MASTER

Strain rate dependency of confor foam

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Award date:
1999

Link to publication
Strain Rate Dependency of Confor Foam

WFW-report 99.031
Anton Janssen

Master's Thesis
December 1999

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Abstract

Finite Element Modeling (FEM) for impact conditions requires accurate material characterization for the applicable conditions. This report describes a research performed at Dutch Organization for Applied Scientific Research (TNO) and Eindhoven University of Technology (TUE) for characterization of pink Confor Foam\(^1\) (CF42) for impact conditions.

Confor Foam is a material applied in crash-dummies and as energy absorbing material in vehicle interiors. In impact conditions, this material is typically loaded in compression to extreme deformations combined with high strain rates. Characteristic for these foams is their pronounced time-dependent behavior and their large damping capacity.

Dynamic impact tests have been performed in compression, with initial strain rates ranging from 80 up to 280 \([1/\text{s}]\). Stresses and strains were calculated from measured forces and recorded images of the drop experiments. Unloading strain rates amounted up to 40\% of the loading strain rates.

Strain distributions in height were also examined. The experimental strain distributions indicate that deviations up to 20\% compared to the average strains occur. Simulations showed marginal dependency.

An existing material model, based on the work of Happee & Klompen was defined as benchmark set BMC set. A material set was determined from the experiments. Next, simulations of both sets were compared with experimental curves. It proved that the EC curves were as accurate as the BMC set for the loading part, and unloading behavior was improved.

\(^1\)Confor is a trademark licensed to Aearo Company
Chapter 1

Introduction

During the last decades the number of road vehicles has increased significantly. Till 1972 this increase in traffic lead to an increase in traffic deaths. After 1972, the number of traffic deaths decreased, caused by successful injury reduction strategies, resulting in improved vehicle safety.

Vehicle safety can be divided into two categories: active and passive safety. Active safety comprises measures, to prevent an accident from happening (e.g. ABS). Passive safety or crash safety comprises the measures to reduce the injury when an accident occurs (e.g. seat belt).

In the last decades much has been done to improve the passive safety of cars. Mechanical human body models (dummies) were developed to assess the response of the human body under crash conditions. Injury criteria were developed to indicate the amount of damage that occupants suffer from collisions. To reduce the costs involved in crash research, more often mathematical models are used instead of experiments.

TNO Delft has developed a mathematical simulation program, called MADYMO. Until recently, the mathematical models in MADYMO consisted entirely of rigid bodies. This multi body technique requires less CPU-time, but contrary to e.q. FEM does allow the assessment of deformation of bodies.

The required CPU-time for FEM calculations was drastically reduced in the recent years. As a consequence, this diminished the advantage of the rigid body approach. In resent releases of MADYMO the user is offered the possibility to use he multi body technique, FEM, or a combination of both.

In order to make an accurate mathematical model of the dummy, one has to have an accurate description of the material behavior of the materials used in the dummy. This information in the form of stress-strain relations, is commonly obtained using standard material tests, such as tensile, compression and shear tests.
One of these materials is Confor Foam. Characteristic for this foam is its pronounced
time-dependent behavior and its large damping capacity (see the next section).

The goal of his study was to obtain the stress-strain relations for Confor Foam with
a so-called drop experiment. In a drop experiment a mass is dropped on a sample
under influence of gravity, without influencing the mass. This in contrast to other
techniques, which prescribe the motion of the mass. The drop experiment, compared
to these other techniques, is cheap to perform, but a disadvantage is the non-constant
strain-rate.

In this study, first a literature study was performed to know whether it is valid to use
the drop test on Confor-foam and what experience exists on previous experiments.
Besides that, a stress-strain relation that successfully describes strain rate dependent
material behavior is needed.

Experiment were executed on the foam with different impact speeds. The results of
these tests were analyzed to achieve displacements, displacement fields and forces.
The forces were measured directly, the displacements and displacement fields were
determined using a technique which calculates the displacement of the mass in subse-
quent images of the foam sample. From those displacement and displacement fields,
strains and strain-rates were calculated and evaluated. The forces were used to cal-
culate stresses. These strains and stresses were combined into stress-strain curves
which were used to model the foam in MADYMO. The drop tests were simulated
with MADYMO and compared to the experiment results.

In the next chapter the experiments and stress-strain models that are available in
the literature are discussed. The experimental setup of the test and its results are
displayed in chapter 3. The modeling of the experiment is discussed in chapter 4 and
in chapter 5 the experiment is simulated modeled in MADYMO and compared to
the experimental data. Finally, conclusions and recommendations are presented in
chapter 6.

1.1 Materials

This section is mostly taken from TNO14.

The behavior of solid foams can be described as highly nonlinear and strain-rate
dependent with high energy dissipation characteristics and hysteresis in cyclic loading.
Low density combined with high energy dissipation capacity make foams attractive
for energy absorbing functions in automotive applications. However, its discontinuous
nature makes it difficult to construct constitutive equations that accurately describe
the mechanical behavior of foam.

Foams are typically used under compression (Figure 1.1). At small strains the me-
chanical behavior is close to linear elastic, followed by a large order of magnitude
1.1. MATERIALS

Figure 1.1: Typical stress-strain curve for solid foam material

reduction in slope. Then, there is a long region in which the slope changes gradually. This stage corresponds to the collapse of cells. After the cells have collapsed, the final stage of densification is reached in which the collapsed cells come in contact with one another causing a sharp increase in the stress. This phenomenon is also known as bottoming out.

Strain-rate dependency is a very important factor which must be considered when modeling the characteristics of foam. If the dynamic stress-strain relationship from experiments is used directly in a simulation without considering strain rate effects, the foam model will almost certainly be either stiffer or softer than the real foam.
Chapter 2

Literature review

The objective of the performed literature study was bipartite:

1. to obtain information of drop experiments executed on strain-rate dependent material.
2. to obtain information of constitutive models which describe strain-rate dependent material behavior.

With this information it was decided

- Whether it is valid to use the drop test on strain-rate dependent, low density foam such as Confor-foam
- Which constitutive model is best suited for modeling Confor foam, assuming that the constitutive model is or can be made available for MADYMO.

The constitutive models were often tested on other experiments than the drop experiment. Therefore, first a review of the most frequently used experimental methods is given.

2.1 Experimental methods

There are a few commonly used techniques to assess strain rate sensitivity:

1. The Drop test,
2. The Split Hopkinson pressure bar (SHPB), also known as the Kolsky method,
3. The hydraulic compression machine.
A comprehensive description of the different techniques can be found in Bragov & Lomunov\(^3\); Hamouda & Hashmi\(^6\); Hill & Sjöblom\(^7\) and Hsiao et al.\(^8\). There is also the Expanding ring test. It is rarely used because it is difficult to perform the test and it requires a very precise displacement measurement (see Hamouda & Hashmi\(^5\)). Zhang & Moore\(^9\) mentioned a 'servo-hydraulic testing machine', but no further information about the characteristics of this machine was given.

### 2.1.1 Drop test

Figure 2.1 shows the basic setup of a drop test. It typically consists of a drop tower with a massive foundation, an impacting unit (or drop weight), and optionally stop blocks. In the test, the unit is raised to a predetermined height and then dropped. The impacting unit is guided during its fall on the specimen. The impact force can be measured in three different positions: a) with an accelerometer mounted on the top of the drop weight, or b) with a force transducer between the sample and end cap (calibration steel) and c) between the sample and the ground (not shown in figure 2.1). It is reported that option (c) gives best results: in situation (a) and (b), additional wave propagations in the impactor and the entire system affect the measurement.

Hsiao et al.\(^8\) performed a drop test on 72-ply unidirectional laminates (carbon-epoxy composite). The specimens were 2.54 [cm] long (high) and 1.27 [cm] wide. They used a mass of 4.66 [kg] falling from a height of 2.44 [m], producing strain rates from 10 to several hundred per second. The major problem was the presence of superimposed vibrations. As a solution, rubber sheets, from 2.54 [mm] to 7.62 [mm] thick, were placed over the top end cap to minimize ringing due to impact. Fiber cork vibration
2.1. EXPERIMENTAL METHODS

2.1.1 Specimen

Damping pads were placed between the floor and the entire drop tower apparatus to reduce the transmitted waves.

2.1.2 Split Hopkinson pressure bar

Figure 2.2 shows the basic setup of a SHPB test. The dynamic stress-strain relationship in uniaxial compression of a material is obtained by sandwiching a small sample between two elastic bars of equal cross-sectional area and elasticity, called the incident and the transmission bar, respectively. An elastic stress pulse is caused in the incident bar by a striking bar of a given cross-sectional area and material properties. By ensuring that all the three bars remain elastic, (plastic) deformation is induced in the (usually) ductile sample. The stress in the sample is obtained by measuring the transmitted pulse, the strain of the sample is calculated from the pulse reflecting from the sample back into the incident bar. The pulses are measured by means of a strain gauge attached to each bar.

The SHPB technique permits testing at higher strain rates exceeding 1000 [1/s]. Contact surface conditions are very critical. Specimens must be short to minimize wave propagation effects. The use of thin ring specimens under dynamic internal or external pressure minimizes the wave propagation effects, but is expensive and complex and cannot be used for thick samples.

Hill & Sjöblom\(^7\) and Thiruppukuzhi & Sun\(^{13}\) both performed SHPB tests on laminates. Strain rates from \(1.10^{-1}\) up to \(1.10^4\) [1/s] were measured for strains up to 0.1 [-].

Sawas & Brar\(^{11}\) performed SHPB tests on both polycarbonate, elastomer and PU foam. The materials were tested with strain rates from 500 to 1600 [1/s]. Strains up to 0.7 [-] were measured.

2.1.3 Hydraulic compression machine

Figure 2.3 shows the basic setup of the hydraulic compression test at TUE. The static part of the test rig is connected to a height adjustable platform. A sample is placed
between the two compressive plates, which are connected to the static and dynamic part of the system. The relative position of these plates is measured with a LVDT. The force is measured by a Kistler piezoelectric force transducer. A draw bolt was added as a load limiter to prevent the overloading of the load cell.

The machine has two modes of operation: a low speed mode and a high speed mode. In the low speed mode, the sample can be loaded using a programmed loading path. The velocities that can be achieved in this mode are limited. In the high speed mode, the machine can reach speeds up to $12 \text{ [m/s]}$. Compared to the low speed mode the control in loading path is ‘considerably less’.

