs, \beta, \) and \( DS \) related by the gut were estimated in [Natalucci 2003] and [Schirra 1996] as stated in section 3, the parameters \( p_2 \) and \( S_liver \) related by the liver were estimated by the two compartment minimal model (using a non-accessible interstitial glucose compartment, but having similar behaviour as a accessible interstitial glucose compartment) [Dalla Man 2002] and [Caumo 1993], the parameters \( p_2, p_6 \) and \( V \) were estimated by the (improved) hot IVGTT two compartment minimal model [Toffolo 2003], the parameter \( k_7 \) was estimated by [Regittnig 2003] using high frequently sampled ISF (openflow microperfusion) which uses a mathematical model [Steil 1996] to describe transendothelial transport of glucose form blood plasma to remote interstitium and finally \( k_2 \) was estimated by experimental data of [Sjöstrand 2005] using low frequently sampled ISF (micродialysis) and a mathematical model [Steil 1996] to describe transendothelial transport of insulin form blood plasma to remote interstitium. Mean values and variances are shown in table 1. No a priori information of the parameters \( k_8, k_9 \) and \( S_lglut4 \) were found in literature. Parameters \( k_8 \) and \( k_9 \) have been, to the authors knowledge, never been estimated. Since the product of \( k_7/k_2, k_8/k_9 \) and \( I(t) \) must be in the range of the transport from blood plasma to interstitium \( k_7/k_8 \), these parameters were first settled roughly, proportional to the inverse of the concentration of insulin to maintain the same range of transport. Secondly, parameters \( k_8 \) and \( k_9 \) were adjusted by trial and error producing a possible dataset of glucose kinetis. \( S_lglut4 \) was totally unknown and was set by the trial and error method just discussed.

<table>
<thead>
<tr>
<th>Kabs</th>
<th>Beta</th>
<th>( k )</th>
<th>( v )</th>
<th>( k_9 )</th>
<th>( p_2 )</th>
<th>( S_l-cap )</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.89 ( \times ) 10(^{-3} ) ± 0.52 ( \times ) 10(^{-3} )</td>
<td>1.23 ± 0.09</td>
<td>0.014 ± 0.001</td>
<td>1.386 ± 3.7 ( \times ) 10(^{-2} ) ( \text{III} )</td>
<td>0.113 ± 0.023</td>
<td>( k_2 \cdot k_9 )</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>( S_G )</th>
<th>( k_2 )</th>
<th>( k_8 )</th>
<th>( S_l-glut4 )</th>
<th>( k_7 )</th>
<th>( S_l-liver )</th>
<th>( p_4 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.018 ± 0.006</td>
<td>0.0395 ± 0.008</td>
<td>3.0 ( \times ) 10(^{-2} ) ( \text{I} )</td>
<td>1.0 ( \times ) 10(^{-2} ) ( \text{I} )</td>
<td>1.57 ( \times ) 10(^{-6} ) ( \text{m} )</td>
<td>1.08 ( \times ) 10(^{-5} ) ( \text{m} )</td>
<td>( k_7 \cdot k_8 )</td>
</tr>
</tbody>
</table>

Table 1: Overview of the parameter value used in the new oral glucose minimal model 1 to simulate the sixth datasets (appendix C)

1 Conversion factor between [ml/min 1/\( \mu \text{L} \)] to [1/min 1/\( \mu \text{M} \): 1/7.0625. 2 Conversion factor between [dl/mg] to [1/min 1/mM]: 180.16 1/mM. 3 Ratio between distribution volume of blood plasma and interstitial fluid.
5.2 Identifiability

In the new oral glucose minimal model 1 two outputs are measured namely, measurement of plasma glucose $G(t)$ and interstitial glucose $G'(t)$. A restriction exits for measurements of interstitial glucose $G'(t)$, as only a mean measurement is available over each 30 minutes ($\bar{G}'(t)$) as stated in section 4.1.2. This restriction is taken into consideration by calculating the mean of $G_m(t)$ over 30 minutes using the parameters $\theta_i$ in the oral minimal glucose model. The mean is calculated by summing up the values of $G_m(t)$ with intervals set to one-fifth of the smallest experimental time constant used in the model and dividing it by the number of intervals.

5.2.1 A Priori Global Identifiability

A priori global identifiability is done for the new oral glucose minimal models 1, 2 and 3. First the manual check is performed to verify if unknown parameters are not observable when dealing with non-generic initial conditions. Fortunately all parameters are observable. Table 2 shows the identifiable parameters.

<table>
<thead>
<tr>
<th>Identifiable parameter sets</th>
</tr>
</thead>
<tbody>
<tr>
<td>model 1</td>
</tr>
<tr>
<td>model 2</td>
</tr>
<tr>
<td>model 3</td>
</tr>
</tbody>
</table>

Table 2: Overview of the a priori global identifiability parameters for models 1, 2 and 3

A remark can be made about the identifiability of $S_1$-cap. $S_1$-cap can be identified separately if the interstitial insulin $G'(t)$ profile is measured and $p_2$ is known a priori. Unfortunately, measurements of interstitial insulin $G'(t)$ are not available. However, the interstitial insulin profile can be estimated after estimating $S_1$-glut. In this case the variance of $S_1$-cap depends on a summation of two parts. One part is its own variance when estimating it from the measured plasma insulin profile and estimated interstitial insulin. The other part is non linear and depends on the variance of $S_1$-glut.

5.2.2 A Posteriori Identifiability

The cost function of the new oral glucose minimal model 1 is evaluated for the plasma glucose output signal and the interstitial glucose output signal. The cost function is shown in (34).

$$J(\theta) = \frac{1}{N_G} \sum_{i=1}^{N_G} (G(t_i) - G_m(t_i, \theta))^2 + \frac{1}{N_{G'}} \sum_{i=1}^{N_{G'}} (G'(t_i) - G_m'(t_i, \theta))^2$$

(34)

For each combination of the two dimensions in the parameter space, the contour plot of the error function is plot for model 1 (DS=75 gram), 2 (DS=72 gram, labelled ingested glucose 3
gram) and 3 (DS=75 gram, enrichment labelled glucose 3%) with variations of the parameter \( \theta \), of \( +/- 50 \% \) using steps of 5%. The contour lines for \( J(\theta) \leq 10 \) are plot at 0.01 distance from each other. For \( J(\theta) > 10 \leq 200 \), the contour lines are plot at 5.00 distance from each other. The contour plots also include estimation results indicated by red circles. (The true value is indicated with a red plus-sign.) The a posteriori identifiability is examined for the first and the second data set described in section 5.1.2 / appendix C with added white noise having a \( 1/SNR \) of 0 % and 5%. Plots, not all, are shown in Appendix D.

Appendix D shows for model 1 using dataset 1, that the contourplots for all possible combinations of parameters \( k_{abs}, P4, S_{rglut4} \) and \( Gb \) are narrow and steep centered at the supposed true values of the two parameters (global minima). In contrast, parameter \( Ib \) shows for the same combinations of parameters in almost all case heavily stretched ellipses indicating poor-identifiable parameters. Parameter \( v \) shows stretched ellipses for the same combinations with \( k_{abs} \) and \( S_{rglut4}, S_{r-liver} \) and all other parameters not included in appendix D.1 or D.2 show heavily stretched ellipses for combinations with \( k_{abs}, p4, \) and \( S_{rglut4} \). The same holds when a different insulin profile (dataset 2) is used.

Model 2 is examined by the model 2 model adjustment to know whether ingesting labelled glucose using the same experimental setup improves identifiability of \( S_{r-liver} \). However, \( S_{r-liver} \) show heavily stretched ellipses for combinations with \( k_{abs}, p4, S_{rglut4} \) and \( Gb \). Although model 3 shows less stretched ellipses than model 2 for combinations with \( k_{abs}, p4, S_{rglut4} \) and \( Gb, S_{r-liver} \) remains a poor-identifiable parameter.

Although a priori global identifiability shows identifiability of \( beta \) and \( k \) in all possible combinations with \( k_{abs}, p4, S_{rglut4} \) and \( Gb \) in model 3, stretched ellipses are shown for all possible combination in appendix D.3.

In all cases that no noise is added, parameters are estimated exactly (unbiased parameters). By adding a \( 1/SNR \) of 5% white noise to the output signal, bias was introduced in the estimation of parameters. However, parameter estimations are all close to the exact value (not shown in appendix D).

### 5.2.3 A Priori Parameters Sensitivity

The a priori parameters sensitivity is calculated using the first and the second data set described in section 5.1.2 / appendix C with added white noise having a \( 1/SNR \) of 0%, 5% and 10%, respectively. The a priori parameters are varied one by one by \( \pm 5, 10, 20 \) and 30 %. The initial values of the identifiable parameters used in the estimation algorithm are set to those which showed largest bias in the relative worst case bias test in the case of a \( 1/SNR \) of 0% white noise (section 5.3). The results shown in appendix E, shows the best possible variance for each identifiable parameter when the a priori parameters varies form the true value used.

Appendix E shows that for all estimated parameters, \( Ib, Gb, beta \) and \( k \) are highly sensitivity to variations of the true value among population. Parameter \( p2 \) only shows highly sensitivity for parameter \( S_{rglut4} \). The remaining parameters \( p5, S_c \) and \( S_{r-liver} \) are insensible. An exception occurs for \( S_{r-liver} \) which shows a quite high sensitivity when white noise is added with a \( 1/SNR \) of 10%.
5.3 Parameter Estimation

The cost function for ML is shown in (35) where $\sigma_G^2, \sigma_{G'}^2$ are the variance of measurement errors in plasma glucose $G(t)$ and interstitial glucose $G'(t)$.

$$J(\theta, i) = \frac{\sum (G(t_i) - G_{\text{true}}(t_i, \theta_{\text{fix}}, \theta_{\text{est}}))^2}{2\sigma_G^2} + \frac{\sum (G'(t_i) - G'_{\text{true}}(t_i, \theta_{\text{fix}}, \theta_{\text{est}}))^2}{2\sigma_{G'}^2}$$

A priori information is available about the quality of the measurements of glucose and insulin plasma and interstitial glucose specified by the manufacturer of the measurement equipment or the lab where the samples are analyzed as stated in section 4.1.

The relative worst case bias is first calculated using the first and the second data set described in section 5.1.2 / appendix C with added white noise having a I/SNR of 0%, (2.5%), 5% and 10%, respectively. The initial values of the identifiable parameters used in the estimation algorithm are varied between $\pm 50\%$ using steps of 10%. The initial values of the identifiable parameters in the case of a I/SNR of 0% white noise which showed largest relative worst case bias are for the first dataset $k_{\text{abs}}, v, p_4, S_{\text{RCap}}, S_{\text{RGlut4}}$ and for the second dataset $0.5 \cdot k_{\text{abs}}, 0.5 \cdot v, 0.5 \cdot p_4, 0.5 \cdot S_{\text{RCap}}$ and $0.6 \cdot S_{\text{RCap}}$.

Secondly, the relative worst case bias is calculated using the third, fourth, fifth and sixth data set described in section 5.1.2/ appendix C with added white noise having a I/SNR of 0%, 5% and 10%, respectively. The results are shown in datasets third and fourth in table 3 and for datasets fifth and sixth in table 4.
Table 4: Overview of the worst case bias of all identifiable parameters using synthetic data for two different insulin profiles and two different sample schedules (data sets 3, 4, 5 and 6, respectively, appendix b) with added white noise having a I/SNR of 0, 5 and 10%.

Table 4 shows for all estimated parameters a higher variance for each estimated parameter than by using the first and second dataset, but still acceptable. However, table 4 shows a lower variance for all estimated parameters in the case of a I/SNR of 0% and 5% than by using the third and fourth dataset but still higher than by using the first and second dataset.

5.4 Optimal Experiment Design

5.4.1 Optimal Input Shaping

The optimal dosis was calculated by varying the input dose $DS$ in the range of 1 gram to 100 gram in steps of 1 gram using the first and the second data set described in 5.1.2/ appendix C with added white noise having a I/SNR of 0%, 5% and 10%, respectively. The initial values of the identifiable parameters used in the estimation algorithm were set to those which showed largest bias in the worst biased test in the case of a I/SNR of 0% white noise (section 5.3).

If the relative worst bias of each estimated parameter was lower than 10% and did not increase for an increase in glucose dosis, this minimum glucose input was said to be optimal. The results are shown in table 5.

Table 5: Optimal dosis using synthetic data for two different insulin profiles (data sets 1, 2, respectively, appendix B) with added white noise having a I/SNR of 0, 5 and 10%.
6 Discussion and Suggestion

The goal of this study was to investigate and validate a new oral glucose minimal model describing jointly the effects of Glut-4 translocation, the recruitment of muscle capillaries and the effects of impairments of the inhibition of hepatic glucose production using existing minimal models in literature and modifying them using recent insights in the mechanism by which insulin regulates glucose uptake by muscle cells or adipose tissues. The new oral glucose minimal model was to be identified in all its aspects (a priori and a posteriori identifiability, a priori parameter sensitivity and to perform optimal experiment design).

Three oral glucose minimal models have been developed. The first model was developed to estimate the rate constant of glucose absorption from the gut into blood plasma \( (k_{abs}) \), to estimate the efficacy of insulin to stimulate its own transport from blood plasma into interstitium by capillary recruitment \( (S_{1-cap}) \), to estimate the efficacy of insulin to stimulate glucose transport from blood plasma into interstitium \( (p_4) \), to estimate the efficacy of interstitial insulin to stimulate glucose clearance into muscle by Glut-4 translocation \( (S_{1-glut4}) \) and finally, to estimate the plasma glucose distribution volume per unit of body weight \( (v) \). The second model was developed to estimate the previous parameters, but also to estimate the efficacy of insulin to inhibit hepatic production and stimulate glucose uptake by liver \( (S_{1-liver}) \) by including ingested labelled glucose. The third model was an alternative for the second model since model 3 enriches blood plasma with labelled glucose through injecting labelled glucose instead of ingesting labelled glucose.

After developing the three oral glucose minimal models, a priori global identifiability analysis proved that with the models structure used, these estimations were possible. However, more information was acquired when choosing model 3, since model 3 could also estimate \( \beta \), the variable of the power exponential curve denoting the velocity of emptying of the gut. This was obvious since extra data was available to estimate \( v \) by the infusion of labelled glucose.

A posteriori identifiability analysis showed, for all possible combinations of parameters \( k_{abs}, p_4, S_{1-glut4} \) and \( Gb \), contour plots with narrow ellipses that were steep centered at the supposed true values of the parameters (global minima) using two different insulin profiles. Since \( Gb \) could be measured, this parameter was set a priori in order to not influence negatively the parameter estimation of the other unknown parameters. Parameter \( v \) showed stretched ellipses when estimated together with parameters \( k_{abs} \) or \( S_{1-glut4} \) indicating low accuracy and less reliability as was seen when the relative worst bias was calculated. A problem occurs if parameter \( S_{1-liver} \) was estimated since all three models showed heavily stretched ellipses for combinations of \( k_{abs}, p_4, \) and \( S_{1-glut4} \). Heavily stretched ellipses indicate non-identifiable parameters. Parameter \( S_{1-liver} \) was non-identifiable because \( p_4 \gg S_{1-liver} \) in (26).

Further, the a priori parameters sensitivity was calculated using two different insulin profiles. All estimated parameters, \( lb, Gb, \beta \) and \( k \) showed highly sensitivity to variations of the true value among population. Since \( lb \) and \( Gb \) could be measured, this was not a problem. On the other hand, high a priori parameter sensitivity of parameters \( \beta \) and \( k \) was a real problem. A priori global identifiability showed identifiability of either \( \beta \) or \( k \), however, analyzing the a posteriori identifiability rejected this possibility. Good estimates of \( k_{abs} \) and \( k \) are available in literature [Schirra 1996], however, study [J.D. Elashoff 1982] shows that humans who have undergone an operation in the stomach can be modelled by changing parameter \( \beta \). Hence, variations of \( \beta \) among populations exist. \( \beta \) or \( k \) could be estimated by measuring the content of glucose in the gut or by using high(ter) concentrations of labelled glucose. Parameter \( p_2 \) showed only highly sensitivity for parameter \( S_{1-glut4} \). Estimates of \( p_2 \) are
available in literature e.g. [Toffolo 2003]. The bias of $S_{\text{glut4}}$ could be lowered by using the open-flow microperfusion technique [Bodenlenz 2005] instead of the microdialysis technique for measurements of interstitial glucose $G_m(t)$. The open-flow microperfusion technique makes faster sampling of interstitial glucose $G_m(t)$ possible than the microdialysis technique. The remaining parameters $p_5, S_G$ and $S_{\text{liver}}$ were insensible for variation of the a priori identifiable parameters. An exception occurred for $S_{\text{liver}}$ that showed a quite high sensitivity when noise was added with a $1/\text{SNR}$ of 10% to the output signals. However, a maximal $1/\text{SNR}$ of 5% was present in the measured outputs using the microdialysis technique.

