A model for moisture-induced dimensional instability in printing paper

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KEYWORDS: Buckling, Hygroexpansion, Modelling, Out-of-plane deformation, Printing paper

SUMMARY: The dimensional stability of printing paper is strongly related to changes in moisture content. This represents a major issue in the field of digital ink-jet printing, where moisture induced reversible and irreversible deformations may compromise printing quality and runnability. This paper proposes a two-dimensional hygro-mechanical model for paper, that focuses on the prediction of moisture induced out-of-plane deformations, due to inhomogeneous moisture variations in the plane. The model is based on a discrete network of beams. The adopted constitutive model, whose input parameters are calibrated to available experimental data for homogeneous moisture cycling, allows to describe typical irreversible phenomena related to the history of the production of paper, such as the release of dried-in strains. The deformation of paper in the wet and in the dry state predicted by the model is compared with experiments, in which the paper is subjected to a moisture cycle under different types of mechanical constraint. The results of the model capture well the experimental response of paper in terms of buckling patterns and of out-of-plane displacement wavelengths and amplitudes.

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Paper sheets are composed of hydrophilic cellulose fibres that exhibit significant deformations upon moisture content variation. As fibres expand more in the transverse direction with respect to their axis than in the longitudinal direction and the fibre orientation distribution is anisotropic, the resulting hygroexpansion of paper is anisotropic.

The hygroexpansive strains due to a repeated moisture cycle in a machine-produced sheet in machine direction (MD) and in cross direction (CD) have been studied in earlier experimental works, (e.g. Uesaka et al. 1992; Nanri, Uesaka 1993; Larsson, Wågberg 2008). Hygro-expansion in CD is much larger than in MD, but seems to be (almost) fully reversible, whereas in MD a shrinkage occurs after wetting and drying the paper for the first time. This is due to the release of dried-in strain that results from drying under stress in MD during paper manufacturing. Fig 1 schematically shows the hygroexpansive behavior used in this study. The assumption has been made that in CD the hygroexpansive response is linear and reversible. In MD an irreversible shrinkage is observed in the first moisture cycle. This is followed by linear, reversible hygroexpansion in subsequent cycles.

During printing, the moisture distribution in the paper sheet is generally inhomogeneous both in the plane of the sheet and through the thickness. Together with the intrinsic material inhomogeneity due to the network structure and the release of dried-in strains induced during manufacturing, this may result in reversible and irreversible out-of-plane deformation of the printed sheet. Predicting and controlling this moisture-induced deformation of paper sheets, therefore, is a major challenge in water-based ink-jet printing.

Several works in the literature describe the anisotropic and non-linear response of paper under pure mechanical loading (see among the others Mäkelä, Östlund 2003; Beldie 2001; Xia et al. 2002; Beex et al. 2009; Wallmeier et al. 2015; Borgqvist et al. 2015). Visco-elastic models have been formulated to describe the time-dependent aspects of paper deformation (Uesaka et al. 1980; Lu, Carlsson 2001; Tjahjanto et al. 2015). The history dependent dimensional stability of paper has been studied through hygro-visco-elastic models presented (Roylance et al. 1980; Uesaka et al. 1989; Lif et al. 2005; Lif 2006). In some studies (Lipponen et al. 2008; Erkkilä et al. 2013; Erkkilä et al. 2015), out-of-plane deformations are simulated through elasto-plastic material models with moisture-dependent material properties. These descriptions, however, are not focused on modeling the release of dried-in strains upon wetting, despite the fact that these strains may play a relevant role in the moisture induced deformation.

The objective of this paper is to develop, calibrate and validate a model for the behavior of printed paper sheets that describes the moisture induced out-of-plane defor-
mation and its dependence on the release of dried-in strains. As an assumption, only deformations caused by moisture content variations in the plane are considered and time effects are neglected. Moisture gradients through the thickness could be included in further work. The developed model consists of a network of elastoplastic beams, whose properties are a function of the beam orientation (i.e. in MD or in CD), providing an anisotropic response. Moisture-dependent mechanical properties are assumed; kinematic hardening plasticity, with a moisture-dependent yield stress, allows to represent the presence of dried-in strain and its release during a moisture cycle. The assumption of linearity has been assumed in most of the relations. The results of this study are therefore valid only in a limited moisture content range that can be obtained by varying the relative humidity of the air (between 40% RH and 100% RH), corresponding to a moisture variation between 0.043 to 0.11.

