A fast all-optical 2R regenerator based on POLIS

Nambale, C.

Award date:
2007
A fast all-optical 2R regenerator based on POLIS

By C. Nambale
A fast all-optical 2R regenerator based on POLIS

By C. Nambale

Master of Science Thesis
carried out from 17/01/07 to 23/08/07

Supervisor:
Dr. J.J.G.M. van der Tol

Graduation Professor:
Prof. Dr. ir. M.K. Smit

The Faculty of Electrical Engineering of Eindhoven University of Technology disclaims all responsibility for the contents of traineeship and graduation reports.
Summary

Optical communication systems are playing an increasingly important role in today’s communications networks. This is because they provide huge capacity, allowing networks to support bandwidth hungry applications like video, online games and peer-to-peer applications. Like in all communication channels, signal degradation occurs in fiber optic cables; the physical communication channel used in most optical communication systems. Signal degradation necessitates the amplification of the signal every 50-200 km depending on the type of fiber cabling in use. In addition to signal power degradation, there is an accumulation of noise and dispersion due to various reasons. This means that the signal has to be re-amplified and re-shaped in order to improve the extinction ratio and the Optical Signal-to-Noise Ratio (OSNR). A device that re-amplifies and re-shapes a signal is called a 2R regenerator. A 2R regenerator based on interference effects of linearly polarized light and nonlinear effects in Semiconductor Optical Amplifiers (SOAs) is proposed. The regenerator is realized using a novel photonic integration method that takes advantage of the different behavior of strained quantum well materials towards Transverse Electric (TE) and Transverse Magnetic (TM) polarizations. A study of the SOA ultrafast nonlinear processes is carried out. These effects are described using mathematical models that are then used to numerically demonstrate the operation of the 2R regenerator.
Contents

1 Introduction 1
  1.1 Optical regeneration ................................................. 2
    1.1.1 Optical losses in fibers ........................................ 2
    1.1.2 Shape degradation .............................................. 3
  1.2 2R regeneration .................................................... 4
  1.3 Optical amplifiers .................................................. 5
    1.3.1 Principle of operation of optical amplifiers .................. 5
  1.4 Quantum wells ..................................................... 7
    1.4.1 Strained Quantum wells ........................................ 8
  1.5 POLIS ............................................................. 9
  1.6 Structure of the report ............................................ 10

2 Physics of semiconductor optical amplifiers 11
  2.1 Introduction to optical gain ....................................... 11
  2.2 SOA ultrafast carrier phenomena .................................. 12
    2.2.1 Spectral Hole Burning (SHB) .................................... 12
    2.2.2 Carrier-Carrier Scattering ..................................... 13
    2.2.3 Carrier-Phonon scattering ..................................... 13
    2.2.4 Two-Photon absorption ......................................... 13
    2.2.5 Carrier Heating ................................................ 14
    2.2.6 Ultrafast nonlinear refraction ................................ 15
  2.3 Ultrafast carrier dynamics in SOAs ................................ 16
  2.4 SOA dynamics - the rate equations ................................ 17
    2.4.1 Recombination terms ............................................ 18
    2.4.2 The pump term .................................................. 19
    2.4.3 The spontaneous emission term ................................ 19
    2.4.4 The stimulated emission term ................................ 20
    2.4.5 The optical loss term ......................................... 21
    2.4.6 The phenomenological rate equations ........................... 21
    2.4.7 SOA model ...................................................... 22
    2.4.8 Solutions for TE and TM polarized pulses ..................... 24
    2.4.9 Internal loss ................................................... 24
    2.4.10 Amplified Spontaneous emission ................................ 24
    2.4.11 High repetition rates ......................................... 25
3 The 2R regenerator
3.1 Introduction ........................................... 27
  3.1.1 TE and TM polarization modes ................. 27
3.2 The polarization converters .......................... 28
  3.2.1 Mathematical modeling of the polarization converter ..... 28
3.3 Principle of operation of the 2R regenerator ........... 31
  3.3.1 An approximation for the transmission .......... 34

4 Device Operation
4.1 The SOA ................................................. 39
4.2 SOA characteristics .................................. 40
  4.2.1 SOA gain dynamics ............................... 40
  4.2.2 Pulse phase dynamics ............................ 41
  4.2.3 Output pulse shape and size .................... 42
  4.2.4 Effect of ASE .................................... 42
  4.2.5 Effect of a pulse train .......................... 43
4.3 Polarization converters ............................... 44
4.4 The 2R regenerator .................................. 46
  4.4.1 Power characteristic ............................. 46
  4.4.2 Pulse trains ..................................... 49
  4.4.3 Performance in an Additive White Gaussian Noise channel .... 55
  4.4.4 Noise cancelation control using a phase shifter ........ 56

5 Conclusions and Recommendations ....................... 59
  5.1 Conclusions .......................................... 59
  5.2 Recommendations ................................... 60

A The SOA model 1 ......................................... 67
  4.1 Discretization ..................................... 70

B The SOA model 2 ......................................... 71
  4.1 Conditions for the simulations .................... 75
<table>
<thead>
<tr>
<th>Symbol/Abbreviation</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>2R</td>
<td>Re-amplification and Re-shaping</td>
</tr>
<tr>
<td>AR</td>
<td>Active Region</td>
</tr>
<tr>
<td>ASE</td>
<td>Amplified Spontaneous Emission</td>
</tr>
<tr>
<td>AWGN</td>
<td>Additive White Gaussian Noise</td>
</tr>
<tr>
<td>BER</td>
<td>Bit Error Rate</td>
</tr>
<tr>
<td>CH</td>
<td>Carrier Heating</td>
</tr>
<tr>
<td>CR</td>
<td>Carrier Reservoir</td>
</tr>
<tr>
<td>DCF</td>
<td>Dispersion-Compensating Fibre</td>
</tr>
<tr>
<td>EDFA</td>
<td>Erbium Doped Fiber Amplifier</td>
</tr>
<tr>
<td>FPA</td>
<td>Fabry-Perot Amplifier</td>
</tr>
<tr>
<td>FWHM</td>
<td>Full Width at Half Maximum</td>
</tr>
<tr>
<td>fs</td>
<td>Femtosecond</td>
</tr>
<tr>
<td>Gbps</td>
<td>Gigabits per second</td>
</tr>
<tr>
<td>GVD</td>
<td>Group Velocity Dispersion</td>
</tr>
<tr>
<td>K-K</td>
<td>Kramers-Kronig</td>
</tr>
<tr>
<td>hh</td>
<td>Heavy hole</td>
</tr>
<tr>
<td>lh</td>
<td>Light hole</td>
</tr>
<tr>
<td>MZI</td>
<td>Mach-Zehnder Interferometer</td>
</tr>
<tr>
<td>NRZ</td>
<td>Non-Return-to-Zero</td>
</tr>
<tr>
<td>O/E/O</td>
<td>Optical-Electrical-Optical</td>
</tr>
<tr>
<td>OH</td>
<td>Hydroxyl</td>
</tr>
<tr>
<td>OSNR</td>
<td>Optical Signal-to-Noise Ratio</td>
</tr>
<tr>
<td>QD-SOA</td>
<td>Quantum Dot Semiconductor Optical Amplifier</td>
</tr>
<tr>
<td>QW-SOA</td>
<td>Quantum Well Semiconductor Optical Amplifier</td>
</tr>
<tr>
<td>PC</td>
<td>Polarization Converter</td>
</tr>
<tr>
<td>PMD</td>
<td>Polarization Mode Dispersion</td>
</tr>
<tr>
<td>POLIS</td>
<td>POLarization based Integration Scheme</td>
</tr>
<tr>
<td>PRBS</td>
<td>Pseudorandom Bit Sequence</td>
</tr>
<tr>
<td>ps</td>
<td>Picosecond</td>
</tr>
<tr>
<td>RZ</td>
<td>Return-to-Zero</td>
</tr>
<tr>
<td>SHB</td>
<td>Spectral Hole Burning</td>
</tr>
<tr>
<td>SLA</td>
<td>Semiconductor laser Amplifier</td>
</tr>
<tr>
<td>SOA</td>
<td>Semiconductor Optical Amplifier</td>
</tr>
<tr>
<td>SPM</td>
<td>Self Phase Modulation</td>
</tr>
<tr>
<td>TE</td>
<td>Transverse Electric</td>
</tr>
<tr>
<td>THz</td>
<td>Terahertz</td>
</tr>
<tr>
<td>TM</td>
<td>Transverse Magnetic</td>
</tr>
<tr>
<td>TPA</td>
<td>Two Photon Absorption</td>
</tr>
<tr>
<td>TWA</td>
<td>Traveling Wave Amplifier</td>
</tr>
<tr>
<td>UNR</td>
<td>Ultrafast Nonlinear Refraction</td>
</tr>
<tr>
<td>WDM</td>
<td>Wavelength Division Multiplexing</td>
</tr>
<tr>
<td>Symbol</td>
<td>Definition</td>
</tr>
<tr>
<td>--------</td>
<td>--------------------------</td>
</tr>
<tr>
<td>$c$</td>
<td>Speed of light</td>
</tr>
<tr>
<td>$q$</td>
<td>Elementary charge</td>
</tr>
<tr>
<td>$h$</td>
<td>Planck's constant</td>
</tr>
<tr>
<td>$k$</td>
<td>Boltzmann's constant</td>
</tr>
</tbody>
</table>
Chapter 1

Introduction

Optical communication systems have played an increasingly important role in telecommu-
nications since the invention of lasers. Today most of the long-haul, and increasingly even
short-haul broadband transmission, is accomplished with optical communication networks.
This is because the high frequency of the optical carrier avails a very large bandwidth.
As an example, at 1.5 $\mu m$ wavelength the carrier frequency is 200 THz, therefore, using the
Shannon-Hartley theory of information

$$C = B \log_2 \left(1 + \frac{S}{N}\right)$$

(1.1)

this implies that a 1 Tbit/s communication channel only occupies about 1% of the available
bandwidth. In Equation (1.1), $C$ is the channel capacity in bits per second, $B$ is the channel
bandwidth in Hertz and $S/N$ is the signal-to-noise ratio.

Although transmission through optical fibers is less attenuated than, for example: elec-
trical signals through copper, or radio signals through wireless communication media.
Signal power degradation through various processes like, material absorption, splicing
losses and Rayleigh scattering takes place. As a result of this degradation the optical
signal has to be re-amplified periodically. This can be achieved using any of the several
types of optical amplifiers, like Raman or Erbium doped fiber amplifiers. Unfortunately,
optical amplification introduces noise due to the spontaneous emission that occurs during
the amplification process. When the spontaneous emission noise is amplified in subsequent
amplifier stages, Amplified Spontaneous Emission (ASE) becomes an issue. After a number
of amplifier stages the ASE becomes larger than the intrinsic photodetector noise. The
OSNR is degraded by a factor of at least 3 dB at each amplifier stage, for ideal amplifiers,
but this may be as large as 6-8 dB for practical amplifiers [1]. Other optical phenomenon
like Group Velocity Dispersion (GVD) and Polarization Mode Dispersion (PMD) lead to
pulse widening as the signal travels along the length of the fiber cable. Re-shaping is
thus required to narrow the pulse width to counter the effects of dispersion and also to
improve the optical signal-to-noise ratio of the transmission. Lastly, re-synchronization
which involves re-timing by reducing the timing jitter of the signal may be performed.
Devices that perform all the functions mentioned above are called 3R regenerators, usually
only the first two functions are necessary, and devices that perform this operation are called
2R regenerators.

Traditionally, regeneration has been attained by converting the signal from the optical
domain to the electrical domain, performing the regeneration, and converting it back to the
optical domain. This process requires a number of components to complete, and because there are many components, the regeneration process becomes a factor in determining the power consumption of the device. This makes Optical-Electrical-Optical (O/E/O) conversion an inefficient and power hungry operation. Optical regeneration mitigates this problem by eliminating the need for conversion into the electrical domain. It also reduces the number of components that are needed to regenerate a signal, thereby reducing the cost of regeneration.

In addition to the drawbacks mentioned above, the demand for more bandwidth from communication networks is always on the increase. As of the year 2003 Internet traffic was judged to be growing at a rate that doubled every year, meaning that the annual growth was between 70 and 150% [2]. Today there are about 215 million fixed broadband subscribers, making up about 56.2% of all the internet users [3]. Between 2000 and 2004 the number of fiber-to-the-home users increased from 50,000 to 3 million. This shows that there is a considerable growth in network access demand which results in an increased demand for greater communication capacity. Therefore, optical networks are being pushed to operate at higher speeds requiring more complicated electronic devices. These complex devices are more difficult to manufacture and therefore more expensive. At a certain critical speed it is no longer possible, with the current technology, to perform regeneration within the electrical domain in an acceptable time frame. All-optical regeneration is the only option in such cases.

In this report we study an all-optical 2R regenerator based on semiconductor optical amplifiers (SOA) and polarization converters.

1.1 Optical regeneration

Optical regeneration is necessary because of the fact that as the optical pulses used in communication systems travel along optical fibers they lose some energy due to scattering and absorption. Furthermore, pulse broadening occurs due to dispersion effects in the fibers.

1.1.1 Optical losses in fibers

Though optical communication through other media exists the most common transmission channel for optical communications is optical fiber. The fiber optic field developed rapidly in the sixties mainly for the purpose of transmitting images. These early fibers were extremely lossy (loss>1000 dB/km). Through the years, these losses have been reduced to about 0.2 dB/km near a wavelength of 1.55 \( \mu m \) [4]. As one may expect, the losses are wavelength dependent. The main causes of fiber loss are material absorption and Rayleigh scattering.

Material absorption

Material absorption occurs when energy from the incoming light pulse is lost as vibration of atoms through the interaction of the electric field of the light pulse and the crystal structure of the fiber cable. In silica fiber, vibrational resonances are in the ultra violet and infrared regions of the optical spectrum. In these regions, incident pulses are heavily absorbed by the fiber cable. There is an operating window between 1.0 \( \mu m \) and 2.0 \( \mu m \) where the absorption is very low. This window is appropriate for optical communications
and it is the most common operating window of most optical communication systems. However, even in this wavelength window impurities in the fiber can lead to a significant amount of absorption. The most important contributory impurity in fibers cables is the OH ion whose vibrational resonance lies at wavelengths between, 1.3 and 1.5 \( \mu m \). Modern fiber cables are produced using processes that ensure that the OH ions are limited to one part in 100 million [5].

**Rayleigh scattering**

As the light travels in a fiber optic cable it interacts with the silica molecules of the cable. These inelastic collisions between the light and the silica molecules scatters light in all directions. This is known as Rayleigh scattering. It is akin to the scattering that happens in the earth’s atmosphere leading to the various colors of the sky at different times of the day. Rayleigh scattering is wavelength dependent and is of the order of \( \lambda^4 \) and is dominant at the shorter wavelengths [5]. This loss is intrinsic to the fiber, and therefore it sets the ultimate limit on fiber loss.

**Other losses**

Others causes of optical signal attenuation include: losses while coupling the fiber cable to devices or other fiber cables. These are sometimes referred to as splicing losses. Losses through bends, core-cladding scattering and imperfections in the crystal structure of the silica.

Because of these effects repeated amplification of the signals is necessary between the transmitter and the receiver. Amplifications of the signal using optical devices like rare earth doped fiber amplifiers and semiconductor optical amplifiers means that there is amplified spontaneous emission after each amplification stage. Because ASE contributes to noise there is a marked decrease in the signal-to-noise ratio of the signal at each stage. This means that regenerative mechanisms are required to improve the extinction ratio of the digital signal.

### 1.1.2 Shape degradation

#### Group velocity Dispersion

Even with today’s modern lasers, the linewidth of the output signal contains a considerable number of frequency components or colors. Even if the optical signal from the laser was very pure, modulating it leads to a creation of additional colors. Since the refractive index of materials \( n(\omega) \) is generally a function of the frequency then \( \Delta n \propto \Delta \lambda \propto \Delta \omega \). Given the guided wave propagation constant \( \beta(\omega) = n(\omega) \frac{c}{\omega} \) if a Taylor series expansion is made around the central frequency \( \omega_0 \)

\[
\beta(\omega) = \beta_0 + \beta_1(\omega - \omega_0) + \frac{1}{2} \beta_2(\omega - \omega_0)^2 + ..., \tag{1.2}
\]

where

\[
\beta_m = \left( \frac{d^m \beta}{d\omega^m} \right)_{\omega = \omega_0} \quad (m = 1, 2, 3...). \tag{1.3}
\]
The parameters $\beta_1$ and $\beta_2$ are related to $n$ and its derivatives through the relations [4]

\begin{align}
\beta_1 &= \frac{1}{v_g} = \frac{n_g}{c} = \frac{1}{c} \left( n + \omega \frac{dn}{d\omega} \right), \quad (1.4) \\
\beta_2 &= \frac{1}{c} \left( \frac{2}{d\omega} \frac{dn}{d\omega} + \omega \frac{d^2 n}{d\omega^2} \right), \quad (1.5)
\end{align}

$n_g$ is the group index and $v_g$ is the group velocity. Equations (1.4), (1.5) show that the group velocity is frequency dependent. This phenomenon, called the group velocity dispersion, has temporal effects on the optical pulse. It leads to pulse broadening, since the different colors travel at different group velocities. $\beta_2$ represents the dispersion of the group velocity and is responsible for pulse broadening. It is related to the group velocity by the dispersion parameter $D$

\begin{equation}
D = \frac{d\beta_1}{d\lambda} = -\frac{2\pi c}{\lambda^2} \beta_2 \approx \frac{\lambda}{c} \frac{d^2 n}{d\lambda^2}. \quad (1.6)
\end{equation}

GVD is usually mitigated with passive optical schemes like the use of dispersion-compensating fibers (DCF) or dispersion-shifted fibers [1]. 2R regeneration also helps in mitigating the effects of GVD, since it induces pulse narrowing.

### Polarization Mode Dispersion

Another form of dispersion whose effect is not as significant as GVD is polarization mode dispersion (PMD), which is caused by imperfections in the fabrication of the fiber cable. This causes the differently polarized signals in the fiber to propagate at different velocities. It’s effects are only significant in extremely long haul links and very at high data rates.

