MASTER

Spare parts inventory control for a single-echelon, multi-location model with lateral transshipments

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Spare parts inventory control for a single-echelon, multi-location model with lateral transshipments.

by

R.J.M. van Sommeren

in partial fulfilment of the requirements for the degree of

Master of Science
in Operations Management and Logistics

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I. Abstract

This Master Thesis describes the development of an inventory control model for spare parts at Vanderlande Industries (VI). The inventory control model determines the base stock levels of spare parts at VI’s local warehouses, such that total system costs are minimized under system-focused customer service level restrictions. Two case studies are conducted to analyze the influence of the new planning method. The developed model is applicable for all VI’s local warehouse and the planning tool is useful for building business cases for customer requirements.
II. Preface

This Master Thesis report is the result of my graduation assignment at Vanderlande Industries in Veghel. This assignment represents the final assignment of my study Industrial Engineering and Management Sciences at Technische Universiteit Eindhoven.

First of all, I would like to thank my supervisor of the university, Geert-Jan van Houtum. His support and advice helped me through several stages of my project. The several discussion sessions we had were enjoyable, but were moreover very useful for the progress of my project. Secondly, I would like to thank my second supervisor, Tarkan Tan, for his feedback on my reports.

Furthermore, I want to thank my supervisors of Vanderlande Industries. I would like to thank Katja Kleinveld for all her time and effort she invested in my project. Her comments and her added value to the project are greatly appreciated by me and our discussion sessions were very pleasant. Next, I want to thank my second supervisor, Harold Bol. His constructive criticism on the model helped me improving different aspects of my model and was very helpful during the project. Furthermore, I would like to thank everyone who has helped me with providing the necessary data and information.

Finally, I would like to thank my family and friends who have supported me during my five years of study at the university.

Ruud van Sommeren

Veghel, August 2007
III. Executive Summary

Vanderlande Industries (VI) is located in Veghel (the Netherlands) and is an international organization with a worldwide reputation in design, implementation and servicing of innovative material handling solutions. VI is one of the leading suppliers and integrators of baggage handling systems at airports, material handling systems for distribution centres and express parcel sorting facilities.

Due to increasing worldwide competition and shrinking profit margins, high-technology-product manufacturers are forced to find new ways to differentiate themselves from their competitors. Providing a fast, high-quality after-sales service to your customers contributes to this. Service is provided as part of the contract between the customer and the manufacturer, and therefore, a logistics network capable of serving customers in a time-responsive manner is crucial for a successful after-sales service. Providing service to their customers is one of the top priorities for VI and therefore the department of Supply Chain Management Services was founded.

Customers of VI are increasingly demanding a higher availability of their systems. VI provides their customers the requested service by maintaining the systems. Both corrective and preventive maintenance are executed by service engineers to maximize the performance of the systems. Once a part breaks down, the performance of the system decreases. Corrective maintenance is carried out in case of failures of parts. This requires the availability of spare parts. The failure rates of spare parts are low and these parts can be very expensive. It is therefore unprofitable for customers to keep spare parts inventory individually. A customer can choose for the option that VI takes care of the spare part provisioning. A service level agreement between the customer and VI, for the availability of spare parts within a certain time frame, is agreed upon. This target service level should be met by VI against reasonable costs. Currently, a classical single-item approach is used to determine the base stock levels of the spare parts. The department Supply Chain Management Services wanted to develop a method that determines the base stock levels of spare parts at the local warehouses that uptimes system uptime against minimal supply chain costs.

The research assignment was formulated as follows:

“Develop a planning algorithm which determines the base stock levels of the spare parts at VI’s local warehouses and minimizes total system costs, subject to customer service requirements.”

This research assignment fits in the overall strategy of the Supply Chain Management Services Department. The Supply Chain Management Services Department wants to increase the provided service level to their customers and decrease the overall supply chain costs. The use of consignment stock and spare parts availability for customers are concepts with which VI wants to professionalize its service logistics. The developed planning method determines the base stock levels of spare parts at VI’s local warehouses. The determined base stock levels of spare parts provide a certain spare parts availability level requested by customers.

The central warehouse located in Veghel is left out of the scope of the assignment. A method has been developed that determines the base stock levels of the spare parts in the local warehouses.
The objective for VI is to optimize the up-time of customers systems. The developed model was derived from the model of Kranenburg [9]. A new customer service measure has been introduced: time-based aggregate fill rates. This required a few adjustments in the optimization procedure of Kranenburg. The new service measure enables VI to provide a clear overview of spare part availability in a certain time frame. The developed model takes item criticality and item batch sizes into account.

Two case studies are conducted, one for a customer with two VI systems and one for a customer with fourteen VI systems. These case studies are conducted to determine the effect of the new planning method.

The main conclusions drawn from the research are:

- The planning tool obtained a decrease in inventory value compared to the current inventory value in both case studies. The customer with two VI systems can achieve a reduction of 53% in inventory value under the new planning method compared to the current planning method. The other case study shows a reduction of 31% in inventory value compared to the current planning method. This reduction in inventory value does not influence the service level i.e., the aggregate fill rates remain the same. The new planning tool is also able to determine new base stock levels that generate higher service levels than the current base stock levels, while the costs for inventory remain constant.

In the case studies we have illustrated that the multi-item approach shows explicit advantages compared to the single-item approach. The objective of spare part inventory is shifted from item availability to total system availability. By using a multi-item approach, expensive items with low service levels are compensated by cheaper items with high service levels. This enables a close-to-optimum spare part availability subject to a certain budget.

- The developed planning tool is a useful tool to build business cases around customer requirements. A business case consists of a number of locations, either defined as a regular or main local warehouse, different criticality groups, etc. Several business cases can be modelled and drawn up, such that they can easily be compared with each other on the basis of the results of the planning tool. The different scenarios in the case studies are examples of how a business case can be designed. The mutual results are used to determine the ‘best’ situation with respect to the number of main local warehouses and/or customer service levels. The planning tool is a useful tool to support the decision making processes in the design of a new spare part provisioning structure.

The main recommendations are:

- The developed planning tool is a useful tool to determine the base stock levels at VI’s local warehouses. The new planning method has shown remarkable cost savings in inventory value compared to the current planning method. Another advantage is that the service measure enables VI to provide a clear overview of spare part availability in a certain time frame. It is therefore recommended to implement the new planning tool in practice.
The output of the new planning method is based on the available information. The more reliable this information is the more reliable the output of the new planning method. The reliability of the item’s failure rate influences the performance of the output of the new planning method. Resale items are left out of the scope of the research assignment due to the low availability of information for these items. Each resale item should be assigned with one unique VI item number, such that reliable information can be gathered on the failure rate. It is reasonable to assume that the new planning method is suited for resale items and that comparable or even better results can be achieved due to inventory pooling. The introduction of a Service Management System, which is currently under development by VI, can provide this information.
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0. Introduction

This report presents the results of my Master Thesis project on spare parts inventory control at Vanderlande Industries (VI). Providing service to their customers is becoming one of the top priorities for VI. Customers of VI are increasingly demanding a higher availability of their systems. VI provides the requested service by maintaining the systems. Service engineers of VI execute corrective and preventive maintenance to maximize the performance of the system. Once a part breaks down, the performance of the system decreases. Corrective maintenance is carried out in case of failures of parts. This requires the availability of spare parts.

A model has been developed that determines the base stock levels of spare parts at VI’s local warehouses. The model aims to minimize total system costs subjected to customer requirements. This report describes the developed model and the impact it has on the provisioning of spare parts to customers.

This report consists of eight chapters. Chapter 1 provides a brief description of Vanderlande Industries and subsequently describes the research assignment. Finally, the project outline and contents of the rest of the report are presented in Section 1.4
1. Company description and research assignment

This chapter provides a brief overview of the company Vanderlande Industries. Vanderlande Industries (VI) is located in Veghel (the Netherlands) and is an international organization with a world wide reputation in design, implementation and servicing of innovative material handling solutions. VI is one of the leading suppliers and integrators of baggage handling systems at airports, material handling systems for distribution centres and express parcel sorting facilities.

Section 1.1 introduces Vanderlande Industries by providing some financial results and product characteristics. Section 1.2 describes the department Supply Chain Management Services. The research assignment and the demarcation of the project are presented in Section 1.3. Finally, the project outline is presented in Section 1.4.

1.1. Introduction

Vanderlande Industries was founded in 1949 from a small company by E. van der Lande, whose primary business was service and repair of machines for the textile industry. The very first product was the “Vanderlande Elevator”, being a simple tool for lifting and transporting heavy objects like barrels and the like. From that time onwards, the company developed into a business dedicated to transportation and materials handling systems. After several joint ventures, Vanderlande Industries became an international organization. The complete Research and Development department is located in Veghel, the Netherlands, but the company has sales, project and service branches in many different countries all over the world (See [6]).

Figure 1 shows the net sales of the last ten years of VI. A large proportion of the net sales consists of very large contracts of a small number of customers. These contracts are worth hundreds of millions of euros and are shown separately.

Figure 1 Net sales in the period 1998 – 2007 (millions in €)

1 VI Annual report 2007
The order intake at VI is divided in four sections, namely Baggage Handling, Distribution, Express Parcel and Services. The percentages of the average order intake over the years 2002-2006 are presented in Figure 2. The large contracts are excluded in this figure.

![Figure 2 Percentages of average order intake FY2002-2006](Image)

The different sections can be defined as follows:

- **Baggage handling**: Vanderlande Industries designs, builds and services leading baggage handling systems for airports of all sizes. VI baggage handling systems are located in several large international airports, like Amsterdam Airport Schiphol, Hong Kong Int. Airport, London Heathrow Airport, etc. VI baggage handling systems are spread at different airports throughout the world.

- **Distribution**: Vanderlande Industries provides automated handling systems for order selection and sortation in distribution centres. These systems are located in different distribution centres at i.e. Daimler Chrysler, Argos, Colombini, etc.

- **Express Parcel**: Vanderlande Industries offers today’s widest range of technologies for parcel handling and documents. VI’s parcel automation systems provide innovative, end-to-end logistics solutions in depots of all sizes: from the world’s largest automated sorting hub handling well over 100,000 parcels per hour to small local depots with throughputs of some thousands of parcels per day. Examples of customers with VI Express Parcel systems are UPS, TNT and DHL.

- **Services**: Full operational support is part of VI’s commitment to customer’s productivity. VI can provide all the required service facilities through the operational lifetime of customer’s system, giving them the assurance of the optimum return on investment (Life-Cycle-Return). In other words, the highest possible productivity, reliability and system availability, with the minimum maintenance costs. Providing service to its customers is one of the top priorities for VI and VI is increasingly investing in Services to increase its effectiveness.

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2 VI internal document: 0610 Comp Pres EN.ppt

3 [Http://www.Vanderlande.com](http://www.Vanderlande.com)
1.2. Supply Chain Management Services

Due to increasing worldwide competition and shrinking profit margins, high-technology-product manufacturers are forced to find new ways to differentiate themselves from their competitors. Providing a fast, high-quality after-sales service to your customers contributes to this.

Research is conducted by Olivia and Kallenberg who investigated the challenges related to the transition process from manufacturer to service provider [11]. They state that providing service requires organizational principles, structures and processes that are new to the product manufacturers. These new capabilities need to be developed and the organization needs to manage the business model changes from transaction- to relationship based.

Service is provided as part of the contract between the customer and the manufacturer, and therefore, a logistics network capable of serving customers in a time-responsive manner is crucial for a successful after-sales service. Service Parts Logistics (SPL) is a critical part in this process. SPL includes activities such as designing a responsive network of part stocking facilities, deciding inventory ordering policies, stocking parts, and dispatching the required parts from facilities to the customers in need. A major challenge in SPL is to provide the service at customer’s request within the committed time frame and against reasonable costs [3]. Providing service to their customers is one of the top priorities for VI. The department Supply Chain Management Services is responsible for providing spare parts to customers. This department is investing in spare parts inventory control and supply chain management. These initiatives show us that VI is changing its organizational structures and processes, such that it can manage a relationship based business model. The initiatives are required to cope with the challenges in the transition process from manufacturer to service provider.

1.3. Research assignment

Customers of VI are increasingly demanding a higher availability of their systems. VI provides their customers the requested service by maintaining the systems. Both corrective and preventive maintenance are executed by service engineers to maximize the performance of the systems. Corrective maintenance is by far the largest proportion of the maintenance performed by VI.

Once a part breaks down, the performance of the system decreases. Corrective maintenance is carried out in case of failures of parts. This requires the availability of spare parts. The failure rates of these spare parts are low and these parts can be very expensive. It is therefore unprofitable for customers to keep spare parts inventory individually. A customer can choose for the option that VI takes care of the spare part provisioning. A service level agreement between the customer and VI, for the availability of spare parts, is agreed upon. This target service level should be met by VI against reasonable costs. VI needs to determine the base stock levels of the spare parts that are put on stock. The department Supply Chain Management Services wants to develop a method that determines the number of spare parts put in inventory at the local warehouses. This inventory needs to satisfy a certain service level requested by the customer. The cost for the provisioning of spare parts needs to be as low as possible.

The research assignment is formulated as follows:

“Develop a planning algorithm which determines the base stock levels of the spare parts at VI’s local warehouses and minimizes total system costs, subject to customer service requirements.”
This research assignment fits in the overall strategy of the Supply Chain Management Department. The Supply Chain Management Department wants to increase the provided service level to their customers and decrease the overall supply chain costs. The use of consignment stock and spare parts availability for customers are concepts with which VI wants to professionalize its service logistics. The developed algorithm can therefore be of great help to determine the base stock levels of spare parts put in inventory against minimal supply chain costs. The determined base stock levels of spare parts provide a certain spare parts availability that customers request.

Two case studies are conducted, one for a customer with two VI systems and one for a customer with fourteen VI systems. Information is available on the historical demand of spare parts by these customers. The analysis is based on the parts with a unique VI item number and those parts denoted as a spare part by VI (See Section 2.2). The method is also applicable to determine the base stock of resale items. However, due to low information availability of the resale items, they are not taken into account in this research.

The central warehouse located in Veghel is left out of the scope of the assignment. Including the central warehouse would increase the complexity of the assignment to a level beyond the scope of a Master thesis. We develop a method that determines the base stock levels of the spare parts in the local warehouses.

### 1.4. Project outline

This section describes the project outline and presents a brief overview of each chapter. Chapter 2 discusses the current situation of spare parts provisioning at VI. This chapter also discusses the strategic directions that VI wants to take related to spare parts provisioning.

Chapter 3 discusses the developed model for the new planning method of spare parts. A model description is discussed that relates to the model developed by Kranenburg [9]. A new customer service measure is introduced. The time-based aggregate fill rate is used as discussed by Caggiano et al. [4].

Chapter 4 describes the derivation of the heuristic optimization procedure. Three algorithms are presented that evaluates and optimizes the model. A greedy heuristic is used to optimize the base stock levels of spare parts.

Chapter 5 and 6 discusses two case studies. In Chapter 5, a case study on a customer with fourteen locations is presented. Each location has a VI system. Chapter 6 presents a case study on a customer with two VI systems. Both case studies evaluate the new planning method and provide some additional analyses on a number of scenarios.

Chapter 7 briefly describes the implementation of the new planning tool in the department Supply Chain Management Services.

Finally, Chapter 8 presents the conclusions and recommendations of the project.
2. Provisioning of spare parts

This chapter describes how VI currently works and how it wants to operate in the near future related to spare parts provisioning. In Section 2.1 the current situation of spare parts provisioning is presented. VI is currently changing the structure of spare parts provisioning by using inventory pooling. Note that the introduction of Strategic Part Centres (SPS) was during this master thesis as described in Section 2.2. The current situation as presented in Section 2.1 is therefore outdated for some situations. We do however present it in this Master Thesis, because it was relevant at the start of the Master thesis. Other characteristics of the future situation that need to be taken into account in our model are discussed in Section 2.3.

2.1. Current situation of spare parts provisioning

Vanderlande Industries is an organization that mainly works in projects. Once a new system is sold to a customer, the system lay-out is designed according to customer requirements. VI has an own R&D department that develops new components and improvement for VI systems. When a new system is sold to a customer, the R&D and Engineering department define certain items of the new system as a spare part. They believe that these items, possibly, need repair or replacement during the lifecycle of the system.