Happee & Klompen$^6$ performed a hydraulic compression test on a PU foam and two Confor foams. The materials were tested at strain rates ranging from $3 \cdot 10^{-2}$ to $4.2 \cdot 10^1 \text{ [1/s]}$ for the PU-foam and from $3.3 \cdot 10^{-2}$ to $3.9 \cdot 10^2 \text{ [1/s]}$ for the Confor foams. Strains up to 0.95 [-] were measured. It was concluded that the system introduces noise for high strain rates. Adaptions must be made to experiment for higher strain rates.

### 2.2 Constitutive models

The constitutive models are divided in three groups: nonlinear visco-elastic, nonlinear visco-plastic and empirical models.
2.2. CONSTITUTIVE MODELS

2.2.1 Viscoelastic constitutive models

- Zhang & Moore\textsuperscript{19,20} investigated a nonlinear viscoelastic (NVE) constitutive model for high density polyethylene. In (1997\textit{b}) experiments were described, in (1997\textit{a}) the models were constructed. The NVE model is constructed by a combination of one spring with six spring-dashpot (Kelvin) elements in series. By selecting appropriate values for the elastic modulus and the viscosity of the Kelvin elements, each element dominates one particular time scale so that the multi-Kelvin model can reproduce the linear rheology of high density polyethylene (HDPE) for a long time period. So:

\[
\varepsilon = \varepsilon^e + \varepsilon^\nu
\]

where \(\varepsilon^e = \dot{\sigma}/E_0\) and \(E_0\) is the stiffness of the independent spring. \(\varepsilon^\nu\) is represented by

\[
\varepsilon^\nu = \sum_{i=1}^{6} \left( \frac{\sigma}{E_i \tau_i} - \frac{\varepsilon^\nu_i}{\tau_i} \right)
\]

where \(\tau_i = \eta_i/E_i\) denotes the retardation time, \(E_i\) is the spring stiffness and \(\eta_i\) is the dashpot viscosity for the \(i\)-th Kelvin element. \(E_0, E_i\) and \(\tau_i\) are defined as functions of stress to describe the nonlinear behavior of HDPE.

2.2.2 Visco-plastic constitutive models

- Zhang & Moore\textsuperscript{19,20} also investigated a viscoplastic (VP) constitutive model for high density polyethylene (see above). The VP model is developed using a framework which is proposed to characterize the inelastic behavior of metals. The total strain rate is considered to be decomposable into elastic and inelastic components:

\[
\dot{\varepsilon} = \dot{\varepsilon}^e + \dot{\varepsilon}^p
\]

Inelastic strain rate is introduced into the state in addition to the inelastic work to reproduce the highly nonlinear and time dependent response of HDPE. For the uniaxial case, this results in

\[
\varepsilon^p = C \left( \frac{\sigma}{X} \right)^n \quad (n \geq 1)
\]

where \(C\) is a scalar factor for the uniaxial case and the state variable \(X\) is defined as a function of the state history and inelastic strain rate.

Tests were conducted in a single axis servo-hydraulic test machine (MTS System Corp.) on cylindrical HDPE specimens with 12.7 [mm] diameters and heights of 25.4 [mm]. True strain up to 0.20 [-] and strain rates up to 0.1 [1/s] were tested.
2.2. CONSTITUTIVE MODELS

- Thiruppukuzhi & Sun\textsuperscript{13} report a more general 3-D viscoplasticity model. They made (amongst other things) the following assumptions:

1. No plastic deformation in the fiber direction.
2. Transversely isotropic composite material.
3. The behavior of the material in tension an compression is the same.
4. Small deformations, allowing additive decomposition of elastic and deviatoric strain.

The total strain rate is expressed in terms of the elastic and viscoplastic components of the strain rate:

\[ \dot{\varepsilon} = \dot{\varepsilon}^e + \dot{\varepsilon}^p \]

The viscoplasticity model is proposed as

\[ \varepsilon = \chi(\dot{\varepsilon}^p)^m(\sigma)^n \]

were the coefficients \(\chi\), \(m\) and \(n\) are to be determined based on experimental data.

SHPB experiments were performed to verify the viscoplasticity model in the high strain rate region. Strain rates in the order of 1000 [1/s] were achieved. The viscoplasticity constitutive equations were incorporated into the FEM program ABAQUS through its user material routine UMAT/VISCO3D. The model predictions were found to be quite good. This indicates that the extrapolation of the viscoplasticity model from low strain rates to high strain rates is valid, though only for low strain values (less than 0.04 [-]).

- Fu Chang et al.\textsuperscript{4} introduced a constitutive equation based on the similarity on the experimental stress-strain curve between visco-plasticity materials and the foam material up to the densification stage. The bottoming out phenomenon of the foam model is simulated by adding an additional term to the state variable expression. The model can be used for compressive, tension and shear loading. The model showed good correlation in compression loading with strain rates from 4E-3 to 8E-2 [1/s] for PU foam (density 88 [kg/m\(^3\)]).

2.2.3 Constitutive models based on empirical laws

- Both Zhang et al.\textsuperscript{21} and Sherwood & Frost\textsuperscript{12} use an empirical power law to account for strain rate effects:

\[ \sigma_c(\varepsilon) = \sigma_0(\varepsilon) \left( \frac{\dot{\varepsilon}}{\dot{\varepsilon}_0} \right)^{a+be} \]
with \(\sigma_c(\varepsilon)\) the compressive stress, \(\sigma_0(\varepsilon)\) the engineering stress-engineering strain relation at an arbitrary quasi-static engineering strain rate \(\dot{\varepsilon}_0\). \(a\) and \(b\) are material constants. Zhang et al.\(^{21}\) implemented the equation in LS-DYNA3D via a user defined material subroutine. Simulated and measured stress-strain responses of PP foam under uniaxial compression at strain rates ranging from \(1 \cdot 10^{-6}\) to \(4 \cdot 10^{-6}\) [1/s] agree well. The unloading behavior of the foams has not been accurately modeled. Sherwood & Frost\(^{12}\) added a function to account for density and temperature effects. They successfully matched experimental and simulated data for strain rates ranging from \(4.2E-3\) to \(8.4E-2\) [1/s], for strains up to \(0.80\) [-].

- A similar approach is reported by Happee & Klompen\(^6\). They used a modified Cowper-Seymonds relation:

\[
g(\dot{\varepsilon}) = c_3 \left(1 + \left(\frac{\dot{\varepsilon}}{c_1}\right)^{\frac{1}{c_2}}\right)
\]

to account for strain rate effects. Herein, \(c_1\), \(c_2\) and \(c_3\) are (material) constants which must be determined from the experiment. They incorporated this function into MADYMO using the FOAM model. Simulations and experiments agreed good. When refining the mesh, unstable behavior arose for high deformations.

### 2.3 Discussion

The drop test has, in comparison to the other methods, some advantages: it is inexpensive, can accommodate different specimen geometries and allows easy variation of strain rate. Strain rates up to a few hundreds can be measured. However, the system is sensitive to the contact conditions between the impactor and specimen and also sensitive to the noise from ringing and vibrations. It is noted that, without a damping system, the measurements are distorted through oscillations, which include rigid body accelerations of the system and shock waves resulting from impact.

The SHPB can deal with large strain rates (up to \(10.000\) [1/s] is reported). The test requires expensive equipment and is sensitive to contact conditions. Material dimensions must be small. However, this raises questions about the homogeneity and uniaxiality of the induced stress in the specimen, and thus a study of wave propagation effects in the testing materials is required.

The hydraulic compression test is performed with strain rates comparable to the rates achieved in drop tests. The unstable behavior that occurred when simulating the foam with a refined mesh, might be caused by the absence of any energy absorption in the system, as neither damping nor hysteresis was modeled.
2.4. CONCLUSIONS

The presented viscoplastic and viscoelastic models are not suited for (engineering) strains larger than 0.1 - 0.2 [\cdot]. For strains up to 0.95 [\cdot], good achievements are obtained with an empirical scaling function.

2.4 Conclusions

- It was reported that the drop test is suited to perform strain-rate dependent material research on low density PU-foam.

- An empirical scaling function -already modeled in MADYMO- will be used to describe the stress-strain material behavior.
Chapter 3

Measurements

The goal of the experiments is to determine the (strain-rate dependent) stress-strain relationship so that the material can be modeled. These stresses and strains need to be determined from the experimentally obtained forces and displacements respectively.

In an experiment, it is often assumed that strains (and strains) are uniformly distributed. To investigate whether this assumption is valid for the foam drop experiment, a second technique is used which calculates a displacement field from subsequent images of the deforming foam sample. From these displacement fields, strain fields are calculated.

In the following sections, first the experimental setup and the steps needed to process the experimental data are described. After that, the two above-mentioned techniques are specified. These techniques are tested and finally the results of the drop experiments are presented and discussed.

3.1 Experimental Setup

The drop test experiment setup is shown schematically in Figure 3.1. The construction consists of a 4 [m] high tube with a diameter of 10 [cm], through which a drop weight with a mass of 5 [kg] falls on the sample. The height of the mass above the sample can be adjusted, and by that its impact speed is estimated and adapted. The collision between the drop weight and the foam is recorded with a Kodak high-speed video camera. A Dedocool 250 [W] lamp illuminates the foam. A sensor was added to the system, as the amount of video-frames the recorder can hold is limited. Just before the mass hits the specimen, this sensor triggers the video-camera. A 30 [kN] Kistler force transducer measures the force in $x, y$ and $z$ direction, with the $z$ axes in vertical direction.
Both the signals from the force transducer and the trigger are processed by a Difa D.TAC100 amplifier (sample frequency 102.4 [kHz]) and stored in Matlab MAT format. The movies are decoded with a Kodak Ektapro Motion Analyzer and saved on harddisk as individual pictures in (uncompressed) TIF format. Only the pictures in which the drop weight makes contact with the foam sample are stored, due to the limited storing capacity of the disk.

To estimate the impact speed it is assumed that all of the potential energy of the drop weight is transformed into kinetic energy. Thus:

$$\frac{1}{2}mv_0^2 = mgh \rightarrow v_0 = \sqrt{2gh}$$  \hspace{1cm} (3.1)

with $m$ the mass of the drop weight, $v_0$ the impact speed, $g$ the gravity acceleration (9.8 [m/s$^2$]) and $h$ the height of the drop mass above the sample. As the height of the tube is 4 [m], the maximum speed (according to this equation) equals 8.9 [m/s]. Theoretically, the lowest impact speed is 0 [m/s]. At this speed however, the impact forces in the force transducers signal are too low to discern them from the noise level.
3.1. EXPERIMENTAL SETUP

Figure 3.2: Mass of drop experiment. At surface 1 the mass triggers the trigger. Surfaces 2 and 3 generate discontinuities in the trigger signal. (see section 3.3.2). Due to this limitation, the minimum impact speed is roughly 2 [m/s].