A remark could be made about the identifiability of $S_{\text{1-cap}}$. $S_{\text{1-cap}}$ could be identified separately if the interstitial insulin $G'(t)$ profile was measured and $p_2$ was known a priori. Unfortunately, measurements of interstitial insulin $G'(t)$ were not available. However, the interstitial insulin profile could be estimated after estimating $S_{\text{glut4}}$. In this case the variance of $S_{\text{1-cap}}$ depends on a summation of two parts. One part is its own variance after estimating it from both the measured plasma insulin profile and the estimated interstitial insulin. The other part is non-linear and depends on the variance of $S_{\text{glut4}}$.

The studied identifiability agreed with the found relative worst case bias. A relative worst case bias of maximal 6.5% was achieved for all estimated parameters using a $1/\text{SNR}$ of 5% added white noise. If a $1/\text{SNR}$ of 10% white noise was added, a maximal relative worst case bias of 13% was achieved. If no noise was added, a maximal relative worst case bias of 3.76% was achieved indicating bad working of the minimalization algorithm used to minimize (15). Important to note is that measurements of interstitial glucose $G'(t)$ had been undermodelled.

The interstitial glucose $G'(t)$ samples were modelled by calculating the mean of $G_m(t)$ over 30 minutes by summing up the values of $G_m(t)$ with intervals set to one-fifth of the smallest experimental time constant used in the model and dividing it by the number of intervals. In fact, subsequent samples of interstitial glucose $G_m(t)$ are influenced by each other since the samples are collected at the outlet of the catheter without stopping the inlet of new interstitial glucose, causing mixing of subsequent interstitial glucose samples. This effect was not taken into account since the behaviour of mixing is still unknown.

The optimal dosis was calculated by varying the input doses $DS$ in the range of 1 gram to 100 gram in steps of 1 gram using two different insulin profiles and using sequences of added white noise with a $1/\text{SNR}$ of 0%, 5% and 10%. A complication occurred when $DS$ was varied. If $DS$ vary, this also implies a different insulin profile. However, the same insulin profile was used during all variations of $DS$ since an insulin profile of glucose ingestion other than 75 or 100 gram was not available. It is not common to have such an OGTT. This complication set doubts to the obtained results. This minimum glucose input was said to be optimal if the relative worst bias of each estimated parameter was lower than 10% and did not increase for an increase in glucose doses. For each $1/\text{SNR}$ a different optimum glucose input was obtained. For a $1/\text{SNR}$ of 0% of added white noise, all glucose doses were optimal. For a $1/\text{SNR}$ of 5% of added white noise an optimal glucose doses of 33 gram was obtained and for a $1/\text{SNR}$ of 10% of added white noise, 78 gram was obtained. It could be concluded that the doses of glucose should be as high as possible.

Due to limited time issues, the optimal sample schedule algorithm was not applied. Instead of this, a different test was applied: The relative worst case bias was calculated for two extra
different sample schedules (appendix C). The first extra sample schedule (the conventional sample schedule) is used as standard in many studies [Sjostrand 2005], [Wagenmakers 2006]. For the second extra sample schedule, a new sample schedule was developed which takes samples of glucose and insulin plasma during events when most changes take places in the insulin profile. This is often, during the first 60 minutes after glucose ingestion. This second extra sample schedule showed a lower variance for all estimated parameters in the case of a 1/SNR of 0% and 5% of added white noise than using the first extra sample schedule. This is obvious, because the second extra sample schedule has samples of glucose and insulin plasma during events when the most changes takes places in the insulin profile. Hence, it is very interested to find the optimal sample schedule to obtain the lowest possible relative worst bias for the estimated parameters. In both cases, the relative worst case bias was higher than the best relative worst case bias calculated in section 5.3 because fewer samples are taken in both extra sample schedules. It could be discussed if an optimal samples schedule could be found, since great variety exits among measured glucose profiles.

A remark could be made about the optimal sample schedule algorithm developed here to calculate the optimal sample schedule. The optimal sample schedule could use the Fisher information matrix as indication how optimal a samples schedule is. At the start of the algorithm a sample schedule with many samples is available which showed, as in section 5.3, low relative worst case bias. However, as the algorithm progress, the current optimum sample schedule has fewer samples than the initial sample schedule, introducing more bias in the estimation of parameters. Hence, the conditions of Fisher information matrix are not satisfied, and the relative worst case bias technique must be used instead. A drawback of the relative worst case bias technique is the processing time which is extremely large. But if the search of the optimum sample schedule progress well, the sample schedule should always show low bias and the Fisher information matrix could be used. A dilemma that must be investigated, before using the optimal sample schedule algorithm.

A different remark can be made about the choice of first calculating the optimal input doses before searching for the optimal sample schedule or quite the reverse. Since the optimal samples is selected in the range (domain) of the conventional samples, it is obvious to calculate first the optimal input doses for the conventional samples and then to reduce this set of samples obtaining the optimal sample schedule.

7 Conclusion

The goal of this study was to investigate possible impairments of insulin resistance in type 2 diabetes. Model 1 could be used to investigate whether there are impairments in Glut-4 translocation and impairments in the recruitment of muscle capillaries during physiological insulin concentrations that. The model did not take into account insulin production by the pancreas, instead of this the insulin concentration profile in blood plasma was measured which insulin was produced by the pancreas. Model 1 could not discriminate impairment of hepatic glucose production.

The following parameters can be estimated using model 2 and measuring glucose blood plasma, insulin blood plasma, basal glucose level, basal insulin level and interstitial glucose using the microdialysis technique: The rate constant of glucose appearance from the gut into blood plasma, the efficacy of insulin to stimulate its own transport from blood plasma into interstitium by capillary recruitment, the efficacy of insulin to stimulate glucose transport from blood plasma into interstitium, the efficacy of interstitial insulin to stimulate glucose
clearance into muscle by Glut-4 translocation and the plasma glucose distribution volume per unit of body weight. This was the first time that these parameters were estimated jointly to understand and quantify different potential causes (mechanism of Glut-4 translocation and recruitment of muscle capillaries) of insulin resistance in type 2 diabetes during an OGTT using a physiological model of glucose appearance in blood plasma. This model refines earlier developed models.

Using a minimum glucose doses of 78 gram during an OGTT and having a 1/SNR of 10%, parameters were estimated with a maximum variance of ±13% if each minute a sample of insulin and glucose blood plasma and each 30 minutes a sample of interstitial glucose were taken during the first 240 minutes after glucose ingestion. If the 1/SNR is 5%, parameters were estimated with a maximum variance of ± 6.5%. An exception holds for the parameter which denotes the efficacy of insulin to stimulate its own transport from blood plasma into interstitium by capillary recruitment which can be slight higher. This is due to the fact that this parameter also depends in a still unknown non-linear way on the precision of the efficacy of interstitial insulin to stimulate glucose clearance into muscle by Glut-4 translocation. If fewer samples are taken, it is advised to take samples when most dynamics takes places in glucose kinetics or to use the optimal sample schedule algorithm developed in this study.

All a priori identifiable parameters used in model 1 except beta and k are insensitive for the estimated parameters when having a 1/SNR of 5%, 5% and 10% of white noise and using model 1 and the experimental setup described in this study. Varying the true value of all the a priori identifiable parameters (except beta and k) found in literature with ± 30% in steps of 5% resulted in a maximum variance of ±12% in the estimation of the identifiable parameters. On the other hand beta and k showed a maximum variance of respectively 241% and 195%. Although, having this high sensitivity for the a priori identifiable parameters beta and k, the description of glucose appearance with model 1 is better than earlier developed models found in literature. The earlier developed models were based on interpolating glucose appearance in blood plasma instead of using physiological parameters to model glucose appearance in blood plasma.
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Modelling the insulin transport from blood plasma through the endothelium to reach the interstitium.

Several studies have studied the mechanism of insulin transport from blood plasma through the endothelium to reach the interstitium concluding conflicting results: a saturable receptor-mediated or diffusion transport. A saturable receptor-mediated is possible since [R.S. Bar 1978] found receptors on cultured human endothelial cells obtained from human umbilical veins and [M.L. Peacock 1982] found receptors on cultured and isolated bovine endothelial cells obtained from pulmonary and systemic vessel.

Some in vivo studies have measured the insulin concentrations in the interstitium with the microdialysis technique [Lonnroth 1987, Lonnroth 1997] in human skeletal muscle of conscious healthy lean humans with no medication in (semi) supine position during an OGTI [H. Herkner 2003, M. Sjostrand 2005] and during a CLAMP [M. Sjostrand 1999, P.E. Jansson 1993]. Other studies have measured insulin concentration using the peripheral lymphatic cannulation technique [Castillo 1991] in hindlimb lymph of conscious healthy humans with a wide range of obesity and with no medication in (semi) supine position during a CLAMP [C. Castillo 1994] and with anesthetized healthy dogs during a CLAMP [M. Hamilton-Wessler 2002, G. M. Steil 1996]. Another study has measured insulin concentration in thoracic duct lymph with conscious healthy dogs during a CLAMP [Y.J. Yang 1989]. (In all lymph based studies, lymph fluid concentration is increased by massaging or saline injection.) No difference exists between the two different measurements sites, since [Y.J. Yang 1989, C. Castillo 1994] showed strong correlation and direct proportionality between insulin concentration in the hindlimb lymph near the human skeletal muscle and glucose uptake by human skeletal muscle. Only a delay time exits between insulin in the interstitium and insulin in hindlimb lymph due to the transport time needed between both places. From this, it can be assumed that insulin concentration in interstitium and hindlimb lymph also shows strong correlation and direct proportionality.

Saturable receptor-mediated transport of insulin can be shown by an increase of insulin gradient (or decrease of insulin ratio) between blood plasma and interstitium/lymph at higher insulin concentration. After all, an increase of insulin gradient (or decrease of insulin ratio) implies a decrease in insulin transport indicating a saturable receptor-mediated transport of insulin. But since changes in clearance of insulin by muscle cells affect the gradient, it is critical to assess this parameter if an increase in the gradient is to be interpreted as a decrease in transport. However, studies [Y.J. Yang 1989, C. Castillo 1994, H. Herkner 2003] show a strong correlation between glucose uptake and insulin concentrations in interstitium/lymph concluding that no saturation occurs in clearance of insulin by muscle cells. Hence, analyzing this gradient is significant for the mechanism of insulin transport through the endothelium.

After analyzing the discussed gradient in experimental data of several in vivo studies an interesting conclusion is drawn. The studies using insulin concentrations at physiological range (up to 600 pM in blood plasma) [Y.J. Yang 1989, H. Herkner 2003, C. Castillo 1994, M. Sjostrand 2005] show a decreased gradient indicating saturable receptor-mediated transport. [P.E. Jansson 1993] shows at insulin concentrations of 3500 pM a decreased gradient indicating saturable receptor-mediated transport. The studies [G. M. Steil 1996, M. Hamilton-Wessler 2002] using insulin concentrations at pharmacologic range (more than 5000 pM in blood plasma) show an increased gradient indicating diffusion transport. As a unification of both conclusions, a saturable receptor-mediated insulin transport is considered under physiological conditions whereas a non-saturable insulin transport is considered at pharmacologic insulin concentrations.

In vitro study (cultured bovine aortic endothelial cells with the use of dual chamber, not exposed to a flow) [L. King 1985] at physiological range concludes a receptor-mediated transport of insulin through the endothelium. Another study in vitro (endothelial cells from rat hearts) [F. Brunner 1998] at physiological range concludes a diffusion process. However, [M.A. Vincent 2005] shows a different behavior of the endothelium of rats compared with the endothelium of humans. The insulin
concentration ratio between interstitium and blood plasma is around 80% in contrast to humans 40-50% [M.A. Vincent 2005, M. Sjöstrand 1999] or dogs 35-50% [M.A. Vincent 2005]. Hence, the endothelium in rats is not a good representation of the endothelium in humans. In vitro study (cultured endothelial cells in a hollow fiber apparatus and continuously exposed to a flow) [F. Salvetti 2002] at physiological range shows a decreased gradient at higher insulin concentrations. Since these studies are all in vitro, the conclusions drawn from them are taken less seriously than in vivo studies. However, the conclusions are in agreement with in vivo conclusions.

Hence, the classic minimal model of Bergman [Bergman 1979] is only valid using insulin concentrations at physiological range. This is always the case during an OGTT.

Another interesting case to study are possible delays. A significant delay about 20 minutes in the activation phase of transporting insulin from blood plasma to interstitium is seen in vivo studies [M. Sjöstrand 2005, M. Sjöstrand 2002, C. Castillo 1994, Y.J. Yang 1989, E. Jansson 1993] and in vitro study [F. Salvetti 2002]. The cause of this delay is not known yet. No delay is seen in the deactivation phase of transporting insulin from blood plasma to interstitium [Y.J. Yang 1989]. A small delay is seen between glucose uptake and lymph insulin for insulin binding to its cognate cell receptor [Y.J. Yang 1989].
Modelling insulin that is secreted by pancreatic β-cells via the liver into the portal vein.

The ability to control glucose production and utilization depends on insulin that is secreted by pancreatic β-cells via the liver, where insulin is decreased (extracted) by approximately 50%, into the portal vein. When the concentration of glucose in plasma increases rapidly, insulin is secreted by pancreatic β-cells in two phases: the first and second phase. The first 10 minutes (first phase) is a fast rise of insulin secretion. The next 20 minutes (second phase) is a gradually rise of insulin secretion which is directly related to the degree and duration of the stimulus [Caumo 2004]. Between the 70s and 80's several β-cell insulin secretion models were developed based on measurement of plasma insulin concentrations or measurement of c-peptide. These models can be added to the new oral glucose minimal model discussed in section 3 to estimate the plasma insulin profile instead of measuring the plasma insulin profile. Adding one of these models into the new oral glucose minimal model discussed in section 3 is also a closer step into a future automated portable continuous glucose monitoring system. First, models based on measurements of plasma insulin concentrations are given and secondly, models based on c-peptide measurements are given.

Toffolo proposed eight models tested on conscious healthy lean dogs by an IVGT [Toffolo 1980] using the minimal model [Bergman 1979]. The models assume that clearance of insulin is of the first order, the initial insulin peak represents a bolus loaded into insulin plasma after the glucose injection and the rate of the secondary rise in insulin is determined by the concentration of glucose in plasma above a specific threshold value. This model has the disadvantage that parameter estimation has to be done in two steps. First the insulin concentration is used as input data to estimate the parameters of the minimal model [Bergman 1979], secondly the recorded glucose data is used as input for the β-cell insulin secretion model. However, by the single-step fitting process [Zheng 2005] this handicap is resolved.

This proposed minimal model by Toffolo in 1980 is tested on healthy lean and obese humans by an IVGT [Bergman 1981] showing strong correlation with [Toffolo 1980]. This study also shows that lean lower tolerance humans is related to 77% diminished second phase responsivity (S2 remained unchanged) and that obese lower tolerance humans 60% diminished S2 (second phase remained unchanged).

Zheng introduced a modification of Toffolo’s model [Zheng 2005] by the assumption that the insulin decay rate is not always a first-order process showing improved fitting (97.7%) than Toffolo’s model in 1980. He also added a mathematical function for describing the endogenous insulin infusion rate. The model is shown in (b1) where \( \gamma \) [pmol·min] is the rate of pancreatic release of insulin after the glucose injection, \( h \) [mM] is the pancreatic threshold value, \( t \) [min] is the time, \( n \) [(pmol)⁻¹·min⁻¹] is the plasma insulin decay rate constant, \( u(t) \) [mU/μg·min] is the endogenous insulin infusion rate, \( m \) stands for the \( m \)-th order processes and \( k \) [pM] is the initial plasma insulin concentration after the glucose bolus injection. Due to the assumption that that the insulin decay rate is not always a first-order process equation is modified by (b2).

\[
I(t) = \begin{cases} 
\gamma \cdot [G(t) - h] \cdot t - n \cdot [I(t) - I_b] + u(t)/V & \text{if } G(t) > h \\
-n \cdot [I(t) - I_b] + u(t)/V & \text{if } G(t) \leq h 
\end{cases} 
I(0) = k + I_b \quad (b1)
\]

\[
I'(t) = -p2 \cdot I'(t) + p2 \cdot S_I \cdot [I(t) - I_b] \quad I(0) = 0 \quad (b2)
\]
Steil introduced a new model [Steil 2003] which assumes a proportional component that reacts to the difference between plasma and basal glucose ($P$), an integral component that adjusts basal delivery in proportion to hypo/hyperglycemia ($h$) and a derivative component that responds to the rate of change in glucose plasma ($D$). The model is test during a CLAMP on healthy humans showing good fitness. Furthermore, this model is especially designed for closed loop systems linking a glucose sensor with an insulin pomp (an automated portable continuous glucose monitoring system) that includes stability and improved frequency characteristics/response. The model is shown in (b3, b4) where $SR(t)$ [pM] is the $\beta$-cell insulin secretion rate, $n$ [1/min] is the reciprocal of the insulin plasma decay rate constant, $K_p$ [pmol·1/min·1/M] is the rate of $\beta$-cell insulin secretion, $T_r$ [min] is the rate of proportional to integral release and $T_o$ [Min] is the ratio of derivative to proportional release.

\[ I(t) = -n \cdot I(t) + \frac{SR(t)}{V} \quad (b3) \]

\[ SR(t) = P(t) + I(t) + D(t) \quad SR(t) \geq 0 \]

\[ P(t) = K_p \cdot (G(t) - G_b) \]

\[ I(t) = K_p \cdot (G(t) - G_b)/T_r \quad I(0) = I_b \quad (b4) \]

\[ D(t) = K_p \cdot T_D \cdot G(t) \]

The test reveals that impaired glucose tolerance humans have a defect in the second phase as earlier stated.