Customers are increasingly demanding a high uptime of their systems. A system failure can result in huge costs and a negative impact on the customer image, for example, in case an airport has delays in the departure of airplanes due to a VI system failure.

Once a part breaks down, the performance of the system decreases. Corrective maintenance is carried out in case of failures of parts. This requires the availability of spare parts. Normal replenishment of spare parts takes a few weeks which is far too long to satisfy customer requirements for an on time spare part availability. Spare part inventory is required to obtain an on time spare part availability. Therefore, on-site inventory is required at the customer. Currently, the spare part provisioning is arranged according to the structure depicted in Figure 3.

![Figure 3 Current situation of spare part provisioning](image-url)
Spare part inventories are located at the customer sites to satisfy customer requirements for timely spare part availability. We distinguish three different types of customer warehouses:

1. Customers make use of a consignment stock, which means that VI remains owner of the spare parts inventory. Customers pay an annual fee for the availability of spare parts. VI controls the new spare parts delivery and the inventory at the customer.
2. Customers become owner of the spare parts inventory, control this inventory and have a service contract with VI. VI does not have any insight into the current state of the spare part inventory at most customers. VI Service Engineers can advise the customer to put certain spare parts on stock.
3. Customers become owner of the spare parts inventory, control this inventory, but do not have a service contract with VI. VI does not have any insight into the current state of the spare part inventory at most customers.

If a system is sold to a customer, a spare part package is offered to the customer. The spare part package is based on the configuration of the system. The configuration of the system represents all parts present in the system. Calculations rules are set for each spare part individually that determine a recommended quantity. The single-item approach currently used by VI is a classical inventory management approach and is primarily directed towards a high availability of individual items. The calculation rules are set by the R&D and Engineering department based on experience and intuition once the part is defined as a spare part. These calculation rules are not adjusted, based on historical information of failure rates. A service support engineer alters these recommended quantities, based on his own experience, the item criticality and price. This initial spare part package is recommended to the customer. This process is visualized in appendix II.

All spare parts are identified by their level of criticality, called the priority code. The priority code indicates the influence of the part, on the system performance, once it fails. The criticality of the item is determined by the engineering and R&D department. VI has identified three criticality levels and can be classified as shown in Table 1.

<table>
<thead>
<tr>
<th>Classification</th>
<th>Definition</th>
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<td>A: High priority</td>
<td>Parts in the critical path of the system: failure result in a total system stop</td>
</tr>
<tr>
<td>B: Medium priority</td>
<td>Failure can cause a significant reduction of the system performance or result in a partial stoppage of the system</td>
</tr>
<tr>
<td>C: Low priority</td>
<td>Failure can cause a minor reduction of the system performance</td>
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</table>

The criticality of a spare part is determined by its position in the system and the type of spare part. If a section of a system occurs multiple times, the items in this section become less critical. E.g. a system has 50 check-in desks and a motor of one check-in fails, the total system’s performance reduces only 2%. If, however, the system has only two check-in desks and one of them fails, the total system performance reduces with 50%! The failure of an item and the reduced system’s performance as a consequence is therefore dependent on the criticality of the item in the system. Customers can demand a higher availability of spare parts with a higher priority code. We therefore take spare part’s criticality into account in our model.

Once a customer does not have stock on hand, an emergency shipment is executed from either the central warehouse or from a direct supplier to fulfill demand.
2.2. Future situation of spare parts provisioning

VI wants to reduce supply chain costs, while upholding a certain customer service degree. The supply chain costs include inventory costs, transportation costs and storage costs. If VI manages the spare part inventory, VI can optimize the spare part provisioning and reduce supply chain costs.

VI wants to reduce supply chain costs by making use of inventory pooling. By introducing a Strategic Part Centre (SPC) close to a cluster of customers, VI wants to increase the spare parts availability within a certain time frame and decrease total supply chain costs. These SPCs hold spare parts inventory for multiple customers.

Once demand occurs at a customer and the customer’s warehouse does not have stock on hand, an emergency shipment from the SPC is executed. If the SPC has no stock on hand, an emergency shipment from the central warehouse or from direct suppliers is performed. The lead time of an emergency shipment from a SPC is smaller than from the central warehouse. Using a SPC therefore increases the service provided to customers. The SPCs are used for emergency shipments only, not for regular replenishments of customer’s warehouses.

It is possible that multiple SPCs are combined in a cluster of customers. If demand occurs and the customer does not have stock on hand, the SPCs are checked for availability. Each customer is assigned to one SPC, which is checked first for availability of the spare part. If this SPC does not have the spare part on stock, another SPC is checked for availability. Each SPC has a sequence defined for all other SPC to be checked. Once none of the SPCs have the spare part on stock, an emergency shipment from the central warehouse or from direct suppliers is executed. This situation is visualized in Figure 4.

A condition to benefit from shared stock is that there exists commonality between the different customer systems. The commonality represents the similarities between different systems and the parts that are used in more than one system. Kranenburg [7] shows that an increase in the commonality percentage leads to an increased expected cost saving if we use shared stock. We therefore need to analyze the information on the installed base of the different locations to see whether there is some commonality between these locations. These analyses are discussed in the two case studies.
2.3. Characteristics of future situation

In the previous sections we have described the situation of VI that needs to be modelled. This section discusses several other characteristics that need to be taken into account in our model.

As stated before, VI wants to reduce supply chain costs, but upholding a certain customer service degree. Currently, VI uses a single-item approach to determine its spare parts inventory. As discussed extensively in literature, a multi-item approach outperforms a single-item approach with respect to spare parts inventory control in most situations based on the objective function (See [12] and [13]). As VI wants to introduce shared stock to reduce supply chain costs for its customers while the system’s uptime is a constraining factor, it is favourable to use a multi-item approach to maximize the reduction in supply chain costs.

Customers require a certain availability of spare parts and both VI and the customer need a service measure that is able to measure the provided spare parts availability. Currently, no clear service measure is used to determine the spare parts availability within a certain time frame. Therefore, a new service measure needs to be implemented that is measurable in practice.

As stated before, the item’s criticality is taken into account in our model. Currently, VI identifies three different priority codes that classify the item’s criticality as is shown in Table 1. Customers need to be able to request a higher service degree for items with a higher priority code.

Within VI, parts are defined as a VI-item or as a resale item. The most important difference between the two part types is that VI-items have a unique VI identification code, while resale-items do not. VI-items can either be produced by the VI factory internally or bought from external suppliers. These VI-items are part of a section assembled by VI. Resale items are part of an assembled product, which is entirely bought at a supplier. Currently, VI does not give a VI identification code to each item of this assembled product. It is therefore difficult for VI to track the demand of resale items. In this research, the items with a VI-item number are included in our
analysis and the resale items are excluded, because of the scarcely available information of historical data for them.

VI has a dataset available of the orders by customers in the last few years. Most of the items are ordered one-by-one, while some items are ordered in batches. The batch sizes can differ in size and it is difficult to analyze the underlying principle of a batch order. An item can be ordered in a batch, but is replaced one-by-one. The rest of the batch is put on stock. In some cases multiple items are replaced at once and the whole batch is needed for replacement. It is, however, almost impossible to analyze the underlying principle of an item ordered in batches. There exists variation in the size of the batch per item, but it is difficult to determine the underlying distribution of the batch size. We need to take care of the items ordered in batches in our model.
3. New planning model for spare parts inventory

This chapter describes a new planning model of spare parts inventory. In Section 3.1, we discuss the literature that corresponds to our problem. Section 3.2 describes the model. The service measure and the method to determine the batch size are described in Section 3.3 and Section 3.4, respectively. Section 3.5 and Section 3.6 discuss the system costs and the problem under consideration. Finally, in Section 3.7, we discuss the critical assumptions we make in our model.

3.1. Literature

This section briefly describes the available literature that is related to the spare part provisioning situation of VI.

Kranenburg [9] describes a network in which inventory pooling is done by means of lateral transhipments. Kranenburg discusses a single-echelon structure in which lateral transhipments are allowed between local warehouses. He distinguishes two types of local warehouses; main and regular local warehouses. Both warehouses can receive lateral transhipments, while only the main local warehouses can be the source of a lateral transhipment. This structure can be characterized as partial pooling, which can be defined as ‘a part of the locations have the ability to act as provider of a lateral transhipment.’ If demand occurs, the local warehouse immediately delivers the item if the local warehouse has stock on hand. If the local warehouse has no stock on hand, a lateral transhipment occurs from a main local warehouse. If this main local warehouse does not have stock on hand, the other main local warehouses are checked. The order of checking the main local warehouse’s availability is predetermined. If no main local warehouses have stock on hand, the central warehouse sends an emergency shipment. The model assumes that the central warehouse has infinite supply of items and can always deliver the requested parts. The aggregate mean waiting time is used as a service measure. Aggregate mean waiting time can be defined as the average time required to fulfil an arbitrary request, and is a weighted average over average delays of individual items.

Wong et al. [15] conducted an analysis on a multi-item, continuous review model of a multi-location inventory system of repairable spare parts. Lateral transhipment and emergency shipment are two extra flexible transportation modes available in the system. Once a part fails, it is removed, sent to repair and immediately replaced by a ready-for-use item at the local warehouse. If a local warehouse does not have stock on hand, a lateral transhipment is used to deliver the part from another local warehouse. A closest neighbour sourcing rule is used to determine which local warehouse is used as the source. The closest neighbourhood rule was also discussed by Kukreja et al. [9]. The authors assume that complete pooling is applied. This means that a local warehouse offers its entire available inventory for lateral transhipments, once a request occurs from another local warehouse. If none of the local warehouses have stock on hand, an emergence shipment takes place from an external supplier. The objective is to determine the optimal base stock settings, such that the average total costs are minimized. The objective function is restricted to a waiting time constrained defined as the average waiting time per request for a ready-for-use part.

Wong et al. [17] consider a two-echelon, multi-item spare parts inventory system and allow several transportation modes in the system. Both lateral transhipment and direct deliveries are used to serve the customer in case of stock outs. A central warehouse supplies the local warehouses with items on a one-for-one replenishment basis. The central warehouse is re-supplied by a plant and it is assumed that this plant has infinite supply. If demand occurs at a
local warehouse, this demand is satisfied from stock on hand, if this item is available. If the local warehouse is unable to supply the item directly, a lateral transhipment from another local warehouse takes place. If none of the other local warehouses have stock on hand a direct emergency shipment takes place from the central warehouse. In case the central warehouse is out of stock, a direct emergency shipment takes place from the plant. The objective is to determine the base stock levels such that it minimizes the total system cost, subjected to a required customer service level. This service level is measured with an average waiting time across items at each local warehouse. A heuristic is developed to determine the base stock level of each item type at the central warehouse and the local warehouse. A trade-off is required between inventory costs and transportation costs.

Other literature is considered as well, but not discussed in this Master thesis project. Due to the use of the SPCs (explained in Section 2.2), VI’s situation for spare parts provisioning shows remarkable resemblance with the model described by Kranenburg [9]. Kranenburg makes use of partial pooling that can be defined as ‘a part of the locations have the ability to act as provider of a lateral transhipment’. Other models make use of full pooling i.e. all locations have the ability to act as provider of a lateral transhipment.

### 3.2. Model description

In this section, the model description is presented. We base our model on the model presented by Kranenburg [9]. Due to an increasing demand by customers of VI for the availability of their systems, these systems require maintenance and servicing. Besides the use of preventive maintenance, corrective maintenance is required upon failure of parts. This corrective maintenance requires the availability of spare parts. A service level agreement is agreed between VI and the customer and a target service level should be met by VI. The time-based aggregate fill rate is used as a service measure and is explained in Section 3.3. The target service measure is set for a group of parts, e.g., for all parts with a priority code A, located in a customer’s system. These groups represent all items with the same criticality level of a customer’s system. A group of items is denoted as a demand source of these items.

The set of demand sources is denoted as \( N \). The parts in a demand source are denoted as Stock Keeping Units (SKU-s). It is possible that one SKU is present in more than one demand source. The performance of the system reduces once a SKU breaks down. The breakdown of a SKU can cause a corresponding system to be non-operational or a reduction in the system’s performance. The SKU’s criticality represents the expected performance reduction of the system caused by a SKU breakdown. Once the defective SKU is replaced, the machine is fully operational again. \( I \) denotes the set of SKU-s, numbered \( l=1,\ldots,|I| \). We assume that the demand for each SKU \( i \in I \) of demand source \( n \in N \) follows a compound Poisson process with a constant rate \( m_{i,n} \). This assumption is checked and discussed in Section 3.7. The total demand rate for demand source \( n \) is denoted as \( M_n := \sum_{i \in I} m_{i,n} \).

Each demand source \( n \) is assigned to exactly one local warehouse and this local warehouse is the first candidate to provide spare parts. The situation presented has a number of local warehouses that each experience direct demand and \( J \) denotes the set of local warehouses, numbered \( j=1,\ldots,|J| \). The subset of demand sources that is assigned to local warehouse \( j \) is denoted as \( N_j(\subseteq N) \). The total demand for SKU \( i \) at local warehouse \( j \) is denoted as \( M_{i,j} := \sum_{n \in N_j} m_{i,n} \).
A base stock policy is used to control the inventories at all local warehouses. $S_{i,j} (\in \mathbb{N}_0 = \mathbb{N} \cup \{0\})$ denotes the base stock level for SKU $i \in I$ in local warehouse $j \in J$. $S_i := (S_{i,1}, \ldots, S_{i,J}), i \in I$, denotes the vector of base stock levels for SKU $i$. Once demand occurs at a local warehouse, it is satisfied immediately if the local warehouse has stock on hand. A one-for-one replenishment policy is applied, so if demand is satisfied from a local warehouse, immediately a new spare part is requested from the central warehouse. The associated transportation time is denoted as $t_{i}^{re}$ for SKU $i \in I$.

A customer can request a certain service level for the availability of spare parts. The service measure for spare parts availability is discussed in Section 3.3. We denote $\beta_{i,j}^1(S_i)$ as the expected fraction of demand of SKU $i \in I$ at local warehouse $j \in J$ that is delivered immediately upon request, i.e. from the stock in local warehouse $j \in J$ itself, with a given vector $S_i$. The corresponding waiting time as well as the corresponding cost is zero.

If the local warehouse does not have stock on hand, a lateral transhipment takes place. We distinguish two types of local warehouses, regular and main local warehouses. Regular warehouses are allowed to receive lateral transhipments, while main warehouses are allowed to both send and receive lateral transhipments. A lateral transhipment is defined as an emergency shipment of a SKU $i \in I$ from a main local warehouse to another local warehouse. $\alpha_{i,j,k}(S_i)$ is denoted as the expected fraction of demand of SKU $i \in I$ at local warehouse $j \in J$ that is delivered by means of lateral transhipment, with a given vector $S_i$. We denote $K(\subseteq J)$ as the subset of main local warehouses. $\alpha_{i,j,k}(S_i)$ is denoted as the expected fraction of demand of SKU $i \in I$ at local warehouse $j \in J$ that is delivered by means of lateral transhipment from main local warehouse $k$, with a given vector $S_i$. A lateral transhipment from main local warehouse $k \in K, k \neq j$ to local warehouse $j \in J$ has transportation cost and time, $C_{j}^{lat}$ and $t_{j}^{lat}$, respectively.

Each regular local warehouse $j \in J$ is assigned to one main local warehouse $k \in K$ which is checked first for availability of a spare part, in case of a need for lateral transhipment. This main local warehouse is denoted as $k_j$. For each main local warehouse $k \in K$, a sequence is defined for all other mains to be checked. This pre-specified order for main local warehouse is represented in vector $\sigma(k) := (\sigma_1(k), \ldots, \sigma_{|K|-1}(k))$. Each main local warehouse other than $k$ appears only once in this pre-specified order. We denote $K(k, \tilde{k})(\subseteq K)$ as the subset of main local warehouses with a lower position number than main $\tilde{k}$ (all predecessors) in the pre-specified order for main local warehouse $k$.

