MASTER

Safety stock allocation for a heterogeneous item set
an assortment approach for multi-item inventory control

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Safety stock allocation for a heterogeneous item set: An assortment approach for multi-item inventory control

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Abstract

Managing an assortment of items as opposed to a single item, complicates safety stock decisions. Accumulating individually set (single item) parameters does not add up to an efficient solution for achieving a service target with minimal inventories (or to spend a given budget to maximize service). A non-myopic approach should recognize that an item’s cost of safety stock, as well as its effect on the (aggregate and individual) service level, differs across items. Strikingly, many inventory holding entities measure and report an aggregate service measure (such as the achieved service level averaged over all items) ex-post, but neglect inclusion of this aggregate dimension in their decision making ex-ante. Taking the effects of heterogeneous item characteristics into account reveals opportunities to allocate safety stock efficiently. Drawing conclusions from the single item case, this study aims to make an explicit link between the individual and aggregate. A procedure is developed that optimizes inventory performance through efficient allocation of safety stock over the set of all items in an assortment. The improvement over alternatives is evaluated using empirical data from PACCAR Parts, a major spare parts provider in automotive.
Management summary

Inventory holding entities face the challenge of providing high customer service levels while keeping the inventory investment to a minimum. PACCAR is a global technology leader in the design, manufacture and customer support of high-quality light-, medium- and heavy-duty trucks under the Kenworth, Peterbilt and DAF nameplates. PACCAR Parts (former DAF Parts) operates a network of Parts Distribution Centre’s (PDC’s) that offers aftersales support to Kenworth, Peterbilt and DAF dealerships around the world.

The stock control and supply department (SCS) of PACCAR Parts manages a large number of stock keeping units (SKU’s or simply items) and is continuously looking to improve inventory performance. New markets, new products, and increased presence of competition means additional pressure on- and importance of service levels, holding cost and inventory turnover.

Safety stock increases service but detoriates turnover and holding cost performance, posing a tradeoff. SCS experienced difficulties in deciding which of many items should get much safety stock and for which ones a small amount suffices. (See appendix I for intuition on safety stock).

The existence of this allocation challenge became particularly perceptible through the phenomenon of a small group of items of items accounting for a major part of the customer service. Focusing on independent demand inventories subject to stochastic demand, the research following question was formulated:

How can safety stock be allocated over a large and diverse assortment in order to achieve an aggregate target service level while minimizing the expected overall inventory holding cost in an environment with a fixed order quantity, budget constrained inventory under weighted fill rate service levels?

The assortment is diverse in the sense that the items have heterogeneous characteristics regarding their:

- Demand patterns
- Supplier lead times
- Replenishment order quantities
- Unit value or holding cost
- Share in the aggregate service level

Literature is extensive for the single item case. Like in many organizations, SCS utilizes a simple logic for computing the best moment of placing a replenishment order (the reorder point) to achieve a prespecified single item target fill rate. For determining this target fill rate SCS uses a two way demand-value categorization matrix with manually filled target fill rates. Accumulating individually set (single item) parameters is not the best way to manage an assortment because the link with aggregate effects is then merely a consequence. Aggregate performance is the total inventory investment and assortment- (aggregate) service level. This thesis aims to show that assortment performance can be included in decision making ex-ante (as is common for the single item situation), rather than being an ex-post consequence of many, independently set, parameters.

The challenge when managing an assortment is choosing the set of item-level target fill rates such that an aggregate service level is achieved. This reveals opportunities to let one item’s higher service compensate for another item’s lower service. Clearly, this can have a major impact on performance due to the heterogeneous characteristics just listed (such as low-cost
high-service items compensating for high-cost low-service items). In contrary to the single item case, literature on the multi-item safety stock allocation problem is scarce. After clarifying the research design, current inventory management practice (in general, as well as its manifestation at PACCAR Parts) is evaluated. In practice, many companies utilize ABC-like methods to approach the allocation problem (Cohen et al 1997). ABC-like methods are demonstrated to have some inherent weaknesses and are shown to lead to undesirable consequences.

The model development chapter starts by investigating definitions of customer service, arguing for a weighted fill rate service measure to represent aggregate customer service. Investigation of results from single item inventory theory allow formulating ex-ante expressions for the expected fill rate and expected on hand inventory as a function of the reorder point. More specifically, given the item characteristics (those variables over which management and planners have little or no control), the assortment performance (aggregate service and holding cost) is suggested to be determined solely by the set of item-level reorder points i.e. the decision variables.

The moderating effects of the item characteristics on individual as well as aggregate performance are evaluated. In addition to the single item relations, the multi-item case poses three additional considerations:

1. Safety stock for item x can be less or more expensive than for item y as item value and item holding costs differ across items.

2. The service level for item x can contribute less or more to the aggregate fill rate than item y as an item’s share in the aggregate performance differs across item.

3. In an efficient solution procedure, the amount of safety stock to be allocated to an item depends on the amount allocated to this and other items, due to diminishing returns of safety stock additions.

A simple iterative optimization procedure is suggested that finds optimal combinations of reorder points based on marginal analysis. This allows the computation of a service-investment curve on which each point represents an efficient trade-off between aggregate inventory investment and aggregate service. The performance of the procedure is tested using assortment data from PACCAR Parts, and compared to several ABC-like methods. The performance improvements over the current methods can lead to an approximate 30% reduction in the expected inventory holding cost amounting to an approximate € 800,000 savings, more than 1% increase in the expected aggregate fill rate, or a combination.
Preface and acknowledgements

This Master Thesis is the result of my graduation project in the Master program Operations Management & Logistics at Eindhoven University of Technology (TU/e). The project was performed at the Stock Control & Supply department of PACCAR Parts.

I would like to thank Bart van den Brink for giving me the opportunity to perform this project at the Stock Control & Supply department and Luke Verhagen for providing day to day doses of guidance, practical input and conversation during my project. Furthermore, many thanks to all of my direct colleagues at DAF and PACCAR Parts for the support and daily company.

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1. Introduction

This chapter functions as a short introduction into this report and the case company PACCAR Parts. The first section starts by providing some descriptive information about the company including its history and strategy. Section 1.2 describes PACCAR Parts’ logistics situation, in terms of high level physical flows and decision making structure. Section 1.3 describes some of the challenging characteristics present in the truck and trailer parts aftersales market. Section 1.4 outlines the structure of this report.

1.1 Company introduction

This research is conducted at PACCAR Parts in the stock control and supply department. In order to depict the challenges of this research in a practical situation, this section and the next starts with the introduction of the PACCAR business environment and its logistics context.

1.1.1 PACCAR Inc.

Paccar Inc, founded in 1905, is a global technology leader in the design, manufacture and customer support of high-quality light- medium- and heavy-duty trucks under the Kenworth, Peterbilt and DAF nameplates.

1.1.2 DAF Trucks manufacturing

DAF Trucks N.V. is a wholly owned subsidiary of the North-American corporation PACCAR Inc. DAF Trucks’ core activities are focused on the development, production, marketing and sale of medium and heavy-duty commercial vehicles.

The Public Limited Company preceding DAF was founded in 1928 by Hubert Jozuf van Doorne, by the name of Huub van Doorne, Machinefabriek en reparatie-inrichting, which after 1932 mainly built trailers under the name Van Doorne’s Aanhangwagen Fabriek N.V. (D.A.F), which grew rapidly and had 30 employees after its first year. Later the company developed itself into a truck factory changing the name to Van Doorne’s Automobiel Fabriek. From 1958 DAF started the production of a passenger car with the, by Huub Van Doorne’s engineered, Variomatic transmission. In 1975 the passenger car division was sold, after which DAF’s strategy focussed on building trucks. DAF Trucks N.V. was established in 1993 after a collapsing vehicle market. In 1996, PACCAR acquired the major shares of DAF Trucks N.V., placing DAF within the PACCAR organization.

Another core activity of DAF Trucks N.V. focuses on the marketing and sale of light trucks manufactured by Leyland Trucks Ltd. in the UK, which, since 1996, is likewise a wholly owned subsidiary of PACCAR Inc. All DAF and Leyland products are backed up by a full range of services. DAF Trucks also produces components for third parties, ranging from axle assemblies to complete engines for busses, coaches and special vehicles.

Before 1996 DAF Trucks N.V. operated its own service organisation for servicing DAF trucks. After the acquisition of DAF by PACCAR Inc, this service organisation set course towards increased after sales activities, including parts for other truck brands as well as general truck and trailer parts.
1.1.3 PACCAR Parts

PACCAR Parts is a subsidiary of PACCAR and operates a network of Parts Distribution Centre’s (PDC’s) that offers aftersales support to Kenworth, Peterbilt and DAF dealerships around the world. Their goal is to assure timely delivery of the parts that truck owners need to keep trucks in top condition. The stock control department of PACCAR Parts aims to employ a technologically-advanced set of systems for inventory control and order placement to provide reliable service to DAF dealerships.

1.2 PACCAR Parts’ logistic situation

After the formal company introduction this section will inform about the physical characteristics of the logistics network (1.2.1) and the decision making units involved (1.2.2).

1.2.1 Logistics network

From a global perspective, external suppliers supply parts to the DAF and PACCAR production facilities as well as to the main PACCAR Parts Distribution Centres (PDC’s) often using coordinated transport facilities, in addition, production facilities provide the PACCAR parts warehouses with manufactured parts. The largest of five European distribution centres is the Central PDC (CPDC) in Eindhoven. For most of the downstream parties in Europe, inventories originate from the Central PDC (CPDC) in Eindhoven, which is responsible for supplying dealers in the western Europe region from their own distribution centre as well as replenishing the other PDC’s. Because the PDC in Leyland is located near a production facility (for the DAF LF truck model and right-steering variants), PDC Leyland replenishes the CPDC with products and parts originating from the Leyland production facility, but besides this, functions as a regular PDC. PDC Moscow is a, distribution centre planned to become operational in the near future. Figure 1 depicts the main physical flows in the European network.

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**Figure 1.1 PACCAR Parts’ European network**
1.2.2 Decision making

With regard to the decision making, inventory decisions for the Europe region are managed from Eindhoven. An exception here is PDC Leyland, who is responsible for managing their own inventories, but replenishes parts with non-UK origins through CPDC Eindhoven. In this report, the management of inventories entails decisions about the timing and quantities orders placed for parts at the suppliers. Although in many small and medium sized enterprises (SME’s) this function is integrated with the purchasing function (pricing, sourcing and managing supplier-relations), we restrict the relevant decisions in this report to the former. Prices and long term contracts are mainly a (centralized) PACCAR-Purchasing function that maintains the network of suppliers and (PACCAR) manufacturers. Capacity decisions (i.e. size of warehouses and budget for inventory cost) are to be considered extraneous from the stock control department’s point of view, although it is the designated department to provide insights in these storage requirements. Although not the primary goal, the insights and models from this report are expected to provide valuable support for this kind of decision making.

1.3 The truck and trailer parts aftersales market

The product sold by PACCAR Parts can generally be viewed as “Truck part, trailer part and accessory service” which boils down to providing availability of truck parts and accessories to truck owners who consume these products. Here the essence of the product is presented in the form of a service because the quality of the physical parts is in general more determined by the PACCAR suppliers and manufacturers (which can be PACCAR production facilities) than by PACCAR Parts. The actual strength of PACCAR Parts falls in the notion that it holds inventories large enough (combined with the network to source high quality items quickly) to reliably provide truck owners with truck parts and accessories promptly when demanded. Similar to other capital goods, idleness of a truck can bring large cost to its owner due to obstruction of its money-making activities, making the truck and trailer aftersales market an important one.

The function of the former DAF Aftersales has changed since the acquisition of DAF Trucks N.V. by PACCAR Inc. While before it could be considered a purely a service oriented organization (providing spare parts for the DAF-fleet), the company now follows a strategy that includes generating revenue through after-sales and increasing market share beyond the DAF fleet. While before demand was already very irregular as typically spare parts environments, this section describes some of the changes in the environment that places additional emphasis on inventory management.

1.3.1 New markets

PACCAR Inc is continuously expanding its influences towards emerging economies, and its vehicles travel increasingly many parts of the world. This has the consequence that demand for spare parts arises in many of these new areas, and in many different ways. While dealers in the Europe region can be replenished within a short period of time and can therefore suffice with small amounts of stock, new markets are ordering less frequently and in larger quantities, with the consequence of increasingly volatile demand patterns. In addition, changing geographical dispersions of demand may dictate future changes in the distribution network such as warehousing locations and strategies.

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1 Throughout this report we will see that demand can often be regarded as “sales (volume)”. But as will be clarified in section 3.1.1, demand refers to the requests for the delivery of products placed by a downstream party which need not necessarily be a (end)consumer.
1.3.2 New products

DAF and other truck brands are frequently introducing new trucks into the truck market, with even better specifications and complying with the latest emission regulations. This puts pressure on inventory control. Firstly it entails an increased assortment breadth. Secondly, for old models, PACCAR Inc has an obligation to provide spare parts for 10-15 years after the last sale of a truck model. When suppliers stop providing these older parts, a final buy or all-time buy problems arises which often results in long term occupation of part of the inventory holding capacity.

PACCAR Parts is increasingly active in providing commercialized common truck and trailer related parts (TRP) that can not only fit DAF Trucks but all brands of trucks (all makes), and include accessories (figure 1.2), these SKU’s do not necessarily have all spare-parts characteristics present. The product assortment and emphasis on inventory management is increasing even more by the inclusion these parts. One way to categorize the origins or types of parts demand is depicted in figure 1.2. For the remainder of this report the assortment consists of a set of items that are non-repairable parts and are to be delivered from stock. As will be made clear in the following section this entails dealing with a diverse portfolio of products and accompanying characteristics. Figure 1.3 shows the fractions of sales for different product types.

![Figure 1.2 Categorization of the different part types at PACCAR Parts.](image)

1.3.3 Competition

The markets that PACCAR Parts is serving are very divers, expanding and subject to change. Although part of the after sales market for genuine DAF parts is of the pure warranty service type (captive), another important part of the revenue comes from parts demand that is generated by damage events, wear-parts consumption and non-warranty defects and is increasingly commercializing. The truck and trailer parts (TRP) market is more competitive than the fraction of the after sales market that is “captive” due to truck warranties.
Figure 1.4 shows the degree exposure to competitors. *Imitation* competitors “imitate” original DAF truck parts to sell them for lower prices, often offering lower quality. *Brand*-items are items from Original Equipment Manufacturers that PACCAR Parts supplies to truck owners in order to complete the assortment for one-stop purposes, these brands often also have their own distribution network for generating revenue selling directly to customers or through trading organizations, making its revenue more sensitive to competition.

### 1.4 Report outline

Following this introduction chapter, the subsequent chapter (2) provides the problem description as experienced by PACCAR Parts, placing it within the context of the multi-item inventory problem, also introduced in this chapter. Product of this chapter are then the research design, including the assignment and scope and the project planning phases. Chapter 3 evaluates the current methods for managing the multi-item problem, in use at PACCAR Parts, as well as other frequently suggested approaches. Concluding that current practice has some serious deficits, chapter 4 is concerned with model development. Defining performance and modeling the mechanics involved, a solution procedure is developed to obtain combinations of parameters that constitute an efficient solution to the multi-item inventory problem. Chapter 5 evaluates the solutions obtain by several solution procedures. Conclusions are drawn in chapter 6. Figure 1.5 summarizes the report outline.
2. Problem description and research approach

The previous section described the PACCAR Parts environment and some of the aspects that add additional complexities to the on-going challenge of managing inventory in the most efficient as well as effective way. This section defines the problem PACCAR Parts has to deal with in their decision making (section 2.1) but also makes the general challenge, what from here will be named, the “multi-item inventory problem” explicit and explains why it is an important research issue for the general case of companies managing inventories (section 2.2). Section 2.3 reports the research design.

2.1 PACCAR Parts problem description

The Stock control department manages a large number of SKU’s and utilizes the overall fill rate (a service measure to express availability), holding cost, and inventory turnover as key indicators of performance.

An important share of item level performance is determined by the amount of safety stock. A way to increase the availability of an item is to add safety stock which functions as a buffer for deviations from the expected demand. Clearly, providing an item with “extra” units of (safety-) stock, will improve its availability but detories holding cost- and inventory turnover performance for this item. The general objective of inventory management is to find an efficient point in this tradeoff.

Similar to most stock holding entities, PACCAR Parts utilizes rules to determine the amount of safety stock needed to achieve a specific target fill rate (e.g. 97% off the shelf availability) on item level. In the initial consultations it became clear that aggregate service measures put different weights on individual items. More specifically, management and planning staff favors providing high service on the fast-moving cheap items and use this to compensate for residual items. The main reason for this is that it is simply infeasible (due to the large cost and storage capacity consequences) to provide a high service level to every single item. This kind of considerations are aggregate-, or equivalently, assortment-considerations. It is recognized by PACCAR Parts that, to achieve a predefined assortment service level, not all item-level target service levels need to be equal and this is demonstrated by their current methods (to be discussed in chapter 3). However, PACCAR Parts lacks an accurate method to relate the item-level safety stock to the aggregate performance.

In the current situation, several item classes are used for assigning target service levels to items and safety stocks were added accordingly in order to achieve these target service levels. Management faces difficulties setting the targets and expected opportunities for performance increase by allocating inventories in a way that achieves a better overall service level with less stock investment. Additionally, the assortment is very diverse in terms of demand characteristics, replenishment characteristics, and unit (holding) cost. Increasing assortment size, assortment diversity and significant investment in safety stocks (in our data set the safety stock accounts for approximately 63% of the total holding cost) motivated the stock control department of PACCAR Parts to evaluate their methods for safety stock allocation.

To summarize, the stock control department has certain policies and rules in place to translate an individual items’ service target into a safety stock quantity in an inventory policy but lacks an accurate method for relating the item-level safety stocks of a large set of items to the aggregate effects on the assortment service-level and total inventory investment.
2.2 The safety stock allocation problem

The indicators of performance used at PACCAR Parts (fill rate and inventory investment), are commonly used among stockholding entities because their combination reflects the downside as well as the upside of inventories. Cohen et al (1997) confirm the presence of this cost-service trade-off in general, in their benchmark among service parts logistics companies. It therefore makes sense to formulate this decision making problem in a general form, and shortly highlight some of the challenges it poses.

2.2.1 Formulation

The problem description in the previous section (2.1) comprises an important part of the collection of inventory management research that can be categorized as treating multi-item inventories. An overview of literature related to multi item inventories in the area of spare parts is provided in Van den Berg (2011). This literature review identifies five main components of an inventory control policy which are summarized in appendix II. This study focusses on the fifth component: determination of the moment of ordering. For a single item this moment is called the reorder point\(^2\) denoted as “\(s\)”. (When dealing with multiple items, the index \(i\) is added). Used throughout this report is the simple, well established, relation between the reorder point and the amount of safety stock given in (2.1). Readers with less experience in (single item) inventory management are referred to appendix I for an intuitive illustration of this relation.

\[ s = \text{Expected demand during leadtime} + \text{safety stock} \tag{2.1} \]

The safety stock allocation problem can qualitatively be described as minimizing the total holding cost, while maintaining the desired aggregate service level. For use in an inventory control policy for a discrete environment, the reorder point should be an integer value. The remainder of this study is aimed at finding quantitative expressions for the terms formulated in (2.3)-(2.5) and a method to obtain solutions to the problem. For an assortment of \(N\) items:

Minimize:

\[ \text{Total holding cost} \tag{2.3} \]

Subject to:

\[ \text{Aggregate service level} \geq \text{Aggregate service level target} \tag{2.4} \]

\[ \text{reorder points are integer values} \tag{2.5} \]

Aggregate performance aims at achieving a high aggregate service level (or equivalently, assortment-service level) with low total holding cost. When the assortment consists of \(N\) items, aggregate performance is a function of \(N\) single item performances that do not necessarily contribute proportional amounts to the holding cost or service level.

\(^2\) Appendix V describes several ordering mechanisms arguing it is safe to focus on the popular and fairly general \(R,s,Q\) ordering mechanism, which best characterizes the ordering mechanism at PACCAR Parts, without loss of generalizability to other ordering mechanisms.
Instead of choosing a desired service level target as a starting point and minimizing the total holding cost needed to achieve this target, the problem can be turned around to maximize performance while spending a given holding cost budget. In essence this is the same problem and the optimization section in chapter 4 will show it requires only a minor change in the procedure.

Safety stock decisions are not independent of the replenishment order quantity. Because stockouts typically occur at the end of a replenishment cycle, the number of cycles (determined by the replenishment order quantity) influences the safety stock requirements. The scope of this study (2.3.2) will point out that the replenishment order quantities are predetermined. Thus, the replenishment order quantity is taken into account as a characteristic, but will not be a decision variable in this problem.

2.2.2 Challenges

It is known that service does not come for free, it is paid for in terms of safety stock. Larger amounts of safety stock (a higher reorder point) lead to less disservice, or equivalently a higher service level.

Single item inventory theory is extensive and provides methods for finding the reorder point such that this item will achieve a pre-specified target service level. One way to obtain a solution for the multi-item safety stock allocation problem (2.3)-(2.5) is to take the target aggregate service level (e.g., 97%) for item 1, use single item inventory theory to find the reorder point that achieves this target, and repeat this process for all items in the assortment. This method is known as the item approach. It’s main disadvantage is that the total inventory investment is not obvious from the item-level decisions (Sherbrooke 2004).

Instead of “fixing” the item service level for all items as in the item approach, one might assign half of the items to group 1 having a 96% target service level and the other half to group 2 with a 98% target to obtain the same service “on average”. Whether this will help performance (in terms of cost versus service) depends on the composition of the two groups.

This is where the challenges of working with a diverse assortment come into play. In real life assortments, items have heterogeneous characteristics. Total holding cost might reduce (compared to the item approach) when group 1 includes all “cheap” items. Furthermore, due to some items accounting for a relatively large share of total demand (i.e. a strong or weak presence of the 80-20 or Pareto-effect. Chen et al, 1993), one of two groups might represent a larger share of the assortment service. In addition, a number of other item-level characteristics may cause one group to require more safety stock (in units) than the other to obtain the same service level. These aspects complicate decision making and a more structured evaluation is needed than the just described trial and error type of approach. Clearly there is no actual obligation to divide the assortment in any number of groups.

