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Inverse simulations in vehicle dynamics

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Summary

This report describes two methods to perform inverse simulations in vehicle dynamics using the dynamic analysis software package DADS. In inverse simulations the input signal (steering angle) and the response of the vehicle must be calculated, given a desired path the vehicle should follow. The input signal can be used as judging criterion for the road holding properties of the vehicle.

The first method discussed is feedback linearization, based on systematical substitution of the equations of motion in the pathconstraint. The second method discussed is output feedback control with a controller gain that goes to infinity.

A 3 d.o.f vehicle model was used to investigate computation times, accuracy and stability of the methods. Output feedback control requires large computation times for a controller gain that reaches infinity. Besides this, the controller can cause instabilities if it is not carefully designed, especially for a nonlinear system. Feedback linearization does not have these disadvantages. However, implementation in DADS requires more effort.

Because of the disadvantages of the controller, feedback linearization is used to perform inverse simulations.

A 3 d.o.f. vehicle model of a YA4440 military truck was used to investigate the usefulness of the input signal in inverse simulations as judging criterion for the instationary road holding properties of a vehicle. For this simple vehicle model the objective judgement, based on minimum steering effort, correlated well with the subjective judgement.

Feedback linearization was implemented in a 16 d.o.f. DADS vehicle model of a FT95-tractor-semitrailer combination, to validate the method in 3d-simulations. The influence of tires, load and rear suspension damping on the instationary road holding properties were investigated. The judging criterium based on minimum steering effort also proved its usefulness for this more complex vehicle model. However, it was found that this criterium needs to be extended with criteria based on the vehicle response.
Chapter 1

Introduction

An important aspect in handling of vehicles concerns the road holding properties of a vehicle. Road holding can be defined as the extent to which the combination of driver and vehicle is able of driving trajectories that occur in practice. Road holding properties of a vehicle are important, because they determine the safety of the vehicle under realistic driving conditions. We divide road holding into two parts:

1. passive road holding. This characterizes the driving limits under normal driving conditions. In normal, every day driving the driver, as the controller of the vehicle, adjusts to his driving conditions (for example wet vs. dry road), the type of manoeuvre he is driving and the dynamics of the vehicle he is driving. He behaves like an adaptive controller and makes sure the vehicle will follow a desired path. The driving limits are mainly determined by saturation effects of the vehicle. Because we may assume that the driver is able to follow a predetermined path we do not have to model him (or, at most, we can model him very simply), e.g. if we investigate the passive road holding of a vehicle, only the vehicle dynamics are important

2. active road holding. This characterizes the driving limits under extreme conditions (for example a sudden evasive manoeuvre at high speed). In this case, the driver has not enough time to adjust and has a great influence on the stability of the vehicle. Thus, the total system driver/vehicle is of importance.

According to Mitschke (1990) the driver/vehicle system feedback contains three sources of information:

1. visual information. The driver sees the path the vehicle has to follow over a certain preview-time.

2. movement information. The driver feels the movement of the vehicle via his balancing organs. Most important sources of information are lateral, yaw, and roll accelerations of the vehicle.

3. tactile information. The driver feels the aligning moment on the front wheels through his steer.
CHAPTER 1. INTRODUCTION

Over the years extensive efforts have been made to model the driver. The simplest models (McRuer, 1971) contain a gain, an equalization term (with a pole and a zero) and a time delay, that represents the neuro-muscular system of the driver. By adjusting the gain and the equalization term the driver is capable of stable driving over a long range of manoeuvres and vehicle speeds. More detailed information of the human control system can be found in McRuer's publications.

Many tests have been designed to judge the road holding of vehicles. We can roughly divide these tests in two categories:

1. open loop tests. Input are the steering angle and the driving/braking torque. Output is the response of the vehicle. The driver has no influence on the movement of the vehicle. Thus, only the passive road holding is judged.

2. closed loop tests. Input is the desired path of the vehicle. Output are the steering angle and the driving/braking torque as well as the vehicle response. We subdivide closed loop tests in two categories:

   (a) closed loop tests with an "ideal driver". Here the vehicle must exactly follow a predetermined path. The driver is assumed to be able to generate the necessary input signal to do so. In these tests only the passive road holding is judged.

   (b) closed loop tests with a "realistic driver". In these tests the driver has the freedom to drive an "optimal" path. The total driver/vehicle system is of interest. With these tests the active road holding can be judged.

The general judging criteria in road holding tests are (Mitschke, 1990):

1. the vehicle must be easy to drive, but has to keep its driver alert.
2. external disturbances, like windgusts, may not cause vehicle responses that surprise the driver.

3. the driver must be able to easily recognize the driving limits of the vehicle.

The control effort of the driver (point 1) and the "stability" of the vehicle (points 2 and 3) are the most important judging criteria.

At DAF the road holding properties of trucks are mainly determined subjective. In future DAF wants to be able to judge the road holding already in the design phase of its vehicles. For this purpose the dynamic analysis software package DADS is available.

It is known that calculation of the vehicle response to a given input signal can be done relatively easy with DADS. However, inverse simulations, where the input signal and the response of the vehicle must be calculated, given a predefined path, require much more effort. A method which enables us to perform inverse simulations with DADS is presented in this report.

The method will be validated with a 3 degrees-of-freedom model of a YA4440 military truck and a 16 degrees-of-freedom model of a FT95 tractor-semitrailer combination.

In chapter 2 the most important road holding tests are summarized. Objective judging criteria and the relation between vehicle behaviour and subjective judgement is discussed. A choice for tests in simulations will be made.

Chapter 3 contains two methods to solve the so-called TPPC-problem (trajectory prescribed path control problem (Campbell (1989)) in inverse simulations. A choice for one of the methods is made.

DAF proposed the idea, that the input signal in closed loop tests contains valuable information about the road holding properties of a vehicle. In chapter 4 the correlation between the input signal (steering angle at the kingpin) and subjective judgement of a YA4440 military truck is investigated, using a simple bicycle model.

In chapter 5 a more realistic model of a FT95 tractor-semitrailer combination is used to validate the method from chapter 3. The influence of tires, load and suspension damping on the instationary road holding properties of the combination is investigated.

Finally, conclusions and recommendations for further investigation are given in chapter 6 and 7 respectively.
Chapter 2

Road holding tests

In this chapter the four most important manoeuvres in road holding tests will be discussed

2.1 Steady-state-turning

With the steady-state-turning test (quasi-) stationar curve driving is judged. In this test the vehicle must follow a circular path while slowly increasing its velocity. This manoeuvre is a closed loop test with an ideal driver. Most important goal of the steady-state-turning test is to determine the under/oversteer character of the vehicle.

At DAF the following criteria have been proposed to judge the stationary curve driving of trucks (Vogels, 1989):

1. the vehicle should be gradually understeering (Figure 2.1).
2. let $\eta$ be the understeer coefficient, defined as:
   
   \[
   \eta = \frac{1}{\tau} \frac{d\delta_d}{da_d}
   \]

   with the steering angle at the steering wheel $\delta_d$, the lateral acceleration $a_d$ and the transduction ratio of the steering system $\tau$.
   
   For lateral accelerations beneath 2 m/s$^2$ $\eta$ should be smaller than $75.10^{-4}$ rad.s$^2$/m and larger than $0$ rad.s$^2$/m. If $\eta < 0$ rad.s$^2$/m the vehicle is oversteering.

3. the lateral acceleration at which the vehicle breaks out should be smaller than the lateral acceleration at which it rolls over.

   Because of the high center of gravity of a laden truck this often does not occur in practice.

4. a maximum increase of the roll angle of 1° per 0.1 g increase of the lateral acceleration is good. A maximum increase of the roll angle of 2° per 0.1 g increase of the lateral acceleration is acceptable.

5. the lateral acceleration at which the vehicle breaks out or rolls over should be significantly larger than lateral accelerations achieved in normal every day driving.
In literature (Mitschke, 1990) the slip angle $\alpha_d$ at the driver's seat is considered to be very important in the subjective judgement of the vehicle. Mitschke (1990) found from literature that small slip angles are desirable and that there exists close correlation between the gradient $d\alpha_d/d\delta_d$ and the subjective judgement of the driver. Mitschke (1990) also proposes the torque at the steering wheel as judging criterium in the steady-state-turning test.

### 2.2 J-turn

The J-turn is an open loop test to determine the instationary road holding properties of a vehicle. In a J-turn the steering wheel is turned linear in time and then fixed at a constant angle. The response of the vehicle to this input signal is judged.

According to Mitschke (1990) a vehicle is subjectively judged as good if:

1. the yaw gain $|\dot{\theta}/\delta_d|$, with $\dot{\theta}$ the yaw rate of the vehicle, is large and
2. the peak response time, defined as the time between starting of the manoeuvre and reaching the maximum yaw rate in the manoeuvre, is small.