$\beta_{i,j}^2(S_i)$ is denoted as the expected fraction of demand of SKU $i \in I$ at local warehouse $j \in J$ that is delivered immediately upon request, i.e. from the stock in local warehouse $j \in J$ itself, or delivered by a lateral transhipment from main local warehouse $k_j \in K, k \neq j$, therefore $\beta_{i,j}^2(S_i) = \beta_{i,j}^1(S_i) + \alpha_{i,j,k_j}(S_i)$.

$\beta_{i,j}^3(S_i)$ is denoted as the expected fraction of demand of SKU $i \in I$ at local warehouse $j \in J$ that is delivered immediately upon request, i.e. from the stock in local warehouse $j \in J$...
itself, or delivered by a lateral transshipment from a main local warehouse \( k \in K, k \neq j \) therefore \( \beta_{i,j}^3(S_i) = \beta_{i,j}^3(S_i) + A_{i,j}(S_i), \) where \( A_{i,j}(S_i) := \sum_{k \in K, k \neq j} \alpha_{i,j,k}(S_i). \)

\( \beta_{n}^1(S), \beta_{n}^2(S) \) and \( \beta_{n}^3(S) \) are used to indicate the time-based aggregate fill rate of each demand source \( n \) at a given vector \( S := (S_1, ..., S_{|n|}) \). This service measure is discussed in Section 3.3.

\[
\begin{align*}
\beta_{n}^1(S) &= \sum_{i \in I} \frac{m_{i,n}}{M_n} \beta_{i,j}^1(S_i) & j \in J, & n \in N_j \\
\beta_{n}^2(S) &= \sum_{i \in I} \frac{m_{i,n}}{M_n} \beta_{i,j}^2(S_i) & j \in J, & n \in N_j \\
\beta_{n}^3(S) &= \sum_{i \in I} \frac{m_{i,n}}{M_n} \beta_{i,j}^3(S_i) & j \in J, & n \in N_j
\end{align*}
\]

If both the regular and the main local warehouses do not have stock on hand, an emergency shipment takes place from the central warehouse. We denote \( \theta_{i,j}(S_i) \) as the expected fraction of demand of SKU \( i \in I \) at local warehouse \( j \in J \) that is delivered from the central warehouse as an emergency shipment. The corresponding transportation cost and time is \( C_{j}^{em} \) and \( t_{j}^{em} (\leq t_{i}^{reg}) \), respectively. \( C_{j}^{em} \) is the extra transportation cost for an emergency shipment compared to a regular shipment.

For each SKU \( i \in I \) at local warehouse \( j \in J \), it holds that

\[
\beta_{i,j}^1(S_i) + A_{i,j}(S_i) + \theta_{i,j}(S_i) = 1 \quad (3.1)
\]

Infinite supply at the central warehouse is assumed such that the central warehouse can always supply the requested items. The described situation is visualized in Figure 5.

![Figure 5 Situation provisioning of spare parts with one main local warehouse.](image-url)
3.3. Service Measure

Customers require a certain service if they purchase a new system at VI. A part of the provided service is the provisioning of spare parts. Customers demand a certain availability of spare parts. But how do we measure the availability of spare parts for the different customers? There are different service measures available that are widely discussed in literature. In the research proposal, we have discussed several service measures.

The use of the time-based aggregate fill rate as a service measure is discussed with the management of the Supply Chain Management Services department. It is agreed that the time-based aggregate fill rates is a good service measure for our model. Especially the ability to determine the actual time-based aggregate fill rates in practice per local warehouse influences this decision. The impact this service measure has on the assumption that the central warehouse has infinite supply, as discussed in Section 3.7, influenced the decision making as well. We briefly discuss other service measures as well.

A discussed service measures is the time-based aggregated fill rate by Caggiano et al. [4]. In this article, the authors discuss a multi-echelon, multi-item supply chain with time based customer service level agreements. The authors measure the customer service degree with the time-based aggregate fill rate. The authors define the time-based aggregate fill rate as: “the probability that an arriving order for the item at the base can be fulfilled within the transportation time along the replenishment path from that location to the base.” The service measure used is the fraction of demand that is delivered within a pre-determined time frame. In our model we have three different time frames. $\beta_{n}^{obj,1}$ is the target fraction of demand of demand source $n \in N$ that is delivered immediately upon request. The time frame used for $\beta_{n}^{obj,1}$ is equal to zero hours. $\beta_{n}^{obj,2}$ is the target fraction of demand of demand source $n \in N$ that is delivered immediately upon request or delivered by a lateral transhipment from local warehouse $k_j$. Warehouse $k_j$ is the first main local warehouse that is checked for availability, once warehouse $j$ doesn’t have stock on hand. The time frame used for $\beta_{n}^{obj,2}$ is equal to $t_{lat}^{k_j}$. $\beta_{n}^{obj,3}$ is the target fraction of demand of demand source $n \in N$ that is delivered immediately upon request or delivered by a lateral transhipment from another main local warehouse. $\beta_{n}^{obj,3}$ has a time frame of $\bar{t}_{lat}^{j}$. The remaining fraction of demand ($100\% - \beta_{n}^{obj,3}$) is supplied by an emergency shipment from the central warehouse and the time frame is equal to $t_{em}^{j}$. Figure 6 visualizes the time-based aggregate fill rate.

A service measure related to the time-based fill rate is the aggregate mean waiting time for parts. Aggregate mean waiting time can be defined as the average time required to fulfil an arbitrary request, and is a weighted average over average delays of individual SKUs [8]. This service measure is discussed widely by several authors e.g. Kranenburg [8] and Wong et al. ([16] & [17]). The use of this service measure requires reliable lead times for the different transportation modes. Especially the reliability of the lead time of an emergency shipment is an issue in the situation of VI.

A third service measure which can be used is the downtime waiting for parts (DTWP). The DTWP is defined as: “percentage of the total operational time that a system is down, because a part is not directly available.” This service measure is also used in the Msc Thesis of Paul Enders.
[4]. Using the DTWP as a service measure in the situation of VI is more problematic, we require information on the influence of an item failure on the system performance.

![Diagram: Time-based aggregate fill rate]

**Figure 6 Service Measure: Time-based aggregate fill rate**

### 3.4. Batch size

We assume a compound Poisson process for the demand rate per unit time. A compound distribution is a distribution where orders arise as a Poisson arrival process, but orders will be of different sizes [1]. Some items are ordered in batches as discussed in Section 2.3. We therefore need to determine a factor that represents the batch size, \( b_i \). As stated in Section 2.3, it is difficult to determine the underlying distribution of the batch size. In the available data, we observe some extreme outliers in the batch size. Most likely, these large batches are ordered due to preventive maintenance. We therefore determine factor \( b_i \) by adding the difference between the minimum
and the average with the average of the batch size. This is shown in Figure 7. The sensitivity of the factor is discussed in Section 4.4.

Figure 7 Method to determine the factor \( b_i \) that represents the batch size

### 3.5. System costs

This section describes the expected total system cost per time unit for the provisioning of spare parts. The system cost comprises the transportation cost, the inventory cost, and the storage cost.

The expected total transportation cost incurred per time unit for SKU \( i \in I \) is

\[
\sum_{j \in J} M_{i,j} \left( \sum_{k \in K, k \neq j} C^t_{i,j,k}(S_i) + C^m_j \theta_{i,j}(S_i) \right)
\]

The inventory cost is the value of the total inventory per time unit. The total inventory cost per time unit for SKU \( i \in I \) is:

\[
\sum_{j \in J} h P_i S_{i,j} b_i,
\]

where \( h \) is the depreciation factor of an item per time unit and \( P_i \) is the price of item \( i \in I \).

The storage cost is the cost related to e.g. renting the \( m^3 \) space at a warehouse for putting the items on stock. The total storage cost per time unit for SKU \( i \in I \) equals

\[
\sum_{j \in J} C^s_{i,j} S_{i,j} b_i,
\]

where \( C^s_{i,j} \) is the cost for storing one unit of SKU \( i \in I \) at warehouse \( j \in J \).

The total expected cost per time unit for the spare parts provisioning for SKU \( i \in I \) is

\[
C_i(S_i) = \sum_{j \in J} C^h_{i,j} S_{i,j} + \sum_{j \in J} M_{i,j} \left( \sum_{k \in K, k \neq j} C^t_{i,j,k}(S_i) + C^m_j \theta_{i,j}(S_i) \right),
\]

where \( C^h_{i,j} = (h P_i + C^s_{i,j}) b_i \)

Note that the factor \( b_i \) is used in the above formulas is only used in Algorithm 3 until step 6. Once step 6 is executed, the factor \( b_i \) is excluded to determine costs, because factor \( b_i \) is inherent of parameter \( S_{i,j} \).

### 3.6. Problem

The objective for VI is to minimize the expected total system cost, under the condition that the expected aggregate fill rates, \( \beta^1_{n}(S) \), \( \beta^2_{n}(S) \) and \( \beta^3_{n}(S) \), for an arbitrary request from each
demand source \( n \in N \) exceed the target aggregate fill rates \( \beta_{n}^{obj.1} \), \( \beta_{n}^{obj.2} \) and \( \beta_{n}^{obj.3} \). The problem (P) can be formulated as follows:

\[
\text{Problem (P):} \quad \min \sum_{i \in I} C_i(S_i)
\]

\[
s.t.
\]

\[
\sum_{i \in I} \frac{m_{i,n}}{M_n} \beta_{i,j}^{1}(S_i) \geq \beta_{n}^{obj.1}, \quad j \in J, \quad n \in N_j
\]

\[
\sum_{i \in I} \frac{m_{i,n}}{M_n} \beta_{i,j}^{2}(S_i) \geq \beta_{n}^{obj.2}, \quad j \in J, \quad n \in N_j
\]

\[
\sum_{i \in I} \frac{m_{i,n}}{M_n} \beta_{i,j}^{3}(S_i) \geq \beta_{n}^{obj.3}, \quad j \in J, \quad n \in N_j
\]

\( S_{i,j} \in \mathbb{N}_0, \quad i \in I, \quad j \in J \)

3.7. Critical assumptions

This section describes the critical assumptions that are assumed in our model. Each assumption is explained and justified.

The demand rate of items per time unit follow a compound Poisson process.

The model assumes a Poisson arrival process of items per time unit. A Poisson process is widely accepted in literature for describing the number of failures of an item per unit time. Usually, the demand for spare parts is low and irregular and this fits the Poisson distribution. This is also applicable for the situation of VI.

A Chi-Square test is conducted as a goodness-of-fit test to verify whether the time between successive arrivals is exponentially distributed [14]. If this is the case, we cannot reject the hypotheses that the number of arrivals per time unit is Poisson distributed. The demand data of two customers in the period 2005-2006 has been used to verify this. A significance level of 5% has been used, which indicates that we cannot reject the hypotheses that the number of arrivals come from a Poisson distribution with 95% or higher confidence. A minimum number of observations are required to conduct the Chi-square test. Of the data available, 38 items have sufficient data to conduct a Chi-square test. For 34 items (89.5%) we cannot reject the hypothesis that the arrivals follow a Poisson distribution. We were unable to fit a distribution to the demand pattern of the other four items.

We can conclude that the Poisson distribution is a good representation of the number of arrivals per time period in the situation of VI. Although we could not conduct a goodness-of-fit test for all items and customers, there is no reason to believe that arrivals of these items do not follow a Poisson arrival process.

The central warehouse has infinite supply of spare parts.

Infinite stock is assumed at the central warehouse, such that the central warehouse can always supply demand, either in regular or emergency shipments. The influence of this assumption on the model’s results is caused by the reliability of the lead times of the regular shipments. The lead
times of a regular shipment per item is based on the lead times that VI issue to its customers. It is reasonable to believe that the lead times of regular shipment are reliable. A sensitivity analysis is conducted to determine the influence of the lead time on the service level. This analysis is discussed in Section 5.4.2.

Due to the service measure time-based aggregate fill rate, the reliability of the lead times of emergency shipments does not influence the model’s results. $\theta_{i,j}(S_t)$ is the fraction of demand delivered by an emergency shipment. The agreed time frame for emergency shipments determines the lead time of an emergency shipment. However, this agreed time frame does not influence the model’s results.

**Failure rates of an item with multiple criticality levels are the same.**

It is possible that some items have more than one priority code in the same system. Demand occurs at a local warehouse of an identical item, but from different demand sources. In practice, these failure rates can be different between demand sources. To determine the failure rates of items we use historical data. However, the information of historical demand does not distinguish the item’s criticality. The failure rate of an item in one demand source is derived from the overall demand of the item. We need to assume that the failure rates of an item with multiple criticality levels are the same. However, due to the low failure rates and low number of items with more than one priority code, this assumption has a very small impact on the model’s result and is therefore reasonable to state.

**Transportation times and costs are constant for each local warehouse.**

Constant transportation time and cost for each local warehouse is assumed. Alfredsson and Verrijdt [1] have shown that their model is to a large extent insensitive to the lead time distribution. It is reasonable to believe that is also the case in our model.

Transportation costs depend on the distance between the warehouses and the weight of the transport. The distance between the two warehouses is constant, but the weight of the transportation depends on the weight of the item. It is however difficult to determine the weight of all items. We therefore take an average weight and corresponding costs for transportation. The transportation costs are therefore not the same as in practice. This will influence the model’s result, but not in a significant way. The sensitivity of the transportation costs is discussed in Section 5.4.3.

**One-for-one replenishment policy**

A one-for-one replenishment policy is assumed. If a spare part is used for corrective maintenance, immediately a new spare part is requested from the central warehouse. This way of ordering new spare parts will not happen in practice, but shows some resemblance due to the low demand rate. The lead time of a normal replenishment is influenced once this assumption does not hold in practice. The sensitivity of the lead time of normal replenishments is discussed in Section 5.4.2.
4. Derivation of the heuristic optimization procedure

This chapter describes the heuristic optimization procedure to solve the problem formulated in Section 3.6. In Section 4.1 we describe the method to evaluate the performance of the system. The optimization procedure is described in Section 4.2. The implementation of the heuristic in a software tool is described in Section 4.3. Section 4.4 discusses the model verification and validation.

4.1. Evaluation

We can evaluate the choice of the base stock levels for all main and regular local warehouses exactly. However, this can be very time-consuming due to the number of local warehouses. We therefore use an approximate evaluation technique that leads to accurate results as is concluded by Kranenburg [9].

The approximate evaluation of the performance of the system can be determined in the same way as by Kranenburg [9]. We therefore use the same algorithm with only some alterations. We use a regular replenishment lead time per item instead of a regular replenishment lead time constant for all items. A second adjustment made to the algorithm is that we also determine \( A_{i,j}(S_i) \) for regular local warehouses, instead for only the main local warehouses. These adjustments will not influence the accuracy of the approximate evaluation, because the evaluation method stays the same as the method described by Kranenburg. Extra calculation steps are executed, but do not influence other calculation steps. The performance of the system is evaluated per item. The approximate evaluation of the performance of the system is formally described in Algorithms 1 and 2. An explanation of Algorithms 1 and 2 is given in appendix III.

Algorithm 1

Step 1 For all regular local warehouses \( j \in J \setminus K \), \( \beta_{i,j}^1(S_i) := 1 - L(S_{i,j}, M_{i,j}, t_{i,j}^{reg}) \).

Step 2 For all main local warehouses \( k \in K \), \( \tilde{M}_{i,k} := M_{i,k} + \sum_{j \in K} (1 - \beta_{i,j}^1(S_i)) M_{i,j} \).

Step 3 For all main local warehouses \( k \in K \), determine \( \beta_{i,k}^1(S_i) \), \( \alpha_{i,k,k}(S_i) \), \( \tilde{k} \in K, \tilde{k} \neq k \), and \( \theta_{i,k}(S_i) \), using Algorithm 2.

Step 4 For all regular local warehouses \( j \in J \setminus K \), if \( K = \emptyset \), then \( \theta_{i,j}(S_i) = (1 - \beta_{i,j}^1(S_i)) \).

