Dissipativity based control synthesis approach for suppressing oscillations in electrical power systems

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Award date:
2007

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Dissipativity Based Control Synthesis Approach for Suppressing Oscillations in Electrical Power Systems

by

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Master of Science thesis

Project period: April 2007
Report Number: 07A/03
Commissioned by:
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Dissipativity Based Control Synthesis Approach for Suppressing Oscillations in Electrical Power Systems

Graduation paper
March 5, 2007

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Abstract—In this paper a control synthesis approach is presented for supplementary controllers in electrical power systems, with the goal of damping low frequency oscillations. The approach is based on restricted dissipativity and the Generalized Kalman-Yakubovich-Popov (GKYP) lemma. The proposed synthesis approach is used to design a “Power system stabilizer”. The resulting controller is tested in simulations on a standard benchmark case of a multi-machine power system.

I. INTRODUCTION

Power systems are often characterized by badly damped electromechanical oscillations with frequencies ranging from 0.1Hz to 2.0Hz. The stability of this oscillating mode has become a source of concern, since the economical forces in today’s competitive market environment often tend to push the system closer to its stability limits. Efficient damping of the power swings is therefore crucial for keeping the system online [1]. The cause of the badly damped oscillations is twofold. Improvement of transient stability, for example high gain voltage regulation, will decrease the damping of local plant modes in the range of 0.8Hz till 2.0Hz [2]. As a second cause, if large groups of generators are coupled by weak links that are characterized by heavy power transfers, it is likely that inter-area modes of oscillation in low frequency range (0.1Hz-0.8Hz) will occur [2].

In comparison to expensive upgrade of the system interconnections, the use of supplementary controllers, for instance on Flexible AC Transmission Systems (FACTS) [3], is the only practical method of damping inter-area oscillations. The topic of damping electromechanical oscillations has been extensively studied. The most common approaches for design of those controllers include optimal control design techniques like for instance $H_\infty$, $H_2$, multi-objective control, etc. For an overview of the state-of-the-art of this techniques with application to the power system control, see [2] and the references therein. Although the resulting controller is optimal in a certain sense, those design techniques have several drawbacks. One of the drawbacks is that the order of the resulting controller is equal to the order of the so called generalized plant. The generalized plant includes the dynamics of the system that is to be controlled, complemented with a set of weighting filters used to specify a desired shape of the closed loop frequency responses. For power systems applications, the order of the generalized plant is always high, making the resulting controllers impractical for implementation. Furthermore, the choice of suitable weighting filters is often a hard problem and requires a lot of trial-and-error iterations even for a skilled control designer. On the other side, classical control techniques, like PID-type controllers involve tuning based on graphical, frequency domain approaches. PID controllers have an advantage of low complexity of the resulting controller. However, these techniques are easily applicable only for SISO systems and in general do not result in optimal controllers.

Because of the large amount of literature on this topic, in this introduction we briefly address only the references that are directly related to the synthesis approach presented in this paper. In [4] and [5] a quantitative feedback theory (QFT) technique was used to tune controllers like the “Power System Stabilizer” (PSS) [6] and the supplementary controllers for various FACTS devices. The goal was to render the subsystem, e.g. the generator or the FACTS device, passive in a desired frequency range. The results of [4] and [5] have shown that this dissipativity based design procedure produces good candidate controllers. The control design objective in this paper is the same as in [4] and [5], and the novelty is that, instead of using graphical techniques applicable only to SISO systems, we achieve the desired dissipativity properties by utilizing the Generalized Kalman-Yakubovich-Popov (GKYP) lemma [7]. This approach permits a direct treatment of multiple frequency-domain inequalities (FDI) specifications, completely avoiding approximations associated with frequency gridding or frequency weights [8]. The resulting synthesis conditions are given as a finite-dimensional convex optimization problem described by linear matrix inequalities (LMIs), which can be efficiently solved by dedicated software, e.g. [9].

Depending on network topology and operating points the frequency of the badly damped oscillated mode changes, but, as already mentioned, is almost exclusively in the frequency range 0.1-2.0 Hz. Making the subsystem, e.g. the generator or the FACTS device, passive for that particular frequency range has a positive effect on damping the oscillations. The advantage is that the passivity based design can be performed based on the subsystem model alone, without building the overall system model, as is for instance often done in [2]. The overall system model, although necessary for the optimal solution, is characterized by its large scale and large uncertainties, and is often impractical or even useless for controller design. The resonances in the frequency band of interest are influenced by changing power system configurations and loading patterns [4]. It is hard to take into
account all possible variations. For that reason, the design procedure described in this paper does not rely on identifying the resonances precisely.

The contribution of this paper is the utilization of the GKYP lemma to translate the dissipativity requirements into solvable LMIs with the aim to suppress oscillations in a restricted frequency range in power systems. In comparison with graphically design procedures in [4] and [5], the synthesis approach presented in this paper is more straightforward, suitable for MIMO systems, results in low order controllers and can be extended with additional design requirements. An optimization procedure with the aim to increase the dissipation of energy in a restricted frequency range is derived for the specific case of a synchronous generator.

It is possible to extend the presented synthesis approach to other supplementary controllers, on for example FACTS devices, to damp low frequency oscillations in power systems.

A. Notation

We use the following notation. The field of real numbers and the field of complex numbers are denoted by \( \mathbb{R} \) and \( \mathbb{C} \), respectively, while \( \mathbb{R}^{m \times n} \) and \( \mathbb{C}^{m \times n} \) denote \( m \times n \) matrices with elements in the corresponding fields. The transpose and complex conjugate transpose of the matrix \( A \) are denoted by \( A^T \) and \( A^* \), respectively. The set of all \( n \times n \) Hermitian and symmetric matrices are denoted by \( \mathbb{H}_n \) and \( \mathbb{S}_n \), respectively. The matrix inequalities \( A \succ B \) and \( A \succeq B \) mean \( A - B \) are Hermitian and \( A - B \) is positive definite and positive semi-definite, respectively.

