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Local network effects on hygroscopic expansion in digital ink-jet printing

Emanuela Bosco, Ron H.J. Peerlings, and Marc G.D. Geers

KEYWORDS: Hygro-expansion, Fibrous network, Printing, Local deformation, Local stresses

SUMMARY: Dimensional stability of paper is a key problem in the field of digital ink-jet printing. In the literature, this phenomenon is mostly approached through continuum models representing the overall response of paper. However, if the length scale of the applied wetting is comparable to the characteristic length scale of the microstructure a continuum description may be not sufficient to correctly capture the response of the material. The present work explores this question by proposing a two dimensional fibrous network model, which investigates the hygro-expansion phenomena due to the printing process and their effects on the local response of the paper's microstructure. The proposed description incorporates several microscale features, such as the fibre hygro-elastic properties, orientation, areal coverage. Different moisture patterns are applied to the fibrous networks, allowing to study their behaviour as a function of the coverage, the anisotropic orientation and the size of the wet regions relative to the fibre length. A comparison with a homogenized continuum model is finally performed. This reveals that, for low coverages, a microstructural approach is substantially more accurate, whereas for denser networks the continuum model is a valid alternative, even when the size of the wetting pattern becomes comparable to the fibre length.

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Moisture induced deformations may lead to dimensional stability issues that are critical in the area of printing applications. Among the available printing technologies, digital ink-jet printing is nowadays extremely popular due to its high print quality, application flexibility, and speed of the printing process. Ink-jet printing consists essentially in jetting droplets of ink of typical radius of 25-35 μm from small nozzles directly to a specified position on a paper media to create an image (Le, 1998). Ink-jet inks are primarily made of water containing colorants, either dye or pigments. Considering a printing head operating at a typical resolution of 300 dots per inch (dpi), it can be estimated that about 0.5 g/m^2 water are transferred in the paper. Assuming a paper of grammage of 40 g/m^2 , this yields approximatively an increase in moisture content of 0.0125 (calculation based on Trollsås 1995; Ketoja et al. 2001). This leads to an overall hygro-

expansive strain up to 1mm/m, which is visible at naked eye. Digital ink-jet printing is versatile in its applications; therefore, different paper types are used, from standard office paper, to high-quality coated or glossy paper. The goal of ink-jet printer manufacturers is to make printing independent from the paper type used. The realization of this objective is mostly measured by the output quality achieved on standard office paper, which will be considered throughout this study.

During printing, paper is subjected to inhomogeneous moisture content changes, both in the plane and through the thickness of the sheet. The sheet level response is driven by the interaction between the applied moisture pattern and the underlying phenomena occurring at the microstructural level: the anisotropic swelling of a single fibre, its influence on the fibrous network deformation through the bonding regions, and finally how these mechanisms affect the macroscale behaviour.

The dimensional stability of paper has been extensively investigated in the literature. From the experimental point of view, the reader may refer among others to Uesaka et al. 1992, Nanri and Uesaka 1993, Niskanen et al. 1997, Ketoja et al. 2001, Hashemi et al. 2003, Torgnysdotter and Wågberg 2006, Larsson and Wågberg 2008, Erkkilä et al. 2015a. Modelling approaches mostly rely on phenomenological material models at the sheet level. They are either based on orthotropic, moisture dependent elasticity (Kulachenko et al. 2007), on visco-elasticity to describe time-related aspects of paper deformation (Roylance et al. 1980; Uesaka et al. 1989; Lif et al. 2005; Lif 2006), or on elasto-plasticity with moisture dependent material properties (Lipponen 2008; Erkkilä et al. 2013; Erkkilä et al. 2015b), possibly accounting for the release of dried in strains due to the manufacturing process (Van der Sman et al. 2016). These continuum models provide adequate estimates of the overall response of paper due to moisture content variations. However, they are intrinsically unable to capture local effects (e.g. local stress and strain distributions) related to the material's heterogeneous microstructure.

Particularly for high-resolution digital ink-jet printing operations, the size of a single character/letter or of a certain printing pattern is often comparable to (or even smaller than) the characteristic length of a single fibre, on average 2-3 mm. This may induce local effects, which can be observed only with a detailed microstructural model. In this regard, in the literature, network models are used to represent the paper's underlying fibrous microstructure. Most of them focus on the description of the mechanical response only (e.g. Bronkhorst 2003; Hägglund, Isaksson 2008; Kulachenko, Uesaka 2012). Moisture related effects are included in Strömbro and Gudmundson 2008 and in Sellén and Isaksson 2014; these works, however, assume uniform moisture variations over the microstructure.