Otherwise, \( \alpha_{i,j,k}(S_i) \) is defined as:

\[
\alpha_{i,j,k}(S_i) := \begin{cases} 
(1 - \beta_{i,j}^1(S_i)) \beta_{i,j,k}^1(S_i), & k = k_j, \\
(1 - \beta_{i,j}^1(S_i)) \alpha_{i,k,j,k}(S_i), & k \in K, k \neq k_j,
\end{cases}
\]

\( \theta_{i,j}(S_i) := (1 - \beta_{i,j}^1(S_i)) \theta_{i,j,k}(S_i) \)

and \( A_{i,j}(S_i) := 1 - (\beta_{i,j}^1(S_i) + \theta_{i,j}(S_i)) \)

Algorithm 2

Step 1 For all main local warehouses \( k \in K \), \( \theta_{i,k}(S_i) := L(\sum_{k \in K} S_{i,k}, \sum_{k \in K} \tilde{M}_{i,k} t_{i,k}^{reg}) \).

Step 2 For all main local warehouses \( k \in K \), \( \beta_{i,k}^1(S_i) := 1 - L(S_{i,k}, \tilde{M}_{i,k} t_{i,k}^{reg}) \), and
\[ A_{i,k}(S_i) := 1 - (\beta_{i,k}^{i}(S_i) + \theta_{i,k}(S_i)). \]

**Step 3** For one main local warehouse \( k \in K \) that has \( S_{i,k} > 0 \):

**Step 3-a** \[
\hat{M}_{i,k} := \frac{A_{i,k}(S_i)\hat{M}_{i,k}}{1 - (\prod_{\hat{k}\in K,k \neq \hat{k}}(1 - \beta_{i,\hat{k}}^{i}(S_i))) \prod_{i\in K,k \neq i}(1 - \beta_{i,\hat{k}}^{i}(S_i))} \quad \hat{k} \in K, \hat{k} \neq k
\]

with the last product term defined as 1 if \( K(\hat{k}, k) = \phi \), and

\[
\hat{M}_{i,k} := \hat{M}_{i,k} + \sum_{\hat{k}\in K,k \neq k} \hat{M}_{i,k}.
\]

**Step 3-b** \[
\beta_{i,k}^{1}(S_i) := 1 - L(S_{i,k}, \hat{M}_{i,k} \bar{t}_i^{reg}), \quad \text{and}
\]

\[
A_{i,k}(S_i) := 1 - (\beta_{i,k}^{1}(S_i) + \theta_{i,k}(S_i)).
\]

**Step 4** Repeat Step 3 for all other mains that have \( S_{i,k} > 0 \).

**Step 5** Repeat Steps 3 and 4 until \( \hat{M}_{i,k} \) does not change more than \( \epsilon \) for each \( k \in K \), with \( \epsilon \) small.

**Step 6** For all main local warehouses \( k \in K \), \( \alpha_{i,k}(S_i) := \beta_{i,k}^{1}(S_i)\hat{M}_{i,k} \bar{t}_i^{reg} \).

\[
\hat{M}_{i,k} := \hat{M}_{i,k} + \sum_{\hat{k}\in K,k \neq k} \hat{M}_{i,k}.
\]

### 4.2. Optimization

This section discusses the optimization of the system by finding a feasible policy for Problem (P) while the system costs are minimized. The optimization procedure is comparable to the procedure described by Kranenburg [9], except a different service measure is used. A greedy heuristic is provided, which determines the base stock levels using the approximate evaluation method discussed in Section 4.1. A greedy heuristic repeatedly executes a procedure which tries to maximize the return per unit cost increase. The method searches for an increase of \( S_{i,j}, i \in I, j \in J \) that leads to ‘the biggest bang for a buck’. A maximum on \( \beta_{i,j}(S_i) \) is implemented such that the base stock level \( S_{i,j} \) does not increase anymore once \( \beta_{i,j}(S_i) \) is greater than 0.998. This prevents a very high base stock level for very cheap items. Increasing the base stock level of these cheap items even more once \( \beta_{i,j}(S_i) \geq 0.998 \), would most likely not lead to an increase in service level in practice. The method makes sure that items are put on stock in regular local warehouses that receives direct demand or in main local warehouses if more than one warehouse experiences demand.

The greedy method determines the inventory for each SKU \( i \in I \) in all local warehouses \( j \in J \). We can describe this in five steps. In the initial step, we set all base stock levels \( S_{i,j} := 0, i \in I, j \in J \).

Secondly, we increase the base stock levels if and as long as it does not increase total system cost. If an increase for an SKU \( i \) of a base stock level \( S_{i,j} \) would lead to a cost decrease, we increase the base stock level that gives us the largest cost decrease. Since cost and fill rates do depend on \( S_i \) only, we execute this step for each SKU \( i \in I \) separately.

In the third step, we iteratively increase \( S_{i,j}, i \in I, j \in J \), that provides us with the largest increase in aggregate fill rate \( \beta_{n}^{1}(S) \) per unit cost increase, until \( \beta_{n}^{1}(S) \) is met.
In step four we execute the same procedure as in step three, as long as our current solution is not feasible. We iteratively increase $S_{i,j}$, $i \in I$, $j \in J$, that provides us with the largest increase in aggregate fill rate $\beta_n^2(S)$ per unit cost increase, until $\beta_n^{obj,2}, n \in N$ is met.

Step five is optional and depends on the number of main local warehouses. This step is executed if the number of main local warehouses is greater than one. We execute the same procedure as in step three, as long as our current solution is not feasible. We iteratively increase $S_{i,j}$, $i \in I$, $j \in J$, that provides us with the largest increase in aggregate fill rate $\beta_n^3(S)$ per unit cost increase, until $\beta_n^{obj,3}, n \in N$ is met.

In the sixth and last step, we multiply $S_{i,j}$, $i \in I$, $j \in J$ with a factor $b_i$ as discussed in Section 3.4.

In our algorithmic description we denote $e_j$ as a row vector of size $|I|$ with the $j$-th elements equal to 1 and all other elements equal to 0. We define $\Delta c(i,j) := c_i(S_i + e_j) - c_i(S_i)$, $i \in I$, $j \in J$ as the difference in cost if the base stock level for SKU $i$ at local warehouse $j$ would be increased by one, at a given vector $S_i$. The decrease in distance to the set of feasible policies if for SKU $i \in I$ and local warehouse $j \in J$, the base stock level $S_{i,j}$ will be increased by one, is defined as $\Delta \beta^1(i', j')$, $\Delta \beta^2(i', j')$ and $\Delta \beta^3(i', j')$, where

$$\Delta \beta^1(i', j') := \sum_{j \in J} \sum_{n \in N} \left[ \beta_{n}^{obj, 1} - \frac{m_{i,n}}{M_n} \beta_{i,j}^{1}(S_i) \right]^+ - \sum_{j \in J} \sum_{n \in N} \left[ \beta_{n}^{obj, 1} - \frac{m_{i,n}}{M_n} \beta_{i,j}^{1}(S_i) - \frac{m_{i,n}}{M_n} \beta_{i,j}^{1}(S_i + e_j) \right]^+, \quad i \in I, \ j \in J,$$

$$\Delta \beta^2(i', j') := \sum_{j \in J} \sum_{n \in N} \left[ \beta_{n}^{obj, 2} - \frac{m_{i,n}}{M_n} \beta_{i,j}^{2}(S_i) \right]^+ - \sum_{j \in J} \sum_{n \in N} \left[ \beta_{n}^{obj, 2} - \frac{m_{i,n}}{M_n} \beta_{i,j}^{2}(S_i) - \frac{m_{i,n}}{M_n} \beta_{i,j}^{2}(S_i + e_j) \right]^+, \quad i \in I, \ j \in J,$$

$$\Delta \beta^3(i', j') := \sum_{j \in J} \sum_{n \in N} \left[ \beta_{n}^{obj, 3} - \frac{m_{i,n}}{M_n} \beta_{i,j}^{3}(S_i) \right]^+ - \sum_{j \in J} \sum_{n \in N} \left[ \beta_{n}^{obj, 3} - \frac{m_{i,n}}{M_n} \beta_{i,j}^{3}(S_i) - \frac{m_{i,n}}{M_n} \beta_{i,j}^{3}(S_i + e_j) \right]^+, \quad i \in I, \ j \in J,$$

with $[a]^+ := \max(0,a)$

Finally, we define the ratio’s $R_1(i, j)$, $R_2(i, j)$ and $R_3(i, j)$, $i \in I$, $j \in J$ as
\[ R_1(i, j) := \frac{\Delta \beta^1(i, j)}{\Delta C(i, j)} \quad i \in I, j \in J \]
\[ R_2(i, j) := \frac{\Delta \beta^2(i, j)}{\Delta C(i, j)} \quad i \in I, j \in J \]
\[ R_3(i, j) := \frac{\Delta \beta^3(i, j)}{\Delta C(i, j)} \quad i \in I, j \in J \]

In Algorithm 3 we describe our optimization algorithm formally.

**Algorithm 3**

**Step 1** Set \( S_{i,j} := 0 \), \( i \in I, j \in J \)

**Step 2** For each SKU \( i \in I \):

a. Calculate \( \Delta C(i, j) \), \( j \in J \)

b. While \( \min \{ \Delta C(i, j) \} \leq 0 \):

   1. Determine \( \hat{j} \) such that \( \Delta C(i, \hat{j}) \leq \Delta C(i, j) \), \( j \in J \)
   2. Set \( S_{i,j} := S_{i,j} + 1 \)
   3. Calculate \( \Delta C(i, j) \), \( j \in J \)

**Step 3** a. Calculate \( R_1(i, j) \), \( i \in I, j \in J \)

b. While \( \max \{ R_1(i, j) \} > 0 \):

   1. Determine \( \hat{i} \) and \( \hat{j} \) such that \( R_1(\hat{i}, \hat{j}) \geq R_1(i, j) \), \( i \in I, j \in J \)
   2. Set \( S_{i,j} := S_{i,j} + 1 \)
   3. Calculate \( R_1(i, j) \), \( i \in I, j \in J \)

**Step 4** a. Calculate \( R_2(i, j) \), \( i \in I, j \in J \)

b. While \( \max \{ R_2(i, j) \} > 0 \):

   1. Determine \( \hat{i} \) and \( \hat{j} \) such that \( R_2(\hat{i}, \hat{j}) \geq R_2(i, j) \), \( i \in I, j \in J \)
   2. Set \( S_{i,j} := S_{i,j} + 1 \)
   3. Calculate \( R_2(i, j) \), \( i \in I, j \in J \)

**Step 5** While \( |K| > 1 \):

a. Calculate \( R_3(i, j) \), \( i \in I, j \in J \)

b. While \( \max \{ R_3(i, j) \} > 0 \):

   1. Determine \( \hat{i} \) and \( \hat{j} \) such that \( R_3(\hat{i}, \hat{j}) \geq R_3(i, j) \), \( i \in I, j \in J \)
   2. Set \( S_{i,j} := S_{i,j} + 1 \)
   3. Calculate \( R_3(i, j) \), \( i \in I, j \in J \)

**Step 6** Set \( S_{i,j} := S_{i,j} \times b_i \), \( i \in I, j \in J \)

### 4.3. Model implementation

The algorithms described in Section 4.1 and 4.2 are programmed in Borland Delphi 2005. Borland Delphi is a high-level, compiled, strongly typed language in a sophisticated Windows
programming environment. It supports structured and object-oriented design and the Borland Delphi language is based on Object Pascal\(^4\). The main reason to use Borland Delphi was, because part of the program was already available. Kranenburg has developed this program during his PhD study. Using and expanding this program was more logical than starting all over again.

A second reason to use Borland Delphi as programming language is that the language can easily be understood by others. The visual interface and the programming language are clear and understandable. This can be an advantage when the model needs to be implemented and/or used in further research.

The spare parts planning program needs input information to determine the base stock levels of the spare parts. These input files are implemented in Microsoft Excel. Information is required on the number of local warehouses, item characteristics, etc. For a more detailed description of the input files, we refer to the ‘Spare parts planning manual’.

### 4.4. Model verification and validation

We need to examine whether the translation from the conceptual model to the implemented program model is done correctly. Verification ensures that the software program satisfies or matches the concept model (low-level checking)\(^5\). We need to determine whether we have built the software model right. The processing steps are carefully analyzed and tested step by step. The software program is executed in a simple test environment. The results of the software model are compared with the results of the conceptual model and show no differences. This verifies our model.

Validation checks that the software model satisfies or fits the intended usage (high-level checking). We check whether the results of the model are comparable to the results achieved in practice. A test is conducted to check if we have built the right software program. This is done through dynamic testing. Dynamic analysis refers to the examination of the physical response from the software program to variables that are not constant and change with time. A simulation is conducted with real-life demand of a customer in the period January 2005 – December 2006. One regular local warehouse is taken under consideration and \(\beta_{n}^{obj,2}\) is set to 98%. After simulation with real-life demand we observe a service level of 97.8%.

No simulation is performed with a main local warehouse. Including a main local warehouse in the simulation with real-life data would increase the complexity of the simulation model. Due to time restrictions and agreements with persons involved we conducted no simulation with a main local warehouse. There is however no reason to believe that the model is less valid if main local warehouses are included. The model’s results of situations with main local warehouses are according to expectations.

\(^4\) Http://delphi.about.com

\(^5\) Http://en.wikipedia.org/wiki/Verification_and_Validation
To validate the method to determine the factor $b_i$ for the batch size is difficult. We have compared three different methods to determine $b_i$, with the current method to determine $b_i$. The other three methods to determine $b_i$:

- **Round (average)** := Rounds the average batch size to the nearest integer.
- **Roundup (average)** := Rounds the average batch size up to the nearest integer.
- **Max** := Takes the maximum batch size.

We compare the service levels achieved with the different $b_i$, using a simulation with real-life data. $\beta_n^{obj,2}$ is set to 98%. We observe a low service level for round(average) and roundup(average), 82.86% and 85.39% respectively. If we use the maximum of the batch size we observe a service level of 99.01%, but total system costs increase enormously. From these results, we can state that the current method of determining the factor $b_i$ for the batch size is appropriate and that the factor $b_i$ has a large influence on the actual service level.
5. Case study: Customer with fourteen locations

This chapter describes the characteristics of the case study of a customer with fourteen different locations. Each location has a VI system and a warehouse for spare part inventory. We analyze the current situation of all the customers’ warehouses for which we want to implement our planning algorithm. First, we discuss the characteristics of the customer in Section 5.1. In Section 5.2 we evaluate the new planning method. We evaluate the current planning method with the new planning method in an equivalent situation. This way, we can determine the influence of a multi-item approach compared to a single-item approach. Section 5.3 discusses several scenarios such that we can determine the influence of inventory pooling. In Section 5.4 we describe the sensitivity of several input parameters.

5.1. Characteristics

The fourteen locations have VI systems and are maintained by VI Service Engineers. In the current situation, one location is used as a main local warehouse and is allowed to send lateral transhipments. This main local warehouse has two separate spare part inventory warehouses. One warehouse is used for the spare part provisioning of the system located in this location. The other warehouse is used for lateral transhipments only. The customer has therefore one main local warehouse and fourteen regular local warehouses, while the main local warehouse does not have a demand source assigned to it. The main local warehouse does therefore not experience direct demand. All other locations receive lateral transhipments from the main local warehouse.

Once demand occurs at one of the locations, it is satisfied immediately if stock is on hand. If not, a lateral transhipment from the main local warehouse to the particular location occurs, if this main local warehouse has stock on hand. If the main local warehouse does not have the item on stock an emergency shipment occurs from the VI central warehouse or from a direct supplier. A one-for-one replenishment is assumed, so the customer places a new order at VI, once an item is used for maintenance.

Section 5.1.1 describes the commonality between the different locations of the customer. Section 5.1.2 and 5.1.3 describes the current stock levels and the criticality of the items in the installed base. We discuss the available data in Section 5.1.4.

5.1.1. Commonality

This paragraph describes the commonality between the different locations. The commonality between locations is presented in Table 2. We have in total 1220 different items, with a VI item number and denoted as a spare part, in the installed base. The price of the most expensive item is about 10^6 times the price of the cheapest item. We observe from Table 2 that 53.8% of all items are present in at least two different locations and almost 10% in at least six different locations. From the level of commonality we can conclude that it is worthwhile considering shared stock for this customer. If there was no commonality present between the different locations it makes no sense to use a main local warehouse and to make use of lateral transhipments.

