Vibration behaviour of slender footbridges due to synchronized pedestrian loading

Altememy, O.A.N.

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VIBRATION BEHAVIOUR OF SLENDER FOOTBRIDGES DUE TO SYNCHRONIZED PEDESTRIAN LOADING

OSAMEH ALTEMEMY
IN PARTIL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE OF MASTER OF SCIENCE
ARCHITECTURE, BUILDING AND PLANNING

RESEARCH NUMBER:  O-2015.103
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# Vibration behaviour of slender footbridges

## Graduation research

### MASTER THESIS

‘Vibration behaviour of slender footbridges due to synchronized pedestrian loading’

<table>
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<tr>
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<th>OSAMEH ALTEMEMY</th>
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</thead>
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<td>VUURVLINDERSTRAAT 13</td>
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<td></td>
<td>5641 DK EINDHOVEN</td>
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</table>

**GRADUATION COMMITTEE**

- PROF. DR. IR. J. MALJAARS
- IR. B.W.E.M. VAN HOVE
- PROF. IR. H.H. SNIJDER
Summary

This research aims to investigate the accuracy of the most advanced design guideline for slender footbridges, the EUR 23984 EN, concerning synchronization effects between pedestrians and the corresponding increase in dynamic loading, and to investigate the occurrence of the phenomenon synchronization between pedestrians and vertical bridge vibrations. The EUR 23984 EN states that the current load-and response model, takes into account the synchronization effects between pedestrians, and that the synchronization between pedestrians and bridge is negligible. The interaction between pedestrians and bridge vibrations may occur for vertical bridge vibrations larger than 1.5 m/s², which are outside acceptable limits for serviceability of footbridges.

Experiments that were performed in the van Musschenbroek Laboratory at the University of Technology Eindhoven, showed that synchronization between pedestrians could be quantified, including in step frequency distributions for streams, in contrast to the synchronization between pedestrians and bridge. It turned out that the synchronization effects between pedestrian and bridge occurs for some pedestrians and only for a couple of steps, but is negligible when analyzing the overall picture. The maximum occurred acceleration during the experiments was 0.8 m/s², thus it is unsure if this synchronization effect is negligible for accelerations between 0.8 m/s² and 1.5 m/s².

The accuracy of the load model of the EUR 23984 EN was investigated by first of all, developing a time step simulation with a Monte Carlo approach, which takes into account the variation of the force characteristics, and which is then used to assess the load model of EUR 23984 EN with it. This analytical model with probabilistic approach, is combined with the experiments: measured values are used as input for the Monte Carlo Simulation. The Monte Carlo Simulations was validated first, by recalculating the experiments that have taken place in the laboratory: the calculated accelerations were compared with the measured acceleration.

A number of calculations were performed with the load model of EUR 23984 EN, and with the Monte Carlo Simulation. The differences between the values were extremely large: the EUR 23984 EN approaches the reality too conservatively. These large differences were further investigated by analyzing the different parameters of the deterministic load model separately. This analysis, which can be found in the analytical research of this report, clarifies the large difference between the EUR 23984 EN, and the reality i.e. Monte Carlo Simulation. The EUR 23984 EN gives safe values, but designing with the EUR 23984 EN will give no optimized design.
Preface

This Master thesis comprises of a research project about vibration behaviour of slender footbridges due to synchronized pedestrian loading, conducted at Eindhoven University of Technology. The start of this research project was challenging, since dynamics is a new field of knowledge for me. Besides that, it was also difficult to find the right direction: the supervisors gave me the freedom to choose a direction and to develop the research questions and goals. Afterwards, I am very thankful for that. It has helped me to extend my knowledge in dynamics in general and especially in conducting a research.

I would firstly like to thank my family and friends who have supported me during the graduation period in general. Special thanks goes to Professor dr. ir. Johan Maljaars, chairman of the graduation committee. His knowledge in dynamics, research in general, and his contribution in reading and improving my papers, has been important for my thesis. In addition, I would like to thank ir. B.W.E.M. van Hove for her supervision during the graduation period, and for helping arranging participants for the experiments that were conducted in the van Musschenbroek Laboratory. At last, I would like to thank to Professor ir. Snijder for his supervision.


Osameh Altememy
# Symbols

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Meaning</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>B</td>
<td>Width of bridge</td>
<td>m</td>
</tr>
<tr>
<td>L</td>
<td>Span of bridge</td>
<td>m</td>
</tr>
<tr>
<td>S</td>
<td>Loaded surface of bridge</td>
<td>m²</td>
</tr>
<tr>
<td>d</td>
<td>Density of pedestrians on a surface</td>
<td>P/m²</td>
</tr>
<tr>
<td>P</td>
<td>Pedestrian</td>
<td>-</td>
</tr>
<tr>
<td>P₀</td>
<td>Dynamic force amplitude due to a single walking pedestrian</td>
<td>N</td>
</tr>
<tr>
<td>P₀ × cos(2πft)</td>
<td>Harmonic load due to a single pedestrian</td>
<td>N</td>
</tr>
<tr>
<td>p(t)</td>
<td>Distributed surface load</td>
<td>kN/m²</td>
</tr>
<tr>
<td>G</td>
<td>Static weight of pedestrian</td>
<td>N</td>
</tr>
<tr>
<td>fₛ</td>
<td>Step frequency of a single pedestrian</td>
<td>Hz</td>
</tr>
<tr>
<td>fₛ,mean</td>
<td>Average step frequency of a pedestrian stream</td>
<td>Hz</td>
</tr>
<tr>
<td>fₙ</td>
<td>Fundamental natural frequency for mode</td>
<td>Hz</td>
</tr>
<tr>
<td>m</td>
<td>Mass per length</td>
<td>kg/m</td>
</tr>
<tr>
<td>m*</td>
<td>Modal mass</td>
<td>kg/m</td>
</tr>
<tr>
<td>k</td>
<td>Stiffness</td>
<td>N/m</td>
</tr>
<tr>
<td>k*</td>
<td>Modal stiffness</td>
<td>N/m</td>
</tr>
<tr>
<td>c</td>
<td>Damping constant</td>
<td>-</td>
</tr>
<tr>
<td>ζ</td>
<td>Structural damping ratio</td>
<td>-</td>
</tr>
<tr>
<td>a</td>
<td>Acceleration</td>
<td>m/s²</td>
</tr>
<tr>
<td>v</td>
<td>Velocity</td>
<td>m/s</td>
</tr>
<tr>
<td>u</td>
<td>Displacement</td>
<td>m</td>
</tr>
<tr>
<td>ϕ</td>
<td>Phase of harmonic force</td>
<td>-</td>
</tr>
<tr>
<td>n</td>
<td>Number of pedestrians on the loaded surface S (n = S × d)</td>
<td>-</td>
</tr>
<tr>
<td>n'</td>
<td>Equivalent number of pedestrians on a loaded surface S</td>
<td>-</td>
</tr>
<tr>
<td>Ψ</td>
<td>Reduction coefficient</td>
<td>-</td>
</tr>
</tbody>
</table>

### Subscripts

| exp | Experimental value | - |
| 95% | 95th percentile value, 95% response limit | - |
| mean | Average | - |
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1. Introduction

Nowadays slender footbridges are common, because of improvements of materials, building methods and the involvements of architects. A slender design of a footbridge is possible because of the low statical loads due to walking pedestrians. But, as a result of the increasing slenderness, these footbridges have a low stiffness, mass, damping and natural frequency. The decrease of stiffness and mass lead to more sensitivity to dynamic loads. Slender footbridges have natural frequencies that near the average step frequencies of walking pedestrians, which could result in undesirable vibrations of the bridge deck at the crossing of the pedestrians.

In addition to that, undesirable vibrations could increase when synchronization effects between pedestrians occur. In crowded situations, walking pedestrians may adopt unintentionally the same step pattern as other pedestrians around them [1]. This effect increases when the walking behaviour of these pedestrians interacts with the bridge vibrations. Once pedestrians notice vibrations, they tend to walk with the same frequency as the vibration frequency of the bridge, in order to maintain their balance.

In past years, two footbridges illustrated the complexity of this problem: Millennium Bridge in London and the Solferino Bridge in Paris. Pedestrians walking over one of these experienced undesired vibrations at the opening ceremony, which lead to their closure. Modifications to the structures had to be made afterwards; damping devices where installed, which turned out to be an effective but costly solution [2].

This has led to the research of the synchronization phenomenon and the development of synchronized loading models. Extensive testing took place within two European research projects, namely SYNPEX [3] - and HIVOSS projects [4], which where the basic of the design guideline EUR 23984 EN ‘Design of Lightweight Footbridges for Human Induced Vibrations’. The EUR 23984 EN is the most advanced European design guideline, which is used to predict the vibration behaviour of slender footbridges and to check the comfort criteria.

The EUR 23984 EN provides a load- and response model, which takes into account the synchronization effect between walking pedestrians. This effect is taken into account, by converting a random pedestrian stream, with random pedestrian characteristic values, into a perfect, synchronized pedestrian stream, with deterministic values.
The load model represents the equivalent pedestrian stream, a harmonic load:

\[ p(t) = P \cdot \cos(2\pi f_s t) \cdot n' \cdot \Psi \quad \text{where } P \text{ is the component of the force due to a single pedestrian, which is a deterministic value, } f_s \text{ is the step frequency, } n' \text{ is the equivalent number of pedestrians representing a random stream, and where the } \Psi \text{ is the reduction coefficient taking into account the probability that the footfall frequency approaches the critical range of natural frequencies.} \]

The product of \( n' \cdot \Psi \) takes into account the synchronization between pedestrians. In this approach there is assumed that all equivalent pedestrians \( (n') \) walk with the same step frequency \( f_s \), and in addition to that there is assumed that the step frequency \( f_s \) of these equivalent pedestrians is equal to the natural frequency \( f_1 \) of the bridge. The reduction factor \( \Psi \) reduces the equivalent dynamic loading if the average step frequency \( f_{s,\text{mean}} \) of the (real) random stream far away from the bridge frequency \( f_1 \).

However, this load- and response model does not take into account the synchronization effects between pedestrians and structure. The force characteristics are based on ‘normal’ walking behaviour: the influence of the vibration structure on the step frequency is not taken into account. There is no additional (magnification) factor that takes into account the probability that the average step frequency \( f_{s,\text{mean}} \) of the pedestrian stream, may move towards the bridge frequency \( f_1 \).

According to the SYNPEX project, pedestrians may synchronize with harmonic vertical vibrations of 1,5 m/s² and higher. Vertical vibrations of 1,5 m/s² are according to the SYNPEX project outside comfort limits of slender footbridges, which are based on comfort questionnaires that are answered by pedestrians walking on several bridges. If vertical vibrations occur outside acceptable limits for serviceability of footbridges, pedestrians might be disturbed and stop walking. As for vertical vibrations lower than 1,5 m/s², these would be absorbed by legs and joints which provide a certain amount of damping so that the center of gravity is not affected by vertical vibrations. Pedestrians react much more sensibly to lateral vibrations compared to vertical ones. Lateral synchronization occurs already at small vibration amplitudes, whereby pedestrian keep on walking synchronized with the vibration and hence turn a random excitation into a resonant one. Therefore, the design guideline provides a check of criteria for lateral lock-in. In this check, the predicted lateral accelerations, must not exceed the trigger acceleration amplitude, when the pedestrian-structure interaction (lock-in) begins [4].

The EUR 23984 EN [1], which is based on the SYNPEX, states on the other hand that the discomfort is unacceptable for accelerations higher than 2,5 m/s². Thus, for accelerations between 1,5 m/s² and 2,5 m/s², synchronization may occur. Furthermore, the experiments in which has been observed that synchronization between pedestrian and vertical vibration occurs for vibrations larger than 1,5 m/s², was limited to single pedestrians and not pedestrian streams.
1.1 Problem definition

The EUR 23984 EN provides a load- and response model which takes into account the synchronization effects between pedestrians via the equivalent number of pedestrians \( n' \) and via the reduction coefficient \( \Psi \). This load model does not take into account the synchronization effects between pedestrians and structure that may occur.

The following questions then arise: how accurate is the approach of the equivalent number of pedestrians and the reduction coefficient \( \Psi \) in taking into account the synchronization between pedestrians in different traffic situations, and is it safe or too conservative? Besides that, is the synchronization between pedestrians and vertical vibrations negligible? If not, is this approach safe enough if this synchronization effect occurs?

1.2 Goal

The goal is to investigate how accurate the approach of the equivalent number of pedestrians \( n' \) is, and to investigate how accurate the reduction coefficient \( \Psi \) is, with respect to the load model in EUR 23984 EN.

1.3 Scope

The following research boundaries will be held by investigating the accuracy of this load- and response model. The focus will be on excessive vertical vibrations that could cause discomfort of walking pedestrians. The bridges that will be treated, are simple supported bridges, of which the structures are considered as single degree of freedom structures. Furthermore, this research is limited to pedestrians that walk behind each other in one line.

1.4 Approach

A literature study has been performed to identify and select research evidence that are relevant to this subject. This literature study contains theory of footbridge dynamics, but also the design guideline, its background document and other scientific articles and studies concerning pedestrian-induced dynamic loads are studied. The literature survey can be found in Annex B.

The subsequent step of this research is the performance experiments in the van Musschenbroek laboratory of the department of the Built Environment faculty at the TU/e. These experiments were performed to measure the pedestrian force characteristics and to quantify the synchronizations effects that could occur in footbridge engineering.

These measured values were used in the following step, the analytical research. In this part of the research, the load- response model of the EUR 23984 EN will be assessed with an analytical dynamic load model with a Monte Carlo approach.
2. Footbridge dynamics

2.1 Dynamic loads induced by pedestrians

Dynamic forces induced by one pedestrian are generated by the movement of the body mass and the put-down, rolling and push-off of the feet. These forces are called human ground reaction forces. When these forces are induced by walking, then they form an almost periodic excitation [5].

During walking, a single pedestrian produces a dynamic time varying force which has components in three directions: vertical, horizontal-lateral and horizontal-longitudinal. The single pedestrian walking force, has been studied for many years. The vertical component of the force, has been most investigated. It has been regarded as the most important of the three forces because it has the highest magnitude. Other human-induced forces that are important for footbridges are forces due to running and some forms of deliberate vandal loading, like jumping and bouncing. Large groups of pedestrians have seldom been formally investigated [6].

2.1.1 Load model for a single pedestrian

Continuous walking or running force can be obtained artificially, by combining individual foot forces, which are assumed to be identical. As shown in figure 2-1, during running there are periods when both feet are off the ground, leading to zero force recorded. As for normal walking, there are some time periods when both feet are on the ground, which gives an overlapping between the left and right leg in the walking time history [6].

![Figure 2-1 Typical pattern of running and walking forces (Sources: Galbrath and Barton [7])](image)

Figure 2-1 gives an example of a force-time history of the vertical force imposed by a pedestrian, during normal walking. A period of 1.25 seconds is shown in which two steps are combined. The total force in the period when both feet are on the ground, is the sum of the individual force of the left and right foot, resulting in a force pattern which has a constant and a fluctuating part. In footbridge dynamics, the constant part is usually ignored and the focus is on the fluctuating part of the force [2].
This time-domain force model for walking pedestrians is based on the assumption that both human feet produce exactly the same force and that the pedestrian induced force is approximately periodic. The periodic force $F_{p,\text{vert}}(t)$ with a period $T$ can be presented by a Fourier series \[1\]:

$$F_{p,\text{vert}}(t) = G[1 + \sum_{i=1}^{n} \alpha_{i,\text{vert}} \sin(2\pi i f_s t - \Phi_i)] \quad [2-1]$$

- $F_{p,\text{vert}}$ vertical periodic force due to walking
- $G$ pedestrian's body weight [N]
- $\alpha_{i,\text{vert}}$ Fourier coefficient of the $i^{th}$ harmonic for vertical forces, i.e. dynamic load factor (DLF)
- $f_s$ step frequency [Hz]
- $\Phi_i$ phase shift of the $i^{th}$ harmonic

The step frequency $f_s$, phase shift $\Phi$ and the dynamic load factor $\alpha$ is explained hereafter.
2.1.2 Step frequency

The step frequency is the number of steps per second that a pedestrian makes during walking. The step frequencies depend on different aspects such as the person's length, age, walking speed and so on. In other words, step frequencies vary among pedestrians. Different authors expressed step frequencies by normal distributions. According to a research of Matsumoto [9], normal walking follows a normal distribution with a mean value of 2.0 Hz, and a standard deviation of 0,173 Hz.

![Figure 2-3 Probability density of step frequencies regarding the walking intention, per person (Source: SYNPEX-project [4])]({})

Butz[10], another author, found the same value of 2.0 Hz for fast walking, but found lower average step frequencies for normal and slow walking pedestrians. This has been confirmed with tests within the SYNPEX project. The following step frequency distributions were found for slow, normal and fast walking intentions in Figure 2-3.
2.1.3 Dynamic load factor $\alpha$ and phase shift

The dynamic load factor $\alpha$ is the fraction of the static load of a pedestrian that acts as a dynamic load, and is dependent on the step frequency. Researchers who have tried to quantify the dynamic load factor, have found different values.

The phase shift is the time difference between the dynamic amplitudes due to the footsteps of pedestrians, which is an important characteristic with regard to the synchronization effects. Researchers also found different values for the phase shift. Bachmann gives a value of $\pi/2$, while the experiments within the SYNPEX project give a value of 0, as shown in Table 2-1. The background document of SYNPEX [4], where the results of the SYNPEX are presented, does not give a clear definition of phase shift. Besides, the terms phase shift and phase angles are mixed up.

In extensive, time-step calculations, a uniform distribution is used for the phase shift $[0, 2\pi]$. This assumption has been made, because each individual pedestrians enters a footbridge on a different moment. Therefore, there are initial phase shifts between pedestrians [11].

Table 0-1 Fourier coefficients and phase shift by different authors for walking and running (Source: Report ‘Design of Lightweight Foot Bridges for Human Induced Vibrations’ [1])

<table>
<thead>
<tr>
<th>Authors</th>
<th>Dynamic load factors / Phase angles</th>
<th>Comment</th>
<th>Type of activity and load direction</th>
</tr>
</thead>
<tbody>
<tr>
<td>Blanchard et al.</td>
<td>$\alpha_1 = 0.257$</td>
<td>$f_p = 2.0 - 2.4 \text{ Hz}$</td>
<td>Walking – vertical</td>
</tr>
<tr>
<td>Bachmann et al.</td>
<td>$\alpha_1 = 0.4 / 0.5; \alpha_2 = \alpha_3 = 0.1$</td>
<td>$f_p = 2.0 / 2.4 \text{ Hz}$</td>
<td>Walking – vertical</td>
</tr>
<tr>
<td></td>
<td>$\alpha_1 = \alpha_2 = \alpha_3 = 0.1$</td>
<td>$f_p = 2.0 \text{ Hz}$</td>
<td>Walking – lateral</td>
</tr>
<tr>
<td></td>
<td>$\Phi_2 = \Phi_3 = \pi/2$</td>
<td>$f_p = 2.0 - 3.0 \text{ Hz}$</td>
<td>Running – vertical</td>
</tr>
<tr>
<td>Kerr</td>
<td>$\alpha_1, \alpha_2 = 0.7; \alpha_3 = 0.2$</td>
<td>$\alpha_1$ is frequency dependant</td>
<td>Walking – vertical</td>
</tr>
<tr>
<td>Young</td>
<td>$\alpha_1 = 0.37 (f_p - 0.95) \leq 0.5$</td>
<td>Mean values Fourier coeff.</td>
<td>Walking vertical</td>
</tr>
<tr>
<td></td>
<td>$\alpha_2 = 0.054 + 0.0088 f_p$</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\alpha_3 = 0.026 + 0.015 f_p$</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\alpha_4 = 0.01 + 0.0204 f_p$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>EC5, DIN1074</td>
<td>$\alpha_1 = 0.4; \alpha_2 = 0.2$</td>
<td></td>
<td>Walking vertical</td>
</tr>
<tr>
<td></td>
<td>$\alpha_1 = \alpha_2 = 0.1$</td>
<td></td>
<td>Walking – lateral</td>
</tr>
</tbody>
</table>
2.1.4 Synchronization

Synchronization effects are divided into two types: synchronization between pedestrians and synchronization between pedestrian and structure. Both result in an increase in the dynamic force.

Interaction between pedestrians is more likely when the density of pedestrians increases. Pedestrians in high density streams, are not able to walk freely and are forced to adopt the same walking speed as the rest of group. Image an opening ceremony of the bridge, where the walking speed of slow pedestrians in front could affect the walking speed of pedestrians behind. Or an example where hurried pedestrians in crowded situations, force other pedestrians to walk faster.

The synchronization between pedestrians, is a loading problem, in contrast to the synchronization between pedestrian and structure, which is a loading- and response problem. Once pedestrians notice vibrations, they tend to walk with the same frequency as the vibration frequency of the bridge, in order to maintain their balance. This results in an increase in the dynamic force, which increases the response and so on. The interaction between pedestrian and structure is a form of second order effect.

2.1.4.1 Synchronization between pedestrians

The correlation between the mean step frequency and the mean walking speed of single pedestrians indicate that synchronization between pedestrians occurs when the pedestrian density increases. Figure 2-4 show that the average step frequency becomes smaller when the walking speed decreases. But also the standard deviation of the step frequency becomes smaller. The standard deviation is approximately 0.1 Hz for low densities and 0.05 Hz for high densities.
Oeding [12] also gives identical values for the average step frequency and standard deviation, in different streams: 0.2 ; 0.5 ; 1.0 and 1.5 pedestrians per m². The results of Oeding are shown in Table 2-2. Similar conclusions have been made in the PEDIGREE-research [13], where the stepping behaviour of pedestrians walking in line was investigated.

<table>
<thead>
<tr>
<th>Pers/m²</th>
<th>( \bar{v}_w ) [m/s]</th>
<th>( f_{w,m} ) [Hz]</th>
<th>( \sigma_f ) [Hz]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2</td>
<td>1.45</td>
<td>1.93</td>
<td>0.089</td>
</tr>
<tr>
<td>0.5</td>
<td>1.30</td>
<td>1.81</td>
<td>0.076</td>
</tr>
<tr>
<td>1.0</td>
<td>1.04</td>
<td>1.61</td>
<td>0.054</td>
</tr>
<tr>
<td>1.5</td>
<td>0.79</td>
<td>1.41</td>
<td>0.033</td>
</tr>
</tbody>
</table>

2.1.4.2 Synchronization between pedestrian and bridge structure

**Vertical vibrations**

Other tests within SYNPEX where focused on the synchronization effects between pedestrian and vertical vibrations [4]. Pedestrians, who wear a pressure sensor in their shoe (Type FSR), were asked to walk on a vertically vibrating platform. The signal of the pressure sensors in combination with the bridge vibration can give an indication whether synchronization takes place or not.

In Figure 2-5, the first-, second-, third- and fourth graph respectively represent the vertical ground reaction force of the 1\(^{st}\) and 2\(^{nd}\) foot combined [N], the continuous bridge vibration [m/s], the vibration velocity of the 1\(^{st}\) foot measured with the pressure sensor [m/s], and the vibration velocity of the 2\(^{nd}\) foot measured with the pressure sensor [m/s], against time [s]. As the pressure sensor is relatively small compared to the foot sole, it cannot measure the complete contact time.

It is shown in Figure’s 2-5 and 2-6, that for the identification of synchronization it is necessary to compare the complete contact time in relation to the velocity. Therefore, besides the signals of the pressure sensors also the measured ground reaction forces are compared regarding the positive impulse loading.
In figure 2-5, the experiment is shown where the bridge is excited with a frequency $f_p=1.55$ Hz and the pedestrian has a step frequency $f_s=1.55$ Hz.

![Graph showing vibration behaviour](image)

Figure 2-5 Exciting the bridge in resonance, $f_s=f_p=1.55$ Hz (Source: SYNPEX [4])

The same experiment is repeated but now with a step frequency of $f_s=1.40$ Hz, see Figure 2-6.

Lines are drawn for both experiments, between the minimum of the vibration velocity of the bridge (which is the maximum displacement) and the vertical human ground reaction forces, to see whether synchronization takes place. In figure 2-5, there can be seen that the minimum of the vibration velocity (the maximum displacement of the bridge) again and again matches the maximum dynamic load due to the footsteps of the pedestrian.
For the experiment where the pedestrian has step frequency 1.55 Hz (Figure 2-5), there can be seen that the maximum displacement of the bridge matches the maximum impulse of the ground reaction forces. This can be seen for both 1st and 2nd foot. If the maximum displacement and the maximum impulse of the foot match, then synchronization takes place. As for the pedestrian with step frequency 1.40 Hz (Figure 2-6), it is clear that the maximum displacement of the bridge does not match the maximum ground reaction (see the lines) force for both 1st and 2nd foot.

As there are only four load cells and not all load cells are hit during one crossing, one cannot identify whether persons show a stable synchronization behaviour over a longer time period or just a short one that can also take place by chance. The evaluation of the measured pressure sensor signals and the force cells signals combined with the evaluation of step frequencies showed no stable synchronization behaviour of the test persons at considerably large accelerations (1.5 m/s²), as it is shown in Figure 2-7. Synchronization might occur with larger accelerations.
If larger accelerations occur, pedestrians might be disturbed and stop walking compared to lateral synchronization, which occurs already at small vibration amplitudes and pedestrians keep on walking synchronized with the vibration and hence turn a random excitation into a resonant one.
2.2 Dynamic response

2.2.1 Discrete systems

A pedestrian bridge can be considered as an oscillator if a dynamic load is applied on it. An oscillator can have one or more degrees of freedom (n-DOF). Most simply supported slender footbridges have second natural frequencies \((n=2)\) that are larger than the excitation range of \(1.25 \text{ Hz} - 4.6 \text{ Hz}\). As a result of this, the focus of this dynamic problems for this type of footbridges is on the first vibration mode. This offers the possibility to represent a continuous model by a system with only a single degree of freedom (SDOF). For the structure as shown in Figure 2-8, it can be the deflection in the middle.

\[
F = m \frac{d^2x}{dt^2} = m \ddot{x} \tag{2-2}
\]

The equation of motion for a discrete system such as in Figure 2-9 can be described using the second law Newton:

\[
F = m \frac{d^2x}{dt^2} = m \ddot{x} \tag{2-2}
\]
M₁, k₁ and c₁ represent the modal mass, modal stiffness and modal damping constant respectively.

When the mass is pushed downwards, we will find the following equilibrium:
\[-F_{\text{spring}}(t) - F_{\text{damping}}(t) = m₁ \cdot ẍ(t)\] [2-3]

\[F_{\text{spring}}(t) = k₁ \cdot x(t)\]
\[F_{\text{damping}}(t) = c₁ \cdot ẋ(t)\]

Inserting in equation 2-3 gives:
\[-k₁ \cdot x(t) - c₁ \cdot ẋ(t) = m₁ \cdot ẍ(t)\]
\[m₁ \cdot ẍ(t) + c₁ \cdot ẋ(t) + k₁ \cdot x(t) = 0\]

This equation is the second order equation that describes the behaviour of this structure with SDOF [14].

The modal damping constant c is calculated using a specified damping ratio (ζ). This damping ratio is often based on experimental tests on footbridges. The value refers to an equivalent amount of damping present in the structure and is a combination of different damping mechanisms such as friction in connections and structural damping in the main structure. The influence of the amount of damping on the vibration behaviour is shown in Figure 2-10.

![Figure 2-10 Damped and undamped vibrations (Source: www.roymech.co.uk)](image)

In undamped systems, the system oscillates at its own natural frequency. This vibration is mathematically speaking continuous. When adding an amount of damping, the vibrations dissipates in time. This can be seen in underdamped systems, where the system oscillates with the amplitude gradually decreasing to zero.

When a system is strong damped, the system returns to equilibrium as quickly as possible, while critically damped systems return to equilibrium without oscillating.

If the damping ratio is known, the modal damping constant can be determined as follows:
\[c = \zeta \frac{2 \sqrt{k₁ m₁}}{}\] [2-4]
The following equations give the natural frequency of the SDOF system without \( f_1 \) and with damping \( f_{1,D} \):

\[
f_1 = \frac{1}{2\pi} \sqrt{\frac{k_1}{m_1}} \quad [2-5]
\]

\[
f_{1,D} = f_1 \sqrt{1 - \zeta^2} \quad [2-6]
\]

In order to transfer a continuous model, with an infinite number of natural frequencies, to a SDOF-model, the modal properties are required. These properties can be obtained using the equivalent mode shape. The modal mass for a simple supported beam with a uniformly distributed mass is described with the following equation [1] :

\[
m_1 = \int_0^L m(x) \Phi_1^2(x) \, dx = \frac{1}{2} m L \quad [2-7]
\]

where \( m(x) \) represents the mass per unit length and \( \Phi_1(x) \) represents the mode shape of the 1st natural vibration mode.

The modal stiffness \( k_1 \) can now be determined using the following equation:

\[
k_1 = (2\pi f_1)^2 m_1 \quad [2]
\]

### 2.2.2 Steady state response of a SDOF-system

Vibrations occur with a transient and steady-state phase, as shown in Figure 2-11. The transient phase shows an increase of the amplitude of the response. At a certain point, the maximum amplitude of the response remains the same which is called the steady state phase.

![Figure 2-11 Transient- and steady state phase](Source: Blauwendraad, J., Dynamica van Systemen, University of Technology Delft [14])

Most design procedures related to footbridges are based on the steady state response of a SDOF-system under harmonic excitation. Based on the static deflection, the dynamic response can be calculated using the following formula:

\[
u_{\text{max}} = u_{\text{static}} \cdot \text{DAF} \quad [2-8]
\]

where \( u_{\text{static}} \) is the static deflection and DAF is the dynamic amplification factor.