The paper is organized as follows. First, a phenomenological model is presented that captures the hygro-mechanical time-independent response of paper. The developed model is next calibrated for standard office paper, using available experimental data on hygroexpansion and tensile behavior. Subsequently, experiments are performed and compared to model predictions, to evaluate the model’s performance. Finally, conclusions are presented.

Materials and Methods

Discrete network model

The proposed model is focused on capturing irreversible moisture-induced deformations of paper sheets, by describing both the moisture-dependent non-linear mechanical response of paper and its coupling with hygroexpansion. To limit the model to these essential features, it is assumed that: i) the properties of the paper sheet are homogeneous, both in-plane and through the thickness; ii) the response in the thickness direction is not of interest and is therefore not modeled; iii) temperature effects are much smaller than moisture effects and are therefore neglected; iv) time effects are not taken into account, as the time scale of the deformation process considered in this work is much larger than the typical relaxation times of paper (Uesaka et al. 1979; Lu, Carlsson 2001).

Beam network model

In this study, a discrete network model of the paper sheet is developed, as such a model allows for describing the desired anisotropic yield surface and strain hardening in a simple manner using available commercial software resources. The discrete network model describes a paper sheet as a regular network of beams in MD and CD, as illustrated in Fig 2 (left). This regular network is built by periodically repeating a square unit cell of dimension \( L \times L \), which consists of two perpendicular beams rigidly connected in the center of the cell, see Fig 2 (right). The two beams have a rectangular cross-section with beam widths \( w^{MD} \) and \( w^{CD} \) and beam thicknesses \( t^{MD} \) and \( t^{CD} \) for the beams in MD and CD, respectively. The assumption of thin beams is made; the shear strain in the beams is not taken into account.

Constitutive equations

The general constitutive equations for a single beam are presented below. They will be used to describe the response of the beams both in MD and in CD, by specifying the corresponding material properties for each of the directions.

The proposed constitutive law is based on a moisture-dependent elasto-plastic linear kinematic hardening model. This is schematically illustrated in Fig 3 (top) as the series of an elastic element, a plastic element and a hygroexpansive element. The dried-in strain is modeled as an initial plastic strain. The kinematic hardening law enables describing internal stresses at zero applied stress, as the yield surface shifts during plastic flow. An illustrative 1D sketch of how the yield surface varies as a function of the moisture content is shown in Fig 3 (bottom). In the initial state, at a given moisture content \( x_1 \), the yield surface is centered at \( \sigma = 0 \). While loading the material beyond the elastic range, the original yield surface (A–B) maintains its initial size but is shifted upwards to (A′–B′).

\[
\begin{align*}
\text{Fig 2} & \quad \text{(left): 2D periodic beam network model of a paper sheet and (right) the unit cell consisting of two perpendicular beams, from which the beam network can be constructed.}
\end{align*}
\]
The material is next unloaded, reaching a zero stress-state after which the moisture content is progressively increased. Due to the moisture dependence of the yield stress, the yield surface shrinks. At a certain point, corresponding to moisture level \( x_0 \), the zero stress-state lies on the yield surface \( (A'' \rightarrow B'') \), and the plastic strain can thus be released, eventually to zero if reaching the moisture content \( \chi_0 \), at which the yield stress vanishes \( (A'' \rightarrow B'') \).

The total axial strain \( \varepsilon \) is given by the sum of the elastic strain \( \varepsilon_e \), plastic strain \( \varepsilon_p \) and hygroexpansive strain \( \varepsilon_h \):

\[
\varepsilon = \varepsilon_e + \varepsilon_p + \varepsilon_h
\]

[1]

The constitutive equations for the elastic strain and hygroexpansive strain are given as:

\[
\varepsilon_e = \frac{\sigma}{E(\chi)} \quad \text{with} \quad i = MD, CD
\]

[2]

\[
\varepsilon_h = B^i \Delta \chi 
\]

[3]

where \( \sigma \) is the applied stress, \( E(\chi) \) is the Young's modulus of the beam, which depends on the moisture content \( \chi \), \( B^i \) is the coefficient of hygroexpansion and \( \Delta \chi = \chi - \chi^* \) is the difference between the current moisture content \( \chi \) and a certain reference moisture content \( \chi^* \). The reference moisture content \( \chi^* \) can be chosen freely and is defined as the moisture content at which the hygroexpansive strain is zero.