<table>
<thead>
<tr>
<th># Locations</th>
<th># Items</th>
<th>Percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>&gt; 0</td>
<td>1220</td>
<td>100,00%</td>
</tr>
<tr>
<td>&gt; 1</td>
<td>656</td>
<td>53,77%</td>
</tr>
<tr>
<td>&gt; 2</td>
<td>426</td>
<td>34,92%</td>
</tr>
</tbody>
</table>
We observe that the average ICP per item decreases as the commonality of an item increases. Research by Kranenburg [7] shows us that the total spare parts provisioning costs does not decrease significant if the more common items are relative cheap.

### 5.1.2. Current stock levels

At the moment, every location has stock on hand. However, neither VI nor the customer has clear insights in the actual stock on hand. To determine the current stock levels, we use the calculation rules that are used to draw up spare part packages. We assume a one-for-one replenishment such that the current stock levels are the same as the original spare part packages. The installed base of the customer is known, so from the installed base information and the calculation rules, we determine the current stock levels. The stock on hand at the main local warehouse is known. This stock is used for lateral transshipments to other locations.

### 5.1.3. Criticality

Information is available on the current installed base of the systems at the locations. The installed base contains information on the number of items in the system. The priority code, that represents the item’s criticality, is known for most items. Some items are not classified with a priority code and we assign priority code A to these items. Priority code A is denoted as the most critical level and we are therefore certain that we have the required service level. Some priority code groups contain a limited number of items and are therefore removed. These items are placed in a group with a higher priority code. The different groups per location are shown in Table 4.

### 5.1.4. Data structure

Information is available on the demand occurred by the locations in the period January 2004 – June 2006. The dataset contains 221 different items. The dataset with information on historical demand is somewhat polluted, due to ordering for items for preventive maintenance. Especially the items ordered in batches are most likely used for preventive maintenance. It is however impossible to determine whether orders are for corrective maintenance or preventive maintenance.

The price of the most expensive item is about 0.6*10^5 times the price of the cheapest item. The commonality of these 221 items is presented in Table 3. We observe a higher commonality in items that have been ordered in the period January 2004 – June 2006 than the installed base. This is obvious, because items with a high commonality have a higher probability of being ordered than items with no commonality.

| > 3  | 302 | 24.75% |
| > 4  | 250 | 20.49% |
| > 5  | 196 | 16.07% |
| > 6  | 116 | 9.51%  |
| > 7  | 61  | 5.00%  |
| > 8  | 41  | 3.36%  |
| > 9  | 24  | 1.97%  |
| > 10 | 7   | 0.57%  |
| > 11 | 4   | 0.33%  |
Table 3 Commonality the items sold in Jan 2004- Jun 2006

<table>
<thead>
<tr>
<th># Locations</th>
<th># Items</th>
<th>Percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>&gt;0</td>
<td>221</td>
<td>100.00%</td>
</tr>
<tr>
<td>&gt;1</td>
<td>166</td>
<td>75.61%</td>
</tr>
<tr>
<td>&gt;2</td>
<td>125</td>
<td>56.56%</td>
</tr>
<tr>
<td>&gt;3</td>
<td>104</td>
<td>47.06%</td>
</tr>
<tr>
<td>&gt;4</td>
<td>91</td>
<td>41.18%</td>
</tr>
<tr>
<td>&gt;5</td>
<td>67</td>
<td>30.32%</td>
</tr>
</tbody>
</table>

This section describes the method to determine \( m_{i,n} \). Information available on the demand of items is limited. To extend the information for analysis we make use of the installed base of the customer. First, we determine the failure rate per SKU \( i \in I \). The failure rate of SKU \( i \in I \) is determined by dividing the total arrivals by the number of items in the total installed base. Failure rates are assumed to be equal between different demand sources. Secondly, the failure rate is multiplied by the number of items of SKU \( i \in I \) in demand source \( n \in N \). Be aware that the rate \( m_{i,n} \) defines the arrival rate of order lines per item per time unit for demand source \( n \in N \).

We determine the demand rates \( m_{i,n} \) using the following formula:

\[
m_{i,n} = \frac{\text{Total arrivals of order lines of SKU } i \text{ per day}}{\text{#items of SKU } i \text{ in installed base}} \times \text{#items of SKU } i \text{ in demand source } n
\]

\( i \in I, n \in N \).

In the near future, VI can derive more reliable demand rates by using historical demand rates of other customers and the corresponding installed bases. This requires the use of a more reliable dataset that is currently available.

### 5.2. Evaluation of new planning method

In our analysis we compare the current performance with the performance of the new model. We evaluate the model with two different methods. First, we compare the inventory value of the current planning method with the inventory value of the new planning method if the service levels are equal. Secondly, we determine the difference in service level if we keep the investment in inventory of spare parts equal. In both situations, we have fourteen regular local warehouses and one main local warehouse. This is equivalent to the current situation. This way, we analyze the influence of the multi-item approach compared to the single-item approach on the inventory value.

#### 5.2.1. Comparison of inventory investment

Figure 8 describes the method to compare the inventory investment of the current method with the new planning method. First, we determine the current base stock level, the demand rate and the factor \( h_i \) per item. We perform a simulation and determine the aggregate fill rates as a result of the current stock levels. These aggregate fill rates are the aggregate fill rate objectives for our new model. This ensures an equivalent or higher service level in the new planning method compared to the current method. We use the spare parts planning model to determine the optimal base stock levels at the corresponding inventory value. Finally, we compare the investments in inventory of both situations.
In this situation, we have one main local warehouse that is allowed to send lateral transshipments and fourteen regular local warehouses. We determine the aggregate fill rates of the current base stock levels using algorithm 1 and 2, as discussed in Section 4.1. Table 4 shows the aggregate fill rates with the current base stock levels per group. The groups are denoted as group[location number],[criticality group], e.g. group 1.A. These aggregate fill rates are used as objectives for our new planning method. This way, we are certain that the service level in our new planning method is as least the same as with the current planning method.

Table 4 Aggregate fill rates with the current base stock levels.

<table>
<thead>
<tr>
<th>Warehouse</th>
<th>$\beta_1^S(S)$</th>
<th>$\beta_2^S(S)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Group 1.A</td>
<td>81.05%</td>
<td>81.89%</td>
</tr>
<tr>
<td>Group 2.A</td>
<td>78.22%</td>
<td>80.61%</td>
</tr>
<tr>
<td>Group 2.B</td>
<td>98.00%</td>
<td>99.00%</td>
</tr>
<tr>
<td>Group 3.A</td>
<td>69.23%</td>
<td>73.40%</td>
</tr>
<tr>
<td>Group 4.A</td>
<td>81.19%</td>
<td>85.68%</td>
</tr>
<tr>
<td>Group 5.A</td>
<td>84.98%</td>
<td>88.31%</td>
</tr>
<tr>
<td>Group 5.B</td>
<td>83.94%</td>
<td>90.92%</td>
</tr>
<tr>
<td>Group 6.A</td>
<td>69.18%</td>
<td>72.15%</td>
</tr>
<tr>
<td>Group 6.B</td>
<td>67.43%</td>
<td>69.73%</td>
</tr>
<tr>
<td>Group 6.C</td>
<td>86.68%</td>
<td>93.03%</td>
</tr>
<tr>
<td>Group 7.A</td>
<td>77.96%</td>
<td>78.05%</td>
</tr>
<tr>
<td>Group 7.B</td>
<td>98.47%</td>
<td>99.41%</td>
</tr>
<tr>
<td>Group 8.A</td>
<td>81.15%</td>
<td>81.72%</td>
</tr>
<tr>
<td>Group 8.B</td>
<td>74.58%</td>
<td>83.88%</td>
</tr>
<tr>
<td>Group 9.A</td>
<td>99.56%</td>
<td>99.65%</td>
</tr>
<tr>
<td>Group 10.A</td>
<td>72.63%</td>
<td>73.15%</td>
</tr>
<tr>
<td>Group 10.B</td>
<td>75.02%</td>
<td>84.33%</td>
</tr>
<tr>
<td>Group 11.A</td>
<td>79.74%</td>
<td>80.00%</td>
</tr>
<tr>
<td>-----------</td>
<td>--------</td>
<td>--------</td>
</tr>
<tr>
<td>Group 11.B</td>
<td>63.47%</td>
<td>79.30%</td>
</tr>
<tr>
<td>Group 12.A</td>
<td>90.02%</td>
<td>95.74%</td>
</tr>
<tr>
<td>Group 13.A</td>
<td>85.19%</td>
<td>87.27%</td>
</tr>
<tr>
<td>Group 13.B</td>
<td>94.21%</td>
<td>94.61%</td>
</tr>
<tr>
<td>Group 14.A</td>
<td>90.17%</td>
<td>90.31%</td>
</tr>
</tbody>
</table>

One important note needs to be made for the aggregate fill rates of the current stock. In practice these aggregate fill rates will be higher. The polluted data set and especially the ordering of items in batches reduces the aggregate fill rates determined by algorithm 1 and 2. Due to the orders for items for preventive maintenance, the number and size of item’ batches will be larger than in reality. This influences the aggregate fill rates of the current stock.

The new planning method is used to determine the base stock levels at the fourteen regular local warehouses and the main local warehouse. We observe a reduction of 31.3% in the total inventory value if we use the new planning method. This reduction in inventory value is for the items with a VI item number and those that have experienced demand in the period January 2004 – June 2006. The results are shown in Table 5.

<table>
<thead>
<tr>
<th>Planning Method</th>
<th>Inventory Costs (normalized)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Current Method</td>
<td>100</td>
</tr>
<tr>
<td>New Method</td>
<td>68.7</td>
</tr>
</tbody>
</table>

Some items have a higher base stock level determined by the new planning method than the current base stock level. Investment is required to level up the base stock level. An investment of 18% of the current inventory value is required to level up the base stock levels.

Most items’ base stock levels determined in the new planning method are lower than the current base stock levels. The items’ stock level that is higher than the optimal base stock level needs to be lowered. It takes a certain period of time to lower the current stock levels to the optimal base stock levels. Table 6 shows the percentage of the value that is recovered in a certain period, if we assume a constant demand rate. We can conclude that 71.5% of the decrease in inventory value is recovered in 2 years.

<table>
<thead>
<tr>
<th>Period</th>
<th>Percentage recovery inventory value</th>
</tr>
</thead>
<tbody>
<tr>
<td>6 months</td>
<td>31.3%</td>
</tr>
<tr>
<td>1 year</td>
<td>49.9%</td>
</tr>
<tr>
<td>2 years</td>
<td>71.5%</td>
</tr>
</tbody>
</table>

Some items have experienced demand in the period January 2004 – June 2006, but are not put on stock in the new planning method. These items have low demand rates and high price levels. The planning model computes that placing these items on stock will give a small effect on the aggregate fill rate, related to the item’s price level. 18 different items have a base stock level
equal to zero. If we set the base stock level of these items in the main local warehouse equal to one, the inventory value decreases by 25.1% compared to the current inventory value.

The reduction in inventory value is mainly caused by the more expensive items. Most expensive items have lower base stock levels in the new planning method than the current planning method, while the cheaper items have a higher base stock level. The 20% most expensive items take care of 98.5% of the reduction in inventory value, while these items need only 6% of the total investment.

No relation is found between the calculation rules that currently determine the base stock levels and the reduction in inventory value. We can therefore not conclude that the current calculation rules are wrongly determined.

The main reason for the reduction in inventory value is due to the used approach. The calculation rules, that are currently used to determine the base stock levels per item, are comparable to a single-item approach, while our model determines the base stock levels following a multi-item approach. The multi-item approach takes care that the more expensive items are put relatively less on stock than the relative cheap items. Low service levels of expensive products are compensated by high service levels of cheaper products.

5.2.2. Comparison of service level

Secondly, we compare the expected service level due to the current base stock levels and due to optimal base stock levels. The investment in spare parts inventory remains constant. The current aggregate fill rates are shown in Table 4. We use an iterative approach to determine the new aggregate fill rates, such that the inventory value remains constant. We assign the groups with a priority code A with a higher $\beta_{i}^1(S)$ and $\beta_{i}^2(S)$ than the groups with priority code B and subsequently priority C. The increases in aggregate fill rates are shown in appendix IV. The average increase in aggregate fill rate is per group about 12%.

5.3. Scenarios

We will analyze the case with several scenario settings. The characteristics of the scenarios are the same as in the previous analysis. These scenarios are used to determine the influence of lateral transhipments and the number of main local warehouses on the total annual costs. The following scenarios will be analyzed in our case study:

1. All locations are denoted as regular local warehouses. No lateral transhipments are allowed between the warehouses. We can compare this scenario with a scenario that makes use of a main local warehouse.
2. One location is denoted as a main local warehouse and all other warehouses are denoted as regular local warehouses. This scenario is comparable to the current way of working at the customer.
3. Multiple locations are denoted as main local warehouses. This scenario determines the influence of multiple main local warehouses on the total system cost.
We compare the scenarios on total system costs by varying $\beta_n^{obj,3}(S)$. $\beta_n^{obj,1}(S)$ has a constant relative difference of 5% to $\beta_n^{obj,3}(S)$, but other differences show comparable results. In scenario one and two $\beta_n^{obj,2}(S) = \beta_n^{obj,3}(S)$, because the number of main local warehouses is less than two. In scenario three, $\beta_n^{obj,1}(S) = \beta_n^{obj,2}(S)$, such that there is no restriction in the location that sends the lateral transhipment. The restriction on the first main local warehouse to be checked becomes redundant. The depreciation factor $h$ is set to 17.5%. The total annual costs for the spare part provisioning are normalized and shown in Figure 9.

![Figure 9 Total annual costs for the different number of main local warehouses](image)

We observe that the influence of main local warehouses on the total annual costs increases as $\beta_n^{obj,3}(S)$ increases, compared to a situation with only regular local warehouses. The number of main local warehouses also influences the total annual costs, but this influence remains almost constant as $\beta_n^{obj,3}(S)$ increases.

A drawback from this situation is that we have no insight in which main local warehouse sends the lateral transhipment to the regular local warehouse. As described in Section 3.2, each regular local warehouse is assigned to a main local warehouse $k_j$. This main local warehouse $k_j$ is checked first for availability in case of need for a lateral transhipment. Further analysis is conducted to see what percentage of lateral transhipments is send from main local warehouse $k_j$. In case of two main local warehouses we observe a ratio of about 50:50 between lateral transhipments send from main local warehouse $k_j$ and the other main local warehouse. In case of three main local warehouses, this ratio even increases in favour of the other main local warehouses. We can therefore conclude that the commonality between the regular and main local

32
warehouses is a more decisive factor than the sequence in which the main local warehouses are checked.

To improve service to the customers, a higher percentage of lateral transhipments from main local warehouse \( k_j \) is preferred. \( \beta^1_n(S) \) - \( \beta^2_n(S) \) is the fraction of the aggregate fill rate send by main warehouse \( k_j \). The analysis on the scenarios described above has been conducted with different parameter settings. Again, we increase \( \beta^{obj,3}_n(S) \), but set \( \beta^{obj,1}_n(S) \) and \( \beta^{obj,2}_n(S) \) with a relative difference of 5% and 1%, respectively.

The results are somewhat remarkable. If we compare the situation with one main local warehouse to a situation with two main local warehouses, we observe a small increase in total annual costs in the situation with two main local warehouses. This can be explained as follows: Instead of a situation of thirteen regular local warehouses assigned to one main local warehouse that is restricted to \( \beta^{obj,3}_n(S) \). We have two situations with each having six regular local warehouses assigned to one main local warehouse that are restricted to \( \beta^{obj,2}_n(S) \) and in combination restricted to \( \beta^{obj,3}_n(S) \). Due to the small difference between \( \beta^{obj,2}_n(S) \) and \( \beta^{obj,3}_n(S) \), the effect of lateral transhipments between the two main local warehouses is minimal. The cost increase is therefore caused by the lower number of regular local warehouses assigned to the main local warehouse. The larger the number of regular local warehouses assigned to a main local warehouse, the greater the profits gained related to a situation with only regular local warehouses.