The static deflection of a SDOF system and the dynamic amplification factor can be obtained from the following equations:
where $\alpha$ is the dynamic load factor (Fourier coefficient of the $i^{th}$ harmonic) and $G$ is the body weight. The dynamic load factor $\alpha$ represents the fraction of the static load of a pedestrian that acts as the dynamic load.

$$\text{DAF} = \frac{1}{\sqrt{(1-g^2)^2+(2\,\zeta\,g)^2}} \quad [2-10]$$

where $g = \frac{f_s}{f_1}$, the ratio between the step frequency $f_s$ and the natural frequency $f_1$ of the bridge, and $\zeta$ is the damping of the system. The concept of a dynamic amplification factor (DAF) is used to describe the ratio between the maximum load effect when a bridge is loaded dynamically, and the maximum load effect when the same load is applied statically to the bridge [19].

Dynamic amplification is obtained as a function of $g$ and $\zeta$. It may be represented by a set of curves parameterized by $\zeta$. Some of these curves are provided in Figure 2-12 for some specific values of the critical damping ratio $\zeta$. These curves show a peak for the value of $g_r = \sqrt{1 - 2\zeta^2}$, characterizing the resonance [11].

![Figure 2-12 Resonance phenomenon (Source: Wikipedia: Vibrations [15])](image)

The maximum response in terms of acceleration [m/s$^2$] can be obtained by multiplying the calculated dynamic displacement with $(2\pi f_s)^2$. This is equal to differentiating the displacement signals:

$$a_{\text{max}} = u_{\text{static}} \, \text{DAF} \, (2\pi f_s)^2 \quad [2-11]$$

The maximum response amplitude in terms of accelerations can be used to define the steady
state response in the time domain. The time domain response of the SDOF-system can be defined by:

\[
a(t) = a_{\text{max}} \sin(2\pi f_t t + \Phi) \quad [2]
\]

The phase shift \(\Phi\) gives the difference between the moment where the force is maximum and the response.

Figure 2-13 gives phase angles based on the ratio between the forcing frequency and the natural frequency of the bridge \(f\) and the damping ratio. The phase shift is \(1/2\pi\) (90) degrees in case of resonance.

![Figure 2-13 Phase angle](Source: Wikipedia: Vibrations [15])
3. Standards and guidelines

3.1 Eurocode

Eurocode 0

Eurocode 0 [18] defines comfort criteria for slender footbridges. The comfort criteria is defined in terms of maximum acceptable acceleration of the bridge deck. The amplitude of the vibrations are directly related to the acceleration. The Eurocode prescribes that an assessment of the vibration comfort criteria should be performed, if the natural frequency of the bridge is within the critical range [1.25 Hz – 2.3 Hz].

However, the Eurocode 0 does not propose rules to perform an assessment of the vibration comfort criteria for footbridges. It refers to the JRC-document EUR 23984 EN ‘Design of Lightweight Foot Bridges for Human Induced Vibrations’ [1], to calculate the occurring acceleration of the bridge deck and to check the comfort vibration criteria of slender footbridges.

Eurocode 5

Eurocode 5 [16] provides rules to determine the accelerations of footbridges subjected to dynamic forces. The formulas for vertical vibration, for bridges with natural frequencies lower than 2.5 Hz and bridges between 2.5 Hz and 5.0 Hz, are shown below:

\[ a_{vert} = \frac{200}{m L \xi} \left[ \frac{m/s^2}{s} \right] \quad \text{for } f_{1,vert} < 2.5 \text{ Hz} \]  
\[ a_{vert} = \frac{100}{m L \xi} \left[ \frac{m/s^2}{s} \right] \quad \text{for } 2.5 \text{ Hz} < f_{1,vert} < 5.0 \text{ Hz} \]  

These formulas are derived as follows. The acceleration in the mid-span of the bridge can be calculated with the following formula if 1st resonant frequency is assumed to be equal to the step frequency \( f_s \) (\( f_s = f_1 \)):

\[ a = u_{static} \left( \frac{1}{2 \zeta} \right) (2\pi f_s)^2 \]  

Substituting [2-5] and [2-9] gives:

\[ a = \frac{\alpha G}{k_1} \left( \frac{k_1}{m_1} \right)^2 \left( \frac{1}{2 \zeta} \right) (2\pi f_s)^2 \]  

This equation can be simplified by substituting \( m_1=1/2 mL \):

\[ a = \frac{\alpha G}{m L \xi} \]  

where \( \alpha G \) is the deterministic value of amplitude of the dynamic force. Eurocode 5 gives a value of 200 N for \( \alpha G \).
As for the response of a pedestrian stream consisting of \( n \) random pedestrians, the Eurocode 5 gives the following formula:

\[
a_{\text{vert}, \ n} = 0.23 \ a_{\text{vert}} \ n \ k_{\text{vert}}
\]  \[3-6\]

Where \( n \) is the number of pedestrians, which should be taken as \( 0.6xA \) where \( A \) is the area of the bridge, and reduction coefficient \( k_{\text{vert}} \) which is given in Figure 3-1. \( k_{\text{vert}} \) is the factor which reduces the response as the bridge frequency is far away from the average step frequency of the (real) stream. The factor 0.23 is considering the amount of synchronization between pedestrians. \( k_{\text{vert}} \) reduces

The reduction factor \( \Psi \) reduces the equivalent dynamic loading if the average step frequency \( f_{s,\text{mean}} \) of the (real) random stream far away from the bridge frequency \( f_1 \).

![Figure 3-1 Relation between the vertical fundamental frequency \( f_{\text{vert}} \) or \( f_1 \) and the coefficient \( k_{\text{vert}} \) (Source: Eurocode 5 [16])](image)

### 3.2 Design guidelines

Three design guidelines are used often in Europe to predict the vibration behaviour of slender footbridges and to check their comfort level: HIVOSS [3], French Technical guide of Setra [11] and the EUR 23984 EN [1].

The French guideline ‘Guide to assessing pedestrian induced vibratory behavior of footbridges’ was prepared in 2006 within the framework of the Setra (service d’Etudes techniques des routes et autoroutes) working group on ‘Dynamic behaviour of footbridges’, led by Charles and Hooproah [11]. In 2008 the HIVOSS, Human induced Vibrations of Steel Structures, was introduced. This design guideline was based on steel footbridges, but it could be generalized for footbridges with other materials. The methodology of these two guidelines have much in common, especially in the determination of synchronized pedestrian loading. There are some differences regarding damping ratios, but these are small differences. EUR 23984 EN was introduced in 2009 and is based on the HIVOSS and the research project SYNPEX.
3.3 EUR 23984 EN

The report ‘Design of Lightweight Foot Bridges for Human Induced Vibrations’ [1] is the most advanced design guideline for slender footbridges in Europe. It is prepared under the JRC (Joint Research Centre) of the European Commission and based on the research project SYNPEX (Advanced Load Models for Synchronous Pedestrian Excitation and Optimized Design Guidelines for Steel Bridges), and is funded by the Research Fund for Coal and Steel.

The design steps of the EUR 23984 EN [1] are shown in the figure 3-2 down below.

![Design steps EUR 23984 EN](Source: Report ‘Design of Lightweight Foot Bridges for Human Induced Vibrations’)

Figure 3-2 Design steps EUR 23984 EN (Source: Report ‘Design of Lightweight Foot Bridges for Human Induced Vibrations’)
Illustration design steps

**Step 1:** The first step is to determine natural frequencies of the footbridge.

**Step 2:** Step 2 is to check whether an assessment of the vibration comfort criteria should be performed. The critical ranges for natural frequencies $f_i$ of footbridges with pedestrian excitation are:

- For vertical vibrations:
  
  \[1.25 \text{ Hz} < f_i < 2.3 \text{ Hz}\]

- For lateral vibrations:
  
  \[0.5 \text{ Hz} < f_i < 1.2 \text{ Hz}\]

Footbridges with frequencies for vertical vibrations in the range

\[2.5 \text{ Hz} < f_i < 4.6 \text{ Hz}\]

might be excited to resonance by the 2\textsuperscript{nd} harmonic of pedestrian loads. In that case, the critical frequency range for vertical vibrations expands to:

\[1.25 \text{ Hz} < f_i < 4.6 \text{ Hz}\]

Mostly for simply supported footbridges, higher harmonics are not considered. The second natural frequency mostly is outside the critical range \[1.25 – 4.6 \text{ Hz}\]. This means that the excessive vibrations are expected at the center of the bridge.

Lateral vibrations are not affected by the 2\textsuperscript{nd} harmonic of pedestrian loads.

**Step 3:** Based on an agreement between the client and the designer, an expected pedestrian density and a comfort class can be chosen. This is called a design situation. The pedestrian densities, which are divided into different traffic classes, are used to determine the pedestrian loading. The comfort classes are acceleration limits that must be met.

3a) Assessment of traffic classes

In step 3a several significant design situations are specified. Each design situation is defined by an expected traffic class. For example, there are design situations which might occur once in the lifetime of a footbridge, like the inauguration of the bridge, and other design situations that will occur daily, such as commuter traffic.

Table 3-1 gives an overview of some typical traffic situations which may occur on footbridges. Based on an agreement between the designer and owner, an expected traffic class can be chosen. This is of importance for determining the dynamic loads.
Table 3-1 Pedestrian traffic classes and densities (Source: Report ‘Design of Lightweight Foot Bridges for Human Induced Vibrations’)

<table>
<thead>
<tr>
<th>Traffic Class</th>
<th>Density $d$ ($P = \text{pedestrian}$)</th>
<th>Description</th>
<th>Characteristics</th>
</tr>
</thead>
<tbody>
<tr>
<td>TC 1*</td>
<td>$\text{group of } 15P; \quad \frac{d}{d} = 15 P / (3B) L$</td>
<td>Very weak traffic</td>
<td>$B =$width of deck; $L =$length of deck</td>
</tr>
<tr>
<td>TC 2</td>
<td>$d = 0,2 \text{ P/m}^2$</td>
<td>Weak traffic</td>
<td>Comfortable and free walking Overtaking is possible Single pedestrians can freely choose pace</td>
</tr>
<tr>
<td>TC 3</td>
<td>$d = 0,5 \text{ P/m}^2$</td>
<td>Dense traffic</td>
<td>Still unrestricted walking Overtaking can intermittently be inhibited</td>
</tr>
<tr>
<td>TC 4</td>
<td>$d = 1,0 \text{ P/m}^2$</td>
<td>Very dense traffic</td>
<td>Freedom of movement is restricted Obstructed walking Overtaking is no longer possible</td>
</tr>
<tr>
<td>TC 5</td>
<td>$d = 1,5 \text{ P/m}^2$</td>
<td>Exceptionally dense traffic</td>
<td>Unpleasant walking Crowding begins One can no longer freely choose pace</td>
</tr>
</tbody>
</table>

3b) Assessment of comfort classes.

In step 3b the acceleration limit $a_{\text{lim}}$ has to be determined, and this is the limit that must be met. Four comfort classes are recommended by this guideline, see Table 3-2.
Table 3-2 Defined comfort classes with common acceleration ranges (Source: Report ‘Design of Lightweight Foot Bridges for Human Induced Vibrations’)

<table>
<thead>
<tr>
<th>Comfort class</th>
<th>Degree of comfort</th>
<th>Vertical $a_{\text{limit}}$</th>
<th>Lateral $a_{\text{limit}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>CL 1</td>
<td>Maximum</td>
<td>$&lt; 0,50 \text{ m/s}^2$</td>
<td>$&lt; 0,10 \text{ m/s}^2$</td>
</tr>
<tr>
<td>CL 2</td>
<td>Medium</td>
<td>$0,50 - 1,00 \text{ m/s}^2$</td>
<td>$0,10 - 0,30 \text{ m/s}^2$</td>
</tr>
<tr>
<td>CL 3</td>
<td>Minimum</td>
<td>$1,00 - 2,50 \text{ m/s}^2$</td>
<td>$0,30 - 0,80 \text{ m/s}^2$</td>
</tr>
<tr>
<td>CL 4</td>
<td>Unacceptable discomfort</td>
<td>$&gt; 2,50 \text{ m/s}^2$</td>
<td>$&gt; 0,80 \text{ m/s}^2$</td>
</tr>
</tbody>
</table>

Note that this is not the criteria check for lateral lock-in! This is only the comfort vibration criteria.

**Step 4**: In step 4 the amount of present damping will be determined. The damping depends both on the intrinsic damping of construction materials, which is of distributed nature, and on the local effect of bearings or other control devices.

It is necessary to consider early in the design stadium the dynamic behaviour of the structure, even though the final damping system is not determined and an assumption must be made. The vibration behaviour here only gives an indication of the actual behaviour. If it is clear in an early stage that the response of the structure is located in the critical zone, a quick change will be made in the design. It is also necessary afterwards, to check if the predicted accelerations, with the assumed damping parameters, are similar to the constructed bridge. Based on the actual dynamic behaviour due to on site experiments, it can be stated if the chosen damping system is superfluous or not. Table 3-3 gives the minimum and average damping ratios according to the EUR 23984 EN.

Table 3-3 Damping ratios for service loads (left) and for large vibrations (right) (Source: Report ‘Design of Lightweight Foot Bridges for Human Induced Vibrations’)

<table>
<thead>
<tr>
<th>Construction type</th>
<th>Minimum $\xi$</th>
<th>Average $\xi$</th>
<th>Damping ratio $\xi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reinforced concrete</td>
<td>0,8%</td>
<td>1,3%</td>
<td>5,0%</td>
</tr>
<tr>
<td>Prestressed concrete</td>
<td>0,5%</td>
<td>1,0%</td>
<td>2,0%</td>
</tr>
<tr>
<td>Composite steel-concrete</td>
<td>0,3%</td>
<td>0,6%</td>
<td>2,0%</td>
</tr>
<tr>
<td>Steel</td>
<td>0,2%</td>
<td>0,4%</td>
<td>4,0%</td>
</tr>
<tr>
<td>Timber</td>
<td>1,0%</td>
<td>1,5%</td>
<td>7,0%</td>
</tr>
<tr>
<td>Stress-ribbon</td>
<td>0,7%</td>
<td>1,0%</td>
<td></td>
</tr>
</tbody>
</table>
3.4 SDOF response model for pedestrian streams according to EUR 23984 EN

The load and response model that is provided in the design guideline, represents a pedestrian stream. A pedestrian stream consists of n “random” pedestrians that walk independently from each other. Based on a so called traffic class, the number of pedestrian’s n on the bridge can be determined. The stream consisting of n “random” pedestrians will be converted into an idealized stream consisting of n’ equivalent number of pedestrians. Hence, the synchronization effect is taken into account in EUR 23984 EN, by determining the idealized stream consisting of n’ perfectly synchronized pedestrians.

Now the equivalent stream can be modelled as a deterministic load. The dynamic load representing the stream will be converted into a distributed harmonic load. In fact, this method is based on the behaviour of one single pedestrian, multiplied by the equivalent number of pedestrians and the reduction coefficient taking into account the probability that the footfall frequency approaches the critical range of natural frequencies under consideration.

The following load model represents the equivalent pedestrian stream, a harmonic load:

\[ p(t) = P \cdot \cos (2\pi f_s t) \cdot n' \cdot \Psi \]  \[3-7\]

Where \( P \cdot \cos (2\pi f_s t) \) is the harmonic load due to a single pedestrian

\( P \) is the vertical component of the force due to a single pedestrian walking step frequency \( f_s \)

\( f_s \) is the step frequency, which is assumed equal to the footbridge natural frequency under consideration.
n’ is the equivalent number of pedestrians on the loaded surface S.

$\Psi$ is the reduction coefficient taking into account the probability that the footfall frequency approaches the critical range of natural frequencies under consideration.

<table>
<thead>
<tr>
<th></th>
<th>Vertical</th>
<th>Longitudinal</th>
<th>Lateral</th>
</tr>
</thead>
<tbody>
<tr>
<td>P [N]</td>
<td>280</td>
<td>140</td>
<td>35</td>
</tr>
</tbody>
</table>

Reduction coefficient $\Psi$

![Figure 3-4](Source: EUR 23984 EN [])

As can be seen in Figure 3-4, the equivalent number of pedestrians is relative to the pedestrian density. For pedestrian density lower than 1.0 pedestrian per m$^2$, the structural damping plays a role in the equation for the equivalent number of pedestrians.

The SETRA also provides a reduction coefficient, which has an identical figure as the reduction the EUR 23984 EN, but different natural frequency ranges. According to the SETRA, bridges with natural frequencies between 1.0 Hz and 2.6 Hz, are in the critical range of resonance.

The derivation of the equivalent number of pedestrians can be found in the literature survey. The equation of n’ and the value 280 N follow from extensive experiments and statistical analyses [SYNPEX 4]. The equation of n’ is natural frequency independent, and therefore the effect of the natural frequency on the response is taken into account with the reduction coefficient $\Psi$. As the equation of n’ and the value of 280 N are deterministic, and the derivation of the reduction coefficient $\Psi$ has not been found either in the design guidelines and background documents, the question remains to what extent can the dynamic behaviour of slender footbridges due to random streams be approached by n’ and $\Psi$. 
4. Experimental research into the force characteristics and synchronizations effects of walking pedestrians

4.1 Purpose of the experiments

The experiments have been performed in the van Musschenbroek laboratory of the department of the Built Environment faculty at the TU/e, in cooperation with two master students Bram Crutzen and Tim Krijntjes. They have done this as a part of their master research project. The experiments have taken place between February and April 2015.

The purpose of these experiments is to measure pedestrian force characteristics, and to quantify the synchronization effect that may occur. In previous experiments within the SYNPEX project the influence of the bridge vibrations on the step frequencies have not been taken into account. Pedestrian characteristics with affected walking behaviour are not provided by the SYNPEX EUR 23984 EN. Besides that, the synchronization between walking pedestrians and vertical vibrations is investigated for single pedestrians and not for pedestrian streams. Additional experiments will be performed to investigate the equivalent dynamic loading due to pedestrian streams.

Therefore, the experiments are divided into two series of experiments: the walking behaviour of pedestrians in different traffic situations, and the measurement of equivalent dynamic loading.

First series

In the first series of experiments, three pedestrian streams are composed. The pedestrian density of these streams are chosen according to the Traffic Classes of the EUR 23984 EN. Pedestrians within these streams, are asked to walk on 4 slender ‘footbridges’ with different natural frequencies. During the walking over, pedestrian force characteristics of pedestrians are measured, with the aim of determining statistical distributions, where synchronization effects are considered. Table 4-1 shows the pedestrian density belonging to the traffic classes of EUR 23984 EN.

<table>
<thead>
<tr>
<th>Traffic Class</th>
<th>Pedestrian density d</th>
<th>Number of persons</th>
</tr>
</thead>
<tbody>
<tr>
<td>Traffic Class 1</td>
<td>0,25 P/m²</td>
<td>3</td>
</tr>
<tr>
<td>Traffic Class 2</td>
<td>0,5 P/m²</td>
<td>6</td>
</tr>
<tr>
<td>Traffic Class 3</td>
<td>1,0 P/m²</td>
<td>12</td>
</tr>
</tbody>
</table>

The first traffic classes consisted of the graduate and the two master students. For the execution of the experiments with Traffic Classes 2 and 3, bachelor students of the Built Environment faculty were invited to participate in these experiments: in total 16 bachelor students participated in the experiments. Because of limited materials and place available in the laboratory, one bridge structure has been built. This bridge structure is called the standard bridge structure, which has a natural frequency of 2,82 Hz.
To obtain other natural frequencies, this standard bridge is stiffened one time to increase the natural frequency. This ‘bridge’ is called the reference bridge. Furthermore, the standard bridge is made heavier two times, by adding dead load (masses), to decrease the natural frequency. The standard bridge is made heavier with 200 kg and 400 kg, distributed over 3 points of the bridge: standard bridge-200 and standard bridge-400. The specifications of the bridge structures will be regarded in the next paragraph. The natural frequencies of the considered bridges are shown in Table 4-2.

Table 4-2 Varied natural frequencies

<table>
<thead>
<tr>
<th>Bridge</th>
<th>Description</th>
<th>Unloaded natural frequency [Hz]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_{1,\text{ref}}$</td>
<td>Reference bridge</td>
<td>2.92 Hz</td>
</tr>
<tr>
<td>$f_{1,1}$</td>
<td>Standard bridge</td>
<td>2.82 Hz</td>
</tr>
<tr>
<td>$f_{1,2}$</td>
<td>Standard bridge-200</td>
<td>2.56 Hz</td>
</tr>
<tr>
<td>$f_{1,3}$</td>
<td>Standard bridge-400</td>
<td>2.3 Hz</td>
</tr>
</tbody>
</table>

On each stream-bridge combinations (3 x 4), the following pedestrian force characteristics are be measured during the first series of experiments:

- Static weight $G$ of participants
- Step frequencies $f_s$ of walking participants
- Walking speed $v$ of participants
- Phase shift $\phi$
- Acceleration $a$ at the mid span of the bridge due to walking participants
- Displacement $u$ at the mid span

The participants were asked to enter the bridge on command, herewith the pedestrian were distributed equally over the bridge in length direction, as can be seen in Figure 4-1. Each crossing over was repeated 3 times. The makes the number of crossing overs: $4 \exp \cdot 3$ streams $\cdot$ 4 bridges $= 48$.

Figure 4-1 Participants according to Traffic Class 3 cross over the standard bridge-400

Furthermore, the damping and the natural frequency of the bridge structures with and-without pedestrian mass are measured, with the so-called knock-test. The participants were asked to stand on the bridge, equally distributed over the length, as shown in Figure 4-2. Then the bridge structure is pushed out its equilibrium, by hitting the bridge with a wooden stick downwards.
Second series

In the second series of experiments, 8 pedestrian 'streams' were composed: 1P (pedestrian), 2P, 3P, 4P, 5P, 6P, 7P and 8P. These streams are not based on a traffic class. In this series of experiments, the focus is on the equivalent dynamic loading, which is an essential part of the synchronization effects between pedestrians. The equivalent dynamic loading in this experiment, is the ratio between the dynamic loads induced by streams 2P to 8P, and the dynamic loads induced by reference person 1P. These experiments are performed on the reference bridge in first instance, so that the bridge vibrations has no influence on the step behaviour of the participants. This experiment was not repeated in first instance. Afterwards, there has been chosen to repeat this experiment, but now with streams 1P up to 5P, because of lack of participants.

As part of the second series of the experiments, the dynamic load factor $\alpha$ is determined, to verify the values of the EUR 23984 EN.
4.2 Experimental setup

Bridge structure
The bridge structure has a width of 1.2 meters and a span of 12 meters. It consists of two steel girders and wooden plates as bridge deck. Each main girder consists of several Meccano girders: 2 x 3.3 meters, 3 meters and 2.4 meters. These Meccano girders are HE B 150 profiles with holes. In Figure 4-3 three coupled HE B 150 Meccano girder is shown.

These Meccano girders are connected to each other in length direction with steel M16 bolts. The wooden plates are connected to the main girders with M16 bolts, as illustrated in Figure 4-4.

Test experiments had to be performed, to see which natural frequencies are achievable with these available materials in the laboratory.
Setup and measuring instruments
In figure 4-5, the definitive test set up of the bridge is shown. This figure contains the reference bridge: standard bridge reinforced with two 600 mm long Meccano girders. The bridge girders are connected to a cross beam that is shown in Figure 4-6, which is simply supported on 4 load cells. These load cells are used to measure the pedestrian induced dynamic forces at the four support points.

![Setup Bridge (Reinforced)](image)

Furthermore, two accelerometers are placed at the mid-span of the bridge, at the left and right side of the bridge. These accelerometers are used to measure the accelerations due to the walking pedestrians, but also to determine the damping and natural frequency of the bridge on the basis of the knock-test signal. In addition to that, a LVDT displacement meter is placed in the mid-span of the bridge, at the left side, to control the outcome of the accelerometer.

![Figure 4-6 Setup of the experiment: frontal view](image)

A video camera is used to film the walking behaviour of pedestrians. On the basis of the video images, the step frequency, walking speed and the phase of pedestrians is determined afterwards. A scale is used for the static weight G of the pedestrians.
In Figures 4-7 and 4-8 down below, the definitive bridge structure is shown and the locations of the measuring instruments are summarized.

Figure 4-7 Locations measuring instruments

Figure 4-8 Definitive bridge structure

1 = Load cell 18  
2 = Acceleration sensor 1 + LVDT  
3 = Load cell 17  
4 = Load cell 10  
5 = Acceleration sensor 2  
6 = Load cell 13
4.3 Results of the first series of experiments

4.3.1 Natural frequencies

The pedestrian mass was taken into account in the natural frequency, by placing the pedestrians equally distributed on the bridge and performing a knock test. The vibration in time is measured with the accelerometer. An example of this acceleration signal is shown in Figure 4-9. It is an assumption that the mass of the walking pedestrians can be represented by pedestrians that stand on the bridge, equally distributed over the length.

![Image](image.png)

Figure 4-9 Knock test of the unloaded reference bridge

After pushing the bridge out of its equilibrium with this knock, the bridge starts to vibration in its natural frequency. The natural frequency is determined by counting the number of peaks in a certain time. The results of the loaded and unloaded natural frequencies are summarized in Table 4-3.

<table>
<thead>
<tr>
<th>Situation</th>
<th>Loaded natural frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_{1,\text{ref}}$</td>
<td>2.92 Hz</td>
</tr>
<tr>
<td>$f_{1,\text{ref}}$ loaded with TC 1</td>
<td>2.61 Hz</td>
</tr>
<tr>
<td>$f_{1,\text{ref}}$ loaded with TC 2</td>
<td>2.39 Hz</td>
</tr>
<tr>
<td>$f_{1,\text{ref}}$ loaded with TC 3</td>
<td>2.08 Hz</td>
</tr>
<tr>
<td>$f_{1,1}$</td>
<td>2.82 Hz</td>
</tr>
<tr>
<td>$f_{1,1}$ loaded with TC 1</td>
<td>2.52 Hz</td>
</tr>
<tr>
<td>$f_{1,1}$ loaded with TC 2</td>
<td>2.31 Hz</td>
</tr>
<tr>
<td>$f_{1,1}$ loaded with TC 3</td>
<td>2.01 Hz</td>
</tr>
<tr>
<td>$f_{1,2}$</td>
<td>2.56 Hz</td>
</tr>
<tr>
<td>$f_{1,2}$ loaded with TC 1</td>
<td>2.33 Hz</td>
</tr>
<tr>
<td>$f_{1,2}$ loaded with TC 2</td>
<td>2.16 Hz</td>
</tr>
<tr>
<td>$f_{1,2}$ loaded with TC 3</td>
<td>1.9 Hz</td>
</tr>
<tr>
<td>$f_{1,3}$</td>
<td>2.3 Hz</td>
</tr>
<tr>
<td>$f_{1,3}$ loaded with TC 1</td>
<td>2.18 Hz</td>
</tr>
<tr>
<td>$f_{1,3}$ loaded with TC 2</td>
<td>2.04 Hz</td>
</tr>
<tr>
<td>$f_{1,3}$ loaded with TC 3</td>
<td>1.82 Hz</td>
</tr>
</tbody>
</table>
The decrease of the natural frequency as a result of the increase in mass, can also be determined analytically. The influence of the dead load on the natural frequency can be determined with the following formula:

\[ f_{1, \text{loaded}} = f_1 \cdot \frac{1}{\sqrt{\frac{m + m_{\text{add}}}{m}}} \]  \[ \text{[4-1]} \]

where \( m \) is the mass of the bridge and \( m_{\text{add}} \) is the additional mass of the pedestrians.

For example, the natural frequency of the standard bridge loaded with TC 2 is calculated and compared with the measured values:

\[ m_{\text{add}} = \text{Pedestrians} \cdot \frac{G}{L_{\text{bridge}}} \]
\[ m_{\text{add}} = 6 \cdot \frac{75}{12} = 37,5 \text{ kg/m} \]
\[ m_{\text{bridge}} = 98 \text{ kg/m} \]
\[ f_{1, \text{loaded}} = 2,82 \cdot \frac{1}{\sqrt{\frac{98 + 37,5}{98}}} = 2,4 \text{ Hz} \]

The difference between the calculated and the measured value of 2,31 Hz is relatively small.

### 4.3.2 Damping ratios

The damping ratio is the percentage actual damping of the critical damping. On the basis of the response signal of the knock test, the logarithmic decrement \( \delta \) has to be determined first:

\[ \delta = \frac{1}{n} \cdot \ln \frac{x_1}{x_2} \]  \[ \text{[4-2]} \]

where \( n \) is the number of peaks between the considered peaks \( x_1 \) and \( x_2 \).

The ramping ratio \( \zeta \) is calculated with the following equation:

\[ \zeta = \frac{\delta}{\sqrt{(2\delta)^2 + \delta^2}} \]  \[ \text{[4-3]} \]

The damping ratios are determined for all bridge-traffic class combination. The results are summarized in Figure 4-4 down below.

<table>
<thead>
<tr>
<th>Situation</th>
<th>Damping 1st measurement [%]</th>
<th>Damping 2nd measurement [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f_{1,\text{ref}} )</td>
<td>-</td>
<td>2,25 %</td>
</tr>
<tr>
<td>( f_{1,\text{ref}} ) loaded with TC 1</td>
<td>2,67 %</td>
<td>2,4 %</td>
</tr>
<tr>
<td>( f_{1,\text{ref}} ) loaded with TC 2</td>
<td>2,25 %</td>
<td>2,44 %</td>
</tr>
<tr>
<td>( f_{1,\text{ref}} ) loaded with TC 3</td>
<td>2,59 %</td>
<td>3,2 %</td>
</tr>
<tr>
<td>( f_{1,1} )</td>
<td>2,97 %</td>
<td>2,32 %</td>
</tr>
<tr>
<td>( f_{1,1} ) loaded with TC 1</td>
<td>2,79 %</td>
<td>2,88 %</td>
</tr>
<tr>
<td>( f_{1,1} ) loaded with TC 2</td>
<td>1,85 %</td>
<td>1,35 %</td>
</tr>
<tr>
<td>( f_{1,1} ) loaded with TC 3</td>
<td>2,22 %</td>
<td>1,91 %</td>
</tr>
<tr>
<td>( f_{1,2} )</td>
<td>2,59 %</td>
<td>2,82 %</td>
</tr>
</tbody>
</table>
The measured damping ratios are 5 to 10 times higher than the damping ratio for steel (for service loads) that is given by EUR 23984 in Table 3.3. The bridge has a higher damping, because it is composed of different materials (steel and wood plates) which are connected with bolts.

