MASTER

Numerical assessment of the design imperfections for steel beam lateral torsional buckling

van der Aa, R.P.

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R.P. van der Aa
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Name: R.P. van der Aa
Student number: 0635325
E-mail address: r.p.v.d.aa@student.tue.nl
                      robvanderaa@hotmail.com
Home address: De Sitterlaan 56
Zip code 5505 AD Veldhoven

1st graduation supervisor: Prof. ir. H.H. (Bert) Snijder
Position: Full professor
Chair: Steel structures

2nd graduation supervisor: Ir. B.W.E.M. (Dianne) van Hove
Position: Assistant professor
Chair: Aluminum structures and steel structures

3rd graduation supervisor: Dr.ir. H. (Herm) Hofmeyer
Position: Associate Professor
Chair: Applied Mechanics
Preface

This report has been written to conclude the graduation project. The graduation project has been carried out as a part of the master Architecture, Building and Planning with the specialization Structural Design. It has been carried out at the Department of the Built Environment at Eindhoven University of Technology, the Netherlands.

I would like to thank my graduation committee: Prof. ir. H.H. Snijder, ir. B.W.E.M. van Hove and dr.ir. H. Hofmeyer for their supervision.

Finally, I would like to thank my family and friends for their contribution. Sincere gratitude is expressed to my parents and my brother. I thank my parents for their great support and my brother for his interest and advice.

Rob van der Aa
Eindhoven, June 2015
Abstract

Eurocode 3 provides design rules for the assessment of lateral torsional buckling in clause 6.3 of EN 1993-1-1. But when a structural designer wants to make a second order analysis himself, a Geometrically and Materially Nonlinear Analyses with Imperfections (GMNIA) of beams may be performed. Eurocode 3 prescribes the size and shape of the geometric imperfection in clause 5.3.4(3) of EN 1993-1-1. The shape is prescribed as an equivalent initial bow along the weak axis of the section, excluding torsion of the cross-section. The size is prescribed as half the value presented in Table 5.1 of EN 1993-1-1. But when searching for background information on this rule, almost none can be found. The Dutch National Annex gives an alternative for Table 5.1, which is similar to equation (5.10) of EN 1993-1-1. This alternative is tested and the results of it are compared with the appropriate lateral torsional buckling curves.

Excluding the torsion of the cross-section in the imperfection shape is questionable, since torsion in the imperfection shape makes a beam more susceptible to lateral torsional buckling. As another alternative the shape of the imperfection can also be taken equal to the lateral torsional buckling mode, including torsion of the cross-section. The results of this alternative are also compared with the appropriate lateral torsional buckling curves.

For the next generation of Eurocodes, Taras derived new lateral torsional buckling design rules as an alternative to those of the current EN 1993-1-1. Using these design rules and their derivation, a formula is obtained, that describes explicitly the imperfection size for a lateral torsional buckling mode. This obtained formula is tested and the results are compared with the new design rules of Taras.

The three different approaches for imperfection shape and size are all tested by using a finite element program. An implicit static finite element model for lateral torsional buckling has been developed and verified with several other models from the literature. With the model, the three different approaches for imperfection shape and size were applied in GMNIA calculations to evaluate the lateral torsional buckling resistances. The approaches were tested for two types of cross-section and three types of load cases. The two types of cross-sections are the IPE and the HEA sections. For the final approach also HEB sections were treated. The three different load cases used are the constant bending moment, the point load in the middle on the top flange and the line load on the top flange.

From the results it is concluded that the first alternative is not sufficient and gives very poor results. The alternative needs major adjustments to become sufficient to use. The second alternative, using the lateral torsional buckling mode as imperfection shape, produces better results in comparison with the first alternative. However, it is still not sufficient and also needs major adjustments to become sufficient to use. The final alternative, does lead to correct lateral torsional buckling resistances if allowed to be slightly modified. If not modified a slight overestimation of the lateral torsional buckling resistance of five percent remains for some cases, which might be acceptable.
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<td>Normalized warping function</td>
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1. Introduction

1.1 Motivation

The capacity of a loaded beam is not just determined by its plastic moment but also by its resistance against instability. Lateral torsional buckling is a type of instability that can occur when a beam is loaded by bending. This is why there are design rules that take lateral torsional buckling into account and prevent this type of instability from happening. But what is lateral torsional buckling? A good way to describe this phenomenon is by using the following situation. Assume an I-shaped beam that is loaded by a bending moment around its strong axis. The beam will start bending around this axis. At a certain loading the compressed top flange will buckle sideways and the beam will also start to bend in the weak direction, with this also a rotation of the cross-section occurs. Figure 1.1 shows how the beam will deform when it fails in lateral torsional buckling (LTB).

![Figure 1.1 A beam subjected to lateral torsional buckling](image)

Thanks to the design rules, which are given in Eurocode 3 [1], structural engineers and designers do not have to make full non-linear numerical calculations every time they need to assess the lateral torsional buckling resistance of a steel structure. However, recent studies [2], [3] and [4] have shown that part of these design rules are not safe to use. That is why Taras is currently developing new design rules and buckling curves for lateral torsional buckling [5]. Proposals for adjustments to Eurocode 3 can be found in §2.8 [6]. There is a reasonable chance that these new rules will be adopted in Eurocode 3 in 2019. But until that time the current design rules from Eurocode 3 are used for checking lateral torsional buckling.

Of course it is nearly impossible to capture every possible situation in practice by these design rules or design tables. Eurocode 3 also gives structural designers the option to make a second order analysis themselves taking account for lateral torsional buckling. For carrying out such analyses, certain assumptions for the size and shape of the geometric imperfection, can be found in Eurocode 3 in paragraph 5.3.4 member imperfections. But when searching for background information on this rule, almost none can be found. So some research on this paragraph from the Eurocode is necessary.
1.2 Definition of the problem

As stated in the motivation, there are design rules adopted to check lateral torsional buckling. But when a structural designer wants to make a second order analysis himself, Eurocode 3 gives the possibility by prescribing the structural designer the size and shape of the geometric imperfection. As stated in Eurocode 3 paragraph 5.3.4 Member imperfections [1]:

(3) For a second order analysis taking account of lateral torsional buckling of a member in bending the imperfections may be adopted as $ke_{0,d}$ where $e_{0,d}$ is the equivalent initial bow imperfection of the weak axis of the profile considered. In general an additional torsional imperfection need not to be allowed for.

NOTE The National Annex may choose the value of $k$. The value $k = 0,5$ is recommended.

Originally this formula can be found in the DIN 18800 [7], which used to be the German code. There is very little background information on what this formula $ke_{0,d}$ was based. That makes this an unreliable formula, which should be properly tested.

e_{0,d} can be found in Table 2.8 in §2.9: design values of initial bow imperfection $e_{0}/L$. But the Dutch Annex of the Eurocode gives an alternative for $e_{0,d}$. It is stated that the following formula has to be used:

$$e_{0,d} = \alpha (\bar{\lambda} - 0,2) \frac{M_{Rk}}{N_{Rk}}$$  \hspace{1cm} (1.1)

Where:

$\alpha$ Imperfection factor for relevant buckling curve

$\bar{\lambda}$ Relative slenderness

$M_{Rk}$ Characteristic moment resistance of the critical cross section

$N_{Rk}$ Characteristic resistance to normal force of the critical cross section

Formula (1.1) is derived for buckling, so not especially for lateral torsional buckling. So the question is whether $ke_{0,d}$ with $k = 0,5$, is a safe approach in comparison with the current design rules for lateral torsional buckling. The derivation of formula (1.1), combined with part of the derivation of the buckling rules, is given in Appendix A.

Furthermore, it is stated that the additional torsional imperfection may be neglected. It may be assumed that this is stated simply because, at the time the rule was set up, the input for this type of calculations was prepared by hand. Additional torsion would make those calculations too complex. Also, as can be seen in the derivation of formula (1.1), at the starting point of the derivation, it is assumed the l-shaped profile only has an equivalent initial bow imperfection around the weak axis. So formula (1.1) is based on such an imperfection.

Taras and Boissonnade show in respectively [5] and [8] that additional torsion to the imperfection shape makes a beam more susceptible to lateral torsional buckling. However, both state the influence is minimal. Though, in §7.2 it is showed that the influence of torsion actually is load case depended and that the influence for certain load cases is very significant. It is a realistic assumption that a beam also has a rotational imperfection. With the help of a finite element program, giving a beam its additional torsional imperfection can easily be arranged for nowadays. So it is also interesting to see if
In member Formula mode, to beam very

\[ \alpha_{LT}(\bar{\lambda} - 0.2) \frac{M_{Rk}}{N_{Rk}} \]

\[ (1.2) \]

Where:

- \( \alpha_{LT} \) Imperfection factor for Taras’ lateral torsional buckling curve
- \( \bar{\lambda} \) Relative slenderness
- \( M_{Rk} \) Characteristic moment resistance of the critical cross-section
- \( N_{Rk} \) Characteristic resistance to normal force of the critical cross-section

At first sight this formula looks pretty similar to formula (1.1), however the imperfection factors are very different. For formula (1.1) the imperfection factor \( \alpha \) should be taken from Table 2.10 in §2.9. Which imperfection factor should be chosen is given in Table 2.9 in §2.9. For formula (1.2) \( \alpha_{LT} \) should be taken form Table 2.6 presented in §2.8.

Formula (1.2) has been derived from the assumption that the imperfection mode of the I-shaped beam is based on its lateral torsional buckling mode. Since the formula is specially derived for lateral torsional buckling it can be assumed that the outcome of the formula does not need to be reduced anymore, which means \( k = 1 \).

1.3 Objectives

The main goal of this research is to determine whether Eurocode 3 [1] with respect to its rules on member imperfections, provides a safe approximation for lateral torsional buckling. The main goal consists of three sub goals:

1. Determine whether Eurocode 3 paragraph 5.3.4 Member imperfections, with \( ke_{0,d} \) being the amplitude of an imperfection that is based on the weak axis flexural buckling mode, combined with formula (1.1) is in accordance with the lateral torsional buckling curves. If this is not the case the goal is to develop a new value for \( k \), that makes it in accordance with the lateral torsional buckling curves.

2. Determine whether Eurocode 3 paragraph 5.3.4 Member imperfections, with \( ke_{0,d} \) being the amplitude of an imperfection that is based on the lateral torsional buckling mode, combined with formula (1.1) is in accordance with the lateral torsional buckling curves. If this is not the case the goal is to develop a new value for \( k \), that makes it in accordance with the lateral torsional buckling curves.

3. Determine whether formula (1.2) being the amplitude of an imperfection that is based on the lateral torsional buckling mode, is in accordance with Taras’ buckling curves. If this is not the case \( \alpha_{LT} \) that Taras has developed, will be modified in accordance with Taras’ lateral torsional buckling curves.
If a new value for \( k \) needs to be designed, because the current rule on member imperfection is not safe, the approach is to modify \( k \) in a way that the rule on member imperfections becomes as precise as possible. This means it can be an option to develop a formula for \( k \), so it gives a better fit. However this new formula for \( k \) cannot get too extensive or too complicated. So a balance needs to be found between a precise formula and a not too complicated formula.

### 1.4 Scope

This research project is about lateral torsional buckling of steel I-shaped members. Within this research only hot-rolled sections are treated. Besides that only class 1 and 2 cross-sections are taken into account within this research. The focus is put on global lateral torsional buckling, which means local buckling effects were omitted. Cross-sections of class 1 and 2 are defined by Eurocode 3 [1] as:

- Class 1 cross-sections are those which can form a plastic hinge with the rotation capacity required from plastic analysis without reduction of the resistance.
- Class 2 cross-sections are those which can develop their plastic moment resistance, but have limited rotation capacity because of local buckling.

So for the section modulus the plastic section modules should be taken. For the area of the section the gross area should be taken. This means the holes for connections do not need to be taken in account for.

The research starts with two types of steel sections: IPE- and HEA-sections without fillet radius. At the end also HEB-sections without fillet radius are researched. The cross-section of an I-shaped steel section is shown in Figure 1.2c below.

![Figure 1.2 Sections of an I-shaped profile](image)

In Figure 1.2a a regular IPE-profile is shown. In this case the beam is modeled with shell elements, so the section becomes what is shown in Figure 1.2b. When these shell elements are shown with their given thickness, the section in Figure 1.2c is formed. As can be seen the ends of the web overlap with the flanges. For this research the cross-section shown in Figure 1.2c is close enough to the reality to use it. This is further elaborated in §2.11. Using Figure 1.2c as cross-section does mean that the section-properties of this section also should be used in the design rules.
Within this research there are three load cases that will be treated. These three load cases are shown in Figure 1.3.

(a) load case 1  
(b) load case 2  
(c) load case 3  

Figure 1.3 The three load cases

Load case 1 has in-plane bending moments at the beam ends, which means there is a constant bending moment over the beam. Load case 2 is a point load situated on the exact middle of the beam. Load case 3 is an equally distributed load over the entire beam. For load case 2 and load case 3 the load case is put on the top flange of the beam.

1.5 Approach

The modeling of lateral torsional buckling is done with ABAQUS [9], a finite element method computer program. With this program two types of numerical simulations are performed:

- LBA; Linear Buckling Analysis, which are used to determine the eigenmodes for perfect elastic lateral torsional buckling and to determine the elastic critical moment for lateral torsional buckling.
- GMNIA; Geometrically and Materially Non-linear Analysis with Imperfections, which are used to obtain realistic numerical values for lateral torsional buckling.

Beams with different slendernesses are modeled as single-span members with in-plane, out-of-plane and torsional restraints at the supports. The beams are loaded with a constant bending moment over the beam. First an LBA is carried out, from this simulation the elastic critical moment and the buckle mode are obtained. The elastic critical moment is used to arrive at the relative slenderness $\lambda_{LT}$. The obtained buckle mode is used for the imperfection shape of the beam.

For the first sub goal the beam is given an imperfection shape that is based on its weak axis flexural buckling mode with an amplitude of $ke_{0,d}$. For the second sub goal the beams are given an imperfection shape based on the lateral torsional buckling mode, determined by a LBA, also with an amplitude of $ke_{0,d}$. Important to notice is that because different load cases are used, the lateral torsional buckling modes will differ as well. Subsequently, these beams are loaded and $M_R$, the moment at which failure occurs, is determined using a GMNIA calculation. Once the moment at which failure due to lateral torsional buckling occurs is known, the reduction factor $\chi_{LT}$ can be determined.

Once $\chi_{LT}$ and $\lambda_{LT}$ are known these values can be compared with the current lateral torsional buckling curves. These are shown in §2.3.1 General method. As is shown in §2.6.2 and §2.7.2 the General method gives good results. This means that if the reduction factor, calculated with a finite element method program using the description in Eurocode 3 paragraph 5.3.4 in combination with the Dutch National Annex $(\chi_{LT,FEM})$, is greater than the reduction factor, calculated with the General Method from Eurocode 3 $(\chi_{LT,EC3})$ the moment capacity of the cross-section of the beam is not reduced enough to take lateral torsional buckling in account. So when comparing the $\chi_{LT}$-values, cases obtained from the GMNIA, should lay below its lateral torsional buckling curve, obtained from the General Method, to be on the safe side. Figure 1.4 shows two reduction factors $(\chi_{LT,FEM})$, one on the unsafe side of the lateral torsional buckling curve and one on the safe side.
So if the calculated reduction factors ($\chi_{LT,FEM}$) are on the unsafe side the value $k$ should be adjusted and the procedure should be repeated until the values are on the safe side.

To reach the final sub goal the same approach is used with some adjustments. As stated before, formula (1.2) should be used. This time the results have to be compared with the new design curves for lateral torsional buckling, created by Taras. The formulas of these lateral torsional buckling curves are shown in the Proposal for amended rules for member buckling and semi-compact cross-section design [6]. This proposal is also explained in paragraph 2.5. Also in this case the values obtained from the GMNIA calculations should lay below their lateral torsional buckling curve in order to call formula (1.2) safe to use.

1.6 Outline of report

Chapter 2: Literature study
In this chapter the literature that is used for this research is discussed. The current lateral torsional buckling rules from the Eurocode are treated. Some discussion points on these lateral torsional buckling rules are discussed. Also Taras’ proposal for new lateral torsional buckling rules is presented.

Chapter 3: Finite element model
This chapter gives information about the finite element model. Within this chapter the material properties, boundary conditions, load cases and the system imperfections are explained.

Chapter 4: Illustration of numerical simulations
Within this chapter is shown which numerical simulations are used and how the numerical simulations are applied.

Chapter 5: Method for obtaining results
This chapter informs how the results from the numerical simulations are used. This is explained with the help of an extensive example calculation. After this the finite element model is validated with two other finite element models.

Figure 1.4 Comparing $\chi_{LT,FEM}$ with $\chi_{LT,EC3}$
Chapter 6: Rule on member imperfection comparing with EC3
In this chapter the first sub goal is treated. So first the rule on member imperfection is compared with the General method from EC3. After this the rule is adjusted.

Chapter 7: Rule on member imperfection including torsion
This chapter treats the second sub goal. So the rule on member imperfection is used without neglecting the additional torsional imperfection. After this the rule is further adjusted.

Chapter 8: Taras’ lateral torsional buckling curves
First the Taras’ lateral torsional buckling curves are compared with the General method. After that formula (1.2) is tested and the results are compared with the lateral torsional buckling curves Taras proposed.

Chapter 9: Discussion
This chapter discusses the obtained results and proposals

Chapter 10: Conclusion and recommendations
At the end the research is closed with conclusions and recommendations.
2. Literature study

2.1 Introduction
In this literature study the literature that is used for this research project is discussed. The subject of this research project as shown on the front page is: Imperfection study for lateral torsional buckling of steel I-shaped beams. The literature study starts with a general part on lateral torsional buckling. After this the design rules that check lateral torsional buckling are explained. This is followed by an investigation that questions the current buckling rules. Then the proposed new design rules for lateral torsional buckling designed by Taras are summarized. After this, it is treated what the Eurocode states on imperfections, that are necessary for a second order lateral torsional buckling analysis. Then some background information and some discussion on this design rule is provided.

2.2 Lateral torsional buckling in general
Lateral torsional buckling is caused by compression in the flange of a beam in bending. When the compression gets too large in this flange, it will buckle over the weak axis, so bending in the weak direction occurs. There is tension in the tension flange, so this flange wants to stay straight. This causes the beam to rotate as well. This can be seen in Figure 2.1 below. Because of the rotation a lateral deflecting moment occurs. Due to the lateral deflection a torsional moment arises. As a result of the additional torsion again the lateral deflecting moment gets larger. So the lateral moment and the torsional moment keep increasing each other. With that the deformation keeps increasing, which means the beam becomes instable.

![Figure 2.1 The arising of lateral torsional buckling](image)

Assume this I-section shaped beam is perfectly straight, made from a linear elastic material. The moment lateral torsional buckling occurs, for such a beam, is called the elastic critical moment. In practice this will never be the critical point at which lateral torsional buckling really occurs. Beams are never fully straight, they do not act fully elastic, there are residual stresses and the beams usually are not loaded centric. So in practice the moment at which lateral torsional buckling occurs, should always be lower than the elastic critical moment.
2.3 Eurocode on lateral torsional buckling

In Eurocode 3 [1] lateral torsional buckling is taken into account by the reduction factor $\chi_{LT}$. This factor is used to reduce the moment capacity $M_{pLrd}$ of a cross-section calculated by:

$$M_{pLrd} = \frac{W_{pl} \cdot f_y}{\gamma_M}$$  \hspace{1cm} (2.1)

In Eurocode 3 there are two different methods to calculate $\chi_{LT}$. The general method and the specific method, within this specific method also a modification factor may be used to modify the reduction factor. The derivation of the general method is shown in Appendix A.

2.3.1 General method

In Eurocode 3 section 6.3.2.2 the general method can be found:

$$\chi_{LT} = \frac{1}{\phi_{LT} + \sqrt{\phi_{LT}^2 - \bar{\lambda}_{LT}^2}} \leq 1,0$$  \hspace{1cm} (2.2)

Where:

$$\phi_{LT} = 0,5 \left[ 1 + a_{LT} (\bar{\lambda}_{LT} - 0,2) + \bar{\lambda}_{LT}^2 \right]$$  \hspace{1cm} (2.3)

$$a_{LT} = \text{Imperfection factor (see Table 2.1)}$$

$$\bar{\lambda}_{LT} = \sqrt{\frac{W_{pL} f_y}{M_{cr}}}$$  \hspace{1cm} (2.4)

$M_{cr} = \text{Elastic critical moment for lateral torsional buckling (paragraph 2.3.4)}$

| Table 2.1 Recommended values for imperfection factors for lateral torsional buckling curves |
|-------------------------------|---|---|---|---|
| **Buckling curve** | **a** | **b** | **c** | **d** |
| Imperfection factor $\alpha_{LT}$ | 0,21 | 0,34 | 0,49 | 0,76 |

Which buckling curve should be chosen depends on the height to width ratio of the I-shaped sections. Table 2.2, shows which buckling curve should be chosen.

| Table 2.2 Recommended values for lateral torsional buckling curves |
|---------------------------------------|-----------------|-----------------|
| **Section** | **Limits** | **Buckling curve** |
| Rolled I-sections | $h/b \leq 2$ | a |
| | $h/b > 2$ | b |
| Welded I-sections | $h/b \leq 2$ | c |
| | $h/b > 2$ | d |
| Other sections | - | d |
2.3.2 Specific method

This method is designed for lateral torsional buckling of rolled or equivalent welded sections. The reduction factor $\chi_{LT}$ may be calculated with:

$$\chi_{LT} = \frac{1}{\phi_{LT} + \sqrt{\phi_{LT}^2 - \beta \bar{\lambda}_{LT}^2}} \leq 1,0 \text{ also } \chi_{LT} \leq \frac{1}{\bar{\lambda}_{LT}^2} \quad (2.5)$$

Where:

$$\phi_{LT} = 0,5 \left[ 1 + \alpha_{LT} (\bar{\lambda}_{LT} - \bar{\lambda}_{LT,0}) + \beta \bar{\lambda}_{LT}^2 \right] \quad (2.6)$$

**NOTE** The parameters $\bar{\lambda}_{LT,0}$ and $\beta$ and any limitation of validity concerning the beam depth or h/b ratio may be given in the National Annex. The Dutch National Annex follows the recommended values for rolled sections or equivalent welded sections:

$\bar{\lambda}_{LT,0} = 0,4$

$\beta = 0,75$

The lateral torsional buckling curve should be chosen from Table 2.3.

<table>
<thead>
<tr>
<th>Section</th>
<th>Limits</th>
<th>Buckling curve</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rolled I-sections</td>
<td>h/b $\leq 2$</td>
<td>b</td>
</tr>
<tr>
<td></td>
<td>h/b $&gt; 2$</td>
<td>c</td>
</tr>
<tr>
<td>Welded I-sections</td>
<td>h/b $\leq 2$</td>
<td>c</td>
</tr>
<tr>
<td></td>
<td>h/b $&gt; 2$</td>
<td>d</td>
</tr>
</tbody>
</table>

2.3.3 Modified specific method

This method continues where the specific method stops. This method takes the moment distribution and the corresponding beneficial effect of reduced plastic zones into account. Because of this the reduction factor $\chi_{LT}$ may modified:

$$\chi_{LT,mod} = \frac{\chi_{LT}}{f} \leq 1 \quad (2.7)$$

Where:

$$f = 1 - 0,5(1 - k_c) \left( 1 - 2,0(\bar{\lambda}_{LT} - 0,8)^2 \right) \leq 1,0 \quad (2.8)$$

$k_c$ is taken from Table 2.4

NOTE The parameters $\bar{\lambda}_{LT,0}$ and $\beta$ and any limitation of validity concerning the beam depth or h/b ratio may be given in the National Annex. The Dutch National Annex follows the recommended values for rolled sections or equivalent welded sections:
### Table 2.4 Correction factors $k_c$

<table>
<thead>
<tr>
<th>Moment distribution</th>
<th>$k_c$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Psi = 1$</td>
<td>1,0</td>
</tr>
<tr>
<td>$-1 \leq \Psi \leq 1$</td>
<td>$\frac{1}{1,33 - 0,33\Psi}$</td>
</tr>
<tr>
<td></td>
<td>0,94</td>
</tr>
<tr>
<td></td>
<td>0,90</td>
</tr>
<tr>
<td></td>
<td>0,91</td>
</tr>
<tr>
<td></td>
<td>0,86</td>
</tr>
<tr>
<td></td>
<td>0,77</td>
</tr>
<tr>
<td></td>
<td>0,82</td>
</tr>
</tbody>
</table>

### 2.3.4 Elastic critical moment for lateral torsional buckling from Dutch National Annex

In the Dutch National Annex D.4 of Eurocode 3 it is given how to calculate the elastic critical moment for lateral torsional buckling. This can be done using the formula:

$$M_{cr} = k_{red} \cdot \frac{C}{l_g} \sqrt{EI_z \cdot GI_t}$$

(2.9)

Where:

$k_{red} = 1,0$ ← for standard rolled and welded section

$$C = \frac{\pi C_1 l_g}{l_{LTB}} \left( 1 + \frac{\pi^2 S^2}{l_{LTB}^2 (C_2^2 + 1)} + \frac{\pi C_2 S}{l_{LTB}} \right)$$

(2.10)

$$S = \frac{EI_w}{\sqrt{GI_t}}$$

(2.11)

Values for $C_1$ and $C_2$ can be found in table 2.5
Table 2.5 Values for $C_1$ and $C_2$

<table>
<thead>
<tr>
<th>Load case</th>
<th>$C_1$</th>
<th>$C_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1.png" alt="Image" /></td>
<td>1,0</td>
<td>0</td>
</tr>
<tr>
<td><img src="image2.png" alt="Image" /></td>
<td>1,13</td>
<td>0,45</td>
</tr>
<tr>
<td><img src="image3.png" alt="Image" /></td>
<td>1,35</td>
<td>0,55</td>
</tr>
</tbody>
</table>

However formula (2.9) should only be used when certain conditions are met:

- double-symmetric I-shaped section at which the load may not be applied higher than 0,1 times the height of the profile above the upper flange;
- only single-span members with in-plane, out-of-plane and torsional restraints at the supports can be considered (“end fork” conditions);
- the ratio between the length $L_g$ of the beam and the height $h$ of the profile is greater than or equal to 5: $L_g / h \geq 5$.

Also important to notice is the factor $k_{\text{red}}$. As stated earlier this factor should be taken as 1 for rolled and welded sections. This means in those cases the cross-section of the beam is assumed to retain its shape.

