Eindhoven University of Technology

MASTER

The impact of additive manufacturing on service supply chains

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The Impact of Additive Manufacturing on Service Supply Chains

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ABSTRACT

Additive Manufacturing (AM), also known as 3D printing, is a relatively new production method where material is added layer for layer, instead of cutting away unnecessary material, as is done with traditional means of manufacturing. With AM, lead time of new spare parts for service supply chains can be reduced from a couple of weeks to one day. This has significant supply chain effects, due to the fact that inventories can be reduced. On the other hand, investments in this technology need to be done.

Inspired by the trade-off between lower inventory costs on the one hand, and investing in AM on the other hand, we have developed several mathematical models that can help a company with expensive spare parts to calculate their expected inventory and backorder levels. One of our models includes waiting time at the printer and can be used for a high-demand, multi-item situation.

We have found appropriate input parameter values for our model, and have run several numerical investigations. Based on our modeling choices, we can conclude that AM is typically not beneficial for low-demand, single-item situations. This is the case for both in-house printing, as well as outsourced printing. For a multi-item situation however, where the printer is sufficiently utilized, AM can be cheaper than traditional means of manufacturing.
This thesis concludes my graduation project, as well as my time as a student at the Eindhoven University of Technology. I had a wonderful time where I learned a lot, and I made a lot of new friends.

I would like to start with thanking my mentor and first supervisor, Frank Karsten. Thank you for your guidance and your weekly feedback. Those weekly meetings really helped me out with focusing on what was important. I also want to thank Rob Basten. Thanks to your fresh views and with your help, I was able to assess my own work better and it helped me structure my thesis. Frank and Rob, your criticism could be harsh at times, but it was also fair and just. I have learned a lot from it.

I would like to thank Nils Knofius for providing extra feedback on my work. I wish you all the best with your research. I am sure that your work will be an important contribution to the research that is done on additive manufacturing in supply chains. I also want to thank Johnny Peeters for taking the time to enlighten me on the process of additive manufacturing. Your information helped me to find realistic input parameters for my model.

I want to thank my friends for providing me with the necessary distraction from work. I have a lot of good memories from being together with you guys, and I hope that there will be many more to come. I finally want to thank my family for their support. You guys are always there for me.
SUMMARY

Over the last couple of years we have heard more and more stories about 3D printers and 3D printing technologies. The formal term for 3D printing is Additive Manufacturing (AM). When an object is made by means of AM, a digital design of the desired object is sent to a 3D printer that can print the object layer for layer. The main difference with between AM and traditional manufacturing is that AM adds material to the final product, while the traditional method removes unnecessary material.

AM could have a huge impact on the capital goods industry. A capital good is a durable good that is used to create value. Examples of capital goods are passenger jets, wafer steppers and windmills. Capital goods typically have high downtime costs. This is why it is important to keep their downtimes as low as possible. In order to do that, it is vital to keep enough critical spare parts in stock. However, it is also costly to keep these items in stock, mainly because of their high price. There always is a trade-off between expected inventory and expected downtime costs. The more parts are kept in stock, the lower the expected downtime costs, but the higher the expected inventory costs. A certain amount of safety stock of parts is always kept because it typically takes weeks or months to resupply inventory of parts.

With the introduction of AM this could drastically change. Instead of waiting weeks for a new part, it could in theory be resupplied the very next day. This would bring about a huge decrease in inventory, and thus inventory costs. The necessary investments in AM technology however have to be made. In this master’s thesis we looked at how this situation can be quantified with a mathematical inventory model. We also did a numerical investigation that uses this model to see under which circumstance the investment may be worth the effort.

We started with a literature study, where we reviewed the (S-1,S) inventory model with backorders. In this model, failures occur according to a Poisson process. Every time when a failure occurs, a part is taken from stock and a new one is ordered. If there are no parts in stock when demand occurs, a backorder is created. These backorders represent the downtime of a machine. Downtime costs can become very high and are to be avoided as much as possible. This can be done by maintaining a sufficient amount of inventory. This does however increase inventory costs. There always is a trade-off between expected inventory costs on the one hand, and expected downtime/penalty costs on the other.

The longer it takes to resupply inventory, the higher the optimal base stock level becomes. In the aerospace and semiconductor industry it is normal to see lead times of a couple of weeks to a couple of months. As mentioned before however, if we could print the parts ourselves instead, we would see lead times of only one or two days. This drastically reduces the need to keep inventory. On the other hand however, we do need to make the necessary investments AM technology.

The goal of our study was to formulate a mathematical inventory model that can aid a company with low-demand, expensive spare parts in deciding on the use of additive manufacturing in their spare parts inventory policy. Besides describing this model, we also wanted to perform numerical investigations in order to see when investing in AM would be economically feasible. In order to do that we also needed to find realistic value ranges for our input parameters, such as the speed at which 3D printers operate and the amount of money that needs to be paid for a 3D printer. These values were found by studying literature on AM and by means of interviewing an employee who works in the AM industry.
We have formulated models for several situations, and we performed numerical investigations for all of them. These situations were:

1. A low-demand, single item situation where printing is done in-house
2. A low-demand, single item situation where printing is outsourced
3. A high-demand, multi-item situation where printing is done in-house

In the first situation we assume that print times are deterministic. We only have one stock keeping unit (SKU) with a demand of only a few items per year. Due to the fact that a 3D printer in this case is only used a couple of days per year due to very low demand, there is virtually no waiting time for a print job. Hence, the supply system behaves like a so-called \( M|\delta|\infty \) system. We can apply Palm's theorem to calculate expected inventory and backorder levels, which allows us to optimize the stock level. We have performed a numerical investigation with this basic model, with the realistic parameter values that we have found and compared the expected annual costs under an optimal base stock level with those of a setting without AM. It turns out that buying a printer that is used for only a few days per year is not worth the investment.

This is why we looked at an alternative model where the only difference was that the printing was done by a third party. In this second model we removed the fixed AM costs (mainly printer costs) and replaced them with variable AM costs. The third party that does the printing charges a markup on each part that is printed. It goes without saying that the more parts need to be printed, the less attractive it is to outsource AM, because of the markup that has to be paid for every part. The total AM lead time also increases a little bit due to the fact that somebody else (at a different location) now needs to print the item. We also performed a numerical investigation with this model and even though the differences in costs between the regular situation (without AM) and a situation where a part is 3D printed are much smaller, AM is still more expensive and not worth the investment. Outsourcing AM can be interesting in some cases, for instance when a relatively low markup is used or when the lead times of the regular supply process are much longer than normal.

We finally also looked at a third situation where we are dealing with a high-demand model that allows for multiple items. Because with \( \geq 100 \) parts per year, the printer has a much higher utilization, and because orders arrive stochastically, congestion at the printer is inevitable. We had to include waiting times somehow. This is however very complicated and that is why we needed to find an approximation method that includes waiting times, but still would remain easy to understand. We had three different candidate approximation methods that could calculate expected on-hand inventory and backorder levels. We chose an approximation method based on an \( M|\delta|\infty \) system, where we used a different processing time. Instead of using just the print time (net processing time), we chose gross processing times where waiting time is also included. It turns out that this method is approximation is fairly accurate for a utilization up till 82%.

We have also performed a numerical investigation with our chosen approximation method for the high-demand situations, with the realistic parameter values that we found earlier. This time however AM actually IS the better choice! In other words, if there is a high enough demand (probably multi-item) and we consider holding costs, backlogging costs, and potentially 3D printer purchase costs, then investing in additive manufacturing can in many cases turn out to be cheaper than ordering new parts the 'old-fashioned' way.
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1 INTRODUCTION

Additive manufacturing (AM), better known as 3D printing, has gained a lot of attention over the last couple of years. Stories about 3D printed food, organs, and even houses all contributed to the hype it has become today (Perry, 2015). Even though the technology is still in its infancy phase, there is a rapid development going on. Between 2007 and 2011 the sales of 3D printers grew 200 to 400 percent annually (McKinsey, 2013). McKinsey (2013) has estimated that the economic impact of additive manufacturing in 2025 could be between $230 billion and $550 billion per year.

Additive Manufacturing is the process where a three-dimensional object is made by using a so called 3D printer. A digital file (for instance a CAD file) has the information on the measurements of an object. This information is sent to the printer which sequentially deposits material layer for layer, hence the name additive manufacturing. Instead of cutting away unnecessary material, as is done in traditional manufacturing, this method adds material to the final product.

Even though there are still many disadvantages to additive manufacturing such as the relatively high cost of printers, the limited amount of materials to work with, and the printing speed compared to traditional methods, there is a lot of potential for this technology. Additive manufacturing is a technology that also has quite some benefits over traditional manufacturing. It is highly customizable, since the machine itself does not have to be altered in order to change a design. It uses less material, since it only prints that what is necessary, instead of cutting off material in order to shape it. It could reduce the need to keep an inventory of parts, since one can print it whenever one would need it. Since this technology is relatively slow, utilizing it for fast moving machine parts would not be very beneficial compared to traditional methods. However using 3d printing for slow movers with relatively high value could be interesting (Holmström et al., 2010).

One of the industries that could hugely benefit from this new development is the capital goods industry. A capital good is a durable good that is used in the production of goods or services. An example of a capital good is a passenger jet. Passenger jets have a very high purchase price, but can be used for a couple of decades if maintained properly. If a capital good however fails to function due to a breakdown of a critical component, costs can become very high in a very short amount of time. In order to minimize these downtime costs, one has to perform corrective maintenance as soon as possible. The availability of spare parts has a very large impact on the speed in which a repair can be performed. This is why owners of capital goods keep inventory of spare parts worth millions of euros in total. A certain amount of safety stock of parts is always kept because it takes weeks or months to resupply inventory of parts.

This could however change with the introduction of AM. Instead of waiting weeks for a new part, it could in theory be resupplied the very next day. This would drastically decrease the need for inventory. On the other hand however, one has to be willing to make an investment in a 3D printer. There is a tradeoff between investing in AM on the one hand and benefiting from less inventory costs on the other hand. In this master’s thesis we will look at how this situation can be quantified with a mathematical inventory model. We will also do a numerical investigation that uses this model to see under which circumstance the investment may be worth the effort.
The remainder of this thesis is organized as follows. We will start with a literature review in chapter 2. We will review the literature on inventory models and see if there is any literature on the use of AM in (service) supply chains yet. We will also identify the research gap. In the next chapter, we will present the research questions of this thesis, based on the research gap that we have found in the preceding chapter. We will then give an overview and explanation of the values of input parameters that will be used throughout this thesis chapter 4. This chapter also mentions the context and some limitations of our models. Chapter 5 describes the basic low-demand, single-item situation, where we assume infinite production capacity. Chapter 6 contains a model which is a variation of the basic model of chapter 5. The difference here is that instead of having an in-house 3D printer, the print job is outsourced. Chapter 7 will extend the basic model towards a high-demand model with finite production capacity that thus has to deal with waiting times. We will conclude this thesis in chapter 8 where we also state the limitations of this work, and where we explore future research opportunities.
2 LITERATURE REVIEW

The literature review consists of two different parts. In the first part we will review the necessary inventory theory that this thesis is founded upon. In the second part we will review the literature on additive manufacturing in service supply chains. The research questions of this thesis, which can be found in the next chapter, are the result of the gap that we will identify in our literature study.

2.1 THE (S-1,S) MODEL

In order to understand the possible implications of additive manufacturing on service supply chains, it is necessary to understand the basics of inventory models regarding spare parts. In this chapter we will describe a so called single-item, single location situation under a base stock policy. This model is also known as the (S-1,S) model. We will start with describing the basic model, followed by the underlying assumptions that were made. After that, an evaluation is given. We will conclude this section with the optimization of this model. The following is a description of this model, which is a summarized version of the multi-item, single location model by Van Houtum & Hoen (2010).

2.1.1 BASIC MODEL

Consider a single warehouse where several spare parts are kept on stock to serve an installed base of machines of the same type. The machines consist of multiple components, which may be classified as critical and non-critical components. When a critical component of a machine fails, the whole machine goes down, while a machine can continue its functioning (i.e., to a sufficiently large extent) upon the failure of a non-critical component. We limit ourselves to the inventories of the spare parts for the critical components. When a critical component fails in a given machine, then the failed part is replaced by a spare part from the warehouse if available or as soon as possible. We assume an infinite time horizon $[0,\infty)$. In the remainder, we focus on a single component.

Demands occur according to a Poisson process with a constant rate $m (> 0)$. The rate $m$ denotes the demand rate for all machines together. A demand is fulfilled immediately if possible, and otherwise the demand is backordered and fulfilled as soon as possible. Immediately after each demand, a new part is ordered at a supplier or the failed part is sent into a repair shop to be turned into an as-new part. The time that it takes for the new part to arrive at the warehouse is called lead time, and consists of waiting time and repair or production time. Lead times are independent and identically distributed, and the average lead time is denoted by $t (> 0)$. Because each failed part is immediately re-ordered, the inventory position of an item, defined as the physical stock minus backordered demand plus parts in the pipeline, is constant. This constant amount is denoted by $S \left( \in \mathbb{N} \cup \{0\} \right)$ where $\mathbb{N} := \{1,2,\ldots\}$.

Instead of saying that each failed part is immediately re-ordered, we may also say that the stock is controlled by a continuous-review base stock policy, with base stock level $S$. Base stock level $S$ represents the initial stock and is a decision variable.
2.1.2 **OVERVIEW OF ASSUMPTIONS**

We summarize and discuss the main assumptions made above:

(i) *Demands occur according to independent Poisson processes.*

The assumption of independence is justified when a failure of a component does not lead to additional failures of other components in the same machine. In general this is true. The assumption of Poisson processes is justified either when lifetimes of components are exponential or when lifetimes are generally distributed and the number of machines that is served by the warehouse is sufficiently large.