All knock tests can be found in Annex 8.

4.3.3 Walking speed

The average speed of walking pedestrians within pedestrian streams according to TC 1, 2, 3, are shown in Table 4-5 down below. It is striking that the walking speed increases, when the pedestrian density increases. Other measurements, such as Oeding’s (Table 2-2, subparagraph 2.1.4.1), show that the walking speed decreases when the pedestrian density increases.

An explanation of the increase of walking speed, is that the pedestrians want to follow the person in front of them. If the first pedestrian increases his speed, other pedestrian could also increase their speed. In these experiments, the participants were asked to walk freely. In reality, both decrease as increase in walking speed could occur at high pedestrian densities. It depends on the interaction between persons. If a pedestrian in the middle of bridge comes is in a hurry, and walks very close to the pedestrian in front of him, the pedestrian in front of him could increase his walking speed, so the other pedestrians in front will adopt the same walking speed.

<table>
<thead>
<tr>
<th>Traffic Class</th>
<th>Average walking speed</th>
<th>Standard deviation speed</th>
</tr>
</thead>
<tbody>
<tr>
<td>Traffic Class 1</td>
<td>1.27 m/s</td>
<td>0.060 m/s</td>
</tr>
<tr>
<td>Traffic Class 2</td>
<td>1.49 m/s</td>
<td>0.069 m/s</td>
</tr>
<tr>
<td>Traffic Class 3</td>
<td>1.53 m/s</td>
<td>0.105 m/s</td>
</tr>
</tbody>
</table>

In figure 4-10, the correlation between average step frequencies of pedestrians and walking speed is shown. The correlations in these experiments appears not so strong.
4.3.4 Step frequency distributions for pedestrians within different traffic classes

Table 4-6 summarizes the results of the step frequency distributions of the pedestrian streams according to the Traffic Classes, and on bridges with different natural frequencies.

When analyzing the step frequency distributions of TC 1, 2 and 3, on the reference bridge, where the bridge vibration should not affect the walking behaviour of pedestrians, the standard deviation of the step frequency decreases when the pedestrian density increases.

Furthermore, the average step frequency increases when the pedestrian density increases, because of the increase in walking speed. Thus, on the basis of these results, synchronization between pedestrians is visible. Figure 4-11 gives the results of TC 1, 2 and 3 on the reference bridge.
For all step frequency distributions, see Annex.

As for the so called ‘affected step frequency’ distributions, i.e. when the 1st natural frequency of the bridge approaches the average step frequency and when the bridge vibrations could ‘force’ the walking pedestrians to match the natural frequency, the results are not straightforward. The average step frequency adapts in some cases to the 1st natural frequency, when the natural frequency is in the critical range. At that moment, the standard deviation is often lower.

In traffic class 1, the standard deviation of the step frequency decreases only on the loaded bridge (400 kg) with natural frequency \( f_{1,3} \). In traffic class 2, the standard deviation decreases on the standard bridge with \( f_{1,1} \) and the loaded bridge (200 kg) with \( f_{1,2} \), and then again increases on the loaded bridge (400 kg) with \( f_{1,3} \).

As for traffic class 3, the standard deviation increases on the standard bridge and the two loaded bridges. On the basis of these distributions, synchronization between pedestrians bridge structure cannot be seen clearly. On the recorded video’s, there can be seen that synchronization occurred between bridge structure \( f_{1,2} \) and some pedestrian occurs, but not for the whole crossing over. Explanations: the pedestrians who do not walk in synchrony, damp/affect the bridge vibration somehow, so that the synchronous walking pedestrians go out phase.

Therefore, this analysis is extended by determining the number of synchronized pedestrians in time while crossing the bridge. This has been done for Traffic Class 3, on the standard bridge and on the reference bridge. Definition of synchronization pedestrians: pedestrians who walk in synchrony with the vibration of the bridge.
When comparing Figures 4-12 and 4-13, there can be seen that in that pedestrians on the standard bridge synchronize more in the beginning of the crossing over, in contrast to the reference bridge. This can be explained as follows. The natural frequency of the standard bridge (2 Hz) approaches the average step frequency of pedestrians (1.94 Hz) more than the natural frequency of the reference bridge (2.08 Hz). Therefore, the first pedestrians walking over the standard bridge, excite the bridge at the beginning at most, where after the subsequent pedestrians damp the bridge vibrations. This explains why the synchronization effects decrease in Figure 4-11.

The reference bridge show the opposite: synchronization effects increase in time. The first pedestrians are not exciting the bridge at the beginning. The bridge vibrations increases slowly when more pedestrians enter the bridge.
In figure 4-14, the step frequency distribution of 1 single pedestrian is shown, who walked in first instance on the reference bridge (2,92 Hz), where the influence of the bridge vibration on the pedestrian in minimum. When this pedestrian walked on the standard bridge-400 (2,41 Hz), his average step frequency increased and the standard deviation of the step frequency decreased. In other words, his step frequency increased towards the frequency of the bridge. Synchronization between pedestrians and bridge structure could be seen with distributions of single pedestrians.

In this particular experiment, the single pedestrian walked within Traffic Class 1. The two distributions are based on one crossing over (15 steps).
4.3.5 Phase shift

In extensive, time step calculations, the phase shift chosen randomly between 0 and 2\(\pi\) for pedestrians walking out of phase. In this paragraph the phase shift between walking pedestrians is determined, to verify the earlier mentioned assumption.

To determine the phase shift on the basis of the videos of the participants walking on the bridge, first the phase shift between pedestrians have to be determined, using 1 reference person and associated reference moment. The reference moment is the time when this reference person, has his foot straight on the bridge deck, as can be seen in figure 4-15. Logically, there will be a ‘time’ difference/shift between the moment person 2 puts her feet straight on the bridge deck and the moment that the reference person had her feet on the bridge deck. On the basis of this time difference, for each other person, the phase shift will be calculated.

![Figure 4-15 Illustration of the phase shift between pedestrians](image)

The following phase shifts between pedestrians are calculated for traffic class 2 and 3, and each traffic class on the stiffest bridge \((f_{1,\text{ref}})\) and on the most flexible bridge\((f_{1,3})\). The results are shown in Tables 4-7 and 4-8.

<table>
<thead>
<tr>
<th>Pedestrian</th>
<th>Phase shift TC 2, bridge (f_{1,\text{ref}})</th>
<th>Phase shift TC 2, bridge (f_{1,3})</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pedestrian 1</td>
<td>1,25 (\pi)</td>
<td>0,25 (\pi)</td>
</tr>
<tr>
<td>Pedestrian 2</td>
<td>1,5 (\pi)</td>
<td>(\pi)</td>
</tr>
<tr>
<td>Pedestrian 3</td>
<td>REF</td>
<td>REF</td>
</tr>
<tr>
<td>Pedestrian 4</td>
<td>1,5 (\pi)</td>
<td>1,25 (\pi)</td>
</tr>
<tr>
<td>Pedestrian 5</td>
<td>2 (\pi)</td>
<td>(\pi)</td>
</tr>
<tr>
<td>Pedestrian 6</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>
In both Traffic Classes, one can see that the distribution between the phase shift is rather a uniform distribution than a normal or any other type of distribution. This is illustrated in Figures 4-16 and 4-17.

<table>
<thead>
<tr>
<th>Pedestrian</th>
<th>Phase shift TC 3, bridge $f_{1,\text{ref}}$</th>
<th>Phase shift TC 3, bridge $f_{1,3}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pedestrian 1</td>
<td>-</td>
<td>0,5 $\pi$</td>
</tr>
<tr>
<td>Pedestrian 2</td>
<td>-</td>
<td>$\pi$</td>
</tr>
<tr>
<td>Pedestrian 3</td>
<td>1,75 $\pi$</td>
<td>1,25 $\pi$</td>
</tr>
<tr>
<td>Pedestrian 4</td>
<td>$\pi$</td>
<td>REF</td>
</tr>
<tr>
<td>Pedestrian 5</td>
<td>0,25 $\pi$</td>
<td>1,5 $\pi$</td>
</tr>
<tr>
<td>Pedestrian 6</td>
<td>$\pi$</td>
<td>0,25 $\pi$</td>
</tr>
<tr>
<td>Pedestrian 7</td>
<td>REF</td>
<td>0,25 $\pi$</td>
</tr>
<tr>
<td>Pedestrian 8</td>
<td>1,5 $\pi$</td>
<td>-</td>
</tr>
<tr>
<td>Pedestrian 9</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Pedestrian 10</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Pedestrian 11</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Pedestrian 12</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

This analysis should be extended by determining the phase shift between pedestrians and bridge vibration. The phase shift between pedestrians and bridge vibrations is difficult to see with the video images. It is recommended to measure step frequencies with pressure sensors in the shoes of the participants. On the basis of the pressure sensors (load signals), and the bridge vibrations of the bridge, the phase shift between persons and bridge vibration can be determined.
4.4 Results of the second series of experiments

The results of the second series of experiments, which contains pedestrian induced dynamic loads by streams 1P up to 8P and 1P up to 5P, can be found in Annex A.4.2. In Annex A.4.1, the dynamic induced forces by streams TC 1, TC 2 TC 3 can be found, but these signals are not used for any purpose. The analysis of the equivalent number of pedestrians and the dynamic load factor is based on the signals in Annex A.4.2.

4.4.1 Dynamic load factor

Additional experiments have been performed to check the dynamic load factor $\alpha_{\text{EUR}}$. A pedestrian is asked to walk several times over the reference bridge, but each time with a different step frequency. The pedestrian is first asked to walk with a low step frequency, than in the following crossing over with a higher step frequency and so on. The increase in step frequency, goes hand in hand with an increase in the response of the bridges. In Figure 4-18, one can see the response (acceleration) of the reference bridge due to a walking pedestrian, with step frequency $f_s=1.57$ Hz and $f_s=1.69$ Hz.

The dynamic load factor $\alpha$ can be checked in two ways. The first way is to calculate $\alpha_{\text{exp}}$, and to compare it with $\alpha$ that is given by EUR 23984 EN. Using equations 2-9, 2-10 and 2-11, $\alpha_{\text{exp}}$ can be defined as follows:

$$\alpha_{\text{exp}} = \frac{a_{\text{max}} k_1}{\text{DAF} G_{\text{ped}} (2\pi f_s)^2}$$

Another way to check the dynamic load factor $\alpha_{\text{EUR}}$ is to calculate $\text{DAF}_{\text{exp}}$, and to compare $\text{DAF}_{\text{exp}}$ with $\text{DAF}$ that is given in [eq. 2-10]. $\text{DAF}_{\text{exp}}$ is defined as follows:

$$\text{DAF}_{\text{exp}} = \frac{a_{\text{max}} k_1}{\alpha_{\text{EUR}} G_{\text{ped}} (2\pi f_s)^2}$$

Acceleration of the bridge $a_{\text{max}}$, the modal stiffness $k_1 (f_1)$ and the pedestrian weight $G_{\text{ped}}$ are measured values and $\alpha$ is assumed to be correct according to EUR 23984 EN.
The second way will be used to check the dynamic load factor $\alpha$, to find out if the dynamic load factor $\alpha$ is only depending on the step frequency $f_s$. The dynamic amplification factor $DAF_{exp}$ is determined for a walking pedestrian with $f_s=1.57$ Hz:

$$a_{\text{max}} = 0.09 \text{ m/s}^2$$

$$k_1 = (2 \pi f_1)^2 \cdot m_1 = (2 \pi f_1)^2 \cdot \left( (m_{\text{bridge}}+m_{\text{ped}}) \frac{1}{2} L \right)$$

$$k_1 = (2 \pi 2.8)^2 \cdot \left( (98+7.5) \frac{1}{2} 12 \right) = 197725 \text{ N/m}$$

$$\alpha_{\text{EUR}} = 0.0115 \cdot f_s^2 + 0.2803 \cdot f_s - 0.2902$$

$$\alpha_{\text{EUR}} = 0.0115 \cdot 1.57^2 + 0.2803 \cdot 1.57 - 0.2902 = 0.178$$

$$DAF_{exp} = \frac{0.09 \cdot 197725}{0.178860 \cdot (2\pi \cdot 1.57)^2} = 1.19$$

$$DAF = \frac{1}{\sqrt{(1-g^2)^2 + (2 \zeta g)^2}}$$

$$DAF = \frac{1}{\sqrt{(1-\frac{f_s^2}{f_1^2})^2 + (2 \zeta \frac{f_s}{f_1})^2}} = \frac{1}{\sqrt{(1-\frac{1.57^2}{2.8})^2 + (2 \cdot 0.025 \cdot \frac{1.57}{2.8})^2}} = 1.46$$

The calculation of $DAF_{exp}$ has been repeated for 5 different step frequencies. For each crossing over, the average step frequency has been determined on the basis of the video images. The results are shown in Table 4-9.

<table>
<thead>
<tr>
<th>Crossing over nr.</th>
<th>$f_{s,\text{mean}}$ [Hz]</th>
<th>$a_{\text{max}}$ [m/s$^2$]</th>
<th>$DAF_{exp}$</th>
<th>$DAF$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.57</td>
<td>0.09</td>
<td>1.19</td>
<td>1.46</td>
</tr>
<tr>
<td>2</td>
<td>1.69</td>
<td>0.14</td>
<td>1.33</td>
<td>1.57</td>
</tr>
<tr>
<td>3</td>
<td>1.86</td>
<td>0.22</td>
<td>1.38</td>
<td>1.79</td>
</tr>
<tr>
<td>4</td>
<td>2</td>
<td>0.28</td>
<td>1.42</td>
<td>2.04</td>
</tr>
<tr>
<td>5</td>
<td>2.13</td>
<td>0.37</td>
<td>1.38</td>
<td>2.36</td>
</tr>
<tr>
<td>6</td>
<td>2.37</td>
<td>0.57</td>
<td>1.39</td>
<td>3.49</td>
</tr>
</tbody>
</table>

It is clear that equation 2-10, which describes the dynamic amplification factor $DAF$, gives higher values than $DAF_{exp}$. $DAF$ according to equation 2-10, increases sharply when the step frequency increases. As for $DAF_{exp}$, there is a slight increase visible when the step frequency increases from 1.57 Hz up to 2 Hz. Then, $DAF_{exp}$ starts to decrease. The bridge response does not increase constantly, while the terms $\alpha_{\text{EUR}}$ and $(2\pi f_s)^2$ do increase. This clarifies the decrease for $f_s=2.13$ Hz and $f_s=2.37$ Hz. The difference between the two is shown in Figure 4-19.
Comparison between $\text{DAF}_{\text{exp}}$ and $\text{DAF}$

- $\text{DAF} = 3.2113f_s^2 - 10.206f_s + 9.6095$
- $\text{DAF}_{\text{exp}} = -0.6812f_s^2 + 2.8833f_s - 1.6317$

Figure 4-19 Comparison between the $\text{DAF}_{\text{exp}}$ and $\text{DAF}$
5. Analytical research on the synchronous load- and response model according to EUR 23984 EN

5.1 Analytical solution of Fryba with a Monte Carlo Approach

During the analytical research, a dynamic, time-step load model has been developed to assess the load-and response model of the EUR 23984 EN. This dynamic load model, with a probabilistic approach to take into account the variation in the pedestrian force characteristics, is developed on the basis of the analytical solution obtained by Fryba [17]. This solution, represents a harmonic force moving at a constant speed over a simply supported beam. The analytical solution of Fryba is based on the following assumptions:

- The vibration behaviour is described by the Bernoulli-Euler’s differential equation
- Constant mass per unit length and cross-section along the bridge span
- Damping is proportional to the velocity of vibration
- Simply supported boundary conditions

The moving harmonic force is described with the following equation [17]:

\[
\begin{align*}
    u &= \frac{L}{2} \left( u_{st} \left[ \frac{1}{g^2} - 1 \right]^2 + \frac{4 \zeta^2}{g^2} \sin(\omega_s t + \phi) \sin(\omega t) + 2 \frac{\omega}{\omega_s} (\cos(\omega_s t) \cos(\omega t) - e^{-\omega_b t} \cos(\omega_1 t)) \right] \\
    &+ \frac{2 \omega_s}{\omega_1} \omega_c \cos(\omega_1 t) \\
\end{align*}
\]

Where:

- \( u_{st} \) is the static deflection at mid span due to the maximum amplitude of the harmonic force applied
- \( g \) ratio between step frequency and natural frequency \( \frac{\omega_s}{\omega_1} \)
- \( \zeta \) damping ratio
- \( Z = 1 + \frac{1}{g^4} + \frac{1}{g^2} + \frac{2 \omega^2}{\omega_s^2} \)
- \( \omega \) circular frequency = \( \pi v / L \)
- \( \omega_b \) circular frequency of damping = \( \omega_1 \cdot \zeta \)
- \( \omega_s \) step frequency = \( f_1 \cdot 2\pi \)
- \( \omega_1 \) natural frequency = \( f_1 \cdot 2\pi \)
- \( v \) walking speed
- \( \phi \) phase shift
- \( L \) bridge span
This equation is used to calculate the dynamic displacement due to single pedestrians. The displacement signal is then transformed into an acceleration signal. Then the sum of the acceleration signals due to single pedestrians is added to obtain the sum of the response. An example of summing up the accelerations signals due to single pedestrians is shown in Figure 5-1.

This analytical solution is combined with a Monte Carlo Simulation. A Monte Carlo Simulation, repeats a calculation a considerable number of times, each time choosing the values for the variables according to the predefined probability distributions. The (repeated) calculated responses are then recorded. The 95th percentile value is chosen as the statistical response.

However, the Monte Carlo Simulation is made in Excel. In Excel it was not possible to choose the variables according to probability distributions. Therefore, the selecting was random based from the measured experimental results. Even though the selecting procedure is random from the measured results, an overview of the normal distributions of the measured values is given down below.
Vibration behaviour of slender footbridges
Graduation research

Variables Monte Carlo Simulation

Normal distribution for static weight G
\(G_{\text{mean}} = 710 \text{ N}\)
\(\sigma_G = 95 \text{ N}\)

Normal distributions for walking speed \(v\):
TC 1: \(v_{\text{mean}} = 1,27 \text{ m/s} ; \sigma_v = 0,06 \text{ m/s}\)
TC 2: \(v_{\text{mean}} = 1,49 \text{ m/s} ; \sigma_v = 0,069 \text{ m/s}\)
TC 3: \(v_{\text{mean}} = 1,53 \text{ m/s} ; \sigma_v = 0,105 \text{ m/s}\)

Normal distributions for step frequency \(f_s\):
Not affected walking behaviour:
TC 1 + Reference bridge: \(f_{s,\text{mean}} = 1,85 \text{ Hz} ; \sigma_f = 0,185 \text{ Hz}\)
TC 2 + Reference bridge: \(f_{s,\text{mean}} = 1,91 \text{ Hz} ; \sigma_f = 0,113 \text{ Hz}\)
TC 3 + Reference bridge: \(f_{s,\text{mean}} = 1,94 \text{ Hz} ; \sigma_f = 0,105 \text{ Hz}\)

Affected walking behaviour:
TC 1 + Standard bridge: \(f_{s,\text{mean}} = 1,84 \text{ Hz} ; \sigma_f = 0,187 \text{ Hz}\)
TC 2 + Standard bridge: \(f_{s,\text{mean}} = 1,96 \text{ Hz} ; \sigma_f = 0,099 \text{ Hz}\)
TC 3 + Standard bridge: \(f_{s,\text{mean}} = 1,88 \text{ Hz} ; \sigma_f = 0,119 \text{ Hz}\)

TC 1 + Standard bridge-200: \(f_{s,\text{mean}} = 1,89 \text{ Hz} ; \sigma_f = 0,186 \text{ Hz}\)
TC 2 + Standard bridge-200: \(f_{s,\text{mean}} = 1,91 \text{ Hz} ; \sigma_f = 0,091 \text{ Hz}\)
TC 3 + Standard bridge-200: \(f_{s,\text{mean}} = 1,92 \text{ Hz} ; \sigma_f = 0,123 \text{ Hz}\)

TC 1 + Standard bridge-400: \(f_{s,\text{mean}} = 1,89 \text{ Hz} ; \sigma_f = 0,16 \text{ Hz}\)
TC 2 + Standard bridge-400: \(f_{s,\text{mean}} = 1,99 \text{ Hz} ; \sigma_f = 0,106 \text{ Hz}\)
TC 3 + Standard bridge-400: \(f_{s,\text{mean}} = 2,01 \text{ Hz} ; \sigma_f = 0,134 \text{ Hz}\)

Uniform distribution for \(\varphi\):
\([0,2\pi]\)

Number of repetition:
500
5.1.1 Validation model

In order to verify the response found with the Monte Carlo Simulation, a comparison is made with the measured response at the mid span of the bridge. For the validation, calculations have been made with the reference bridge and the standard bridge-400.

In Figures 5-2, the response of the reference bridge due to different groups of pedestrians is shown, and the responses that were measured during the experiments. An approximation curve have been made to compare the calculated and measured responses. For the validation of the Monte Carlo simulation, the 50th percentile value is chosen as the response limit. The following values and distributions were held for the variables $m$, $\zeta$, $f_s$, $G$, $v$ and $\phi$.

Distributions that are dependent on Traffic Class and bridge frequency, are bolted.

<table>
<thead>
<tr>
<th>1 Pedestrian</th>
<th>2 Pedestrians</th>
<th>3 Pedestrians</th>
</tr>
</thead>
<tbody>
<tr>
<td>L=12 meters</td>
<td>L=12 meters</td>
<td>L=12 meters</td>
</tr>
<tr>
<td>$m = m_{pedestrian} + m_{bridge}$</td>
<td>$m = m_{pedestrian} + m_{bridge}$</td>
<td>$m = m_{pedestrian} + m_{bridge}$</td>
</tr>
<tr>
<td>$m = 71/12 + 98 = 103$ kg/m</td>
<td>$m = 142/12 + 98 = 109,8$ kg/m</td>
<td>$m = 213/12 + 98 = 115,8$ kg/m</td>
</tr>
<tr>
<td>$\zeta = 0,025$</td>
<td>$\zeta = 0,025$</td>
<td>$\zeta = 0,025$</td>
</tr>
<tr>
<td>$G_{mean} = 710$</td>
<td>$G_{mean} = 710$</td>
<td>$G_{mean} = 710$</td>
</tr>
<tr>
<td>$\sigma_G = 95$ N</td>
<td>$\sigma_G = 95$ N</td>
<td>$\sigma_G = 95$ N</td>
</tr>
<tr>
<td>$\varphi = [0,2\pi]$</td>
<td>$\varphi = [0,2\pi]$</td>
<td>$\varphi = [0,2\pi]$</td>
</tr>
<tr>
<td>$v_{mean} = 1,27$ m/s</td>
<td>$v_{mean} = 1,27$ m/s</td>
<td>$v_{mean} = 1,27$ m/s</td>
</tr>
<tr>
<td>$v_{\sigma} = 0,06$ m/s</td>
<td>$v_{\sigma} = 0,06$ m/s</td>
<td>$v_{\sigma} = 0,06$ m/s</td>
</tr>
<tr>
<td>$f_s;mean = 1,85$ Hz</td>
<td>$f_s;mean = 1,85$ Hz</td>
<td>$f_s;mean = 1,85$ Hz</td>
</tr>
<tr>
<td>$f_s;\sigma = 0,185$ Hz</td>
<td>$f_s;\sigma = 0,185$ Hz</td>
<td>$f_s;\sigma = 0,185$ Hz</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>4 Pedestrian</th>
<th>5 Pedestrians</th>
<th>6 Pedestrians</th>
</tr>
</thead>
<tbody>
<tr>
<td>L=12 meters</td>
<td>L=12 meters</td>
<td>L=12 meters</td>
</tr>
<tr>
<td>$m = m_{pedestrian} + m_{bridge}$</td>
<td>$m = m_{pedestrian} + m_{bridge}$</td>
<td>$m = m_{pedestrian} + m_{bridge}$</td>
</tr>
<tr>
<td>$m = 284/12 + 98 = 122$ kg/m</td>
<td>$m = 355/12 + 98 = 127,6$ kg/m</td>
<td>$m = 426/12 + 98 = 133,5$ kg/m</td>
</tr>
<tr>
<td>$\zeta = 0,025$</td>
<td>$\zeta = 0,025$</td>
<td>$\zeta = 0,025$</td>
</tr>
<tr>
<td>$G_{mean} = 710$</td>
<td>$G_{mean} = 710$</td>
<td>$G_{mean} = 710$</td>
</tr>
<tr>
<td>$\sigma_G = 95$ N</td>
<td>$\sigma_G = 95$ N</td>
<td>$\sigma_G = 95$ N</td>
</tr>
<tr>
<td>$\varphi = [0,2\pi]$</td>
<td>$\varphi = [0,2\pi]$</td>
<td>$\varphi = [0,2\pi]$</td>
</tr>
<tr>
<td>$v_{mean} = 1,49$ m/s</td>
<td>$v_{mean} = 1,49$ m/s</td>
<td>$v_{mean} = 1,49$ m/s</td>
</tr>
<tr>
<td>$v_{\sigma} = 0,069$ m/s</td>
<td>$v_{\sigma} = 0,069$ m/s</td>
<td>$v_{\sigma} = 0,069$ m/s</td>
</tr>
<tr>
<td>$f_s;mean = 1,91$ Hz</td>
<td>$f_s;mean = 1,91$ Hz</td>
<td>$f_s;mean = 1,91$ Hz</td>
</tr>
<tr>
<td>$f_s;\sigma = 0,113$ Hz</td>
<td>$f_s;\sigma = 0,113$ Hz</td>
<td>$f_s;\sigma = 0,113$ Hz</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>12 Pedestrian</th>
</tr>
</thead>
<tbody>
<tr>
<td>L=12 meters</td>
</tr>
<tr>
<td>$m = m_{pedestrian} + m_{bridge}$</td>
</tr>
<tr>
<td>$\zeta = 0,025$</td>
</tr>
</tbody>
</table>
$G_{\text{mean}} = 710$
$\sigma_G = 95 \text{ N}$
$\varphi = [0, 2\pi]$ 
$v_{\text{mean}} = 1,53 \text{ m/s}$
$\sigma_v = 0,105 \text{ m/s}$
$f_{s, \text{mean}} = 1,94 \text{ Hz}$
$\sigma_f = 0,105 \text{ Hz}$

It is clear that the Monte Carlo simulations give higher values than measured. One can see that the measured response increases less quickly than the calculated response. The calculated response has a more constant increase. This can be explained as follows. The analytical load model does not take into account the fact that the legs of the pedestrians absorb a part of vibrations.

This provides a certain amount of damping to the structure. In the video images there has been observed that the bridge vibrations stopped when 3 pedestrians in the middle of the line walked out of phase. Therefore, it is naturally that the calculated response is higher than the measured response. Another explanation is that the given value of the dynamic load factor $\alpha$ is conservative. Hand calculations showed earlier that the dynamic load factor $\alpha_{\text{exp}}$ determined on the basis of experimental results, gives lower values. Therefore, the calculated response with the Monte Carlo Simulation will then be lower.

The analysis has also been performed for the standard bridge-400. The results are shown in Figure 5-3.
5.2 Comparison with EUR 23984 EN and Eurocode 5

The deterministic load-and response models of the Eurocode 5 [16] and the EUR 23984 EN [1] have been compared with the Monte Carlo simulation. This is the first step in the assessment of the model of the EUR 23984 EN. The next step is to assess the equivalent number of pedestrians \( n' \) and the reduction coefficient \( \Psi \) separately.

One calculation is worked out with both EUR 23984 EN and Eurocode 5. The load scenario: Traffic Class 2 on standard bridge-200 with \( f_{1,2} \).

**EUR 23984 EN**

The following load model represents the equivalent pedestrian stream, a harmonic load:

\[
p(t) = P \cdot \cos(2\pi f_s \cdot t) \cdot n' \cdot \Psi
\]

The acceleration is calculated with the following equation:

\[
a_{\text{max}} = \frac{p^{*}}{m^{*}} \cdot \frac{1}{2\zeta}
\]

where \( p^{*} \) is the generalized load = \( 2/\pi \cdot p(t) \cdot L \)

\( m^{*} \) is the generalized modal mass = \( ½ \cdot m \cdot L \)
Vibration behaviour of slender footbridges
Graduation research

ζ is the structural damping ratio

Based on a simple supported beam, with a distributed mass \( m \, [\text{kg/m}] \), stiffness \( k \), length \( L \), and a uniformly distributed load \( p(x) \cdot \sin(\omega t) \), the parameters are defined as follows. The mode shapes \( \Phi(x) \) of the bending modes are assumed to be represented by a half sine function \( \Phi(x) = \sin \left( m \cdot \frac{x}{L} \cdot \pi \right) \), whereas \( m \) is the number of half waves.

Calculation

\[
\begin{align*}
P &= 280 \, \text{N} \\
f_s &= 2,25 \, \text{Hz} \\
n &= 6 \\
\zeta &= 2,5 \% (0,025) \\
n' &= 10,8 \cdot \sqrt{0,025 \cdot 2} = 3,74 \\
\Psi &= 0,25 \ (\text{according to Figure 3-4}) \\
p(t) &= 22 \, \text{N/m} \\
p^* &= 22 \cdot 2/\pi \cdot 12 = 168 \\
m^* &= \frac{1}{2} \cdot (98+38+18) \cdot 12 = 924 \, \text{kg} \\
a_{\text{EUR 23984 EN}} &= 4,127 \, \text{m/s}^2
\end{align*}
\]

Earlier there has been said that the measured damping ratios were relatively higher than the values that are given by the EUR 23984 EN. This calculation will be repeated, but now with \( \zeta = 0,0025 \).