This means the vehicle must react fast and strong to actions taken by its driver.

At DAF, the J-turn is not performed. Subjective judgements of vehicles in J-turns and values of desired peak response times and yaw gains are not available.

### 2.3 Sine-steer

The sine-steer test is an open loop test to determine the instationary road holding properties of a vehicle. It is often used as an alternative of the J-turn, if the latter can not be performed due to dangerous weather conditions (like snow) or a small test square. The sine-steer test is also used to get a relatively simple description of the vehicle dynamics in the frequency
domain. In the sine-steer test the steering wheel is turned as a sine function of time. The response of the vehicle to this input signal is judged with bodeplots.

Mitschke (1990) found from literature that a vehicle, when driving on a curved path (which is more or less equivalent with the sine-steer test), is subjectively judged as good if the yaw gain is large and the vehicle contains strong damping properties. Furthermore, the amplitude of the yaw rate may not decrease at low frequencies. The decrease of the amplitude of the yaw rate, at higher frequencies, should be small, so the driver will not be surprised by the changing road holding properties of the vehicle.

At DAF the sine-steer test is not performed. Subjective judgements of vehicles in sine-steer tests and objective data are not available.

2.4 Lane-change

The lane-change (Figure 2.2), an overtake manoeuvre at high speed, has also been designed to judge the instationary road holding properties of a vehicle. At DAF, the test is performed at the highest speed the vehicle can achieve in the manoeuvre.

![Figure 2.2: Lane-change trajectory](image)

The lane-change is a closed loop test with a realistic driver. The driver chooses, within certain limits, the, to his opinion, optimal path to perform the manoeuvre. However, due to the fact that the test is performed by skilled drivers with preknowledge of the vehicle behaviour, only the passive road holding of the vehicle is judged. Therefore we assume that in simulations the lane-change can be driven with an ideal driver if we choose a realistic path.

At DAF (van Zoest, 1988) the following criteria are proposed for an objective judgement of the instationary road holding properties:

1. the average signal power of the steering angle at the kingpin, defined as:

   \[ \frac{1}{T_d} \int_{\tau=t_b}^{t_e} \delta_k^2(\tau) d\tau \]  \hspace{1cm} (2.2)

   with \( T_d \) the drive-through time and \( \delta_k \) the steering angle at the kingpin, should be as small as possible.

2. the lateral acceleration at which the vehicle breaks out or rolls over in the stationary situation should be significantly greater than the maximum lateral acceleration in the lane-change.
At this moment the lane-change is judged only subjectively at DAF. Via simulation of the lane-change with an ideal driver the instationary road holding properties can be judged objective.

2.5 Choice of tests in simulations

Because of the complicity of the human controller, closed loop tests with a realistic driver will not be simulated. Only the passive road holding of vehicles is judged in the simulations.

The major disadvantage of open loop tests is that they are not performed in practice at DAF. Therefore we can not compare results from open loop simulations with results from real tests. A second disadvantage of open loop tests is that every vehicle variant will drive a different path with the same input signal. This makes it more difficult to objectively compare vehicle variants. Closed loop tests with an ideal driver do not have these disadvantages. Therefore we will judge the passive road holding through simulation of closed loop tests with an ideal driver. In simulations the steady-state-turning and the lane-change (with a predefined path) will be used.
Chapter 3

Methods to solve the TPPC-problem

3.1 The TPPC-problem

If we want to simulate closed loop tests with an ideal driver with a dynamic analysis software-package like DADS, we must solve the so-called TPPC-problem (trajectory-prescribed-path-control-problem (Campbell, 1989)). For each step in the numerical integration of the system equations the input signal must be calculated, given a predefined path and the current state of the system.

Only manoeuvres with a predefined traction/braking torque will be simulated, so the steering angle (at the kingpin) is the only input signal. This strongly simplifies the problem.

The TPPC-problem can mathematically be formulated as follows:

Let the regarded dynamic system (vehicle) be given by the equations of motion:

\[ M^*(q) \ddot{q} = k^*(q, \dot{q}, u, t) , \quad q(t_0) = q_0 , \quad \dot{q}(t_0) = \dot{q}_0 , \quad (3.1) \]

where \( q(t) \in \mathbb{R}^n \) is the column with the generalized coordinates, \( M^*(q) \) is the massmatrix, \( k^*(q, \dot{q}, u, t) \in \mathbb{R}^n \) is a column with the generalized forces and \( u(t) \) is the input signal.

If there exists at least one input signal \( u(t) \) with which the desired path can be followed, Equations (3.1) and (3.2) form a solvable system of \( n+1 \) equations in \( n+1 \) unknowns \( q(t) \) and \( u(t) \).

In §3.2 the equations of motion (3.1) are rewritten in a form that is commonly used in multibody dynamics. In §3.3 two methods are discussed to solve the TPPC-problem.
CHAPTER 3. METHODS TO SOLVE THE TPPC-PROBLEM

3.2 Description of multibody dynamic systems

A commonly used method to describe the dynamics of multibody systems is to combine the Newton-Euler equations of motion of the free moving bodies with a system of algebraic equations that represents the constraints of the system.

In the dynamic analysis software package DADS each body contains 7 generalized coordinates: the cartesian coordinates $x$, $y$ and $z$, that determine the position of the center of gravity of the body in space and the Euler-parameters $e_0$, $e_1$, $e_2$ and $e_3$, that determine the orientation of the body. Each body also introduces a constraint that describes the relation $e_0^2 + e_1^2 + e_2^2 + e_3^2 = 1$ between the Euler-parameters.

The equations of motion of a system with $NB$ bodies can be written as:

$$M(q)\ddot{q} - k(q, \dot{q}, u, t) = 0 \quad (3.3)$$

where $q(t) \in \mathbb{R}^{7NB}$ is the column with the generalized coordinates, $M(q)$ is the mass matrix, $k(q, \dot{q}, u, t) \in \mathbb{R}^{7NB}$ is the column with the generalized forces and $u(t)$ is the input signal.

The combined set of kinematic, driving and Euler-parameter constraints of the system can be written in the form:

$$\Phi(q, t)\delta q = 0, \quad \Phi \in \mathbb{R}^{NCx7NB}, \quad \Phi_{ij} = \frac{\partial \phi_i}{\partial q_j} \quad (3.4)$$

with $\phi \in \mathbb{R}^{NC}$, where $NC$ is the number of constraints.

The path constraint (3.2) is not incorporated in Eqn. (3.4). This constraint must be fulfilled by a proper choice of the input signal.

Let $\delta q$ be an infinitesimal variation of the generalized coordinates $q$. This variation is called kinematically admissible if the varied generalized coordinates $q + \delta q$ satisfy the constraints, i.e. if $\Phi(q + \delta q, t) = 0$. Since $\delta q$ is infinitesimal this means that $\delta q$ is kinematically admissible if and only if:

$$\Phi(q, t)\delta q = 0 \Rightarrow \Phi \in \mathbb{R}^{NCx7NB}, \quad \Phi_{ij} = \frac{\partial \phi_i}{\partial q_j} \quad (3.5)$$

In multibody dynamics systematic distinction is made between constraint forces $k^C(q, \dot{q}, t)$ and applied forces (gravity forces and spring-, damper- and actuator forces) $k^A(q, \dot{q}, u, t)$:

$$k(q, \dot{q}, u, t) = k^C(q, \dot{q}, t) + k^A(q, \dot{q}, u, t) \quad (3.6)$$

According to the third law of Newton (action = reaction) the total virtual work of the constraint forces is zero for any kinematically admissible variation $\delta q$. Therefore

$$\delta q^T k^C(q, \dot{q}, t) = 0 \quad (3.7)$$

for all $\delta q$ that satisfy $\Phi(q, t)\delta q = 0$. It can be shown that this is possible if and only if $k^C$ can be written as

$$k^C = -\Phi^T \Delta \quad (3.8)$$

where $\Delta \in \mathbb{R}^{NC}$ is a column with the Lagrange multipliers.

Combination of this result with Eqns. (3.3), (3.4) and (3.6) leads to

$$M(q)\ddot{q} - k^A(q, \dot{q}, u, t) + \Phi^T(q, t)\Delta = 0, \quad \Phi(q, t) = 0 \quad (3.9)$$
CHAPTER 3. METHODS TO SOLVE THE TPPC-PROBLEM

Eqn. (3.9) forms a system of \( 7NB \) differential and \( NC \) algebraic equations in \( q \) unknowns and \( A \). To solve this system in DADS, Eqn. (3.4) is twice differentiated:

\[
\begin{align*}
\Phi(q,t) & = -v(q,t), \quad v_i = \frac{\partial v_i}{\partial t} \\
\Phi(q,t) & = -\gamma(q,\dot{q},t), \quad \gamma = \Phi^T + \dot{\Phi}
\end{align*}
\] (3.10) (3.11)

Eqn. (3.11) is equivalent to Eqn. (3.4) under the conditions, that

\[
\begin{align*}
\phi(q(t_0),t_0) & = 0 \\
\Phi(q(t_0),t_0) & = -v(q(t_0),t_0)
\end{align*}
\] (3.12) (3.13)

This finally results in the combined equations:

\[
\begin{bmatrix}
M(q) & \Phi^T(q,t) \\
\Phi(q,t) & 0
\end{bmatrix}
\begin{bmatrix}
\ddot{q} \\
\dot{q}
\end{bmatrix}
= \begin{bmatrix}
kA(q,\dot{q},u,t) \\
-\gamma(q,\dot{q},t)
\end{bmatrix}, \quad q(t_0) = q_0, \quad \dot{q}(t_0) = \dot{q}_0
\] (3.14)

where \( q_0 \) and \( \dot{q}_0 \) have to satisfy (3.12) and (3.13).