### 2.4 Basic parameters for FE-model

Instead of using the lateral torsional buckling curves, it is also possible to use finite element models to simulate the behavior of members with respect to lateral torsional buckling. There are two methods available for including imperfections in the FE-model, being the equivalent imperfection method and the plastic zone theory. For the equivalent imperfection method only a geometric imperfections is used to cover all types of imperfections. The input for this method is found in paragraph 5.3.4(3) of Eurocode 3 and will be scrutinized within this research. This method is most commonly used when a structural designer wants to make a second order calculation with respect to lateral torsional buckling.

Within the plastic zone theory residual stresses and geometric imperfections are used to get realistic values out of the simulations. The size and shape of the residual stresses and the geometric imperfections that should be used are found in [10]. These values are widely accepted in the scientific community and therefore can be used to check the lateral torsional buckling curves, as is done by Bruins [3] and Rebele [4], which is described in respectively §2.6 and §2.7. Also they are used to design new lateral torsional buckling rules as Taras has done, which is described in §2.8.
The basic parameters for plastic zone theory as they are stated in [10] consist of the stress-strain law, residual stresses and geometric imperfections. The basic parameters are showed in respectively Figure 2.2, Figure 2.3 and Figure 2.4.

2.5 A closer look at the basic parameters

In [8] a closer look is taken to the basic parameters and these are compared with other options. This comparison is done with the use of an FE-model that is made with the FEM software FINELg.

2.5.1 FE-model

The model consists of quadrangular 4-nodes plate-shell finite elements with typical features (Corotational total Lagrangian formulation, Kirchhoff’s theory for bending). Because shell elements are used, an error is made. The actual cross-section has fillets and these are not accounted for by the shell elements. This leads to large differences between the torsional properties of the FE-model and those of the actual cross-section. For these differences, compensation elements were used. Elastic-plastic rectangular hollow section elements were placed on the web-flange intersection.
The beams are simply supported at the beam ends. So called end fork conditions are applied. In-plane, out-of-plane and torsional restraints at the supports are present. In Figure 2.5 the support conditions are shown.

![Figure 2.5 Support conditions](image)

All models are loaded with a constant bending moment. Figure 2.6 shows how the forces are introduced at end of the beams.

![Figure 2.6 Applied loads at flange tip](image)

Two types of beam sections are modeled within the research, being the IPE500 and the HEB300.

### 2.5.2 Stress-strain law

Three stress-strain constitutive laws are shown in Figure 2.7. These are frequently used in numerical studies, and are generally assumed to represent the actual behavior of structural steel in a suitable way.

![Figure 2.7 Investigated stress-strain laws](image)

Figure 2.8 gives the result of the comparison between the three different stress-strain laws used in the FE-model.
2.5.3 Residual stresses

Three different types of distributions are considered here. The three different distributions of the residual stresses are shown in Figure 2.9.

Type A and B are for hot-rolled sections and type C is for welded sections. These three type of distributions are compared in Figure 2.10.

As can be seen in Figure 2.10, the differences between the triangular and parabolic distribution are negligible. This is not the case for the welded distributions, as Figure 2.10 clearly shows.
2.5.4 Geometrical imperfections

Experimental measurements show that within Europe’s production, a realistic average value of steel member’s initial bow imperfection amplitude lies around L/1000, where L is the length of the member. Four types of geometric imperfection are showed in Figure 2.11.

![Figure 2.11 Four types of geometrical imperfections](image)

From the four types of geometrical imperfections type 1 is the eigenmode obtained from an LBA. In Figure 2.12 the comparison between the influences of the four types of geometrical imperfections is shown.

![Figure 2.12 Influence of type of imperfection - left graph: IPE500 - right graph: HEB300](image)

All results are close, except for the case of type 4. This indicates that the eigenmode obtained from an LBA may be seen as adequate to initiate lateral torsional buckling. As stated before within the research of Boissonnade the only load cases that is treated is the constant bending moment showed in Figure 2.6. Though in §7.2 is explained that the influence of additional torsion actually is load depended.

2.5.5 Conclusion and recommendations for FEM modeling

As a summary of the performed investigations, the following features may be recommended for the modeling of lateral torsional buckling:

- No strain-hardening may be considered in the stress-strain law, which is contradiction with the basic parameters described in §2.4;
- For hot-rolled profiles, parabolic or triangular residual stresses patterns are recommended, with a reference yield stress \( f_y \) equal to 235 N/mm²;
- Use of eigenmodes as initial imperfection shape is suitable, provided a careful scaling of the initial amplitude (L/1000 is recommended).
2.6 Discussion on current lateral torsional buckling rules by Bruins

Part of Bruins’ master thesis [3] shows that two of the three current lateral torsional buckling methods from the Eurocode (§2.3.1, §2.3.2 and §2.3.3) are actually unsafe to use. To get to this conclusion, finite element calculations have been carried out with Ansys.

2.6.1 FE-model

The model of the I-shaped beam consists of shell elements. As was discussed in §2.5.1 an error is made, because shell elements are used. For this reason, compensation elements were used. Elastic-plastic rectangular hollow section elements were placed on the web-flange intersection.

Two types of supports were used. The first one is a simply supported beam, as can be seen in Figure 2.13a. In-plane, out-of-plane and torsional restraints at the supports are present. This is the end fork condition. The stiff elements at the end of the beam as shown in the figure are used to prevent distortion of the cross-section, but they allow for warping. The second one is a fixed support, as can be seen in Figure 2.13b.

![Figure 2.13 Types of supports](image)

This model is tested with three kind of load cases, shown in Figure 2.14.

![Figure 2.14 Load cases](image)

A steel grade of S235 was used with a yield strength of 235 N/mm². A bilinear stress-strain diagram was used with a Young’s modulus of elasticity of 2,1·10⁵ N/mm².

As mentioned before a beam is never fully straight and residual stresses are present in the beam. The residual stresses are divided over the cross-section according to the idealized pattern given in [10]. The pattern is shown in Figure 2.3. The maximum residual stress is a third of the yield stress. In order to model the out-of-straightness of the beam the first elastic critical lateral torsional buckling shape is used. The amplitude of this buckling shape was taken as L/1000 according to [10].
2.6.2 Comparison FE-model versus design rules EC3

With a geometrically and materially non-linear analysis with imperfections (GMNIA) one can determine the lateral torsional buckling resistance using the FE-model. These values can be used to determine the reduction factor $\chi_{L,T,FEM}$. This value can be compared with the calculated value using the general method from the Eurocode, which is the reduction factor $\chi_{L,T,GM}$. Three different beam lengths with the three different load cases are compared. In Figure 2.15 the results of this comparison are shown in a diagram. The black line shows the perfect match with the general method. If the results from the FE model lay above this line, the results are on the safe side. When the results lay below the line the results are on the unsafe side, which means that the general method is for those cases unsafe to use. The two grey lines show a 5% over- and underestimation.

![Diagram](image)

**Figure 2.15** Comparison FE-element model versus General method

As can be seen in Figure 2.15 most cases are on the safe side, the rest of the cases lay very close to the black line. So the General Method can be considered safe to use. A same kind of comparison can be made for the Specific Method and for the Modified Specific Method. These results are shown in Figure 2.16.

![Diagram](image)

**Figure 2.16** Comparison FE-element model versus Specific Methods

The results clearly show that for load case 1 and 2 the Specific Methods are unsafe to use. Though, for load case 3 the Specific method can be used safely. The diagram on the right side shows that in all but one cases the Modified Specific Method is unsafe to use.
2.7 Discussion on current lateral torsional buckling rules by Reb elo

Part of Reb elo’s article [4] shows that the modified specific method for lateral torsional buckling from the Eurocode (§2.3.3) is unsafe to use. It is stated that the General method in combination with the correction factor \( f \) should be used. To get to this conclusion, finite element calculations have been carried out with Safir.

2.7.1 Parametric study

A simply supported beam with fork supports was studied. Hot rolled sections were analyzed with steel grades of S235 and S460. The chosen sections were HEA500, IPE220 and IPE500. The following relations between the height \( h \) and the width \( b \) are respectively: \( h/b < 2 \), \( h/b = 2 \) and \( h/b > 2 \). Each of these profiles were then loaded with two types of loads showed in Figure 2.17 with \( \psi = 1 \) or \( \psi = -1 \).

![Simply supported beam subjected to non-uniform bending](image)

Figure 2.17 Simply supported beam subjected to non-uniform bending

In the numerical simulations, a lateral geometric imperfection of sinusoidal type with a maximum value of \( L/1000 \) is given by the following expression:

\[
y(x) = \frac{l}{1000} \sin \left( \frac{\pi x}{l} \right)
\]

(2.12)

where \( l \) is the length of the beam. An initial rotation around the \( x \) axis with a maximum value of \( L/1000 \) radians at mid span was also introduced.

The residual stresses are divided over the cross-section according to the idealized pattern given in [10]. The pattern is shown in Figure 2.3.

2.7.2 Comparison FE-model versus design rules EC3

The evaluation of the lateral torsional buckling reduction factor using the three methods, the General Case (GC), the Modified Special Case (SC) and the General Case/\( f \) (GC/\( f \) ), was made and the results were compared with the numerical results obtained with the finite element program SAFIR.

The results for the three sections are showed in Figure 2.18, Figure 2.19 and Figure 2.20.
Figure 2.18 Numerical results for the HEA500

Figure 2.19 Numerical results for the IPE220
These figures compare the numerical results obtained with the program SAFIR, the General Case, the Modified Special Case with $\lambda_{LT,0} = 0.4$ and $\beta = 0.75$ and the General Case/f. They are representative of the values $h/b < 2$, $h/b = 2$ and $h/b > 2$, for hot rolled sections and for the steel grade S235 and S460. The figures show that the Modified Special Case gives some unsafe results. On the other hand the General Case/f shows a good agreement with the numerical results and is always on the safe side. The results also show that the General Case is generally over-conservative for non-uniform bending moment diagrams.

### 2.8 Taras on lateral torsional buckling rules

As a result of research [3] and [4] about the current lateral torsional buckling curves, Taras developed new lateral torsional buckling rules [5]. Taras carried out calculations with a FE-model and obtained numerical lateral torsional buckling curves. His FE-model had residual stresses divided over the cross-section as is shown in Figure 2.4. To model the out-of-straightness of the beam Taras used the first elastic critical lateral torsional buckling shape with an amplitude of $L/1000$. He also made analytical lateral torsional buckling curves, by deriving these along the lines of the specific Ayrton-Perry formulation [11]. In the end he calibrated the analytical results to the numerical results via curve fitting and made a proposal for new lateral torsional buckling rules [6]. Part of his proposal is presented in this paragraph. The derivation of this proposal can be found in the Appendix B.
The new lateral torsional buckling curves derived by Taras can be represented as follows:

\[ \chi_{LT} = \frac{\varphi}{\Phi_{LT} + \sqrt{\Phi_{LT}^2 - \varphi \cdot \bar{\lambda}_{LT}^2}} \leq 1.0 \]  

(2.12)

Where:

\[ \Phi_{LT} = 0.5 \left[ 1 + \varphi \left( \alpha_s \cdot \alpha_{LT} \left( \bar{\lambda}_{e0} - 0.2 \right) + \bar{\lambda}_{LT}^2 \right) \right] \]  

(2.13)

\[ \bar{\lambda}_{LT} = \frac{W_y f_y}{M_{cr}} \]  

(2.14)

\( M_{cr} \) is the elastic critical moment for lateral torsional buckling

\[ W_y = W_{pl,y} \] for Class 1 or 2 cross-sections

The values of \( \alpha_{LT}, \alpha_s, \varphi \) and \( \bar{\lambda}_{e0} \) should be obtained from the Tables 2.6 and Table 2.7

<table>
<thead>
<tr>
<th>Table 2.6 Buckling curve coefficients for lateral torsional buckling</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Rolled I-sections</strong></td>
</tr>
<tr>
<td>-----------------</td>
</tr>
<tr>
<td>Rolled I-sections</td>
</tr>
<tr>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Table 2.7 Factor ( \varphi )</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>LOAD CASE</strong></td>
</tr>
<tr>
<td>----------------</td>
</tr>
<tr>
<td>M = [ ] ( \psi \cdot M )</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
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<tr>
<td></td>
</tr>
</tbody>
</table>

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2.9 Eurocode on member imperfections

In §2.3.1 is explained how to use the buckling curves to check lateral torsional buckling. When it is not possible to use these current design rules on lateral torsional buckling or the structural designer wants do a second order lateral torsional buckling analysis himself, Eurocode 3 gives conditions for the imperfection that should be taken. As stated in Eurocode 3 [1] §5.3.4 Member imperfections:

(3) For a second order analysis taking account of lateral torsional buckling of a member in bending the imperfections may be adopted as \( ke_{0,d} \) where \( e_{0,d} \) is the equivalent initial bow imperfection of the weak axis of the profile considered. In general an additional torsional imperfection need not to be allowed for.

**NOTE** The National Annex may choose the value of \( k \). The value \( k = 0.5 \) is recommended.

\( e_{0,d} \) can be found in Table 2.8. Which buckling curve should be used can be found in Table 2.9.

| Table 2.8 Design values of initial local bow imperfection \( e_{0}/L \) |
|-----------------------------------|-----------------|-----------------|
| Buckling curve acc. to Table 2.9 | Elastic analysis | Plastic analysis |
|                                  | \( e_{0}/L \)   | \( e_{0}/L \)   |
| \( a_0 \)                        | 1/350           | 1/300           |
| \( a \)                          | 1/300           | 1/250           |
| \( b \)                          | 1/250           | 1/200           |
| \( c \)                          | 1/200           | 1/150           |
| \( d \)                          | 1/150           | 1/100           |

<table>
<thead>
<tr>
<th>Table 2.9 Recommended values for flexural buckling curves</th>
</tr>
</thead>
<tbody>
<tr>
<td>Section</td>
</tr>
<tr>
<td>----------</td>
</tr>
<tr>
<td>hot-rolled sections</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
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<tr>
<td></td>
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<td></td>
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<tr>
<td></td>
</tr>
<tr>
<td>( h/b )</td>
</tr>
<tr>
<td>( &gt; 1,2 )</td>
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</tbody>
</table>
As stated before in §1.2, the Dutch National Annex of Eurocode 3 gives an alternative for \( e_{0,d} \). It was stated that the formula (1.1) has to be used. The derivation of the formula (1.1) can be found in appendix A.

\[
e_{0,d} = \alpha (\tilde{\lambda} - 0,2) \frac{M_{Rk}}{N_{Rk}}
\]

(1.1)

Where:

- \( \alpha \) Imperfection factor for relevant buckling curve
- \( \tilde{\lambda} \) Relative slenderness
- \( M_{Rk} \) Characteristic moment resistance of the critical cross-section
- \( N_{Rk} \) Characteristic resistance to normal force of the critical cross-section

<table>
<thead>
<tr>
<th>Table 2.10 Recommended values for imperfection factors for buckling curves</th>
</tr>
</thead>
<tbody>
<tr>
<td>Buckling curve</td>
</tr>
<tr>
<td>Imperfection factor</td>
</tr>
</tbody>
</table>

The relevant imperfection factor can be found in Table 2.10. Which imperfection factor should be chosen can be found in Table 2.9.

### 2.10 Discussion on member imperfections rule EC3

The current rule on member imperfection in Eurocode 3 is explained in §2.9. In this paragraph it is discussed where the rule on member imperfections originates from. Furthermore it is discussed what the vision of Lindner and Kindmann, two German experts on lateral torsional buckling, is on the member imperfections rule.

**2.10.1 Background information on member imperfections**

As stated before there is very little information on Eurocode 3 paragraph 5.3.4 Member imperfections. Before the Eurocode existed, the rule was already found in the DIN 18800 [7]. With the emerging of the Eurocode the rule was adopted from DIN 18800. DIN 18800 adopted this rule from the work of Friemann [12]. Friemann carried out some hand calculations and compared these with the lateral torsional buckling rule from DIN 18800, which differs from the General method used in the Eurocode described in §2.3.1. Formula (2.15) describes the lateral torsional buckling rule that can be found in DIN 18800.

\[
\chi_{LT} = \left( \frac{1}{1 + \chi_{LT}^{2\pi}} \right)^{\frac{1}{n}}
\]

(2.15)

\( n \) should be taken as 2,5 as is shown in Table 2.11

<table>
<thead>
<tr>
<th>Table 2.11 Coefficient ( n ) for determination of ( \chi_{LT} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Section</td>
</tr>
<tr>
<td>Rolled</td>
</tr>
<tr>
<td>Welded</td>
</tr>
</tbody>
</table>
In Figure 2.21 the difference is shown between the reduction factors calculated with DIN 18800 or with the General Method of Eurocode 3 [1].

![Figure 2.21 Comparing reduction factors EC3 and DIN](image)

As can be seen in Figure 2.21, the $\chi_{LT}$ factor calculated with formula (2.16) from DIN 18800 reduces the bending moment less than the $\chi_{LT}$ factor calculated with the General Method from Eurocode 3. The difference between the two methods is large. Nowadays it is known that the General Method from Eurocode 3 is the method to use. In §2.6.2 this was shown by Bruins. This means that the lateral torsional buckling curve suggested by DIN 18800 is less conservative. So the calculations done by Friemann were actually compared with an unconservative curve. Also the fact that he had to do all his calculations by hand gives enough reason to seriously doubt the rule on member imperfection. In addition, the fact that it were hand calculations also explains why it is stated that the torsional imperfections could be neglected; additional rotational imperfections would make his calculation too complex to carry out.

### 2.10.2 Lindner and Kindmann on member imperfections rules in EC3

Table 2.9, states that for flexural buckling cross-sections with $h/b \leq 1.2$ are more unfavorable than those with $h/b > 1.2$. For lateral torsional buckling this is reversed, according to Table 2.2, cross-sections with $h/b \leq 2$ are more favorable than those with $h/b > 2$. When structural designers have to make a second order analysis taking account of lateral torsional buckling, the Eurocode states in the paragraph on member imperfection (see §2.9), to use Table 2.8, to determine $\varepsilon_0/L$. According to Table 2.8, Table 2.9 has to be used to determine the associated buckling curve, this applies to both flexural buckling and lateral torsional buckling. The reference to table 2.8 which is done in §2.9 can therefore be seen as incorrect from a principle point of view. As was shown in the previous paragraph the Dutch National Annex gives an alternative for this table, but as stated before this is also questionable for lateral torsional buckling.
Investigations by Lindner and by Kindmann show that new values for the imperfections for lateral torsional buckling have to be chosen depending on $h/b \leq 2$ and $h/b > 2$. Both only used a line load at the upper flange as a load case. Kindmann [13] [14] made ultimate load calculations, using plastic zones theory, residual stresses and an initial bow imperfection over the weak axis of the profile with an amplitude of L/1000. Kindmann did this calculation for different rolled sections HEA, HEB, HEM, HEAA, IPE in a range of $h = 200$ to 1000 mm and relative slendernesses between 0.6 and 2.0. These calculations were carried out with a finite element method computer program. He compared these results with the results of the equivalent imperfection method, which Kindmann obtained using the partial internal forces method to calculate this. Within this calculation only an initial bow imperfection over the weak axis of the profile is added. Kindmann has investigated what the magnitude of the amplitude of the initial bow imperfection should be to get similar results as he arrived at using the plastic zone theory.

Lindner used a similar approach as Kindmann. He used the ultimate load results from Kindmann and made own calculations using the equivalent imperfection method taking into account bending stresses $\sigma_{my}$, $\sigma_{gz}$ and warping stresses $\sigma_{mv}$. The results were not as bad as those from Kindmann but where in the same range. Allowing some overestimation a table was proposed which was accepted in the German National Annex to EN 1993-1-1, see below Table 2.12. [15] So in the German code this table replaces Table 2.8 and Table 2.9, but the italic part shown in paragraph 2.6 remains.

| Table 2.12 Design values of initial local bow imperfection $e_0/L$ in the German NA |
|---------------------------------|-----------------|-----------------|-----------------|
| Cross-section                   | Dimensions      | Elastic analysis | Plastic analysis |
| 1 Rolled I-Section              | $h/b \leq 2.0$  | 1/500            | 1/400           |
| 2                               | $h/b > 2.0$     | 1/400            | 1/350           |
| 3 Welded I-Section              | $h/b \leq 2.0$  | 1/400            | 1/350           |
| 4                               | $h/b > 2.0$     | 1/300            | 1/200           |

For the range $0.7 \leq \bar{\lambda}_{LT} \leq 1.3$ the values should be doubled.

The values in this table are based on different kind of profiles. The table only makes a distinction between two categories, namely larger or smaller than $h/b = 2.0$. So all the different profiles are divided only in two categories, which means that the table cannot be that precise on every section. That was also the approach both Kindmann and Lindner used when they started their research in response to the current table in Eurocode 3, which is Table 2.8 in this report. Both Kindmann and Lindner state at the end of their paper that more research is necessary in this field but that it is clear that the table form Eurocode 3 is not safe to use.

2.10.3 Discussion on research by Kindmann and Lindner

At the bottom of Table 2.12 it is stated that for the range $0.7 \leq \bar{\lambda}_{LT} \leq 1.3$ the values in the table should be doubled. The range $0.7 \leq \bar{\lambda}_{LT} \leq 1.3$ is the range that is most susceptible to lateral torsional buckling. This actually means that Kindmann and Lindner state that a larger imperfection should be used in a range that is more susceptible to lateral torsional buckling. This is a questionable statement because if a beam is susceptible to lateral torsional buckling, why would it need an even larger imperfection? It would already by nature be more susceptible to lateral torsional buckling.
2.11 Fillet radius and compensation elements

In §2.6.1 about the master thesis of Bruins, there is mentioning of compensation elements which replace the fillet radius. The fillet radius is missing, because the I-shaped profile is modeled with shell elements. Taras [5] compared the $\chi_{LT}$-values of an IPE 240 profile with the fillet radius and an IPE240 profile without the fillet radius. The result of this comparison is shown in Figure 2.22.

![Figure 2.22 Influence of the fillet radius](image)

As can be seen in the figure the difference between the two curves is very small for the IPE240. There is a maximum difference between the curves of 2.5%. The maximum difference lays around a relative slenderness of $\bar{\lambda}_{LT} = 1.0$.

The beam with a length of 3500 mm lays around that point. The cross-sectional plastic moment capacity $M_{pl}$ lays around 6% higher for a beam with the fillet radius ($r=15$) than a beam without the fillet radius ($r=0$). The elastic critical moment of such a beam lays around 12% higher for a beam with fillet radius than a beam without fillet radius. So the omitting of the fillet radius has influence on the relative slenderness $\bar{\lambda}_{LT}$. For this case the relative slenderness $\bar{\lambda}_{LT}$ of the beam with the fillet radius is 3% lower than the relative slenderness $\bar{\lambda}_{LT}$ without the fillet radius.

The lateral torsional buckling strength $M_R$ of an IPE240 beam of 3500 mm is approximately 10% higher with the fillet radius than the beam without the fillet radius. This difference is not the same for the cross-sectional plastic moment capacity $M_{pl}$. So omitting the fillet radius also influences the reduction factor $\chi_{LT}$. For this case the reduction factor $\chi_{LT}$ of the beam with the fillet radius is around 5% higher than the reduction factor $\chi_{LT}$ without the fillet radius.

So omitting the fillet radius influences the relative slenderness $\bar{\lambda}_{LT}$ and the reduction factor $\chi_{LT}$. In this research lateral torsional buckling curves are compared with other lateral torsional buckling curves. So for this research not beams with the same lengths are compared, but beams with the same relative slendernesses $\bar{\lambda}_{LT}$ are compared. If that is done the maximum difference between two reduction factors $\chi_{LT}$ is 2.5%.
The section that is used for the research of this report was shown in Figure 1.2c and has the overlap between the web ends and the flanges. So this section actually gives higher $\chi_{LT}$-values than the section without the filet radius, but it still gives smaller $\chi_{LT}$-values than the section with the filet radius. So if the section used in this research would be implemented in Figure 2.22, the line would be between $r=0$ and $r=15$.

Although this paragraph only treats the omitting of the fillet radius for an IPE240 beam, it definitely shows that for this research it makes sense to omit the fillet radius. Since including the fillet radius in the model means that compensation elements need to be designed. The dimensioning of the compensation elements consist of calculations and several iterations. This is a time consuming job. It is not worth doing this procedure to reduce the small deviations between the lateral torsional buckling curves.
3. Finite element model

3.1 Introduction

This chapter gives information about the finite element model and the idea behind the finite element model. The model should represent a steel I-shaped beam and is used for calculations with respect to lateral torsional buckling. The steel I-shaped beams are modeled using shell elements. Because shell elements are used, the fillet radius is not taken into account for, as was already stated in §1.4 and is shown in Figure 1.2. Though, there are options to compensate the missing of the fillet radius. In this research however this is not necessary. In §2.11 is described why for this research it is allowed to omit the fillet radius. So the model consist of three strips, two replacing the flanges and one strip replacing the web.

Because of symmetry reasons only half of the beam needs to be modeled. The failure modes due to the three load cases, extensively described in §3.5, are all symmetric over the cross-section. Also the imperfection modes which are described in §3.6 are symmetric over the cross-section. The geometry of the model is shown in Figure 3.1. The right half of the model is faded in the figure, because that is not part of the actual FE-model. So the actual FE-model consist of the left half of the geometry. The finite element program used is the software package Abaqus 6.12 [9]

![Figure 3.1](image1.png)

Figure 3.1 The geometry of the model

3.2 Used elements and mesh

As was stated in the introduction the model consists of shell elements. The elements used are called S8R-elements in Abaqus. It means the shell elements are quadrilateral eight node elements with six degrees of freedom (DOFs) at each node (translation and rotation) and four integration points as is showed in Figure 3.2. These shell elements are based on the Mindlin shell theory, which means they incorporate bending and shear deformation. The elements provide for arbitrarily large rotations but only small strains. There are five integrations points along the thickness of the shell element.
Eight node elements are more efficient than four node elements. Since eight node elements are able to accurately describe a bending moment, because the displacement function is quadratic. This is in contrast with the four node elements, where the displacement function is linear.

![Figure 3.2 8-node reduced integration element](image)

Irregular meshes of S8R elements converge very poorly because of severe transverse shear locking; therefore, this element is recommended for use in regular mesh geometries for thick shell applications. [9]

For determining the correct mesh refinement a mesh study is performed. The study is performed on a beam with an HEA100 section and a length of 2000mm. Four different mesh refinements are compared. The elements used in the mesh all have the same length-to-width ratio of 4:1. The coarsest possible mesh is shown in Figure 3.3a, where only 30 elements are used. The finest mesh contains 1920 elements and is shown in Figure 3.3d.