Subsequently, we compare the situation with one main local warehouse to a situation with three main local warehouses. In the situation of three main local warehouses, we assign three or four regular local warehouses to one main local warehouse restricted to \( \beta^{obj,2}_n(S) \). The combination of the three main local warehouses is restricted to \( \beta^{obj,3}_n(S) \). Depending on the height of \( \beta^{obj,3}_n(S) \), we observe a small reduction in the total annual costs! This is a contradictory to the observation made in the situation with two main local warehouses. After some analysis, it shows that the commonality between the regular local warehouses and main local warehouses is greater in the situation with three main local warehouses than in the situation with two local warehouses, which was coincidentally. This proves that the commonality between different the local warehouses is also of significant importance.

The choice for the number of main local warehouses mainly depends on the commonality between the related warehouses, the number of regular local warehouses assigned to a main local warehouse and the time and costs of lateral transhipments. These parameters are situation depended. The planning tool is therefore a useful tool to build business cases for customers and for VI strategic decisions-making processes. The choice for the number of main local warehouses can be derived from the results determined by the planning tool. In the decision making process several other parameters need to be taken into account, e.g. set up costs for a main local warehouse. These constant costs are not taken into account in our model.

### 5.4. Sensitivity of parameters

This section discusses the sensitivity of different parameters on the results of the model. We use this case study with one main local warehouse to determine the sensitivity of different parameters
settings. In Section 5.4.1, we discuss the sensitivity of the demand rates on the expected service level and the total annual costs. The sensitivity of lateral transhipments and the emergency shipments is discussed in Section 5.4.3. The effect of storage costs is explained in Section 5.4.4.

5.4.1. Sensitivity of parameter demand rates

This section discusses the sensitivity of the demand rates on the expected service level and the expected total annual costs. First we analyze the effects on the expected service level and the expected total annual costs if the demand rates of all item deviate with the same proportion. These effects are shown in Figure 10 and Figure 11. We determine the deviation in service level by comparing the current service level with the service level resulting from the deviated demand rate, while the base stock levels are kept constant. We use algorithm 1 and 2, described in Section 4.1, to determine the service levels. Beta 1 and Beta 2 in Figure 10 stands for $\beta^1_n(S)$ and $\beta^2_n(S)$.

![Sensitivity arrival rates](image)

Figure 10 Effects of deviation of demand rates on the Service Level

We determine the deviation in total annual costs by comparing the current total annual costs with the annual costs resulting from the deviated demand rates. The ‘new’ demand rates are the new input for the planning tool and with the tool we determine the ‘new’ total annual costs.

![Sensitivity arrival rate](image)

Figure 11 Effects of deviation of demand rates on total annual costs

We observe only small deviations in the service level and the total annual costs, due to a deviation in the demand rate.
An analysis is conducted where we randomly assign a deviation to the demand rate per item. Deviations are between +50% and -50% of the original demand rates. We observe almost no deviation in the expected service level compared to the original aggregate fill rates of the total situation. Small deviations occur between different groups, ranging from -0.50% to +0.50% related to the original aggregate fill rate.

These small effects on service level are due to the relative high base stock level for the low demand rates. The Erlang loss formula absorbs these deviations and is slightly influenced due to a variation in the demand rate.

**5.4.2. Sensitivity of parameter lead time**

This section describes the sensitivity of the parameter lead time. The sensitivity of the lead time represents the sensitivity of the assumption that the central warehouse has infinite supply. VI issues a lead time per item to its customers for normal replenishments. This issued lead time can deviate in practice. The influence of the deviation on the service level is discussed in this section.

Each item’s lead time is assigned with a certain deviation. As can be derived from the probability of the Erlang loss model $\beta^1_{i,j}(S_i) := 1 - L(S_i, M_{i,j}, t^\text{reg}_i)$, the sensitivity of the lead time is comparable to the sensitivity of the demand rate. Figure 12 shows the deviation in service level if we alter the demand rate. Beta 1 and Beta 2 in Figure 12 stands for $\beta^1_{i}(S)$ and $\beta^2_{i}(S)$.

Another analysis is conducted where the deviations are randomly assigned from +50% to -50%. Negligible deviations in the average service level between groups are observed. Per group a maximum of +0.52% and -0.77% in service level is observed.

Only small deviations are observed and we can draw the same conclusion as in the previous section. The small effects on service level are due to the relative high base stock level for the low parameter $M_{i,j} t^\text{reg}_i$. The Erlang loss formula absorbs these deviations and is slightly influenced due to a variation in the parameter $M_{i,j} t^\text{reg}_i$.

![Sensitivity lead time](image-url)  
**Figure 12** Effects of deviation in lead time on the Service Level
5.4.3. Sensitivity of parameter transportation costs

The influence of both the costs for lateral transshipments and for emergency shipments are analyzed and discussed in this section. A restriction on the costs for a lateral transhipment is that it needs to be lower than the costs for an emergency shipment. This is a condition for a correct use of the model.

The analysis shows us that an increase in the cost for a lateral transhipment has a small impact on the total annual costs. We have altered the costs for a lateral transhipment from €0.01 to the costs of an emergency shipment, where the emergency shipment is set for €500. The analysis is done for several situations. We observe only a small increase in the total annual costs, if we increase the costs for a lateral transhipment. This is due to the small percentage of lateral transshipments in the various situations.

If we increase the costs for emergency shipments we see that the total annual costs increase significantly. In our analysis, the cost for a lateral transhipment is set for €0.01. We increase the costs for an emergency shipment from €0.02 to €1000 and see an average increase of 30% in the total annual costs. This increase is mainly caused by an increase of inventory value. At first, the transportation costs increase until a certain costs for emergency shipments. If we increase the costs for emergency shipments even more, the total annual costs increase due to an increase in inventory value. This switchover point depends on the average price of items. First, it is more profitable to leave the inventory constant such that the cost for transportation increases, until this switching point. If the costs for emergency shipments are at a certain height, it is more profitable to increase the inventory such that the number of emergency shipments decreases.

5.4.4. Sensitivity of parameter storage costs

In the analysis of our case studies, the storage costs were set to €0.00, because no reliable information was available for analysis. However, the model is capable of implementing storage costs. In this section, we analyse the effect of storage costs on the base stock levels and the total annual costs. The size of the item sets the height of the storage costs and, most likely, relates the price of the item to its size. If we follow this reasoning, the storage costs have almost no effect on the base stock levels and the transportation costs. The total annual costs increase linear with the increase of storage costs. This observation can be derived from the formulas presented in Section 3.5.

If the storage costs do not relate to the items’ price, the storage cost will influence the base stock levels, but the exact degree is not analyzed. This is situation depended and the planning tool is therefore a useful tool to analyze this per situation.
6. Case study: Customer with two locations

This chapter describes the analysis conducted on a customer with two locations. In Section 6.1, we describe the characteristics of the customer under consideration. We evaluate the new planning method in Section 6.2. In the evaluation, we use an equivalent situation to the current situation, such that we can determine the influence of the multi-item approach compared to the single-item approach. Section 6.3 discusses various scenario settings that have been analyzed in this case study. This way, we can determine the influence of inventory pooling. Section 6.4 briefly describes the analysis on the sensitivity of several input parameters.

6.1. Characteristics

This section describes the characteristics of the customer under consideration. Vanderlande Industries services two systems, one at each location, denoted here as location 1 and location 2. Both systems have an independent warehouse in which VI holds spare parts on stock. Once an item fails at a system, a new part is satisfied immediately from the local warehouse. If the warehouse does not have stock on hand, an emergency shipment occurs from the VI central warehouse or from a direct supplier. In Section 6.1.1, we discuss the commonality between the two systems. The current stock levels and the criticality of the items in the installed base are discussed in Section 6.1.2 and 6.1.3. Section 6.1.4 describes the data structure of demand on spare parts for this customer.

6.1.1. Commonality

This section discusses the commonality between the two locations. Commonality is defined as the fraction of items used in both systems. Location 1 has 805 items with a VI item number and denoted as a spare part in its installed base. This is 56.7% of the total installed base of location 1. Location 2 has in total 246 items in the installed base, identified with a VI number and denoted as a spare part. This is 39.0% of the items in the installed base of location 2. The price of the most expensive item is about 0.6*10^6 times the price of the cheapest item.

If we analyze the commonality based on the VI numbering, we find 113 items that are in both systems. This is 45.9% of all the items in location 2 and 14.0% of the items in location 1. The average prices of the common items are comparable to the other items in the installed base. Based on this information on commonality, we can conclude that an analysis on shared stock is recommendable.

6.1.2. Current Stock levels

Information is available on the exact number of spare parts on stock. Both systems have an own warehouse with spare parts on stock. The inventory of spare parts is maintained by the service engineers of VI. VI is owner of the stock and the customer pays an annual fee for the stock on hand. The spare part inventory consists of both resale items and VI items. 907 different spare parts with a VI item number are on stock, this 41% of all items put on stock and 48% of the total stock value.

6.1.3. Criticality

The priority code of the spare parts is derived from the information on the installed base. Table 7 shows the priority codes of the items in the system. It is possible, that an item has two different
priority codes. This is caused by the different sections of the system in which this item is present. Observe that both groups of different priority codes are equally divided.

Table 7 Priority codes of items in the installed base

<table>
<thead>
<tr>
<th>Priority code</th>
<th>Location 1</th>
<th>Location 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>404</td>
<td>162</td>
</tr>
<tr>
<td>B</td>
<td>407</td>
<td>175</td>
</tr>
</tbody>
</table>

6.1.4. Data structure

A detailed dataset containing information on the demand of items in the period January 2005 – December 2006 is available. In this period, the customer has ordered 233 different items with a VI item number denoted as a spare part. Out of these 233 items, 49 items (21.0%) are common items and are located in both systems. The average price of these common items is comparable to the average price of the other items. We can therefore conclude that these 233 items are representative for the total installed base. The items with a certain demand rate are used in the analysis. If an item does not have a demand rate, the heuristic will not put an item on stock. The analysis conducted in this case study has in total 233 items.

The method to determine $m_{i,n}$ is the same as discussed in Section 5.1.4. We determine the demand rates $m_{i,n}$ using the following formula:

$$m_{i,n} = \frac{\text{Total arrivals of order lines of SKU } i \text{ per day} \times \text{#items of SKU } i \text{ in demand source } n}{\text{#items of SKU } i \text{ in installed base}}$$

$i \in I, n \in N$.

In the near future, VI can derive more reliable demand rates by using historical demand rates of other customers and the corresponding installed bases. This requires the use of a more reliable dataset that is currently available.

6.2. Evaluation of new planning method

In our analysis we compare the current performance with the performance of the new planning method. We evaluate the model with two different methods. First, we compare the inventory value of the current planning method with the new planning method if the service levels are kept constant. Secondly, we determine the difference in service level if we keep the investment in inventory of spare parts equal. Both warehouses are denoted as regular local warehouses and no lateral transshipments are allowed between the two warehouses in this evaluation. This is equivalent to the current situation at the customer. This way, we determine the influence of the multi-item approach compared to the single-item approach.

6.2.1. Comparison of inventory investment

Figure 8 describes the method to compare the inventory investment of the current method with the new planning method. First, we determine the current base stock level, the demand rate and $b_i$ per item. We perform a simulation and determine the aggregate fill rates as a result of the current stock levels. We use algorithm 1 and 2, discussed in Section 4.1, to determine the aggregate fill rates. These aggregate fill rates are the aggregate fill rate objectives for our new model. This
ensures an equivalent or higher service level in the new planning method compared to the current method. We use the spare parts planning model to determine the optimal base stock levels and the corresponding inventory value. Finally, we compare the investments in inventory of both situations.

Currently, no lateral transhipments are performed between the two warehouses. We therefore denote these warehouses as regular local warehouses in this comparison. The group of items in location 1 with priority code A is denoted as location 1-A, the group of items in location 1 with priority code B is denoted as location 1-B, etc. We determine the aggregate fill rates using Algorithms 1 and 2, as discussed in Section 4.1. The performance of the system with the current stock levels is shown in Table 8.

**Table 8 Aggregate fill rates with the current base stock levels**

<table>
<thead>
<tr>
<th>Group name</th>
<th>$\beta_n^2(S)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>location 1-A</td>
<td>98.38%</td>
</tr>
<tr>
<td>location 1-B</td>
<td>96.08%</td>
</tr>
<tr>
<td>location 2-A</td>
<td>92.01%</td>
</tr>
<tr>
<td>location 2-B</td>
<td>95.99%</td>
</tr>
</tbody>
</table>

These aggregate fill rates are used to set $\beta_n^{\text{obj},2}(S)$ for group $n \in N$. We are therefore certain that the aggregate fill rates due to the new planning method is at least the same as the current aggregate fill rate.

Due to the new planning method, we observe a decrease in inventory value of 53.2%! Note that this decrease is not for the current total inventory value, but for the items with a VI item number defined as a spare part and who have experienced demand in the period January 2005 – December 2006. This result is shown in Table 7.

**Table 9 Results for case study for customer with 2 locations**

<table>
<thead>
<tr>
<th>Planning Method</th>
<th>Inventory Costs (normalized)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Current Method</td>
<td>100</td>
</tr>
<tr>
<td>New Method</td>
<td>46.8</td>
</tr>
</tbody>
</table>

Most items’ base stock levels determined in the new planning method are lower than the current base stock levels. Some items, however, have a higher base stock level and investment is required to level up the base stock level. An investment of 2.4% of the current inventory value is required to level up the base stock levels.

The item’s base stock levels that are higher than the optimal base stock level need to be lowered. It takes a certain period of time to lower the current stock levels to the optimal base stock levels. Table 10 shows the percentage of the value that is recovered in a certain period, if we assume a constant demand rate. We can conclude that 82.6% of the decrease in inventory value is recovered in 2 years.
Table 10 Time it takes to recover the inventory value after lowering the current stock levels to optimal base stock levels

<table>
<thead>
<tr>
<th>Period</th>
<th>Percentage recovery inventory value</th>
</tr>
</thead>
<tbody>
<tr>
<td>6 months</td>
<td>34.3%</td>
</tr>
<tr>
<td>1 year</td>
<td>54.8%</td>
</tr>
<tr>
<td>2 years</td>
<td>82.6%</td>
</tr>
</tbody>
</table>

In the new planning method, some items have a base stock level equal to zero. These items have experienced a demand in the period January 2005 – December 2006, but due to their low demand rate and high price level, these items are not put on stock. The model computes that placing these items on stock will give a small effect on the aggregate fill rate, related to the item’s price level. If we set the base stock level of these items equal to one, the inventory value decreases by 43.2% compared to the current inventory value.

Expensive items with low failure rates (e.g. once a year) have a relative high base stock level, while their regular replenishment lead time is comparable to other items. The new planning method assigns lower base stock levels to the expensive items with low failure rates. This results in remarkable cost savings, as shown above.

The difference in base stock levels for the expensive items is due to the multi-item approach. By using a multi-item approach, expensive items with low service levels are compensated by cheaper item with high service levels. A single-item approach determines the base stock level based on the availability per item, irrespective of the item’s price.

6.2.2. Comparison of service level

Secondly, we compare the expected service level due to the current base stock levels and optimal base stock levels if the investment in spare parts inventory remains constant. The current aggregate fill rates are shown in Table 8.

We use an iterative approach to determine the service levels per group $n$ such that the inventory value remains constant. We assign the groups with a priority code A with a higher $\beta_n^{obj.2}(S)$ than the groups with priority code B. We observe an enormous increase in service level, while the investment in inventory remains constant. Table 11 shows the new aggregate fill rates per group $n$.