II. BACKGROUND AND PRELIMINARIES

For completeness of presentation, in this section we briefly recall the notion of dissipativity of dynamical systems, the KYP lemma and its generalization. For a detailed introduction to the subject we refer to the textbook [10]. For GKYP lemma see [7].

A. Dissipativity

A dissipative dynamical system is characterized by the property that the amount of energy which the system can supply to its environment cannot exceed the amount of energy that has been supplied to it [11]. In other words, when time evolves, a dissipative system absorbs a fraction of the supplied energy and transforms it into energy losses like heat and electro-magnetic radiation. Formal definition of the dissipativity is given as follows [11], [12].

Consider a continuous time, time-invariant dynamical system \( \Sigma \) described by ordinary differential equations

\[
\dot{x}(t) = f(x(t), u(t)),
\]

\[
y(t) = g(x(t), u(t)),
\]

where \( x(t) \in \mathbb{R}^n \), \( u(t) \in \mathbb{R}^m \) and \( y(t) \in \mathbb{R}^p \), denote the state, input, and output vector, respectively. Let \( s : \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R} \) be a mapping and assume that for all \( t_0, t_1 \in \mathbb{R} \) and for all input-output pairs \((u, y)\) satisfying (1) the composite function \( s(u(t), y(t)) \) is locally absolutely integrable. The mapping \( s \) is referred to as the supply function. The system \( \Sigma \) is dissipative with respect to the supply function \( s \) if there exists a storage function \( V : \mathbb{R}^n \to \mathbb{R} \) such that

\[
V(x(t_0)) + \int_{t_0}^{t_1} s(u(t), y(t)) \, dt \geq V(x(t_1))
\]

for any \( t_0 \leq t_1 \) and all signals \((u, x, y)\) which satisfy (1). The supply function \( s \) is interpreted as the supply delivered to the system. The time integral in (2) has therefore interpretation of the work that has been done on the system (i.e. the work done by the system, if its value is negative). A storage function \( V \) generalizes the notion of an energy function for a dissipative system.

Passivity is a specific case of dissipativity, and, for \( n_y = n_x \), it is specified by the supply function \( s(u, y) = u^T y \). This particular supply function is widely encountered in applications since it denotes the (physical) power that is supplied to a system, e.g. product of voltages (as inputs) and currents (as outputs) in electrical circuits. In the case of a synchronous generator, in this paper we will consider the supply function \( s(T, \phi) = T \phi \), where \( T \) denotes a torque acting on a generator rotor and \( \phi \) denotes the rotor angular velocity.

For a linear, time-invariant dynamical system \( G(s) := (sI - A)^{-1}B + D \), with \( A \in \mathbb{R}^{n_x \times n_x} \), \( D \in \mathbb{R}^{n_y \times n_x} \), and a quadratic supply rate of the form

\[
s(u, y) = \begin{pmatrix} u \\ y \end{pmatrix}^T \begin{pmatrix} Q & S \\ S^T & R \end{pmatrix} \begin{pmatrix} u \\ y \end{pmatrix},
\]

the dissipativity condition (2) is equivalent to the frequency domain inequality (FDI)

\[
\begin{pmatrix} I & 0 \\ G(j\omega) & -S \end{pmatrix} \begin{pmatrix} -Q & S \\ -S^T & -R \end{pmatrix} \begin{pmatrix} I \\ -G(j\omega) \end{pmatrix} \leq 0, \forall \omega \in \mathbb{R} \cup \{\infty\}.
\]

Here, the matrix

\[
\Pi := \begin{pmatrix} -Q & S \\ -S^T & -R \end{pmatrix}
\]

is a real symmetric matrix, i.e. \( \Pi \in \mathbb{S}^{n_x+n_y} \), which is partitioned to confirm with \( u \) and \( y \). We will refer to \( \Pi \) as the supply matrix.

It is easy to see that passivity is specified by a supply rate with

\[
\Pi = \begin{pmatrix} 0 & \begin{pmatrix} -I \\ -I \end{pmatrix} \end{pmatrix},
\]

which, in the frequency domain, yields the celebrated positive real condition for a transfer matrix:

\[
G(j\omega) + G(j\omega)^* \succeq 0, \quad \forall \omega \in \mathbb{R} \cup \{\infty\}.
\]

B. Kalman-Yakubovich-Popov lemma

The Kalman-Yakubovich-Popov lemma states that the FDI (4) holds for all \( \omega \in \mathbb{R} \cup \{\infty\} \) if and only if the LMI

\[
\begin{pmatrix} -Q & S \\ -S^T & -R \end{pmatrix} \geq 0,
\]

with \( (Q, S, R) \) being a solution of the LMI (4).
(I 0)\(W\) (I 0) + (0 I)\(\Pi\) (0 I) \leq 0, \quad (8a)

\[W = \begin{pmatrix} -Y & X + j\omega_0 Y \\ X - j\omega_0 Y & -\omega_0^2 Y \end{pmatrix},\]  

where \(\omega_0 := (\omega_0 + \omega_2)/2\) and \(X, Y \in \mathbb{H}_n, Y \succeq 0\).