![Figure 3.3 Four different mesh refinements](image)

All four models contain the same material properties, boundary conditions and system imperfection, which are discussed in detail in the remainder of this chapter. The load case is a constant bending moment (load case 1). Using the LBA and the GMNIA, which are being treated in chapter 4, the critical elastic moment and the failure load of each model are determined. The results of the LBA are compared which each other in Figure 3.4a and also the analytical value is shown, calculated with formula (2.9) presented in §2.3.4. The results from the GMNIA are compared with each other in Figure 3.4b. The same mesh convergence study is also done for some other type of elements, other than the S8R element. The results of it are shown Appendix C.
The figure shows that the results of the three models shown in Figure 3.3b-d are very close to each other. So there could be continued with the 120 elements model, but to make sure a reliable mesh refinement is used the model that has a mesh refinement that is one step finer is chosen. So the model that consist of 480 elements is used. It is not necessary to use the model that consists of 1920 elements, this will only increase the computational time.

3.3 Material properties

For carrying out FE calculations with respect to the ultimate strength theory [10] describes the stress-strain law that should be used, as is showed in §2.4. It is generally assumed to represent the actual behavior of structural steel in a suitable way. However, in [8] is showed that a bilinear stress-strain law without strain-hardening is effective for this research. This is further elaborated in §2.5.

The bilinear stress-strain diagram is shown in Figure 3.5. For the elastic properties the Young’s modulus of elasticity has been set to 2,1 \cdot 10^5 \text{ N/mm}^2. The Poisson’s ratio has been set to 0,3. As for the plastic properties a yield strength of 235 N/mm$^2$ is used.

In Appendix J.3 one shell element is tested to verify whether the correct stress-strain law has been entered in the FE-model.
3.4 Boundary conditions

At the end of the beam “end fork conditions” are introduced. This condition means there are in-plain, out-of-plain and torsional restraints added. Also distortion of the cross-section is prevented. Only warping is not constrained by the boundary conditions. Distortion of the cross-section is prevented at the beam end by the use of kinematic coupling. With kinematic coupling it is possible to enter these restrictions to the beam edges. In Appendix D.1 is shown what restrictions are allocated. In Figure 3.6 the boundary conditions are shown.

![Figure 3.6 The conditions at the end of the beam](image)

Instead of using kinematic coupling it is also possible to use stiff beam elements to prevent distortion of the cross-section as is showed in [3]. More elaboration on these two ways is given in Appendix D.

Since only half of an I-shaped beam is modeled, boundary conditions are needed at the symmetry line. For the nodes at the symmetry line $U_x$, $UR_y$ and $UR_z$ are fixed.

3.5 Load cases

As was stated in chapter 1 at the scope, there are three load cases that will be treated. These three load cases are shown in Figure 3.7.

![Figure 3.7 The three load cases](image)

Load case 1 has in-plane bending moments at the beam ends, which means there is a constant bending moment over the beam. For load case 2 and load case 3 the load case is put on the top flange of the beam. Load case 2 is a point load situated on the exact middle of the beam. Load case 3
is an equally distributed load over the entire beam. When geometrical imperfections are added over the beam length, load cases 2 and 3 become eccentric load cases. Which means the beam will be more susceptible to lateral torsional buckling.

3.5.1 Load case 1

The in-plane bending moment is applied at the middle of the edge of the beam as is shown in Figure 3.8. The moment acts around the y-direction, to create the in-plane bending moment. Although the moment is put directly on the beam there will not be a peak load. The applied moment will be spread out over the edge of the beam, due to the kinematic coupling that is applied at the beam edge.

![Figure 3.8 In-plane bending moment on FE-model](image)

3.5.2 Load case 2

The point load is added on the middle of the beam on the top flange. As can be seen in Figure 3.9 there is a stiffener added below the point load. The stiffener is added to prevent distortion of the cross-section. This is necessary to prevent local failure from occurring, especially for beams with a very low relative slenderness.

![Figure 3.9 Point load on FE-model](image)
3.5.3 Load case 3

The line load is put at the middle of the top flange of the FE-model as is shown in Figure 3.10. The line load consists of point loads that are added on the nodes. So the size of the line load depends on the size of the shell elements. The size of the line load can be calculated with formula 3.1.

\[ q_{\text{line load}} = \frac{2F}{\text{shell length}} \]  

(3.1)

![Figure 3.10 Line load on FE-model](image)

3.6 System imperfections

As is stated in §1.2 according to the Eurocode the imperfection shape should be taken as the initial bow imperfection with respect to the weak axis of the profile. The shape of this imperfection can be taken from the flexural buckling shape over the weak axis of the profile. So the FE-model is loaded in compression and by performing an LBA the flexural buckling shape over the weak axis is obtained. A cross-section over the middle of the beam length, of this buckling mode with the original shape is shown on Figure 3.11a.

This is not the only imperfection shape that will be used. Also an imperfection shape with a lateral and rotational imperfection is used. This imperfection shape is based on the lateral torsional buckling shape of the profile. By performing an LBA for the FE-model that is loaded with the investigated load case, the lateral torsional buckling shape is obtained. A cross-section over the middle of the beam length of this buckling mode with the original shape is shown on Figure 3.11b.

![Figure 3.11 The two types of imperfection shapes](image)
Formula (1.1) is based on the imperfection shape showed in Figure 3.11a, so it make sense to also use this as the imperfection shape, when using this formula. This reasoning is used to get to sub goal 1.

However, formula (1.1) is derived for flexural buckling and is now used for lateral torsional buckling calculations. The imperfection shape showed in Figure 3.11a is based on the flexural buckling mode. Because formula (1.1) in this case is used for lateral torsional buckling calculation, it makes sense to use an imperfection shape that is also based on the lateral torsional buckling mode, showed in Figure 3.9b. This is the reasoning behind sub goal 2.

Formula (1.2) is derived especially for lateral torsional buckling and is also based on the imperfection shape based on the lateral torsional buckling mode. This is the logic behind sub goal 3.

In this research these are the only two imperfections shapes treated. It could however be an interesting study to also look at other imperfection shapes. Though, keeping the goal of this research in mind it does not make sense to do that within this research.
4. Procedure of typical numerical simulations

4.1 Introduction

Within this chapter the general operation of the numerical simulations done within this research is explained. As stated earlier in paragraph 1.5 two types of numerical simulations will be performed with the FEM software Abaqus:

- LBA; Linear Buckling Analysis, which is used to determine the elastic critical moment and the eigenmodes for perfect elastic lateral torsional buckling.
- GMNIA; Geometrically and Materially Non-linear Analysis with Imperfections, which is used to obtain realistic numerical values for lateral torsional buckling.

4.2 Linear buckling analysis

As stated above an LBA is used to determine eigenmodes. The flexural buckling mode for the first sub goal and the lateral torsional buckling mode to investigate the second and third sub goal of this research. In this research only the first eigenmodes need to be used, as was explained in §3.6. This mode is used as the imperfection shape for the GMNIA calculation.

To carry out a linear buckling analysis, Abaqus gives two options for the buckle step. There needs to be chosen between Lanczos or Subspace eigensolver. For this research Subspace eigensolver is chosen because only one eigenmode needs to be obtained and because the FE-model contains kinematic coupling constraints [9].

The first lateral torsional buckling mode of an IPE240 beam with a length of 3600 mm subjected to load case 1 is shown in Figure 4.1.

![Figure 4.1](image)

With a buckling mode comes an elastic critical moment ($M_{cr}$) for elastic lateral torsional buckling. The value of this elastic critical moment can be compared with a value obtained from formula (2.9) from the Dutch National Annex to the Eurocode, stated in §2.3.4. Table 4.1 shows a comparison between the elastic critical moments from the IPE240 model from Abaqus loaded with a constant bending
moment and the corresponding elastic critical moments from formula (2.9). This formula is derived and therefore the output of the formula can be seen as analytical.

Table 4.1 Comparing the \( M_{cr} \)-values of an IPE240 beam

<table>
<thead>
<tr>
<th>Length (mm)</th>
<th>Abaqus (kNm)</th>
<th>Analytical (kNm)</th>
<th>Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>1800</td>
<td>233.86</td>
<td>237.68</td>
<td>-1.63%</td>
</tr>
<tr>
<td>2400</td>
<td>144.29</td>
<td>145.31</td>
<td>-0.71%</td>
</tr>
<tr>
<td>3600</td>
<td>77.43</td>
<td>77.46</td>
<td>-0.03%</td>
</tr>
<tr>
<td>7200</td>
<td>31.57</td>
<td>31.50</td>
<td>0.22%</td>
</tr>
<tr>
<td>10800</td>
<td>20.01</td>
<td>19.98</td>
<td>0.16%</td>
</tr>
<tr>
<td>14400</td>
<td>14.72</td>
<td>14.71</td>
<td>0.094%</td>
</tr>
</tbody>
</table>

In this case the beam is loaded with a constant bending moment. The values for \( M_{cr} \) from the Eurocode deviate very little from those obtained with the FE-model. As can be seen in the table the beams with the shortest span give the largest deviation, although the deviation is still less than two percent. This can be explained when a closer look is taken to the lateral torsional buckling shape of a shorter a beam. In Figure 4.2 half of the lateral torsional buckled shape is shown. The one the left is a relative short beam and the one on the right is a relative long beam.

![Figure 4.2](image)

As can be seen the web of the shorter beam is deformed. Instead of a straight web, the web has an s-curved shaped. The larger beam still has a straight web. When one looks at the formula in the Eurocode for determining the elastic critical moment for lateral torsional buckling (2.9), there is a factor \( k_{red} \) in the formula. This is a reduction factor depending on the deformability of the cross-section. The values in Table 3.1 are calculated with formula (2.9) with \( k_{red} = 1 \) which is recommended for standard rolled and welded sections. This means it is assumed that the cross-section of the beams will not deform. This explains why in table 3.1 the first two values deviate the most. However, the difference in general is reasonably small.

The table clearly shows that the FE-model acts as it should, when it comes to linear buckling. Some extra validity tests on the linear elastic behavior of the FE-model are showed in Appendix J. So it can be concluded that the FE-model works well linear elastic.
4.3 Geometrically and Materially Non-linear Analysis with Imperfections

A GMNIA is performed to describe the behavior of a beam due to a certain loading. A GMNIA consist of geometrical and material nonlinearities. Geometric nonlinearities takes into account the change in geometry as the structure deforms (equilibrium is found in the deformed shape). Materially nonlinearity describes the nonlinear material behavior which has been explained in §3.3. Abaqus can visualize how the beam deforms due to loads. It is also possible to obtain a load-displacement diagram from the calculations. From this diagram the failure load can be determined, which is used for the research.

The system imperfections used in the GMNIA calculation are obtained from the LBA calculation. The amplitude of the imperfection is calculated from either formula (1.1) or formula (1.2). The amplitude of the imperfection is showed in Figure 3.11, this means the amplitude is the horizontal displacement of the top flange.

Within a GMNIA the results are calculated using an iteration method. For this research the modified Riks method is used in Abaqus. The Riks method is recommended for unstable collapse and post buckling analysis, because the Riks method is able to calculate a snapback in force and/or displacement. The method includes nonlinear effects of large displacements and is often used to follow the eigenvalue buckling analysis.

The Riks method uses the load magnitude as an additional unknown, it solves simultaneously for loads and displacements. Therefore another quantity must be used to measure the progress of the solution. Abaqus uses the “arc-length” $l$, along the static equilibrium path in load-displacement space. This approach provides solutions regardless of whether the response is stable or unstable. [9]

Within Abaqus the option is given to enter the size of the increments. Figure 4.3 shows what the difference is between small increments and large increments. In both cases a beam with an IPE240 section with the length of 1800mm is modeled. Both have been given an imperfection based on their lateral torsional buckling mode.

![Figure 4.3 Load-displacement diagrams IPE240 length 1800](Image)

Figure 4.3b shows that when small increments are used the behavior of the beam due to the loading is smoothly displayed in the load-displacement diagram. When too large increments are entered the behavior does not come out as smooth and is therefore less realistic. However, the difference
between the two failure loads is only 0.02%, which is negligible. Even though the failure load is the main value that should be obtained from the GMNIA calculations, it does not mean that the large increments should be used. Because when background on a failure load is needed, it could be helpful to investigate the behavior of the beam. So the increments should be chosen rather on the smaller side. The determining of the size of the increments is an experience based process.

Since Figure 4.3b gives a realistic behavior of the beam it is interesting to look further into this diagram and connect these with some images that show the stresses in the beam. These images also show for which elements the yield stress is reached. This is made visible with the color black. This is shown in Figure 4.4. Besides the stresses, also the displacement is visible. The displacement is enlarged twenty times.

![Graph and images showing behavior and stress distribution of IPE240 beam](image)

**Figure 4.4** Behavior of an IPE240 beam with a length of 1800mm

The picture on the bottom left of Figure 4.4 shows the deformation and the stresses when the beam still behaves fully elastic. Mainly in-plane bending occurs and little out-of-plain bending occurs. Both proceed linear.
The picture on the bottom right of Figure 4.4 shows the deformation and the stresses at the point where the beam starts to yield in the outermost fiber, indicated by the black elements. From this point the behavior becomes nonlinear, which can be seen in the graph in Figure 4.4. Also from this point the out-of-plain displacement starts to increase rapidly with respect to the in-plain displacement.

The picture on the top left of Figure 4.4 shows the deformation and the stresses at the point where the failure load is reached. At this point the in-plane displacement is still larger than the out-of-plain displacement.

The final picture on the top right of picture 4.4 shows the deformation and the stresses at a point after the failure load was reached. Both flanges are almost entirely yielding. At this point the out-of-plain displacement is larger than the in-plane displacement.
5. Method for obtaining results

5.1 Processing the results from the simulations

To compare the determined failure load from the FE-model with that from the General method from the Eurocode, the approach explained in §1.5 should be used. Which means $\tilde{\lambda}_{LT,FEM}$ and $\chi_{LT,FEM}$ should be determined. For determining the relative slenderness $\tilde{\lambda}_{LT,FEM}$:

$$\tilde{\lambda}_{LT,FEM} = \sqrt{\frac{M_{pl}}{M_{cr}}}$$  \hspace{1cm} (5.1)

Where:

$$M_{pl} = W_{pl,y} \cdot f_y$$  \hspace{1cm} (5.2)

$M_{cr}$ is the elastic critical moment for lateral torsional buckling which is calculated with a LBA simulation.

For determining the reduction factor $\chi_{LT,FEM}$ a GMNIA simulation needs to be performed. The beam will be given an imperfection shape based on either its flexural buckling mode or its lateral torsional buckling mode, determined by an LBA. The size of the imperfection is given by $k e_{0.4}$. From the results of this simulation the moment at which failure occurs, $M_R$, can be determined. $M_R$ can be taken from the load-displacement diagram shown in Figure 5.1 that is produced by performing a GMNIA. When $M_R$ is known, the reduction factor can be calculated with:

$$\chi_{LT,FEM} = \frac{M_R}{M_{pl}}$$  \hspace{1cm} (5.3)

![Load-displacement diagram](image)

**Figure 5.1 Load-displacement diagram**

Important to notice is that the maximum load obtained from the load-displacement diagram should always be less than the elastic critical lateral torsional buckling load obtained from the LBA. Since the failure load from an LBA is obtained from an FE-model that contains no imperfections and acts fully elastic.
5.2 Extensive illustration of the calculation

Within this research many different section and lengths are used. To prevent having to make a new model every single time another section or length is required, a Python script has been written with parametric input, this script can be found in Appendix E. In this script the input parameters are the dimensions of the cross-section, the length, the imperfection size and the mesh refinement.

In this paragraph the calculation of $\bar{\lambda}_{LT,FEM}$ and $\chi_{LT,FEM}$ are extensively explained by means of one example. The beam that is used for this example is an IPE240 section with a length of 3400 mm. The beam is loaded with load case 1, a constant bending moment. The imperfection shape that is used, is based on the lateral torsional buckling shape obtained from the LBA. So this example calculation is used for the second sub goal. Since for the first sub goal the imperfection shape should be based on the weak axis flexural buckling mode.

The dimensions of the cross-section of the IPE240 FE-model are showed in Appendix F. Also the calculated section properties of the FE-model are shown in Appendix F.

In the Python file the dimensions of the cross-section, the length and the mesh refinement can be filled in. After running the script in Abaqus, the LBA is performed. The result of the LBA is shown in Figure 5.2. The legend shows the values of the displacement in the Y-direction (U2).

![Figure 5.2 Buckling shape of IPE240 beam with a length of 3400 mm](image)

Just beneath the lateral torsional buckling shape the eigenvalue is shown, this value is also the elastic critical moment ($M_{cr}$). To calculate the $\bar{\lambda}_{LT,FEM}$ also the bending restistance moment $M_{pl}$ of the IPE240 section is necessary.

$$M_{pl} = W_{pl,y} \cdot f_y = 352.9 \cdot 10^3 \cdot 235 = 82.962 \cdot 10^6 \text{ Nmm} = 82.962 \text{ kNm}$$

So the relative slenderness of the FE-model then becomes:

$$\bar{\lambda}_{LT,FEM} = \frac{M_{pl}}{M_{cr}} = \frac{82.962}{84.099} = 0.99$$
Before the GMNIA is performed, first the size of the imperfection $e_0$ needs to be determined with formula (1.1).

$$e_0 = \alpha (\bar{\lambda} - 0.2) \frac{M_{Rk}}{N_{Rk}} = \alpha (\bar{\lambda}_z - 0.2) \frac{W_{pLz}}{A}$$

(1.1)

Where:

$$\bar{\lambda}_z = \frac{A* f_y}{\sqrt{N_{cr,z}}}$$

(5.4)

So calculating $e_0$ starts with determining the elastic critical buckle force $N_{cr}$:

$$N_{cr,x} = \frac{\pi^2 * E I_z}{l^2} = \frac{\pi^2 * 2.1 * 10^5 * 282.7 * 10^4}{3400^2} = 506.85 \cdot 10^3 \text{ N}$$

After this the slenderness $\bar{\lambda}_z$ can be calculated:

$$\bar{\lambda}_z = \frac{A * f_y}{\sqrt{N_{cr,x}}} = \frac{3779.24 * 235}{\sqrt{506.85 \cdot 10^3}} = 1.32$$

Now $e_0$ is calculated:

$$e_0 = \alpha (\bar{\lambda}_z - 0.2) \frac{W_{pLz}}{A} = 0.34 (1.32 - 0.2) \frac{72772}{3779} = 7.36 \text{ mm}$$

With $k = 0.5$ the imperfection size eventually becomes 3.68 mm and can be put in the Python file. Both the imperfection shape as well as the imperfection size is known. All the necessary input to run a GMNIA is obtained. From the GMNIA Figure 5.3 is obtained.
The maximum load is reached at 57.6 kNm. This is the failure moment $M_R$, which is used to calculate the reduction factor:

$$\chi_{LT,FEM} = \frac{M_R}{M_{pl}} = \frac{57.638}{82.962} = 0.695$$

$\chi_{LT,FEM}$ and $\bar{\lambda}_{LT,FEM}$ are determined, which means these values can be compared with the current lateral torsional buckling curves. This is shown in Figure 5.4.

**Figure 5.4** Comparison $\chi_{LT,FEM}$ with $\chi_{LT,EC3}$ for one beam length

Figure 5.4 shows two diagrams. The left diagram displays the reduction factor calculated with the FE-model $\chi_{LT,FEM}$ and its lateral torsional buckling curve, determined with the General method described in §2.3.1. The diagram on the right also shows the comparison between the reduction factor calculated with the FE-model $\chi_{LT,FEM}$ and the lateral torsional buckling curve. The diagonal black line shows a perfect match between the reduction factor calculated with the FE-model and the General Method. If $\chi_{LT,FEM}$ lays above this black line it is on the unsafe side. The two grey dashed lines show a 5% over- or underestimation. With the help of these kind of diagrams is shown whether $\chi_{LT,FEM}$ is on the safe side, which means that the rule on member imperfection with $ke_{0,d}$ being the amplitude of an imperfection that is based on the lateral torsional buckling mode, combined with formula (1.1) is in accordance with the lateral torsional buckling curves.

Figure 5.4 only shows the reduction factor for one case, namely the IPE240 section loaded by load case 1 with a length of 3400 mm, which equals a relative slenderness around 1.0. By calculating $\chi_{LT,FEM}$ for different relative slendernesses for the IPE240 section, a clear image occurs whether the rule on member imperfection is safe to use for all relative slendernesses.

To determine the reduction factor for other relative slendernesses, other lengths should be chosen and the same procedure should be performed as described in this paragraph. Figure 5.5 shows the results of the $\chi_{LT,FEM}$ for eight different $\bar{\lambda}_{LT,FEM}$ in comparison with the lateral torsional buckling curves calculated with the General Method.
Conclusions could be drawn from Figure 5.5, but this is just the results of one type of profile with one kind of load case. As was stated in the scope there are three load cases that are treated within this research. To investigate the other two load cases the same procedure can be used as described before. The only difference is that the python script should be slightly modified in a way that the other load cases are put on the FE-model. All three python scripts are shown in Appendix D. Figure 5.6 shows the result of all three load cases.

The two diagrams clearly show the comparison between $\chi_{LT,FEM}$ and $\chi_{LT,EC3}$. Some conclusion could be drawn from the figure with respect to the rule on member imperfections, but that is extensively covered in chapter 6. Some load-displacement diagrams of different points from Figure 5.6 are shown in Appendix G.
5.3 Influence of the cross-sectional resistance around the weak axis

A remarkable aspect that can be noticed from Figure 5.6 is that for cases with a large relative slenderness the maximum load obtained from the GMNIA calculations is higher than the elastic critical moment obtained from the LBA, represented by the hyperbola \(1/\lambda_{LT}^2\) (\(1/\lambda_{LT}^2\) is equal to \(M_{cr}/M_{pl}\)). This can be explained by looking at the deformation-path of the middle section of a beam with a high relative slenderness. In Figure 5.7 this is shown for an IPE240 section with a length of 14400 mm, which has a relative slenderness of 2.37. The beam is loaded with a constant bending moment. At the right side of Figure 5.7 the corresponding load-displacement diagram is shown. On the y-axis of this diagram the load proportionality factor is put. An LPF-displacement diagram shows what happens to the displacement if the load is slowly increased. When the maximum load is reached the graph stops. The 1 on the vertical axis equals 100% of the load put on the beam, which is equal to its elastic critical moment. So the figure shows that that maximum load obtained from the GMNIA calculations is greater than the elastic critical moment.

![Diagram showing deformation path and load-displacement diagram](image)

**Figure 5.7** Deformation path of the mid span beam section of an IPE240 section with a length of 14400 mm

Figure 5.7 shows that for a beam with a high slenderness, large displacements and rotations occur before the failure load is reached. At a certain point when the rotation is large enough, the cross-sectional resistance against bending around the weak axis starts to have an influence.

For an IPE240 section with a relative slenderness larger than circa 2.9, the rotating of the middle section of the beam will eventually become 90°. The beam is still loaded over its strong axis at the beam ends, but at the middle the beam is now only being loaded over its weak axis. So in this case the beam will not fail due to lateral torsional buckling, but when its weak axis moment capacity is reached. The GMNIA buckling curve will get an asymptote with the value of \(M_{pl,x}/M_{pl,y}\), which in case for the IPE240 profile is \(\lambda_{LT} = 0.20\).
As shown in Figure 5.7, the IPE240 profile with a length of 14400 mm, fails when the rotation of the middle section of the beam is 60°. The beam is still loaded over its strong axis at the beam ends, which means the moment capacity for bending over the y-axis in the middle of the beam lays between $M_{pl,y}$ and $M_{pl,z}$. It actually lays around 50% of $M_{pl,y}$. However this does not mean that $\chi_{LT} = 0.50$, because in this case lateral torsional buckling is still of influence on the beam. Eventually the reduction factor becomes $\chi_{LT} = 0.25$.

The point where the influence of the weak axis moment capacity is really starting to matter is hard to determine. This is difficult because the influence of the weak axis moment capacity gradually increases when the relative slenderness is increased. The only conclusion that can be drawn is that when the GMNIA buckling curve reaches the asymptote at $\chi_{LT} = 0.20$ that bending over the weak axis is the reason of failure.

Figure 5.9 tries to show how the influence of the cross-sectional resistance around the weak axis of the beam is gradually increasing. In the figure the $\chi_{LT}$ values of load case 1 are shown. Also a line is shown that shows the ratio between the moment capacity over the y-axis of the rotated mid-section of the beam at time of failure ($M_{pl,\theta}$), shown in Figure 5.8a, and the moment capacity of the IPE240 profile over its strong axis ($M_{pl,y}$), shown in Figure 5.8b. As Figure 5.8 shows both plastic moments are calculated over the y-axis. The calculation of the $M_{pl,\theta}/M_{pl,y}$ ratio is explained in Appendix H.

The influence of the cross-sectional resistance around the weak axis is at its lowest when the $M_{pl,\theta}/M_{pl,y}$ ratio is equal to 1. The influence is at its highest when the $M_{pl,\theta}/M_{pl,y}$ is equal to $M_{pl,z}/M_{pl,y}$, which in case for the IPE240 profile is $\chi_{LT} = 0.20$. 

![Figure 5.8 Cross-sections for calculating $M_{pl,\theta}$ (a) and $M_{pl,y}$ (b)](image-url)
Figure 5.9 The influence of cross-sectional resistance around the weak axis for an IPE240 section

The figure shows that the influence of the cross-sectional strength of the weak axis really starts to have an influence around a relative slenderness of 1.4. So for an IPE240 profile it can be said that when the relative slenderness is higher than approximately 1.4 the beam does not fail just because of pure lateral torsional buckling anymore. A beneficial effect of weak axis cross-section resistance is also present.

The figure shows just for an IPE240 profile at what moment the cross-sectional resistance around the weak axis starts to be of influence. This means this figure can only be used for an IPE240 profile. Every profile has its own $\frac{M_{pl,z}}{M_{pl,y}}$ ratio curve. For all profiles this line will be an S-curve that starts at $\chi_{LT} = 1.0$ and ends with an asymptote at $\chi_{LT} = \frac{M_{pl,x}}{M_{pl,y}}$. If this ratio is relative high the beneficial influence of the cross-sectional resistance around the weak axis will start earlier than just was shown for an IPE240 profile. For example an HEA300 profile has a $\frac{M_{pl,z}}{M_{pl,y}}$ ratio of 0.48, which means the curve ends at $\chi_{LT} = 0.48$. Figure 5.10 shows the influence of the cross-sectional resistance around the weak axis for an HEA300 section.