Table 11 Aggregate fill rates determined with new planning method and with current inventory value

<table>
<thead>
<tr>
<th>Group name</th>
<th>$\beta_n^{obj.2}(S)$</th>
<th>$\beta_n^2(S)$</th>
<th>Increase aggregate fill rate compared to current method</th>
</tr>
</thead>
<tbody>
<tr>
<td>location 1-A</td>
<td>99.90%</td>
<td>99.90%</td>
<td>1.52%</td>
</tr>
<tr>
<td>location 1-B</td>
<td>99.75%</td>
<td>99.75%</td>
<td>3.67%</td>
</tr>
<tr>
<td>location 2-A</td>
<td>99.90%</td>
<td>99.91%</td>
<td>7.90%</td>
</tr>
<tr>
<td>location 2-B</td>
<td>99.75%</td>
<td>99.76%</td>
<td>3.77%</td>
</tr>
</tbody>
</table>

The aggregate fill rates shown in Table 11 are relative high for service levels used in practice. Aggregate fill rates around 98% are more common service levels in practice. Table 12 and Table 13 show the decrease in inventory value if we use more realistic aggregate fill rates.
Table 12 Aggregate fill rates with realistic service level objectives (1)

<table>
<thead>
<tr>
<th>Group name</th>
<th>$\beta_{n}^{obj-2}(S)$</th>
<th>$\beta_{n}^{2}(S)$</th>
<th>Increase aggregate fill rate compared to current method</th>
</tr>
</thead>
<tbody>
<tr>
<td>location 1-A</td>
<td>98.00%</td>
<td>98.00%</td>
<td>-0.38%</td>
</tr>
<tr>
<td>location 1-B</td>
<td>98.00%</td>
<td>98.00%</td>
<td>1.92%</td>
</tr>
<tr>
<td>location 2-A</td>
<td>98.00%</td>
<td>99.13%</td>
<td>7.12%</td>
</tr>
<tr>
<td>location 2-B</td>
<td>98.00%</td>
<td>98.16%</td>
<td>2.17%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Group name</th>
<th>$\beta_{n}^{obj-2}(S)$</th>
<th>$\beta_{n}^{2}(S)$</th>
<th>Increase aggregate fill rate compared to current method</th>
</tr>
</thead>
<tbody>
<tr>
<td>location 1-A</td>
<td>99.00%</td>
<td>99.00%</td>
<td>0.62%</td>
</tr>
<tr>
<td>location 1-B</td>
<td>97.00%</td>
<td>97.01%</td>
<td>0.93%</td>
</tr>
<tr>
<td>location 2-A</td>
<td>99.00%</td>
<td>99.00%</td>
<td>6.99%</td>
</tr>
<tr>
<td>location 2-B</td>
<td>97.00%</td>
<td>97.03%</td>
<td>1.04%</td>
</tr>
</tbody>
</table>

It is however difficult to determine an optimal or good distribution between groups of the overall service level, i.e. the difference between groups with Priority code A and B. This distribution between groups is situation and customer depended, but the service level and total annual costs can be determined using the new planning method.

Again, we can conclude that the multi-item approach results in higher aggregate fill rates against an inventory value budget than a single-item approach.

6.3. Scenarios

In this section we analyze several scenario settings. At the moment, two systems are serviced by VI. Both systems have the disposal of a warehouse. In this analysis we denote these warehouses as main local warehouses or as regular local warehouses. The model determines the optimal base stock values and the corresponding system costs. Scenario one is comparable to the current situation of spare parts provisioning. The following scenarios will be analyzed in our case study:

1. Both warehouses are denoted as regular local warehouses. This scenario is comparable to the current way of working at the customer. At the moment, hardly any transportation of spare parts occurs between the warehouses of location 1 and location 2.
2. Both warehouses are denoted as main local warehouses. This allows lateral transshipments between the warehouses.
3. Both warehouses of location 1 and location 2 are denoted as a regular local warehouse. A warehouse is created outside the warehouse and is denoted as a main local warehouse. This warehouse is allowed to send lateral transshipments to the other regular local warehouses.
4. One regular local warehouse at the customer supplies the demand for both systems. Both systems are assigned to this warehouse for supply of spare parts.

These scenarios are used to determine the influence lateral transshipments and the situation in which main local warehouse are positioned. We compare the scenarios on total system costs by varying $\beta_{n}^{obj-2}(S)$. Costs for transportation are described in appendix IV. The depreciation factor $h$
is set to 17.5%. Figure 13 represents the annual total costs resulting from $\beta_n^{obj,2}(S)$ of the different scenarios. The annual total costs are normalized. The difference between $\beta_n^{obj,2}(S)$ and $\beta_n^{obj,1}(S)$ is set to 4%, but other differences show comparable results.

![Figure 13 Total annual costs for the different scenarios](image)

**Figure 13 Total annual costs for the different scenarios**

We observe only small differences in annual total costs between the different scenarios. We can conclude that the introduction of main local warehouses and the corresponding lateral transhipments reduces the total annual costs. But due to the low number of warehouses assigned to a main local warehouse in this case study the reduction in annual total costs is limited. We do see a larger reduction in costs, in terms of percentage, between the scenarios when $\beta_n^{obj,2}(S)$ increases. Again, we can conclude that the tool is useful to check the influence of different parameter settings and situational factors.

### 6.4. Sensitivity of parameters

An analysis is conducted to determine the sensitivity of several input parameters. An analysis is conducted on the sensitivity of the following parameters: the demand rate, the lead time of normal replenishments, transportation costs and storage costs. Comparable results with only negligible deviations are observed as were described in Section 5.4. We therefore refer to Section 5.4 for the sensitivity of these input parameters.
7. Implementation

This chapter describes the issues that need to be resolved, before we can implement the developed planning method. It is agreed with the management of VI that no analysis is required for the costs associated with the implementation. No timetable is required that describes the moments in which implementation milestones need to be finished. Several steps need to be executed, before the actual application of the software program can start.

First of all, we need to create a basis of support in the department Supply Chain Management Services. Eventually, this department will make use of the new planning method and software tool to determine the base stock level of items at the local warehouses. Secondly, the application of the planning tool needs to be clear for the persons who will use the tool in the near future. This is explained in Section 7.1. Methods and tools need to be developed such that the required information can be obtained as is explained in Section 7.2. Finally, the base stock levels of items for a customer are determined with the new planning method.

7.1. Creation for basis of support

The creation for a basis of support for the new planning method is an important pre-condition for the success of the implementation and application of the planning tool. Creating a basis for support already started during the Master thesis project. The development of the planning tool is consulted and discussed with responsible persons in the department Supply Chain Management Services.

Interactive discussions sessions were planned with several employees of the department Supply Chain Management Services, who eventually are planned to work with the new planning tool. During these discussion sessions the relevance and the benefits of the planning tool are explained.

Persons working with the planning tool need to be trained, such that they can work with the planning tool. First of all, a ‘spare parts planning’ manual is developed, that describes the use of the planning tool step by step.

Special training sessions are conducted, to explain the use of the planning tool in detail. Examples are shown, derived from the case studies described in this report. These training sessions contributed to the awareness of the planning model and the actual requirements of the input files.

7.2. Methods to obtain required information

As is explained in the ‘spare parts planning’ manual, the input files require specific information in a particular format; an example is the demand rate of items per location. Other examples are: item characteristics, such as the price and regular replenishment lead times, location characteristics, etc. This information needs to be obtained from the several datasets available at VI. Some information needs to be obtained from the customers under considerations.

At the moment it is a time consuming and complex task to obtain all the relevant information required to use the planning tool. It is therefore advised, to develop certain methods and tools that obtain the required information from the available datasets. At the moment, determining the demand rate per item is the most complex issue to be resolved. The implementation of a Service Management System can resolve this issue in the future.
Information needs to be obtained for the structure of spare part provisioning. The position of main local warehouses in a cluster of customers and the customer service level requirements need to be determined.

7.3. Actual use of planning tool

Analysis needs to be conducted on the supply chain from VI’s suppliers to the central warehouse and the central warehouse itself. Once this analysis is conducted, VI has a clear picture of their complete supply chain. An MSc student will conduct this analysis.

After resolving the above issues, the planning tool is ready for usage. Once the persons using the planning tool are fully trained and understand the underlying principles, they are capable of using the new planning tool. Once all input information is entered and the program run is finished, the responsible person needs to check the output. Some items have a base stock level of zero, while these items can be very critical for the system performance. The program user can alter these base stock levels by hand, based on experience and intuition.
8. Conclusions and Recommendations

This chapter discusses the main conclusions and recommendation that follow from the research project. The research assignment has been discussed in Section 1.3 and was formulated as follows:

“Develop a planning algorithm which determines the base stock levels of the spare parts at VI’s local warehouses and minimizes total system costs, subject to customer service requirements.”

A new planning method has been developed and its implications have been discussed throughout the report. In this chapter, we discuss the main conclusions and recommendations that follow from this research assignment. Some directions for further research are discussed as well.

8.1. Conclusions

The main conclusions based on this project are described in this section.

- The planning tool obtained a decrease in inventory value compared to the current inventory value in both case studies. The customer with two VI systems can achieve a reduction of 53% in inventory value under the new planning method compared to the current planning method. The other case study shows a reduction of 31% in inventory value compared to the current planning method. This reduction in inventory value does not influence the service level i.e., the aggregate fill rates remain the same. The new planning tool is also able to determine new base stock levels that generate higher service levels than the current base stock levels, while the costs for inventory remain constant.

  In the case studies we have illustrated that the multi-item approach shows explicit advantages compared to the single-item approach. The objective of spare part inventory is shifted from item availability to total system availability. By using a multi-item approach, expensive items with low service levels are compensated by cheaper items with high service levels. This enables a close-to-optimum spare part availability subject to a certain budget.

- The developed planning tool is a useful tool to build business cases around customer requirements. A business case consists of a number of locations, either defined as a regular or main local warehouse, different criticality groups, etc. Several business cases can be modelled and drawn up, such that they can easily be compared with each other on the basis of the results of the planning tool. The different scenarios in the case studies are examples of how a business case can be designed. The mutual results are used to determine the ‘best’ situation with respect to the number of main local warehouses and/or customer service levels. The planning tool is a useful tool to support the decision making processes in the design of a new spare part provisioning structure.

- The model of Kranenburg [9] is suitable for the spare part provisioning situation of VI. The concept partial pooling and the main local warehouses correspond to the use of SPCs around a cluster of customers. A new service measure is introduced, the time-based aggregate fill rate of Caggiano et al. [4] which required adjustments in the model of
Kranenburg. This service measure enables VI to provide a clear overview of spare part availability in a certain time frame.

8.2. Recommendations

This section presents the recommendations and directions for further research based on the research conducted.

8.2.1. Main recommendations

- The developed planning tool is a useful tool to determine the base stock levels at VI’s local warehouses. The new planning method has shown remarkable cost savings in inventory value compared to the current planning method. Another advantage is that the service measure enables VI to provide a clear overview of spare part availability in a certain time frame. It is therefore recommended to implement the new planning tool in practice.

- The output of the new planning method is based on the available information. The more reliable this information is the more reliable the output of the new planning method. The reliability of the item’s failure rate influences the performance of the output of the new planning method. A distinction needs to be made on the demand of items for corrective and preventive maintenance. Currently, no clear distinction is made between these different demand rates. This especially influences the factor $b_i$ for the batch size, because it is assumed that most orders in batches are due to preventive maintenance. The introduction of a Service Management System, which is currently under development by VI, can provide this information.

- An analysis has been conducted on items with a VI item number and denoted as a spare part. Resale items are left out of the scope of the research assignment due to the low availability of information for these items. Each resale item should be assigned with one unique VI item number, such that reliable information can be gathered on the failure rate. The introduction of a Service Management System, which is currently under development, can provide this information. If reliable information is available on the failure rates of resale items, we can use the new planning method to determine the base stock levels of the resale items. It is reasonable to assume that the new planning method is suited for the resale items and that comparable or even better results can be achieved due to inventory pooling.

8.2.2. Additional recommendations

- Further research needs to be conducted to develop a new planning method that determines the base stock levels at the central warehouse. An analysis should determine the effects of this new planning method for the central warehouse on the service levels of the local warehouses. The supply chain of VI suppliers to the central warehouse needs to be analyzed as well. An MSc student will conduct this analysis.

- VI should determine the optimal frequency of recalculating the base stock levels and the optimal length of the demand horizon needs to be determined. A rolling horizon of three
years and a recalculation of the base stock levels every three months are examples. Currently, there is not enough data available to determine these parameters.

- A Service Level Measurement system needs to be developed, such that the time-based aggregate fill rates can be determined in practice. The reliability of the model can be monitored and checked more thoroughly than in this research. Another advantage is that both VI and the customer can see whether the agreed service levels have been met. The introduction of a Service Management System, which is currently under development by VI, can provide this information.
9. References


10. Appendix

I. Notation

\[ J \] := the set of local warehouses, where \( j = 1, \ldots, |J| \).
\[ K(\subseteq J) \] := the subset of main local warehouses, where \( k = 1, \ldots, |K| \).
\( k_j \) := The first warehouse that is checked for availability, once warehouse \( j \in J \) doesn’t have stock on hand.
\[ I \] := the set of (non-empty) SKU-s, where \( i = 1, \ldots, |I| \).
\[ N \] := the (non-empty) set of demand sources, where \( n = 1, \ldots, |N| \).
\( N_j (\subseteq N) \) := the subset of demand sources that is assigned to local warehouse \( j \in J \).
\( m_{i,n} \) := demand rate of SKU \( i \) in demand source \( n \), \( i \in I \) and \( n \in N \).
\( M_{i,j} \) := total demand rate for SKU \( i \) at local warehouse \( j \), \( i \in I \) and \( j \in J \)
\[
M_n := \text{total demand rate for demand source } n, n \in N \left[ := \sum_{n \in N} m_{i,n} \right]
\]
\( t_{i,reg} \) := Mean replenishment lead time from the central warehouse to a local warehouse for SKU \( i \in I \).
\( t_{k,j}^{lat} \) := Mean lateral transhipment lead time from main local warehouse \( k_j \) to local warehouse \( j \in J \).
\( \bar{t}^{lat}_j \) := Average lateral transhipment lead time from all main local warehouses \( k \in K \) to local warehouse \( j \in J \). \( k \neq k_j \)
\( t_j^{em} \) := Mean emergency shipment lead time from the central warehouse to local warehouse \( j \in J \).
\( C_j^{em} \) := the costs for an emergency replenishment from the central warehouse to a local warehouse
\( C_j^{lat} \) := the costs for a lateral transhipment from a main local warehouse to a local warehouse.
\( C_{i,j} \) := the costs for storing SKU \( i \in I \) on stock at warehouse \( j \in J \).
\( \beta_n^{obj,1} \) := target fraction of demand of demand source \( n \in N \) that is delivered immediately upon request.
\( \beta_n^{obj,2} \) := target fraction of demand of demand source \( n \in N \) that is delivered immediately upon request or delivered by a lateral transhipment from main local warehouse \( k_j \).
\[ \beta_n^{obj} := \text{target fraction of demand of demand source } n \in N \text{ that is delivered immediately upon request or delivered by a lateral transshipment from a main local warehouse.} \]

\[ S_{i,j} := \text{base stock level for SKU } i \in I \text{ in local warehouse } j \in J . \]

\[ S_i := \text{vector of the base stock levels for SKU } i \in I \ (:= (S_{i,1}, \ldots, S_{i,n})) \]

\[ S := \text{vector of the base stock levels } (:= (S_1, \ldots, S_n)) \]

\[ \beta_n^1(S) := \text{expected fraction of the demand of demand source } n \in N \text{ that is delivered immediately upon request.} \]

\[ \beta_n^2(S) := \text{expected fraction of the demand of demand source } n \in N \text{ that is delivered immediately upon request or delivered by a lateral transshipment from main local warehouse } k_j. \]

\[ \beta_n^3(S) := \text{expected fraction of the demand of demand source } n \in N \text{ that is delivered immediately upon request or delivered by a lateral transshipment from a main local warehouse.} \]

\[ \beta_{i,j}^1(S_i) := \text{expected fraction of the demand of SKU } i \in I \text{ at local warehouse } j \in J \text{ that is delivered immediately upon request, i.e. from the stock in local warehouse } j \in J \text{ itself, also called the item fill rate.} \]

\[ \beta_{i,j}^2(S_j) := \text{expected fraction of the demand of SKU } i \in I \text{ at local warehouse } j \in J \text{ that is delivered immediately upon request, i.e. from the stock in local warehouse } j \in J \text{ itself, or delivered by a lateral transshipment from main local warehouse } k_j. \]

\[ \beta_{i,j}^3(S_j) := \text{expected fraction of the demand of SKU } i \in I \text{ at local warehouse } j \in J \text{ that is delivered immediately upon request, i.e. from the stock in local warehouse } j \in J \text{ itself, or delivered by a lateral transshipment from a main local warehouse.} \]

\[ \alpha_{i,j,k}(S_i) := \text{expected fraction of demand of SKU } i \in I \text{ at local warehouse } j \in J \text{ demand that is delivered from main local warehouse } k \text{ by means of lateral transshipment, } k \in K, k \neq j. \]

\[ A_{i,j}(S_j) := \text{expected fraction of demand of SKU } i \in I \text{ at local warehouse } j \in J \text{ that is delivered by means of lateral transshipment.} \]

\[ \theta_{i,j}(S_i) := \text{expected fraction of demand of SKU } i \in I \text{ at local warehouse } j \in J \text{ that is delivered from the central warehouse as an emergency shipment.} \]

\[ \sigma(k) := \text{vector for the pre-specified order for main local warehouse } k \in K \ (:= (\sigma_1(k), \ldots, \sigma_{K-1}(k))) \]

\[ h := \text{Percentage for holding a SKU on stock for one year (depreciation factor)} \]

\[ P_i := \text{The cost price for SKU } i \in I . \]

\[ b_i := \text{The factor for the batch size of SKU } i \in I . \]
II. Appendix ‘Spare Part Packages’ (Flow diagram)

Customer buys new project

Installed base is determined

Spare parts package is determined and proposed to customer

Customer purchases spare parts package?