(ii) *The demand rate is constant.*

The single warehouse serves multiple machines. When one or more machines fail and failed parts cannot be provided immediately, then some machines may be down for a while and the demand rates for a given SKU decreases accordingly. However, when the fraction of machines that is down is always sufficiently small, either because downtimes are short in general or because downtimes occur only rarely, then the decrease in demand rate is small, and thus it is reasonable to assume a constant demand rate.

(iii) *Lead times are independent and identically distributed.*

This assumption is justified if planned lead times have been agreed with suppliers (or departments). It then is the responsibility of the supplier to meet the planned lead times. In practice, planned lead times often occur because deliveries are executed by an external company.

(iv) *A one-for-one replenishment strategy is applied.*

This is justified as long as there are no fixed ordering costs or fixed ordering costs are small relative to the prices of the parts (or, thinking of the EOQ rule, relative to price multiplied by demand rate). If fixed ordering costs are relevant, then fixed order quantities may be appropriate to assume and one may follow an \((s, Q)\) instead of a basestock policy. This extension is described in van Houtum & Hoen (2010, Section 8.3).

**Example 1**

We now describe an illustrative example that is used for clarification. A manufacturer of capital goods keeps spare parts on stock in a single warehouse to support a reasonably large number of installed machines. All spare parts demands are fulfilled from this warehouse. We consider one stock keeping unit (SKU) where the average number of failures per year is 15 \((m = 15)\). The average repair lead times are equal to 2 months or \(\frac{1}{6}\) year \((t = \frac{1}{6})\).
2.1.3 Evaluation

In this section, we evaluate the steady-state behavior for a given base stock policy $S$, demand rate $m$ and average lead time $t$.

Consider an arbitrary SKU, and assume that the base stock level $S$ is given. The demand fulfilment process of this SKU is depicted by the Petri net in Figure 2-1. On the left-hand side in this figure, demands for ready-for-use parts arrive with rate $m$. The failed parts follow the upper stream in the figure. That is, they first go into the ordering process which takes on average $t$ time units. Then they arrive in a queue with ready-for-use parts. Actually this queue represents the physical stock, also called stock on hand. The demands for ready-for-use parts follow the lower stream. That is, these requests are sent to the warehouse, where they are fulfilled immediately if there is enough stock on hand and after some delay otherwise. Delayed requests are fulfilled according to a First-Come-First-Serve (FCFS) discipline. When both a request and a ready-for-use part are available, they merge (i.e., the transition on the right-hand side in the figure “fires”) and leave the system.

The steady state of the system may be described by $(X, OH, BO)$, where $X$ denotes the number of parts in the order pipeline, $OH$ denotes the stock on hand of ready-for-use parts, and $BO$ denotes the number of backordered demands. The amount $X$ represents the number of parts in the order pipeline, also called pipeline stock. Notice that $(X, OH, BO)$ constitutes a partial description because lead times are generally distributed, and thus for a full description one also has to denote how long parts are in the pipeline already.

It holds that

$$OH = (S - X)^+,$$
$$BO = (X - S)^+,$$

where $x^+ = \max\{0, x\}$ for any $x \in \mathbb{R}$. These equations imply that

$$OH - BO = S - X$$
or, equivalently, that

\[ X + OH - BO = S \]

This latter equation is known as the stock balance equation and shows that the number of parts in the upper stream of the Petri net in figure 2-1 is always \( S \) more than the number of requests in the lower stream. In the remainder, we will write \( X(m, t), OH(m, t, S), \) and \( BO(m, t, S) \) to make the dependence on these input parameters explicit.

In our model, demand occurs according to a Poisson process and each ordered part stays on average a time \( t \) in the pipeline. The order pipeline may be seen as a queueing system with infinitely many servers and average service times \( t \). In other words, the pipeline is an \( M|G|^\infty \) queueing system and thus we may apply Palm’s Theorem (Palm, 1938):

**Palm’s Theorem:** If jobs arrive according to a Poisson process with rate \( m \) at a service system and if the times that the jobs remain in the service system are independent and identically distributed according to a given general distribution with mean \( t \), then the steady-state distribution for the total number of jobs in the service system is Poisson distributed with mean \( mt \).

Application of this theorem to the pipeline leads to the following

(i) The pipeline stock \( X(m, t) \) is Poisson distributed with mean \( m \cdot t \), i.e.,

\[ P\{X(m, t) = x\} = \frac{(m \cdot t)^x}{x!} e^{-m \cdot t}, \quad x \in \mathbb{N}_0 \]

(ii) The distribution of the stock on hand \( OH \) is given by

\[ P\{OH(m, t, S) = x\} = \begin{cases} \sum_{y=0}^{\infty} P\{X(m, t) = y\} & \text{if } x = 0 \\ P\{X(m, t) = S - x\} & \text{if } x \in \mathbb{N}, \ x \leq S \end{cases} \]

(iii) The distribution of the number of backordered demands \( BO \) is given by

\[ P\{BO(m, t, S) = x\} = \begin{cases} \sum_{y=0}^{S} P\{X(m, t) = y\} & \text{if } x = 0 \\ P\{X(m, t) = x + S\} & \text{if } x \in \mathbb{N} \end{cases} \]

(i), (ii) and (iii) contain the main results for the evaluation of a given policy. From here on, we easily obtain relevant service measures, among which the mean backorder position, which is given by

\[ E[BO(m, t, S)] = \sum_{x=S+1}^{\infty} (x - S)P\{X(m, t, S) = x\} = m \cdot t - S + \sum_{x=0}^{S} (S - x)P\{X(m, t) = x\}, \quad S \in \mathbb{N}_0 \]
Notice that the latter expression for \( E[BO(S)] \) is most appropriate for computational purposes as it avoids complications because of sums with infinitely many terms.

The mean number of stock on hand is

\[
E[OH(m, t, S)] = \sum_{x=1}^{S} P[X(m, t) = S - x] \cdot x, \quad S \in \mathbb{N}_0
\]

**Example 1 (continued)**

We will now calculate the expected number of backorders given a certain \( S \). Assume the situation illustrated earlier in example 1. Assume that the time between order arrivals is exponentially distributed. In that case we are dealing with a Poisson distribution and we can use Palm’s theorem for our calculations. We want to know what the expected number of backorders are. Suppose that we choose the following order-up-to level: \( S = 6 \). Filling in these values in the formula for \( E[BO(m, t, S)] \) gives:

\[
E \left[ BO \left( 15, \frac{1}{6}, 6 \right) \right] = 0.0199
\]

2.1.4 Optimization

When we keep inventory, we have to pay holding costs \( h \ (> 0) \) for each part we have on stock per time unit. When backorders occur, we pay a penalty \( p \ (> 0) \) over each backordered part per time unit. We can describe the total expected costs per time unit in steady state, \( C(m, t, S) \), as:

\[
C(m, t, S) = h \cdot E[OH(m, t, S)] + p \cdot E[BO(m, t, S)]
\]

Our goal is to minimize the total costs of a chosen base stock policy. We want to find a balance between holding costs and penalty costs. This problem can be described as follows:

\[
\min C(m, t, S), \quad S \in \mathbb{N}_0
\]

Given a certain demand rate and average lead time, there is a base stock level where the expected total costs for that item is minimal. The smallest base stock level for which this is the case will be denoted with \( S^* \).

**Example 1 (continued)**

Assume the situation illustrated earlier in example 1. Assume that the holding costs are € 500 per year \( (h = 500) \) and that penalty costs are € 50,000 per year \( (p = 50,000) \). The expected costs are then minimal iff an order-up-to level of 7 is chosen, so \( S^* = 7 \). The costs in this case are \( C \left( 15, \frac{1}{6}, 7 \right) = € 2,540 \).

The total expected costs for different values of \( S \) can be seen in Table 2-1.

<table>
<thead>
<tr>
<th>S-level</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>( C(m, t, S) )</td>
<td>€ 4,378</td>
<td>€ 2,756</td>
<td>€ 2,540</td>
<td>€ 2,825</td>
<td>€ 3,268</td>
</tr>
</tbody>
</table>

**Table 2-1** The total expected costs for different values of \( S \)
ADDITIVE MANUFACTURING IN SERVICE SUPPLY CHAINS

Since additive manufacturing in (service) supply chains is a relatively new topic, there has not appeared much scientific literature on it yet. In recent years however, a few scientific papers have discussed this topic up to some extent. The oldest papers are of a conceptual nature. More recent papers also attempt to quantify the impact of additive manufacturing on supply chains. In order to find relevant papers on this topic, the following keywords were used in Google Scholar: 3d printing, additive manufacturing, inventory theory, supply chain, spare parts. The papers found this way were used to find older work that preceded those papers.

2.2.1 CONCEPTUAL PAPERS

Holmström et al. (2010) is a conceptual paper. It mentions the possibility to use additive manufacturing in spare parts supply chains. The purpose of this paper is to start building a theoretical foundation for follow-up research. A summation of the (potential) benefits of AM is given, such as the higher amount of customization and flexibility of the design, and the possibility for producing small batches.

Moreover, two different network setups are discussed: A centralized versus a decentralized approach. In the centralized approach spare parts are produced on demand from regional distribution centers. These centers supply the service locations. The service center is the final storage location of the spare part before it is used for maintenance. In the decentralized approach, also known as the distributed approach, spare parts are printed at the service locations, instead of at the regional distribution center. Because of the fact that printing is done at service locations, service response times decrease. This setup demands more printers, which at the moment of writing are still quite expensive.

Lindemann et al. (2012) attempts to break down the costs of AM in production by means of analysis and simulation combined with a technique called activity based costing. It compares this process to traditional manufacturing. The purpose of this paper is to create a better understanding of the cost structure of AM. They conclude that this new technology has great potential, especially in the medical and airline industry. However the technology today is still very expensive. According to them, about 74% of the costs come down to the purchase price of 3D printers. It seems that the price of printers has to drastically decrease in order to make this technology economically feasible.

2.2.2. PAPERS WITH QUANTITATIVE INVENTORY MODELS

Based on the work of Holmström et al. (2010), another paper was written by Khajavi et al. (2014). The authors try to analyze and estimate the costs and benefits of 3D printing for a specific spare part in the aerospace industry. The centralized versus the decentralized approach of Holmström et al. (2010) is tested by means of Monte Carlo simulation. Several assumptions about input variables such as costs and lead times are made. Based on these values a Monte Carlo simulation is run and this simulation calculates the most cost-effective network setup. Their conclusion is that the centralized approach would be better given the current circumstances such as purchase price of machines. However, they also make assumptions on the technology and prices of additive manufacturing in near future. With these future input values another simulation is run. This time however, the decentralized approach is favored over the centralized one.

Liu et al. (2014) mentions that AM technology is especially interesting for parts that are relatively expensive and have a low/average demand rate. They also present a slightly ambiguous inventory model, where they try to show with the help of a tool called a SCOR model, that the required safety inventories
are lower when using AM. We do not know exactly how they modeled. It is basically a black box where they use some input values, where afterwards some output parameters roll out. They also do not describe what the behavior pattern of arrivals is. Moreover, they seem to be using fast movers instead of slow movers. Liu et al. (2014) is not very useful for follow-up research since its results are very hard to verify and reproduce, because there is no solid mathematical basis, and because of the assumptions on demand made in this paper.

Wullms (2014) studied the possibilities for the use of AM in spare parts supply chains. He focused his master thesis on last-time buy situations at Philips Healthcare in Best. He made a mathematical model in order to quantify the effect of including additive manufacturing to the spare parts supply chain in the last time buy process. His model considers a combination of the classic last time buy decision and a base stock policy using additive manufacturing in the last phase of the remaining service period. It calculates the optimal order up to level at the last time buy moment and the optimal base stock level using additive manufacturing. The thesis concludes that the main advantage of using additive manufacturing in the last time buy decision is preventing safety stocks.

2.3 IDENTIFICATION OF THE RESEARCH GAP
The papers that were studied conceptualized the possible impact of AM in service supply chains. Some papers even tried to calculate the costs of a policy that includes this option of 3D printing. However no paper combined the possibility of additive manufacturing with mathematically backed inventory theory. Moreover most of the papers consist of scenario modelling where data is put in a black box resulting in a certain outcome. However the processes behind the modelling are not quite clear.

Wullms (2014) did use a quantitative inventory model where AM was included. However he focused on the last-time buy situation, which has a finite horizon with only one ordering possibility. The combination of quantitative inventory models describing a situation with an infinite horizon and continuous ordering possibilities that includes AM does not exist yet. This is a gap in the known literature.

Combining mathematical models with a 3D printing option can be very interesting. Even though the technology might not be real yet, it could form a starting point for further research, perhaps when the technology is more accessible and more widely available. Using formal inventory models to explain the possible benefits of additive manufacturing has its benefits over simulations. One of them being that the inventory models are very transparent whereas Monte Carlo simulations for instance can be considered as a black box that produces results. The underlying processes of a mathematical model are much clearer to understand in comparison with a simulation. This helps others to verify and reproduce the results. A sound mathematical model thus makes the research more robust. This is why it would be a good idea to try to build a formal inventory model that incorporates the possibility of additive manufacturing, in order to see what predictions it gives about their performances in comparison to regular scenarios that do not consider additive manufacturing.
3 RESEARCH QUESTIONS

As we have seen in the previous Chapter, there are only a couple of scientific papers on the topic of additive manufacturing in (service) supply chains. In fact, there are no papers yet that use quantitative mathematical models to explain the impact of AM on service supply chains. This is where the gap in the literature exists. We can partially fill that gap by making a quantitative mathematical inventory model that includes the potential possibilities of additive manufacturing. Once we have built a model, we need to run this model in order to see what the impact of additive manufacturing can be. In other words we want to see whether or not application of AM can lower costs in service supply chains. In order to do that, we need to formulate models that can help us with this decision and we need to find realistic values for the input parameters of these models.