The plastic strain is determined by the yield criterion, which for linear kinematic hardening is:

\[
[\sigma - K^i(\chi)\varepsilon_p] \leq \sigma^*_i(\chi) \quad \text{with} \quad i = MD, CD
\]

[4]

with \( K^i(\chi) \) the moisture dependent hardening modulus, which describes the relation between stress applied and plastic strain introduced. \( \sigma^*_i(\chi) \) is the moisture-dependent yield stress. For a kinematic hardening yield criterion, the value of the yield stress determines the radius of the yield surface, whereas the product \( K^i(\chi)\varepsilon_p \) determines the location of its center. Although in the literature (Erkkilä et al. 2013) more sophisticated hardening laws are presented, it will be assumed for simplicity that a linear kinematic hardening law is sufficient for capturing the major trends in deformation caused by moisture influences.

A linear relation between yield stress and moisture content is chosen:

\[
\sigma^*_i(\chi) = \sigma^*_i(0) \left[ 1 - \frac{x}{x_0} \right] \quad \text{with} \quad i = MD, CD
\]

[5]

In this relation, \( \sigma^*_i(0) \) is the yield stress at \( \chi = 0 \), which is different in MD and CD, and \( x_0 \) is the moisture content (assumed to be equal in MD and CD) at which the yield stress becomes zero.

Experimental results also show a moisture-dependence of the Young's modulus \( E^i \) and hardening modulus \( K^i \) (Erkkilä et al. 2013). Both moduli decrease with increasing moisture content. A linear relationship is again assumed. The moisture-dependence of the Young's modulus and hardening modulus are given by:

\[
E^i(\chi) = E^i_0 \left[ 1 - \frac{x}{x^i_0} \right] \quad \text{with} \quad i = MD, CD
\]

[6]

\[
K^i(\chi) = K^i_0 \left[ 1 - \frac{x}{x^i_K} \right] \quad \text{with} \quad i = MD, CD
\]

[7]

Here, \( E^i_0 \) and \( K^i_0 \) are the Young's modulus and hardening modulus at \( x = 0 \), both different in MD and CD, and \( x^i_0 \) and \( x^i_K \) are the moisture contents at which these moduli become zero, again assumed to be equal in MD and CD.

**Model calibration**

The required beam properties can be derived from sheet-level experiments. In the following, subscripts \( xx \) and \( yy \) will indicate the experimentally determined sheet level properties while, as mentioned earlier, MD/CD superscripts refer to the beam level properties to be used as an input for the numerical model. The material used in measurements is paper from a production machine, with an average thickness of 0.1 mm and a basis weight of 80 g/m². Experimental data are available, consisting of a moisture sorption isotherm, results of homogeneous hygroexpansion experiments in MD and CD, and results of uniaxial tensile tests on standard office paper samples in MD, at 50% RH and 75% and in CD, at 50% RH, 75% RH and 85% RH. The moisture sorption isotherm, shown in Fig 11 of Appendix A, is determined at a temperature of 23°C. Relative humidity is cycled from 25%, via 30%, 40%, 55% and 70%, to 85% and backwards. Although the moisture sorption isotherm is different in absorption and desorption, this hysteresis is not of particular interest in this study. A single exponential expression, determined by fitting the experimental data, is used to represent it:

\[
\chi = 0.022776 e^{0.0159RH}
\]

[8]

To determine the mechanical properties of the paper sheet, the Young's modulus \( E \), the hardening modulus \( K \) and the yield stress \( \sigma_y \) are extracted from each tensile experiment. The Young's modulus is determined by the maximum slope of an 8th order polynomial fitted through the data points. Then the elastic strain, obtained by dividing the stress \( \sigma \) by the experimentally fitted \( E \), is subtracted from the total strain and a linear function is fitted to describe the plastic behavior. The hardening modulus \( K \) equals the slope of this line, whereas its value at zero plastic strain gives \( \sigma_y \). The strain at which yielding occurs can be calculated with the yield stress and the Young's modulus. Note that for determining the hardening modulus all data points over the entire plastic strain range have been used.