Calculation (\( \zeta = 0,0025 \))

\[
\begin{align*}
P &= 280 \, \text{N} \\
f_s &= 2,25 \, \text{Hz} \\
n &= 6 \\
\zeta &= 0,25 \% (0,0025) \\
n' &= 10,8 \cdot \sqrt{0,0025 \cdot 2} = 1,32 \\
\Psi &= 0,25 \ (\text{according to Figure 3-4}) \\
p(t) &= 7,7 \, \text{N/m} \\
p^* &= 7,7 \cdot 2/\pi \cdot 12 = 58,82 \\
m^* &= \frac{1}{2} \cdot (98+38+18) \cdot 12 = 924 \, \text{kg} \\
a_{\text{EUR 23984 EN}} &= 12,73 \, \text{m/s}^2
\end{align*}
\]

According to the EUR 23984 EN, the response becomes approximately 3 times larger when the damping ratio decreases from 0,025 to 0,0025.

EUROCODE 5

The Eurocode 5 presents the following formula to determine vertical accelerations induced by one single pedestrian, and for bridges up to 2,5 Hz.

\[
a_{\text{vert}} = \frac{200}{m \cdot L \cdot \zeta}
\]
The group effect, is taken into account by the following formula.

\[ a_{\text{vert},n} = 0.23 \cdot n \cdot k_{\text{vert}} \] where \( n \) is the number of pedestrians and \( k \) is the reduction coefficient according to the Eurocode 5 (Figure 3-1)

**Calculation**

\[
\begin{align*}
    m &= 98 + 38 + 18 = 154 \text{ kg/m} \\
    L &= 12 \text{ m} \\
    \zeta &= 0.025 \\

    a_{\text{vert}} &= \frac{200}{154 \cdot 12 \cdot 0.025} = 4.33 \text{ m/s}^2 \\

    a_{\text{vert},n} &= 0.23 \cdot 6 \cdot 4.33 \cdot 1 = 5.98 \text{ m/s}^2
\end{align*}
\]

Here again, the calculation will be repeated, with \( \zeta = 0.0025 \).

**Calculation (\( \zeta = 0.0025 \))**

\[
\begin{align*}
    m &= 98 + 38 + 18 = 154 \text{ kg/m} \\
    L &= 12 \text{ m} \\
    \zeta &= 0.0025 \\

    a_{\text{vert}} &= \frac{200}{154 \cdot 12 \cdot 0.0025} = 43.3 \text{ m/s}^2 \\

    a_{\text{vert},n} &= 0.23 \cdot 6 \cdot 4.33 \cdot 1 = 59.8 \text{ m/s}^2
\end{align*}
\]

According to the Eurocode 5, the response becomes approximately 10 times larger when the damping ratio decreases from 0.025 to 0.0025.

**MONTE CARLO SIMULATION**

**Input bridge**

\[
\begin{align*}
    f_1 &= 2.25 \text{ Hz} \\
    m &= 154 \text{ kg/m} \\
    \zeta &= 0.025
\end{align*}
\]

**Input pedestrian stream**

\[
\begin{align*}
    f_{s,\text{mean}} &= 1.91 \text{ Hz} \\
    \sigma_f &= 0.091 \text{ Hz} \\
    G_{\text{mean}} &= 710 \text{ N} \\
    \sigma_G &= 95 \text{ N} \\
    \varphi &= [0,2\pi] \\
    \alpha &= 0.0115f_s^2 + 0.2803f_s - 0.2902
\end{align*}
\]

**Calculation**

\[ a_{\text{sim,95\%}} = 2.29 \text{ m/s}^2 \]
In Figure 5-4 the histogram of the Monte Carlo simulations results is shown.

![Histogram of Monte Carlo Simulation Results](image)

Here again, the calculation will be repeated, with $\zeta = 0,0025$.

**Calculation ($\zeta = 0,0025$.)**

$a_{\text{sim},95\%} = 2,8 \text{ m/s}^2$

**COMPARISON**

The results are summarized below:

Results ($\zeta=0,025$):

- $a_{\text{EUR 23984 EN}} = 4,13 \text{ m/s}^2$
- $a_{\text{EUROCODE 5}} = 5,98 \text{ m/s}^2$
- $a_{\text{sim},95\%} = 2,29 \text{ m/s}^2$

The results show that the EUR 23984 EN and the Eurocode 5, both present a conservative approach, to guarantee the comfort of walking pedestrians on slender footbridges.

As for the second calculations, the decrease of damping ratio from 2,5 % to 0,25 % results in 1,3 times larger response according to the Monte Carlo Simulations, 3 times larger response according to the EUR 23984 EN and 10 times larger response according to the Eurocode 5.

Results ($\zeta=0,0025$):

- $a_{\text{EUR 23984 EN}} = 12,73 \text{ m/s}^2$
- $a_{\text{EUROCODE 5}} = 59,8 \text{ m/s}^2$
- $a_{\text{sim},95\%} = 2,8 \text{ m/s}^2$

The EUR 23984 EN approaches the reality more in this matter. The influence of the damping on the response becomes larger, when the 1$^{\text{st}}$ natural frequency approaches the step frequency of pedestrians.
This is not taken into account by the Eurocode 5. The response is calculated by dividing the dynamic load due to a pedestrian, by the mass multiplied by the length of the bridge, multiplied by the damping ratio ($\frac{200}{m\cdot L \cdot \zeta}$). According to this formula, the response becomes 10 times larger if the damping ratio is decreased 10 times, independently from the 1st natural frequency of the bridge and the step frequency of the pedestrian.

In the following calculations, the damping ratio of 2,5 % (0,025) will be held.

These same calculations have been repeated for other traffic situations, were the damping ratio of 2,5 % (0,025) will be held. After all, this is the average of the measured damping ratios, so it represents the reality. The results of these calculations are shown in Table 5-1. In some traffic situations, the natural frequency of the bridge exceeds the 2,3 Hz, so the reduction coefficient $\Psi$ is 0 according to the EUR 23984 EN. This means that $a=0$.

<table>
<thead>
<tr>
<th>Traffic situation</th>
<th>Acceleration mid span of bridge [m/s$^2$]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>EUR 23984 EN</td>
</tr>
<tr>
<td>Traffic Class 1 on $f_{1,\text{ref}}$</td>
<td>0</td>
</tr>
<tr>
<td>Traffic Class 1 on $f_{1,1}$</td>
<td>0</td>
</tr>
<tr>
<td>Traffic Class 1 on $f_{1,2}$</td>
<td>0</td>
</tr>
<tr>
<td>Traffic Class 1 on $f_{1,3}$</td>
<td>3,19</td>
</tr>
<tr>
<td>Traffic Class 2 on $f_{1,\text{ref}}$</td>
<td>0</td>
</tr>
<tr>
<td>Traffic Class 2 on $f_{1,1}$</td>
<td>0</td>
</tr>
<tr>
<td>Traffic Class 2 on $f_{1,2}$</td>
<td>4,44</td>
</tr>
<tr>
<td>Traffic Class 2 on $f_{1,3}$</td>
<td>10,11</td>
</tr>
<tr>
<td>Traffic Class 3 on $f_{1,\text{ref}}$</td>
<td>10,39</td>
</tr>
<tr>
<td>Traffic Class 3 on $f_{1,1}$</td>
<td>12,23</td>
</tr>
<tr>
<td>Traffic Class 3 on $f_{1,2}$</td>
<td>18,38</td>
</tr>
<tr>
<td>Traffic Class 3 on $f_{1,3}$</td>
<td>17,91</td>
</tr>
</tbody>
</table>

In the next two paragraphs, the aim is to investigate why the responses calculated with the EUR 23984 EN are extremely higher than the Monte Carlo Simulation, even though the measured damping ratios of 2,5 % are used in the EUR 23984 EN and Eurocode 5 calculations. Therefore, the reduction coefficient $\Psi$ and the equivalent number of pedestrians will be assessed separately.
5.3 Assessment of the reduction coefficient $\Psi$

The EUR 23984 EN [1] presents a conservative approach, to guarantee the comfort of walking pedestrians on slender footbridges. In this section the equivalent number of pedestrians $n'$ and the reduction coefficient will be assessed separately to find out the source of the difference.

From previous calculations, one can conclude that the equivalent number of pedestrians that takes into the account synchronization between pedestrians approaches the reality conservatively. And thereby, the response calculated with that load model is higher than the Monte Carlo Simulation.

But, in addition, the values for the reduction coefficient provided by the EUR 23984 EN, has also its contribution in the responses. In this paragraph, the reduction coefficient $\Psi$ according to the standard will be compared with the Monte Carlo Simulation.

The function of the reduction coefficient $\Psi$ can be described as follows. Consider the pedestrian stream and bridge structure from previous calculation, with an average step frequency $f_{s,mean} = 1,91$ Hz and standard deviation $\sigma_f = 0,091$ Hz and $f_{1,2} = 2,16$ Hz. The load model converts this pedestrian stream into a number of synchronously walking pedestrian with a step frequency that is equal to 1$^{st}$ natural frequency of the bridge, so in other words $f_{1,2} = f_s = 2,25$ Hz. But in reality, the average step frequency is 1,91 Hz, so the response calculated with $f_{1,2} = f_s = 2,25$ Hz, will be reduced with a factor $\Psi$ to represent the stream correctly.

Reduction coefficient for not affected walking behaviour

The reduction coefficient can be re-calculated with the Monte Carlo Simulation, per Traffic Class. For each of the Traffic Classes, the response of an (imaginary) bridge will be calculated, whose natural frequency will be varied between 3,0 Hz and 1,0 Hz, in small steps. The calculated responses will then be divided by the highest response, to normalize all calculated responses. The normalized values are in fact the values for the reduction coefficient and will be plotted with be belonging natural frequencies. The natural frequency with the highest response, has a reduction coefficient of $\Psi=1,0$.

First the reduction coefficients will be calculated for Traffic Classes 1, 2 and 3, for not affected walking behaviour i.e. there is no interaction with bridge vibration. In Figure 5-5, the calculated responses due to 3 Pedestrians, Traffic Class 1, are plotted with the associated natural frequencies. The following values and distributions were held to calculate the reduction coefficient:

3 Pedestrians
L=12 meters
$m= m_{\text{pedestrian}} + m_{\text{bridge}}$
$m= 213/12 + 98 = 115,8$
$\zeta= 0,025$
$G_{\text{mean}}= 710 ; \sigma_G= 95 N$
$\varphi= [0,2\pi]$
$v_{\text{mean}}= 1,27 \text{ m/s} ; \sigma_v= 0,06 \text{ m/s}$
$f_{s,\text{mean}} = 1.85 \, \text{Hz} \quad \sigma_f = 0.185 \, \text{Hz}$

Figure 5-5 Calculated responses due to Traffic Class 1 – Not affected walking behaviour

Figure 5-6 shows the normalized values. These normalized values represent the extent to which the response decreases if the natural frequency is shifted away from the average step frequency of pedestrians.

Figure 5-6 Analytical reduction coefficient $\Psi$ for Traffic Class 1 - Not affected walking behaviour
Comparison with design guidelines EUR 23984 EN and SETRA
This figures is compared with the reduction coefficient figure presented by the EUR 23984 EN [1] and the SETRA [11], and one can see that the figure is more or less within the limits of the SETRA.

Both EUR 23984 EN [1] and SETRA [11] give a conservative values for bridges with natural frequencies between 1,7 Hz and 2,1 Hz (Ψ=1,0 = no reduction). EUR 23984 EN seems un-conservative for f₁ > 2,2 Hz.

This analysis will be repeated, but now with a damping ratio of ζ= 0,0025. The aim is to investigate if the damping has an influence on the reduction coefficient Ψ. Figure 5-7 shows that the damping has an influence on the response of the bridge when the 1st natural frequency of the bridge is in the critical range. Besides, the peak is at the same location: approximately 1,95 Hz.

Figure 5-7 Calculated responses due to Traffic Class 1 – ζ=0.025 and ζ=0.0025 – Not affected walking behaviour

Figure 5-8 shows the analytically obtained reduction coefficient Ψ for damping ratio ζ= 0,025 and ζ= 0,0025. Both series will be divided by its own highest response, to normalize them separately.
One can see that the damping has negligible influence on the form of the reduction coefficient and on the peak of the reduction coefficient. Therefore, for the remaining analysis, the damping ratio will not be varied: the average of the measured damping ratio will held, because after it all, it represents the bridge in the laboratory.

This analysis has further been performed for Traffic Classes 2 and 3.

As can be seen in Figure 5-9, the figure of the reduction coefficient is dependent of the figure of the step frequency distributions.
For instance, the reduction coefficient for Traffic Class 2 (Figures 5-10), has a lower deviation in contrary to Traffic Class 1 (Figure 5-6), which has a higher deviation. The same can be seen in the step frequency distributions of TC 1 and 2.

![Analytical reduction coefficient Ψ for Traffic Class 2 - Not affected walking behaviour](image1)

Comparison with design guidelines EUR 23984 EN and SETRA

The same can be seen in Figures 5-11 and 5-12, where the reduction coefficient for Traffic Class 3 is presented, which also has a smaller deviation in comparison to Traffic Class 1.

![Response due to 12 pedestrians - Traffic Class 3](image2)

Figure 5-11 Calculated responses due to Traffic Class 3 – Not affected walking behaviour
The EUR 23984 EN and the SETRA both use one reduction coefficient figure that covers all common step frequencies. To approach the occurring accelerations more accurate (less conservative), the reduction coefficient belonging to the step frequency distributions should be used.

An additional example is treated to explain the difference of the reduction coefficient figures in Traffic Class 1 and Traffic Class 3.

Consider two pedestrian streams which both have approximately the same average step frequency $f_{s,\text{mean}} = 1.85$, but the first stream has a high standard deviation and the second stream has a low standard deviation (pedestrians are somehow in synchrony). Both streams walk on a bridge with natural frequency $f_1 = 2.1$ Hz. In that case, the first stream will cause a higher structural response, even though the pedestrians do not walk in synchrony, but because of the higher standard deviation, statistically, the step frequency distribution of this stream reaches the bridge more than the second stream with a low standard deviation (pedestrians in synchrony). Therefore, the stream with a higher standard deviation has higher values for the reduction coefficient. The conclusions that follows from that, is that the synchronization between pedestrians could only cause high responses if the bridge frequency is near the average step frequency of the stream!

As has been said before, the EUR 23984 EN uses one conservative figure for the reduction coefficient. Figures 5-6, for examples, shows that the actual critical range of resonance is approximately between 1.9 Hz and 2.0 Hz, which is much smaller than the 1.7 - 2.1 Hz that is given by the EUR 23984 EN. The EUR 23984 EN gives a bigger range to cover the different possibilities where the locations of the peaks could be. Besides that, with this bigger range...
provided by the EUR 23984 EN, the synchronization between pedestrians (pushing the step frequency towards the natural frequency) could be taken into account.

The reduction coefficients $\Psi$ above were calculated on the basis of the ‘not affected’ step frequency distributions, i.e. no interaction with bridge vibration. This will also be done for the ‘affected’ step frequency distributions.

However, the reduction coefficient with the ‘forced’ step frequency distributions, can only be calculated for values bridges between $3,0 \, \text{Hz}$ and $1,82 \, \text{Hz}$. The bridge with the lowest stiffness in the laboratory has a natural frequency of $1,82 \, \text{Hz}$, so step frequency distributions for affected walking behaviour are only available to $1,82 \, \text{Hz}$.

**Reduction coefficient for affected walking behaviour**

In previous calculations, the reduction coefficient was calculated using the step frequency distributions for not affected walking behaviour. For example, the step frequency distributions for Traffic Class 1 ($f_{s,\text{mean}} = 1,85 \, \text{Hz} ; \sigma_f = 0,185 \, \text{Hz}$) does not change for all natural frequencies between $3,0 \, \text{Hz}$ and $1,0 \, \text{Hz}$.

In this analysis, the intention is to take into account the interaction between pedestrians and vibration, by using the ‘affected’ step frequency distributions that were measured during the experiments. For Traffic Class 1, the following step frequency distributions were held for the following natural frequency ranges:

- Between $3,0 \, \text{Hz}$ and $2,52 \, \text{Hz} \rightarrow f_{s,\text{mean}} = 1,85 \, \text{Hz} ; \sigma_f = 0,185 \, \text{Hz}$
- Between $2,52 \, \text{Hz}$ and $2,33 \, \text{Hz} \rightarrow f_{s,\text{mean}} = 1,84 \, \text{Hz} ; \sigma_f = 0,187 \, \text{Hz}$
- Between $2,33 \, \text{Hz}$ and $2,18 \, \text{Hz} \rightarrow f_{s,\text{mean}} = 1,89 \, \text{Hz} ; \sigma_f = 0,186 \, \text{Hz}$
- Between $2,18 \, \text{Hz}$ and $1,7 \, \text{Hz} \rightarrow f_{s,\text{mean}} = 1,89 \, \text{Hz} ; \sigma_f = 0,16 \, \text{Hz}$

Figure 5-13 shows the calculated response due to Traffic Class 1, with affected and not affected step frequency distribution.
The reduction coefficients with the red, dotted line, which is shown in Figure 5-14, is determined using the step not affected frequency distributions of Traffic Class 1, while the reduction coefficients with the black, solid line, is for affected walking behaviour.

![Figure 5-14 Analytical reduction coefficient Ψ for Traffic Class 1 - Affected walking behaviour Comparison with design guidelines EUR 23984 EN and SETRA](image)

The same analysis for affected and not affected walking behaviour has been repeated for Traffic Classes 2 and 3 and the following results were found.

![Figure 5-15 Calculated responses due to Traffic Class 2 - Affected and not affected](image)

In Figures 5-14, 5-16 and 5-18, one can see that the dotted and solid line are more or less the same for high bridge frequencies (not affected by bridge vibration), but then the solid line start to deviate when the critical range of resonance has been reached.
Earlier the conclusion was made that the reduction coefficient is dependent on the step frequency distributions. This conclusion can still be made on the basis of these results. The dotted and solid line have the same path at the beginning, but in the critical range of resonance, they show different peaks and different deviation.

The second conclusion is that both the SETRA and the EUR 23984 EN provide reduction coefficients that cover the synchronizations effects that may occur. Both peaks and the deviations are within the safe lines of the guidelines.
Figure 5.18 Analytical reduction coefficient $\Psi$ for Traffic Class 3 - Affected and not affected walking. Comparison with design guidelines EUR 23984 EN and SETRA.
5.4 Assessment of the approach of the equivalent number of pedestrians

Previous paragraph showed that the EUR 23984 EN [1] gives high values for the reduction coefficient $\Psi$, so it has its contribution in the high responses. In this section, the equivalent number of pedestrians $n'$ according to the EUR 23984 EN will be assessed. Thereafter, the equivalent dynamic loading according the EUR 23984 EN, where $n'$ and $\Psi$ are multiplied, will be assessed.

Synchronization between walking pedestrians is taken into account in the EUR 23984 EN, by converting a random pedestrian stream into a perfect synchronized equivalent stream. The synchronized stream, consisting of a equivalent number of pedestrians $n'$, can be determined using the following equation:

$$n' = 10.8 \cdot \sqrt{\xi \cdot n} [1],$$

for pedestrian densities to $1P/m^2$

As a part of the experiments, dynamic loading has been measured with load cells at the supports of the bridge. The bridge structure is simply supported at 4 load cells, on each corner of the bridge. The earlier mentioned equation has been tested with the following procedure.

Pedestrian streams consisting of 2P, 3P, 4P, 5P, 6P, 7P and 8P have crossed the reference bridge $f_{1,\text{ref}}$. These persons were distributed equally over the bridge. Besides that, a reference person 1P walks over the bridge. The dynamic reaction forces of reference person 1P, and the streams 2P till 8P, have been measured with the 4 load cells. The reaction forces in the 4 load cells were summed to obtain the maximum dynamic loading.

Thereafter, the dynamic reaction force due to the streams 2P till 8P have each been divided by the dynamic reaction forces due to 1P. This rate between total dynamic loads of the streams and the total dynamic loads of the reference person, is $n'_{\exp}$. $n'_{\exp}$ has been compared to $n'$ of the EUR 23984 EN.

The last step is to determine the equivalent dynamic loading. The design guidelines gives a value of 280 N that represents the dynamic amplitude due to one single footstep. This deterministic value, multiplied by the equivalent number of pedestrians $n'$, represents a random stream. The equation of $n'$ and the value 280 N follow from extensive experiments and statistical analyses. The question now arises: to which extend does the product of 280 N · $n'$ approach different kind of traffic situations. The product of 280 N · $n'$, is compared to the product of $n'_{\exp}$ · $G_{1P}$ · $\alpha_{1P} + ((1- \Psi_{\exp}) \cdot n'_{\exp} \cdot G_{1P} \cdot \alpha_{1P})$, which is the actual equivalent dynamic loading. $G_{1P}$ is the static weight of reference person 1P during the experiment, $\alpha_{1P}$ is the dynamic load factor of reference person 1P, and $\Psi_{\exp}$ is the reduction coefficient obtained from the Monte Carlo Simulations and experimental results. The second part of the equation (after the +), is the correction of the dynamic loading if the natural frequency of the considered bridge is far away from the critical point i.e. where $\Psi=1.0$. This will be clarified further in the calculations.
Table 5-2 below, shows the loaded and unloaded natural frequencies of the reference bridge $f_{1,\text{ref}}$.

<table>
<thead>
<tr>
<th>Load scenario</th>
<th>Natural frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unloaded frequency</td>
<td>$f_{1,\text{ref}} = 2.92$ Hz</td>
</tr>
<tr>
<td>Loaded with 1P</td>
<td>$f_{1,\text{ref}} = 2.87$ Hz</td>
</tr>
<tr>
<td>Loaded with 2P</td>
<td>$f_{1,\text{ref}} = 2.78$ Hz</td>
</tr>
<tr>
<td>Loaded with 3P</td>
<td>$f_{1,\text{ref}} = 2.67$ Hz</td>
</tr>
<tr>
<td>Loaded with 4P</td>
<td>$f_{1,\text{ref}} = 2.58$ Hz</td>
</tr>
<tr>
<td>Loaded with 5P</td>
<td>$f_{1,\text{ref}} = 2.51$ Hz</td>
</tr>
<tr>
<td>Loaded with 6P</td>
<td>$f_{1,\text{ref}} = 2.42$ Hz</td>
</tr>
<tr>
<td>Loaded with 7P</td>
<td>$f_{1,\text{ref}} = 2.36$ Hz</td>
</tr>
<tr>
<td>Loaded with 8P</td>
<td>$f_{1,\text{ref}} = 2.3$ Hz (critical zone has been reached)</td>
</tr>
</tbody>
</table>

**Assessment**

Equivalent number of pedestrians $n'_\text{exp}$ according to our approach, is determined by dividing the peak of the dynamic loads due to the streams > 1P, by the peak of the dynamic load due to reference person 1P. In Figures 5-19 and 5-20, the total dynamic loads due to stream 2P and the reference person 1P are shown.

$$n'_\text{exp} = \frac{D_{\text{total},2P}}{D_{\text{total},1P}} = \frac{2.23 \text{ kN}}{1.3 \text{ kN}} = 1.71$$

![Graph showing dynamic loads](image)

The 0 value that is shown in Figure 5-19, is the static weight of the bridge.
This has also been done for the other groups. The results are shown in Table 5-3 and Figure 5-21.

Table 5-3 Equivalent number of pedestrians: experimental values versus EUR 23984 EN

<table>
<thead>
<tr>
<th>Persons</th>
<th>(n')</th>
<th>(n'_{\text{exp}})</th>
<th>error</th>
</tr>
</thead>
<tbody>
<tr>
<td>1P</td>
<td>1</td>
<td>1</td>
<td>0 %</td>
</tr>
<tr>
<td>2P</td>
<td>1.71</td>
<td>2.16</td>
<td>26.32 %</td>
</tr>
<tr>
<td>3P</td>
<td>2.21</td>
<td>2.65</td>
<td>19.9 %</td>
</tr>
<tr>
<td>4P</td>
<td>2.28</td>
<td>3.05</td>
<td>33.77 %</td>
</tr>
<tr>
<td>5P</td>
<td>2.96</td>
<td>3.42</td>
<td>15.54 %</td>
</tr>
<tr>
<td>6P</td>
<td>3.35</td>
<td>3.74</td>
<td>11.65 %</td>
</tr>
<tr>
<td>7P</td>
<td>3.5</td>
<td>4.04</td>
<td>15.43 %</td>
</tr>
<tr>
<td>8P</td>
<td>4.4</td>
<td>4.32</td>
<td>-1.8 %</td>
</tr>
</tbody>
</table>

In all groups, except for group 8P, the EUR 23984 EN overestimates the equivalent number of pedestrians.
Statistical analysis

This analysis is extended afterwards, by repeating each of the mentioned experiments 10 times. The 95th percentile value of the total dynamic load due to 1P and the streams (up to 5P) will be determined, to calculate $n'_{\text{exp},95\%}$ and $D_{\text{exp},95\%}$. Due to lack of participants afterwards, the experiments were done to streams up to 5P on the reference bridge with $f_{1,\text{ref}}$. The results are shown in Table 5-4.

<table>
<thead>
<tr>
<th>Persons</th>
<th>$n'_{\text{exp},95%}$</th>
<th>$n'$</th>
<th>error</th>
</tr>
</thead>
<tbody>
<tr>
<td>1P</td>
<td>1</td>
<td>1</td>
<td>0 %</td>
</tr>
<tr>
<td>2P</td>
<td>1.59</td>
<td>2.16</td>
<td>35.85 %</td>
</tr>
<tr>
<td>3P</td>
<td>2.5</td>
<td>2.65</td>
<td>6 %</td>
</tr>
<tr>
<td>4P</td>
<td>3.24</td>
<td>3.05</td>
<td>-5.86 %</td>
</tr>
<tr>
<td>5P</td>
<td>3.92</td>
<td>3.42</td>
<td>-12.76 %</td>
</tr>
</tbody>
</table>

As can be seen in Table 5-4 and Figure 5-22, the $n'_{\text{exp},95\%}$ is higher than $n'$ for groups 4P and 5P.

**Figure 5-22** Difference between $n'$ and $n'_{\text{exp},95\%}$, statistical values, reference bridge

Equivalent dynamic loading

Now the equivalent dynamic loading $D_{\text{model}}$ according to the EUR 23984 EN will be determined, and compared with the calculated equivalent dynamic loading $D_{\text{exp},95\%}$ on the basis of the measured values. As an example this will be done for group 2P.

$$D_{\text{exp}} = n'_{\text{exp}} \cdot G_{1P} \cdot \alpha_{1P} + \left( (1 - \Psi_{\text{exp}}) \cdot n'_{\text{exp}} \cdot G_{1P} \cdot \alpha_{1P} \right)$$

where:

- $G$ = static weight of reference person = 900 N
- $\alpha$ = 0.0296 $f_s^2 + 0.1319 f_s - 0.2341$
\[ \alpha = 0.0296 \cdot 2^2 + 0.1319 \cdot 2 - 0.2341 \]
\[ \alpha = 0.16 \]

\[ \Psi_{\text{exp}} = 0.25 \text{ (see Figure 5-23)} \]

\[
\text{eq. } D_{\text{exp}} = n'_{\text{exp}} \cdot G_{1P} \cdot \alpha_{1P} + ((1- \Psi_{\text{exp}}) \cdot n'_{\text{exp}} \cdot G_{1P} \cdot \alpha_{1P})
\]
\[
\text{eq. } D_{\text{exp}} = 1.59 \cdot 900 \text{ N} \cdot 0.16 + ((1- 0.25) \cdot 1.59 \cdot 900 \text{ N} \cdot 0.16)
\]
\[
\text{eq. } D_{\text{exp}} = 400.68 \text{ N}
\]

The equivalent dynamic loading is corrected (by the term after the +), because the reference bridge has a natural frequency that is far from the critical point. The critical point is where \( \Psi_{\text{exp}} = 1.0 \), see Figure 5-23. The equivalent dynamic loading according to the EUR 23984, and calculated with the product 280 \cdot n', is at its critical point (\( \Psi=1.0 \)). Therefore, the comparison between eq.\( D_{\text{model}} \) and eq.\( D_{\text{exp}} \) is only fair, if eq.\( D_{\text{exp}} \) (product of 1.59 \cdot 900 \text{ N} \cdot 0.16) is increased with 75 % of its value, so that it has reached its critical point.

Equivalent number of persons \( n' \) according to EUR 23984 EN:
\[
n'= 10.8 \cdot \sqrt{0.02} \cdot n
\]
\[
n'= 10.8 \cdot \sqrt{0.02} \cdot 2
\]
\[
n'= 2.16
\]

Equivalent dynamic loading according to EUR 23984 EN:
\[
\text{eq. } D_{\text{model}} = 280 \cdot 2.16 = 604.8 \text{ N}
\]

This calculation, has been done for the other pedestrians streams 3P to 5P. The results are shown in 5-24.
Figure 5-24 Difference between the measured equivalent loading (statistical) and the equivalent according to EUR 23984 EN, reference bridge

Standard bridge-400
The same analysis has been performed for the standard bridge-400, which has a lower natural frequency \(f_{1,3}\). Compared to the reference bridge, \(n'_{\text{exp},95}\) of bridge-400 is significantly lower. This can be explained as follows. The standard bridge-400 has a lower natural frequency and approaches the average step frequency of the pedestrians. Reference pedestrian 1P already excites the standard bridge-400 much, whereby the ratio between 1P and streams (2P up to 5P) is not so high. This results in a low value for \(n'_{\text{exp},95}\), that is much lower than the \(n'\). The comparison between the \(n'_{\text{exp},95}\) and \(n'\) for the standard bridge-400 is shown in Table 5-5.