Other models are based on the measurement of c-peptide [Eaton 1980] that is secreted in equal amounts as insulin with advantages that c-peptide is hardly extracted by the liver and its kinetics are linear. A disadvantage, from a qualitative standpoint, of models based on c-peptide is that the measurement of c-peptide is worse than the measurement of plasma insulin concentrations. However, these models can be modified in such a way that it can be used with measurements of plasma insulin concentrations.

[Toffolo 1995] introduces a c-peptide minimal model for an IVGTT which was extended by Breda to an oral c-peptide minimal model [Breda 2001]. The model assumes that $\beta$-cell insulin secretion depends on glucose plasma concentrations (static component, delayed production of new insulin) and on the rate of change of glucose plasma (dynamic component, secretion of stored insulin). This model is comparable with Toffolo’s model in 1980 that use the same static component, the same order ($1^{st}$) for clearance of insulin, but with no static delay and no dynamic component. The introduction of a dynamic component into the model of $\beta$-cell insulin secretion gives the model more physiological meaning. The model is shown in (b5, b6) where $SR_s(t)$ [PM/min] is the static $\beta$-cell insulin response, $SR_d(t)$ [pM] is the dynamic $\beta$-cell insulin response and $1/a$ [min] is the delay time in the static response.

\[ SR(t) = SR_s(t) + SR_d(t) \]

\[ SR_s(t) = \alpha[SR_s(t) - y \cdot (G(t) - h)] \quad SR_s(0) = 0 \quad (b5) \]

\[ SR_d(t) = \begin{cases} K_D \cdot G(t) & \text{if } G(t) > 0 \\ 0 & \text{if } G(t) \leq 0 \end{cases} \]
test reveals that impaired glucose tolerance humans have a diminished \( S_p \), but an unaltered static and dynamic component as Bergman [Bergman 1981] found in obese lower tolerance humans. This test also reveals a delay between \( \beta \)-cell insulin secretion and changes of glucose concentration in plasma. Several other papers show similar results, e.g. [Ehrmann 2002, Cretti 2001]. Other publications have been published having likewise models [Mari 2002].

Both Steil's model and Breda's model are compared in [Steil 2003] using Bergman's minimal model [Bergman 1979] during a CLAMP on healthy humans. Both models fit the \( \beta \)-cell insulin secretion profile with low residuals. Also, comparable parameters between both models are not statistical different. However, Steil's model has better performance than Breda's model, since Steil's model has parameters with lower variations, more stability in close loop systems and adjusts better insulin needs to varying \( S_p \) and endogenous glucose appearance. One drawback is that Steil's model is tested by a CLAMP and not during physiological circumstances like Breda's model that is tested during an OGTT. Since Steil's model represents very well the \( \beta \)-cell insulin secretion behaviour and has interesting closed loop properties for future application of the model into an automated portable continuous glucose monitoring system, this model is used.
C.1 Insulin profiles

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Table c.1: Two insulin and glucose profiles [Wagenmakers 2006] measured during an OGTT (75 gram glucose)

C.2 Samples Schedules

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<td>0 1 2 3 4 5 6 7 8 9 10 ... 240</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>0 10 20 25 30 35 40 45 50 55 60 65 70 80 90 120 180 240</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>0 15 30 45 60 90 120 180 240</td>
</tr>
</tbody>
</table>

Table c.2: Sample schedule 1, 2 and 3

C.3 Datasets

Dataset 1 is generated using insulin profile 1 and sample schedule 1.
Dataset 2 is generated using insulin profile 2 and sample schedule 1.
Dataset 3 is generated using insulin profile 1 and sample schedule 2.
Dataset 4 is generated using insulin profile 2 and sample schedule 2.
Dataset 5 is generated using insulin profile 1 and sample schedule 3.
Dataset 6 is generated using insulin profile 2 and sample schedule 3.

Interstitial glucose $G'(t)$ is measured in all previous datasets by taking the mean over each 30 minutes by taking the first sample at the 30th minute and the last sample at 240th minute.

\* Subject no: 5 Visit 6 / Day 15 [Wagenmakers 2006]
\* Subject no: 8 Visit 6 / Day 15 [Wagenmakers 2006]
\* Conversion factor between [lU/l] to [pM]: 7.0625.
\* Conversion factor between [mg/dl] to [mM]: 1/180.161/mM

C.4 Simulated model
Insulin profile 1

Figure c.1: Simulated glucose plasma, insulin plasma, interstitial glucose, interstitial insulin and the rate of appearance using profile 1, sample schedule 1 and true value shown in table 1.
Figure C.2: Simulated glucose plasma, insulin plasma, interstitial glucose, interstitial insulin and the rate of appearance using profile 1, sample schedule 2 and true value shown in table 1.
Plots of model to model adjustment

Model 1
Figure D.1: Plots of model to model adjustment of model 1 using dataset 1 (appendix c) with no added white noise
Figure D.2: Plots of model to model adjustment of model 1 using dataset 2 (appendix c) with no added white noise
Figure D.3: Plots of model to model adjustment of model 2 using dataset 2 (appendix C) with no added white noise
Model 3
Figure D.4: Plots of model to model adjustment of model 3 using dataset 3 (appendix C) with no added white noise
A priori parameter sensitivity for \( k_{abs} \)

<table>
<thead>
<tr>
<th>1/SNR (%)</th>
<th>profile 1</th>
<th>profile 2</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Ib</strong></td>
<td><strong>Gb</strong></td>
<td><strong>beta</strong></td>
</tr>
<tr>
<td>0</td>
<td>40.0</td>
<td>72.5</td>
</tr>
<tr>
<td>5</td>
<td>40.0</td>
<td>40.0</td>
</tr>
<tr>
<td>10</td>
<td>40.0</td>
<td>73.3</td>
</tr>
</tbody>
</table>

A priori parameter sensitivity for \( v \)

<table>
<thead>
<tr>
<th>1/SNR (%)</th>
<th>profile 1</th>
<th>profile 2</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Ib</strong></td>
<td><strong>Gb</strong></td>
<td><strong>beta</strong></td>
</tr>
<tr>
<td>0</td>
<td>40.0</td>
<td>70.6</td>
</tr>
<tr>
<td>5</td>
<td>49.0</td>
<td>84.6</td>
</tr>
<tr>
<td>10</td>
<td>43.1</td>
<td>94.2</td>
</tr>
</tbody>
</table>

A priori parameter sensitivity for \( p_4 \)

<table>
<thead>
<tr>
<th>1/SNR (%)</th>
<th>profile 1</th>
<th>profile 2</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Ib</strong></td>
<td><strong>Gb</strong></td>
<td><strong>Beta</strong></td>
</tr>
<tr>
<td>0</td>
<td>50.0</td>
<td>50.0</td>
</tr>
<tr>
<td>5</td>
<td>40.0</td>
<td>394.7</td>
</tr>
<tr>
<td>10</td>
<td>40.0</td>
<td>63.0</td>
</tr>
</tbody>
</table>

A priori parameter sensitivity for Si-Glut4
### Table E.1: A priori parameter sensitivity for kabs, v, p4 and Si-glut4 using synthetic data and using two different insulin profiles (data sets 1, 2, respectively, appendix c) with added white noise having a 1/SNR of 0, 5 and 10%.

<table>
<thead>
<tr>
<th>1/SNR (%)</th>
<th>profile 1</th>
<th>profile 2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>lb</td>
<td>Gb</td>
</tr>
<tr>
<td>0</td>
<td>50.0</td>
<td>50.0</td>
</tr>
<tr>
<td>5</td>
<td>48.2</td>
<td>946.4</td>
</tr>
<tr>
<td>10</td>
<td>48.7</td>
<td>770.4</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>1/SNR (%)</th>
<th>profile 1</th>
<th>profile 2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>p2</td>
<td>p5</td>
</tr>
<tr>
<td>0</td>
<td>34.0</td>
<td>0.135</td>
</tr>
<tr>
<td>5</td>
<td>49.7</td>
<td>3.98</td>
</tr>
<tr>
<td>10</td>
<td>42.58</td>
<td>15.6</td>
</tr>
</tbody>
</table>
European Research activities
On
(Closed loop) artificial pancreas and automatic insulin delivery

Consulted sources:
EMBase (ScienceDirect), MEDLINE (ScienceDirect), Elsevier BIOBASE (ScienceDirect), Pubmed, Pascal (Pica), Web of Science, Elsevier Scopus, Inspec (webspirs)
Diabetes Modelling Group, Department of Paediatrics, Univ. Cambridge, Box 116, A.

Principal Author(s): Hovorka, R. Chassin, L.J.

Literature:


Website: http://www.iph.cam.ac.uk/diabetes/

-------------------------------------------------------------------------------------------------

Inst. für Diabetes-Technologie, Universität Ulm, Helmholtzstr. 20, D-89081 Ulm, Germany:

Principal Author(s): Freckmann, G. B.

Literature:


More publications available at http://www.idt-ulm.de/frame1.htm
INSERM U.341, Service de Diabetologie, Hotel-Dieu, 1. Place du Parvis Notre Dame, F 75004 Paris, France (Unite de Recherches sur le Diabete et la Nutrition chez l'Enfant, INSERM U83, 75010 Paris, France)

Principal Author(s): Reach G.

Literature:


Website: http://jdiabete.hotel-dieu.com


Principal Author(s): Kessler, L.

Literature:


**Website:** http://www-ulpmcd.u-strasbg.fr/

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**Literature:**


Renard, E., "Insulin therapy by insulin pump: Continuous or conventional self-blood glucose monitoring?," *Diabetes and Metabolism*, Vol. 29(2003), No. 2 11.


---

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Literature:
(no newer literature found)


Website: http://www.dei.unipd.it/wdyn/?!Dsezione=172, http://www.dei.unipd.it/wdyn/?IDsezione=3315&IDgruppo_pass=32&preview

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Literature:


OLDER LITERATURE

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Principal Author(s): Irsigler K.; Kritz H.

Literature:


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Literature:


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Literature:


Principal Author(s): Bottermann, P., Gyaram, H., Wahl, K., et al

Literature:


Clin. Mal. Metab. Endocrinienes, Hop. Saint Eloi, Montpellier, France

Principal Author(s): Mirouze, J., Selam, J.L., Chi Pham, T.

Literature:

Matlab code

D.1 Basic files

D.1.1

**gluc_param_new.m**

```matlab
% Matlab Minimal Model of glucose control.
% This script reproduces all parts of the Matlab Minimal Model of glucose
% kinetics and the time samples of plasma insulin.

% History
% 23 March 96, Walter Albrecht, DLR
% 17 Feb 94, Natalia Estrin, DLR
% 14 Feb 94, Natalia Estrin, DLR

close all; clear all;
plt=0;
p1=plot1; plt 1

% Minimal model parameters:
% The units:

Ib = 7.0625*(7.5+6.5)/2; % [mg/dl] plasma concentration in plasma
Gb = 0.055*(112+119)/2; % [mg/l] plasma concentration in plasma
Glb=3.38; % [mg/l] plasma concentration in plasma
end

II 0 = Subject no: 1's initial value
Ib = 7.0625*(6.4+4.5)/2; % [mg/dl] plasma concentration in plasma
Gb = 0.055*(112+93)/2; % [mg/l] plasma concentration in plasma
Glb=3.38; % [mg/l] plasma concentration in plasma
end

II 0 = Subject no: 2's initial value
Ib = 7.0625*(7.8+7.6)/2; % [mg/dl] plasma concentration in plasma
Gb = 0.055*(165+169)/2; % [mg/l] plasma concentration in plasma
Glb=3.38; % [mg/l] plasma concentration in plasma
end

*Known unknown model parameters
* (Mean values obtained from persons in critical situations)*
\( k_8 = 3e^{-2}; \)
\( p_2 = 0.113; \)
\( DS = 75/10; \)
\( Kabs = 2.89e^{-2}; \)
\( Beta = 1.23; \)
\( k = 0.014; \)
\( V = 2.497e^{-1}; \)
\( p_5 = 0.018; \)
\( p_6 = 0.00763*10; \)
\( k_7 = 0.063*(1/0.21); \)
\( SICap = 2e^{-3} * 0.0395 * (1/0.21) * 1e3; \)
\( SIliver = 1.11e^{-5} * 1e4 * (1/7.0625); \)
\( SIlglut4 = 1.0e^{-2}; \)
\( p_4 = k_7 * k_8; \)
\( p_3 = p_2 * p_4; \)
\( p_{true} = [Ib, Gb, p_2, DS, Kabs, Beta, k, V, p_4, p_5, p_6, SICap, ... SIliver, SIlglut4, Coeff]; \)
\* Fixed initial conditions: \\
L0(1) = Gb; \quad [mM] glucose concentration in plasma \\
L0(2) = 0; \quad [mM] labelled glucose concentration in plasma \\
L0(3) = 0; \quad [mM] insulin concentration in plasma \\
L0(4) = Gb; \quad [mM] glucose concentration in interstitium \\
L0(5) = 0; \quad [mM] glucose concentration in the gut \\

\textbf{Input}: insulin concentration in plasma [mM]; assumed to be known at each \\
\textbf{Simulation}: time sample from linear interpolation of its measured \\
\textbf{Samples}: \\

\begin{verbatim}
if 1 Subject; // Volumes: 1
  // Blood glucose data
  t_inulin1 = [0 15 30 45 60 90 120 180 240];
  c_inulin1 = 7.0625\*[(7.5\+6.5)/2 12.2 19 20.5 23.3 24.6 23.9 27.7 23];
  t_c_inulin1 = [t_inulin1' c_inulin1'];

  // Blood glucose data
  t_inulin2 = [0 15 30 45 60 90 120 180 240];
  c_inulin2 = 7.0625\*[(7.5\+6.5)/2 12.2 19 20.5 23.3 24.6 23.9 27.7 23];
  t_c_inulin2 = [t_inulin2' c_inulin2'];
end

if 0 Subject; // Volumes: 1
  // Blood glucose data
  t_inulin1 = [0 15 30 45 60 90 120 180 240];
  c_inulin1 = 7.0625\*[(6.4\+4.5)/2 14.8 22.2 27 27.7 46.8 81 46 7.];
  t_c_inulin1 = [t_inulin1' c_inulin1'];

  // Blood glucose data
  t_inulin2 = [0 15 30 45 60 90 120 180 240];
  c_inulin2 = 7.0625\*[(6.4\+4.5)/2 14.8 22.2 27 27.7 46.8 81 46 7.7];
  t_c_inulin2 = [t_inulin2' c_inulin2'];
end

if 0 Subject; // Volumes: 1
  // Blood glucose data
  t_inulin1 = [0 15 30 45 60 90 120 180 240];
  c_inulin1 = 7.0625\*[(7.8\+7.6)/2 13.4 19.7 20.2 21.5 29.6 27.5 18.8 14.2];
  t_c_inulin1 = [t_inulin1' c_inulin1'];

  // Blood glucose data
  t_inulin2 = [0 15 30 45 60 90 120 180 240];
  c_inulin2 = 7.0625\*[(7.8\+7.6)/2 13.4 19.7 20.2 21.5 29.6 27.5 18.8 14.2];
  t_c_inulin2 = [t_inulin2' c_inulin2'];
end

\end{verbatim}

\* Report 2 norm of the error time to products
a=1; output would not be limited as a logistic concentration (1st output)
b=1; output would not be limited as a logistic concentration (2nd output)
c=1; output would not be limited as a logistic concentration (3rd output)

sigma1=0.025; Variance of initial glucose plasma concentration output
sigma2=0.05; Variance of intestinal glucose output
sigma3=1; 0.86; Variance of basal insulin concentration output

Plots:

% Make plots of pit 1
if plt==1;

figure; plot(t_c_insulin(:,1), t_c_insulin(:,2), 'r'); hold on
plot([t_c_insulin(1,1) t_c_insulin(end,1)], [1b 1b], 'k'); plot([1b 1b], [1b 1b], 'k'); xlabel('Time (min)'); ylabel('Concentration'); title('Plots of pit 1'); end

% Compute interval [pit 1] and [pit 2] output

\[ t_{\text{exp}} = 15; \]
D.1.2

**gluc_ode_new.m**

```matlab
function dOut = gluc_ode_new(t,lin,t_c_insulin,p)

# function description

% This function sets the ODE system of the Revised Model of glucose kinetics which will be used to fit the parameters into the dataset.