As a counterpart to the time domain, dissipativity interpretation of FDI (4), in [13] it was shown that the restricted frequency inequality is equivalent to "restricted dissipativity". The notion of restricted dissipativity is defined in the time domain, and states that the system is dissipative for all the inputs \(u(t)\) that drive the system with an appropriate "speed". For instance, if the FDI in (7) holds for all \(\omega\) within a low frequency range (and not necessarily for all \(\omega \in \mathbb{R} \cup \{\infty\}\)), then the system is passive for all inputs \(u(t)\) that drive the system "slowly", where a precise, time domain definition of the "slowness" is given in [13].

III. MODEL AND PROBLEM DEFINITION

Synchronous generators are essential components of an electrical power system. Although an electrical power system is composed of an extremely large number of different dynamical systems, e.g. a large variety of industrial loads, residential loads, FACTS devices, etc., it is the synchronous generators and their controllers that are mainly responsible for forming many of the crucial properties of the power system's dynamics. The badly damped, low frequency oscillation mode considered in this paper, is an example of one such property.

A block diagram of a synchronous generator model is presented in Fig. 1. Here, we will only briefly describe its main components and for detailed description we refer to [6], chapter 12. A precise model of a generator is nonlinear and in this paper we will use its LTI approximation, which is accurate enough and a widely used model in this type of studies. In Fig. 1 we use the symbol \(\Delta\) in the names of all of the signals to emphasize the fact the model is linearized, and that all signal amplitudes in the figure refer to the deviations from some nominal value that defined the operating point.

The rotor of a generator is modeled as a rotating inertia with a damping, with the accelerating torque \(\Delta T_r := \Delta T_m - \Delta T_e\) as an input and angular velocity \(\Delta \omega\) as an output. Signal \(\Delta T_r\) represents the electromagnetic torque formed in the generator, while \(\Delta T_m\) represents the driving mechanical torque. Symbols \(H\) and \(K_D\) denote inertia constant of a rotor (for precise definition see [6], pp. 129) and a damping constant, respectively. The exciter provides ac power to the rotor field winding. Its goal is to control the generator terminal voltage \(\Delta E_f\) to its reference value, which is on Fig. 1 denoted as \(\Delta V_{ref}\). The power system stabilizer (PSS) provides a supplementary stabilizing signal to control the excitation of the rotor field winding to improve the dynamic performance of the system. Later in this section we will give more detailed, insightful description of the PSS function in terms of dissipativity. The flux linkage equations block contains the model of the electromagnetic phenomena in the generator, with a developed electromagnetic torque \(\Delta T_e\) as output. The signal pair \((\Delta V, \Delta \omega)\) are interconnecting port signals on the interface with the rest of the power system.

Analysis of the badly damped oscillation mode of a typical power system network shows that the angular velocities
without a priori defined structure, such that the following requirements are met: i) the closed-loop system is stable; ii) the closed-loop transfer function $G(s)$ from $\Delta T_m$ to $\Delta \omega$ is positive real in some pre-defined frequency interval $\Omega := [\omega_1, \omega_2]$, $\omega_1 < \omega_2$, i.e. $G(j\omega) + G(j\omega)\ast \geq 0$ for all $\omega \in \Omega$. \hfill \Box

In this paper, the structure imposed on the PSS controller is its a priori defined (low) order. In the numerical example presented in Section VI, we will show that a PSS controller that meets the requirements stated in Problem III.1, when utilized in a power system, gives good performance in terms of damping oscillations.

IV. PASSIVITY BASED PSS SYNTHESIS

Structured controller synthesis is a hard problem, which is, despite the large number of research publications on the topic, still not fully solved. For recent results and an overview on this subject, see [14]. In this section we present one possible approach for solving Problem III.1, which is based on the GKYP lemma. We will consider the 2nd order PSS controller given by its transfer function

$$PSS(s) = \frac{\beta_1 s^2 + \beta_2 s + \beta_3}{\alpha_1 s^2 + \alpha_2 s + \alpha_3},$$

(10)

where $\beta := (\beta_1, \beta_2, \beta_3)$ denotes the set of free parameters which are to be designed, while $\alpha := (\alpha_1, \alpha_2, \alpha_3)$ is a fixed set of parameters. The reason why the parameters $\alpha$ are not considered as variables in the synthesis problem, is that in that case the design objective generally cannot be formulated as an LMI feasibility problem. On the other hand, by fixing the PSS poles and exploiting the freedom in choosing its zeros, the synthesis problem can be formulated in terms of LMIs.

Of course, by fixing the poles of the PSS we do not use the complete freedom for synthesis of the controller. Even with this limited freedom in choosing the controller parameters, for the example considered in this paper, the proposed synthesis approach has shown to be successful in achieving the desired objectives. For choosing the poles of the PSS, we can take, for instance, the poles from a model reduced controller that is original designed by $H_m$, or other full order control synthesis approaches.

Fig. 4 presents an aggregated block diagram of a synchronous generator model, where blocks $P$ and $K$ from Fig. 4 correspond to the dashed-line blocks $P$ and $K$ of Fig. 1. The PSS controller is therefore a part of the subsystem $K$. The
restricted passivity objective from Problem III.1 is given by the following restricted FDI:

\[
\left( \begin{array}{c}
I \\
(P(I + KP)^{-1})^* \\
0 \\
-1
\end{array} \right) \left( \begin{array}{c}
0 \\
-I \\
1 \\
0
\end{array} \right) \left( \begin{array}{c}
I \\
(P(I + KP)^{-1}) \\
0 \\
-1
\end{array} \right) \preceq 0, \forall \omega \in \Omega.
\]  
(11)

After the congruence transformation with \((I + KP), \) inequality (11) is shown to be equivalent to the following FDI:

\[
P^*(K^* + K)P \preceq -(P + P^*), \quad \forall \omega \in \Omega.
\]  
(12)

In the case of \(P\) being positive real for all \(\omega \in \Omega,\) it is apparent that a sufficient condition for (12), and therefore a sufficient condition for (11), is given by

\[
K^* + K \succeq 0, \quad \forall \omega \in \Omega.
\]  
(13)

The above result is a simple generalization of the main passivity theorem for feedback connection to restricted passivity of feedback interconnection of LTI systems. The main passivity theorem [15] states that the negative feedback connection of two passive systems is passive. Although the frequency domain is often suitable for defining desired synthesis specifications, like for instance condition (13), the resulting FDIs are hard to verify in its original form. Utilizing the GKYP lemma, FDIs are replaced by equivalent, finite dimensional LMI feasibility conditions, which are more suited for both analysis and controller synthesis.