The goal of our research project is to develop insight into the possibility of including 3D printers in a spare parts supply chain. We want to do this by formulating a mathematical inventory model that can help companies to make a decision on whether or not to make use of additive manufacturing in their service supply chain.

There are three main research questions. The first deals with finding realistic value ranges that can serve as input for our models. The other two are about modeling an inventory supply system in such a way that we can decide if AM is worth the investment. One deals with low-demand situations, and the other deals with high-demand situations. In the latter we have to deal with congestion at the 3D printer. The second and third main research questions have sub questions, which are denoted with letters (a and b).

3.1 INPUT PARAMETERS

Our model will contain several input parameters, such as printing speed or 3D printer purchase price. Because we want to know if an investment in AM is worth the money, we first need to find realistic values that can serve as input parameters for our models. Our first main research question is:

1. Which value ranges are realistic for our model’s input parameters?

Once we know which values our input parameters can have, we can test under what kind of circumstances the application of additive manufacturing can save time and/or money. We will use these parameter values to answer Research Questions 2a, 2b and 3a. We need to get our data from (scientific) literature and field research, such as interviews. Once we have built the models and we have all the necessary data on input parameters, we can run numerical investigations with them in order to understand which factors do and which do not play an essential role in the decision process on the utilization of AM. Research Question 1 is answered in Chapter 4.
3.2 LOW DEMAND, INFINITE CAPACITY
We will start by formulating a mathematical model in a low-demand setting. This means that we will formulate a model for a situation where, due to relatively low total demand, a 3D printer is busy for only a small fraction of the time, such that there is virtually no queue upon the arrival of a new order. If that is the case, then waiting times can be neglected. It would be as if we had infinite capacity at our printer. Our second main research question will be:

2. **What model can aid a company with expensive spare parts in deciding on the use of additive manufacturing in its spare parts inventory policy in a low-demand setting?**

Spare parts of capital goods are typically expensive and have a low demand (Cohen et al., 1997). These are the kind of items that we will focus on throughout this thesis. The focus of this ‘basic’ model will be on a single-item setting. However, it can also be used in a multi-item setting as long as the utilization of the 3D printer remains low enough to keep neglecting waiting time.

If a company decides that it wants to use AM to make their spare parts, it can choose to either print the parts in-house, or to outsource the print job to a third party. Research Questions 2a and 2b deal with scenarios of in-house printing and outsourced printing respectively.

2a. **Is investing in additive manufacturing beneficial in case of low-demand?**

This sub question will only look at costs and benefits when demand is low enough to ignore waiting time at the printer. A numerical investigation is performed with the input values that we found in Chapter 4 where we searched for realistic input parameter value ranges for our model. We will answer question 2a in Chapter 5.

It might be the case that the required investments for buying a 3D printer are too high for printing only a couple of parts per year. In that case it is worth looking at other scenarios where we have a high demand (question 3a) or where AM is outsourced to a third party. Hence the next sub question:

2b. **Is investing in additive manufacturing beneficial in case of low-demand if we outsource the print job to a third party?**

If we outsource the print job, then it is not necessary to purchase a 3D printer anymore. On the other hand however, we can expect that the price paid per part is higher (due to a markup that is demanded by the third party), and that lead times are longer. We will also perform a numerical investigation so that we can answer this sub question. The answer for Research Question 2b can be found in Chapter 6.
3.3 HIGH DEMAND, FINITE CAPACITY

Besides looking at a scenario where we only print a couple of parts per year, we will also look at a setup where we have high demand. This means that the demand is high enough to cause congestion at the printer up to a point where we can no longer neglect waiting times. Assuming that we already have answered the second main question, we already have a working model for low-demand situations. We now want make an extension in such a way that we can also incorporate queueing somehow. The third main question including its sub questions are answered in Chapter 7. Our third and final main Research Question is:

3. **How do we extend our existing ‘basic’ model with waiting times?**

Once we have modeled waiting time at the printer in our extended model, we can also see if AM is beneficial in case of high demand. Our first sub question for this scenario thus is:

3a. **Is investing in additive manufacturing beneficial in case of high-demand?**

Question 3a will be answered in a similar fashion as question 2a.

Due to the fact that it is unlikely that a high demand for parts is caused by just a single SKU, it is realistic to assume that we are dealing with a multi-item setup for a high-demand scenario. That is why we will look at how we calculate expected on-hand inventory and backorder levels for non-identical items. Non-identical items are items that have different processing/printing times, depending on which SKU they are. Hence our final sub question:

3b. **How do we calculate expected on-hand inventory and backorder levels when we are dealing with non-identical items?**
4 INPUT PARAMETER VALUES, CONTEXT AND LIMITATIONS

In this chapter we will describe the input parameter values that will be used later on in our thesis when we will perform numerical investigations. We start by giving an overview table of the value ranges that all our input parameters have, followed by a detailed explanation for each parameter. Some of the sources that we used, are in U.S. dollars instead of euros. In that case we converged the dollars to euros with the exchange rate on 1 September 2015. We finish this chapter by giving some context and limitations of the models that will be introduced in the following chapters. This chapter will also be the answer to Research Question 1: Which value ranges are realistic for our models’ input parameters?

4.1 INPUT PARAMETER VALUES

An overview of all input parameters for our models is given in Table 4-2. It contains the range of values that each parameter can have (min and max value). Their typical/standard value is given in the fifth column. These values were found by studying (scientific) literature and by interviewing Johnny Peeters, who is an employee of a company specialized in AM.

Some parameters are present in all every model of this thesis: penalty costs, regular lead time, and annual demand per SKU. We use holding costs, annual AM costs and AM lead time (in-house) in chapters 5 and 7. We use AM lead time (outsourced), markup and part price in chapter 6, and the number of SKUs in chapter 7.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Min value</th>
<th>Max value</th>
<th>Standard value</th>
<th>Unit of measurement</th>
</tr>
</thead>
<tbody>
<tr>
<td>Holding costs</td>
<td>$h$</td>
<td>€400</td>
<td>€28k</td>
<td>€4k</td>
<td>Euros per part per year</td>
</tr>
<tr>
<td>Penalty costs</td>
<td>$p$</td>
<td>€126k</td>
<td>€630M</td>
<td>€80M</td>
<td>Euros per part per year</td>
</tr>
<tr>
<td>Annual AM costs</td>
<td>$C_{AM}$</td>
<td>€40k</td>
<td>€170k</td>
<td>€80k</td>
<td>Euros per year</td>
</tr>
<tr>
<td>Regular lead time</td>
<td>$t_{Reg}$</td>
<td>0.083</td>
<td>0.500</td>
<td>0.167</td>
<td>Years</td>
</tr>
<tr>
<td>AM lead time (in-house)</td>
<td>$t_{AM}$</td>
<td>0.000</td>
<td>0.005</td>
<td>0.003</td>
<td>Years</td>
</tr>
<tr>
<td>AM lead time (outsourced)</td>
<td>$t_{AM_{out}}$</td>
<td>0.014</td>
<td>0.058</td>
<td>0.014</td>
<td>Years</td>
</tr>
<tr>
<td>Annual Demand per SKU</td>
<td>$m$</td>
<td>1</td>
<td>18</td>
<td>5</td>
<td>Items per year</td>
</tr>
<tr>
<td>Markup</td>
<td>$\gamma$</td>
<td>8</td>
<td>44</td>
<td>20</td>
<td>%</td>
</tr>
<tr>
<td>Part price</td>
<td>$C_{part}$</td>
<td>€1.6k</td>
<td>€112k</td>
<td>€16k</td>
<td>Euros</td>
</tr>
<tr>
<td>Number of SKUs</td>
<td>$</td>
<td>I</td>
<td>$</td>
<td>1</td>
<td>72</td>
</tr>
</tbody>
</table>

Table 4-1 Input parameter value ranges. We let $k$ denote thousands and $M$ denote millions.
Holding costs
Holding costs consist of warehouse expenses, insurance costs, and lost interest (because one cannot retrieve any interest on money that is already spent on spare parts). We will assume an inventory carrying cost of 25% (Huribut, 2004; Kranenburg & Van Houtum, 2009). This means that the annual holding costs are 25% of the total value of the inventory.

Karsten (2009) did a study on realistic parameter values for spare parts inventory situations. He studied several scientific papers in order to find an appropriate range of values to work with, and made a table with an overview of parameter values used in recent literature. This table can be found in appendix 5 on page 73 of Karsten’s Master’s Thesis (2009). The values that he eventually chose for his model were mainly based on the work of Kranenburg and Van Houtum (2009). Values of parts ranged from €2,000 to €100,000 per part. This comes down to the fact that the holding costs vary between €500 and €25,000 per part. Based on the findings of Karsten (2009) for proper parameter value ranges, we will set our standard value of holding costs to €4,000 per part per year, with a minimal value of €400 and a maximal value of €28,000 per year. Holding costs are represented by $h$ and their unit of measurement is in euros per part per year.

Penalty costs
Penalty costs are paid when backorders occur. When we are dealing with backorders, we are dealing with downtime of machines. In this thesis, downtime costs, penalty costs and backorder costs are all the same thing, and we will use these terms interchangeably. The costs of downtime can vary very much between different industries. Downtime costs for a truck are a lot lower than those of an airplane, and the semiconductor industry sees even higher downtime costs.

The cost per hour of downtime per aircraft is around $10,000 (Pohl, 2013), which is about €8,950 or €78.4 million per year. The airline industry is an industry that is often used as a reference for calculations for capital goods. This is why we choose the airline industry for our standard downtime costs and we will set them to €80 million per part per year on average. Obviously some industries, such as the semiconductor industry, have higher downtime costs. Other industries, such as road transportation companies have lower ones. In general, more technology means higher costs (Pohl, 2013). We will consider the downtime costs of trucks as the cheapest form of downtime for a capital good. These costs are around $385 per day or $140,000 per year (Automotive Fleet, 2013), which is about €343 per day or €126,000 per year. We will set the minimum possible value for downtime at €130,000 per part per year. We estimate that the downtime costs are the highest in the semiconductor industry where it is known that downtime costs can be several tens of thousands of euros per hour (Kranenburg, 2006). The downtime costs for customers of ASML are €72,000 per hour or €630.72 million per year (Linssen, 2015). We will set the maximum penalty costs at €630 million per part per year. Penalty costs are represented by $p$ and their unit of measurement is in euros per part per year.

Annual 3D printing costs
The biggest driver of AM costs are the depreciation costs of a machine. The depreciation cost depend on the purchase costs and the lifetime of a printer. With a utilization of 83% a printer is expected to operate for 8 years (Roland Berger, 2013). Due to the fact that in our standard scenarios, the utilization is much lower than 83%, it is realistic to assume a lifetime of 10 years (Peeters, 2015). The prices of 3D metal printers range from $125,000 to $1.4 million (Whitefield, 2014), which is €111,000 to €1.24 million. Roland Berger (2013) states that these printers cost about €500,000 on average. This standard value is also confirmed by an employee of an employee of Additive Industries, a company that develops 3D printers
We assume that a 3D printer has no value at the end of its lifetime. Working with a lifetime of 10 years, we let our annual depreciation vary between €11,000 and €125,000, with a typical value of €50,000 per year. Other major fixed costs drivers are maintenance and rent of €24,000 and €3,640 respectively on a yearly basis (Roland Berger, 2013).

Variable printing cost consist mainly of labor and material costs (Roland Berger, 2013). However due to the fact that these costs are significantly lower than fixed costs, and because we want to keep our AM cost breakdown as clear as possible, we will ignore variable AM costs in this thesis. Because we will later on make the assumption that there is no difference between individual part prices (Section 5.1), regardless of how they were manufactured, we can also make the argument that these variable costs are already included in the individual part price.

Based on the fixed annual AM costs data, we will vary the range of annual 3D printer costs between €40,000 and €170,000, with a standard value of €80,000 per year. Annual AM costs are represented by $C_{\text{AM}}^{\text{Ann}}$ and the unit of measurement is in euros per year.

**Lead times regular model**
We assume that under normal circumstances a new part has to be re-ordered from the supplier of spare parts once a part has failed. We will choose our boundaries for the lead times based on lead times that are normal in the aerospace and semiconductor industry. Silicon Experts, a provider of data on components and parts for the electronics industry, analyzed the lead times of semiconductors between July 2011 and August 2012. The average lead time was about 11 weeks (Siliconexpert Technologies, 2012). De Cos Juez F.J. et al. (2010) did an analysis on the lead times of metallic components in the aerospace industry. According to their data, about 50% of the components have lead time of about 75 days. According to them, lead times of less than 20 days are unfeasible and lead times of 200 days are possible, but uncommon. We will choose to vary the lead times of the regular model between 1 month and 6 months, with a standard value of 2 months. Regular lead times are represented by $t_{\text{Reg}}$ and the unit of measurement is years.

**Lead times AM model (in-house)**
The AM lead time is solely dependent on the total time it takes to build a part. Order and delivery times are not included due to the fact that we have a printer available on site. The printing of one object typically takes about half a day (Peeters, 2015). Since this is a trial-and-error process, the total printing time can take up to a couple of days when a new design is introduced. However, when the part is known and has been printed a couple of times the production time reduces due to a learning curve. There is also a lot of manual labor required in order to get a part ready. The printer has to be prepared, calibrated and cleaned out afterwards for instance. Someone also needs to separate the printed object from the base plate since these are attached to each other once the printing process is finished (Peeters, 2015). Since we want to be on the safe side, we will assume that the entire process of preparations, printing and finishing the product will take a full day. In the future, the printing times are expected to decrease drastically (Roland Berger, 2013; Khajavi et al., 2014; Peeters, 2015).