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3 A replenishment cycle is the time elapsed between receiving a replenishment orders in inventory and (after occurrence of demand and placement of a new order) receiving the next, see appendix I for an illustration.  
4 Service is a concept. Several realizations and definitions of service are evaluated in appendix VII, in section 4.1 the expressions for service are made explicit and from there the *fill rate* will be regarded as the service measure of choice.  
5 If there is, for example due to some extraneous restriction, the cost of such a restriction can be quantified using this study.
A method that considers safety stock decision making with regard to the set of items as a whole (i.e. the assortment) instead of accumulating item level-decisions is, from here, named an assortment approach. Conceptually, it is closely related to the system approach introduced by Sherbrooke (2004) but focuses on the situation of safety stock decision making for a warehouse subject to stochastic demand, rather than the availability/uptime of a specific good in the field.

This study aims to show how one can obtain efficient solutions to problem (2.3)-(2.5) that can lead to significant savings in total holding cost, increased aggregate service, or a combination. The multi-item problem is related to the single item problem in the sense that every multi-item solution is computed from a number of item approach solutions for a particular set of parameters (Sherbrooke 2004). Thus, to solve for an assortment solution it is necessary to solve a series of single item problems.

The remainder of this chapter is devoted to clarifying the research objective and steps taken in approaching the problem.

### 2.3 Research design

After introducing PACCAR Parts and the challenges it faces, the safety stock decision making requirements turned out to be closely related to a specific case of the general multi item safety stock allocation control problem. More specifically, considering the project scope, to be outlined in section 2.3.2, the problem can be characterized as a multi item safety stock allocation problem for a heterogeneous assortment under a periodic review ordering policy with batch-size replenishment order quantities and an aggregate fill rate constraint. This problem, for which to the author’s knowledge so far no decision models are available in literature, is the main subject of this research. The next subsection (2.3.1) defines the research objective and research question. Subsequently subsection 2.3.2 outlines the scope of the project. The project approach methodology is discussed in section (2.4).

#### 2.3.1 Research assignment

PACCAR Parts is currently managing over 290,000 SKU’s, of which more than 28,000 are delivered from stock. Clearly this is a multi-item environment in which items (each with its own value, sales volume, order quantities and leadtime) compete for a limited budget of safety stock. PACCAR Parts faces the challenge of investing its inventory budget in a way that maximizes their overall service towards customers. The stock control department has certain policies and rules in place to translate a service requirement to parameters (reorder point) in an inventory policy (to be discussed in chapter 3), but lacks an accurate method for relating the parameters and characteristics of a large set of items to the aggregate effects on the assortment service level and inventory investment. Desired service levels/fill rates per item are based on a manually filled matrix with two criteria: Sales volume and item value. Due to a changing environment, increasingly large and diverse assortment, and increased pressure on inventory control, management expected performance improvement opportunities to be present by evaluating their method of allocating safety stock and was eager to quantify possible performance increases when revising their approach to the safety stock allocation problem.
The goal of this research is twofold:

1. For a multi-item inventory assortment under periodic review and predetermined replenishment batch sizes consisting of a wide range of SKU’s each having its own characteristics, design a procedure for allocating safety stock such that:
   - The expected overall holding cost are minimized
   - The aggregate fill rate is at least equal to the target fill rate set by management.

2. Apply the decision model to the situation of PACCAR Parts and evaluate the results and implications for performance.

Alternatively, the problem can be "inverted" to spend a given holding cost budget in a way that maximizes the aggregate fill rate. Management expects to identify areas for improvement by differentiating items with regard to service, and seeks to gain insights into which type of items should be given a high or equivalently low service objective in order to meet the aggregate service objective with lower inventory investment, increase the aggregate service with the same budget or a combination. The research question is stated as:

*Research question:*

*How can safety stock be allocated over a large and diverse set of items in order to achieve an aggregate fill rate target, while minimizing the expected total inventory holding cost, in an environment with a periodic review ordering policy and predetermined replenishment quantities?*

Past research that treats the multi-item inventory problem from a assortment point of view instead of an item focus is scarce. New in this research is use of fill rates as a service level as opposed to a cost criterion or cycle service level (P1 or probability of no stock out in a replenishment cycle), and consideration of other than continuous review base stock policies. Choosing a target aggregate fill rate is much more intuitive to industry users such as PACCAR Parts than trying to define a “cost per backorder” or cycle service level (which are only partially related to the service experienced by customers). In addition, this research explicitly takes the aggregate effects of setting individual inventory parameters into account, to comply with the practical situation of dealing with finite storage capacity and aggregate service. Besides application to the specific situation of PACCAR Parts, the design of a method that addresses the multi item safety stock allocation problem for a heterogeneous assortment under a periodic review ordering policy with batch-size replenishment order quantities from an assortment perspective, will complement existing literature.

### 2.3.2 Scope

*Internal customers*

For PACCAR Parts, when focusing on the customer, inventory located at the dealers determines the service a truck owner will perceive. In essence, the dealers have the same multi item inventory control problem as the PDC’s.

Lateral transshipments between dealers and between PDC’s are physically possible and could improve performance. Recent research within PACCAR Parts has devoted considerable attention to the multi-echelon coordination possibilities (Van Willenswaard, 2006). However, due to some practical considerations such as the fact that the dealers are independent instead
of PACCAR-owned and the local orientation of IT-architecture, the possibilities regarding implementations are limited. This study focusses on the single-site situation of the central distribution center in Eindhoven assuming it is an independent demand inventory situation.

**Backorder environment**

The sales environment on dealer-level is in practice a mix of backorder- and lost-sales items. Stock-outs in a lost-sales environment can eventually lead to customers losing loyalty when they are frequent. These effects are difficult to quantify (Schwartz 1970). This study is not about the cost of disappointing customers and consequences of customer loyalty. This is why decisions to include or not include items in the assortment are considered beyond the scope of this project, negative safety stock is considered infeasible. Dealers are regarded as the (internal) customers, and service is perceived in terms of the number of units delivered from stock. Lost sales are not considered, unsatisfied demand is backordered.

**Order quantities**

One of the components of an inventory policy constitutes a logic for determining order sizes. In many organizations order sizes are dictated by packaging or transportation quantities (e.g. full pallet), or a minimum order quantity is set by the supplier. In case of PACCAR, at least half of the active items have an order quantity or order quantity multiple imposed, and employ an EOQ-similar logic for the remaining cases. Setup cost, as in the fixed cost of placing an order, are considered in this (EOQ) logic only and are assumed absent in subsequent decision making. Periodic review policies where the review period is equal across items allows some savings in setup cost (coordinated replenishments) as replenishment orders are placed in one run. This might have been an argument to use a periodic review ordering mechanism. Being one of the research conditions in this study, the periodic review as well as the item-level replenishment order quantities are assumed predetermined.

It should be noted that the separate determination of the replenishment order quantities (for example using EOQ-like methods or (packaging-) imposed quantities) is a widely used assumption in practice (see Cohen et al 1997; Zhang et al 2001, Teunter et al 2010). Furthermore, Zheng (1992) finds that the relative increase of the costs incurred, by using the quantity determined by the EOQ instead of the optimal from the stochastic model is small, and vanishes when the ordering costs are large relative to other costs (i.e. the case of large replenishment order quantities).

**Demand process and cross selling**

Present in practice and recognized in literature are the effects of cross selling. In some situations, especially in situations where items are consumed in pairs or combinations, demand can be correlated. In general, items having the same characteristics with respect to demand, value and other criteria, would be allocated approximately equal amounts of safety stock (relative to their variance). In these cases the effect of cross selling is expected to be very small. When differentiating items on characteristics that are very different between pairs or combinations, the cross selling effect might have its influences. In case low service levels are set for one of the items that are part of a combination, the customer might refrain from buying the other item(s) in the combination. Because cross selling information can be hard to infer from sales data, and PACCAR Parts in most cases creates “kits” for items that are part of a combination by nature, the effect of cross selling is assumed to be small. An extension of the model to take account of cross selling remains for future research. For this study, cross selling effects are assumed absent. More specifically, demand for items is assumed to be characterized as a sequence of independent and identically distributed random variables. This
implies that demand is a random variable with a known distribution that is independent across periods.

2.4 Project plan

After defining the research objective in the previous section, the next step is framing the project in steps that guide the research process. The first subsection introduces several conceptually high level phases for the project to follow. Subsequent subsections each describe the contents of one step. Lower level activities are then summarized in the activity flow diagram in appendix III.

2.4.1 Research phases

As a basic process, the research design consists of only a few high level steps, that each include a number of lower level steps and activities. The phases are depicted in figure 2.4 and are based on Wijnen (2001).

![Research design project phases](image)

The next sections will each clarify the purpose and content of each phase in the order depicted in figure 2.4. The sections shortly discuss the contents of each phase, starting with the definition phase.

2.4.2 Definition

The definition phase is the phase that explores the research objectives and purposes of the design. Input for the definition phase are the literature study and the research proposal. The definition phase includes the formulation of research sub-questions of which the answers will help achieving the research objective. Input for the definition phase are the less well researched areas of literature, as identified in the literature study, in combination with managerial challenges occurring in the practical field of business, as indicated by the case company. For this project the following research sub-questions are identified. The remainder of this subsection motivates each question separately. Appendix III shows in which project phase they come into play.

1. Which ordering mechanisms are suitable for managing fixed order quantity, periodic review inventory situations subject to stochastic demand?
2. Which variables\(^6\) and characteristics\(^7\) influence an individual item’s inventory performance?
3. How can performance be expressed \textit{ex-ante}?
4. How can safety stocks be allocated over an assortment, based on the identified item characteristics as to maximize performance?

---

\(^6\) Variables are considered as input to the inventory process over which organisations can decide.

\(^7\) Characteristics are input to the inventory process over which the organisation is assumed to have little or no direct control, examples of item characteristics are the demand process, item value, or supply lead-time.
5. Which item characteristics in general are determining for obtaining a higher item target fill rate? Can they be captured in a categorization scheme or heuristic procedure?

As a starting point, an initial characterization of the inventory process is required. The first subquestion will therefore focus on a short evaluation of ordering mechanisms (also called ordering policies).

Decision making is concerned with manipulating certain variables in a process in order to, with proper consideration of the characteristics, steer performance. A requirement for decision making that is very relevant in our study is therefore to distinguish variables from characteristics. Variables can be influenced by management, characteristics are those aspects of the process over which management has little or no control but do influence performance and should be therefore be taken into account. Research sub question 2 is focused on the identification of the relevant variables and characteristics.

An important contribution from the field of single item inventory theory is the ability to not only measure an items service level and holding cost after one or more cycles of depletion and replenishment of inventory (that is, after the event or “ex-post”), but to use expressions that indicate what the performance is expected to be, before the event or “ex-ante”. Although most modern inventory holding entities employ the “ex-ante” expressions on item level, management of aggregate performance is dominated by ex-post measurements. Research sub question 3 is aimed towards the development of sensible ex-ante expressions for aggregate performance, making use of the results provided by single item inventory theory.

Research sub question 4 is concerned with choosing values for the variables we have control over (identified in question 2), using the expressions for performance provided by the answer to question 3, (which include the influences of the relevant characteristics) in order to maximize performance. Because approximate, simplified schematic approaches to the allocation problem such as ABC-like methods are currently dominating industrial practice (Cohen et al 1997), research question 5 is aimed at identifying the most important conclusions and possibly capturing them in a schematic- or otherwise simplified procedure.

Answers to the research questions should provide an understanding of the requisites for designing the model. Appendix III shows a more detailed overview of the activities included in each phase.

2.4.3 Design

The design phase contains the actual modeling of the decision making problem. The design phase is focused mainly on the aggregate inventory processes. The modeling activities will focus on developing methods to control the input variables such that the performance metrics that measure the output will be most favorable. From analyzing the results, insights into the most important characteristics of a good solution are derived which can optionally be used to develop a ABC-comparable heuristic procedure, which in turn is tested and compared on feasibility and performance. Appendix III shows a more detailed overview of the activities that will be included in this phase.

2.4.4 Verification

The verification process involves testing whether the proposed input will produce feasible results, and compares them to other approaches. Having a design in place, real life assortment
data can be used to verify and quantify the savings when applying the safety stock allocation method. Optionally, the same can be done using some other approaches for comparison. Appendix III shows a more detailed overview of the activities included in this phase.

2.4.5 Implementation and evaluation

Although not necessarily part of the master thesis report, implementation of the model and evaluation of new situation should make sure cost savings and/or performance increases are realized. The evaluation phase should include monitoring the performance and actualization of the model in time.
3. Current inventory management practice

The previous chapters should provide an idea of the company and the safety stock allocation problem as the subject of this research. Section 3.1 describes the method for determining the item level parameters currently employed at PACCAR Parts. Section 3.2 zooms in on PACCAR Parts’ current logic for safety stock allocation and demonstrates the aggregate dynamics involved, by means of some quantitative descriptives about the assortment.

3.1 PACCAR Parts’ approach for determining the item-level inventory parameters

This section provides a brief description of PACCAR Parts’s current inventory policy. The equations in this section are mainly derived from the (IT-)system description provided by PACCAR Parts and a preceding study at PACCAR Parts (Valkenburg, 2000;1999). The contents of this section are not to be confused with propositions for an inventory control method, but functions merely as a description of the current working of the system. Successive subsections describe: the order- and demand registration, computation of the replenishment order size, computation of the reorderpoint and finally the logic for choosing the target service level.

3.1.1 Registration of placed, outstanding and received replenishment orders

PACCAR Parts registers the inventory position (or “Economische Voorraad”) for each item \( i \) denoted as \( IP \) (Inventory Position) using the equation:

\[
IP = OH + OO - BO - COM
\]

(3.1)

The inventory position at any point in time consists of the amount of inventory physically on hand \( OH \) in the distribution center, \( OO \) the amount of units outstanding at the supplier, \( BO \) the amount of backorders, and \( COM \) the amount committed or reserved. Information on the on-hand inventory and reception of replenishment orders is automated using a barcode registration system.

3.1.2 Registration of demand

Demand is registered by means of orders placed by dealers. Because of the size of the order line database, the IT-system aggregates these individual orders to a value that contains the total demand for item \( i \) for each 4-week period \( t \), denoted as \( x_{t,i} \). Smoothing methods are used to generate estimates on the mean and mean average deviation (MAD, a measure of dispersion) of the demand process for the next \( \tau \) periods denoted as \( \hat{X}_{t,i}(\tau) \) and \( \hat{MAD}_{t,i}(\tau) \). Subsequently these two estimates are used to characterize the demand process in subsequent inventory control decisions.

3.1.3 Order size computation


An item’s order quantity at PACCAR Parts is determined as the maximum of the EOQ (Camp 1922, Harris 1913) rounded upward to the nearest case pack and respects a minimum order quantity. The order size computation is then determined by:
\[ Q_i = \max \left( \sqrt{\frac{2X_{t,i}(13)c_i^h}{c_i^h + Q_i^{cp}}} \cdot Q_i^{min} \right) \]  

(3.2)

Where

- \( Q_i \): Order quantity in units for item \( i \)
- \( Q_i^{cp} \): Case pack quantity in units for item \( i \)
- \( Q_i^{min} \): Minimum order quantity in units for item \( i \)
- \( X_{t,i}(13) \): Estimated yearly demand, determined in period \( t \)
- \( c_i^h \): Holding cost per unit per year for item \( i \)
- \( c_i^o \): Ordering cost for item \( i \)
- \([x]\): \( x \) rounded up towards the nearest integer

This expression coincides with what in Silver’s (1998) notation is \( Q^* = \sqrt{\frac{2AD}{vr}} \), but with a mean (estimated) yearly demand \((D)\) or equivalently \( X_{t,i}(13) \). Although not economical in the view of inventory efficiency, case pack multiples and minimum order quantities are very common exogenous constraints in practice.

### 3.1.4 Reorderpoint computation

The replenishment order quantity noted as \( Q_i \) is determined in the previous subsection and is taken into account when calculating the reorder point. To find the reorder point that would achieve a prespecified service level, estimators of the expected mean demand and variance of the demand process are required. These are provided by the forecasting system in the form of \( X_{t,i}(\tau) \), which is the expected average demand for subsequent \( \tau \) periods, and \( M\hat{D}_{t,i}(\tau) \) which is an estimator for the expected mean average deviation from this average for subsequent \( \tau \) periods.

The following logic is then used to determine the individual reorder points:

\[ s_i = \bar{X}_{t,i} \left( \frac{L_{i}^{supply} + L_{i}^{pack} + R}{4} \right) + k_i M\hat{D}_{t,i} \left( \frac{L_{i}^{supply} + L_{i}^{pack} + R}{4} \right) \]  

(3.3)

Where

- \( s_i \): The reorderpoint for item \( i \)
- \( \bar{X}_{t,i} \): The estimator for the mean demand for item \( i \) over the next \( \tau \) (4-week) periods made in period \( t \).
- \( M\hat{D}_{t,i} \): The estimator for the mean average deviation for item \( i \) over the next \( \tau \) (4-week) periods made in period \( t \).
- \( R \): The length of the review period in weeks
- \( L_{i}^{supply} \): The number of weeks required for the supplier to deliver product \( i \).
- \( L_{i}^{pack} \): The number of weeks needed for (re)packaging and receiving, currently set to (assumed constant and the same for each item).
- \( k_i \): The safety factor
The first term on the right side of (3.3) represents the expected demand during leadtime, while
the second term represents the safety stock, making it of the same form as (2.1). Note that a
period is 4 weeks and $\hat{X}_{t}(\tau)$ is period-based. Leadtimes are given in weeks, arguing for the
division by four.

What is left is to find the value of the safety factor $k_i$, for use in (3.3), such that the resulting
reorderpoint is expected to achieve the item target fill rate $slp_i$ (the % of demand satisfied
from shelf). Choosing a value for $slp_i$ determines the amount of safety stock item $i$ gets and
setting a $slp_i$ for each item $i$ constitutes the approach taken to allocate safety stock. This is
described in the next subsection.
The safety factor $k_i$ is currently determined by computing $SF_i$ using (3.5) and finding the
accompanying value of $k_i$ using the safety factor table in appendix IV.

\[
SF_i = \frac{(1 - \frac{slp_i}{100})Q_i}{MAD_{t_i}(\frac{L_{supply} + L_{pack}}{4} + R)}
\]

(3.4)

Where

$slp_i$  The predefined target service level permillage

This method is a heuristic alternative based on the decision rule for the (s,Q) ordering policy under continuous review proposed by Silver (1998) using a P2 fill rate and
normally distributed forecast errors (For an explanation of the most common ordering policies
see appendix V). Although the ordering policy at PACCAR Parts is referred to as an (R,s,nQ)
policy, this (s,Q) decision rule is used as a heuristic to approximate the reorder point in an
(R,s,nQ) policy. Because the review period is relatively short and demand during review
periods rarely exceeds the replenishment order quantity, no large differences are expected.
Use of this decision rule is motivated by fast and easy computations.

In addition to the described logics, planners and managers have the ability to make changes to
the system parameters based on experience. At the end of every period, the settings will revert
to the updated parameters determined by the described logics.
3.1.5 Target service levels

The previously discussed choices for a reorder point in combination with setting a target Service Level Permillage, \( slp_i \), informs about how safety stock is allocated across items and constitutes the approach taken for the safety stock allocation problem. PACCAR Parts currently determines the \( slp_i \) using two factors, the estimated mean demand \( \hat{X}_{t,i}(1) \) and the item value \( v_i \) the resulting \( slp_i \) is then used in the calculation of the reorderpoint. Table 3.1 shows \( slp_i \) for each category, the values are decided by management (the matching fill rate target \( P2 = \frac{slp_i}{1000} \)) and constitute the method for allocating safety stock over the items.

<table>
<thead>
<tr>
<th>Target service levels</th>
<th>Demand volume: ( \hat{X}_{t,i}(1) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( v_i )</td>
<td>( \hat{X}_{t,i}(1) &lt; \vartheta_1 )</td>
</tr>
<tr>
<td>( v_i &lt; \epsilon_1 )</td>
<td>995</td>
</tr>
<tr>
<td>( \epsilon_1 \leq v_i &lt; \epsilon_2 )</td>
<td>960</td>
</tr>
<tr>
<td>( \epsilon_2 \leq v_i )</td>
<td>800</td>
</tr>
</tbody>
</table>

Table 3.1 Current SLP table logic for setting target service levels at PACCAR Parts.

\( \epsilon_{1,2} \) and \( \vartheta_{1,2} \) are the value and demand volume thresholds respectively, they are constant and the same for all items (\( \vartheta_1 < \vartheta_2 , \epsilon_1 < \epsilon_2 \)). The values remain undisclosed for confidentiality reasons.

These two-dimensional SLP classes provide an intuitive method that allows more discrimination than the classical (one dimensional) ABC classification based on demand volume \( \hat{X}_{t,i}(1) \) or demand value \( (v_i \hat{X}_{t,i}(1)) \) criterion. In addition, a number of items, get their safety stock determined as prespecified number of weeks demand (set manually). Appendix VI provides an overview of (ABC-) criterion procedures suggested in literature. Classifications like this reduce the decision problem to choosing a target service level per category (instead of individually). Subsequently, this target is fixed for all items belonging to the category. Although reducing the number of decisions, classification does not solve the allocation problem. Where normally performance is determined by the set is reorder points, it is now determined by choosing the category thresholds and targets which in turn translate into reorder points. The allocation problem is then shifted from choosing reorderpoints to choosing targets (and thresholds). Clearly, classification in itself is not a solution method. It is hard to infer a good value for the thresholds or the category service target because the aggregate effects of setting choosing this value are not transparent. The relevance of these aggregate effects is demonstrated in the next section (3.2).

3.1.6 PACCAR Parts approach conclusion

In conclusion of this section, inventory control at PACCAR Parts is based on a set of semi-objective logics for heuristically determining the inventory control parameters. From here the focus is on calculations for the reorder points, which of course, determine the amount of safety stock and expected service level as a consequence. The next section evaluates the implications that come with the current procedure and identifies the improvement potential.
3.2 Problem implications and potential for improvement

In the problem definition by PACCAR Parts and description of the current procedure it became clear that the stock control department has certain policies and rules in place to translate a service requirement to parameters (reorder point) in an inventory policy, but does not have an accurate method for relating the parameters and characteristics of a large set of items to the aggregate effects on the service level and inventory investment. A review on current literature on the multi-item inventory control problem (Van den Berg 2011) indicated that the currently suggested approaches for multi item safety stock decisions have its limitations. There is only a scarce amount of literature available that treats this problem, and the use of traditional ABC categories is widespread. PACCAR Parts and other industry organizations are expected to suffer performance loss from not properly addressing important safety stock decisions. This section starts by demonstrating the intransparancy of categorization schemes. Subsequently the results are compared to some of the effects found in literature. Using a related study, the improvement potential is estimated.