Over the last years numerous softwarepackages (including DADS) were developed that compose and solve the equations of motion of the form (3.14). To solve Eqn. 3.14 the input signal must be a function of the generalized coordinates, their derivatives and/or time.

In the following section two methods are discussed to calculate the input signal as a function of time, given both the path the vehicle must follow and the system state.

3.3 Methods to solve the TPPC-problem

3.3.1 Feedback linearization

Feedback linearization is based on systematical substitution of the equations of motion in the pathconstraint (3.2) or derivatives of this equation, until the input signal can be calculated as a function of time. If the vehicle can follow the desired path this method will lead to an exact solution of the input signal.

We recall the vehicle model from the previous section:

\[
\begin{bmatrix}
M(q) & \Phi^T(q,t) \\
\Phi(q,t) & 0
\end{bmatrix}
\begin{bmatrix}
\ddot{q} \\
\dot{q}
\end{bmatrix}
= \begin{bmatrix}
kA(q,\dot{q},u,t) \\
-\gamma(q,\dot{q},t)
\end{bmatrix}, \quad q(t_0) = q_0, \quad \dot{q}(t_0) = \dot{q}_0
\] (3.15)

under the conditions:

\[
\begin{align*}
\phi(q_0,t_0) & = 0 \\
\Phi(q_0,t_0) & = -v(q_0,t_0)
\end{align*}
\] (3.16) (3.17)

The path the vehicle must follow can be written as

\[
\psi(q,t) = 0
\] (3.18)

Twice differentiating Eqn. (3.18) yields:

\[
\begin{align*}
\Psi^T(q,t) & = -v\psi(q,t), \quad \Psi_i = \frac{\partial \psi}{\partial q_i}, \quad v_\psi = \frac{\partial \psi}{\partial t} \\
\Psi^T(q,t) & = -\gamma\psi(q,\dot{q},t), \quad \gamma_\psi = \Psi^T + \dot{\psi}
\end{align*}
\] (3.19) (3.20)
The requirement $\psi(q, t) = 0 \forall t \geq t_0$ is equivalent with Eqn. (3.20) under the conditions:

\[
\begin{align*}
\psi(q_0, t_0) &= 0 \\
\Psi^T(q_0, t_0) \dot{q}_0 &= -v_\psi(q_0, t_0) 
\end{align*}
\]

(3.21)  (3.22)

If Eqns. (3.16), (3.17), (3.21) and (3.22) are satisfied we may add (3.20) to the equations of motion. The total system of equations that has to be solved becomes:

\[
\begin{bmatrix}
M(q) & \Phi^T(q, t) \\
\Psi^T(q, t) & 0 \\
\end{bmatrix}
\begin{bmatrix}
\ddot{q} \\
\lambda \\
\end{bmatrix}
= 
\begin{bmatrix}
\dot{\gamma}^A(q, \dot{q}, u, t) \\
-\gamma(q, \dot{q}, t) \\
-\gamma_\\psi(q, \dot{q}, t) \\
\end{bmatrix}, \quad q(t_0) = q_0, \quad \dot{q}(t_0) = \dot{q}_0
\]

(3.23)

If the matrix on the left hand side of Eqn. (3.23) has full row rank a row $C^T(q, t)$, 

$C \in R^{(7NB+NC+1)}$, $\text{rank}(C) = 1$, $C^T = [C_1^T \ C_2^T \ C_3]$, can be determined such that:

\[
\begin{bmatrix}
C_1^T(q, t) \\
C_2^T(q, t) \\
C_3(q, t) \\
\end{bmatrix}
= 0^T
\]

(3.24)

In our case the constraints will never be redundant, thus the matrix on the left hand side of Eqn. (3.23) will always have full row rank.

One possible way to determine the row $C^T$ is to use the QR-factorization of the matrix on the left hand side of Eqn. (3.23). For convenience this matrix is abbreviated as $G(q, t)$. QR-factorization of this matrix is possible if it has full rank, which is always the case. QR-factorization of $G$ yields:

\[
G = Q \begin{bmatrix}
R \\
0^T
\end{bmatrix}
\]

with $Q \in R^{(7NB+NC+1) \times (7NB+NC+1)}$ an orthogonal matrix, $R \in R^{(7NB+NC) \times (7NB+NC)}$ an upper triangular matrix and $0^T \in R^{(7NB+NC)}$ a zero row. Using the orthogonality of $Q$ we can find the row $C^T$ by taking the $(7NB+NC+1)$-th row of $Q^T$.

Once we have found $C^T$ we premultiply Eqn. (3.23) with this row. This leads to:

\[
C_1^T(q, t) \dot{\gamma}^A(q, \dot{q}, u, t) - C_2^T(q, t) \gamma(q, \dot{q}, t) - C_3(q, t) \gamma_\\psi(q, \dot{q}, t) = 0
\]

(3.26)

It is known that, given the system state, from Eqn. (3.26) $u$ can be solved, provided that the Jacobian:

\[
C_1^T(q, t) K^A(q, \dot{q}, u, t), \quad K^A \in R^{7NB}, \quad K_i^A = \frac{\partial \dot{\gamma}^A}{\partial u}
\]

(3.27)

is not equal to zero (Figure 3.1). If the Jacobian $C_1^T K^A = 0$ three situations are possible:

1. no solution for $u$ exists with which the path can be followed. In our case this situation will occur if the saturating tireforces on the frontwheels of the vehicle reach their limits (Figure 3.2).

2. Multiple solutions for $u$ exist with which the path can be followed (Figure 3.3). In this case solving of Eqn. (3.26) will give great numerical problems. We assume that this case will not occur for the manoeuvres we want to simulate.
the input signal is not present at all in Eqn. (3.26) (Figure 3.4) This situation will occur when the input signal acts on a different body than the pathconstraint. In this case the above described method may be repeated with a new pathconstraint, namely Eqn. (3.26). An example of this situation can be found in appendix A.

Figures 3.1 to 3.4: Possible forms of Eqn. (3.26): $f(q, \dot{q}, u, t) = r(q, \dot{q}, t)$
with $f = C_1^T k^A$ and $r = C_2^T \gamma + C_3 \gamma \psi$
CHAPTER 3. METHODS TO SOLVE THE TPPC-PROBLEM

Some extra remarks can be made:

- Eqn. (3.20) is numerically unstable. It is possible, even if the initial conditions satisfy Eqns. (3.21) and (3.22) the final solution for \( \psi \) strongly violates the condition \( \psi = 0 \). To prevent this we may modify the equation \( \dot{\psi} = 0 \) to:

\[
\ddot{\psi} + 2\alpha \dot{\psi} + \beta^2 \psi = 0, \quad \alpha = \beta, \quad \alpha > 0, \quad \beta > 0
\]  

(3.28)

which is numerically stable. Eqn. (3.28) does not affect the system response if the initial conditions satisfy Eqns. (3.21) and (3.22).

- another method to avoid numerical difficulties is to explicitly substitute the equations \( \psi = 0 \) and \( \dot{\psi} = 0 \) into the equations of motion. This method is described in appendix B.

- A simpler method to derive an equation from which the input signal can be calculated would be the following:

Rewrite Eqn. (3.9) into the form:

\[
\ddot{q} = M^{-1}(k^A - \Phi^T \Delta)
\]  

(3.29)

Substitution of (3.29) in (3.20) yields:

\[
\Psi^T M^{-1}(k^A - \Phi^T \Delta) = -\gamma \psi
\]  

(3.30)

from which the input signal can be determined. However, if the input signal acts on a different body than the pathconstraint it is neccessary to consider derivatives of Eqn. (3.30). However, differentiation of (3.30) will lead to derivatives of \( \Delta \) that complicate the problem.

- in general, the input signal has to be determined numerically, using an iterative procedure.