The objective of this paper is to investigate the local response of the fibrous network due to non-uniform moisture content variations, consistent with an idealized printing operation. This is done by extending the concepts of previous works (Bosco et al. 2016a; Bosco et al. 2016b), in which the network was subjected to uniform wetting. A random network model has been developed, characterized by an anisotropic orientation probability density function. The fibres are modelled as two dimensional transversely isotropic elements; this allows to describe the coupling occurring in the bonding regions between the longitudinal and transverse hygro-mechanical properties that strongly influence the overall response. The present analysis considers a hygro-elastic constitutive behaviour. This hypothesis implies that the deformed material is able to return to the original undeformed configuration once the moisture content returns to its initial value, i.e. paper follows the same deformation path during the wetting and the drying stages of a moisture cycle. The focus is thus on freely dried paper, which shows a reversible response upon a moisture cycle (see e.g. Uesaka et al. 1992; Nanri and Uesaka 1993). The printing operation is modelled by considering moisture content variations in the plane; moisture gradients through the thickness may be included in further work. It is assumed that the moisture penetrates instantaneously in the material, i.e. diffusion effects are neglected. Thermal effects are ignored. The network response of paper is investigated by applying several moisture patterns and by studying the corresponding local deformation and stress distributions. The influence of the anisotropic fibre orientation and the length of the wetting pattern with respect to the fibre length on the network behaviour are addressed. A comparative study between the proposed network model and a homogenized continuum solution is performed. This allows to determine their range of applicability as a function of the network properties and the applied moisture pattern characteristics.

Methodology

Fibrous network model

Consider a reference system (x, y) oriented along the in-plane principal directions of paper, the machine direction (MD) and the cross direction (CD), respectively. The paper fibrous network is described through a two dimensional square periodic representative volume element (RVE) aligned with the x and y direction, with edge L and area Q . Each fibre is modelled as a rectangle, characterized by a length l and a width w . The number of fibres is characterized by the coverage $\bar{c}$, which is defined as the ratio between the total fibre area and the area of the representative volume element,

$$\bar{c} = \frac{n_f l w}{Q} \quad [1]$$

with n_f the number of fibres contained in Q . It is assumed that for all fibres $l = 3 \text{ mm}$ and $w = l/100 = 30 \mu\text{m}$ (Niskanen 2008); the size of the RVE edge is taken as $L = 2l = 6 \text{ mm}$. This size is sufficient to guarantee that the RVE is representative of the effective material behaviour (Bosco et al. 2016a). Note that, in a real paper

network, the length and the width of fibres are not constant but follow a certain probability distribution. The choice here adopted, a part for simplicity reasons, is justified by the fact that the overall response is essentially governed by the total amount of bonding area within the network (Bosco et al. 2015b). At a fixed level of coverage, this area does not change much, whether considering or not a probability distribution for the fibres' length and width. Therefore, whereas small local changes in the stress and strain may appear, the overall response of the network is not influenced.

The position of the fibres within the domain Q is defined through a uniform random point field, whereas their orientation is given by a wrapped Cauchy orientation distribution (Cox 1952):

$$f(\vartheta) = \frac{1}{\pi} \frac{1-q^2}{1+q^2-2q \cos(2\vartheta)} \quad [2]$$

where ϑ ($-\pi/2 \leq \vartheta \leq \pi/2$) is the angle between MD and the fibre axis and the parameter q ($0 \leq q < 1$) quantifies the anisotropy of the orientation distribution. For $q = 0$, the fibres have a random orientation distribution; in the limit $q \rightarrow 1$ all the fibres are aligned along MD. The periodicity of the RVE is obtained by trimming the portions of the fibres that fall outside the area Q along a certain edge and copying them inside Q at the opposite edge.