3.2.1 Relating individual parameters to aggregate effects

The previous section demonstrated the current method for determining the reorder points, including a decision logic for setting the target service levels for each individual item (table 3.1). Compared to the pure item approach or ABC analysis, the current method at PACCAR Parts, is already performing better as it discriminates on both unit cost and demand volume independently (while the item approach does not discriminate at all, and ABC methods, although possibly determined by multiple factors, discriminate on only one value). When taking a closer look at the decision rule (in the form of the SLP-table 3.1), we are able to extract some important information by comparing it to the assortment. The result is not in every case self-explaining, emphasizing the intransparency between setting the parameters and the aggregate effects. Recall from subsection 3.1.5 that the current method assigns the items to a class, and fixes the target service level for all items in the class. This is illustrated in table 3.2

<table>
<thead>
<tr>
<th>Value $v_i$</th>
<th>$\bar{x}_{t,i}(1) &lt; \vartheta_1$</th>
<th>$\vartheta_1 \leq \bar{x}_{t,i}(1) &lt; \vartheta_2$</th>
<th>$\vartheta_2 \leq \bar{x}_{t,i}(1)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$v_i &lt; \varepsilon_1$</td>
<td>Class 1, Service target: 99.5%</td>
<td>Class 4, Service target: 99.5%</td>
<td>Class 7, Service target: 99.8%</td>
</tr>
<tr>
<td>$\varepsilon_1 \leq v_i &lt; \varepsilon_2$</td>
<td>Class 2, Service target: 96%</td>
<td>Class 5, Service target: 97.5%</td>
<td>Class 8, Service target: 98.5%</td>
</tr>
<tr>
<td>$\varepsilon_2 \leq v_i$</td>
<td>Class 3, Service target: 80%</td>
<td>Class 6, Service target: 85%</td>
<td>Class 9, Service target: 94%</td>
</tr>
</tbody>
</table>

Table 3.2 Current SLP table logic for setting target service levels at PACCAR Parts
To provide a descriptive indication about the distribution of items over the different categories, table 3.3 indicates the fraction of items assigned to each of the 9 categories. For example, 9% of items is categorized in the low-value high-volume category. The amount of items assigned to the slow-moving categories (56%) signifies PACCAR Parts can be characterized as a typical spare parts company with many slow-moving items.

<table>
<thead>
<tr>
<th>Item assignment</th>
<th>Demand volume: $\tilde{X}_{t,i}(1)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value $v_i$</td>
<td>$\tilde{X}_{t,i}(1) &lt; \theta_1$</td>
</tr>
<tr>
<td>$v_i &lt; \mathcal{E}_1$</td>
<td>37%</td>
</tr>
<tr>
<td>$\mathcal{E}_1 \leq v_i &lt; \mathcal{E}_2$</td>
<td>16%</td>
</tr>
<tr>
<td>$\mathcal{E}_2 \leq v_i$</td>
<td>3%</td>
</tr>
</tbody>
</table>

Table 3.3. Distribution of the assortment over each of the SLP categories.

A similar table, showing the fraction of total demand represented by each category, can be generated and is shown in table 3.4. This table shows that, although table 3.3 showed the largest part of items was assigned to the slow movers category, the far greatest fraction of demand is generated by the fast moving category, and especially the fast movers with relatively low unit value. High-volume low-cost items represent 75.2% of the total demand volume. As we will see in section 4.1, this effect is of great importance when considering aggregate performance. When for example controlling inventories to operate at a demand weighted service level, the target set in the upper right category, will determine 75.2% of the overall service level. These tables illustrate that, Although rarely treated by methods suggested in literature, the question: “What will be the right target service level for this (category of) item(s)?” is not to be taken lightly. It is what determines performance.

Setting target service levels is not a decision to be taken in a myopic way and should be based on an aggregate view on the assortment, that is, decision making should be related to effects on service and inventory investment. Furthermore it can be seen that these 9 classes (or the 3 considered in traditional ABC analysis) are very broad and intransparant categories.

<table>
<thead>
<tr>
<th>Demand volume</th>
<th>Demand volume: $\tilde{X}_{t,i}(1)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value $v_i$</td>
<td>$\tilde{X}_{t,i}(1) &lt; \theta_1$</td>
</tr>
<tr>
<td>$v_i &lt; \mathcal{E}_1$</td>
<td>2,9%</td>
</tr>
<tr>
<td>$\mathcal{E}_1 \leq v_i &lt; \mathcal{E}_2$</td>
<td>0,3%</td>
</tr>
<tr>
<td>$\mathcal{E}_2 \leq v_i$</td>
<td>0,0%</td>
</tr>
</tbody>
</table>

Table 3.4 The fraction of total demand falling in each of the SLP categories.

The previous tables made clear that the upper right category (fast moving, inexpensive products) is determining the far most important part of demand. When service is weighted by demand as suggested by many authors (Teunter 2010; Thonemann 2002; Zhang 2001 and many more), the major part of service is determined in this class. Notice that having a 99,9%

---

8 Due to rounding, the percentages will not necessarily add up to 100.
service level on the lower left category would be noticed by nearly no-one, but would require a large safety stock investment as it concerns expensive items. Since inventory performance is a holding cost versus service tradeoff, the next question that might arise is where in the table the current holding cost are concentrated. When using the approximate calculation in which the average on hand inventory is determined by half of the order quantity plus safety stock (more on these kind of relations is found in section 4.2), table 3.5 shows the distribution of inventory investment in the different SLP categories. The distribution is very similar when only considering the cost of safety stock (the maximum deviation is less than 2%).

<table>
<thead>
<tr>
<th>Total holding cost</th>
<th>Demand volume: $\bar{X}_{t,i}(1)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_i$</td>
<td>$\bar{X}_{t,i}(1) &lt; \vartheta_1$</td>
</tr>
<tr>
<td>$v_i &lt; \varepsilon_1$</td>
<td>0.4%</td>
</tr>
<tr>
<td>$\varepsilon_1 \leq v_i &lt; \varepsilon_2$</td>
<td>1.6%</td>
</tr>
<tr>
<td>$\varepsilon_2 \leq v_i$</td>
<td>2.6%</td>
</tr>
</tbody>
</table>

Table 3.5 fraction of total holding cost in each SLP category

What can be concluded from analyzing above tables is that:

- The major fraction of item numbers are slow-moving, low value items
- The major fraction of service is determined by the fast moving, low value items
- The major fraction of current inventory cost is determined by fast moving, medium value items.
- Setting an input value for the target SLP (table 3.1) and determining the class thresholds ($\varepsilon_{1,3}$ and $\vartheta_{1,3}$) has very large but unknown consequences for the overall service and total holding cost.

3.2.2 Demand skewness

One of the phenomena that the previous section (more specifically, comparison of table 3.3 and 3.4) has pointed out is that of demand skewness across items. When the distribution of total demand over items is skewed, there is a small portion of items that is responsible for a large part of the total demand for items. From the tables in the previous subsection it is clear that significant demand skewness can be expected. The demand skewness in the PACCAR Parts assortment is such that 20% of the items accounts for approximately 85% of the total demand. This skewness, often referred to as the Pareto- or 80/20 phenomenon (Chen et al, 1993) is typical in many organizations. In developing the optimization model in section 4.4, ways to make use of this skewness are suggested.

3.2.3 Cost skewness

A phenomena similar to demand skewness is the unit cost skewness. In many organizations but especially in spare parts, item value can range from a few cents to thousands of euros. The cost skewness in the PACCAR Parts assortment is very large. Thonemann et al (2002) finds that the relative improvement from using an assortment approach over an item approach is largest in situations with large skewness of unit cost.

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9 This skewness should not be confused with the skewness of a probability distribution. If a small fraction of the items accounts for a large fraction of the demand, that assortment is said to have large skewness and vice versa. (see Thonemann 2001).
3.2.4 Improvement potential

Suppose one would want to improve the assortment service level using the SLP table (table 3.2). Clearly this entails increasing one of the target service levels in table 3.2. It is then unknown which of the 9 classes is the best candidate for an increase. Increasing any of the 9 classes would improve service, but not to the same degree, also the cost of this increase (in terms of inventory investment) is not the same across categories.

In- or decreasing the SLP target in a specific category is like turning a large group of knobs at once (of which the size and composition is unknown), without being able to see its effect or that of the other choices available, and without knowing whether turning one (or a combination of) the others leads to a larger service increase at less cost.

This emphasizes the importance of being able to relate individual service levels and reorderpoints to overall performance in order to make reasoned decisions that help performance. Thonemann et al (2002) developed a method to assess the relative improvement when moving from an item- to an assortment approach. Making several assumptions that include compound poisson demand and a base stock \((s - 1, S)\) ordering mechanism, he develops a continuous approximation for the system cost as a function of the demand- and cost-skewness.

Utilizing Thonemans approach it was found that for assortments with the demand- and cost-skewness of the PACCAR Parts assortment, relative improvements (the percentage improvement in inventory investment when moving from an item approach to a system/assortment approach) are approximately around 21%. The assumptions are not the same as those underlying PACCAR’s situation and PACCAR Parts’ allocation method is somewhat more sophisticated than the pure item approach. However, this does illustrate a potential for improvement, and together with the problem statement confirms expectations for a performance increase when using an assortment approach.

3.3 Summary

This chapter began with clarifying PACCAR Parts’ methods for determining the inventory parameters. Together with several assortment descriptions it became clear that the current safety stock allocation procedure has some important limitations that prevent efficient allocation of safety stock. A preliminary analysis, together with assortment information indicated a large potential for improvement when approaching safety stock allocation from an assortment perspective.
4. Model development

Elaborating on the conclusions drawn from inventory management practice and theory, the previous chapter indicated that the impact of safety stock allocation on performance can be large. The current chapter is devoted to the design of a safety stock allocation model that treats the problem of safety stock allocation from an assortment perspective. In order to maximize performance, it is important to first carefully define it. Recall that, on a conceptual level, the optimization problem to be solved by the safety stock allocation model, is given in (2.3) - (2.5). This chapter is aimed at translating this conceptual problem into quantitative form. Moving from the conceptual towards the operational, the following formulation describes our problem.

Minimize:

\[ \sum_{i=1}^{N} c_i Y_i(s_i) \]  \hspace{1cm} (4.1)

Subject to:

\[ \beta(S) \geq OSL^{target} \]  \hspace{1cm} (4.2)
\[ \beta_i(s_i) \geq SL^{min}_i \]  \hspace{1cm} (4.3)
\[ s_i \in \{ \mathbb{Z}^+ \} \]  \hspace{1cm} (4.4)

Where :

- \( Y_i(s_i) \): The average on hand inventory for item \( i \) as a function of the reorder point \( s_i \).
- \( c_i \): The unit holding cost for holding item \( i \).
- \( \beta_i(s_i) \): The expected service level for item \( i \) as a function of the reorder point \( s_i \).
- \( \beta(S) \): The expected aggregate service level as a function of the set \( S \) of reorder points \( s_i \).
- \( OSL^{target} \): The target aggregate service level for the assortment.
- \( SL^{min}_i \): The minimal item service level.
- \( \mathbb{Z}^+ \): The set of positive integer numbers \( \{0,1,2,\ldots\} \).
- \( i \): The item(number) index. \( i \in \{1,2,\ldots,N\} \).

(4.1) represents the total holding cost by summing over all items: the holding cost per item \( c_i \) multiplied by its expected on hand inventory \( Y_i(s_i) \). Regarding holding cost (4.1), the individual items are only interdependent in the sense that they consume the same “budget” for inventory holding cost.

(4.2) ensures that the aggregate service level \( \beta(S) \) complies with its target \( OSL^{target} \) set by management. The aggregate service level \( \beta(S) \) is a function of the collection of item level reorder points \( s_i \) defined as the set \( S \).

\( SL^{min}_i \) in (4.3) is the minimum item level service to be achieved by every item \( i \), it is set by management. For use in an inventory control policy, the individual reorder points \( s_i \) should be integer values, this is indicated in (4.4).

Note that the average on hand inventory \( Y_i(s_i) \) includes safety stock as well as cycle stock (determined by the replenishment order quantity). Although we have no control over the cycle stock since \( Q_i \) is predetermined, the safety stock is related to the order quantity. The replenishment order quantity is necessarily present as a characteristic in the service
expressions (4.2 and 4.3) (see 2.2.1). For consistency it is retained in the total holding cost (4.1) as well. Hence, for the scope of this problem, total holding cost consists of both safety stock and cycle stock.

The setup of this chapter is as follows. Section 4.1 and 4.2 are about finding mathematical expressions for the service measures ($\beta_i(s_i)$ and $\beta(S)$), and expected inventory $Y_i(s_i)$ respectively. These mathematical expressions are well-established in inventory management literature. Using the contents of the first two sections, the problem formulations can be written in mathematical form which is presented in section 4.3. Subsequently, the nature of the problem and several solution methods are evaluated. Section 4.5 describes the optimization procedure for the allocation problem. The section concludes in 4.6.

4.1 Customer Service: $\beta_i(s_i)$ and $\beta(S)$

To solve the safety stock allocation problem, a measure of service has to be agreed upon. This service measure has to represent the upside of inventories in our tradeoff. The first subsection (4.1.1) evaluates three categories of service objectives originating from inventory management theory (this section is strongly related to the question “What is service?”). The modeling process consists of defining which behavior of the inventory process is or is not taken into account. This is why subsection 4.1.2 discusses the main assumptions that are relevant and commonly differ across modeling approaches. 4.1.3 then suggests mathematical expressions for expressing the expected service level as a function of our decision variables. The section concludes in 4.1.4.

4.1.1 Service objectives

When demand is uncertain, chances are that at a some point in time the on hand inventory is insufficient to fulfil arriving customer demand and a backorder occurs. Safety stocks are used to create a buffer for avoiding the undesired consequences of a stock out. In order to be able to decide on the desired amount of safety stock, one needs to define what these safety stocks are to achieve. In most cases a service objective is a statement about the probability, frequency and magnitude of backorders occurring. This subsection is about what exactly is the measure to use when setting a target. Before evaluating service objectives, distinguishing ex-ante and ex-post service is in order.

Ex-post performance is measuring the actual performance “after the event”. During or after one or more periods of replenishment and depletion of demand, data can be collected on the number of demands satisfied from stock, the average inventory and inventory turnover during this past period. Ex-post service measures are a valuable tool for evaluating performance once events have occurred, but its usability is very limited when making decisions for a subsequent future period.

Ex-ante performance is an expectation of what future performance will most probably look like. It is a function of the decision variables. Ex-ante expressions make several assumptions on the nature of the event that is, the mechanisms with which depletion and replenishment of demand are expected to progress. Having and using ex-ante service expressions allows gaining valuable insights about the decisions to be made for the future. When using an accurate ex-ante service expression, it is possible to know what the ex-post service measure

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10 Some environments one experiences a lost sale rather than a backorder, see also section 4.1.2.
are most likely to report, out front. This is of value when making decisions for a future period. Clearly, ex-ante expressions are more valuable than ex-post measures.

Some popular inventory service objectives that are used in practice are summarized in appendix VII (largely based on Silver 1998) and can be classified in four categories, a short discussion of each is found in the next four headings.

**Myopic category**
Myopic approaches (category 1) include setting the safety stock as fixed number of weeks average demand, or fixing the safety factor k for the whole assortment. Service measures under this approach (for example number of backorders) are only evaluated retrospectively (i.e. ex-post) and this category contains methods of guess-and-observe rather than setting a target that complies with a service measure.

**Penalty cost category**
Safety stocks based on minimizing penalty cost (category 2) involves evaluating ex-ante expressions. This category can provide cost-optimal solutions. It assumes decision making revolves around minimizing the sum of penalty and holding cost, it is therefore important to include all important aspects of performance in one of these two cost factors. Penalty cost and customer service approaches can often be showed to lead to identical solutions. The main disadvantage of the penalty cost category is a) the penalty cost of a backorder being hard to quantify and b) not being intuitive: the effects of choosing a value for the, somewhat ambiguous, penalty cost is less evident than choosing for example a target % of demand to be satisfied from stock.

**Customer service category**
Among safety stocks based on customer service (category 3) the most well-known measures are the “cycle service level” called P1, and the “fill rate” called P2. The P1 is the probability that the on hand inventory is positive when the replenishment order arrives (ending the cycle) and can be viewed as the probability that no stockout occurs per replenishment cycle. This definition of service can be kind of misleading when the number of replenishment cycles is not the same across items (an item with a small order quantity may have many more stockouts per year, than an item with the same P1 but large order quantity because it has less cycles in a year). Furthermore the size of the stockout is not necessarily evident from the probability of one occurring. The advantage of this measure is easy calculation of the expected P1 service level (ex-ante) from only the safety factor k. This ease of computations could explain its popularity.

We found that, although requiring slightly more complex in calculations, the fill rate (P2) best represents customer service and is most intuitive to both users and customers. This is in line with findings from a benchmarking analysis in the service parts logistics by Cohen et al (1997) who find the fill rate being one of the most commonly used measures for parts availability.

The (single) item fill rate (P2) is measured as:

\[
\beta_i = \frac{\text{demand for item } i \text{ satisfied from stock}}{\text{total demand for item } i}
\]  

(4.5)

Single item inventory theory is extensive in providing ex-ante expression for the fill rate, under various assumptions.
**Multi item service category**

Being one of the main messages across this report, it is emphasized that a distinction between the *item* service level and the assortment- or *aggregate* service level (category 4) is in order. Customers ordering items, experience service on the whole range of items that is requested. Having a 99% service level on item A and 70% on item B will work out great if a customer orders mainly item A but if customers are ordering item B only, they will probably experience insufficient service. Note that averaging the service levels only represents customer service when item A and B are demanded equally many times. This is why weighting the service levels based on demand better represents the aggregate customer service.

An intuitive and popular *ex-post* service measure among practitioners to evaluate assortment performance, through analyzing historical information, is the total amount of demand satisfied from stock as a fraction of total demand occurred in a past time interval, the *aggregate fill rate*:

\[
\beta = \frac{N \sum_{i=1}^{N} \text{demand for item } i \text{ satisfied from stock}}{\sum_{i=1}^{N} \text{total demand for item } i} \quad (4.6)
\]

More generally, when defining the total demand for item \(i\) as \(W_i\), the (assortment- or equivalently) aggregate service level \(\beta\) (without index) for this measure can be written in the following simple form:

\[
\beta = \frac{1}{\sum_{i=1}^{N} W_i} \sum_{i=1}^{N} W_i \beta_i \quad (4.7)
\]

Where \(W_i / \sum_{i=1}^{N} W_i\) are the weights, in this case defined as the demand for item \(i\), and \(\beta_i\) the fill rate for an individual item (item- or item-level fill rate). This means that if the weights \(W_i\) are known, the aggregate fill rate, like the item-level fill rate can be computed *ex-ante*. It is expected that most of the common item-level service measures can be represented in aggregate form by weighting the individual measures on a common factor.

Strikingly, many inventory holding entities report aggregate measures *ex-post*, but neglect inclusion of this aggregate dimension in their decision making *ex-ante*. *Ex-ante* expressions for *item-level* performance are widely treated in inventory literature and accepted in practice. As demonstrated in this subsection, the step towards converting them into (ex-ante) aggregate service measures is a simple one, which can be of great value in decision making.

The next subsections are aimed atformulating *ex-ante* expressions for the aggregate service level to be used for determining the inventory parameters in a way that achieves the (ex-post) performance target. Initially, these calculations are all grounded in traditional single item inventory theory, however, it will be demonstrated that the step towards the aggregate brings additional considerations and opens up opportunities for allocating safety stock in a more efficient way.
4.1.2 Assumptions on the inventory process

Predicting the service levels requires modelling of the inventory process, which in turn requires some simplifications of reality in the form of assumptions. Appendix IX discusses the assumptions made for the safety stock allocation model in general, as well as the assumptions for the model realization for PACCAR Parts. This section highlights some assumptions that are determining which expressions are to be chosen for \( \beta(s_i) \) and \( \beta(S) \) for the situation of PACCAR Parts. Each of the following headings lists one of the assumptions that commonly differ among fill rate functions suggested in literature, in order: the demand distribution used, estimation procedure for the demand process, undershoot of the reorder level, and stochastic lead times.

Ordering mechanisms

It is required to model the process of inventory depletion and replenishment in order to find out what the expected service level will be, as a function of the input parameters. Since safety stock calculations depend on the order policy used, some understanding on ordering policies may be required. An overview of the most common ordering mechanisms can be found in Appendix V.

An evaluation of the current programming logic of PACCAR Parts, performed by the author in light of this study, made clear that the ordering mechanism at PACCAR Parts is of the \((R,s,nQ)\) type, but that the probability of the undershoot exceeding the replenishment quantity negligible. Although methods that explicitly account for the replenishment of multiple \(Q\)’s at once (instead of one at a time) exist (see Donselaar and Broekmeulen, 2010) these require more extensive computations and programming and it was decided not to include this scenario in computations as they would offer only a minimal accuracy increase. Hence, subsequent sections model the ordering policy at PACCAR Parts as an \((R,s,Q)\) ordering mechanism.

It is not the goal of this research to develop new expressions for single item performance metrics under certain ordering policies or assumptions, nor to discuss all type of measures under all kinds of policies and assumptions, let alone provide normative statement about superior ordering policies in different situations. As mentioned in the earlier chapters, many companies have a calculation method in place that they value (simple or sophisticated) to link the parameters they set to an expected service level. The development of these expressions is widely researched in existing single item literature (e.g. Zipkin 2000, Silver 1998, Tijms 1994, De Kok 1991).

For many ordering mechanisms it can be difficult to find the reorder point that matches a prespecified target fill rate, especially for the fill rate service measure. However the good news is that the optimization procedure suggested in section 4.5 does not require these kind of reverse fill rate calculations, but only the “standard” expression for the service level as a function of the reorder point. This way the “barrier” of working complicated inverse loss functions (Teunter et al 2010) is avoided.

The safety stock allocation model is compatible with any ordering policy, provided that an expression for the service level is available. (Note that practically every ordering policy treated in literature comes with an expression for the service level under this policy).