For a synchronous generator, \(P\) is a stable first order system and is therefore always passive. This implies that the above stated results can be directly used in formulating a sufficient condition for a controller \(K\) to fulfill the objectives stated in Problem III.1. With \((A_K, B_K, C_K, D_K)\) denoting the matrices of a state-space realization of \(K, \) i.e. \(K(s) = C_K(sI - A_K)^{-1}B_K + D_K,\) the following condition is equivalent to (13):

\[
\begin{bmatrix}
(* & *)
\end{bmatrix}^T 
\begin{bmatrix}
-Y & X + j\omega Y \\
* & -\omega Y\end{bmatrix} \begin{bmatrix}
0 & I \\
A_K & B_K
\end{bmatrix} + 
\begin{bmatrix}
0 \\
-C_K
\end{bmatrix} + 
\begin{bmatrix}
0 & -C_K
\end{bmatrix} \preceq 0,
\]  
(14)

with \(\omega := (\omega_1 + \omega_2)/2,\) is feasible for some \(X, Y \in \mathbb{H}^n, Y \succeq 0.\)

Note that if \(A_K\) and \(B_K\) are fixed, while \(C_K\) and \(D_K\) are treated as variables, the matrix inequality (14) is an LMI in the decision variables \((X, Y, C_K, D_K).\) As a next step, we shown that the design of PSS zeros can be formulated in this form. For that purpose, let the connection of the exciter and the “flux linkage” block from Fig. 1 be given by a state-space realization as follows:

\[
\begin{bmatrix}
\dot{x} \\
\Delta T_e
\end{bmatrix} = 
\begin{bmatrix}
A & B_1 & B_2 \\
C_1 & D_1 & D_2
\end{bmatrix} 
\begin{bmatrix}
x \\\n\Delta T_{pss}
\end{bmatrix}.
\]  
(15)

Furthermore, let PSS (10) given by a state-space realization

\[
\begin{bmatrix}
\dot{x}_{pss} \\
\Delta T_{pss}
\end{bmatrix} = 
\begin{bmatrix}
A_{pss} & B_{pss} \\
C_{pss} & D_{pss}
\end{bmatrix} 
\begin{bmatrix}
x_{pss} \\\n\Delta T_{pss}
\end{bmatrix}.
\]  
(16)

One possible realization of the PSS (10) is given by its observer canonical form as follows: \(A_{pss} = \begin{bmatrix}
-\alpha_2/\alpha_1 & 1 \\
-\alpha_3/\alpha_1 & 0
\end{bmatrix},\)

\(B_{pss}(\beta) = \begin{bmatrix}
\beta_{a_1} - \beta_{a_2} \\
\beta_{a_2} - \beta_{a_3}
\end{bmatrix}^T, C_{pss} = \begin{bmatrix}
1 & 0
\end{bmatrix}\) and \(D_{pss}(\beta) = \frac{\beta}{\alpha_1}.\) Note that \(B_{pss}(\beta)\) and \(D_{pss}(\beta)\) are affine in the parameters \(\beta.\) We have the following expression for the state-space realization matrices of \(K:\)

\[
\begin{bmatrix}
A_K & B_K \\
C_K & D_K
\end{bmatrix} := 
\begin{bmatrix}
A & B_1C_{pss} & B_1D_{pss}(\beta) + B_2 \\
0 & A_{pss} & B_{pss}(\beta)
\end{bmatrix}.
\]  
(17)

Note that the variable \(\beta\) is entering \(K\) through the matrix \(B_K\) and therefore the matrix inequality (14) is not an LMI. However, it can be transformed into an LMI as follows. Define the conjugate system of \(K(s)\) as \(K^-(s) := K^T(-s) = B_K^T(-sI - A_K)^{-1}C_K + D_K.\) Since \((K(j\omega))^* = K^-(j\omega)\) for all \(\omega \in \mathbb{R},\) condition (13) can be equivalently written as

\[
K^-(j\omega) + K^-(j\omega)^* \succeq 0, \quad \forall \omega \in \Omega.
\]  
(18)

Now, since a realization of \(K^-(s)\) is given with the following set of state-space matrices

\[
\begin{bmatrix}
\lambda_K & B_K \\
C_K & D_K
\end{bmatrix} := 
\begin{bmatrix}
A_K^T & -C_K \\
-B_K & D_K
\end{bmatrix},
\]  
(19)

the GKYP lemma yields an LMI condition equivalent to (18). This condition is further used to design appropriate values of \(\beta.\)

V. ADDITIONAL CONSTRAINTS FOR SYNTHESIS

Along the same lines as above, additional desired (restricted) dissipativity properties can be added as control synthesis objectives.