These rough estimated velocities cannot be used in the modeling of the experiment, as their accuracy is not high enough:

1. The use of formula 3.1 leads to an overestimated speed, because drag and friction are neglected. Since both effects are proportional to the square of the velocity, the error increases at higher speeds.

2. The height of the drop weight above the sample is measured with a precision of 0.05 [m]. The lowest impact speed in the experiment is roughly 2 [m/s]. Then, the height of the mass above the sample is 0.2 [m], resulting in a relative error of 25%.

As a solution, the trigger signal is used to calculate the speed of the mass just before impact. For that purpose, the height of the drop weight that generates the trigger signal is introduced as measuring height, which has a value of 50 [mm] (see Figure 3.2).

When the weight passes the trigger, a signal similar to Figure 3.3 is generated. The parts of the signal correspond to the parts of the drop weight in Figure 3.2. Assuming a constant velocity over the time from \( t_1 \) to \( t_3 \), the initial drop speed is calculated with the formula

\[
v_0 = \frac{50 \cdot 10^{-3}}{\Delta T} \text{ [m/s]} \quad (3.2)
\]

with \( \Delta T = t_3 - t_1 \) the time intervals [s] in Figure 3.3.

Actually, this calculated speed is smaller than the real impact speed, as the drop weight falls a further 60 [mm] before it hits the sample. Therefore, a correction must be made for this difference. Assuming

a) a linear increase of the velocity and
b) a small difference between the measuring height of the drop weight and the distance of the trigger above the sample, the actual impact speed is approximated as follows:

\[
\begin{align*}
  v_3 &= v_1 + g \Delta T \\
  v_m &= \frac{v_1 + v_3}{2} = \frac{\Delta h}{\Delta T} = v_0 \\
  \Rightarrow v_3 &= \frac{g}{2} \Delta T + v_0 = \frac{g}{2} \Delta T + \frac{\Delta h}{\Delta T}
\end{align*}
\]

with \(v_m\) the average speed between \(t_1\) and \(t_3\) [m/s], \(v_1\) and \(v_3\) the actual speed of the drop weight [m/s] when it triggers the trigger sensor \((t = t_1)\) and at impact respectively \((t \approx t_3)\), \(\Delta T\) the time interval [s] from Equation 3.2 and \(\Delta h = 50\) [mm] the measuring height of the drop weight.

The sensor is positioned in such a way that it just more than 50 [mm] above the sample (see Figure 3.4). So the measured speed is not influenced by velocity changes due to the weight hitting the sample.

3.2 Data processing

There are two data streams available from the experiment: the signal of the force transducer and the TIF-images.

The signal of the force transducer is filtered to remove noise and then it is averaged over the runs in a single test. As a result of a (small) difference in impact speed between runs in a single test, the force-time curves are shifted in time from one
3.2. DATA PROCESSING

Figure 3.4: Position of the trigger relative to drop weight at maximum compression.

another. Averaging those shifted runs may introduce errors. With the aid of the trigger signal from the previous paragraphs, it is possible to correct for these time shifts. The time shifts are calculated as follows:

\[ ts_{ij} = \Delta T_{ij} - \Delta T_{ij} \]  \hspace{1cm} (3.4)

with \( \Delta T \) the time intervals [s] in Figure 3.3, \( i \) the number of the run and \( j \) the number of the experiment. The time shifts are applied to the force signals, so that the signals in a test are shifted towards the first run.

The images are used to calculate two quantities: the displacement of the mass and the displacement field of the foam surface. The former is used to calculate strains, which in turn are necessary to describe the stress-strain relation. The latter is used to investigate the assumption that the strains are uniformly distributed in the foam sample. In the next paragraphs, first the calculation of the mass displacement is treated, were after the determination of the displacement field is discussed.

Mass displacement

The displacement of the upper side of the sample is approximated by estimating the displacement of the drop weight, assuming that the sample and the drop weight make contact to each other during impact. Figure 3.5(a) shows an arbitrary image of a foam sample under impact loading. Both the sample and the drop weight are illuminated by the spotlight. The metal drop weight however reflects more light than the foam sample. Therefore, the drop weight is lighter (i.e. has a lower gray value) in the image than the sample.
3.2. DATA PROCESSING

Figure 3.5: Determination of mass displacement. a) Image of a foam sample, b) Row difference of (a) and c) Row totals of b. In (c), horizontally the row totals are plotted, vertically the row number.

The shown image is actually a matrix representing the gray value of the pixels. The spatial derivative in z direction of the gray value (i.e. the row difference) is a measure for changes in brightness. The larger the value, the larger the change in brightness.

The number of the highest row total indicates the position of the top of the sample. From these positions, displacements are calculated. Figure 3.5 shows an example of this procedure: the left figure is an image of a sample, the middle image is the row difference and the right figure shows the row totals.

The error estimate of the displacement is at most half a pixel. With 128 pixels corresponding to approximately 30 mm, this error equals roughly 0.12 mm. Determining the first image in which the drop weight hits the foam sample, is crucial to this method. An error of one image results in a time shift of 1/(frame rate). It also results in a mismatch in the (maximal) displacement, which is larger for high impact speeds.

Displacement Field

To approximate the displacement field of the foam surface, another technique is used. The program, written by M. Geers of Eindhoven University of Technology, is based on contrast distribution. The user defines a rectangular grid of virtual markers in the first image (see Figure 3.6). The contrast distribution within a given window size \((w_{i_x}, w_{i_z})\) around these markers is settled. The program then tries to follow those windows within a user defined search area \((s_{a_x}, s_{a_z})\) around the marker in the subsequent images. The \(x\) and \(z\) values of the window size and search area may differ. The weighted coordinates of the markers are calculated and recorded.

The method depends on a good contrast distribution. The following precautions were made to get a as high as possible contrast in the image:

1. The diaphragm of the camera is adjusted to record with maximum contrast.
3.2. DATA PROCESSING

2. The lamp is placed at a small angle with the surface of the foam. In this way, the rough surface generates more contrast than when the lamp is placed perpendicular to the surface.

Another possibility to improve the contrast was to spray some black dye on the foam. Although this works with (white) polystyrene foams, it is not used on the Confor foam because the pink foam shows quite dark gray already on a black-and-white camera. White dye could be more suitable, but was not available at the time of the experiment.

Finally, the force-time curve and the displacement-time curve of the mass are synchronized to plot them against each other. To synchronize the signals it is assumed that the force has a maximum value when the foam is maximal compressed. Both the starting and ending time of the force signal are extracted from the starting and ending time of the displacement curve.

3.2.1 Evaluation of the marker tracking program

To test the accuracy of program that is used to calculate the displacement field, a so-called null experiment is executed. A sample is filmed without the mass dropping on it, resulting in 200 subsequent images. Next the program is run on this data.

The influence of the size of the window and search area is investigated with run 1 from test 1. All possible combinations of \((5 \leq wi_x, z \leq 17)\) and \((9 \leq sa_x, z \leq 25)\) are analyzed to attain the highest number of complete trajectories.

Figure 3.6: Window size \((wi_x, wi_z)\) and search area \((sa_x, sa_z)\) definitions for a given marker \(i\)
3.2. DATA PROCESSING

Figure 3.7: Mean error (solid lines) and standard deviation (dotted lines) for low (left image) and high (right image) values of the search area and windows size.

In the null-experiment, up to 98 of the 100 markers could be followed during the full 200 images. Special attention must be given to the position of the upper and lower markers. Putting them too close to the upper and lower boundary of the sample causes substantial loss of these markers during the first part of the tracking.

The error in the estimate of the position was calculated as follows:

\[ r_{ij} = \sqrt{(x_{ij} - x_i)^2 + (z_{ij} - z_i)^2} \]  \hspace{1cm} (3.5)

with \( x \) and \( z \) the position of the marker, \( i \) the image number and \( j \) the number of the marker. Figure 3.7 shows the mean error and its confidence interval for two typical values for \( w_{i_{\text{sz}}} \) and \( s_{a_{\text{zy}}} \). Increasing \( s_{a_{\text{zy}}} \) and \( w_{i_{\text{sz}}} \) decreases the error.

The figure also shows that the markers tend to drift away from their origin. Correlation between the markers initial position and this direction of the drift was not found, so a randomly distributed drift is assumed.

From the tests which were executed to estimate the influence of window size and search area, the following conclusions are drawn:

a) Square windows and search areas (\( w_{i_x} = w_{i_y} \) and \( s_{a_x} = s_{a_z} \)) give better result than an unequal combination. Whether this is true for unequal pixel densities [pixels/mm] in the image for \( x \) and \( z \) direction, was not investigated.

b) A combination of 11×17 or 13×19 pixels for \( w_{i_{\text{sz}}} \) and \( s_{a_{\text{sz}}} \) respectively, returned the most valid trajectories for both low and high impact speeds. Lower values of \( w_{i} \) and \( s_{a} \) returned lesser trajectories, whereas higher values of \( w_{i} \) and \( s_{a} \) led to a relatively higher number of mismatched trajectories.

Based on the results from the previous tests, a value of 13 × 19 pixels for \( w_{i_{\text{sz}}} \) and \( s_{a_{\text{sz}}} \) was used in the image processing process.
3.2.2 Experimental protocol

Pink Confor Foam (CF-42, density 98 [kg/m³]) samples of 50x50x25 [mm] (width x depth x height) were cut using a bandsaw at high speed to prevent the samples from deforming during the cutting process. The foam was tested at 5 different speeds, each test was repeated 5 times. The different settings are displayed in Table 3.1. The first three tests are recorded with a frame-rate of 9000 [1/s]. At the highest impact speed test, this frame-rate is not high enough to record a substantial amount of video frames, these test is recorded with a recording rate of 27000 [1/s]. As a consequence, the image quality drops from 256x256 to 128x64 [pixels].

During the tests, it is noticed that the images of the samples in the highest speed tests have less contrast at large deformations. It is expected that less markers are tracked. To investigate whether this is due to the higher impact speed (and therefore larger compression) or the lower pixel density, a fifth test was done at a lower impact speed and a lower pixel density.