The first research subquestion: “Which ordering mechanisms are suitable for managing fixed order quantity periodic review inventory situations subject to stochastic demand?” Is easily

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11 The undershoot is the amount by which the inventory position is below the reorder point at the moment of ordering, see also the undershoot heading in further in this section.
answered by simple evaluation of existing inventory management theory on this subject and points us to the (R,s,Q) mechanism. Hence the ordering mechanism at PACCAR Parts is modeled assuming an (R,s,Q) ordering mechanism in subsequent sections.

**Demand distribution**

The most widely applied distribution for inventory control is the Normal distribution (Teunter 2010) and through the years some handy methods are developed for its computation (such as a rational approximations and tables (such as appendix IV), for the inverse loss function). Using the normal distribution is however often dissuaded from in inventory management literature because of its positive probability on negative demands in case of a large coefficients of variation

Burgin (1977) finds that in the field of inventory control, demand frequency distributions have the following general characteristics:

1. They exist only for non-negative values of demand
2. As the mean demand of items increases, the observed demand distributions change from monotonic increasing to unimodel skewed to the right and finally to normal type distribution.

Although other distributions exist that are nonnegative such as the negative exponential distribution, the gamma distribution is nonnegative and accommodates the just mentioned range through its modulus parameter. Burgin (1977) shows that the gamma distribution includes the negative exponential as a particular case and tends to normality as the modulus tends to infinity. Many authors suggest the nonnegative and skewed gamma distribution as a suitable alternative to the normal distribution (Aviv and Federgruen 2001; Bagchi et al. 1986; Das 1976). It will be demonstrated in subsequent section, that the gamma distribution loss function can be computed in closed form.

As it is an item-level decision belonging to the domain of single item inventory, extensive research towards modeling the demand process in a way that best represents reality is considered beyond our scope. For the case of PACCAR Parts, it was decided in consultation that the gamma distribution is a suitable candidate for modeling the demand process due its properties mentioned in this paragraph.

**Registration of demand**

Expressions for the service level on hand inventory require the first two moments of the demand process for computation. The way these can be obtained are now shortly discussed. The following is based on Janssen et al (1996). Three methods of treating demand information are evaluated:

- Aggregate information method (AIM)
- Discrete time method (DTM)
- Compound renewal method (CRM)

The choice for one of these methods depends on the way information about the demand during leadtime can be obtained and stored.

The AIM starts monitoring demand when a replenishment order is placed, and sums all arriving demands, up until the replenishment order has arrived, to obtain a figure for the mean and variance of actual historical demand during leadtime.

The DTM assumes that the time axis is divided into disjunct time units of time (e.g. days, weeks or months) and sums all historical demands arriving during each time unit to obtain an
estimate of the mean and variance of demand per time unit which can then be converted to the demand over the replenishment period.

The CRM does not divide time axis into disjunct intervals, but describes the demand process as a (continuous) compound renewal process, a generalization of a compound poisson process (Sahin, 1983). The CRM requires historical information of the first two moments of both interarrival times of customer and the demand sizes of each customer.

Janssen et al (1996) conclude that the AIM performs very well but shifts the problem of determining the demand during leadtime to data collection by sampling the length of the leadtime and leadtime demand in an aggregate way. This method requires many occurrences of a replenishment and this information is unavailable in most practical cases. The CRM registers the demand quantities and inter-arrival times separately, allowing it to better cope with erratic demand, but underperforms in cases where inter arrival times are large compared to the review period and when very variable. Furthermore performance is degraded in the presence of short leadtimes and small order quantities. The DTM has the simplest and most often utilized method of data collection. A disadvantage that comes with this method of data collection is that it cannot account for inter arrival times and is therefore less accurate when the coefficient of variation of the inter arrival times is very different from one. A better option in this case, according to the authors might be to integrate the forecast rather than using estimates for the moments of the variables. Further research is needed here.

In short, the choice for a method is dictated by the method of data collection which for the PACCAR Parts resembles the DTM. The fill rate expressions used can however be easily substituted with that of the CRM if future (IT-) developments enable this kind of data collection and the reader is referred to Janssen for estimates on the potential accuracy increases. It should be noted that collecting the inter-arrival and order quantity information separately can improve accuracy and as an additional benefit, the use of many (more sophisticated) forecasting methods typically require this data. See also chapter 4 in the literature review by Van den Berg (2011).

Undershoot of the reorder level

Most simple modeling approaches assume that at the moment of ordering, the inventory position is exactly equal to the reorder point. This is however not always the case in practice, especially when periodic- instead of continuous review is utilized (but also if customers order multiple items at once). In a realistic case, the inventory position at the moment of ordering will be \( s_t - u \), where \( u \) is a nonnegative integer. For the case of PACCAR Parts the useful approximation by Tijms (1994) for the first two moments of the stochastic random variable representing the undershoot \( U_t \) proved useful, enabling us to include the undershoot in our model.

Replenishment order quantities

Most expression for the expected service level assume that when an order is placed the size is always equal to \( Q \). For the case of PACCAR Parts this was not an issue as the number of cases with such small order quantities in relation to the review period demand is negligible. According to De Kok (1991) the relations even hold when \( Q \) is around the expected review period demand. Donselaar and Broekmeulen (2010) derives formulae for the fill rate that explicitly account for the ordering of multiple \( Q \)’s in (their case the case pack size), that can be used in cases where the replenishment quantities frequently consist of more than one \( Q \).

Stochastic leadtimes
Methods that take stochastic leadtimes into account (Silver et al, 1998; De Kok 1991) boil down to increasing the variation of demand during leadtime with that of the leadtime itself. Including the variation of leadtime demand in our expression for the service level is possible when the leadtime distribution is available and will always result in larger amounts of safety stock needed for the same service (since an additional form of uncertainty is introduced that before was assumed absent). As an advantage, including stochastic leadtimes will provide an additional characteristic for the allocation process, and its inclusion is especially interesting when lead time variations are very heterogeneous across the assortment. For PACCAR Parts, we had to settle using constants leadtimes as leadtime variation is not registered.

4.1.3 Mathematical expressions for $\beta_i(s_i)$ and $\beta(S)$

The problem statement given in equation (4.1)-(4.4) includes the service level functions $\beta(S)$ and $\beta_i(s_i)$, subsection 4.1.1 indicated that service is best represented using the fill rate. This section is devoted to providing operational expressions that, given the item characteristics, enable computing its value. $\beta_i(s_i)$ consists of single item service measures, the computations of $\beta(S)$ are therefore discussed first.

A function for the single item fill rate as a function of $s_i$

Notation

- $s_i$: The reorderpoint for item $i$
- $\beta_i(s_i)$: The single item fill rate as a function of $s_i$
- $Q_i$: The replenishment order quantity for item $i$
- $f_X(.)$: The probability density function for random variable $X$
- $E[X^y]$: The $y$'th moment of random variable $X$
- $G_X(.)$: Loss function for a given distribution of random variable $X$
- $D_i$: Random variable representing the demand per time unit for item $i$
- $U_i$: undershoot of the reorderpoint for item $i$
- $V_i$: demand during lead time for item $i$
- $Z_i$: depletion of stock from the moment of crossing the reorderpoint up until receipt of the replenishment order. (for item $i$)
- $R$: The review period as a multiple of the time unit

We are looking for an ex-ante expression for the fill rate in an $(R, S, Q)$ ordering policy under the assumptions given in the previous subsection. Starting from the definition of the fill rate for the more common continuous review $(s, Q)$ mechanism (4.4), subsequent equations work towards finding mathematical expressions for $\beta_i(s_i)$ for our case.

The exact definition for the item fill rate under an $s, Q$ ordering logic under continuous review can be defined as (4.8) (adapted from Zhang et al, 2001). Note that although the form remains, some modifications are introduced in what follows.

$$
fill \text{ rate} (s_i) = 1 - \frac{1}{Q_i} \left( \int_{s_i}^{\infty} (y - s_i) f_{V_i}(y) \, dy - \int_{s_i + Q_i}^{\infty} (y - s_i + Q_i) f_{V_i}(y) \, dy \right) \quad (4.8)
$$

The terms between brackets on the right hand side are loss functions, which can be conceptualized as the expected leadtime demand in excess of the reorder point $s_i$. The loss function of random variable $X$ called $G_X(.)$ is defined as:
\[ G_X(x) = \int_x^\infty (y - x)f_X(y) \, dy \]  

(4.9)

Where \( f_X(\cdot) \) is the probability density function of random variable \( X \). \( V_i \) is the random variable representing the demand during the lead time. \( D_i \) is the random variable representing the demand per time unit for item \( i \) with known first two moments. Assuming that the lead time \( L \) is a constant and integer number of periods then:

\[ E[V_i] = L_i E[D_i] \]  

(4.10)

\[ E[V_i^2] = L_i E[D_i^2] \]  

(4.11)

Suppose one would want to incorporate stochastic leadtimes and \( L \) becomes a random variable with its first two moments \( E[L_i] \) and \( E[L_i^2] \) then:

\[ E[V_i] = E[L_i] E[D_i] \]  

(4.12)

\[ E[V_i^2] = E[L_i](E[D_i^2] - E[D_i])^2 - E[L_i^2] E[D_i]^2 \]  

(4.13)

Equation (4.8) does not take the undershoot into account, it is included here using the well-known approximation (Tijms 1994) for the first two moments of the stochastic random variable representing the undershoot \( U_i \). The distinction between the \((s, Q)\) and \((R, s, Q)\) (continuous- versus periodic review) model is made only in calculation of \( U_i \) (see De Kok 1991). In the \((R, s, Q)\) model, the first two moments of \( U_i \) are based on the demand per review period \( D_i^R \) while in the \((s, Q)\) model it is based on the demand per arriving customer \( D_i^C \). The former is used and the moments can be determined by using the same demand distribution as for the demand, consistent with the discrete time method.

\[ E[D_i^R] = R E[D_i] \]  

(4.14)

\[ E[D_i^R^2] = R E[D_i]^2 \]  

(4.15)

The undershoot becomes (Tijms 1994):

\[ E[U_i] = \frac{E[D_i^R^2]}{2E[D_i^R]} \]  

(4.16)

\[ E[U_i^2] = \frac{E[D_i^R^3]}{3E[D_i^R]} \]  

(4.17)

In order to compute (4.17), a value for \( E[D_i^R^3] \) is required and it is obtained by fitting a distribution on the first two moment (note that the third moment is specific for the probability distribution chosen). In case normal distributed demand:

\[ E[D_i^R^3] = \frac{1}{3}(1 + 3c_{D_i^R}^2)E[D_i^R]^2 \]  

(4.18)

---

12 Throughout this section the relation of the moments is used: \( \text{variance}(X) = \sigma^2(X) = E[X^2] - E[X]^2 \).
Where \( cv_{D_i}^R \) the coefficient of variation of \( D_i^R \), is the ratio of its standard deviation to its mean:

\[
cv_{D_i}^R = \frac{1}{E[D_i^R]} \sqrt{E[D_i^{2R}] - E[D_i^R]^2} \tag{4.19}
\]

If one would choose to assume demand is distributed according to a gamma distribution, a more realistic assumption, then through fitting a gamma distribution we obtain:

\[
E[D_i^{3R}] = (1 + cv_{D_i}^R)(1 + 2cv_{D_i}^R)E[D_i^R]^2 \tag{4.20}
\]

As argued in subsection 4.1.2, we use (4.20) for our case. The undershoot at the moment of ordering \( U_i \) and the demand during leadtime \( V_i \) together determine the depletion of stock from the moment of crossing the reorderpoint up until receipt of the replenishment order. From arithmetic for stochastic random variables it is known that the mean and variances of \( Z_i \) can be obtained from simply summing its components \( U_i \) and \( V_i \) using their independence. Hence we can replace \( V_i \) in (4.8) with:

\[
Z_i = U_i + V_i \tag{4.21}
\]

Using the result from Tyworth et al (1996) allows the following computation of the loss function (4.9) in case of gamma distributed demand:

\[
G_{z_i}(x) = \alpha \beta \left( 1 - \Gamma_{\alpha+1, \beta}(x) \right) - x(1 - \Gamma_{\alpha, \beta}(x)) \tag{4.22}
\]

With a gamma distribution fitted on the first two moments of \( Z_i \):

\[
\beta = \frac{E[Z_i^2] - E[Z_i]^2}{E[Z_i]} \tag{4.23}
\]

\[
\alpha = \frac{E[Z_i]^2}{E[Z_i^2] - E[Z_i]^2} \tag{4.24}
\]

Where \( \Gamma_{\alpha, \beta}(\cdot) \) is the cumulative gamma distribution function (available in spreadsheet software). Although we use the former for PACCAR Parts, the loss function \( G_{z_i}(x) \) under the assumption of normal distributed demand can be computed as (for derivation see Silver and Smith 1981):

\[
G_{z_i}(x) = \sqrt{E[Z_i^2] - E[Z_i]^2} \left[ \phi \left( \frac{x - E[Z_i]}{\sqrt{E[Z_i^2] - E[Z_i]^2}} \right) - \left( \frac{x - E[Z_i]}{\sqrt{E[Z_i^2] - E[Z_i]^2}} \right) \left( 1 - \Phi \left( \frac{x - E[Z_i]}{\sqrt{E[Z_i^2] - E[Z_i]^2}} \right) \right) \right] \tag{4.25}
\]

Where \( \phi(x) \) is the probability density function and \( \Phi(x) \) the cumulative distribution function of the standard normal distribution, which are both standard functions in spreadsheet software such as Microsoft Excel.

We now have a mathematical expression for \( \beta_i(s_i) \):

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\[
\beta_i(s_i) = 1 - \frac{1}{Q_i} \left( G_{s_i}(s_i) - G_{s_i}(s_i + Q_i) \right)
\]  
(4.26)

As argued in 4.1.1 (second heading) the aggregate fill rate can be defined as:

\[
\beta(S) = \frac{1}{\sum_{i=1}^{N} W_i} \sum_{i=1}^{N} W_i \beta_i(s_i)
\]  
(4.27)

Where demand weights are used i.e.:

\[
W_i = E[D_i]
\]  
(4.28)

To obtain the expected demand-weighted aggregate fill rate.

### 4.1.4 Service goals conclusion

This section discussed several service objectives for the single item as well as aggregate service level. Besides arguing that the fill rate is most representative for the single item service level, aggregation based on a weighted average is suggested to represent the service realized by a set of items. Section 4.1.2 highlighted the main assumptions often made in inventory models. Section 4.1.3 suggested methods for explicitly computing the item- and aggregate fill rate accommodating undershoot, stochastic lead times and gamma as well as of normal demand distributions. It can be seen that obtaining a certain target aggregate fill rate can be achieved by many combinations of individual reorder points. What is left is the finding out which combination brings the least amount of inventories. Before moving towards a solution procedure, computation of \( Y_i(s_i) \) is required. The next section is aimed at quantifying this downside of holding inventories and follows several steps similar to this section.
4.2 On hand inventory: $Y_i(s_i)$

The problem statement given in equation (4.1)-(4.4) includes the function $Y_i(s_i)$, stating it represents the expected on hand inventory. This section is devoted to its computation. This section treats the downside of the tradeoff (Cohen et al, 1997) between service-level and inventory investment. Section 4.2.1 briefly summarizes the negative consequences that come with inventories. Section 4.2.2 describes which of these consequences will be considered and suggests some simple expressions for estimating the expected holding cost. Conclusions are given in 4.2.3.

4.2.1 Carrying cost and the downside of inventories

The function $Y_i(s_i)$ in (4.1)-(4.4) is multiplied by $c_i$, the yearly cost of holding 1 unit in inventory. In industry, these cost have to be estimated. Some suggestions are made in this and following subsection.

Some very common measures of financial performance are *Operating profit, current assets or current ratio (liquidity), inventory turnover and return on investment*. Excess inventory investments have a direct negative effect on all of these measures. According to Silver et al (1998), the cost of carrying items in inventory includes the opportunity cost of money invested, the expenses of running a warehouse, handling and counting expenses, cost of storage equipment, deterioration of stock, damage, theft, obsolescence insurance and taxes. Regarding financial reporting, this carrying cost is in most cases proportionally related to the item value and the most common way of inventory costing is to assume this is the case in general.

The most common convention of costing (Silver et al, 1998) is to determine the single item holding cost over a certain period as the item value multiplied by a factor $r$ representing the fractional holding cost of keeping one € of inventory for one period.

4.2.2 Mathematical expressions

Considering the previous, we consider the expected holding cost for a set of $N$ items to be given by:

$$
\text{Expected holding cost for a set of } N \text{ items} = \sum_{i=1}^{N} c_i Y_i(s_i) \quad (4.29)
$$

Where the $Y_i(s_i)$ is the expected on hand inventory for item $i$. And $c_i$ the holding cost per for item $i$. Consisting of:

$$
c_i = rv_i \quad (4.30)
$$

With $r$ the prespecified fractional charge of holding one € of inventory for a prespecified period (for example year) and $v_i$ the value of item $i$. (A common)

It should be noted that using the fractional charge is not a requirement for using the results from this report. $c_i$ can be filled in for each item individually by any considerations that seem to best represent the costs mentioned in section 4.2.1.

Janssen et al (1998) and De Kok (2010) suggest that the expected on hand inventory can be formulated as:
\[ Y_i(s_i) = \frac{H_{Z_i}(s_i + Q_i) - H_{Z_i}(s_i)}{2Q_i} \]  

(4.31)

With

\[ H_{Z_i}(x) = \int_0^x (x - y)^2 f_{Z_i}(y) \, dy \]  

(4.32)

For example, under gamma demand \( H_{Z_i}(x) \) can be computed as (De Kok, 2010)

\[ H_{Z_i}(x) = s_i^2 \Gamma_{\alpha,\beta}(x) - 2s_i E[Z_i] \Gamma_{\alpha+1,\beta}(x) + E[Z_i]^2 + (E[Z_i] - E[Z_i]^2) \Gamma_{\alpha+2,\beta}(x) \]  

(4.33)

With \( \alpha, \beta \) as in (4.23), (4.24).

Although this is the accurate way of determining the on hand stock, Janssen et al (1998) and others found that in many cases (more specifically, for the higher service levels) the expected on hand stock can be approximated by the linear function:

\[ Y_i(s_i) = \frac{Q_i}{2} + s_i - E[Z_i] \]  

(4.34)

This is also the most widely used expression in practice. (4.30) makes only one adjustment to the one based on the approximate model of Zhang et al, 2001; Teunter et al 2010, by, instead of using the pure lead-time demand \( E[V_i] \), the use of \( E[Z_i] \) includes the expected undershoot (see 4.13). Substitution of either (4.34) or (4.33) for \( Y_i(s_i) \) in (4.29) provides us with a closed form expression for the total holding cost. This is the performance indicator for the downside of holding inventory, to be used in the optimization procedure.

### 4.2.3 On hand inventory conclusion

This section provided some insights in carrying cost and the downside of holding inventories, subsection 4.2.1 suggested using a fractional charge per unit per euro per year for calculating the cost of holding items in stock, although item holding cost may be substituted by other costing methods. To calculate the expected holding cost, section 4.2.2 suggests summing expressions for the expected on hand inventory (based on established inventory theory) over all \( N \) items to obtain the assortment holding cost.

The mathematical expressions for \( Y_i(s_i) \), \( \beta_i(s_i) \) and \( \beta(S) \) are now operational. The next step is to rewrite the problem and search for a procedure to find the composition of \( S \) that realizes an assortment fill rate \( \beta(S) \) with the lowest total inventory investment. This is done in subsequent sections.
4.3 Mathematical problem formulation

Section 4.1 and 4.2 suggested mathematical expressions for \( \beta_i(s_i) \) and \( Y_i(s_i) \) in problem formulation (4.1)-(4.4) to compute the expected fill rate and on hand inventory in closed form for the case of PACCAR Parts. Substituting the expressions leads to the following minimization problem:

Minimize:

\[
\sum_{i=1}^{N} c_i \frac{H_{Z_i}(s_i + Q_i) - H_{Z_i}(s_i)}{2Q_i}
\]  

(4.35)

Subject to:

\[
\frac{1}{\sum_{i=1}^{N} D_i} \sum_{i=1}^{N} D_i \left( 1 - \frac{1}{Q_i} [G_{Z_i}(s_i) - G_{Z_i}(s_i + Q_i)] \right) \geq OSLtarget
\]

(4.36)

\[
1 - \frac{1}{Q_i} (G_{Z_i}(s_i) - G_{Z_i}(s_i + Q_i)) \geq SLmin \quad \forall \ i
\]

(4.37)

Where:

\[
G_{Z_i}(x) = \int_{x}^{\infty} (y - x)f_{Z_i}(y) \, dy
\]

(4.39)

\[
H_{Z_i}(x) = \int_{0}^{x} (x - y)^2f_{Z_i}(y) \, dy
\]

(4.40)

\( s_i \)
- The reorder point for item i

\( c_i \)
- The unit holding cost for holding item i

\( OSLtarget \)
- The target aggregate fill rate for the assortment

\( SLmin \)
- The minimal item fill rate.

\( \mathbb{Z}^+ \)
- The set of positive integer numbers \{0,1,2,…\}

\( i \)
- The item(number) index. \( i \in \{1,2 \ldots N\} \)

\( Z_i \)
- The random variable representing the demand occurring in the interval between crossing the reorderpoint and receipt of the replenishment order.

\( D_i \)
- Random variable representing the demand per time unit.

\( f_X(\cdot) \)
- The probability density function for random variable X
4.4 Solution methods

This section is concerned with finding a solution method to solve the problem provided in the previous section. No reasonable situations exist in which the service level is linearly related to the reorder point and this is also holds for our definition of $\beta_i(s_i)$. This implies the problem (4.35)-(4.40) can be categorized as a non-linear integer optimization problem (NLIP). According to Frontline solvers (developers of optimization software under which @Risk and Excel solver) integer variables make an optimization problem far more difficult to solve. Memory and solution time may rise exponentially as you add more integer variables. Even with highly sophisticated algorithms and modern supercomputers, there are models of just a few hundred integer variables that have never been solved to optimality.” (Fylstra 2011). Fortunately, our problem turns out to possesses some properties that can reduce its computational complexity.

