Propeller efficiency at full scale: measurement system and mathematical model design

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Propeller efficiency at full scale
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Contents

1 Introduction 1
  1.1 Motivation ........................................... 1
  1.2 Design objectives .................................... 4
  1.3 Design concept ...................................... 4
  1.4 Thesis layout ....................................... 5

2 Propeller efficiency: definition, current practice, and challenges at full scale 9
  2.1 Power and efficiency .................................. 10
  2.2 Current practice ..................................... 11
    2.2.1 Propeller designer ............................... 12
    2.2.2 Ship operator .................................. 12
    2.2.3 Model test ...................................... 13
    2.2.4 CFD simulations ................................. 17
  2.3 Need for propeller efficiency at full scale .............. 18
  2.4 Challenges at full scale ............................. 18

3 Design aspects of propeller efficiency at full scale 23
  3.1 Approach ............................................. 23
  3.2 Realizability ....................................... 28

4 Design of mathematical models 31
  4.1 Coordinate systems .................................. 32
  4.2 Ship trajectory ...................................... 35
  4.3 Ship propulsion ..................................... 39
    4.3.1 Shaft ........................................... 40
    4.3.2 Engine .......................................... 42
    4.3.3 Propeller ....................................... 43
    4.3.4 Surge ........................................... 46
    4.3.5 Thrust and torque ............................... 49
    4.3.6 Advance velocity ................................ 53
  4.4 Summary of the ship propulsion model .................. 55
## CONTENTS

5 Analysis of the ship propulsion model 57
  5.1 Dimensionalization ........................................... 58
  5.2 The stationary state model ................................. 61
    5.2.1 Solutions of the stationary state model .............. 61
    5.2.2 Model test data ....................................... 63
  5.3 Time scale analysis ........................................ 66
  5.4 Qualitative validation of the model ...................... 71

6 Estimation of model parameters 75
  6.1 Control and response ....................................... 78
    6.1.1 Stationary control .................................... 78
    6.1.2 Zero-thrust control ................................. 81
    6.1.3 Harmonic control .................................... 82
  6.2 Resistance exponent from stationary control ............ 84
  6.3 Resistance factor from zero-thrust control ............... 86
  6.4 Thrust deduction from stationary control ................. 88
  6.5 Wake fraction from stationary control .................... 89
  6.6 Wake fraction from harmonic control ..................... 94
    6.6.1 Preliminaries ......................................... 95
    6.6.2 Stationary control .................................... 96
    6.6.3 Harmonic control ..................................... 97
    6.6.4 The parameter estimation scheme ..................... 99
    6.6.5 Proof of principle .................................... 100
    6.6.6 Discussion ........................................... 106
  6.7 Design of measurement scheme ............................. 107
    6.7.1 List of variables ..................................... 107
    6.7.2 Data collection system ............................... 108
    6.7.3 Propulsion tests ...................................... 110

7 Full scale measurements 115
  7.1 In situ data collection system ............................ 115
  7.2 Illustrations of raw signals .............................. 117
  7.3 Features of raw signals .................................. 123
    7.3.1 Trend ................................................. 123
    7.3.2 Oscillation ........................................... 124
    7.3.3 Noise ............................................... 125

8 Processing of measurements 127
  8.1 Signal decomposition ...................................... 127
    8.1.1 General formulation ................................. 128
    8.1.2 Practical implementation .............................. 132
  8.2 Signal reconciliation ....................................... 134
    8.2.1 General formulation of signal reconciliation ......... 135
    8.2.2 Reconciliation of navigation signals ................. 138
Chapter 1

Introduction

1.1 Motivation

A ship propeller is a mechanical device that transfers energy; it consumes the power produced by the engine and puts this power into thrust power. The power transfer is enhanced by the particular geometry of the propeller. The propeller consists of a revolving shaft with several blades attached to it. The propeller is connected to the engine by the propeller shaft. As part of the power transmission, the propeller shaft rotates due to a torque produced by the engine. The geometry of the propeller blades is designed in such a way that when rotated the propeller produces a force called thrust. The thrust pushes the ship through the water.

The word efficiency originates from the Latin efficere, which means to accomplish. The English word efficiency is in use since the 16th century by engineers. In engineering, efficiency is the ratio of the useful work performed by a machine or in a process to the total energy expended or heat taken in, according to The New Oxford Dictionary of English (Ed: Clarendon Press, Oxford). Given that there are always losses in the process, efficiency is always smaller than 1.