<table>
<thead>
<tr>
<th>Test number</th>
<th>Speed [m/s]</th>
<th>Recording rate [s⁻¹]</th>
<th>Image size in pixels (X×Y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 - 5</td>
<td>2</td>
<td>9,000</td>
<td>256×128</td>
</tr>
<tr>
<td>6 - 10</td>
<td>4</td>
<td>9,000</td>
<td>256×128</td>
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<tr>
<td>11 - 15</td>
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<tr>
<td>16 - 20</td>
<td>8</td>
<td>27,000</td>
<td>128×64</td>
</tr>
<tr>
<td>21 - 25</td>
<td>3</td>
<td>27,000</td>
<td>128×64</td>
</tr>
</tbody>
</table>

Table 3.1: Experimental settings. The speed is estimated with formula 3.1

3.3 Drop experiments

In this section, first the the force signal is treated. Next the displacements and the displacement fields are shown. Finally the force and displacement curves are synchronized and plotted.

3.3.1 Determination of the impact speed

The impact speeds were determined from the trigger signal. Figure 3.8 shows the signal from the trigger. For clarity only the first test is shown. The other test results are depicted in appendix A.1, Figures A.1 - A.5.

The average speeds, calculated with the equation 3.3, are 2.0, 4.1, 5.7, 7.0 and 3.0 m/s respectively for tests 1, 2, 3, 4 and 5.
3.3. DROP EXPERIMENTS

3.3.2 The Force signal

Next we look at the Force-Time diagram. The raw (that is, the unprocessed) data of test number 1 is shown in Figure 3.9. It shows a 50 [N] ripple and a high-frequency noise. The ripple is probably caused by the 50 [Hz] Voltage of the Dedocool lamp voltage cables.
3.3. DROP EXPERIMENTS

Figure 3.10: Power Spectrum Magnitude for run 1. A frequency of 1 equals the Nyquist frequency ($= 51.2 \text{ [kHz]}$), or half the sampling rate

The Power Spectral Density of run one (see Figure 3.10) indicates that it might be possible to lower the noise of the signal by applying a low-pass filter. The noise was removed by applying a 8th order low-pass Butterworth filter to the data.

The ripple could not be removed from the force signal by applying a high pass filter to the data. This is due to the pulse of force signal, which amounts roughly 0.02 [s]. The accompanying frequencies are in the order size of 25 [Hz] and multiples of this, which is of the same order size as the ripple frequency. Filtering the signal with a high pass filter to remove the ripple will also remove significant frequencies of the force signal.

Another possibility to reduce the ripple is averaging the five runs. Before averaging the signals, first the time shifts from Table 3.2 are applied to the force signals. Figure 3.11 shows the average signal for the first test. It shows that the ripple is lowered to 30 [N]. It was decided to use the averaged filtered force-time signal in the calculation process.

The remaining force-time signals are depicted in appendix A.2, Figures A.6 to A.10. The force signal from test 4 (impact speed is 7.0 [m/s]) cannot be used for calculation purposes, as it is leveled by the limits of the force sensor.
3.3. DROP EXPERIMENTS

<table>
<thead>
<tr>
<th>speed</th>
<th>1st run</th>
<th>2nd run</th>
<th>3rd run</th>
<th>4th run</th>
<th>5th run</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.0</td>
<td>0.00</td>
<td>-0.20</td>
<td>-0.50</td>
<td>-0.20</td>
<td>-0.20</td>
</tr>
<tr>
<td>4.1</td>
<td>0.00</td>
<td>0.08</td>
<td>0.00</td>
<td>0.08</td>
<td>0.00</td>
</tr>
<tr>
<td>5.7</td>
<td>0.00</td>
<td>0.07</td>
<td>0.07</td>
<td>0.07</td>
<td>0.07</td>
</tr>
<tr>
<td>7.0</td>
<td>0.00</td>
<td>-0.35</td>
<td>-0.06</td>
<td>-0.06</td>
<td>-0.20</td>
</tr>
<tr>
<td>3.1</td>
<td>0.00</td>
<td>0.00</td>
<td>-0.20</td>
<td>0.06</td>
<td>-0.07</td>
</tr>
</tbody>
</table>

Table 3.2: Time shifts in milliseconds for the five runs per test. The time-shifts are relative to the value of the first run.

Figure 3.11: Measurement of Force vs time, test 1, after averaging over five runs

3.3.3 Displacements, strains and strain rates

The results of the displacement calculations are show in Figure 3.12. At increasing impact speeds, it shows a gradually increase of both initial slope and maximum displacement. Increasing the impact speed from 5.7 to 7.0 [m/s] does not seem to generate larger compression. Apparently, the bottoming out stage is reached.

From the displacements, engineering strains are calculated \( \varepsilon = (l - l_0)/l_0 \). The derivative of the (engineering) strain is the (engineering) strain rate. The calculated strain rates are shown in Figure 3.13. For the higher impact speeds, the strain rates indeed are constant within a margin of 5%.

The initial strain rates are approximated with the formula: \( \dot{\varepsilon} \approx v_0/0.025 \) [1/s]. They are -80, -164, -228, -280 and -120 [1/s] for the five tests respectively. The initial strain
3.3. DROP EXPERIMENTS

rate is about 10% smaller than the approximated strain rate. This might be caused by

1. An overestimated impact speed, caused by either a mismeasurement of the mass height or an aberrance in the trigger signal,

2. An underestimated height of the foam sample,

3. A velocity change of the mass at impact

The difference was not caused by one of the first two possibilities. The last one could not be checked, as the images just before impact were not saved. The simulation of the experiment might provide more information.

To compare the several experiments, the strain rates and experiment times were divided by their most negative values. These normalized strain rates versus normalized times are plotted in Figure 3.14. This figure shows an interesting characteristic: For all tests, the unloading strain rate amounts to approximately 40% of the maximal loading strain rate. In chapter 5 this correspondence is used to determine the unloading curve.

3.3.4 Force versus displacement

The force-displacement curves are plotted in Figure 3.15. The test at the highest impact speed is not plotted, as the accompanying force was not correctly measured.
3.3. DROP EXPERIMENTS

The figure shows good agreement for the four curves. At increasing impact speed, the force is increased. The curve of the lowest impact speed is, compared to the other three curves, a little shifted towards lower displacements. This is probably due to an error in the synchronization process.

3.3.5 Displacement field

The number of complete trajectories diminished rapidly at increasing impact speeds. At 4.0 [m/s], 9 trajectories were found. At impact speeds of 5.7, 7.0 and 3.0 [m/s] no trajectories were found. As the 3 [m/s] experiment brings up less trajectories than the 4 [m/s] experiment, it was concluded that this was due to lower pixel density.

The greater part of the markers was lost at compression more than 0.7 [-]. Obviously, the contrast was too low to track the markers. The number of 9 trajectories is too low to compute strains.

The displacement field results from the trajectories of the markers. Typical trajectories of an experiment with an impact speed of 2 [m/s] are shown in Figure 3.16. It shows that the trajectories are mainly parallel to the gravity direction. Trajectories of markers with equal initial y-coordinates travel close paths, almost independent of the markers x-position. In the further calculation, this x-dependency is neglected.

Happee & Klompen\(^6\) indicated that this pink Confor foam shows isotropic behavior. In isotropic materials, the principal planes of stress and strain coincide. To investigate whether the x, y and z axis coincide with the principal axis, it is examined whether
3.3. DROP EXPERIMENTS

Figure 3.14: Normalized strain rates versus normalized times, for different impact speeds

Figure 3.15: Force versus displacement for different impact speed. The y-axis is scaled logarithmically to show the whole region of the forces. Solid = 2 [m/s], dotted = 3.1 [m/s], dashed = 4 [m/s], dot-dashed = 5.7 [m/s]
3.3. DROP EXPERIMENTS

Shear deformation

To investigate the presence of shear deformations, the behavior of rectangular surfaces is investigated. When these squares keep their rectangular shape, shear deformations are not present. A mesh is generated by connecting markers with an identical initial \(z\)- or \(x\)-coordinate. Meshes for various times are plotted in Figure 3.17.

Perpendicular axis remain approximately perpendicular during impact. Therefore, shear deformations can be neglected. The foam sample has symmetry axes at \(x = y = 25 \text{ [mm]}\) and \(z = 12.5 \text{ [mm]}\) (half the sample dimensions), thus the principal axes are parallel to the \(x,y\) and \(z\) axes. Therefore the strains in \(x\), \(y\) and \(z\) direction are principal strains and can be analyzed independently of each other. The displacement in \(x\) direction, however, not zero. Therefore, lateral contraction influences are present, albeit small.

Both Figure 3.16 and Figure 3.17 indicate that the \(z\)-displacement of the markers is only dependent of the initial \(z\)-coordinate, whereas the \(x\)-displacement is dependent of both \(z\) and \(x\) coordinates. Therefore, the \(z\)-strain is not dependent on the \(x\)-coordinate and needs no further examination.

\textbf{z-strain distribution} To investigate whether the \(z\)-strain varies of the height of the foam, the following procedure is executed on the five runs of test 1. Only complete trajectories are processed.
3.3. DROP EXPERIMENTS

1. The y-position is averaged over all markers with the same (initial) z-coordinate. This results in (maximal) 10 mean positions, \( p_1, \ldots, p_{10} \). The numbers 1 \ldots 10 indicate the linear increasing height of the foam sample.

2. The averaged positions are transformed into the displacements \( d_1, \ldots, d_{10} \). The displacements are approximated by a least squares polynomial fit to remove possible noise.

3. Up to 9 engineering strains are calculated with the following formula:

\[
\varepsilon_{zi} = \frac{(d_{i+1} - d_i) - (d_{i+1,0} - d_{i,0})}{d_{i+1,0} - d_{i,0}} \quad i \in [1:10]
\]

with the index \( 0 \) the initial displacement (at \( t = 0 \)).

The engineering strains between \( p_1 \) and \( p_2 \) and between \( p_8 \) and \( p_9 \) could not be calculated, as not enough complete trajectories were found for the accompanying markers. Figure 3.18 shows the (averaged) engineering strains. The solid line represents the average strain over the total height, calculated in section 3.3.3. Line-pieces close to

Figure 3.17: Mesh plots of recovered markers for test 1. (a) \( t = 0 \) ms (uncompressed), (b) \( t = 4.7 \) ms, (c) \( t = 9.3 \) ms, (d) \( t = 14 \) ms (maximum compression, 82%), (e) \( t = 19.6 \) ms and (f) at 25.1 ms.
3.4 Conclusions & Discussion

- Drop test were successfully executed on pink Confor foam at impact speeds of 2.0, 3.1, 4.0, 5.7 and 7.0 [m/s].