This section starts by briefly listing methods that are often used for solving the problem (4.4.1). Concluding that these methods are either infeasible or unable to find the best combination within reasonable time, subsection 4.4.2 turns to a more detailed investigation of the relations between $\beta_i(s_i)$, $Y_i(s_i)$ and $\beta(S)$. Subsection 4.4.3 gives the main findings from these investigations. 4.4.4 then introduces the idea of marginal analysis as a solution method and argues why it leads to efficient solutions. The section concludes in 4.4.4.

4.4.1 Common solution methods

The problem given in (2.4)-(2.8) can be characterized as a nonlinear integer constrained optimization problem (NLIP). These kind of problems are known to be difficult to solve, and the following procedures are found to be frequently suggested in literature.

- Full enumeration
- Lagrangian relaxation
- Search procedures
- ABC methods

The methods are now briefly evaluated on their ability to produce feasible and efficient or possibly optimal solutions to problem (4.35)-(4.40) within reasonable computation time. As a lower level criterion, a method that is easily implementable is preferred.

**Full enumeration**

This procedure consists of evaluating all possible combinations for $S$ and their expected performance, in order to find the combination with the best expected performance. The enumeration option quickly becomes infeasible for practical use as the reorder points $s_i$ can take on many integer values on its domain. The assortment needs to consist of only a small number of items to make the scale of an enumeration infeasible for computation within reasonable time.

**Lagrange multipliers**

An optimization technique suggested for similar problems is the use of Lagrange multipliers to relax the constraints by including them in the objective function and introducing a “penalty” for its violation. Hopp, et al (1997) formulate a problem very similar as a Lagrangian. They intend to approximate the expressions for the service and on hand inventory with simpler approximations that, given suitable Lagrange multipliers, can compute the inventory parameters. Several of disadvantages of the method are indicated, one important one being one being that lagrangian relaxation is slow for large problems. Solving a lagrangian relaxation problem requires a search procedure for finding the value of the
multipliers, which on itself is a large scale optimization problem (Zhang et al, 2001). Lagrangian relaxation will not produce feasible solutions without rounding off the decision variable values. To the authors knowledge a general (Lagrange) approach for solving problem formulations like (4.35)-(4.40) to optimality is non-existent.

Search procedures
Search procedures are the collection of methods that aims to search possible combinations in a more or less intelligent way such that not all combinations have to be computed. This is done by discarding candidates with little or no potential to become the optimal solution. This collection of methods includes branch and bound methods and (meta-)heuristics such as gradient-, genetic- and evolutionary strategies. These methods are especially used for very complex functions and do not guarantee finding a global optimum. These “bulldozer” approaches (which come with significant computation times) are considered overkill for our problem as it has some favourable properties that allows for a more subtle approach: (1) The problem has only one type of decision variable ($s_i$) and (2) The service level and holding cost are both increasing with $s_i$ on the domain considered.

ABC methods
Whether motivated by the above mentioned deficiencies of alternative solutions, ease of use, or imitation behaviour, ABC methods are the most widely applied method in industry (See Cohen et al 1997). It should be noted that, although it is listed in the solution methods section, ABC is a classification method, rather than a solution method. More specifically, ABC methods are methods to classify items in (most often 3) classes, and subsequently fix the target service levels per class. Subsequently, through a “reverse” service level calculation, the item-level reorder point is sought for that approximately achieves this target (Notice the similarity with the PACCAR Part’s SLP method in section 3.1.5.). The ABC methods known to the author are summarized in appendix VI. ABC methods do not provide a procedure for determining the targets (and thus the safety stock). The targets have to be set manually. This process in itself is an optimization problem. Because of the huge popularity of ABC-like methods it would be pitiful not to compare them to a more accurate solution procedure. In chapter 5 the process for finding the target service levels per class is going to be mimicked using the enumeration solution method on (ABC-) class-level. This enables providing an indication of the cost difference resulting from using an ABC method.

Conclusions
During the study it became clear that the solution methods in this subsection are complying insufficiently with the requirements for a solution method for our problem. They are not able to provide feasible, efficient (possibly optimal) solutions within reasonable computation time. For this reason we turned to a closer investigation of the relationship between $Y_i(s_i)$ and $\beta_i(s_i)$ that is reported in the next section.
4.4.2 Item characteristics and the service-investment curve

To answer the second research subquestion: “Which variables and characteristics influence an individual item’s inventory performance?”, the expressions formulated in the previous subsections are examined. Recall that the input for the single item expressions for the case of PACCAR Parts originate from the following four characteristics.

- The first two moments of the demand process $E[D_t]$ and $E[D_t^2]$
- The replenishment order quantity $Q_i$
- The replenishment leadtime $L_t$
- The unit holding cost $c_i$

These characteristics are considered predetermined for the inventory process, we cannot decide on them. This includes the order quantities $Q_i$ because these, as argued in section 2.3.2, are predetermined. Furthermore all expression are formulated as a function of the reorder point because, to determining the amount of safety stock we may choose the reorder point. Clearly this is our decision variable.

In the section following, the moderating effects of each of the characteristics on the relation between on hand inventory and fill rate are demonstrated on item level. This is done by fixing remaining factors and examining the relation between on hand inventory $Y_i(s_i)$ and fill rate $\beta_i(s_i)$ as a function of $s_i$.

The holding cost $c_i$ are omitted as it, in the single item case, represents only a scaling factor. The base situation for comparison has a coefficient of variation $(cv_l)^{13}$ for $D_l$ of 1.5, an order quantity of 3 times the period demand and a lead-time of 4 periods$^{14}$. For obtaining insights, equations (4.31) and (4.32) in combination with (4.26)-(4.28) (including the more realistic assumptions) are used to prevent drawing conclusions from what might be the bias from an approximation. The single item service level $\beta_i(s_i)$ is plotted against the on hand inventory $Y_i(s_i)$ (as a multiple of $D_l$ to preserve identical scales across graphs) to compare which characteristics provide the “better” curve, that is, a curve that achieves a high service level with less on hand inventory.

The main purpose of the following subsections is demonstrating some of the observations that led to the decision of using the solution procedure in the next section. The graphs in this section are on item-level, the x-axis represents the fill rate, the y-axis represents the average on hand inventory.

---

$^{13}$The coefficient of variation of $D_l$ is the ratio of its standard deviation to its mean.

$$cv_l = \frac{1}{E[D_l]} \sqrt{E[D_l^2] - E[D_l]^2}$$

$^{14}$The scale of the graph remains unchanged across plots to allow comparison. Every plot includes the base case curve. $(cv_l = 1.5, Q/\bar{D} = 3, L = 4)$
**Order quantity $Q_i$**

Items with large order quantities require less safety stock for the same fill rate. The reason for this is that backorders typically occur at the end of the replenishment cycle. When there are less replenishment cycles, then opportunities for backorders to occur are more rare. (consider the extreme policy of receiving a year-supply replenishment size each 1st of january, backorders will probably occur only in December. For week-supply replenishment sizes, each end of the week there will be a probability of a backorder to occurring). This is why items with large order quantities generally need less safety stock for the same fill rate.

On the other hand, order quantities are paid for in terms of *cycle* stock (approximately half of the order quantity is physically on-hand), cycle stock is the other component also present in $Y_i(s_i)$.

![Characteristics: Replenishment order quantity $Q_i$](image)

*Figure 4.1 Base case service-inventory curve for varying values of $Q_i/D_i$.*

Figure 4.1 shows the base situation but with separate curves for different values of $Q_i$. Small order quantities have the best service-inventory curve and performance worsens with large $Q_i/D_i$ (nonlinearly). However, differences are small for practical cases and become negligible as $cv_i$ and $L_i$ increase (the safety stock occupies an increasingly dominant part of the on-hand inventory in these situations). It can be seen that the cycle stock coming with large (small) replenishment order quantities stock is somewhat compensated for by requiring less (more) safety stock.

**Replenishment leadtime $L_i$**

Safety stock functions as a buffer for the demand uncertainty during the leadtime. One would expect this uncertainty to decrease with $L_i$ as demand is expected to vary less over a short period of time than over a longer period.

In figure 4.2 the service-inventory curve is plotted for different values of $L_i$. 

![Figure 4.2](image)
It can be seen that an item with a short leadtime has the best service-inventory curve. As the next heading will show, a large moderator in this relation is the coefficient of variation. Furthermore it can be seen that increasing the leadtime in a linear manner as illustrated by the different curves in the figure first has a large impact but the effect of additional leadtime decreases. This is explained by the well-known effect (Silver et al., 1998) of the standard deviation of demand during lead-time increasing with the square root of the lead time.

**Coefficient of variation**

Safety stock functions as a buffer for the variation in demand occurring between the moment the inventory position and the replenishment order is received, so one would expect the best service inventory curve for items whose demand distribution is least variable. The often used measure of variability is $CV_i$ which can be obtained by dividing the standard deviation of $D_i$ by its mean. Figure 4.3 makes clear that the coefficient of variation has a dominant effect on the service-investment curve.
Diminishing returns
Next to the effects of the characteristics, the figures in the previous headings also demonstrate the effect of diminishing returns. An item, irrespective of its characteristics needs infinite many units of safety stock to bring the expected fill rate to 1. This has an important consequence if we consider the multi-item case.

At a certain point, the cost of increasing the expected fill rate of a single item (through safety stock additions), no matter how good its curve, will exceed the cost of increasing another item in the assortment.

For illustration suppose one would want to increase the service on an assortment consisting of just two item by increasing the safety stock. Let $\Delta_{i,t}$ denote the change in on hand inventory in for increase number $t$ for item $i$ in figure 4.4. Suppose item 1 and 2 have the same mean demand but different $L_i$, $Q_i$ or $c_{v_i}$, such that one has a better curve. Both have a reorder point that provides them with a 92% fill rate. $\Delta_{1,1}$ and $\Delta_{2,1}$ represent the change in on hand inventory required to bring each to a 93% fill rate in increase number 1. Clearly item 2 has the “best” curve and the cost of going from 92% to 93% is smallest for item 2. Now suppose this increase for item 2, since it is cheapest, is performed multiple times. At some iteration, increasing item 2 becomes more expensive, owing to diminishing returns (the “cost” is $\Delta_{2,t}>\Delta_{1,1}$) and item 1 becomes a more attractive option for the next increase even though it does not have the better curve. After some increases for item 1, item 2 again becomes attractive. The main message here is that it is impossible to identify a single or group of superior A-item(s) that is always the best candidate for a service increase, due to diminishing returns.

![Figure 4.4 An example of the evolution of $\Delta_{i,t}$ after $t$ iterations. The figure shows that due to diminishing returns, item 2 is not always the best candidate for a safety stock increase.](image-url)
4.4.3 Findings from the service-investment investigations

The previous subsections suggested that certain items, for example those with less variation, shorter leadtimes or smaller order quantities, have a better service-investment curve than others. Furthermore, depicting the relations as a curve makes clear that adding a fixed amount of safety stock has a large effect when the item is low on the service (x-) axis, but persistent addition of safety stock brings diminishing returns when moving up the curve.

Managing the assortment as a whole requires, in addition to the single item relations, considering that in the multi-item case:

1. Safety stock for item \( x \) can be less or more expensive than for item \( y \) as \( c_i \) differs across items

2. The service level for item \( x \) can contribute less or more to the aggregate fill rate than item \( y \) as \( W_i \) differs across item.

3. In an efficient solution, the amount of safety stock to be allocated to an item depends on the amount already allocated to this and other items, due to diminishing returns.

First, although in the single item case \( c_i \) was merely used for multiplication with the average inventory, the multi-item problem allows compensating low safetystock on expensive items with high safetystock on inexpensive items.

Second, when using the weighted fill rate as argued in section 4.1.1, items that have similar (single item) characteristics can be assigned different weights. This has the effect of safety stock influencing the aggregate fill rate to a different degree.

The third point suggests that besides the form of the curve (i.e. finding the “best” curve), the location on the curve is determining for whether adding safety stock is well invested (finding the best move on a curve). This constitutes an important weakness of ABC-like methods.

4.4.4 Marginal analysis

A problem very related to ours is the knapsack problem (Kellerer et al 2004) renowned in combinatorial optimization and computer science. It constitutes the problem of filling a backpack/knapsack with limited (weight-) capacity from a set of available packages that that each have different weights and value, in order to maximize the total value fitted in the knapsack. The knapsack problem is also an NLIP problem. Conceptually, the knapsack (weight-) capacity is analogous to the budget available, and the value (i.e. the measure to maximize on) similar to our aggregate fill rate. Marginal analysis has been suggested for solving the knapsack problem with varying success. The basic idea constitutes evaluating the value-to-weight ratio and repeatedly adding the item with the highest ratio to the backpack until it is full. This “greedy” approach is inefficient in cases where the supply of each item is limited and the backpack is small, because the method only looks one step ahead and disregards opportunities to “puzzle” with fitting of remaining items and the space left over.

Taking into account the findings from the service-investment investigations, the author argues, despite of its disadvantages when using it as a solution method for knapsack problems in general, marginal analysis is fit to solve the problem under consideration (4.35)-(4.40). For our problem the “packages” are as small as an increment of 1 in the reorder point.
on a specific item \( i \), and the “capacity” is generally very large in comparison to the increment size. This is why the disadvantages of the greedy algorithm are expected to be negligible. Sherbrooke (2004) suggested using marginal analysis to maximize the up-time of technical systems subject to random failures. The optimization procedure suggested in the next section relies on a very similar logic, which is surprisingly simple.

4.4.5 Solutions methods conclusions

The first subsection of 4.4 initially treated four categories of solution methods. The first three being general methods to solving non-linear integer problems, the fourth was the immensely frequent utilized ABC methods which turned out to be a classification method rather than a solution method. As these four methods complied insufficiently with the requirements for a solution method for our problem 4.4.2 turned to a more detailed investigation of the relevant problem relations identifying some important properties of the problem. Marginal analysis, although only partly suitable as a method for solving the general knapsack problem, gratefully makes use of the properties of our problem and its drawbacks seem very insignificant for the practical application of our model.
4.5 Optimization procedure

This section is concerned with the design of a method for finding the combination of decision variable values \((s_i \text{ for every } i \in N)\) that leads to the highest performance in terms of the performance measures defined in previous sections.

A note on the formulation of the optimization problem might be required to convince users of its applicability. Literature shows some quite diverse approaches for setting safety stocks. Many methods such as traditional ABC analysis attempt to separate choosing the target service levels from the actual computation of reorder points by utilizing the reverse fill rate calculations (to find the reorder point that is expected to achieve a prespecified target fill rate). This is not done here, is it shown that the reorder point is the main parameter underlying both the expected fill rate and the on hand inventory and it is chosen to evaluate these performance indicators in the straightforward manner, as a consequence.

This section follows a bottom up oriented approach, using the reorder points as the decision variables, and using the characteristics combined with the expressions developed in the 4.1 and 4.2 to estimate its effects.

Input to the optimization model are the item characteristics identified in research question 2 together with the assumptions about the inventory process. Primary output of the procedure are item-level reorder points, secondary output are the expected effects resulting, such as the (item- as well as aggregate-) fill rates and holding cost.

Before running the procedure an initial set \(S\) has to be chosen. This constitutes initializing each \(s_i\) to match the minimal required item fill rate \(SL_{\text{min}}\) using the initialization steps given in appendix VIII. As suggested in Teunter et al (2010), firms typically do not plan for orders to come in late and it is reasonable to assume on nonnegative safety stocks. As such it practical to initialize \(s_i = [E[Z_i]]\), and (if applicable) increase each \(s_i\) until a minimal item service level \(SL_{\text{min}}\) (which is optional and can be set by management) is achieved, the exact initialization procedure is described in appendix VIII.

Given an expression for the expected item fill rate \(\beta_i(s_i)\) and average on hand inventory \(Y_i(s_i)\) we can formulate the following algorithm for our problem based on marginal analysis (Sherbrooke, 2004). An conceptual explanation of the steps is provided afterwards.
Optimization

Step 1

\[ C(S) = \sum_{i=1}^{N} c_i Y_i(s_i) \]

\[ \beta(S) = \frac{\sum_{i=1}^{N} W_i \beta_i(s_i)}{\sum_{i=1}^{N} W_i} \]

\[ \Delta_i^H(s_i) = c_i(Y_i(s_i + 1) - Y_i(s_i)) \]

\[ \Delta_i^F(s_i) = \frac{W_i(\beta_i(s_i + 1) - \beta_i(s_i))}{\sum_{i=1}^{N} W_i} \]

\[ \Delta_i(s_i) = \frac{\Delta_i^F(s_i)}{\Delta_i^H(s_i)} \]

Step 2

\[ u = \arg\max_{i \in N} (\Delta_i(s_i)) \]

Step 3

If \( \beta(S) < OSL^{target} \) then:

\[ s_u = s_u + 1 \]

\[ C(S) = C(S) + \Delta_u^H(s_u) \]

\[ \beta(S) = \beta(S) + \Delta_u^F(s_u) \]

\[ \Delta_u^H(s_u) = c_u(Y_u(s_u + 1) - Y_u(s_u)) \]

\[ \Delta_u^F(s_u) = \frac{W_u(\beta_u(s_u + 1) - \beta_u(s_u))}{\sum_{i=1}^{N} W_i} \]

\[ \Delta_u(s_u) = \frac{\Delta_u^F(s_u)}{\Delta_u^H(s_u)} \]

Go to step 2

Else: END
Step 1 calculates the total holding cost \( C(S) \) and the aggregate fill rate \( \beta(S) \) that results from the initial combination of reorder points. Additionally, \( \Delta_i^F(s_i) \) is calculated for each item individually. It is to be conceptualized as the increase of the aggregate fill rate \( \beta(S) \) in case the reorderpoint \( s_i \) of item \( i \) would be increased by 1. In a similar vein, \( \Delta_i^H(s_i) \) is calculated as the increase in the total holding cost (in euro) when \( s_i \) would be increased by one. Subsequently \( \Delta_i(s_i) \) is the ratio of the former two, to be conceptualized as the amount of incremental aggregate fill rate per euro in case the reorderpoint \( s_i \) of item \( i \) would be increased by 1.

Step 2 consists of finding the maximum value of \( N \) different \( \Delta_i(s_i) \)'s, and returning its index \( i \). This means finding the item whose unit increase would bring the most incremental fill rate, per euro. The index of this “winner item” is \( u \).

In step 3 the “winner” resulting from step 2 then gets its unit increase. His reorderpoint \( s_u \) is increased by one and the new \( \Delta_u(s_u) \) is calculated for this item. \( \Delta_i(s_i) \) remains unchanged for the non-winners. The value of \( C(S) \) and \( \beta(S) \) is updated since, through the addition of one to item \( u \), the total holding cost as well as the aggregate service had a small increase.

Step 2 and 3 are then repeated in an iterative manner, until the aggregate target fill rate \( OSL_{target} \) is achieved.

### 4.5.1 Optimality

It should be noted that, depending on the assortment characteristics, this procedure in nearly every practical case will overshoot the target aggregate service level \( OSL_{target} \) by a very small amount. This is the disadvantage inherent to marginal analysis as mentioned in 4.4.2. Although strictly taken this would imply a suboptimal solution to the formal problem, the difference is negligible in most practical cases, that is, it diminishes as \( \Delta_i^F(s_i) \) decreases. Furthermore, this procedure is optimal for the resulting \( \beta(S) \). As before, this procedure is very similar when spending a finite budget \( C_{target} \) to maximize the aggregate fill rate. More specifically, one simply substitutes \( \beta(S) \geq OSL_{target} \) for \( C(S) + \Delta_u^H(s_u) \leq C_{target} \) in the first line of step 3. A similar deficit (in the form of a very small fraction of unused budget) will occur, which similarly can be regarded as negligible by most practitioners, the solution is optimal for the resulting \( C(S) \).

### 4.5.2 Aggregate service-inventory curves

One advantage of the procedure described in the previous subsection is that the steps are separable, enabling the generation of an exchange curve (instead of a single solution) adding just diminutive computation time.

The procedure suggested boils down to executing the optimization procedure described in the previous step for a low value of \( OSL_{target} \), saving \( \beta(S), C(S) \) as well as the decision variable values (the set \( S \)), increasing the \( OSL_{target} \) according to a predefined step size, and repeating this process multiple times depending on the step size.

Hence, to generate an exchange curve with \( k \) points instead of a single solution the following procedure can be followed:
Step 1:
Initialize
\[ x = 1 \]
Save: \( C(x) = C(S) \)
Save: \( \beta(x) = \beta(S) \)
Save: \( S \)
\( OSL_{initial} = \beta(S) \)
\[ x = 2 \]

Step 2:
\[ OSL_{target} = OSL_{initial} + \frac{(1 - OSL_{initial})}{k} (x - 1) \]
Perform optimization (section 4.4.3)
Save: \( C(x) = C(S) \)
Save: \( \beta(x) = \beta(S) \)
Save: \( S \)

Step 3:
if \( x < k \) then \( x = x + 1 \), go to step 2
else stop

Step 1 Stores the \( \beta(S) \) and \( C(S) \) resulting from the initialization procedure (appendix VIII) as the first datapoint.
For each subsequent datapoint, the optimization procedure (described in 4.4.3) is followed using \( OSL_{target} \) as the target aggregate fill rate. This target is increased by the stepsize at each round. The stepsize is \( (1 - OSL_{initial})/k \).
\( x \) is the integer variable counting the number of rounds. The optimization procedure runs \( k - 1 \) times. Because \( S \) is retained, the solution \( S \) functions as the starting state for the next round.
This procedure is identical to running the optimization at once with \( OSL_{target} = 1 - (1 - OSL_{initial})/k \), but stores intermediate points.

The result is a table similar to table 4.2, which can be plotted on a graph to obtain the exchange curve (similar to figure 4.4 but for now for the aggregate case). Note that at least the most recent set of reorderpoints \( S_{x-1} \) is to be retained from step 2. Saving each intermediate set of reorderpoints \( S_x \) enables quickly obtaining the item-level parameters once a point on the curve is chosen by management without rerunning the computation.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( OSL_{target} )</th>
<th>( \beta(x) )</th>
<th>( C(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>..</td>
<td>..</td>
<td>€ ..</td>
</tr>
<tr>
<td>2</td>
<td>..</td>
<td>..</td>
<td>€ ..</td>
</tr>
<tr>
<td>3</td>
<td>..</td>
<td>..</td>
<td>€ ..</td>
</tr>
<tr>
<td>..</td>
<td>..</td>
<td>..</td>
<td>€ ..</td>
</tr>
<tr>
<td>..</td>
<td>..</td>
<td>..</td>
<td>€ ..</td>
</tr>
<tr>
<td>..</td>
<td>..</td>
<td>..</td>
<td>€ ..</td>
</tr>
<tr>
<td>( k )</td>
<td>..</td>
<td>..</td>
<td>€ ..</td>
</tr>
</tbody>
</table>

Table 4.2 Output generated by the curve-generation procedure, \( C(x) \) and \( \beta(x) \) can be plotted on a graph to obtain the aggregate service investment curve.