A. Gain reduction

In this subsection we present the implementation of the constraint that the resulting PSS will have the maximal gain below some pre-defined value \(\gamma\) on a specified frequency range \(\Omega.\) This can be formulated as an FDI, i.e.

\[
\begin{bmatrix}
I \\
(PSS(j\omega))\end{bmatrix} \begin{bmatrix}
-\gamma^2 I & 0 \\
0 & I
\end{bmatrix} \begin{bmatrix}
PSS(j\omega)
\end{bmatrix} \succeq 0,
\]  
(20)

\(\forall \omega \in \Omega.\)

We define the conjugate system of the PSS (16) as \(PSS^*,\) which realization is given by

\[
\begin{bmatrix}
\lambda_{pss} & B_{pss} \\
C_{pss} & D_{pss}
\end{bmatrix} := 
\begin{bmatrix}
-A_{pss}^T & -C_{pss} \\
-B_{pss}(\beta) & D_{pss}(\beta)
\end{bmatrix}.
\]  
(21)

However, the translation from (20) to an LMI is not straightforward, because with the specific supply matrix in (20), the FDI will not result in an LMI directly when we follow the GKYP lemma. The utilization of the Schur complement [10] will translate, in this specific case, the resulting nonlinear
matrix inequality to an LMI. The resulting LMI equivalent to (20) is
\[
\begin{pmatrix}
M_{11} & M_{12} \\
M_{21} & M_{22}
\end{pmatrix} \preceq 0,
\]
with
\[
\begin{align*}
M_{11} &= \begin{pmatrix} * & * \end{pmatrix}^T W \begin{pmatrix} 1 & 0 \\ \bar{A}_{\text{pass}} & \bar{B}_{\text{pass}} \end{pmatrix} \begin{pmatrix} 0 & 0 \\ 0 & -\tau^T \end{pmatrix}, \\
M_{21} &= \bar{C}_{\text{pass}}(\beta) D_{\text{pass}}(\beta), \\
M_{22} &= -I,
\end{align*}
\]
where \( W \) denotes the multiplier representing the frequency range \( \Omega \), and is given by (9).

If the considered frequency range is equal to \( \mathbb{R} \cup \{\infty\} \), this is nothing else than putting the bound on the \( H_\infty \) norm of \( \mathcal{L} \). The integral in (25) corresponds to the dissipated energy by the system \( G \) during \( k \) periods of the input-signal. If we, instead of (2), use the strict dissipativity inequality \([10]\), pp. 38, then (25) becomes
\[
\int_0^{t_0+kT} s(u(t),y(t))dt \geq \varepsilon^2 \int_0^{t_0+kT} \|u(t)\|^2 dt,
\]
with \( \varepsilon > 0 \). The maximum \( \varepsilon \) can be seen as a measure of the dissipated energy with respect to the input-energy. According to (26), maximizing \( \varepsilon \) during synthesis will maximize the energy dissipated by the system \( G \). From (26) and (23) it follows that
\[
G(j\omega)^* + G(j\omega) \geq \varepsilon, \quad \text{for some } \omega \in \Omega.
\]

B. Closed-loop stability

In this subsection we propose a condition to ensure the resulting closed-loop system is stable, and therefore solve part (i) of Problem III.1.

Since \( P \) is passive in the case of a synchronous generator, for the closed-loop to be stable, it is sufficient to ensure that \( K \) is strictly passive, i.e. that it is strictly positive real for all frequencies including infinity \([15]\). Indeed, in that case the synthesis objective (ii) of Problem III.1 is automatically fulfilled. Still, with \( K \) being passive, one can use the GKYP lemma to optimize the restricted passivity properties of a closed loop system on a desired frequency band \( \Omega \), as explained in the next subsection.

C. Increasing dissipation of energy

In this subsection we show that it is beneficial to add an optimization to Problem III.1 with objective to maximize \( \varepsilon \) subject to \( G(j\omega)^* + G(j\omega) \geq \varepsilon \), \( \forall \omega \in \Omega \). We will prove that maximizing \( \varepsilon \) for a certain frequency band \( \Omega \) increases the dissipation of energy by the system \( G \) when it is excited with an input signal with frequency \( \omega \in \Omega \). First we derive a general condition for one frequency and further we extend this condition for a frequency band in the case of a synchronous generator.

Consider the linear, time-invariant dynamical system \( G(s) = C(sI-A)^{-1}B + D \), with \( A \in \mathbb{R}^{n \times n}, D \in \mathbb{R}^{n \times n} \). Let \( \omega \in \Omega, \omega > 0 \) be such that \( \det(j\omega I - A) \neq 0 \) and consider the (complex) input \( u(t) = U_0 e^{j\omega t} \) with \( U_0 \in \mathbb{R}^n \). Define \( x(t) = (j\omega I - A)^{-1}Bu_0 e^{j\omega t} \) and \( y(t) = Cx(t) + Du(t) \), then
\[
y(t) = G(j\omega)U_0 e^{j\omega t} \quad \text{is periodic with } \tau = \frac{\pi}{\omega}.
\]
With this input, the supply function for passivity is
\[
s(u(t),y(t)) = y(t)^*u(t) + u(t)^*y(t) = U_0^* (G(j\omega)^* + G(j\omega)) U_0,
\]
and is independent of time \( t \).

If the system \( G \) is a stable, dissipative system with storage function \( V \) and is excited with a \( \tau \)-periodic input signal, then
\[
V(x(t_0)) = V(x(t_0 + k\tau)),
\]
for all \( k \in \mathbb{Z} \). From (2) and (24) it follows that
\[
\int_0^{t_0+k\tau} s(u(t),y(t))dt \geq 0.
\]
Because \( \varepsilon \) is positive, (27) ensures \( G \) to be passive for a frequency \( \omega \in \Omega \) and by maximizing \( \varepsilon \) the dissipated energy by \( G \) is maximized for an input signal with frequency \( \omega \in \Omega \).