- The force as function of time was determined for all tests. Vibrations disturbed the signal at higher impact speeds. It was not possible to filter these vibrations out of the signal. The experimental setup must be further investigated to improve the quality of the force transducer signal.

- The displacement of the foam top was determined. From these displacements, strains and strain rates were calculated. It appeared that the strain rates became constant at the higher impact speeds. Further, the unloading strain rates amounted to 40% of the loading strain rates, for all impact speeds. This is used in Chapter 5, where the experimental unloading curve is determined.

Figure 3.18: ystrain

the surface undergo larger strains than line-pieces close to the bottom of the foam. The difference between the local strain and the average strain increases up to 0.2 [-]. By assuming uniform distributed strain, errors up to 20% are made.

The strain in the direction of the compression divided by the strain in perpendicular directions results in an estimate of the Poisson's ratio. From visual inspection of the images, the highest (average) strains were determined to 0.7 and 0.08 [-], leading to a ν of approximately 0.11 [-].
The displacement field was successfully calculated for a test with an impact speed of 2 [m/s]. It showed strain dependency in load direction for the $z$ components of the strain. Therefore, strains were not uniformly distributed in the sample. Errors up to 20% can be introduced when this still is assumed. In Chapter 5 it is examined whether simulations give the same result.

It was not possible to calculate the displacement field at higher impact speeds than 2.0 [m/s]. This is due to the contrast of the foam surface, which is too low at high impact speeds. Spraying white dye on the foam surface might improve its contrast, but must be further examined.
Chapter 4

Material Modeling

The drop experiment as described in the previous chapter, is simulated in MADYMO. This process is divided in two phases.

First, the experiment is simulated with material parameters derived from data of Happee & Klompen. The parameters were adapted to match the drop experiment more closely. An optimization technique to computationally fit the material parameters to the experimental data is explained and evaluated.

Second, the material parameters are fitted to the experimental data using this optimization strategy. This second part, which consist of the practical part is reported in the next chapter.

4.1 The MADYMO Model

MADYMO (MAthematical DYnamic MOdel) is a computer package which is used to simulate crash situations. It combines in one simulation program the capabilities offered by multi-body (for the simulation of the gross motion of the system of bodies connected by kinematical joints) and finite element techniques (for the simulation of structural behavior and assessment of deformation of structures). Both techniques are used in the drop experiment model.

The MADYMO model of the experiment consists of an inertial space (the ground), one body (the drop weight) and one FEM-model, the foam sample (see Figure 4.1). The FEM model is connected to both the inertial space and body with a so-called support. The upper support connects the drop weight to the foam sample. The lower support prevents the foam sample from falling down due to the influence of gravity. Gravity acts upon the whole system in negative z-direction.

In the MADYMO Model the following choices are made:
4.1. THE MADYMO MODEL

Figure 4.1: MADYMO model of drop experiment and axes definitions. The upper support connects the body (drop mass) to the foam sample. The lower support connects the foam sample to the ground. The FEM model represents the foam, in this example with a 5x5x5 mesh. The FEM model is symmetrical in x and y direction. The dimensions are 50x50x25 [mm], \((x \times y \times z)\)

1. The sample is loaded only in (negative) z-direction. As the FOAM model uses un-coupled strains \((\nu = 0, \text{ see paragraph 4.2})\), there is no force in \(x/y\) direction due to lateral expansion. So, \(x\) and \(y\) mode displacement were not prescribed.

2. As both Multi-body and FEM structures are used in the model, an Euler integration method is chosen.

Furthermore, there are two parameters that influence the accuracy of the FEM model:

1. The number of integration points is chosen by means of the element type, which can be SOLID1 (reduced integration with 1 integration point at the center of the element) or SOLID8 (8 integration points per element). Both elements are 8-node brick elements and are based on a tri-linear displacement interpolation. The SOLID1 element requires less CPU-time but introduces so-called zero-energy or hourglass modes: modes in which the element can deform without dissipating energy. This can be suppressed by a hourglass parameter, at the expense of energy. The SOLID8 element is more accurate and has no hourglass problem, but requires 10% more CPU-time.

2. The number of elements. Theoretically, 1 element occupying the whole sample should be able to describe the FEM model accurately, as the FOAM model assumes \(\nu = 0\). However, not only the whole displacement and forces are of interest, but also the displacement field within the sample. A finer mesh is required to achieve this. Raising the number of elements however, affects the
4.2. THE FOAM MATERIAL MODEL

Courant time step. The Courant time step is defined as

\[ t_c = L \sqrt{\frac{E}{\rho}} \]  \hspace{1cm} (4.1)  

with \( L \) the smallest element size, \( E \) the Young's module and \( \rho \) the density of the element. The time step in a FEM model must be smaller than this Courant time step.

The influence of the both parameters is researched in paragraph 4.3.2. The foam sample is modeled with material model FOAM, which is described in the next paragraphs.

4.2 The FOAM material model

The FOAM material model uses an experimental stress-strain curve rather than a material law. It is available for brick elements only. The model is based on the following two assumptions:

- There is no coupling between stresses and strains of different principal directions, hence lateral contraction is neglected.
- Strain rate effects can be characterized by a strain rate dependent scaling factor.

Consequently, the stress-strain curve can be determined from uni-axial compression and tension tests for different loading rates. Mathematically, the stress-strain relationship has the following form

\[ \sigma = g(\dot{\varepsilon}) \sigma_r \]

where \( g \) is a scaling factor that depends on the effective strain rate \( (\dot{\varepsilon}) \), and \( \sigma_r \) is a user specified reference stress curve. This curve represents the quasi-static uni-axial behavior of foam under both compression and tension. The reference stress curve must be defined in terms of nominal stresses versus logarithmic strains. The effective strain in the scaling factor is defined as

\[ \dot{\varepsilon} = \sqrt{\text{tr}(\dot{\varepsilon} \cdot \dot{\varepsilon}^T)} \]

Two analytical laws are available for scaling up the user defined stress-strain curve:

1. The Cowper-Seymonds model:

\[ g(\dot{\varepsilon}_n) = 1 + \left( \frac{\dot{\varepsilon}_n}{c_1} \right)^{\frac{1}{c_2}} \]  \hspace{1cm} (4.2)  

2. The Johnson-Cook model:

\[ g(\dot{\varepsilon}_n) = 1 + c_2 \ln \left( \max \left(1, \frac{\dot{\varepsilon}_n}{c_1} \right) \right) \quad (4.3) \]

where \( c_1 \) and \( c_2 \) are user specified positive constants. Since these empirical laws are based on nominal strain rates a transformation of logarithmic strain rates to nominal strain rates must be carried out. Taking the derivative of the effective strain with respect to time and using the relationship between nominal and logarithmic strains, the nominal strain rate is obtained as

\[ \dot{\varepsilon}_n = \frac{\varepsilon_{se}}{\varepsilon} \text{tr} \left( \varepsilon \cdot \dot{\varepsilon}^T \right); \quad s = \text{sign} \left[ \min(\varepsilon) + \max(\varepsilon) \right] \]

In addition to the scaling function, the hysteresis model also influences the material model. The following hysteresis model is chosen because it agrees best with the physical behavior of foams (see Figure 4.2).

1. If the independent variable \( x \) reaches a maximum value \( x_{\text{max}} \), unloading will take place along the hysteresis slope \( s_l \) until the unloading curve is reached.

2. Continued unloading then proceeds downward along the unloading curve.

3. Reloading takes place immediately along the hysteresis slope \( s_l \) until the loading curve is reached again.

4. Further loading beyond \( x_{\text{max}} \) proceeds along the loading curve until a new \( x_{\text{max}} \) is reached. Any subsequent loading and unloading will occur in the same way.

The loading curve must be above the unloading curve for all \( x \). The hysteresis loop is only traced in clockwise direction as shown in the figure. For negative values of \( x \) a similar strategy is followed.
4.3 Verification of the MADYMO Model

To verify the MADYMO model, independent input data available through Happee & Klompen\textsuperscript{6} (HAKL model) was used as material model. In this model, the Cowper-Seymonds model is used for scaling up of the stress-strain curves. As they were not able to determine an unloading curve, they used a model without hysteresis, i.e. $Y_l(x) = Y_u(x)$. Remarkably, the density of the foam is modeled ten times too high (1000 [kg/m$^3$]), probably to stabilize the simulations.

With this model, the influence of the meshsize and the number of integration points is checked.

### 4.3.1 Influence of mesh size

To investigate the influence of the mesh size, the number of elements per direction was gradually increased from 1 to 10. The total number of elements therefore ranges from 1 to 1000 ($10^3$). The model was tested at two impact speeds: 2 [m/s] and 7 [m/s].

The 1 and 8 element model was stable for both speeds with a timestep of $1 \times 10^{-6}$ [s]. Increasing the number of elements above 8 resulted in numerical instabilities. The instabilities are larger for higher number of elements. Decreasing the time step to $1 \times 10^{-8}$ resolved most of these instabilities, although the Courant time step was satisfied even with the smaller time steps. A similar problem was reported by Happee & Klompen\textsuperscript{6}.

This phenomenon might be caused by:

1. An overestimate of the Courant time step: this time step is calculated at the beginning of the simulation. During the simulation the density increases as the deformation increases, so the actual Courant time step is lower. This could explain the numerical instabilities that happen at large deformations.

2. A lack of unloading curve: the force as a function of the displacement collapses when the displacement is over its maximum value. This results in discontinuous force fields. Decreasing the time step does not prevent this force to collapse, but inertia effects are not negligible with these smaller time steps. These inertia effects provoke sufficient forces to make the FEM-system stable.

Some of these instabilities might be due to hourglass modes. Increasing the hourglass parameter indeed decreased some of the instabilities, but resulted in modified force-displacement curves.
4.3.2 Influence of number of integration points

The same tests were performed with SOLID8 in stead of SOLID1 elements. Numerical instabilities were noticed, although they can not be attributed to hourglass effects. The SOLID8 tests were more stable (with the same time-step) compared to the SOLID1 tests.

To resolve these instabilities, it was decided to suppress all $x$ and $y$ node displacements in all models.

Based on the information presented in the previous paragraphs, it was decided to use 1 SOLID8 element for the modeling of the foam sample.