Figure 5.10 The influence of the cross-sectional resistance around the weak axis for a HEA300 profile
As predicted both curves end at $\chi_{LT} = 0.48$. As can be seen in the figure the influence of the cross-sectional resistance around the weak axis starts already at a relative slenderness of approximately 1.0.

The cases where the cross-sectional resistance around the weak axis of a section starts to influence the reduction factor $\chi_{LT}$, are cases that should not be included within this research, because this research is about pure lateral torsional buckling. The lateral torsional buckling curves from Eurocode 3 described in §2.3 or the lateral torsional buckling curves from Taras described in §2.8 are based on pure lateral torsional buckling. The results from the GMNIA are compared with these curves, so the part where the cross-sectional resistance around the weak axis of a section starts to influence the reduction factor $\chi_{LT}$ should therefore not be included in this research.

Also this is only the case for extremely long beams, that are probably not of any practical relevance. In practice usually beam lengths are between 20 times and 30 times the height of the section. For example the beam of 14400 mm at beginning of the paragraph is 60 times the beam height of the IPE240 section, which means the beam is extremely long. Though, for this research it is useful to know when the influence starts. That point could be seen as the end point of the range of relative slendernesses that need to be studied.

To determine the end of the range it is possible to make a $M_{pl,0}/M_{pl,Y}$ ratio curve for every case that is investigated within this research. Since this is a time consuming job, a different criteria is proposed. This proposal is showed in Table 5.1.

<table>
<thead>
<tr>
<th>Limits</th>
<th>$\chi_{\text{min}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$h/b \leq 1.2$</td>
<td>$M_{pl,x} / M_{pl,Y} \leq 1.5$</td>
</tr>
<tr>
<td>$h/b &gt; 1.2$</td>
<td>$M_{pl,x} / M_{pl,Y} \leq 2$</td>
</tr>
</tbody>
</table>

The proposal for this criteria is not mathematically derived, but determined empirical accordingly to the results presented in chapter 8. The realization of Table 3.2 is further elaborated in Appendix I. Because Table 5.1 is empirical determined, the $\chi_{\text{min}}$ values are approximate values.

### 5.4 Comparison of the FE-model with literature

It is customary to compare numerical models with experiments and literature before the numerical research can be undertaken. This is done to validate the numerical model used. Several comparisons were made, only part of it will be shown in this paragraph. Some other tests are shown in Appendix J. In this paragraph results of the FE-model are compared with results from the FE-model of Bruins [3] and with the results of Taras [6].

Bruins compared his FE-model with results from an experimental research. This research was a master project by Swart & Sterrenburg [16] performed at Eindhoven University of Technology. They performed experiments on the influence of an uncoupled concrete slab on the load bearing capacity of a steel beam. Within this research two experiments have been performed; the first one without the concrete slab and the second one with a concrete slab. Bruins compared his FE-model with the first
experiment and found satisfying results. So by comparing the FE-model with the FE-model of Bruins, the FE-model is implicitly validated by experiments performed by Swart & Sterrenburg.

The thesis of Bruins shows the result of an IPE240 beam with three different lengths, being 3600 mm, 5400 mm and 7200 mm. Bruins used an imperfection shape based on the lateral torsional buckling shape, with an imperfection size of l/1000. Furthermore there were residual stresses added over the cross-section. How the residual stresses are divided is shown in Figure 2.3. The residual stresses have a maximum stress of one third of the yield stress, so that is one third of 235 N/mm². Bruins used three types of load cases in his thesis, shown in Figure 2.14, two of those load cases are being used for the comparison. These load cases are the point load in the middle of the beam and the line load over the entire beam.

In §2.6.1 it was mentioned that Bruins uses compensation elements. As stated before in paragraph §2.11 the FE-model for the research of this thesis has no compensation elements. So to make a good comparison, in this case the FE-model was adapted and the compensation elements that are described in the thesis of Bruins were added.

Figure 5.11 shows the comparison of the results from the FE-model with the results from the FE-model from Bruins.

![Curve a](#) Euler curve
![Point Load Bruins](#) Point Load Bruins
![Point Load FE-model](#) Point Load FE-model
![Line Load Bruins](#) Line Load Bruins
![Line Load FE-model](#)

\[ \tilde{\lambda}_{LT} = \frac{M_{pl}/M_{cr}}{\sqrt{\frac{M_{pl}/M_{cr}}{1}}} \]

<table>
<thead>
<tr>
<th>( \tilde{\lambda}_{LT} )</th>
<th>Curve a</th>
<th>Euler curve</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>1.0</td>
<td>0.8</td>
</tr>
<tr>
<td>0.5</td>
<td>0.6</td>
<td>0.5</td>
</tr>
<tr>
<td>1.0</td>
<td>0.4</td>
<td>0.3</td>
</tr>
<tr>
<td>1.5</td>
<td>0.2</td>
<td>0.1</td>
</tr>
<tr>
<td>2.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
</tbody>
</table>

Figure 5.11 Comparison FE-model with Bruins

As can be seen in Figure 5.11 there is very little difference between the two results. Which means the comparison gives satisfying results.

The same results from the FE-model can also be compared with results from the research of Taras. In Figure 5.12 the comparison between the results of the FE-model and the Taras lateral torsional buckling rules is shown. Taras lateral torsional buckling rules are shown in §2.8.
For the validation of Taras’ FE-model, he made a comparison between LBA results from his model with elastic buckling loads from Trahair [17]. He also made a comparison between GMNIA calculations from his FE-model with column buckling curves as they were developed and published by Beer & Schulz [18]. So in this instance the FE-model is also implicitly validated to the results from that literature.

### 5.5 Overview of the different parameters used in the research

This report can be divided in three researches as result of the three sub goals presented at the objectives in §1.3. The results of the three different researches are presented in chapter 6, 7 and 8. Table 5.1 gives an overview of different parameters per research.

<table>
<thead>
<tr>
<th>Table 5.2 Overview of the different parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>Load types</td>
</tr>
<tr>
<td>Cross-sections</td>
</tr>
<tr>
<td>Relative slendernesses</td>
</tr>
<tr>
<td>Imperfection size</td>
</tr>
<tr>
<td>Imperfection shape</td>
</tr>
<tr>
<td>Comparing results with</td>
</tr>
<tr>
<td>Objectives</td>
</tr>
</tbody>
</table>
6. Testing rule on member imperfection

6.1 Introduction

As stated in §1.3 the main goal of this research is to determine whether the Eurocode 3 rule on member imperfections provides a safe approximation for lateral torsional buckling. The main goal is subdivided in three sub goals. The investigation to reach the first sub goal is showed in this chapter.

The first sub goal investigates what is written in the current Eurocode in combination with the Dutch National Annex. For this sub goal the model was given an imperfection shape based on the weak axis flexural buckling mode, a buckling shape without torsion. The size of the amplitude is determined with formula (1.1) and the k-factor has a value of 0.5 which is recommended by the Eurocode. The relative slenderness $\tilde{\lambda}_{LT}$ is determined using the LBA. With the results contained from the GMNIA, the reduction factor $\chi_{LT}$ can be determined. The values are compared with the general method for lateral torsional buckling as stated in the Eurocode. The general method is also described in §2.3.1 of this thesis.

6.2 Rule on member imperfection versus EC3 buckling curves

As stated in the scope of this report, three load cases will be used in this model. These three load cases were shown in paragraph 3.5. In Figure 6.1 the results for an IPE240 section, subjected to the three load cases, are shown. Also curve a from the General method from Eurocode 3 is shown.

The left diagram displays the reduction factor calculated with the FE-model $\chi_{LT,FEM}$ with curve a from the general method. The diagram on the right shows the comparison between the reduction factor calculated with the FE-model $\chi_{LT,FEM}$ and the lateral torsional buckling curve from Eurocode 3 $\chi_{LT,EC3}$. The diagonal black line shows a perfect match between the reduction factor calculated with the FE-model and Eurocode 3. If $\chi_{LT,FEM}$ lays above this black line it is on the unsafe side. The two grey dashed lines show a 5% over- or underestimation.
As can be seen in Figure 6.1 all but one case lay above the corresponding buckling curve. This clearly shows that the rule on member imperfections is currently unsafe to use. Which means the rule should be adjusted. Of course this is just the result for one type of section. So before adjusting the rule on member imperfection can start, other profiles need to be looked into first. Figure 6.2 shows the results of an IPE600 section that is subjected to the three load cases in comparison with curve b.

The graphs show that the current rule on member imperfection is also not safe to use for an IPE600 section. The $\chi_{LT,FEM}$ deviates over 10% from curve b from the General method. So the results for the IPE600 section are even worse than the results for the IPE240 section.

Both cases treated so far were IPE sections. Figure 6.3 shows the result for an HEA300 section if the current rule on member imperfections is used.
Only few cases are shown in the graphs, because the influence of cross-sectional resistance around the weak axis starts around a relative slenderness of 1.0 for an HEA300 section. The phenomenon is thoroughly discussed in §5.3. Nevertheless, the graph on the right side clearly shows that the current rule on member imperfection is also not safe to use for an HEA300 section.

From the graphs shown in this paragraph the conclusion can be drawn that the rule on member imperfection in combination with formula (1.1) is unsafe to use.

6.3 Discussion on results

From the results showed in the previous paragraph several things can be noticed. Starting with the fact that the three load cases differ quite a lot at low slendernesses. Secondly, the order of the three load cases is the same for all profiles. In all cases load case 2 is the upper line, load case 3 is the middle line and finally load case 1, which is always the lowest line. This means that load case 2 for this research is the decisive load case. Which can be explained when a look is taken to the different bending moments of the three load cases shown in Figure 6.4.

![Figure 6.4 Bending moment diagrams of the three load cases](image)

By comparing the area of the bending moment diagrams of the three different load cases conclusion are drawn. The bending moment of load case 1 has the biggest area, so load case 1 is loaded more than the other two load cases. This explains why load case 1 is the lowest line in all the graphs. The bending moment diagram of load case 2 has the lowest area, which explains why load case 2 is highest line. This conclusion could also have been drawn when looking at Table 2.4 and Table 2.7. Both tables show that the reduction factor varies per load case.

Load cases 2 and 3 become eccentric load cases when geometrical imperfections are added over the beam length. Which means these cases will be more susceptible to lateral torsional buckling. However the imperfections are apparently not big enough that this becomes of major influence, especially for the low slendernesses.

6.4 Adjusting the k-value

As was stated in §1.3 on the objectives, when the rule on member imperfection in combination with formula (1.1) is unsafe to use, the k-value needs to be adjusted. Figure 6.5 shows what happens to the results of an IPE240 section when the k-value is increased to 1.1.

All cases lay around the black diagonal. So also some cases lay above the black line, the deviation is always very little. The only case that has a deviation over 5% is load case 2 with a relative slenderness around 0.6. For an IPE240 section, this relative slenderness is equal to a length of 1600 mm. Which means this case is probably not of any practical relevance, since dividing the beam length by the section height is only 6.67. Increasing the k-value even further to get such a point on the safe side of the line would mean all other cases are going to get a reduction factor that is way too conservative. So for an IPE240 section the k-value of 1.1 is good enough to combine with the rule on member imperfection and formula (1.1).
However this does not mean that increasing the k-factor to 1.1 for all other sections is going to solve sub goal 1. For example for an IPE600 section using a k-factor of 1.1 is not high enough. Figure 6.6 shows the result for an IPE600 section if the k-factor is increased to 1.6. Although the k-factor for an IPE600 section is increased more than for the IPE240 section it should be noted that in this case the load cases need to be adjusted to lateral torsional buckling curve b instead of lateral torsional buckling curve a.

By increasing the k-factor to 1.7 all cases now lay around the black diagonal black line, which is shown on the right graph in Figure 6.6. For the IPE600 section the cases with a low relative slenderness deviate the most. In this case load case 2 and 3 both deviate to the unsafe side around 5% of the Eurocode. There can be discussed whether these points are of any practical relevance, since both cases are very short beams. A relative slenderness of 0.8 equals a beam length of 3800 mm for load...
case 2. For load case 3 a relative slenderness of 0.8 equals a beam length of 3400 mm. This gives length-to-height ratios of respectively 6.33 and 5.67. So from Figure 6.6 can be concluded that increasing the k-factor to 1.7 for an IPE600 section is a good enough adjustment for the rule on member imperfection combined with formula (1.1).

So far two IPE sections are discussed. Figure 6.7 shows the results for an HEA300 section, where the k-factor is increased to 1.3.

In this instance, again load case 2 with the short relative slenderness is on the unsafe side as is shown in Figure 6.7 on the right graph. This point deviates around 5% of lateral torsional buckling curve a. All other cases are either on or below the black diagonal line. So by increasing the k-factor to 1.3 for HEA300 sections the rule on member imperfection combined with formula (1.1) is adjusted enough.

6.4.1 Discussion on adapting the k-factor

It is possible to get all the cases on the safe side by making the k-factor large enough. However the cases with very low slendernesses especially for load case 2 need very high k-factor to get that case on the safe side. The problem is that cases around a relative slenderness of 1.0 do not need such an increase on the k-factor. And the cases that lay around a relative slenderness of 1.0 are the most realistic cases. That is why sometimes with the adjusting of the k-factor there are cases left which deviate at the unsafe side around 5% from the general method form Eurocode 3. Also the paragraph shows that load case 2 is still the decisive load case.

6.4.2 Overview of several IPE and HEA sections

From the results in this chapter so far it is hard to determine what the new k-value should be. So some kind of overview of several IPE and HEA sections and their reduction factor $\chi_{LT, FEM}$ is needed, to get more insight on how to adjust the k-value. It is possible to make an overview of IPE and HEA sections which are all loaded with load case 2 and are given the same relative slenderness. Load case 2 is chosen, since this is the decisive load case. As for the relative slenderness 0.9 is chosen. Figures 6.5, 6.6 and 6.7 show, when the results are satisfying at a relative slenderness of 0.9 that for other
relative slendernesses the results are also satisfying. In the overview the vertical axis gives the reduction factor and the horizontal axis should represent the characteristics of the cross-sections. For example the height-to-width ratio of the section. Figure 6.8 shows the overview of several IPE and HEA sections.

The sections are from HEA100 to HEA1000 and from IPE80 to IPE600. There are some linearities that can be detected, but unfortunately also some non-linearities. Instead of using the height-to-width ratio of the section, it is also possible to try other characteristics. For example the $I_y/I_z$ ratio, this is shown in Figure 6.9.
Looking at the different linearities showed in Figure 6.9 there are 4 categories, based on the height-to-width ratio, that can be set up. These are also shown in the figure, being:

- $h/b \leq 1.0$
- $1.0 < h/b \leq 1.2$
- $1.2 < h/b \leq 2.0$
- $h/b > 2.0$

The figure also shows that the IPE and HEA section show different linearities, which means they should be separately treated. So different $k$-factors need to be developed for the different type of sections.

### 6.4.3 Developing $k$-value for IPE sections

For the IPE sections two linearities occur, which are separated by the height-to-width ratio of 2.0. Because the $X_{LT,FEM}$-values show a linear relation with the $I_y/I_z$-ratio of a section, it is expected that the $k$-values, which are needed to get the $X_{LT,FEM}$-values close to the lateral torsional buckling curves from Eurocode 3, also show a linear relation with the $I_y/I_z$-ratio. Figure 6.10 shows which $k$-values are needed to get the $X_{LT,FEM}$-values close to their associated curve. The numbers above the cases show which IPE-section it is about.

![Figure 6.10 Several IPE sections with a relative slenderness of 0.9 and their matching k-value](image)

With the $k$-values showed in Figure 6.10 it is possible to design two formulas for the $k$-values for IPE sections; one formula for sections with $h/b \leq 2.0$ and one formula for sections with $h/b > 2.0$. The design of the formulas is done with the help of Figure 6.11. The figure shows the relation between the $I_y/I_z$-ratio and the $k$-value.
Figure 6.11 shows there is a linear relation between $I_y$-$I_z$-ratios and the k-factors. So from this figure a proposal can be obtained that is applicable for IPE sections. For IPE sections formula (1.1) can be used in combination with the k-value obtained from Table 6.1:

Table 6.1: Recommended k-values for IPE sections

<table>
<thead>
<tr>
<th>Section</th>
<th>Limits</th>
<th>k-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rolled I-section</td>
<td>h/b ≤ 2</td>
<td>$k = -0.34\left(\frac{I_y}{I_z}\right) + 5.6$</td>
</tr>
<tr>
<td></td>
<td>h/b &gt; 2</td>
<td>$k = -0.017\left(\frac{I_y}{I_z}\right) + 2.15$</td>
</tr>
</tbody>
</table>

### 6.4.4 Developing k-value for HEA sections

Figure 6.9 shows that for the HEA sections four linearities occur, which are divided in four categories:

- h/b ≤ 1.0
- 1.0 < h/b ≤ 1.2
- 1.2 < h/b ≤ 2.0
- h/b > 2.0

At first sight the $\chi_{LT,FEM}$-values show a linear relation with the $I_y$-$I_z$-ratio of a section. It is expected that the k-values, which are needed to get the $\chi_{LT,FEM}$-values close to the lateral torsional buckling curves from Eurocode 3, also show a linear relation with the $I_y$-$I_z$-ratio, as also was showed for the IPE section in the previous paragraph.

For determining the k-values for HEA sections a similar approach is used as for the IPE sections. First the two categories 1.2 < h/b ≤ 2.0 and h/b > 2.0 will be treated. Figure 6.12 shows which k-values are needed to get the $\chi_{LT,FEM}$-values close to their associated curve. The numbers above the cases show which HEA-section it is about.
Several HEA sections with a relative slenderness of 0.9 and their matching k-value

With the k-values showed in Figure 6.12 it is possible to design two formulas for the k-values for HEA sections in a similar way as has been done for IPE sections.

One formula is designed for sections with $1.2 < h/b \leq 2.0$ and one formula for sections with $h/b > 2.0$. The design of the formulas is done with the help of Figure 6.13.
For determining the k-values for the HEA sections for the other two categories being \( h/b < 1.0 \) and \( 1.0 < h/b \leq 1.2 \) a similar approach is used. Figure 6.14 shows which k-values are needed to get the \( \chi_{LT,FEM} \)-values close to their associated curve. The numbers above the cases show which HEA-section it is about.

![Figure 6.14](image) Several HEA sections with a relative slenderness of 0.9 and their matching k-value

With the k-values showed in Figure 6.14 it is possible to design two formulas for the k-values for HEA sections in a similar way as has been done before. One formula should be designed for sections with \( h/b < 1.0 \) and one formula should be designed for sections with \( 1.0 < h/b \leq 1.2 \). The design of the formulas is done with the help of Figure 6.15.

![Figure 6.15](image) Designing formulas for k-values for HEA sections
For HEA sections with h/b < 1.0 the relation between $l_y/l_z$-ratios and the k-value is not as linear as for the other categories. This is why some k-values in this category will be a bit conservative. Nevertheless, the relation is linear enough and therefore a similar approach is used as for the other categories.

With the help of Figure 6.13 and 6.15 linear relation between $l_y/l_z$-ratios and the k-values have been found. So with these linear relations a proposal is obtained that is applicable for HEA sections. For HEA sections formula (1.1) can be used in combination with the k-value obtained from Table 6.2:

<table>
<thead>
<tr>
<th>Section</th>
<th>Limits</th>
<th>k-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rolled I-section</td>
<td>h/b ≤ 1.0</td>
<td>$k = -6.9*{(l_y/l_z)} + 20.4$</td>
</tr>
<tr>
<td></td>
<td>1.0 &lt; h/b ≤ 1.2</td>
<td>$k = -0.11*{(l_y/l_z)} + 1.65$</td>
</tr>
<tr>
<td></td>
<td>1.2 &lt; h/b ≤ 2.0</td>
<td>$k = -0.041*{(l_y/l_z)} + 1.7$</td>
</tr>
<tr>
<td></td>
<td>h/b &gt; 2.0</td>
<td>$k = -0.008*{(l_y/l_z)} + 2.2$</td>
</tr>
</tbody>
</table>

6.5 Conclusion

In this chapter the rule on member imperfection in combination with formula (1.1) is checked by comparing the results from the FE-model with the lateral torsional buckling curves from the Eurocode. The FE-model was given an equivalent initial bow imperfection around the weak axis of the section. The size of the imperfection was determined with formula (1.1) in combination with the recommended value of 0.5 for the k-value. In this chapter is shown that this is unsafe to do.

The recommended value of 0.5 for the k-value has proven to be far too low. Research shows that a way larger k-value is needed. The k-values that need to be used according to this research are given in Table 6.1 for IPE sections and in Table 6.2 for HEA sections. The correct linear formula for the k-value is chosen depending on the height-to-width ratio of the section and then the k-value can be calculated.
7. Testing rule on member imperfection with torsion

7.1 Introduction

This chapter describes an investigation on what is written in Eurocode 3 on member imperfection in combination with formula (1.1), but now the additional torsional imperfection is not neglected. This research is to reach the second sub goal. The model is given an imperfection shape based on its lateral torsional buckling mode. The size of the amplitude is determined with formula (1.1) and the k-factor has a value of 0.5 which is recommended by the Eurocode. The relative slenderness $\bar{\lambda}_{LT}$ is determined using the LBA. With the results contained from the GMNIA, the reduction factor $\chi_{LT}$ can be determined. These values are compared with the general method for lateral torsional buckling as stated in Eurocode 3, which is also described in §2.3.1 of this thesis.

7.2 Influence of additional torsional imperfection

As is stated in Eurocode 3 paragraph 5.3.4, the additional torsional imperfection does not need to be allowed for. In §2.5.4 of this thesis is showed by Boissonnade [8] that the additional torsional imperfection is of very little influence on the reduction factor $\chi_{LT}$. However it was only tested for one load case, being a constant bending moment over the beam, similar to load case 1 of this research. The three load cases are shown in §3.5. Figure 7.1 shows what the influence is of additional torsional imperfection for load case 2 and load case 3 for an IPE240 section.

![Figure 7.1](image)

The figure makes clear that for load case 2 and 3 neglecting the additional torsional imperfection influences the moment at which failure occurs. The influence of the additional torsional imperfection for load case 2 and load case 3 is large with respect to the influence it has on load case 1. The reason for this is that load case 2 and load case 3 are both loaded eccentric when the geometrical imperfection is present and load case 1 is not.
7.3 Rule on member imperfection including torsion versus EC3 buckling curves

In Figure 7.2 the results for an IPE240 section, subjected to the three load cases, are shown. Also curve a from the General method from Eurocode 3 is shown. To compare the result from the FE-model with results from the Eurocode the same type of figures are used as in the previous chapter. The diagram on the right shows the comparison between the reduction factor calculated with the FE-model $\chi_{LT,FEM}$ and the lateral torsional buckling curve from Eurocode 3 $\chi_{LT,EC3}$. The diagonal black line shows a perfect match between the reduction factor calculated with the FE-model and Eurocode 3. If $\chi_{LT,FEM}$ lays above this black line it is on the unsafe side. The two grey dashed lines show a 5% over- or underestimation.

Figure 7.2 shows, that almost all cases are on the unsafe side of the corresponding buckling curve. However when comparing these results with the results from Figure 6.1 from the previous chapter, the results in Figure 7.2 lay closer to buckling curve a. In this case most results lay between the black line and the dashed line. This means there is only a maximum deviation of 5%, which could be acceptable. If such a deviation is not acceptable, the k-value should be adjusted. However, not as much as was shown in the previous chapter.

Before the k-value is going to be adjusted, other profiles are going to be looked into. As is done in the previous chapter, first the IPE600 section and HEA300 section are presented and discussed. Figure 7.3 shows the results for an IPE600 section that is subjected to the three load cases in comparison with curve b.
Figure 7.3 shows that for the IPE600 section the rule on member imperfection with the additional torsional imperfection is also not safe to use for this section. The deviation for all cases from curve b is over 5%, which is not acceptable. Though, the results are better than the results shown in Figure 6.2.

Finally, the HEA300 section is discussed. Figure 7.4 shows the results for the HEA300 section that is subjected to the three load cases in comparison with curve a.

Because of the phenomena, discussed in §5.3, only few cases are shown in Figure 7.4. The few cases are all reasonably close to their corresponding buckling curve, however they are still on the unsafe side. From the three figures showed in this paragraph can be concluded that the rule on member imperfections from Eurocode 3 with the additional torsional imperfection in combination with formula (1.1) is unsafe to use and that the k-value needs to be adjusted.
7.4 Discussion on results

The results which have been shown in the previous paragraph can be compared with the results from §6.2. In the figures from the previous paragraph, the results from the different load cases lay closer to each other than in the results shown in §6.2. This is a result of additional torsional imperfection, as is shown in §7.2, since it has a big influence on load case 2 and load case 3.

For small slendernesses the order the results of the three load cases are in, is similar to the results presented in §6.2. Load case 2 starts as the upper line, load case 3 starts as the middle line and load case 1 starts as the bottom line. Therefore load case 2 is the decisive load case for small slendernesses. As the slenderness becomes larger the line that represents the results from load case 1 slowly becomes the upper line and therefore becomes the decisive load case. Again this can be explained by what is described in §7.2. Because the imperfection size is small for small slendernesses, the eccentricity does not play a major role yet. When the slenderness gets larger, the imperfection size grows and with that the eccentricity for load case 2 and load case 3 gets larger. This can be seen in the results.

7.5 Adjusting the k-value

As was concluded in §7.3 the k-value needs to be adjusted. From the figures which have been shown in this chapter and with the experience from the last chapter, it make sense to use a similar approach as has been demonstrated in §6.4.2 till §6.4.4.

7.5.1 Overview of several IPE and HEA sections

In §6.4.2 a graph is made that gave an overview of several IPE and HEA sections. This is shown in Figure 6.9. In this paragraph the same procedure is followed. So a graph is made were the \( l_y/l_z \) ratio of a section is put on the x-axis and the reduction factor calculated with the FE-model \( \chi_{LT,FEM} \) is put on the y-axis. Again all beams are loaded with load case 2 and have a relative slenderness of 0.9, for the same reasons as described in §6.4.2. Figure 7.5 gives the overview of several IPE and HEA sections.

![Figure 7.5 Overview of several IPE and HEA sections with relative slenderness of 0.9](image-url)
Given the different linearities shown in Figure 7.5 the same 4 categories as proposed earlier are used:

\[ h/b \leq 1.0 \]
\[ 1.0 < h/b \leq 1.2 \]
\[ 1.2 < h/b \leq 2.0 \]
\[ h/b > 2.0 \]

Again the figure shows a clear separation between the IPE and the HEA sections. So the IPE section and the HEA sections are separately treated.