Table 5-5 95\(^{th}\) percentile value of the equivalent number of pedestrians versus EUR 23984 EN, standard bridge-400 with \(f_{1,3}\)

<table>
<thead>
<tr>
<th>Persons</th>
<th>(n'_{\text{exp},95})</th>
<th>(n')</th>
<th>error</th>
</tr>
</thead>
<tbody>
<tr>
<td>1P</td>
<td>1</td>
<td>1</td>
<td>0 %</td>
</tr>
<tr>
<td>2P</td>
<td>1,19</td>
<td>2,16</td>
<td>81,52 %</td>
</tr>
<tr>
<td>3P</td>
<td>1,22</td>
<td>2,65</td>
<td>117,21 %</td>
</tr>
<tr>
<td>4P</td>
<td>1,49</td>
<td>3,05</td>
<td>104,7 %</td>
</tr>
<tr>
<td>5P</td>
<td>1,8</td>
<td>3,42</td>
<td>90 %</td>
</tr>
</tbody>
</table>

Even though \(n'_{\text{exp},95}\) is lower \(n'\) for the standard bridge-400, the difference between eq. \(D_{\text{model}}\) and eq. \(D_{\text{exp}}\) should smaller, because the dynamic load factor is higher for the standard bridge-400.
The equivalent dynamic loading eq. \(D_{\text{exp},95}\) on the standard bridge-400 is calculated for group 2P:
Vibration behaviour of slender footbridges
Graduation research

\[ D_{\text{exp}} = n'_{\text{exp}} \cdot G_{1P} \cdot \alpha_{1P} + (1 - \Psi_{\text{exp}}) \cdot n'_{\text{exp}} \cdot G_{1P} \cdot \alpha_{1P} \]

where:
\[ G = \text{static weight of reference person} = 740 \text{ N} \]

The dynamic load factor \( \alpha_{1P} \) for 1P on the standard bridge-400 can be calculated as follows:

\[ \alpha_{1P} = \frac{G_{\text{bridge}} + D_{\text{pedestrian}}}{G_{\text{bridge}}} - 1 \]

\[ \alpha_{1P} = \frac{11.76 + 5.3}{11.76} - 1 = 0.43 \]

\[ \Psi_{\text{exp}} = 0.8 \] (obtained from Figure 5-26)

\[ D_{\text{exp}} = 1.19 \cdot 740 \text{ N} \cdot 0.43 + (1 - 0.8) \cdot 1.19 \cdot 740 \text{ N} \cdot 0.43 \]

\[ D_{\text{exp}} = 422 \text{ N} \]

Equivalent number of persons \( n' \) according to the EN-code:
\[ n' = 10.8 \cdot \sqrt{0.02} \cdot 2 \]
\[ n' = 2.16 \]
Equivalent dynamic loading according to the EUR:
\[ \text{eq. } D_{\text{model}} = 280 \cdot 2,16 = 604,8 \, \text{N} \]

The calculation is repeated for the other groups on the standard bridge-400 and the results are shown in Figure 5-27.

Figure 5-27 Difference between the measured equivalent loading (statistical) and the equivalent according to EUR 23984 EN, standard bridge-400

Figure 5-28 shows that the difference between the responses calculated with the Monte Carlo Simulation and the EUR 23983 EN, have a similar pattern as the difference between the equivalent dynamic loadings: the difference becomes larger if the pedestrian density increases.

Figure 5-28 Difference between the measured equivalent loading (statistical) and the equivalent according to EUR 23984 EN, standard bridge-400
6. Conclusions & Recommendations

6.1 Conclusions

**Force characteristics and synchronization effects**

According to the experiments conducted in the van Musschenbroek laboratory, the increase of the walking speed of pedestrians goes hand in hand with the increase of the pedestrian density. The measurements of Oeding’s [12] and the Synpex show the opposite: the walking speed decreased when more pedestrians were involved in the experiments. However, the experiments conducted in the van Musschenbroek laboratory were limited to pedestrians walking in one line. An explanation for the increase in speed, is that the first person feels forced to walk faster, knowing that much more pedestrians are going to enter the bridge.

The measurement of the step frequencies of walking pedestrians that were not affected by the bridge vibrations, i.e. walking on the stiff reference bridge, show similar results as the Synpex: the standard deviation of the step frequencies decreases when the pedestrian density on a bridge increases. For Traffic Classes 1, 2 and 3 respectively the following step frequency normal distributions were found: N[1.85, 0.185], N[1.91, 0.113] and [1.94, 0.105] in Hz. A form of synchronization between pedestrians in not affected walking behaviour is thereby quantified. Similar results have been shown in the Synpex [4] and the Pedigree project.

As for walking pedestrians that were affected by the bridge vibrations, the results of step frequency distributions are not straightforward. For pedestrians walking on bridges with lower natural frequencies, the normal distributions do not show a form of synchronization between pedestrians and bridge interaction: the standard deviation does not decrease constantly when the pedestrian density increases. On the basis of the video images, similar conclusions can be made. Synchronization between pedestrians and bridge vibration occurs for some pedestrians and for a short period of time: stable synchronization does not occur for most pedestrians, thereby synchronization between pedestrian and bridge cannot be quantified with step frequency distributions of streams (averages and standard deviations).

The analysis of the synchronization between pedestrians and bridge structures is therefore extended by determining the number of synchronization pedestrians in time while crossing over the bridge. This has been done for a certain pedestrian density (12 pedestrians) on the stiff reference bridge and the standard bridge with a lower natural frequency. The observation was that the number of pedestrian’s synchronization with the standard bridge was higher than with the reference bridge: 3 synchronized pedestrians against 2 synchronization pedestrians.

Another observation is that the moment of synchronization is also dependent on the bridge frequency. The 3 pedestrians on the standard bridge began synchronizing after 7.5 seconds while the 2 pedestrians began synchronizing after 12 seconds. The standard bridge has a lower natural frequency than the reference bridge, thus approaches the average step frequency of pedestrians more. Therefore, when the first pedestrians cross over the standard
bridge, they start to excite the bridge heavily, whereby they start to synchronize with the vibration rhythm. Then in time, other pedestrians enter the bridge, walking out of phase of the bridge vibration, whereby the bridge vibrations starts to damp and the synchronization effects start to decrease. As for the stiff reference bridge, which has a higher natural frequency, more walking pedestrians are needed to excite the bridge: the synchronization effects occur therefore later in time.

Earlier there has been said that the synchronization between pedestrians and bridge cannot be quantified with step frequency distributions of streams. Therefore as a last analysis, the step frequency distribution of one single pedestrian was determined, who crossed the reference bridge (2.92 Hz) and the standard bridge-400 (2.41 Hz). Figure 4-14 shows that the standard deviation becomes smaller for a single pedestrian on standard bridge-400 and the average step frequency becomes higher. The latter can be clarified: the standard bridge-400 starts to vibrate, so it forces the pedestrian (normally approximately 1.9 Hz) to adjust his step frequency towards the natural frequency of the bridge.

As can be concluded from these observations, 3 out of 12 pedestrians synchronized with the vertical bridge vibration when they walked on a bridge with lower natural frequency. The maximum acceleration that occurred during the experiments, was 0.8 m/s². The Synpex stated that for accelerations larger than 1.5 m/s², stable synchronization between pedestrians and vertical vibrations could occur. As it was not possible to decrease the natural frequencies of the bridges more with weights, because of strength and deflection reasons, unfortunately the conclusions can only be based on accelerations up to 0.8 m/s². Synchronization between pedestrians and bridge is negligible for accelerations up to 0.8 m/s².

The phase shift between pedestrians shows scattered results: distribution is rather uniform than normal. The Synpex [4] and Setra [11], give assumptions that the phase shift is uniform distributed between 0 and 2π. This assumption could be reasonably. Furthermore, the dynamic load factor αexp that is determined on the basis of the measured values, gives lower values than the α of the EUR 23984 EN. In the calculation of the dynamic load factor α, the effect of the bridge was taken into account, because the dynamic loads were measured at the supports of the bridge. In contrast to α of the EUR 23984 EN, where the dynamic load due to pedestrians was measured with pressure sensors that were placed in the shoes of the pedestrians.

**Assessment of the EUR 23984 EN**

The validation of the Monte Carlo Simulation showed differences between the accelerations calculated with the Monte Carlo, where the median values were obtained, and the measured response at the mid span of the bridge. Even though the median values were obtained and not the 95th percentile values, the Monte Carlo Simulation gave higher values than the experiments, when the pedestrian density increases. The accelerations calculated with the Monte Carlo simulations increases somehow constantly when the pedestrian density increases, while the measured increases less. In the video images there has been observed that the pedestrians damp the vibration somehow more when the pedestrian density increases, since the chance on out of phase walking becomes bigger. In some cases when the pedestrian density was high, the bridge vibration was even damped to its equilibrium for some time. This affect, where the footsteps of the pedestrians absorb the vertical vibrations, is not taken into account in the Monte Carlo Simulation. Therefore, the Monte Carlo
Simulation show a constant increase in acceleration when adding more pedestrians, while measured values show that the increase becomes less when the pedestrian density is higher.

The comparison between the accelerations calculated with the Monte Carlo Simulation and accelerations calculated with the Eurocode 5 and the EUR 29384 EN showed large differences. These differences become larger, when using the damping ratios that are given by the EUR 23984 EN (\(\zeta=0,002\) for steel) instead of the measured damping ratio (\(\zeta=0,025\)). This is then investigated further by assessing the reduction coefficient \(\Psi\) and the equivalent number of pedestrians \(n'\) separately, and then assessing the equivalent dynamic loading finally, to find the source of these differences.

The assessment of the reduction coefficient \(\Psi\) showed that the EUR 23984 EN and the SETRA give high values for the reduction coefficient \(\Psi\). With the Monte Carlo Simulation, reduction coefficients \(\Psi_{\text{exp}}\) were determined for each of the three Traffic Classes, first for not affected walking behaviour, and then for affected walking behaviour. The conclusion is that the reduction coefficient is dependent on the step frequency distribution belonging to the Traffic Class: the form of \(\Psi_{\text{exp}}\) is identical to the form of the normal distribution. Matter for course, Traffic Classes 2 and 3 have less spread, so the reduction coefficient \(\Psi_{\text{exp}}\) belonging to these traffic classes is less conservative than the reduction coefficient \(\Psi_{\text{exp}}\) belonging to Traffic Class 1. Even though Traffic Class 1 has a high spread, it is still safe and has lower values than the \(\Psi\) of EUR 23984 EN and the SETRA. Traffic Class gives a critical range of natural frequencies approximately between 1,9 Hz and 2,0 Hz, while the EUR 23984 EN gives a bigger range: between 1,7 and 2,1 Hz. The EUR 23984 EN gives such a big range, to cover all possible step frequencies of pedestrians that may exist, and in addition to that: the synchronization effects between pedestrians and bridge is covered by giving this big range.

In addition, to investigate the influence of the structural damping on the reduction coefficient, the calculation of the reduction coefficient for Traffic Class 1 has been repeated, but now with a damping ratio of \(\zeta=0,0025\) instead of the measured damping ratio of \(\zeta=0,025\). It seems that the damping ratio has no influence on the form or the peak of the reduction coefficient. The assumption of the EUR 23984 EN and the SETRA, that damping has no influence on the reduction coefficient, seems reasonable.

The equivalent number of pedestrians \(n'_{\text{exp}}\) calculated on the basis of the measured results, cannot be compared with the \(n'\) of the EUR 23984 EN, because the latter is natural frequency independent, while the \(n'_{\text{exp}}\) is dependent on which bridge has been considered. For example, the ratio \(n'_{\text{exp}}\) between the dynamic loading due to 3 Pedestrian and the dynamic loading due to 1 Pedestrian, is lower on the standard bridge-400 than the reference bridge, because 1 Pedestrians excitates the standard bridge-400 more than the reference bridge. Therefore, a useful comparison is the comparison between the equivalent dynamic loading according to the EUR 23983 EN, eq. D_{model}, and the measured equivalent dynamic loading eq. D_{exp}. Eq. D_{model} is calculated by 280 \(\cdot\) \(n'\), whereby the values are at the critical point \(\Psi=1\) i.e. the equivalent dynamic loading is at its maximum. The measured equivalent dynamic loading eq. D_{exp} is calculated with \(n'_{\text{exp}} \cdot G_{1P} \cdot \alpha_{1P} + ((1-\Psi_{\text{exp}}) \cdot n'_{\text{exp}} \cdot G_{1P} \cdot \alpha_{1P})\), whereby the second part of the equation corrects the equivalent dynamic load so that it reaches the critical point (\(\Psi_{\text{exp}}=1\)).
Figure 5-27 shows the difference between the normalized equivalent dynamic loading and the equivalent dynamic loading according to the EUR 23984 EN, on the reference and the standard bridge-400. These large difference in both figures have similar shapes: the difference becomes larger when the pedestrian density increases. This clarifies finally, the extremely high differences between the responses calculated with the Monte Carlo Simulation (which could considered seen as the reality) and the responses calculated with the EUR 23984 EN. In figure 5-28, the difference between the two is shown. The pattern of the difference is the same is the difference in equivalent dynamic loading: the difference increases when more pedestrians are involved.

### 6.2 Recommendations

The analysis of the synchronization between pedestrians and bridge should be extended by analyzing bridges with lower natural frequencies, and thus higher vibration amplitude. In previous experiments, the highest amplitude of acceleration that was reaches was 0.8 m/s². The bridge with the lowest natural frequency that could be realized in the van Musschenbroek laboratory had a unloaded natural frequency of approximately 2.3 Hz.

Another aspect which requires some attention, is the actual contribution of the mass of the walking pedestrians on the natural frequency of the bridge. Up to now, the mass of the walking pedestrians was considered as dead load, which is in reality untrue. This assumption has to be investigated more accurately.

Furthermore, the measurements of the step frequencies and the dynamic loads for determining the dynamic load factor, could be done better with pressure sensors. The calculations on the basis of the video images could give more scattered values than pressure sensors.

As for the analytical part of this research, a Monte Carlo Simulation should select variables from statistical distributions (average, standard deviation). In the current Monte Carlo Simulation which was made in Excel, the selection procedure was random: variables were chosen randomly from the measured values belonging to associated Traffic Class-Bridge combination.

Last but not least, this Monte Carlo Simulation could be extended by analyzing the correlation between the different quantities. In the current Monte Carlo Simulations, conclusions can only be made on the three pedestrian densities and on the bridges with the same properties as the bridges in the lab. With correlations influencing the selecting procedure of the Monte Carlo Simulations, conclusions can be made on other pedestrians streams and bridges with other properties.
References


Appendix

A: Experimental results

B: Literature survey
Appendix A: Experimental results
A.1. Step frequency distributions

Step frequency distribution
Traffic Class 1
On reference bridge with $f_{1,\text{ref}}$

Distribution based on 89 samples

<table>
<thead>
<tr>
<th>Traffic Class 1</th>
<th>1.56</th>
<th>1.60</th>
<th>2.07</th>
<th>1.82</th>
<th>1.90</th>
<th>1.62</th>
<th>2.03</th>
<th>1.94</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.62</td>
<td>1.72</td>
<td>1.93</td>
<td>1.87</td>
<td>1.88</td>
<td>1.67</td>
<td>1.90</td>
<td>1.74</td>
<td></td>
</tr>
<tr>
<td>1.74</td>
<td>1.72</td>
<td>1.94</td>
<td>1.90</td>
<td>1.65</td>
<td>1.77</td>
<td>2.00</td>
<td>1.84</td>
<td></td>
</tr>
<tr>
<td>1.54</td>
<td>1.90</td>
<td>2.22</td>
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**Step frequency distribution**
Traffic Class 1
On standard bridge with $f_{1,1}$

Distribution based on 87 samples.

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**Step frequency distribution**
Traffic Class 1
On loaded (200 kg) bridge with $f_{1,2}$

Distribution based on 91 samples.

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**Step frequency distribution**

Traffic Class 1

On loaded (400 kg) bridge with $f_{1,3}$

Distribution based on 88 samples.

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Step frequency $f_s$, mean = 1.89 Hz

$\sigma_f = 0.16$ Hz
Step frequency distribution
Traffic Class 2
On reference bridge with $f_{1,\text{ref}}$

**Distribution based on 105 samples.**

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**Step frequency distribution**
Traffic Class 2
On standard bridge with $f_{1,1}$

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Distribution based on 102 samples.

**Step frequency distribution**
Traffic Class 2 on $f_{1,1}$

$f_{s,\text{mean}} = 1.96 \text{ Hz}$

$\sigma_f = 0.099 \text{ Hz}$
**Step frequency distribution**

Traffic Class 2

On loaded (200 kg) bridge with $f_{1,2}$

Distribution based on 108 samples.

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Step frequency distribution
Traffic Class 2
On loaded (400 kg) bridge with \( f_{1,3} \)

Distribution based on 96 samples.

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Step frequency distribution
Traffic Class 2 on \( f_{1,3} \)

\( f_{s,\text{mean}} = 1.99 \text{ Hz} \)

\( \sigma_f = 0.106 \text{ Hz} \)
Step frequency distribution
Traffic Class 3
On reference bridge with $f_{1,\text{ref}}$

**Step frequency distribution**
Traffic Class 3 on $f_{1,\text{ref}}$

$f_{s,\text{mean}} = 1.94 \text{ Hz}$
$
\sigma_f = 0.105 \text{ Hz}$

Distribution based on 119 samples.

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<td>2.00</td>
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<td>1.79</td>
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<td>2.07</td>
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<td>1.82</td>
<td>2.04</td>
<td></td>
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<td></td>
</tr>
<tr>
<td>1.79</td>
<td>1.90</td>
<td>1.84</td>
<td>2.00</td>
<td>1.93</td>
<td>2.11</td>
<td>1.93</td>
<td>2.03</td>
<td></td>
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<td></td>
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<td>1.82</td>
<td>1.90</td>
<td>1.88</td>
<td>2.11</td>
<td>2.00</td>
<td>2.03</td>
<td>1.97</td>
<td>2.07</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.97</td>
<td>1.74</td>
<td>2.00</td>
<td>1.82</td>
<td>2.07</td>
<td>2.11</td>
<td>1.93</td>
<td>2.14</td>
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<td>1.90</td>
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<td>2.07</td>
<td>1.93</td>
<td>2.07</td>
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<td>2.06</td>
<td>2.03</td>
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<td>2.16</td>
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<td></td>
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<td>1.88</td>
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<td>1.94</td>
<td>1.95</td>
<td></td>
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<tr>
<td>1.84</td>
<td>1.85</td>
<td>1.82</td>
<td>2.06</td>
<td>2.07</td>
<td>1.97</td>
<td>1.82</td>
<td>1.96</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.72</td>
<td>2.00</td>
<td>1.90</td>
<td>1.93</td>
<td>2.06</td>
<td>1.79</td>
<td>1.90</td>
<td>1.90</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2.06</td>
<td>1.90</td>
<td>1.88</td>
<td>2.00</td>
<td>2.07</td>
<td>1.96</td>
<td>1.96</td>
<td>2.00</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.72</td>
<td>1.90</td>
<td>1.96</td>
<td>1.97</td>
<td>1.93</td>
<td>1.90</td>
<td>1.90</td>
<td>1.93</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.79</td>
<td>1.79</td>
<td>1.79</td>
<td>1.79</td>
<td>1.90</td>
<td>1.88</td>
<td>2.06</td>
<td>1.97</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
**Step frequency distribution**

Traffic Class 3

On standard bridge with $f_{1,1}$

Distribution based on 118 samples.

<table>
<thead>
<tr>
<th>Step frequency TC 3 on $f_{1,1}$ [Hz]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.88</td>
</tr>
<tr>
<td>1.76</td>
</tr>
<tr>
<td>1.90</td>
</tr>
<tr>
<td>1.79</td>
</tr>
<tr>
<td>2.06</td>
</tr>
<tr>
<td>1.85</td>
</tr>
<tr>
<td>1.89</td>
</tr>
<tr>
<td>1.89</td>
</tr>
<tr>
<td>1.67</td>
</tr>
<tr>
<td>1.87</td>
</tr>
<tr>
<td>1.77</td>
</tr>
<tr>
<td>2.00</td>
</tr>
<tr>
<td>1.90</td>
</tr>
<tr>
<td>2.05</td>
</tr>
<tr>
<td>1.87</td>
</tr>
</tbody>
</table>
Step frequency distribution
Traffic Class 3
On loaded (200 kg) bridge with $f_{1,2}$

Step frequency distribution
Traffic Class 3 on $f_{1,1}$

$\bar{f}_s = 1.92 \text{ Hz}$

$\sigma_f = 0.123 \text{ Hz}$

Distribution based on 117 samples.

| Step frequency TC 3 on $f_{1,2} [\text{Hz}]$ |
|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| 1.88            | 1.87            | 1.88            | 1.76            | 2.00            | 1.94            | 2.00            | 1.97            |
| 1.76            | 1.88            | 1.79            | 1.77            | 2.15            | 1.88            | 1.90            | 1.84            |
| 1.90            | 1.76            | 1.79            | 1.90            | 1.98            | 1.88            | 2.04            | 1.76            |
| 1.79            | 1.82            | 1.82            | 1.92            | 1.89            | 1.76            | 2.11            | 2.11            |
| 2.06            | 1.76            | 1.72            | 1.87            | 1.79            | 1.85            | 1.92            | 2.11            |
| 1.85            | 1.97            | 1.91            | 1.90            | 1.87            | 1.75            | 2.03            | 1.88            |
| 1.89            | 1.80            | 2.03            | 1.86            | 1.79            | 1.79            | 1.97            | 2.07            |
| 1.89            | 1.92            | 1.90            | 1.84            | 1.87            | 1.67            | 2.03            | 1.86            |
| 1.67            | 1.90            | 1.87            | 1.76            | 1.74            | 1.62            | 1.86            | 2.00            |
| 1.87            | 2.06            | 1.97            | 1.77            | 1.85            | 1.90            | 1.97            | 1.88            |
| 1.77            | 1.56            | 1.72            | 1.82            | 1.79            | 1.90            | 2.14            | 1.97            |
| 2.00            | 1.87            | 1.47            | 1.91            | 2.00            | 2.00            | 1.97            | 2.00            |
| 1.90            | 1.77            | 1.86            | 1.69            | 1.82            | 1.96            | 2.03            | 1.96            |
| 2.05            | 1.87            | 1.79            | 1.69            | 1.77            | 2.11            | 1.88            | 1.85            |
|                | 1.87            | 2.00            | 1.93            | 2.03            | 2.00            | 1.87            |
Step frequency distribution
Traffic Class 2
On loaded (400 kg) bridge with $f_{1,3}$

Distribution based on 118 samples.

### Step frequency TC 3 on $f_{1,3}$ [Hz]

<table>
<thead>
<tr>
<th>f_s [Hz]</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.03</td>
</tr>
<tr>
<td>2.07</td>
</tr>
<tr>
<td>1.90</td>
</tr>
<tr>
<td>2.19</td>
</tr>
<tr>
<td>1.96</td>
</tr>
<tr>
<td>1.97</td>
</tr>
<tr>
<td>1.88</td>
</tr>
<tr>
<td>1.90</td>
</tr>
<tr>
<td>1.82</td>
</tr>
<tr>
<td>1.88</td>
</tr>
<tr>
<td>2.00</td>
</tr>
<tr>
<td>2.14</td>
</tr>
<tr>
<td>1.97</td>
</tr>
<tr>
<td>2.07</td>
</tr>
<tr>
<td>2.14</td>
</tr>
<tr>
<td>2.04</td>
</tr>
<tr>
<td>1.96</td>
</tr>
<tr>
<td>2.11</td>
</tr>
<tr>
<td>1.98</td>
</tr>
</tbody>
</table>

$f_{s,\text{mean}} = 2.01$ Hz
$\sigma_f = 0.134$ Hz
A.2. Walking speed

Following table contains the average walking speed and standard deviation of the walking speed of streams according to Traffic Class 1, 2 and 3.

<table>
<thead>
<tr>
<th>Traffic Class</th>
<th>Average walking speed</th>
<th>Standard deviation speed</th>
</tr>
</thead>
<tbody>
<tr>
<td>Traffic Class 1</td>
<td>1.27 m/s</td>
<td>0.060 m/s</td>
</tr>
<tr>
<td>Traffic Class 2</td>
<td>1.49 m/s</td>
<td>0.069 m/s</td>
</tr>
<tr>
<td>Traffic Class 3</td>
<td>1.53 m/s</td>
<td>0.105 m/s</td>
</tr>
</tbody>
</table>

The average values and standard deviations are based on the following results.

Walking speed results for TC 1, based on 27 samples.

<table>
<thead>
<tr>
<th>Walking speed TC 1 [m/s]</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1.27</td>
<td>1.3</td>
</tr>
<tr>
<td>1.24</td>
<td>1.23</td>
</tr>
<tr>
<td>1.28</td>
<td>1.35</td>
</tr>
<tr>
<td>1.2</td>
<td>1.29</td>
</tr>
<tr>
<td>1.26</td>
<td>1.15</td>
</tr>
<tr>
<td>1.25</td>
<td>1.37</td>
</tr>
<tr>
<td>1.22</td>
<td>1.35</td>
</tr>
<tr>
<td>1.236</td>
<td>1.2</td>
</tr>
<tr>
<td>1.33</td>
<td>1.33</td>
</tr>
<tr>
<td>1.21</td>
<td>1.33</td>
</tr>
<tr>
<td>1.34</td>
<td>1.19</td>
</tr>
<tr>
<td>1.35</td>
<td>1.32</td>
</tr>
<tr>
<td>1.27</td>
<td>1.317</td>
</tr>
</tbody>
</table>
**Walking speed results for TC 2, based on 54 samples.**

<table>
<thead>
<tr>
<th>Walking speed TC 2 [m/s]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.42 1.58 1.49 1.63</td>
</tr>
<tr>
<td>1.43 1.52 1.59 1.43</td>
</tr>
<tr>
<td>1.48 1.45 1.61 1.39</td>
</tr>
<tr>
<td>1.46 1.56 1.52 1.43</td>
</tr>
<tr>
<td>1.41 1.4 1.48 1.57</td>
</tr>
<tr>
<td>1.43 1.45 1.54 1.56</td>
</tr>
<tr>
<td>1.52 1.53 1.43 1.61</td>
</tr>
<tr>
<td>1.54 1.37 1.48 1.5</td>
</tr>
<tr>
<td>1.5 1.41 1.59 1.42</td>
</tr>
<tr>
<td>1.49 1.4 1.38 1.44</td>
</tr>
<tr>
<td>1.54 1.4 1.51 1.5</td>
</tr>
<tr>
<td>1.45 1.54 1.57 1.52</td>
</tr>
<tr>
<td>1.54 1.53 1.6 1.44</td>
</tr>
</tbody>
</table>

**Walking speed results for TC 3, based on 48 samples.**

<table>
<thead>
<tr>
<th>Walking speed TC 3 [m/s]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.41 1.35 1.43 1.64</td>
</tr>
<tr>
<td>1.41 1.39 1.5 1.75</td>
</tr>
<tr>
<td>1.42 1.34 1.54 1.7</td>
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<tr>
<td>1.5 1.38 1.5 1.59</td>
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<tr>
<td>1.55 1.46 1.56 1.66</td>
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<tr>
<td>1.62 1.49 1.58 1.61</td>
</tr>
<tr>
<td>1.49 1.49 1.51 1.65</td>
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<tr>
<td>1.55 1.45 1.47 1.55</td>
</tr>
<tr>
<td>1.56 1.46 1.56 1.41</td>
</tr>
<tr>
<td>1.64 1.48 1.58 1.41</td>
</tr>
<tr>
<td>1.61 1.38 1.7 1.42</td>
</tr>
<tr>
<td>1.66 1.46 1.69 1.5</td>
</tr>
<tr>
<td>1.37 1.49 1.68 1.55</td>
</tr>
</tbody>
</table>
A.4. Load cells

The dynamic load due to walking pedestrians was measured at 4 support points of the bridge. In figures down below, the dynamic loads measured at load cells 10, 13, 17 and 18. Furthermore, the total dynamic loads in the mentioned load cells is added together and shown in figures below. Values that were measured during experiments were calibrated and plotted in Matlab.

A.4.1. Dynamic loads due to streams (Traffic Classes)

Dynamic loads due to streams (Traffic Classes)

Traffic Class 1

On reference bridge with \( f_{1,\text{ref}} \)
Traffic Class 1
On standard bridge with $f_{1,1}$
Traffic Class 1
On standard bridge-200 with $f_{1,2}$
Traffic Class 1
On standard bridge-400 with $f_{1,3}$
Traffic Class 2
On reference bridge with $f_{1,\text{ref}}$

Load cells total:
Traffic Class 2
On standard bridge with $f_{1,1}$

Load cells total:
Traffic Class 2
On standard bridge-200 with $f_{1,2}$

Load cells total:
Traffic Class 2
On standard bridge-400 with $f_{13}$

Load cells total:
Traffic Class 3
On reference bridge with $f_{1,\text{ref}}$

Load cells total:
Traffic Class 3
On standard bridge with \( f_{1,1} \)

Load cells total:
Traffic Class 3
On standard bridge-200 with $f_{1,2}$

Load cells total:
Traffic Class 3
On standard bridge-400 with $f_{1,3}$

Load cells total:
A.4.2: Dynamic loads due to streams 1P to 8P/5P (Equivalent dynamic loading)
Dynamic loads due to streams 1P to 8P

These measurements were done to calculate the equivalent dynamic loading, for the assessment of the EUR 23984 EN. The total dynamic loads measured in the load cells 10, 13, 17 and 18, due to streams 1P to 8P on the reference bridge, are shown in figures below.