4.4 Introduction to the optimization process

The goal of optimization is to minimize a given function, the objective function, by influencing one or more so-called design variables. The design variables span the design space. The values of these variable are limited by a set of restrictions, the constraints. Often, the mathematical relationship between objective function and design variables is not known explicitly but is computed.

MADYMO optimization module MADYMIZER uses a midrange multi-point optimization process, also known as sequential linear programming (SLP). Midrange approximation methods attempt to use local function approximations with a wider range of applicability. The approximation is only valid in a subregion of the whole design space. Therefore model functions can remain simple, that is linear functions are applied. Each design variable is evaluated at two levels.

In MADYMIZER, a design variable is varied with a difference step forward or backward depending on the search direction, while other design variables remain at start point level. Within the optimization process all design variables are scaled between 0 and 1. Linear approximations of objective functions and constraints are built. These are based upon the objective and constraint function values, calculated at the $n+1$ (including start point) design points in the subregion. The function values of previous cycles are discarded. This process is shown in figure 4.3 for a 1-dimensional case.

After a solution for the subregion is found, it is checked for convergence. The condition for acceptance of the solution in a subregion is:

1. The difference between the approximate model (either the continuous or discrete approximated optimum) values and the actual calculated values must be less than the error tolerance.

2. The largest constraint violation must be less than the allowed violation violation
If these two conditions are not met, the estimated solution is rejected and the sub-region is reduced in size and the cycle is repeated. The acceptance of an solution concludes one optimization cycle. The next step is to determine if the optimization process has converged. The process repeats itself until converge occurs or until the user-defined maximum number of optimization cycles is reached. The process converges when the following conditions are satisfied:

1. The relative change in objective function values of two successive solutions is less than the objective tolerance. The default value for the objective tolerance is 0.1.
2. All the approximation errors are within acceptable limits.
3. The largest constrain violation must be less than the allowed violation.

4.5 Verification of the optimization process

In chapter 5, MADYMIZER is used to fit the two parameters \( c(1) \) and \( c(2) \) from equation 4.3 to the experimental data. To examine the accuracy of MADYMIZER the following tests are performed:

- **Robustness.** MADYMO was run with arbitrary, yet feasible values for the two design parameters. These values are hereafter called the reference values (of the design parameters). After completion, the force-time curve became the
reference curve. Next, new (different) values were assigned to the two parameters. After that, an optimization run with MADYMIZER was done on these two new values. The goal was to minimize the RMS difference between the reference curve and the force-time curve of the actual design variables. At the optimum, the two design variables should be equal to the two reference values of the parameters. The difference between the found optimum and the reference values is a measure of the (maximal) accuracy the MADYMIZER module can achieve.

- **Influence of noise** In the second test noise was added to the reference curve. The above optimization process is performed with exactly the same values for the design variables. In this way it is possible to examine MADYMIZER'S liability to noise.

The objective function used in these tests equals

\[ L_{\text{obj}} = \sum_{i=1}^{n} (F_{\text{exp}}(i) - F_{\text{sim}}(i))^2, \]

with \( F_{\text{exp}}(i) \) the experimental force, \( F_{\text{sim}}(i) \) the simulated force, \( i \) the counter and \( n \) the number of samples in the experimental force-time curve.

### 4.5.1 Optimization Experiments

The MADYMO input file is shown in Appendix C. The reference values for \( c(1) \) and \( c(2) \) are 0.1 and 2.0 respectively.

All starting points converged to the reference values, without a restart in the found optimum. The number of iterations varied from 17 to 22. The theoretical solution was \((0.10, 2.0)\), with an objective function of 0 (see Table 4.1).

<table>
<thead>
<tr>
<th>Starting point</th>
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<th>Optimum</th>
<th>Obj.Funct. ([\text{N}])</th>
<th># cycles</th>
</tr>
</thead>
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<td>((0.15, 1.89))</td>
<td>(1.630\times10^6)</td>
<td>21</td>
</tr>
<tr>
<td>((2.00, 2.00))</td>
<td>(2.868\times10^8)</td>
<td>((0.18, 1.90))</td>
<td>(1.612\times10^6)</td>
<td>20</td>
</tr>
<tr>
<td>((3.00, 0.10))</td>
<td>(1.298\times10^8)</td>
<td>((0.16, 1.93))</td>
<td>(1.712\times10^6)</td>
<td>17</td>
</tr>
<tr>
<td>((0.10, 4.00))</td>
<td>(6.897\times10^6)</td>
<td>((0.12, 2.02))</td>
<td>(1.603\times10^6)</td>
<td>22</td>
</tr>
</tbody>
</table>

Table 4.1: *Found optima for different starting values. The exact optimum is \((0.10, 2.0)\), with an objective function of 0*

It was possible to reduce the objective tolerance (default 0.1). Setting this value to 0.01 did not result in more accurate approximations of neither the reference curve, nor the reference values. Starting the process again in the optimum neither resulted in a better solution.
Next, white noise with a magnitude of 3 and 10 [N] was added to the reference curve (the force-time curve) to examine the influence of noise. The same starting values as above were used. All runs resulted in comparable optima as above. More iterations were necessary and sometimes a restart in the found optimum was needed to achieve the final optimum (see Table 4.2). This indicates the presence of local minima.

<table>
<thead>
<tr>
<th>Starting point</th>
<th>Noise [N]</th>
<th>Optimum</th>
<th>Obj.Funct. [N]</th>
<th># cycles</th>
<th>restart</th>
</tr>
</thead>
<tbody>
<tr>
<td>(0.10, 1.00)</td>
<td>3</td>
<td>(0.16, 1.99)</td>
<td>1.712×10^6</td>
<td>25</td>
<td>No</td>
</tr>
<tr>
<td>(2.00, 2.00)</td>
<td>3</td>
<td>(0.15, 1.84)</td>
<td>1.660×10^6</td>
<td>34</td>
<td>Yes</td>
</tr>
<tr>
<td>(3.00, 0.10)</td>
<td>10</td>
<td>(0.19, 1.91)</td>
<td>1.625×10^6</td>
<td>28</td>
<td>Yes</td>
</tr>
<tr>
<td>(0.10, 4.00)</td>
<td>10</td>
<td>(0.11, 2.00)</td>
<td>1.583×10^6</td>
<td>27</td>
<td>No</td>
</tr>
</tbody>
</table>

Table 4.2: Found optima for different starting values with white noise of 3 and 10 [N]. The exact optimum is (0.10, 2.0), with an objective function of 0

4.6 Conclusions & Discussions

- The drop experiment was successfully modeled using MADYMO. Problems arose at large compressions. Lowering the time step to 1.10^{-8} [s] made the simulation more stable.

- The use of SOLID8 elements is preferred above SOLID1 elements, as this results in a more stable simulation.

- Increasing the number of elements resulted in unstable solutions, even when SOLID8 elements were used. Adding hysteresis to the material model might resolve these instabilities.

- MADYMIZER was successfully applied to estimate two material parameters from a test bed. Noise did influence the number of iterations, but the process converges to the same optimum as without any additive noise.
Chapter 5

Parameter Estimation

In the Chapter 3 it was measured that the local strains deviate up to 20% of the global strains. In the first section, it is examined whether simulations result in similar differences.

In the second part of this chapter, the material parameters of the Confor foam are determined. First, data based upon the work by Happee & Klompen\(^6\) (HAKL curves) was used to generate reference curves which serve as benchmark curves (BMC) to compute the HAKL data to the experimental ones. Next, a reference stress-strain curve (RSC) was determined from the experimental data. In chapter 3 it was shown that for the bottoming out phase was reached for large compressions. Besides that, the belonging force-time signal of the experiment with \(v = 5.7 \, [\text{m/s}]\) is not usable as the it is leveled by the limits of the force sensor. Therefore, the experiment with an impact speed of 5.7 [m/s] was used to determine the RSC.

Strain rate dependency is included with the use of a scaling function. This scaling functions only scale for strain rates higher than the reference strain rate. Therefore, the RSC must be at lowest strain rate. Accordingly, \(c1\) and \(c2\) were determined using the optimization process, as described in the previous chapter. Next, simulations of the experiments were executed with this new material data (EC curves). Finally the BMC, EC and experimental curves were compared to each other.

5.1 Strain distribution

To examine whether MADYMO simulations result in similar difference in local strains as was found in Chapter 3, a simulation was run with 1000 elements (10 \(\times\) 10 \(\times\) 10 elements). As both the foam sample and the loading are symmetrical in \(x\) and \(y\) direction, only a quarter mesh was modeled to decrease CPU-time. The material data was based on the HAKL data. The resulting \(z\)-strains are displayed in Figure 5.1. Although the loading part does show some dependency in \(z\) direction, the differences
5.2. Definition of the benchmark curves (BMC)

The foam in the following optimization processes was represented by a single SOLID1 element. The time step was set to $1 \times 10^{-6}$ [s].

Figure 5.2 shows both the results of the simulations of the drop experiments with the HAKL data and the experimental curves, grouped per impact speed. All four subfigures indicate that the simulated peak forces are lower than the experimental forces.

This is possibly due to an improper scaling function. The HAKL data uses a Cowper-Seymonds (CS) scaling function, though this function is less suited for simulations with large range of strain rates than the Johnson-Cook (JC) scaling function. To examine whether the JC scaling function improves the BMC, simulations were executed with the modified scaling function (MHAKL, JC instead of CS scaling function). The results are also displayed in Figure 5.2. The force of the MHAKL curves approximates the experimental data better than the force of the HAKL model.
5.2. **DEFINITION OF THE BMC**

After reaching the maximal force, unloading is faster in simulation. Choosing a (better) unloading curve might diminish this deficiency.

It is, however, doubtful whether the scaling parameters $c_1$ and $c_2$ of Happee & Klompen still are representative of the drop experiment, as they were optimized using a different scaling function and a different experiment. An altered $c_1$ and $c_2$ might represent the drop experiment more accurate than the original values by Happee & Klompen. To examine this assumption, an optimization of $c_1$ and $c_2$ (as explained in the previous chapter) was executed.

![Simulations of the drop experiments with HAKL data (dotted) and MHAKL data (dashed) compared to experimental data (solid)](image)

Figure 5.2: Simulations of the drop experiments with HAKL data (dotted) and MHAKL data (dashed) compared to experimental data (solid)
5.2. **DEFINITION OF THE BMC**

5.2.1 Optimizing the simulated force

The simulations are fitted to the experiment by minimizing the square difference of the simulated force-time signal and the force-time signal obtained from the drop test (see also Section 3.3.2):

\[
L_{\text{obj}} = \sum_{i=1}^{n} (F_{\text{exp}}(i) - F_{\text{sim}}(i))^2 ,
\]

with \(L_{\text{obj}}\) the object function, \(F_{\text{exp}}(i)\) the experimental force, \(F_{\text{sim}}(i)\) the simulated force, \(i\) the counter and \(n\) the number of samples in the experimental force-time curve.