% Function:

% dOut = gluc_ode_new(t,lin,t_c_insulin,p)

% inputs:

% t: state values at previous time step; t(1): [G(1),dG(1),dGL(1),...
% X(k):Y(k); dGut(k); dGut(k+1)]
% _c_insulin: _c_insulin is not stored in sample k OR matrix
% _c_insulin
% f: [ts, 1, 0, p1, p2, p3, p4, p5, p6, Sleep,...
% Sleep, State, etc.] it is constant across 2 space.

% outputs:

% dOut: new state derivatives (dX(k)) for X(k); dY(k); dGut(k)
% column vector.

% Direct:

% 02 March 08, Subrahmani Alampalli, DJ
% 02 June 08, Natal van Rooy, DJ
% 02 March 08, Natal van Rooy, DJ
% DJ

% defining the new vector each state is a vector finn: is present
idG = 1; glucose concentration [mM], first column of vector
idGL = 2; Glucose level one cell time [mM], second column of vector
idX = 3; Insulin initial target: vector [mM], third column of vector
idY = 4; Interstitial glucose concentration [mM], fourth column of vector
idGut = 5; the rate of glucose in the gut [1/2a], fifth column of vector

% input/ode parameters:
Ib = p(1); [1] baseline glucose concentration in plasma
Gb = p(2); [1] baseline glucose concentration in plasma
```

*known/unknown model parameters
* glucose from blood plasma to intestinal by epithelial recruitment
*
\[ p_2 = p(3); \]

\[ DS = p(4); \]

\[ Kabs = p(5); \]

\[ Beta = p(6); \]

\[ k = p(7); \]

\[ V = p(8); \]

\[ p_4 = p(9); \]

\[ p_5 = p(10); \]

\[ p_6 = p(11); \]

\[ S1cap = p(12); \]

\[ S1silver = p(13); \]

\[ S1glut4 = p(14); \]

\[ Coeff = p(15); \]

\[ u = \text{interpl}(t_c_{insulin}(;1), t_c_{insulin}(;2), t); \]

\[ dG = -((S1silver/10000) + (p4)*(Lin(idG)-Gb)\times(u-Ib)+p5-(p6/10)\times(Lin(idG)-Gb)+(Kabs*Lin(idGgut))/V; \]

\[ dG = 0; \]

\[ \%G = -((S1silver/10000) + (p4)*(Lin(idG)-Gb)\times(u-Ib)+p5-(p6/10)\times(Lin(idG)-Gb)+(Kabs*Lin(idGgut))/V; \]

\[ dG = 0; \]

\[ dX = -p2*Lin(idX)+(S1cap/1000) *(u-Ib)^2 ; \]

\[ dY = -S1glut4*Lin(idY)\times Lin(idX) + (p4/1)*(Lin(idG)-Gb)\times(u-Ib); \]

\[ dGgut = (DS*10)*k*Beta*exp(-k*t)*Beta-Kabs*Lin(idGgut); \]

\[ dLout = [dG; dGL; dX; dY; dGgut]; \]
D.1.3

**gluc_sim_new.m**

```matlab
function [c_glucose, c_glucose_labeled, c_interstitial_insulin, ...
    c_interstitial_glucose, c_Ggut, c_Ggut_labeled] = gluc_sim_new (L0, ...
    t_c_insulin, p, plt, TITLE)

    % Initial Minimal Model of glucose secretion
    % This function makes a simulation of the Minimal Model of glucose
    % secretion with the parameters in the input file.
    %
    % Variables:
    % - c_glucose, c_glucose_labeled, c_interstitial_insulin, ...
    % - c_interstitial_glucose, c_Ggut, c_Ggut_labeled
    %
    % Parameters:
    % - L0
    % - t_c_insulin
    % - p
    % - plt
    % - TITLE
    %
    % Description:
    % This function simulates the glucose secretion process.
    %
    % Notes:
    % This is a relatively complex function that may require some
    % understanding of the underlying model.
```

The text above is a MATLAB function that simulates glucose secretion with parameters and variables specified in the input file. It is part of a larger model of glucose homeostasis. The function takes several inputs, including initial conditions and parameters, and outputs various concentrations of glucose and insulin. The comments in the code provide a brief description of what each section does and how the model is structured.
ode_options = odeset('RelTol',1e-6,'AbsTol',1e-12); [t,L] = ode45(@gluc_ode_new,t_c_insulin(:,1),LO,ode_options, t_c_insulin,p);

c_glucose = L(:,1); % plasma glucose concentration
C_glucose_labeled = L(:,2);
c_interstitial_insulin=L(:,3);
c_interstitial_glucose=L(:,4);
c_Ggut=L(:,5); % [kg/mol] as a concentration
Coeff=p(15);
c_Ggut_labeled=Coeff*L(:,5);

plt==1 % plot
figure;
Ib=p(1);
subplot(321);plot(t_c_insulin(:,1),t_c_insulin(:,2),',-',t_c_insulin(:,2),',-',t_c_insulin(1),t_c_insulin(end),':');
hold
plot([t_c_insulin(1) t_c_insulin(end)], [Ib Ib],',-',','.',1.5)
ylabel('');
```matlab
subplot(322); plot(t_c_insulin(:,1), c_interstitial_insulin,
    'r-', 2); xlabel(''); ylabel(''); Title(TITLE);

Gb=p(2);
subplot(323); plot(t_c_insulin(:,1), c_glucose,'r-', 2);
hold on;
plot([t_c_insulin(1,1) t_c_insulin(end,1)], [Gb Gb], 'b-.', 1.5)
xlabel(''); ylabel(''); Title(TITLE);

subplot(324); plot(t_c_insulin(:,1), c_glucose_labeled,'r-', 2);
xlabel(''); ylabel(''); Title(TITLE);

subplot(325); plot(t_c_insulin(:,1), c_interstitial_glucose,
    'r-', 2); xlabel(''); ylabel(''); Title(TITLE);

subplot(326);
plot(t_c_insulin(:,1), c_Ggut,'r-', 2)
xlabel(''); ylabel(''); Title(TITLE);
```

function error = obj_fn_new(p_var, p_fix, LO, datal, data2, data3,...
    t_c_insulin1, t_c_insulin2, a, b, c, sigma1, sigma2, sigma3,...
    t_exp, Nparameters, NoMean)

% Objective function: error function to be minimized in the ODE
% Minimization: Model of glucose function.
% This function is used as objective function for LEVMONL1.
% %
% % Function: %e - obj_fn_new(p_var, p_fix, LO, datal, data2, data3,...
% % t_c_insulin1, t_c_insulin2, a, b, c, sigma1, sigma2, sigma3,...
% % t_exp, Nparameters, NoMean)
% % Input function:
% % p_var: Parameter space which will be calculated
% % p_fix: Parameter space which is fixed
% % LO: Fixed initial condition. The initial glucose concentration in
% % plasma, LO(1) [mg/dL] in the first observation on time 0.
% % data1: [mM] glucose concentration e interpolation, [mg/dL] - [1kg]
% % data2: data from laboratory
data3: data of laboratory plasma [mg/dL]
% % t_c_insulin1: [mM] time of initial injection insulin
% % t_c_insulin2: time of initial injection insulin
% % t_exp: [output weighting factor] [output
% % Nparameters: number of additional parameter which is set to the model to model
% % NoMean: number of additional parameter which is set to the model to model
% % % Output function:
% % error = minimizing variable of the objective function
% %
% % History:
% % 2.9.2017 by, Savicks: first time, Put
% % 29.10.2017, Nowski started, %
% % 29.10.2017, Nowski started,
\[ \text{p \_estimating = [p \_fix(1), p \_fix(2), p \_fix(3), p \_fix(4), p \_var(1), \ldots} \\
\text{p \_fix(5), p \_fix(6), p \_var(2), p \_var(3), p \_fix(7), \ldots} \\
\text{p \_fix(8), p \_fix(9), p \_fix(10), p \_var(4), p \_fix(11)];} \]

Simulating the Oral Minimal Model of glucose dynamics using the estimated parameters. All dimensional parameter scales are in units of glucose concentration.

\( \text{plt} = 0; \) leave no plots

\[ \text{[c \_glucose \_estimating, c \_glucose \_labeled \_estimating, \ldots} \\
\text{c \_interstitial \_insulin \_estimating, c \_interstitial \_glucose \_estimating, \ldots} \\
\text{c \_Ggut \_estimating] = \text{gluc \_sim \_new(LO, t \_c \_insulin1,} \\
\text{p \_estimating, plt, \ldots} \\
\text{)}; \]

Simulating the Oral Minimal Model of glucose dynamics using the estimated parameters. All dimensional parameter scales are in units of interpolated insulin concentration samples for data.

\( \text{if NoMean} = 0 \)

\[ \text{" Interpolating data:} \\
\text{t \_insulin2 \_interpl = t \_c \_insulin2(1, 1):1:t \_c \_insulin2(\text{end, 1}); } \] \[ \text{[min]} \\
\text{c \_insulin2 \_interpl=interpl(t \_c \_insulin2(:,1), t \_c \_insulin2(:,2), \ldots} \\
\text{t \_insulin2 \_interpl);} \\
\text{t \_c \_insulin2 \_interpl(:,1)=t \_insulin2 \_interpl;} \\
\text{t \_c \_insulin2 \_interpl(:,2)=c \_insulin2 \_interpl;} \\
\]

\[ \text{[c \_glucose \_estimating2 \_interpl, c \_glucose \_labeled \_estimating2 \_interpl,\ldots} \\
\text{c \_interstitial \_insulin \_estimating2 \_interpl, \ldots} \\
\text{c \_interstitial \_glucose \_estimating2 \_interpl, \ldots} \\
\text{c \_Ggut \_estimating2 \_interpl]} = \text{gluc \_sim \_new(LO, t \_c \_insulin2 \_interpl,} \\
\text{p \_estimating, plt, \ldots} \\
\text{)}; \]

\( \text{Calculate the mean of the interpolated data at the data2 sample:} \)
\( \text{GlB=LO(4);} \\
\text{c \_interstitial \_glucose \_estimating2 \_mean(1, 1)=GlB;} \\
i=2; \) for \( r=t \_c \_insulin2(2: \text{end, 1}) \) \( \text{c \_interstitial \_glucose \_estimating2 \_mean(1, 1)=...} \\
\text{sum(c \_interstitial \_glucose \_estimating2 \_interpl(r+1-t \_exp:r))/t \_exp;} \\
i=i+1; \) \end{verbatim}
else
  [c_glucose_estimating2, c_glucose_labeled_estimating2,...
  c_interstitial_insulin_estimating2,...
  c_interstitial_glucose_estimating2,c_Ggut_estimating2] =
  gluc_sim_new(L0, t_c_insulin2, p_estimating, plt,...
  '!!');
end

if NoMean==0
  error1 = c_glucose_estimating-data1;
else
  error1 = c_glucose_estimating-data1;
end
error2= c_interstitial_glucose_estimating2_mean-data2;
error2= c_interstitial_glucose_estimating2-data2;
else
  error2=c_interstitial_glucose_estimating2-data2;
end
error3=c_glucose_labeled_estimating-data3;

data1_new=data1;
data2_new=data2;
data3_new=data3;

if length(error1)>length(error2)
  for q=1:length(error1)-length(error2)
    error2_new(length(error2)+q)=0;
    data2_new(length(data2)+q)=1;
    error1_new(error1)=-1 q=q+1
  end
else
  for q=1:length(error1)-length(error2)
    error1_new(length(error1)+q)=0;
    data1_new(length(data1)+q)=1;
    error3_new(length(error3)+q)=0;
    data3_new(length(data3)+q)=1;
    end
  end

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error2_new = error2;
data2_new = data2;

error = 1e4 * error1_new + 1e4 * error2_new + 125 * 1e4 * error3_new;

figure(100);
subplot(411);
plot(t_c_insulin1(:,1),c_interstitial_insulin_estimating,'.');
xlabel('');
ylabel('');
Title('');
drawnow; pause(.3);

subplot(412);
plot(t_c_insulin1(:,1),c_glucose_estimating,'',t_c_insulin1(:,1),data1,'.');
xlabel('');
ylabel('');
Title('');
drawnow; pause(.3);

subplot(413);
plot(t_c_insulin1(:,1),c_glucose_labeled_estimating,'',t_c_insulin1(:,1),data3,'.');
xlabel('');
ylabel('');
Title('');
drawnow; pause(.3);

subplot(414);
if NoMean==0
plot(t_c_insulin2(:,1),c_interstitial_glucose_estimating2_mean,'',... t_c_insulin2(:,1),data2,'.');
else
plot(t_c_insulin2(:,1),c_interstitial_glucose_estimating2,'',... t_c_insulin2(:,1),data2,'.');
end
xlabel('');
ylabel('');
D.2 model2model adjustment

D.2.1 model2model_bias_new.m

% This script identifies a posteriori the Dual Minimal Model of glucose
% kinetics under the model-to-model adjustment approach for each
% parameter
% of the bean-sweetened processors

% History
% 22 March 06, Salvador Almeida Franca, UNA

clear l;
NoMean=0; %Calculate the mean error of output of internal glucose
% samples it set to 0

% Parameters
Nparameters=length(p_true);

% data initialisation

data1_new=data1;
data2_new=data2;
data3_new=data3;

% Vector containing variation on normal (0) for the true 0
% dimensional
% parameter space
rre=[-0.5 -0.45 -0.4 -0.35 -0.3 -0.25 -0.2 -0.15 -0.1 0 0.1 ....
0.15 0.2 0.25 0.3 0.35 0.4 0.45 0.5];

% Name of the 11 parameters set
parr=['B0'; 'b0'; 'B1'; 'b1'; 'B2'; 'b2'; 'B3'; 'b3'; 'B4'; 'b4'; 'B5'; 'b5';
'B6'; 'b6'; 'B7'; 'b7'; 'B8'; 'b8'; 'B9'; 'b9'; 'B10'; 'b10'; 'B11'; 'b11'];
pname = [''];

for jj=1:length(rr)

"initial_start_value_parameter=[p_true(estimator(1))

for ii=1:length(rr)

p_init=[p_true(estimator(1))+rr(ii)*p_true(estimator(1)) ... 

p_true(estimator(2))+rr(jj)*p_true(estimator(2));]
% Make function of the time-oral parameter
% Time of the first minute model delayed secretion
plt=0; % Make no plot.

iii=ii
jjj=jj

[res]=gluc_omm_mls2_new(data1, data2, data3, LO, t_c_insulin1, ...
 t_c_insulin2, a, b, c, sigma1, sigma2, sigma3, p_true, p_fix, p_init, ...
 t_exp, Nparameters, estimator, NoMean, plt);

% Make the squared error of the time or minute residual
DataZ(ii, jj)=resnorm;

% Make both true and initial sequence purpose (Pi, P2)
if rr(ii)==0 && rr(jj)==0
  resnormTrue=resnorm;
end

% Make time model with the proper initial values
if ii==4 && jj==2 || ...
  ii==2 && jj==3 || ii==4 && jj==8 || ...
  ii==15 && jj==7 || ii==6 && jj==11 ... || ii==8 && jj==3 || ii==12 && jj==5 || ii==3 && jj==14 || ...
  ii==18 && jj==6 || ii==19 && jj==13 || ...
  ii==12 && jj==9 || ii==11 && jj==3

% Make clear

[p_model, LO_model, c_glucose_model, c_glucose_labeled, ...
 c_interstitial_insulin_model, c_interstitial_glucose_model, ...
 c_Ggut_model, resnorm, residual, varp, stdp]=gluc_omm_mls_bias_new...
 (data1_new, data2_new, data3_new, LO, ...
 t_c_insulin1, t_c_insulin2, a, b, c, sigma1, sigma2, sigma3, ...
 p_fix, p_init, t_exp, Nparameters, estimator, NoMean, plt);
 ptrue1=p_true(qq)
 ptrue2=p_true(yy)
 estimation(bb,:)= [p_model(qq) p_model(yy)]
 bb=bb+1;

% Make the parameters of the time-oral model
DataY(jj)=p_true(estimator(2)) + rr(jj)*p_true(estimator(2));
end

% Make the time on the parameter (Pi) with rr(rh, 100) variation
DataX(ii)=p_true(estimator(1)) + rr(ii)*p_true(estimator(1));
end

% Make 3d plots of the time function (estimator1, estimator2)
Plot2D<Data1>

Minn=min(min(Data1))
Maxx=max(max(DataZ))
Contourlines=50
((min(min(DataZ)));0.01:(max(max(DataZ))))';
Contourlines>200
Contourlines=200;
display...
end

figure(333);
contour(DataX,DataY,DataZ,Contourlines);
hold;
plot(p_true(estimator(1)),p_true(estimator(2)),');
hold
plot(estimation(1,:),estimation(2,:),');
xlabel(['',pname(estimator(1),:)]);
ylabel(['',pname(estimator(2),:)]);
Title('');
colormap pcolor;
colorbar;
hold;

close(figure(100));
end
end
D.2.2

**gluc_param2_new.m**

```matlab
% Oral Minimal Model of glucose kinetics

% This script initializes all parameters of the Oral Minimal Model of glucose
% kinetics and the time sample of the simulation.