#### 1.2 2R regeneration

Regeneration of the signal can be achieved by passing it through an optical gate with a nonlinear transfer function. Figure 1.1 shows an ideal transfer function and the improvement of the extinction ratio of a signal. The 2R regenerator works by suppressing the noise peaks around the logical ones and zeros in digital transmissions. The regenerator considerably enhances parts of the signal that have a medium peak due to the shape of the middle part of the transfer function. This means that there is an improvement in both the extinction ratio and the optical signal to noise ratio. However it should be noted that regenerators like these, based on a single transfer function do not lead to an improvement in the bit error rate (BER) although they reduce the degradation of the BER [6].

Most traditional all-optical regenerators are based on the interferometric switch concept. These include 2R regenerators based on the Mickelson (MI) and Mach-Zehnder Interferometer (MZI) [7, 8]. These set-ups require two physical paths to provide the two optical signals that are required for the interference. This makes these configurations expensive in terms of chip area. The fast all-optical 2R regenerator based on POLIS is dependent on the interferometric switch concept as well. The nonlinear transfer is achieved through the use of orthogonally polarized optical paths instead of two interferometer arms. In Section 3, the construction and operation of this device is discussed in greater detail.
1.3 Optical amplifiers

When amplification of an optical signal is performed in a device within the optical domain this device is known as an optical amplifier. Optical amplifiers can use the same physical mechanism as a laser. In amplifiers unlike lasers, optical feedback is avoided through anti-reflection coatings. They also have a higher pumping rate or longer interaction length than lasers, in order to obtain a reasonable signal gain. The longer interaction length is satisfied by most rare-earth doped fiber amplifiers, well as semiconductor optical/laser amplifiers (SLA) satisfy the condition for a high material gain through high pumping. This is study focusses on the use of the traveling wave amplifier (TWA), where the input signal traverses the active medium only once. TWA require anti-reflection coatings so that the signal does not resonate in the amplifier cavity. Only the TWA is considered because of its superior gain bandwidth, signal gain saturation and noise figure.

1.3.1 Principle of operation of optical amplifiers

The principle of operation of EDFA and SOAs is similar. Due to its importance in understanding the operation of the 2R regenerator, a brief description is appropriate for the semiconductor optical amplifier.

The allowed energy levels of atoms are obtained though the solution of Shrödinger’s equation using the appropriate electronic potentials. Taking the simple starting point of an infinite potential well, the one dimensional time-independent Shrödinger’s equation can be written as

\[ -\frac{\hbar^2}{2m} \frac{d^2\psi(x)}{dx^2} - E\psi(x) = 0, \]

where \( m \) is the mass of an electron, \( \hbar \) is Planck’s constant divided by \( 2\pi \), \( x \) is the dimension and \( E \) is the energy. A solution of this second order linear differential equation is

\[ \psi(x) = B \sin(kx), \]

where \( k^2 = \frac{2mE}{\hbar^2} \).

The solution of \( \psi(x) \) describes the motion of the electrons at different energy levels. In solids like crystals, the infinite potential well becomes a group of periodic potential wells.
which will lead to the splitting of the energy levels into bands. From above it is evident that

\[ E_n = \frac{\hbar^2 k^2}{2m} = \frac{p^2}{2m}. \]  

(1.9)

This equation is used to describe the parabolic energy to momentum diagrams that are often used to describe the conduction and valence bands.

Consider an amplifier with two energy bands \(^1\), the conduction band and the valence band. For semiconductors at \( T=0 \) K the valence band is filled and the conduction band is empty. When thermal or other energy is added to the system, electrons in the valence band may be excited into the conduction band. Likewise under certain conditions electrons may loose energy resulting in an electron leaving the conduction band and recombining with a hole in the valence band. This process may be accompanied by the generation of a photon (light) or another form of energy.

\[ \text{CB} \]
\[ \text{VB} \]

(a) (b) (c)

Figure 1.2: Electron transitions in an Amplifier: (a) Spontaneous emission, (b) Stimulated emission, (c) Absorption

Figure 1.2 illustrates the different types of transitions that occur in semiconductor amplifiers or lasers. (a) shows the transition of an electron from the conduction band to the valence band leading to the generation of a single photon. This process called spontaneous emission, is the principle cause of amplifier noise in optical systems since the photons generated by this process are incoherent. The second illustration details the transition of an electron from the conduction band to the valence band due to an incident photon, as shown in (b). The photon produced by this stimulated transition is coherent with the original photon; it has the same wavelength, phase and direction as the incident photon. By the generation of the second coherent photon the active medium has amplified the incident light. The third process illustrated in (c) shows the incident photon being destroyed. Its energy is used to promote an electron from the valence band into the conduction band. This process is known as absorption. For amplification to take place, population inversion must occur in the active region. This means that the number of carriers in the conduction band has to be higher than the number of carriers in the valence band. This is achieved by electrical pumping as illustrated in Figure 1.3, or by optical pumping via absorption. Note that each photon above has an energy \( hf \) equal or greater than the band gap energy \( E_g = E_c - E_v \). Here \( h \) is Planck's constant and \( f \) is the frequency of the incident light.

\(^1\)The materials treated in this report are considered as two band systems; an approach suitable for gaseous and solid-state amplifiers.
1.4 Quantum wells

Quantum well semiconductor devices have attracted considerable interest due to their pump and gain advantages over conventional bulk semiconductors. When the active region of a semiconductor laser or optical amplifier is decreased until it reaches the size of the de-Broglie wavelength, its physical properties are changed. Such a thin slab of semiconductor material is called a quantum well.

Materials can be categorized by the degrees of freedom of the confined electrons. Bulk semiconductors have a 3-D confinement because the electron has three degrees of freedom. Quantum wells have a 2-D confinement, quantum wires a 1-D and quantum dots 0-D.

Because conventional bulk material devices have dimensions that are much larger than the electron wavelength, discrete energy levels can be approximated as a continuum of electron energies and wavelengths. However, once the active region width falls below 20nm, quantum effects emerge and the permitted energies are quantized between $E_{\text{act}}$ and $E_{\text{clad}}$, the active and cladding region energies respectively. Due to the smaller effective mass of the electrons compared to the holes, the quantization in the conduction band is much larger than the quantization in the valence band. In the valence band sub-band appear. Their energies are split into ‘heavy-hole’ (hh) and ‘light-hole’ (lh) sub-bands named so because of the difference in the effective mass of the ‘heavy-holes’ and the ‘light-holes’. The first ‘light-hole’ level typically lies further from the valence band edge than the first ‘heavy-hole’ sub-band. Later on, it is shown that this influences the gain properties of the quantum well devices with respect to the TE and TM polarizations. Transitions are only possible at energies $E_{Q1}$, $E_{Q2}$, as shown in Figure 1.4. Only the lowest states are occupied, except at high carrier-injection levels. Because the transverse physical dimensions of the
active region are very small the carriers are confined to two dimensions forcing them to occupy a planar region. The density of states function becomes a staircase as opposed to $\sqrt{E - E_c}$ for bulk material. Figure 1.5 shows the density of states in a bulk and quantum well structure. The density of states in the conduction band of quantum wells are filled in a different manner from that in bulk materials, with a step like function as opposed to the continuous function of bulk material. Filling occurs at discrete defined levels, which means that fewer density of states are needed for a given energy level in quantum well material than in bulk material. Therefore, it is easier to reach population inversion in quantum wells, meaning that injection currents are reduced. The different density of states function also means that the gain spectrum of quantum well devices is different, in that the gain peak is much higher, with the gain spectrum being compressed.

The spectral width of spontaneous emission and gain should be narrower in the quantum well devices than for bulk material, though electron-electron collisions still broaden the energy bands. This is especially the case at higher injection currents as additional energy levels are filled.

Because of the step-like density of states distribution of electrons and holes in quantum well material, the energy distribution does not vary a lot with temperature. Therefore the injection current required for transparency in quantum well SOAs is more stable than that of bulk semiconductor SOAs.

Transition probabilities, which result in gain in quantum wells, depend on the direction as well as the magnitude of momentum ($k$) of the charge carriers. Therefore the polarization of the electric field and its orientation with respect to the quantum wells also helps to determine whether TE or TM polarized modes are favored. This means that processes which break the lattice symmetry, such as strain in the quantum wells, will lead to different gain for TE and TM polarization. This anisotropic behavior is an advantage in this version of 2R regenerator as will be described later in Section 3.3.

Quantum well devices have similar recombination mechanisms to those of bulk materials. This means that the equations used to simulate the bulk semiconductors can be applied to quantum well devices as well [9].

1.4.1 Strained Quantum wells

Although the gain in unstrained quantum well SOAs favors the TE mode, due to its larger confinement factor, the quantum wells are strained to further enhance the TE gain and
supress the TM gain. The thin quantum wells layers allow for very large compressive or tensile strains to be applied to the material without catastrophic damage to the lattice. Strain is applied to semiconductor material by epitaxially growing a semiconductor layer on a substrate with a different lattice constant. If the lattice constant of the grown layer is larger than that of the substrate then compressive strain is generated in the grown layer. If the opposite is true tensile strain is produced in the grown layer. With the decrease in symmetry of the crystal structure, the degeneracy shown in Figure 1.6 is removed from the energy sub-bands at the gamma point \( \mathbf{k} = \mathbf{0} \). The allowed transitions from the conduction band to the heavy-hole band favor TE polarization over TM polarization. The approximate anisotropy factor [9] that accounts for polarization behavior is shown below in Table 1.1

<table>
<thead>
<tr>
<th>Polarization</th>
<th>Heavy-hole</th>
<th>Light-hole</th>
</tr>
</thead>
<tbody>
<tr>
<td>TE</td>
<td>1.5</td>
<td>0.5</td>
</tr>
<tr>
<td>TM</td>
<td>0</td>
<td>2</td>
</tr>
</tbody>
</table>

Table 1.1: Strengths of transitions in relation to the polarization of the exciting light.

Compressive strain will lead to the TE mode being favored because the light-hole band is pushed further away from the valence band edge due to the strain as shown in Figure 1.7. In the case of tensile strain the heavy-hole band is pushed further away from the valence band edge (to lower energies) and therefore this strain will favor TM polarization, see Figure 1.8.

1.5 POLIS

POLarization based integration scheme (POLIS) is a technique that was developed to take advantage of the different behavior of strained quantum well materials towards TE and TM polarizations. A direct consequence of this is that, transparency, absorption or gain for one polarization state happens at a different wavelength from that of the other polarization. This is because both polarizations interact differently with the hh and lh bands as explained in the previous section. By carefully controlling the amount of strain in the quantum well, fabrication can be achieved whereby the TE polarization interacts with the active components of the chip while it is absorbed in the passive components. The active and passive components are transparent to TM polarization. This makes it possible
Figure 1.7: Transitions in compressively strained material.

Figure 1.8: Transitions due in tensile strained material.

to integrate lasers, detectors, waveguides, polarization converters and many other devices on the same wafer, significantly reducing manufacturing costs, time and the amount of labor required for fabrication of the optical components.

Materials grown on a substrate of indium phosphide (InP) with strained quantum wells of indium gallium arsenide (InGaAs) and an InGaAsP waveguide layer [10] have been reported with these properties.

1.6 Structure of the report

This report contains results of a study of the ultrafast nonlinear processes in semiconductor amplifiers with the aim of using them in the design of a 2R regenerator, as first described by Milan Marell [11]. This work differs from his work in that by taking into account the ultrafast processes in the design, the operation of the device can be extended to include operation with bit rates of up to 40 Gbps. This study, based on polarization dependent optical paths, is expected to reduce the chip area of the 2R regenerator since it does not depend on interference effects from two different physical paths, which is a requirement with MZI and MI type devices.

Chapter 2 introduces the physics behind the ultrafast processes in the SOA. A model describing the behavior of the gain and phase response is presented in this section. Chapter 3 deals with the design and operation of the regenerator. The chapter starts with a description of the behavior of each component of the regenerator separately, finally the operational behavior of the device as a whole is presented. In Chapter 4, results from the simulations of the regenerator under different operating conditions are presented and discussed. Conclusions and recommendations are made in Chapter 5.
Chapter 2

Physics of semiconductor optical amplifiers

2.1 Introduction to optical gain

Among the energy states the lowest state is thermally the most stable. At 0 K the electrons will occupy this lowest energy state. When electrons are excited by thermal or light energy or electron beams, the electrons in the atom absorb these energies and transit to higher energy states. These transitions lead to a less stable system and therefore the excited electron will return to lower energy levels through a process known as relaxation. These relaxations could be between energy levels in the same band e.g. in the conduction band, in this case the transitions are known as intraband transitions. Intraband transitions are usually caused by an exchange of energy by carrier-carrier scattering or carrier-phonon scattering. The transitions could also happen between the conduction band and the valence band through an interband relaxation process which could be radiative, i.e. followed by the emission of a photon, where the energy of the photon corresponds to the difference in energy between the initial and final energy level. They transitions also can be non-radiative, followed by the emission of phonons whereby the energy of the excitation is lost through some means other than light. The average time that the electron spends in the excited state is known as the lifetime of that state.

In semiconductor material the number of electrons per unit volume, \( N \), allowed in a given region of the \( k \) space is given by [12]

\[
N = \frac{k}{3\pi^2},
\]

where \( k \) is the magnitude of the electron wave-vector. The density of electronic levels as a function of energy in the conduction band is:

\[
\rho_c(E) = \frac{dN}{dE} = \frac{1}{2\pi^2} \left( \frac{2m^*_e}{\hbar^2} \right)^{\frac{3}{2}} E^{\frac{1}{2}},
\]

where \( E = \hbar^2 k^2 / 2m^* \) for a parabolic band, \( m^* \) is the effective mass of either an electron or a hole, \( \hbar \) is Planck’s constant divided by \( 2\pi \).

Given that the probability that a given energy state in the conduction band is occupied
by an electron is

\[ f_c(E) = \frac{1}{1 + e^{\frac{E-E_{pc}}{k_B T_c}}} \]  

(2.3)

with \( E_{pc} \) the quasi-Fermi energy level, the density of the carriers in the conduction band is given by

\[ N = \int f_c(E) \rho_c(E) dE, \]

(2.4)

and similarly,

\[ P = \int (1 - f_v(E)) \rho_v(E) dE, \]

(2.5)

for the holes in the valence band.

Taking a transition from energy level \( E_1 \) in the conduction band to \( E_2 \) in the valence band the rate of absorption is

\[ r_{21} = B' f_v(E_2)[1 - f_c(E_1)] S(E_{21}), \]

(2.6)

with \( S(E_{21}) \) the photon flux density, energy \( hf = E_1 - E_2 \) and \( B' \) the transition probability. Similarly the rate of stimulated emission is given by

\[ r_{12} = B f_c(E_1)[1 - f_v(E_2)] S(E_{21}), \]

(2.7)

where \( B = B' \), with the net direction of the stimulated emission plus absorption being determined by the relative carrier concentrations in the two energy levels.

The incident photon experiences gain if the rate of stimulated emission exceeds the rate of absorption i.e \( r_{12} > r_{21} \). This is known as the Bernard-Duraffourg condition. The gain can be described as

\[ g(E_{21}) = \frac{r_{12} - r_{21}}{S(E_{21}) v_g} = \frac{B [f_c(E_1) - f_v(E_2)]}{c/n_g}. \]

(2.8)

This relationship will be encountered again in the discussion of the dynamic properties of SOA. For charge neutrality the number of electrons should equal the number of holes. However for gain to exist the probability that energy level \( E_1 \) is occupied should be greater than the probability that \( E_2 \) is occupied i.e. \( f_c(E_1) > f_v(E_2) \).

### 2.2 SOA ultrafast carrier phenomena

Ultrafast light pulses traveling through an SOA affect the operation of SOA by changing the gain and refractive index of the amplifier. The phenomena that are responsible for this behavior in the SOA are described below.

#### 2.2.1 Spectral Hole Burning (SHB)

When an SOA is impinged upon by a light pulse at a certain wavelength the stimulated emission and absorption are limited to the range of wavelengths in that pulse. This means that only carriers at certain energy levels are affected by this interaction. Thus, only carriers of those energy levels are depleted by this interaction leading to a burning of a spectral hole in the carrier distribution. This spectral hole instantaneously reduces the
gain at the corresponding wavelength. In an SOA biased in the gain region SHB leads to a gain compression that lasts about 115 ± 10 fs [13]. The implication of this is that in semiconductor materials stimulated transitions disturb Fermi distribution. The width of the spectral hole depends on the \( k \)-vector selectivity and the homogenous dephasing rate thought to be extremely fast at high carrier densities. SHB is observed in the gain and absorption regions. In the absorption region an optical beam generates carriers in only a limited energy range, thus bleaching a hole in the absorption. At the transparency point there is no net stimulated transitions and therefore no SHB.

### 2.2.2 Carrier-Carrier Scattering

Carrier-carrier scattering determines the exchange of energy between carriers, mainly by collisions. Photoexcited carriers collide with carriers at thermal equilibrium, exchanging energy until the system returns to equilibrium. These carrier interactions can be electron-electron, hole-hole or electron-hole scattering. The differences in the effective masses between electrons and holes reduce the energy exchange between them [15]. It should be noted that the scattering rate is very much dependent on the density [16]. For semiconductors biased at gain this phenomenon will usually occur within 30-150 fs after a high intensity pulse has created a spectral hole in the semiconductor [17].

### 2.2.3 Carrier-Phonon scattering

When excited carriers collide with phonons (vibrations of the semiconductor lattice), they lose some of their energy and momentum to the lattice. This interaction determines the thermal relaxation of a photoexcited semiconductor and also affects the transport and gain properties of the semiconductor. Carrier-Phonon scattering leads to carrier heating (cooling) and it occurs on a timescale of 1 ps [18]. Changes in carrier temperatures cause changes in gain, because gain is a sensitive function of carrier temperature. An increase of a few degrees leads to a decrease in gain of several percent [19]. Heated electrons and holes cool to lattice temperature in a duration that is manifested as a gain recovery timescale. Carrier temperature increase is also seen to lead to an increase in refractive index due to a change in carrier density [20].