### 7.5.2 Developing k-value for IPE sections

For the IPE sections two linearities occur, which are separated by the height-to-width ratio of 2.0. Of course different k-values are needed here then presented in §6.4.3, but the procedure used here is exactly the same. Figure 7.6 shows which k-values are needed to get the \( \chi_{LT,FEM} \)-values close to their associated curve.

![Figure 7.6](image)

*Figure 7.6* Several IPE sections with a relative slenderness of 0.9 and their matching k-value

From Figure 7.6 the k-values are used to design two formulas for the k-values for IPE-sections. One formula is designed for sections with \( h/b \leq 2.0 \) and one formula for sections with \( h/b > 2.0 \). The designing of the formulas is done with Figure 7.7.
Designing formulas for k-values for IPE sections

From this figure a proposal is obtained. For IPE sections formula (1.1) is used in combination with Table 7.1 to determine the amplitude of the imperfection and the shape of the imperfection is based on its lateral torsional buckling mode.

Table 7.1 Recommended k-values for IPE sections

<table>
<thead>
<tr>
<th>Section</th>
<th>Limits</th>
<th>k-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rolled I-section</td>
<td>h/b ≤ 2</td>
<td>( k = -0.13 \left( \frac{I_y}{I_z} \right) + 2.44 )</td>
</tr>
<tr>
<td></td>
<td>h/b &gt; 2</td>
<td>( k = -0.017 \left( \frac{I_y}{I_z} \right) + 1.44 )</td>
</tr>
</tbody>
</table>

7.5.3 Developing k-value for HEA sections

For developing the k-values for HEA sections exactly the same procedure is used as in §6.4.4. So first the two categories \( 1.2 < h/b \leq 2.0 \) and \( h/b > 2.0 \) will be treated. Figure 7.8 shows which k-values are needed to get the \( X_{LT,FEM} \)-values close to their associated curve.
With the k-values which are shown in Figure 7.8 it is possible to design two formulas for the k-values for HEA sections similar to the procedure for IPE sections. The design of the formulas is done with Figure 7.9.

The k-values for the HEA sections for the other two categories being h/b < 1.0 and 1.0 < h/b ≤ 1.2 are now determined. Figure 7.10 shows which k-values are needed to get the $\chi_{LT,FEM}$-values close to their associated curve.

With the k-values shown in Figure 7.10 two formulas for the k-values for HEA sections are designed. Figure 7.11 shows the design of the formulas.
Designing formulas for k-values for HEA sections

With the help of Figure 7.9 and 7.11, linear relations between $I_y/I_z$-ratios and the k-factors have been found. So with these linear relations a proposal is obtained for the k-value that is applicable for HEA sections. For HEA sections formula (1.1) can be used in combination with the k-value obtained from Table 7.2 to determine the amplitude of the imperfection if the imperfection shape is based on its lateral torsional buckling mode:

![Figure 7.11 Designing formulas for k-values for HEA sections](image)

$$k = -1.69 \left( \frac{I_y}{I_z} \right) + 5.39$$

$$k = -0.11 \left( \frac{I_y}{I_z} \right) + 1.05$$

<table>
<thead>
<tr>
<th>Section</th>
<th>Limits</th>
<th>k-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rolled I-section</td>
<td>$h/b \leq 1.0$</td>
<td>$k = -1.69 \left( \frac{I_y}{I_z} \right) + 5.39$</td>
</tr>
<tr>
<td></td>
<td>$1.0 &lt; h/b \leq 1.2$</td>
<td>$k = -0.11 \left( \frac{I_y}{I_z} \right) + 1.05$</td>
</tr>
<tr>
<td></td>
<td>$1.2 &lt; h/b \leq 2.0$</td>
<td>$k = -0.027 \left( \frac{I_y}{I_z} \right) + 0.94$</td>
</tr>
<tr>
<td></td>
<td>$h/b &gt; 2.0$</td>
<td>$k = -0.008 \left( \frac{I_y}{I_z} \right) + 1.21$</td>
</tr>
</tbody>
</table>

### 7.6 Conclusion

In this chapter, the rule on member imperfection without neglecting the additional torsional imperfection is checked by comparing the results from the FE-model with the lateral torsional buckling curves from the Eurocode. The FE-model was given an imperfection based on its first lateral torsional buckling mode. The size of the imperfection was determined with formula (1.1) in combination with the recommended value of 0.5 for the k-value. In this chapter it is shown that this is unsafe to do.

With the additional torsional imperfection a k-value of a 0.5 is still too low. The new k-values are all around 1.0. Eventually the same method as was used in the previous chapter is used to arrive at new k-values. The recommended k-values are shown in Table 7.1 for the IPE sections and in Table 7.2 for the HEA sections.
8. Taras’ lateral torsional buckling curves

8.1 Introduction

This chapter discusses whether formula (1.2), being the amplitude of an imperfection that is based on the lateral torsional buckling mode, is in accordance with Taras’ lateral torsional buckling curves. It is expected that this is the case since formula (1.2) is derived from the Taras’ lateral torsional buckling curves. The derivation of formula (1.2) is written out in Appendix B. If the two are not in accordance with each other, $\alpha_{LT}$ is modified in accordance with Taras’ lateral torsional buckling curves. The Taras’ lateral torsional buckling curves are shown in §2.8. From the objectives shown in §1.3, this is the third sub goal.

8.2 Comparing Taras’ lateral torsional buckling curves with EC3

The approach from Taras is different as compared to the approach from the General method from Eurocode 3. Where in the Eurocode all rolled I-shaped are divided into two groups based on their height to width ratio being greater or smaller than 2.0, Taras developed his curves in a way that every single section gets its own curve that is also dependent on the load. This means the Taras’ curves give accurate values, were the Eurocode gives more conservative values, since there are only two lines that represent all I-shaped sections. In Figure 8.1 a comparison is given for several sections between curves from Eurocode 3 and the curves from Taras. Also different load cases are used.

The figure shows there is a scatter within curve a and curve b for the accompanying sections. While in the last two chapters the curves from the Eurocode were used to fit the k-value, Figure 8.1 shows that for most cases this is too conservative. For few cases it is actually unconservative.
8.3 Testing formula (1.2) with Taras’ curves

Formula (1.2) is shown again below. As stated before the difference with formula (1.1) is the imperfection factor which is shown in Table 8.1.

\[
e_0 = \alpha_{LT}(\bar{\lambda} - 0.2) \frac{M_{Rk}}{N_{Rk}}
\]

(1.2)

Table 8.1 Imperfection factor for formula (1.2)

<table>
<thead>
<tr>
<th>Cross-section</th>
<th>Limits</th>
<th>(\alpha_{LT})</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rolled I-section</td>
<td>(h/b &gt; 1.2)</td>
<td>(0.12 \cdot \sqrt{\frac{W_{el,y}}{W_{el,z}}} \leq 0.34)</td>
</tr>
<tr>
<td></td>
<td>(h/b \leq 1.2)</td>
<td>(0.16 \cdot \sqrt{\frac{W_{el,y}}{W_{el,z}}} \leq 0.49)</td>
</tr>
</tbody>
</table>

The testing of formula (1.2) is done for all three load cases. As stated before there are different curves for different load cases. In Figure 8.2 the results for an IPE240 section are shown.
The four diagrams show the reduction factor calculated with the FE-model, compared to their associated Taras curve. The first three diagrams show the result per load case. The fourth diagram shows the results for all load cases. The diagonal black line shows a perfect match between the reduction factor calculated with the FE-model and Taras curves. If $\chi_{LT,FEM}$ lays above this black line it is on the unsafe side. The two grey dashed lines show a 5% over- or underestimation.

Figure 8.2 shows that formula (1.2) works well for the IPE240 section. All cases lay very close to their associated curve. So by using formula (1.2) results are safe but not too conservative, which is desirable. The formula is also tested on other IPE section and in all cases the results were satisfying in a similar way as for an IPE240 section. This is shown in Appendix K.1

Off course formula (1.2) should also be checked for HEA sections. Figure 8.3 shows the results for the HEA600 section in a similar way as for the IPE240 section.

Figure 8.3 Comparison $\chi_{LT,FEM}$ with $\chi_{LT,Taras}$ for an HEA600 section
Figure 8.3 shows that formula (1.2) gives satisfying results for the HEA600 section as well. All cases lay either on or very close to their associated curve. The formula was also tested on other HEA section with h/b > 1.2. These other results are shown in Appendix K.2. In all cases the results are similar as is shown in Figure 8.3, which means the results are satisfying.

From the results so far it is noticed that if load case 1 gives satisfying results, the other two load cases give satisfying results as well. So for the remainder of this research it suffices to only check the results for load case one.

Until now formula (1.2) is only checked for sections with h/b > 1.2. For the sections with h/b ≤ 1.2 the HEA300 section is investigated in Figure 8.4. Only few cases are shown in the graph. As stated before the influence of cross-sectional resistance around the weak axis starts around a relative slenderness of 1,0 for an HEA300 section. This phenomenon is discussed in §5.3. Cases where this occurs are omitted resulting in a limited number of results.

![Graph showing comparison of formula results for HEA300 section.](image)

**Figure 8.4** Comparison $\chi_{LT,\text{FEM}}$ with $\chi_{LT,\text{Taras}}$ for an HEA300 section

Figure 8.4 shows that formula (1.2) does not work as well for sections with h/b ≤ 1.2 as for sections with h/b > 1.2. The results of the HEA300 section show that the given imperfection is slightly too small and therefore the obtained failure load gets too high. However, the maximum deviation is around 5%, which is acceptable. This maximum deviation of 5% applies to all HEA sections with h/b ≤ 1.2.

If such a deviation up to 5% is unwanted the $\alpha_{LT}$ value from Table 8.1 should be slightly modified for this category. By enlarging the factor, a more suitable formula for $\alpha_{LT}$ is found after several trials. So Table 8.1 is now adjusted into Table 8.2.

<table>
<thead>
<tr>
<th>Cross-section</th>
<th>Limits</th>
<th>$\alpha_{LT}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rolled I-section</td>
<td>h/b &gt; 1,2</td>
<td>$0,12 \cdot \sqrt{\frac{W_{el,y}}{W_{el,z}}} \leq 0,34$</td>
</tr>
<tr>
<td></td>
<td>h/b ≤ 1,2</td>
<td>$0,21 \cdot \sqrt{\frac{W_{el,y}}{W_{el,z}}} \leq 0,49$</td>
</tr>
</tbody>
</table>
The influence on the results for the HEA300 sections caused by slightly adjusting $\alpha_{LT}$ is shown in Figure 8.5.

![Figure 8.5](image1.png)  
*Figure 8.5 Comparison $\chi_{LT,FEM}$ with $\chi_{LT,Taras}$ for an HEA300 section*

The same test is also performed for other HEA sections with $h/b \leq 1.2$ and all gave satisfying results. Some more results are shown in Appendix K.2. So with the small adjustment the formula is now safe to use.

Formula (1.2) in combination with Table 8.2 is now tested for both IPE sections as for HEA sections and in both cases very satisfying results came up, as was shown in this paragraph. The formula should also work for other cross-sections, since there are cross-section properties included in the formula. For a final check also an HEB100 sections is treated in this research. This is shown in Figure 8.6.

![Figure 8.6](image2.png)  
*Figure 8.6 Comparison $\chi_{LT,FEM}$ with $\chi_{LT,Taras}$ for an HEB100 section*
The figure shows that the formula also works for HEB sections. Also some other HEB sections were tested, they all gave similar results and are shown in Appendix K.3.

8.4 Conclusion

Within this chapter it is discussed whether formula (1.2), being the amplitude of an imperfection that is based on the lateral torsional buckling mode, is in accordance with Taras’ lateral torsional buckling curves. The results were really satisfying. Only a small adjustment had to be made to $\alpha_{LT}$ value for sections with $h/b \leq 1.2$. The formula is tested for IPE, HEA and HEB sections. Formula (1.2) in combination with Table 8.2 gives really exact results for these sections and is therefore a good proposal to replace the current rule on member imperfection from Eurocode 3.
9. Discussion

As was shown previously the current rule on member imperfections in combination with formula (1.1) is not as safe as it should be, as was shown among others with Figure 9.1. By adapting the k-factor it was possible to adjust the rule in a way that it can be considered safe again. However the results of the three load cases were somewhat far apart from each other, which also can be seen in Figure 9.1.

By fitting load case 2, a point load on the middle of the top flange, to the associated lateral torsional buckling curve, the other load cases are reduced more than needed. So load case 1, a constant bending moment, and load case 3, a line load on the top flange, give somewhat over-conservative results, because a distinction was not made between the different load cases, when determining a new k-value.

By adding the torsional imperfection to the imperfection shape, the results of the three load cases came much closer to each other. After fitting the results of these load cases to their associated lateral torsional buckling curve, the non-decisive load cases are not as over-conservative as the ones explained in the part above. So the proposal with the additional torsional imperfection is definitely better than the proposal without the additional torsional imperfection.

The rule on member imperfection from the Eurocode is compared with the lateral torsional buckling curves from the General method. As discussed earlier Taras derived new lateral torsional buckling curves that are able to prescribe the adequate reduction factor \( \chi_{LT} \) depending on the cross-section and the load case. In Figure 9.2 these curves are compared with the curves from the General method.
In Figure 9.2 a scatter around curve a en b from the General method is shown, which makes sense since there are only two curves that represent all I-shaped cross-sections. So to get a more exact rule on member imperfection, it is better to check with the lateral torsional buckling curves Taras has developed. Which is not done for formula (1.1).

From the derivation of the Taras’ lateral torsional buckling curves formula (1.2) has been derived. This formula is tested and checked using the lateral torsional buckling curves, which Taras has developed. So here the results are compared with the Taras lateral torsional buckling curves. The results are extremely positive. Only for cross-sections with h/b ≤ 1.2 there is a deviation of 5% to the unsafe side. By performing a small adjustment the results are fitted to the associated curve.

The proposals with new k-values, resulting from the first two imperfection approaches, give in most cases over-conservative values, which is a result of having to fit three load cases to one curve with one k-value that represents all load cases. And the fact that the curve is a lower limit that represents all the I-shaped cross-sections with either h/b ≤ 2.0 or h/b>2.0. From the three approaches for imperfections described, the results of the final imperfection shape and size are the most valuable.
10. Conclusions and recommendations

10.1 Conclusions

1. Eurocode 3 paragraph 5.3.4 Member imperfections, where \( k e_{0,d} \) is the amplitude of an imperfection that is based on the weak axis flexural buckling mode, combined with formula (1.1) is not in accordance with the lateral torsional buckling curves, when \( k = 0.5 \).

\[
e_{0,d} = \alpha(\bar{\lambda} - 0.2) \frac{M_{Rk}}{N_{Rk}}
\]  

(1.1)

Where:
- \( \alpha \) Imperfection factor for relevant buckling curve
- \( \bar{\lambda} \) Relative slenderness
- \( M_{Rk} \) Characteristic moment resistance of the critical cross section
- \( N_{Rk} \) Characteristic resistance to normal force of the critical cross section

To make it in accordance with the lateral torsional buckling curves the value for \( k \) should be:

<table>
<thead>
<tr>
<th>Table 10.1 Recommended k-values for IPE sections</th>
</tr>
</thead>
<tbody>
<tr>
<td>Section</td>
</tr>
<tr>
<td>Rolled I-section</td>
</tr>
<tr>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Table 10.2 Recommended k-values for HEA sections</th>
</tr>
</thead>
<tbody>
<tr>
<td>Section</td>
</tr>
<tr>
<td>Rolled I-section</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
</tbody>
</table>

2. Eurocode 3 paragraph 5.3.4 Member imperfections, where \( k e_{0,d} \) being the amplitude of an imperfection that is based on the lateral torsional buckling mode, combined with formula (1.1) is not in accordance with the lateral torsional buckling curves, when \( k = 0.5 \).

To make it in accordance with the lateral torsional buckling curves the value for \( k \) should be:

<table>
<thead>
<tr>
<th>Table 10.3 Recommended k-values for IPE sections</th>
</tr>
</thead>
<tbody>
<tr>
<td>Section</td>
</tr>
<tr>
<td>Rolled I-section</td>
</tr>
<tr>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Table 10.4 Recommended k-values for HEA sections</th>
</tr>
</thead>
<tbody>
<tr>
<td>Section</td>
</tr>
<tr>
<td>Rolled I-section</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
</tbody>
</table>
3. Formula (1.2), being the amplitude of an imperfection that is based on the lateral torsional buckling mode, is in accordance with Taras’ buckling curves after a small modification to $\alpha_{LT}$. To replace paragraph 5.3.4 Member imperfections from Eurocode 3, the following is proposed:

For a second order analysis taking account of lateral torsional buckling of a member in bending the amplitude of the imperfection may be adopted as $e_o$, the shape of the imperfection should be based on its lateral torsional buckling mode.

$$e_o = \alpha_{LT} (\bar{\lambda} - 0,2) \frac{M_{Rk}}{N_{Rk}}$$  \hspace{1cm} (1.2)

Where:

- $\alpha_{LT}$ Imperfection factor should be taken from Table 10.5
- $\bar{\lambda}$ Relative slenderness
- $M_{Rk}$ Characteristic moment resistance of the critical cross section
- $N_{Rk}$ Characteristic resistance to normal force of the critical cross section

**Table 10.5** Imperfection factor for formula (1.2)

<table>
<thead>
<tr>
<th>Cross-section</th>
<th>Limits</th>
<th>$\alpha_{LT}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rolled I-section</td>
<td>h/b&gt;1,2</td>
<td>$0,12 \cdot \sqrt{\frac{W_{el,y}}{W_{el,x}}} \leq 0,34$</td>
</tr>
<tr>
<td></td>
<td>h/b\leq1,2</td>
<td>$0,21 \cdot \sqrt{\frac{W_{el,y}}{W_{el,x}}} \leq 0,49$</td>
</tr>
</tbody>
</table>

However if a 5% overestimation is allowed for, then Table 10.6 is proposed instead of Table 10.5.

**Table 10.6** Imperfection factor for formula (1.2)

<table>
<thead>
<tr>
<th>Cross-section</th>
<th>Limits</th>
<th>$\alpha_{LT}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rolled I-section</td>
<td>h/b&gt;1,2</td>
<td>$0,12 \cdot \sqrt{\frac{W_{el,y}}{W_{el,x}}} \leq 0,34$</td>
</tr>
<tr>
<td></td>
<td>h/b\leq1,2</td>
<td>$0,16 \cdot \sqrt{\frac{W_{el,y}}{W_{el,x}}} \leq 0,49$</td>
</tr>
</tbody>
</table>
10.2 Recommendations

10.2.1 Testing other rolled I-section profiles

The proposal was tested for the rolled I-sections IPE, HEA and HEB. This research can be extended by also testing for other rolled I-section profiles. Though, the ones which were tested within this research are the sections most commonly used. But it is still valuable to test other sections as well.

10.2.2 Testing for welded I-section profiles

Besides other rolled I-sections it is also interesting to derive and test a proposal for several welded I-sections.

10.2.3 Testing other load cases

All sections treated within this research are all tested for three different load cases, being the constant bending moment, the point load on the top flange and the line load on the top flange. It is useful to extend the research with other load cases. Non-uniform bending moments could be tested as well.
References


Appendix A. Derivation of buckling curves

For the derivation of the buckling curves and the formula $e_0$, the Ayrton-Perry formulation [11] for column buckling is used. It starts with the first yield condition using second-order internal forces. The internal forces result from a sinusoidal pre-deformation about the weak bending axis.

$$\frac{N_b}{A \cdot f_y} + \frac{N_b \cdot e_0}{W \cdot f_y} \cdot \frac{1}{1 - \frac{N_b}{N_{cr}}} = 1,0$$  \hspace{1cm} (A.1)

Some variables, known to Eurocode users, can be introduced here:

$$\chi = \frac{N_b}{A \cdot f_y} = \frac{N_b}{N_{pl}} \quad \bar{\lambda} = \frac{A \cdot f_y}{\sqrt{N_{cr}}}$$  \hspace{1cm} (A.2)

After some rearranging and simplifying equation (A.1) can be rewritten as:

$$\chi + \eta \cdot \frac{\chi}{1 - \chi \cdot \bar{\lambda}^2} = 1$$  \hspace{1cm} (A.3)

With $\eta$ being the imperfection parameter:

$$\eta = \frac{A \cdot e_0}{W}$$  \hspace{1cm} (A.4)

Equation (A.3) can be rewritten so that a quadratic equation of the form $ax^2 + bx + c = 0$ arrises:

$$\bar{\lambda}^2 \cdot \chi^2 - (1 + \eta + \bar{\lambda}^2) \cdot \chi + 1 = 0$$  \hspace{1cm} (A.5)

Another variable from the Eurocode is introduced here:

$$\phi = \frac{1}{2} \cdot (1 + \eta + \bar{\lambda}^2)$$  \hspace{1cm} (A.6)

Equation (A.5) can now be solved like any other quadratic equation:

$$\chi = \frac{2 \cdot \phi \pm \sqrt{4 \cdot \phi^2 - 4 \cdot \bar{\lambda}^2}}{2 \cdot \bar{\lambda}^2}$$  \hspace{1cm} (A.7)

Only the smallest solution for $\chi$ is relevant here. After some rewriting the equation that is currently present in the Eurocode is obtained:

$$\chi = \frac{\phi - \sqrt{\phi^2 - \bar{\lambda}^2}}{\bar{\lambda}^2} = \frac{\phi - \sqrt{\phi^2 - \bar{\lambda}^2}}{\bar{\lambda}^2} \cdot \frac{\phi + \sqrt{\phi^2 - \bar{\lambda}^2}}{\phi + \sqrt{\phi^2 - \bar{\lambda}^2}} = \frac{\bar{\lambda}^2}{\phi + \sqrt{\phi^2 - \bar{\lambda}^2}}$$  \hspace{1cm} (A.8)

Resulting in:

$$\chi = \frac{1}{\phi + \sqrt{\phi^2 - \bar{\lambda}^2}} \leq 1,0$$  \hspace{1cm} (A.9)
The calibration between the different buckling curves was carried out via a curve fitting procedure for
the imperfection parameter \( \eta \). This term was rewritten with a calibration factor \( \alpha \), which can be
found in table 2.10, and a plateau value \( \lambda_0 \), set to 0.2.

\[
\eta = \alpha \cdot (\lambda - 0.2)
\]

(A.10)

So the \( \Phi \) variable now becomes:

\[
\Phi = \frac{1}{2} \cdot [1 + \alpha \cdot (\lambda - 0.2) + \lambda^2]
\]

(A.11)

From this derivation it is possible to derive the formula for the imperfection amplitude \( e_0 \) that was
already mentioned in paragraph 1.2 as formula (1.1):

\[
\eta = \frac{A \cdot e_0}{W} = \alpha \cdot (\lambda - 0.2) \rightarrow e_0 = \alpha \cdot (\lambda - 0.2) \frac{W}{A} \cdot \frac{f_y}{f_y} = \alpha \cdot (\lambda - 0.2) \frac{M_{Rk}}{N_{Rk}}
\]

(A.12)
Appendix B. Derivation Taras’ LT-buckling curves

In order to derive new lateral torsional buckling rules similar steps can be taken as has been done with the derivation of the column buckling rules shown in the previous section. In this section a derivation along the lines of the Ayrton-Perry [11] formulation is used. Second order section loads as well as a first-yield criterion for the definition of an ultimate buckling load is used. The beam is a single-span member that has a double-symmetric cross-section. The beam is loaded with a constant bending moment. The assumption of initial lateral and torsional imperfections \( v_0 \) and \( \theta_0 \) of sinusoidal shape lead to the following second-order equilibrium equations:

\[
N_{cr,z} \cdot \ddot{v} - M_y \cdot \ddot{\theta} = M_y \cdot \ddot{\theta}_0 \tag{B.1}
\]

\[
-M_y \cdot \ddot{v} + \frac{M_{cr}^2}{N_{cr,z}} \cdot \ddot{\theta} = M_y \cdot \ddot{v}_0 \tag{B.2}
\]

With:

\( M_{cr}, N_{cr,z} \) being the elastic critical buckling loads for lateral torsional-buckling and flexural buckling.

As can be seen in Figure B.1 there is a coupling of the two degrees of freedom.