Assume \( G \) is a closed-loop system, connected as in Fig. 4. Along the same lines as in section IV, we will derive a condition for system \( K \) to fulfill (27). With \( G = P(I + KP)^{-1} \), (28) can be written as
\[
(I + KP)^{-1} \begin{pmatrix} \varepsilon I & -I \\ -I & 0 \end{pmatrix} (I + KP)^{-1} \leq 0
\]
for some \( \omega \in \Omega \).

If we assume \( P \) and \( K \) both SISO-systems (what is the case for a synchronous generator), we can write \( K(j\omega) = a(j\omega) + jb(j\omega) \) and \( P(j\omega) = c(j\omega) + jd(j\omega) \), \( a, b, c, d \in \mathbb{R} \). From (28) it follows
\[
\begin{align*}
\left( a(j\omega) - \frac{c(j\omega)}{c(j\omega)^2 + d(j\omega)^2} \right)^2 + \\
\frac{b(j\omega) - \frac{d(j\omega)}{c(j\omega)^2 + d(j\omega)^2}}{1} \leq \frac{1}{1 + \varepsilon^2}.
\end{align*}
\]

Because the system \( P(j\omega) \) is known, the interpretation of (29) is that for a fixed \( \omega \), the frequency response \( K(j\omega) \) in the complex plane must lie inside a circle with radius \( \frac{1}{1 + \varepsilon^2} \) and its center at
\[
(a_0(\omega), b_0(\omega)) = \left( -\frac{c(\omega)}{c(\omega)^2 + d(\omega)^2} + \frac{1}{\varepsilon^2} \cdot \frac{d(\omega)}{c(\omega)^2 + d(\omega)^2} \right).
\]

For the rest of this section we concentrate on the case of a synchronous generator, i.e. we take \( P(j\omega) = \frac{T_0}{T_0+c(j\omega)} \) with \( T_0, T_1, T_2 \in \mathbb{R} \). Note that in this particular case \( a_0(\omega) \) in (30) is independent of \( \omega \). Furthermore, we will show that for this \( P \), in some cases it is possible to derive easily verifiable sufficient conditions for (27) to hold for all \( \omega \in \Omega \), where \( \Omega = \{ \omega \mid \omega_1 \leq \omega \leq \omega_2 \} \) for some \( \omega_1 < \omega_2 \).

For this \( P \) the set \( (a_0(\varepsilon), b_0(\varepsilon)) \) presents a line segment in the complex plane. Next example will illustrate this graphically.
Example. \( P = \frac{1}{2H^2 + KD^2} \) with \( H = 3.5 \) and \( KD = 10 \). We take \( \varepsilon = 0.01 \), \( \Omega = [\omega_1, \omega_2] = [1.0, 2.0] \) Hz. Fig. 5 shows circles that are described by (29) for a finite set of frequencies \( \omega \in \Omega \) including \( \omega_1 \) and \( \omega_2 \). The upper and lowest circle corresponds to \( \omega_1 \) and \( \omega_2 \), respectively. If we assume that the intersection of the circles corresponding to \( \omega_1 \) and \( \omega_2 \) have a nonempty interior \( \mathcal{X} \), then the intersection of the circles corresponding to all \( \omega \in \Omega \) is \( \mathcal{X} \).

Fig. 5. Circles correspond to inequality (29) for a finite set of frequencies in the frequency range [1.0, 2.0] Hz.

From the example it is clear that a sufficient condition for (27) to hold for all \( \omega \in \Omega \) is that \( K(j\omega) \in \mathcal{X} \) for all \( \omega \in \Omega \). This implies that it is sufficient to check feasibility of the following inequalities

\[
\begin{align*}
\left( a(\omega) + \frac{c_i}{c_i^2 + d_i^2} - \frac{1}{\varepsilon} \right)^2 + \left( b(\omega) - \frac{d_i}{c_i^2 + d_i^2} \right)^2 & \leq \frac{1}{\varepsilon^2} \quad \forall i = 1, 2, \quad (31)
\end{align*}
\]

where \( P(j\omega_1) = c_1 + jd_1 \) and \( P(j\omega_2) = c_2 + jd_2 \), for all \( \omega \in \Omega \).

Equations (31) can be written as FDIs for the conjugate system \( K^*(j\omega) = (K(j\omega))^* \):

\[
\begin{align*}
\left( I + K^*(j\omega) \right)^* \begin{bmatrix} \sigma_i^2 - r^2 & -\sigma_i \\ -\sigma_i^* & -1 \end{bmatrix} \left( I + K(j\omega) \right) & \preceq 0, \quad \forall i = 1, 2, \quad \forall \omega \in \Omega \quad (32)
\end{align*}
\]

with \( \sigma_i = \left( -\frac{c_i}{c_i^2 + d_i^2} + \frac{1}{\varepsilon} \right) + j \left( \frac{d_i}{c_i^2 + d_i^2} \right) \) and \( r = \frac{1}{\varepsilon} \).