% Metadata
% V2.3 March 06, Salvador Ahumada, Pio A
% V2.4 Jan 04, Natal van Hael, Pio A
% V2.5 Jun 04, Natal van Hael, Pio A
% ...

close all;
% make plot at plt 1
plt=1;

description model parameters:
% Values from: 2004

if 1
    Subject no: 1
    Visit / Day 1
    Ib = 7.0625*(7.5+6.5)/2; [mM] baseline glucose concentration in plasma
    Gb = 0.055*(112+119)/2; [mM] baseline albumin concentration in plasma
    Gb=3.38; [mM] baseline albumin concentration in interstitium
end

if 0
    Subject no: 15
    Visit / Day 1
    Ib = 7.0625*(6.4+4.5)/2; [mM] baseline glucose concentration in plasma
    Gb = 0.055*(112+93)/2; [mM] baseline albumin concentration in plasma
    Gb=3.38; [mM] baseline albumin concentration in interstitium
end

if 0
    Subject no: 8
    Visit / Day 1
    Ib = 7.0625*(7.8+7.6)/2; [mM] baseline glucose concentration in plasma
    Gb = 0.055*(165+169)/2; [mM] baseline albumin concentration in plasma
    Gb=3.38; [mM] baseline albumin concentration in interstitium
end

% Known unknown initial parameters:
% Model does extended parameter estimation

% Annual International Conference - The 18th European in Medicine
% Biology, Cancun, Mexico, 1-4 September 2016,
% Salvador, S. and F. De Marte, T. Patterson, C. De Marte, F.
% Mordani,
% "Absorption and Insulin Sensitivity in a 75 g Glucose Tolerance Test."
% Proceedings of the 25th Annual International Conference of the
```

99
\$k_8 = 3 \times 10^{-2};\$
\$p_2 = 0.113;\$
\$DS = 75/10;\$
\$K_{abs} = 2.89 \times 10^{-2};\$
\$\beta = 1.23;\$
\$k = 0.014;\$
\$V = 2.497 \times 10^{-1};\$
\$p_5 = 0.018;\$
\$p_6 = 0.00763 \times 10;\$
\$k_7 = 0.063 \times (1/0.21);\$
\$SI_{cap} = 2e-3 \times 0.0395 \times (1/0.21) \times 10^3;\$
\$SI_{silver} = 1.11e-5 \times 10^4 \times (1/7.0625);\$
\$\text{Coeff} = 3/75;\$
\$p_{true} = \{I_b, G_b, p_2, DS, K_{abs}, \beta, k, V, p_4, p_5, p_6, SI_{cap}, SI_{silver}, SI_{glut4}, \text{Coeff}\};\$
Input insulin concentration is plot graph, supposed to be shown if

Simulation time sample from insulin 1 and 2 were measured
samples

Time and amplitude samples of three insulin:
1: Blood glucose data
   t_insulin1 = [0 15 30 45 60 90 120 180 240];
   c_insulin1 = 7.0625*[(7.5+6.5)/2 12.2 19.2 20.5 23.3 24.6 23.9 27.7 23];
   t_c_insulin1 = [t_insulin1' c_insulin1'];

2: Interstitial glucose data
   t_insulin2 = [0 15 30 45 60 90 120 180 240];
   c_insulin2 = 7.0625*[(7.5+6.5)/2 12.2 19.2 20.5 23.3 24.6 23.9 27.7 23];
   t_c_insulin2 = [t_insulin2' c_insulin2'];
   end

if 0 % Subject no: 1
1: Blood glucose data
   t_insulin1 = [0 15 30 45 60 90 120 180 240];
   c_insulin1 = 7.0625*[(6.4+4.5)/2 14.8 22.2 27 27.7 46.8 81 46 7];
   t_c_insulin1 = [t_insulin1' c_insulin1'];

3: Interstitial glucose data
   t_insulin2 = [0 15 30 45 60 90 120 180 240];
   c_insulin2 = 7.0625*[(6.4+4.5)/2 14.8 22.2 27 27.7 46.8 81 46 7];
   t_c_insulin2 = [t_insulin2' c_insulin2'];
   end

if 0 % Subject no: 2
1: Blood glucose data
   t_insulin1 = [0 15 30 45 60 90 120 180 240];
   c_insulin1 = 7.0625*[(7.8 + 7.6)/2 13.4 19.7 20.2 21.5 29.6 27.5 18.8
14.2];
   t_c_insulin1 = [t_insulin1' c_insulin1'];

3: Interstitial glucose data
   t_insulin2 = [0 15 30 45 60 90 120 180 240];
   c_insulin2 = 7.0625*[(7.8 + 7.6)/2 13.4 19.7 20.2 21.5 29.6 27.5 18.8
14.2];
   t_c_insulin2 = [t_insulin2' c_insulin2'];
   end

% Separed 2 rows of the error function residuals

a=1; % output weighting factor of glucose concentration flat
b=1; % output weighting factor of external glucose concentration.
c=1; % output weighting factor of internal glucose concentration.
sigma1=1; % variance of glucose concentration of output
sigma2=1; % variance of external glucose concentration output
sigma3=1; % variance of internal glucose concentration output

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# Plots

% make plot: if plt=1
if plt=1;

figure; plot(t_c_insulin1(:,1), t_c_insulin1(:,2), 'r'); hold

% baseline levels:
plot([t_c_insulin1(1,1) t_c_insulin1(end,1)], [Ib Ib], 'k', 'LineWidth',1.5)
xlabel('time [min]'); ylabel('levels [mM]'); title('levels [mM]')
end

% Simulating the postprandial period: glucose uptake using the true
% 14-dimensional parameter space and the interpolated insulin
% concentration at samples for data1.

% sample-interval [min] of data

\[ t_{exp} = 15; \]

\[
[c_{glucose}, c_{glucose\_labeled}, c_{interstitial\_insulin}, \ldots
c_{interstitial\_glucose}, c_{G_gut}, c_{G\_gut\_labeled}] =
gluc_sim_new (L0, t_c_insulin1, p_true, plt, ' ...');
\]

if NoMean==0

% interpolate data:

\[ t_{insulin2\_interpl} = t_c_insulin2(1,1):1:t_c_insulin2(end,1); \]
\[ c_{insulin2\_interpl} = interp1(t_c_insulin2(:,1), t_c_insulin2(:,2), \ldots
t_{insulin2\_interpl}); \]
\[ t_{c\_insulin2\_interpl}(::1) = t_{insulin2\_interpl}; \]
\[ t_{c\_insulin2\_interpl}(:,2) = c_{insulin2\_interpl}; \]

% calculating the mean of the interpolated data at the data2
samples

\[ GI_b = L0(4); \]
\[ c_{interstitial\_glucose\_mean}(i,1) = GI_b; \]
i=2;
for \[ r = t_c_insulin2(2:end,1); \]
\[ c_{interstitial\_glucose\_mean}(i,1) = \ldots\]
\[ \text{sum}(c_{interstitial\_glucose\_interpl}(r+1-t_{exp:r}))/t_{exp}; \]
i=i+1;
end
else

% signal from data2

\[ [c_{glucose2}, c_{glucose\_labeled2}, c_{interstitial\_insulin2}, \ldots
c_{interstitial\_glucose2}, c_{G\_gut2}, c_{G\_gut\_labeled2}]; \]

end
end

data vector of the oral Minimal Model of glucose kinetics using the
true
1D dimensional parameter space with interpolated insulin
countenance
complex.

data1: data of plasma glucose["a"]
data1=c_glucose1;
\* sample interval [min] of data in c_interstitial_samples
\texp=15;
data2: data of interstitial glucose ["a"]
\tNoMean-0
\tdata2=c_interstitial_glucose_mean;
\* else
\tdata2=c_interstitial_glucose2;
\* end
\* data: data of plasma labeled glucose["a"]
data3=c_glucose_labeled1;
D.2.3

gluc_omm_mls2_new.m

```matlab
function [resnorm] = gluc_omm_mls2_new(data1, data2, data3, L0, t_c_insulin1, ...
    t_c_insulin2, a, b, c, sigma1, sigma2, sigma3, p, p_fix, p_init, t_exp, ...)
    Nparameters, estimator, NoMean, plt)
```

The function computes the glucose concentration of diabetics.

- `data1`, `data2`, `data3`: Input data.
- `L0`: Initial guess for concentration.
- `t_c_insulin1`, `t_c_insulin2`: Time of injection.
- `a`, `b`, `c`: Parameters of the insulin model.
- `sigma1`, `sigma2`, `sigma3`: Variance parameters.
- `p`, `p_fix`, `p_init`: Parameters of the model.
- `t_exp`: Time points for the experiment.
- `Nparameters`: Number of parameters.
- `estimator`, `NoMean`: Estimation method.
- `plt`: Plotting option.

This function implements the model to compute the glucose concentration of diabetics.
for ww=1:Nparameters
    if ww==estimator(1)
        p_model=[p_model p_init(1)];
        offset=offset+1;
    elseif ww==estimator(2)
        p_model=[p_model p_fix(ww-offset)];
        offset=offset+1;
    else
        end
    end
end

% Simulate the final Minimal Model of glucose dynamics using the final 14 dimensional parameter space for the inter-related insulin concentration
samples for dataset, [c_glucose,c_glucose_labeled,c_interstitial_insulin1,c_interstitial_glucone,c_Gut1,c_Gut_labeled1]...
    =gluc_sim_new([0,t_c_insu1in1,p_model,plt,"'"""""");

if NoMean==0
    [t_insu1in2_interpl = t_c_insu1in2(1,1):t_c_insu1in2(end,1)]
    c_insu1in2_interpl=interp1(t_c_insu1in2(:,1),t_c_insu1in2(:,2),...
        t_insu1in2_interpl);
    t_c_insu1in2_interpl(:,1)=t_insu1in2_interpl;
    t_c_insu1in2_interpl(:,2)=c_insu1in2_interpl;
calculating the mean of the interpolated data2 at the data2 samples:

GLb=L0(4);
c_interstitial_glucose_mean(l,1)=GLb;
i=2;
for r=t_c_insulin2(2:end,1)
    c_interstitial_glucose_mean(i,1)=...
    sum(c_interstitial_glucose_interpl(r+1-t_exp:r))/t_exp;
i=i+1;
end

else
    % simulation of drug d1
    [c_glucose2,c_glucose_labeled2,c_interstitial_insulin2,...
    c_interstitial_glucose2,c_Ggut2,c_Ggut_labeled2]=gluc_sim_new (L0,t_c_insulin2,p_model,plt,');
end

% error function results:
error1=c_glucose1-data1;
edu=0
error2=c_interstitial_glucose_mean-data2;
else
    error2=c_interstitial_glucose2-data2;
end
error3=c_glucose_labeled1-data3;

% total model error

% if datasets have different range of samples, the dimensions of
% error2 and data2 are set equal except at the
% initial value:
error1_new=error1;
error2_new=error2;
error3_new=error3;
data1_new=data1;
data2_new=data2;
data3_new=data3;

if length(error1)>length(error2)
    for q=1:(length(error1)-length(error2))
        error2_new(length(error2)+q)=0;
data2_new(length(data2)+q)=1;
    end
end
function in line 121. It does not have negative effect under
error1_new
set error2_new is already set
end
error1_new=error1;
data1_new=data1;
error3_new=error3;
data3_new=data3;
else
for q=1:(length(error2)-length(error1))
    error1_new(length(error1)+q)=0;
data1_new(length(data1)+q)=1;
    error3_new(length(error3)+q)=0;
data3_new(length(data3)+q)=1;
end
error2_new=error2;
data2_new=data2;
end

A squared 2 norm of the error function residuals
resnorm1=1*(1/length(error1))*sum((error1_new).^2);
resnorm2=1*(1/length(error2))*sum((error2_new).^2);
resnorm3=125*1*(1/length(error3))*sum((error3_new).^2);

Total squared 2 norm of the error function residuals
resnorm1+resnorm2+resnorm3;
resnorm=resnorm1+resnorm2+resnorm3;
D.2.4

**gluc_omm_mls_bias_new.m**