To obtain the constants for linear moisture-dependence, the values for each mechanical property in MD and CD are plotted against moisture content. There are four tensile experiments available per combination of paper direction and relative humidity. First, for every single tensile experiment the values of \( E, K \) and \( \sigma_y \) in MD and CD are determined. Next, from the four obtained values of the parameter of interest the moisture-dependence is determined. To this aim, a linear relation is fitted to both the MD and CD data points, under the constraint that they intersect the horizontal axis at the same point: \( \chi_E, \chi_K \) and \( \chi_0 \). The line through the MD data intersects the vertical axis at \( E_{xx}^0, K_{xx}^0 \) and \( \sigma_{y,xx}^0 \), the line through the CD data intersects this axis at \( E_{yy}^0, K_{yy}^0 \) and \( \sigma_{y,yy}^0 \). The fit of the model to the experimental tensile curves is shown in Fig 4. The shear modulus is found using the following empirical relationship for paper (Baum et al. 1981):

\[
\sigma_{xy} = 0.387\sqrt{E_{xx}(\chi)E_{yy}(\chi)}
\]

[9]

The experimental data of the hygroexpansion experiment and the model fit are shown in Fig 5. The
hygroexpansion coefficients $\beta^{CD}$ and $\beta^{MD}$ are found by determining the slope of a linear fit through the data points in the hygroexpansion curve. In MD, the data points for the fitting are referred to the drying stage, as after the release of dried-in strains the material response is (almost) linear reversible, and attributable to hygroexpansion. In CD, the data points in wetting or in drying can be alternatively used to determine the hygroexpansion coefficient, as no shrinkage occurs and the response is reversible.

An additional parameter for the model is the total amount of dried-in strain in MD, $\varepsilon_0$. The assumption is made that all dried-in strain is released when the moisture content of the paper reaches $\chi_0$. A straightforward way of determining $\varepsilon_0$ would be to increase the moisture content to $\chi_0$ and back to the initial value and then measure the shrinkage that occurs. An alternative strategy is used here. The moisture cycle considered in Fig 5 reaches a moisture content lower than $\chi_0$. A shrinkage ($\varepsilon_{\text{shrink}}$ in Fig 5) is measured as the difference between the strains measured at the lowest moisture content; however, $\varepsilon_{\text{shrink}}$ is smaller than the total amount of dried-in strain that would have occurred if reaching $\chi_0$. The total amount of dried-in strain $\varepsilon_0$ can be then found by adding to the measured shrinkage $\varepsilon_{\text{shrink}}$ the remaining amount of dried-in strain, referred to as $\varepsilon_{\text{dried-in}}$ in Fig 5. $\varepsilon_{\text{dried-in}}$ is equal to the plastic strain that would have occurred if wetting from the highest moisture content in the cycle up to $\chi_0$, in a zero stress state, during yielding. As $\sigma_\chi^{MD}(\chi)$, and $K^{MD}(\chi)$ are fitted, this remaining dried-in strain can be calculated from the yield criterion as

$$\varepsilon_{\text{dried-in}} = \frac{\sigma_\chi^{MD}(\chi)}{K^{MD}(\chi)}$$

Finally, a relationship between the network and sheet-level properties is established to convert the sheet properties found to beam properties (see Appendix B). The beam parameters are dependent on the unit cell size $L$. For a unit cell dimension of 1 mm x 1 mm, the beam parameters are summarized in Table 1. Note that the values for the Poisson’s ratios are indeed unrealistic in terms of lateral contractions. However, the Poisson’s ratios in this model merely serve to give the discrete model the correct torsional stiffness, equivalent to the sheet level one.

**Model validation**

The model predictions are compared to experimental results for two relevant test cases, illustrated in Fig 6. A paper sheet is mechanically constrained by one or two clamps and the free paper surface is subjected to a moisture cycle. The mechanical constraint has a similar effect as a dry region in the sheet would have and thus simulates an inhomogeneous moisture content in the plane of the paper.

The number of constraints and the orientation of the constraints with respect to the MD direction of the paper are different in each of the test cases. Different buckling patterns are therefore expected in each case.