By utilizing the GKYP lemma and the Schur complement [10], the FDIs in (32) can be written as

\[
\begin{align*}
M = \begin{bmatrix} M_{11} & M_{12}^T \\ M_{21} & M_{22} \end{bmatrix} \preceq 0,
\end{align*}
\]

with

\[
\begin{align*}
M_{11} &= \begin{bmatrix} * & *^T \\ * & * \end{bmatrix} W \begin{bmatrix} I & 0 \\ 0 & -K^* \end{bmatrix} + \begin{bmatrix} * & * \\ * & * \end{bmatrix} \begin{bmatrix} \Pi_{11}(\varepsilon) & \Pi_{12}(\varepsilon)^* \\ \Pi_{12}(\varepsilon) & 0 \end{bmatrix} \begin{bmatrix} 0 & I \\ \bar{C}_K(\beta) & \bar{D}_K(\beta) \end{bmatrix}, \\
M_{21} &= (\bar{C}_K(\beta) \quad \bar{D}_K(\beta)), \\
M_{22} &= -\Pi_{22},
\end{align*}
\]

\[
\begin{align*}
\Pi_{11}(\varepsilon) &= |\sigma_i(\varepsilon)|^2 - r(\varepsilon)^2 \\
\Pi_{12}(\varepsilon) &= -\sigma_i(\varepsilon)^* \\
\Pi_{22} &= I \\
\forall i &= 1, 2
\end{align*}
\]

where \( W \) denotes the multiplier representing the frequency range \( \Omega \), given by (9), and \( \beta \) is the set of free variables defined in section IV.

Unfortunately (33) will not result in an LMI if \( \varepsilon \) is treated as a variable. For that reason, an optimization with the aim to maximize the dissipated energy by \( G \) for all \( \omega \in \Omega \) will not result in a convex optimization problem. In the next section we will give an example to illustrate the usefulness of the optimization described in this section.

VI. NUMERICAL EXAMPLE

In this example we have considered the two area network depicted in Fig. 6. This particular system is a widely used benchmark case of a multi-machine power system. The complete model and the parameters for the system can be found in [6], pp. 813. To obtain a LTI model of the system, the nonlinear model is linearized in a steady state operating point, which is derived from standard power flow calculations [16]. All generators are represented by 6th order state-space models with exciters and PSSs as proposed by [6]. The restricted dissipativity based synthesis approach presented in this paper was tested by designing the PSS controller for generator 1. The poles for that PSS are chosen to be the one from [6]. The resulting PSS is given by the following

Fig. 6. Two Area Network
Fig. 7. The value \( G(j\omega) + G(j\omega)^* \) for generator 1 with different controllers. Dashed lines indicate the borders of the frequency interval [0.1,2]Hz.

Fig. 8. Zoomed in from Fig. 7 at the the frequency interval [0.1,2]Hz.

Fig. 9. Bode plots of different PSS controllers.

Fig. 10. Simulated trajectories of the rotor speed.

The value \( G(j\omega) + G(j\omega)^* \), where \( G(s) \) denotes the closed-loop transfer function from \( \Delta T_\text{m} \) to \( \Delta \omega \) of generator 1, is plotted in Fig. 7 and Fig. 8. Different plots in the figures correspond to a different PSS controller, as it is indicated in the legend. The label “classical PSS”, which is used in the legend, refers to the PSS controller proposed in [6]. The label “passivity based PSS” stands for the PSS (34).

Fig. 9 presents the bode diagram of PSS (34) and the bode diagram of the classical PSS. Since in [6] the PSS is given with the so-called “washout filter” (for explanations see [6], pp. 770), for comparison of the two controllers in Fig. 9 the same washout filter was added to the PSS (34). Fig. 10 presents simulated trajectories of the rotor speed from generator 1, following a disturbance caused by a temporary (1sec) addition and removal of load (35MW and 5MVAr) at bus 9. In Fig. 11 the rotor speed of all generators is plotted for different PSS controllers on generator 1. The example clearly illustrates the effectiveness of the proposed control synthesis approach.

For an additional simulation the passivity based PSS is designed for all four generators separately. The resulting PSSs are applied to the generators and in Fig. 12 the rotor speeds are plotted following the same disturbance as before.

A. Maximization of energy dissipation in a restricted frequency range

To improve performance, we will increase the dissipated energy in a frequency band by the generator as described in section V-C. The frequency range \( \Omega = [0.1,2.0] \text{Hz} \) is split up in four frequency ranges, namely \( \Omega_1 = [0.1,0.17] \text{Hz} \), \( \Omega_2 = [0.17,0.24] \text{Hz} \), \( \Omega_3 = [0.24,0.95] \text{Hz} \) and \( \Omega_4 = [0.95,2.0] \text{Hz} \), so \( \Omega = \{\Omega_i, \forall i = 1,2,3,4\} \). The objective for the nonlinear optimization is \( \max(w^T e) \), with \( e = [e_1, e_2, e_3, e_4]^T \) denoting the optimization variables and \( w = [w_1, w_2, w_3, w_4]^T \) denoting the weights. There are different possibilities to select \( w \). One possible choice is to take the fraction \( \frac{E_i}{w_j} \) equivalent to

\[
PSS(s) = \frac{299.5s^2 + 3546s + 612.9}{s^2 + 50.19s + 9.26}. \tag{34}
\]
length $\Omega_1$. This will result in maximizing the area under the graph in Fig. 8. On the other side, if information about the power spectrum of a response of the network is known, it would be suitable to choose $w_j$ equivalent to $\text{mean power in } \Omega_j$.

So $w_j$ is chosen larger than $w_j$ if there is relative more power in the spectrum of the range $\Omega_j$ then in the range $\Omega_i$. The latter choice is taken in this example.

For the non-linear optimization the Nelder-Mead method [17] is used. The used weights are $w = [3.11, 2.39, 0.5497, 0.0001]$. The optimization results in a PSS given by the transfer function

$$\text{PSS}(s) = \frac{300.4s^2 + 6123s + 554.7}{s^2 + 50.19s + 9.26}.$$  \hspace{1cm} (35)$$

and is applied to generator 1. The resulting values of the optimization variables are $\varepsilon = [1.00, 1.21, 1.24, 4.14] \cdot 10^{-4}$. The value $G(j\omega) + G(j\omega)^*$ after optimization is not increased for all frequencies in the range $\Omega$ in comparison with the not optimized case. However the value is larger for the frequency $\omega = 0.15$Hz which is the worst damped mode in the two area network. The results of the simulated trajectories of the rotor speed for the cases with and without optimization are in Fig. 14.