The main interest of this work is to analyse the hygro-expansive response of paper, in which both the longitudinal and the transverse behaviour of a single fibre have a strong influence. For this reason, unlike most of the works in the literature considering fibres as uni-axial elements (e.g. Bronkhorst 2003; Häggglund, Isaksson 2008; Strömbro, Gudmundson 2008; Sellén, Isaksson 2014), each fibre is modelled as a two dimensional ribbon-like element. In the bonds, which are defined by the regions of overlap of two or more fibres, the fibres are assumed to be fully kinematically coupled, i.e. they are subjected to the same strain.

Under these assumptions, the generation of a finite element mesh for the network geometry may be challenging. A simplified procedure is therefore adopted, which is based on the definition on the RVE of a regular grid of square finite elements. The finite element edge l_e is defined with respect to the fibre width w as $l_e = w/\varphi$, where φ is an integer $\varphi \geq 1$. A finite element is associated to a given fibre if its geometrical center is located inside the fibre area $l \times w$. This results in an approximation of the fibre boundaries through a staircase shape. If φ is sufficiently large ($\varphi \geq 5$), this does not affect the results in a significant manner.

Hygro-elastic problem

The aim of this work is to study the local effects produced by printing on a paper fibrous network. The printing process is idealized by assuming a local variation of the moisture content χ within the microstructure, representing different printing patterns.

Assuming that the moisture penetration within the fibres is instantaneous, i.e. neglecting moisture transport, the response of the material is governed by the equilibrium equation

$$\text{div}(\sigma) = 0 \quad \text{in } Q \quad [3]$$

where, according to the chosen constitutive behaviour, the stress can be written as

$$\boldsymbol{\sigma}(\mathbf{x}) = {}^4\mathbf{C}(\mathbf{x}):(\boldsymbol{\varepsilon} - \boldsymbol{\beta}(\mathbf{x})\chi) \quad [4]$$

The infinitesimal strain tensor $\boldsymbol{\varepsilon} = 1/2(\nabla\mathbf{u} + \nabla\mathbf{u}^T)$ is obtained from the displacement field $\mathbf{u}$, which is the unknown of the problem. ${}^4\mathbf{C}(\mathbf{x})$ and $\boldsymbol{\beta}(\mathbf{x})$ are the elasticity and the hygro-expansive tensors, respectively, that collect the local values of the fibre and bond constitutive properties depending on their positions and orientations. In particular, a transversely isotropic hygro-elastic constitutive law is assumed for the fibres (Bergander, Salmén 2002, Borodulina et al. 2015). The material properties are taken from several works in the literature (Niskanen 2008; Bergander, Salmén 2002; Strömbro, Gudmundson 2008; Sellén, Isaksson 2014); Table 1 collects their values. The constants E_ℓ and E_t are respectively defined as the longitudinal and transverse elastic stiffnesses, relative to the fibre axis, whereas $G_{\ell t}$ is the in-plane shear modulus. For the in-plane Poisson's ratios $\nu_{\ell t}$ and $\nu_{t\ell}$ the following relation holds: $\nu_{t\ell} = \nu_{\ell t} E_t/E_\ell$. Finally, β_ℓ and β_t denote the longitudinal and the transverse hygro-expansive coefficients of the fibre.

For the bonds, having assumed full kinematical compatibility, the elasticity tensor and the hygro-expansion tensor are obtained by performing a Voigt average in the thickness direction (for details, see Bosco et al. 2015a).

Eq 3 and 4 are completed with periodic boundary conditions, requiring that the difference in the displacement field of corresponding points on opposite boundaries (left and right/top and bottom) remains constant during the simulation. The hygro-elastic problem is finally solved considering a free-expansion condition. The numerical solution is based on the finite element method and is performed with a lab-developed MATLAB code. Bilinear quadrilateral elements have been used.

Results

Local deformation and stress

The hygro-expansive response of two fibrous networks with different coverages is shown in Fig 1. The deformation is driven by an in-plane, checker-board moisture (printing) pattern. In this case, the “wave” length of the printing pattern equals two times the fibre length. The case of a uniform fibre orientation ($q = 0$) is first considered. Two networks of different coverages ($\bar{c} = 1$ and $\bar{c} = 10$) have been examined. Reasonable values of coverage for printing paper, estimated from typical basis weights, may be in the range $\bar{c} = [5 - 10]$ (Niskanen 2008). A previous analysis on a broad set of coverages (Bosco et al. 2015a), revealed that for $\bar{c} \geq 5$, the overall hygro-expansive network response reaches a plateau value and is almost independent from the value of the coverage. The choice of $\bar{c} = 10$ is thus intended to represent the response of a wider coverage range. For sparser networks ($\bar{c} < 5$), however, there is a strong dependence of the overall hygro-expansion from the coverage value. The value of $\bar{c} = 1$, despite low for

Table 1 - Material parameters used in the fibre constitutive model, specified with respect to the fibre local coordinate system (ℓ, t) in which the ℓ -axis is coincident with the fibre axis.