### 2.2.4 Two-Photon absorption

Two Photon Absorption (TPA) is the excitation of an electron to a higher energy level by two photons of the same wavelength. In the TPA process the transition from the ground state (1) to the excited state(2) is accomplished via an intermediate state that is represented by a virtual energy level as shown in Figure 2.1a. One Photon Absorption (OPA) shown in the same figure happens via real energy levels 2 and 3. If the electron is absorbed from the second energy level to the third as shown in Figure 2.1b, free carrier absorption (FCA) has occurred instead of TPA.

Two-photon absorption is not exhibited at low pulse intensities because the absorption cross section \( \sigma \) that describes this process increases linearly with laser intensity according to,

\[
\sigma = \sigma_2 I, \quad (2.9)
\]

where \( \sigma_2 \) is a coefficient that describes TPA. Because the atomic transition rate due to TPA is given by \( R = \sigma I/h\omega \) which becomes \( R = \sigma_2 I^2/h\omega \) the effects of TPA scale as a
At higher pump intensities, TPA is manifested by carrier heating which leads to changes in the non-linear refractive index and gain. Its effects are instantaneous with the arrival of the high intensity pulse at the SOA. It should be noted that TPA can lead to gain suppression because it causes carrier heating, however this effect is negligible for most SOAs biased in the gain regime. On the other hand it may enhance the gain through the creation of free carriers. Experiments and numerical simulations show that the effect of TPA increases with the reduction in pulse width [22].

2.2.5 Carrier Heating

When the carriers have achieved a Fermi distribution, the temperature of this distribution may differ from that of the lattice temperature. When electron and hole distributions are at a temperature higher than the lattice temperature the gain of the SOA is reduced. Heating the carrier distribution compresses the gain and shifts the gain peak to shorter wavelengths (higher energy).

There are several means by which carrier distributions are heated. FCA, for example, where a photon is absorbed by a conduction band electron that is then excited to a higher energy level in the conduction band creating highly energetic carriers. The same can happen for holes in the valence band. Through carrier-to-carrier scattering, the hot carriers transfer their energy to the rest of the distribution leading to a heating of the total carrier distribution. Carrier heating unlike SHB, reduces the gain across the entire range of energies. This is because the electron and hole quasi-Fermi energy levels shift to higher energies further in the absorption regime [14].

Carrier heating also occurs when there are energy changes due to stimulated transitions. Carriers occupying energy levels below the average energy are ‘cold’, whereas those occupying levels above the average energy are ‘hot’. Photons with energy equal to or higher than the bandgap energy ($E_g$) incident on this system stimulate emissions, removing the cold electrons and holes from the distribution. Removing the cold carriers effectively heats the distribution.
2.2.6 Ultrafast nonlinear refraction

Usually spectral broadening due to Self Phase Modulation (SPM) in an SOA is attributed to saturated gain that leads to a reduction in carrier density, and hence an increase in the refractive index. However, even at transparency where there is no gain, some effects of SPM are visible in SOAs. The recovery time for this non-linearity is governed by intraband relaxation for which a recovery time of the order of 1 ps is expected [23]. Ultrafast nonlinear refractive index change is therefore governed by several overlapping processes.

For a semiconductor whose absorption \( \alpha(\omega) \) or gain \( g(\omega) \) at a given frequency is known, the refractive index \( n(\omega) \) can be obtained by the Kramers-Kronig (K-K) relation which connects the real part of an analytic complex function to an integral containing the imaginary part of the same function, and vice-versa,

\[
n(\omega, 0) = \frac{c}{\pi} \int_0^\infty \frac{\alpha'(\omega', 0)}{\omega'^2 - \omega^2} d\omega'
\]

or

\[
n(\omega, 0) = \frac{c}{\pi} \int_0^\infty \frac{g'(\omega', 0)}{\omega'^2 - \omega^2} d\omega'.
\]

In nonlinear cases, there is a perturbed change of the refractive index due to a photonic optical field intensity \( I \), therefore

\[
\Delta \alpha_{NL}(\omega, I) = \alpha(\omega, I) - \alpha(\omega, 0).
\]

The changes in gain due to a change in the carrier density \( N \) can be characterized by:

\[
\Delta g_0 = g_0(\omega, N_1) - g_0(\omega, N_2).
\]

Therefore the total refractive index is thus given by [12]:

\[
n(\omega, I) = n(\omega, 0) + \Delta n_{NL}(\omega, I),
\]

where the change in the refractive index \( \Delta n(\omega, I) \) from the change in the absorption \( \Delta \alpha(\omega, I) \) or gain is:

\[
\Delta n_{NL}(\omega, I) = \frac{c}{\pi} \int_0^\infty \frac{\Delta \alpha_{NL}(\omega', I)}{\omega'^2 - \omega^2} d\omega'.
\]

For photon energies near the bandgap increasing the carrier density leads to a decrease in refractive index. The relationship showing how the complex refractive index is affected by carrier density is described using the linewidth enhancement factor

\[
\alpha_N = -\frac{\delta n/\delta N}{\delta n'/\delta N} = -\frac{4\pi \delta n/\delta N}{\lambda \delta g/\delta N},
\]

\( n \) and \( n' \) are the real and imaginary refractive index respectively. This shows that the linewidth enhancement factor is wavelength dependent. Notice that the denominator of the last term is the differential gain or the gain coefficient \( \delta g/\delta N = a_N \). For small signal gain \( g \approx a_N(N - N_{tr}) \), where \( N_{tr} \) is the carrier density at transparency.

Nonlinear index changes associated with carrier temperature can also be characterized by an effective parameter \( \alpha_T \),

\[
\Delta g_0 = g_0(\omega, T_1) - g_0(\omega, T_2).
\]
Using the Kramer-Krönig transform it is seen that increasing the carrier temperature leads to a gain compression and an increase in the refractive index. These temperature induced changes are related via [12]:

\[ \alpha_T = -\frac{\delta \chi / \delta T}{\delta \chi / \delta Teq} = -\frac{4\pi \delta n / \delta T}{\delta g / \delta T} \] (2.18)

The long-lived refractive index change exists in two regimes: gain and absorption. There are other refractive index phenomena that exist in all regimes, including transparency. This includes:
a) A transient increase in refractive index lasting around 600 fs and consistent with delayed carrier heating.
b) An instantaneous decrease in refractive index after around 150 fs attributed to electronic and virtual processes like TPA [12], [24].

2.3 Ultrafast carrier dynamics in SOAs

A semiconductor optical amplifier biased in the gain region is first assumed to be in a steady state, with no incident light in the active region. Carriers in the SOA in this state achieve a Fermi distribution as shown in Figure 2.2a, for the conduction band electrons. Note that due to electrical pumping by carrier injection, the carrier distribution is no longer in thermal equilibrium because there are many electrons in the conduction band and many holes in the valence band. In this case the Fermi level \( (E_F) \) cannot describe the distribution functions of the electrons and holes. To remedy this it is assumed that the electrons in the conduction band, and the holes in the valence band, are governed by separate distribution functions defined by quasi-Fermi levels \( (E_{Fc}) \) and \( (E_{Fv}) \) for the conduction band and the valence band respectively [25]. The carrier concentrations are then given by

\[ n = N_c \exp \left( -\frac{E_c - E_{Fc}}{k_B T} \right) \] (2.19)

\[ p = N_v \exp \left( -\frac{E_{Fv} - E_v}{k_B T} \right) \] (2.20)

\( n \) and \( p \) are the electron and hole concentrations, respectively. \( E_c \) and \( E_v \) are the energy of the bottom of the conduction band and the top of the valence band, respectively. \( k_B \) is Boltzmann’s constant and \( T \) is the absolute temperature in Kelvin units. \( N_c \) and \( N_v \) are the effective density of states for the electrons in the conduction band and the holes in the valence bands respectively.

The Fermi distribution functions of electrons in the valence band with energy \( E_1 \), and that of electrons with energy \( E_2 \) in the conduction band are then given by the following expressions

\[ F_v(E_1) = \frac{1}{1 + e^{\left( \frac{E_1 - E_{Fv}}{k_B T} \right)}} \] (2.21)

\[ F_c(E_2) = \frac{1}{1 + e^{\left( \frac{E_2 - E_{Fc}}{k_B T} \right)}} \] (2.22)
The distribution function for holes in the valence band would be given by $1 - F_v(E_1)$, because holes are simply missing electrons.

![Figure 2.2: Conduction band carrier dynamics in a SOA when an intense pulse excites it.](image)

At time $t = 0$ an intense, spectrally narrow and short light pulse is incident in the active region of the SOA. This pulse causes stimulated emission, whereby carriers in a spectrally narrow region are depleted through the interband recombination process. This is because the light causes the stimulated transitions only between specific energy levels $h\nu \geq E_c - E_v$. A spectral hole is formed in the carrier distribution, as shown in Figure 2.2b.

Note that it has been assumed that the spectral range of the light pulse described above is within the gain band of the SOA. When the spectral hole is formed it is immediately followed by a reduction in gain at the corresponding wavelength of the SOA, due to the reduction in the number of free carriers that are available for stimulated emission. Within 30-150 fs [26] the spectral hole will be washed out by carrier to carrier scattering, creating a quasi-Fermi distribution at an elevated temperature. This means that even when carriers have achieved a Fermi distribution, the temperature of this distribution may be higher than the lattice temperature Figure 2.2c.

The quasi-Fermi distribution is at an elevated temperature because the stimulated transition have removed the relatively cold electrons and holes, effectively heating the distribution [12]. The cooling of the quasi-Fermi distribution to lattice temperature occurs via carrier-phonon scattering, this process takes approximately 2 ps [17], and it is shown Figure 2.2d.

The full recovery of the gain profiles requires the injection of carriers to replace those lost through the stimulated transitions. Depending on the design of the active medium and the biasing point of the the SOA, this process takes the order of a few nanoseconds to complete Figure 2.2e.

### 2.4 SOA dynamics - the rate equations

Rate equations provide the most fundamental descriptions of photonic devices like lasers and semiconductor amplifiers. They describe the variation of different qualities within the device, like the carrier density and the photon density, with time and space. Rate equations are an expression of energy conservation in terms of the number of carriers in an energy level. Elementary reasoning and physical experience lead to these phenomenological
equations, though an effort to describe them in fundamental physical terms is attempted here. The rate equation for the carrier density has a basic form,

$$\frac{dN}{dt} = \text{Generation} - \text{Recombination}, \quad (2.23)$$

where $N$ are the number of carriers in an active region $N = NV_{act}$. This equation can be expanded into the appropriate processes that lead to generation and recombination,

$$\frac{dN}{dt} = -(\text{Stimulated Emission}) + (\text{Absorption}) + (\text{Non-Radiative Recombination}) - (\text{Spontaneous Recombination}). \quad (2.24)$$

A similar equation can be written relating the carrier dynamics to the total number of photons in an active region,

$$\frac{d\Upsilon}{dt} = + (\text{Stimulated Emission}) - (\text{Absorption}) - (\text{Optical Loss}) + (\text{Fraction of spontaneous emission}), \quad (2.25)$$

where $\Upsilon$ is the total number of photons in the modal volume $\Upsilon = SV_{mod}$. The optical losses are due to the scattering of photons from the active volume.

### 2.4.1 Recombination terms

Recombination of carriers can occur in three ways in a semiconductor. Monomolecular (nonradiative), bimolecular (radiative) and Auger recombination (nonradiative). The recombination terms can be given by,

$$R_r = R_{\text{radiative}} + R_{\text{nonradiative}} = AN + BN^2 + CN^3, \quad (2.26)$$

where $A$ is the inverse of the average lifetime of the carriers $\tau_n$, in either the conduction or valence band i.e. the time they can exist before being trapped in the gap states in the absence of other processes. This process depends on only one type of carrier initially, hence it scales linearly with $N$, the carrier density [30]. $B$ is a constant of proportionality due
to a recombination between both types of carriers, hence it scales as $N^2$, if it is assumed that there are equal numbers of electrons and holes. This is almost always the case due to the requirement for charge neutrality. The second term is due to spontaneous emission and it results in the production of a photon. The third term is due to Auger recombination which occurs after interactions of two electrons and a hole, hence the proportionality $N^3$. The energy from this process is nonradiative and it is transferred to phonons heating up the semiconductor.

The effective carrier lifetime can be defined as

\[
\frac{1}{\tau_c} = A + B N + C N^2.
\]  

(2.27)

This means that the total recombination rate can be written as \( R_r = N/\tau_c \).

### 2.4.2 The pump term

The number of electron-hole pairs that contribute to the photon emission process in each unit of volume (cm\(^3\)) of the active region in each second can be related to the current by the equation

\[
J = \eta_i \frac{I}{q V_{act}} ,
\]  

(2.28)

\( J \) is the pump-current number density, \( \eta_i \) is the internal quantum efficiency of the SOA. It is a measure of what percentage of the current that is pumped into the semiconductor is available as carriers that produce photons on recombination in the active region. \( q \) is the elementary charge, and \( V_{act} \) is the active volume.

### 2.4.3 The spontaneous emission term

The rate of spontaneous emission in the active volume that couples to the propagating mode, is

\[
R_{sp} = \beta B N^2 ,
\]  

(2.29)
B is the same as the term used in the bimolecular radiative recombination above. \( \beta \) is the percentage of the emitted photons that couple into the propagating mode.

2.4.4 The stimulated emission term

The stimulated emission term is given by \( R_{stim} \) which is the number of photons produced or absorbed per unit volume in a given time. For stimulated emission or absorption to take place a photon has to be incident on a medium, therefore the change in the total number of photons in the modal volume must be proportional to the number of photons present in the active volume

\[ R_{stim} V_{mod} = V_{mod} \frac{dS}{dt}_{stim} \sim V_{act} S, \]

where \( V_{mod} \) is the volume of the SOA medium which interacts with the laser beam called the "modal volume". \( V_{act} \) is the volume of the active medium and \( S \) is the photon density. If a temporal gain \( g_t \) is defined as a constant of proportionality we get

\[ R_{stim} = \frac{dS}{dt}_{stim} = \frac{V_{act}}{V_{mod}} g_t S = \Gamma g_t S, \]  

(2.30)

\( \Gamma \) is the confinement factor, it is the ratio of the active volume to the modal volume. It gives the percentage of the total optical power that is found in the gain region \( V_{act} \), keeping in mind the fact that part of the optical power will be found in the cladding region of the SOA as shown in the Figure 2.5. \( V_{mod} \) is the actual volume of the optical energy.

![Diagram](image)

Figure 2.5: Photons in the modal volume which lies in the active and cladding regions

The temporal gain \( g_t \) is the temporal gain parameter in units of \( s^{-1} \) and it must depend on the number of excited carriers \( n \) in the semiconductor. Therefore \( g_t = g_t(n) \). It is common to define the change of photons with length rather than time by using the chain rule of differentiation on Equation (2.30).

\[ \frac{dS}{dt} = \frac{dS}{dz} \frac{dz}{dt} = v_g \frac{dS}{dz}, \]  

(2.31)

where \( v_g \) is the group velocity. If \( g = g(n) = g_t/v_g \)

\[ \frac{dS}{dz} = \Gamma g S. \]  

(2.32)
Note that $g$ depends on the number of carriers per unit volume. It is also dependent on $v_g$, which is also dependent on the refractive index of the material $(c/n)$, therefore it is called the material gain. $g$ is a property of the material.

### 2.4.5 The optical loss term

The optical loss from the semiconductor cavity depends on the average photon lifetime. Intuitively the rate of change of photons is equal to the ratio of the number of photons and the photon lifetime. Therefore

$$\frac{dS}{dt} = -\frac{S}{\tau_s},$$

where $\tau_s$ is the photon lifetime.

### 2.4.6 The phenomenological rate equations

The terms mentioned above are combined to form the SOA rate equations.

$$\frac{dN}{dt} = -v_g g S + J - \frac{N}{\tau_e},$$

for good materials $A=C=0$ so the Equation (2.34) becomes

$$\frac{dN}{dt} = -v_g g S + J - B N^2.$$

The photon rate equation is given by,

$$\frac{dS}{dt} = \Gamma v_g g(n) S - \frac{S}{\tau_s} + \beta B N^2.$$  

These equations are the fundamental rate equations used to model SOAs. Depending on the circumstances more terms can be added to account for phenomena like TPA, SHB, UNR etc. Similarly some of the terms that are accounted for therein can be dropped if their effect on the dynamic modeling of the SOA are presumed to be negligible.
2.4.7 SOA model

In the previous sections, an account of various processes that take place within the active region of the SOA when it is excited by a light pulse has been given. There are various mathematical models in literature that can describe these effects in several different ways. However given the complexity of these physical phenomenon and various limitations in describing them mathematically, one has to make an educated choice when it comes to which model to use in their research. In this project, the model that is most appropriate is one which is able to describe the output pulse shape and size, the variation of the SOA gain as the pulse traverses the amplifier, the effect of the physical phenomenon mentioned earlier in this chapter on the output pulse, and the relaxation of the amplifier back to its initial state. To this end two SOA models were considered for this work, one by Tang and Shore [22] and the other by Mecozzi and Mørk [31].

The Tang and Shore model is the more complete model as it can describe most of the ultrafast effects like TPA, FCA, SHB, and CH and ultrafast nonlinear refraction. For this project the motivation for using this model would be the effect of TPA which is not considered in the Model by Mecozzi and Mørk. The Tang model was used considering the rate equations as shown in Appendix A. Comparisons were made for light pulses with different input pulse energy keeping in mind that the 2R regenerator design that is being implemented is for telecom purposes, where the maximum expected signal strength is about 100 mW.

![Figure 2.6: 1 watt pulse, no TPA considered (left) and with TPA (right)](image)

The results from the Figures 2.6 and 2.7 show that TPA starts affecting the pulse shape at an average input energy of about 1PJ for a pulse power of about 1 watt. Below this pulse energy, the effects of TPA do not manifest themselves. This means that for this application of the 2R regenerator this particular model would be overkill. Furthermore, the model does not properly account for the behavior of the amplifier gain after the pulse has left the active region of the amplifier. This means that it is difficult to model a train of pulses with this system of equations, making it difficult to determine the usefulness of the 2R regenerator in an actual transmission simulation.