4.5.3 Optimization conclusions
The optimization section uses the reorder point’s as decision variables, arguing the aggregate fill rate, aggregate holding cost, and individual items service levels all depend on this variable
and are given by the expressions in section 4.1 and 4.2. Common optimization procedures are evaluated. The problem under consideration can be compared to a specific case of the knapsack problem and has some particular properties that make a marginal analysis approach an attractive optimization procedure. Furthermore, because the solutions procedure is “separable”, creating a service-investment curve instead of a single solution is easy and can provide valuable insights for management decision making.

4.6 Conclusion

This concludes the model development chapter. Starting with the analysis of common definitions of performance sections 4.1 and 4.2 provide an answer to the third research question “How can performance be expressed ex-ante?” Using the results, the conceptual problem formulation is translated into a mathematical one in section 4.3. In section 4.4 the most common solution methods for this type of problem are evaluated, concluding that they did not meet the requirements for our application. A more detailed investigation of the nature of our problem lead us to the development of an optimization procedure based on marginal analysis. Subsection 4.4.2 answered the second research question “Which variables and characteristics influence an individual item’s inventory performance?” as well as providing insights in the considerations when moving to the multi-item situation. Section 4.5 elaborates on the optimization procedure and its product answers our fourth research question “How can safety stocks be allocated over an assortment, based on the identified item characteristics as to maximize performance?” Because the optimization procedure is easily “separable” a method to generate an aggregate service-inventory curve is suggested that is expected to prove valuable in decision making. The next section is concerned with evaluating the methods suggested using actual assortment data from PACCAR Parts.
5. Analysis and comparison of quantitative results

In this chapter the performance of the solution procedure is evaluated for a real life assortment. Besides providing the results from optimization, a comparison with widely used ABC-classification methods is performed. Although not really a method, coupling ABC classifications with an optimization procedure based on enumeration allows to visualize the cost savings from finding a good solution to the safety stock allocation problem. Additionally a comparison with the current safety stock realization of PACCAR Parts will provide insights on the potential savings. The first section (5.1) of this chapter describes the experimental setting from which results are obtained. The second section (5.2) compares the results obtained when applying the methods to the assortment data from PACCAR Parts. Conclusions are provided in section 5.3.

5.1 Experimental setting

This section clarifies the setup in which the different methods are tested. The first subsection discusses the data collected. Because the gains from using the assortment approach are expected to depend on the size of the assortment $N$. To evaluate applicability for companies having a smaller assortment, the dataset from PACCAR Parts is subdivided in smaller parts to evaluate the effects. The first subsection describes the datasets. 5.1.2 defines the setting for the optimization procedure. The way in which the ABC procedures are used is explained in section 5.1.3. 5.1.4. explains how the data for the current method of PACCAR Parts is obtained.

5.1.1 Datasets

PACCAR Parts, provided assortment information for testing the methods. Descriptives of the assortment considered are given in table 5.4. The assortment of PACCAR Parts consists of the spare parts that are bought from a supplier, are to be delivered from stock and of which expected demand is positive. Hence items that are remanufactured (repairables) are excluded. To test the situation of a smaller assortment, the assortment is also divided in smaller portions. This is done to provide an indication of what the safety stock allocation method can do for companies (that have characteristics similar to PACCAR Parts) with a smaller assortment. Secondly, it can indicate what happens when implementation is done at planner level. That is, when a planner at the stock control department (responsible for his fraction of the assortment) applies the safety stock allocation method to his own (sub) assortment.

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Datasets 1-3 are drawn from set 4 (the total dataset), using a pseudo-random generator, without replacement. Hence, datasets 1-3 are mutually exclusive. The datasets consist of \( N \) items, each having an index \( i \). Each item has its own value for its mean demand \( D_i \), the leadtime \( L_i \), the holding costs \( c_i \), the standard deviation of demand \( \sigma(D_i) \) and the replenishment order quantity \( Q_i \) (i.e. the item characteristics) implying each data sets consists of \( N \times 5 \) values. Tables 5.1 – 5.4 provide statistics regarding the set composition. The minimum, maximum, median as well as the 25\(^{th} \) and 75\(^{th} \) percentiles are given for each type of parameter. (the \( x^{th} \) percentile is the value below which \( x\% \) of the observation are found).

### Dataset 1

<table>
<thead>
<tr>
<th>N=50</th>
<th>( D_i )</th>
<th>( L_i )</th>
<th>( c_i )</th>
<th>( \sigma(D_i) )</th>
<th>( Q_i )</th>
</tr>
</thead>
<tbody>
<tr>
<td>minimum</td>
<td>0,1</td>
<td>2</td>
<td>€ 0,0</td>
<td>0,09</td>
<td>1</td>
</tr>
<tr>
<td>25(^{th} )ile</td>
<td>0,4</td>
<td>5</td>
<td>€ 0,1</td>
<td>0,55</td>
<td>10</td>
</tr>
<tr>
<td>median</td>
<td>1,7</td>
<td>7</td>
<td>€ 0,7</td>
<td>1,53</td>
<td>30</td>
</tr>
<tr>
<td>75(^{th} )ile</td>
<td>10,2</td>
<td>7</td>
<td>€ 2,5</td>
<td>6,35</td>
<td>100</td>
</tr>
<tr>
<td>maximum</td>
<td>367,3</td>
<td>18</td>
<td>€ 29,6</td>
<td>101,79</td>
<td>10000</td>
</tr>
</tbody>
</table>

### Dataset 2

<table>
<thead>
<tr>
<th>N=500</th>
<th>( D_i )</th>
<th>( L_i )</th>
<th>( c_i )</th>
<th>( \sigma(D_i) )</th>
<th>( Q_i )</th>
</tr>
</thead>
<tbody>
<tr>
<td>minimum</td>
<td>0,0</td>
<td>2</td>
<td>€ 0,0</td>
<td>0,03</td>
<td>1</td>
</tr>
<tr>
<td>25(^{th} )ile</td>
<td>0,4</td>
<td>4</td>
<td>€ 0,3</td>
<td>0,47</td>
<td>6</td>
</tr>
<tr>
<td>median</td>
<td>1,0</td>
<td>7</td>
<td>€ 1,1</td>
<td>1,28</td>
<td>20</td>
</tr>
<tr>
<td>75(^{th} )ile</td>
<td>3,8</td>
<td>7</td>
<td>€ 2,7</td>
<td>2,82</td>
<td>53</td>
</tr>
<tr>
<td>maximum</td>
<td>2692,2</td>
<td>25</td>
<td>€ 223,0</td>
<td>1432,40</td>
<td>15000</td>
</tr>
</tbody>
</table>

### Dataset 3

<table>
<thead>
<tr>
<th>N=5000</th>
<th>( D_i )</th>
<th>( L_i )</th>
<th>( c_i )</th>
<th>( \sigma(D_i) )</th>
<th>( Q_i )</th>
</tr>
</thead>
<tbody>
<tr>
<td>minimum</td>
<td>0,0</td>
<td>2</td>
<td>€ 0,0</td>
<td>0,03</td>
<td>1</td>
</tr>
<tr>
<td>25(^{th} )ile</td>
<td>0,4</td>
<td>4</td>
<td>€ 0,3</td>
<td>0,47</td>
<td>6</td>
</tr>
<tr>
<td>median</td>
<td>1,2</td>
<td>7</td>
<td>€ 1,1</td>
<td>1,28</td>
<td>20</td>
</tr>
<tr>
<td>75(^{th} )ile</td>
<td>4,7</td>
<td>8</td>
<td>€ 4,4</td>
<td>3,89</td>
<td>70</td>
</tr>
<tr>
<td>maximum</td>
<td>11797,1</td>
<td>31</td>
<td>€ 194,8</td>
<td>5105,40</td>
<td>50000</td>
</tr>
</tbody>
</table>

### Dataset 4

<table>
<thead>
<tr>
<th>N=20303</th>
<th>( D_i )</th>
<th>( L_i )</th>
<th>( c_i )</th>
<th>( \sigma(D_i) )</th>
<th>( Q_i )</th>
</tr>
</thead>
<tbody>
<tr>
<td>minimum</td>
<td>0,0</td>
<td>2</td>
<td>€ 0,0</td>
<td>0,03</td>
<td>1</td>
</tr>
<tr>
<td>25(^{th} )ile</td>
<td>0,4</td>
<td>4</td>
<td>€ 0,3</td>
<td>0,47</td>
<td>5</td>
</tr>
<tr>
<td>median</td>
<td>1,2</td>
<td>7</td>
<td>€ 1,1</td>
<td>1,35</td>
<td>20</td>
</tr>
<tr>
<td>75(^{th} )ile</td>
<td>5,1</td>
<td>9</td>
<td>€ 4,3</td>
<td>4,04</td>
<td>80</td>
</tr>
<tr>
<td>maximum</td>
<td>37680,3</td>
<td>31</td>
<td>€ 699,7</td>
<td>8648,78</td>
<td>105000</td>
</tr>
</tbody>
</table>

**Tables 5.1-5.4,** Descriptive statistics of the four datasets.

In these datasets the standard deviation (\( \sigma \)) was estimated by the Mean Average Deviation (MAD) using the relation \( \sigma = \sqrt{\pi/2 \text{MAD}} \) which is shown by Brown (1959) to be an unbiased estimator of the standard deviation for a normal distribution. Jacobs and Wagner (1989) evaluate the “cost” of substituting statistical estimates for the demand parameters. The (wide) ranges over which the item characteristics vary, demonstrate the diverseness of the assortment. Notice how the item cost of the most expensive item is least 600 to 14.000 times that of the cheapest item. Similar effects occur for the other characteristics.

### 5.1.2 Optimization procedure

The optimization procedure as described in section 4.5 is used to obtain solutions for the datasets, \( \beta(S) \) is given by (4.26)-(4.28) and \( Y_i(s_i) \) by (4.31)-(4.33). Using the curve generating procedure from 4.4.4.the service-investment curves are generated.

### 5.1.3 ABC procedures

None of the ABC classification methods include a concrete method to set target service levels in the identified classes. The solution space is reduced by fixing the target service level per class (all items in the same class get an identical target service level). Reducing the number of choices is intuitive, but logically, decisions still have to be made. Computing optimal values involves solving a complex optimization problem. For choosing the three target service levels, (for class A,B and C) manual procedures are suggested in which practitioners try several
combinations through trial and error. It is then reasonable to assume target service levels are
chosen as combinations of integer multiples of 1% (i.e. \{0.8 , 0.81 , 0.82 ... 0.99\}). An
exhaustive enumeration is performed enumerating all combinations. The number of
combinations is \(20^3\). Note that this is a conservative comparison (in favor of criterion
methods) since evaluating 8000 combinations in a structured manner can be considered far
beyond manual. The criterion procedures use simplifications for the \(\beta_i(s_i)\) and \(Y_i(s_i)\)
expressions to allow for the translation of a target service level into a reorderpoint (i.e. the
reverse fill rate calculations). To enable comparison, the performance resulting from the
criterion methods is calculated using the exact expressions (4.26)-(4.28) and (4.31)-(4.33).
The resulting solutions are reported in the next section.

5.1.4 PACCAR Parts procedure
As became clear in section 3.1.5, the procedure currently in use resembles a two way (6-class)
matrix based on demand volume and unit cost. Additionally, a group of items had their safety
stock chosen manually as a multiple of the week demand.
The two-way matrix method is expected to outperform the “ABC”- criterion methods when
the target values are chosen wisely. However, many more combinations are possible and,
even more than in the 3-class versions, the best combination is hard to determine (through
automated enumeration, let alone manually). To summarize, its best combination has better
performance than the other criterion methods but is harder to find. Furthermore, the
disadvantages of using ABC methods (section 3.2) remain. To evaluate the current situation,
the current of set reorder points (the decision variable) was extracted at the same time as the
assortment data.

5.2 Results
Running the computations using the setups described in the previous section provides us with
solutions of which performance can be compared.
5.2.1 reports the results from the optimization procedure. The optimization procedure,
combined with the curve generating method, produces data points that are connected in order
to obtain a the service investment curve that allows graphical comparison.
In subsection 5.2.2 the ABC procedures are evaluated through enumeration, their datapoints
(solutions) are plotted as clouds of dots. Plotting the curves and data points generated with
the different methods provides a concise overview of their performance. The experiment
conclusions are given in 5.2.4.

5.2.1 Optimization procedure
As shown in figure 5.2, the optimization procedure can require quite some iterations
depending on the size and composition of the assortment. However these iterations are very
simple and can be performed fast by a simple desktop pc. More specifically, each iteration requires:
- Two evaluations of the \(Y_i(.)\) function
- Two evaluations of the \(\beta_i(.)\) function
- Two multiplications by a constant; \(c_u\) and \(W_u/\sum_{i=1}^{N} W_i\) (evaluating the sum is a one-
time operation).
- Three additions of a constant (addition of 1 to the reorderpoint, and addition of \(\Delta^H_u\) and
\(\Delta^L_u\) to \(C(S)\) and \(\beta(S)\) respectively).
- One operation for finding the index of a maximum value in an array of \(N\) items.
In practice these operations can be performed in a very small fraction of a second, where the duration of only the last operation depends on the assortment size \( N \).

Figure 5.2 shows an example of a service-investment curve generated by the optimization procedure. Service investment curves are a comprehensive way of illustrating inventory performance and the tradeoff between inventory (cost) and aggregate service. Clearly, the lower this curve, the better (in efficient solutions service comes cheap). For upcoming figures, dataset 3 was considered the base case for comparison as it sufficiently illustrates the effects of a large assortment. Ironically, for much larger \( N \), not the optimization-, but the ABC-procedures require tedious computation times to evaluate. To illustrate the savings for PACCAR Parts, the results from optimization for dataset 4 is reported later in this section.

All upcoming figures have the expected aggregate fill rate \( \beta(S) \) (4.36) on the x-axis and the expected total holding cost (4.35) on the y-axis.

![Optimization solution dataset 3 (N=5000)](image)

\[ \text{Figure 5.1 Inventory versus service level for the solutions obtained using the optimization procedure. The x-axis represents the expected aggregate fill rate, the y-axis the expected total holding cost.} \]

It can be seen that the curve in figure 5.2 is much flatter than the single item curve examples from section 4.3. This stems from the ability to allocate safety stock over items (identified in section 4.4.2.). It can be seen that PACCAR Parts current method provides high service and pays for it with large inventory investments. The current method at PACCAR Parts, achieves a \( \beta(S) \) of 0.9848 with € 592.057 for this sub-assortment. Approximately the same point (a \( \beta(S) \) of 0.985) on the efficient curve is achieved with € 426.839 indicating a saving of approximately 28% over this set of 5000 items.

The solution presented here provides an opportunity to obtain the same service with less holding cost, to moderately increase the aggregate fill rate with the same investment, or a combination in between. Conceptually an aggregate fill rate increase of less than 2% does not seem much, but due to diminishing returns, those fill rates close to 1 are far more expensive. Would the current aggregate fill rate be around 0.85, fill rate increases of around 10% would come “almost for free”.

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Running the optimization procedure for dataset 3 provided good results. Figure 5.3 demonstrates the results for the total assortment (dataset 4).

![Optimization solution data set 4 (N=20303)](image)

**Figure 5.2 Inventory versus service level for the solutions obtained using the optimization procedure on data set 4.**

The current method at PACCAR Parts, achieves a $\beta(S)$ of 0.9852 with € 2.621.463. Approximately the same point (a $\beta(S)$ of 0.985) on the efficient curve is achieved with € 1.795.689 indicating a saving of approximately 30% over this set of 20303 items.

### 5.2.2 Criterion procedures

Using the search procedure suggested in section 5.1.3, the criterion procedures are evaluated. The search procedure does not provide one curve, but rather a cloud of data points, with the points closest to the x-axis being the most efficient. Because of the numerousness of data points, only data points originating from target service levels in decreasing order are plotted. That is, only the combinations that have a target service level for class A that is larger or equal to the target service level of the B-class, and similar for B and C. This reduces the number of data points for each criterion to 1540.

Figure 5.3 shows, for dataset 3, the inventory versus service level for the solutions generated by the ABC procedures compared to the efficient curve and current method at PACCAR Parts. The following methods are compared:

- The optimization procedure (section 4.5)
- PACCAR Parts current method
- ABC 1: the ABC method using the criterion $D_i$
- ABC 2: the ABC method using the criterion $D_i v_i$
- ABC 3: the ABC method using the criterion $D_i/(l_i c_i^2)$
- ABC 4: the ABC method using the criterion $D_i/(c_i Q_i)$
As expected, the demand value criterion leads to the worst performance as it, unlike the other methods, ranks expensive items higher. Second worst criterion is the demand volume criterion, as it lacks any cost information in its ranking. Criterion 3 and 4 are roughly similar, which one of these two is the best option depends on the preferred service level. The curve that would result from a pure item approach can be deduced from this figure, as it is represented by the case where the target service level for all three classes is identical. Connecting the locations where all 4 criterions have a data point (these points are located on the right end of the criterion 2 strokes which are the most horizontal ones\textsuperscript{15}) would generate the item-approach curve.

\textsuperscript{15} This is the most expensive point on each thickmark, confirming the idea of omitting combinations of target service in decreasing order.
As can be seen in figure 5.4, in case of a small assortment \((N = 50)\) certain combinations deviate little from the optimal solution at some service levels, especially those using ABC criterion 3 and 4. For the cost differences to be significant, the assortment has to be sufficiently large. This however depends on the method that is compared with. The traditional demand value and demand volume criterions can still be improved upon by at least 15% even in the case of this smaller assortment.

5.3 Experiment conclusions

This chapter tested six solutions procedures for problem (4.35)-(4.40): The optimization procedure suggested in section 4.5 is compared to the (PACCAR Parts’) current as well as 4 ABC criterion methods. For very small assortments, ABC procedures 3 and 4 are outperformed by only a few percentages making them a good option for practitioners managing a small assortment, provided that they make an effort in finding the best combination for the three targets. For the case of larger assortments differences become larger, for datasets 3 and 4 the savings over the current method were 28% and 31% respectively. The traditional ABC analysis in the form of the demand-value and demand-volume criterions (ABC 1 and ABC 2), are strongly dissuaded from as they lead to needlessly high inventory investments at every service level for all datasets. The criterion procedures have the simplest implementation in a spreadsheet but finding a good solution requires a
search procedure. Evaluating combinations for the criterion procedure requires some programming or many manual actions. In case of large assortments, evaluating the criterion procedure proved very time consuming. The optimization procedure proved to be relatively fast compared to the other procedures, and out of the six methods it is the best method for solving the allocation problem.

6. Implementation

For implementing the ideas provided in this report in operational setting, this chapter suggests some practical considerations.

6.1 Heuristic procedure

The optimization procedure in section 4.5 consists of only a few simple steps and was easily implemented in Microsoft Excel. It should however be noted that within Excel some use of “programming” in the form of a macro (Visual Basic for applications) was required to automate the iterations which would otherwise require a large amount of manual operations (the number of iterations are reported in the legenda of figure 5.1-5.3). This might pose a drawback for some practitioners. For this reason Appendix X develops and tests a heuristic procedure that can obtain near optimal solutions with considerably less iterations. Depending on the assortment size and choice of parameters, this method is more attractive when the iterations are done manually. Tests show that for dataset 3 (an assortment of 5000 items), approximately 95% of savings over the current situation can be achieved in 33 iterations, which is very feasible to do manually. It should however be noted that as the assortment grows, the number of spreadsheet cell computations grows large and computation times might become infeasible. In this case one should abandon spreadsheets and programming becomes the only feasible option. This holds also for the ABC methods, where obtaining solutions for dataset 3 was already a programming challenge.

6.2 Obtaining the relevant characteristics

The assortment information containing the characteristics \(D_i, L_i, c_i, \sigma(D_i)\) and \(Q_i\) where extracted from the database system, obtaining them is not expected to pose a problem for any company. In the safety stock allocation model, these parameters are assumed to remain unchanged over time. This is the basic assumption in current inventory theory and is included in the scope. For more on this limitation see recommendations for future research in subsequent chapter. In practice, however the parameters can change. A method to heuristically relax this assumption is running the calculations periodically (every 4 weeks), this is similar to the current method where, the reorder points, if necessary are, updated every 4 weeks. Changing parameters can pose deviation from the expected performance, this is known and experienced at PACCAR Parts because the current methods use the same assumption. The stock control department accepts that deviation from the expected performance can occur when severely changing the parameters.

6.3 Management input using sub-assortments

The ease with which a curve can be generated in addition to a single solution opens up a new opportunity for meeting practitioners’ preferences. Many real-life organizations have some “soft” factors that they somehow translate to inventory parameters. Multi-criteria classification and sorting methods tap into the subject of determining the relative importance of items through semi-subjective methods. Zopounis (2002) reviews these methods. One common option for inclusion of soft factors is using the multi-criteria or other methods in the
determination of the weight factor $W_i$. This approach includes all the risks that come with moving away from the true customer emphasis. A mismatch between expected and actual customer service can lead to systematic dissatisfaction of customers. Customer service levels should always be investigated empirically. The curve generating method can be of value for management decision making. Soft factors are often a problem because they are, by definition, hard to quantify. Making use of the curve generating method described in the section 5.4.2, a simple procedure can be followed to divide the assortment $N$ in smaller subsets (sub-assortments), that coincide with soft factor categories and perform optimization on each of the sub-assortments sequentially. A useful but preliminary method is described in appendix XI. Further research is needed in this area.

7. Conclusions
This concluding chapter summarizes the main findings and suggests several recommendations regarding improvement of performance. Furthermore, recommendations and challenges for future research are identified for PACCAR Parts, as well as for inventory management research.

7.1 General conclusions
The research assignment focused on the design a procedure for allocating safety stock to a large set of items, including a method to translate the solution into operational ordering policy parameters such that: a) The expected overall holding cost are minimized and b) The aggregate fill rate is at least equal to its target. The secondary assignment was the application of the decision model to the situation of PACCAR Parts to evaluate its potential performance improvement.