New proposal

New spare parts package is determined

Customer purchases spare parts package?

Yes

Yes

Customer purchases spare parts package

Customer doesn’t purchase spare parts package
III. Explanation of algorithm 1 and 2

This section provides an explanation for algorithm 1 and 2 discussed in Section 4.1. These algorithms formally describe the evaluation of the performance of the system. The explanation is taken from Kranenburg [9] with some few alterations as is discussed in Section 4.1.

Kranenburg [9] describes how a policy, i.e., a choice of base stock levels for all main and regular local warehouses, can be evaluated exactly. However, exact evaluation is done numerically, and can be time-consuming or even computationally intractable if the number of local warehouses is large, since each local warehouse constitutes a dimension in the Markov process. (Of course, in the special case that all local warehouses are regulars, i.e., if no lateral transhipment takes place at all, each regular can be analyzed individually and we do not have computational problems.) Since our method should be applicable for real-life instances with many local warehouses, we have to overcome the computational problems of exact evaluation, and therefore, we introduce an approximate evaluation method in this section.

As mentioned in the previous section, evaluation can be done for each SKU \( i \in I \) separately. In the evaluation, for a given policy \( S_i \) for SKU \( i \), i.e., for a given vector of base stock levels in all local warehouses \( j \in J \) for SKU \( i \), we determine \( \beta_{i,j}(S_i) \), \( j \in J \), \( \alpha_{i,j,k}(S_i) \), \( j \in J \), \( k \in K \), \( k \neq j \), and \( \theta_{i,j}(S_i) \), \( j \in J \); the cost \( C_i(S_i) \) then follows from Equation (3.2). The key to our approximate evaluation method is that we reduce the state space of the Markov processes that we have to analyze. We do this reduction in two steps. The first reduction step decouples regular local warehouses from the mains, leaving us separate regulars and a system of mains to analyze. The second reduction step decouples the system of main local warehouses so that each main can be analyzed individually. The approximation method is described more formally in Section 4.1.

Decoupling the regulars from the mains

The first reduction step aims to decouple the regular local warehouses from the main local warehouses. In our model, the connection between a regular local warehouse \( j \in J \setminus K \) and all main local warehouses \( k \in K \) is that the mains can provide a lateral transhipment to the regular local warehouse. A demand for such a lateral transhipment occurs when regular local warehouse \( j \) faces customer demand at a moment that it is out of stock. In our approximation method, we assume that the overflow demand process at regular \( j \in J \setminus K \) behaves as a Poisson process that constitutes an additional demand stream at main \( k_j \). Obviously, in reality this demand process can be burstier than Poisson, as it occurs only at the moments that the regular local warehouse is out of stock, but especially for low demand rates our assumption seems not so bad. Also, notice that in case a regular \( j \) has \( S_{i,j} = 0 \), the overflow demand really follows a Poisson process, since all demand is forwarded.

By making our assumption, we can decouple the regulars from the mains, and analyze each regular individually. This assumption reduces the complexity of the analysis for the partial pooling situation (with both regulars and mains). We do not have to analyze a Markov process with a \(|J|\)-dimensional state space, which is large if \(|J|\) is large, and which can even be computationally intractable in that case. Instead, we can now first straightforwardly determine the fill rates at the regulars, then analyze the mains, for which we are left with a Markov process with
a $|K|$-dimensional state space only. Finally, we can determine the other performance measures of the regulars, using the output of the analysis of the mains. In the remainder of this subsection, we describe how we analyze the regulars.

We determine the probability that a local warehouse faces customer demand at a moment it is out of stock. We make use of the loss probability in an Erlang loss system and is denoted as $L(n, \rho)$.

$$L(n, p) = \frac{\rho^n / n!}{\sum_{x=0}^{n} \rho^x / x!}$$

, where $n$ represents the number of servers in the system and $\rho$ the occupation rate.

For the regular local warehouses, the analysis is done as follows. Each regular $j \in J \setminus K$ is analyzed separately using an Erlang loss model with demand rate $M_i, S_{ij}$ servers, and as mean service time the mean replenishment time $t_i^{\text{reg}}$. This gives us $\beta_{i,j}^1(S_i)$ (that actually only depends on $S_{ij}$, and not on the base stock levels at other local warehouses) as $\beta_{i,j}^1(S_i) \equiv 1 - L(S_{ij}, M_{i,j}, t_i^{\text{reg}})$. Notice that this determination of the fill rates in the regular local warehouses is exact.

At this point, we use our assumption that the demand for lateral transhipment to regular $j$ can be modelled as demand at main $k_j$ that follows a Poisson process, for which we set as parameter $(1 - \beta_{i,j}^1(S_i))M_{i,j}$. We can now analyze the system of mains, where each main $k \in K$ faces demand that follows a Poisson demand process with parameter $\hat{M}_{i,k} = M_{i,k} + \sum_{j \in J, k_j = k} (1 - \beta_{i,j}^1(S_i))M_{i,j}$, i.e., with a parameter that includes both its own demand and the demand for lateral transhipment to the regulars assigned to that main. The analysis of the system of mains provides us with $\beta_{i,k}^1(S_i), k \in K, \alpha_{i,k}^1(S_i), k \in K, \tilde{k} \in K, \tilde{k} \neq k$, and $\theta_{i,k}(S_i), k \in K$. The system of mains could be analyzed exactly, as described in Section 6.3, or approximately, as described in the next subsection.

As a last step, we can determine the remaining performance measures $\alpha_{i,j,k}^1(S_i), k \in K$, and $\theta_{i,j}(S_i)$ for all regulars $j \in J \setminus K$, based on the performance measures that we determined for the mains:

$$\alpha_{i,j,k}^1(S_i) \begin{cases} 
(1 - \beta_{i,j}^1(S_i))\beta_{i,k}^1(S_i), & k = j_j, \\
(1 - \beta_{i,j}^1(S_i))\alpha_{i,k}^1(S_i), & k \in K, k \neq j_j,
\end{cases}$$

$$\theta_{i,j}(S_i) \equiv (1 - \beta_{i,j}^1(S_i))\theta_{i,k}(S_i)$$

and

$$A_{i,j}(S_i) \equiv 1 - (\beta_{i,j}^1(S_i) + \theta_{i,j}(S_i))$$

Decoupling the mains

The second reduction step aims to decouple the main local warehouses, so that each main local warehouse can be analyzed individually. In our model, the connection between the main local
warehouses \( k \in K \) is that lateral transhipment can take place from each main to each other main. A demand for a lateral transhipment occurs when a main \( k \in K \) faces customer demand at a moment that it is out of stock. In our approximation method, we assume that the overflow demand process at main \( k \in K \) behaves as a Poisson process, with an increased rate, that constitutes additional demand streams at all other mains \( l, l \in K, l \neq k \). Again, in reality this demand process could be burstier than Poisson, as it occurs only at the moments that main \( k \) is out of stock. However, by making the assumption that it follows a Poisson process, we can decouple the mains, and analyze each main individually. This assumption reduces the complexity of the analysis of a system of mains. We do not have to analyze a Markov process with a \(|K|\)-dimensional state space, which is large if \(|K|\) is large, and which can even be computationally intractable in that case. Instead, we can now iteratively analyze the mains individually.

As initialization, we calculate \( \theta_i (S_i) , k \in K \) exactly. Given main local warehouses \( k \in K \) with demand rates according to a Poisson process with rates \( \tilde{M}_{i,k} \) and with base stock levels \( S_{i,k} \), we can calculate \( \theta_i (S_i) , k \in K \) as the Erlang loss probability of the aggregate system:

\[
\theta_i (S_i) = L(\sum_{k \in K} S_{i,k} \tilde{M}_{i,k} r_{i,k}), k \in K.
\]

In our iterative analysis of the main local warehouses, we use an Erlang loss model for each main. Below, we explain how we determine the demand rate in the Erlang loss model, and how our procedure iterates. We use an example with \( K = \{1, 2\} \) for ease of exposition. The pre-defined lateral transhipment order for the mains, obviously, is given by \( \sigma (1) = (2) \) and \( \sigma (2) = (1) \).

(i) Determination of the demand rate. Let us arbitrarily choose to consider main 1. We assume that \( \alpha_{i,2}(S_i) \) is known, as a result of the application of an Erlang loss model for main 2: Because we know \( \theta_i (S_i) \), at any approximation of \( \beta_{i,2}^1(S_i) \), the (approximated) lateral transhipment fraction \( \alpha_{i,2}(S_i) \) follows immediately from Equation (3.1).

We want to determine the demand rate in the Erlang loss model for main 1. Let \( \tilde{M}_{i,k} \) denote the total demand rate for SKU \( i \) at main local warehouse \( k \in K \) in the Erlang loss model. The demand rate at main 1 constitutes of two parts.

First, we have a demand rate \( \tilde{M}_{i,k} \) originating from customers from main local warehouse \( k \in K \) itself, including demand for lateral transhipment to all regulars assigned to main 1. In our system with main local warehouses only, it is given that this demand stream follows a Poisson process.

Second, we have additional demand for lateral transhipment from main 1 to main 2. Let \( \tilde{M}_{i,k}, k, \tilde{k} \in K, \tilde{k} \neq k \), denote the additional demand rate at main \( k \) related to lateral transhipment from main \( k \) to main \( \tilde{k} \). So, in our example we are interested in \( \tilde{M}_{i,2,1} \). We know that a fraction \( A_{i,2}(S_i) = \alpha_{i,2,1}(S_i) \) of the demand \( \tilde{M}_{i,2} \) at main 2, eventually is supplied by lateral transhipment from main 1. Furthermore, we know that in the Erlang loss model for main 1, a fraction \( \beta_{i,1}^1(S_i) \) of the observed Poisson demand rate will be fulfilled. To ensure that eventually
the fulfilled demand rate for lateral transhipment from main 1 to main 2 equals $A_{i,2}(S_i)\tilde{M}_{i,2}$, the additional demand rate $\tilde{M}_{i,2,1}$ at main 1 has to be set equal to $A_{i,2}(S_i)\tilde{M}_{i,2}/\beta_{i,1}^1(S_i)$.

Combining the first (own) demand stream and the second demand stream (for lateral transhipment), we obtain $\tilde{M}_{i,1} = \tilde{M}_{i,1} + \tilde{M}_{i,2,1}$.

(ii) The iterative procedure. Initially, for both main local warehouses 1 and 2, we assume that no lateral transhipment takes place between the mains and accordingly, we set the demand rates equal to $\tilde{M}_{i,k} := \tilde{M}_{i,k}$, $k \in K$. We apply an Erlang loss model for each main $k \in K$, with $S_{i,k}$. We determine $\beta_{i,1}^1(S_i)$ and $\beta_{i,2}^1(S_i)$, and thus, by Equation (3.1), we have $A_{i,1}(S_i)$ and $A_{i,2}(S_i)$. As described in (i), we can now determine the new demand rate $\tilde{M}_{i,j}$ for the Erlang loss model representing main 1. Subsequently, we determine the updated $\beta_{i,j}^1(S_i)$ as $\beta_{i,j}^1(S_i) := 1 - L(S_{i,j}, \tilde{M}_{i,j}t_{avg})$, and $A_{i,j}(S_i)$. Next, for main 2, successively determine $\tilde{M}_{i,1,2}$, $\tilde{M}_{i,2}$, $\beta_{i,2}^1(S_i)$, and $A_{i,2}(S_i)$. Continue with 1, etcetera, until $\tilde{M}_{i,1}$ and $\tilde{M}_{i,2}$ each do not change more than $\varepsilon$, with $\varepsilon$ small. We observed that this iterative procedure converges in all cases, but we did not yet formally prove it.

If $|K| > 2$, the determination of the demand rate in the mains is somewhat more complicated, but the same principle is applied as described in (i). For an example with 3 mains 1, 2 and 3, and with $\sigma(1) = (2, 3)$, we describe how $\tilde{M}_{i,1,2}$ and $\tilde{M}_{i,1,3}$ are determined at given values of $A_{i,j}(S_i)$.

We know that a fraction $A_{i,j}(S_i)$ of the demand $\tilde{M}_{i,1}$ at main 1, eventually is supplied by lateral transhipment from another main. Furthermore, we know that in the Erlang loss models for mains 2 and 3, respectively, fractions $\beta_{i,2}^1(S_i)$ and $\beta_{i,3}^1(S_i)$ of the observed Poisson demand rates will be fulfilled. Under the assumption of independency of the stock levels in mains 2 and 3, we have that from the demand for a lateral transhipment to main 1 only a fraction $1 - (1 - \beta_{i,2}^1(S_i))(1 - \beta_{i,3}^1(S_i))$ can be fulfilled. To ensure that eventually the fulfilled demand rate for lateral transhipment to main 1 equals $A_{i,j}(S_i)\tilde{M}_{i,1}$, the additional demand rates $\tilde{M}_{i,1,2}$ and $\tilde{M}_{i,1,3}$ together have to satisfy

$$\tilde{M}_{i,1,2} + \tilde{M}_{i,1,3} = \frac{A_{i,1}(S_i)\tilde{M}_{i,1}}{1 - (1 - \beta_{i,2}^1(S_i))(1 - \beta_{i,3}^1(S_i))} \quad \text{(III.1)}$$

From this demand for lateral transhipment, the correct fraction should be fulfilled by the correct main (2 or 3), according to the pre-defined lateral transhipment order $\sigma(1)$. We first let main 2 face all demand for lateral transhipment to main 1 with the rate as in Equation (III.1). A fraction $\beta_{i,2}^1(S_i)$ of this rate is fulfilled by main 2. We let main 3 face the remaining demand for lateral transhipment to main 1, i.e., demand with rate

56
\[
\frac{A_{i,1}(S_i)\tilde{M}_{i,1}}{1 - (1 - \beta_{i,2}^1(S_i))(1 - \beta_{i,3}^1(S_i))} (1 - \beta_{i,2}(S_i))
\]

, of which a fraction \(\beta_{i,3}^1(S_i)\) is fulfilled by main 3. Together, this implies that the total amount provided to main 1 by means of lateral transhipment from another main equals \(A_{i,1}(S_i)\tilde{M}_{i,1}\) (as can be verified algebraically).
IV. Increase in aggregate fill rates

Table 14 Increase in service level related to the current service levels described in Table 4 (1)

<table>
<thead>
<tr>
<th>Warehouse</th>
<th>$\beta_{n}^{obj,1}(S)$</th>
<th>$\beta_{n}^{1}(S)$</th>
<th>Difference</th>
<th>$\beta_{n}^{obj,2}(S)$</th>
<th>$\beta_{n}^{2}(S)$</th>
<th>Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Group 1.A</td>
<td>92.00%</td>
<td>95.74%</td>
<td>14.69%</td>
<td>96.50%</td>
<td>97.61%</td>
<td>15.72%</td>
</tr>
<tr>
<td>Group 2.A</td>
<td>92.00%</td>
<td>92.01%</td>
<td>13.79%</td>
<td>96.50%</td>
<td>96.81%</td>
<td>16.20%</td>
</tr>
<tr>
<td>Group 2.B</td>
<td>92.00%</td>
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