The experiments were simulated using the reference curve as proposed by Happee & Klompen and the JC-scaling function. \(c1\) and \(c2\) were chosen as design variables. The optimal values are shown in Table 5.1. From the results of the various optimization processes, the following was concluded:

1. For the experiment with the lowest impact speed (2 [m/s]), the optimal simulation agrees slightly better to the experimental data compared to the original HAKL data.

2. For higher impact speeds, the solution returned by the optimization process deteriorated the approximation of the experimental force: the simulated force-time curves are flattened. This was explained by the shape of the force signal (see f.e. Figure 5.1.c). The simulated and experimental force-peaks are shifted relative to each other. Apparently, the timing of the force peak has little influence on the object function.

Concluding, a different goal function is needed for the optimization process.

5.2.2 Optimizing the simulated displacement

The displacement-time signal is, compared to the force-time signal, less impulse-like and more broadened, as shown before. The object function, similar to equation 5.1, is
5.2. DEFINITION OF THE BMC

<table>
<thead>
<tr>
<th>HAKL</th>
<th>Obj.Funct. 1 $v=2$ [m/s]</th>
<th>Obj.Funct. 2 $v=2$ [m/s]</th>
<th>Obj.Funct. 2 $v=5.7$ [m/s]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c_1$</td>
<td>0.1771</td>
<td>0.010</td>
<td>0.0028</td>
</tr>
<tr>
<td>$c_2$</td>
<td>2.0695</td>
<td>1.952</td>
<td>1.9786</td>
</tr>
</tbody>
</table>

Table 5.2: Values for $c_1$ and $c_2$ before (HAKL) and after optimization, based on HAKL input data. Obj.Funct. 1 is the force-oriented object function, Obj.Funct. 2 is the displacement oriented object function.

defined as the least squares difference of the experimental and simulated displacement,

$$L_{obj} = \sum_{j=1}^{m} [d_{exp}(j) - d_{sim}(j)]^2,$$

with $d_{exp}(j)$ the experimental, $d_{sim}(j)$ the simulated displacement, $j$ the counter and $m$ the number of samples in the experimental displacement-time curve.

As it was expected that the lowest and highest impact speeds were most deterministic for the optimization process, only the simulations of those two impact speed experiments were optimized. The results are depicted in Table 5.2.

Both the displacement-time curve of the simulation with the MHAKL data and the experimental data are shown in figure 5.3 for an impact speed of 5.7 [m/s]. It shows that after the maximum displacement at $t = 5$ [s] the foam is less relaxed in the simulation than in the experiment. Choosing a (better) unloading curve might diminish this deficiency.

From the results of the various optimization processes, the following was noted and concluded:

1. Optimizations of simulations with different impact speeds resulted in different optimal values for $c_1$ and $c_2$: (0.0028, 1.9786) respectively (0.0001, 3.7230) for $v = 2$ and 5 [m/s].

2. The simulation with optimal values for $c_1$ and $c_2$ resulted in a better (overall) fit of the simulated and experimental displacement (see Figure 5.3). Optimizing up to the maximum displacement resulted in a better fit up to this maximum displacement, but deteriorated the part from there up to the end of the experiment.

3. It was not possible to achieve a displacement slope for $(t_1 \leq t \leq t_2)$ similar to the experimental one. Again, choosing a (better) unloading curve might diminish this deficiency.

4. For the lowest impact speed ($v=2$ [m/s]), the returned optima for $c_1$ and $c_2$ of the two different object functions agree quite well. Apparently, both object functions lead to the same optimum.
This leads to two conclusions:

1. The MHAKL data set is the best attainable fit to the experimental data based upon the work by Happee & Klompen. Therefore, this data set is used as BMC.

2. With the HAKL RSC curve the experiment can be modeled in good agreement to the experimental data, at least for the loading phase of the experiment. Compared to the experiments, the unloading phase of the simulation is characterized by little relaxation. Adding an unloading curve different to the loading curve might resolve this problem.

In the next paragraphs, the process of determining a RSC from the experimentally obtained data is dealt with.

5.3 Experimental reference stress-strain curve and scaling factors

The force-displacement curve of Section 3.3.4 with impact speed of 5.7 [m/s] was used to determine the RSC, as stated at the introduction of this chapter.
The force is divided by the initial surface, which results in the nominal stress:
\[
\sigma = \frac{F}{A} \approx \frac{F}{A_0} = \frac{F}{0.05^2}
\]
Visual inspection of the images of the deformation process indicated that the difference of \(L_0\) and \(L\) (\(A = L^2\), \(A_0 = L_0^2\)) is less than 1 [mm]. So, it was assumed that \(A_0 \approx A\). The displacement is transformed into the true (or logarithmic) strain,
\[
\epsilon = \ln \left( \frac{L_0}{l} \right) = \ln \left( \frac{d}{l_0} + 1 \right) = \ln \left( \frac{d}{0.025} + 1 \right)
\]
Finally, the stress-strain curve is plotted and the reference curve (loading curve, unloading curve and hysteresis slope) is approximated.

Figure 5.4 shows the above mentioned stress-strain curve. The left figure shows the complete curves, whereas the right figure is zoomed in on the small strain region. The loading function approximates the stress curve up to the maximum stress. The unloading curve was approximated by taking 40% of the loading curve, as the force signal in this part is too noisy to extract the curve directly from the stress-strain curve. The 40% was chosen because in Section 3.3.3 it was concluded that the unloading strain rates amount to 40% of the loading strain rates. The hysteresis slope was calculated by determining the slope of the dot-dashed line in Figure 5.4, which amounts to 55 [MPa].

![Figure 5.4: Experimental and approximated loading curve (dashed), unloading curve (dotted) and hysteresis slope (dot-dashed), for impact speed = 5.7 [m/s]. The left figure shows the complete curves, whereas the right figure is zoomed in on the small strain region.](image)

To examine whether this reference curve is useful and valid, the accompanying drop test is simulated using this curve. The scaling function should equal 1 for all strain
rates. This is accomplished by setting $c_1$ to a larger value than the largest absolute strain rate (which is roughly $230 \, [\text{s}^{-1}]$, see Figure 3.13). Figure 5.5 shows both the experimental data (solid) and the simulations with MHAKL data (dashed) and EC data (dotted).

![Figure 5.5: Experimental and simulated displacement-time curve (left) and force-time curve (right). The solid line represents the experimental data, the dashed line the MHAKL data and the dotted line the EC data.](image)

The following remarks are made:

a The EC displacement curve follows, up to the maximal displacement, the MHAKL curve. Both curves deviate from the experimental curve. This could be caused by an synchronization error of the force-time curves and displacement-time curves.

b The unloading part of the displacement matches the experiment. Especially the unloading-slope shows good agreement. The slope is also influenced by the choice of the hysteresis slope. A higher hysteresis slope resulted in a lower displacement slope.

c Compared to the MHAKL curve, the force-peak in the EC curve is closer to the experimental force peak.

d The force-time curve for the EC shows negative forces for times greater than 7 [ms]. The modeled drop weight is rigidly connected to the foam, so when the displacement rate drops, the foam 'pulls' the drop weight downwards. This results in negative forces.

The next step was to find a set of $c_1$ and $c_2$ with which all four drop experiments can be simulated. For this purpose, the EC curve was scaled down, as the scaling functions only scale up. The scaling factor was set to 0.0142, as the JC scales the reference curve with similar values. Next, an optimization with a displacement oriented object function identical to the one depicted in Section 5.2.2 was performed.
The returned values for $c_1$ and $c_2$ were 0.231 and 1.176 respectively. These newly determined scaling factors constitute, together with the experimentally determined reference stress-strain curve, the EC data set.

5.4 Comparison of experiments and simulations

The results of the simulations are shown in figure 5.7 (the displacements) and Figure 5.6 (the forces).

Figure 5.6: Displacement versus time for simulations of the drop experiments with MHAKL data (dashed) and EC data (dotted) compared to experimental data (solid)

The following remarks are made:
5.4. COMPARISON OF EXPERIMENTS AND SIMULATIONS

Figure 5.7: Force versus time for simulations of the drop experiments with MHAKL data (dashed) and EC data (dotted) compared to experimental data (solid)

a For all impact speeds, the EC displacement curve follows, up to the maximal displacement, the simulation with the MHAKL data. Both curves deviate towards larger displacements compared to the experimental curve.

b For all impact speeds, the unloading part of the displacement matches the experiment. Especially the unloading-slope shows good agreement.

c Compared to the MHAKL curve, the force-peak in the EC curve is closer in time to the experimental force peak, though the force-peaks are higher and further remote from the experimental curves.
d For the highest impact speed, EC displacement curve shows negative forces for times greater than 7 [ms].

5.5 Discussion & Conclusions

a An EC data set, containing a unloading curve and a hysteresis slope additional to a loading curve, was successfully introduced.

b The EC data set is accurate within 10% of the experimental displacements and within 20% of the experimental force-peaks.

c Scaling up with the impact speed does not deteriorate the accuracy.

d Compared to the simulations with MHAKL data, the forces simulated with the EC data were of the same magnitude and shifted 10% in time towards the experimental curves.

e Up to the maximum displacements, the displacements simulated with the EC data coincide with the displacements simulated with the HAKL data.
Chapter 6

Discussion & Conclusions

• The force as function of time was determined for all tests. Vibrations disturbed the signal at higher impact speeds. It was not possible to filter these vibrations out of the signal. The experimental setup must be further investigated to improve the quality of the force transducer signal.

• The displacement of the foam top was determined. From these displacements, strains and strain rates were calculated. It appeared that the strain rates became constant at the higher impact speeds. Further, the unloading strain rates amounted to 40% of the loading strain rates, for all impact speeds. This was used in Chapter 5, where the experimental unloading curve was determined.

• The displacement field was successfully calculated for a test with an impact speed of $2 \, \text{[m/s]}$. It showed strain dependency in load direction for the z components of the strain. Therefore, strains were not uniformly distributed in the sample. Errors up to 20% can be introduced when this still is assumed. It showed that simulations did not result in a similar deviation.