Due to the reasons explained above, the SOA model used in the simulations is taken from Mecozzi and Mørk [31], details of which are presented in Appendix (B). The model presents a set of coupled differential equations based on the semiconductor rate equations,
which describe the effect of photoexcitation on the gain of the SOA. Given an input pulse description, the resulting output pulse can be computed anywhere along the length of the amplifier if the initial condition of the SOA is known.

If the modal gain $g_m(t, z) = h_N + \sum_\beta(h_{CH\beta} + h_{SHB\beta})$, the differential equations that describe the evolution of the transmission with the optical pulse are:

$$
\frac{dh_N}{dt} = -\frac{h_N}{\tau_s} - \frac{1}{S_s \tau_s} [G(t, z) - 1]S(t, 0) + \frac{g_0(z)}{\tau_s},
$$

$$
\frac{dh_{CH\beta}}{dt} = -\frac{h_{CH\beta}}{\tau_{CH\beta}} - \frac{\epsilon_{CH\beta}}{\tau_{CH\beta}} [G(t, z) - 1]S(t, 0),
$$

$$
\frac{dh_{SHB\beta}}{dt} = -\frac{h_{SHB\beta}}{\tau_{SHB\beta}} - \frac{\epsilon_{SHB\beta}}{\tau_{SHB\beta}} [G(t, z) - 1]S(t, 0) - \frac{dh_{CH\beta}}{dt} - y_\beta \frac{dh_N}{dt},
$$

$$
G = \exp(g_m),
$$

$h_N$ represents the part of the gain that is associated with the interband effects, $h_{CH}$ is the gain associated with carrier heating, and $h_{SHB}$ with spectral hole burning. $\beta = c, v$ indicates the conduction or the valence band. For simulations in this study, only the conduction band is considered because the heating effects in the valence band can be ignored due to the much shorter relaxation time of the holes. $\epsilon_{CH}$ is the nonlinear gain suppression factor due to carrier heating and $\epsilon_{SHB}$ is the nonlinear gain suppression factor due to spectral hole burning. $\tau_{SHB}$ and $\tau_{CH}$ are the carrier-carrier scattering time and the temperature relaxation time respectively.

$g_m(t, z)$ is proportional to the integral of the gain coefficient over the active waveguide length. Therefore it depends on the gain dynamics in the SOA. The length of the amplifier is usually a few hundred micrometers, and since the pulse widths are in the picosecond range, group velocity dispersion is not noticeable [32] in the amplifier. Therefore this model ignores it. GVD will only become noticeable if the duration of the pulses reduces to sub-picosecond scales.
2.4.8 Solutions for TE and TM polarized pulses

We assume that with a proper strain in the quantum wells we can eliminate any gain for the TM mode. The differential Equations (2.37) can be stated for the TE polarization as:

\[
\frac{dh_{TE}}{dt} = -h_{TE}^{N} \frac{1}{\tau_{s}}[G_{TE}(t, z) - 1]S(t, 0) + \frac{g_{0}^{TE}(z)}{\tau_{s}},
\]

\[
\frac{dh_{CH}}{dt} = -\frac{h_{TE}^{N}}{\tau_{CH}} \frac{\epsilon_{CH,\beta}}{\tau_{CH}}[G_{TE}(t, z) - 1]S(t, 0),
\]

\[
\frac{dh_{SHB}}{dt} = -\frac{h_{TE}^{N}}{\tau_{SHB}} \frac{\epsilon_{SHB,\beta}}{\tau_{SHB}}[G_{TE}(t, z) - 1]S(t, 0) - \frac{dh_{CH}}{dt} - \frac{dh_{TE}}{dt}. \tag{2.39}
\]

If the gain is considered to be zero for the TM polarized pulse then all the values of \( h \) and \( \epsilon_{ij} \) which are gain dependent above will reduce to zero. This means that the TM polarized pulse propagates without any change through the amplifier.

The resulting changes in phase for the two polarizations treated separately are:

\[
\phi_{TE}(t, z) = \phi_{TE}(t, 0) - \frac{1}{2} \alpha_{N}[h_{N}^{N} - g_{0}(z)] - \frac{1}{2} \sum_{\beta} \alpha_{T_{\beta}} h_{CH,\beta}^{N}, \tag{2.40}
\]

\[
\phi_{TM}(t, z) = \phi_{TM}(t, 0). \tag{2.41}
\]

2.4.9 Internal loss

The theoretical framework for modeling the SOA does not provide an avenue to add the effects from the internal loss. However this can be done by simply considering that the expression for the saturated gain can be written as

\[
g(t) = \int_{0}^{L} [\Gamma g(t, z) - \alpha_{int}]dz, \tag{2.42}
\]

where \( L \) is the length of the amplifier, \( \alpha_{int} \) is the internal loss of the amplifier and \( g(t, z) \) the material gain of the SOA. Numerical solution of this equation is possible by breaking the amplifier into several small sections [33] and calculating an appropriate value of the attenuators \( A_{\Delta z} = \exp(-\alpha_{int}\Delta z) \). These attenuators are a result of the internal loss in each section of the amplifier. The section sizes are chosen so that the amplifier behavior matches known experimental values.

2.4.10 Amplified Spontaneous emission

Spontaneous emission (SE) noise is an important feature that should be considered in the operation of all SOA based devices. In the operation of the 2R regenerator, it is ironic that the regenerator itself will also be a contributor of noise to the optical signal. This section analyzes the ASE noise features of the SOA in order to determine the extent of the additional noise that is introduced by the insertion of the 2R regenerator in the optical channel.

It has been shown [33] that the total power at the amplifier output with the inclusion of the ASE noise contribution is

\[
P_{out} = [G - 1]P_{in}, \tag{2.43}
\]
with

\[ P_{\text{in}} = |E_{\text{in}} + E_{\text{ASE}}|^2, \]  

(2.44)

where \( E_{\text{in}}, E_{\text{ASE}} \) are the input signal field and the ASE noise source respectively. This means that the ASE noise contribution is simulated by an equivalent noise source that is injected into the ideal SOA together with the input signal. The noise field \( E_{\text{ASE}} \) is a band-limited complex white noise. A limitation of this model is that it assumes that the ASE noise is propagating in the direction of the input pulse only. In reality this is not the case since ASE noise propagates in both directions.

Assuming that the power spectral density of the ASE noise is constant in the band of interest, the power of the input noise can be given by

\[ P_{\text{ASE}} = h\omega_0 n_{\text{sp}} B_N, \]  

(2.45)

where \( n_{\text{sp}} \) is the population inversion factor and \( B_N \) is the noise equivalent bandwidth. Using a random number generator, the ASE noise power can be multiplied with the resulting random numbers and fed into the input of the amplifier to simulate the effect of ASE.

### 2.4.11 High repetition rates

Gain recovery from phenomena like TPA and SHB have been considered for single isolated pulses. After a pulse has excited the SOA, the relaxation mechanisms mentioned in the previous sections may not be able to put the SOA back to its original state, before next pulse arrives. This is especially noticeable as the repetition period \( T_r \) becomes smaller than the carrier lifetime, which together with the pumping determines the gain recovery time. The overall transmission amplification reduces with each pulse in a pulse train, because the SOA gain does not recover to the unsaturated value between pulses. The carrier density \( N_i \) just before the arrival of a pulse can thus be presented as

\[ N_i = N_f + (N_0 - N_f) \left[ 1 - \exp \left( \frac{-T_r}{\tau_c} \right) \right], \]  

(2.46)

where \( N_f \) is the carrier density just after the pulse leaves the SOA, \( N_0 \) is the steady state carrier density with no input pulse present in the amplifier. This expression can be presented in terms of the signal gain as

\[ G_i = G_f \left( \frac{G_0}{G_f} \right)^{\left[ 1 - \exp\left( -\frac{T_r}{\tau_c} \right) \right]}. \]  

(2.47)

This equation shows that there is a correlation between the gain just before a pulse enters the SOA and the repetition period. This is a major limitation of SOAs in high speed optical applications. However at optimum speeds the gain recovery is sufficient for the SOA to reach a quasi-equilibrium after a few pulses. The attainment of this quasi-equilibrium means that SOA use is possible in some high speed applications with no significant impact, due to a change in the SOA gain. It should be noted that in digital systems some encoding methods make the attainment of this quasi-equilibrium impossible because of pattern effects caused by the arrival of arbitrary 1's and 0's.
Chapter 3

The 2R regenerator

3.1 Introduction

This overview of the operation of the 2R regenerator will be taken with digital signals in mind. There are two types of digital signal modulation formats for optical signals, the return-to-zero (RZ) format and the non-return-to-zero (NRZ) format. The RZ format has a bit slot that is wide enough, so that the amplitude of a pulse representing a logical ‘1’ returns to zero within the bit slot. With the NRZ format, the bit slot is equal to the pulse duration, meaning that the NRZ occupies less bandwidth but is more susceptible to pulse broadening. Throughout this report, the description of the the logical ‘1’ and ‘0’ is according to the RZ format. Therefore a pulse during the clock duration represents a ‘1’ and no pulse in a clock duration represents a ‘0’.

3.1.1 TE and TM polarization modes

For light to be properly guided in a waveguide it needs to propagate at certain finite and discreet angles corresponding to the stable field conditions. These are known as modes of the waveguide. Consider the simple case of a three slab waveguide shown in Figure 3.1. It is seen that if an electromagnetic wave is launched into the waveguide with the electric field vector everywhere perpendicular to the plane of the zigzag propagation, then the polarization mode is considered transverse electric (TE). Transverse because it is transverse to the interfaces of the waveguide. Similarly in the case where the magnetic field vector is everywhere perpendicular to the plane of propagation of the zigzag electromagnetic wave as shown in Figure 3.2, the polarization mode is considered to be transverse magnetic (TM).

Figure 3.1: TE mode note that z’ and y’ are at an angle to z and y

27
3.2 The polarization converters

The 2R regenerator which will be treated in more detail in the following sections is constructed with two SOA’s and three polarization converters. The polarization converter (PC) is an asymmetric channel waveguide with a high refractive index contrast. If the waveguide is sufficiently narrow the penetration of the waveguide mode in the substrate regions is balanced by their penetration of the asymmetric region of the waveguide section near the slanted facet. Therefore the modes are rotated and they become ‘TE’ and ‘TM’ with respect to the angled facet instead of TE and TM, with respect to the waveguide/substrate interface. The polarization rotation angle is dependent on the material and construction of the slanted angle waveguide of the polarization converter [34]. A compact converter has been reported and it has an almost vertical side-wall (84°) and a slanting side-wall that is in the (111) crystal plane (54.7°)[35].

Due to this construction, hybrid modes containing both TE and TM parts now propagate in the slanted waveguide with enough confinement but with different propagation constants $\beta_0$, $\beta_1$. These hybrid modes are orthogonal to each other and they propagate at an angle with the horizontal. After traveling a half-beat length $L = \frac{\pi}{2(\beta_0 - \beta_1)}$ polarization conversion will take place through constructive interference. This means that an incident TM-only mode is polarized into two hybrid modes, after traveling a distance that is equal to the half beat length of the converter, the hybrid modes interfere to induce a TE-only polarized mode. The opposite conversion takes place if the incident wave is TE mode only.

3.2.1 Mathematical modeling of the polarization converter

Several physical phenomena that involve the coupling of two signals can be represented by coupled mode theory. The problem of the propagation and interaction of optical radi-
ation also falls into this category and we rely on it to simulate the interaction of the two polarizations in the polarization converters.

Assumptions are made that the waveguide fabrication is in such a way that there is no mismatch in the propagation constants of the TE and TM waves. This is the required condition for obtaining hybrid modes under 45° in the converter. Considering that the polarization converter will be grown using POLIS material, a further assumption is that the losses in the polarization converter affect only the TE polarization so that the TM polarization is transmitted without any losses. These assumptions are a result of experimental observations which have shown that losses due to TE propagation in POLIS material is about 181 dB/cm and less than 3 dB/cm for TM [41].

Starting with the coupled mode equations with no polarization mismatch between TE and TM polarization. TE and TM polarization coupled mode equations are stated, rather than equations involving the hybrid modes. This treatment is necessary because we have to account for the absorption of the TE polarization. This is not possible if we construct the coupled mode equations around the hybrid modes.

\[ \frac{dA}{dz} = ikB, \]
\[ \frac{dB}{dz} = ikA - \alpha B, \] (3.1) (3.2)

here \( A \) and \( B \) represent the amplitudes of the TM and TE polarizations respectively. \( \alpha \) is the absorption of the TE polarized light. Substituting (3.1) into (3.2)

\[ \frac{d^2B}{dz^2} + \alpha \frac{dB}{dz} = -k^2B. \] (3.3)

The characteristic equation of the homogenous differential Equation (3.3) is

\[ c^2 + c\alpha + k^2 = 0. \] (3.4)

The roots of the quadratic equation are

\[ c = \frac{-\alpha \pm \sqrt{\alpha^2 - 4k^2}}{2}. \] (3.5)

In order to make the equations more readable a new coefficient \( k' = \sqrt{k^2 - \frac{\alpha^2}{4}} \) is introduced, therefore

\[ c = \frac{-\alpha}{2} \pm ik'. \] (3.6)

The known solution for these equations is \( \exp(\alpha z) \), so that \( A \) and \( B \) can be written as:

\[ B = e^{-\frac{\alpha z}{2}} [C_1 \cos(k'z) + iC_2 \sin(k'z)] \]
\[ ikA = \frac{dB}{dz} + \alpha B, \]
\[ = -\frac{\alpha}{2} e^{-\frac{\alpha z}{2}} [C_1 \cos(k'z) + iC_2 \sin(k'z)] + e^{-\frac{\alpha z}{2}} k'[ -C_1 \sin(k'z) + iC_2 \cos(k'z)], \]
\[ + \alpha e^{-\frac{\alpha z}{2}} [C_1 \cos(k'z) + iC_2 \sin(k'z)], \]
\[ = \frac{\alpha}{2} e^{-\frac{\alpha z}{2}} [C_1 \cos(k'z) + iC_2 \sin(k'z)] + e^{-\frac{\alpha z}{2}} k'[ -C_1 \sin(k'z) + iC_2 \cos(k'z)]. \] (3.7) (3.8)
For an initial condition where only the TE (B) mode is incident on the polarization converter at \( z = 0 \), we have \( A(0) = 0, B(0) = B_{in} \)

\[
B(0) = B_{in} = C_1,
\]

\[
 ikA = e^{-\alpha z} \left[ \frac{\alpha}{2} C_1 \cos(k'z) + k' i C_2 \cos(k'z) \right],
\]

meaning that,

\[
 C_2 = \frac{i \alpha B_{in}}{2k'}.
\]

The general solution for \( A \) and \( B \) is then:

\[
 B = B_{in} e^{-\alpha z} \left[ \cos(k'z) - \frac{\alpha}{2k} \sin(k'z) \right], \tag{3.9}
\]

\[
 A = i \left[ \frac{\alpha^2}{4k'k} + \frac{k'}{k} \right] e^{-\alpha z} B_{in} \sin(k'z). \tag{3.10}
\]

If we take the initial condition as only incident TM, \( B(0) = 0 \) and \( A(0) = A_{in} \) at \( z = 0 \), then from Equation (3.7)

\[
 C_1 = 0,
\]

and

\[
 ikA_{in} = k' i C_2,
\]

meaning that

\[
 C_2 = \frac{k}{k'} A_{in}.
\]

The Equations (3.7) and (3.8) then become:

\[
 B = \frac{k}{k'} e^{-\alpha z} A_{in} \sin(k'z), \tag{3.11}
\]

\[
 ikA = \frac{\alpha}{2} e^{-\alpha z} A_{in} \frac{k}{k'} \sin(k'z) + i e^{-\alpha z} A_{in} \frac{k}{k'} \cos(k'z).
\]

This simplifies to

\[
 A = A_{in} e^{-\alpha z} \left[ \frac{\alpha}{2k'} \sin(k'z) + \cos(k'z) \right]. \tag{3.12}
\]

Thus given a transfer matrix

\[
 \begin{pmatrix}
 A_{out} \\
 B_{out}
 \end{pmatrix} = \begin{pmatrix}
 A_1 & A_2 \\
 B_1 & B_2
 \end{pmatrix} \begin{pmatrix}
 A_{in} \\
 B_{in}
 \end{pmatrix}, \tag{3.13}
\]

with the solutions for the coefficients in the transfer matrix given by:

\[
 A_1 = e^{-\alpha z} \left[ \frac{\alpha}{2k} \sin(k'z) + \cos(k'z) \right], \tag{3.14a}
\]

\[
 A_2 = ie^{-\alpha z} \left[ \frac{\alpha^2 + 4k'^2}{4kk'} \right] \sin(k'z), \tag{3.14b}
\]

\[
 B_1 = ie^{-\alpha z} \frac{k}{k'} \sin(k'z), \tag{3.14c}
\]

\[
 B_2 = e^{-\alpha z} \left[ \cos(k'z) - \frac{\alpha}{2k'} \sin(k'z) \right]. \tag{3.14d}
\]
The transfer matrix takes an input composed of the complex amplitudes of the input TE and TM polarized light at the start of the polarization converters and gives the complex amplitude of the same at any given length \( z \). In order to determine the length of \( z \) that is required for partial polarization or full polarization a simulation of the power transfer from one polarization to the other is done using a hypothetical TM input of 1 and TE of 0.

### 3.3 Principle of operation of the 2R regenerator

![The effect of a nonlinear transfer function](image)

**Figure 3.4:** The effect of a nonlinear transfer function

The 2R regenerator studied here is similar to 2R regenerators based on interferometric switches, which have a nonlinear transfer function. The nonlinear transfer function consists of three regions with two of them (region 1 and 3) in Figure 3.4 corresponding to the mark and space levels being as flat as possible to suppress noise in these states. Region 2, with a sharp slope, helps in improving the extinction ratio. This transfer function affects the input pulse by suppressing those pulses whose power lies in region 1 because of its flat characteristic. Input signals in this region should normally correspond to a space. Signals in region 2 experience the effects of the steep slope. This is the amplification region of the transfer function, meaning that a small input is amplified at the output. This effect leads to an improvement of the extinction ratio thereby improving the optical signal-to-noise ratio.

Region 3 has the same characteristic as region 1. This almost flat region means that a wide variation in the input leads to a very small variation at the output. This ensures suppression of the noise variations that occur in the region of the mark. This corresponds to smoothing the tops of the pulses, thereby eliminating noise.