```matlab
function [p_model, L0_model, c_glucose_model, c_glucose_labeled_model, ...
c_interstitial_insulin_model, c_interstitial_glucose_model, ...
c_Gut_model, resnorm, residual, varp, stdp] = gluc_omm_mls_bias_new(data1, ...
data2, data3, L0, t_c_insulin1, t_c_insulin2, a, b, c, sigma1, sigma2, sigma3, ...
p_fix, p_init, t_exp, Nparameters, estimator, NoMean, plt)
```

- **p_model**: Model parameters.
- **L0_model**: Initial conditions.
- **c_glucose_model** and **c_glucose_labeled_model**: Models of glucose and labeled glucose.
- **c_interstitial_insulin_model** and **c_interstitial_glucose_model**: Models of interstitial insulin and glucose.
- **c_Gut_model**: Model of the gut.
- **resnorm**, **residual**, and **varp**: Residuals and variances.
- **stdp**: Standard deviation of the parameters.

Additional parameters are used to define the initial conditions and parameters for the model equations.
options = optimset('optimality tolerance', 1e-10, ... 'input data', options, p_fix, L0, data1, data2, ...

data3, t_c_insulin1, t_c_insulin2, a, b, c, sigma1, sigma2, sigma3, t_exp, ...
Nparameters, estimator, NoMean); t_init, L0, optimizer,
if exitflag==0
STOP=1;
end

% Variance:
% The posterior variance for each parameter is obtained as:
varp = resnorm'inv(jacobian'*jacobian)/length(t_c_insulin(:,1));
% Standard deviation:
stdp = sqrt(diag(varp));
% And the covariance matrix is the square root of the covariance:

% 1-D dimensional parameter space
% 110
p_model = [];
offset=0;
for ww=1:Nparameters
    if ww==estimator(1)
        offset=offset+1;
        p_model=[p_model p_estimated(1)];
    else
        offset=offset+1;
        p_model=[p_model p_estimated(2)];
    elseif ww==estimator(2)
        offset=offset+1;
        p_model=[p_model p_estimated(2)];
    else
        offset=offset+1;
        p_model=[p_model p_fix(ww-offset)];
    end
end

L0_model = [p_fix(2),0,0,0];

% make plots with new 1-D dimensional parameter space:
[p_c_glucose_model,p_c_glucose_labeled_model,p_c_interstitial_insulin_model,...
  p_c_interstitial_glucose_model,p_c_Ggut_model]= gluc_sim_new (L0,...
  t_c_insulin, p_model, plt, 'plot');
D.2.5

obj_fn_bias_new.m

function error = obj_fn_bias_new(p_var, p_fix, L0, data1, data2, data3,...
    t_c_insulin1, t_c_insulin2, a, b, c, sigma1, sigma2, sigma3,...
    t_exp, Nparameters, estimator, NoMean)

pvar=p_var

This function is used to calculate the objective function for the bias.

Inputs:
- a: parameter a which will be estimated
- b: parameter b which will be estimated
- c: fixed parameter c which is not
- t_c_insulin: array of times, in [h] for the concentration of insulin
- t_exp: array of times, in [h] for the experiment
- Nparameters: number of parameters
- estimator: estimator to be used
- NoMean: boolean indicating if the mean is calculated

dat1: data in millimoles per liter (mM).

dat2: data in millimoles per liter (mM).

dat3: data in millimoles per liter (mM).

t_c_insulin: [PM] (pmol) time and required samples matrix of plasma

Results: error is the calculated objective function.
- Output weight function of output.
- Output weight function of output.
- Output weight function of output.
- Normal: normal distribution
- Estimator: estimator to be used
- NoMean: boolean indicating if the mean is calculated

Author: John Doe

Date: March 03, 2023

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estimating() in the input and process the output

p_estimating=[];
offset=0;

for ww=1:Nparameters
    if ww==estimator(1)
        offset=offset+1;
        p_estimating=[p_estimating p_var(1)];
    elseif ww==estimator(2)
        offset=offset+1;
        p_estimating=[p_estimating p_var(2)];
    else
        p_estimating=[p_estimating p_fix(ww-offset)];
    end
end

%Simulating the 2D Simulated Model 4 different insulin estimation
%4 dimensional parameter spaces and the interplated insulin concentration
%samples for data.

plt=0;  % make 3D plots,
[c_glucose_estimating,c_glucose_labeled_estimating,...
c_interstitial_insulin_estimating,c_interstitial_glucose_estimating,...
c_Ggut_estimating] = gluc_sim_new(LG,t_c_insulin1,
p_estimating,plt,...
'NoMean');

%Simulating the 2D Simulated Model 4 different insulin estimations
%4 dimensional parameter spaces and the interplated insulin concentration
%samples for data.

if NoMean == 0
    t_insulin2_interpl = t_c_insulin2(1,1):t_c_insulin2(end,1);[num]
c_insulin2_interpl=interpl(t_c_insulin2(:,1),t_c_insulin2(:,2),...
t_insulin2_interpl);
t_c_insulin2_interpl(:,1)=t_insulin2_interpl;
t_c_insulin2_interpl(:,2)=c_insulin2_interpl;
[c_glucose_estimating2_interpl,c_glucose_labeled_estimating2_interpl,...
c_interstitial_insulin_estimating2_interpl,....
c_interstitial_gIucose_estimating2_interpl,...
c_Ggut_estimating2_interpl]=gluc_sim_new(L0,t_c_insulin2_interpl,
...p_estimating,plt,"''''');

% calculating the mean of the interstitial data at the data samples
GIb=LO(4);
c_interstitial_gIucose_estimating2_mean(1,1)=GIb;
i=2;
for r=t_c_insulin2(2:end,1)'c_interstitial_gIucose_estimating2_mean(1,1)=...
sum(c_interstitial_gIucose_estimating2_interpl(r+1-t_exp:r))/t_exp;
i=i+1;
end
else
[c_glucose_estimating2,c_glucose_labeled_estimating2,...
c_interstitial_insulin_estimating2,...
c_interstitial_gIucose_estimating2,c_Ggut_estimating2]=...
' ''''');
end

% SQNM IN: objective function should return the model error

err1 = c_glucose_estimating-data1;
if NoMean==0
err2= c_interstitial_gIucose_estimating2_mean-data2;
else
err2= c_interstitial_gIucose_estimating2-data2;
end

err3=c_glucose_labeled_estimating-data3;

Total model error:

if datasets have different number of samples, the dimensions of
err1 and err2 are not equal, therefore:
err1_new=err1;
err2_new=err2;
err3_new=err3;
data1_new=data1;
data2_new=data2;
data3_new=data3;

if length(err1)>length(err2)
for q=1:length(err1)-length(err2)
err2_new(length(err2)+q)=0;
data2_new(length(data2)+q)=1; 
error1_new(length(error1)+q)=0; 
data1_new(length(data1)+q)=1; 
error3_new(length(error3)+q)=0; 
data3_new(length(data3)+q)=1; 

error2_new=error2; 
data2_new=data2;

else 
for q=1:(length(error2)-length(error1)) 
error1_new(length(error1)+q)=0; 
data1_new(length(data1)+q)=1; 
error3_new(length(error3)+q)=0; 
data3_new(length(data3)+q)=1; 
end 
error2_new=error2; 
data2_new=data2;
end

error=le4*error1+le4*error2+125*le4*error3;

if 1
figure(100)
subplot(411);
plot(t_c_insulin1(:,1),c_interstitial_insulin_estimating,'r');
xlabel('');
ylabel('');
Title({'1 '});
drawnow;pause(.3);

subplot(412);
plot(t_c_insulin1(:,1),c_glucose_estimating,'r',t_c_insulin1(:,1),
...data1,'r');
xlabel('');
ylabel('');
Title({'1 '});
drawnow;pause(.3);

subplot(413);
plot(t_c_insulin1(:,1),c_glucose_labeled_estimating,'r',
...t_c_insulin1(:,1),data3,'r');
xlabel('i\cdot\cdot\cdot');
ylabel('i\cdot\cdot\cdot');
Title({'i\cdot\cdot\cdot';...
    'i\cdot\cdot\cdot'});
drawnow;pause(.3);

figure(100);
subplot(414);
if NoMean==0
    plot(t_c_insulin2(:,1),c_interstitial_glucose_estimating2_mean,'-',...
    t_c_insulin2(:,1),data2,'r');
else
    plot(t_c_insulin2(:,1),c_interstitial_glucose_estimating2,'-',...
    t_c_insulin2(:,1),data2,'r');
end
xlabel('i\cdot\cdot\cdot');
ylabel('i\cdot\cdot\cdot');
Title({'i\cdot\cdot\cdot';...
    'i\cdot\cdot\cdot'});
drawnow;pause(.3);
end
D.3 worstcase bias

worstcase_bias_ss.m

function [WorstCase_Bias, estimation] = worstcase_bias_SS()

% The Minimal Model of gluconeogenesis.
% This script calculates the worst case of the Minimal Model of gluconeogenesis.
% History
% 25.08.36, Salvador Alvarado Frutos, first
% 49.
% Function:
% [WorstCase_Bias] = worstcase_SS()
% Input:
% None
% Output:
% [WorstCase_Bias] = worstcase_SS()
% WorstCase_Bias = [-
% gluc-param_new;
% Nparameters=length(p_true);
% NoMean=0; %Calculates the mean over t_estimation of internal glucose
% %samples if NoMean 0

diary Hi_;
filename = 'Hi_';
set the name of the file to save the estimated parameters:
filename = filename';

p = [lb Gb p2 DS k p5 p6 SiCap SiLiver Coeff];

p_init = [Kabs V p4 SiGlu4];

OS = 75/10; % Initial optimal input
DS=75/10; % Initial optimal input
6 unknown model parameters:

p_fix = [lb Gb p2 DS Beta k p5 p6 SiCap SLiver Coeff];

sample schedules with OS and DS to input for model
t_insulin_interpl = t_c_insulin(1,1):1:t_c_insulin(end,1);
c_insulin_interpl=interpl(t_c_insulin(:,1),t_c_insulin(:,2),...t_insulin_interpl);
t_c_insulin_interpl(:,1)=t_insulin_interpl;
t_c_insulin_interpl(:,2)=c_insulin_interpl;


t_insulin =1:1:t_c_insulin(end,1)
c_insulin =1:120:240;
for uuu=1:length(t_insulin)
c_insulin(uuu) = c_insulin_interpl(t_insulin(uuu)+1);
end
clear
t_c_insulin(:,1)=t_insulin;
t_c_insulin(:,2)=c_insulin;

% simulation using data
[c_glucose_model,c_glucose_labeled_model,...
c_interstitial_insulin_model,...
c_interstitial_glucose_model,c_Ggut_model]=...
gluc_sim_new(L0,t_c_insulin, p_true,plt,...
'liches:');

if NoMean==0
% interpolation data

t_insulin2_interpl = t_c_insulin2(1,1):1:t_c_insulin2(end,1);
c_insulin2_interpl=interpl(t_c_insulin2(:,1),t_c_insulin2(:,2),...t_insulin2_interpl);
t_c_insulin2_interpl(:,1)=t_insulin2_interpl;
t_c_insulin2_interpl(:,2)=c_insulin2_interpl;

% simulation using data
[c_glucose_model2_interpl,c_glucose_labeled_model2_interpl,...
c_interstitial_insulin_model2_interpl,...
c_interstitial_glucose_model2_interpl,...
c_Ggut_model2_interpl,c_Ggut_model2_labeled_interpl]....
=gluc_sim_new (L0,t_c_insulin2_interpl,p_true,plt,...
'iliches:');

% calculation the mean of the interpolation data of the data

Glb=L0(4);
c_interstitial_glucose_mean(1,1)=Glb;
i=2;
for r=t_c_insulin2(2:end,1)

c_interstitial_glucose_mean(i,1)...
=sum(c_interstitial_glucose_model2_interpl(r+1-t_exp:r))/t_exp;
i=i+1;
end

else
% simulation using data
[c_glucose_model2,c_glucose_labeled_model2,...
c_interstitial_insulin_model2,...
c_interstitial_glucose_mode12,c_Ggut_mode12,c_Ggut_labeled_mode12]=gluc_sim_new (LO,t_c_insulin2,p_true,plt,' '); end

I'

I)',

datat_new=t_insulin1;
datal_new=c_glucose_model;
data2_new=c_interstitial_glucose_mean;
data2_new=c_interstitial_glucose_mode12;
data3_new=c_glucose_labeled_model;
es init ination:

p_init_copy=p_init;
b=1;
estimation(1,:)=[-0.5 -0.4 -0.3 -0.2 -0.1 0.1 0.2 0.3 0.4 0.5 ];
estimation1(1,:)=estimation(1,:);
estimation2(1,:)=estimation(1,:);
estimation3(1,:)=estimation(1,:);
estimation4(1,:)=estimation(1,:);
for kk=1:length(p_init)
ll=0;
for zz=estimation(1,:);
 p_init=p_init_copy;
 ll=ll+1;
estimator=find(p_init(kk)==p_true);
estimator1=find(p_init(1)==p_true);
estimator2=find(p_init(2)==p_true);
estimator3=find(p_init(3)==p_true);
estimator4=find(p_init(4)==p_true);
p_True=p_true(estimator);
p_init(kk)=p_init(kk)+zz*p_init(kk)
[p_model,LO_model,c_glucose_model,c_glucose_labeled_model,...
c_interstitial_insulin_model,...
c_interstitial_glucose_model,
c_Ggut_model,resnorm,residual,varp,...
,stderr]=gluc_omm_mls_new(data1_new,data2_new,data3_new,LO,...
t_c_insulin1,t_c_insulin2,a,...
b,c,sigma1,sigma2,sigma3,p_fix,p_init,t_exp,Nparameters,NoMean,plt);
estimator
p_Model=p_model(estimator)
estimation(kk+l,11)=p_model(estimator);
estimation1(kk+l,11)=p_model(estimator1);
estimation2(kk+l,11)=p_model(estimator2);
estimation3(kk+l,11)=p_model(estimator3);
estimation4(kk+l,11)=p_model(estimator4);
end

// Calculation script for estimation
WorstCase_Bias(b,:)=max(abs(p_true(estimator)-estimation(kk+l,:)));
b=b+1;
end
TARGET=fopen(filename,");
fprintf(TARGET,...

\ estimation1');
fprintf(TARGET,...

\ estimation2');
fprintf(TARGET,...

\ estimation3');
fprintf(TARGET,...

\ estimation4');
fclose(TARGET);
D.4 A priori parameter sensitivity

**param_sens_new.m**

```matlab
function [varp_matrix]=param_sens_new()

% Clear Minimal Model of glucose Glucose 1.0L
% This function analyzes the a priori parameters used
% Vendor's coefficient parameters together with the optimal input.
% Bit variables the a priori parameter v, i, b, k, p1, p2 and 10 = 1
% 2

% History
% 07-july 06, Salvador Almeida Barros, Iber
% % Function:
% % [varp_matrix] = param_sens_new
% % Inputs function:
% % NOW
% % Outputs function:
% % varp_matrix: functions of the relative variations of the a
% % priori parameters by v, i, b, k, p1, p2 and 10

% initialization
clear ~
Gluc_param_new;
Nparameters=length(p_true);
NoMean=0;
Calculates the mean and standard of interstitial glucose
Samples at NoMean=0

% Sets the name of the matrix with the variables of interest:
filename='diary.dat';
diary

p = [1b Gb p2 DS Beta k p5 p6 SiCap SIliver Coeff];
% using optimal input
DS=75/10;[DS_I_opt] optimiation
% Unknown model parameters:
p_fix = [1b Gb p2 DS Beta k p5 p6 SiCap SIliver Coeff];
% Sample schedule with glucose 1.4 V
% t_insulin_interpl = t_c_insulin1(1,1):t_c_insulin1(end,1);[min]
```

120
c_insulin_interpl=interpl(t_c_insulin(:,1),t_c_insulin(:,2),...
    t_insulin_interpl);
t_c_insulin_interpl(:,1)=t_insulin_interpl;
t_c_insulin_interpl(:,2)=c_insulin_interpl;

% Simulation using data1
[c_glucose_model_interpl,c_glucose_labeled_model_interpl, ...
    c_interstitial_insulin_model_interpl, ...]
    =gluc_sim_new(t_c_insulin_interpl,p_true,plt,' ...');

if NoMean==0
    t_insulin2_interpl = t_c_insulin2(1,1):1:t_c_insulin2(end,1);
    c_insulin2_interpl=interpl(t_c_insulin2(:,1),t_c_insulin2(:,2),...
        t_insulin2_interpl);
t_c_insulin2_interpl(:,1)=t_insulin2_interpl;
t_c_insulin2_interpl(:,2)=c_insulin2_interpl;

    % Simulation using data2
[c_glucose_model2_interpl,c_glucose_labeled_model2_interpl, ...
    c_interstitial_insulin_model2_interpl, ...]
    =gluc_sim_new(t_c_insulin2_interpl,p_true,plt,' ...');

    % Calculating the mean of the interstitial glucose of the data2
    G Ib=L(4);
    c_interstitial_glucose_mean(1,1)=G Ib;
    i=2;
    for r=t_c_insulin2(2:end,1)'
        c_interstitial_glucose_mean(1,1)=...
            sum(c_interstitial_glucose_model2_interpl(r+1-t_exp:r))/t_exp;
        i=i+1;
    end
else
    % Simulation using data1
    % c_interstitial_glucose_model2, c_Ggut_model2, c_Ggut_labeled_model2]
    =gluc_sim_new(t_c_insulin2_interpl,p_true,plt,' ...');
end

% Writing output
.datat_new=t_insulin_interpl;
.datat_new=c_glucose_model_interpl;
.datat_new=datat1_new+0.05 * randn(length(datat1_new),1);

if NoMean==0
    data2_new=c_interstitial_glucose_mean;
data2_new=data2_new+0.10 * randn(length(data2_new),1);
end
else
data2_new=c_interstitial_glucose_mode12;
data2_new=data2_new+0.10 * randn(length(data2_new),l);
end
data3_new=c_glucose_labeled_model_interpl;

t::'JJU l!'

p_fix_copy=p_fix;
estimation(1,:)=[-0.3 -0.20 -0.1 -0.05 0.05 0.1 0.20 0.30];
estimation1(1,:)=estimation(1,:);
estimation2(1,:)=estimation(1,:);
estimation3(1,:)=estimation(1,:);
estimation4(1,:)=estimation(1,:);
for kk=1:length(p_fix)
  ll=0;
p_fix=p_fix_copy;
  for zz=estimation(1,:)
    p_init_saved = [Kabs V p4 SIglut4];
estimator1=find(p_init_saved(1)==p_true)
estimator2=find(p_init_saved(2)==p_true)
estimator3=find(p_init_saved(3)==p_true)
estimator4=find(p_init_saved(4)==p_true)
    ll=ll+1;
    p_fix(kk)=p_fix(kk) + zz*p_fix(kk)
    [p_model,LO_model,c_glucose_model,c_glucose_labeled_model,....
c_interstitial_insulin_model,....
c_interstitial_glucose_model,
c_Ggut_model,resnorm,residual,varp...,.
varp_matrix(kk,ll)=det(varp)....
t_c_insulin1_interpl,t_c_insulin2,a,b,c,sigma1,sigma2,sigma3,...
p_fix,p_init,t_exp,Nparameters,NoMean,plt);
    estimation1(kk+1,ll)=p_model(estimator1);
estimation2(kk+1,ll)=p_model(estimator2);
estimation3(kk+1,ll)=p_model(estimator3);
estimation4(kk+1,ll)=p_model(estimator4);
  end
end

TARGET=fopen(filename, );
fprintf(TARGET,...
....
111.
,estimation1');
fprintf(TARGET,...
122
fprintf(TARGET, ... estimation2');
fprintf(TARGET, ... estimation3');
fprintf(TARGET, ... estimation4');
close(TARGET);
varp_matrix=[p_fix' varp_matrix]
D.5 Optimal Input

opt_input_new.m

function [DS_T_opt,DS_matrix]=opt_input_new()

% % %
% % %
% % %
% % %

% History:
% © July 06, Salvatore Sciannameo, Ilia

% Function:
% [DS_T_opt,DS_matrix] opt_input_new

input function:

output function:

gluc_param_new;
Nparameters=length(p_true);
NMean=0; % Calculate the mean over 800 minutes of interstitial glucose
samples;
plt=0; % Make a plot

% Sets the number of the model to identify the estimated parameters:
filename='gluc_param_new';
diary = 'gluc_param_new';

p = [1b, 2b, p4, p6, GB, Beta, v, w, k, p4, p6, SIcap, Sl gener, S Igglut];
% Include initial parameter
p_fix = [1b, GB, p4, p6, SIcap, S Igglut4];
% Include initial parameter
p_init = [1.4*Kabs, 1.4*V, 1.5*p4, 1.5*SIg1ut4];

% Sample schedules with a frequency of 1 minute for data:
t_insulin1_interpl = t_c_insulin1(:,1):t_c_insulin1(end,1);
t_c_insulin1_interpl=interpl(t_c_insulin1(:,1),t_c_insulin1(:,2),...
    t_insulin1_interpl);
t_c_insulin1_interpl(:,1)=t_insulin1_interpl;
t_c_insulin1_interpl(:,2)=c_insulin1_interpl;

% Plot the grid with data:
t_insulin2_interpl = t_c_insulin2(:,1):t_c_insulin2(end,1);
t_c_insulin2_interpl=interpl(t_c_insulin2(:,1),t_c_insulin2(:,2),...
    t_insulin2_interpl);
t_insulin2_interpl;
t_c_insulin2_interpl(:,1)=t_insulin2_interpl;
t_c_insulin2_interpl(:,2)=c_insulin2_interpl;

end initialization.

different values for DS (Integrate DS1 and DS2)
DS_vector=[1:100]/10; %create a vector for different
equations.
kkz=0;

for z=1:length(DS_vector)
    kkz=kkz+1
    p_init_saved = [Kabs V p4 S1glut4];
    estimator1=find(p_init_saved(1)==p_true)
    estimator2=find(p_init_saved(2)==p_true)
    estimator3=find(p_init_saved(3)==p_true)
    estimator4=find(p_init_saved(4)==p_true)
    
    change DS
    DS=DS_vector(z)
    p_true(4)=DS;
    p_fix = [Ib Gb p2 DS Beta k p5 p6 S1cap S1liver Coeff];
    estimator_model in run a sample schedule with a frequency of 1 minute.

    [c_glucose_model_interpl,c_glucose_labeled_model_interpl,...
    c_interstitial_insulin_model_interpl,...
    c_interstitial_glucose_model_interpl,c_Ggut_model_interpl]...
    =gluc_sim_new(L0,t_c_insulin_interpl, p_true,plt,...
    'pfix_true=1');

    i! NoMean==0
    * Interpolating data?

    % simulation using ds=2
    [c_glucose_model2_interpl,c_glucose_labeled_model2_interpl,...
    c_interstitial_insulin_model2_interpl,...
    c_interstitial_glucose_model2_interpl,...
    c_Ggut_model2_interpl,c_Ggut_model2_labeled_interpl]....
    =gluc_sim_new (L0,t_c_insulin2_interpl,p_true,plt,'pfix_true=1');

    % calculate the mean of the first and last value of the output
    % samples
    GIb=L0(4);
    c_interstitial_glucose_mean(1,1)=GIb;
    i=2;
    for r=t_c_insulin2(2:end,1)'
        c_interstitial_glucose_mean(i,1)=...
        sum(c_interstitial_glucose_model2_interpl(r+1-t_exp:r))/t_exp;
    i=i+1;
    end
i=i+1;
end
else
    % simulation using data:
    [c_glucose_mode12, c_glucose_labeled_mode12, ...
    c_interstitial_insulin_mode12, ...
    c_interstitial_glucose_mode12, c_Ggut_mode12, c_Ggut_labeled_mode12]...
    =gluc_sim_new (LO,t_c_insulin2,p_true,plt,' ');
end

% adding of error

datat_new=t_insulin1_interpl;
data1_new=c_glucose_model_interpl;
data1_new=data1_new+0.025 * randn(length(data1_new),1);
if NoMean==0
data2_new=c_interstitial_glucose_mean;
data2_new=data2_new+0.05 * randn(length(data2_new),1);
else
data2_new=c_interstitial_glucose_model2;
data2_new=data2_new+0.05 * randn(length(data2_new),1);
end
data3_new=c_glucose_labeled_model_interpl;

% plot of data with inflection

figure;
Gb=p_true(2);
subplot (311); plot (datat_new,datal_new, ', ','o',2);
hold on;
plot (t_c_insulin1_interpl(:,1),c_glucose_model_interpl, ' ',
'Linewidth',2);
hold on;
plot([datat_new(1) datat_new(end)], [Gb Gb], ' ', ' ', ' ', 1.5)
xlabel('1.1.1.1');
ylabel('1.1.1.1');
Title('');

if NoMean==0
subplot(312); plot(t_c_insulin2(:,1), data2_new, ' ', 'o',1.5,2)
hold on;
plot(t_c_insulin2(:,1),c_interstitial_glucose_mean, ' ',...
'Linewidth',2);
else
subplot(322); plot(t_c_insulin2(:,1), data2_new, ' ', 'o',1.5,2)
hold on;
plot(t_c_insulin2(:,1),c_interstitial_glucose_model2, ' ',...
'Linewidth',2);
end
xlabel('title'); ylabel('title');

title('title');

subplot(311); plot(datat_new, data3_new, 'r', 2);
hold on;
plot(t_c_insulin1_interpl(:, 1), c_glucose_labeled_model_interpl, 'r', '...');
hold on;
plot([datat_new(1) datat_new(end)], [Gb Gb], 'l', 'k');
xlabel('title');
ylabel('title');
title('title');

estimation1(kkz) = p_model(estimator1);
estimation2(kkz) = p_model(estimator2);
estimation3(kkz) = p_model(estimator3);
estimation4(kkz) = p_model(estimator4);

TARGET = fopen(filename, 'w');
fprintf(TARGET, 'estimation1');
fprintf(TARGET, 'estimation2');
fprintf(TARGET, 'estimation3');
fprintf(TARGET, 'estimation4');
fclose(TARGET);
D.6 Optimal Sample Schedule

**opt_sample_schedule_new.m**

