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Aspects of identification of fluid/structure interactions at impellers in boilerfeed pumps

Mimpen, T.J.C.M.

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SUMMARY

This report describes the work carried out within the framework of the Brite/Euram project called: "Identification of fluid/structure interaction for the development and design of boilerfeed pumps". In the first part of the project, the so called motion dependent interactions at the impeller will be identified. These interactions will be identified using a two stage pump with active magnetic bearings, which will also be used to excite the rotor of the pump.

The identification procedure that will be applied is indirect. This means that forces and displacements will be measured at the magnetic bearings and will be related to the, to be identified forces at the impellers with an analytical model. The frequency response functions of the analytical model will be generated with software based on the transfer matrix method. The software is extended and modified to handle five degrees of freedom.

Forward and backward whirling are typical rotor dynamic phenomena and could influence the identification procedure. From a survey a better insight in these phenomena is obtained. The result from this survey together with other surveys is that the indirect identification procedure is not influenced by the phenomena.

After the design of the test pump was finished, rotor dynamic analyses had to be carried out for a number of reasons. The rotor is modelled and checked on several sensitivities. The influence of the additional stiffness of the bushings and impellers is significant and modal analyses have been carried out to check the influence and the model itself. With the results the model will be adjusted till the results of the models agree with the results from the modal analysis.

Modal parameters of a reference model are compared to the preliminary design, resulting in good similarities. The free-free modes (no bearing stiffness modelled) are calculated as a function of the clearance of the seals and the motion dependent interaction range. This is completed with a study of the influence of the bearing stiffness on the first three modes.
In the studies of the model so far, the undamped natural frequencies are calculated. The stability of the rotor is also determined for the previous mentioned range of clearances and motion dependent interactions. It appears that with maximum bearing damping the rotor becomes unstable for high motion dependent interactions at the impellers, in case the rotor speed is in the upper part of the test speed range.

The rotor dynamic analysis of the final design ends with a review of the excitation capacity of the magnetic bearings. The review shows that sufficient displacement can be applied at the impeller for the interesting frequency range.

At the end of the project, software will be modified to be able to use the gained knowledge for the design of new pumps. A choice will be made between software based on the transfer matrix method and software based on the finite element method. Several numerical test are carried out and results are compared with analytical solutions. More experiments are necessary to make a fundamental decision. At this moment the finite element method seems to be the best choice, especially because of the flexibility of the method.
ACKNOWLEDGEMENTS

I like to express my thanks to everyone who has been of help in the last nine months of my study Mechanical Engineering at the Eindhoven University of Technology. Especially to the company BW/IP Pump Division, who has given me the opportunity to work on this project and offered all facilities to carry out my work.

Also I wish to thank Jan Verhoeven, who has not only been of help on theoretical and practical matters but was also a good companion and source of inspiration. Also Bram de Kraker, who has been my contact and adviser at the University, is thanked for his help and advice. Finally I wish to thank Simone de Bont for typing the text and making the illustrations.

Etten-Leur, May 1991

Theo Mimpen
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<tr>
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<td>rotor speed</td>
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<tr>
<td>$\omega$</td>
<td>excitation frequency</td>
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<td>A</td>
<td>cross-sectional area</td>
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<td>I</td>
<td>moment of inertia of cross section</td>
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<td>E</td>
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<tr>
<td>$I_p$</td>
<td>polar mass moment of inertia</td>
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<tr>
<td>$I_t$</td>
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INTRODUCTION

As a part of the Brite/Euram project called: "Identification of fluid/structure interaction for the development and design of boilerfeed pumps" the author performed some tasks, which are described in this report. The project which started in March 1990, is a cooperation between BW/IP Pump Division (The Netherlands), LMS (Belgium) and the University of Kaiserslautern (Germany).

The main objective is to identify the fluid/structure interaction at the impeller in a boilerfeed pump (see also appendix I). In phase 1 of the tests, two test pumps will be used for the identification of the motion dependent interactions (MDI). The MDI will be identified with respect to five degrees of freedom, three more than in the previous research in this field. During the tests several parameters will be varied like pressure, flow, shroud gap distance etc. to identify their influence on the MDI.

At this moment the manufacturing of all parts is finished and the pumps are being assembled and fine tuned to start testing in June. The test pumps will both run in active magnetic bearings, which will also be used to excite the rotor. The application of magnetic bearings in a pump in this way is unique and is a big challenge in this project.

Because the subtasks, carried out within the framework of the project, differ too much in their subject, the author decided to describe the work in three parts. The first two parts are directly related to tasks of the project, the third part is a preparation of one of the tasks of the project, which is carried out at the end of the project.

The report has however a so called leading thread. Subtask 1 describes the elementary subjects of rotor dynamics. In subtask 2 the rotor dynamic theory is applied to perform the dynamic analyses and in subtask 3 the experience with the theory and analyses is used to compare two basic methods to perform rotor dynamic analyses. These subtasks will be described more extensively.

In the first section the work related to the identification of the so called motion dependent interactions at the impellers is described. The main objective of this subtask is to modify and extend software for generating frequency response functions, which will
be used in the indirect identification procedure.

The second section describes the extensive rotor dynamic analysis of the final pump layout, as it is assembled at this moment. Analyses were carried out to check model accuracy and to calculate the natural frequencies of the pump rotor for free-free conditions. Also to check the stability for several operating conditions and to get an indication of excitation capacity of the magnetic bearings.

In the last section a preparing investigation is described. At the end of the project software will be modified and extended to be able to use the gained knowledge of the project. BW/IP is now using software which is based on the so called transfer matrix method. The University of Kaiserslautern developed a software program based on the finite element method. Both methods have their advantages, but one them will be selected to be extended and modified.

At the time this report is written, the project is nearing the first experimental phase. All the theory worked out so far will be applied and in the next months will be clear what will work in practice.
SUBTASK 1

MODIFICATION OF SOFTWARE FOR GENERATING FREQUENCY RESPONSE FUNCTIONS

The purpose of the tests with the two-stage boilerfeed pump, is to identify the motion dependent interactions (MDI) at the impellers. As described in the introduction of this report, several geometrical and operational parameters will be varied, like pressure, flow and size of the shroud gap. These parameters will influence the MDI, each one in its own way. The work described in this section is related to task 4 of the project.

The important part of the parameter extraction is the indirect identification procedure (see also appendix I). With this procedure the results from the test rig with the unknown MDI are compared with the results from an analytical model. From the differences the dynamic parameters of the MDI can be extracted. To be able to perform the extraction of the parameters, frequency response functions have to be generated from an analytical model. These functions are generated by software, based on the transfer matrix method. The software, which internally uses four degrees of freedom i.e. two radial and two angular displacements, was modified to handle five degrees of freedom. The two angular and the axial displacements are the important extensions to the previous research done in this field by for instance Sulzer and Caltech. The software is now capable of generating frequency response functions for almost all combinations of the displacements and loads.

In this section of the report an introduction is presented about the theory of rotor dynamics. The implementation of the theory into the transfer matrix method is explained in appendix II. Next to the theory an overview is given about the modifications of the software as described in this introduction. Finally two important phenomena in the rotor dynamics, i.e. forward and backward whirling, will be explained. Therefore the results of a survey to test the existence of these phenomena with analytically generated frequency response functions are described. This section ends with some conclusions concerning the work carried out.
1.1 Theory of rotor dynamics

A short overview of the theory of rotor dynamics will be given here. Especially the differences compared to the (more familiar) structural dynamics will be highlighted. The systems, described in this part, are subjected to bending vibrations acting in two perpendicular planes, i.e. the x-z and y-z plane (figure 1.1).

![Figure 1.1 An example of a rotor model](image)

A rotor model consists of several basic elements, of which the most used will be described shortly.

- **Disc elements**

  degrees of freedom (DOFs): x, y, φx, φy

  **parameters:**
  - m [kg]
  - \( I_p \) [kg.m²]
  - \( I_t \) [kg.m²]

  Together with mass effects also gyroscopic effects will determine the vibration behaviour of a disc element. A disc element is assumed not to bend in the x-z and y-z plane.
Beam elements

These elements have a distributed mass with a polar and transverse moment of inertia. Typically these elements have a certain stiffness against bending in the x-z and y-z plane. The stiffness is determined by Hooke's law with the assumption of a linear elastic behaving material and small displacements. Frequently also shear effects have to be taken into account depending on the kind of beam model. For the so called Timoshenko beam these shear effects are characterised by the coefficient

$$a = \frac{12 EI}{GA_s l^2}$$  \hspace{1cm} (1.1)

With $G$ as the shear modulus [N/m²] and $A_s$ as the reduced area of the cross section [m²]. This factor decreases the stiffness and mass effects of a beam. For a Bernoulli beam there is no influence of shear effects, so $a = 0$. Finally it should be noted that also gyroscopic effects play a role of importance.

Mass-damper-spring elements

These elements are characterized by the existence of forces which are related to the acceleration, velocity and position of the rotor. Typically for rotor dynamic systems is the existence of cross coupling between the two radial or angular directions. For example seals and journal bearings have dynamic coefficients, which can be characterized by skew symmetric
stiffness and damping matrixes as a function of the rotor speed. In the scope of the project the MDI at the impeller will be observed with respect to five degrees of freedom so including the axial displacement z.

With these elements a rotor dynamic system can be modelled. Assuming a linear behaviour of the system, the separate elements can be superposed into one equation of motion described by:

\[ M\ddot{q} + D\dot{q} + Kq = f(t) \]  

(1.2)

With

- \( M \) = total mass matrix (including masses and moments of inertia)
- \( D \) = total damping matrix
- \( D = D_d + D_g \) with
  - \( D_d \) = matrix with dissipative terms
  - \( D_g \) = matrix with gyroscopic terms
- \( K \) = total stiffness matrix
- \( q \) = column with the degrees of freedom
- \( f(t) \) = column with external applied loads

In this equation the mass-matrix \( M \) is normally a positive definite matrix, whereas the damping matrix \( D \) and stiffness matrix \( K \) can be non-symmetric.