In the device mentioned by Wolfson et. al. [7] the arms of the Michelson interferometer
Figure 3.5: A 2R regenerator based on a Mach-Zehnder interferometer

have asymmetrically biased SOAs with different injection currents. The same principle can be used with a Mach-Zehnder interferometer shown in Figure 3.5. The input signal is split equally into the two arms of the interferometer. If for example the SOA in the upper arm is biased at a higher current than in the lower arm, then the SOA 1 in the upper arm will have a higher gain and a lower saturation power than the lower SOA 2. Because the phase change in the amplifiers is a function of the saturation of the amplifier due to Self Phase Modulation SPM, the phase shift in the upper amplifier (SOA 1) will start at lower input power due to its lower saturation power, meaning that the total phase difference between the signals from the two arms will be similar to that of a decision gate.

Figure 3.6 shows the variations in the phase shift as the input power of the incoming light pulse increases and the subsequent phase difference is shown in Figure 3.7. This means that the power transfer function of the regenerator will be nonlinear since it will follow the shape of the phase difference between the signals from the two arms. Depending on the phase difference between the signals from the two arms at the output coupler of the MZI, there is constructive interference or destructive interference. The SOAs are biased so that at the output coupler of the MZI, the stronger signals will have a phase difference that will constructively interfere and the weaker signals will destructively interfere. This means that the logically ones (strong pulse) will be enhanced, whereas the ASE noise (the weaker pulses) will be destroyed. It should be noted that due to the difference in gain between the two arms of the MZI the destructive interference cannot be absolute. This is a disadvantage the the 2R regenerator studied in this work does not suffer, since the SOA’s have equal biasing. In this project, the difference in phase shift is achieved by splitting the optical power unequally between the two optical paths.

The fast all-optical 2R regenerator based on POLIS operates with the same principles
expounded above. However instead of the two interferometer arms the signal is converted into two polarizations, TE and TM polarization, by polarization converters. The two amplifiers and the three polarization converters are arranged in a cascade. All the components of this device are realized with quantum well material that has been strained, so that it delivers no gain to the TM polarization. This configuration is taken from the work of Milan Marell [11] which will be described briefly.

In stage 1 the incoming TM polarization is converted into TE and TM polarizations. The polarization converter introduces a $\pi/2$ phase shift between the two polarizations. In the second stage in the first SOA (SOA 1), only the TE polarized light will experience any gain due to the nature of the POLIS SOA. Depending on the power of the optical pulse, there will be a nonlinear phase shift as the pulse traverses the amplifier. After SOA 1 a full polarization converter completely switches the polarizations of the two optical paths in stage three. Power from the TE polarized light is now TM polarized and power from the TM polarized pulse is now fully TE. In stage 4 the pulse now traverses SOA 2 and TE polarized light is amplified. At the exit of the SOA 2 the partial polarization converter now combines the two polarizations into one mode, TM in this case because the POLIS waveguides heavily attenuate (over 100 dB/cm [10]) the TE mode. The attenuation of the TE polarization is especially useful because the process of the regeneration may not have fully canceled out any unwanted noise. Now part of it is also filtered out by this attenuation. Note that the noise that is in the TM polarization will not be filtered.

When the input signal is fed into the first polarization converter which converts the signal into two orthogonally polarized waves, the amount of power that is polarized to either the TE or the TM mode depends on the design requirements of the regenerator, although certain ratios are better than others [36].

If the intention is to avoid complicated bias circuitry then the conversion between TE and TM modes should not be a 50-50 split so that the two polarized modes can
experience different phase changes in the two SOAs. This means that two optical paths are created from the single incoming data stream and because the split ratio is unequal the two optical paths experience a different phase shift as they pass through the regenerator. Finally when the two paths are recombined at the output of the 2R regenerator in the last polarization converter, there is constructive or destructive interference depending on the phase difference between the two optical paths. This phase difference is a direct result of the power intensity of each optical path in the two SOAs, this in turn determines the degree of nonlinearity that the amplifier is driven to, and hence the degree of phase shift. The difference in phase change is necessary since it is responsible for the nonlinear transfer function of the regenerator. For a 50-50 split to work the two amplifiers will have to be biased with different currents so that there is a different nonlinear phase change in each SOA at the same input light intensity for both optical paths. This is because each optical path experiences gain in only one SOA. Then the interference effects in the polarization converter can proceed as explained before.

The full setup of the system should introduce a phase shift between the two polarizations of $\pi/2$ degrees at the injection of the pulse into the last polarization converter. As a direct result, weak pulses which are assumed to be essentially noise, will not experience any nonlinear phase change in the SOAs, meaning that at the last polarization conversion when the two polarizations are combined the interference effect in the output partial polarization converter is such that at the output only TE polarized light is received, this is similar to obtaining the light at only the cross-output port in an MZI and only destructive interference at the bar-output port. For pulses that carry a sufficient amount of energy i.e. pulses that represent a logical '1' there is nondestructive interference in the last partial converter because these higher intensity pulses cause a nonlinear phase shift in the two SOAs, so that the phase difference between the two polarization modes at the output converter is not exactly $\pi$ radians. In this way the logical '1's are enhanced and the noise is suppressed, leading to an enhancement of the extinction ratio and the OSNR.

As can be seen from the preceding paragraphs there are similarities between the MZI and the cascaded configuration of the 2R regenerator. The input and output partial polarization converters in the cascaded 2R regenerator have a similar function to the input and output couplers of the MZI. The two distinct optical paths that are created by the conversion of the incoming light into orthogonal polarizations in the 2R regenerators is similar to the splitting of the light pulse between the two arms of the MZI. This particular setup of the 2R regenerator is similar to the self switching effect that has been suggested for the MZI in [37].

### 3.3.1 An approximation for the transmission

Assuming that the polarization converters and the SOA's are ideal and that only one of the SOA's is driven into it's saturation region in the operation of the regenerator it is possible to give a mathematical description of the transmission of the regenerator. If the transmissions for the TE and TM polarizations through the device are taken as:

$$P_{out}^{TE} = T_{TE} \cdot P_{in}^{TM},$$
$$P_{out}^{TM} = T_{TM} \cdot P_{in}^{TM}. \quad (3.15)$$

Writing the output amplitude at the end of each stage of the regenerator:

$$P_{out}^{TE} = T_{TE} \cdot P_{in}^{TM},$$
$$P_{out}^{TM} = T_{TM} \cdot P_{in}^{TM}. \quad (3.16)$$

34
Stage 1
Consider that the first partial polarization converter keeps $(1-X)\%$ of the incoming signal power as TM polarization and converts the rest of the signal power to TE polarization.

\[
\begin{align*}
P_{TE_1} &= P_{TM_1}^{TM}(X), \\
P_{TM_1} &= P_{TM_1}^{TM}(1 - X),
\end{align*}
\]

alternatively, in terms of the amplitude

\[
\begin{align*}
A_{TE_1} &= i\sqrt{P_{TM_1}^{TM}(X)}, \\
A_{TM_1} &= \sqrt{P_{TM_1}^{TM}(1 - X)},
\end{align*}
\]

Stage 2

\[
\begin{align*}
A_{TE_2} &= i\sqrt{P_{TM_1}^{TM}(X)e^{(g/2)L}e^{i\Delta \phi_{1NL}}}, \\
A_{TM_2} &= \sqrt{P_{TM_1}^{TM}(1 - X)},
\end{align*}
\]

where $g$ and $L$ are the gain and the length of the SOA respectively. This development is for the case where most of the light stays as TM polarization in this stage.

Stage 3
In this stage the polarization converter interchanges the power between the TE and TM polarizations.

\[
\begin{align*}
A_{TE_3} &= i\sqrt{P_{TM_1}^{TM}(1 - X)}, \\
A_{TM_3} &= i\sqrt{P_{TM_1}^{TM}(X)e^{(g/2)L}e^{i\Delta \phi_{1NL}}}.
\end{align*}
\]

Stage 4
At the second SOA:

\[
\begin{align*}
A_{TE_4} &= i\sqrt{P_{TM_1}^{TM}(1 - X)e^{(g/2)L}e^{i\Delta \phi_{2NL}}}, \\
A_{TM_4} &= \sqrt{P_{TM_1}^{TM}(X)e^{(g/2)L}e^{i\Delta \phi_{1NL}}},
\end{align*}
\]

Stage 5

\[
\begin{align*}
A_{TE_5} &= i\sqrt{P_{TM_1}^{TM}(1 - X)(1 - Y)e^{(g/2)L}e^{i\Delta \phi_{2NL}}} \\
&+ i\sqrt{P_{TM_1}^{TM}(X)(Y)e^{(g/2)L}e^{i\Delta \phi_{1NL}}}, \\
A_{TM_5} &= \sqrt{P_{TM_1}^{TM}(X)(1 - Y)e^{(g/2)L}e^{i\Delta \phi_{1NL}}} \\
&- \sqrt{P_{TM_1}^{TM}(1 - X)(Y)e^{(g/2)L}e^{i\Delta \phi_{2NL}}},
\end{align*}
\]
In order to find the transmission coefficients the amplitudes are multiplied with their complex conjugates in order to get the expression with the power. The TE and TM transmissions $T_{TE}, T_{TM}$ are

$$T_{TE} = (1 - X)(1 - Y)e^{(g - \Delta g_2)L} + XYe^{(g - \Delta g_1)L}$$
$$+ 2\sqrt{X(1 - X)Y(1 - Y)}e^{gL}e^{-\frac{\Delta \phi_{1NL}^2 + \Delta \phi_{2NL}^2}{2}}\cos(\Delta \phi_{1NL} - \Delta \phi_{2NL}), \quad (3.18)$$

$$T_{TM} = Y(1 - X)e^{(g - \Delta g_2)L} + X(1 - Y)e^{(g - \Delta g_1)L}$$
$$- 2\sqrt{X(1 - X)Y(1 - Y)}e^{gL}e^{-\frac{\Delta \phi_{1NL}^2 + \Delta \phi_{2NL}^2}{2}}\cos(\Delta \phi_{2NL} - \Delta \phi_{1NL}). \quad (3.19)$$

The transmission of the TM polarization can be plotted as a function of the phase change. It should be remembered that it is the phase change in the two SOA’s that is responsible for the non-linear switching behavior of the 2R regenerator.

The nonlinear phase shift can be related to the gain saturation via the Henry factor

$$\Delta \phi_{NL} = \frac{\alpha \Delta g L}{2}. \quad (3.20)$$

It is also assumed that the the degree of nonlinearity is only due to the intensity of the pulse, so that $\Delta g_1 = \frac{\Delta g_2 X}{1 - X}$ and $\Delta \phi_{1NL} = \frac{\Delta \phi_{2NL} X}{1 - X}$.

This result can be substituted into the Equation (3.19) in order to get the variation of the transmission with the nonlinear phase change,

$$T_{TM} = Y(1 - X)e^{-\frac{\Delta \phi_{2NL}}{\alpha}X}$$
$$+ X(1 - Y)e^{-\frac{\Delta \phi_{2NL}}{\alpha}(1 - X)}$$
$$+ 2\sqrt{X(1 - X)Y(1 - Y)}e^{-\frac{\Delta \phi_{2NL}}{\alpha}(1 - X)}$$
$$\times e^{-\frac{\Delta \phi_{1NL}}{\alpha(1 - X)} \cos(\frac{\Delta \phi_{2NL}(1 - 2X)}{1 - X})}. \quad (3.21)$$

Because we are interested in the nonlinear switching effects, the common factor $e^{gL}$ is ignored. With this expression it is now possible to simulate the transmission through the regenerator with different amounts of polarization conversion between the TE and TM polarization at the first and the last partial polarization converter. This plot of this equation shows the nonlinear transfer function through the regenerator with different ratio of TE and TM polarizations after the first and last polarization converters.

The Figures 3.9, 3.10 and 3.11 show the TM transmission curve against the nonlinear phase shift for different polarization conversion ratios. The ratios are $X$ fixed at 20%, $Y$ fixed at 20% and $X = Y$ for various values, respectively. From these plots it is clear that the ratio of the conversion between TM and TE at both the first and the last partial polarization converter have an effect on the general behavior of the regenerator. When the ratios of conversion in both partial polarization converters are not equal the transfer function does not extend from the origin i.e. from a transmission power of zero. This means that a small amount for amplification will still take place for the low power signals that do not induce a nonlinear phase shift. These low power signals are usually the contribution of noise on the channel.

Figure 3.11 shows that the best split ratio is $X = 20\%$ and $Y = 20\%$ because it gives the steepest curve and therefore will give the best extinction ratio improvement.
Figure 3.9: Transmission with $X=20\%$ and various values of $Y$.

Figure 3.10: Transmission with $Y=20\%$ and various values of $X$.

Figure 3.11: Transmission with $X=Y$ for various values.
Chapter 4

Device Operation

This chapter presents the results from the simulations of both the individual components of the regenerator and the 2R regenerator as a whole. The device is studied under various conditions including, operation under noisy conditions.

4.1 The SOA

The SOA study is carried out with the amplifier biased in the gain region. Table 4.1 lists the various parameters and operating conditions at which the SOA was simulated.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
<th>Units</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Differential gain</td>
<td>$a$</td>
<td>$3.76 \times 10^{-20}$</td>
<td>$m^2$</td>
<td>[38], [31]</td>
</tr>
<tr>
<td>Confinement factor</td>
<td>$\Gamma$</td>
<td>0.3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Length of active region</td>
<td>$z$</td>
<td>250</td>
<td>$\mu m$</td>
<td></td>
</tr>
<tr>
<td>Cross-sectional area</td>
<td>$A$</td>
<td>0.4</td>
<td>$(\mu m)^2$</td>
<td></td>
</tr>
<tr>
<td>Carrier lifetime</td>
<td>$\tau_s$</td>
<td>200</td>
<td>ps</td>
<td>[39], [31]</td>
</tr>
<tr>
<td>Group refractive index</td>
<td>$n_g$</td>
<td>3.56</td>
<td></td>
<td>[31]</td>
</tr>
<tr>
<td>CH nonlinear gain suppression factor</td>
<td>$\epsilon_{CH}$</td>
<td>$1 \times 10^{-23}$</td>
<td></td>
<td>[31]</td>
</tr>
<tr>
<td>SHB nonlinear gain suppression factor</td>
<td>$\epsilon_{SHB}$</td>
<td>$0.5 \times 10^{-23}$</td>
<td></td>
<td>[31]</td>
</tr>
<tr>
<td>Carrier-carrier relaxation time</td>
<td>$\tau_{SHB}$</td>
<td>70</td>
<td>fs</td>
<td>[39], [31]</td>
</tr>
<tr>
<td>Temperature relaxation time</td>
<td>$\tau_{CH}$</td>
<td>700</td>
<td>fs</td>
<td>[39], [31]</td>
</tr>
<tr>
<td>Pulse wavelength</td>
<td>$\lambda$</td>
<td>1.55</td>
<td>$\mu m$</td>
<td></td>
</tr>
<tr>
<td>Henry factor related to carrier density</td>
<td>$\alpha_N$</td>
<td>4</td>
<td></td>
<td>[38], [31]</td>
</tr>
<tr>
<td>Henry factor related to CH</td>
<td>$\alpha_{CH}$</td>
<td>3</td>
<td></td>
<td>[31]</td>
</tr>
<tr>
<td>Transparency current</td>
<td>$I_{tr}$</td>
<td>5</td>
<td>mA</td>
<td></td>
</tr>
<tr>
<td>Injection current</td>
<td>$I$</td>
<td>120</td>
<td>mA</td>
<td></td>
</tr>
<tr>
<td>Spontaneous noise coefficient</td>
<td>$n_{sp}$</td>
<td>1</td>
<td></td>
<td>[33]</td>
</tr>
<tr>
<td>Noise equivalent bandwidth in Hertz</td>
<td>$B_N$</td>
<td>$1 \times 10^{-12}$</td>
<td>Hertz</td>
<td>[33]</td>
</tr>
</tbody>
</table>

Table 4.1: Amplifier parameters

These values are copied directly from literature. In cases where the reference is not stated the value was derived from other parameters found in the literature. Some of the parameters like length and area of the active region are design choices. The SOA model itself is compared to the experimental observation that are listed in the paper by P. Borri
and others and found to be in close agreement with the results therein.

4.2 SOA characteristics

In this section, the effects of ultrashort pulses on the behavior of the SOA is examined. When an SOA biased in the gain region is excited by a light pulse, various phenomena take place, changing the properties of both the pulse and SOA.

4.2.1 SOA gain dynamics

In order to study the dynamic behavior of the SOA under photoexcitation, pulses of input energy $0.12156 \times 10^{-12}$ J (high) with peak intensity of 57 mW, $0.52156 \times 10^{-13}$ J (middle) with peak intensity of 24 mW and $0.12156 \times 10^{-15}$ J (low) with a peak intensity of 0.057 mW are simulated using the SOA model presented in Section 2.4.8.

The input pulses are gaussian pulses described by the equation

$$P_{in}(\tau) = \frac{E_{in}}{\tau_0 \sqrt{\pi}} \exp \left( -\frac{\tau^2}{\tau_0^2} \right). \tag{4.1}$$

$\tau_0$ is related to the Full Width at Half Maximum (FWHM) by $\tau_{FWHM} \approx 1.665 \tau_0$. $\tau$ is the delay of the pulse. The delay is used to describe a moving reference time scale. In effect the observer moves with the pulse observing its slowly changing amplitude. The delay $\tau$ is related to the absolute time frame $t$ by, $\tau = t - z/v_g$. Where $z$ is the distance travelled by the pulse and $v_g$ is the group velocity. The input pulses are of a FWHM of 2 ps which is sufficient for this application. It also lies in the range of operation of the SOA model.

It is observed in Figure 4.1 that the gain of the SOA falls as it is excited by the light. The magnitude of the gain compression depends on the energy of the exciting light pulse. There is a fast recovery of the gain as the strength of the exciting pulse begins to fall. This is attributed to the ultrafast processes, namely the carrier-carrier scattering that takes place in the timescale of 70 fs and the carrier cooling that takes place within 700 fs. After the fast gain recovery, as shown by Figure 4.2. At this point the slow gain recovery
process takes over and this depends on electrical pumping to replace the free carriers that have been lost during the stimulated emission process.