Figure 3.5 gives a flowchart of neccessary computations when we use feedback linearization to calculate the input signal.
CHAPTER 3. METHODS TO SOLVE THE TPPC-PROBLEM

Desired path:
\[ \psi = 0 \]

Differentiate path constraint to:
\[ \psi^T \ddot{q} = -\gamma \psi \]

Use QR-factorization to determine a row
\[ C^T = \begin{bmatrix} C_1^T & C_2^T & C_3 \end{bmatrix} \]
such that:
\[ \begin{bmatrix} C_1^T & C_2^T & C_3 \end{bmatrix} \begin{bmatrix} M & \Phi^T \\ \Phi & 0 \\ 0 & 0^T \end{bmatrix} = 0^T \]

Is \( C_1^T k^A = 0 \)?

\[ \psi := \begin{cases} \text{YES} & \text{Is the input signal } u \text{ present in } C_1^T k^A - C_2^T \gamma - C_3 \gamma \psi = 0? \\ \text{NO} & \text{No solution for } u \text{ possible} \end{cases} \]

Calculate initial conditions \( q_0 \) and \( \dot{q}_0 \)
that satisfy \( \psi_0 = 0 \) and \( \dot{\psi}_0 = 0 \).

Solve:
\[ \begin{bmatrix} M & \Phi^T \\ \Phi & 0 \end{bmatrix} \begin{bmatrix} \ddot{q} \\ \dot{a} \end{bmatrix} = \begin{bmatrix} k^A \\ -\gamma \end{bmatrix}, \quad q(t_0) = q_0, \quad \dot{q}(t_0) = \dot{q}_0 \]

Figure 3.5: Calculations with feedback linearization
3.3.2 Output feedback

A traditional method to follow a desired path in linear constant control theory is to use an output feedback controller. In theory an inverse simulation is performed if the gain of the controlled system reaches infinity.

\[
\frac{Y(s)}{Y_d(s)} = \frac{kH_s(s)}{1 + kH_s(s)} = 1 \text{ for } k \to \infty
\]  

\[
\frac{U(s)}{Y_d(s)} = \frac{1}{H_s(s)} \text{ for } k \to \infty
\]

Figure 3.6: Inverse simulation with output feedback control

For linear constant systems of order 1 or 2 the gain can be increased without destabilizing the system. However if the order becomes larger than 2 a bad choice of the controller can cause instabilities. In such cases a good controller design will be much more complex.

When we use this linear theory for a nonlinear system (such as a vehicle) the danger exist that the nonlinearities destabilize the system. Besides this we can not guarantee convergence for nonlinear systems.

A practical disadvantage is, that the computation time needed in simulations is proportional with the highest frequency in the system. For large controller gains this will result in very large computation times.

However, implementation in a dynamic analysis software package is very easy, so it is worth trying.
3.4 Comparision of methods

A 3 degrees-of-freedom vehicle model (Figure 3.7) was used to investigate several aspects of the methods from the previous section when they are implemented in DADS. The saturating tireforces (Figure 3.8) were modeled with the empirical Magic Formula (Leurs, 1991). In appendix C the equations of motion of the vehicle model are derived and the Magic Formula is described.

$$\begin{align*}
  m\ddot{x} &= -F_f(\alpha_f) \sin(\delta + \theta) - F_r(\alpha_r) \sin \theta + F_l \cos \theta \\
  m\ddot{y} &= F_f(\alpha_f) \cos(\delta + \theta) + F_r(\alpha_r) \cos \theta + F_l \sin \theta \\
  J\ddot{\theta} &= aF_f \cos \delta - bF_r(\alpha_r)
\end{align*}$$

In this section we will discuss computation time and accuracy of the two methods. Possible instabilities due to the nonlinearities in the system, when using an output feedback controller, are also discussed.
3.4.1 Computation time and accuracy

The double lane-change from Figure 3.9 was used to compare computation time and accuracy of the two methods from §3.3. The manoeuvre is driven without traction force at the highest possible speed (for this model approximately 12 m/s). The results are given in appendix D.1.

From Figures D.1 to D.6 we can conclude the following:

* the input signal (steering angle at the kingpin) calculated with the output feedback controller converges to the input signal calculated with feedback linearization for a very large gain.

* if we use output feedback control with a large gain a noise signal is present in the input signal. This is probably due to the fact that the numerical integration error in the system state will also occur in the input signal, strongly amplified by the gain. We can remove this noise signal by taking smaller integration steps in the simulation. Another possibility would be to postprocess the data with a lowpass filter.

Let the relative error \( r_u \) in the input signal be defined as:

\[
 r_u = \frac{RMS(\delta^C - \delta^{FL})}{RMS(\delta^{FL})}, \quad RMS = \text{root mean square}
\]  

where \( \delta^C \) and \( \delta^{FL} \) are the input signals calculated with the output feedback controller and feedback linearization respectively.

In Figure D.7 the relative error in the input signal, when using output feedback control, is plotted versus the controller gain. It is clear that the relative error decreases with an increasing gain, if the maximum integration step is reduced in time. However, the computation time increases with increasing gain, as Table D.1 shows.
3.4.2 Destabilization of the system

If we use output feedback control, particularly at high velocity of the vehicle, the system can become unstable. This can simply be demonstrated with the steady-state-turning manoeuvre (Figure 3.10)

![Figure 3.10: Desired path and definition of the deviation $\varepsilon$](image)

If we force the vehicle, coming from straight on driving, suddenly to follow a circular path, a step will occur in the lateral acceleration of the vehicle. This can cause an unacceptable response of the controlled system.

The vehicle model was linearized round its initial conditions. (appendix D.2) The linearized vehicle model is unstable. Two poles of the system are equal to 0. If we use a simple proportional controller the controlled system becomes conditionally stable. For very small gains of the controller it is unstable, for large gains it is stable. The stability of the controlled system depends not only on the controller gain, but also on the vehicle speed (Figures D.8 to D.10).

If the controller gain is too small the linearized system will be unstable and therefore also the nonlinear system will be unstable. This situation is shown in Figure D.11.

For a larger controller gain the system becomes stable (Figure D.12).

For very large controller gains the linearized system is stable. However, the imaginary part of the poles is very large. This causes a strong oscillation in the input signal, which will have the tire forces reach their limit. In this case the nonlinearities destabilize the system (Figure D.13).

From the above it is shown that simply increasing the controller gain is a very dangerous method to perform inverse simulations.

3.5 Choice of method in simulations

Output feedback control has the advantage that it is easy to implement in DADS. However, since we do not have much insight in the vehicle behaviour a priori, it is a dangerous method.
Another disadvantage are the expected large calculation times if we only want to judge the vehicle (and not the combination controller/vehicle). Therefore we will use feedback linearization in inverse simulations. This has the disadvantage that extra software has to be added to the program.
Chapter 4

Objective and subjective judgement of a YA4440

We used a bicycle model of a YA4440 military truck to investigate the correlation between subjective and objective judgement of the instationary road holding properties.

In practice the instationary road holding properties are subjectively judged by the double lane-change (Figure 2.2). Because we expect that also a single lane-change would suffice to judge the instationary vehicle behaviour, we have chosen for this in simulations to save computation time (Figure 4.1). In simulations feedback linearization is used to generate the input signal. The judging criterium in simulations is based on desired minimum steering effort.

![Simulated lane-change](image)

Figure 4.1: Simulated lane-change

Four vehicle variants were simulated:

1. an unladen understeering truck
2. a laden understeering truck
3. a laden lightly oversteering truck
4. a laden heavily oversteering truck

The vehicles have Michelin-XZA tires. The tire forces in simulations are determined with the Magic Formula. Table 4.1 contains more information about the four vehicle variants.

<table>
<thead>
<tr>
<th>variant</th>
<th>a (m)</th>
<th>b (m)</th>
<th>m (kg)</th>
<th>J (kg.m²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.70</td>
<td>2.35</td>
<td>7800</td>
<td>31442</td>
</tr>
<tr>
<td>2</td>
<td>2.15</td>
<td>1.90</td>
<td>11500</td>
<td>52828</td>
</tr>
<tr>
<td>3</td>
<td>3.00</td>
<td>1.05</td>
<td>11500</td>
<td>52828</td>
</tr>
<tr>
<td>4</td>
<td>3.50</td>
<td>0.55</td>
<td>11500</td>
<td>52828</td>
</tr>
</tbody>
</table>

Table 4.1: Simulated YA4440 vehicle variants

The manoeuvre diagrams (figure 4.2) clearly show the under/oversteering character of the variants.

The vehicle variants 1 and 2 were subjectively judged (Van Zoest, 1986). Variant 1 was graded with a 8 and variant 2 with a 7 (on a scale from 1 to 10).

The vehicle variants 3 and 4 are fictive. The maximum admissible vertical load of the rear axle is exceeded for these variants. However, instinctively we feel that the driver will experience the vehicle behaviour as worse for loads that are placed further in the rear of the truck.