To calculate the undamped natural frequencies of the systems the eigenvalue problem of the reduced system is solved:

\[ (M\lambda^2 + K_s)u = 0 ; \quad \lambda = -\omega^2 \]  

(1.3)

The solutions are the undamped natural frequencies \( \omega \), which are real and positive in case the matrix \( M \) and \( K \) are positive definite. For this reason the total stiffness matrix \( K \) of the system is stripped to a symmetric stiffness matrix \( K_s \).
The damped natural frequencies are calculated from the homogeneous part of equation (1.2)

\[ M\ddot{q} + D\dot{q} + Kq = 0 \]  

(1.4)

by solving the eigenvalue problem. The to the eigenvalues corresponding eigenvectors are complex, similar to the situation of structural dynamics with non-proportional damping. In general, in rotor dynamic systems, these eigenvectors have different phase angles. The modeshapes look like "corkscrews", a typical rotor dynamic phenomenon.

The damped natural frequencies are speed dependent because of the gyroscopic effects, the bearings and the seals in the system. An often used diagram to show the speed dependency of the damped natural frequencies is the Campbell-diagram (figure 1.2)

![Campbell diagram](image)

Figure 1.2: The Campbell diagram

In this diagram each line represents a natural frequency with a certain whirl direction of the rotor. Whirling of a rotor is the motion of the centre of the rotor along an often elliptical orbit. In case the direction is equal to the direction of rotation of the shaft this is called forward whirling (FW), in the other case it is called backward whirling (BW) (figure 1.3).
This ends the overview of the theory of rotor dynamics. For a more detailed overview will be referred to E. Krämer [1], M. Lalanne and G. Ferraris [2] and to W. Diewald [3].

The implementation of the theory into the transfer matrix method is described in appendix II and more extensively in Hurty and Rubinstein [4]. This method is the base of the program BRITE1, which generates user specified frequency response functions (FRFs). This program is described in the next part.

1.2 Software modifications

As explained in the introduction, the software was originally only capable of generating FRFs for radial displacement-radial force combinations. As measurements will be carried out with respect to five degrees of freedom, the original program has been modified and extended to generate also the FRFs for the other displacement-load combinations. Note that with the described theory, two groups of FRFs can be generated independently from each other. That means all combinations between radial/angular displacements and forces/moments are generated separately from the axial force-axial displacement relation. The matrix $H_{ij}$ which relates loads at station $j$ with displacements at station $i$ is presented in equation (1.5).
In previous research on fluid/structure interactions at the impeller, only the radial displacements of the impeller were treated as degrees of freedom. As there is nothing reported for the relations between axial displacement and radial forces/moments, as well for the relations between the axial force and radial/angular displacements, eight relations in matrix $H^{ij}$ are set to zero. This does not mean that those relations will be zero in the actual situation on the test rig. As stated before, in the indirect identification procedure all known dynamic effects will be modelled and presented by FRFs.

Concerning the relation between the axial displacement and axial force, only for the mechanical seals some values are reported for a standard type of seal. In the test rigs new developed mechanical seals will be used and these will be tested to determine the dynamic coefficients. This way the known dynamic effects in axial direction can be modelled. To model the relation, the rotor is assumed to behave stiff in axial direction. A simple one DOF system with a mass, damper and spring can then represent the dynamic behaviour in axial direction due to axial excitation (figure 1.4).

Assuming the rotor to behave stiff results in one value for the $Z/F_z$ - relation independently from the station where the load is applied and the station where the displacement is observed. Finally should be remarked that already from numerical fluid flow calculations with a bulk flow model appeared that at the impeller a $Z/F_z$ relation exists. This resulted from work performed by the University of Kaiserslautern described in the project task 3.
With these changes the program can now calculate any displacement at station $i$ due to a load at station $j$. Using unit loads, the matrix with transfer functions can be generated column wise. This process of generating FRFs is shown in equation (1.6).

$$ x^d = H \begin{bmatrix} f_{j}^{\text{unit}} \\
\vdots \\
0 \end{bmatrix} \quad (1.6) $$

So with the unit load $f_j^{\text{unit}} = 1$ at station $j$ the responses $x^d$ can be calculated at the desired stations. These responses correspond with column $j$ of the H-matrix so $H_{ij} = x_1^d$, $H_{2j} = x_2^d$ etc. In this way all columns of $H$ can be calculated.

With the described modifications and extensions of the software, the user is now capable of generating any FRF of the system. The user has to specify:

- the model;
- the stations where the unit loads are applied and the required direction of each unit load;
- the stations where the excitations are observed and the required direction of each observation;
- the excitation frequency range as the equations of motion are directly
influenced by the excitation frequency;
- the rotor speed range as the gyroscopic effects and some of the
dynamic coefficients (for instance of journal bearings) depend on the
rotor speed.

The program generates an output file with the desired FRFs for each excitation
frequency and rotor speed. Figure 1.5 shows the flow chart of the BRITE1 program.

![Flow chart of BRITE1 program](image)

Figure 1.5: Flow chart of BRITE1 program
Finally some remarks will be made towards the numerical stability of the program. During tests of the program with a simple one mass-damper-spring system with four DOFs, problems occurred with the generated H-matrixes. It appeared that for a certain frequency range some elements of the H-matrix were oscillating. The reason for this was found in the solving procedure to solve linear equations, used in the BRITE1 program. As described in appendix II a part of the transfer matrix is used to determine the so called state vector at the left station from the equation

\[ K x_L + f_{\text{applied}} = 0 \]  

Depending on the system calculated, the matrix K in this equation can become ill conditioned. Referring to the test with the four DOFs system, this matrix was ill conditioned for a frequency range between 0 and 10 Hz. This problem is due to the difference in magnitude of the components in the matrix K. The elements outside the diagonal of the matrix K are dominant for this frequency range.

1.3 Forward/backward whirling phenomena in relation to the analytical derived FRFs

During discussions about the FRFs generated with the software, some doubts existed whether forward backward whirling phenomena would interfere with the analytical and experimental FRFs used in the indentification procedure. For this reason and to get a better insight in the forward/backward whirling phenomena, a survey was carried out. In this survey a two DOFs system is considered as presented in figure 1.6.

\[ \begin{align*}
  b_{xx}, b_{xy}, b_{yx}, b_{yy} & \quad k_{xx}, k_{xy}, k_{yx}, k_{yy} \\
\end{align*} \]

\[ y \quad x \]

\[ m \]

Figure 1.6: Simple mass-damper-spring system with two DOF
The gyroscopic effects are not included and cross coupled stiffness and damping are assumed to be equal but opposite in sign. The equation of motion can be written as:

\[ M \ddot{q} + D \dot{q} + K q = f \]

with

\[ q = \begin{bmatrix} x \\ y \end{bmatrix} ; \quad b = b_{xy} = -b_{yx} \text{ and } k = k_{xy} = -k_{yx} \]

According to the harmonic theory this can be rewritten to:

\[
\begin{bmatrix}
  k_{xx} - \omega^2 m + j\omega b_{xx} & k + j\omega b \\
  -k - j\omega b & k_{yy} - \omega^2 m + j\omega b_{yy}
\end{bmatrix}
\begin{bmatrix}
  \ddot{x} \\
  \ddot{y}
\end{bmatrix} =
\begin{bmatrix}
  \ddot{f}_x \\
  \ddot{f}_y
\end{bmatrix} - K(\omega) \ddot{q} = \ddot{f}
\]

with \( x, y, \ddot{f}_x \) and \( \ddot{f}_y \) as complex amplitudes.

The matrix \( K(\omega) \), sometimes called dynamic stiffness matrix, can be calculated by inverting the transfer matrix \( H(\omega) \). The transfer matrix \( H(\omega) \) is determined by the BRITE1 program as described in the previous section. The process of inverting the matrix \( H(\omega) \) is also done in the indirect identification procedure to perform the parameter extraction. In the next part the formulation with the \( H(\omega) \) matrix is used to describe the forward/backward whirling phenomena.

Now it is possible to determine the whirl direction by observing the position vector \( \ddot{q} \) and the velocity vector \( \dot{q} \). As the harmonic theory is used the position vector can be
calculated from:

\[ q = Re \left\{ \vec{q} e^{j\omega t} \right\} \]  \hspace{1cm} (1.10)

and the velocity vector from:

\[ \dot{q} = Re \left\{ \vec{q} j\omega e^{j\omega t} \right\} \]  \hspace{1cm} (1.11)

The whirl direction is determined by the vector product of the position vector and the velocity vector as shown in figure 1.7.

Figure 1.7: Determination of whirl direction

This will be illustrated with examples from the simple mass-damper-spring system of figure 1.6. Assuming values for the parameters \( m, b_x, b_y, k_x, k_y \) and \( k \) (conversion table is provided at the end of the report):

\[
\begin{align*}
  m &= 10 \text{ lbs} \\
  b_{xx} = b_{yy} &= 0.5 \text{ lbs.s/in} \quad b = 0.1 \text{ lbs.s/in} \\
  k_{xx} = k_{yy} &= 1000 \text{ lbs/in} \quad k = 50 \text{ lbs/in}
\end{align*}
\]

From this system the eigenvalues can be determined, which are

\[
\begin{align*}
  \lambda_{1,2} &= -2.3 \pm 30.9 \text{ j Hz} \\
  \lambda_{3,4} &= -0.8 \pm 31.6 \text{ j Hz}
\end{align*}
\]
With the corresponding eigenvectors the whirl direction can be determined. For \( \lambda_{1,2} \) the whirl direction is backward, for \( \lambda_{3,4} \) the whirl direction is forward. Also from a forced response of the system the whirl direction was determined for a given frequency range. Three different excitation forces were used:

1) Rotating force with same direction of rotation as rotor
2) Rotating force with opposite direction of rotation as rotor
3) Unilateral force fixed in space in x-direction

The whirl direction for these three cases was determined for \( 10 < \omega < 50 \text{ Hz} \). For the first two cases the whirl direction is respectively forward for the first and backward for the second case. This can also be determined with the Bode-diagram shown in figure 1.8.

![Bode diagram for rotational force excitation (forward)](image)

Figure 1.8: Bode diagram for rotational force excitation (forward)

for the forward whirl case. In this diagram the response in x and y direction are plotted for the given frequency range. The phase angle of the X/Ff response is always leading on the phase angle of the Y/Ff response, as can be seen in the phase angle diagram of
figure 1.8. As the lines of the phase angles do not intersect, the rotor remains in a forward whirl. In case the system is not symmetric in x-direction and y-direction, the whirl direction can change over the frequency range. If for instance $k_x = 1000$ lbs/in and $k_y = 2000$ lbs/in, the Bode diagram changes as shown in figure 1.9. The whirl direction changes from forward to backward at $\omega \approx 31$ Hz and at $\omega \approx 43$ Hz it changes back to forward. At the intersections of the phase angles, the $X/Ff$ response and $Y/Ff$ response have equal phase angles, what means that the centre of the rotor is moving along a straight line. The angle of the line is determined by the ratio of the magnitudes of the $X/Ff$ response and $Y/Ff$ response.