![Image](image_url)

**Figure 4.2: Gain dynamics of the SOA.**

### 4.2.2 Pulse phase dynamics

Because of the Kramers-Kronig relationships, when the gain of the SOA changes due to a change in the concentration of free carriers and carrier temperature in the active region, there is an accompanying change in the refractive index. Consequently this change in the refractive index of the active region leads to a change in the phase of the exciting pulse. Note that the input pulse has a uniform phase of zero radians. When the phase of the output pulse is simulated it is evident that each part of the pulse experiences a different phase shift. This nonlinear phase shift corresponds to the power in that part of the pulse and the saturation of the amplifier. For a pulse strength of $0.12156 \times 10^{-15}$ J, there is no noticeable phase shift. The nonlinear behavior of the SOAs is very important because it determines the amount of phase shift that different parts of the pulse and the different

![Image](image_url)

**Figure 4.3: Phase of the output pulses. Input pulses have different strength.**
pulse strengths experience in the SOA. This makes it possible to setup the regenerator device to discriminate the various parts of the pulses according to their strengths by taking advantage of the interference effects.

4.2.3 Output pulse shape and size

The dynamic character of the gain of the SOA means that an incident pulse experiences a change in shape and strength. Figure 4.4 shows an amplified output pulse from the SOA. The gain compression results in a greater amplification for the leading edge of the pulse since it is amplified by a larger gain. Other phenomena like TPA and FCA also have an influence on the pulse shape. However, for pulses with the intensity described above, these effects are negligible.

![Figure 4.4: Size comparison of an input E = 0.12156 \times 10^{-12} and its output pulse.](image)

![Figure 4.5: Normalized Output pulse shape for different input pulse strengths.](image)

Figure 4.5 shows that the effect on the shape of the pulse becomes more pronounced, as the strength of the input pulse increases. For pulses of lower strength, the gain compression in the SOA is negligible, therefore the whole pulse experiences the same gain as it traverses the amplifier. In this case the output pulse shape is similar to the input pulse shape. When the strength of the input pulse is increased the gain compression starts to affect the shape of the pulse because the leading edge of the pulse experiences a larger gain than the trailing edge. The output pulse reflects this with the leading edge having a shorter rise time and the trailing edge a longer fall time.

4.2.4 Effect of ASE

Following the discussion in Section 2.4.10 the presence of ASE during the amplification process means that noise is added to the output pulse. To test the effect of this the input pulses shown in the previous section are simulated with ASE, for different SOA lengths. The strengths of the input pulses determine the effect of ASE on the output pulse. Figure 4.6 shows that ASE effects on SNR are negligible. However, a normalized output pulses shown in Figure 4.7 shows significant distortion for the 0.057 mW pulse. Generally these results show that the effect of ASE is negligible in terms of SNR for a single SOA.
4.2.5 Effect of a pulse train

It is important to understand the behavior of the SOA, not only with excitation of a single pulse, but when multiple pulses excite it. This a more realistic representation of an actual communication system, since the device will continually be excited by a train of pulses representing logical ‘1’s and no pulse representing a logical ‘0’s1. In this simulation, a series of pulses each 25 ps seconds apart are sent through the SOA to simulate an environment with a 40 Gbps operating speed. Note that this simulation involves a stream of 60 pulses of equal power representing a logical ‘1’, even though this a worst case scenario in a real telecommunication scenario.

Figure 4.8 shows that the gain compression as successive pulses excite the SOA becomes smaller and smaller. By the 59th and 60th pulse there is almost no noticeable change in

1This depends on the encoding technique used in the transmission
the gain as the pulses goes through the SOA. It should be noted as shown in Figure 4.9, that the gain does not return to its original value by the end of each excitation. The plot of the peak gain Figure 4.10, in each excitation shows more clearly the variation of the gain with each passing pulse. This plot shows an exponential decay; after a number of pulses the gain of the SOA hardly changes under excitation. The variation of the phase, Figure 4.11, shows that the phase behaves like the gain, with nonlinear changes. In this it is not so important that the actual phase change is not the same for each subsequent pulse. This is because the operation of the 2R regenerator depends on phase difference between the pulses from the two optical paths. The phase difference depends more on the relative values of the phase from the two optical paths rather than the exact phase of each part of the pulses. The actual value of the gain amplifier after each passing pulse is important, because in this application it determines the the strength of the output pulse and the amount of amplification achieved.

Figure 4.10: Variation of the peak gain of the SOA for a pulse train.

Figure 4.11: Phase change experienced by each pulse.

4.3 Polarization converters

The polarization converter properties are mainly a function of the length of the device and the absorption of the material. In this study an assumption has been made that the propagation constants of both the TE and TM are equal. In reality this is only possible if the fabrication of the device is very accurate. This should not be too much of an issue because taking the behavior of the whole device into consideration, it is apparent that due to the interchange of the polarizations at the halfway point of the regenerator, it can be assumed that the propagation constant differences cancel out for the device as a whole. As such the polarization converter is modeled with no propagation mismatch.

The polarization converter is initially modeled as an ideal converter with a coupling coefficient of $9.47 \times 10^{-3}$ and no TE polarization absorption.

This simulation shows 100% conversion from one polarization to the other after a length of about 165.3 $\mu$m as shown by Figure 4.12. The Figure 4.13 shows that the phase difference between the TE and TM polarizations is always $\pi$ radians. At the length where full polarization conversion takes place there is a phase jump. Simulations with a
Figure 4.12: Output power of TE and TM polarization versus length for the ideal polarization converter.

Figure 4.13: Output phase TE and TM polarization versus length for the ideal polarization converter.

Figure 4.14: Output power of TE and TM polarization versus length for a PC with TE absorption.

Figure 4.15: Output phase of TE and TM polarized pulses versus length for a PC with TE absorption.

TE absorption of 0.00209 $\mu m^{-1}$ [41] in Figure 4.14 show that full conversion where one polarization is completely changed to another, is still a possibility in this situation. The phase behavior of the two pulses is that same as that of the ideal converter as shown by Figure 4.15. There is a significant loss of power for both polarizations along the length of the polarization converter. Note that with the absorption taken into account, the polarization conversion length giving the maximum TE power output occurs at 156.3 $\mu m$. The length that gives and minimum TM power output is 178.5 $\mu m$. Since the operation of the 2R regenerator depends on the difference in phase shift between the two SOAs we choose a full converter length of 156.3 $\mu m$, thereby obtaining the maximum TE power input for the second SOA.
4.4 The 2R regenerator

After simulating the behavior of the individual components of the 2R regenerator, the whole setup is now studied. The aim of this section is to learn about the characteristics of the regenerator under different operation conditions.

4.4.1 Power characteristic

![Figure 4.16: Input and output TM polarized pulse power, with input pulses of low peak intensity.](image1)

![Figure 4.17: Output phase of the pulses from the two optical paths for an input pulse of low peak intensity.](image2)

![Figure 4.18: The phase difference between the output pulses from the two optical paths.](image3)

![Figure 4.19: Normalized input and output TE and TM power with an input pulse of low peak intensity.](image4)

The 2R regenerator is examined with varying input pulses with the same strength as those used for the SOA in Section 4.2.

Although theory predicts that for the low intensity input pulses the output pulses experience a weakening of the amplitude, this does not happen. Instead there is a small
amplification of the pulse as shown by Figure 4.16. Figure 4.17 shows that there is hardly any change in phase across the whole pulse, for the two pulses that have followed different optical paths in the regenerator. Due to this lack of change in the magnitude of the phase difference between output pulses as shown by Figure 4.18, the width of the normalized output pulses are the same for both optical paths and for the input pulse, see Figure 4.19.

Figure 4.20: Output phase of the pulses from the two optical paths for a medium peak intensity input pulse.

Figure 4.21: The phase difference between the TE and TM output pulse before the last converter.

Figure 4.22: Normalized input and output TE and TM power for an input pulse of medium peak intensity.

Figure 4.23: Input and output TM polarized pulse power, with an input pulse medium peak intensity of 24 mW.

When the input energy is increased, the nonlinear phase shift of the TE and TM output pulses from the two optical paths becomes unequal see Figure 4.20. This happens because the two SOAs in the regenerator are driven to different levels of saturation. This is due to the fact that, the initial split of the pulse intensities between the two optical paths was unequal. This leads to a phase difference between the two paths that causes a nonlinear transfer function, see Figure 4.21. There is a much more exaggerated phase
difference than that for the low energy input pulses in Figure 4.18. Figure 4.22 shows the
effect of this uneven phase change; the normalized TM output pulse is now narrower than
the input pulse and the output TE pulse. Figure 4.23 shows that the regenerator further
amplifies input pulses at this input pulse intensity.

Figure 4.24: Output phase of the pulses from
the two optical paths for a high energy input
pulse.

Figure 4.25: Normalized input and output TE
and TM power with high energy input pulse.

At a peak intensity of 57 mW the second SOA (SOA 2) is driven further into its
saturation region compared to the first SOA (SOA 1) Figure 4.24. Thus the output pulses
from one of the optical paths, exhibits a big phase shift especially in the mid-section of
the pulse. This phase shift is responsible for the interference effects that cause the output
TM to be be significantly reshaped, with a narrower characteristic than either the input
pulse or the output TE pulse as shown by Figure 4.25. The amplifying function of the
regenerator is still in effect with the output pulse being stronger than the input pulse
Figure 4.26.

Figure 4.26: Input and output TM polar­
ization power with high peak intensity input
pulse of 57 mW.

Figure 4.27: Transmission vs input pulse
power.
If the plot of the output power is done for a whole range of input pulses, a plot of the transmission curve can be obtained. The result is a nonlinear transfer curve showing that the 2R regenerator application is possible. The figures showing the input and output pulses above show that reshaping and amplification are taking place.

Figure 4.27 shows that the regenerator can operate with pulse narrowing characteristics up to an upper bound of 60mW. Above this input power, the output pulse width will start to grow with increasing input power.

**Optimization**

With the model in place it is possible to optimize the output transfer shape of the 2R regenerator so that pulse compression is achieved for the low power pulse. Figures 4.28 and 4.29 show the optimized transfer function and the attenuated output pulse for a low intensity input, respectively.

![Figure 4.28: An optimized regenerator transfer function with attenuation of low intensity input pulses](image)

![Figure 4.29: The suppressed output pulse.](image)

**4.4.2 Pulse trains**

As already noted in Section 4.2.5. The gain of the SOA falls when it is exposed to a train of pulses. This is because the free carriers that are lost during the stimulated emission process, cannot be replaced fast enough before the next pulse arrives. In order to study the behavior of the regenerator under pulse train conditions, the output pulse is plotted for the 1st, 5th, and 60th pulse, with input pulses having the same input energy as used in the previous section.

Figure 4.30 shows that there is a drop in the output pulse power, as more pulses go through the amplifier. The drop in output pulse power is caused by the slow gain recovery of the amplifiers. This scenario shows that for a transmission system this will cause pattern effects, because there is never a stable output pulse power. The normalized output pulses show that the pulse narrowing gets better as more pulses go through the amplifier as shown by Figure 4.31. Though the amplification application is lost for a long pulse train, the shaping function is still ok and this should be the case because the unequal
phase shift of the output pulses from the two optical paths is still present, as displayed by Figure 4.32. The cause of the loss of amplification is mostly due to the high saturation of the second SOA (SOA 2) as shown by the gain compression in Figure 4.33.

A similar condition is evident when a train of input pulses of medium strength 24 mW is sent through the amplifier, see Figures 4.34, 4.35, 4.36. Due to the high gain compression in the second SOA (SOA 2) there is a reduction in the overall power of the output pulses as the SOAs are subjected to more pulses.

The drop between the first and fifth pulse is much higher than the drop between the fifth and sixtieth pulse. This follows the results obtained for the single SOA, where it was noted that there is a smaller and smaller gain compression with each subsequent pulse. Figure 4.37 shows that by the tenth pulse the SOAs are in a quasi-equilibrium, which means that there is little variation in output pulse power after the first 10 pulses. This
could be used as the number of training pulses needed in a real communication system, to get the device into a useable state. However, it should be noted that in real communication scenarios ones and zeros are present. Therefore training the device for inputs of only logical ones is not sufficient.

Figure 4.38 shows that the lower energy input pulses do not show any appreciable decay in the output power of each subsequent pulse. This is in keeping with the fact that the SOAs are not saturated by the low input energy pulses. At the same time, due to the fact that the difference in nonlinear phase shift between the two optical paths is negligible, there is no pulse narrowing of the output pulses Figure 4.39.
Effect of a higher injection current on a pulse train

In order to mitigate the gain compression in the SOAs caused by a pulse train, so as to avoid the decaying power of subsequent output pulses, a train of pulses is sent through a device with the SOAs at a higher injection current of 200mA. This improves the power of the output pulses, so that even pulse number 60 is stronger than the input pulse as shown in Figure 4.40. Note that this affects the pulse width of the first pulse, making it broader than the input pulse Figure 4.41. This is due to the fact that, at this injection current the first SOA in the device is also saturated by the input pulse so that in the end the phase difference for all the regions of the pulses from the two optical paths is the same. This shows that the higher injection current increases the overall gain of the SOAs, while it changes the nonlinear phase shift of the output pulses, so that they are more or less equal for the two optical paths. This is due to the fact that both SOAs in the 2R regenerator are now saturated at lower input pulse energies. This destroys the pulse narrowing function.
Effect of a phase shifter on a pulse train

A component that can possibly be used to improve the 2R-regeneration of a pulse train on the 2R regenerator is a phase shifter, which is added anywhere along the optical path, but before the last polarization converter. The additional phase shift allows the flexibility to optimize the phase difference between the pulses from the two optical paths. Figure 4.42 shows the effect of a -0.504 rad phase shifter. The phase shifter affects the operation of the device by increasing the phase difference between the output pulses from the two optical paths. Since the phase shift introduced by the phase shifter is the same across the whole pulse, the pulse narrowing function is lost as the amount of phase shift from the phase shifter becomes comparable to, or exceeds, the unequal nonlinear phase shift induced by the SOAs on the output pulses. This loss of the pulse narrowing is illustrated.
in Figure 4.43.

The phase shifter will be considered in more detail in a later section, where its effect on the noise performance of the device will be detailed.

**Pulse train eye diagram**

![Figure 4.44: Eye diagrams for an input pulse trains of peak intensity: 57 mW (a) and 24 mW (b).](image)

Figure 4.44: *Eye diagrams for an input pulse trains of peak intensity: 57 mW (a) and 24 mW (b).*

Eye diagrams are simulated for pulse trains of different input pulse intensities to gain an insight into the communication capabilities of the regenerator, as shown by Figures 4.44a and 4.44b. The input information is made up of a Pseudorandom Bit Sequence (PRBS) of $2^7 - 1$ which is sufficient for 40 Gbps simulations. Furthermore, this PRBS is less computationally intensive than the other bit sequences.

![Figure 4.45: Eye diagram for an input pulse trains of 4 Gbps.](image)

Figure 4.45: *Eye diagram for an input pulse trains of 4 Gbps.*

The random binary bits of the same input energy are also simulated at 4 Gbps as
shown by the eye diagram in Figure 4.45. The eye diagram shows a good eye opening, this shows that the device operates sufficiently well, even with the gain recovery delay of the SOAs. It is evident that the pattern effects are an issue for the 2R regenerator, if it is to operate at 40 Gbps. As a comparison at a speed of 4 Gbps, which repetition rate is of the order of the reciprocal of the carrier lifetime in the SOAs, the pattern effects are not a problem. In this case the SOAs are able to recover their original gain in between pulses.

### 4.4.3 Performance in an Additive White Gaussian Noise channel

![Figure 4.46: Pulse with AWGN. (a) The input pulse (b) The output pulse number 60.](image)

In Section 2.4.10, the ASE noise performance of the device was introduced. In addition to the internally generated noise from the device, the communication channel is modeled with Additive White Gaussian Noise (AWGN). The noise is simulated as explained in Section 2.4.10 by adding an electric field component for the noise in series with the pure gaussian pulse inputs. The AWGN with an SNR of 30 dB, is generated with a random number generator. The random number generator is always reset to the same state so that the comparison is fair.

Figure 4.46 shows the input and output pulses in an AWGN channel. The intensity of the input pulses is 57 mW which is a high energy pulse. The figures show that the leading edge of the output pulse has as significant noise reduction. The fact that the leading edge of the pulse benefits most from the pulse reshaping implies that the amount of nonlinear phase shift between the pulses of the two optical paths is the determining factor in this reshaping. Unfortunately for the trailing edge of the pulse which is exposed to an already saturated SOA there is very little noise cancelation. This fact is further reinforced by the fact that for the low intensity input pulses of 0.057 mW, there is no noise cancelation for both the front and the back of the pulses Figure 4.47. This is because the low energy input pulses do not get any nonlinear phase shift after crossing the amplifier.
4.4.4 Noise cancelation control using a phase shifter

In Section 4.4.2 a phase shifter was used to mitigate the effects of a pulse train on the amplifying function of the regenerator. In this section a similar attempt is made with the noise reduction ability of the 2R regenerator. The input pulse is shown together with the output pulse number 60 for the phase shifts 0.252, -0.252 and 0.504 radians Figure 4.48.

In Figure 4.48b the positive phase shift induces pulse breakup. This is because the phase shift changes the interference between the pulses from the two optical paths so that the point of destructive interference is now in middle of the pulse, and the constructive interference is at the flanks of the pulse. This is further highlighted by the noise suppression around the breakup point. The negative phase shift of -0.504 radians almost removes any interference between the TE and TM pulses from the two optical paths in the last polarization converter, so that there is no suppression of noise at the output of the regenerator.
Figure 4.48: Pulses from an AWGN channel. (a) The input pulse (b) The output pulse number 60 phase shift 0.252 rad (c) Output pulse number 60 phase shift -0.252 rad. (d) Output pulse number 60 phase shift -0.504 rad.
Chapter 5

Conclusions and Recommendations

5.1 Conclusions

A 2R regenerator based on two SOA’s and three polarization converters, arranged in a cascade has been studied. The aim of this design is to realize a single chip device with a reduced chip-area compared to conventional designs.