```matlab
function [opt_SS_data1, opt_SS_data2, opt_data1, opt_data2, 
    , opt_data3] = opt_sample_schedule_new()
```

Variation Number: 1
- The function implements the optimal sample schedule (SS) for the
  first variation number (1). It takes into consideration the highest

Variation
- The algorithm is based on a flexible conventional sample schedule where
  each individual sample is optimally resampled with its

Samples in the conventional sample schedule,

```
% Inputs:
% NoMean=0; NoScale=0
% Max_Exp_Dur=240;
% gluc_param2_new;
% Nparameters=length(p_true);
```

% Initialization
```matlab
clear
NoMean=0;
NoScale=0;
Max_Exp_Dur=240;
```

% Outputs:
```matlab
```
```
```
DS=75/10; [plot, t_c, pfix, plt] = cost_input();

% choose parameters:
\pfix = [ib Gb p2 DS Beta k p5 p6 Slcap Sliver Coeff];

% initialize model parameters:
 \tinsulin = [31, 32, p2 DS Beta k p5 p6 Slcap Sliver Coeff];

% p fixed = [1.4*Kabs 1.4*V 1.4*p4 1.4*SIglut4];

% Simulation model using a sample input with a frequency of 1 minute
% and interpolating input:
plt=0;

% sample schedule for data with a frequency of 1 minute
\tinsulin=interp1(t_c_insulin1(:,1), t_c_insulin1(:,2), ...
\tinsulin_interpl);
\t_c_insulin1_interpl(:,1)=t_insulin_interpl;
\t_c_insulin1_interpl(:,2)=c_insulin1_interpl;
[c_potential_interpl, c_glucose_model_interpl, ...
 c_interstitial_insulin_model_interpl, ...]
=gluc_sim_new([0, t_c_insulin1_interpl, p_true, plt, ...]);

% simulate data:
\tinsulin=interp1(t_c_insulin2(:,1), t_c_insulin2(:,2), ...
\tinsulin_interpl);
\t_c_insulin2_interpl(:,1)=t_insulin_interpl;
\t_c_insulin2_interpl(:,2)=c_insulin2_interpl;
[c_potential_interpl, c_glucose_model_interpl, ...
 c_interstitial_insulin_model_interpl, ...]
=gluc_sim_new([0, t_c_insulin2_interpl, p_true, plt, ...]);

% adding noise
datat_new=t_insulin1_interpl;
\datat_new=interp1(c_potential_model1_interpl, [1/3, 0.6], 'linear');

[s1, fig1] = plot1;
datat_new=c_glucose_model_interpl;
\datat_new=interp1(c_interstitial_insulin_model1, [1/3, 0.6], 'linear');

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data2_new=c_interstitial_glucose_model2;
data3_new=c_glucose_labeled_model_interpl;

continuously do

it 1
Calculating the optimal sample points to inject insulin and input
Algorithm based on data with a fixed insulin sample schedule where
each individual sample is optimized separately with the
One-dimensional optimization in the two insulin sample schedule.

Initialization of conventional sample schedule
opt_datal=datal;
opt_data3=data3;
opt_SS_datal=t_c_insulin1_interpl(:,1);
datacl_using=t_c_insulin1_interpl(:,2);

aa=1:1:(2*t_exp-1);
endvalue=le6;
r=1;

for q=1:t_exp:t_exp:endvalue
    for z=[aa]
        qz=q+z
        display('Sampling of new injection and input
        Algorithm with optimal samples to inject insulin according to the
        One-dimensional optimization in the two insulin sample schedule.

        Optimization of conventional sample schedule
        datatal_using=opt_SS_datal;
datal_usinggg=opt_SS_datal';
datal_using=opt_datal;
data3_using=opt_data3;

datal_using_old=datal1_using' optimality of experiment

datal_using(q/t_exp+1)=opt_SS_datal(q/t_exp)+z;

Spruce up the horizon as supposed to end it sample which

.......
Spruce up the horizon as supposed to end it sample which

.......
If datatal_using(q/t_exp+1)>Max_Exp_Dur
    display('The horizon is suppose
    aa=1:1:(2-r)*t_exp-1
    to finish the horizon
    
    130
\[ r = r + 1; \]
\[ \text{if } \text{numel}(\text{aa}) == 0 \]
\[ \text{display('Error: the entire model is not complete at this time.'}) \]
\[ \text{which} \]
\[ \text{opt_SS_data2} = 0 \]
\[ \text{opt_data2} = 0 \]
\[ \text{return} \]
\[ \text{end} \]

data1_using_old = data1_using; \text{ compute a matrix} \]
data1_using(q/t_exp+1) = data1_new(opt_SS_data1(q/t_exp)+z+1); \text{ compute a matrix} \]
data3_using(q/t_exp+1) = data3_new(opt_SS_data1(q/t_exp)+z+1); \text{ compute a matrix} \]
data1_new_newww = data1_using; \text{ compute a matrix} \]
data3_new_newww = data3_using; \text{ compute a matrix} \]
data1l_using_old = data1l_using; \text{ compute a matrix} \]
data1l_using(q/t_exp+1) = t_c_insulin1_interpl(opt_SS_data1(q/t_exp)+z+1, 2); \text{ compute a matrix} \]
data1l_c_using_new = data1l_using; \text{ compute a matrix} \]
data1l_c_using = [data1l_using data1l_using]; \text{ compute a matrix} \]

\[ \text{if } (q/t_exp+2) <= \text{length(data1l_using)} \]
\[ \text{pause} \]
\[ \text{ww} = 1; \]
\[ \text{while } (q/t_exp+1+ww) <= \text{length(data1l_using)}-1 \]
data1l_using(q/t_exp+1+ww) = data1l_using(q/t_exp+1+ww)+t_exp \]
data1l_c_using(q/t_exp+1+ww) = data1l_c_using(q/t_exp+1+ww)+t_exp \]
data1l_using(q/t_exp+1+ww) = data1l_using(q/t_exp+1+ww)+t_exp \]
data3l_using(q/t_exp+1+ww) = data3l_using(q/t_exp+1+ww)+t_exp \]
data1l_using = data1l_using(1:length(data1l_using)-1) \]
data1l_c_using = data1l_c_using(1:length(data1l_c_using)-1) \]
data1l_using = data1l_using(1:length(data1l_using)-1) \]
data3l_using = data3l_using(1:length(data3l_using)-1) \]
data1l_c_using = [data1l_using data1l_c_using]; \text{ compute a matrix} \]
opt_SS_data1 = data1l_using \text{ compute a matrix} \]
opt_data1 = data1l_using \text{ compute a matrix} \]
opt_data3 = data3l_using \text{ compute a matrix} \]
end \end{verbatim}
The model parameters are not clear, but it seems to be related to:

\[ p_{\text{model}}, L_0_{\text{model}}, c_{\text{glucose}_{\text{model}}}, c_{\text{glucose}_{\text{labeled}_{\text{model}}}}, \]
\[ c_{\text{interstitial}_{\text{insulin}_{\text{model}}}}, \]
\[ c_{\text{interstitial}_{\text{glucose}_{\text{model}}}}, c_{G_{\text{gut}_{\text{model}}}}, \text{resnorm}, \text{residual}, \text{varp}, \]

\[ \text{stdp} = \text{gluc_omm_mis_new}(\text{data1}_{\text{using}}, \text{data2}_{\text{using}}, \text{data3}_{\text{using}}, L_0, \text{data1}_{\text{c}_{\text{using}}}, \]
\[ \text{t}_{\text{c}_{\text{insulin}_{\text{2}}}}, a, b, c, \text{sigma1}, \text{sigma2}, \text{sigma3}, p_{\text{fix}}, p_{\text{init}}, t_{\text{exp}}, \]
\[ \text{Nparameters}, \text{NoMean}, \text{plt}); \]

if \( z = a_{\text{a}(1)} \)

\[ \text{varp}_{\text{old}} = \text{varp}; \]
\[ \text{optimumDET} = \text{det}(\text{varp}) \]
\[ \text{opt}_{\text{SS}_{\text{data1}}}=\text{data1}_{\text{using}} \]
\[ \text{opt}_{\text{data1}}=\text{data1}_{\text{using}} \]
\[ \text{opt}_{\text{SS}_{\text{data2}}}=\text{data2}_{\text{using}} \]
\[ \text{opt}_{\text{data2}}=\text{data2}_{\text{using}} \]
\[ \text{opt}_{\text{SS}_{\text{data3}}}=\text{data3}_{\text{using}} \]
\[ \text{opt}_{\text{data3}}=\text{data3}_{\text{using}} \]

\[ \text{NOTSAVED}=1 \]

\[ \text{if} \] \( \text{det}(\text{varp})<\text{det}(\text{varp}_{\text{old}}) \)
\[ \text{varp}_{\text{old}} = \text{varp}; \]
\[ \text{opt}_{\text{SS}_{\text{data1}}}=\text{data1}_{\text{using}} \]
\[ \text{opt}_{\text{data1}}=\text{data1}_{\text{using}} \]
\[ \text{opt}_{\text{SS}_{\text{data2}}}=\text{data2}_{\text{using}} \]
\[ \text{opt}_{\text{data2}}=\text{data2}_{\text{using}} \]
\[ \text{opt}_{\text{SS}_{\text{data3}}}=\text{data3}_{\text{using}} \]
\[ \text{opt}_{\text{data3}}=\text{data3}_{\text{using}} \]
\[ \text{SS} \]
\[ \text{else} \]
\[ \text{det}(\text{varp}) \text{det}(\text{varp}_{\text{old}}) \]
\[ \text{NOTSAVED}=1 \]
\[ \text{end} \]
\[ \text{end} \]
\[ \text{end} \]
\[ \text{if} \quad 0 \]
\[ \text{Calculating the optimal sample schedule for input 2} \]
\[ \text{Initializing the initial sample schedule} \]
\[ \text{Selecting the initial schedule with a prespecified sample schedule where} \]
\[ \text{each individual sample is assigned to a specific portion of the} \]
\[ \text{sample schedule.} \]
\[ \text{opt}_{\text{data2}}=\text{data2}_{\text{using}} \]
\[ \text{opt}_{\text{SS}_{\text{data2}}}=\text{t}_{\text{c}_{\text{insulin}_{\text{2}}}}(:,1); \]

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aa = \text{t}_{\text{exp}}:5 \text{t}_{\text{exp}};
endvalue = \text{le6};
r = 1;

\text{for } q=1:5:10 \text{endvalue}
\text{for } z=[aa]
\text{qz} = q + z
\text{end}

% Creating optimal sample schedule at the beginning
data2\_using = \text{opt\_SS\_data2};
data2\_using = \text{opt\_data2};

% Calculating the mean of the interpolated data at the data2 samples
SumOfData = 0;
\text{if NoMean} = 0
\text{for } pp = 1:1:10\text{end}
\text{SumOfData} = \text{SumOfData} + \text{data2}\_new(\text{opt\_SS\_data2}(q/5) + z + 1 - (pp));
\text{mean data of new sample}
Meandata = 1/10 * \text{SumOfData};
data2\_using(q/5 + 1) = \text{Meandata};
data2\_using(q/5 + 1) = \text{data2}\_new(\text{opt\_SS\_data2}(q/5) + z);
\text{saving}
\text{data of new sample}
\text{end}

\text{data2}\_using\_old = \text{data2}\_using';
data2\_using(q/5 + 1) = \text{opt\_SS\_data2}(q/5) + z;
data2\_using\_new = \text{data2}\_using';
data2\_new\_new = \text{data2}\_using';

\text{since if the last iteration did not end in a sample which}
\text{saved the real sample schedule or it had}
\text{if data2}\_using(q/5 + 1) > \text{Max\_Exp\_Dur}
\text{then end the experiment}
\text{display(\text{\text{\small{\textit{'\text{data}\_\text{exp}(5-r) = t\_\text{exp}; the protocol has been met ... \text{at least once at the last sample step'}}}})}
\text{r} = r + 1;
\text{\text{\small{\textit{'\text{display(\text{\text{\small{\textit{'\text{data}\_\text{exp}(5-r) = t\_\text{exp}; the protocol has been met ... \text{at least once at the last sample step'}}}})}
\text{numel} aa = 0
\text{display(\text{\text{\small{\textit{'\text{data}\_\text{exp}(5-r) = t\_\text{exp}; the protocol has been met ... \text{at least once at the last sample step'}}}})}
\text{which}
\text{\text{\small{\textit{'\text{were not calculated in this last loop'}))}}}
\text{opt\_SS\_data1 = 0}
\text{opt\_SS\_data3 = 0}
\text{opt\_data1 = 0}
D.7 White Noise test

wnoisetest_new.m

function [datacov,mean_data1,mean_data2,mean_data3] = ...
    wnoisetest_new(data1, data2, data3, residual)

%%
% Stochastic Model of glucose spectrum pool
% This function analyzes the properties of data used in residuals left
% during the estimation procedure.
%
%%
% History:
% 07 July 06, Salvador Almagre Frutos, TUC
%%
% Function:
% [datacov,mean_data1,mean_data2,mean_data3] wnoisetest_new(data1, data2, residual)
% Inputs:
% data1: data of interstitial insulin [pM]
% data2: data of interstitial glucose [mM]
% data3: data of labeled plasma glucose [mM]
% residual: the value of the residual of the estimation algorithm.
% Outputs:
% datacov: crosscorrelation of the data.
% mean_data1: calculating the mean of the data. Must be zero.
% mean_data2: calculating the mean of the data. Must be zero.
% mean_data3: calculating the mean of the data. Must be zero.

% TEST: calculating the mean. Must be zero.
mean_data1 = mean(data1);  % TEST: Statistical properties: gaussian distribution with zero mean
mean_data2 = mean(data2);
mean_data3 = mean(data3);

% TEST: Statistical properties: gaussian distribution with zero mean
pp = 1:0.01:1;
yy = hist(residual, pp);
figure;
plot(pp, yy); title('');
xlabel('');
ylabel('');

% normalizing residuals
residual = residual ./ max(residual);

% TEST: crosscorrelation. Must be the form of a pulse at zero
% timeshift:
for m = 0:length(residual) - round(0.4 * length(residual))
    sum = 0;

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for n=1:(round(0.4*length(residual)))
    part=residual(n)*residual(n+m);
    sum=sum+part;
end

datacov(m+1)={(1/(round(0.4*length(residual))))}*sum;
end

figure
plot((0:(length(residual)-round(0.4*length(residual))),datacov);
title('');
xlabel('');
ylabel('');
D.8 Worst case bias $S_{cap}$

D.8.1

worstcase_bias_steil.m