The unilateral force shows a backward whirl for $10 < \omega < 27.1$ Hz. At the frequency $\omega = 27.1$ Hz the whirl direction switches from backward to forward whirling. In figure 1.10 the Bode diagram is shown with respectively backward and forward whirling. The
phase angle of the $Y/F_x$ response is leading on the phase angle of the $X/F_x$ response for $10 < \omega < 27.1$ Hz, what means backward whirling. This situation switches at $\omega = 27.1$ Hz.

From this survey a much better insight in these typical rotor dynamic phenomena is obtained. Towards the concerns of the generated FRFs the next remarks can be made from this and other surveys as can be seen in Verhoeven [5]. By generating the analytical FRFs with an unit force, as described in the previous part, and measuring the FRFs with uncorrelated excitation signals the MDI coefficients can be extracted correctly. The only condition is that these coefficients have to be independent from the whirling sense, with other words the MDI must behave linear.
1.4 Conclusions, remarks and recommendations

Finally, to end the description of subtask 1, some remarks and conclusions will be summarized here. The main purpose of this subtask, to modify and extend software for generating FRFs, has been carried out without significant problems. The numerical problems in the program however have been experienced already from a simple testproblem and could also appear with the real models.

The typical forward and backward whirling phenomena, have been identified in two ways. The Bode diagram of a forced response, has appeared to be an easy way to identify the whirl direction.

FRFs generated with an analytical model by the BRITE1 program, together with the measurement procedure of LMS lead to correct identification of the impeller MDI coefficients, on the condition that the MDI behave linear.
This paragraph describes the analyses of the two stage pump, which will be used to identify the motion dependent interactions (MDI) at the impeller. After finishing the design of the test pump (task 1 of the project), it was necessary to perform a series of dynamic analyses for several reasons:

- To check the dynamic behaviour of the rotor in relation to the tests.
  One of the project objectives is to have a resonance generated by the fluid/structure interactions in the measurement frequency range. In this way accurate curve fitting will be possible and mass, damping and stiffness terms can be derived.
  The undamped natural frequencies of the rotor for free - free conditions are calculated, a) to compare the design with the preliminary design, b) to judge its behaviour in the measurement frequency range and c) to check the design requirements for the magnetic bearings. Furthermore stability has to be checked to ensure the possibility to perform the tests.

- To identify sensitivities of the model.
  To be able to extract parameters in the indirect identification procedure it is important to have a very accurate model. Some model assumptions can lead to unacceptable errors and if certain known dynamic effects are not modelled these could be identified as the MDI at the impellers.

- To identify the real dynamic effects at the seals.
  The dynamic effects at the balance sleeve and wearings are a function of several parameters. The dynamic coefficients have to be calculated in order to create an accurate model.

- To calculate the remaining load capacity for excitation at the magnetic bearings.
  The total load capacity of the active magnetic bearings (AMBs) is required to carry the static and dynamic load. The maximum AMB excitation load capacity is calculated from an operating load analysis and the total AMB load capacity. The excitation load capacity is converted in displacement excitation capacity.
and compared to the ratios of displacements at AMB and impeller.

- To build models to be used by LMS for the actual measurements.

The analytical models are used in the indirect identification procedure (appendix I) to identify forces. These models are generated by BW/IP.

The results from the analyses are the final check before the experiments on the test rig start. It should give an indication where to expect problems, so alternatives can be thought of before the actual experiments start.

This section starts with the preparation of the analyses, followed by an overview of the model sensitivities and finally a description of the analyses and its results.

2.1 The model

The model, presented in appendix III, consists of 49 stations. These stations include those necessary for the dynamic analysis and 9 stations which represent the sensor locations, necessary for the indirect identification procedure. In this model several assumptions are made to convert a continuous system into a discrete system. First of all the components connected to the shaft (impellers, mechanical seals etc.) are modelled as stations with lumped masses and moments of inertia. In case these elements supply the shaft with additional stiffness, an effective shaft diameter is introduced.

Regarding the mass distribution of these elements, two options to model these elements were considered. When a rotor is modelled, a continuous distributed mass will be modelled as a discrete distributed mass. The same is valid for the components on the shaft. The mass effects of the components can be modelled by lumping all the mass and moments of inertia at one station located at the centre of gravity of the component or by lumping at all stations of which the component consists. To investigate the influence of the way of lumping mass at one or more stations, two models were analyzed and compared.

The experiments at the test rig will be done in a variable speed range of 1900 RPM up to 2980 RPM. For each rotor speed the dynamic rotor behaviour will differ because of
the change in flow and pressure in the pump.

2.2 Motion dependent interactions

At the balance sleeve, eye wearrings and hub wearrings of the impellers hydrostatic effects will occur. These short seal effects play an important role in the dynamic behaviour of the rotor.

For all these components the MDI force coefficients are calculated for the maximum and minimum rotor speeds. These coefficients are a function of pressure difference, flow and geometrical parameters (see figure 2.1).

![Figure 2.1: Balance sleeve and wearrings](image)

2.2.1 Balance sleeve

The pressure difference in this case is equal to the pressure difference between the first stage and the suction, so half the pressure difference of the pump (see also figure 2.1). Using the finite element program "TURSEAL" the direct and cross coupled stiffness and damping terms are calculated for the minimum and maximum rotorspeed. The cross coupled damping terms are negligible.

2.2.2 Hub wearring

At each of the two hub wearrings, located at the back of impellers, there is a pressure difference equal to one half of the pressure difference between the two stages, so one
fourth of the total pump pressure difference. The dynamic coefficients are calculated with the TURSEAL program.

2.2.3 Eye wearring

The pressure difference of the eye wearrings is more complex to calculate. For this case several evaluations were made and finally the pressure difference over the eye wearring was calculated with the formula of Stepanoff [6]:

\[
\Delta p_{\text{eye}} = \Delta p_{\text{impeller}} - \frac{1}{2} \rho c_3^2 - \frac{1}{2} \rho \left( \frac{1}{2} u_2^2 - \frac{1}{2} u_r^2 \right)
\]  

(2.1)

with \( \Delta p = \) pressure difference [N/m²]
\( c_3 = \) equivalent fluid velocity in the volute [m/s]
\( u_2 = \) circumferential velocity at impeller tip [m/s]
\( u_r = \) circumferential velocity at wearring [m/s]
\( \rho = \) density of the fluid [kg/m³]

This formula is build up as follows: From the total pressure difference of the impeller, the velocity pressure of the fluid which will be converted into static pressure in the volute is subtracted \( (c_3) \) and the pressure loss along the shroud \( (u_2 \text{ and } u_r) \) is taken into account. Although this formula seems to be a reasonable approximation, there are still uncertainties about the calculation of the individual parts, especially the second part in this formula.

From BW/IP field experience is known that the pressure loss along the eye wearring is roughly 70% of the total pressure difference of the impeller. Comparing this to the 77% calculated with the formula the results are not very close. Also from results calculated by the University of Kaiserslautern with the bulk flow model (task 3 in the project [7]) it appeared that the pressure loss along the wearring is a lot different (less than 60%) compared to our calculations.

Realizing that the fluid behaviour in the shroud gap is idealised (no secondary effects taken into account) and that special flow fields are reported by Maroti [8], it is difficult to calculate the dynamic behaviour of the eye wearrings with equation (2.1). From this point it would be better to consider these effects as unknown and measure these together with the motion dependent interactions of the impeller itself. If in future
a better calculation of the eye wearing effects is possible, the results of the test rig can be used again to extract the parameters separately for the impeller.

2.2.4 Impeller
To carry out the dynamic analyses for final design verification, an indication of the impeller motion dependent interactions is necessary. Therefore a range has been defined, which includes these effects reported by Caltech (Acosta et al [9]) and by Sulzer (Bolleter et al [10]). The motion dependent interactions reported by Sulzer are in general higher compared to those of Caltech.

The MDI range is defined by doubling the Sulzer values and by halving the Caltech values. For the analyses three levels in this range are used: low, average and high. The values are calculated for the actual test impeller and vary with speed. An important aspect is that these dynamic coefficients also include mass and even (small) cross coupled mass effects. Furthermore the direct stiffness terms are negative and the cross coupled damping is significant.

For both test rig results (Sulzer and Caltech) uncertainties exist, but using this wide range the real existing effects are probably included. Only the force coefficients related to two radial DOFs are included as the axial and moment coefficients are unknown and cannot be included in the existing rotor dynamic software. The cross coupled mass effects have to be neglected for the same reason.

From this point the model is complete and ready for analyses. First the important model sensitivities will be investigated with a reference model.

2.3 Undamped natural frequencies and mode shapes of the reference model
To investigate the model sensitivities, first a reference model will be defined. In this reference model bushings are modelled with maximum (stiffness) diameter, the mass effects of the bushings and impellers are modelled in the centre of gravity of each element. The total model has 49 stations including the stations for the sensor locations. The dynamic effects at the seals are modelled and at the bearings are omitted.
The undamped natural frequencies are calculated with the program "MODAL1". This program calculates these frequencies with the determinant search method [11]. The results are presented in table 2.1.

**Table 2.1** Natural frequencies of final design and preliminary design no. 1

<table>
<thead>
<tr>
<th></th>
<th>final design (reference model) [Hz]</th>
<th>preliminary design (no. 1) [Hz]</th>
</tr>
</thead>
<tbody>
<tr>
<td>mode 1</td>
<td>9.3</td>
<td>8.7</td>
</tr>
<tr>
<td>mode 2</td>
<td>25.9</td>
<td>25.9</td>
</tr>
<tr>
<td>mode 3</td>
<td>50.8</td>
<td>54.9</td>
</tr>
</tbody>
</table>

The corresponding mode shapes are plotted in figure 2.2.