In figure 4.3 the average signal power of the steering angle at the kingpin is plotted as a function of the vehicle's speed in the lane-change. At high speed the results from the simulations confirm our expectations very well. The average signal power at the kingpin is smaller as the vehicle is subjectively judged better. Besides this, the maximum speed with
which the lane-change can be driven in simulations increases as the vehicle is subjectively judged better.

![Graph showing average signal power at the kingpin in lane-change simulations](image)

**Figure 4.2: Average signal power at the kingpin in lane-change simulations**

The simulations show that there is a good correlation between subjective judgement in practice and objective judgement in simulations (for this simple vehicle model). In the future meaningful values for the input signal (and vehicle response) have to be searched for to judge the instationary road holding properties of a vehicle. However, judging the instationary road holding of vehicles by means of inverse simulations seems a good starting point for optimizing the road holding properties of a vehicle.
Chapter 5

Road holding of a FT95 tractor-semitrailer

A 16 degrees-of-freedom dynamic model of a FT95 tractor-semitrailer combination was built with DADS (figure 5.1). The model is used to validate the method of §3.3 in 3D-simulations and to globally investigate the influence of the diverse truck-elements on the road holding properties.

Figure 5.1: Vehicle model of a FT95 tractor-semitrailer combination
5.1 Model description

The front suspension of the truck contains leafsprings. Vertical and lateral stiffnesses of the springs are modeled. Kinematic rollsteer of the front axle, due to the front suspension, is (globally) taken into account. The front suspension contains nonlinear viscous dampers. Coulomb-friction of the leafsprings is not modeled. The leafsprings are modeled as linear springs and bump is not taken into account.

The rear suspension of the tractor, as well as the trailer, contains airsprings. These are modeled as nonlinear springs. Damping is taken into account with nonlinear viscous dampers. Again, bump is not taken into account.

A static FEM-analysis was done to determine the torsion stiffness of the tractor. In the model this is represented by a linear torsional spring between the front and the rear of the chassis of the tractor. The trailer also contains a torsion stiffness.

Roll stabilizers on the front and the rear of the tractor are modeled through roll stiffnesses between axle and chassis. The construction of the trailer suspension has a high roll stiffness, which is also modeled.

The cabine suspension is not modeled: in the model the cabine is rigidly attached to the chassis.

The vehicle has Michelin-XZA tires. The vertical tire characteristics are modeled as linear springs and dampers. The lateral tire forces are modeled with the Magic Formula. In the tire model longitudinal forces and the aligning moment are not taken into account.

The steering system is not modeled. We assume that there is a constant transmission ratio between the steering angle at the kingpin and the steering wheel.

Because of the simplicity of the model it can easily be used to investigate the influence of the various components on the road holding properties of the tractor-semitrailer combination. In the future the model should be refined to get more reliable results. Then special attention should be paid to:

- modeling of the leafspring. At this moment the behaviour of the leafsprings in road holding tests is investigated at DAF.

- modeling of the steering system. If the stiffness of the steering system is very low we can not ignore this subsystem of the tractor. At this moment the influence of the stiffness of the steering system on the road holding properties is investigated at DAF.

- flexibility of the chassis. In the model the flexibility of the chassis is discretized with torsion stiffnesses. However the exact behaviour of the chassis in road holding tests is not known. This could be investigated by combining FEM-analysis results with multibody dynamics. In DADS the component-mode-synthesis method is used for this purpose.

- modeling of the tires. The tire model should be expanded with the earlier mentioned aspects. Cross-correlation of longitudinal and lateral tireforces should be investigated.

A detailed description of the model can be found in appendix E.
5.2 Implementation of feedback linearization in DADS

To omit the problem of multiple differentiation of the pathconstraint (entering the loop in Figure 3.1) we prescribe the path of the center of gravity of the front axle. In the model the tires are not modeled as separate bodies, thus the pathconstraint acts on the same body as the input signal.

The matrix that has to be factorized using QR-decomposition is very large (approximately 100x99 in our case). Besides it is very difficult to calculate this matrix with the information that is available in DADS. It can be proven (appendix F) that in our case the matrix can be reduced to a 3x2-matrix. With help of a simple modeling trick we are able to carry out this reduction in DADS (appendix F).

The equation from which the input signal is determined is numerically solved with a standard algorithm from the NAG-software library.

5.3 Instationary road-holding properties

A single lane-change (Figure 5.2) was simulated to investigate the instationary road holding properties of the tractor-semitrailer combination.

The influence of the load, tires and rear suspension damping of the tractor on the road holding of the combination was investigated. The average signal power at the kingpin (A.s.p.) was used to judge the road holding of the combination.

5.3.1 Influence load

Two vehicle variants were simulated:

- unladen vehicle
- maximum laden vehicle with the load placed 2.6 m above the ground
Results of the simulations can be found in appendix G.1.

We expect the laden vehicle to be judged worse than the unladen vehicle.

The a.s.p. of the laden vehicle is lower than the a.s.p. of the unladen vehicle. This is caused by the fact that the laden vehicle is less understeering than the unladen vehicle. However the maximum speed the laden vehicle can achieve in the manoeuvre is lower than the maximum speed of the unladen vehicle.

The maximum speed of the laden vehicle is determined by the roll of the vehicle. The maximum speed of the unladen vehicle is determined by the saturating tire forces.

Besides a larger roll of the vehicle also the yaw of the trailer increases if the load increases. This causes the traject the trailer drives to deviate more from the traject the tractor drives.

To get a better correlation with subjective judgement the judging criterium based on minimum steering effort could be extended with roll and yaw of the vehicle.

### 5.3.2 Influence tires

Two vehicle variants were simulated:

- maximum laden vehicle with the load placed 1.6 m above the ground
- vehicle with better tires: factor $D(F_n)$ of the Magic Formula doubled.

Results of the simulations can be found in appendix G.2.

As we would expect the vehicle with the better tires requires less steering effort and can perform the lane-change at higher speed.

### 5.3.3 Influence suspension damping

Three vehicle variants were simulated:

- maximum laden vehicle with the load placed 1.6 m above the ground
- suspension damping on the rear of the tractor increased with a factor 4
- suspension damping on the rear of the tractor decreased with a factor 4

Results of the simulations can be found in appendix G.3. Suspension damping on the rear of the tractor has hardly any influence on the steering effort. This is caused by the fact that the frequency of the manoeuvre is very low. If the manoeuvre would be driven on an uneven road the suspension damping will probably have a much bigger influence.

### 5.4 Steady-state-turning

For the standard vehicle a steady-state-turning manoeuvre was driven. Results of the simulation can be found in appendix G.4. A strange effect happened in this manoeuvre. At some times the input signal would stepwise change and after a while come back to the expected curve.

This is probably caused by the following. To calculate the input signal with feedback linearization the reaction forces on the front axle are required. We derive them by connecting a dummy element to the front axle with a bracket joint. For this bracket joint DADS can provide the Lagrange multipliers, from which we can derive the reaction forces. At some times
these Lagrange multipliers become zero. However the reaction forces reported by DADS in
the output file did have the correct value. This strange situation is probably due to the
internal program information flow. In the near future the reaction forces will be directly
available during the integration. This should resolve the problems. Another way to solve the
problems would be to explicitly calculate the spring and damper forces on the front axle from
the system state. In this case the software that has to be written will be more complex.
Chapter 6

Conclusions

The main conclusions of the investigation, described in this report, are the following:

- Inverse simulations form a valuable method in optimization of the road holding properties in the design phase of a vehicle.

- Usage of a controller with a very large gain is a dangerous method to perform inverse simulations if we do not have much insight in the vehicle dynamics.

- Usage of feedback linearization to calculate the input signal (steering angle at the kingpin) of a vehicle, given a desired path, works well if the input signal and the path constraint act on the same body in the system.

- The average signal power at the kingpin is a good criterium to judge the instationary road holding properties of a vehicle. However, this criterium needs to be extended with other criteria, like yaw and roll response and maximum speed of the vehicle, to get a better correlation with subjective judgement of the vehicle.

- When driving on an equal road the rear suspension damping of a tractor has hardly any influence on the road holding.
Chapter 7

Recommendations for further investigation

The following recommendations may be useful for further investigation:

- The possibility of implementation of feedback linearization in a multibody software package should be investigated for systems where the input signal and the path constraint act on different bodies in the system (for example: if the steering system is also modeled).

- The possibility of implementation of feedback linearization in a multibody software package should be investigated for a system with multiple input signals and constraints (for example implementation of the desired path and desired velocity that have to be satisfied by a proper choice of steering angle and traction/braking torque).

- The maximum speed achieved in lane-change simulations is significantly lower than in reality. A better choice of the lane-change trajectory in simulations could lower this discrepancy.

- In this report the path the vehicle had to drive was exactly prescribed. However, in practice the driver has the freedom to drive an "optimal" path in the lane-change. Different lane-change trajectories should be simulated to investigate the influence of the chosen path on the desired input signal and the judgement of the vehicle.