The development of 3D printers is going at a tremendous pace, and printer manufacturers expect that lead times will improve significantly in the upcoming couple of years due to the fact that the technology is still in its infancy phase. The build speeds will go from 10 cm$^3$/h in 2013 to an estimated 40 cm$^3$/h in 2018 and 80 cm$^3$/h in 2023 (Roland Berger, 2013). This is an increase of 700% in 10 years. There is a lot of potential for improvement, on both quality of the final product and the printing lead times. Khajavi et al. (2014) assume that future AM machines will increase their speeds six times, compared to the speeds we
are working with today. A 12 hour job would in that case only take 2 hours. However Khajavi et al. (2014) do not specify what they mean with future.

If we are dealing with complex parts that have just been introduced, it may take up to three days in order to get the job done. This is mainly due to trial and error and learning how the quality can be improved (Peeters, 2015). If we consider that we have machines of the future (2020 – 2025) at our disposal, build times can be reduced to only two hours. We will not neglect the print times since they are too much of a significant factor. If we had a lead time of zero, we would never keep any stock and we would not have any backorder costs. This is not a realistic scenario. We will choose a range from two hours to three days, with a standard value of one day. AM lead times are represented by $t_{AM}$, and the unit of measurement is in years.

**Lead times AM (outsourced)**

In the case of outsourcing the print jobs, we will see different lead times than in the standard AM scenario where all the printing is done by the end user. Due to the fact that in the case of outsourcing the printing is not done by the end user anymore, a third party has to supply the spare parts. This will increase the lead times ($t_{AM}$) on the resupplying process. This is because of some extra steps that need to be taken before the part is available. A purchase order has to be made, the job has to be scheduled at the AM company, and the delivery needs to be done. The delivery time in most instances takes between 5 and 21 days (Wellington, 2013). These are the boundaries that we will set for this input parameter. We will assume a standard lead time of 6 days, based on a white paper published by Stratasys (n.d.), a company that develops 3D printers. AM outsourced lead times are represented by $t_{out}^{AM}$, and the unit of measurement is in years.

**Markup**

When we choose to outsource AM, we pay a higher price per part than we would under normal conditions where we would print the part ourselves. We now pay a higher price for every purchased part instead of paying a fixed annual amount of additive manufacturing costs. This is due to the fact that a third party has to supply the printed parts, instead of printing them oneself. The company that will now print the parts, will calculate a markup ($\gamma \geq 0$) on each purchased part. The markup is calculated as follows:

$$\gamma = \frac{\text{Selling Price}}{\text{Marginal costs}} - 1$$

The selling price is the price that is charged from the customers by the supplier. Marginal cost is the change in the total cost that arises when the quantity produced is incremented by one unit. That is, it is the cost of producing one more unit of a good (O'Sullivan & Sheffrin, 2003).

Due to the fact that we do not have markup percentages for additive manufacturing companies at our disposal, we have to estimate what a realistic markup percentage would be. Martins et al. (1996), did a study on markups in manufacturing industries in 14 OECD countries between 1980 and 1992. A small selection of capital goods industries is presented in the table below. Each of the 14 OECD countries has its own markup per industry. The lowest out of the 14 different values that could be found for each industry is depicted in the second column of the table. The same goes for the highest value in the third column.
Based on the study of Martins et al. (1996), we will let the markup vary between 8% and 44% with a standard value of 20%.

**Price per part**

Let the standard price of a part be $C_{part}$. This is the price that would normally be paid for a part when there was no outsourcing. As we mentioned earlier in the paragraph on holding costs, the annual holding costs per part are 25% of the value of a part. To remain consistent, we will choose our range for this input parameter based on the holding costs. We will only multiply the amount by four. The price per part will thus vary between €1,600 and €112,000 with a standard value of €16,000.

**Annual demand**

As mentioned earlier on in this section on holding costs, Karsten (2009) did a study on realistic parameter values for spare parts inventory situations. Besides holding costs, we can also find the ranges that we used for demand in several scientific papers. The ranges vary from 0.5 to 50 in Kranenburg and Van Houtum (2009). However in Kukreja et al. (2001) the ranges are between 1 and 6 parts per annum. Enders (2004) varies demand between 0.61 and 96 per year in his master thesis. We will choose to vary demand between 1 and 18 with a standard value of 5 parts per year per SKU. The demand rate is represented by $m$ and the unit of measurement is parts per year.

**Number of SKUs**

We looked at the literature and found that organizations can have a large number of service parts to be stocked, varying between 2,500 and 300,000 with prices ranging from $270 on average with exceptions up to several hundreds of thousands of dollars (Cohen et al., 1997). Wong et al. (2007) used real life historical data of a company selling agricultural equipment that kept about 2,370 critical SKUs on stock. The demand rates varied from 0.0002 to 13.79 per year. Obviously, we cannot work with thousands of SKUs in our numerical testbed due to the limited capacity of our printer. We also have to keep in mind that not all items may be printable.

Wong et al. (2007) did a computational experiment for the purpose of testing heuristic methods in order to solve their optimization problem. The set up different test beds with two different number of SKUs: one where $|I| = 20$ and one where $|I| = 100$. Since we chose a standard demand of 5 per year per SKU, we obviously cannot have 100 SKUs, since we chose a printing time of exactly one day. This is why we will limit ourselves to 20 SKUs, so $|I| = 20$.

With a standard annual demand of 5 per SKU, we can have 72 SKUs at most, and with a standard number of 20 SKUs we can have a maximum demand of 18 per item per year, with the chosen capacity restrictions. This guarantees stability because in order to have a stable system, the utilization of the printer always has to be lower than 1 ($\rho < 1$) (Kulkarni, 1999). With the chosen parameter values, this will always be the case.
4.2 CONTEXT AND LIMITATIONS
This section gives some more context on the situation and explains the most important limitations of our models. The last two only apply for the high-demand model.

**Printer type: Powder bed fusion**
Since we are focusing on high-end spare parts that can be used in jet engines for instance, it is very likely that we will have to deal with metal parts. Not all AM machines are capable of printing metal. Most printers that are sold in the market, are in fact polycarbonate printers. Since these plastics are not an option for the applications that we have in mind, our model is constructed with metal printers in mind.

The most common technique used for metal printing is also known as Powder bed fusion. Powder bed fusion is the accepted ASTM\(^1\) term for an additive manufacturing process where a point heat source selectively fuses or melts a region of a powder bed. The process is also known as direct metal laser sintering or selective laser melting (Roland Berger, 2013). These are also the kind of printers that are used by Additive Industries in Eindhoven. These printers have a metal base plate on which the final product is printed upon. A very thin layer of metal powder is sprinkled on this base plate after which a high power laser melts the powder into a certain shape. After each layer is built, a new layer of powder is sprinkled upon the base plate and the final object. The laser keeps melting or sintering powder upon the final object. This process is also known as scanning. One layer of powder is about 30 micrometers thick. One can imagine that this process has to be repeated a lot of times until the printed object is finished (Peeters, 2015).

Even though we will for this thesis assume that we only make use of metal printers, this model is still applicable if another type of printer is used.

**Printability of parts**
There is a lot of variety in the complexity, size, and demand of spare parts. A simple part with a high demand, like a screw or a bolt, would not be suitable for 3D printing. The parts with a relatively high complexity and a low demand are suitable for 3D printers (Holmström et al., 2010). However, not everything can be 3D-printed. The outmost complex parts will still have to be made by means of traditional methods. Size is also a limitation for 3D printers. For example, at the moment a build chamber cannot be higher than 50 cm. Making items bigger than that, would not be stable with the current technology (Peeters, 2015). Additive Manufacturing is not a technology that will replace existing ones. It rather complements them. We have to assume that only a part of the total amount of spare parts are printable. It will not fully replace the entire inventory system.

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\(^1\) American Society for Testing and Materials (ASTM), www.astm.org
In-house versus outsourcing
We will assume a model where the company that needs the printed parts owns the printer and thus does not need to outsource. The assumption is that a companies are cautious with sharing intellectual properties with third parties such as printing companies. Another pro for insourcing is that lead times are smaller, since one for instance does not need to transport a part from a printing company to one’s own site, or make purchase orders.

In a variation of our basic model as presented in Chapter 6, we will look at an outsourcing option. The company then does not own printers anymore. They do however pay a markup on the price of a printed part, and the lead times will increase. This is due to the fact that the company will most likely have an agreement with the owner of the printers on a lead time, which will almost certainly be much longer than the original lead times. Fixed printer costs however are gone.

Service region
We are dealing with a company that builds and maintains capital goods. Examples of such companies are Boeing and ASML. These are companies that operate on the global market. Their service region is the whole world. Even though we are assuming a worldwide service region in this thesis, the model is also applicable for a very local setup.

Limited capacity due to having only one 3D printer (high-demand)
Because we chose to only regard a situation where there is just one 3D printer at our disposal, we have a very limited capacity. Assuming that printing a spare part takes exactly one day, we can have an average total demand of no more than 364 items, since the utilization has to be lower than 1. This may be not realistic, since most organizations have much a much higher number of SKUs to manage (Cohen et al., 1997; Wong et al., 2007). This problem can easily be solved with the use of multiple printers. However, this is outside our scope so we limit ourselves in the numerical experiment to only a couple of hundred items at most.

No variance in processing times due to identical items and deterministic printing times (high-demand)
We limit ourselves to identical items, meaning that there is no variance in processing times over the different SKUs. The assumption that the printing time for one specific item should be more or less the same every time it is printed can be justified, as mentioned in section 5.1. The assumption of identical printing times for all items may not fit reality, but one reason for assuming identical printing times is that we can have a clear numerical investigation with $|I|$ as a parameter without muddling it. In section 7.5, we will relax the assumption of identical SKUs and show how we would calculate expected on-hand inventory and backorder levels in a scenario where the lead times per SKU are different.
5 AM SERVICE SUPPLY CHAIN MODEL WITH UNLIMITED PRINTER CAPACITY AND LOW DEMAND

In this chapter we will review a single-item, low-demand situation. This model will also be referred to as our basic model. We will start with explaining the model assumptions in section 5.1. The model choices and mathematical formulas are given in section 5.2. After that we will perform a numerical investigation in chapter 5.3, with the input values from chapter 4. We do this so that we can see under what circumstances AM is cheaper than normal. With that information we can answer Research Question 2a. Section 5.4 concludes this chapter.

5.1 MODEL STRUCTURE

In this section we will explain the model assumptions.

i) Demand is Poisson distributed

We assume that a demand can occur at any time and that the distribution of the mean time to failure for a part is exponentially distributed. This gives us a demand that occurs in a Poisson fashion. We refer back to section 2.1.2, (i) and (ii) for justification of these assumptions.

ii) Printing speeds are deterministic

Because we only print one type of part in the basic model, printing times should be about the same every time that we make that specific part. This is why we assume that we are dealing with deterministic processing times.

iii) Waiting times are neglected

We assume a low demand, such that the printer is idle for most of the time, making waiting times negligible. We refer to Appendix A for a detailed justification of this assumption.

iv) Backorders, no lost sales

Backorders occur when demand cannot be satisfied directly from stock. A backorder has to be placed, and the machine will be down until the backordered part has arrived and has been installed. The party that uses the capital good in order to create value loses money as long as the machine is down. For example, the money that could have been made by flying an airplane with passengers and goods, is now lost due to the fact that the plane is inoperative. A model with backorders is very common in scientific literature on spare parts (Wong et al., 2007; Van Houtum & Hoen, 2010; Karsten & Basten, 2014), and we will thus also assume a model with backorders.

v) No batching in 3D printing

Even though it is possible to produce in batches of more than one item (Peeters, 2015), we will still work with batches of one due to the relatively high holding costs that we are facing.

vi) The supply system behaves like an M|G|∞ Continuous-Time Markov Model

Based on the assumptions thus far, we can also assume that the supply system behaves like an M|G|∞ model.
vii) Parts are consumable
This model assumes that a failed part will not be repaired. If we would repair a part and also print new parts, we would increase the total amount of parts in the system, but for simplicity our model assumes that parts are consumable. Alternatively, in case of repairable parts, the model also applies if printing costs are equal to repair costs. In that case you would never choose for repair, since repair lead times would almost never be as low as printing times while the costs would still be the same.

viii) Quality of printed parts
We assume that the quality of an additive manufactured item is identical to an item made by means of traditional manufacturing. This means that parts that were made by traditional means have the same failure rate as 3D printed parts. If this was not the case, we would expect different demands for the regular/traditional model and the AM model. Even though difference in quality might be interesting to look at, we will for now focus on comparing the two situations where the demands are equal. The U.S. Federal Aviation Administration (FAA) already cleared the first 3D printed metal part to fly (General Electric, 2015). Although this does not mean that they have equal quality, it does mean that the quality of a 3D printed part is acceptable by the standards of the authority that oversees U.S. civil aviation safety.

ix) Regular parts have the same price as AM parts
We assume that if we print a part, we pay just as much for that part as when we would order a new part that was made by non-AM means of production. If we would deal with mass production, this assumption is not a reasonable one to make since we have no economies of scale in AM. However, due to the low demand of the kind of parts that we are dealing with here, there are no benefits of mass production. A recent study by Thomas & Gilbert (2014) for the U.S. National Institute of Standards and Technology shows that AM can be cost effective for manufacturing small batches. They however also say that it is very hard to measure AM costs and that there are only a limited amount of studies on this subject. A study from Allen (2006) showed that AM can be 30% cheaper than traditional manufacturing if the buy:fly ratio is 12:1 or higher. This is the ratio of the weight of the component as installed in the aircraft referred to the weight of the bought-in material prior to machining. This is typically the case for components with high raw material costs and complex shapes (Allen, 2006). Allen (2006) also states that in the future, the ratio only has to be 3:1 for AM to break even. Typical titanium aero engine components have buy:fly ratios varying from 6:1 to 20:1 (Allen, 2006). In short, AM in general is more expensive than traditional means of production, but can be cheaper if we are dealing with low volumes, high raw material costs and complex shapes.

Because the focus of our study is not on traditional versus AM production costs, but rather on the trade-off between less inventory versus 3D printer costs, we will leave the difference in production costs between the two methods out of our scope.