![First three mode shapes](image)

**Figure 2.2** Mode shapes of reference model

The same conditions are used in this reference model as those used for the model of preliminary design no. 1 (see first 6 month progress report BW/IP 01 page 15 [12]). The natural frequencies of this model are lower for the first, equal to the second and higher for the third mode. Despite an increase in mass at both ends and an increase in
length compared to the preliminary model, the first mode is higher. The reason is that the balance sleeve, located at approx. 38 inch from the left end (see figure 2.2), has now a stiffness value of approximately 55000 lbs/in compared to 32000 lbs/in the preliminary model. The reason for the change in stiffness is the use of a short seal in the final design instead of a long seal as planned in the preliminary design. For the second mode the effect of the balance sleeve is neutralized by the extra mass and length and there is no change in frequency. From a comparison of the third mode can be concluded that the effect of increase of mass becomes more significant at higher frequencies and therefore the third mode has a lower frequency compared to the preliminary model.

2.4 Model sensitivities

Inherent to modelling is making assumptions about the real situation. Some of these assumptions can be made without any doubts, but for some cases this should be checked. Also numerical effects, due to the method and program used, should be checked.

Three different sensitivities were checked:

1. Numerical accuracy.
2. Influence of added stiffness of bushings and impellers on the dynamic behaviour of the rotor.
3. Influence of local distribution of mass of the bushings and impeller on the dynamic behaviour.

2.4.1 Numerical sensitivity

For most of the analyses the rotor dynamic program "MODAL1" is used. This program is based on the modal theory, by using a reduced number of the eigenvalues/eigenvectors of the model to calculate the dynamic behaviour. The eigenvalues however are calculated with the transfer matrix method (see appendix II). With this method a numerical deviation will be transferred through the model and can therefore be expanded enormous when the model is numerically unstable.
To check this, the reference model with 49 stations was stripped to 40 stations by omitting the stations where the sensors are located. Recalculating the natural frequencies and comparing them to those of the reference model results in table 2.2.

Table 2.2 Undamped natural frequencies of reference model and stripped model

<table>
<thead>
<tr>
<th>Mode</th>
<th>Reference Model [Hz]</th>
<th>Stripped Model [Hz]</th>
<th>Difference [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>9.3</td>
<td>9.3</td>
<td>0.0</td>
</tr>
<tr>
<td>2</td>
<td>25.9</td>
<td>25.9</td>
<td>0.0</td>
</tr>
<tr>
<td>3</td>
<td>50.8</td>
<td>50.7</td>
<td>0.2</td>
</tr>
</tbody>
</table>

This leads to the conclusion that the numerical accuracy is not influenced by the number of stations.

2.4.2 Influence of added stiffness of bushings and impellers

The bushings of the AMB, the balance sleeve and the impellers supply the shaft with additional stiffness. Especially the bushings of the AMB, because of their length and diameter, influence the dynamic behaviour of the rotor. It is however difficult to predict the exact contribution to the shaft stiffness. The bushing stiffness is transferred to the shaft stiffness by the connection. In an ideal situation, the bushing being a part of the shaft, the bending stress distribution, which represents the shaft and bushing stiffness, will behave like figure 2.3.

**Figure 2.3 Stress distribution with maximum stiffness diameter**
However if the contact pressure between the shaft and the bushing is relative small, the bending stress will reach a lower ultimate value at the outer diameter of the bushing. The stress distribution will progress as presented in figure 2.4.

Figure 2.4 Stress distribution (stiffness diameter < maximum stiffness diameter)

Whether the contact is intensive or not will be determined by the fit and the operating speed (because of centrifugal effects). Furthermore the bushing is not fitted to the shaft for the whole bushing length. This will also influence the stiffness of the rotor.

To get an indication of the influence of the bushings and other stiffening elements two cases were modelled and compared to the reference model. In the reference model the bushings of the AMB are modelled with maximum bushing diameter. The balance sleeve and impellers are modelled using a BW/IP engineering rule, were the shaft is increased in diameter by 0.3 times the length of the bushing to a maximum of the bushing diameter.

The first case to compare is the shaft without additional stiffness from the bushings. The second case is a model with additional stiffness from the balance sleeve and impeller but without additional stiffness from the AMB-bushings. Results from these analyses are presented in table 2.3
From the results for the two limit cases, i.e. model with maximum contribution of bushing stiffness and without contribution, can be concluded that the difference is only approximately 2%.

As expected the influence on the first mode (rocking mode) is less than on the second and third mode, because these modes have much more bending. The reason that the second mode is equally influenced as the third mode, although the third mode is the first real bending mode, can also be explained. As one can see in figure 2.2, the main difference in bending curvature is located at the middle of the rotor for both modes. The AMB-bushings, located at the shaft ends, are the main contributors to the additional stiffness and therefore the natural frequencies are almost equally influenced.

From the second test case, one can see that the main contribution to the additional shaft stiffness is supplied by the AMB bushings as expected because of their length and diameter.

With these results and to check the accuracy of the shaft model, was decided to carry out a modal analysis of the rotor for free - free conditions without rotation. This way two situations will be analyzed: The rotor with and without the bushings and impellers. The numerical results from MODAL1 analyses will be compared with experimental results. From this comparison the numerical model can be adjusted if necessary.

The rotor was first analyzed using the finite element program "GIFTS" and the "MODAL1" program by calculating the free-free modes for the non-rotating rotor (without seals and impellers). Because it was not certain that a rotor with free-free conditions could be analyzed the results of both programs were compared.
To calculate the undamped natural frequencies of a free-free rotor with GIFTS an eigenvalue problem has to be solved. With MODAL1 this problem is solved differently and the rigid body modes of the system will cause numerical problems when a real free-free rotor is analyzed. This can be omitted by using very weak springs to connect the model to the world. Now the rigid body modes will occur as modes with a very low frequency and the first three bending modes (which are of interest) will not change. The results of the analyses with GIFTS and MODAL1 were very close to each other and because the MODAL1 program is used for all the analyses, the MODAL1 program will also be used to compare these results with those from the experiments (sling tests).

The sling test was carried out by using elastic strings to carry the rotor and exciting the rotor with a hammer. The results of the sling test of the bare rotor, presented in table 2.4 together with the results from MODAL1, have a tolerance of $\pm 0.625$ Hz.

### Table 2.4 Free-free modes of rotor without bushings and impellers

<table>
<thead>
<tr>
<th></th>
<th>MODAL1 [Hz]</th>
<th>Sling test [Hz]</th>
<th>difference [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>mode 1</td>
<td>76.4</td>
<td>77.5</td>
<td>1.4</td>
</tr>
<tr>
<td>mode 2</td>
<td>207.2</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>mode 3</td>
<td>387.4</td>
<td>388.8</td>
<td>0.4</td>
</tr>
</tbody>
</table>

Considering the tolerance of the experiment results, the differences are small. The second mode could not be measured with an acceptable error and was omitted. Therefore the results of the analysis and sling test of the rotor with impellers and AMB bushings is of more importance. The thrust disc of the axial AMB was left out and was replaced by a dummy bushing. The results of the analysis and sling test are presented in table 2.5.

### Table 2.5 Free-free modes of rotor with bushings and impellers

<table>
<thead>
<tr>
<th></th>
<th>MODAL1 [Hz]</th>
<th>Sling test [Hz]</th>
<th>Difference [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>mode 1</td>
<td>54.5</td>
<td>55.0</td>
<td>0.9</td>
</tr>
<tr>
<td>mode 2</td>
<td>175.9</td>
<td>175.0</td>
<td>0.5</td>
</tr>
<tr>
<td>mode 3</td>
<td>318.5</td>
<td>313.8</td>
<td>1.5</td>
</tr>
</tbody>
</table>
Also this table show good similarities between the analytical model and the experiment. As stated before it is most important to create an accurate model. Therefore the model will be adjusted to get the free-free modes of the analytical model closer to the results of the sling test, keeping in mind that the tolerance of the test is still \( \pm 0.625 \) [Hz].

2.4.3 Influence of mass distribution

In the reference model the mass effects of the bushings, mechanical seals and impellers are lumped to the centre of gravity of each of these elements. These effects could however also be lumped to more stations of each element. A survey was made by calculating a model with better distributed masses and moments of inertia, keeping in mind that the resulting mass and moments of inertia have to be the same as in the reference model. Table 2.6 shows the results from this calculation and a comparison with the reference model.

<table>
<thead>
<tr>
<th></th>
<th>reference model [Hz]</th>
<th>model with distributed mass effects [Hz]</th>
<th>difference [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>mode 1</td>
<td>9.3</td>
<td>9.3</td>
<td>0.0</td>
</tr>
<tr>
<td>mode 2</td>
<td>25.9</td>
<td>25.9</td>
<td>0.0</td>
</tr>
<tr>
<td>mode 3</td>
<td>50.8</td>
<td>50.8</td>
<td>0.0</td>
</tr>
</tbody>
</table>

It appeared that the influence is negligible for the first three modes. For mode four and five the differences are 0.6% and 0.9% respectively; still acceptable differences. The main reason that this effect is not of influence, is the strong stiffness effects at the balance sleeve and wearings.

With these results it is not necessary to model the mass effects of the bushings and impeller to more stations than only the centre of gravity.
2.5 Free - free modes as function of MDI and running clearance (CL)

In the preliminary study (see first 6 month progress report BW/IP-01 [9]) the measurement frequency range was set from 0 to 120 Hz. As indicated before one of the objectives is to have resonance frequencies induced by the fluid/structure interactions in this frequency range. The quality of the curve fit and the total measuring time will be influenced by the measuring frequency range. For this reason it is important to find out how the natural frequencies are influenced by the MDI at the impeller. The running clearances could be used to control the position of the natural frequencies in the measurement frequency range.

The analyses are carried out with free - free conditions, i.e. the rotor modelled with the MDI at the seals and impellers but without the magnetic bearings. This way the undamped natural frequencies can be judged in the way it is done when a pump is designed. Furthermore the undamped natural frequencies, used as design requirements for the magnetic bearings, can be checked with the preliminary model.

In this survey the impeller MDI - values were set to three levels of the impeller MDI - range: Maximum value called HIGH, minimum value called LOW and average value called AVG. The annular seal clearances were varied from design values (CL = 100%), to 150% (CL = 150%) and finally to 200% (CL = 200%) for the maximum and minimum rotor speed. In figure 2.5 the results are plotted for the third mode at 2980 RPM. In this case the critical speed of 2980 RPM could result in problems for the measurements. The other figures with frequency - MDI - CL relations are presented in appendix IV, for the minimum and maximum operating speed of the test rig.