Parameter	Value	Unit	Reference
E_ℓ	35	GPa	Sellén, Isaksson 2014
E_t	5.83	GPa	Ratio E_t/E_ℓ . Bergander, Salmén 2002
$G_{\ell t}$	3.5	GPa	Ratio $G_{\ell t}/E_\ell$. Bergander, Salmén 2002
$\nu_{\ell t}$	0.3	-	Bergander, Salmén 2002
$\nu_{t\ell}$	0.05	-	Transverse isotropy $\nu_{t\ell} = \nu_{\ell t} E_t/E_\ell$
β_ℓ	0.013	-	Strömbro, Gudmundson 2008
β_t	0.26	-	Ratio β_t/β_ℓ . Niskanen 2008

real printing paper, has therefore additionally been considered as a representative case for this class of networks, in which relevant local effects may arise. These coverages correspond to a grammage of $b_p = 100 \text{ g/m}^2$ for $\bar{c} = 10$, and $b_p = 10 \text{ g/m}^2$ for $\bar{c} = 1$. These values are calculated by using relation $\bar{c} = b_p/b_f$, in which the fibre basis weight is assumed as $b_f = 10 \text{ g/m}^2$, as given in Niskanen 2008.

Fig 1a and 1c illustrate the undeformed configurations of the two networks for $\bar{c} = 1$ and $\bar{c} = 10$, respectively. Note that the staircase shape of the fibres' boundaries, mentioned in Section Methodology, is not visible at naked eye because the discretization used is very fine. The magenta areas -top left and bottom right- are wetted (W), whereas the cyan ones -top right and bottom left- do not experience a moisture variation (D). The deformed configurations upon wetting for a moisture content variation $\Delta\chi = 0.01$ in the wet areas are shown in Fig 1b and 1d. The deformation is enlarged by a factor of twenty. Note that due to the adopted linear hygro-elastic behaviour, there is a linear relation between the applied moisture and the obtained displacement field. For this reason, for a larger moisture content variation $n\Delta\chi$, the displacement (stress and strain) field scales linearly with n . In the real printing process, the absolute value of the moisture content variation matters as non-linear or history effects play a role. These features are not yet included in the present model. The non-uniform printing profile determines the deformed shape of the networks, in which the expansion of the wet zones is constrained by the dry ones. Whereas qualitatively the deformation of the two networks is similar, the denser network ($\bar{c} = 10$) shows a larger overall swelling. This is quantified by the normalized average hygro-expansive strains in MD and CD: $\alpha_{MD} = \langle \varepsilon_{xx} \rangle / \Delta\chi$ and $\alpha_{CD} = \langle \varepsilon_{yy} \rangle / \Delta\chi$. For $\bar{c} = 1$, $\alpha_{MD} = 0.0165$ and $\alpha_{CD} = 0.016$; for $\bar{c} = 10$, $\alpha_{MD} = 0.0277$ and $\alpha_{CD} = 0.0264$. The small difference between the two directions is due to the stochastic nature of the networks. Note also that, despite the fact that the overall response is essentially isotropic, the anisotropic response of individual fibres can be clearly noticed, with a much larger transverse deformation than the longitudinal one. Finally, it is remarked that bonds essentially govern the overall expansion of the network.

Fig 1 - Network deformation for a chess-like printing pattern for two different network coverages. The magenta areas, denoted with W , are wetted, whereas the cyan ones, denoted with D , do not experience a moisture variation. The deformation is enlarged of 20 times.

This is due to the fact that the free-standing parts of the fibres swell only little in the longitudinal direction and their lateral swelling is not transferred to the network and thus does not contribute.