5.2 MODEL CHOICES AND MATHEMATICAL FORMULAS
The mathematical description of our inventory model, also referred to as the basic model as described in this chapter, is based on the single-item, single-location model as described in chapter 2.

In the general single-item, single-location model, we had a couple of input parameters that we will also use in our own model. The annual demand is denoted by \( m (>0) \). We also had two cost parameters. The penalty costs that were paid per year for each backorder denoted by \( p (>0) \) and the holding costs per item per year \( h (>0) \). We will make a distinction between the lead times in the regular scenario (i.e. without additive manufacturing) and the lead times when we do have a 3D printer at our disposal (print
time). The lead times in the regular scenario are denoted as $t^{Reg} (> 0)$. The lead times in the AM scenario are denoted as $t^{AM} (> 0)$. Because we have to make additional costs when we make use of a 3D printer we also have to incorporate this in our model. Our annual AM costs will be denoted by $C^{AM}_{Ann} (≥ 0)$.

Due to the fact that we now have two different scenarios; a scenario that includes AM, and one that does not, we also have different cost compositions for each scenario. The total expected costs per time unit in steady state in the scenario with AM is denoted by $C^{AM}(S)$. In the regular scenario these costs are denoted by $C^{Reg}(S)$.

\[
C^{Reg}(S) = h \cdot E[OH(m, t^{Reg}, S)] + p \cdot E[BO(m, t^{Reg}, S)], \; S \in \mathbb{N}_0
\]

\[
C^{AM}(S) = h \cdot E[OH(m, t^{AM}, S)] + p \cdot E[BO(m, t^{AM}, S)] + C^{AM}_{Ann}, \; S \in \mathbb{N}_0
\]

Here we used the on-hand stocks and backorders as derived in section 2.1.3.

The optimal costs are defined by $C^{Reg*} = C^{Reg}(S^{Reg*})$ and $C^{AM*} = C^{AM}(S^{AM*})$

$S^{Reg*}$ is the optimal basestock level for the regular scenario and is defined as the smallest element of

\[\arg\min_{S \in \mathbb{N}_0} C^{Reg}(S)\]

$S^{AM*}$ is defined analogously for the scenario with AM and thus is the smallest element of

\[\arg\min_{S \in \mathbb{N}_0} C^{AM}(S)\]

### 5.3 RESULTS OF A NUMERICAL INVESTIGATION

In chapter 4 we have seen what the ranges are for the values of the input parameters. We will now see what happens when insert these values in our model. First, we will look at what the expected costs and optimal base stock levels are if all parameters are set to their standard values. After this first analysis we will fix all parameters to their standard value, with the exception of one parameter which we will vary between its minimum and maximum value. We will then analyze what the behavior of the output parameters, S-levels and the expected costs, are. This can give some insight in the feasibility of procuring a 3D printer.

**The standard situation**

It can be seen in Table 5-1 what the output parameters are when all input parameters are set to their standard values. It is clear that the AM option is far too expensive to even consider as an alternative option.

<table>
<thead>
<tr>
<th></th>
<th>Regular</th>
<th>AM</th>
</tr>
</thead>
<tbody>
<tr>
<td>Optimal S-level</td>
<td>6</td>
<td>2</td>
</tr>
<tr>
<td>Costs</td>
<td>€ 23,056</td>
<td>€ 87,979</td>
</tr>
</tbody>
</table>

**Table 5-1 Output values in the standard situation**
Varying demand
When we vary our demand between 1 and 18, and keep the other parameters at their standard values, we can see that the expected costs for the regular model are gradually increasing, while the expected costs for the AM Model are more or less stable. The optimal order-up-to levels also drastically increase in the regular model, while the AM model has fairly stable optimal S-Levels.

Figure 5-1 Cost with varying annual demand

Figure 5-2 S-Levels with varying annual demand
**Varying holding costs**

Once we start increasing holding costs, we see that the expected costs for the regular model increase rapidly, while the AM model only sees a very limited increase in expected costs. The rapid increase in costs are due to the fact that holding costs per part increase, but the S-levels remain fairly stable. At a certain point it even becomes cheaper to choose the AM option. The S-levels do not change very much for both situations. For companies with very high holding costs (≥ €24,000), AM can be an interesting option, even in this basic single-item setup.

![Figure 5-3 Cost with varying holding costs](image1)

**Figure 5-3 Cost with varying holding costs**

![Figure 5-4 S-levels with varying holding costs](image2)

**Figure 5-4 S-levels with varying holding costs**
Varying penalty costs
We do not see much of a difference if we increase the penalty costs in the standard situation. This is probably due to the fact that the chances on a high number of backorders are very low, and thus do not make much of a difference for the expected costs. The optimal base stock levels however more than double in the regular situation. There is only a minor increase of S-Levels in the AM situation.

![Costs with varying penalty costs](image1)

**FIGURE 5-5 COST WITH VARYING PENALTY COSTS**

![S-Levels with varying penalty costs](image2)

**FIGURE 5-6 S-LEVELS WITH VARYING PENALTY COSTS**
**Varying regular lead times**
When we vary the regular lead times, we see a minor steady increase in expected costs for the regular model and a strong increase in S-Levels for the regular model. The AM model is obviously not influenced by this input parameter and thus remains the same.

**Figure 5-7 Costs with varying regular lead times**

**Varying AM lead times**
Varying the AM lead times between our chosen boundaries makes almost no difference. Costs and S-levels remain more or less the same.

**Figure 5-8 Costs with varying AM lead times**
Varying annual printing costs
Once annual costs of printer increase, we see a sharp linear incline in the expected costs. However, the S-Levels remain the same for all cases.

**Figure 5-9 Cost with varying annual printer costs**

5.4 CONCLUSION
It is quite obvious that Additive Manufacturing is not a feasible option in the current setup of a single-item, single-echelon situation with these chosen parameter values. This is due to the fact that AM printers are still relatively expensive. Only when the holding costs become very high or if demand rates are higher, 3D printers might be a good solution. For the other situations, it is most likely too expensive to purchase such a machine. Its utilization would also be very low. However it could be interesting once we have more items to print. It also might be interesting to consider outsourcing the print job. We will elaborate on outsourcing in the next chapter, and on higher demand rates in Chapter 7.
6 OUTSOURCING AM WITH LOW DEMAND

As we concluded in the previous chapter, investing in AM is typically not feasible with very low demand rates and when printing is done in-house. However, there is also a possibility to outsource the print job. One no longer has to invest in a printer of half a million euros. However, lead times will increase due to extra transportation and handling times and the price paid per part is also higher due to the fact that a markup has to be paid to a third party that does the printing. In this chapter we will see what model changes are applied and we will review if outsourcing 3D printing to a third party is feasible. This chapter answers Research Question 2b.

6.1 MODEL STRUCTURE

The model structure remains largely the same. There are however two structural changes. The AM costs are no longer fixed, but variable. The fixed costs of having a 3D printer are gone, but an extra markup has to be paid for the 3D-printed part to a third party that prints the parts. We also let the holding costs depend on the purchase price of a part and on the markup percentage.

Printing costs
In this model, one does not need to purchase a 3D printer. This of course saves about half a million euros on purchasing costs. However, one does need to pay a markup on the parts that are ordered. We let the markup vary in order to see what the maximum markup can be, before outsourcing becomes too expensive.

Lead times AM
Due to the fact that the printing job is not done by the end user anymore, a third party has to supply the spare parts. This will increase the lead times ($t_{AM}$) on the resupplying process. We will let the lead times fluctuate between a minimum and a maximum in order to see what happens to the output variables.

Impact on holding costs
Due to the fact that the price paid per part is higher for this variation, the holding costs will also increase, since holding costs depend on the value of a part.

6.2 MODEL CHOICES AND MATHEMATICAL FORMULAS

The mathematical formula for calculating costs in the AM scenario is:

$$C_{AM}(S) = \frac{C_{part} \cdot (1 + \gamma)}{4} \cdot E[OH(m, t_{out}^{AM}, S)] + p \cdot E[BO(m, t_{out}^{AM}, S)] + C_{part} \cdot \gamma \cdot m, \quad S \in \mathbb{N}_0$$

The main difference between the costs calculation of the AM model in this chapter and the previous chapter, where we had printing done in-house, is that the fixed component ($C_{Ann}^{AM}$) is removed and replaced by a variable component ($C_{part} \cdot \gamma \cdot m$). This variable component represents the markup that is paid for each purchased spare part. Because we assumed in Chapter 4 that holding costs are 25% (or $\frac{1}{4}$) of the value of a part, the holding costs will increase if the purchase price of a part, or the markup percentage increases. So instead of using $h$, we now use $\frac{C_{part} \cdot (1 + \gamma)}{4}$ to represent the holding cost parameter instead. The lead times are also different. We use $t_{out}^{AM}$, instead of $t^{AM}$, due to the fact that lead times for AM are assumed to be different in case of outsourcing.
6.3 RESULTS OF A NUMERICAL INVESTIGATION

We will now conduct a numerical investigation analogous to the one in section 5.3. All input parameters are set to their standard value except for one that we will vary. The values of the input parameters can be found in section 4.1. This time however, we will review what will happen if we let the markup percentage and the purchase price of a part vary instead of the printer purchase price. We will only depict graphs when we see different behavior than in section 5.3. All the other graphs can be found in Appendix E.

The standard situation

In the standard situation, all parameters are set to their standard values. Now that we have removed the costs of buying a printer, we can already see that the costs for the AM scenario have drastically gone down. The outsourced AM model is still more expensive but the gap between the two is much smaller than it used to be in the basic model.

<table>
<thead>
<tr>
<th></th>
<th>Regular</th>
<th>AM</th>
</tr>
</thead>
<tbody>
<tr>
<td>Optimal S-level</td>
<td>6</td>
<td>3</td>
</tr>
<tr>
<td>Costs</td>
<td>€ 23,056</td>
<td>€ 30,824</td>
</tr>
</tbody>
</table>

TABLE 6-1 OUTPUT VALUES IN THE STANDARD SITUATION (OUTSOURCED)

Varying demand

When we let demand vary, we can observe a very different behavior in comparison to the scenario where AM was done in-house. In that case, a higher demand was more beneficial for the use of AM. However, due to the fact that a markup is paid on every printed part, a higher demand does not favor the use of outsourced AM. The higher the demand, the more expensive AM becomes in comparison to the regular scenario without AM. The behavior of the S-levels remains more or less the same, even though the S-levels are slightly higher for AM compared to the previous chapter, since the lead times have increased with a couple of days.

FIGURE 6-1 COSTS WITH VARYING ANNUAL DEMAND
Varying penalty costs
As we already saw in Section 5.3, penalty costs only have a very small influence on the output variables. We see a similar behavior of the total costs and the optimal S-levels in the case of outsourcing.

Varying regular lead times
Since outsourcing AM does not influence the regular model, we see exactly the same behavior as we saw earlier with the case of insourcing. The difference however, is that we now have an intersection of the two curves, making AM the preferred choice once regular lead times become longer than 0.30 years, whereas in the case of insourcing, AM would never be the preferred choice with regular lead times varying in the domain of this investigation.

![Costs with varying regular lead times](image)

**Figure 6-2 Costs with varying regular lead times**

Varying AM lead times
We see similar behavior of the costs and the S-levels as we did in the previous chapter, even though we have much higher AM lead times. The maximum AM lead time of 21 days is still a lot lower than the average lead time of 2 months in the case of the regular scenario.

Varying AM markup percentage
As mentioned earlier in Section 4.1, we let the markup percentage vary between 8% and 44%. We can see that under the current assumptions, AM is the preferred choice as long as the markup is lower than 14%, which is below our chosen standard value of 20%. S-levels remain constant for all markup values.
When we vary the purchase price of a part, we can see that this has a large impact on the total cost for the AM scenario. This is due to the fact that the price of a part also impacts the markup amount and the holding costs. The S-levels are constant for part prices up to about €100,000. The optimal base stock level for AM decreases with one for higher part prices, due to the increase in holding costs.
6.4 CONCLUSION
We can see that under standard circumstances, outsourcing AM for low demand scenarios is still more expensive compared to the regular situation without any additive manufacturing. However, the difference in costs between the AM and the regular scenario is much smaller due to the fact that annual printer costs are no longer part of the equation. Instead, the price per part becomes higher and the AM lead times increase. AM can be the cheaper alternative in the case when the parameters are slightly adjusted in favor of additive manufacturing. This can for instance be the case if we lower the markup on parts to 14%, if the average demand is only a couple of parts per year, or if the regular lead times become 0.30 years of higher, all while keeping all other parameters at their typical values. Compared to the basic model, where investing in AM is almost always out of the question, AM can in some cases be interesting if the printing are outsourced.

In chapter 5 and 6 we have described the basic single-item, single location model with a variation where the print jobs are outsourced to a third party. Even though chapter 5 and 6 focus on a single-item situation, the model can also be applied for a multi-item situation, as long as the utilization remains low enough so that waiting time can be neglected. In that case, total costs for the multi-item setting is simply the sum of the costs for all individual items.

We have seen that with the current test bed, AM is almost never the preferred choice. This is due to the fact that the printing costs are still relatively high, even if we outsource the print jobs to a third party. Due to the low demand rates, the printer tends to be utilized less than 10% of the time for our SKU. This of course does bring up the question what would happen if demand would be much higher, for instance if would have more than one SKU to deal with? In the next chapter, we construct a model to handle these high-demand situations.
7 AM SERVICE SUPPLY CHAIN MODEL WITH LIMITED PRINTER CAPACITY AND HIGH DEMAND

In this chapter we will extend our basic model, where we assumed low demand and infinite capacity, to a model where we have high demand and limited printer capacity. We assume that the demand rate goes up to an extent where congestion at the printer is inevitable and no longer negligible. This is why we have to include waiting time in our model. This chapter answers Research Questions 3, 3a and 3b.