The simulations have shown that the device can act as a re-amplifier and re-shaper within an operating range of up to 60 mW. For the low energy input pulses, the simulations show that the device hardly amplifies the output pulse. This is a critical property that should help the device to have a good noise suppression characteristic, assuming that the noise is usually in the low energy input range. For medium and high energy pulses the device amplifies the input pulses. This is the required behavior for extinction ratio improvement. Pulse narrowing has also been demonstrated for all pulses, apart from those of the low input energy range. This is the expected behavior, because for the low input energy pulses the phase hardly changes across the whole pulse. This means that the phase is uniform across the whole pulse, and therefore there is no accompanying pulse narrowing.

The pulse narrowing makes the device an important re-shaping tool, capable of mitigating the effects of GVD and PMD. In addition to simulating the characteristics of the output pulses, the transmission for the device was simulated, and it confirmed the devices 2R regeneration capabilities because of its nonlinear shape. This nonlinear transfer action acts as a logic gate that suppresses signals whose strength is below a critical value, whilst at the same time it enhances those signals that are above this critical value in strength.

The 2R regenerator was also simulated with signals that contained Additive White Gaussian Noise. The front of the signal pulses in general attained a significant amount of noise reduction compared to the midsections and the trailing edges of the pulse. This is similar to the pulse width narrowing mentioned above. Basically the same mechanisms that leads to the pulse narrowing is responsible for this behavior. The noise reduction on the pulse relies on the transient dynamics of the phase difference of the pulses traversing the device. It was also noted that a phase shifter can be used to redistribute the noise suppression characteristics, to parts of the pulse other than the leading edge.

When an optical signal goes through the regenerator there is a change in pulse shape. This magnitude of this change depends on a number of factors including the energy of the input pulse and the bias current of the SOAs in the 2R regenerator. The pulse shape
change causes slight changes in the position of the pulse in the train. It is evident that after a number of regeneration stages the signal will get a significant amount of jitter. Therefore it is necessary that 3R regeneration is done after a few 2R regeneration stages.

The dynamic behavior of the regenerator has also been examined. Even though there are ultrafast effects like carrier-carrier scattering in the SOA which contribute to the rapid recovery from the gain compression, for an input pulse train, the SOAs do not attain their original gain in between pulses. Full recovery depends on the much slower electrical pumping mechanism. This means that the full recovery of the SOAs is dependent on the carrier lifetime and the carrier injection in the SOA. This slow recovery has a number of side-effects on the behavior of the 2R regenerator. The output pulses for a train of input pulses show a rapid fall in output power. This has been shown to lead to pattern effects when a pseudo random bit sequence was sent through the device. Although the SOAs do achieve a quasi-equilibrium after a number of pulses this equilibrium is only achievable with a constant train of equidistant pulses. This scenario is a rare occurrence in real communication systems. Besides, the gain at which the quasi-equilibrium is attained is significantly below the initial gain of the device, and therefore a loss of amplification is still experienced.

Various attempts have been made to try and compensate for the slow gain recovery in the SOA. Pumping the SOAs with higher injection currents showed that the input pulse trains were amplified even for the later pulses. However, it was seen that there was a loss of the suppression characteristics of the device with the higher pumping. At the same time there is still a difference in the strength of the first output pulse and the later pulses, that will contribute to pattern effects.

A phase shifter was added to the setup to mitigate the effects of the slow gain recovery. The output pulses are increased in strength with the appropriate value of phase shift, however, the draw back of this is that the pulse narrowing function is lost. The phase shifter also improves on the noise suppression mechanism of the device. Good noise suppression was shown at both the leading and trailing edge of the output pulses for a phase shift of -0.252 rad. However, an optimum value for the phase shift is difficult to determine, due to the fact that different phase shifts lead to different advantages as shown by the results.

5.2 Recommendations

Although it is generally accepted that the models that are used for modeling bulk SOAs are also sufficient to model QW-SOAs [9], in this case it might be better to concentrate on a model that is specifically for QW-SOAs. This is because this application relies on the fast recovery of the SOA, and some of the fast recovery mechanism in quantum wells like the replenishing of carriers from the barriers or carrier reservoirs into the quantum well [27] [42], are not taken into account. A number of publications [39] [42] show that the gain recovery for ultrashort pulses of a few hundred femtoseconds is significantly faster than for picosecond pulses. Again due to the limitations of our model which is not valid for pulses that are below the picosecond duration, it is impossible to use these ultrashort pulses.

A number of devices are being designed with the promise that these devices will have faster recovery rates than QW-SOAs. Amongst these are semiconductor amplifiers with carrier reservoirs and Quantum Dot amplifiers (QD-SOA). The QD-SOA is a possible alternative, since it can be manufactured with POLIS characteristics. Additionally there
are several schemes that have been proposed to improve the recovery rate of the amplifiers available today. And these include, using a continuous wave optical holding beam to enhance the recovery rate of SOAs [43]. With the correct holding beam power and proper biasing, a gain recovery time as short as 10 ps has been achieved.
Bibliography


[16] Shah, "Ultrafast spectroscopy of semiconductors and semiconductor nanostructures," see pp[133].


[38] N. Storkfelt, M. Yamaguchi, B. Mikkelsen and K. E. Stubkjaer, “Recombination constants and $\alpha$ factor in 1.5 $\mu$m MQW optical amplifiers taking carrier overflow into account,” *Electronics Letters*, vol. 28, September 1992, see pp[1774-1776].


Appendix A

The SOA model 1

The model for the optical amplifier cavity is taken from Tang and Shore [22]. This model takes into account effects from SHB, TPA, CH and UNR. The model is modified so that the two polarization modes are distinguished using the assumption that in the strained quantum well material that's being considered there is no gain for the TM mode in the SOA. This is after making the assumption that the amount of compressive strain that is applied is big enough to cause only interactions with TE polarized light [44]. This peculiarity is due to the fact that the selection rules favor transitions from the conduction band (CB) to the heavy-hole band (hh). The TE polarized light which is aligned parallel to the well layer is favored in this transition compared to the TM mode which has its electric field normal to the well layer [9]. This means that amplification of TM polarized light which is a result of transitions from the CB to the light-hole band (lh) is nonexistent.

Using the wave equation

\[ \nabla^2 E - \frac{\varepsilon}{c^2} \frac{\partial^2 E}{\partial t^2} = 0, \]  

(A.1)

to describe the propagation of the electromagnetic field within the SOA. Where the dielectric constant \( \varepsilon \) is given by

\[ \varepsilon = n_0^2 + \chi, \]  

(A.2)

with \( n_0^2 \) the background refractive index being a function of the transverse coordinates \( x \) and \( y \) accounting for dielectric waveguiding in semiconductor laser amplifiers. The susceptibility \( \chi \) represents the contribution of charge carriers inside the active region and it is a function of carrier density and temperature. A phenomenological model has been suggested by Agrawal [32]

\[ \chi = -\frac{\tilde{n}c}{\omega_0} (\alpha + i)g, \]  

(A.3)

where \( \alpha \) is the linewidth enhancement factor, \( g \) is the gain per unit length, \( \tilde{n} \) is the effective mode index, \( \omega_0 \) is the pulse center frequency. \( g \) is defined as [22]

\[ g = \frac{g_l - \epsilon_{TPA} S^2}{1 + \epsilon S}, \]  

(A.4)

where \( \epsilon_{TPA} \) is the absorption factor due to TPA and the non linear gain compression \( \epsilon = \epsilon_T + \epsilon_{SHB} \) and \( g_l = a_N (N - N_{tr}) \). \( a_N \) and \( N_{tr} \) are the general gain coefficient and the carrier density at transparency respectively.
Substituting (A.4) into (A.3) and taking into account the carrier density and temperature dependence of the susceptibility [22], Equation (A.3) becomes

$$\chi(N,T) = -\frac{\tilde{n}c}{\omega_0} \left[ g_{1e} - \frac{\epsilon_{TPA} S^2}{1 + \epsilon S} + a_N g_{1e} - \frac{\epsilon_{TP} S + \epsilon_{TPA} S^2}{1 + \epsilon S} \right], \tag{A.5}$$

where $\alpha_N$ and $\alpha_T$, the traditional and temperature linewidth enhancement factors and $\beta$ can be $c$ or $v$ for conduction or valence band respectively.

Using Equation (A.1), (A.2) and (A.5) in a procedure suggested by Agrawal and Olsson in [32], equations describing the propagation of an optical pulse in the SOA is obtained:

\[
\begin{align*}
\frac{\partial A^{TE}(z,t)}{\partial z} + \frac{1}{v_g} \frac{\partial A^{TE}(z,t)}{\partial t} &= \frac{1}{2} \left[ \frac{G - \epsilon_2}{1 + \epsilon_1} \right] A^{TE}(z,t) \\
&\quad - \frac{j}{2} \left[ \alpha_N G - \alpha_{Tc} \frac{\epsilon_1 G}{1 + \epsilon_1} \right] \frac{1}{\sigma} \left[ A^{TE}(z,t) - A^{TE}(z,t) - \frac{1}{2} \alpha_{int} A^{TE}(z,t) \right], \tag{A.6}
\end{align*}
\]

\[
\begin{align*}
\frac{\partial A^{TM}(z,t)}{\partial z} + \frac{1}{v_g} \frac{\partial A^{TM}(z,t)}{\partial t} &= - \left( \Gamma_2 \beta_2 + j \Gamma_0 \frac{\omega_0}{c} n_2 \right) \frac{1}{\sigma} \left[ A^{TM}(z,t) + A^{TE}(z,t) \right] \\
&\quad - \frac{1}{2} \alpha_{int} A^{TM}(z,t), \tag{A.7}
\end{align*}
\]

where

\[
\begin{align*}
S^{TE/TM} &= \frac{|A^{TE/TM}|^2}{\hbar \omega_0 \sigma v_g}, \\
G &= \Gamma a_N(N - N_{tr}), \tag{A.8} \\
\epsilon_1 &= \frac{\epsilon}{\hbar \omega_0 \sigma v_g}, \\
\epsilon_2 &= \frac{\Gamma \epsilon_{TPA}}{(\hbar \omega_0 \sigma v_g)^2},
\end{align*}
\]

here $A(z,t)$ is the slowly varying envelope of the optical pulse and the mode cross section $\sigma = wd/\Gamma$. $\epsilon_1$ is the gain compression factor due to SHB and CH $\epsilon = \epsilon_T + \epsilon_{SHB}$ and $\epsilon_2$ is the gain compression factor due to TPA. The last three terms of Equations (A.7) and (A.6) account for direct effects of TPA ($\beta_2$), UNR ($n_2$), and internal loss ($\alpha_{int}$) on the propagating pulse.

The formulation of Equations (A.7) and (A.6) is similar to that derived by Tang and Shore [22], although it accounts for the two polarizations separately. We assume that the TE and TM modes propagate through the SOA independently, though they affect each other indirectly through carrier dynamics [45]. Notice that though we assume minimal interaction between the two polarizations, TPA is assumed to be due to both polarizations,
subsequently the UNR is also a result of carrier transitions due to both polarizations. With the assumption that there is almost no gain for the TM polarization, we consider the gain compression due to SHB and CH to be a result of the power from the TE polarization only. This model allows the flexibility of dropping the TPA term depending on the intensity or width of the incident pulse.

Simplifying by using a moving coordinate frame according to \( \tau = t - z/v_g \) and using the relation between the intensity and the amplitude, 
\[
A^{TE/TM}(z, \tau) = \sqrt{P^{TE/TM}(z, \tau)} e^{i\phi^{TE/TM}(z, \tau)}
\]
so that 
\[
\dot{A}(z, \tau) = \frac{\dot{P}(z, \tau)}{2\sqrt{P(z, \tau)}} e^{i\phi(z, \tau)} + i\phi(z, \tau) \sqrt{P(z, \tau)} e^{i\phi(z, \tau)}.
\]
(A.9)

Equation (A.9) is essentially the same as Equations (A.6) and (A.7) therefore the imaginary parts of Equations (A.6) and (A.7) are equal to the spatial derivative of the phase, while twice the real part multiplied by the optical power is equal to the spatial derivative of the optical power. We write the previous equations in terms of intensity and phase. We use simplifying nomenclature for reasons of clarity, where 
\[
P^{TE/TM} = P^{TE/TM}(z, \tau), \quad \phi^{TE/TM} = \phi^{TE/TM}(z, \tau), \quad A^{TE/TM} = A^{TE/TM}(z, \tau), \quad P = P^{TE} + P^{TM}
\]
\[
\frac{\partial P^{TE}}{\partial z} = \left[ G - \epsilon_2 P^2 \right] \frac{1 + \epsilon_1 P^{TE}}{1 + \epsilon_1 P^{TE}} - 2\Gamma_2 \beta_2 \frac{1}{\sigma} P \cdot P^{TE},
\]
(A.10)
\[
\frac{\partial P^{TM}}{\partial z} = \alpha_{int} P^{TM} - 2\Gamma_2 \beta_2 \frac{1}{\sigma} P \cdot P^{TM},
\]
(A.11)
\[
\frac{\partial \phi^{TE}}{\partial z} = -\frac{1}{2} \left[ \alpha_N G - \alpha_{T_e} \epsilon_1 G P^{TE} - \epsilon_2 P^2 \right] - \Gamma_2 \frac{\omega_0}{c} n_2 \frac{1}{\sigma} P,
\]
(A.12)
\[
\frac{\partial \phi^{TM}}{\partial z} = -\Gamma_2 \frac{\omega_0}{c} n_2 \frac{1}{\sigma} P.
\]
(A.13)

From the phenomenological rate equations
\[
\frac{\partial N}{\partial t} = \frac{I}{qV_{act}} - \frac{N}{\tau_e} - v_g g S + \frac{\Gamma_2}{\Gamma} v_g \beta_2 S^2.
\]
(A.14)

The last term on the RHS is a modification to the phenomenological rate equations and shows the change in carrier density that is a result of TPA. \( \Gamma_2 \) is the confinement factor for TPA and \( \Gamma \) is the traditional confinement factor.

Subtracting \( N_{tr}/\tau_e \) from the first term and adding it to the second term on the right-hand side (RHS) of Equation (A.14), multiplying both the left-hand (LHS) side and the RHS by \( \Gamma a_N \), substituting \( g \) and \( S \) with the equivalent expressions from Equations (A.4) and (A.8)
\[
\frac{\partial G}{\partial t} = \Gamma a_N \left[ \frac{I}{qV} - \frac{N_{tr}}{\tau_e} - \frac{N + N_{tr}}{\tau_e} - v_g P^{TE} \right] \frac{1}{\hbar \omega_0 \sigma v_g} \cdot \frac{G - \epsilon_2 P^2}{1 + \epsilon_1 P^{TE}} + \frac{\Gamma_2}{\Gamma} v_g \beta_2 \frac{P^2}{(\hbar \omega_0 \sigma v_g)^2},
\]
(A.15)
\[
\frac{\partial G}{\partial \tau} = \frac{G_0 - G}{\tau_e} - \frac{1}{E_{sat}} \frac{G - \epsilon_2 P^2}{1 + \epsilon_1 P^{TE}} + \Gamma_2 \beta_2 P^2,
\]
(A.16)
where \( G_0 = \Gamma a_N N_{tr} (I/I_0 - 1) \) is the small signal gain, \( I_0 = q V N_{tr}/\tau_e \) is the injected current at transparency, the coefficient \( \beta_2' = a_N \beta_2/[(\hbar \omega_0 \sigma)^2 v_g] \) and the saturation energy of the amplifier \( E_{sat} = \hbar \omega_0 / a_N \).
A.1 Discretization

If \( u^n_j = u(t_n, x_j) \) and \( u^{n+1}_j = u(t_n, x_j + \Delta x) \) then using a finite difference discretization based on the Lax method [46] where:

\[
\frac{\partial u}{\partial z} = \frac{u^{n+1}_j - u^{n-1}_j}{2 \Delta z}, \tag{A17}
\]

\[
\frac{\partial u}{\partial t} = \frac{u^z_{n+1} - u^z_{n-1}}{2 \Delta t}, \tag{A18}
\]

Equations (A.10), (A.11), (A.12), (A.13) and (A.16) become:

\[
\frac{p_T^{Ez+1} - p_T^{Ez-1}}{2 \Delta z} = \left[ G_n^z - \epsilon_2 P_j^{Ez}^2 \left( 1 + \epsilon_1 P_j^{Ez} \right) - \alpha_{int} \right] P_j^{Ez} - \frac{2 \Gamma_2^z \beta_2}{\sigma} P_j^{Ez} \cdot P_j^{Ez}, \tag{A19}
\]

\[
\frac{p_T^{Mz+1} - p_T^{Mz-1}}{2 \Delta z} = -\alpha_{int} P_j^{TMz} - \frac{2 \Gamma_2^z \beta_2}{\sigma} P_j^{TMz}, \tag{A20}
\]

\[
\frac{\phi_j^{Ez+1} - \phi_j^{Ez-1}}{2 \Delta z} = -\frac{1}{2} \left[ \alpha_N G_n^z - \alpha_{TC} \frac{\epsilon_1 G_n^z P_j^{Ez} - \epsilon_2 P_j^{Ez}}{1 + \epsilon_1 P_j^{Ez}} \right] - \Gamma_2^z \frac{\omega_0 n_2}{c \sigma} P_j^{Ez}, \tag{A21}
\]

\[
\frac{\phi_j^{TMz+1} - \phi_j^{TMz-1}}{2 \Delta z} = -\Gamma_2^z \frac{\omega_0 n_2}{c} \frac{1}{\sigma} P_j^{Ez}, \tag{A22}
\]

\[
\frac{G_n^{z+1} - G_n^{z-1}}{2 \Delta \tau} = \frac{G_0 - G_n^z}{\tau_e} - \frac{1}{E_{sat}} \left[ \frac{G_n^z - \epsilon_2 P_j^{TMz}^2}{1 + \epsilon_1 P_j^{TMz}} P_j^{TMz} + \Gamma_2^z \beta_2 P_j^{TMz} \right]. \tag{A23}
\]

Here the subscript \( z \) represents the different sections that the amplifier has been divided into. \( n \) represents the time at which the the gain, power and phase had a certain value. This means that for each calculation the gain \( G_n^z \) of an amplifier section at a certain time instance is found. The incoming pulse has also been divided into a number of sections each with a value \( j \). So for each run of the program the value of the power or phase of the light pulse at the next time instance is calculated.