The stress distribution in the two networks for a moisture content variation $\Delta\chi = 0.01$ is shown in Fig 2, (top) for $\bar{c} = 1$ and (bottom) for $\bar{c} = 10$, in terms of the hydrostatic stress $\sigma_h = 1/3 \text{tr}(\sigma)$, i.e. the average of the stress components in the principal directions of paper. The hydrostatic stress has been used instead of one of the single stress components (e.g. σ_{xx} or σ_{yy}) to minimize the influence of possible anisotropy due to the randomness of the fibre orientation. The stress in the bonds should be interpreted as the average value through the thickness of the stresses of the single fibre layers. Note that, since the model is two dimensional, the thickness direction cannot be explicitly represented and therefore it is assumed that in any region in which more than one fibre is present they are all bonded to each other. The peak values of the stress are larger in the low

coverage compared to high coverage one. This can be explained by the fact that a denser network is more homogeneous and the stresses are therefore more uniformly distributed. It can also be observed that, due to the applied moisture distribution and to the consequent mutual constraint, the dry areas are on average subjected to tension, while the wet zones are in compression. This can be explained by the fact that, as the moisture content increases, the wet regions expand. However, their expansion is constrained by the dry areas that maintain the original shape and therefore force the wet regions to expand less than they would like. This originates a compressive stress in the wet regions. On the contrary, the dry areas are forced to partially accommodate the expansion of the wet areas; therefore, they are in tension. Due to the hypothesis of linear elasticity, the stresses are significant already for a low moisture content variation. In reality, such stresses might be partially relaxed by buckling of the compressive (wet) parts of the sheet, thus causing dimensional stability issues.

Fig 2- Distribution of the hydrostatic stress σ_h (in GPa) in the two networks of coverages $\bar{c} = 1$ and $\bar{c} = 10$, corresponding to a moisture content variation $\Delta\chi = 0.01$.

Influence of the wetting pattern and comparison with the homogenized solution

The influence of the characteristic length of the wetting pattern with respect to the fibre length in relation to the network coverage is investigated next. To this aim, three wetting profiles are considered, as shown in Fig 3, in which periodic bands of different width, parallel to the x -axis, have been wetted. These patterns are characterized by the parameter $\lambda = l/l_{wet}$, where l is the fibre length and l_{wet} the width of the wet band. The fibre orientation is assumed to be uniform ($q = 0$). Figure 4 shows the strain field $\langle \varepsilon_{yy} \rangle_x / \Delta\chi$ calculated as the average of the strains along lines parallel to the x -axis, as a function of the nondimensional coordinate $y/(\lambda L)$. Magenta dots, blue crosses and cyan diamonds refer to $\lambda = [1, 1/2, 1/4]$, respectively. The top diagram is for a network with $\bar{c} = 1$, the bottom one for $\bar{c} = 10$.

The wet areas are characterized by expansive strains, while the dry zones slightly shrink due to the Poisson's effect. This is particularly evident in the low coverage network. Here, near the boundaries with the wet regions, due to the heterogeneous nature of the network, wet and dry regions interact, deviating from the average continuum response. An estimate of this effect can be

Fig 3- Considered wetting patterns, with $\lambda = l/l_{wet} = [1, 1/2, 1/4]$.

Table 2 - Material parameters used in the homogenized model, computed through homogenization of the network response.

$\bar{c} = 1$			$\bar{c} = 10$		
Parameter	Value	Unit	Parameter	Value	Unit
E_{xx}	10.3	GPa	E_{xx}	20.0	GPa
E_{yy}	10.6	GPa	E_{yy}	18.4	GPa
G_{xy}	3.8	GPa	G_{xy}	7.9	GPa
ν_{xy}	0.29	-	ν_{xy}	0.29	-
ν_{yx}	0.3	-	ν_{yx}	0.27	-
β_{xx}	0.035	-	β_{xx}	0.055	-
β_{yy}	0.034	-	β_{yy}	0.054	-

obtained by calculating the intersection between the network response and the average continuum one in the dry region. For the network with $\bar{c} = 1$, the intersection is located at coordinates $y/(\lambda L) = [0.75, 1.2, 1.4]$, for corresponding values of $\lambda = [1, 1/2, 1/4]$. This shows that the boundary effect increases as λ decreases, i.e. as the fibres get longer relative to the wetting pattern. In the wet zones, the influence of the different wetting patterns is negligible. Moreover, for $\bar{c} = 10$, the effect is much less pronounced.