We will start in 7.1 with the explanation of the model structure, where we will also name the differences between the basic model, and our extended model. Due to the fact that is very difficult to calculate the expected on-hand inventory and backorder levels when we have to include waiting times, we have to find a way to approximate those levels. This is done in Section 7.2. In Section 7.3, we will introduce multiple SKUs, which changes this model from a single-item, to a multi-item model. A numerical investigation is done in 7.4 in order to find out under which circumstances it is beneficial to invest in AM. Up till that point we assume that all items have the same printing time. This assumption is relaxed in 7.5 where we show how to calculate expected on-hand inventory and backorder levels in case of non-identical items with different lead times. This chapter is concluded in 7.6.

7.1 MODEL STRUCTURE

The model structure as described in the basic model in chapter 5 remains largely the same. This is an M|D|1 setup. The big difference with the basic model is that we cannot simply neglect waiting times, as we did up till this point. This is why we have to include waiting time in our model.

i) All items have identical printing times
For the moment we will assume that all items in the system, regardless of SKU, have identical printing times. We will relax this assumption in Section 7.4, where we calculate expected on-hand inventory and backorder levels for non-identical items, with different printing times.

ii) We will not neglect waiting time at the printer anymore
Because we want to see what happens with higher printer utilizations, it is no longer realistic to neglect waiting time.

iii) The supply system is an M|D|1 Continuous-Time Markov Model
In the basic model, we ignored waiting times, so we could assume that the supply process feeding the stock point would behave like an M|D|∞ model. Due to the fact that we no longer disregard waiting times, we can no longer speak of an M|D|∞ system. Based on i) and ii), we can say that we are dealing with an M|D|1 system. So, this is a system where all items have the same printing time, but where capacity is limited. In the next section we will explain in more detail how we will include waiting times in our model and how we will calculate expected on-hand inventory and backorders.

iv) No prioritization of items
We will serve all items on a first-come-first-serve basis. No priority will be made to items whose current stock level is low. This assumption is made for the sake of simplicity.
7.2 APPROXIMATING EXPECTED ON-HAND INVENTORY AND BACKORDER LEVELS WITH FINITE PRINTER CAPACITY

This section is dedicated to finding an appropriate approximation method in order to calculate expected on-hand inventory and backorder levels when we are dealing with a printer with finite capacity. We will start in 7.2.1 by explaining why we need to approximate, instead of making exact calculations. In 7.2.2 we explain how we can calculate actual on-hand inventory and backorder levels. In 7.2.3 we explain several approximation options that we can choose, which we will benchmark against actual values in 7.2.4. That is also when we will choose the best approximation option, which we will use in the remaining part of this chapter.

7.2.1 THE PROBLEM OF FINITE CAPACITY

Now that we include waiting times, we need to adjust our basic M|D|∞ model to incorporate queues. Unfortunately, there is no simple formula to calculate the exact expected on-hand-stock and expected backorders for an M|D|1 supply system with waiting times. It is possible to derive them after determining the steady-state distribution of the number of waiting customers in an M|D|1 queue via advanced queueing theory methods, as we will do shortly. However, this method is complex, numerically unstable, and does not extend well to a setting where items have non-identical printing times.

The problem of finite serving capacity is well known in literature. Sleptchenko (2002) reviewed multiple methods from earlier literature that deal with finite capacity, and compared their outcomes with those of a simulation in order to see how well the different methods perform. Some methods, such as the Jackson network, are very accurate, but are very complex and hard to work with. Other methods, such as the one used by Diaz & Fu (1997), are much easier to work with, but are less accurate. Diaz and Fu (1997) used an approximation where they still would pretend as if they were working with an M|G|∞ supply system, but replaced the net processing times (without waiting time) with gross processing times (where waiting time is included). The approximation method of Diaz & Fu (1997) will be also be reviewed in this work shortly.

The goal of this extension is to provide a relatively simple to understand formula that is still sufficiently accurate for a setting where printing times are deterministic and items are identical. We want to keep it simple so that it can be still be comprehended by employees of companies that want to apply this formula. This is why we have to approximate the expected on-hand inventory and backorder levels. We have to make a decision on how we can derive an easy to understand formula, without compromising too much accuracy.

7.2.2 CALCULATING STATIONARY M|D|1 PROBABILITIES

In order to find out what the actual steady state expected backorder and on-hand inventory levels are, we can also calculate them by means of Taylor expansions (Nakagawa, 2005). We will start by determining the pipeline stock, which can be regarded as a M|D|1 queue. Once we know how to calculate the stationary probabilities of the length of this queue, given a certain base stock level $S \in N_0$, we can calculate the expected on-hand inventory and backorder levels.

Assume that we have an M|D|1 queue with arrival rate $m \in (0,1)$ and service rate 1. So, we measure time such that the average service time is one time unit. The arrival process is Poisson. We can calculate the stationary distribution on an M|D|1 queue as follows. Let $\pi_n$ be the steady-state probability of having a queue length of $n \in N_0$. In that case we have the following formulas for the steady state probabilities of the queue length (Nakagawa, 2005).
\[ \pi_0 = 1 - m \]
\[ \pi_1 = (1 - m)(e^m - 1) \]
\[ \pi_n = (1 - m)(e^{mn} + \sum_{k=1}^{n-1} e^{mn} (-1)^{n-k} \left( \frac{(kn)^{n-k}}{(n-k)!} + \frac{(kn)^{n-k-1}}{(n-k-1)!} \right)), \quad n \geq 2 \]

Let \( P[OH(m, S) = x] \) denote the steady state probability of having \( x \) items on hand. In that case

\[ P[OH(m, S) = x] = \pi_{S-x} \cdot x, \quad x \in N_0, \quad x \leq S \]

We can now determine the expected inventory level

\[ E[OH(m, S)] = \sum_{x=0}^{S} x \cdot P[OH(m, S) = x], \quad S \in N_0 \]

Let \( P[BO(m, S) = x] \) denote the steady state probability of having \( x \) items backordered. In that case

\[ P[BO(m, S) = x] = \pi_{S+x} \cdot x, \quad x \in N_0 \]

We can now determine the expected amount of backorders

\[ E[BO(m, S)] = \sum_{x=1}^{\infty} x \cdot P[BO(m, S) = x], \quad S \in N_0 \]

These values are also verified by means of simulations as can be seen in Appendix C.

The problem with this formula is that it includes alternating additions of positive and negative numbers of very large absolute values, which makes it very difficult to use this formula. We also have to emphasize that this method only works because we are dealing with a single-item scenario here. This would not work if we would deal with multiple items, since we cannot make a clear distinction between different SKUs in the queue, which makes it impossible to calculate the expected on-hand inventory and backorder levels for each SKU. This is why we need to find approximations that do not have these problems.
7.2.3 APPROXIMATION METHODS
In this section we will describe three approximation methods, which we will benchmark in Section 7.2.4 against the actual values, as calculated in Section 7.2.2.

Option 1: \(M|G|\infty\) approximation of supply system
In our basic model, as described in chapter 5, we had an \(M|G|\infty\) system because we ignored waiting times. Even though this assumption of no waiting times is not entirely true, it is safe to assume for low utilizations. We have to test how this system behaves when the workload becomes (much) higher. The expected inventory and backorders were given by \(E[OH(m, t^{AM}, S)]\) and \(E[BO(m, t^{AM}, S)]\) for any \(S \in \mathbb{N}_0\), using the functions as defined in section 2.1.3.

Option 2: \(M|G|\infty\) approximation of supply system with adjusted processing times
Another method is to include waiting time in the gross processing time in a similar fashion as Diaz & Fu (1997). The total lead time for the AM model consists of waiting time in the queue \(E[W]\), and the actual printing time \(t^{AM}\). The waiting time in the queue depends on the utilization \(\rho = m \cdot t\). The new formula for the gross AM lead time \(\tau(>0)\), where waiting time is incorporated now becomes:

\[\tau = E[W] + t^{AM}\]

Where \(E[W]\) is derived from Kulkarni (1999)

\[E[W] = \frac{t^{AM}}{2} \cdot \frac{\rho}{1 - \rho}\]

All the calculations for expected inventory and backorder levels remain the same as in the basic \(M|G|\infty\) model, however we replace \(t^{AM}\) with \(\tau\) so that the new processing times now have the expected waiting time included. This is an approximation because Palm’s theorem is applied even though successive lead times are actually not independent: if one print job has to wait in the queue for a long time, then the next one probably has to do so as well.

Option 3: \(M|M|1\) approximation of supply system
Another option could be to pretend that printing times are exponentially distributed. In that case, it is relatively simple to calculate expected inventory and backorders. The formulas for expected inventory and backorders are derived from Zipkin (2000). The expected on-hand inventory in steady state is

\[S - \frac{\rho}{1 - \rho} (1 - \rho^S), \quad S \in \mathbb{N}_0\]

and the expected backorder position in steady state is

\[\left(\frac{\rho}{1 - \rho}\right)^S, \quad S \in \mathbb{N}_0\]
7.2.4 Benchmarking Approximation Methods and Choosing the Best One

Now that we know how we can calculate what the actual expected inventory and backorder levels are, we can use them as a benchmark for the three different approximation methods. The benchmarking procedure will be as follows.

We will use the basic $M|G|\infty$ model to calculate what the optimal base stock levels are under standard conditions (i.e., print time of exactly one day or $365^{-1}$ years) with varying demand from 10 to 350 and step size 10. We will use these base stock levels to calculate what the expected inventory and backorder levels are according to the different approximation methods. We will also use these $S$-levels as an input value for the calculations of the actual values of the expected inventory and backorder levels (Section 7.2.2). We will then compare their values with different demand rates in order to see which approximation method is the most accurate.

A comparison of the three different approximations with the actual values can be seen in Table 7-1. In the final two columns, the values that were found by means of simulation are also given for the sake of validation. The simulation procedure is described in Appendix C. Due to the fact that expected backorders are in many cases extremely low, we will only compare the expected on-hand inventory levels of the different approximations to the actual values (If we would compare the approximated expected backorder levels with the actual values, we would sometimes get differences of more than 3000%, which is not realistic anymore). In Table 7-2 we can see how much percent the expected inventory deviates from reality for each of the three cases.

When we look at Table 7-2, we can conclude that Option 1 performs poorly. Approximation Option 3 is most accurate for very large utilizations, but is worse than Option 2 approximation for $\rho \leq 88\%$. Option 2 has virtually no deviation from the actual value when $\rho \leq 68\%$. At an 82% utilization, it still deviates less than 10% from the actual value. We conclude that, for our purposes, Option 2 is the most accurate of these three and we will thus choose to continue our modeling with this approximation.
<table>
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<th>Annual Demand</th>
<th>S-Levels for AM Utilization</th>
<th>Option 1</th>
<th>Option 2</th>
<th>Option 3</th>
<th>Actual Value</th>
<th>Simulation</th>
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<td>3.763</td>
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Table 7-1 Comparison of different approximation options to the actual and simulated values.
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<th>Option 3</th>
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<td>268%</td>
<td>-96%</td>
<td>-35%</td>
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Table 7-2 Comparison of expected on-hand inventory levels of the approximation options to the actual values
7.3 INCLUDING MULTIPLE STOCK KEEPING UNITS

Now that we know how to approximate expected on-hand inventory and backorder levels when waiting time has to be included, we can introduce multiple SKUs. We start by describing our input parameters in 7.3.1. Sub section 7.3.2 explains how we can calculate utilization and waiting time in a multi-item setup, followed by 7.3.3 where we explain how costs for both AM and non-AM scenarios are calculated. This section ends with 7.3.4 where we explain how to choose the optimal base stock levels.

7.3.1 INPUT PARAMETER OVERVIEW

Because we are working with a multi-item system now, we have to make a distinction between different stock keeping units (SKUs). The set of SKUs is denoted by $I$. For each SKU $i \in I$, demands occur according to a Poisson process with a constant rate $m_i (> 0)$. The total demand rate for all SKUs together is denoted by $M = \sum_{i \in I} m_i$. We will for now assume that all different SKUs have identical print times $t^{AM} (> 0)$. The mean lead time in the regular scenario is $t^{Reg} (> 0)$. We also have holding costs $h_i (> 0)$ per item on hand per unit time, and penalty costs $p_i (> 0)$ per backorder per unit time. The annual AM costs are $C^{AM}_{Ann} (> 0)$. In Section 7.5 we will review the case of SKUs with non-identical print times.

7.3.2 CALCULATING UTILIZATION AND WAITING TIME IN A MULTI-ITEM SETUP

Now that we have introduced a notation for multiple items, we can also approximate what the steady state expected backorder and the inventory levels are for each SKU. We first need to calculate the utilization of the printer $\rho$.

$$\rho = M \cdot t^{AM}$$

In this case it does not matter which item is chosen for the calculation of the utilization, since all items have the same deterministic printing time and we print them on first-come-first-serve basis. The expected time in the queue is:

$$E[W] = \frac{t^{AM}}{2} \cdot \frac{\rho}{1 - \rho}$$

$E[W]$ is derived from Kulkarni (1999). The actual expected AM lead time $\tau$ for any item $i$ thus becomes:

$$\tau = E[W] + t^{AM}$$
7.3.3 Costs
In the regular scenario, for any vector of base stock levels \( S = (S_i)_{i \in I} \in \mathbb{N}_0^I \), the expected costs per unit time in steady-state are

\[
\bar{C}_{\text{Reg}}(S) = \sum_{i \in I} h_i \cdot E[OH(m_i, t^{\text{Reg}}_i, S_i)] + p_i \cdot E[BO(m_i, t^{\text{Reg}}_i, S_i)]
\]

In the AM scenario, for any vector of base stock levels, the expected costs per unit time in steady-state are

\[
\bar{C}_{\text{AM}}(S) = \sum_{i \in I} h_i \cdot E[OH(m_i, \tau, S_i)] + p_i \cdot E[BO(m_i, \tau, S_i)] + C_{\text{Ann}}^A
\]

7.3.4 Optimization
For every SKU in both scenarios there is an optimal base stock level. The definition of these base stock levels is similar to the ones given for the basic single-item model in chapter 5.2.