Note that in this figure the MDI - value is the unknown parameter and the CL value can be set to control the natural frequencies.

The results from this survey did not lead to changes in the design requirements for the magnetic bearings.
Figure 2.5 Undamped natural frequency of third mode at 2980 RPM

2.6 Undamped natural frequencies as a function of AMB stiffness

So far the models have been calculated for free - free conditions, i.e. the AMB's have not been modelled in dynamic sense. In figure 2.6 the undamped natural frequencies are plotted as a function of the magnetic bearing stiffness. Together with the previous analyses the dynamic behaviour can be judged now. From this figures it appears that till 5600 lbs/in, the maximum equivalent AMB - stiffness defined by Glacier, only the first mode is influenced by the equivalent AMB - stiffness. The model used for these calculations was provided with high motion dependent interactions at the impeller (MDI=HIGH) and clearances were at design value (CL = 100%).

2.7 Stability survey

To be able to perform the measurements at the test rig, the rotor has to be stable in a dynamic sense. The magnetic bearings supply the rotor with additional positive damping, quantified with results from the analyses with the preliminary design. The maximum damping value of 340 lbs.s/in was then defined as design requirement for the magnetic
Figure 2.6 Natural frequency as a function of AMB-stiffness at 1900 and 2980 RPM

bearings. The damping of the magnetic bearing is a function of frequency and is defined by Glacier. From data supplied by Glacier it appeared that for the frequency range of interest (0 - 120 Hz) the control system of the magnetic bearing will maintain the maximum value of 340 lbs-s/in.

To perform the tests at the rotor operating speeds, it has to be ensured that the rotor is behaving stable. For the same MDI-range and clearances-range a survey was carried out with MODAL1 by performing transient analyses of the rotor with an unbalance at the impellers. Stability was examined by observing the progress of the amplitudes. This was again carried out with maximum and minimum operating speed of the rotor.

The results of this survey are presented in table 2.7 and 2.8.
Table 2.7  Stability survey at 1900 RPM

<table>
<thead>
<tr>
<th>MDI</th>
<th>CL</th>
<th>100%</th>
<th>150%</th>
<th>200%</th>
</tr>
</thead>
<tbody>
<tr>
<td>LOW</td>
<td>stable; synchronous</td>
<td>stable; synchronous</td>
<td>stable; synchronous</td>
<td></td>
</tr>
<tr>
<td>AVG</td>
<td>stable; synchronous</td>
<td>stable; synchronous</td>
<td>stability on threshold; slightly subsynchr.</td>
<td></td>
</tr>
<tr>
<td>HIGH</td>
<td>stable; synchronous</td>
<td>stable; synchronous</td>
<td>stability on threshold; subsynchronous</td>
<td></td>
</tr>
</tbody>
</table>

Table 2.8  Stability survey at 2980 RPM

<table>
<thead>
<tr>
<th>MDI</th>
<th>CL</th>
<th>100%</th>
<th>150%</th>
<th>200%</th>
</tr>
</thead>
<tbody>
<tr>
<td>LOW</td>
<td>stable; synchronous</td>
<td>stable; synchronous</td>
<td>stable; synchronous</td>
<td></td>
</tr>
<tr>
<td>AVG</td>
<td>stability on threshold; slightly subsynchronous</td>
<td>stability on threshold; subsynchronous</td>
<td>stability on threshold; subsynchronous</td>
<td></td>
</tr>
<tr>
<td>HIGH</td>
<td>unstable; subsynchronous</td>
<td>unstable; subsynchronous</td>
<td>unstable; subsynchronous</td>
<td></td>
</tr>
</tbody>
</table>

Observing these results makes clear that at the operating speed of 1900 RPM hardly any problem occurs. Clearances of 200% of the design value should not be necessary under normal conditions and for CL = 100% and 150% the rotor is behaving stable for the complete MDI-range.

At the operation speed of 2980 RPM problems can occur in the complete clearances-range. Only if MDI = LOW, stability is ensured. For MDI = AVG the rotor is acting on the border between stability and instability.

If serious problem will occur during testing, alternatives have to be provided to ensure stability of the rotor. Additional positive damping by decreasing the annular seal clearances (CL) is one of these alternatives. A second and easy alternative is reducing the test speed envelope. A third alternative is to couple the x- and y-direction in the control loop of the AMB, to create tangential forces opposite to the whirling direction of the rotor.
2.8 Capacity for excitation at AMB

One of the tasks of the magnetic bearings is to excite the rotor. On top of that the load capacity of the magnetic bearing is necessary to carry the static load and the dynamic load due to normal operation. Figure 2.7 shows an example of the division of the total capacity of the AMB.

![Graph showing division of load capacity of AMB]

**Figure 2.7 Division of load capacity of AMB**

The determination of the static and dynamic load is described next.

**Static load**
The static load is determined by the weight of the rotor, excluding mass effects of the MDI, and the hydraulic load as defined by Agostenelli [13]. This hydraulic load exist during operation and is not motion dependent, therefore it will be treated as a static load. The hydraulic load can be calculated using:
\[ F_r = K_r \cdot b \cdot d \cdot \Delta p \]  \hspace{1cm} (2.2)

With 
- \( F_r \) = force \([N]\)
- \( K_r \) = force coefficient \([-\]
- \( b \) = width of impeller at outlet \([m]\)
- \( d \) = impeller diameter \([m]\)
- \( \Delta p \) = pressure difference of stage \([N/m^2]\)

The direction of this force is according to Agostenelli unknown. As a worst case the direction of this force is assumed to act in the same direction as the gravity.

**Dynamic load (normal operation)**

The dynamic bearing load at normal operation acting on the magnetic bearings is calculated with the PRS program from BW/IP. This program calculates the response of a system due to an user defined force spectra. The force spectra acting at several locations of the rotor are simulated and the reaction forces and displacements are calculated with this program. In this case two force spectra are defined, one at each impeller, representing the unsteady hydraulic forces. The coupling is assumed to be aligned perfect so no forces will contribute from the couplings. The force spectrum at an impeller can be represented by four major parts according to Verhoeven [14]: a linear force spectrum from 0 to 250 Hz, a force at 0.1 - 0.2 x operating speed, a force at 1 x operating speed and a force at 3 x operating speed (vane passing), presented in figure 2.8.

![Figure 2.8 Force spectrum at impeller](image)

The magnitude of these forces are calculated with the same formula as presented for
calculation of the hydraulic static load, with different $K_r$ values for each force case.

Adding the static load and dynamic load together, the remaining load capacity for excitation for the inboard and outboard bearing can be calculated for the model with $\text{MDI} = \text{AVG}$ and $\text{CL} = 100\%$. The outboard bearing is the bearing at the non-driving end, the inboard bearing is located at the coupling end. Figure 2.9 shows the excitation capacity for the inboard magnetic bearing operating at 2980 RPM. In appendix V you will find the excitation load capacity for the inboard and outboard bearings for the

![Graph showing the excitation capacity for the inboard magnetic bearing operating at 2980 RPM.](image)

**Figure 2.9** Remaining excitation load capacity of inboard bearing (2980 RPM)

minimum and maximum operating speeds of the test rig.

From the excitation load capacity it is possible to calculate the maximum displacements at the bearings, which can be applied to excite the test rig rotor. Figure 2.10 the maximum displacement diagram for the outboard AMB is presented as an example. Appendix V shows diagrams with the maximum displacement of the magnetic bearings as a function of frequency. From these figures it is obvious that at higher frequencies
only small displacements can be generated at the magnetic bearing.

It should be noted that the calculations so far are carried out for a worst case situation. The situation in y-direction is clearly better because the static load is small (only hydraulic) is acting in this direction.

The information about the maximum displacements at the magnetic bearings must be related to the required displacements at the magnetic bearings for excitation at the impellers. To complete this survey, the effect of several excitation scenarios on the radial and angular displacements at the impellers is calculated.

Four scenarios were used:

1. Force on inboard bearing
2. Force on outboard bearing
3. Force on inboard and outboard bearing acting in the same direction
4. Force on inboard and outboard bearing acting in opposite direction.
With the rotor speed set to 2980 RPM the ratios of the radial displacements of the magnetic bearing versus the impellers are presented in the figures in appendix V. Figure 2.11 shows the ratio of the radial displacements at the impeller versus the radial displacement at the inboard and outboard AMB. The excitation force is acting at the inboard AMB. Also the ratios between the angular displacements at the magnetic bearing versus the impellers are presented in appendix V. From these figures several conclusions can be drawn. To generate radial displacements at the impeller, it is easier to excite at the inboard bearing compared to the outboard bearing. With a force at the outboard bearing reasonable angular displacements at the impeller are created, but with scenario four this effect is stronger.

Finally it can be concluded that with the used scenarios sufficiently large radial and angular displacements can be generated at the impellers and the displacements at the magnetic bearings are of the same magnitude as those at the impellers. It can be expected that the noise ratios of the signals at the magnetic bearings will be of the same magnitude as those at the impellers.
One thing that has to be remarked is the existence of a resonance frequency at about 70 Hz. The undamped natural frequencies are not recognizable for two reasons. First of all the ratios are calculated what means that a resonance in both displacements at the same frequency will not be observed. Secondly the damping in the system is significant. The resonance frequency of about 70 Hz is calculated with the undamped natural frequencies, but turns up in the damped system.

2.9 Conclusions, remarks and recommendations

Originally it was planned to consider the dynamic coefficients of the eye wearings as known coefficients. From the evaluations and calculations of the University of Kaiserslautern it appears that these coefficients cannot be determined with an accuracy that is acceptable in the analytical model. The eye wearings should be considered as unknown and be measured together with the MDI at the impellers.

The comparison with the preliminary design showed good similarities concerning the undamped natural frequencies. The objective to have resonance frequencies in the measurement frequency range is feasible.

From the sensitivity study appeared that the numerical accuracy and the distribution of the mass of the bushings will not influence the results obtained from the models. The additional stiffness from the bushings however is acknowledged and the models will be adjusted with the results from the sling tests to create accurate models.

The free - free modes as a function of the MDI and the clearances, showed a possible problem for the third mode at 2980 RPM. For the design value of the clearance (CL=100%) the free - free mode can cause problem for the measurements if the MDI appear to be high. The best solution in that case is to adjust the speed envelop.