- The influence of the vehicle response in the lane-change on the correlation between subjective and objective judgement should be investigated. Possible other criteria than the input signal that influence the subjective judgement of the vehicle are roll and yaw of the vehicle.

- The sensitivity of the type of road holding test that is chosen in simulations on the judgement of the vehicle should be investigated.

- In this report only the steady-state-turning manoeuvre and the lane-change were used. However other road holding aspects could probably also be investigated with inverse simulations. Possible tests to simulate are for example braking in a curve and cornering on an uneven road. Therefore, the usefulness of simulating these tests with feedback linearization should be investigated.
• The model needs further extension. Special attention should be paid to modeling of the leafsprings, the tires and the chassis.
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Inverse simulations in vehicle dynamics: Appendices

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Appendix A

Example feedback linearization

For a simple mass-spring-system (Figure A.1) the input $u$ is calculated with feedback linearization. The input acts on mass 2, and we want mass 1 to follow a desired path in time:

Figure A.1: Mass-spring-system

$q_1(t) = f(t)$. The equations of motion for this system are:

$$M \ddot{q} = k$$  \hspace{1cm} (A.1)

with $q^T = [q_1 \ q_2]$, $M = \begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix}$ and $k = \begin{bmatrix} k_2(q_2 - q_1) - k_1q_1 \\ -k_2(q_2 - q_1) + u \end{bmatrix}$.

The path constraint (desired path) of the system is written as:

$$\dot{\psi} = q_1 - f(t) = 0$$  \hspace{1cm} (A.2)

This constraint differentiated twice and written in matrix notation, yields:

$$[1 \ 0] \ddot{q} - \ddot{f}(t) = 0$$  \hspace{1cm} (A.3)

$$[1 \ 0] \dddot{q} - \dddot{f}(t) = 0$$  \hspace{1cm} (A.4)

We can add (A.4) to (A.1). This gives:

$$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix} \begin{bmatrix} \dddot{q}_1 \\ \dddot{q}_2 \end{bmatrix} = \begin{bmatrix} k_2(q_2 - q_1) - k_1q_1 \\ -k_2(q_2 - q_1) + u \end{bmatrix}$$  \hspace{1cm} (A.5)
APPENDIX A. EXAMPLE FEEDBACK LINEARIZATION

A row $\mathbf{c}^T$ with which we premultiply Eqn. (A.5) such that the left hand side equals 0 is:

$$\mathbf{c}^T = [1 \ 0 \ -m_1]$$  \hspace{1cm} (A.6)

The right side of (A.5) premultiplied with $\mathbf{c}^T$ yields:

$$k_2(q_2 - q_1) - k_1q_1 - m_1 \ddot{f}(t) = 0$$  \hspace{1cm} (A.7)

It is obvious that we cannot determine the input signal from this equation. Therefore we will repeat the entire process again with the equation above as the new constraint.

Twice differentiating this equation gives:

$$[-(k_1 + k_2) \ k_2] \ddot{q} = f^{(3)}(t)$$  \hspace{1cm} (A.8)

$$[-(k_1 + k_2) \ k_2] \dddot{q} = f^{(4)}(t)$$  \hspace{1cm} (A.9)

Adding (A.9) to (A.1) yields:

$$\begin{bmatrix}
  \frac{m_1}{m_2} & 0 \\
  0 & \frac{m_2}{m_2}
\end{bmatrix}
\begin{bmatrix}
  \ddot{q}_1 \\
  \ddot{q}_2
\end{bmatrix}
= \begin{bmatrix}
  k_2(q_2 - q_1) - k_1q_1 \\
  -k_2(q_2 - q_1) + u
\end{bmatrix}$$  \hspace{1cm} (A.10)

Adding (A.9) to (A.1) yields:

$$\mathbf{c}^T = \begin{bmatrix}
  \frac{1}{m_1} & -\frac{k_2}{m_2(k_1 + k_2)} & \frac{1}{k_1 + k_2}
\end{bmatrix}$$  \hspace{1cm} (A.11)

Premultiplication of (A.10) with $\mathbf{c}^T$ gives:

$$\frac{1}{m_1}(k_2(q_2 - q_1) - k_1q_1) + \frac{-k_2}{m_2(k_1 + k_2)}(-k_2(q_2 - q_1) + u) + \frac{m_1}{k_1 + k_2} f^{(4)}(t) = 0$$  \hspace{1cm} (A.12)

From (A.12) we can simply determine $u$:

$$u = \frac{m_1 m_2}{k_2} f^{(4)} - \frac{(m_2(k_1 + k_2)^2}{m_1} q_1 + \frac{(m_2(k_1 + k_2)}{m_1} q_2$$  \hspace{1cm} (A.13)
Appendix B

Transformation of generalized coordinates

With this method the path constraint is explicitly taken into account by substituting it in the transformed equations of motion. However, implementation in DADS is far from simple.

The equations of motion of the system can be written as:

\[ M(q) \ddot{q} = k(q, \dot{q}, u, t), \quad q(t_0) = q_0, \quad \dot{q}(t_0) = \dot{q}_0, \]  

(B.1)

where \( q(t) \in \mathbb{R}^n \) is the column with the generalized coordinates, \( M(q) \) is the mass matrix, \( k(q, \dot{q}, u, t) \in \mathbb{R}^n \) is a column with the generalized forces and \( u(t) \) is the input signal. Differentiation of the path constraint:

\[ \psi(q, t) = 0 \]  

(B.2)
yields:

\[ \Psi^T(q, t) \dot{q} = -\nu_\psi(q, t), \quad \Psi \in \mathbb{R}^n, \quad \Psi_i = \frac{\partial \psi}{\partial q_i}, \quad \nu_\psi = \frac{\partial \psi}{\partial t} \]  

(B.3)

If \( \text{rank}(\Psi) = 1 \) it is -at least locally- possible to write the \( n \) generalized coordinate \( q \) as a function of \( n-1 \) new variables \( \eta \in \mathbb{R}^{n-1} \). This means that there exists a column function \( \Omega(\eta, t), \Omega \in \mathbb{R}^n \) such that \( \psi(\Omega(\eta, t), t) = 0 \). For all admissible \( q \) there exists an \( \eta \) such that:

\[ q(t) = \Omega(\eta, t) \]  

(B.4)

Twice differentiating (B.4) yields:

\[ \dot{\eta} = L(\eta, t) \dot{\eta} + r(\eta, t), \quad L \in \mathbb{R}^{(n-1)\times n}, \quad \dot{\eta}_i = \frac{\partial \Omega_i}{\partial \eta_j}, \quad r_i = \frac{\partial \Omega_i}{\partial t} \]  

(B.5)

\[ \ddot{\eta} = L(\eta, t) \ddot{\eta} + h(\eta, \dot{\eta}, t), \quad h = \dot{L} \dot{\eta} + \ddot{\eta} \]  

(B.6)

By substituting Eqns. (B.4), (B.5) and (B.6) in (B.1) we can rewrite the equations of motion into the form:

\[ M(\eta)L(\eta, t)\ddot{\eta} = -M(\eta)h(\eta, \dot{\eta}, t) + k(\eta, \dot{\eta}, u, t) \iff L\ddot{\eta} = -\dot{\eta} + M^{-1}k \]  

(B.7)

The system (B.7) contains \( n \) equations in the \( n-1 \) unknowns \( \eta \) and the unknown \( u \).

The transformed equations of motion (B.7) can be solved as follows: Determine the QR-factorization of the matrix \( L \). This gives:
APPENDIX B. TRANSFORMATION OF GENERALIZED COORDINATES

\[ Q_L = \begin{bmatrix} R \\ 0^T \end{bmatrix} \]  

(B.8)

where \( Q \) is an orthogonal \( nxn \)-matrix, \( R \) is an \((n-1)x(n-1)\) upper triangular matrix and \( 0^T \) an \((n-1)\) zero row. Premultiplication of (B.7) with \( Q \), where:

\[ Q = \begin{bmatrix} Q_1 \\ Q_2 \end{bmatrix}, \quad Q_1 \in \mathbb{R}^{(n-1)xn}, \quad Q_2 \in \mathbb{R}^{1xn} \]  

(B.9)

leads to the following equations:

\[ Q_1 \ddot{\eta} = Q_1 \dot{\eta} + Q_1 M^{-1}k \]  

(B.10)

\[ 0 = Q_2 \dot{\eta} + Q_2 M^{-1}k \]  

(B.11)

From (B.9) \( u \) can be solved if the Jacobian

\[ Q_2 M^{-1}K, \quad K(\eta, \dot{\eta}, u, t) \in \mathbb{R}^n, \quad K_i = \frac{\partial k_i}{\partial u} \]  

(B.12)

is not equal zero.

If the input signal acts on a different body in the system than the pathconstraint, the transformation should be repeated with (B.9) the new constraint and (B.8) the new system equations.