Total dynamic loads due to 1P
Total dynamic loads due to 2P

Total dynamic loads due to 3P
Total dynamic loads due to 4P

Total dynamic loads due to 5P
Total dynamic loads due to 6P

Total dynamic loads due to 7P
Total dynamic loads due to 8P
Dynamic loads due to streams 1P to 5P (for statistical analysis)

The same experiment has been done for streams 1P to 5P, but now each experiment is repeated 10 times. The 95th percentile value of the total dynamic load due to 1P and the streams (up to 5P) will be determined, to calculate $n'_{\text{exp},95\%}$ and eq.$D_{\text{exp},95\%}$. Due to lack of participants afterwards, the experiments were done to streams up to 5P on the reference bridge $f_{1,\text{ref}}$. Besides that, the same experiments were also done on bridge with $f_{1,3}$, so the number of experiments are 100. The results are shown in table below.

Walking on reference bridge with $f_{1,\text{ref}}$

<table>
<thead>
<tr>
<th>Measurement</th>
<th>Total dynamic loads on bridge with $f_{1,\text{ref}}$ [kN]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1P</td>
</tr>
<tr>
<td>Measurement 1</td>
<td>1,56 kN</td>
</tr>
<tr>
<td>Measurement 2</td>
<td>1,55 kN</td>
</tr>
<tr>
<td>Measurement 3</td>
<td>1,69 kN</td>
</tr>
<tr>
<td>Measurement 4</td>
<td>1,3 kN</td>
</tr>
<tr>
<td>Measurement 5</td>
<td>1,48 kN</td>
</tr>
<tr>
<td>Measurement 6</td>
<td>1,58 kN</td>
</tr>
<tr>
<td>Measurement 7</td>
<td>1,71 kN</td>
</tr>
<tr>
<td>Measurement 8</td>
<td>1,69 kN</td>
</tr>
<tr>
<td>Measurement 9</td>
<td>1,83 kN</td>
</tr>
<tr>
<td>Measurement 10</td>
<td>1,53 kN</td>
</tr>
<tr>
<td>$D_{\text{exp},95%}$</td>
<td>1,7 kN</td>
</tr>
</tbody>
</table>

Walking on reference bridge with $f_{1,3}$

<table>
<thead>
<tr>
<th>Measurement</th>
<th>Total dynamic loads on bridge with $f_{1,3}$ [kN]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1P</td>
</tr>
<tr>
<td>Measurement 1</td>
<td>5,25 kN</td>
</tr>
<tr>
<td>Measurement 2</td>
<td>5,7 kN</td>
</tr>
<tr>
<td>Measurement 3</td>
<td>5,5 kN</td>
</tr>
<tr>
<td>Measurement 4</td>
<td>5,92 kN</td>
</tr>
<tr>
<td>Measurement 5</td>
<td>5,4 kN</td>
</tr>
<tr>
<td>Measurement 6</td>
<td>5.69 kN</td>
</tr>
<tr>
<td>Measurement 7</td>
<td>5.24 kN</td>
</tr>
<tr>
<td>Measurement 8</td>
<td>4.52 kN</td>
</tr>
<tr>
<td>Measurement 9</td>
<td>4.55 kN</td>
</tr>
<tr>
<td>Measurement 10</td>
<td>6 kN</td>
</tr>
<tr>
<td>$D_{exp, 95%}$</td>
<td>5.96 kN</td>
</tr>
</tbody>
</table>
A.5. Accelerations (for validation Monte Carlo Simulation)

Accelerations were measured at the mid span of the bridge, with two accelerometers on the left and right side of the bridge. The two accelerometers were sticked on the left and right bridge girders.

**Accelerations**

**Reference bridge with** $f_{1,\text{ref}}$

1 Pedestrian

![Graph](image1)

2 Pedestrians

![Graph](image2)

3 Pedestrians

![Graph](image3)
4 Pedestrians

5 Pedestrians

12 Pedestrians
Accelerations
Standard bridge-400 with $f_{1,3}$

1 Pedestrian

2 Pedestrians

3 Pedestrians
4 Pedestrians

5 Pedestrians

6 Pedestrians
12 Pedestrians
A.6. Displacements

Traffic Class 1
On reference bridge with $f_{1,\text{ref}}$

Traffic Class 1
On standard bridge with $f_{1,1}$

Traffic Class 1
On standard bridge-200 with $f_{1,2}$
Traffic Class 1
On standard bridge-400 with $f_{1.3}$
Traffic Class 2
On reference bridge with $f_{1,\text{ref}}$
A.7. Static weight participants

The static weight of the participants was measured with a scale at the laboratory before the start of the experiments. The normal distribution can be shown in figure below.

<table>
<thead>
<tr>
<th>Participant</th>
<th>Static weight G [kg]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Participant 1</td>
<td>74.6 kg</td>
</tr>
<tr>
<td>Participant 2</td>
<td>69.4 kg</td>
</tr>
<tr>
<td>Participant 3</td>
<td>69 kg</td>
</tr>
<tr>
<td>Participant 4</td>
<td>57.3 kg</td>
</tr>
<tr>
<td>Participant 5</td>
<td>70.4 kg</td>
</tr>
<tr>
<td>Participant 6</td>
<td>75.5 kg</td>
</tr>
<tr>
<td>Participant 7</td>
<td>72.6 kg</td>
</tr>
<tr>
<td>Participant 8</td>
<td>77 kg</td>
</tr>
<tr>
<td>Participant 9</td>
<td>64.3 kg</td>
</tr>
<tr>
<td>Participant 10</td>
<td>61 kg</td>
</tr>
<tr>
<td>Participant 11</td>
<td>74 kg</td>
</tr>
<tr>
<td>Participant 12</td>
<td>59 kg</td>
</tr>
<tr>
<td>Participant 13</td>
<td>84.9 kg</td>
</tr>
<tr>
<td>Participant 14</td>
<td>76.4 kg</td>
</tr>
<tr>
<td>Participant 15</td>
<td>58.3 kg</td>
</tr>
<tr>
<td>Participant 16</td>
<td>93.1 kg</td>
</tr>
</tbody>
</table>

\[
G_{\text{mean}} = 71.05 \text{ kg} \\
\sigma_G = 9.48 \text{ kg}
\]
A.8. Knock test signals

Reference bridge with $f_{1,\text{ref}}$
Unloaded

Reference bridge with $f_{1,\text{ref}}$
Loaded with TC 1
Reference bridge with $f_{1,\text{ref}}$
Loaded with TC 2

Reference bridge with $f_{1,\text{ref}}$
Loaded with TC 3
Standard bridge with $f_{1,1}$
Unloaded

![Vibration displacement 1](image1)

Standard bridge with $f_{1,1}$
Loaded with TC 1

![Vibration displacement 1](image2)
Standard bridge with $f_{1,1}$
Loaded with TC 2

![Vibration displacement 1](image1)

Standard bridge with $f_{1,1}$
Loaded with TC 3

![Vibration displacement 2](image2)
Standard bridge-200 with $f_{1,2}$
Unloaded

Standard bridge-200 with $f_{1,2}$
Loaded with TC 1
Standard bridge-200 with $f_{1,2}$
Loaded with TC 2

Standard bridge-200 with $f_{1,2}$
Loaded with TC 3
Standard bridge-400 with $f_{1,2}$
Unloaded

![Acceleration vs. Time Graph](image1)

Standard bridge-400 with $f_{1,2}$
Loaded with TC 1

![Acceleration vs. Time Graph](image2)
Standard bridge-400 with $f_{1,2}$
Loaded with TC 2

Standard bridge-400 with $f_{1,2}$
Loaded with TC 3
Appendix B: Literature survey

Introduction
This report is a literature survey on the graduation research project ‘Vibration behaviour of slender footbridges due to synchronized pedestrian loading’. It serves as an introduction to the research and focuses on the research questions that were prepared in the graduation plan. The aim is to select and to identify research evidence that are relevant to these questions.

Synchronization effects may occur when the density of pedestrians on a bridge increases. In crowded situations, walking pedestrians may adopt unintentionally the same step pattern as other pedestrians around them. The step frequency of pedestrians follows the same pattern, which increases the risk of undesirable vibrations. This effect increases more when the walking behaviour of these pedestrians interacts with the structure. Once pedestrians notice vibrations, they tend to adopt the same forcing frequency as the vibration frequency of the bridge, in order to maintain their balance.

The EUR 23984 EN ‘Design of Lightweight Foot Bridges for Human Induced Vibrations’ [1], provides a simplified load model, where the synchronization effect is taken into account via the equivalent number of pedestrians $n'$, and the structure-pedestrian interaction is taken into account via the reduction coefficient $\Psi$. The aim of this research is to check whether this approach takes into account the synchronization effects correctly.

This literature survey is divided into four parts. First of all, the determination of pedestrian-induced loads will be considered in Chapter 1. Different kind of dynamic models representing pedestrians will be discussed. Furthermore, the response of dynamic systems will be studied on the basis of a Single Degree of Freedom System (SDOF). Chapter 2 includes two experiments on synchronization effects. Subsequently, guideline EUR 23984 EN, including the simplified load model will be regarded in chapter 3.

The content of this literature survey is a combination of theory of dynamics, codes of practice, researches and experiments that are relevant to the different parts of this research project.

1. Literature study on dynamic loads induced by pedestrians

1.1 General dynamic loading
Dynamic loads are time-varying forces that can move or change when acting on a structure. Wind gusts, earthquakes, passing vehicles and walking pedestrians on a bridge cannot be modelled as static loads because of the rapid load fluctuations. In these cases a quasi-static analysis is not sufficient. Rapid load fluctuations are associated with large values of accelerations, whereby the product of the mass and the acceleration of a structure is not negligible. In contrast to static loads, where accelerations in the Newton's Second Law are negligible. [2]

Dynamic loads can be classified as two kinds: periodic loads and non-periodic loads. Examples of dynamic loads are harmonic loads, exponential loads, step loads, pulse loads
and random loads. These loads will be considered in this paragraph and conclusions will be made regarding the main subject, namely pedestrian induced loads.

1.1.1 Periodic loads
Periodic loads are loads that repeat itself at equal time intervals. A single form of periodic loads is either a sine or a cosine function. The figure below shows the gradient of a single form of a periodic load.

![Simple harmonic motion](image1)

**Figure 1: Simple harmonic motion (Source: Structural Dynamics of earthquake engineering, Rajasekaran [3])**

The type of periodic loads shown in figure 2 are non-harmonic periodic loads. A non-harmonic periodic load can be described with a function \( f(t) \), which repeats itself after a time \( T \). For all values of \( t \) the function will be:

\[
f(t + T) = f(t)
\]  

[1-1]

\( T \) is called the period of the function \( f(t) \).

![Periodic loadings](image2)

**Figure 2: Periodic loadings (Source: Dynamica van Systemen, Blauwendraad [2])**

Periodic loadings can be represented by a summing sufficient number of harmonic terms in a Fourier series. Under certain assumptions regarding the properties of the periodic function \( f(t) \), it is possible to write the function as an infinite sum of sine and cosine functions with periods \( T, T/2, T/3, \ldots, T/n \).

\[
f(t) = \sum_{n=0}^{\infty} \left( a_n \cos n\omega t + b_n \sin n\omega t \right)
\]  

[2]

With \( \omega = \frac{2\pi}{T} \)  

[1-2]

By way of illustration, Figure 3 shows how a periodic square wave which can be approached by summing sine- and cosine functions. Function \( S_5 \) for example can be described as follows.

\[
S_5(t) = \frac{4}{\pi} \left( \sin \omega t + \frac{1}{3} \sin 3\omega t + \frac{1}{5} \sin 5\omega t \right)
\]  

[1-3]
1.1.2 Non-periodic loads

Loads that do not seem to repeat are called non-periodic loads. Non-periodic loads may be of comparatively long duration. In this case the response is usually modelled as being static or quasi static. Wind forces for example are often converted into equivalent static forces. Non-periodic loads may also be of short duration and are then called impulsive loads. Loads that are applied stepwise, are called step loads. Pedestrian induced forces can be categorized as step loads.

Deterministic versus non-deterministic loads

Dynamic loads may also be classified as deterministic and non-deterministic. If the magnitude, point of application of the load and the variation of the load with respect to time are known, the loading is said to be deterministic. The analysis of a system to such loads is defined as a deterministic analysis.

On the other hand, if the variation of load with respect to time is not known, the loading is referred to as random or stochastic loading. The corresponding analysis is termed as non-deterministic analysis. Pulse loads, representing a falling person or a car crashing into a building, or a blast load, as shown in Figure 4, are examples of random loadings. Earthquake ground motions are also an example of random loadings, see Figure 5. [3]
1.2 Dynamic loads induced by one pedestrian

Dynamic forces induced by one pedestrian are generated by the movement of the body mass and the put-down, rolling and push-off of the feet. These forces are called human ground reaction forces. When these forces are induced by walking, then they form an almost periodic excitation.

During walking, a single pedestrian produces a dynamic time varying force which has components in three directions: vertical, horizontal-lateral and horizontal-longitudinal. The single pedestrian walking force, has been studied for many years. The vertical component of the force, which will be the focus in this research, has been most investigated. It has been regarded as the most important of the three forces because it has the highest magnitude. Other human-induced forces that are important for footbridges are forces due to running and some forms of deliberate vandal loading, like jumping and bouncing. Large groups of pedestrians have seldom been formally investigated. [4]

1.2.1 Early investigations on pedestrian induced dynamic loads

One of the oldest investigations on pedestrian-induced forces was performed by Harper et al [5]. Measurements of pedestrian-induced forces were conducted with the aim to investigate the friction and slipperiness of a floor surface. They measured horizontal and vertical forces from a single footstep using a force plate [6]. The shape of the vertical force with two peaks of the kind shown in figure 6 was recorded. Other researchers such as Galbraith and Barton [7], Blanchard et al. [8] and Kerr [9] confirmed the general shape of the force time history.

In another research into walking forces which has been done in the field of biomechanics, Andriacchi et al. [10] measured similar to Harper et al. [5], single step walking forces in all three directions by means of a force plate. The typical shapes found are shown in Figure 6.

Andriacchi et al. also reported that the dynamic effect of the forces changes with the walking speed. Increasing walking velocity led to increasing step length and peak force magnitude.
Figure 6: Typical shapes of walking forces in (a) vertical, (b) lateral and (c) longitudinal direction (Source: Andriacchi et al.[10])

Hereby, the nature of human-induced dynamic forces and their dependence on many parameters is very complex. For example, a research on multiple walking speed-frequency relations [11], has shown that tests with control of only one of the parameters, such as the pacing frequency, speed, or step length, each produce different relationships between the walking speed and the step frequency.

In the research on human-induces forces by Galbraith and Barton, a single step vertical force on an aluminum plate was measured, ranging from slow walking to running. It was notable that the shape of running force differed from the walking force in having only one peak. The subject weight and step frequency were identified as important parameters which increase led to higher peak amplitudes of the force.

Continuous walking or running force can be obtained artificially, by combining individual foot forces, which are assumed to be identical. As shown in figure 7, during running there are periods when both feet are off the ground, leading to zero force recorded. As for normal walking, there are some time periods when both feet are on the ground, which gives an overlapping between the left and right leg in the walking time history.
Vibration behaviour of slender footbridges
Graduation research

Figure 7: Typical pattern of running and walking forces (Source: Galbraith and Barton [7])

Figure 8 an example of a force-time history of the vertical force imposed by a pedestrian, during normal walking. A period of 1.25 seconds is shown in which two steps are combined. The total force in the period when both feet are on the ground, is the sum of the individual force of the left and right foot. [12]

Within the SYNPEX project [14], a recent research with the aim of the development of advanced load models for synchronous pedestrian excitation, pedestrian induced forces were also measured. The ground reactions forces were measured by means of loads cells. Furthermore, the walking behaviour of single pedestrians and groups were measured by means of pressure taps.

Typical vertical ground reactions forces due to slow- and fast walking that were measured, are shown in figure 9 down below.
b) Slow walking        c) Fast walking

Figure 9: Measured vertical human ground reaction forces, one step (Source: SYNPEX-project [14])

In figure 10 the characteristic properties of vertical ground reaction force are shown. The vertical ground reaction forces are normalized with the body weight G.

- the duration of the loading $t_{\text{end}}$;
- the gradient of the force $m_A$ when the force increases;
- first maximum $F_{\text{max},1}/G$ during rolling up;
- the minimum $F_{\text{min}}/G$ during the support phase;
- the second maximum $F_{\text{max},2}/G$ at the terminal support phase when the foot lifts off;
- the gradient of the force $m_B$ during load relieving.

The gradient are defined:

$$m_A = \frac{F_{\text{min}}/G - 0.1}{t_2 - t_1}$$

$$m_B = \frac{0.1 - F_{\text{min}}/G}{t_4 - t_3}$$

Step frequencies of crossing pedestrians were measured and evaluated assuming that the duration of a step with the right foot is the same as for the left foot. This is done for the three
walking intentions slow, normal and fast walking. The mean values of the three crossings for each walking intentions are determined for each test persons. The step frequencies are distributed normally, and the belonging mean values and standard deviations are shown in figure 11.

![Figure 11: Probability density of step frequencies regarding the walking intention, per person (Source: SYNPEX-project)](image)

The step frequencies of a person crossing the platform vary with a standard deviation of approximately 0.1 Hz. This confirms that people do not walk perfectly periodic. These measured mean values correspond well with the observation of Pachi on footbridges [15], where a mean value of 1.80 Hz resp. 1.86 Hz and a standard deviation of 0.1 Hz is given.

1.2.2 Load model for a single pedestrian

There are two types of time-domain force models: force models with a deterministic approach and force models with a probabilistic approach. With the deterministic approach, the intention is to establish one general force model for each type of human walking. While, the probabilistic approach takes into account the fact that some parameters which influence human force, such as the previously mentioned step frequency, body weight and so on, are random variables whose statistical nature should be considered in terms of their probability distribution functions.

In both cases, the time-domain force models for walking pedestrian are based on the assumption that both human feet produce exactly the same force and that the pedestrian induced force is approximately periodic.

**Deterministic force model for a single pedestrian**

The periodic force $F_p(t)$ with a period $T$ can be represented by a Fourier series [1]:

$$F_{p,\text{vert}}(t) = P[1 + \sum_{i=1}^{n} a_{i,\text{vert}} \sin(2\pi i f_i t - \phi_i)]$$  \[1-4\]

$F_{p,\text{vert}}$  vertical periodic force due to walking

$P[N]$  pedestrian's body weight

$a_{i,\text{vert}}$  Fourier coefficient of the $i^{th}$ harmonic for vertical forces, i.e. dynamic load factor (DLF)
f_s [Hz] step frequency
Φ_i phase shift of the i\textsuperscript{th} harmonic
n total number of contributing harmonics

The periodic force moves with a constant speed along the bridge. Within the earlier mentioned SYNPEX research project, the relationship between step frequency and walking speed is found by measurements for a step frequency range of 1.3 to 1.8 Hz:

\[ v_s = 1.271 f_s - 1 \]  

[1-5]

In standards such as the EN 1995-2 [16] the body weight P is given as 700 or 800 N.

Furthermore, Fourier coefficients or dynamic load factors have been measured in various researches [4]. The measured dynamic load factors scatter, because human ground reaction forces are influenced by a variety of factors (e.g. walking speed, individual physiological body properties). Table 1 lists dynamic load factors (DLF) and phase angles from some authors.

<table>
<thead>
<tr>
<th>Authors</th>
<th>Dynamic load factors / Phase angles</th>
<th>Comment</th>
<th>Type of activity and load direction</th>
</tr>
</thead>
<tbody>
<tr>
<td>Blanchard et al.</td>
<td>( \alpha_1 = 0.257 ) ( \alpha_2 = \alpha_3 = 0.1 ) ( \Phi_2 = \Phi_3 = \pi/2 )</td>
<td>( f_p = 2.0 ) – 2.4 Hz</td>
<td>Walking – vertical</td>
</tr>
<tr>
<td>Bachmann et al.</td>
<td>( \alpha_1 = 0.4/0.5; \alpha_2 = \alpha_3 = 0.1 ) ( \alpha_1 = \alpha_2 = \alpha_3 = 0.1 ) ( \Phi_2 = \Phi_3 = \pi/2 ) ( f_p = 2.0 ) Hz ( f_p = 3.0 ) Hz</td>
<td>Mean values Fourier coeff.</td>
<td>Walking – vertical, lateral, long, vertical &amp; lateral</td>
</tr>
<tr>
<td>Kerr</td>
<td>( \alpha_1, \alpha_2 = 0.7; \alpha_3 = 0.2 )</td>
<td>( \alpha_1 ) is frequency dependant</td>
<td>Walking – vertical</td>
</tr>
<tr>
<td>Young</td>
<td>( \alpha_1 = 0.37 (f_p - 0.95) \leq 0.5 ) ( \alpha_2 = 0.054 + 0.0088 f_p ) ( \alpha_3 = 0.026 + 0.015 f_p ) ( \alpha_4 = 0.01 + 0.0204 f_p )</td>
<td>Mean values Fourier coeff.</td>
<td>Walking – vertical</td>
</tr>
<tr>
<td>EC5, DIN1074</td>
<td>( \alpha_1 = 0.4; \alpha_2 = 0.2 ) ( \alpha_1 = \alpha_2 = 0.1 )</td>
<td>Mean values Fourier coeff.</td>
<td>Walking – vertical</td>
</tr>
</tbody>
</table>
1.3 Dynamic response

In this paragraph an introduction into the dynamics is given to increase understanding of different procedures to calculate the dynamic response of bridges. This introduction into the dynamics is limited to discrete systems. Most simply supported slender footbridges have second natural frequencies (n=2) that are larger than the excitation range of 1.25 Hz – 4.6 Hz that is mentioned earlier. As a result of this, the focus of this dynamic problems for this type of footbridges is on the first vibration mode. This offers the possibility to represent a continuous model by a system with only a single degree of freedom (SDOF).

1.3.1 Discrete systems

A pedestrian bridge can be considered as an oscillator if a dynamic load is applied on it. An oscillator can have one or more degrees of freedom (n-DOF). The dynamic behaviour of a simple supported structure can be described with one degree of freedom. For the structure as shown in Figure 12, it can be the deflection in the middle.

\[ m= \text{modal mass of the structure} \]
\[ k= \text{modal stiffness, e.g. the stiffness of the main girders of a bridge} \]
\[ c= \text{modal damping constant of the structure} \]
\[ F(t)= \text{external force on the footbridge, pedestrian-induced load} \]
\[ x(t)= \text{displacements of the mass in the time} \]

Figure 12: Girder as a system with one degree of freedom (Source: Technical guide 'Assessment of vibrational behaviour of footbridges under pedestrian loading, by Serive d'Etudes Techniques des Routes et Autoroutes' [18])

<table>
<thead>
<tr>
<th>SYNPEX findings</th>
<th>Walking - vertical</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \alpha_1=0.0115f_s^2+0.2803f_s-0.2902 )</td>
<td></td>
</tr>
<tr>
<td>( \Phi_2 = 0 )</td>
<td></td>
</tr>
<tr>
<td>( \alpha_2=0.0669f_s^2+0.1067f_s-0.0417 )</td>
<td></td>
</tr>
<tr>
<td>( \Phi_2=-99.76f_s^2+478.92f_s-387.8[^\circ] )</td>
<td></td>
</tr>
<tr>
<td>( \alpha_3=0.0247f_s^2+0.1149f_s-0.1518 )</td>
<td></td>
</tr>
<tr>
<td>( \Phi_3= 813.12f_s^3-5357.6f_s^2+11726f_s-8505.9[^\circ] )</td>
<td></td>
</tr>
<tr>
<td>If ( f_s &lt; 2.0 \text{ Hz} )</td>
<td></td>
</tr>
<tr>
<td>( \Phi_3= 813.12f_s^3-5357.6f_s^2+11726f_s-8505.9[^\circ] )</td>
<td></td>
</tr>
</tbody>
</table>
The equation of motion for a discrete system such as in Figure 13 can be described using the second law Newton:

\[ F = m \frac{d^2x}{dt^2} = m\ddot{x} \quad [1-6] \]

Figure 13: Girder as a system with one degree of freedom

\[ M_1, k_1 \text{ and } c_1 \text{ represent the modal mass, modal stiffness and modal damping constant respectively.} \]

When the mass is pushed downwards, we will find the following equilibrium:

\[ -F_{\text{spring}}(t) - F_{\text{damping}}(t) = m \cdot \ddot{x}(t) \quad [1-7] \]

\[ F_{\text{spring}}(t) = k_1 \cdot x(t) \]
\[ F_{\text{damping}}(t) = c_1 \cdot \dot{x}(t) \]

Inserting in equation 1-7 gives:

\[ -k_1 \cdot x(t) - c_1 \cdot \dot{x}(t) = m_1 \cdot \ddot{x}(t) \]
\[ m_1 \cdot \ddot{x}(t) + c \cdot \dot{x}(t) + k_1 \cdot x(t) = 0 \]

This equation is the second order equation that describes the behaviour of this structure with SDOF [27].

The modal damping constant \( c \) is calculated using a specified damping ratio (\( \zeta \)). This damping ratio is often based on experimental tests on footbridges. The value refers to an equivalent amount of damping present in the structure and is a combination of different damping mechanisms such as friction in connections and structural damping in the main structure. The influence of the amount of damping on the vibration behaviour is shown in Figure 14.
In undamped systems, the system oscillates at its own natural frequency. This vibration is mathematically speaking continuous. When adding an amount of damping, the vibrations dissipates in time. This can be seen in underdamped systems, where the system oscillates with the amplitude gradually decreasing to zero.

When a system is strong damped, the system returns to equilibrium as quickly as possible, while critically damped systems return to equilibrium without oscillating.

If the damping ratio is known, the modal damping constant can be determined as follows:

\[ c = \zeta \left( \frac{2}{\pi} \right) \sqrt{k_1 m_1} \]  \[ \text{[1-8]} \]

The following equations give the natural frequency of the SDOF system with \((f_1)\) and without damping \((f_{1,D})\):

\[ f_1 = \frac{1}{2\pi} \sqrt{\frac{k_1}{m_1}} \]  \[ \text{[1-9]} \]
\[ f_{1,D} = f_1 \sqrt{1 - \zeta^2} \]  \[ \text{[1-10]} \]

In order to transfer a continuous model, with an infinite number of natural frequencies, to a SDOF-model, the modal properties are required. These properties can be obtained using the equivalent mode shape. The modal mass for a simple supported beam with a uniformly distributed mass can be described in the following equation [12]:

\[ m_1 = \int_0^L m(x) \Phi_1^2(x) \, dx = \frac{1}{2} m L \]  \[ \text{[1-11]} \]

where \(m(x)\) represents the mass per unit length and \(\Phi_1(x)\) represents the mode shape of the 1st natural vibration mode.

The modal stiffness \(k_1\) can now be determined using the following equation:

\[ k_1 = (2 \pi f_1)^2 m_1 \]
1.3.2 Steady state response of a SDOF-system

Vibrations occur with a transient and steady-state phase, as shown in Figure 15. The transient phase shows an increase of the amplitude of the response. At a certain point, the maximum amplitude of the response remains the same which is called the steady state phase.

![Figure 15: Transient- and steady state phase](Source: Blauwendraad, J., *Dynamica van Systemen*, University of Technology Delft [2])

Most design procedures related to footbridges are based on the steady state response of a SDOF-system under harmonic excitation. Based on the static deflection, the dynamic response can be calculated using the following formula:

\[ u_{\text{max}} = u_{\text{static}} \cdot DAF \]  

where \( u_{\text{static}} \) is the static deflection and DAF is the dynamic amplification factor.

The static deflection of a SDOF system and the dynamic amplification factor can be obtained from the following equations:

\[ u_{\text{static}} = \alpha \frac{G}{k_1} \]  

where \( \alpha \) is the dynamic load factor (Fourier coefficient of the \( n \)-th harmonic) and \( G \) is the body weight. The dynamic load factor \( \alpha \) represents the fraction of the static load of a pedestrian that acts as the dynamic load.

\[ DAF = \frac{1}{\sqrt{1 - g^2 + (2 \zeta g)^2}} \]  

where \( g = \frac{f_n}{f_1} \), the ratio between the forcing frequency and the natural frequency of the bridge, and \( \zeta \) is the damping of the system.

Dynamic amplification is obtained as a function of \( g \) and \( \zeta \). It may be represented by a set of curves parameterized by \( \zeta \). Some of these curves are provided in Figure 16 for some specific values of the critical damping ratio \( \zeta \). These curves show a peak for the value of \( g_r = \sqrt{1 - 2\zeta^2} \), characterizing the resonance [18].
The maximum response in terms of acceleration can be obtained by multiplying the calculated dynamic displacement with \((2\pi f_s)^2\). This is equal to differentiating the displacement signals:

\[
\text{\(a_{\text{max}} = u_{\text{static}} \cdot \text{DAF} \cdot (2\pi f_s)^2\)} \quad [1-15]
\]

The maximum response amplitude can be used to define the steady state response in the time domain. The time domain response of the SDOF-system can be defined by:

\[
\text{\(a(t) = a_{\text{max}} \sin(2\pi f_s t + \Phi)\)} \quad [1-16]
\]

The phase shift \(\Phi\) gives the difference between the moment where the force is maximum and the response.

Figure 17 gives phase angles based on the ratio between the forcing frequency and the natural frequency of the bridge \(f\) and the damping ratio. The phase shift is \(1/2\pi\) \((90)\) degrees in case of resonance.
Figure 17: Phase angle (Source: Wikipedia: Vibrations)
2. Literature study on the synchronization effects

The synchronization effects are examined in two different kinds of studies: research on the stepping behaviour of pedestrians walking in line, performed within the PEDIGREE [19] project, and research on the human interaction behaviour and induced forces on moving structures concerning synchronization/lock in effect, within SYNPEX [14] project.