![Figure B.1 Coupling of the two degrees of freedom](image)

Out of this the following equation can be obtained:

\[
\ddot{v}_0 = \frac{M_{cr}}{N_{cr,z}} \cdot \ddot{\theta}_0 \tag{B.3}
\]

By combining equations (B.1) and (B.2) and combining equation (B.2) and (B.3), new equations for the deformed amplitudes \( \ddot{v} \) and \( \ddot{\theta} \) are found:

\[
\ddot{\theta} = \ddot{\theta}_0 \cdot \frac{M_y}{M_{cr} - M_y} \tag{B.4}
\]

\[
\ddot{v} = \ddot{\theta}_0 \cdot \frac{M_{cr}}{N_{cr,z}} \cdot \frac{M_y}{M_{cr} - M_y} \tag{B.5}
\]
In order to determine the second order internal forces (out-of-plane bending moment and warping moment) the following expressions can be used:

\[ M_z = EI_z \cdot \frac{\pi^2}{l_z^2} \cdot \ddot{v} \]  
\[ M_\omega = EI_\omega \cdot \frac{\pi^2}{l_\omega^2} \cdot \ddot{\theta} \]  

By using (B.4), (B.5), (B.6) and (B.7) the following equations can be obtained:

\[ M_z = \ddot{\theta}_0 \cdot \frac{M_y}{1 - \frac{M_y}{M_{cr}}} \]  
\[ M_\omega = EI_\omega \cdot \frac{\pi^2}{l_\omega^2} \cdot \ddot{\theta}_0 \cdot \frac{M_y}{M_{cr}} - M_y \cdot \frac{l_\omega}{l_z} \cdot \ddot{\theta}_0 \cdot \frac{M_y}{1 - \frac{M_y}{M_{cr}}} \]  

The out-of-plane bending moment and warping moment can also be expressed in terms of the imperfection amplitude, with the relationship shown in Figure B.1.

\[ \ddot{\theta}_0 = \frac{\ddot{e}_0}{M_{cr}/N_{cr,x} + h/2} \]  

The maximum stress equation can now be written for the outermost fiber of the flange in compression and it can be compared to the yield strength stress. Where \( \omega_{max} \) is the normalized warping function:

\[ \frac{M_y}{W_{y,el}} + \frac{M_z}{W_z} + \frac{M_\omega}{I_\omega} \cdot \omega_{max} \]
\[ = \frac{M_y}{W_{y,el}} + \frac{M_y}{1 - \frac{M_y}{M_{cr}}} \cdot \frac{e_0}{M_{cr}/N_{cr,x} + h/2} \times \left[ \frac{1}{W_{y,el}} + \frac{N_{cr,x}}{M_{cr}} \cdot \frac{l_\omega}{l_z} \cdot \omega_{max} \right] \leq f_y \]  

In this case the beam has a double-symmetric I cross-section, which means the following relationships are applicable:

\[ W_{z,el} = \frac{l_z}{b/2} \quad \omega_{max} = \frac{h \cdot b}{4} \]  

Equation (B.11) can now be modified:

\[ \frac{M_y}{W_{y,el} \cdot f_y} + \frac{M_y}{W_{y,el} \cdot f_y} \cdot \frac{1}{1 - \frac{M_y}{M_{cr}}} \cdot \frac{\ddot{e}_0}{A \cdot (M_{cr}/N_{cr,x} + h/2)} \leq f_y \]  

This expressions (B.11) must be divided by the yield stress in order to obtain a dimensionless equation. Also the second term on the left-hand side can be expanded with \( W_{y,el}/W_y \) and \( A/A \):

\[ \frac{M_y}{W_{y,el} \cdot f_y} + \frac{M_y}{W_{y,el} \cdot f_y} \cdot \frac{1}{1 - \frac{M_y}{M_{cr}}} \cdot \frac{A \cdot \ddot{e}_0}{A \cdot (M_{cr}/N_{cr,x} + h/2)} \left[ 1 + \frac{N_{cr,x}}{M_{cr}} \cdot \frac{h}{2} \right] \leq 1.0 \]  

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The normalized slenderness and buckling reduction factors can now be introduced:

\[
\lambda_{LT,el} = \frac{M_y}{W_{y,el} \cdot f_y}, \quad \bar{\lambda}_z = \frac{A \cdot f_y}{N_{cr,z}}, \quad \bar{\lambda}_{LT,el} = \frac{W_{y,el} \cdot f_y}{M_{cr}}
\]  

(B.15)

Taking equations (B.15) in (B.14) and simplifying leads to:

\[
\lambda_{LT,el} + \frac{A \cdot \bar{\delta}_0 \cdot \bar{\lambda}_{LT}^2}{W_{z,el} \cdot \bar{\lambda}_z^2} \cdot \frac{\lambda_{LT,el}}{1 - \lambda_{LT,el} \cdot \bar{\lambda}_{LT,el}^2} \leq 1.0
\]  

(B.16)

Introducing \( \eta^* \):

\[
\eta^* = \frac{A \cdot \bar{\delta}_0 \cdot \bar{\lambda}_{LT}^2}{W_{z,el} \cdot \bar{\lambda}_z^2}
\]  

(B.17)

By substituting (B.17) into (B.16) a equation arises that is equal to the buckling formula:

\[
\lambda_{LT,el} = \frac{1}{\Phi_{LT,el} + \sqrt{\Phi_{LT,el}^2 - \bar{\lambda}_{LT,el}^2}} \leq 1.0
\]  

(B.18)

With:

\[
\Phi_{LT,el} = \frac{1}{2} \cdot \left[ 1 + \eta^* + \bar{\lambda}_{LT,el}^2 \right]
\]  

(B.19)

The result of this second-order derivation is almost identical to the ones given by the General Method formulas (2.2) and (2.3), shown in paragraph 2.3.1.

The specific formulation derived will be calibrated to the representative numerical LT buckling curves that were obtained with the FE-model. Formula (B.17) will be replaced by a new calibrated factor \( \eta^* \). A common, basic convention for the new calibration efforts is that the considerations in this section make use of the plastic cross-sectional capacity, even though formula (B.18) was derived for a purely elastic cross-sectional interaction. This means that the indices “el” are dropped in formula (B.17), (B.18) and (B.19).

While at first sight this might appear to be erroneous, it is justified by the fact that a different, plastic interaction would only add certain constants to the above derivation, provided that the cross-sectional interaction between internal force components and the corresponding resistances is still assumed to be linear. Due to plasticity-induced bifurcation phenomena at higher slenderness, enhanced by the presence of residual stresses, the actually applicable cross-sectional interaction at failure varies at each slenderness, lying somewhere between the plastic and elastic interaction. This means that the above-mentioned additional constants stemming from a plastic cross-sectional interaction would add little to the accuracy of the formulation—they would essentially only complicate the formulation. This is just as true for column buckling as for lateral torsional buckling. As was shown in Appendix A, this effect is taken into account by the calibration factor \( \alpha \) in equation (A.10). For the lateral torsional buckling case, it is proposed to do the same.
\( \eta^* \) can be rewritten as:
\[
\eta^* = \eta \cdot \frac{\tilde{\lambda}_{LT}^2}{\tilde{\lambda}_z^2}
\]  
(B.20)

Again after a curve fitting procedure \( \eta \) can be replaced. This time by:
\[
\eta = \alpha_{LT} (\tilde{\lambda}_z - 0.2)
\]  
(B.21)

With:
\( \alpha_{LT} \) being the imperfection factor, see Table B.1

| Table B.1 Recommended values for imperfection factors for lateral buckling curves |
|----------------------------------|----------------------------------|
| h/b                             | Hot-rolled I                     |
| \( \leq 1.2 \)                   | \( \alpha_{LT} = 0.16 \cdot \frac{W_{y,el}}{W_{z,el}} \leq 0.49 \) |
| \( >1.2 \)                      | \( \alpha_{LT} = 0.12 \cdot \frac{W_{y,el}}{W_{z,el}} \leq 0.34 \) |

Taras’ lateral torsional buckling curves then become:
\[
\chi = \frac{1}{\Phi_{LT} + \sqrt{\Phi_{LT}^2 - \tilde{\lambda}_{LT}^2}} \leq 1,0
\]  
(B.22)

With:
\[
\Phi_{LT} = 0.5 \left[ 1 + \alpha_{LT} (\tilde{\lambda}_z - 0.2) \frac{\tilde{\lambda}_{LT}^2}{\tilde{\lambda}_z^2} + \tilde{\lambda}_{LT}^2 \right]
\]  
(B.23)

The derivation is done for a uniform bending moment. Because the shape of the lateral torsional buckling curve also depends on the type of bending moment that is put on the beam, Taras added an “over-strength” factor \( \varphi \) to formulas (B.22) and (B.23). The factor \( \varphi \) depends on the load case that is used as is shown in Table 2.7. In order to incorporate \( \varphi \) in formulas (B.22) and (B.23), all terms that contain \( M_{pl} \) need to be replaced with \( M_{pl} \cdot \varphi \). This is done in formula (B.24), which originated from formula (B.16).
\[
\chi_{LT} \cdot \frac{\varphi}{\Phi_{LT}} + A \cdot e_0 \cdot \Phi_{LT}^2 \cdot \frac{\tilde{\lambda}_{LT}^2}{\tilde{\lambda}_z^2} \cdot \frac{\chi_{LT}}{\varphi} \cdot \frac{\tilde{\lambda}_{LT}}{\tilde{\lambda}_z} \cdot \left[ 1 - \frac{\tilde{\lambda}_{LT}}{\tilde{\lambda}_z} \cdot \varphi \cdot \tilde{\lambda}_{LT}^2 \right] \leq 1,0
\]  
(B.24)

After this the same derivation can be done as has been shown above in formulas (B.16) till (B.24). The final proposal eventually became:
\[
\chi_{LT} = \frac{\varphi}{\Phi_{LT} + \sqrt{\Phi_{LT}^2 - \varphi \cdot \tilde{\lambda}_{LT}^2}} \leq 1,0
\]  
(B.25)

Where:
\[
\Phi_{LT} = 0.5 \left[ 1 + \varphi \left( \alpha_{LT} (\tilde{\lambda}_z - 0.2) \frac{\tilde{\lambda}_{LT}^2}{\tilde{\lambda}_z^2} \right) + \tilde{\lambda}_{LT}^2 \right]
\]  
(B.26)
From this derivation it is possible to derive the formula for the imperfection amplitude $e_0$, specially for lateral torsional buckling, that was already mentioned in paragraph 1.2 as formula (1.2):

$$
\eta = \frac{A \cdot e_0}{W} = \alpha_{LT}(\bar{\lambda}_z - 0.2) \rightarrow e_0 = \alpha_{LT}(\bar{\lambda}_z - 0.2) \frac{W}{A} \cdot \frac{f_y}{f_y} = \alpha_{LT}(\bar{\lambda} - 0.2) \frac{M_{Rk}}{N_{Rk}} \tag{B.27}
$$
Appendix C. Mesh convergence study

Abaqus makes a distinction in three types of shell elements, being the thick conventional shell elements, the thin conventional shell elements and the general purpose shell element. In this mesh convergence study the thick shell element and the thin shell element are compared. The thick conventional shell is based on Mindlin plate theory, which includes transverse shear deformation. The thin conventional shell is described by classical (Kirchhoff) shell theory.

From every category one type of element from the Abaqus library is chosen:

- Thick shell: S8R 8-node quadratic element with 6 D.O.F. per node
- Thin shell: S8R5 8-node quadratic element with 5 D.O.F. per node

The two elements are compared with a three point bending test, for an extremely short beam. The study is performed on a beam with an HEA100 section and a length of 300mm. Several different mesh refinements are used. All models contain the same material properties, boundary conditions and system imperfection. The test is shown in Figure C.1.

![Figure C.1 Three point bending test](image)

The results of the three point bending tests are compared with each other in Figure C.2. The figure shows that eventually for both type of elements the results converge to approximately the same value, which is good. This means that the transverse shear deformation is not decisive.

![Figure C.2 Comparison results different mesh refinements for different elements](image)
Appendix D. Preventing distortion of the cross-section

In this Appendix two ways will be shown to prevent distortion of the cross-section at the beam ends. The cross-section should be allowed to warp. First it is shown how this is made possible with kinematic coupling. Subsequently it is shown that it is also possible by attaching stiff beam elements against the beam ends. Finally the results of the two methods are compared.

D.1 Kinematic coupling on beam ends

In §3.4 it is shown that for this research kinematic coupling is used to prevent distortion of the cross-section, with allowing the cross-section to warping. To use kinematic coupling in Abaqus [9] a control point on the beam edge is selected. The next step is selecting the rest of the nodes on the associated beam edge. So if the control point on the upper flange is selected, the other nodes on the upper flange need to be selected. These nodes are now constrained to the rigid body motion of the control point, in other words the nodes are now coupled to the control point. After this step it can be specified to the coupled nodes which degrees of freedom (Ux, Uy, Uz, URx, URy and URz) are constrained to the rigid body motion of the control point.

For this research the beam edges of the profile should act infinitely stiff. But the flanges should be allowed to warp. This means for the kinematic coupling that is put on the edge of the web that rotating around the Z axis (URz) should be allowed. In Figure D.1 is shown what the degrees of freedom for each constraint should be.

![Figure D.1 Control points and kinematic coupling restraints](image)

An argument could be made to also allow bending in the x-direction (Ux and URx not restrained) because this would be more realistic. Eventually this will not be of any influence if this would be allowed. To get a beam edge of an IPE240 profile to bend in the direction of the extension of the beam, would take a lot of force on the beam edge, which is not the case within this research. In the next paragraph it is elaborated more on this subject.
In geometrically nonlinear analysis steps, the coordinate system in which the constrained degrees of freedom are specified will rotate with the control point regardless of whether the constrained degrees of freedom are specified in the global coordinate system. Radial motion in this case is defined as motion normal to the structure’s axis, this axis is rotating with the reference node. Therefore, the reference node will refer to the axis that rotates with the structure. [9]

**D.2 Stiff beam elements on beam ends**

Another option for meeting with the boundary conditions instead of using kinematic coupling is to use stiff beam elements and attach these to the beam edges. For example Rick Bruins [3] uses this option in his research. However these stiff beam elements should not be torsional stiff, because warping should not be restrained. This can be done by giving these beam elements a cross-section in which the section properties can be specified directly. In Abaqus the following properties should be filled in: Area (A), Moment of inertia in the 1-direction (I_{11}), Moment of inertia in the 2-direction (I_{22}) and Torsional constant (J). Which is the 1-direction and which is the 2-direction can be selected on the beam elements, as is shown on Figure D.2.

![Figure D.2 Stiff beam elements against beam ends](image)

\[
\begin{align*}
A &= \infty \\
I_{11} &= \infty \\
I_{22} &= \infty \\
J &\approx 0,0
\end{align*}
\]

For this research high bending stiffness and low torsional stiffness is required. So in theory the properties A, I_{11} and I_{22} should get infinitely high values and J should get a value that is equal to zero. However, Abaqus gives errors when difference between the torsional stiffness and the bending stiffness are too large. Also, it is not necessary to give values that are that far apart. When the bending stiffness values are relatively high and the rotational stiffness has a relatively low value it is good enough. By experimenting with the values, eventually the right values can be found. This method is shown here for an IPE240 section.
As a starting point for the values the stiff beam element was given values that are approximately the same as the values for an IPE300 section. The torsional factor was then lowered to \(1.0 \, \text{mm}^3\), because the IPE240 section should be able to warp at the beam ends. Therefore a relatively low value was given to the torsional constant. The experimenting with the values starts with the following values:

\[
A = 5 \cdot 10^3 \, \text{mm}^2 \\
I_{11} = 8 \cdot 10^7 \, \text{mm}^4 \\
I_{22} = 6 \cdot 10^6 \, \text{mm}^4 \\
J = 1.0 \, \text{mm}^4
\]

After the starting values are chosen, one section property at a time can be changed. So for example, it is started with increasing and decreasing the area, with this the other properties remain their starting value. In Table D.1 it is shown what the influence is of increasing and decreasing the area on the elastic critical moment. The Table clearly shows that when the area is bigger than \(1 \cdot 10^3 \, \text{mm}^2\) the elastic critical moment remains the same. This means from that point the area is large enough. Although it is not needed for this research, it is also interesting to see what happens if the value for the area is lowered. It can be expected that when the area is lowered to a minimum value, the elastic critical moment would also find a value that remains the same, because the stiffness of the beam elements would be so low as if there were no stiff beam elements on the beam ends of the IPE240. But Abaqus starts to give errors at this point.

With the changing of a property the elastic critical moment changes. In Table D.1 it is shown what the influence of each section property is on the elastic critical moment, when one property is changed at a time.

<table>
<thead>
<tr>
<th>Table D.1</th>
<th>The influence on changing one section property at a time</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(A)</td>
</tr>
<tr>
<td></td>
<td>mm(^2)</td>
</tr>
<tr>
<td>1.0E+05</td>
<td>31.498</td>
</tr>
<tr>
<td>1.0E+04</td>
<td>31.498</td>
</tr>
<tr>
<td>1.0E+03</td>
<td>31.498</td>
</tr>
<tr>
<td>1.0E+02</td>
<td>31.497</td>
</tr>
<tr>
<td>1.0E+01</td>
<td>31.494</td>
</tr>
<tr>
<td>1.0E+00</td>
<td>31.464</td>
</tr>
<tr>
<td>1.0E-01</td>
<td>31.170</td>
</tr>
<tr>
<td>1.0E-02</td>
<td>error</td>
</tr>
</tbody>
</table>

As can be seen in the table when the properties are high or low enough a maximum or minimum occurs. Except for the \(I_{22}\) value. It seems as if the that value has no influence on the elastic critical moment. The stiffness in the 2-direction prevents the beam edge from bending in the extension direction of the IPE240 profile. A lot of force is needed to get the beam edge to deform in that direction. With the load cases that will be used within this research that is not going to happen. It could be possible if a profile was taken that has extremely wide flanges, but that is not the case within...
this research. So for the elastic critical moment it does not matter which $I_{22}$ value is chosen. For the other values it does, but the table clearly shows which values should be chosen. The values that were chosen for the IPE240 profile are shown below.

\[
\begin{align*}
A &= 1 \cdot 10^4 \text{ mm}^2 \\
I_{11} &= 1 \cdot 10^9 \text{ mm}^4 \\
I_{22} &= 1 \cdot 10^9 \text{ mm}^4 \\
J &= 1.0 \text{ mm}^4
\end{align*}
\]

### D.3 Comparison kinematic coupling and stiff beam elements

In Figure D.3 the two methods are compared in a LPF-displacement diagram (LPF: load proportionality factor). An LPF-displacement diagram shows what happens to the displacement if the load is slowly increased. When the maximum load is reached the graph stops. The 1 on the vertical axis equals 100% of the load put on the beam, which is equal to its elastic critical moment. The diagram is from an IPE240 profile with a length of 7200 mm, which is loaded under a constant bending moment. The load (LPF=1) put on the beam, which is the elastic critical moment, is a constant bending moment of $31,57 \cdot 10^6 \text{ Nmm}$. Both in-plane and out-of-plane displacement are shown.

As can be seen in Figure D.3 the two methods give an almost exact match. In this research it is chosen to continue with the kinematic coupling method, because this method is less time consuming and slightly more precise than the method with the generalized profile. Having to choose for each case the right values for the section properties of the stiff beam elements, makes the method with the stiff beam elements more time consuming than the method using kinematic coupling. The method that uses kinematic coupling can use the same settings for all profiles.
Appendix E. Python script FE-model

E.1 Python script for LC1 on FE-model

```python
### PARAMETERS ###
height=240
width=120
tf=9.8
tw=6.2

w=width
h=height-tf

length=3400

ML=50 ### MeshLengthe ###
MFN=4 ### MeshFlangeNumber ###
MWN=8 ### MeshWebNumber ###

### PART ###
mdb.models['Model-1'].ConstrainedSketch(name='__profile__', sheetSize=500.0)
    mdb.models['Model-1'].sketches['__profile__'].Line(point1=(0.0, 0.0), point2=(0.0, 0.5*h))
mdb.models['Model-1'].sketches['__profile__'].Line(point1=(0.0, 0.5*h), point2=(-0.5*w, 0.5*h))
mdb.models['Model-1'].sketches['__profile__'].Line(point1=(0.0, 0.5*h), point2=(0.5*w, 0.5*h))
mdb.models['Model-1'].sketches['__profile__'].Line(point1=(0.0, 0.0), point2=(0.0, -0.5*h))
mdb.models['Model-1'].sketches['__profile__'].Line(point1=(0.0, -0.5*h), point2=(-0.5*w, -0.5*h))
mdb.models['Model-1'].sketches['__profile__'].Line(point1=(0.0, -0.5*h), point2=(0.5*w, -0.5*h))

mdb.models['Model-1'].Part(dimensionality=THREE_D, name='BEAM', type=DEFORMABLE_BODY)
    mdb.models['Model-1'].parts['BEAM'].BaseShellExtrude(depth=0.5*length, sketch=mdb.models['Model-1'].sketches['__profile__'])
del mdb.models['Model-1'].sketches['__profile__']

### MATERIAL ###
    mdb.models['Model-1'].Material(name='STEEL')
    mdb.models['Model-1'].materials['STEEL'].Elastic(table=((210000.0, 0.3), ))

### SECTION ###
    mdb.models['Model-1'].HomogeneousShellSection(idealization=NO_IDEALIZATION,
        integrationRule=SIMPSON, material='STEEL', name='FLANGE', numIntPts=5,
        poissonDefinition=DEFAULT, preIntegrate=OFF, temperature=GRADIENT,
        thickness=tf, thicknessField='', thicknessModulus=None, thicknessType=UNIFORM, useDensity=OFF)
```

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mdb.models['Model-1'].HomogeneousShellSection(idealization=NO_IDEALIZATION, integrationRule=SIMPSON, material='STEEL', name='WEB', numIntPts=5, poissonDefinition=DEFAULT, preIntegrate=OFF, temperature=GRADIENT, thickness=tw, thicknessField='', thicknessModulus=None, thicknessType=UNIFORM, useDensity=OFF) # CHANGE

### ASSIGN SECTION ###
mdb.models['Model-1'].parts['BEAM'].Set(faces= mdb.models['Model-1'].parts['BEAM'].faces.getSequenceFromMask(('[#1d ]', ), name='Set-1')
mdb.models['Model-1'].parts['BEAM'].SectionAssignment(offset=0.0, offsetField= '', offsetType=MIDDLE_SURFACE, region= mdb.models['Model-1'].parts['BEAM'].sets['Set-1'], sectionName='FLANGE', thicknessAssignment=FROM_SECTION)

mdb.models['Model-1'].parts['BEAM'].Set(faces= mdb.models['Model-1'].parts['BEAM'].faces.getSequenceFromMask(('[#2 ]', ), name='Set-2')

mdb.models['Model-1'].parts['BEAM'].SectionAssignment(offset=0.0, offsetField= '', offsetType=MIDDLE_SURFACE, region= mdb.models['Model-1'].parts['BEAM'].sets['Set-2'], sectionName='WEB', thicknessAssignment=FROM_SECTION)

### MESH ###
mdb.models['Model-1'].parts['BEAM'].seedEdgeByNumber(constraint=FINER, edges= mdb.models['Model-1'].parts['BEAM'].edges.getSequenceFromMask(('[#1ba05 ]', ), number=MWN) # CHANGE
mdb.models['Model-1'].parts['BEAM'].seedEdgeBySize(constraint=FINER, deviationFactor=0.1, edges= mdb.models['Model-1'].parts['BEAM'].edges.getSequenceFromMask(('[#2442a ]', ), size=ML) # CHANGE

mdb.models['Model-1'].parts['BEAM'].generateMesh()

mdb.models['Model-1'].parts['BEAM'].setElementType(elemTypes=(ElemType(elemCode=S8R, elemLibrary=STANDARD), ElemType(elemCode=STRI65, elemLibrary=STANDARD)), regions={
    mdb.models['Model-1'].parts['BEAM'].faces.getSequenceFromMask(('[#1f ]', ))})

### ASSEMBLY ###
mdb.models['Model-1'].rootAssembly.DatumCsysByDefault(CARTESIAN)

mdb.models['Model-1'].rootAssembly.Instance(dependent=ON, name='BEAM-1', part= mdb.models['Model-1'].parts['BEAM'])

mdb.models['Model-1'].rootAssembly.Set(name='Set-1', vertices= mdb.models['Model-1'].rootAssembly.instances['BEAM-1'].vertices.getSequenceFromMask( ('[#10 ]', )))

### BOUNDARY CONDITIONS ###
mdb.models['Model-1'].DisplacementBC(amplitude=UNSET, createStepName='Initial', distributionType=UNIFORM, fieldName='', localCsys=None, name='BC-1',
region = mdb.models['Model-1'].rootAssembly.sets['Set-1'], u1=SET, u2=SET,
  u3=UNSET, ur1=UNSET, ur2=UNSET, ur3=SET)
mdb.models['Model-1'].rootAssembly.Set(edges=
  mdb.models['Model-1'].rootAssembly.instances['BEAM-1'].edges.getSequenceFromMask(  
  ('[#128c4 ]', ), name='Set-2', vertices=
  mdb.models['Model-1'].rootAssembly.instances['BEAM-1'].vertices.getSequenceFromMask(  
  ('[#2acc ]', ), ))
mdb.models['Model-1'].DisplacementBC(amplitude=UNSET, createStepName='Initial',
  distributionType=UNIFORM, fieldName='',
  localCsys=None, name='BC-2', region=mdb.models['Model-1'].rootAssembly.sets['Set-2'],
  u1=UNSET, u2=UNSET,
  u3=SET, ur1=SET, ur2=SET, ur3=UNSET)

### CONSTRAINTS - KINEMATIC COUPLING ###
mdb.models['Model-1'].rootAssembly.Set(name='m_Set-3', vertices=
  mdb.models['Model-1'].rootAssembly.instances['BEAM-1'].vertices.getSequenceFromMask(  
  ('[#2 ]', ), ))
mdb.models['Model-1'].rootAssembly.Set(edges=
  mdb.models['Model-1'].rootAssembly.instances['BEAM-1'].edges.getSequenceFromMask(  
  ('[#8000 ]', ), ), name='s_Set-3')
mdb.models['Model-1'].Coupling(controlPoint=
  mdb.models['Model-1'].rootAssembly.sets['m_Set-3'], couplingType=KINEMATIC,
  influenceRadius=WHOLE_SURFACE, localCsys=None, name='LF-1', surface=
  mdb.models['Model-1'].rootAssembly.sets['s_Set-3'], u1=ON, u2=ON, u3=ON,
  ur1=ON, ur2=ON, ur3=ON)
mdb.models['Model-1'].rootAssembly.Set(name='m_Set-5', vertices=
  mdb.models['Model-1'].rootAssembly.instances['BEAM-1'].vertices.getSequenceFromMask(  
  ('[#2 ]', ), ))
mdb.models['Model-1'].rootAssembly.Set(edges=
  mdb.models['Model-1'].rootAssembly.instances['BEAM-1'].edges.getSequenceFromMask(  
  ('[#1 ]', ), ), name='s_Set-5')
mdb.models['Model-1'].Coupling(controlPoint=
  mdb.models['Model-1'].rootAssembly.sets['m_Set-5'], couplingType=KINEMATIC,
  influenceRadius=WHOLE_SURFACE, localCsys=None, name='LF-2', surface=
  mdb.models['Model-1'].rootAssembly.sets['s_Set-5'], u1=ON, u2=ON, u3=ON,
  ur1=ON, ur2=ON, ur3=ON)
mdb.models['Model-1'].rootAssembly.Set(name='m_Set-7', vertices=
  mdb.models['Model-1'].rootAssembly.instances['BEAM-1'].vertices.getSequenceFromMask(  
  ('[#10 ]', ), ))
mdb.models['Model-1'].rootAssembly.Set(edges=
  mdb.models['Model-1'].rootAssembly.instances['BEAM-1'].edges.getSequenceFromMask(  
  ('[#100 ]', ), ), name='s_Set-7')
mdb.models['Model-1'].Coupling(controlPoint=
  mdb.models['Model-1'].rootAssembly.sets['m_Set-7'], couplingType=KINEMATIC,
  influenceRadius=WHOLE_SURFACE, localCsys=None, name='LW', surface=
  mdb.models['Model-1'].rootAssembly.sets['s_Set-7'], u1=ON, u2=ON, u3=ON,
  ur1=ON, ur2=OFF, ur3=ON)
mdb.models['Model-1'].rootAssembly.Set(name='m_Set-9', vertices=
  mdb.models['Model-1'].rootAssembly.instances['BEAM-1'].vertices.getSequenceFromMask(  
  ('[#2acc ]', ), ))
 mdb.models['Model-1'].rootAssembly.Set(edges=
    mdb.models['Model-1'].rootAssembly.instances['BEAM-1'].edges.getSequenceFromMask(
    ('[#10 ]', ), ), name='s_Set-9')
mdb.models['Model-1'].Coupling(controlPoint=
    mdb.models['Model-1'].rootAssembly.sets['m_Set-9'], couplingType=KINEMATIC,
    influenceRadius=WHOLE_SURFACE, localCsys=None, name='UW', surface=
    mdb.models['Model-1'].rootAssembly.sets['s_Set-9'], u1=ON, u2=ON, u3=ON,
    ur1=ON, ur2=OFF, ur3=ON)

mdb.models['Model-1'].rootAssembly.Set(name='m_Set-11', vertices=
    mdb.models['Model-1'].rootAssembly.instances['BEAM-1'].vertices.getSequenceFromMask(
    ('[#20 ]', ), ))
mdb.models['Model-1'].rootAssembly.Set(edges=
    mdb.models['Model-1'].rootAssembly.instances['BEAM-1'].edges.getSequenceFromMask(
    ('[#1000 ]', ), ), name='s_Set-11')
mdb.models['Model-1'].Coupling(controlPoint=
    mdb.models['Model-1'].rootAssembly.sets['m_Set-11'], couplingType=KINEMATIC
    , influenceRadius=WHOLE_SURFACE, localCsys=None, name='UF-1', surface=
    mdb.models['Model-1'].rootAssembly.sets['s_Set-11'], u1=ON, u2=ON, u3=ON,
    ur1=ON, ur2=ON, ur3=ON)

mdb.models['Model-1'].rootAssembly.Set(name='m_Set-13', vertices=
    mdb.models['Model-1'].rootAssembly.instances['BEAM-1'].vertices.getSequenceFromMask(
    ('[#20 ]', ), ))
mdb.models['Model-1'].rootAssembly.Set(edges=
    mdb.models['Model-1'].rootAssembly.instances['BEAM-1'].edges.getSequenceFromMask(
    ('[#200 ]', ), ), name='s_Set-13')
mdb.models['Model-1'].Coupling(controlPoint=
    mdb.models['Model-1'].rootAssembly.sets['m_Set-13'], couplingType=KINEMATIC
    , influenceRadius=WHOLE_SURFACE, localCsys=None, name='UF-2', surface=
    mdb.models['Model-1'].rootAssembly.sets['s_Set-13'], u1=ON, u2=ON, u3=ON,
    ur1=ON, ur2=ON, ur3=ON)