The following remarks can be made:

- in general it will be very difficult to determine the function \( Q(\eta, t) \), especially if the system has to be transformed more than once.

- the input signal \( u = u(\eta, \dot{\eta}, t) \) has to be determined iteratively in general.
Appendix C

Bicycle model
C.1 Derivation of the equations of motion

Transformation of reference frames

\[ \begin{align*}
\tilde{e}^1_x &= \cos \theta \tilde{e}_x^0 + \sin \theta \tilde{e}_y^0 \\
\tilde{e}^2_x &= -\sin \theta \tilde{e}_x^0 + \cos \theta \tilde{e}_y^0 \\
\tilde{e}^3_y &= \cos \delta \tilde{e}_x^0 + \sin \delta \tilde{e}_y^0 \\
\tilde{e}^4_y &= -\sin \delta \tilde{e}_x^0 + \cos \delta \tilde{e}_y^0
\end{align*} \]  

(C.1)  
(C.2)  
(C.3)  
(C.4)

Tire forces

\[ \begin{align*}
\cos \alpha_f &= \frac{\tilde{e}^2_y \cdot \tilde{v}_f}{|\tilde{v}_f|}, \quad |\alpha_f| \leq \frac{\pi}{2} \\
\cos \alpha_r &= \frac{\tilde{e}^3_y \cdot \tilde{v}_r}{|\tilde{v}_r|}, \quad |\alpha_r| \leq \frac{\pi}{2} \\
F_f(\alpha_f) &= -f_f(\alpha_f) \text{sign}(\tilde{e}^2_y \cdot \tilde{v}_f) \\
F_r(\alpha_r) &= -f_r(\alpha_r) \text{sign}(\tilde{e}^3_y \cdot \tilde{v}_r)
\end{align*} \]  

(C.5)  
(C.6)  
(C.7)  
(C.8)

The functions \( f_f \) and \( f_r \) are determined by fitting the so-called Magic Formula to experimental data.

equations of motion

\[ \begin{align*}
m(\ddot{x} \tilde{e}_x^0 + \ddot{y} \tilde{e}_y^0) &= F_r(\alpha_r) \tilde{e}^3_x + F_f(\alpha_f) \tilde{e}^2_y + F_r(\alpha_r) \tilde{e}^4_y \\
m\ddot{x} &= -F_f(\alpha_f) \sin(\delta + \theta) - F_r(\alpha_r) \sin \theta + F_i \cos \theta \\
m\ddot{y} &= F_f(\alpha_f) \cos(\delta + \theta) + F_r(\alpha_r) \cos \theta + F_i \sin \theta
\end{align*} \]  

(C.9)  
(C.10)  
(C.11)

\[ \begin{align*}
J \ddot{\theta} &= a \tilde{e}^2_x \cdot F_f(\alpha_f) \tilde{e}^2_y \cdot \tilde{e}_z - b \tilde{e}^4_y \cdot F_r(\alpha_r) \tilde{e}^4_y \\
&= aF_f(\alpha_f) \tilde{e}^2_y \cdot \tilde{e}^2_y - bF_r(\alpha_r) \\
&= aF_f(\alpha_f) \cos \delta - bF_r(\alpha_r)
\end{align*} \]  

(C.12)

C.2 Magic Formula

With the Magic Formula the saturating tireforces are described as:

\[ f(\alpha) = D(F_n) \sin(C \arctan(B\nu)) \]  

(C.13)

with

\[ \nu = (1 - E)\xi + E/B \arctan(B\xi) \]  

(C.14)

\[ \xi = \alpha + Sh \]  

(C.15)

The parameters \( B, C, D(F_n) \) (as function of the normalforce acting on the tire in the tire-road surface), \( E \) and \( Sh \) are found by fitting the Magic Formula to experimental data.
Appendix D

Figures belonging to chapter 3

D.1 Computation time and accuracy

Figure D.1: Lane-change with output feedback: $k = 1$, max.int.step = 0.005s
Figure D.2: Lane-change with output feedback: $k = 10$, max.int.step = 0.005s

Figure D.3: Lane-change with output feedback: $k = 100$, max.int.step = 0.005s
APPENDIX D. FIGURES BELONGING TO CHAPTER 3

Figure D.4: Lane-change with output feedback: $k = 1000$, max.int.step = 0.005s

Figure D.5: Lane-change with output feedback: $k = 1000$, max.int.step = 0.001s
Figure D.6: Lane-change with output feedback: $k = 10000$, max.int.step = 0.005s

Figure D.7: Relative error in the input signal for different values of the gain
APPENDIX D. FIGURES BELONGING TO CHAPTER 3

D.2 Stability of the controlled system

The equations of motion are linearized round the initial conditions: \( x = 0, \dot{x} = v, y = 0, \dot{y} = 0, \theta = 0, \dot{\theta} = 0 \). For constant vehicle speed \( \dot{x} = v \) only two of the linearized equations of motion are important (these are derived in Pacejka, 172):

\[
\begin{align*}
\dot{y} & = \frac{1}{v} \left( C_f a + C_r b \right) \dot{\beta} - \left( C_f + C_r \right) \theta = C_f \delta \\
J \ddot{\theta} & = -\frac{1}{v} \left( C_f a^2 + C_r b^2 \right) \ddot{\beta} - \left( C_f a - C_r b \right) \dot{\theta} + \frac{1}{v} \left( C_f a - C_r b \right) \dot{y} = C_f a \delta
\end{align*}
\]

where \( C_f \) and \( C_r \) are the lateral tire stiffnesses on the front and the rear of the vehicle.

Adding a controller \( u = k(y_d - y) \) and writing in state space notation gives:

\[
\begin{bmatrix}
\dot{y} \\
\dot{\theta}
\end{bmatrix} = \begin{bmatrix}
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix} \begin{bmatrix}
y \\
\theta \\
\dot{y} \\
\dot{\theta}
\end{bmatrix} + \begin{bmatrix}
0 \\
kC_f/m
\end{bmatrix} \begin{bmatrix}
y_d
\end{bmatrix}
\]

where \( C_1 = C_f + C_r, C_2 = C_f a - C_r b \) and \( C_3 = C_f a^2 + C_r b^2 \).
Figure D.8: Poles of the linearized controlled system for different controller gains, $v = 1 \text{ m/s}$

Figure D.9: Poles of the linearized controlled system for different controller gains, $v = 10 \text{ m/s}$
Figure D.10: Poles of the linearized controlled system for different controller gains, \( v = 20 \, \text{m/s} \)

Figure D.11: Steady-state-turning with output feedback: \( k = 1, v_0 = 10 \, \text{m/s} \)
Figure D.12: Steady-state-turning with output feedback: $k = 10$, $v_0 = 10 \text{ m/s}$

Figure D.13: Steady-state-turning with output feedback: $k = 10000$, $v_0 = 10 \text{ m/s}$
Appendix E

3d-Model of a FT95-tractor-semi-trailer combination

Figure E.1: Global model
E.1 Front suspension of the tractor

The front of the tractor contains leafsprings. The vertical stiffness of the leafsprings is modeled by TSBA-elements. The lateral stiffness by BUSHING-elements. Since we expect that the vehicle does not reach its bump/rebound in road holding tests the springs are modeled linear. Hysteresis in the leafspring is neglected. The dampers are modeled by nonlinear TSDA-elements.

The REV-REV-joint between front axle and chassis allows two relative d.o.f. between axle and chassis. With this joint rollsteer is globally modeled.

The roll stabilizer is modeled as a roll stiffness. It is modeled by a BUSHING-element between axle and chassis.
The front suspension is very roughly modeled. DADS contains a LEAFSPRING-element with which the proper leafspring characteristics can be taken into account. However it requires a FEM-analysis of the leafspring in advance.

**E.2 Rear suspension of the tractor**

![Diagram of the rear suspension](image)

**Figure E.4: Modeling of the rear suspension**

The rear of the tractor is suspended by 4 airsprings. These are modeled as nonlinear TSDA-elements. The dampers are also modeled as nonlinear TSDA-elements.

![Airspring and damper characteristics rear suspension](image)

**Figure E.5: Airspring and damper characteristics rear suspension**

The roll stabilizer is modeled by a BUSHING-element.

The rear axle has 2 relative d.o.f. to the chassis. It is attached to the chassis by 4 DISTANCE-constraints.
E.3 Trailer suspension

The 3 axles of the trailer are replaced by 1 axle in the model. The trailer axle has 2 relative d.o.f. to the trailer. It is attached to the trailer by 4 DISTANCE-constraints. The trailer suspension contains nonlinear airsprings and nonlinear dampers, both modeled by TSDA-elements.