The research question was formulated as:

*How can safety stock be allocated over a large and diverse assortment in order to achieve an aggregate target service level while minimizing the expected overall inventory holding cost in an environment with a fixed order quantity, budget constrained inventory under weighted fill rate service levels?*

After evaluating PACCAR Parts’ current realization of the allocation problem in chapter 3, the fourth chapter consecutively investigated single- and multi-item performance indicators and their relations. To find the answer to the research question, the moderating effects of the item characteristics on individual as well as aggregate performance are evaluated. The basic take-away regarding the management of an assortment should be the following:

In addition to the single item relations, the multi-item case poses three additional considerations:

1. Safety stock for item $x$ can be less or more expensive than for item $y$ as item value and item holding costs differ across items

2. The service level for item $x$ can contribute less or more to the aggregate fill rate than item $y$ as an item’s share in the aggregate performance differs across item.
3. **In an efficient solution procedure, the amount of safety stock to be allocated to an item depends on the amount allocated to this and other items, due to diminishing returns of safety stock additions.**

A procedure was suggested that optimizes aggregate performance through efficient allocation of safety stock. Chapter 5 evaluated the procedure and demonstrated its superiority to traditional ABC procedures. One of the major advantages is that hardly any concessions regarding the fill rate and on-hand inventory expressions are required. The strength of the optimization procedure comes from its ability to regard every aspect that is included in the fill rate- (and holding cost) function, rather than “decoupling” the relations and using the characteristics of a “good curve” to identify an “A” category. Furthermore, diminishing returns, which occur when repeatedly increasing safety stock for a static group of items, are avoided because the effect of a safety stock increase is evaluated in an iterative manner.

The decision model was applied to the assortment of PACCAR Parts, identifying an improvement potential of 31% reduction of expected total holding cost for the assortment considered while remaining the same expected aggregate fill rate.

### 7.2 Recommendations for PACCAR Parts

For PACCAR Parts, the results section showed major performance improvements with large savings. Besides using the model to realise the improvements and gain valuable insights regarding aggregate performance, consideration of the following notes is recommended.

As is the case in the current situation, the degree to which the expected performance is realised, depends on the correctness of the parameters. It is therefore essential to pursue having system parameters that represent reality. Deviation from the assumed lead times, demand process and ordering behaviour will, like in the single item case, lead to a deviation in performance.

One definition of *aggregate* is “A total considered with reference to its constituent parts”. Although they are not the main subject of this report, it should be emphasized that single-item inventory control methods are the main ingredient of multi-item inventory control. Valuable opportunities for PACCAR Parts to improve performance and accuracy through the individual item control are:

- Obtaining demand information through the collection of customer-interarrival times (or time between orders) and customer order sizes separately to allow for investigations towards more accurate estimation of the expected fill rates and on-hand inventory.

- Collection of data to estimate the distribution of the leadtime (length) as a replacement for the constant leadtimes-assumption.

Subsequently, this more detailed information can be utilized in aggregate decision making. One can imagine that the lead time variation and customer order size (variation) moderate the effectiveness of safety stock. To include these relations, the safety stock allocation method requires no alterations, adaptation of the on hand inventory- and fill rate functions suffices.

The implementation cost of the safety stock allocation model are mainly IT-related, furthermore it is clear that savings are not obtained immediately as excess stock needs to be depleted. The implementation of the calculations can vary from complete integration with the
inventory management system to simple external calculations using Microsoft Excel. The lightest implementation consists of querying the assortment (in a similar way as for this study), running the procedure, and performing a read-in of the output.

It is shown that, finding optimal solutions for most assortments (using the optimization procedure) is a matter of minutes. Because PACCAR Parts, like most practical organisations, is expected to be more than willing to spend some desktop-pc computation time when they have the potential of saving large amounts of inventory cost, the author recommends the optimization procedure over other (ABC) methods.

7.3 Recommendations for future research

In appendix XI an attempt is made to combine the optimization procedure with methods to include management input regarding soft factors. Using the aggregate service-investment curves for decision making has the potential of providing valuable insights for managerial decisions. Specifically, the situation where items can belong to more than one subcategory is an interesting area for future research. The model proposed in this study might prove useful in multi-echelon situations as well. Future research might reveal similar methods to aid decision making when inventories are to be allocated over multiple locations.

A topic for further research is relaxing the characterization of demand as a sequence of independent identically distributed random variables. The methods suggested here and in other literature rely on this assumption to allow use of results obtained from renewal theory. In practice these results are used as a heuristic for environments in which demand processes vary significantly in time. Often, inventory parameters are coupled with a forecasting method. The effects on accuracy are to the authors knowledge insufficiently addressed in literature. Strijbosch et al (2000) investigate the case where demand parameters are estimated using a forecast, rather than assumed known. They however consider only the stationary demand patterns, arguing events which influence the demand pattern strongly, should be accounted for by the 'management-by-exception' principle. This seems to be custom in practice also. In their study, Hopp et al (1997) identified the related problem of determining whether a dynamic forecasting model coupled with a stationary inventory model is an effective way to capture evolving demand patterns. Investigation of this kind are beyond the scope of this study but the effects nevertheless poses a limitation to this and other studies and an opportunity for research.

Another useful extension of this study would be relaxing the assumption of uncorrelated demand to cope with the effects of cross-selling (see 2.3.2).

To conclude, this study demonstrates how modern inventory holding entities like PACCAR Parts can obtain major improvements on inventory performance by evaluating their approach to safety stock allocation.
8. References


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9. Appendices

Appendix I The safety stock – reorderpoint relation
This appendix provides a simple illustration of common single item inventory processes, proofs and assumptions are omitted for brevity.

Suppose customers demand 2 units/week ($D = 2$), inventory is replenished by the supplier 16 units at a time ($Q = 16$) and the supplier delivers 2 weeks after an order is placed ($L = 2$).

Placing an order when the inventory position is 4 (2 weeks x 2 units/week) should leave us with exactly 0 units in inventory each end of a replenishment cycle (the time between 2 replenishment shipments). The 4 is called the reorder point (sometimes also referred to as the reorder level).

In real-life customers do not consume products at a deterministic rate and chances are that the stock on hand runs short (or is not entirely depleted) at the end of the cycle when customer have ordered more (less) than the average. The size of this shortage (residue) depends on how variable demand for products is.

Demand variability can be buffered for by increasing the reorder point and thereby expecting a residue in case average demand. This is a buffer for the case where the demand during leadtime is larger than the average 4 units. The following figure illustrates the case where customer demand is again 2 units/week on average, but fluctuates moderately in a random fashion around its mean. To cope with the variability the reorderpoint is increased to 8. This increase is what on average will remain at the end of the cycle, but protects the downside when demand is beyond its average (which normally is expected in half of the cases.)
Note that, without the 4 unit increase of the reorderpoint, a backorder would have occurred in week 8. These 4 units are safety stock. More generally, the height of the reorderpoint minus the average leadtime demand (L x D) (that is the 2-week demand amounting to 4 units in the deterministic case) is called safety stock. Since we have no control over the demand process, this implies the amount of safety stock is only realized by a choice for the reorderpoint value. This example should have illustrated the following relation:

\[
\text{reorderpoint} = \text{Expected demand during leadtime} + \text{safety stock}
\]

Furthermore, it is clear that (assuming supplier replenishment order size is a given) the amount of backorders (or equivalently the service level) as well as the average amount of on hand inventory is only determined by the value of our decision variable, the reorder point. (ordering early (high reorder point) increases the probability of a residue, decreasing the probability of a backorder and vice versa). Single item inventory theory is concerned with finding the amount of safety stock such that disservice occurs in only \(X\%\) of the cases (under all kinds of different assumptions on the demand process, service definitions and ordering logics). Clearly the more variable the demand quantities, the more safety stock is needed to achieve this performance.
Appendix II Components of an inventory control policy

Firms that manage inventory for a large assortment of items typically cannot do this manually. This is why most practical inventory managers have an inventory control policy (or equivalently stocking policy) in place that can either be simple or sophisticated but consists minimally of the following components:

1. Registration of on hand inventory and placed, outstanding and received replenishment orders.
2. Registration of (characteristics of) demand occurred in the past
3. A method to predict expected future demand using (2)
4. A logic that determines the order sizes
5. A logic that determines the moment of ordering (ordering policy) using (1), (3) and (4)

These components will always be required present, implicitly or explicitly, by heart, by paper or using an automated system. Publications that consider inventories typically take on one (or a combination) of these components. Safety stocks decisions are located in the 5th component and the remainder of this report will focus on this component, while utilizing the decisions made in the previous components.

The first component (registration of replenishment orders (1)) hardly requires any explanation and consists merely of registering replenishment orders to prevent that new replenishment orders are placed without regarding the replenishment orders that are in the pipeline and soon to be received. Clearly, in order to manage inventory, registration of the amount of items physically present in inventory (on hand inventory) is required.

The second component of an inventory control policy; registration of demand (2) can range from very detailed (registering demand sizes and inter-arrival times separately on a continuous timeline) to a high level of aggregation measuring only the average demand over large time intervals (for example month). Janssen et al (1996) provides important insights on the consequences of using different methods of demand data collection and aggregation of this information.

Predicting (the distribution of) future demand (3), often referred to in literature as predicting LTD (Lead Time Demand) is the subject of a large amount of forecasting literature. Van den Berg (2011) provides a brief overview of forecasting literature. Note that predicting (forecasting) future demand beyond the lead-time horizon does not necessarily help in deciding on the point of reordering for the current replenishment cycle. Willemain et al (2004) supports this, stating that calculating the order quantity normally requires forecasts of the average demand per period. In contrast, calculating the correct reorder point requires estimates of the entire distribution of demand over the interval, known as the lead time, between the generation of a replenishment order and its arrival in inventory. This is why forecasting methods for the purpose of inventory control focus on short range forecasting techniques. A complete investigation of the range of forecasting techniques falls beyond the scope of this research and the forecast is considered input for our model.

---

16 The conceptualization of the components is mainly based on Silver et al (1998).
The logic that determines the order sizes (4) often is either dictated by packaging or transportation quantities (for example full truckload or pallet quantities) or can be chosen as desired. In the case that the order sizes can be chosen, the most classical cost trade off in order size is between ordering and holding cost such as considered in the classical EOQ (Ford 1913). Furthermore there is a relation between choices for the order quantity (4) and the moment of ordering (5) when considering the effects on the service level since large order quantities last longer (longer replenishment cycles) and stockout opportunities typically occur at the end of a replenishment cycle. This divides inventory literature in situations where the order size is considered a given on the one hand, and literature that optimizes the moment of ordering and size of the order simultaneously on the other.

The logic that determines the moment at which an order is placed (5) in most cases consists of a procedure combining the information about the demand during lead time ($X_L$) and order size (Q). In general, ordering policies either:

1. Trigger a replenishment order when the inventory position (On hand inventory + amount of inventory on order – backorder – committed inventory) is smaller or equal to a certain reorder point $s$ (lower case). Or:

2. Trigger a replenishment order at the end of every pre-specified interval and order enough to raise the inventory position to or beyond a certain order-up-to-level $S$ (capital).

These two pure policies will from now be referred to as the order-point and order-quantity policies respectively. In the first, the replenishment moment varies while in the second the replenishment quantity varies. A method to determine the amount of safety stock (SS) is implicit in this fifth component of the inventory control policy. The reorderpoint $s$ in the order-point type of policy can be captured by the following relationship:

$$s = X_L + SS \tag{2.1}$$

Where the safety stock SS serves as a buffer for stochastic variations in the demand during leadtime. In a similar vein, the order up to level $S$ up to which is ordered every R periods can be capture by the following relationship:

$$S = X_{R+L} + SS \tag{2.2}$$

The order-quantity policy can also be adapted to generate an order less often than every review period by ordering up to $S$ only if the inventory position at the point of review is below a certain threshold. De Kok (1991) provides analytical expressions for this so called $R,s,S$ policy. Combinations of these to policies are also common, such as the $R,s,Q$ policy in which inventory position is reviewed every R periods and an order of $Q$ is placed when it is below $s$. In some cases (when demand during the review period exceeds $Q$) an order of $Q$ will not place the inventory position above $s$, and thus a multiple ($n$) of $Q$ is ordered.

As the safety stock (SS) is a buffer against stochastic variations in demand during the leadtime, it is often expressed as a safety factor $k$ multiplying the standard deviation of demand during leadtime:

$$SS = k \times \sigma_{X_L}$$

For cases in which demand and leadtime are deterministic and known, logically there is no need for safety stock and this reduces the situation to a deterministic lot sizing problem. The reader is referred to Silver et al (1998) chapter 5 and 6 for these situations.
SS = kσₜ

Using this, many researchers developed methods and expressions to determine the value of the safety factor \( k \) such that a certain service level is obtained, often by assuming that demand (during leadtime) follows a certain statistical distribution, the most well-known method is based on the assumption that demand follows a normal distribution. The service level is a measure of inventory performance and is most commonly measured using either the probability that a stockout occurs in a replenishment cycle (also called the cycle service level or \( P1 \)-measure) or using the fraction of demand that is met routinely (also called the fill rate or \( P2 \)-measure); that is without backorders or lost sales (Silver et al, 1998). More on customer service levels in appendix VII.

\[18\] More service measures are discussed in Silver et al 1998, in Appendix VII the most common flavors are summarized.
Appendix III Project activity flow

Multi item inventory control for a budget constrained, periodic review spare parts environment

- Literature review
  - Literature gaps
  - Potential research objectives
  - Business Challenges
  - PACCAR Parts business situation
  - Sub question 1: Which ordering mechanisms are suitable for managing fixed order quantity inventory situations?

- Problem verification
  - Define order mechanism
  - Define performance aspects
  - What limits performance?
  - Define constraints
  - Sub question 2: Which variables and characteristics influence inventory performance?

- Design
  - Compare modeling alternatives for modeling inventory processes
  - Model inventory processes
  - Performance metrics
  - Sub question 3: How can we express performance?

- Design optimization model
  - Obtain solutions for sample problems
  - Evaluate results and feasibility
  - Observe results from alternatives or heuristics for comparison

- Extract important criteria
  - Design heuristic procedure

- Test model using real life input
  - Gather empirical data
  - Test heuristic procedure using real life input
  - Compare performance
  - Test alternatives using real life input

- Implement model/heuristic

- Evaluation

- Evaluate
## Appendix IV Safety factor table

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Appendix V Ordering mechanisms

Basic ordering policies
This section will show how inventories that are subject to stochastic demand behave under each of the basic ordering policies and introduce some notation. To cope with uncertainty in demand one can choose to vary the moment of placing an replenishment order, vary the quantity of a replenishment order, or both. Next to this, inventory policies are either reviewed continuously (i.e. every time a demand occurs, the inventory position is reviewed) or periodically, in which case the inventory position is reviewed every $R$ time units. This section is mainly based on Silver (1998).

The first subsection (3.2.1) discusses a continuous review, fixed order quantity policy, the $(s, Q)$ policy (by some authors referred to as the $(Q, r)$ policy). Section (3.2.2) introduces the continuous order up to policy or $(s, S)$ policy. A periodic review version of this order up to policy is the $R, S$ to be discussed in section (2.3.3). The fourth combination, which consists of a fixed order quantity but periodic review is the $(R, s, Q)$ and will be considered in section (2.3.4). Figure V.1 provides an overview of the policies and sections.
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*Table V.1 Overview of the most common ordering mechanisms (De Kok, 2009)*

$s$ Reorderpoint  
$S$ Order-up-to level  
$Q$ (predetermined) replenishment quantity  
$R$ Review period

*Table V.1 Summary of ordering policies (De Kok 2010)*

**$(s, Q)$ Policy**

The $(s, Q)$ policy reviews inventory continuously, and places an order of size $Q$ when the inventory position drops below $s$.

**$(s, S)$ Policy**

The $(s, S)$ policy reviews inventory continuously and places an order that is exactly enough to bring the inventory position to $S$ (capital) when the inventory position gets below $s$ (lower case). The specific case where $s = S - 1$, i.e. where a replenishment order of 1 is placed after every demand occurring, is called a $(s - 1, S)$ policy or base stock policy (Johansen 2005).

**$(R, S)$ Policy**

The $(R, S)$ policy reviews inventory periodically every $R$ time units and places an order that is exactly enough to bring the inventory position to $S$. This policy places a replenishment order every review period. An adaption to this method is the $(R,s,S)$ policy which reviews inventory periodically and places a replenishment order only if the inventory level is less than $s$ (lower case). In this case a replenishment order is placed to bring the inventory position to $S$ (capital). This adaptation allows replenishment less frequent than every $R$ time units.

**$(R,s,Q)$ Policy**

The $(R,s,Q)$ policy reviews inventory periodically every $R$ time units and places an order of size $Q$ when the inventory position drops below $s$. According to Janssen(1996) both a supplier and the seller enjoy advantages when using this policy because:

- Because of periodic review, (inbound-) ordering and transportation can be coordinated, reducing cost.
- Because of fixed order quantities and moments, a supplier knows the timing of potential orders and can, owed to fixed order quantities, make more efficient production schedules.

Furthermore the $R,s,Q$ inventory model coincides with the time phased reorder point in a MRP (Material Requirements Planning) system (Janssen, 1996).

**$(s,nQ)$ and $(R,s,nQ)$ policies**
In case the order quantities are fixed such as in the \((s, Q)\) and \((R, s, Q)\) policies, some situations can occur where placing an order of size \(Q\) does not raise the inventory position above \(s\) and a multiple \(n\) of \(Q\) is required. When \(Q\) is sufficiently large compared to the average demand per review period, the probability of this occurring is negligible, and most analysis’ make this assumption. Policies that explicitly take the possibility of ordering \(n\) times \(Q\) into account are called \((s, nQ)\) and \((R, s, nQ)\) policies.

The answer to the research sub question ‘Which ordering mechanisms are suitable for managing fixed order quantity inventory situations subject to stochastic demand?’ is easily answered by evaluating basic inventory theory. Ordering mechanisms subject to stochastic demand that respect fixed order quantities are the order-point mechanisms. The \((s, Q)\) mechanism only orders amounts of \(Q\) but only works under the condition that undershoot of the reorderpoint is smaller than this quantity. Depending on whether a multiple of the fixed order quantity is allowable, the \((s, nQ)\) and \((R, s, nQ)\) can accommodate fixed order quantities.

The class of ordering mechanisms that uses an order-up-to level (instead of a reorderpoint) also fits the model proposed in this study. Substituting \(S_i\) for \(s_i\) in the problem formulation and using accompanying expressions for \(B(s)\) and \(Y(s)\) to be found in existing single item inventory theory (for example Silver; 1998)

**Ordering policy conclusion**

This section briefly summarized the basic ordering policies. Section 3.2 makes clear that the ordering policy at PACCAR Parts is of the \((R, s, nQ)\) type, but as the review period is short, and the probability of demand during a review period exceeding the replenishment quantity negligible, the decision rule for setting reorderpoints is currently approximated by that of the \((s, Q)\) policy (Silver 1998).

It is not the goal of this research to develop new expressions for single item performance metrics under certain ordering policies or assumptions, nor to discuss all type of measures under all kinds of policies and assumptions, let alone provide normative statement about superior ordering policies in different situations. As mentioned in the earlier chapters, most companies have a calculation method in place that they value (simple or sophisticated) to link the parameters they set to an expected service level. The development of these expressions is widely researched in existing literature (e.g. De Kok 1991).

For many policies it can be very difficult to find the reorder point that matches a prespecified target fill rate, especially for the fill rate service measure. However the good news is that the optimization procedure suggested in section 4.5 does not require these kind of calculations, but only the “standard” expression for the service level as a function of the reorderpoint. This way the “barrier” of working complicated inverse loss functions (Teunter 2010) is avoided. The methods are compatible with any of the previously mentioned ordering policies, provided that an expression for the service level is available. (Note that practically every ordering policy treated in literature comes with an expression for the service level under this policy).

Although from here the focus is on (re-)order-point policies, Silver (1998) suggests that the periodic review order-up-to (order quantity) or \(R, S\) situation is exactly equivalent to the \((s,Q)\) situation if one replaces \(s\) by \(S\), \(Q\) by \(DR\), and \(L\) by \(R + L\).
Appendix VI ABC procedures

The following criterions are suggested in literature, for ranking the items such that giving the top (A-class) the most safety stock (highest target service level) supposedly leads to the most service for the least inventory cost.

Criterion 1: the $D_i$ traditional volume criterion  
Criterion 2: the $D_i\nu_i$ traditional demand value criterion  
Criterion 3: the $D_i/(l_ic_i^2)$ criterion by Zhang et al (2001)  
Criterion 4: the $b_iD_i/(c_iQ_i)$ criterion by Teunter et al (2010)

The procedure for using the criterions, can be summarized follows:

1. Rank the assortment on the criterion value in descending order
2. Mark the upper 20% of items as A-items, subsequent 30% as B-items, and lower 50% as C-items
3. Use a “reverse” computation method to translate a target service level into a reorderpoint.
4. Choose a target service for each of the 3 ABC classes.
Appendix VII Customer service measures

This section describes some popular measures of inventory performance that are used in practice and is mainly based on Silver (1998). When demand uncertain, chances are that at a certain point in time the on hand inventory is insufficient to fulfill arriving customer demand and a backorder (or lost sale) occurs. Safety stocks are used to create a buffer for avoiding the undesired consequences of a stockout. In order to be able to decide on the desired amount of safety stock, one needs to define what these safety stocks are to achieve.

Silver (1998) identifies four categories of service objectives;

1. Safety stocks established through the use of a myopic approach
2. Safety stocks based on minimizing (penalty) cost
3. Safety stocks based on customer service
4. Safety stocks based on aggregate considerations.

The four categories of service objectives are clarified in the following four subsections.

Safety stocks established through the use of a myopic approach

As seen in section 2.2.1 and equation (1) and (2), the safety stock should be the product of a safety factor \( k \) multiplied by the standard deviation of demand during leadtime \( \sigma_L \). Items with highly variable demand patterns require more safety stock to obtain the same service and therefore allocating safety stocks only on the basis of forecasted average demand is not the way to go. It should be clear that not respecting some essential differences such as variability fails to prevent stockouts for more variable demand items, and certainly does not allocate safety stocks in a good way. Two myopic methods are (Silver 1998):

- Setting the safety stock equal to a number of weeks of forecasted average demand (also known as the equal time supplies method)
- Setting a fixed safety factor directly \( k \) for all items (for example determining the safety stock as 2 times the standard deviation of demand, for all items.)