### STEP ###
mdb.models['Model-1'].BuckleStep(maxIterations=300, name='BUCKLE', numEigen=2,
    previous='Initial', vectors=4)

### LOADS ###
mdb.models['Model-1'].rootAssembly.Set(name='Set-15', vertices=
    mdb.models['Model-1'].rootAssembly.instances['BEAM-1'].vertices.getSequenceFromMask(
    ('[#10 ]', ), ))
mdb.models['Model-1'].Moment(cm1=-1.0, createStepName='BUCKLE',
    distributionType=UNIFORM, field='', localCsys=None, name='Load-1', region=
    mdb.models['Model-1'].rootAssembly.sets['Set-15'])

### ADD NODE FILE (Model-1) ###
mdb.models['Model-1'].keywordBlock.synchVersions(storeNodesAndElements=False)
mdb.models['Model-1'].keywordBlock.insert(87, \n"*NODE FILE\nU")
### JOB ###
```
mdb.Job(atTime=None, contactPrint=OFF, description="", echoPrint=OFF,
   explicitPrecision=SINGLE, getMemoryFromAnalysis=True, historyPrint=OFF,
   memory=90, memoryUnits=PERCENTAGE, model='Model-1', modelPrint=OFF,
   multiprocessingMode=DEFAULT, name='LBA_IPE240_3400_MC_LTS_k50',
   nodalOutputPrecision=SINGLE,
   numCpus=1, numGPUs=0, queue=None, scratch="", type=ANALYSIS,
   userSubroutine="", waitHours=0, waitMinutes=0)
```

### MODEL 2 ###
```
mdb.Model(modelType=STANDARD_EXPLICIT, name='Model-2')
```

### PART 2 ###
```
mdb.models['Model-2'].ConstrainedSketch(name='__profile__', sheetSize=500.0)
```

```
mdb.models['Model-2'].sketches['__profile__'].Line(point1=(0.0, 0.0), point2=(
   0.0, 0.5*h))
```

```
mdb.models['Model-2'].sketches['__profile__'].Line(point1=(0.0, 0.5*h), point2=(
   -0.5*w, 0.5*h))
```

```
mdb.models['Model-2'].sketches['__profile__'].Line(point1=(0.0, 0.5*h), point2=(
   0.5*w, 0.5*h))
```

```
mdb.models['Model-2'].sketches['__profile__'].Line(point1=(0.0, 0.0), point2=(
   0.0, -0.5*h))
```

```
mdb.models['Model-2'].sketches['__profile__'].Line(point1=(0.0, -0.5*h),
   point2=(-0.5*w, -0.5*h))
```

```
mdb.models['Model-2'].sketches['__profile__'].Line(point1=(0.0, -0.5*h),
   point2=(0.5*w, -0.5*h))
```

```
del mdb.models['Model-2'].sketches['__profile__']
```

### MATERIAL 2 ###
```
mdb.models['Model-2'].Material(name='STEEL')
```

```
mdb.models['Model-2'].materials['STEEL'].Elastic(table=((210000.0,
   0.3),))
```

```
mdb.models['Model-2'].materials['STEEL'].Plastic(table=((235.0,
   0.0),))
```

### SECTION 2 ###
```
mdb.models['Model-2'].HomogeneousShellSection(idealization=NO_IDEALIZATION,
   integrationRule=SIMPSON, material='STEEL', name='FLANGE', numIntPts=5,
   poissonDefinition=DEFAULT, preIntegrate=OFF, temperature=GRADIENT,
   thickness=tf, thicknessField="", thicknessModulus=None, thicknessType=
   UNIFORM, useDensity=OFF) # CHANGE
```

```
mdb.models['Model-2'].HomogeneousShellSection(idealization=NO_IDEALIZATION,
   integrationRule=SIMPSON, material='STEEL', name='WEB', numIntPts=5,
   poissonDefinition=DEFAULT, preIntegrate=OFF, temperature=GRADIENT,
   thickness=tw, thicknessField="", thicknessModulus=None, thicknessType=
   UNIFORM, useDensity=OFF) # CHANGE
### ASSIGN SECTION 2 ###
```
mdb.models['Model-2'].parts['BEAM'].Set(faces=
    mdb.models['Model-2'].parts['BEAM'].faces.getSequenceFromMask(('[#1d ]', ),
    ), name='Set-1')
 mdb.models['Model-2'].parts['BEAM'].SectionAssignment(offset=0.0, offsetField="
", offsetType=MIDDLE_SURFACE, region=
    mdb.models['Model-2'].parts['BEAM'].sets['Set-1'], sectionName='FLANGE',
    thicknessAssignment=FROM_SECTION)
 mdb.models['Model-2'].parts['BEAM'].Set(faces=
    mdb.models['Model-2'].parts['BEAM'].faces.getSequenceFromMask(('[#2 ]', ),
    ), name='Set-2')
 mdb.models['Model-2'].parts['BEAM'].SectionAssignment(offset=0.0, offsetField="
", offsetType=MIDDLE_SURFACE, region=
    mdb.models['Model-2'].parts['BEAM'].sets['Set-2'], sectionName='WEB',
    thicknessAssignment=FROM_SECTION)
```

### MESH 2 ###
```
mdb.models['Model-2'].parts['BEAM'].seedEdgeByNumber(constraint=FINER, edges=
    mdb.models['Model-2'].parts['BEAM'].edges.getSequenceFromMask(('[#1ba05 ]',
    ), ), number=MFN) ################CHANGE#################
 mdb.models['Model-2'].parts['BEAM'].seedEdgeByNumber(constraint=FINER, edges=
    mdb.models['Model-2'].parts['BEAM'].edges.getSequenceFromMask(('[#1d0 ]',
    ), ), number=MWN) ################CHANGE#################
 mdb.models['Model-2'].parts['BEAM'].seedEdgeBySize(constraint=FINER, deviationFactor=0.1, edges=
    mdb.models['Model-2'].parts['BEAM'].edges.getSequenceFromMask(('[#2442a ]',
    ), ), size=ML) ################CHANGE#################
 mdb.models['Model-2'].parts['BEAM'].generateMesh()
```

### ASSEMBLY 2 ###
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```
mdb.models['Model-2'].DisplacementBC(amplitude=UNSET, createStepName='Initial',
  distributionType=UNIFORM, fieldName='', localCsys=None, name='BC-2',
  region=mdb.models['Model-2'].rootAssembly.sets['Set-2'], u1=UNSET, u2=UNSET,
  u3=SET, ur1=SET, ur2=SET, ur3=UNSET)

### CONSTRAINTS - KINEMATIC COUPLING 2 ###

mdb.models['Model-2'].rootAssembly.Set(name='m_Set-3', vertices=)
  mdb.models['Model-2'].rootAssembly.instances['BEAM-1'].vertices.getSequenceFromMask(('[#2 ]',),))

mdb.models['Model-2'].rootAssembly.Set(edges=)
  mdb.models['Model-2'].rootAssembly.instances['BEAM-1'].edges.getSequenceFromMask(('[#800 ]',), name='s_Set-3')

mdb.models['Model-2'].Coupling(controlPoint=)
  mdb.models['Model-2'].rootAssembly.sets['m_Set-3'], couplingType=KINEMATIC,
  influenceRadius=WHOLE_SURFACE, localCsys=None, name='LF-1', surface=
  mdb.models['Model-2'].rootAssembly.sets['s_Set-3'], u1=ON, u2=ON, u3=ON,
  ur1=ON, ur2=ON, ur3=ON)

mdb.models['Model-2'].rootAssembly.Set(name='m_Set-5', vertices=)
  mdb.models['Model-2'].rootAssembly.instances['BEAM-1'].vertices.getSequenceFromMask(('[#2 ]',))

mdb.models['Model-2'].rootAssembly.Set(edges=)
  mdb.models['Model-2'].rootAssembly.instances['BEAM-1'].edges.getSequenceFromMask(('[#1 ]',), name='s_Set-5')

mdb.models['Model-2'].Coupling(controlPoint=)
  mdb.models['Model-2'].rootAssembly.sets['m_Set-5'], couplingType=KINEMATIC,
  influenceRadius=WHOLE_SURFACE, localCsys=None, name='LF-2', surface=
  mdb.models['Model-2'].rootAssembly.sets['s_Set-5'], u1=ON, u2=ON, u3=ON,
  ur1=ON, ur2=ON, ur3=ON)

mdb.models['Model-2'].rootAssembly.Set(name='m_Set-7', vertices=)
  mdb.models['Model-2'].rootAssembly.instances['BEAM-1'].vertices.getSequenceFromMask(('[#10 ]',))

mdb.models['Model-2'].rootAssembly.Set(edges=)
  mdb.models['Model-2'].rootAssembly.instances['BEAM-1'].edges.getSequenceFromMask(('[#100 ]',), name='s_Set-7')

mdb.models['Model-2'].Coupling(controlPoint=)
  mdb.models['Model-2'].rootAssembly.sets['m_Set-7'], couplingType=KINEMATIC,
  influenceRadius=WHOLE_SURFACE, localCsys=None, name='LW', surface=
  mdb.models['Model-2'].rootAssembly.sets['s_Set-7'], u1=ON, u2=ON, u3=ON,
  ur1=ON, ur2=OFF, ur3=ON)

mdb.models['Model-2'].rootAssembly.Set(name='m_Set-9', vertices=)
  mdb.models['Model-2'].rootAssembly.instances['BEAM-1'].vertices.getSequenceFromMask(('[#10 ]',))

mdb.models['Model-2'].rootAssembly.Set(edges=)
  mdb.models['Model-2'].rootAssembly.instances['BEAM-1'].edges.getSequenceFromMask(('[#10 ]',), name='s_Set-9')

mdb.models['Model-2'].Coupling(controlPoint=)
  mdb.models['Model-2'].rootAssembly.sets['m_Set-9'], couplingType=KINEMATIC,
  influenceRadius=WHOLE_SURFACE, localCsys=None, name='UW', surface=
  mdb.models['Model-2'].rootAssembly.sets['s_Set-9'], u1=ON, u2=ON, u3=ON,
  ur1=ON, ur2=OFF, ur3=ON)
mdb.models['Model-2'].rootAssembly.Set(name='m_Set-11', vertices=
    mdb.models['Model-2'].rootAssembly.instances['BEAM-1'].vertices.getSequenceFromMask(
        ('[#20 ]', ), ))
mdb.models['Model-2'].rootAssembly.Set(edges=
    mdb.models['Model-2'].rootAssembly.instances['BEAM-1'].edges.getSequenceFromMask(
        ('[#1000 ]', ), ), name='s_Set-11')
mdb.models['Model-2'].Coupling(controlPoint=
    mdb.models['Model-2'].rootAssembly.sets['m_Set-11'], couplingType=KINEMATIC
    , influenceRadius=WHOLE_SURFACE, localCsys=None, name='UF-1', surface=
    mdb.models['Model-2'].rootAssembly.sets['s_Set-11'], u1=ON, u2=ON, u3=ON,
    ur1=ON, ur2=ON, ur3=ON)

mdb.models['Model-2'].rootAssembly.Set(name='m_Set-13', vertices=
    mdb.models['Model-2'].rootAssembly.instances['BEAM-1'].vertices.getSequenceFromMask(
        ('[#20 ]', ), ))
mdb.models['Model-2'].rootAssembly.Set(edges=
    mdb.models['Model-2'].rootAssembly.instances['BEAM-1'].edges.getSequenceFromMask(
        ('[#200 ]', ), ), name='s_Set-13')
mdb.models['Model-2'].Coupling(controlPoint=
    mdb.models['Model-2'].rootAssembly.sets['m_Set-13'], couplingType=KINEMATIC
    , influenceRadius=WHOLE_SURFACE, localCsys=None, name='UF-2', surface=
    mdb.models['Model-2'].rootAssembly.sets['s_Set-13'], u1=ON, u2=ON, u3=ON,
    ur1=ON, ur2=ON, ur3=ON)

### STEP 2 ###
mdb.models['Model-2'].StaticRiksStep(initialArcInc=0.02, maxArcInc=0.035,
    maxNumInc=100, minArcInc=1e-10, name='Riks', nlgeom=ON, previous='Initial')

### LOADS 2 ###
mdb.models['Model-2'].rootAssembly.Set(name='Set-15', vertices=
    mdb.models['Model-2'].rootAssembly.instances['BEAM-1'].vertices.getSequenceFromMask(
        ('[#10 ]', ), ))
mdb.models['Model-2'].Moment(cm1=‐8.40991e7, createStepName='Riks',
    distributionType=UNIFORM, field='', localCsys=None, name='Load-1', region=
    mdb.models['Model-2'].rootAssembly.sets['Set-15'])

### ADD IMPERFECTION (Model-2) ###
mdb.models['Model-2'].keywordBlock.synchVersions(storeNodesAndElements=False)
mdb.models['Model-2'].keywordBlock.insert(74,
    '\n*IMPERFECTION, FILE=LBA_BEAM, STEP=1\n2, 3.68')

### JOB 2 ###
mdb.Job(atTime=None, contactPrint=OFF, description='',
    echoPrint=OFF, explicitPrecision=SINGLE, getMemoryFromAnalysis=True, historyPrint=OFF,
    memory=90, memoryUnits=PERCENTAGE, model='Model-2', modelPrint=OFF,
    multiprocessingMode=DEFAULT, name='GMNIA_IPE240_3400_MC_LTS_k50',
    nodalOutputPrecision=SINGLE,
    numCpus=1, numGPUs=0, queue=None, scratch='', type=ANALYSIS,
    userSubroutine='', waitHours=0, waitMinutes=0)
E.2 Python script for LC2 on FE-model

The Python script for load case 2 is pretty similar to the Python script for load case 1. The part about the stiffener needs to be added before the material part. Some other parts needs to be replaced, which are following parts: Assign section, Assembly, Mesh and Loads

### PART STIFFENER ### (model 1 & 2)

```python
mdb.models['Model-1'].ConstrainedSketch(name='__profile__', sheetSize=500.0)
mdb.models['Model-1'].sketches['__profile__'].Line(point1=(0.5*w, 0.5*h),
point2=(0.5*w, 0.0))
```

```
mdb.models['Model-1'].sketches['__profile__'].Line(point1=(0.5*w, 0.0), point2=(
0.5*w, -0.5*h))
```

```
mdb.models['Model-1'].sketches['__profile__'].Line(point1=(0.5*w, -0.5*h),
point2=(-0.5*w, -0.5*h))
```

```
mdb.models['Model-1'].sketches['__profile__'].Line(point1=(-0.5*w, -0.5*h),
point2=(-0.5*w, 0.0))
```

```
mdb.models['Model-1'].sketches['__profile__'].Line(point1=(-0.5*w, 0.0), point2=(
-0.5*w, 0.5*h))
```

```
mdb.models['Model-1'].sketches['__profile__'].Line(point1=(-0.5*w, 0.5*h),
point2=(0.5*w, 0.5*h))
```

```python
mdb.models['Model-1'].Part(dimensionality=THREE_D, name='Stiffener',
type=DEFORMABLE_BODY)
mdb.models['Model-1'].parts['Stiffener'].BaseShell(sketch=
mdb.models['Model-1'].sketches['__profile__'])
del mdb.models['Model-1'].sketches['__profile__']
```

### ASSIGN SECTION ### (model 1 & 2)

```python
mdb.models['Model-1'].parts['Profile'].Set(faces=
    mdb.models['Model-1'].parts['Profile'].faces.getSequenceFromMask((['#1e', ], ),
    name='Set-1'))
mdb.models['Model-1'].parts['Profile'].SectionAssignment(offset=0.0,
    offsetField='', offsetType=MIDDLE_SURFACE, region=
    mdb.models['Model-1'].parts['Profile'].sets['Set-1'], sectionName='Flange',
    thicknessAssignment=FROM_SECTION)
mdb.models['Model-1'].parts['Profile'].Set(faces=
    mdb.models['Model-1'].parts['Profile'].faces.getSequenceFromMask((['#1', ], ),
    name='Set-2'))
mdb.models['Model-1'].parts['Profile'].SectionAssignment(offset=0.0,
    offsetField='', offsetType=MIDDLE_SURFACE, region=
    mdb.models['Model-1'].parts['Profile'].sets['Set-2'], sectionName='Web',
    thicknessAssignment=FROM_SECTION)
mdb.models['Model-1'].parts['Stiffener'].Set(faces=
    mdb.models['Model-1'].parts['Stiffener'].faces.getSequenceFromMask((
    ['#1', ]), name='Set-1'))
mdb.models['Model-1'].parts['Stiffener'].SectionAssignment(offset=0.0,
    offsetField='', offsetType=MIDDLE_SURFACE, region=
    mdb.models['Model-1'].parts['Stiffener'].sets['Set-1'], sectionName=
    'Flange', thicknessAssignment=FROM_SECTION)
```

### ASSEMBLY ### (model 1 & 2)

```python
mdb.models['Model-1'].rootAssembly.DatumCsysByDefault(CARTESIAN)
mdb.models['Model-1'].rootAssembly.Instance(dependent=ON, name='Profile-1',
```
```python
part = mdb.models['Model-1'].parts['Profile']

mdb.models['Model-1'].rootAssembly.Instance(dependent=ON, name='Stiffener-1',
part = mdb.models['Model-1'].parts['Stiffener'])

mdb.models['Model-1'].rootAssembly._previewMergeMeshes(instances=(
    mdb.models['Model-1'].rootAssembly.instances['Profile-1'],
    mdb.models['Model-1'].rootAssembly.instances['Stiffener-1']),
keepOnlyOrphanElems=True, nodeMergingTolerance=1e-06)

mdb.models['Model-1'].rootAssembly.InstanceFromBooleanMerge(domain=BOTH,
instances=(mdb.models['Model-1'].rootAssembly.instances['Profile-1'],
mdb.models['Model-1'].rootAssembly.instances['Stiffener-1']),
keepIntersections=ON, mergeNodes=BOUNDARY_ONLY, name='Part-1',
nodeMergingTolerance=1e-06, originalInstances=SUPPRESS)

### MESH ### (model 1 & 2)

mdb.models['Model-1'].parts['Part-1'].seedEdgeBySize(constraint=FINER,
deviationFactor=0.1, edges=
    mdb.models['Model-1'].parts['Part-1'].edges.getSequenceFromMask((
    '#265800', ), size=ML) ##########CHANGE##########

mdb.models['Model-1'].parts['Part-1'].seedEdgeByNumber(constraint=FINER, edges=
    mdb.models['Model-1'].parts['Part-1'].edges.getSequenceFromMask((
    '#198264', ), number=MFN) ##########CHANGE##########

mdb.models['Model-1'].parts['Part-1'].seedEdgeByNumber(constraint=FINER, edges=
    mdb.models['Model-1'].parts['Part-1'].edges.getSequenceFromMask((
    '#259b', ), number=MWN) ##########CHANGE##########

mdb.models['Model-1'].parts['Part-1'].generateMesh()

mdb.models['Model-1'].parts['Part-1'].setElementType(elemTypes=(ElemType(
    elemCode=S8R, elemLibrary=STANDARD), ElemType(elemCode=STRI65,
    elemLibrary=STANDARD)), regions=(
    mdb.models['Model-1'].parts['Part-1'].faces.getSequenceFromMask((
    '#7f', ), ), ))

mdb.models['Model-1'].rootAssembly.regenerate()

mdb.models['Model-1'].rootAssembly.Set(name='Set-1', vertices=
    mdb.models['Model-1'].rootAssembly.instances['Part-1-1'].vertices.getSequenceFromMask(
    ('#200', ), ))

### LOADS ### (model 1 & 2)

mdb.models['Model-1'].rootAssembly.Set(name='Set-15', vertices=
    mdb.models['Model-1'].rootAssembly.instances['Part-1-1'].vertices.getSequenceFromMask(
    ('#4', ), ))

mdb.models['Model-1'].ConcentratedForce(cf2=-0.5, createStepName='Buckle',
distributionType=UNIFORM, field='', localCsys=None, name='Load-1', region=
    mdb.models['Model-1'].rootAssembly.sets['Set-15'])
```
E.3 Python script for LC3 on FE-model

The FE-model for load case 3 is the same as the FE-model for load case 1. The only thing different is the load. So the load needs to be replaced.