The first study focuses on the longitudinal interactions between pedestrians walking in line. In this study there were four research teams involved, in Rennes (INRIA), in Toulouse (IMT, CRCA), and in Orsay (LPT). The experiments were performed at University Rennes, with the help of the laboratory M2S from Rennes. Besides the interactions between pedestrians, the experiments within the SYNPEX project also focuses on the pedestrian-structure interaction.

2.1 Research on the stepping behaviour of pedestrians walking in line

In human crowds, interactions among individuals give rise to a variety of self-organized collective motions that help the group to effectively solve the problem of coordination. However, it is still not known exactly how humans adjust their behaviour locally, nor what the direct consequences on the emergent organization are. One of the mechanisms of adjusting individual motions is the stepping dynamics. This research [19] presents a quantitative analysis on the stepping behaviour in a one-dimensional pedestrian flow, studied under controlled laboratory conditions. It focuses on the relationship between the step length (respectively step duration) and the velocity and pedestrian density, and it also focuses on step synchronization.

In crowded situations, were pedestrians are separated by a small distance, they cannot walk freely. The distance between them can become of the same order as the longitudinal displacement due to steps. Accelerations and decelerations occur on time scales that are similar to the stepping period. Steps cannot be ignored when dealing with high density flows. It was observed in another research of Seyfried [20], that at high densities, people were walking in lockstep in order to optimize the use of the available space in front of them.

Normally interactions between walking pedestrians take place in a two-dimensional space, where they change their velocity in direction and modulus. There are situations, however, where interactions are mostly longitudinal, for example, when people are walking along a narrow corridor. This study is limited to one-dimensional pedestrian flows. One-dimensional pedestrian flows involve purely longitudinal interactions which induce only changes in velocity.

Such experiments are easier to interpret if periodic boundary conditions are used, i.e. pedestrians walk on a closed line. Transients may occur when pedestrians enter or exit an experimental set-up. Furthermore, the global density of the pedestrians is constant in a closed system, while it is difficult to control it in an open system. Such experiments have been performed in recent years, e.g. experiments by Seyfried et al. [20][21][22], where pedestrians followed an oval path. Similar studies have been reported in references [23] [24]. The experiments have either been performed by using video analysis of the individual
trajectories along one straight portion of the set-up[22], or by measuring times at which participants pass a given measuring point [24].

In this new experiment [19], pedestrians were asked to follow circular paths. These pedestrians were tracked with a high precision motion capture device. As a result of being able to cover a larger range of densities than in previous experiments, there is found that the behavior of a pedestrian following another one was exhibiting two transitions, when the distance between the pedestrians was becoming respectively less than 1,1 and 3 meters. Furthermore, there is found that the size of steps is directly related to the space available in front of the pedestrian, and that the step frequency is far less sensitive to the local density. Besides, the effect of the flow densities on the synchronization of steps among the consecutive pairs of pedestrians is examined. There is found, as expected, that a certain amount of synchronization occurs at high densities.

More surprisingly, some synchronization at lower densities has been found, and also occurrence of an “anti-synchronization phenomenon”, i.e. a consecutive pair of pedestrian can be synchronized in such a way that when one of them is stepping with the left leg, the other if stepping with the right leg, and vice versa.

2.1.1 Experiment
The aim of the experiment [19] was to study the longitudinal interactions between pedestrians walking in line, without overpassing each other, along a circular path. The study focusses on the extraction of the stepping behaviour in the various dynamic regimes (free flow, jammed, etc.). The experiment was performed inside a ring corridor formed by inner and outer circular walls of radii 2 and 4.5 meters respectively, see Figure 18.
The participants, volunteers who were unaware of the goal of the experiment, were told to walk in a “natural way”, in line along either the inner or outer wall. As a result, two types of pedestrian’s trajectories were obtained: along the inner circular path the observed average radius of the trajectories was 2.4 m, and along the outer circular path, the observed radius was 4.1 m.

The experiment involved up to 28 pedestrians, 20 males and 8 females. The average global pedestrian density was varied from 0.31 to 1.86 P/m, by varying both the number of participants involved, and the length of the circular trajectory. The pedestrians were equipped with 4 markers, one on the left shoulder, two on the right shoulder, and one on the top of the head. Motion was tracked by 12 infra-red cameras (VICON MX-40 motion capture system). The raw data were turned into 3D markers trajectories using the reconstruction software VICON IQ, with a frequency of 120 frames per second [25].

The trajectories of the markers belonging to the same pedestrian were aggregated in order to give one single three-dimensional trajectory for each pedestrian: it’s radial, angular and height coordinates are given as a function of time during the whole duration of experiment. The corresponding velocities were easily calculated.
2.1.2 Step measurements

The body of a pedestrian is swaying when the body weight is shifted from one leg to the other. The swaying of the body results in oscillations that can be seen on the three coordinates of a given pedestrian. The angular coordinate oscillations are entangled with the average forward motion of pedestrians, and the height data can be spoiled with spurious motion of the pedestrian head. There is found that the radial coordinate is the one yielding the best signal for the detection of the stepping cycles, see Figure 19.

Figure 19: The blue line represents the non-filtered radial coordinate of one participant (in mm), walking along the inner circle trajectory of average radius 2.4 m. The dashed red line is the radial coordinate. The tin vertical lines, mark the local extrema – magenta circles are local maxima and green diamonds are local minima. (Source: Properties of pedestrians walking in line: Stepping behavior, Phys. Rev. E 86 [19])

Stepping induces some lateral body movement clearly visible on the radial coordinate. Besides, as pedestrians are walking along circular paths, the radial coordinate is decoupled from the forward motion.

The high precision of the experimental measurements, allowed to extract data on the step length, step duration, and synchronization phenomenon between two successive pedestrians.

The step duration is defined as the time $\Delta t_s$ passed between the consecutive local minimum and maximum of oscillations. The step length is then the distance that a pedestrian has traveled along the circle during this time.

It is defined as $l_s=\Delta \theta_s <R>_s$, where $\Delta \theta_s$ is the angle covered during time $\Delta t_s$, and $<R>_s$ the average of the radius of a circular trajectory along which pedestrian is walking during step duration $\Delta t_s$. Analysis is performed on the set of all steps made by each pedestrian during 52 experiments, each lasting about 1 minute.

Results

In Figure 20 the results for the relationship between the step length (respectively step duration) and the velocity and density are shown. The values of velocity and density were obtained as averages of the velocity and density during a given step. For two successive pedestrians, a density for the pedestrian in the back is obtained as the inverse of the
distance that is available in front of him, i.e. to his predecessor.

![Figure 20: Dependence of the step length (left) and duration (right) as a function of the velocity (top) and density (bottom) (Source: Properties of pedestrians walking in line: Stepping behavior, Phys. Rev. E 86 [19])](image)

The most striking observation is that the step length is overall proportional to the velocity, up to velocities very close to zero. The step length decreases when the velocity decreases. Within a jammed regime, when pedestrians have a vanishing step length- and velocity, they continue to sway and shift their body weight from one leg to the other without moving forward.

A linear fit for velocities between 0.2 to 1.1 m/s gives the following formula for the step length:

\[ l_s = 0.065m + 0.724v \]  

[2-1]

The velocity of a pedestrian is mechanically produced by the footsteps, so that \( v = \frac{l_s}{\Delta t_s} \).

As said before, it can be states that the velocity is proportional to the step length, and as a consequence, the step duration \( \Delta t_s \) should be constant. Figure 20 (b) shows that when the velocity is larger than 0.6 m/s, the step duration is then mostly constant, with a value around 0.8 seconds. This differs from the measurements of step frequencies in the SYNPEX [14] project, were a value around 0.55 seconds was found.

Another observation is that the pedestrians rather adapt their velocity through their step length rather than step frequency. Note that, in Figure 20 (a) and (b), the data obtained both along the inner and outer circle fall on top of each other. This seems to indicate that there is little influence of geometry on the stepping behavior.

With respect to the dependences of the steps characteristics with the density (see Figure 20 c), the average velocity converges towards a finite nonzero value when the density becomes large, and the average step length saturates around 0.1m at large densities. By contrast, the step duration changes much less within different walking regimes. The step duration varies only for around 20% as a function of density. The saturation of step frequency at low densities, or high velocities, indicates that increasing the step frequency beyond a certain value is not comfortable for pedestrians.
2.1.3 Step synchronization

In earlier research of Seyfried [20] it was noticed that pedestrians tend to synchronize their footsteps, when they do not have much space to walk. They adapt their step frequency by squeezing the front leg into the hole left by the front leg of the predecessor. This phenomenon is called walking in lockstep. Also in this experiments, the aim is to investigate the occurrence of this tendency. The definition of full synchronization is when two successive pedestrians walk with the same step frequency and in phase.

The synchronization measurements include two stages. In the first stage, the data is analyzed to select pairs of successive pedestrians for which the stepping frequencies were not too different. The second step, is to measure the phase shift between close minima (maxima) of the stepping cycles of these two successive pedestrians.

For each detected minimum (maximum) on the radial coordinate of the predecessor (occurring at time \(t_0\)), the analysis consists of the following steps:

- The individual and local frequency of the ‘leader’ over 3 periods, defined from the two maxima (minima) just before and after the given minimum (maximum), will be measured. \(T\) is the average period over these three cycles.

- Determination whether there is a minimum (maximum) in the radial coordinate of the follower within the time range \([t_0 - T/4, t_0 + 3T/4]\). This minimum occurs at time \(t_1\).

- Evaluation of the individual and local frequency \(1/T'\) for the follower, exactly as it was done for the leader

- If \(T'\) and \(T\) differ less than 24 %, the current steps of this pair of pedestrians will be selected. The rejection ratio due to too large difference in frequencies is up to 20 %. This underlines that at least 20 % of the pedestrians are not synchronized with their leader – as synchronization requires first to have the frequency.

- Then the phase will be measured:
  \[
  \Phi = \frac{2\pi(t_1 - t_0)}{T}
  \]

  In a similar way, the phase \(\Psi\) separating a minimum (maximum) in the radial coordinate of the leader, from a maximum (minimum) in the radial coordinate of the follower, will be measured. In this way, the expectation is to obtain more precise measurements for the anti-synchronization phenomena.

- Finally, the density will be measured. It is defined as the inverse of the distance between the centers of mass of the two pedestrians. As this distance oscillates with the steps, it was more relevant to evaluate it on the filtered data. However, the results presented in Figure 21 are similar when non-filtered data are used.

Figure 21 shows the normalized histograms of \(\Phi\) obtained for various local density ranges. A peak is observed around phase \(\Phi=0\) at large densities (beyond 1,25 P/m). This indicates the existence of the synchronization phenomenon. Also when the density is lower, a peak around zero is still observed, though it is smaller than for higher densities.
Surprisingly, another peak appears around $\Phi = \pi$. This second peak corresponds to anti-synchronization, i.e. walking in phase with the opposite legs. When anti-synchronization occurs at high densities, the expectation could be that pedestrians would be located at different distances from the wall, so that the left leg of one pedestrian is more or less aligned with the right leg of the other. However, any visible effect of this type in the data is not observed.

Anti-synchronization mostly disappears when the density is large, while it survives at densities as large as $\rho < 0.5$ P/m, for which there are clearly no steric constraints between the pedestrians.
In figure 22 there is checked if anti-synchronization is also visible when the phase shift $\Psi$ between the local extrema of the opposite kind (minimum and maximum) in the radial coordinate is measured. This corresponds to the steps made by the left leg of one and the right leg of the other of the two consecutive pedestrians. In this case, anti-synchronization should appear as a peak around zero. Indeed there is a second peak located around $\Psi = 0$ for the lowest densities as seen in Figure 22. On the other hand, synchronization can be seen again, but this time as a peak around $\Psi = \pi$.

These results show that both synchronization and anti-synchronization effects occur at low enough densities where the pedestrians are not bound by the steric constraints. Pedestrians are probably sensitive to the stepping oscillations that they perceive visually when watching their predecessor, they could synchronize naturally. It is still an open question to determine precisely to which visual signal pedestrians are most sensitive.
2.2 Research on the human interaction behaviour and induced forces on moving structures concerning synchronization, within SYNPEX project

Because human synchronization behaviour is not yet fully understood and analyzed, the objectives of the SYNPEX project [14] was to perform experiments to investigate the synchronization effects in footbridge engineering. Based on extensive measurements on a test rig, inter alia, a practical load model was derived that can be used for the determination of vibrations for sinusoidal mode shapes. The experiments that were performed and the results will be discussed in this paragraph.

The experiments that will be discussed can be divided into two parts: walking behaviour and associated force characteristics of single pedestrians on a fixed- and vertically vibrating platform, and walking behaviour of streams.

2.2.1 Walking behaviour on a fixed- and vertically vibrating platform

The 12 m long test platform represents a section of a footbridge and is able to vibrate vertically. It is composed of a grid of Sigma profiles $\Sigma 170$ and is covered with commercially available steel grating, which provides an additional bracing and serves as a walkway (12 m x 3 m) for test persons. Podiums with stairs are located at both front sides of the platform to provide an easy access on the platform, as seen in Figure 23.

![Figure 23: Pedestrian crosses the platform (Source: SYNPEX [14])](image)

The steel grating is covered with a non-slip and nontransparent coating, so that test persons feel safe when they cross the platform. The cross beams of the platform are supported on vertical springs, which defines the vertical natural frequency. It is important that the platform vibrates with frequencies that are in the range of walking frequencies. The springs also allow for sufficient swing travel to excite the platform in the relevant amplitude range. An electrical motor with crank mechanism drives the platform to vibrations.
Figure 24: Supports with vertical springs (Source: SYNPEX [14])

Four load cells of size 400 mm × 600 mm are embedded in the walkway to measure vertical the human ground reaction forces. The cells are covered with the same nontransparent material as the rest of the walkway to avoid that persons target on them. The load cells are shown in Figure 25.

Figure 25: Arrangement of the load cells (Source: SYNPEX [14])

They are spaced in such a way that they are very likely to be hit by footsteps. The longitudinal distance from the centers of the cells is between 750 to 760 mm, which is a usual step length. The lateral distance is 200 mm.

The amplitude of the platform is measured with an accelerometer PCB 3701. The sampling rate of the force plates and the accelerometer is 600 Hz. The persons wear in their right shoe a pressure sensor (Type FSR). The duration of a double step is measured, which is used for the determination of step frequencies and synchronization behaviour on the vibrating platform. The sampling rate of the pressure sensors is 10 Hz. These signals are transmitted via Bluetooth to a computer. In addition video films for documentation purposes are recorded during the experiments.
Results of walking behaviour on a fixed ground
The step frequencies of the persons on the fixed platform are evaluated assuming that the duration of a step with the right foot is the same as for the left foot. The mean values of the three crossings for each walking intention are determined for each test persons. The step frequencies are distributed normally. The normal distributions of step frequency for slow, normal and fast walking, typical ground reaction forces and the characteristic properties of vertical ground reaction forces were discussed earlier in sub paragraph 1.2.1.

As seen in Figure 26, a slight correlation between step frequency and body height is visible for normal and fast walking.

![Figure 26: Step frequencies versus body height (Source: SYNPEX [14])](image)

Characteristic properties of vertical ground reaction forces, such as duration of the foot contact with the ground, the normalized force maxima and the minimum as well as the gradient at the beginning and at the end show a strong correlation with step frequency, see Figure 27. Also the point of time of the maxima and minimum related to the duration of the foot contact are shown.

Linear regression functions based on the least square method are determined to describe the mean characteristic properties in dependence of step frequency.
Figure 27: Characteristic properties of the vertical ground reaction forces in dependence of step frequency, in following order: $F_{\text{max},1}/G$; $F_{\text{max},2}/G$; $F_{\text{min}}/G$; $t_{\text{end}}$; $t_{\text{end}}/t_{\text{end}}$; $t_{\text{max},2}/t_{\text{end}}$; $m_A$; $m_B$ (Source: SYNPEX [14])
Results of walking behaviour on a vertically vibrating platform
For the determination of the existence of vertical synchronization it is necessary to look carefully into the different motion phases of walking. In Figure 24 the different phases of walking, the corresponding movement of the center of gravity and the vertical ground reaction forces are shown.

It can be seen that the movement of the center of gravity is assumed to be identical to the vibration of the platform. The force of a single footfall introduces partly a positive and partly a negative impact loading to the platform. Also, it is characteristic that, if the movement of the center of gravity is synchronized with the vibration, the part of the positive impact loading of the ground reaction force is slightly shifted to the rear part of the ground reaction force pattern. Compare the red circles in Figures 28 and 29.
Figure 29: Movement and vertical ground reaction forces if forces are placed in resonance with the bridge vibration (Source: SYNPEX [14])

Comparing Figure 28 with Figure 29, where the energy input is shown when the foot are placed with the natural frequency and excite the bridge in resonance, one sees that the difference between synchronization of the body’s center of gravity compared to resonant excitation is very small.

It turns out that the signal of the pressure sensors in combination with the bridge vibration can give an indication whether a synchronization takes place.

This is shown in Figure 30, where the first-, second-, third- and fourth graph respectively represent the vertical ground reaction force of the 1st and 2nd foot combined, the continuous bridge vibration, the vibration velocity of the 1st foot measured with the pressure sensor, and the vibration velocity of the 2nd foot measured with the pressure sensor.

As the pressure sensor is relatively small compared to the foot sole, it cannot measure the complete contact time, see the circle in the third graph in Figure 30.
It is shown in Figure’s 30 to 32, that for the identification of synchronization it is necessary to compare the complete contact time in relation to the velocity. Therefore, besides the signals of the pressure sensors also the measured ground reaction forces are compared regarding the positive impulse loading.

In figure 31, the experiment is shown where the bridge is excited with a frequency $f_p=1,55$ Hz and the pedestrian has a step frequency $f_s=1,55$ Hz. The same experiment is repeated but now with a step frequency of $f_s=1,40$ Hz, see Figure 32. Lines are drawn for both...
experiments, between the minimum of the vibration velocity of the bridge (which is the maximum displacement or the maximum acceleration) and the vertical human ground reaction forces, to see whether synchronization takes place.

For the experiment where the pedestrian has step frequency 1.55 Hz (Figure 31), there can be seen that the maximum displacement of the bridge match the maximum impulse of the ground reaction forces. This can be seen for both 1st and 2nd foot. If the maximum displacement and the maximum impulse of the foot match, then **synchronization takes place**. As for the pedestrian with step frequency 1.40 Hz, it is clear that the maximum displacement of the bridge does not match the maximum ground reaction force for both 1st and 2nd foot.

![Image](image.png)

**Figure 32:** Walking with a smaller step frequency compared to vibration frequency, $f_s=1.40$ Hz, $f_v=1.55$ Hz (Source: SYNPEX [14])

As there are only four load cells and not all load cells are hit during one crossing, one cannot identify whether persons show a stable synchronization behaviour over a longer time period or just a short one that can also take place by chance. The evaluation of the measured pressure sensor signals and the force cells signals combined with the evaluation of step frequencies showed no stable synchronization behaviour of the test persons at considerably large amplitudes (ca. 10 mm), as it is shown in Figure 33. Synchronization might occur with higher amplitudes.
These amplitudes are outside acceptable limits for serviceability of footbridges. If these amplitudes occur, pedestrians might be disturbed and stop walking compared to lateral synchronization, which occurs already at small vibration amplitudes and pedestrians keep on walking synchronized with the vibration and hence turn a random excitation into a resonant one.

While the ground reaction forces lead to the assumption of synchronization, the signals of the pressure sensors indicates a temporarily walking with the right phase but no stable synchronization. As the walking behaviour already shows slight variations in step frequency when walking on a fixed ground, it is hardly possible to distinguish stable or unstable synchronization behaviour on the basis of the data.

2.2.2 Walking behaviour of streams
For the simulation and investigation of bridge responses due to streams and groups, it is important to determine the correlation among pedestrians in dependence of density and walking speed. This has been investigated in the SYNPEX project [14].

Test set-up
A walking path of 30 m length and a width of 1.5 m is marked on the cinder track on a sports field. 18 persons participate in this test and allow to have a sufficiently large pedestrian group to draw conclusions about the walking properties of unidirectional pedestrian streams. The density varies between 1.2 P/m² to 3 P/m². The step frequencies are measured with pressure sensors (see sub paragraph 2.2.1)
Five persons are equipped with sensors and are randomly distributed in the group. Walking along the path is performed several times. After each passage the pressure sensors are exchanged between persons, so that step frequencies of different persons are measured.

For the determination of the walking speed the experiments are recorded on a video film. Two different walking intentions, ‘strolling’ and ‘normal walking’, are given during the tests and the density is varied.

The experiments are not only performed with the large group of 18 persons but also with small groups of 4 resp. 5 persons.

**Results**

Figure 34 shows the measured values of step frequency and walking speed for small and large groups, which move as unidirectional traffic. A strong correlation between mean step frequency and walking speed is identified and is independent of the group size.

The correlation between standard deviation of the step frequencies and the walking speed is weaker.

The following conclusions are drawn:

- the standard deviation of the step frequencies is approximate $\sigma_f = 0.08 - 0.1$ Hz for low densities and high walking speeds
- the standard deviation is approximate $\sigma_f = 0.05$ for large densities and low walking speeds

![Figure 34: Correlation between $f_s$ and $v_m$ for small and large groups, without bridge interaction (Source: SYNPEX [14])](image)

These values are compared with the measured values of Oeding’s, see Table 2. The correlation formula between $f_s$ and $v_m$, approaches the measured values of Oeding’s well.

**Table 2: Summary of step frequencies according to density classes (Source: Verkehrsbelastung und dimensionierung von Gehwegen und anderen Anlagen des Fussgängerverkehrs, Oeding [26])**
### Vibration behaviour of slender footbridges

**Graduation research**

<table>
<thead>
<tr>
<th>Pers/m²</th>
<th>$v_m$ [m/s]</th>
<th>$f_{sm}$ [Hz]</th>
<th>$\sigma_f$ [Hz]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2</td>
<td>1.45</td>
<td>1.93</td>
<td>0.089</td>
</tr>
<tr>
<td>0.5</td>
<td>1.30</td>
<td>1.81</td>
<td>0.076</td>
</tr>
<tr>
<td>1.0</td>
<td>1.04</td>
<td>1.61</td>
<td>0.054</td>
</tr>
<tr>
<td>1.5</td>
<td>0.79</td>
<td>1.41</td>
<td>0.033</td>
</tr>
</tbody>
</table>
3. Standards and guidelines

3.1 Eurocode
The Eurocode [17] defines comfort criteria for slender footbridges. The comfort criteria is defined in terms of maximum acceptable acceleration of the bridge deck. The amplitude of the vibrations are directly related to the acceleration. The Eurocode prescribes that an assessment of the vibration comfort criteria should be performed, if the natural frequency of the bridge lies within the critical range [1,25 Hz – 2,3 Hz] that is mentioned earlier.

However, the Eurocode does not propose rules to perform an assessment of the vibration comfort criteria for footbridges. It refers to the JRC-document EUR 23984 EN ‘Design of Lightweight Foot Bridges for Human Induced Vibrations’ [1], to calculate the occurring acceleration of the bridge deck and to check the comfort vibration criteria of slender footbridges.

3.2 Design guidelines
Three design guidelines are used often in Europe to predict the vibration behaviour of slender footbridges and to check their comfort level: HIVOSS, French Technical guide of Setra and the EUR 23984 EN [1].

The French guideline ‘Guide to assessing pedestrian induced vibratory behavior of footbridges’ was prepared in 2006 within the framework of the Setra (service d’Etudes techniques des routes et autoroutes) working group on ‘Dynamic behaviour of footbridges’, led by Charles and Hoorpah [18]. In 2008 the HIVOSS, Human induced Vibrations of Steel Structures, was introduced. This design guideline was based on steel footbridges, but it could be generalized for footbridges with other materials. The methodology of these two guidelines have much in common, especially in the determination of synchronized pedestrian loading. There are some differences regarding damping ratios, but these are small differences.

EUR 23984 EN [1] was introduced in 2009 and is based on the HIVOSS and the research project SYNPEX [14]. It is the most advanced design guideline and therefore, the design steps of EUR 23984 EN will be regarded in the next paragraph.

3.3 EUR 23984 EN
The report ‘Design of Lightweight Foot Bridges for Human Induced Vibrations’ [1] is the most advanced design guideline for slender footbridges in Europe. It is prepared under the JRC (Joint Research Centre) of the European Commission and based on the research project SYNPEX (Advanced Load Models for Synchronous Pedestrian Excitation and Optimized Design Guidelines for Steel Bridges), and is funded by the Research Fund for Coal and Steel.

The design steps of the EUR 23984 EN [1] are shown in the figure below.
Figure 35: Design steps EUR 23984 EN (Source: Report 'Design of Lightweight Foot Bridges for Human Induced Vibrations')

Illustration design steps
Step 1: The first step to determine natural frequency(s) of the footbridge.

Step 2: Step 2 is to check whether an assessment of the vibration comfort criteria should be performed. The critical ranges for natural frequencies $f_i$ of footbridges with pedestrian excitation are:

- For vertical vibrations: $1.25 \text{ Hz} < f_i < 2.3 \text{ Hz}$
- For lateral vibrations: $0.5 \text{ Hz} < f_i < 1.2 \text{ Hz}$

Footbridges with frequencies for vertical vibrations in the range $2.5 \text{ Hz} < f_i < 4.6 \text{ Hz}$
might be excited to resonance by the 2nd harmonic of pedestrian loads. In that case, the critical frequency range for vertical vibrations expands to:

\[ 1.25 \text{ Hz} < f_i < 4.6 \text{ Hz} \]

Mostly for simply supported footbridges, higher harmonics are not considered. The second natural frequency mostly is outside the critical range \([1.25 – 4.6 \text{ Hz}]\). This means that the excessive vibrations are expected at the center of the bridge.

Lateral vibrations are not affected by the 2nd harmonic of pedestrian loads.

**Step 3**: Based on an agreement between the client and the designer, an expected pedestrian density and a comfort class can be chosen. This is called a design situation. The pedestrian densities, which are divided into different traffic classes, are used to determine the pedestrian loading. The comfort classes are acceleration limits that must be met.

3a) **Assessment of traffic classes**

In step 3a several significant design situations will specified. Each design situation is defined by an expected traffic class. For example, there are design situations which might occur once in the lifetime of a footbridge, like the inauguration of the bridge, and other design situations that will occur daily, such as commuter traffic.

Table 3 gives an overview of some typical traffic situations which may occur on footbridges. Based on an agreement between the designer and owner, an expected traffic class can be chosen. This is of importance for determining the dynamic loads.
Table 3: Pedestrian traffic classes and densities (Source: Report 'Design of Lightweight Foot Bridges for Human Induced Vibrations')

<table>
<thead>
<tr>
<th>Traffic Class</th>
<th>Density ( d ) ((P = \text{pedestrian}))</th>
<th>Description</th>
<th>Characteristics</th>
</tr>
</thead>
<tbody>
<tr>
<td>TC 1(^*)\</td>
<td>group of 15 P; ( d = 15 P / (3L) )</td>
<td>Very weak traffic</td>
<td>((B=\text{width of deck}; L=\text{length of deck}))</td>
</tr>
<tr>
<td>TC 2</td>
<td>( d = 0.2 \text{ P/m}^2 )</td>
<td>Weak traffic</td>
<td>Comfortable and free walking (\text{Overtaking is possible}) (\text{Single pedestrians can freely choose pace})</td>
</tr>
<tr>
<td>TC 3</td>
<td>( d = 0.5 \text{ P/m}^2 )</td>
<td>Dense traffic</td>
<td>Still unrestricted walking (\text{Overtaking can intermittently be inhibited})</td>
</tr>
<tr>
<td>TC 4</td>
<td>( d = 1.0 \text{ P/m}^2 )</td>
<td>Very dense traffic</td>
<td>Freedom of movement is restricted (\text{Obstructed walking}) (\text{Overtaking is no longer possible})</td>
</tr>
<tr>
<td>TC 5</td>
<td>( d = 1.5 \text{ P/m}^2 )</td>
<td>Exceptionally dense traffic</td>
<td>Unpleasant walking (\text{Crowding begins}) (\text{One can no longer freely choose pace})</td>
</tr>
</tbody>
</table>

3b) Assessment of comfort classes.
In step 3b the acceleration limit \( a_{\text{lim}} \) has to be determined, and this is the limit that must be met. Four comfort classes are recommended by this guideline, see Table 4.
Table 4: Defined comfort classes with common acceleration ranges (Source: Report ‘Design of Lightweight Foot Bridges for Human Induced Vibrations’)

<table>
<thead>
<tr>
<th>Comfort class</th>
<th>Degree of comfort</th>
<th>Vertical $a_{\text{limit}}$</th>
<th>Lateral $a_{\text{limit}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>CL 1</td>
<td>Maximum</td>
<td>&lt; 0,50 m/s²</td>
<td>&lt; 0,10 m/s²</td>
</tr>
<tr>
<td>CL 2</td>
<td>Medium</td>
<td>0,50 - 1,00 m/s²</td>
<td>0,10 - 0,30 m/s²</td>
</tr>
<tr>
<td>CL 3</td>
<td>Minimum</td>
<td>1,00 - 2,50 m/s²</td>
<td>0,30 - 0,80 m/s²</td>
</tr>
<tr>
<td>CL 4</td>
<td>Unacceptable discomfort</td>
<td>&gt; 2,50 m/s²</td>
<td>&gt; 0,80 m/s²</td>
</tr>
</tbody>
</table>

Note that this is not the criteria check for lateral lock-in! This is only the comfort vibration criteria.

**Step 4:** In step 4 the amount of present damping will be determined. The damping depends both on the intrinsic damping of construction materials, which is of distributed nature, and on the local effect of bearings or other control devices.