These equations form an explicit difference scheme in that the values known at this instant in time are used to calculate the values at the next time instant. In order to use this scheme a few assumptions have to be made about the arriving wave packet and the gain. The initial gain \( G_0 \) of the amplifier is calculated using the biasing current and the current required to achieve transparency. It is then assumed that before the light pulse enters the active region of the amplifier, the gain of the amplifier is uniform, meaning that all the sections have the same gain \( G_0 \).

The incoming light pulse is assumed to be a gaussian pulse. The pulse is sampled at different values of \( \tau \) and it is assumed that the first part of the pulse enters the amplifier at a time \( t = 2 \). The power of those parts of the pulse that are still in the waveguide are not recalculated because the gain is assumed to be zero in the waveguide.

To simplify the program \( \Delta t \) and \( \Delta x \) are chosen in such a way that for each change in the time step a section of the pulse is assumed to advance into the next section of the amplifier.
Appendix B

The SOA model 2

Starting with rate equations derived from the semiclassical density matrix equations by adiabatic elimination of the interband polarization [31], we have:

\[
\frac{\partial n_\beta}{\partial t} = -\frac{n_\beta - \bar{n}_\beta}{\tau_{SHB\beta}} - v_\beta g S, \quad (B.1)
\]

\[
\frac{\partial N}{\partial t} = \frac{I}{eV} - \frac{N}{\tau_s} - v_\beta g S, \quad (B.2)
\]

\[
\frac{\partial T_\beta}{\partial t} = \left(\frac{\partial U_\beta}{\partial T_\beta}\right)^{-1}_N \left\{ \frac{\sigma_\beta N \hbar \omega_0}{g} + \left(\frac{\partial U_\beta}{\partial N}\right)_{T_\beta} - E_{\beta,0} \right\} v_\beta g S - \frac{T_\beta - T_L}{\tau_{CH\beta}}, \quad (B.3)
\]

where \( g \) is

\[
g = \frac{a_N}{v_\beta} (n_c + n_v - N_0), \quad (B.4)
\]

a function of the local densities. \( t \) is the moving coordinate frame where \( t = t' - z/v_\beta \) and \( t' \) is the time and \( z \) the distance along the length of the amplifier and \( v_\beta \) the group velocity. \( \tau_{SHB\beta} \) is the carrier-carrier scattering time, \( \tau_{CH\beta} \) the temperature relaxation time, \( T_L \) the lattice temperature, \( T_\beta \) is the carrier temperature, \( \tau_s \) the carrier lifetime and \( I \) the current with \( V \) the volume of the active region and \( \sigma_\beta \) the free-carrier absorption coefficient. \( S \) and \( N \) take on the usual meanings of carrier density and photon density with \( n_\beta \) is the local carrier density with \( \bar{n}_\beta \) the quasi-equilibrium density. \( \beta \) can be the conduction or valence band.

The quasi-Fermi carrier densities \( \bar{n}_\beta \) can be separated into a carrier density dependent linear part and a contribution due to the carrier temperature change

\[
\bar{n}_\beta(N, T_\beta) = \bar{n}_{\beta,l}(N) + \frac{\partial \bar{n}_\beta}{\partial T_\beta} \Delta T_\beta, \quad (B.5)
\]

where \( \bar{n}_{\beta,l} \) are the "Fermi densities" at the lattice temperature and \( \Delta T_\beta = T_\beta - T_L \).

Equation (B.1) can be written as

\[
\frac{\partial (n_\beta - \bar{n}_\beta)}{\partial t} = -\frac{n_\beta - \bar{n}_\beta}{\tau_{SHB\beta}} - v_\beta g S - \frac{\partial \bar{n}_\beta}{\partial t}, \quad (B.6)
\]
The influence of free carrier absorption is neglected in Equation (B.3) because the
device operating point is far above transparency. The temperature change obeys
\[
\frac{\partial \Delta T_\beta}{\partial t} = -\frac{\Delta T_\beta}{\tau_{CH,\beta}} + K_\beta v_g g S,
\]  
(B.7)
given that
\[
K_\beta = \left( \frac{\partial U_\beta}{\partial T_\beta} \right)^{-1} \left[ \frac{\partial U_\beta}{\partial N} - E_{\beta,0} \right].
\]  
(B.8)

To simplify the equations, the carrier density changes are defined as
\[
\Delta n_{SH,\beta} = n_\beta - \bar{n}_\beta,
\]  
(B.9)
and
\[
\Delta n_{CH,\beta} = \bar{n}_\beta - \bar{n}_{\beta,l} = \frac{\partial \bar{n}_\beta}{\partial T_\beta} \Delta T_\beta = \frac{v_g}{a_N} \frac{\partial g}{\partial T_\beta} \Delta T_\beta.
\]  
(B.10)
from the results of Equation (B.4) and (B.5).

Subtracting and adding \( N_{tr}/\tau_s \) on the right hand side of Equation (B.2) gives
\[
\frac{\partial (N - N_{tr})}{\partial t} = -\frac{N - N_{tr}}{\tau_s} - v_g g S(t, z) + \frac{I}{eV} - \frac{N_{tr}}{\tau_s},
\]  
(B.11)
Using Equation (B.5) and (B.10) \( \frac{\partial \bar{n}_\beta}{\partial t} \) can be written as
\[
\frac{\partial \bar{n}_\beta}{\partial t} = \frac{\partial \bar{n}_{\beta,l}}{\partial N} \cdot \frac{\partial N}{\partial t} + \frac{\partial (\bar{n}_\beta - \bar{n}_{\beta,l})}{\partial T_\beta} \cdot \frac{\partial T_\beta}{\partial t}.
\]

Substituting the result above and Equation (B.9) into Equation (B.6) and noting that the
nonlinear gain suppression factor due to SHB is \( \epsilon_{SH,\beta} = a_N \tau_{SH,\beta} \)
\[
\frac{\partial \Delta n_{SH,\beta}}{\partial t} = -\frac{\Delta n_{SH,\beta}}{\tau_{SH,\beta}} - \frac{v_g \epsilon_{SH,\beta}}{a_N \tau_{SH,\beta}} g S(t, z) - \left[ \frac{\partial \bar{n}_{\beta,l}}{\partial N} \cdot \frac{\partial N}{\partial t} + \frac{\partial (\bar{n}_\beta - \bar{n}_{\beta,l})}{\partial T_\beta} \cdot \frac{\partial T_\beta}{\partial t} \right].
\]  
(B.12)
Substituting for \( \Delta T_\beta \) in Equation (B.7) using (B.10) noting that the nonlinear suppression factor due to carrier heating is \( \epsilon_{CH,\beta} = -a_N K_\beta \tau_{CH,\beta} \frac{\partial \bar{n}_\beta}{\partial t} \) gives
\[
\frac{\partial \Delta n_{CH,\beta}}{\partial t} = -\frac{\Delta n_{CH,\beta}}{\tau_{CH,\beta}} - \frac{v_g \epsilon_{CH,\beta}}{a_N \tau_{CH,\beta}} g S(t, z).
\]  
(B.13)

The gain in Equation (B.4) can be divided into a linear component attributed to
the change in carrier density and a nonlinear gain component due to carrier heating and spectral hole burning:
\[
g(t) = \frac{a_N}{v_g} (\bar{n}_{c,l} + \bar{n}_{e,l} - N_0) + \frac{a_N}{v_g} \sum_\beta [(n_\beta - \bar{n}_\beta + (\bar{n}_\beta - \bar{n}_{\beta,l})],
\]  
(B.14)
\[
= g_l(N) + \frac{a_N}{v_g} \sum_\beta (\Delta n_{SH,\beta} + \Delta n_{CH,\beta}),
\]  
(B.15)
\[
= g_l(N) + \sum_\beta (\Delta g_{SH,\beta} + \Delta g_{CH,\beta}),
\]  
(B.16)
note that $g_t = a(N - N_{tr})$

Equation (B.12) can be rewritten as

$$\frac{\partial \Delta n_{SHB\beta}}{\partial t} = - \frac{\Delta n_{SHB\beta}}{\tau_{SHB\beta}} - \frac{\nu_g \epsilon_{SHB\beta}}{a_N \tau_{SHB\beta}} g S(t, z) - \left[ \frac{\partial \Delta n_{CH\beta}}{\partial t} + \frac{\nu_g}{a_N} \frac{\partial (N - N_{tr})}{\partial t} \right], \quad (B.17)$$

where

$$y_{\beta} = \frac{a_N}{a v_g} \frac{\partial n_{\beta}}{\partial N}. \quad (B.18)$$

Using the results obtained above, Equations (B.11) (B.12) (B.13) and (B.17) become:

$$\frac{\partial g_t}{\partial t} = \frac{g_t}{\tau_s} - \frac{1}{S_s \tau_s} g S(t, z) + \frac{a(N_{st} - N_{tr})}{\tau_s}, \quad (B.19)$$

$$\frac{\partial \Delta g_{SHB\beta}}{\partial t} = - \frac{\Delta g_{SHB\beta}}{\tau_{SHB\beta}} - \frac{\epsilon_{SHB\beta}}{\tau_{SHB\beta}} g S(t, z) - \left( \frac{\partial \Delta g_{CH\beta}}{\partial t} + y_{\beta} \frac{\partial g_t}{\partial t} \right), \quad (B.20)$$

$$\frac{\partial \Delta g_{CH\beta}}{\partial t} = - \frac{\Delta g_{CH\beta}}{\tau_{CH\beta}} - \frac{\epsilon_{CH\beta}}{\tau_{CH\beta}} g S(t, z). \quad (B.21)$$

$N_{st}$ is the unsaturated value of the carrier density. This is the value of the carrier density that is a result of pumping and spontaneous carrier recombination. This is before the optical pulse enters the active region of the amplifier.

$$N_{st} = \frac{I \tau_s}{e V}. \quad (B.22)$$

Saturation photon density

$$S_s = \frac{1}{v_g d \tau_s}, \quad (B.23)$$

this is related to the saturation power by

$$P_s = \hbar \omega A_{eff} v_g S_s = \kappa S_s. \quad (B.24)$$

Given that the propagation equation neglecting the effects of waveguide internal loss becomes

$$\frac{\partial S(t, z)}{\partial z} = \Gamma g S(t, z). \quad (B.25)$$

The solution of this equation will be

$$S(t, z) = S(t, 0) G(t, z). \quad (B.26)$$

Defining

$$G(t, z) = \exp[g_m(t, z)], \quad (B.27)$$

given that

$$g_m(t, z) = \Gamma \int_0^z dz' g(t, z'), \quad (B.28)$$

and

$$h_N = \Gamma \int_0^z dz' g(N), \quad (B.29)$$

$$h_{SHB\beta} = \Gamma \int_0^z dz' \Delta g_{SHB\beta}, \quad (B.30)$$

$$h_{CH\beta} = \Gamma \int_0^z dz' \Delta g_{CH\beta}. \quad (B.31)$$
If \( g_m(t, z) = h_N + \sum_\beta (h_{CH\beta} + h_{SHB\beta}) \) the differential equations that describe the evolution of the transmission with the optical pulse are then:

\[
\frac{dh_N}{dt} = -\frac{h_N}{\tau_s} - \frac{1}{S_N \tau_s} [G(t, z) - 1] S(t, 0) + \frac{g_0(z)}{\tau_s},
\]
\[
\frac{dh_{CH\beta}}{dt} = \frac{h_{CH\beta}}{\tau_{CH\beta}} - \frac{c_{CH\beta}}{\tau_{CH\beta}} [G(t, z) - 1] S(t, 0),
\]
\[
\frac{dh_{SHB\beta}}{dt} = \frac{h_{SHB\beta}}{\tau_{SHB\beta}} - \frac{c_{SHB\beta}}{\tau_{SHB\beta}} [G(t, z) - 1] S(t, 0) - \frac{dh_{CH\beta}}{dt} - \frac{dh_N}{dt}, \tag{B.32}
\]

where \( g_0(z) = \Gamma g_l(N_{st})z = \Gamma a(N_{st} - N_{tr})z \). It should be noted that \( h \) is a dimensionless quantity well as \( g_m \) is measured in \( m^{-1} \). From the earlier definition

\[
G(t, z) = \exp \left[ h_N + \sum_\beta (h_{CH\beta} + h_{SHB\beta}) \right]. \tag{B.33}
\]

The phase of a pulse propagating through the waveguide changes with the carrier density and temperature. It can be described by

\[
\frac{\partial \phi}{\partial z} = \Gamma \frac{\partial k}{\partial N} \Delta N + \sum_\beta \Gamma \frac{\partial k}{\partial T_\beta} \Delta T_\beta + \Gamma \Delta k_{neq}, \tag{B.34}
\]

\( k = 2\pi n/\lambda \) is the propagation constant, \( n \) is the effective index of the waveguide and \( \lambda \) is the wavelength in a vacuum. The last term accounts for the non-equilibrium part of the phase dynamics.

Given the linewidth enhancement factor

\[
\alpha_R = -2 \frac{\partial \Delta k / \partial R}{\partial \Delta g / \partial R}, \tag{B.35}
\]

with \( R = N, T_e, T_v \) using this result in Equation (B.34)

\[
\frac{\partial \phi}{\partial z} = -\frac{1}{2} \Gamma \left[ \alpha_N g_l(N) \Delta N + \sum_\beta \alpha_{T_\beta} \Delta g_{CH\beta} \right] + \Gamma \Delta k_{neq}, \tag{B.36}
\]

\( \alpha \) is the differential gain and \( \Delta N = N - N_{st} \) is due to stimulated emission and can be written as \( \Delta N = N - N_{tr} + N_{tr} - N_{st} \), using this result together with Equation (B.10) we obtain

\[
\frac{\partial \phi}{\partial z} = -\frac{1}{2} \Gamma \left[ \alpha_N g_l(N) - \alpha_N g_l(N_{st}) + \sum_\beta \alpha_{T_\beta} \Delta g_{CH\beta} \right] + \Gamma \Delta k_{neq}, \tag{B.37}
\]

where \( \Delta g_{CH\beta} = a_N / v_g \Delta n_{CH\beta} \). This analysis neglects optical nonlinearities due to TPA and optical stark effect, which produce instantaneous absorptive and refractive nonlinearities. This is justified if the pulsewidth of the pulses is larger than 1 ps.

Defining:

\[
h'_N = \sum_\beta \frac{\tau_s}{\tau_s - \tau_{SHB\beta}} y_\beta h_N - \sum_\beta \frac{\tau_{SHB\beta}}{\tau_s - \tau_{SHB\beta}} S_\beta B_\beta, \tag{B.38}
\]

\( \beta \)
\[ h'_{CH\beta} = \sum_{\beta} \frac{\tau_{CH\beta}}{\tau_{CH\beta} - \tau_{SHB\beta}} h_{CH\beta} - \frac{\tau_{SHB\beta}}{\tau_{CH\beta} - \tau_{SHB\beta}} \epsilon_{CH\beta} \frac{A_{\beta}}{B_{\beta}}, \quad (B.39) \]

\[ h'_{SHB\beta} = \epsilon_{SHB,\beta} \frac{A_{\beta}}{B_{\beta}}, \quad (B.40) \]

with

\[ A_{\beta} = h_{SHB\beta} - \frac{\tau_{SHB\beta}}{\tau_{CH\beta} - \tau_{SHB\beta}} h_{CH\beta} - \frac{\tau_{SHB\beta}}{\tau_{s} - \tau_{SHB\beta}} y_{\beta} h_{N}, \quad (B.41) \]

\[ B_{\beta} = \epsilon_{SHB\beta} - \frac{\tau_{SHB\beta}}{\tau_{CH\beta} - \tau_{SHB\beta}} \epsilon_{CH\beta} - \frac{\tau_{SHB\beta}}{\tau_{s} - \tau_{SHB\beta}} y_{\beta} S_{s}. \quad (B.42) \]

Noting that

\[ g_{m} = h_{N} + \sum_{\beta} [h_{CH\beta} + h_{SHB\beta}] = h'_{N} + \sum_{\beta} [h'_{CH\beta} + h'_{SHB\beta}]. \quad (B.43) \]

The primed quantities may be considered proportional to the effect of changes in carrier temperature, density and SHB on the gain and index. A delay appears in the effect on the gain and index when changes to the carrier temperature and density occur. The primed quantities naturally account for this delay in the establishment of the quasi-equilibrium which delay is equivalent to \( \tau_{SHB\beta} \).

The phase as described in Equation (B.37) becomes

\[ \phi(t, z) = \phi(t, 0) - \frac{1}{2} \alpha_{N} [h'_{N} - g_{0}(z)] - \frac{1}{2} \sum_{\beta} \alpha_{T_{\beta}} h'_{CH\beta}, \quad (B.44) \]

with the last term on the RHS of Equation (B.37 being absorbed in the primed coefficients.

**B.1 Conditions for the simulations**

The phase and gain at any length \( L \) of the amplifier can be numerically calculated using the Equations (B.32) and (B.44). The initial conditions of the simulations are \( h_{N} = g_{0}(z) \) and \( h_{SHB\beta} = h_{CH\beta} = 0 \). The effects of heating and SHB are ignored for the valence band because the heavier hole mass typical leads to much shorter relaxation times than those of the electrons. It should also be noted that because \( \sum_{\beta} \partial \bar{n}_{\beta,1}/\partial N = (v_{g}/a_{N})(\partial g/\partial N) = a v_{g}/a_{N} \) so that

\[ \sum_{\beta} y_{\beta} = 1. \quad (B.45) \]

With the gain and phase temporal variation at the location \( z \), the electrical field at that location becomes (output field for \( z=L \))

\[ E(t, z) = E(t, 0) \exp \left[ \frac{1}{2} g_{m}(t, z) + i \phi(t, z) \right]. \quad (B.46) \]
The author would like to thank the members of the OED group for the many helpful comments and hints. Most thanks go to Dr. J.J.G.M. van der Tol for the many discussions, comments and all-round assistance and encouragement.