**Experimental procedure**

The experiments are performed in an environment with controllable relative humidity. The initial “dry” reference state is chosen to be at the relative humidity of ambient air that is measured to vary between 54% and 56%, with a maximum error of 2%, by a Sensirion SHT21 humidity and temperature sensor. The same paper type, on which calibration is based, has been used.

To wet the paper sample, it is placed inside a custom-made sample chamber with high constant relative humidity. The chamber contains a saturated potassium sulfate ($K_2SO_4$) solution, resulting in a constant 97% relative humidity at a temperature of 23°C (Rockland 1960). The top of the chamber has a flat glass window,
through which the sample can be imaged and measured. After opening and closing the chamber, it takes less than 15 minutes for the relative humidity to return to its equilibrium humidity value.

The height profile of each sample is measured in both the wet state and after drying, using a Sensofar PLu 2300 optical profilometer. Each height profile measurement is performed at least three hours after changing the relative humidity, to allow the moisture content of the paper to reach an equilibrium state and to allow time effects to vanish.

### Numerical procedure

For each test case, a simulation in MSC Marc is performed. The unit cell size is chosen such that the result does not change upon decreasing the cell size. First, a uniaxial force, corresponding to a stress \(\sigma^{MD}(\chi) = K^{MD}(\chi) e_{\text{dried-in}}\), is applied in MD and released to introduce the dried-in plastic strain \(e_0 = 0.003\) in the MD beams, as established in the model calibration. In a subsequent load case, the moisture cycle and the boundary conditions of the test case are prescribed. The boundary conditions for the clamped regions are zero displacement and zero rotation. The moisture content dependence is modeled as temperature dependency.

The moisture content \(\chi\) is cycled from \(\chi_l = 0.054\) to \(\chi_e = 0.0959\) and back in simulations, the first value corresponding to the ambient relative humidity, the latter (slightly smaller than the higher value of moisture content used in the experiment) being the maximum possible relative humidity for maintaining a positive yield stress. A set of very small out-of-plane forces is applied to the sample during wetting to trigger buckling. This set consists of an equal number of positive and negative forces, having a magnitude between \(1 \times 10^4\) N and \(5 \times 10^6\) N. It has been verified that the locations and magnitudes of these forces do not affect the deformation, provided that the forces are small enough.

### Results

To evaluate the accuracy of the model predictions, the buckling patterns for the two test cases as predicted by the model and observed experimentally are compared, both in the wet and the dry state.

#### Test case 1

In test case 1, one of the edges of the sample parallel to the CD is clamped, see Fig 6 (left). This case has been investigated on two samples of size \((CD \times MD)\) 20 mm x 5 mm and 30 mm x 5 mm. Fig 7 shows the height profiles in the wet state, obtained from experiments and simulations for the sample of 30 mm x 5 mm, where deformations are largest. The free edge deflections of all experiments and simulations done for test case 1 are shown Fig 8.

The experimental results are first discussed. The edge of the 30 mm x 5 mm sample (Fig 7 and 8b) shows a clear and regular buckling pattern in the wet state, that seems to be partially maintained after drying, albeit with a less regular shape. The 20 mm x 5 mm sample (Fig 8a) only shows buckling in one of the three experiments, but after drying, the out-of-plane deformation of all samples becomes more pronounced.

The wavelengths \(\lambda\) and amplitudes \(A\) of the edge deflection of the buckled samples in the wet state are shown in Table 2. The wavelength is determined per sample as the average of the determined distances between the peaks, multiplied by two. The amplitude is the average between the amplitudes of the peaks found. The wavelength of the buckled samples in the wet state varies between 10 mm and 13 mm. The displacement amplitudes are larger in the 30 mm x 5 mm samples than in the 20 mm x 5 mm sample and decrease when getting closer to the free corners of the sample. Table 2 illustrates also the strain in the wet state that, assuming a sinusoidal shape for the buckled profile, has been estimated as \(\frac{A}{\lambda}\). For the dry state it is difficult to establish reliable wavelengths and amplitudes, as the shape is not as regular as in the wet state. However, a regular buckling pattern is maintained in the red curve in Fig 8b, bottom. The wavelength in the dry state is 9.4 mm and is comparable to that in the wet state (10.5 mm) for this experiment. The amplitude has decreased to 0.10 mm.