The local network strains fluctuate around the black solid lines. The latter represent the response of an equivalent, homogeneous medium. This reference response is obtained by considering the cell domain ($L \times L$) and replacing the heterogeneous network structure by homogeneous elements. The material properties of these elements are computed through homogenization of the fibrous network response in uniform wetting conditions. The homogenization procedure, whose details are not relevant for the scope of this paper, has been performed according to the methodology illustrated in Bosco et al. 2016a. The homogenized material properties are specified in Table 2; note that in this case, due to the assumed uniform orientation distribution, the homogenized material properties are essentially isotropic. The continuum model is next subjected to the non-uniform printing patterns shown in Fig 3; the hygro-elastic problem illustrated by Eq 3-4 is finally solved within the finite element procedure.

The wet regions in the lower coverage network, *Fig 4* (top), show a much larger fluctuation in the local strains compared to the average macroscopic solution. Moreover, in the dry regions, the local strains substantially differ from the average, due to the boundary effect discussed earlier. For the higher coverage network, in which the material is more homogeneous, the local strain fluctuations are much smaller. This suggests that for isotropic networks with a coverage that is representative of printing applications ($\bar{c} = 10$), a continuum approach provides an adequate approximation of the network response, whereas for low coverage networks the local response cannot be neglected. Note finally that, as already observed in the previous Section, a higher coverage results into a larger overall expansive strain.

Effect of anisotropy

The effect of the anisotropic fibre orientation distribution on the local network deformation is finally investigated. For this purpose, the network with realistic coverage ($\bar{c} = 10$) is considered, where the fibre orientation distribution is defined by taking $q = 0.5$ in relation [2]. Two wetting patterns are applied to the network, consisting of a wet paper strip (a) parallel to the CD and (b) parallel to the MD. In both the cases, $\lambda = l/l_{wet} = 1$. Figure 5 presents the strains $\langle \varepsilon_{yy} \rangle_x / \Delta\chi$ (magenta dots) and $\langle \varepsilon_{xx} \rangle_y / \Delta\chi$ (blue dots), obtained as the averages of the corresponding strain components along lines parallel to the x and y direction, respectively, shown as a function of the local coordinates $y/(L\lambda)$ and $x/(L\lambda)$. They are related to the printing patterns (a) and (b). The solid black lines represent the homogeneous solution, which employs homogenized material properties. Due to the preferential orientation of the fibres within the network, the number of bonds (i.e. of constraints) is larger along the CD than the MD. Moreover, for an anisotropic orientation distribution, the bonds are composed on average by more fibres oriented along MD. The (larger) transverse expansion of the individual fibre contributes thus more to the resulting bond expansion in CD. For a combination of these reasons, the strains in the cross direction $\langle \varepsilon_{yy} \rangle_x / \Delta\chi$ are therefore larger than the strains in the machine direction $\langle \varepsilon_{xx} \rangle_y / \Delta\chi$.

The strain fluctuations with respect to the homogenous solution follow the same trend. Moreover, the boundary effect that appears in the dry zones at the interface with the wet areas is more pronounced along the CD than along the MD. In any case, the homogeneous model, even if the size of the moisture pattern is comparable with the fibre length, approximates well the full network solution for realistic paper coverages. The overall hygro-expansion coefficients for this anisotropic network have been computed as $\beta_{xx} = 0.0235$, $\beta_{yy} = 0.1156$. Their ratio ($\beta_{yy} / \beta_{xx} = 4.9$) is in the range of [3-5] specified by Niskanen 2008.

Fig 6 shows the stress distribution in terms of σ_{yy} for the wetting pattern (a) and of σ_{xx} for the wetting pattern (b). They correspond to a moisture variation $\Delta\chi = 0.01$. Consistently with the strain profiles and with the higher degree of constraint in the CD, higher stresses arise in the

cross direction than in the machine direction. The stresses are concentrated along paths parallel to CD; these paths should not be confused with fibres, which are mainly oriented in MD. This illustrates that stresses along the CD need to relax, constituting a potential mechanism for out of plane deformation. This aspect will be addressed in future investigations.

Fig 4- Strain field $\langle \varepsilon_{yy} \rangle_x / \Delta\chi$ as a function of the coordinate $y/(L\lambda)$, for the three different wetting lengths. Magenta dots, blue crosses and cyan diamonds refer to $\lambda = [1, 1/2, 1/4]$, respectively, while the black solid line represents the response of a homogenous continuum with equivalent material properties, for $\bar{c} = 1$ (top) and $\bar{c} = 10$ (bottom).