It was not possible to calculate the displacement field at higher impact speeds than $2.0 \, \text{[m/s]}$. This is due to the contrast of the foam surface, which is too low at high impact speeds. Spraying white dye on the foam surface might improve its contrast, but must be further examined.

• An EC data set, containing a unloading curve and a hysteresis slope additional to a loading curve, was successfully introduced. The EC data set accurate within 10% of the experimental displacements and within 20% of the experimental force-peaks. Scaling up with the impact speed does not deteriorate the accuracy. Compared to the simulations with MHAKL data, the forces simulated with the EC data were of the same magnitude and shifted 10% in time towards the experimental curves. Up to the maximum displacements, the displacements simulated with the EC data coincide with the displacements simulated with the HAKL data.
Chapter 7

Recommendations

It was difficult to unambiguously synchronize the force and displacement curves. There are a few possibilities to attain a better synchronization. The ripple could be removed from the force signal by determining the response function of the anvil. With this response function, the real signal is calculated from the measured signal. Another possibility is to record a flash when the mass passes the trigger, so that the images are synchronized to the trigger signal.

At large compressions, markers are easily lost in the tracking process, due to little contrast. Spraying white dye on the foam could enhance the contrast. Recording with a higher pixel density also increases the chance of tracking a marker, although this involves a lower recording rate of the images.

In the experiments it showed that the ratio between $e_x$ and $e_z$ is approximately 0.11. The foam model assumes a Poisson’s ratio of zero. It is recommended to research the influence of a Poisson’s ratio greater than zero to further improve the material behavior of Pink Confor Foam.

In general, in MADYMO energy dissipation is modeled by material damping and hysteresis. Hysteresis was successfully added to the material data. Still, the simulated force peaks were more narrow and higher than the experimental ones. Material damping is not modeled in the FOAM material yet, but could improve the force curves.

It is advisable to simulate other experiments with the EC data set, as is it optimized for drop weight experiments only. Other loading characteristics of the impact must be examined to broaden the use of the EC model.
References


URL: http://www.mathworks.com


URL: http://www.madymo.com

URL: http://www.madymo.com

URL: http://www.madymo.com


Appendix A

Measurements Results

A.1 Trigger Signal

Figure A.1: Raw Trigger Signal data, test 1, run 1 - 5
A.1. TRIGGER SIGNAL

Figure A.2: Raw Trigger Signal data, test 2, run 6 - 10

Figure A.3: Raw Trigger Signal data, test 3, run 11 - 15
Figure A.4: Raw Trigger Signal data, test 4, run 16 - 20

Figure A.5: Raw Trigger Signal data, test 5, run 21 - 25
A.2 Force Sensor

Figure A.6: Average Force, test 1, run 1 - 5
A.2. FORCE SENSOR

**Figure A.7:** Average Force, test 2, run 6 - 10

**Figure A.8:** Average Force, test 3, run 11 - 15
Figure A.9: Average Force, test 4, run 16 - 20

Figure A.10: Average Force, test 5, run 21 - 25
Appendix B

Background to MADYMO

MADYMO (MAthematical DYnamic MOdel) is a computer package which is used to simulate crash situations. It combines in one simulation program the capabilities offered by multibody (for the simulation of the gross motion of the system of bodies connected by kinematic joints) and finite element techniques (for the simulation of structural behavior).

A multi body (MB) system is a system of bodies. Any pair of bodies of the same system can be interconnected by a kinematic joint. A kinematic joint restricts the relative motion of the two bodies it connects. The relative motion allowed by a joint is described by quantities called the joint degrees of freedom (jdof). A system of bodies is defined by the specification of which bodies are connected by kinematic joints, the type of kinematic joints, the geometry, the mass distribution of bodies and initial conditions. Applied loads on bodies can be modeled with force models, e.g. for belts, airbags and contacts of bodies with each other or with their surroundings.

The finite element method (FEM) is used to reduce a continuum to a discrete numerical model. The continuum is divided into relatively simple finite elements representing its shape. These elements can be lines (1D), surfaces (2D), volumes (3D) or combinations thereof. The elements are interconnected at a discrete number of points, the nodes. MADYMO uses a so-called Lagrangian description. This means that the nodes, and therefore also the elements, are fixed to the material and thus move through space with the material.

The system is discretized by interpolating the displacement, velocity and acceleration of any point in an element in terms of the same quantities at the nodes connected to this element. The interpolation functions, the element shape functions, must be such that rigid body motions, these are motions for which the strains are zero, can be described.

The state of stress follows from the strains and the constitutive equation of the material. In dynamic analyses, such as in MADYMO, the equilibrium requirements
for each element are established in terms of the motion of these nodes which can be solved for successive points in time. Therefore, the displacements, the velocities and the accelerations of the nodes are the basic unknowns for a dynamic analysis.
Appendix C

MADYMO data files

1
8 elem SOLID1 pink V0=2 m/s TS=0.00001s TE=0.5s
Valproof_model
GENERAL INPUT
  TO 0.0
  TE 0.026572265625
  INT EULER
  TS 9.765625E-6
END GENERAL INPUT
INERTIAL SPACE
  ondergrond
  PLANES
    0 -0.05 -0.05 0 0.1 -0.05 0 0 0.1 0 0 0 0 pers
END PLANES
END INERTIAL SPACE
SYSTEM 1
valgewicht
CONFIGURATION
  1
END CONFIGURATION
GEOMETRY
  0.0 0.0 0.0 0.025 0.025 0.025
END GEOMETRY
INERTIA
  2.5 1 1 1 0.1 0.1 0.1
END INERTIA
CYLINDERS
  1 0.02 0.04 0.04 0.025 0.025 0.025 2 0.0 0.0 0.0 pers
END CYLINDERS
ORIENTATIONS
  1 1 1 2 1.5708
END ORIENTATIONS
INITIAL CONDITIONS
  0.0 0.0 0.0 0.0 -1.89762
END SYSTEM 1
FEM MODEL
COORDINATES

1  0.00000E+00  0.00000E+00  0.00000E+00
2  2.50000E-02  0.00000E+00  0.00000E+00
3  5.00000E-02  0.00000E+00  0.00000E+00
4  0.00000E+00  2.50000E-02  0.00000E+00
5  2.50000E-02  2.50000E-02  0.00000E+00
6  5.00000E-02  2.50000E-02  0.00000E+00
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9  5.00000E-02  5.00000E-02  0.00000E+00
10 0.00000E+00  0.00000E+00  1.25000E-02
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12 5.00000E-02  0.00000E+00  1.25000E-02
13 0.00000E+00  2.50000E-02  1.25000E-02
14 2.50000E-02  2.50000E-02  1.25000E-02
15 5.00000E-02  2.50000E-02  1.25000E-02
16 0.00000E+00  5.00000E-02  1.25000E-02
17 2.50000E-02  5.00000E-02  1.25000E-02
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20 2.50000E-02  0.00000E+00  2.50000E-02
21 5.00000E-02  0.00000E+00  2.50000E-02
22 0.00000E+00  2.50000E-02  2.50000E-02
23 2.50000E-02  2.50000E-02  2.50000E-02
24 5.00000E-02  2.50000E-02  2.50000E-02
25 0.00000E+00  5.00000E-02  2.50000E-02
26 2.50000E-02  5.00000E-02  2.50000E-02
27 5.00000E-02  5.00000E-02  2.50000E-02

END COORDINATES

ELEMENTS

1  $14  1  2  5  4  10  11  14  13
2  $14  2  3  6  5  11  12  15  14
3  $14  4  5  8  7  13  14  17  16
4  $14  5  6  9  8  14  15  18  17
5  $14  10  11  14  13  19  20  23  22
6  $14  11  12  15  14  20  21  24  23
7  $14  13  14  17  16  22  23  26  25
8  $14  14  15  18  17  23  24  27  26

END ELEMENTS

MATERIALS

TYPE  FOAM
LOADING FUNCTION  1
DENSITY  100.0
RATE DEPENDENT  CWPER
D RATE  $1{1}
PRATE  $1{2}
SET  $13

END MATERIALS

FUNCTION

LOADING

15
-2.6573314E+00 -1.4946944E+06
-2.3903002E+00 -7.4526498E+05
-2.2170901E+00 -4.2324231E+05
-2.0923786E+00 -2.8170319E+05
-1.8845266E+00 -1.4649536E+05
-1.6420323E+00 -7.5696912E+04
-1.2471132E+00 -3.3395334E+04
-9.0069284E-01 -1.7291572E+04
-5.8660508E-01 -9.1147906E+03
-2.8637413E-01 -4.6767909E+03
0.0000000E+00 0.0000000E+00
7.8521940E-02 4.8158003E+03
2.8637413E-01 4.6767909E+03
5.8660508E-01 9.1147906E+03

UNLOADING
22
-2.3025900E+00 -6.0000000E+06
-2.2072700E+00 -4.0000000E+06
-1.9661100E+00 -2.0000000E+06
-1.7719600E+00 -9.0000000E+05
-1.6094379E+00 -2.8000000E+05
-1.3862944E+00 -1.8460000E+05
-1.2456459E+00 -1.1862000E+05
-9.1629073E-01 -1.2186000E+05
-6.9314718E-01 -9.4410000E+04
-5.1028562E-01 -7.8910000E+04
-3.5667949E-01 -6.6690000E+04
-2.3768207E-01 -6.2069000E+04
-2.2314355E-01 -5.7930000E+04
-1.0536052E-01 -4.9340000E+04
-0.0000000E+00 0.0000000E+00
5.1293294E-02 4.1530000E+04
1.0536052E-01 4.9340000E+04
2.3768207E-01 6.2069000E+04
3.5667949E-01 6.6690000E+04
5.1028562E-01 7.8910000E+04

END FUNCTION

SUPPORTS

NUMBER 1
INERTIAL SPACE
DOF D3
SET 1:9
NUMBER 2
SYSTEM 1
BODY 1
DOF D3
SET 19:27

END SUPPORTS

FEMOUT

PRINT RESULTS ELEMENTS

END FEMOUT

FEMHIS

1 PRINT ELEMENT 1 FORCE.Z
END FEMHIS
END FEM MODEL
FORCE MODELS
   ACCELERATION FIELDS
      0 0 0 0 1
   END ACCELERATION FIELDS
FUNCTIONS
   2
   0.0 -9.80665
   1.0 -9.80665
END FUNCTIONS
END FORCE MODELS
OUTPUT CONTROL PARAMETERS
   TSKIN  9.765625E-6
   TSOUT  9.765625E-6
   KIN3
END OUTPUT
END INPUT