\( S^{\text{Reg}*} \) is the optimal base stock vector for the regular scenario and is defined as the smallest element of

\[
\text{argmin}_{S \in \mathbb{N}_0^I} C_{\text{Reg}}(S)
\]

\( S^{\text{AM}*} \) is defined analogously for the scenario with AM and thus the smallest element of

\[
\text{argmin}_{S \in \mathbb{N}_0^I} C_{\text{AM}}(S)
\]

7.4 Results of a Numerical Investigation
In this section we will perform a numerical investigation similar to the one done in the previous two chapters. The input parameter values can be found in Chapter 4. Every input parameter is set at its standard value, with the exception of one, which we vary. Testing both scenarios (AM and regular) with the chosen input parameters gives the following results. We will only show a limited amount of graphs. All other graphs of this model can be found in Appendix F.

The standard situation
In Table 7-3 we can see what the output variables are for both scenarios in the high-demand, multi-item setup when we fix all input variables to their standard values. In the basic model, AM was far too expensive under normal circumstances. In this setup however, AM is the cheaper option. This is due to the fact that AM costs are much lower per item in a multi-item setup. The benefits of shorter lead times, and thus lower inventory however remain.

<table>
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<tr>
<th>Output Variables</th>
<th>Regular</th>
<th>AM</th>
</tr>
</thead>
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<tr>
<td>Optimal S-level per item</td>
<td>6</td>
<td>2</td>
</tr>
<tr>
<td>Total costs</td>
<td>€ 461,112</td>
<td>€ 239,839</td>
</tr>
</tbody>
</table>

Table 7-3 Output values in the standard situation
**Varying demand**

When we let demand vary, we see that AM is always cheaper unless the utilization comes close to 100%. In that case the lead times and thus S-levels ‘explode’. The regular model behaves in a similar fashion as in the basic model.

![Costs with varying annual demand](image1)

**FIGURE 7-1 COSTS WITH VARYING ANNUAL DEMAND**

![S-levels with varying demand](image2)

**FIGURE 7-2 S-LEVELS WITH VARYING DEMAND**
**Varying number of SKUs**

When we let the number of SKUs vary, we see a similar pattern as with varying demand. This is due to the fact that the utilization of the printer varies. We can see that AM becomes cheaper when we have 5 or more different SKUs. However, the closer the utilization comes to 100%, the less it becomes a viable alternative to the regular scenario.

![Costs with varying number of SKUS](image1)

**Figure 7-3 Costs with varying number of SKUS**

![S-levels with varying number of SKUs](image2)

**Figure 7-4 S-levels with varying number of SKUs**
**Varying holding costs**
In the basic model, AM was only interesting when we were dealing with very high holding costs. In this model, we can see that the costs of both models intersect with very low holding costs. This is due to the fact that we are dealing with twenty times as much SKUs, which is beneficial for the AM model because optimal base stock levels for each item in case of AM are lower. The regular scenario is only interesting when holding costs are extremely low. With the S-levels we see a similar pattern as with the basic model.

![Figure 7-5 Costs with varying holding costs](image1)

*Figure 7-5 Costs with varying holding costs*

![Figure 7-6 S-levels with varying holding costs](image2)

*Figure 7-6 S-levels with varying holding costs*
**Varying penalty costs**
Just like in the basic model, penalty costs only have a small impact on the total costs of an inventory policy.

**Varying regular lead times**
Just like with the basic model, varying AM lead times between two hours and two days has virtually no impact on our model at all.

**Varying AM lead times**
As can be expected, increased lead times for our regular model requires higher inventories and thus costs more.

**Varying annual AM costs**
The annual AM costs has absolutely no impact on inventory policies. It does however affect the total costs of the AM policy. Even with a maximum AM costs, AM is still cheaper.

![Costs with varying Annual AM costs](image)

**Figure 7-7 Costs with varying Annual AM costs**
7.5 THE CASE OF NON-IDENTICAL ITEMS WITH DIFFERENT PROCESSING TIMES

Up until now, we always assumed that the printing times would always be the same for all items, even though they might be different SKUs. We will now explain what happens when we have a multi-item setup, where processing/printing times are different, depending on what SKU they are. We will denote the deterministic print time of item \( i \in I \) by \( t_i^{AM} (>0) \). A means to calculate the expected lead time (waiting time in the queue + processing time) will be given, followed by a numerical example.

**Queueing theory**

At the printer, orders for all items in \( I \) arrive, where item \( i \) arrives according to a Poisson process with rate \( m_i \), requiring a print that takes \( t_i^{AM} \) time units. The queue is serviced by a single server, and has infinite waiting room. Such a queue is called an M|G|1 queue (Kulkarni, 1999).

The mean service time \( E[t^{AM}] \) equals

\[
E[t^{AM}] = \sum_{i \in I} t_i^{AM} \left( \frac{m_i}{M} \right)
\]

The variance of the service times equals

\[
\sigma^2 = \sum_{i \in I} \left( t_i^{AM} - E[t^{AM}] \right)^2 \left( \frac{m_i}{M} \right)
\]

The utilization is calculated as follows

\[
\rho = \sum_{i \in I} m_i \cdot t_i^{AM}
\]

The second moment is \( s^2 = E[t^{AM}]^2 + \sigma^2 \)

Once we know the total order arrival rate \( M \) and the utilization of the server (in our case a 3D printer) \( \rho \), we can calculate the expected time in the queue as

\[
E[W] = \frac{Ms^2}{2(1 - \rho)}
\]

The total time spent in the system \( \tau_i \), also known as the lead time for additive manufacturing then becomes

\[
\tau_i = t_i^{AM} + E[W]
\]

We can now calculate the expected inventory and backorder levels with the known formulas from inventory theory, as provided in section 2.1.3, which we will do in Example 2. We have to emphasize that this approximation ignores dependence of successive lead times, as assumed in Section 7.1.
Example 2
Suppose that we have a multi-item setup with two different items: item 1 and 2. We have one printer that gets a printing job once there is a demand for it. Demand (in this case print jobs) for each SKU arrives according to a Poisson arrival process and the printing times are i.i.d. and deterministic. They have the following properties:

<table>
<thead>
<tr>
<th>Item</th>
<th>Printing time $t_i^{AM}$ (days)</th>
<th>Demand $m_i$ (items per year)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>80</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>20</td>
</tr>
</tbody>
</table>

**Table 7-4 Input values for Example 2**

In this case we are dealing with an M|G|1 system. We will now calculate what the expected lead times are for each item with the abovementioned formulas. The time is measured in days. We need to calculate the values of $M$ and $\rho$ first.

$$M = m_1 + m_2 = 80 + 20 = 100 \text{ items per year} = 0.274 \text{ items per day}$$

The printer is busy for 120 days per year ($80 \cdot 1+20 \cdot 2$). The utilization of the printer thus becomes

$$\rho = \frac{120}{365} = 0.329$$

The mean service time equals

$$E[t^{AM}] = \sum_{i \in I} t_i^{AM} \left( \frac{m_i}{M} \right) = 1 \left( \frac{80}{100} \right) + 2 \left( \frac{20}{100} \right) = 1.2 \text{ days}$$

The variance equals

$$\sigma^2 = (t_i^{AM} - E[t^{AM}])^2 \left( \frac{m_i}{M} \right) = 0.8(1 - 1.2)^2 + 0.2(2 - 1.2)^2 = 0.16$$

Now that we know the variance and the mean service time, we can calculate the second moment, after which we will calculate the expected waiting time in the queue.

$$s^2 = E[t^{AM}]^2 + \sigma^2 = 0.16^2 + 1.2^2 = 1.4656 \text{ days}^2$$

$$E[W] = \frac{Ms^2}{2(1-\rho)} = \frac{0.274(1.4656)^2}{2 \left( 1 - \frac{120}{365} \right)} = 0.359 \text{ days}$$

We can see that the average time spent in queue equals 0.359 days which is about 9 hours. The total lead time equals the waiting time plus the service time so:

$$\tau_1 = t_1^{AM} + E[W] = 1.359 \text{ days}$$
$$\tau_2 = t_2^{AM} + E[W] = 2.359 \text{ days}$$
We can now calculate the expected inventory and backorder levels with the known formulas from inventory theory, as provided in section 2.1.3. We can then find out what the optimal base stock levels, $S_1^*$ and $S_2^*$ for item 1 and 2 respectively are with the optimization procedure as described in section 2.1.4. It turns out that the optimal base stock levels for items 1 and 2 are: $S_1^* = 4$ and $S_2^* = 3$. Filling in these input values result in the following inventory and backorder levels.

$$E[OH_1(m_1, \tau_1, S_1^*)] = E[OH_1(80,1.359,4)] = 3.702$$
$$E[OH_2(m_2, \tau_2, S_2^*)] = E[OH_2(20,2.359,3)] = 2.871$$
$$E[BO_1(m_1, \tau_1, S_1^*)] = E[BO_1(80,1.359,4)] = 0.000$$
$$E[BO_2(m_2, \tau_2, S_2^*)] = E[BO_2(20,2.359,3)] = 0.000$$

7.6 CONCLUSION

In the basic model we saw that additive manufacturing is too expensive to regard as a serious alternative to the regular situation, even if we would outsource the printing process. For the input parameters chosen in this extended model however, we can clearly see that in a high-demand multi-item setup AM is the preferred choice. This is due to the fact that we are dealing with far more items now, which has a huge impact on inventory costs. The printer in the basic model was used for less than 10% of the time. This is now no longer the case, making AM relatively less expensive per item. The only limitation of AM is the capacity of the printer. Once the workload becomes close to 100% of the capacity, the waiting times explode and this leads to very high lead times, followed by high inventory costs. One or more extra printers may be an option in that case.

One also has to take into account what the impact of variance in a system can be. If we are dealing with many different items with very different printing times, we will see a high variance in processing times. A high variance leads to longer waiting times in the queue. Once again, one has to be careful that the utilization of the printer is not close to 100%.
8 CONCLUSIONS AND FUTURE RESEARCH

In this chapter we will answer the research questions from chapter 3, summarize the main limitations of our study, and discuss future research topics.

8.1 ANSWERING THE RESEARCH QUESTIONS

The goal of our research project was to develop insight on the possibility of including 3D printers in a spare parts supply chain. We wanted to do this by developing a mathematical inventory model that can help companies to make a decision on whether or not to make use of additive manufacturing in their service supply chain. We first needed to find realistic values that can serve as input parameters for our models.

Our first main research question was:

1. **Which value ranges are realistic for our model's input parameters?**

We had to make a decision on what kind of printer we would use, and chose the powder bed fusion type. This type of printer can print metal parts, which is important in the production of spare parts for capital goods. We interviewed Johnny Peeters, who works at an AM company that is specialized in 3D printing metal parts, and we have studied (scientific) literature to find out what parameter values these printers have, such as their typical printing time and purchase price. The details can be found in Chapter 4. It turns out that AM today is not yet matured, but we expect that the development to faster and more efficient 3D printers will go at a tremendous pace for the next decade. This will lead to an increased application of AM in general, but possibly also in service supply chains.

The first model that we created was a model that assumed a low total demand and infinite printing capacity. We thus had the main second research questions:

2. **What model can aid a company with expensive spare parts in deciding on the use of additive manufacturing in its spare parts inventory policy in a low-demand setting?**

Due to the fact that we assumed such a low demand rate, the expected waiting times were negligible. It was as if we had infinite production capacity. We could thus use Palm’s theorem to calculate expected on-hand inventory and backorder levels, and calculate what the optimal base stock levels had to be if we want to minimize total costs. We wanted to make a cost-benefit analysis where the benefits of the reduced lead times of AM (which means less inventory and thus less holding costs) would be weighed against the costs of AM (mainly the purchase of a 3D printer). This is why we wanted to know:

2a. **Is investing in additive manufacturing beneficial in case of low-demand?**

Apparently, AM is much too expensive in case of printing just one type of item. The demand is so low that the 3D printer is utilized for less than 10% of the time. This clearly is a waste of money. This is why we also wanted to know if we could be beneficial if we outsourced the printing, instead of buying an in-house 3D printer of about a half million euros. The second sub question for Research Question 2 was:

2b. **Is investing in additive manufacturing beneficial in case of low-demand if we outsource the print job to a third party?**

In a setting where the print job is outsourced, the lead times of AM become slightly longer, and a markup is paid for every part that is ordered to the third party that prints the parts. Even though outsourcing is in
many cases much less expensive than buying an in-house 3D printer, it is still more expensive than ordering parts the traditional way, without AM. With higher demand rates, outsourcing becomes much less attractive.

This is why we also looked at a scenario where we have a high demand of parts, such that congestion at the printer is inevitable. We extended our basic model that assumed infinite capacity, to a model that has finite capacity, and thus has to deal with queues. Our final main research question was:

3. **How do we extend our existing ‘basic’ model with waiting times?**

Due to the fact that it is very difficult to calculate the expected on-hand inventory and backorder levels when we have to include waiting times, we found a way to approximate those levels. With this approximation method, we still pretended to have infinite capacity, but we replaced the net printing times with gross printing times. The gross printing times include waiting time in the queue. The higher the utilization of the printer (due to higher demand), the longer the expected waiting time in the queue. This increases the gross production time. For the modeling conditions that we have chosen, this approximation is fairly accurate up till a utilization of the printer of about 82%.

Once again, we performed a numerical investigation. This time to answer the following sub question:

3a. **Is investing in additive manufacturing beneficial in case of high-demand?**

When we did our numerical investigation for the last time, we wanted to know if AM is beneficial for printing expansive spare parts for capital goods in a high-demand setting. This time however we had to conclude that **AM IS CHEAPER THAN TRADITIONAL METHODS OF MANUFACTURING!** This is due to the fact that in case of a high total demand, the holding costs are now so much reduced, that investing in AM is worth the money. One does have to be careful that the utilization of the 3d printer does not reach its maximum, because in that case the expected waiting time in the queue ‘explodes’, which leads to a very high lead time (sometimes even higher than traditional lead times).