Fig 5- Averaged strain fields for the two assumed wetting patterns: parallel to the CD, $\langle \varepsilon_{yy} \rangle_x / \Delta\chi$ (magenta dots) and parallel to the MD, $\langle \varepsilon_{xx} \rangle_y / \Delta\chi$ (blue dots).

Fig 6- Stress distribution (in GPa) in a network with $\bar{c} = 10$, with anisotropic fibre distribution. The stresses refer to two different wetting patterns: σ_{yy} for a wet strip parallel to CD (top) and σ_{xx} for a wet strip parallel to MD (bottom). The applied moisture variation is $\Delta\chi = 0.01$.

Discussion

The analysis of the response of the networks to several moisture patterns presented in the Results section leads to some relevant observations, discussed in the following.

The model adequately captures the local network deformation as a function of the coverage, showing that denser networks have a larger overall expansion and a lower average stress compared to the coarser ones. For both the considered coverages, the stresses appear to be relevant even for low moisture changes. This stress distribution suggests a possible origin of the dimensional stability issues: compressive stresses may relax, triggering out-of-plane deformation and buckling.

The size of the wetting pattern in relation to the fibre length has a clear influence on the network response, particularly in the dry regions and for low coverage networks. A boundary effect has been identified that increases as the size of the wetting pattern decreases. The effect is less evident for realistic paper coverages. More in general, it can be observed that local effects (quantified by the magnitude of the strain micro-fluctuations) are more pronounced in low coverage than in high coverage

networks. This observation is relevant from a practical point of view. It proves rigorously that, for uniformly oriented networks of realistic coverage for printing applications ($\bar{c} = 10$), the continuum model is capable to capture the key aspects of the network deformation. The printing process of paper sheets can be therefore simulated by using the computationally cheaper and easier to implement homogenized description. This holds even when the wave length of the wetting pattern is equal to or smaller than the fibre length, as it frequently occurs in digital ink-jet printing. For coarse fibre distributions ($\bar{c} = 1$), a microstructural description should be used instead; nonetheless as discussed earlier such coverages are not frequent in real printing applications.

The anisotropy of the network orientation leads to a stronger constraint along the CD than along the MD, with corresponding higher stress levels. The comparison with the homogenized solution shows that, also for anisotropic orientations, the continuum model closely captures the high coverage network response. This is also a significant information when simulating real printing processes. The developed network model is nevertheless important, as it allows to calculate the hygro-expansive response of paper based on detailed meso-structural features (individual fibre hygro-elastic properties, geometry, orientation distribution) of the material. The obtained hygro-expansive coefficients for the anisotropic network, with $\bar{c} = 10$, have realistic values compared with experimental data.

The present paper addresses the effects of in-plane moisture induced deformations only. Further work needs to incorporate wetting though the thickness and out-of-plane deformation mechanisms. In view of the above observations, for realistic printing paper coverages, this is expected to be feasible with a macro-scale homogenized model.

The considered hygro-elastic constitutive model can represent the linear reversible response. The irreversibility upon wetting due to the history of paper production should be taken into account by enriching the constitutive description, e.g. along the lines of Bosco et al. 2015b.

Finally, this work can be further extended to model the diffusion processes and their coupling with mechanics, to investigate the time-dependent moisture penetration effects on the resulting overall paper response.

Conclusions

This work focusses on the description of the response of paper fibrous networks due to moisture content variations, which represent an idealized printing operation. In particular, non-uniform in-plane moisture content changes have been considered, whose length scale is comparable to the fibre length. The proposed network model incorporates individual meso-structural features, e.g. fibre geometry, constitutive properties, orientation distribution and network coverage. Simulations have been performed on networks with different coverages and degree of orientation anisotropy, showing that the model is capable to describe local effects related to the material's heterogeneous micro-structure, such as stress and strain micro-fluctuations.

The rigorous analysis of these local responses revealed that a continuum, homogenized model approximates with sufficient detail the behaviour of networks of realistic coverage for printing applications ($\bar{c} = 10$). This verification is important from the practical point of view. The network model is nevertheless relevant, as it allows reconstruct the micro-structural origin of paper hygro-expansive phenomena.

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