The trailer does not have a roll stabilizer, but the suspension construction - at DAF known as a "De Klerk-suspension" - is very roll stiff. This roll stiffness is modeled by a BUSHING-element. The value is estimated according to Vaalburg (1988).
E.4 Torsion stiffness of the tractor and trailer

With the program DRICAP a static FEM-analysis was done to estimate the torsion stiffness of the tractor.

\[ M = 2F \cdot \frac{z}{l} \]
\[ \Delta \phi = \left( \frac{y_1 - y_2}{z_b} - \frac{y_3 - y_4}{z_b} \right) \]
\[ c = \frac{M}{\Delta \phi} \]

Figure E.8: Estimation of the torsion stiffness of the tractor

The torsion stiffness of the trailer was estimated according to Vaalburg (1988). In the model the torsion stiffnesses are discretized by RSDA-elements.

Since we expect that the flexibility of the tractor has great influence on the road holding behaviour this should be properly modeled in the future. The most beautiful way is to use flexible bodies, which combine FEM-analysis with multibody dynamics.
APPENDIX E. 3D-MODEL OF A FT95-TRACTOR-SEMITRAILER COMBINATION

E.5 Tires

Lateral tire forces are modeled with the Magic Formula (Maas, 1991). Maas (1991) fitted the coefficient $D(F_n)$ linearly to measured data. However, this fit is only valid in a small area around $F_n = 30000 N$. If we want to take the influence of weight transfer on the road holding into account the simplest polynomial fit would be:

$$D(F_n) = a_1F_n + a_2F_n^2$$  \hspace{1cm} (E.1)

This polynomial fit was made to the data (Figure E.9).

It should be noted that only a small amount of data was available. To get a more reliable fit it would be better to measure the tire characteristics also at higher normal forces.
Figure E.10: Lateral tireforces

The Magic Formula has been implemented in the DADS TIRE-subroutine. In this subroutine only the BASIC-option is implemented for usage in inverse simulations. At this moment traction forces and the aligning moment are not yet implemented. However, implementation of these tire aspects in DADS can be done very easy. Camber of the tire is also neglected.
E.6 Flow of computations in DADS

**Block Diagram:**

1. **IN48:** Read in user defined data
2. **FRC48:** User subroutine
   - Reduced eq. of motion
   - Calculate matrix C
   - Determine input iterative
     - Change input
       - NO
       - YES
3. **TIRE:** Modified DADS subroutine
   - Calculate tire forces
4. **RPT48:** User defined output
5. **Solve equations of motion**

**Flowchart Notes:**
- **Blockd:** Initialize common-blocks
- **FM3-file:** Model data
- **DADS Model**
- **CONTROL module:** Calculate reaction forces
- **Input of**
- **Response**
Appendix F

Reduction of the equations of motion

It is not necessary to use all the equations of motion when we estimate the input signal. A strongly reduced system will suffice. To reduce the system of equations of motion the following steps are taken:

The total system of algebraic/differential equations of motion can be written as:

\[
\begin{bmatrix}
M_1 & M_2 & \cdots & 0 \\
M_2 & M_{NB} & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
0 & 0 & \cdots & M_{NB}
\end{bmatrix}
\begin{bmatrix}
\ddot{q}_1 \\
\ddot{q}_2 \\
\vdots \\
\ddot{q}_{NB}
\end{bmatrix}
= \begin{bmatrix}
k_1^A \\
k_2^A \\
\vdots \\
k_{NB}^A
\end{bmatrix}
\]

(F.1)

The term \(\Phi^T\Delta\) represents the joint reaction forces working between bodies. We cut body \(i\), on which the pathconstraint acts, lose from the model. To do this we need to know the joint reaction forces working on this body. We can distract these forces from DADS using a simple modeling trick. The joint reaction forces are not directly available during the simulation. However we can simply distract them by rigidly attaching a dummy-element (a rigid body with neglectable mass and inertia properties) between the body from which we want to determine the reaction forces and the other bodies. The Lagrange multipliers of the bracket joint between dummy-element and the body of which we want to know the reaction forces are in fact the reaction forces in this bracket joint. These multipliers are available through the DADS input element. In future releases of DADS the reaction forces will be directly available in the simulation.

We assume that the input signal acts on body \(i\) (in our case body \(i\) will be the front axle of the vehicle).

The equations of motion of this body can be written as:

\[
M_i \ddot{q}_i = k_i^A + k_i^C
\]

(F.2)

with \(k_i^A\) the applied forces and \(k_i^C\) the constraint forces acting on body \(i\).

The equations of motion for body \(i\) can always be written in the diagonal form:
APPENDIX F. REDUCTION OF THE EQUATIONS OF MOTION

\[
\begin{bmatrix}
  x & 0 & 0 & 0 & 0 & 0 \\
  0 & x & 0 & 0 & 0 & 0 \\
  0 & 0 & x & 0 & 0 & 0 \\
  0 & 0 & 0 & 0 & x & 0 \\
  0 & 0 & 0 & 0 & 0 & x
\end{bmatrix}
\begin{bmatrix}
  \ddot{x}_i \\
  \ddot{y}_i \\
  \ddot{z}_i \\
  \dot{w}_{zi} \\
  \dot{w}_{yi} \\
  \dot{w}_{zi}
\end{bmatrix}
= \begin{bmatrix}
  \ddot{x} \\
  \ddot{y} \\
  \ddot{z}
\end{bmatrix}
\]  

(F.3)

where \( x \) denotes a non-zero matrix entry.

\( w_{zi}, w_{yi} \) and \( w_{zi} \) are the rotational velocities round the three principal axes of inertia of body \( i \).

The prescribed path of body \( i \) \( \psi(q) = 0 \) can, in our case, be written as:

\[
\psi(x_i, y_i) = 0
\]  

(F.4)

Twice differentiating (F.4) gives an equation of the form:

\[
\begin{bmatrix}
  x & x & 0 & 0 & 0 & 0 \\
  \dot{x}_i \\
  \dot{y}_i \\
  \dot{z}_i \\
  \dot{w}_{zi} \\
  \dot{w}_{yi}
\end{bmatrix}
= -\gamma \psi
\]  

(F.5)

Thus the matrix that is premultiplied with \( C^T \) has the following form:

\[
\begin{bmatrix}
  x & 0 & 0 & 0 & 0 & 0 \\
  0 & x & 0 & 0 & 0 & 0 \\
  0 & 0 & x & 0 & 0 & 0 \\
  0 & 0 & 0 & x & 0 & 0 \\
  0 & 0 & 0 & 0 & x & 0 \\
  x & x & 0 & 0 & 0 & 0
\end{bmatrix}
\]  

(F.6)

It is obvious that the \( C^T \) must have the form:

\[
C^T = \begin{bmatrix}
  x & x & 0 & 0 & 0 & 0 & x
\end{bmatrix}
\]  

(F.7)

Thus it follows that only the equations of motion in the direction of the generalized coordinates that are present in the pathconstraint are important. We can reduce the system again, now to:

\[
\begin{bmatrix}
  m_i & 0 \\
  0 & m_i
\end{bmatrix}
\begin{bmatrix}
  \ddot{x}_i \\
  \ddot{y}_i
\end{bmatrix}
= \begin{bmatrix}
  k_{x_i}^A + k_{x_i}^C \\
  k_{y_i}^A + k_{y_i}^C
\end{bmatrix}
\]  

(F.8)

From this reduced system we estimate the input signal.

By reducing the equations of motion the matrix that has to be factorized using QR-decomposition has decreased from a \((7NB+NC+1)x(7NB+NC)\)-matrix (\( \approx O(100x99) \) for our vehicle model) to a 3x2-matrix. This, of course, will save a lot of computation time.
Appendix G

Figures belonging to chapter 5

G.1 Influence load

Figure G.1: Average signal power in the lane-change at different vehicle speeds.
Figure G.2: Roll of the tractor at 50 km/h

Figure G.3: Roll of the trailer at 50 km/h
Figure G.4: Driven path of tractor and trailer at 50 km/h

Figure G.5: Sideslip of front tractor tires at 50 km/h
Figure G.6: Sideslip of rear tractor tires at 50 km/h

Figure G.7: Lateral slip of trailer tires in at 50 km/h
Figure G.8: Steering angle at kingpin for the unladen vehicle

Figure G.9: Steering angle at kingpin for the laden vehicle
G.2 Influence Tires

![Figure G.10: Average signal power at different speeds](image)

- [1]: Standard vehicle
- [2]: Vehicle with better tires

Figure G.10: Average signal power at different speeds
Figure G.11: Steering angle at the kingpin for the standard vehicle

Figure G.12: Steering angle at the kingpin for the vehicle with better tires
G.3 Influence rear suspension damping

![Graph showing influence of rear suspension damping]

Figure G.13: Average signal power at different speeds
G.4 Steady-state-turning

Figure G.14: Calculated steering angle in steady state turning
Figure G.15: Reaction forces calculated from Lagrange multipliers

Figure G.15: Reaction forces reported in output file