Because we assumed that all parts had identical printing times for the largest part of this thesis, we also wanted to relax this assumption:

3b. **How do we calculate expected on-hand inventory and backorder levels when we are dealing with non-identical items?**

Non-identical items are items that have different processing/printing times, depending on which SKU they are. We showed in Section 7.5 how we have to deal with non-identical items and variance in processing times.

We have seen that AM is not beneficial for a low demand setting, but it is cheaper when we are dealing with higher demands (probably with multiple items).
8.2 LIMITATIONS
There are several limitations to this study.

**Deterministic and identical processing times**
In our numerical test bed, we always assumed the same deterministic printing times for all items. In the case of one item, this is reasonable, since printing the same item over and over again should always take about the same amount of time. However, with different SKUs, this might no longer be the case. Some items may be bigger or more complex than others which leads to variations in the printing times. We know from queueing theory that variance in processing times leads to longer expected waiting times, which in turn leads to longer throughput times. We have not taken variance of printing times into account when running a numerical experiment. We did however show how to incorporate variance into mathematical expressions. Due to the fact that we did not take variance into account in the numerical test bed, the results are skewed in favor of AM. The true lead times for AM would in reality be higher due to variance.

**Variable AM costs**
Due to the fact that variable AM costs, such as personnel costs, are so low in comparison to the fixed AM costs, we did not take these costs into account in Chapter 5 and 7. However in reality, one would probably see some variable AM costs. When we would increase the total demand, we would see a steeper graph for AM than we see now.

**Single-echelon system**
We have only included a single-echelon inventory system in our scope. However, in reality most systems are multi-echelon systems (Sleptchenko et. al, 2002) where some parts are stored locally, and some centrally. This gives a pooling effect so that the total amount of inventory can be lowered. The literature on AM in supply chains also mentions the possible benefits of having a local printer that prints all items, instead of being supplied from a central hub (Holmström et al., 2010; Khajavi et al., 2014). We have not looked at what the effects of AM are for multi-echelon systems.

**Reparability of parts**
We have assumed that the critical parts that break down are either damaged beyond repair or have repair costs that are just as high as the costs of printing a new part. This is a very strong assumption to make. In reality, this does not have to be the case. It might be much less expensive to send a part to repair than to print an entirely new one. The repair lead times may also be shorter than the lead times of the supply of new parts. Because we did not take these factors into account, the results might be positively skewed in favor of AM.
Quality of printed parts
We assumed that the quality of printed parts are equal to the ones gained by traditional means. Even though we have reason to believe AM parts are of decent quality (General Electric, 2015), we have no proof that this actually is always the case. It is possible that 3D printed parts are of inferior quality in comparison to their ‘normal’ counterparts due to the novelty of the technology. Due to the fact that AM parts might be of inferior quality, these parts would have a smaller mean time to failure, which leads to a higher demand, and thus to higher costs. If that is the case, than our model is too optimistic about AM. We do however expect that the quality of AM products will increase in time, as can be expected of all technologies that reach maturity.

8.3 FUTURE RESEARCH
Now that we have stated our main limitations, we can explore possible future research areas. The first one being a multi-echelon model instead of a single-echelon one. According to literature, extra benefits may occur due to the fact that all parts are readily available in a local service center instead of one central hub that serves an entire region (Holmström et al., 2010; Khajavi et al., 2014).

We have worked in a setting that only contained one printer. Another research possibility could be a setup that has multiple servers or printers. In that case, an additional output parameter could be the amount of printers that an organization needs to purchase.

Instead of assuming deterministic lead times, another model can be developed that allows for more variance in processing times. Instead of deterministic printing times, one could for instance choose for a uniform or lognormal distribution of processing times.

For the sake of simplicity, we have assumed a first-come-first-serve policy. However, in reality, items that are low on stock may get a higher priority than other items.

A study also needs to be done on what makes a part printable. We do not yet know what percentage of the spare parts is actually makeable by means of additive manufacturing and what the criteria are that make this possible.

We made the assumptions that all failing parts have to be re-ordered. However in reality we may see a policy where repairs are done. A model where a combination of repair and AM is included may also lead to insightful information.

Another big challenge are IP rights. We assumed that any part can just be printed if we want to. What happens if anyone with a 3D printer can simply print an item without any restrictions? What kind of agreements have to be made? What are the possibilities of open source designs?

There is a lot of potential for the use of additive manufacturing in the future and we hope and expect that the research on the potential applications of AM will continue. There is a good reason why some have called 3D printing the next industrial revolution (Hopkinson et al., 2006; Berman, 2012).
REFERENCES


APPENDICES

A. CALCULATING EXPECTED WAITING TIME IN THE BASIC MODEL

Consider a single-station queueing system where orders arrive according to a Poisson process and require independent and identically distributed (i.i.d.) service times with mean $t$, variance $\sigma^2$ and second moment $s^2 = t^2 + \sigma^2$. The service times may not be exponentially distributed. The queue is serviced by a single server, and has infinite waiting room. Such a queue is called an M|G|1 queue (Kulkarni, 1999).

The utilization is calculated as follows

$$\rho = m \cdot t$$

Once we know the utilization of the server (in our case a 3D printer) $\rho$, we can calculate the expected time in the queue $E[W]$. Since there is no variance when we have a deterministic printing time, $s^2 = t^2$. In our basic model we have deterministic printing times so we get the following formula for $E[W]$.

$$E[W] = \frac{m \cdot s^2}{2(1-\rho)} = \frac{m \cdot t^2}{2(1-\rho)}$$

Our input parameters have the following values: $m = 5$ items per year. $t = \frac{1}{365}$ year. This gives us a utilization of $m \cdot t = \frac{5}{365} = \frac{1}{73}$. Filling in these values for $E[W]$ gives us an expected waiting time of only 10 minutes which is but a tiny fraction of the total printing time of 24 hours. This is why we can neglect waiting time in our basic model.
B. SUMMARY OF AN INTERVIEW WITH JOHNNY PEEETERS

Johnny Peeters is an industrial engineering student at the University of Technology in Eindhoven. He is also an employee of Additive Industries. Additive Industries is a company that is dedicated to bringing metal additive manufacturing for functional parts from lab to fab by offering a modular 3D printing system and integrated information platform to high-end and demanding industrial markets (Additive Industries, n.d.). The company works together with 8 other partners in a shared facility called Addlab. AddLab is the first 3D printing pilot factory for the production of industrial metal parts in Eindhoven, The Netherlands. It is built on the ambition to develop a broad range of High Tech and high end manufacturing applications for 3D metal printing (Addlab, n.d.).

They are currently developing a machine that uses Selective Laser Melting (SLM) technology. The current SLM technology still requires a lot of manual labor. This is something that Additive Industries would like to see automated so that the total processing time decreases. SLM is not a technology that will be used if a part has a lot of mass. SLM is interesting if complex shapes have to be formed.

Johnny confirmed that a purchase price of € 0.5 M is about normal for SLM printers. The price might go up in the future because the machines are getting far more advanced. Also, it is reasonable to assume that 3D printers have a lifetime of 10 years.

Johnny also explained what the printing process looks like. They start with a design that is inspected and transformed into a print job. The printer has a baseplate on which the object is built. This is done by adding layers of metal powder and melting the layers with a laser. This process is called scanning. The faster the scanner, the more expensive the machine usually is. The powder has a diameter of about 30 μm. This number always needs to be smaller than the actual layer thickness.

Printed objects can be 25 cm in height at the most. Printing larger objects becomes difficult due to the fact that too much heat is built up without having anywhere to go, deforming the final product. To work as efficiently as possible, one has to work in batches if possible. Multiple items can be printed simultaneously on the same base plate. This saves a lot of time, since it can take over half a day just to print an object. With all the manual labor required before and after the print job, the total time for printing a part can become a full day. This is only the case if the part was printed correctly, which is not always the case. In the case of a new design a lot of trial and error occurs before they get it right. The reproducibility of parts is bad at the moment. It is expected that this will improve in the near future.

For more information on 3D printing, Johnny refers to a book called Additive Manufacturing Technologies by Brent Stucker, who is an expert on AM.
C. VERIFICATION OF ACTUAL STEADY STATES IN AN M|D|1 SETTING BY MEANS OF SIMULATION

We can verify our steady state calculations of Section 7.2 by means of simulation. In this part we will explain the simulation procedure.

In order to test the different models with reality, we will calculate what their expected backorders and on-hand inventory levels are for a certain set of input values, \((m, t, S)\) according to our basic \(M|G|\infty\) model, as described in chapter 5. We will then test what the actual average backorders and inventory levels are when we choose these values for our input parameters in our simulation. The simulation has a run length of 10,000 years. We choose to have a large run length, since (multiple) simulations with a short run length tend to suffer from a bias in the point estimator of the steady-state mean (Law, 2007). If a spare part has a lifetime of 30 years, then 10,000 years equals more than 300 times the average lifespan of a part, so it will give a clear picture of what happens to the inventory and backorders in the long run. We will also verify what happens to the output variables if we choose to vary the length of a run. If we choose our simulation horizon long enough, we expect that there will be very little variation in the output variables when we vary the time horizon a couple percent. For example, we should see about the same average amount of backorders in a simulations with a runtime of 9,500, 10,000 and 10,500 years. We will perform this verification after the simulation. The higher the utilization, the more the influence of waiting times become and thus the more deviation from real life we will see in our basic model. This way we can see up to which utilization our basic model is representative for real life situations.

Input values

We will now choose to keep our printer busy up to 96% of the time and see what the optimal order-up-to levels are according to our basic model. We know that the system would ‘explode’ once we reach 100% utilization. 96% utilization is also well over 80%, which would be considered as normal utilization (Roland Berger, 2013). The basic model predicts also the expected inventory and backorder amount.

The simulation runs on a program called Arena by Rockwell. Printing times were fixed to 24 hours and the simulation had a runtime of 10,000 years. We let the demand vary between 10 and 350 items per year in order to see what would happen to the average inventory and backorder levels for a chosen \(S\)-level. The chosen \(S\)-level is the calculated optimal \(S\)-level in the basic \(M|G|\infty\) model, as described in chapter 5. The results can be found in Table 7-1.

Simulation verification

In the Table C-1, we can see what the total number of backorders, the average inventory levels and the average backorder levels are for simulations of different run lengths. We can see that there is no variance in average inventory levels and that there virtually no variance in average backorder levels. The total amount of backorders does differ, as we would expect with different run lengths. With this information, we can conclude that 10,000 years is a representative run length for steady state probabilities.

<table>
<thead>
<tr>
<th>Run time (years)</th>
<th>Number of backorders</th>
<th>Avg. inventory level</th>
<th>Avg. backorder level</th>
</tr>
</thead>
<tbody>
<tr>
<td>9,500</td>
<td>619</td>
<td>0.9865</td>
<td>0.00008600</td>
</tr>
<tr>
<td>10,000</td>
<td>646</td>
<td>0.9865</td>
<td>0.00008563</td>
</tr>
<tr>
<td>10,500</td>
<td>685</td>
<td>0.9865</td>
<td>0.00008641</td>
</tr>
</tbody>
</table>

**Table C-1: Comparison of output variables for simulation of different run lengths**
D. **ALL GRAPHS OF THE UNLIMITED CAPACITY MODEL**

**FIGURE D-1 COSTS WITH VARYING ANNUAL DEMAND**

**FIGURE D-2 S-LEVELS WITH VARYING ANNUAL DEMAND**
Figure D-3 Costs with varying holding costs

Figure D-4 S-levels with varying holding costs
**Figure D-5 Costs with varying penalty costs**

**Figure D-6 S-Levels with varying penalty costs**
**Figure D-7** Costs with varying regular lead times

**Figure D-8** S-levels with varying regular lead times
FIGURE D-9 COSTS WITH VARYING AM LEAD TIMES

FIGURE D-10 S-LEVELS WITH VARYING AM LEAD TIMES
Figure D-11 Costs with varying annual printing costs

Figure D-12 S-levels with varying annual printing costs
E. ALL GRAPHS OF THE OUTSOURCING MODEL

**Figure E-1** Costs with varying annual demand

**Figure E-2** S-levels with varying annual demand
**Figure E-3 Costs with Varying Part Purchase Price**

**Figure E-4 S-Levels with Varying Part Purchase Price**
Figure E-5 Costs with varying penalty costs

Figure E-6 S-levels with varying penalty costs
FIGURE E-7 COSTS WITH VARYING REGULAR LEAD TIMES

FIGURE E-8 S-LEVELS WITH VARYING REGULAR LEAD TIMES
**Figure E-9** Costs with varying AM lead times

**Figure E-10** S-Levels with varying AM lead times
**Figure E-11 Costs with varying markup**

**Figure E-12 S-levels with varying markup**
F. ALL GRAPHS OF THE LIMITED CAPACITY MODEL

**Figure F-1** Costs with varying annual demand

**Figure F-2** S-levels with varying demand
Figure F-3 Costs with varying number of SKUs

Figure F-4 S-levels with varying number of SKUs
**Figure F-5 Costs with varying holding costs**

**Figure F-6 S-levels with varying holding costs**
**Figure F-7 Costs with Varying Penalty Costs**

**Figure F-8 S-levels with Varying Penalty Costs**
FIGURE F-9 COSTS WITH VARYING AM LEAD TIMES

FIGURE F-10 S-LEVELS WITH VARYING AM LEAD TIMES
Costs with varying regular lead times

S-levels with varying regular lead times
**Figure F-13 Costs with Varying Annual AM Costs**

**Figure F-14 S-levels with Varying Annual AM Costs**