The simulations for these tests show a similar trend as the experiments. In the 20 mm x 5 mm and 30 mm x 5 mm samples a buckling pattern is obtained in the wet state that remains in the dried state with a 40% smaller amplitude and 20% smaller wavelength. The decreasing
Table 2 – Wavelengths and amplitudes of the free edge deflection for the buckled samples of test case 1 in wet state. Only one of the three 20x5 mm samples showed a clear buckling pattern in the wet states, therefore only the results for this sample are shown.

<table>
<thead>
<tr>
<th>Size</th>
<th>Experiment</th>
<th>Simulation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(\lambda) [mm]</td>
<td>(A) [mm]</td>
</tr>
<tr>
<td>30 x 5 mm(^2)</td>
<td>10.5</td>
<td>0.27</td>
</tr>
<tr>
<td>20 x 5 mm(^2)</td>
<td>10.7</td>
<td>0.16</td>
</tr>
</tbody>
</table>

Table 3 – Wavelengths and amplitudes of the z-displacement along the center line for the samples of test case 2 in the wet state.

<table>
<thead>
<tr>
<th>Size</th>
<th>Experiment</th>
<th>Simulation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(\lambda) [mm]</td>
<td>(A) [mm]</td>
</tr>
<tr>
<td>25 x 5 mm(^2)</td>
<td>7.8</td>
<td>0.21</td>
</tr>
<tr>
<td>20 x 10 mm(^2)</td>
<td>15.3</td>
<td>0.39</td>
</tr>
</tbody>
</table>

amplitude closer to the free sample corners is also observed in the simulation results, both in the wet and in the dry state. The qualitative description of the deformation by the model is thus in correspondence with the experimental observations, but quantitatively a deviation occurs that is larger than the scatter in the experimental values – particularly in the wavelength.

**Test case 2**

When a paper sample is clamped at two sides, with MD and CD diagonally oriented with respect to the clamps (Fig 6, right), a more complex buckling pattern occurs due to the different hygroexpansivities in MD and CD. The experimental and simulated topographic images of the out-of-plane displacement in the wet state for sample size of 25 mm x 5 mm and 20 mm x 10 mm are shown in Fig 9. From these figures, it can be observed that buckles arise in the wet state diagonally between the clamps, where MD is parallel to the diagonals with zero z-displacements between the buckles. A similar pattern is also observed in the simulation results.

The experimentally observed and simulated height profiles along the center lines of these samples are shown in Fig 10 for both sample dimension. The height profiles of the narrow, 25 mm x 5 mm, sample (Fig 10a) show a pattern containing multiple wavelengths, whereas in the wide, 20 mm x 10 mm, sample (Fig 10b) only one wavelength is observed in the wet state. The wavelengths, amplitudes and estimated strain for both sample in the wet state are shown in Table 3. The wavelength is approximately twice as large for the wide than for the narrow sample and the amplitude of the displacement is also higher for the wide sample. In the dry state, the out-of-plane deformation almost vanishes for both sample sizes. A slight waviness was observed, most clearly in the narrow sample, with wavelengths that are comparable to the wavelengths in the wet state. However, the amplitudes are so small that the identification of peaks is difficult. The simulations reveal similar buckling patterns along the sample line as in the experiments. Also in the simulations the wavelength and amplitude of the z-displacement are larger for the wide sample than for the narrow sample. In the dried state the wavelength of the buckles remains equal to the wet state, but only a small displacement amplitude remains, as also observed in the experiments. Quantitatively, the simulations underpredict the wavelength by approximately 25% and the amplitude by 33%.

**Discussion**

Generally, in the wet state, the qualitative behavior predicted by the model is similar to the deformation observed in experiments. The wavelengths and deflections observed are of the same order of magnitude. Some explanations for the differences between the model and experiments are explored.

In all test cases in the wet state, the displacement wavelength and amplitude are underestimated by the model. One reason may be that the total strain in the wet state is underestimated. Three explanations can be given for this
strain underestimation in the model that may be valid simultaneously. First, the hygroexpansion coefficient of the paper used in the simulations may be underestimated in both directions. Second, the actual difference in moisture content in the experiments was slightly higher than the one used in simulations. This gives a larger hygroexpansion in the experiment and may explain the larger amplitudes in the wet state. Third, in MD, the amount of irreversible shrinkage in the applied moisture cycle may be overestimated. As can be observed from Fig 5, from the onset of the release of plastic strain, the total strain decreases for increasing moisture content. However, it is questionable whether this shrinkage during wetting is representative for the actual behavior of the paper.