```python
### LOADS ### (model 1 & 2)
mdb.models['Model-1'].rootAssembly.Set(name='Set-17', vertices=
    mdb.models['Model-1'].rootAssembly.instances['BEAM-1'].vertices.getSequenceFromMask(  
    ('[#20 ]', )),
)
    mdb.models['Model-1'].ConcentratedForce(cf2=-0.5, createStepName='BUCKLE',
        distributionType=UNIFORM, field='', localCsys=None, name='Load-1', region=
        mdb.models['Model-1'].rootAssembly.sets['Set-17'])

mdb.models['Model-1'].rootAssembly.Set(name='Set-16', vertices=
    mdb.models['Model-1'].rootAssembly.instances['BEAM-1'].vertices.getSequenceFromMask(  
    ('[#40 ]', )),
)
    mdb.models['Model-1'].ConcentratedForce(cf2=-0.5, createStepName='BUCKLE',
        distributionType=UNIFORM, field='', localCsys=None, name='Load-2', region=
        mdb.models['Model-1'].rootAssembly.sets['Set-16'])

mdb.models['Model-1'].rootAssembly.Set(name='NODES', nodes=
    mdb.models['Model-1'].rootAssembly.instances['BEAM-1'].nodes.getSequenceFromMask(  
    mask=('
    #0:2 #ffffff #7ff #0:36 #49220000 #92492492 #24924924',
    ' #249 '),
)
    mdb.models['Model-1'].ConcentratedForce(cf2=-1.0, createStepName='BUCKLE',
        distributionType=UNIFORM, field='', localCsys=None, name='Load-3', region=
        mdb.models['Model-1'].rootAssembly.sets['NODES'])

### ADD NODE FILE (Model-1) ### (model 1)
    mdb.models['Model-1'].keywordBlock.synchVersions(storeNodesAndElements=False)
    mdb.models['Model-1'].keywordBlock.insert(93, '\n*NODE FILE\nU')

### ADD IMPERFECTION (Model-2) ### (model 2)
    mdb.models['Model-2'].keywordBlock.synchVersions(storeNodesAndElements=False)
    mdb.models['Model-2'].keywordBlock.insert(76, \n'*IMPERFECTION, FILE=LBA_BEAM, STEP=1\n1, 3.17') # CHANGE####

```

Appendix F. Dimensions cross-section

![Figure F.1 Dimensions of an I-shaped cross-section](image)

**Table F.1** Dimensions and section properties of an IPE240 section acc. Fig F.1

<table>
<thead>
<tr>
<th>Dimensions:</th>
<th>Formulas:</th>
</tr>
</thead>
<tbody>
<tr>
<td>h</td>
<td>240 mm</td>
</tr>
<tr>
<td>b_f</td>
<td>120 mm</td>
</tr>
<tr>
<td>t_f</td>
<td>9.8 mm</td>
</tr>
<tr>
<td>t_w</td>
<td>6.2 mm</td>
</tr>
<tr>
<td>h_w</td>
<td>230.2 mm</td>
</tr>
<tr>
<td>h-t_f</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Section properties:</th>
<th>Formulas:</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>3779 mm²</td>
</tr>
<tr>
<td>I_y</td>
<td>3746 · 10⁴ mm⁴</td>
</tr>
<tr>
<td>I_z</td>
<td>282.7 · 10⁴ mm⁴</td>
</tr>
<tr>
<td>I_t</td>
<td>93.58 · 10⁴ mm⁴</td>
</tr>
<tr>
<td>W_{el,y}</td>
<td>312.2 · 10⁵ mm³</td>
</tr>
<tr>
<td>W_{el,z}</td>
<td>47.12 · 10⁵ mm³</td>
</tr>
<tr>
<td>W_{pl,y}</td>
<td>352.9 · 10⁵ mm³</td>
</tr>
<tr>
<td>W_{pl,z}</td>
<td>72.77 · 10⁵ mm³</td>
</tr>
<tr>
<td>W_{el,y}</td>
<td>l/(1/2)*h</td>
</tr>
<tr>
<td>W_{el,z}</td>
<td>l/(1/2)*b</td>
</tr>
<tr>
<td>W_{pl,y}</td>
<td>2*(1/8)<em>t_w</em>h_w² + t_f<em>b_f</em>(1/2)*h_w</td>
</tr>
<tr>
<td>W_{pl,z}</td>
<td>2*(1/8)<em>t_w</em>h_w² + 2*(1/8)<em>t_f</em>b_f²</td>
</tr>
</tbody>
</table>
Appendix G. Results of §5.2

In this paragraph the results of the treated cases presented in §5.2 extensive illustration of the calculation, are shown. The results of all three load cases are from that example are showed here. The graphs in the load-displacement diagrams stop when the maximum load is reached.

G.1 Results of load case 1

Table G.1 Results IPE240 section loaded with LC1

<table>
<thead>
<tr>
<th>Length mm</th>
<th>$k^*e_0$</th>
<th>$M_{cr}$ Nmm</th>
<th>$M_R$ Nmm</th>
<th>$\bar{\lambda}_{LT,FEM}$</th>
<th>$\chi_{LT,FEM}$</th>
<th>$\chi_{LT,EC3}$</th>
<th>Diff.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1800</td>
<td>0.5*3.28</td>
<td>2.34E+08</td>
<td>7.32E+07</td>
<td>0.596</td>
<td>0.883</td>
<td>0.892</td>
<td>-1.0%</td>
</tr>
<tr>
<td>2600</td>
<td>0.5*5.32</td>
<td>1.27E+08</td>
<td>6.60E+07</td>
<td>0.809</td>
<td>0.796</td>
<td>0.791</td>
<td>0.7%</td>
</tr>
<tr>
<td>3400</td>
<td>0.5*7.36</td>
<td>8.41E+07</td>
<td>5.76E+07</td>
<td>0.993</td>
<td>0.695</td>
<td>0.671</td>
<td>3.5%</td>
</tr>
<tr>
<td>4500</td>
<td>0.5*10.16</td>
<td>5.68E+07</td>
<td>4.63E+07</td>
<td>1.209</td>
<td>0.558</td>
<td>0.525</td>
<td>6.1%</td>
</tr>
<tr>
<td>5600</td>
<td>0.5*12.96</td>
<td>4.28E+07</td>
<td>3.78E+07</td>
<td>1.393</td>
<td>0.456</td>
<td>0.422</td>
<td>7.5%</td>
</tr>
<tr>
<td>7100</td>
<td>0.5*16.79</td>
<td>3.21E+07</td>
<td>3.01E+07</td>
<td>1.608</td>
<td>0.363</td>
<td>0.330</td>
<td>8.9%</td>
</tr>
<tr>
<td>8600</td>
<td>0.5*20.61</td>
<td>2.57E+07</td>
<td>2.52E+07</td>
<td>1.795</td>
<td>0.303</td>
<td>0.272</td>
<td>10.5%</td>
</tr>
<tr>
<td>10800</td>
<td>0.5*26.22</td>
<td>2.00E+07</td>
<td>2.09E+07</td>
<td>2.036</td>
<td>0.252</td>
<td>0.216</td>
<td>14.5%</td>
</tr>
</tbody>
</table>

Figure G.1 Load-displacement IPE240 length 1800

Figure G.2 Load-displacement IPE240 length 3400

Figure G.3 Load-displacement IPE240 length 5600

Figure G.4 Load-displacement IPE240 length 8600
### G.2 Results of load case 2

#### Table G.2 Results IPE240 section loaded with LC2

<table>
<thead>
<tr>
<th>Length (mm)</th>
<th>$k*e_0$</th>
<th>$F_{cr}$ (N)</th>
<th>$M_{cr}$ (Nmm)</th>
<th>$F_R$ (N)</th>
<th>$M_R$ (Nmm)</th>
<th>$\bar{\lambda}_{LT,FEM}$</th>
<th>$\chi_{LT,FEM}$</th>
<th>$\chi_{LT,EC3}$</th>
<th>Diff.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1600</td>
<td>0.5*2.77</td>
<td>572300</td>
<td>2.29E+08</td>
<td>197213</td>
<td>7.89E+07</td>
<td>0.602</td>
<td>0.951</td>
<td>0.889</td>
<td>6.5%</td>
</tr>
<tr>
<td>2300</td>
<td>0.5*4.55</td>
<td>230733</td>
<td>1.33E+08</td>
<td>121594</td>
<td>6.99E+07</td>
<td>0.791</td>
<td>0.843</td>
<td>0.801</td>
<td>5.0%</td>
</tr>
<tr>
<td>3200</td>
<td>0.5*6.85</td>
<td>105103</td>
<td>8.41E+07</td>
<td>71978</td>
<td>5.76E+07</td>
<td>0.993</td>
<td>0.694</td>
<td>0.670</td>
<td>3.5%</td>
</tr>
<tr>
<td>4400</td>
<td>0.5*9.91</td>
<td>51784</td>
<td>5.70E+07</td>
<td>41279</td>
<td>4.54E+07</td>
<td>1.207</td>
<td>0.548</td>
<td>0.526</td>
<td>4.0%</td>
</tr>
<tr>
<td>5800</td>
<td>0.5*13.47</td>
<td>29040</td>
<td>4.21E+07</td>
<td>25161</td>
<td>3.65E+07</td>
<td>1.403</td>
<td>0.440</td>
<td>0.416</td>
<td>5.4%</td>
</tr>
<tr>
<td>7500</td>
<td>0.5*17.81</td>
<td>17309</td>
<td>3.25E+07</td>
<td>15916</td>
<td>2.98E+07</td>
<td>1.598</td>
<td>0.360</td>
<td>0.334</td>
<td>7.3%</td>
</tr>
<tr>
<td>9500</td>
<td>0.5*22.91</td>
<td>10869</td>
<td>2.58E+07</td>
<td>10544</td>
<td>2.50E+07</td>
<td>1.792</td>
<td>0.302</td>
<td>0.272</td>
<td>9.8%</td>
</tr>
<tr>
<td>12000</td>
<td>0.5*29.28</td>
<td>6897</td>
<td>2.07E+07</td>
<td>7142</td>
<td>2.14E+07</td>
<td>2.002</td>
<td>0.258</td>
<td>0.223</td>
<td>13.9%</td>
</tr>
</tbody>
</table>

**Figure G.5** Load-displacement IPE240 length 1600

**Figure G.6** Load-displacement IPE240 length 3200

**Figure G.7** Load-displacement IPE240 length 5800

**Figure G.8** Load-displacement IPE240 length 9500
G.3 Results of load case 3

Table G.3 Results IPE240 section loaded with LC3

<table>
<thead>
<tr>
<th>Length (mm)</th>
<th>k*e₀</th>
<th>q_qr</th>
<th>M_cr</th>
<th>q_R</th>
<th>M_R</th>
<th>ΔLT, FEM</th>
<th>ΔLT, EC3</th>
<th>ΔDiff</th>
</tr>
</thead>
<tbody>
<tr>
<td>1400</td>
<td>0.5*2.26</td>
<td>959.72</td>
<td>2.35E+08</td>
<td>312.25</td>
<td>7.65E+07</td>
<td>0.594</td>
<td>0.923</td>
<td>0.892</td>
</tr>
<tr>
<td>2200</td>
<td>0.5*4.30</td>
<td>207.86</td>
<td>1.26E+08</td>
<td>109.54</td>
<td>6.63E+07</td>
<td>0.812</td>
<td>0.799</td>
<td>0.789</td>
</tr>
<tr>
<td>3000</td>
<td>0.5*6.34</td>
<td>72.40</td>
<td>8.14E+07</td>
<td>49.53</td>
<td>5.57E+07</td>
<td>1.009</td>
<td>0.672</td>
<td>0.659</td>
</tr>
<tr>
<td>3900</td>
<td>0.5*8.63</td>
<td>30.59</td>
<td>5.82E+07</td>
<td>24.12</td>
<td>4.59E+07</td>
<td>1.194</td>
<td>0.553</td>
<td>0.534</td>
</tr>
<tr>
<td>5100</td>
<td>0.5*11.69</td>
<td>13.08</td>
<td>4.25E+07</td>
<td>11.31</td>
<td>3.68E+07</td>
<td>1.396</td>
<td>0.443</td>
<td>0.420</td>
</tr>
<tr>
<td>6500</td>
<td>0.5*15.26</td>
<td>6.21</td>
<td>3.28E+07</td>
<td>5.69</td>
<td>3.01E+07</td>
<td>1.590</td>
<td>0.363</td>
<td>0.337</td>
</tr>
<tr>
<td>8300</td>
<td>0.5*19.85</td>
<td>2.97</td>
<td>2.56E+07</td>
<td>2.88</td>
<td>2.48E+07</td>
<td>1.800</td>
<td>0.299</td>
<td>0.270</td>
</tr>
<tr>
<td>10400</td>
<td>0.5*25.20</td>
<td>1.52</td>
<td>2.05E+07</td>
<td>1.57</td>
<td>2.12E+07</td>
<td>2.011</td>
<td>0.256</td>
<td>0.221</td>
</tr>
</tbody>
</table>

Figure G.9 Load-displacement IPE240 length 1400

Figure G.10 Load-displacement IPE240 length 3000

Figure G.11 Load-displacement IPE240 length 5100

Figure G.12 Load-displacement IPE240 length 8300
Appendix H. Calculating the $M_{pl,\theta}/M_{pl,y}$ ratio

The difficult part about calculation the $M_{pl,\theta}/M_{pl,y}$ ratio is calculating the $M_{pl,\theta}$. Since $M_{pl,y}$ can be simply calculated by:

$$M_{pl,y} = W_{pl,y} \cdot f_y \tag{3.3}$$

How to calculate $W_{pl,y}$ is shown in Appendix F. $M_{pl,\theta}$ is the moment capacity over the $y$-axis of the rotated mid-section of the beam at time of failure and can be calculated with:

$$M_{pl,\theta} = W_{pl,\theta} \cdot f_y \tag{H.1}$$

The calculation of $W_{pl,\theta}$ is calculated using (G.2):

$$W_{pl,\theta} = 2 \left( \frac{1}{6} \cdot t_w \cdot h_w^2 \cdot \cos \theta + t_f \cdot b_f \cdot \frac{1}{2} \cdot h_w \cdot \cos \theta \right) \tag{H.2}$$

Formula (G.2) can be used for $0^\circ < \theta < 60^\circ$ for an IPE240 section.

The $M_{pl,\theta}$ values calculated with formula (H.1) and (H.2) can be compared with $M_{pl,\theta}$ values generated by FEM-calculations. These values are obtained from the FE-model showed in Figure H.1.

![Figure H.1 FE-model to determine $M_{pl,\theta}$](image)

The FE-model consist of just a beam element, that is given beam properties that correspond to the section properties of the IPE240 section. The length of the beam is 4000 mm. The FEM calculation is deformation driven in the $z$-direction. Besides the boundary conditions on the beam ends it is also necessary to give boundary conditions over the length of the beam, so that only bending over the $y$-axis can occur. The beam element is rotated over the $x$-axis so that the $\theta$ can be set.

Since there are two ways explained in this appendix to determine $M_{pl,\theta}$, it makes sense to compare the outcome of the two ways with each other. For example the IPE240 section with a beam length of 14400 mm is rotated 60° in the middle of the beam, as is showed in §5.3. So $\theta$ is set to 60° and the two ways can be compared, starting with the calculation by hand:

$$W_{pl,\theta} = 2 \left( \frac{1}{6} \cdot t_w \cdot h_w^2 \cdot \cos \theta + t_f \cdot b_f \cdot \frac{1}{2} \cdot h_w \cdot \cos \theta \right) \tag{H.2}$$
The dimensions can be taken from Appendix F and formula (H.2) can be filled in:

\[ W_{pl,\theta} = 2 \left( \frac{1}{6} \cdot 6.2 \cdot 230.2^2 \cdot \cos(60^\circ) + 9.8 \cdot 120 \cdot \frac{1}{2} \cdot 230.2 \cdot \cos(60^\circ) \right) = 176.4 \cdot 10^3 \text{ mm}^3 \]

So \( M_{pl,\theta} \), calculated by hand, with \( \theta \) being 60°, then becomes:

\[ M_{pl,\theta} = W_{pl,\theta} \cdot f_y = 176.4 \cdot 10^3 \cdot 235 = 41.460 \cdot 10^6 \text{ Nmm} = 41.460 \text{ kNm} \]

When in the FE-model \( \theta \) is set to 60° the simulation is started. A load-displacement diagram is obtained from the result of the simulation. The diagram is showed in Figure H.2.

![Load-displacement diagram for an IPE240 beam length 4000 mm rotated 60° over the x-axis](image)

From the figure plastic \( F_{pl,\theta} \) is obtained, which is equal to \( 41.488 \text{ kN} \). With this value \( M_{pl,\theta} \) is determined:

\[ M_{pl,\theta} = \frac{1}{4} \cdot F_{pl,\theta} \cdot l = \frac{1}{4} \cdot 41.488 \cdot 4 = 41.488 \text{ kNm} \]

Comparing this result with the result from the hand calculation there is very little difference between the two results. The comparison between the two ways can also be done for other \( \theta \). Table H.1 shows the result of this comparison.

<table>
<thead>
<tr>
<th>( \theta )</th>
<th>( M_{pl,\theta,\text{hand}} )</th>
<th>( M_{pl,\theta,\text{FEM}} )</th>
<th>Diff</th>
</tr>
</thead>
<tbody>
<tr>
<td>°</td>
<td>kNm</td>
<td>kNm</td>
<td>-</td>
</tr>
<tr>
<td>0</td>
<td>82.920</td>
<td>83.366</td>
<td>-0.54%</td>
</tr>
<tr>
<td>15</td>
<td>80.095</td>
<td>80.525</td>
<td>-0.54%</td>
</tr>
<tr>
<td>30</td>
<td>71.811</td>
<td>72.198</td>
<td>-0.54%</td>
</tr>
<tr>
<td>45</td>
<td>58.634</td>
<td>58.952</td>
<td>-0.54%</td>
</tr>
<tr>
<td>60</td>
<td>41.460</td>
<td>41.488</td>
<td>-0.07%</td>
</tr>
<tr>
<td>75</td>
<td>-</td>
<td>24.101</td>
<td>-</td>
</tr>
<tr>
<td>90</td>
<td>17.101</td>
<td>17.012</td>
<td>0.52%</td>
</tr>
</tbody>
</table>
Table H.1 shows very little difference between the results of the calculations by hand and the results of the FEM simulations. $M_{pl,\text{hand}}$ with $\theta = 90^\circ$ is not calculated with formula G.2, since it is stated that formula (G.2) can be used for $0^\circ < \theta < 60^\circ$ for an IPE240 section. It is calculated with the formula for $W_{\text{pl},z}$ showed in Appendix F.

The $M_{pl,\theta}/M_{pl,y}$ ratio of the IPE240 section with a beam length of 14400 is:

$$\frac{M_{pl,\theta}}{M_{pl,y}} \text{ ratio} = \frac{41.488}{83.366} = 0.498$$

From Table H.1 a graph can be made with the $M_{pl,\theta}/M_{pl,y}$ ratio from $0^\circ$ till $90^\circ$ for an IPE240 section. This graph is showed in Figure H.3.

From this graph Formula (H.3) can be derived. With this formula only the rotation of the midsection at time of failure is needed to determine the $M_{pl,\theta}/M_{pl,y}$ ratio for an IPE240 section.

$$M_{pl,\theta}/M_{pl,y} = 2.013 \cdot 10^{-8} \cdot \theta^4 - 1.583 \cdot 10^{-6} \cdot \theta^3 - 1.236 \cdot 10^{-4} \cdot \theta^2 + 4.273 \cdot 10^{-4} \cdot \theta + 1.0 \quad \text{(H.3)}$$

Formula (H.3) should only be used for an IPE240 section. In §5.3 also the $M_{pl,\theta}/M_{pl,y}$ ratio of an HEA section is showed. That was determined by following the same procedure as is showed in this Appendix.
Appendix I. Developing end of range criteria

In §5.3 is explained that cases where cross-sectional resistance around the weak axis of a section starts to influence the reduction factor $\chi_{LT}$ should not be included within this research. So the point where the influence starts can be seen as the end of the range of relative slendernesses that need to be studied. Since the cases that are loaded with load case 1 show a good match with the Taras lateral torsional buckling curves, the point where the curve from load case 1 starts to deviate from the Taras curves can be seen as the approximate end point. From this point the lateral torsional buckling is influenced too much by cross-sectional resistance around the weak axis of a section.

As is showed in §5.3 eventually all curves determined with the help of the FE-model end find an asymptote at $\chi_{LT} = \frac{M_{pl,z}}{M_{pl,y}}$. This value is unique for every section. This asymptote is used to find the end of the range point. By multiplying the $\chi_{LT}$ value with a factor an unique end of the range criteria is found. After trying different values the factor 2 is working the best. This shown in Figure I.1 where the results of an IPE240 section and the results of an HEA400 section are presented. In both cases is shown that once the line, that represents the cases for load case 1, crosses $\frac{M_{pl,z}}{M_{pl,y}} \cdot 2$ the line starts to deviate from its Taras lateral torsional buckling curves, which means the influence of the cross-sectional resistance around the weak axis of a section starts to matter.

![Figure I.1](image)

From Figure I.1a can be seen why in for example Figure 8.2 only results are shown until the relative slenderness of 1.4. In Figure 5.9 it was already shown that the influence started around a relative slenderness of 1.4 for an IPE240 section when looking at the s-curve that represents $\frac{M_{pl,z}}{M_{pl,y}}$. This means the factor 2 works well for the IPE240 section. For the HEA400 section the factor 2 also works well as can be seen in Figure I.1b. The $\frac{M_{pl,z}}{M_{pl,y}} \cdot 2$ criteria works well for all sections with $\frac{h}{b} > 1.2$. This criterion is also put in Table 5.1.
However for sections with $h/b \leq 1.2$ the factor 2 is too large and another factor needs to be chosen. Eventually the factor 1.5 is chosen. The results are shown in Figure I.2, where the results of an HEA100 section and the results of an HEA300 section are presented.

As Figure I.2 shows, the chosen factor of 1.5 works pretty well for the two HEA sections. In Figure I.2b, using the $\chi_{\text{min}}$ criteria, the influence starts around a relative slenderness of 0.95 for an HEA300. In Figure 5.10 the s-curve that represents $\frac{M_{pl,\theta}}{M_{pl,y}}$ shows that the influence starts around a relative slenderness of 1.0 for an HEA300 section. Since the end of range criteria is an approximate one, the 0.95 can be considered close enough.

So eventually the criteria is used that is shown in Table 5.1, which is also shown here in Table I.1.

<table>
<thead>
<tr>
<th>Limits</th>
<th>$\chi_{\text{min}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$h/b \leq 1.2$</td>
<td>$\frac{M_{pl,x}}{M_{pl,y}} \times 1.5$</td>
</tr>
<tr>
<td>$h/b &gt; 1.2$</td>
<td>$\frac{M_{pl,x}}{M_{pl,y}} \times 2$</td>
</tr>
</tbody>
</table>
Appendix J. Validity tests of the FE-model

J.1 Tests in bending

The tests are performed on an IPE240 beam, with boundary conditions described in §3.4. For this case the beam has a length of 5000mm. On both beam ends a bending moment of 500Nmm is put, that bends the beam around the strong axis of the beam. Figure J.1 shows the result of the bending moments on the beam.

The Figure shows the rotation over the Y-axis. With this rotation and the known inputs it is possible to calculate the moment of inertia over the strong axis. The moment of inertia of the FE-model, can be compared with theoretical value.

\[
I_y = \frac{M \cdot l}{2 \cdot E \cdot \varphi} = \frac{500 \cdot 5000}{2 \cdot 2,1 \cdot 10^5 \cdot 1,587 \cdot 10^{-7}} = 37,507 \cdot 10^6 \text{ mm}^4
\]

\[
I_{y,\text{theory}} = \frac{1}{12} \cdot 6,2 \cdot 230,2^3 + 2 \cdot 9,8 \cdot 120 \left(\frac{1}{2} \cdot 230,2\right)^2 = 37,462 \cdot 10^6 \text{ mm}^4
\]

\[
\text{Difference} = \frac{37,507 \cdot 10^6 - 37,462 \cdot 10^6}{37,462 \cdot 10^6} \times 100 = 0,12\%
\]

As can be seen, the difference between the theoretical value and the value from the FE-model are close to an exact match.
The same procedure can be used to check the moment of inertia over the weak axis. Again, the beam has a length of 5000mm. On both beam ends a bending moment of 500Nmm is put, that bends the beam around the weak axis of the beam. Figure J.2 shows the result of the bending moments on the beam.

\[
I_z = \frac{M \cdot l}{2 \cdot E \cdot \varphi} = \frac{500 \cdot 5000}{2 \cdot 2 \cdot 10^5 \cdot 2 \cdot 105 \cdot 10^{-6}} = 2,828 \cdot 10^6 \text{ mm}^4
\]

\[
I_{z,\text{theory}} = \frac{1}{12} \cdot 6,2 \cdot 230,2^3 + 2 \cdot \frac{1}{12} \cdot 9,8 \cdot 230,2^2 = 2,827 \cdot 10^6 \text{ mm}^4
\]

\[
\text{Difference} = \frac{2,828 \cdot 10^6 - 2,827 \cdot 10^6}{2,827 \cdot 10^6} \times 100 = 0.035\%
\]

As can be seen above the difference between the theoretical value and the value from the FE-model again are close to an exact match.

Figure J.2 IPE240 beam subjected to bending over the weak axis

In this case the figure shows the rotation over the Z-axis. With the rotation and the known inputs it is possible to calculate the moment of inertia over the weak axis. The moment of inertia of the FE-model, can be compared with theoretical value.
J.2 Test in torsion

In this case the beam is 2500mm. For this test different boundary conditions are used. On the right side a clamped support is used, which means in comparison with the simple support also the rotation over the z-axis is restrained. On the left side a torsional moment of 1000Nmm is put. Figure J.3 shows the result of the torsional moment on the clamped beam.

![Figure J.3 IPE240 beam subjected to a torsional moment](image)

In the Figure the rotation over the x-axis is shown. With the rotation and the known inputs it is possible to calculate the torsional moment of inertia and compare this with the theoretical value.

\[
I_t = \frac{M \cdot l}{G \cdot \varphi} = \frac{1000 \cdot 2500}{80769 \cdot 3.379 \cdot 10^{-4}} = 91,602 \cdot 10^3 \text{ mm}^4
\]

\[
I_{t,\text{theory}} = 2 \left( \frac{1}{3} \cdot 120 \cdot 9.8^3 \right) + \frac{1}{3} \cdot 230.2 \cdot 6.2^3 = 93,583 \cdot 10^3 \text{ mm}^4
\]

\[
\text{Difference} = \frac{91,602 \cdot 10^3 - 93,583 \cdot 10^3}{93,583 \cdot 10^3} \times 100 = -2.11\%
\]

As shown, the value from the FE-model is only 2% of from the theoretical value. Important to notice is that the formulae to calculate the theoretical value does not give the exact value, but a good approximation of the value. This somewhat explains the difference between the value of the FE-model and the theoretical value.
J.3 One element under tension

For this test one element from the FE-model is tested. The element is an eight node element with four integration points, that is also shown in §3.2. The element is clamped on the left side and pulled on at the right side. The element is showed on Figure J.4.

![Element with boundary conditions](image)

By putting the element under tension it is possible to check if the stress-strain diagram from the element matches the material properties (§3.3) which are given to the element. The Young’s modulus of elasticity has been set to $2,1 \cdot 10^5$ N/mm$^2$, for the Poisson’s ratio 0,3 has been taken and the yield strength is set at 235 N/mm$^2$. Figure J.5 shows the stress-strain diagram.

![Stress-displacement diagram of the element](image)

The figure clearly shows the bilinear material behavior, with the yield stress of 235 N/mm$^2$. By calculating the slope of elastic part of the diagram it is possible to calculate the Young’s modulus of elasticity. This is done here below.

$$E = \frac{113,431}{5,4 \cdot 10^4} = 210057 \text{ N/mm}^2 \approx 210000 \text{ N/mm}^2$$

As can be seen the calculated value matches the given value.
Appendix K. More results for §8.3

K.1 IPE sections

IPE80

Figure K.1 Comparison $X_{LT,FEM}$ with $X_{LT,Taras}$ for an IPE80 section
Figure K.2 Comparison $\chi_{LT,FEM}$ with $\chi_{LT,Taras}$ for an IPE600 section.
K.2 HEA sections

HEA sections with h/b > 1.2

HEA400

Figure K.3 Comparison $\chi_{LT,FEM}$ with $\chi_{LT,Taras}$ for an HEA400 section
Figure K.4 Comparison $\chi_{LT,FEM}$ with $\chi_{LT,Taras}$ for an HEA1000 section
HEA sections with h/b ≤ 1.2 - without adjustments

HEA100

Figure K.5 Comparison $\chi_{LT,FEM}$ with $\chi_{LT,Taras}$ for an HEA100 section

HEA360

Figure K.6 Comparison $\chi_{LT,FEM}$ with $\chi_{LT,Taras}$ for an HEA360 section
HEA sections with \( h/b \leq 1.2 \) - with adjustments

HEA100

\[ \lambda_{LT} = \sqrt[\frac{M_{pl}}{M_{cr}}} \]

Figure K.7 Comparison \( \lambda_{LT,FEM} \) with \( \lambda_{LT,Taras} \) for an HEA100 section

HEA360

\[ \lambda_{LT} = \sqrt[\frac{M_{pl}}{M_{cr}}} \]

Figure K.8 Comparison \( \lambda_{LT,FEM} \) with \( \lambda_{LT,Taras} \) for an HEA360 section
K.3 HEB sections

HEB sections with h/b > 1.2

HEB400

\[
\lambda_{LT} = \sqrt{\frac{M_{pl}}{M_{cr}}}
\]

Figure K.9 Comparison \(\chi_{LT,FEM}\) with \(\chi_{LT,Taras}\) for an HEB400 section

HEB600

\[
\lambda_{LT} = \sqrt{\frac{M_{pl}}{M_{cr}}}
\]

Figure K.10 Comparison \(\chi_{LT,FEM}\) with \(\chi_{LT,Taras}\) for an HEB600 section
HEB sections with $h/b \leq 1.2$ - with adjustments

HEB300

\[
\lambda_{LT} = \sqrt{\frac{M_{pl}}{M_{CT}}}
\]

Figure K.11 Comparison $\chi_{LT,FEM}$ with $\chi_{LT,Taras}$ for an HEB300 section