It is necessary to consider early in the design stadium the dynamic behaviour of the structure, even though the final damping system is not determined and an assumption must be made. The vibration behaviour here only gives an indication of the actual behaviour. If it is clear in an early stage that the response of the structure is located in the critical zone, a quick change will be made in the design. It is also necessary afterwards, to check if the predicted accelerations, with the assumed damping parameters, are similar to the constructed bridge. Based on the actual dynamic behaviour due to on site experiments, it can be stated if the chosen damping system is superfluous or not.

The minimum and average damping ratios according to Table 5 are recommended.

Table 5: Damping ratios (Source: Report ‘Design of Lightweight Foot Bridges for Human Induced Vibrations’)

<table>
<thead>
<tr>
<th>Construction type</th>
<th>Minimum $\xi$</th>
<th>Average $\xi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reinforced concrete</td>
<td>0,8%</td>
<td>1,3%</td>
</tr>
<tr>
<td>Prestressed concrete</td>
<td>0,5%</td>
<td>1,0%</td>
</tr>
<tr>
<td>Composite steel-concrete</td>
<td>0,3%</td>
<td>0,6%</td>
</tr>
<tr>
<td>Steel</td>
<td>0,2%</td>
<td>0,4%</td>
</tr>
<tr>
<td>Timber</td>
<td>1,0%</td>
<td>1,5%</td>
</tr>
<tr>
<td>Stress-ribbon</td>
<td>0,7%</td>
<td>1,0%</td>
</tr>
</tbody>
</table>

**Step 5:** In step 5 the predicted accelerations will be calculated. The accelerations can be calculated with the SDOF Method, Finite Element Method and the Response Spectra Method. The SDOF Method and the Response Spectra Method are analytical calculations while the Finite Element Method is a numerical calculation method.
As seen in Figure 31, for both SDOF Method and Finite Element Method, a harmonic load-and response model can be used to calculate the response. This load- and response model takes also into account the synchronization effects via the equivalent number of pedestrians \( n' \) and the reduction coefficient \( \Psi \). The approach of this load-and response model will be explained in chapter 4 'Literature study on dynamic response of footbridges'.

**Step 6:** In step 6 the criteria for lateral lock-in has to be checked.

**Step 7:** In the last step, the comfort level of the bridge has to be checked. The response calculated for the specified design situations and the corresponding load models has to be compared with specified comfort limits given in Table 6. If needed, the dynamic behaviour of the footbridge can be improved:
- Modification of the mass
- Modification of frequency
- Modification of structural damping

### 3.4 SDOF response model for pedestrian streams according to EUR 23984 EN

The load and response model that is provided in the design guideline, represents a pedestrian stream. A pedestrian stream consists of \( n \) “random” pedestrians that walk independently from each other. When the density of pedestrians on the bridge increases, synchronization effects may occur.

Based on a so called traffic class, the number of pedestrian’s \( n \) on the bridge can be determined. The stream consisting of \( n \) “random” pedestrians will be converted into an idealized stream consisting of \( n' \) equivalent number of pedestrians. The synchronization effect is taken into account in EUR 23984 EN, by determining the idealized stream consisting of \( n' \) perfectly synchronized pedestrians.

Now the equivalent stream can be modelled as a deterministic load. The dynamic load representing the stream will be converted to a distributed harmonic load. In fact, this method is based on the behaviour of one single pedestrian, multiplied by the equivalent number of pedestrians and the reduction coefficient taking into account the probability that the footfall
The following load model represents the equivalent pedestrian stream, a harmonic load:
\[ p(t) = P \cdot \cos(2\pi f_s t) \cdot n' \cdot \Psi \tag{3-1} \]

Where \( P \cdot \cos(2\pi f_s t) \) is the harmonic load due to a single pedestrian
\( P \) is the component of the force due to a single pedestrian walking step frequency \( f_s \)
\( f_s \) is the step frequency, which is assumed equal to the footbridge natural frequency under consideration
\( n' \) is the equivalent number of pedestrians on the loaded surface \( S \)
\( \Psi \) is the reduction coefficient taking into account the probability that the footfall frequency approaches the critical range of natural frequencies under consideration

In the next two sub paragraphs, the approach of the equivalent number of pedestrian's \( n' \) and the reduction coefficient \( \Psi \) will be explained.

### 3.4.1 Equivalent number of pedestrians \( n' \)

As has been said before, this approach is based on converting a stream consisting of \( n \) "random" pedestrians in to an idealized stream consisting of \( n' \) equivalent number of pedestrians. The two streams are supposed to cause the same effect on a structure, but the idealized, equivalent one can be modelled as a deterministic load.

If a harmonic load \( (F_0 \sin(2\pi ft)) \) is applied to a damped SDOF system, the response can then be given in the following form [14]:
\[ x(t) = \frac{F_0}{4\pi^2 m \sqrt{(f_0^2 - f_s^2)^2 + 4\xi f_0^2 f_s^2}}} \sin(2\pi f_s t - \phi) \tag{3-2} \]

where: \( F_0 \) amplitude of the load
\( m \) system mass
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\[ f_0 \] system natural frequency
\[ f_s \] forcing frequency
\[ \zeta \] damping ratio
\[ \phi \] arctan \( \frac{2 \zeta f_0 f}{f_0^2 - f^2} \)

The response in this form will be used throughout the procedure for the assessment of an equivalent number \( n' \) of pedestrians when using modal analysis.

**Modal analysis**
Consider a beam that is modelled as a system with \( N \) degrees of freedom (Figure 38) and let a loading be represented as point loads on each of the loaded nodes. When a solution to describe the dynamic behaviour of a system is sought by modal analysis, displacements of the nodes are found in the form of superposition of displacements belonging to different representative modes:

\[ y(t) = \sum_{i=1}^{r} x_i(t) \Phi_i, \quad r \leq N \quad [3-3] \]

where:
- \( y(t) \) is the vector of the movements of concentrated masses
- \( \Phi_i \) are the vectors of modal displacement for each mode taken into consideration
- \( x_i(t) \) are the responses of the system for each mode taken into consideration

**Figure 38: \( n \leq N \) harmonic loads (Source: SYNPEX [14])**

If all the loads share the same forcing frequency, \( f \neq f_0i \), the response of the system for one mode only, e.g. mode I, with modal displacements \( \Phi_{ij} \), is:

\[ x_i(t) = \frac{\phi_i^T F_0 / 4 \pi^2 m_i}{\sqrt{(f_{0i}^2 - f_s^2)^2 + 4 \zeta_i^2 f_{0i}^2 f_s^2}} \sin(2 \pi f_0 t - \phi_i) \quad [3-4] \]

where:
- \( \phi_i^T \) vector for modal displacements
- \( F_0 \) vector of load amplitudes \( (F_0^T = (F_1, F_2, \ldots F_j, \ldots F_N)) \)
- \( m_i \) \( \sum_{j=1}^{N} m_i \phi_{ij} \) modal mass
- \( f_{0i} \) frequency for mode \( i \)
- \( f_s \) forcing frequency
- \( \zeta_i \) damping ratio for mode \( i \)
- \( \phi_i \) phase shift for mode \( i \)

**Response to a single harmonic load**
A point load of amplitude \( Q \) and frequency \( f \neq f_0 \) is placed at a node \( j \), which modal displacement is \( \Phi_{ij} \), see Figure 39. Then:

\[ \phi_i^T F_0 = Q \phi_{ij} \quad [3-5] \]
Vibration behaviour of slender footbridges
Graduation research

The maximum displacement for mode I is at the position of the highest modal displacement and can be written as follows:

\[ y_{l,\max} = \max x_i(t) \phi_{l,\max} = \frac{Q \Phi_{i,\max}}{\sqrt{(f_0^2 - f_i^2)^2 + 4 \zeta_i^2 f_{i0}^2 f_i^2}} \]  

[3-6]

**Response to a distributed harmonic load**

First, the deterministic approach will be regarded. In the most general case, the distributed load is represented as \( N \) point loads \( Q_j \sin(2\pi f_j t - \psi_j) \), regularly distributed on half-waves of the mode \( \Phi \), as shown in Figure 40, where:

- the amplitude of the loads are \( Q_j j=1 \) to \( N \)
- each point load had a frequency \( f_j j=1 \) to \( N \)
- each point load has a phase shift \( \psi_j j=1 \) to \( N \)

![Figure 40: N harmonic loads (Source: SYNPEX [14])](image)

If the loaded length is \( l \), the position of each point load is found within the interval \( \left[ \frac{l-1}{N}, \frac{l}{N} \right] \). In order to take into account the mode rank and the distributed character of the loads:

\[ \Phi_I^{T} F_0 = \sum_{j=1}^{N} \frac{\alpha N_{ij}}{l} Q_j \]  

[3-7]

where \( \alpha N_{ij} = \int_{\frac{l}{N}}^{\frac{l}{N}} \Phi_I(x) dx \)

The response is found as a superposition of responses to particular loads, as:

\[ y_{l,\max} = \sum_{j=1}^{N} \left( \frac{\alpha N_{ij} Q_j \sin(2\pi f_j t - \Phi_{ij})}{\sqrt{(f_0^2 - f_j^2)^2 + 4 \zeta_i^2 f_{i0}^2 f_j^2}} \Phi_{i,\max} \right) \]  

Where the phase shift for mode I and point load at node \( j \) is:

\[ \Phi_{ij} = \arctan \left[ \frac{2 \zeta_i f_{i0} f_i \cos \psi_j + (f_{i0}^2 - f_i^2) \sin \psi_j}{(f_{i0}^2 - f_i^2) \cos \psi_j - 2 \zeta_i f_{i0} f_i \sin \psi_j} \right] \]

If the assumptions that all the loads share the same amplitude, but are not necessary in phase \( (Q_j = Q \sin \psi_j) \) is adopted, the response becomes:
$y_{l,\text{max}} = Q \sum_{j=1}^{N} \frac{(\alpha N_{ij} \Phi_{l,\text{max}} / 4 \pi^2 m_i l) \sin(2\pi f_j t - \varphi_{ij})}{\sqrt{(f_{ij}^2 - f_j^2)^2 + 4 \zeta_i^2 f_{ij}^2 f_j^2}}$ \hfill [3-8]

Second, the probabilistic approach will be regarded, to analyze the effect of a pedestrian stream consisting of $n$ random pedestrians. The differences comparing to the case given above are [14]:

- Each point load has a random frequency $f_j$ which follows a normal distribution $N[f_{s,m}, \sigma]$.
- Each point load has a random phase shift $\varphi_j$ which follows a random uniform distribution $U[0, 2\pi]$.
- The response [eq. 3-8] is here a random variable too, because of $f_i$ and $\varphi_j$, and hence its mean value and its standard deviation could be assessed.

If the following is adopted:

- $\lambda_i = f_{ai} / f_{s,m}$ ratio between the frequency for mode $i$ and the mean of forcing frequency (average forcing frequency of single pedestrian in a considered stream).
- $\mu = \sigma / f_{s,m}$ coefficient of variation of the forcing frequency.
- $f_j = f_{s,m} (1 + \mu u_j)$ random frequency of a point load placed at a node $j$.

where $u_j$ is a standardized normal random variable. If instead of displacements, accelerations are considered, each component of the sum in equation [eq. 3-8] should be multiplied by:

$(2 \pi f_j)^2 = (2 \pi)^2 f_{s,m}^2 (1 + \mu u_j)^2$

The absolute maximum acceleration can then be calculated by the following:

$Z_i = \max\{\ddot{y}_{l,\text{max}}(t)\} = (2 \pi)^2 f_{s,m}^2 \frac{Q}{f_{s,m}^2} \times \max \left[\sum_{j=1}^{N} \frac{(\alpha N_{ij} \Phi_{l,\text{max}} / 4 \pi^2 m_i l) (1 + \mu u_j)^4}{(l_i^2 - 1 + 2\mu u_j - \mu^2 u_j^2)^2 + 4 \zeta_i^2 l_i^2 (1 + 2\mu u_j + \mu^2 u_j^2)} \sin(2\pi f_j t - \varphi_{ij})\right]$ \hfill [3-9]

With the phase shift for mode $l$ and point load at node $j$:

$\varphi_{ij} = \arctan\left[\frac{2 \zeta_i l_i (1 + \mu u_j) \cos \psi_j + (\lambda_i^2 - (1 + \mu u_j)^2) \sin \psi_j}{(\lambda_i^2 - (1 + \mu u_j)^2) \cos \psi_j - 2 \zeta_i \lambda_i (1 + \mu u_j) \sin \psi_j}\right] + \psi_j$

Finally, this results in:

$Z_i = (2 \pi)^2 Q z_i$

Remark: For $\lambda_i = 1$, $\mu = 0$ and $\varphi_j = 0$ (deterministic resonant loading case):

$Z_i = (2 \pi)^2 f_{s,m}^2 \frac{Q}{f_{s,m}^2} \times \max \left[\sum_{j=1}^{N} \frac{\alpha N_{ij} \Phi_{l,\text{max}} / 4 \pi^2 m_i l}{2 \zeta_i} \sin(2\pi f_j t - \frac{\pi}{2})\right] = (2 \pi)^2 Q z_i$. 
The last step, is the determination of the equivalent number of pedestrians. The equivalent number of pedestrians in an equivalent, idealized stream – in other words the number of pedestrians, all with footsteps in the frequency for mode i and with no phase shift causing the same behaviour of the structure as the one caused by the random stream of pedestrians – can be obtained by equalizing the absolute maximum accelerations from the following two cases, see Figure 41.

1. Random stream with n pedestrians: \[ Z_i = (2 \pi)^2 Q z_i \] \[ \text{[3-10]} \]
2. Equivalent stream with n' pedestrians: \[ Z_{i,eq} = (2 \pi)^2 Q z_i' \frac{n'}{n} \] \[ \text{[3-11]} \]

Figure 41: Equivalence of streams (Source: SYNPEX [14])

Thus: \[ Z_i = Z_{i,eq} \rightarrow z_i = z_i' \frac{n'}{n} \rightarrow n' = \frac{z_i}{z_i'} n \]

If the approach proposed in SETRA [18] is adopted:

\[ n' = k_{eq} \sqrt{n \cdot \zeta_i} \] \[ \text{[3-12]} \]

then the coefficient \( k_{eq} \) can be obtained as follows:

\[ k_{eq} = \frac{n'}{\sqrt{n} \zeta_i} = \frac{z_i}{z_i'} \frac{n}{\sqrt{\zeta_i}} \] \[ \text{[3-13]} \]

The random feature in equation 3-13 is \( z_i \). The mean value \( E(z_i) \) and the standard deviation \( \sigma(z_i) \) can all be assessed by simulations for different values of intervening parameters:

\[ z_i = \max \left[ \sqrt{\frac{(\alpha N_{ij} \phi_{i,\text{max}} / 4 \pi^2 m_j l)^2 (1+\mu_{ij})^4}{(\lambda_i^2 - 1 - 2\mu u_j - \mu^2 u_j^2)^2 + 4 \zeta_i^2 \lambda_i^2 (1+2\mu u_j + \mu^2 u_j^2)}} \cdot \sin(2\pi f_i t - \phi_{ij}) \right] \] \[ \text{[3-14]} \]

**Response to a distributed harmonic load**

Analyses have been done on the basis of Monte-Carlo simulations carried out on a half-sine mode shape \( \Phi_i \) (Figure 41) in order to represent the random nature of pedestrian loading. In those analyses have been varied the parameters given in Table 6.
Table 6: Elements for the sensitivity analyses (Source: SYNPEX [14])

<table>
<thead>
<tr>
<th>Damping ratio, $\zeta$</th>
<th>0.2%</th>
<th>0.4%</th>
<th>0.6%</th>
<th>0.8%</th>
<th>1%</th>
<th>1.2%</th>
<th>1.4%</th>
<th>1.6%</th>
<th>1.8%</th>
<th>2%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ratio of frequencies, $\omega_n$</td>
<td>min: $\equiv 0.0$</td>
<td>max: 2.0</td>
<td>step: 0.1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Coefficient of variation, $\mu$</td>
<td>2.5%</td>
<td>5.0%</td>
<td>7.5%</td>
<td>10%</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Number of pedestrians, $N$</td>
<td>50</td>
<td>100</td>
<td>200</td>
<td>300</td>
<td>400</td>
<td>600</td>
<td>800</td>
<td>1000</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Histograms of maxima of $z_i$ [eq. 3-14] are firstly obtained on the basis of 2500 simulations for each set of parameters from Table 6. Every simulation consisting of taking $N$ random values of both the standardized normal variable $u_j$ and the phase shift $\psi_j$. A maximum of $z_i$ is taken on a 2-period range (simulations carried out have shown that an 8-period range gives the same results). Coefficient $k_{eq}$ is then calculated [eq. 3-13] on the basis of values of $z_i$ obtained as explained above. Figure 42 gives an example of histogram of $k_{eq}$. Finally, 95th percentile of $k_{eq}$ is obtained.

![Histogram of $k_{eq}$](image)

Figure 42: An example of the obtained histograms (Source: SYNPEX [14])

If a tolerance $\varepsilon$ with the associated confidence level $1-\delta$ is to be reached, a sample size greater or equal to $N_C$ is required:

$$N_C = \frac{P_f (1 - P_f)}{\delta \varepsilon^2}$$

The data that was used in these analyses were as follows:

- $N_C=2500$ simulations
- $P_f=95 \%=0.95$ percentile

The equation above gives tolerances in function of the confidence level chosen, see Table 7.

Table 7: Tolerances (Source: SYNPEX [14])

<table>
<thead>
<tr>
<th>Confidence level, $1-\delta$</th>
<th>90%</th>
<th>95%</th>
<th>99%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tolerance, $\varepsilon$</td>
<td>1.4%</td>
<td>2%</td>
<td>4.4%</td>
</tr>
</tbody>
</table>

The results – 95th percentile – regarding the determination of the coefficient $k_{eq}$ in the case when the mean of point load frequencies $f_i$ is set equal to the natural frequency for mode $i$ (i.e. $\lambda_i = f_{0i} / f_i = 1$) are presented in Figure 43. The values given in the legend, are the values
of damping ratio $\xi_i$ that were mentioned in Table 6. The values are ranging from $\xi_{i, \text{min}} = 0.002 = 0.2\%$ to $\xi_{i, \text{max}} = 0.02 = 2\%$. In figure 43 this has been shown for a coefficient of variation $\mu = 2.5\%$.

\[\mu = 2.5\%\]

Figure 43: Coefficient $k_{eq}$ versus number of pedestrians $N$, for a coefficient of variation $\mu = 2.5\%$ (Source: SYNPEX [14])

Table 8 summarizes the results from figure 43, but also for other values of $\mu$. The highest values are regularly reached for the damping ratio of 0.2 % and the lowest for 2 %, but they are even more dependent on the coefficient of variation of the loading frequency $\mu$. The highest absolute value obtained is $k_{eq} = 13.71$. The safest approach could be to choose the highest value for each coefficient of variation and to interpolate in order to find the value needed.

Table 8: Coefficient $k_{eq}$ versus coefficient of variation and damping ratio (Source: SYNPEX [14])

<table>
<thead>
<tr>
<th>Coefficient of variation, $\mu$</th>
<th>Damping ratio, $\xi_i$</th>
<th>0.2%</th>
<th>0.4%</th>
<th>0.6%</th>
<th>0.8%</th>
<th>1.0%</th>
<th>1.2%</th>
<th>1.4%</th>
<th>1.6%</th>
<th>1.8%</th>
<th>2.0%</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.5%</td>
<td>Max</td>
<td>13.71</td>
<td>13.13</td>
<td>12.48</td>
<td>12.38</td>
<td>11.99</td>
<td>11.73</td>
<td>11.39</td>
<td>11.03</td>
<td>10.79</td>
<td>10.57</td>
</tr>
<tr>
<td></td>
<td>Min</td>
<td>12.89</td>
<td>12.57</td>
<td>12.35</td>
<td>11.55</td>
<td>11.44</td>
<td>11.27</td>
<td>10.93</td>
<td>10.72</td>
<td>10.41</td>
<td>10.19</td>
</tr>
<tr>
<td>5.0%</td>
<td>Max</td>
<td>9.74</td>
<td>9.70</td>
<td>9.32</td>
<td>9.15</td>
<td>9.05</td>
<td>8.93</td>
<td>8.78</td>
<td>8.70</td>
<td>8.49</td>
<td>8.44</td>
</tr>
<tr>
<td></td>
<td>Min</td>
<td>9.24</td>
<td>9.24</td>
<td>8.95</td>
<td>8.84</td>
<td>8.72</td>
<td>8.58</td>
<td>8.56</td>
<td>8.34</td>
<td>8.32</td>
<td>8.21</td>
</tr>
<tr>
<td>7.5%</td>
<td>Max</td>
<td>8.38</td>
<td>7.85</td>
<td>7.92</td>
<td>7.70</td>
<td>7.50</td>
<td>7.57</td>
<td>7.35</td>
<td>7.62</td>
<td>7.41</td>
<td>7.31</td>
</tr>
<tr>
<td></td>
<td>Min</td>
<td>7.78</td>
<td>7.67</td>
<td>7.55</td>
<td>7.39</td>
<td>7.26</td>
<td>7.13</td>
<td>7.08</td>
<td>7.12</td>
<td>7.04</td>
<td>6.93</td>
</tr>
<tr>
<td>10%</td>
<td>Max</td>
<td>7.22</td>
<td>6.90</td>
<td>6.88</td>
<td>6.76</td>
<td>6.74</td>
<td>6.70</td>
<td>6.56</td>
<td>6.51</td>
<td>6.43</td>
<td>6.46</td>
</tr>
<tr>
<td></td>
<td>Min</td>
<td>6.68</td>
<td>6.70</td>
<td>6.50</td>
<td>6.37</td>
<td>6.43</td>
<td>6.34</td>
<td>6.34</td>
<td>6.32</td>
<td>6.12</td>
<td>6.20</td>
</tr>
</tbody>
</table>

The equivalent number of pedestrians are now calculated too (95th percentile) according to equation 3-12. The results, for a coefficient of variation of the forcing frequency $\mu = 2.5\%$ and $\mu = 10\%$, are shown in Figures 44.

From Figure 44 it becomes obvious that the lower the coefficient of variation of the forcing frequency the higher the equivalent number of pedestrians, which is entirely coherent with the chosen approach as such a case approaches the most to the deterministic resonant one (in which the equivalent number of pedestrians would be equal to their real number).
Figure 44: Equivalent number of pedestrians $n'$ versus number of pedestrians $n$, for a coefficient of variation $\mu=2.5\%$ and $10\%$ (Source: SYNPEX [14])

Another study is carried out on the influence of the ratio of frequencies $\lambda_i$ on the coefficient $k_{eq}$. Figure 45 presents results (95th percentile) of the analysis carried out for a fixed number of 400 pedestrians and for the two extreme values only of the damping ratio, $\xi_{i,\text{min}} = 0.002 = 0.2\%$ and $\xi_{i,\text{max}} = 0.02 = 2\%$. 
Figure 45: Equivalent number of pedestrians $n'$ versus number of pedestrians $n$, for a coefficient of variation $\mu=2.5\%$ and $10\%$ (Source: SYNPEX [14])

The influence of the ratio between the frequency for mode I and the mean of the forcing frequency is also expressed in Table 9, $\mu=2.5\%$, $\mu=5\%$, $\mu=7.5\%$ and $\mu=10\%$. 
Recapitulation

According to the previous results, the 95th percentile value of $k_{eq}$ can be obtained by multiplying a value of $k_{eq}$ chosen within the maxima form Table 8 with a coefficient chosen within the maxima from Table 9. This will be illustrated with an example. For instance, if a dense random stream TC 3 (0.5 P/m²) is found on Stade de France Footbridge (length=180 m; width= 11 m), then the entry data for obtaining the equivalent number of pedestrians $n'$, when mode nr. 1 is chosen, are as follows:

- Mean forcing frequency: $f_{s,m} = 1.81$ Hz
- Coefficient of variation: $\mu = 0.076 / 1.81 = 4.2\%$
- Frequency of mode nr.1: $f_1 = 1.8$ Hz
- Ratio of frequencies $\lambda$: $\lambda_1 = 1.8 / 1.81 = 0.99 = 1$
- Number of pedestrians $n$: $n = 11 \times 180 \times 0.5 = 990$ pedestrians
- Damping ratio: $\zeta = 0.24\% = 0.0024$

The equivalent number -95th percentile- of synchronized pedestrians is then:

$$n' = (10.96 \times 1.0) \times \sqrt{990 \cdot 0.0024} = 16.9$$

If, for example, mode nr. 2 is chosen, then:

- Frequency of mode nr.2: $f_2 = 1.95$ Hz
- Ratio of frequencies $\lambda$: $\lambda_2 = 1.95 / 1.81 = 0.99 = 1.08$

$$n' = (10.96 \times 0.549) \times \sqrt{990 \cdot 0.0024} = 9.27$$

It is thus obvious that care is needed in the choice of the range of frequencies for which this kind of calculation makes sense. At last, it is reminded here that the problem of the influence of the structure to the behaviour of the pedestrians IS NOT taken into account with this approach, and it can be an aggravation factor.

### 3.4.2 Reduction coefficient $\Psi$

According to SYNPEX and EUR 23984 EN, the influence of the structure on the footstep behaviour of the pedestrians can be taken into account by multiplying the dynamic load, or the calculated response in terms of accelerations, with a reduction factor. SYNPEX and EUR 23984 EN mention the reduction coefficient $\Psi$ and refer to SETRA. SETRA does also not mention how the reduction coefficient $\Psi$ and its approach is derived.

As shown in figure 46, the reduction coefficient $\Psi$ is 1.0 for bridge structures with natural frequencies between 1.7 and 2.1 Hz. For bridge structures with natural frequencies between
1.25 Hz and 1.7 Hz, interpolation is used to determine the reduction coefficient. The same goes for natural frequencies between 2.1 Hz and 2.3 Hz. Figure 46 shows that the reduction coefficient is 0 for bridge structures with natural frequencies lower than 1.25 Hz and higher than 2.3 Hz. This implies that a comfort criteria check is not needed for bridge structures outside this range.

The basic idea behind this approach is, that slender footbridges with natural frequencies between 1.7 Hz and 2.1 Hz are most sensitive for undesired vibrations, because the mean step frequency of walking pedestrians can be found in that range. However, when analyzing high pedestrian densities such as 1.5 P/m², there can be found that the mean step frequency is 1.41 Hz (Chapter 2.2.2, Table 2). Besides, the standard deviation of the step frequency in such pedestrian densities becomes smaller compared to lower pedestrian densities.

![Figure 46: Equivalent number of pedestrians and reduction coefficient (Source: SYNPEX/SETRA [14,18])](image)
4. Conclusions and recommendations

The objective of this report is to select and identify research evidence that are relevant to the essential subjects of this research project: dynamic loading, synchronization effects, and the dynamic response of slender footbridges.

It is obvious that dynamic induced loads by single pedestrians has been investigated several times, whereof the investigation of Harpet et al. [6] was one of the oldest. Although the nature of induced dynamic forces and their dependence on many parameters (such as walking speed, step frequency, weight, walking intention) is very complex, other researchers such as Galbraith and Barton[7], Blanchard et al [8], Kerr [9] confirmed the general shape of the force time history. The so-called butterfly shape of the walking forces in vertical direction, has also been confirmed during the investigations within the SYNPEX project.

There is an agreement on the deterministic force model [eq 1-4], where continuous walking force can be obtained artificially by combining individual foot forces. But researchers who have tried to quantify Dynamic Load Factors (α_{i,vel}), which are the basis for the common model of perfectly periodic human-induced force, have found different values.

In contrast to the investigations on dynamic induced loads by single pedestrians, investigations on the synchronization effects between pedestrians, and synchronization effects between pedestrians and structure, are relatively new. Both experiments within the PEDIGREE project and the experiments within the SYNPEX project show similar results regarding the synchronization effects between walking persons, when the density of persons increases. But, in the PEDIGREE project there is mentioned that synchronization and anti-synchronization may occur at low densities where the pedestrians are not bound by the steric constrains. Pedestrians are probably sensitive to the stepping oscillations that they perceive visually when watching their predecessor, they could synchronize naturally.

One of the observations that was only mentioned in the PEDIGREE project, is that the pedestrians rather adapt their velocity through their step length rather than step frequency. Furthermore, step length is overall proportional to the velocity and the size of the steps is directly related to the space available in front of the pedestrians.

An important observation that was only mentioned in the SYNPEX project, is that the correlation between the step frequencies and the walking velocity is stronger than the correlation between the standard deviation of the step frequencies and the walking velocity. Furthermore, the investigation of the synchronization effects between pedestrian and structure within the SYNPEX project, show that persons walked temporarily in synchrony with the vibrating bridge but showed no stable synchronization behaviour over a longer time period, because only four load cells and not all load cells are hit during one crossing. Walking temporarily in synchrony could also take place by chance.

In the literature, two types of calculation of the response of slender footbridges were found: deterministic and probabilistic. Even the deterministic calculations that are mentioned in the EUR 23984 EN, such as the spectral method, were based on a probabilistic approach, using a Monte Carlo Simulation for four different traffic scenarios to consider the stochastic properties of the pedestrian-induced forces. The same goes for the other approach, where
the formula for the equivalent number of pedestrians representing a random stream, was derived using Monte Carlo Simulations.

It is also possible to perform an extensive dynamic time-step calculation with a probabilistic approach, where a calculation is repeated n number of times, and where the pedestrian characteristics such as weight, walking velocity, step frequency and so on, will be varied each time. The n-number of calculated responses will be plotted in a histogram and based on a chosen fractile, i.e. 95 % value, the 'probabilistic' response will be determined. This method is not suitable for calculations in the first design stages, but is probably more accurate than earlier mentioned design tools. This time-step calculation method will be used during the next phase of the graduation project, to check the design methods in the EUR 23984 EN.
References literature survey


More information can be found at [http://www.pedigree-project.info](http://www.pedigree-project.info)