Furthermore, boundary conditions may play a role. The displacement close to the clamps is smaller in the simulations than in the experiments. This may cause the modeled wavelengths and amplitudes to be shorter than the ones experimentally observed. Suppressing all rotations at the clamps in the simulation may lead to a stronger constraint than in the actual boundary conditions at the clamped edges in the experiment. Finally, also a difference in paper thickness between simulations and experiments may cause a difference in quantitative results.

In the dry state, one would expect the buckling patterns from the wet state to remain visible, albeit with a smaller amplitude due to the vanishing hygroexpansive strains. For the samples in test case 2, this pattern with a similar wavelength and smaller amplitude is indeed observed in the dry state. For the buckled samples of test case 1, however, this behavior is only observed once. The other samples show an irregular buckling pattern with z-displacements that may be larger than the displacements in the wet state. Only in the center of the sample some positions of peaks still correspond to peak positions in the wet state. An explanation for this difference between experiments and simulations in the dry state may be due to the experimental procedure, or to the occurrence of cockling, which tends to develop in machine produced sheets during RH cycles.

Conclusions
In this study a model for paper is developed, that describes the out-of-plane deformation of paper resulting from an inhomogeneous moisture content in the plane of the sheet. To this end, a paper sheet is modeled as a network of rigidly connected beams that have different properties in MD and CD to account for anisotropy. The mechanical properties of the beams are moisture-dependent and the network expands as it is wetted. This model is calibrated for regular printing paper based on existing experimental data. The comparison between model predictions and experimental results for a number of test cases shows that this relatively simple model succeeds in capturing the major features of the deformation. The buckling patterns in simulations and experiments are similar and their wavelengths and amplitudes are in the same order of magnitude. Trends upon changes in the sample dimension are captured.

The qualitative response is well predicted by the model. The quantitative response, however, can be improved. In future work, a number of assumptions made in this paper can be relaxed to allow for a more accurate description of paper behavior. For example, the assumption of linear hardening is not accurate when considering a large strain variation. A non-linear hardening rule could improve simulation results. Furthermore, the moisture-dependence of mechanical properties is now assumed to be linear. Experimental evidence on the moisture-dependence of the mechanical properties can give more insight in the type of relation that is most valid and thus improve the model’s predictive value. Moreover, a vanishing yield stress at a certain moisture content is physically inadequate. While for the moisture range considered in this work the moisture at which the yield stress is zero is never reached, a different type of relation between yield stress and moisture content should be investigated to generalized the model for a larger moisture content range.

Finally, linking the proposed phenomenological model to the underlying physical mechanisms that govern hygroexpansion at the network level (e.g. along the lines of Bosco et al. 2015a, 2015b) may provide a closer, quantitative agreement between predicted and experimental results.

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Literature


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The values for the thicknesses $t^{CD}$ and $t^{MD}$, width $w$, elastic moduli $E^{MD}$ and $E^{CD}$ and Poisson’s coefficients $\nu^{MD}$, $\nu^{CD}$ of the beams are given as:

$t^{MD} = t_0$

$t^{CD} = t_0$

$w = w^{MD} = w^{CD} = L \sqrt{\frac{1}{E_{xx}} + \frac{1}{E_{yy}}} \sqrt{\frac{G_{xy}}{G_{xx} \left( \frac{1}{E_{xx}} + \frac{1}{E_{yy}} \right)}}$

$E^{MD} = \frac{E_{xy}}{G_{xy} \left( \frac{1}{E_{xx}} + \frac{1}{E_{yy}} \right)}$

$E^{CD} = \frac{E_{yy}}{G_{xy} \left( \frac{1}{E_{xx}} + \frac{1}{E_{yy}} \right)}$

$\nu^{MD} = \frac{wt_0^3 + w^3t_0}{4LG_{xy}t_0^3}E^{MD} - 1$

$\nu^{CD} = \frac{wt_0^3 + w^3t_0}{4LG_{xy}t_0^3}E^{CD} - 1$

with $t_0$ the thickness and $E_{xx}, E_{yy}, G_{xy}$ the experimentally measured elastic moduli and shear modulus of the orthotropic plate, respectively.