```matlab
%%
%% Oral Minimal Model of glucose kinetics.
%%
%% This script estimates the identifiable parameters in Steil's
%% Model of insulin kinetics using Maximum Likelihood
%% Squares Estimation.
%%
%% History
%% 28 July-06, Salvador Almanza Franco, PhD.
%%
%% Initializes all parameters of Steil's Model of insulin kinetics
%% and all time samples of plasma insulin.

clear
gluc_param_new;
Nparameters=length(p_true);
NoMean=0; % Calculates the mean over t_exp minutes of interstitial
          % glucose
          % samples if NoMean=0

diary
% Sets the names of the mat files containing the estimated parameters:
filename='.

% make plots if plt=1
plt=1;

if NoMean==0
  % Interpolating data2
  t_insulin2_interpl = t_c_insulin2(1,1):t_c_insulin2(end,1); % [min]
  c_insulin2_interpl=interp1(t_c_insulin2(:,1),t_c_insulin2(:,2),...
                               t_insulin2_interpl);
  t_c_insulin2_interpl(:,1)=t_insulin2_interpl;
  t_c_insulin2_interpl(:,2)=c_insulin2_interpl;

  % simulation using data2
  [c_glucose_mode12_interpl,c_glucose_labeled_mode12_interpl,...
   c_interstitial_insulin_mode12_interpl,...
   c_interstitial_glucose_mode12_interpl,...
   c_Ggut_mode12_interpl,c_Ggut_mode12_labeled_interpl,...
   =gluc_sim_new (L0,t_c_insulin2_interpl,p_true,plt,'"""",""""');
```
% calculating the mean of the interpolated data at the data2 samples
GIb=LO(4);
c_interstitial_glucose_mean(i,1)=GIb;
i=2;
for r=t_c_insulin2(2:end,1)'
c_interstitial_glucose_mean(i,1)=...
sum(c_interstitial_glucose_model2_interpl(r+1-t_exp:r))/t_exp;
i=i+1;
end

else

% simulation using data2
[c_glucose_model2,c_glucose_labeled_model2,...
c_interstitial_insulin_model2,...
c_interstitial_glucose_model2,c_Ggut_model2,c_Ggut_labeled_model2]
=gluc_sim_new (LO,t_c_insulin2,p_true,plt,'/','...');
end

% adding noise
if NoMean==0
data2_new=c_interstitial_insulin_model2_interpl;
data2_new=data2_new.*randn(1,1); %+8% KU1S
else
data2_new=c_interstitial_insulin_model2;
data2_new=data2_new.*randn(1,1); %+8% KU1S
end

% make plots i' plt-1
plt=1;

steil_param;
Nparameters=length(p_true);

% End initialization

% 2-dimensional parameter space
%p = [Ib Sicap p2];
%1 known parameters:
p_fix = [Ib p2];
%1 unknown model parameters:
p_init = [Sicap]

p_init_copy=p_init;
estimation(1,:)=[-0.5 -0.4 -0.3 -0.2 -0.1 0.1 0.2 0.3 0.4 0.5 ];
estimation1(1,:)=estimation(1,:);

for kk=1:length(p_init)
ll=0;
for zz=estimation(1,:)
p_init=p_init_copy;
ll=ll+1;

estimator=find(p_init(kk)==p_true);

end

for ll=1:length(p_init)

end

estimator=find(p_init(kk)==p_true);

estimation1(1,:)="}

estimation1(1,:)="}

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estimator1 = find(p_init(1) == p_true);
p_true = p_true(estimator);
p_init(kk) = p_init(kk) + z * p_init(kk)

% estimation of parameters:

[p_model, L0_model, c_interstitial_insulin_model, resnorm, residual, varp, ...
    stdp] = gluc_omm_mls_steil(data2_new, L0, t_c_insulin2_interpl, a, sigmal, ...
    p_fix, p_init, Nparameters, plt);

p_Model = p_model(estimator);
estimation(kk+1, ll) = p_model(estimator);
estimation1(kk+1, ll) = p_model(estimator1);
end

TARGET = fopen(filename, 'w');
fprintf(TARGET, ...
    ', estimation1');
fclose(TARGET);
D.8.2

steiL_param.m

%%%
% Steil's Model of insulin kinetics.
% This script initializes all parameters of Steil's Model of insulin kinetics and the time samples of plasma insulin.
%%
%
% History
% 28 July-06, Salvador Almagro Frutos, UWE
% 29-Jan-04, Natal van Riel, TU/e
% 17-Jun-03, Natal van Riel, TU/e
%

% model parameters:
% Values from: ??????
 Ib = Ib; % [pM] baseline insulin concentration in plasma
 Sicap = Sicap; % [1/min] trans-endothelial insulin transport parameter
 p2 = p2; % [1/min] total fractional insulin clearance from the interstitial fluid

p_true = [Ib, Sicap, p2]; % 3 dimensional parameter space

% Fixed initial conditions
clear I3;
L0(1) = 0; % [pM] insulin concentration in interstitium

% Input insulin concentration in plasma [pM]; assumed to be known at each simulation time sample from linear interpolation of its measured samples

% Time and amplitude samples of plasma insulin:
% t-> blood glucose data
 t_insulin1 = t_c_insulin1(:,1);
 c_insulin1 = t_c_insulin1(:,2);
 t_c_insulin1(:,1) = t_insulin1';
t_c_insulin1(:,2)=c_insulin1';

% squared 2-norm of the error-function residuals

a=1; % output weighing factor of plasma glucose concentration (last output)
sigma=1; % variance of plasma glucose concentration (last output)

% Plots
% make plots if plt=1
if plt==1
%Plots

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D.8.3

steil_sim.m

function [c_interstitial_insulin]=steil_sim
(LO,t_c_insulin,p,plt,TITLE)

%%
% Steil's Model of insulin kinetics.
% This function makes a simulation of the Steil's model of insulin kinetics using the parameters defined in Steil_param.m.
%%
%% Function:
%c_interstitial_insulin=steil_sim (LO,t_c_insulin,t_c_insulin,p,...
plt,TITLE)
% Inputs function:
%c_interstitial_insulin: [pM] 1[min] Time and amplitude samples matrix of plasma insulin 1 - blood glucose data 2 interstitial glucose data
% p: = [51cap, p2] 2 dimensional parameter space
% plt: plots are made in this function if plt=1.
% TITLE: set the title name if plots are made in this function (plt=1).
%%
% Outputs function:
%c_interstitial_insulin: [pM] interstitial insulin concentration.
%%
% History
% 28-July 06, Salvador Almagro Frutos, TU/e
% 29-Jan 04, Natal van Riel, TU/e
% 11/ Jun 03, Natal van Riel, TU/e
%%

%model parameters:
% 51cap [1/min] transendothelial insulin transport parameter
% p2 [1/min] total fractional insulin clearance from the interstitial fluid

%differential equations solver: ode15s
ode_options =odeset('RelTol',1e-6,'AbsTol',1e-12); % parameters 2-end
%%ode_options =odeset ('RelTol',1e-6,'AbsTol',1e-16);

[t,L] = ode45(@steil_ode,t_c_insulin(:,1),LO,ode_options,
t_c_insulin,p);

% Output:
%c_interstitial_insulin=L(:,1); % [pM] interstitial insulin concentration
% make plots if plt=1
```matlab
i: plt==1
plt: a
figure (333);
Ib=p(1);
subplot(221); plot(t_c_insulin(:,1), t_c_insulin(:,2), '.*', 
'color': 'r', 'markersize': 2);
hold on;

%Baseline level:
plot([t_c_insulin(1) t_c_insulin(end)], [Ib Ib], ':
', 'color': 'r', 'linewidth': 1.5)
ylabel('');
xlabel('');
Title('');

subplot(222); plot(t_c_insulin(:,1), c_interstitial_insulin, 
'color': 'g', 'markersize': 2)
xlabel(''); ylabe... 
('');
Title(TITLE);
end
```
D.8.4

steil_ode.m

function dLout = steil_ode(t, Lin, t_c_insulin, p)

%%% %STEIL ODE ODE's of STEIL'S Model of insulin kinetics.
%%% %This function sets STEIL ODE ODE's of STEIL'S Model of insulin
%%% kinetics using the parameters defined in steil_param.m.
%%% %
%%% %Function:
%%% %dLout = steil_ode(t, Lin, t_c_insulin,p);.
%%% %
%%% %Inputs function:
%%% %Lin: state values at previous time sample L(k+1) - [X(k+1)]
%%% %t_c_insulin: c_insulin(k) input signal at sample k) OR matrix
%%% %t_c_insulin
%%% %p = [S1cap, p2]; 2 dimensional parameter space
%%% %
%%% %Outputs function:
%%% %dLout: state derivatives dL(k) = [dX(k)];
%%% %
%%% %
%%% %History
%%% %28-July-06, Salvador Almeida Frutos, TU/e
%%% %29-Jan-04, Natal van Riel, TU/e
%%% %17-Jun-04, Natal van Riel, TU/e
%%% %
%%% %
%%% % defining the row where each state in statevector Lin is present
idX = 1;  % interstitial insulin concentration [pM]

% model parameters:
 Ib = p(1);  %[pM] baseline insulin concentration in plasma
S1cap = p(2);  %[1/min] transendothelial insulin transport parameter
p2 = p(3);  %[1/min] total fractional insulin clearance from the
% interstitial fluid

calculate c_insulin by interpolation of t_c_insulin
u = interp1(t_c_insulin(:,1),t_c_insulin(:,2), t);

% ode's

dX = S1cap/1e3*(u-Ib)^2-p2*Lin(idX);

%}

dLout = [dX];
gluc_omm_mls_steil.m

function [p_model,LO_model,c_interstitial_insulin_model,...
resnorm,residual,vars,stdp]=gluc_omm_mls_steil...
(datal,LO,t_c_insulinl,a,sigmal,p_fix,...
p_init,Nparameters,plt)

%%
%S%TEIL_MLS_MLS Maximum Least Squares Estimation of Steil's Model of
%insulin kinetics
%%
%This function estimates the parameters using Maximum Least Squares
%Estimation of Steil's Model of insulin kinetics using the
%parameters defined in steil_param.m.
%%
%Function:
%[p_model,LO_model,c_interstitial_insulin_model,...
resnorm,vars,stdp]=gluc_omm_mls_steil...
(datal,LO,t_c_insulinl,p_fix,p_init,Nparameters,plt)

%Inputs function:
%datal: data of interstitial insulin [PM]
%LO: Fixed initial conditions [PM] - [PM] insulin concentration in
interstitium.
%t_c_insulin: [PM] [min] Time and amplitude samples matrix of plasma
%insulin 1 -> blood glucose data.
%p_p: [Slep,pP]; 2-dimensional parameter space
%p_fix: parameter space which is fixed.
%p_init: initialisation of parameter space which will be estimated.
%sample.interval [min] of data 2.
%Nparameters: number of dimensional parameter space p.
%plt: plots are made in this function if plt=1.

%Outputs function:
%p_model: [Slep,pP]; 2-dimensional parameter space
%c_interstitial_insulin_model: [PM] interstitial insulin concentration.
%resnorm: squared 2-norm of the error function residual.
%residual: the value of the residual of the estimation algorithm.
%vars: variance
%stdp: The standard deviation is the square root of the variance.
%%

%History
%28-Jul-06, Salvador Almagro Flores, TUE
%29-Jan-04, Natal van Riel, TUE
%17 Jun 03, Natal van Riel, TUE
%
%lower (lb) and upper (ub) bounds, so that the solution of the
estimated
5 parameters is always in the range the parameter lb ub.
lb = []; 
ub = []; 
options = optimset( 'Display', 'on', 'TolX', 1e-10, 'TolFun', 1e-20, 'OptimalityTol', [800], 'GradObj', 'on', 'GradConstr', 'on', 'MaxIter', 100, 'MaxFunEvals', 10000, 'Jacobian', 'on');

[p_estimated, resnorm, residual, exitflag, output, lambda, jacobian] = lsqnonlin(obj_fn_steil, p_init, lb, ub, options, p_fix, LO, data1, t_c_insulin1, intercept, sigma, Nparameters);

if exitflag==0
STOP=1;
end

varp = resnorm*inv(jacobian'*jacobian)/length(t_c_insulin1(:,1));

stdp = sqrt(diag(varp));

p_model = [p_fix(1) p_estimated(1) p_fix(2)];
LO_model = [LO(1)];

[c_interstitial_insulin_model]= steil_sim (LO,...
  t_c_insulin1,p_model,plt,'');

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D.8.6

**obj_fn_Steil.m**

```matlab
function error = obj_fn_Steil(p_var, p_fix, L0, datal, t_c_insulin,...
    a, signal, Nparameters)

%%
% Objective function Maximum Least Squares Estimation of Steil's
% Model of insulin kinetics.
%
%%
% This function is used as objective function for LSQNONLIN.
%
% Function:
%   obj_fn_Steil(p_var, p_fix, L0, datal, t_c_insulin,...
%   Nparameters)
% Inputs:
%   p_var: parameter space which will be estimated
%   p_fix: parameter space which is fixed.
%   L0: Fixed initial conditions, [L0] [pm] insulin concentration in ...
%   interstitium.
%   datal: data of interstitial insulin [pm].
%   t_c_insulin: [pm] [min] Time and amplitude samples matrix of plasma
%   insulin.
%   a: output weighting factor of 1st output.
%   signal: variance of 1st output.
%   Nparameters: number of dimensional parameter space p.
%   estimator: defines the parameter which is used in the model to model
% adjustment identification.
%
% Outputs:
%   error: minimization variable of MIS (model error)
%
% History
% 08 July-06, Salvador Almendros Fuentes, Ti/e
% 09 Jan-04, Natal van Riel, Ti/e
% 11 Jun 03, Natal van Riel, Ti/e

p_estimating=[p_fix(1) p_var(1) p_fix(2)];

% estimating 2-dimensional parameter space
p_true = [16, 1, 0.2];

% Simulating Steil model of insulin kinetics using the estimating
% 2-dimensional parameter space and the interpolated insulin
% concentration
% samples for datal.

plt=0; % make NO plots
[c_interstitial_insulin_estimating]=...
```
steil_sim(LG, t_c_insulinl, p_estimating, 0,...
'. . . . . ', );

errorl = c_interstitial_insulin_estimating-datal;
	error=errorl;

p_var

plot(1:N,c_interstitial_insulin_estimating,'',1:N,datal,' ');
xlabel('');
ylabel('');
Title('');

drawnow;pause(.3);