The AMB stiffness hardly influences the free - free modes of the rotor. Only the first mode is almost doubled at the maximum equivalent stiffness of the AMBs.

Stability, in dynamic sense, is a requirement to perform the measurements. At 2980 RPM the rotor is not stable for the MDI levels AVG and HIGH. This problem can be solved be decreasing the clearances at the seals, reducing the speed envelop or coupling the x direction and y direction in the control loop of the AMB. Finally the excitation capacity
survey showed good results. Till approximately 65 Hz a displacement of 40 μm can be applied to the AMBs at 2980 RPM. This should be sufficient looking to the ratios between the displacements at the AMBs and impellers.
SUBTASK 3

COMPARISON OF SOFTWARE BASED ON THE TRANSFER MATRIX METHOD AND THE FINITE ELEMENT METHOD

The main part of the project will (hopefully) result in the identification of the fluid/structure interactions. These interactions will be identified as dynamic coefficients as a function of operating and geometrical parameters of the pump. The first step after the project would be to use the obtained knowledge in the engineering department, to design new high speed pumps. To be able to design these new pumps, the existing rotor dynamic software has to be extended and modified, in order to perform the required rotor dynamic analyses.

BW/IP is using mainly software based on the so called transfer matrix method (TMM). This method was very popular before the finite element method (FEM) was well developed. The TMM is a very effective but inflexible method and is very useful in rotor dynamic analyses. In appendix II you will find a description of this method.

Before the project ends a decision must be made which software program will be modified and extended. To have arguments for the decision a start is made to compare these two methods with each other. To do this, the finite element program TURBO2 of the University of Kaiserslautern (Germany) was transferred to a HP-workstation. This program is specially developed for rotor dynamic analyses and is created by R. Nordmann and W. Diewald. From BW/IP the MODAL1, ROTSTB and the RESP2V3 program, developed by the University of Virginia (USA), were used to perform the similar analyses as with TURBO 2. The MODAL1 program is actually based on the modal theory, but the undamped natural frequencies are calculated with the TMM. The RESP2V3 program, used to calculate unbalance responses, and the ROTSTB program, used to calculate the eigenvalues, are completely based on the TMM.

Four test cases were defined and used to compare the two methods:

1) Massless beam with two point masses on rigid supports. Gyroscopic effects do not act on this rotor.
2) Beam with distributed mass on two rigid supports. Also in this case the gyroscopic effects are not included.
3) Laval rotor, i.e. beam with a disc, in the middle supported by journal bearings with gyroscopic effects included.
4) Vertical pump rotor model which resulted in serious problems when analyzed with RESP2V3 (TMM).

In the next part these cases are described together with some conclusions about the comparison. It should be noted that in this comparison not only the two methods are compared but also the programs based on the TMM are compared to the TURBO2 program.

3.1 Massless beam with two point masses on rigid supports

The model is axisymmetric in the plane perpendicular to the x-direction and has two point masses on equal distance from each support (figure 3.1).

![Massless beam with two discs on rigid supports](image)

Figure 3.1: Massless beam with two discs on rigid supports

The shaft is massless but has a certain stiffness against bending. The shear effects are neglected and the gyroscopic effects are not included as the rotorspeed is equal zero. The analytical solution of the natural frequencies is according to Stepanoff [6] equal to: Both frequencies can be calculated with:
\[ \omega_1^2 = \frac{6 EI}{ma^2(3l - 4a)} \quad \text{and} \quad \frac{\omega_1^2}{\omega_2^2} = \frac{(l - 2a)^2}{l(3l - 4a)} \]  

\begin{align*}
E &= 29 \times 10^6 \text{ lbs/in}^2 \quad d = 2 \text{ in.} \\
m &= 50 \text{ lbs} \quad l = 50 \text{ in.} \\
a &= 15 \text{ in.}
\end{align*}

This results in:

\[ \omega_1 = 228.31 \text{ rad/s} \quad \Rightarrow \quad f_1 = 36.337 \text{ Hz} \]
\[ \omega_2 = 765.75 \text{ rad/s} \quad \Rightarrow \quad f_2 = 121.878 \text{ Hz} \]

The model was calculated with the TURBO2 and MODAL1 program from which the results are presented in table 3.1.

<table>
<thead>
<tr>
<th>Mode</th>
<th>Analytical [Hz]</th>
<th>TURBO [Hz]</th>
<th>Difference analytical-TURBO [%]</th>
<th>MODAL [Hz]</th>
<th>Difference analytical-MODAL [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mode 1</td>
<td>36.337</td>
<td>36.332</td>
<td>0.01</td>
<td>36.350</td>
<td>0.04</td>
</tr>
<tr>
<td>Mode 2</td>
<td>121.878</td>
<td>121.862</td>
<td>0.01</td>
<td>121.910</td>
<td>0.03</td>
</tr>
</tbody>
</table>

The table shows good results for both programs. This could be expected from this relative simple problem. The results mean that the stiffness of the beams is good approximated with the two programs. In contrast with this test case, the next test case is a beam with a distributed mass.

### 3.2 Beam with distributed mass on rigid supports

The model, presented in figure 3.2, is axisymmetric in the plane perpendicular to the x-direction.

The analytical solution of the natural frequencies is derived from Van Campen [15].
Rotary inertia effects are neglected, which is valid for slender beams. The analytical solution can be written as:

\[
    \omega_k = \begin{cases} 
        \frac{(2k - 1)^2 \pi^2}{l^2} \cdot \sqrt{\frac{EI}{\rho A}} & \text{for } k \text{ is even} \\
        \frac{(2k)^2 \pi^2}{l^2} \cdot \sqrt{\frac{EI}{\rho A}} & \text{for } k \text{ is odd}
    \end{cases}
\]  

(3.2)

(3.3)

With certain values for the parameters the natural frequencies can be calculated:

\[
    \begin{align*}
        \omega_1 &= 392.0 \text{ rad/s} \quad \Rightarrow \quad f_1 = 62.39 \text{ Hz} \\
        \omega_2 &= 1568.0 \text{ rad/s} \quad \Rightarrow \quad f_2 = 249.55 \text{ Hz} \\
        \omega_3 &= 3527.9 \text{ rad/s} \quad \Rightarrow \quad f_3 = 561.49 \text{ Hz} \\
        \omega_4 &= 6271.9 \text{ rad/s} \quad \Rightarrow \quad f_4 = 998.20 \text{ Hz}
    \end{align*}
\]

The beam was modelled and calculated with MODAL1 and TURBO2 with increasing numbers of elements. The shear effects are not taken into account in the analytical solutions, so the same is done for the calculations with both programs. This leads to the results presented in table 3.2.
Table 3.2  Natural frequencies calculated with TURBO2 and MODAL1

<table>
<thead>
<tr>
<th></th>
<th>Analytical [Hz]</th>
<th>TURBO [Hz]</th>
<th>Difference TURBO/Analytical [%]</th>
<th>MODAL [Hz]</th>
<th>Difference MODAL/Analytical [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>4 Elements</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mode 1</td>
<td>62.39</td>
<td>60.83</td>
<td>2.5</td>
<td>60.80</td>
<td>2.5</td>
</tr>
<tr>
<td>Mode 2</td>
<td>249.55</td>
<td>227.89</td>
<td>8.7</td>
<td>225.91</td>
<td>9.5</td>
</tr>
<tr>
<td>Mode 3</td>
<td>561.49</td>
<td>471.07</td>
<td>16.1</td>
<td>454.05</td>
<td>19.1</td>
</tr>
<tr>
<td>8 Elements</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mode 1</td>
<td>62.39</td>
<td>61.96</td>
<td>0.7</td>
<td>61.95</td>
<td>0.7</td>
</tr>
<tr>
<td>Mode 2</td>
<td>249.55</td>
<td>242.98</td>
<td>2.6</td>
<td>242.63</td>
<td>2.8</td>
</tr>
<tr>
<td>Mode 3</td>
<td>561.49</td>
<td>530.15</td>
<td>5.6</td>
<td>527.86</td>
<td>6.0</td>
</tr>
<tr>
<td>Mode 4</td>
<td>998.20</td>
<td>906.72</td>
<td>9.2</td>
<td>897.54</td>
<td>10.1</td>
</tr>
<tr>
<td>16 Elements</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mode 1</td>
<td>62.39</td>
<td>62.26</td>
<td>0.2</td>
<td>62.25</td>
<td>0.2</td>
</tr>
<tr>
<td>Mode 2</td>
<td>249.55</td>
<td>247.48</td>
<td>0.8</td>
<td>247.20</td>
<td>0.9</td>
</tr>
<tr>
<td>Mode 3</td>
<td>561.49</td>
<td>551.20</td>
<td>1.8</td>
<td>549.69</td>
<td>2.1</td>
</tr>
<tr>
<td>Mode 4</td>
<td>998.20</td>
<td>966.49</td>
<td>3.2</td>
<td>961.80</td>
<td>3.6</td>
</tr>
</tbody>
</table>

The table shows that TURBO2 produces better estimates for the natural frequencies compared to MODAL1. This can be explained from the difference in the calculation of the mass effects of a beam. In TURBO2 a displacement function is used to create a mass matrix with coefficients, which not only relates radial accelerations with mass but also angular acceleration with mass. This is also done for the rotary inertia effects of a beam. In MODAL1 the mass of a beam is divided into two equal parts and is transferred to the mass stations at the end of a beam. The mass of a beam is then only related to the radial accelerations of the stations. The same approach is carried out for the rotary inertia effects. Although TURBO2 produces better results, the differences with the analytical solutions are still significant even for the model with 16 elements. These differences are due to the rotary inertia effects of the beam, which are neglected in the analytical solution. Even with a L/D ratio of 25 these effects influence the natural frequencies significant. This was checked with MODAL1, which has the possibility to exclude the rotary inertia effects of the beam elements. The natural frequencies are then very close to the analytical solution.
3.3 Laval rotor with journal bearings

The Laval rotor is a well known example in rotor dynamics. It consists of a disc on a beam with distributed mass. In this model the rotor is supported by two journal bearings.