**Fig. 11.** The speeds of all generators with different controllers on generator 1.

**Fig. 12.** The speeds of all generators when passivity based PSSs are applied to all generators.

**Fig. 13.** The value $G(j\omega) + G(j\omega)^*$ plotted for the cases with and without optimization.

**Fig. 14.** Simulated trajectories of the rotor speed of generator 1 with and without optimization.

**VII. Conclusions**

We have presented a control synthesis approach for design of supplementary controllers in electrical power systems with the aim of damping low-frequent oscillations inherent to such systems. The synthesis objective considered in the paper was to render a subsystem, e.g. a synchronous generator, dissipative within a frequency interval which contains the badly damped oscillation mode. To obtain this goal we have utilized the Generalized Kalman-Yakubovich-Popov lemma. The proposed procedure has been applied for design of a PSS controller, which is further implemented on a generator in a standard benchmark case of a multi-machine power system.
The proposed designed methodology is applicable to synthesis of other types of controllers aimed for damping the system oscillations, e.g. controllers of FACTS devices, and is in particular suitable for tuning of existing controllers in the system.

VIII. ACKNOWLEDGMENTS

The research in this paper is submitted to the 16th IEEE Conference on Control Applications (CCA) in 2007 [18].

The author would like to thank Andrej Jokic, Siep Weiland and Prof. Van den Bosch for their helpful comments and support.

REFERENCES


APPENDIX

A. Generator equations

The generator model is taken from [6] (chapter 12). The main equations are in this section. The quantities with a bar are in per unit. The differential equations are
and where
\[ \Delta \omega = \text{rotor speed deviation in pu}, \]
with \( \Delta \omega = \omega - \omega_0 = \frac{\partial \delta}{\partial t} \text{ in rad/s} \)
\[ \delta = \text{rotor angle with respect to reference voltage in rad}, \]
\[ \Phi_{fd} = \text{field rotor flux linkage in pu}, \]
\[ \Phi_{1d}, \Phi_{1q}, \Phi_{2d} = \text{stator flux linkages in pu}, \]
\[ H = \text{Inertia Constant} = \text{stored energy at rated speed in MWs}, \]
\[ \frac{\text{MWs}}{\text{MVA rating}} \]
\[ \bar{T}_m = \text{mechanical torque in pu}, \]
\[ T_c = \text{air-gap torque in pu}, \]
\[ K_d = \text{damping torque coefficient}, \]
\[ \omega_0 = \text{rated speed in rad/s} = 2\pi f_0, \]
with \( f_0 \) the rated frequency of the network in Hz,
\[ \bar{R}_{fd} = \text{field circuit resistance in pu}, \]
\[ \bar{R}_{1d}, \bar{R}_{1q}, \bar{R}_{2d} = \text{stator circuit resistances in pu}, \]
\[ L_{adu} = \text{mutual inductance d-axis in pu}, \]
\[ i_{fd} = \text{field rotor current in pu}, \]
\[ i_{1d}, i_{1q}, i_{2d} = \text{stator currents in pu}, \]
\[ E_{fd} = \text{field voltage and input from the exciter}, \]
\[ \psi_{adu} \text{ and } \psi_{adu} = \text{d- and q-axis mutual flux linkages}. \]

Time \( t \) is in seconds and the rated frequency \( \omega_0 \) is in rad/s.

The air-gap torque equation \( T_c = \psi_{adu} - \psi_{adu} \) completes
the model of the generator. This model is written in state-space form after linearizing round a steady-state operating point.

**B. Power system equations**

The power system is modeled as a state-space model that is
linearized round a steady-state operating point, which usually
follows from a power flow calculation [16]. Voltages and
currents are factorized in d- and q-parts, like in the Park's trans formation, [6] and [16], which transforms the abc-frame to the dq0-frame. All subsystems have voltage-input and
current-output and subsystem \( k \) is modeled as
\[ x_k = A_k x_k + B_k \Delta v \]
\[ \Delta i = C_k x_k - Y_k \Delta v, \] (36)
with \( \Delta v \) the vector of bus voltages, \( \Delta i \) the vector of current
injections and \( x_k \) the state variables of subsystem \( k \). The state
equations of all subsystems in the network are combined into
\[ x = A_D x + B_D \Delta v \]
\[ \Delta i = C_D x - Y_D \Delta v, \] (37)
where \( x \) contains the state-vectors \( x_k \) for all \( k \) subsystems.
The matrices \( A_D \) and \( C_D \) are composed as block diagonal
matrices of all \( A_k \) and \( C_k \) matrices.

The interconnection of the transmission network is described
in the admittance matrix \( Y_N \) and has the relation
\[ \Delta i = Y_N \Delta v, \] (38)
Construction of \( Y_N \) can for instance be found in [6] or [16].
Lines are modeled by a so called \( \Pi \)-model and transformers
by a reactance. Fixed loads in the network are modeled by the
approach in [6] (pp.795-798), and are add to the admittance
matrix.

If we connect \( 38 \) to state-space model \( 37 \),
\[ C_D x - Y_D \Delta v = Y_N \Delta v \] (39)
then
\[ \Delta v = (Y_N + Y_D)^{-1} C_D \]
(40)
gives the state space matrix of the power network.