The first method disregards variability of demand as whole. The second method increases with variability of the demand process but fails to take into account other important characteristics that influence performance. In fact, for myopic methods no performance target or objective is defined at all. These methods entail making decisions without defining what is to be achieved.

Safety stocks based on minimizing (penalty) cost

Safety stocks that are based on penalty cost minimization charge a certain (fractional) monetary penalty for stockouts. The following ways are common examples of manifesting penalty cost (adapted from Silver,1998)

- A fixed penalty is incurred a stockout is incurred (irrespective of magnitude or duration) (\( B1 \) costing)
- Every unit incurs a fraction of its value as a penalty when short (\( B2 \) costing)
- Every unit incurs a fraction of its value per unit time as a penalty when short (\( B3 \) costing)
- Every customer line item short incurs a fraction of its value short when not complete due to a stockout
Safety stocks based on penalty cost minimization are all about trading off penalty cost against holding cost. Low penalty cost in relation to holding cost will have low service as a consequence. The most important disadvantage of this method are the difficulties in finding a value for the penalty cost that is in line with the business strategy. In practice, inventory managers often prefer the more intuitive method of setting a target service level instead of a penalty cost.

### Safety stocks based on customer service

Above mentioned cost objectives focus on minimizing the penalties that come with shortages. A more direct method is to consider minimizing the shortages in itself. This approach is especially popular when no reliable estimates for this penalty cost are available, and as this is often the case, customer service objectives are most common in practice. The customer service objective is most commonly manifested through measuring:

- The probability of no stockout per replenishment cycle (Cycle service level or P1 measure)
- The fraction of demand to be satisfied from shelf (Fill rate or P2 measure)
- The fraction of time during which net stock is positive (Ready rate or P3)
- The average time between stockout occasions (TBS)

The latter two are self-explaining, however the cycle service level (P1) and fill rate (P2) are both often used and some explanation on their difference is in order. A replenishment cycle is the time that passes between the reception of two replenishment orders. The P1 is the probability that the on hand inventory is positive when the replenishment order arrives (ending the cycle) and can be viewed as the probability that no stockout occurs *per replenishment cycle*. This definition of service can be kind of misleading when the number of replenishment cycles is not the same across items (an item with a small order quantity may have many more stockouts *per year*, than an item with the same P1 but large order quantity because it has less cycles in a year). Furthermore the size of the stockout is necessarily evident from the probability of one occurring. The expected (ex-ante) P1 service level is easily determined from only the safety factor $g_1$.

The P2 is a service measure that accounts for the length of a cycle. And is defined as the total demand satisfied from shelf/stock divided by the total demand. Disadvantages include a slightly more complex procedure for determining the expected (ex-ante) fill rate.

### Safety stock based on aggregate considerations

The extent to which existing textbooks and also practice utilize setting safety stocks based on an aggregate service measure is fairly limited. However Silver 1998 identifies two service measures:

- Expected total stockout occasions per year (ETSOPY)
- Expected total value of shortages per year (ETVSPY)

The first (ETSOPY) can be shown to have a rather simple decision rule for safety stock allocation but aiming purely on the total number of stockout occasions does not fully represent customer service and might not be in line with the goals of inventory managers. The expected total value of shortages per year (ETVSPY) selects safety factors based on assuming B2 (same for all items) costing, and selecting the safety factor to minimize the total of carrying and shortage cost. A similar outcome is obtained by specifying the same average time between stockout occasions for every item in the group. Both approaches rely for an important part on the input consisting of either a B2 parameter or a predetermined TBS.
Being one of the main messages across this report, it is emphasized that a distinction between the item service level and the assortment- or aggregate service level (category 4) is in order. Customers ordering items, experience service on the whole range of items that is requested. Having a 99% service level on item A and 70% on item B will work out great if a customer orders mainly item A but if customers are ordering item B only, they will probably experience insufficient service. Note that averaging the service levels only represents customer service when item A and B are demanded equally many times. This is why weighting the service levels based on demand better represents the aggregate customer service.

An intuitive and popular ex-post service measure among practitioners to evaluate assortment performance, through analyzing historical information, is the total amount of backorders divided by total demand occurred in a past time interval.

\[
\beta = \frac{\sum_{i=1}^{N} \text{demand for item } i \text{ satisfied from stock}}{\sum_{i=1}^{N} \text{total demand for item } i} \quad \text{(IV.1)}
\]

This measure is the definition of what from now will be named the aggregate fill rate. PACCAR Parts, agrees with the aggregate fill rate being the most realistic service measure.

When calculating aggregate service, it was noticed that the items frequently having large customer order sizes (the type of items that are ordered in large multiples) have a large influences on aggregate performance. It is common in some studies to convert these multiples into a single unit. In the warehouse operations, order-picking statistics store the number of times an orderline can be completely filled from shelf in one action. This type of measurements somewhat corrects for the absolute value syndrome just described. This “orderline probability” could be an alternative service measure. Literature about its use in inventory management models is fairly limited. Order line probability, defined as the fraction of orderlines delivered completely from stock.

\[
\beta_{i}^{OL} = \frac{OL_{i}^{complete}}{OL_{i}^{incomplete} + OL_{i}^{complete}}
\]

Where the denominator is the total amount of order lines placed for item i and \(OL_{i}^{complete}\) the amount of orderlines that are completely filled for item i. Note that this service measure does not discriminate large from small order lines (i.e. containing many or few units per orderline), and being able to deliver 99 out of 100 units out of an order line will still be regarded as complete disservice. As with the fill rate, we should consider this service measure from a customer perspective and the aggregate order line service level here is simply the total order lines completed divided by the total orderlines placed:

\[
\beta^{OL} = \frac{\sum_{i=1}^{N} OL_{i}^{complete}}{\sum_{i=1}^{N} [OL_{i}^{incomplete} + OL_{i}^{complete}]}\]

This is how orderline service is measured in a retrospective manner only. For use in inventory control an expression for the future expected orderline probability would be required. For a more detailed analysis the reader is referred to the Raesen (2009). To make a prediction for the this service measure requires at least, for each item individually, an estimate of the average and variance of the number of units per orderline. This information is often unavailable (as was the case for PACCAR Parts), in which case practitioners can revert to an
approximation. A simple approximation to the orderline service that is comparable to the weighted fill rate is obtained by weighing the item fill rates by the amount of order lines placed for this item (Valkenburg 1999). We can define the general case in which customer service is weighted by a customer emphasis criterion $W$ as:

$$\beta^W = \frac{\sum_{i=1}^{N} W_i \beta_i}{\sum_{i=1}^{N} W_i}$$

In the case of the mentioned approximation for the orderline service, the set of weight factors $W$ is defined by:

$$W_i = E[OL_i^{\text{incomplete}} + OL_i^{\text{complete}}]$$

And

$$\beta^{OL} \approx \frac{\sum_{i=1}^{N} E[OL_i^{\text{incomplete}} + OL_i^{\text{complete}}] \beta_i^{OL}}{\sum_{i=1}^{N} E[OL_i^{\text{incomplete}} + OL_i^{\text{complete}}]} \approx \frac{\sum_{i=1}^{N} W_i \beta_i}{\sum_{i=1}^{N} W_i}$$

A reason to use the (aggregate) order line probability might be that it somewhat corrects for batch ordering behavior, but at the same time its accuracy is degraded by the subjective behavior of customers placing many consecutive small orders instead of one large one as inter arrival times are not explicit (Raessen (2009) assumes exponentially distributed inter arrival times). For a more elaborate investigation of the relations between volume fill rates, orderline service and order service (the probability that a complete order consisting of multiple items is filled), the reader is referred to Boylan (1994), noting that the safety stock allocation model can accommodate any service expression that is concave and increasing in $s$ (which is expected to hold in most practical cases).

A better way to explicitly account for demand sizes and inter arrival times that do not have a coefficient of variation around 1 is to use the CRM (Compound Renewal Method)(Janssen 1996)(see also 4.1.2). However, this particular CRM research reveals that performance of the CRM method is also degraded when the inter arrival times are very erratic and when $Q$ is relatively small compared to the expected demand size per customer. A better option in this case, according to the authors might be to integrate the forecast rather than using estimates for the moments of the variables. Further research is needed here.

This section highlighted some of the typical service goals for determining safety stocks as found in textbooks. For the purpose of our problem we are looking to allocate safety stock based on aggregate considerations, and a service measure that best represents the service experienced by the customer is sought. Some methods proposed in the previous are examples of service levels with an easy solution such as the P1. It should be noted however that the minimizing the number of stockout occasions do not regard the size of a stockout or type of item. Furthermore, solutions minimizing the expected value of total value of shortages per year would lead to large safety stocks for very expensive items, while an inventory manager would rather see the opposite. This demonstrates the importance of choosing a service goal that represents customer service. In section 4.1.3 expressions for the service level are suggested.
Appendix VIII Initialization procedure

Initialization

Step 1
\( i = 1 \)
\( s_i = \lceil E[Z_i] \rceil \quad \forall i \)

Step 2:
1. If \( \beta_i(s_i) < SL_i^{\min} \) then \( s_i = s_i + 1 \), else step 3
2. Repeat step 2.1

Step 3
1. If \( i = N \) then stop, else \( i = i + 1 \)
2. Go to step 2

Note that under additional assumptions some combinations of an ordering policy and demand distributions allow for faster methods to translate a target service measure into a reorderpoint. Arguing for avoidance of the inverse loss function and initialization not being the main point of this procedure, this simple approach suffices. For individually set minimum service levels, \( SL_i^{\min} \) is substituted by \( SL_i^{min} \).
Appendix IX Assumptions

Assumptions on the safety stock allocation model

- The demand process for each item \( i \) is a sequence of independent and identically distributed random variables with first and second moment \( E[D_i] \) and \( E[D_i^2] \) respectively.
- The expected item service level function is concave increasing for \( s_i \geq s_i^{\text{min}} \)
- The expected net stock function is convex increasing for \( s_i \geq s_i^{\text{min}} \) (linear expressions are allowed as they are both convex and concave)
- The aggregate service level can be obtained from weighting the individual service levels on the customer emphasis weight factor \( W_i \geq 0 \).
- Only nonnegative safety stocks are considered feasible.

Assumptions for the model realization suggested for PACCAR Parts

Input to the safety stock allocation model are functions that represent the single item service level and single item average inventory as a function of the reorder point. All kinds of expressions available in literature can be used provided that they comply with the assumptions required by the safety stock allocation model.

In the realization of the safety stock allocation model for PACCAR Parts the functions that are chosen to represent the service level and the average inventory pose the following assumptions.

Demand process
The demand process for each item \( i \) is a sequence of independent and identically distributed random variables with first and second moment \( E[D_i] \) and \( E[D_i^2] \) respectively.

Unfulfilled demand is backordered
In this report unsatisfied demand is assumed to be backordered. However Tijms (1984) and Silver (1998) suggest that, to approximate the fill rate in a lost sales model with a (s,S) and (s,Q) ordering mechanism respectively, the following relation holds:

\[
1 - p_2^{\text{Backorder}} = \frac{1 - p_2^{\text{Lost sales}}}{p_2^{\text{Lost sales}}}
\]

Van Donselaar (2011) uses this relation to approximate the fill rate for perishable inventory under an \((R,s,nQ)\) ordering mechanism, reporting high accuracy. Clearly an expression for the fill rate under lost sales should be accompanied by an expression for the average inventory under lost sales behaviour to remain accurate.

The order quantities are predetermined
Inventory management often involves approaches based on intuition. However, among objective approaches, one of the most popular ones appear to be using the economic order quantity (EOQ, see Ford 1913) in combination with a method determining a reorder point (Willemain 2004). There is support for fixing the order quantity in safety stock calculations (see Zheng 1992, Hopp, Spearman and Zhang 1997). Furthermore Brown (1967) and Peterson and Silver (1979) and Tijms (1986) provide empirical support for computing the reorder points and order quantities separately. Many practical companies are rather attached to their
fixed order quantities as the EOQ is robust and intuitively appealing, part of the order quantities may also be dictated by packaging (or pallet) multiples.

**Subsequent replenishment orders placed at the supplier cannot overtake**

This assumption is present in practically any research concerning the modeling of inventory processes. It is assumed that a replenishment order placed later cannot arrive earlier. In practice this is not a limiting assumption as suppliers have no reason to let orders for an item cross.

**Leadtimes are known and deterministic**

This assumption can be relaxed by assuming that the leadtime equals an integer number of time units and assuming that the leadtime is an independent random variable to accommodate stochastic leadtimes (see Ross 1983).

**Appendix X Heuristic procedure**

The optimization procedure in section 4.4 consists of only a few simple steps and was easily implemented in Microsoft Excel. Section 4.3.4. made clear that no static (ABC-) classification scheme can be proposed that approximates a solution obtained through optimization (static in the sense that an item is categorized in an category and categories are assumed predetermined in subsequent decisions).

Other studies considering the safety stock allocation problem seem to favor methods of classification for their intuitive nature, and this section suggests something similar as an alternative for the optimization procedure described in the previous section.

During this research, the number of iterations was found to be considerably reduced when:

- The stepsize was moderately increased beyond 1 for items with large \( E[Z_i^2] \).
- Not the (single) largest \( \Delta_i(s_i) \) but the top \( \vartheta \% \) of \( \Delta_i(s_i) \) are selected for an increase.

While this clearly means departing from an optimal solutions, the following procedure can be conceptualized as an ABC-like scheme in which the composition of the A and “residue” classes changes in an iterative manner. It consists of the following steps:

1. Having the 7 columns for assortment information: \( i, L_i, Q_i, E[D_i], E[D_i^2], W_i \) and \( c_i \)
2. Add the columns \( s_i, \Delta_i(s_i), \beta_i(s_i) \) and \( Y_i(s_i) \) with:

   \[
   \begin{align*}
   s_i & \quad \text{Initialized at } E[Z_i] \text{ rounded upwards to the nearest integer} \\
   \Delta_i(s_i), & \quad \text{The delta’s similar to 4.4.3 (see step 3)} \\
   \beta_i(s_i) \text{ and } Y_i(s_i) & \quad \text{The item fill rate- and on hand inventory function as given in equation (4.22) and (4.27)}
   \end{align*}
   \]

Furthermore, the heuristic settings are based on choosing the constants:

\[
\begin{align*}
\vartheta & \quad \text{The top } \vartheta \% \text{ gets a safety stock increase of } \lambda E[D_i] \\
\lambda & \quad \text{The fractional step size}
\end{align*}
\]

3. Compute \( \Delta_i(s_i) \) as:
\[ \Delta_i(s_i) = \begin{cases} 
\frac{W_i(\beta_i(s_i + \lambda E[D_i]) - \beta_i(s_i))}{c_i(Y_i(s_i + \lambda E[D_i]) - Y_i(s_i)) \sum_{i=1}^{N} W_i} & \text{for } \beta_i(s_i + \lambda E[D_i]) < S_t^{\max} \\
0 & \text{otherwise} 
\end{cases} \]

Which is similar to \( \Delta_i(s_i) \) in the optimization procedure but with 1 substituted by \( \lambda E[D_i] \).

4. Rank \( \Delta_i(s_i) \), in descending order
5. For the top \( \% \) increase, \( s_i \) by \( [\lambda E[D_i]] \)
6. Observe \( \beta(S) \) and \( C(S) \) and repeat steps 3-5 until either \( \beta(S) \) or \( C(S) \) achieves its target.

This procedure uses \( \Delta_i(s_i) \), as the “criterion” (similar to for example the demand value-criterion) And a similar method is employed, sorting the assortment descending order on \( \Delta_i(s_i) \). The procedure consists of selecting the top \( \%N/100 \) items (“A-items”) and replacing\( ^{19} \):

\[ s_i = s_i + [\lambda E[D_i]] \]  

(4.31)

At each iteration the assortment is sorted and the reorder points increased according to this logic. Saving intermediate \( \beta(S) \) and \( C(S) \) generates a service-investment curve, that approaches the one generated by the optimization procedure when \( \lambda \to 0 \) and \( \%N/100 \to 1 \).

Instead of repeated sorting, spreadsheet software includes basic functions for determining the \( k \)'th biggest value in an array (use \( k = \%N/100 \) to mark the threshold). Denoting this function as \( f(k, \{\Delta_1, \ldots, \Delta_N\}) \), define \( \gamma_i \) using a simple IF-function:

\[ \gamma_i = \begin{cases} 
1 & \text{for } \Delta_i(s_i) \geq f(k, \{\Delta_1, \ldots, \Delta_N\}) \\
0 & \text{otherwise} 
\end{cases} \]

This enables the use of (4.33) as an alternative for (4.31) to avoid repeated sorting (step 5 can be omitted).

\[ s_i = s_i + \gamma_i [\lambda E[D_i]] \]  

(4.33)

This section suggested a heuristic procedure that can generate good solutions easily. The procedure is comparable to the ABC criterions in the sense that the items are ranked on a criterion, after which this top- or A-item class gets the most safety stock. The difference with static ABC criterions is that the procedure accounts for the effect of diminishing returns, and therefore the assignment of items to categories (the composition of the categories) changes as safety stock is added. Furthermore, inverse calculations for determining the reorder point are not required (as opposed to existing ABC-like procedures). The section evaluates this

\( ^{19} \) Creating a separate column for \( s_i + [\lambda E[D_i]] \) can prove helpful, “replacing” can be done manually, by simple copy-paste, or a macro (recorded) act.
procedure for different values of $\vartheta$, $\lambda$ and $N$ and compares its “performance distance” from the optimal solutions.

**Experimental results for the heuristic procedure**

Recall that the heuristic procedure consist of choosing $\vartheta$ as the fraction of items that gets a safety stock increase (instead of only the single item having the largest $\Delta_i$) and $\lambda$ as the fractional stepsize (instead of increasing the reorder point with 1). The detailed procedure is given in section 4.5. The heuristic solutions are compared for different values of $\vartheta$ and $\lambda$:

$\begin{align*}
\vartheta & \in \{0.05, 0.2, 0.5\} \\
\lambda & \in \{0.05, 0.2, 0.5\}
\end{align*}$

The solutions obtained under the different values for the $\vartheta$ and $\lambda$ parameters are shown in figure X.1. Results are similar for all datasets and for brevity only dataset 3 is reported here.

![Solutions comparison heuristic methods dataset 3 (N=5000)](image)

**Figure 5.3 Inventory versus service level for the solutions obtained through the heuristic procedure.**

These results can seem somewhat counter-intuitive. At glance, one might expect to move further from optimal solutions for high values of the stepsize $\lambda$ (since in the optimization procedure the stepsize is as small as 1 unit) while the graph shows the opposite.
Figure 5.3 shows that when considering only $\theta$, the higher its value, the more performance deteriorates. However, this effect is somewhat corrected for because the increase in each iteration ($\lambda E[D_i]$) is proportional to $E[D_i]$. The $\lambda$ causes this extra “discrimination” within the top $\theta$ group, in favor of fast-movers. This is why figure 5.3 shows that, for large $\theta$, the worst curve is not for the combinations with high values of $\lambda$ but for low ones. Clearly, for low values of $\theta$, the performance deficit compared to the optimized solutions becomes negligible. The heuristic procedure is very attractive for practitioners that value spreadsheet implementation. Although it should be noted that the replacing operation is best implemented using an automated substitution of the $s$-values by their $s_i = s_i + \gamma_i [\lambda E[D_i]]$ replacements. In Microsoft Excel this is easily done by recording this simple act.

The heuristic procedure produces variable results, depending on the choice for $\lambda$ and $\theta$. It is able to approximate the optimal solutions very accurately for low $\theta$ with considerably less iterations than the optimization procedure. The heuristic procedure however requires more computation time per iteration (see figure 5.3 for comparison) but the process can provide some insights for gaining intuition on the dynamics (by observing the changes of the A-class composition).

**Appendix XI Management input using sub-assortments**

The ease with which a curve can be generated in addition to a single solution opens up a new opportunity for meeting practitioners’ preferences. Many real-life organizations have some “soft” factors that they somehow translate to inventory parameters. Multi-criteria classification and sorting methods tap into the subject of determining the relative importance of items through semi-subjective methods. Zopounis (2002) reviews these methods. One common option for inclusion of soft factors is using the multi-criteria or other methods in the determination of the weight factor $W_i$. This approach includes all the risks that come with moving away from the true customer emphasis.

Using the curve generating method described in the previous section, a simple procedure can be followed to divide the assortment $N$ in smaller subsets (sub-assortments), that coincide with soft factor categories and perform optimization on each of the sub-assortments sequentially.

In the first step, the assortment $N$ is divided in subsets. For example, one creates a sub-assortment $R1 \in N$ for the set of items labeled as belonging to “strategy1-category”. The remainder of $N$, the second set, is labeled “strategy2-category” called $R2$.

Second, using the optimization and curve generating procedure described above, an efficient service-investment curve can be generated for $R1$. Management now has a graphical and intuitive overview of the cost of this strategic set $R1$. Management is asked what should be the aggregate service level for this set while presented with the service-cost curve. In a way the question “What is this soft factor worth?” is answered by choosing a point on the curve. The point chosen coincides with a set of reorderpoints $S^{R1}$. The same can be done for $R2$ to obtain $S^{R2}$.
Clearly, performing optimization on the subsets separately leads to suboptimal solutions, depending on the size and composition of the sets (although deficiencies are expected to decrease with the set size).

An attractive alternative option is then to present choosing the point on the R1 curve as representing a minimum. Subsequently, $S^{R1}$ (the reorder points resulting from choosing a point on the R1 curve) is taken as the starting value for the $s_i \in N$. Next, the curve procedure is followed for all items $S^{R1} \cup S^{R2}$.

The strategy items are now guaranteed to have an aggregate service level bigger or equal to the value chosen on the curve, its $s_i$ might be increased further during optimization to achieve $OSL^{target}$.

To accommodate multiple sub-assortments/soft factors one can simply solve each sub-assortment $R_h$ for $h$ sub-assortments in succession, selecting a point on the curve in each step, followed by solving the total assortment in the final stage. The procedure is for subsets that are mutually exclusive, this assumption can possibly be relaxed by taking the maximum of multiple values for $s_i$ when $i$ is in multiple subsets. Future research is needed here.