Both bearings are characterized by a skew symmetric damping and stiffness matrix. The shaft is rotating at 3000 RPM and the gyroscopic effects are included in the calculations. As there exist no analytical solution, the eigenvalue calculations with the TURBO2 and ROTSTB program will be compared. The following more or less practical values are assumed:

\[ E = 29 \times 10^6 \text{ lbs/in}^2 \quad l = 20 \text{ in} \]
\[ m = 10 \text{ lbs} \quad d = 1 \text{ in} \]
\[ I_p = 10000 \text{ lbs.in}^2 \quad \rho = 0.284 \text{ lbs/in}^3 \]
\[ I_t = 1000 \text{ lbs.in}^2 \]

Journal bearings:

\[ b_{xx} = b_{yy} = 0.5 \text{ lbs.s/in} \quad b_{xy} = -b_{yx} = 0.1 \text{ lbs.s/in} \]
\[ k_{xx} = k_{yy} = 1000 \text{ lbs/in} \quad k_{xy} = -k_{yx} = 50 \text{ lbs/in} \]

The results of the calculations are presented in table 3.3 (shear effects are not taken into account).
Table 3.3  Eigenvalues of Laval rotor with journal bearing

<table>
<thead>
<tr>
<th></th>
<th>Damped natural frequencies</th>
<th>Damping exponent</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>TURBO [Hz]</td>
<td>ROTSTB [Hz]</td>
</tr>
<tr>
<td>2 beam elements</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mode 1</td>
<td>3.15</td>
<td>3.15</td>
</tr>
<tr>
<td>Mode 2</td>
<td>33.50</td>
<td>33.67</td>
</tr>
<tr>
<td>Mode 3</td>
<td>34.06</td>
<td>34.33</td>
</tr>
<tr>
<td>4 beam elements</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mode 1</td>
<td>3.15</td>
<td>3.15</td>
</tr>
<tr>
<td>Mode 2</td>
<td>33.51</td>
<td>33.50</td>
</tr>
<tr>
<td>Mode 3</td>
<td>34.07</td>
<td>34.16</td>
</tr>
<tr>
<td>8 beam elements</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mode 1</td>
<td>3.15</td>
<td>3.15</td>
</tr>
<tr>
<td>Mode 2</td>
<td>33.51</td>
<td>33.50</td>
</tr>
<tr>
<td>Mode 3</td>
<td>34.07</td>
<td>34.00</td>
</tr>
</tbody>
</table>

Table 3.3 shows again similar results for both programs. It should be stated however that the ROTSTB only calculates these three eigenvalues of this system with an acceptable error. TURBO2 calculates more eigenvalues due to the different technique applied (QR-decomposition). The TURBO2 program however does not provide in a error estimation, so it is difficult to judge the eigenvalues.

3.4 Vertical pump rotor model from RESPV2V3 program, which resulted in numerical problems

As already indicated in the section of subtask 1 in this report ("software modifications") numerical problems can originate in the TMM. In a particular analysis carried out at BW/IP this numerical problem acted strongly as can be seen in figure 3.4. In this model of a pump operating vertically, the rotor as well as the casing were modelled. Figure 3.4 shows the unbalance response at a station of the motor rotor which is connected to the pump rotor through a stiff coupling. The same model was entered in and calculated with TURBO2. The results, shown in figure 3.5, is now as should be expected from an unbalance response calculation.
Figure 3.4: Magnitude of unbalance response calculated with RESP2V3

Figure 3.5: Magnitude of unbalance response calculated with TURBO2
This type of numerical problems is typical for the TMM. The problems are due to the multiplication of the matrixes and the solving procedure in the program. The TURBO2 program uses numerical procedures which are better controlled (for instance the library "EISPACK" to solve eigenvalue problems).

The unbalance response is calculated in RESP2V3 by directly solving the equations of motion with the harmonic theory. In TURBO2 the modal theory is used to calculate the unbalance response.

3.5 Conclusions, remarks and recommendations

With the tests carried out, it is not completely clear to make a choice between the two methods. With the TURBO2 program (FEM), an additional advantage would be the extensive possibilities of this program. Analyses like unbalance response, stability, critical speed etc. can be carried out with one and the same program. At this moment at BW/IP it would take several programs to carry out these analyses. This can be additional source of errors as was experienced during the analyses carried out for the comparison.

The TURBO2 program has also the possibility to calculate the dynamic coefficients of seals and journal bearings. Furthermore it can vary almost all parameters of the model and can calculate the sensitivities of the parameters. These are very useful tools for the designer and some of these are lacking at this moment at BW/IP.

The calculation of higher modes of a model could give an indication of the accuracy of the two methods. In the test with the Laval rotor it appeared that the ROTSTB program can only calculate the first three eigenvalues of the system with an acceptable error. TURBO2 calculates more eigenvalues but there is no indication what the accuracy of the eigenvalues is. This makes a comparison difficult but from other experiences can be expected that the FEM becomes more inaccurate for higher modes. To reduce CPU-time of analyses of systems with a large number of DOFs, TURBO2 reduces the number of DOFs to a maximum of 20 DOFs. From experiences it is known that to calculate \( n \) number of eigenvalues accurately, the system may be reduced to maximum \( 2n \) DOFs, what means that TURBO2 can only calculate about 10 eigenvalues of a system with a
reasonable error. For this reason it is good to have an error estimation of the calculated eigenvalues.

The flexibility to create a model in a FEM program is higher than in a program based on the TMM. Some of the programs based on the TMM allow to define a second rotor with a different speed, which is set to zero to model the casing. The models of the rotors however are restricted in the choice of the nodal points and the way of connecting the rotors. Figure 3.6 explains this.

Finally some remarks will be made about the results from calculations with the models of the Brite/Euram project. These models were used to calculate the dynamic behaviour as described in subtask 2. These models were analyzed with TURBO2 once including the shear effects and once without shear effects. One would expect more or less similar results because the rotor is slender, so shear effects should not effect the dynamic behaviour much. The results however differed in eigenvalues in this way that the first few eigenvalues disappeared for the case without shear effects. An explanation can not be given, so more work has to be done to find out the reason for this.

As indicated before it is not completely clear what method should be used in future. It is clear however that the TURBO2 program is a good starting point to work from, because it is more flexible and extensive compared to the programs based on the TMM. Also TURBO2 applies the shear effects to the mass and stiffness matrixes (as it should be done), while the TMM programs only adjust the stiffness of a beam with the shear

---

Figure 3.6 Variation in modelling with TMM and FEM

---

[Diagram of rotor and casing models]
coefficient.
Finally can be remarked that no numerical problems occurred so far with the TURBO2 program.
More experiments should be carried out in the future to clarify which is the best suitable to modify and extend.
REFERENCES


53
CONVERSION TABLE

<table>
<thead>
<tr>
<th>US unit</th>
<th>Symbol</th>
<th>Conversion to SI unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>inch</td>
<td>in</td>
<td>$25.4 \times 10^{-3}$ m</td>
</tr>
<tr>
<td>pound</td>
<td>lbs</td>
<td>4.4482 N</td>
</tr>
</tbody>
</table>

Note:

The rotor dynamic software program TURBO2 can be used independently from the unit-system in which the problem is described, only then when the data entered is consistent. This means that the data of the experiments described in subtask 3 had to be modified for the parameters $m$, $I_p$, and $I_t$ as follows ($g$ is the gravity acceleration [in$^2$/s]):

- $m$ [lbs] is converted to $\frac{m}{g}$ [lbs.s$^2$/in] to be consistent with [kg]

- $I_p$, $I_t$ [lbs.in$^2$] is converted to $\frac{I_p}{g}$, $\frac{I_t}{g}$ [lbs.s$^2$/in] to be consistent with [kg.m$^2$]
APPENDIX I

PROJECT DESCRIPTION AND INDIRECT IDENTIFICATION PROCEDURE
PROJECT DESCRIPTION AND INDIRECT IDENTIFICATION PROCEDURE

In this appendix a short description of the Brite/Euram project is presented. A detail of the project aspects, the indirect identification procedure, is explained more extensively.

1. Title

Identification of fluid/structure interaction for the development and design of boilerfeed pumps (BFP).

2. Objectives

The fluid/structure interaction forces of annular clearance seals and impellers are the dominating factors in the dynamic (vibration) characteristics developed by large multistage boilerfeed pumps (BFP). These fluid/structure interaction forces must clearly be defined prior to the product development of a new generation high speed boilerfeed pump, 30% lower in cost and 3 to 5% more efficient. Furthermore, these unknown interaction forces are today the leading cause of unscheduled feed pump outages in power plants world wide. In 1987 the cost of production losses, due to feed pump outages in the ECC power plants were around 145 million ECU. It is now recognized, that centrifugal pump rotor dynamic behaviour is influenced tremendously by fluid/structure interaction forces generated at annular seals and impellers.

These forces can be divided into two groups:

a) Motion dependent forces, generated only when the pump rotor and structure have a motion supplementary to the axial rotation.

b) Pure hydraulic excitation forces, always present during the pumping operation and generated by the axial rotation.

The main aim of the project is to generate sufficient data to quantify both
motion dependent and pure excitation forces developed by fluid/structure interactions of specific boilerfeed pump components. This is accomplished by theoretical fluid flow analysis coupled to experimental research.

- The theoretical fluid flow analysis is conducted towards motion dependent fluid forces developed in annular seals and at the impeller tip and shroud. The main objective is to develop fluid flow models of seals and impellers which can be used for any feed pump configuration.

- The experimental research is conducted towards motion dependent as well as pure excitation fluid forces developed in annular seals and impellers. Experimental research will be done with two test pumps.

- A full scale test boilerfeed pump representing a 50% boilerfeed pump of a 300 mWatt power plant. Experiments will be towards motion dependent and pure excitation forces.

- A small test pump for single component testing. Experiments will be towards motion dependent forces.

INDIRECT IDENTIFICATION PROCEDURE

In order to assure that the principle of indirect identification is correctly understood the basic formulation is first rehearsed.

Upon the shaft of a rotating boilerfeed pump act different forces:

a. bearing forces

b. hydraulic and mechanical internal forces which are independent of the movement of the shaft centre with respect to the casing. These forces are for example mechanical unbalance, hydraulic unbalance due to vane angle error, ...

c. hydraulic forces caused by circumferential and axial fluid flow in seals,
bushings. These forces change when the shaft centre moves with respect to the casing (MDI).

d. External applied forces for testing. By using magnetic bearings, these correspond to imposed bearing forces (a).

In the first part of the project the motion dependent forces (type c) have to be identified.

The logic to do so is as follows: move the shaft position with respect to the casing by applying supplementary external forces. This will change the motion dependent forces, but not the motion independent forces (type b).

As a consequence the change in vibration response of the shaft is only due to the change in external applied force and changed motion dependent forces. As well the vibration responses as the external forces are measured. The only remaining unknowns are the motion dependent forces.

By using a rotor dynamics model of the shaft, the transfer functions between any force input and any response are calculated. These transfer functions, combined with measured forces and responses allow to identify the only remaining unknowns.

This procedure is schematically represented in figure 1. On the right side, the real test pump is depicted. The motion dependent forces are depicted by a spring and damper connected to the impeller. The analytical model does not have these elements since they are treated as external forces.
A simplified formulation is as follows.

The force inputs and displacement outputs are related by the system transfer matrix:

\[ \{ X \} = [ H ] \cdot \{ F \} \]  \hspace{1cm} (1)

The model as described above includes in the system transfer function matrix the elements corresponding to the unknown motion dependent interactions. Hence, \( X \) and \( F \) can be measured but \( H \) is unknown.

By applying additional external forces one can measure the response due to a given additional force input.

\[ \{ \delta X \} = [ H ] \cdot \{ \delta F \} \]  \hspace{1cm} (2)

By additional, we understand a force or displacement supplementary to the normal...
operating forces and displacements.

By using a rotor dynamics model on which we impose the same \( \{\delta F\} \) we calculate a different response \( \{\delta X_{an}\} \).

The calculated response is different since the rotor dynamics model takes not into account the motion dependent interactions (In the rotor dynamics model, they are treated as external forces).

\[
\begin{bmatrix}
\delta X_{an} \\
\delta X_{an}^{MDI}
\end{bmatrix} =
\begin{bmatrix}
H_{an}^{1,1} & H_{an}^{1,2} \\
H_{an}^{2,1} & H_{an}^{2,2}
\end{bmatrix}
\cdot
\begin{bmatrix}
\delta F \\
\delta F^{MDI}
\end{bmatrix}
\]  

(3)

with 'MDI' : motion dependent interaction location

When we impose the external excitation \( \delta F \), with \( \delta F^{MDI} = 0 \) we get :

\[
\begin{bmatrix}
\delta X_{an} \\
\delta X_{an}^{MDI}
\end{bmatrix} =
\begin{bmatrix}
\mathcal{H}_{an}^{1,1} \\
\mathcal{H}_{an}^{1,2}
\end{bmatrix}
\cdot
\begin{bmatrix}
\delta F
\end{bmatrix}
\]  

(4)

The difference between the measured responses and calculated responses is generated by the motion dependent forces. If we take corresponding elements of the calculated transfer function matrix, this writes:

\[
\begin{bmatrix}
\delta X \\
\delta X_{an}
\end{bmatrix} -
\begin{bmatrix}
\delta X_{an} \\
\delta X_{an}^{MDI}
\end{bmatrix} =
\begin{bmatrix}
\mathcal{H}_{an}^{1,2}
\end{bmatrix}
\cdot
\begin{bmatrix}
\delta F^{MDI}
\end{bmatrix}
\]  

(5)

The only unknowns in this formulation are the motion dependent interaction forces \( \{\delta F^{MDI}\} \) which can be solved for. With these forces applied to the rotor dynamics model, we can calculate the responses at the interaction location : \( \{\delta X^{MDI}\} \).

\[
\begin{bmatrix}
\delta X^{MDI}
\end{bmatrix} =
\begin{bmatrix}
H_{an}^{2,1} & H_{an}^{2,2}
\end{bmatrix}
\cdot
\begin{bmatrix}
\delta F \\
\delta F^{MDI}
\end{bmatrix}
\]  

(6)

From forces and responses we generate a dynamic model for the motion dependent interaction. This requires several sets of \( \{\delta X^{MDI}\} \) and \( \{\delta F^{MDI}\} \) in order to allow the full flexibility or stiffness matrix to be computed

\[
\begin{bmatrix}
\{\delta X^{MDI}\}_1 \\
\vdots \\
\{\delta X^{MDI}\}_n
\end{bmatrix} =
\begin{bmatrix}
H^{MDI}
\end{bmatrix}
\cdot
\begin{bmatrix}
\{\delta F^{MDI}\}_1 \\
\vdots \\
\{\delta F^{MDI}\}_n
\end{bmatrix}
\]  

(7)

To summarize:

\[
\begin{bmatrix}
\text{v}
\end{bmatrix}
\]
- impose external forces
- measure force and response
- make rotor dynamics model
- impose force on model, calculate response
- determine response difference
- calculate motion dependent interaction force
- calculate response at motion dependent interaction location
- derive model X/F for motion dependent interaction

The above procedure seems logical and straight forward. However the general theory has to be modified because:

- the magnetic bearings do not allow force excitation: they allow to impose a displacement in a given point, without guaranteeing that other displacements remain zero.

- as discussed above, the entire procedure works with forces of a given amplitude that is measured and then imposed on a model.

It is evaluated to measure instead of absolute inputs and outputs, the relative ratios, or to be more precise, transfer functions (response per unit input).

This allows to make also calculations with unit force excitation and to separate calculations from measurements. The main advantage of calculating transfer functions lies however in noise reduction on the measurements.
APPENDIX II

TRANSFER MATRIX METHOD
TRANSFER MATRIX METHOD

In the transfer matrix approach to the analysis of a beam structure, the system is divided into a finite number of stations, with each station consisting of a point followed by a field section. The representative lumped parameter model consists of rigid bodies at the points connected by massless shaft field segments. All inertia properties are lumped to the masses at the point, such that each mass has 4 degrees of freedom. The field sections between the points are to have elastic behavior according to the beam theory. The stations are numbered in the direction from the left to the right as shown in Figure 1. It should be noted that the last station has no field section. State quantities are chosen to describe the linear displacement $x, y$; the angular displacements $\theta, \phi$; the moments $M_x, M_y$; and the shear forces $V_x, V_y$ at the stations. The state quantities are arranged in a column matrix called the state vector $\{Z\}$. Based on force and moment equilibrium, the transfer equations that relate the state quantities to the right and to the left of each point may be constructed. When cast into the matrix form, they constitute the point transfer matrix $[P]_N$ that represents the jump in the displacement and the forces across the point. Transfer equations for the massless elastic field sections between the points may be generated from the beam theory. The matrix form of these equations establishes the field matrix $[F]_N$.

The general form of the point and the field matrices are shown in the following equations.

\[
\begin{align*}
\begin{bmatrix} D \\ Q \end{bmatrix}^R_N &= \begin{bmatrix} I & O \\ V & I \end{bmatrix}_N \begin{bmatrix} D \\ Q \end{bmatrix}^L_N \quad \Rightarrow \quad \{Z\}^R_N = [P]_N \{Z\}_N \\
\begin{bmatrix} D \\ Q \end{bmatrix}^L_{N+1} &= \begin{bmatrix} F_1 & F_2 \\ O & F_1 \end{bmatrix}_N \begin{bmatrix} D \\ Q \end{bmatrix}^R_N \quad \Rightarrow \quad \{Z\}^L_{N+1} = [F]_N \{Z\}^R_N
\end{align*}
\]

(1)

(2)

The state vectors in these equations are partitioned into displacement $D (x, y, \theta, \phi)$ and force $Q (M_x, M_y, V_x, V_y)$ subvectors. Station number is identified by $N$ at the lower right hand corner. The letter $R$ or $L$ refer to either the right or the left of the point at the
station 1
station 2
station N
station K

Figure 1: Idealized lumped parameter beam model

station.

The submatrix $V$ in the point matrix depends on the inertia and impedance properties at the station. These include forces due to linear and rotary accelerations, gyroscopic moments and bearing reactions. The submatrices $F_1$ and $F_2$ in the field matrix are functions of the shaft elasticity and the amount of internal friction.

By multiplying the point matrices and the field matrices together from the left to the right as in equation (3), the overall transfer matrix $[U]$ is obtained. This matrix relates the state vector at the left of the first point to the state vector at the right of the last point in the system.

$$[U] = [P_K] [F_{K-1}] [P_{K-1}] \cdots [F_1] [P_1]$$

(3)

In the treatment of the forced response problem, an additional row in the matrices is required to take into account of the forcing functions. The extended overall transfer matrix $[U]$ in equation (4) is shown to be divided into $9$ partitioned matrices.
Assuming the structure to have free-free ends, the bending moments and shear forces \( \bar{Q} \), at the first point and the last point are zero. Application of these boundary conditions to equation (4) leads to

\[
\begin{bmatrix}
\bar{U}_{21}
\end{bmatrix} \begin{bmatrix}
D
\end{bmatrix}_1^L + \begin{bmatrix}
\bar{T}_2
\end{bmatrix} = 0
\]

Equation (5) represents a set of simultaneous linear equations from which the state vector \( \{ \bar{Z} \}_1^L \) at the first station is to be calculated. The state vector at the other stations may be generated from \( \{ \bar{Z} \}_1^L \) according to

\[
\begin{bmatrix}
\bar{Z}
\end{bmatrix}_N = \begin{bmatrix}
P_{N-1}
\end{bmatrix} \begin{bmatrix}
\bar{P}_{N-1}
\end{bmatrix} \cdots \begin{bmatrix}
\bar{P}_1
\end{bmatrix} \begin{bmatrix}
\bar{Z}
\end{bmatrix}_1^L
\]

In general, for a damped system, all the matrices except the field matrix are complex. The required solution \( \{ Z \} \) is the real part of the complex state vector \( \{ \bar{z} \} \) multiplied by \( e^{i\omega t} \), where \( \omega \) is the frequency of excitation.
APPENDIX III

MODEL OF TWO STAGE PUMP
APPENDIX IV

FREQUENCY - MDI - CL - PLOTS
Natural frequency mode 1 at 2980 RPM
as a function of HDI and CL

Natural frequency mode 1 at 1900 RPM
as a function of HDI and CL
Natural frequency mode 2 at 2980 RPM
as a function of HDI and CL.

Natural frequency mode 2 at 1900 RPM
as a function of HDI and CL.
Natural frequency mode 3 at 2980 RPM
as a function of HDI and CL

Natural frequency mode 3 at 1900 RPM
as a function of HDI and CL
APPENDIX V

EXCITATION REVIEW
Theta impeller versus Theta AMB OB ratio (force on OB/IB same direction)

Theta impeller versus Theta AMB IB ratio (force on OB/IB same direction)