Designers and manufacturers of propellers define the efficiency of their propellers as the ratio of the power produced by the propeller (thrust) to the power consumed. To quantify propeller efficiency there is consensus in the naval society on how to compute the power produced and the power consumed. Indeed, the power produced is defined as the product of the propeller thrust and the advanced velocity; the power consumed is defined as the product of the propeller torque and the angular velocity. In this thesis we adopt this definition of propeller efficiency.

Propeller efficiency indicates how much energy is lost in the power transmission. Given a scale and a reference the efficiency of the propeller indicates its performance. In other words, efficiency indicates how the propeller performs as a transmitter of power, on a scale from 0 to 1. Naturally, efficient propellers are desirable due to the relationship between propeller efficiency and fuel consumption,
where the fuel consumption is the real indicator of ship performance. In propulsion models the overall efficiency of ship propulsion is defined as the product of a number of efficiencies: engine efficiency, transmission efficiency, propeller efficiency, and hull efficiency. In such model, increase of one of the efficiencies results in overall increase of efficiency. In light of such model a higher propeller efficiency results in a lower fuel consumption for the same output. Full scale ship operations indicates that external conditions have a huge impact on efficient operation, making the propulsion efficiency model more virtual than real.

Still, propeller efficiency is an important design criterion in propeller design. From the beginning of the previous century knowledge of propeller efficiency has increased tremendously by experimenting with scaled versions of the real propellers and by numerical calculations and simulations. Open water tests are nowadays standardized procedures in model basins. During the open water tests a scaled version of a propeller is rotated with a constant angular velocity and translated through the water with a constant advance velocity; propeller thrust and torque are directly measured. Propeller efficiency is calculated from the averaged quantities. The question arises how predictive the model scale propeller efficiency is for the full scale propeller efficiency. The answer is not straightforward. Predictive quality is subject to interpretation. First, there are the propeller scale effects, nicely summarized by J.S. Carlton in (Carlton, 2007, pp. 95-98). Second, a model test environment is controlled, and in that respect incomparable to the uncontrolled environment in which the ship operates at full scale. Model scale tests are described in more details in Section 2.2.3.

Computational Fluid Dynamics (CFD) software packages are being used nowadays in the design phase to simulate the open water tests. CFD simulates the water flow around the full sized propeller blade. Flow velocity of the water and angular velocity of the propeller are input to the simulation. The software calculates the velocity field in the domain around the propeller. From there, interaction between water and propeller blades are calculated in terms of normal and tangential stresses at the surface of the propeller blade. These stresses are then averaged over the propeller blade to compute the thrust and torque. Finally, the propeller efficiency is calculated with the same formula.

At this moment there is no method to calculate propeller efficiency at full scale without making indirect use of the model test procedures and/or the CFD simulations. One of the reasons is the lack of the sensor technology to measure propeller thrust and torque, so that thrust and torque are computed from constitutive laws relating them to speed-through-water and angular velocity. Another reason is that propeller advance velocity, the velocity of water entering the propeller plane, cannot be measured at full scale, so that other ways to retrieve the advance velocity need to be discovered. Besides these technical challenges there are also the interpretational challenges. We should find answers to questions such as: Should propeller efficiency at full scale be compared to propeller efficiency at model scale or from CFD simulation? Are there other variables that influence propeller effi-
1.1. Motivation

ciency at full scale? What is the practical use of measuring propeller efficiency at full scale? The last question is of particular importance at this stage. Practical use of propeller efficiency at full scale has two main beneficiaries: 1. the propeller manufacturer, 2. the ship operator.

**Propeller manufacturer**

A propeller manufacturer wants to sell propellers. Propeller efficiency is a marketing aspect and has become an important criterion in propeller design. Such design is based on historically collected data, model scale test, and CFD simulations. Procedures to determine propeller efficiency from model scale tests and CFD simulations are well established and standardized. They are made consistent with each other.

It is in the interest of the propeller manufacturer that his propeller has the predicted efficiency at full scale as predicted by the open water tests and CFD simulations. There should be consistency between the predicted propeller efficiency and the efficiency measured at full scale.

**Ship operator**

Ship operators want to reduce the costs of their ship operations. There are many ways to reduce the costs. Thinking about propellers, the ship operator wants to buy efficient propellers. This means that the ship operator requires a guarantee from the propeller manufacturer that the propeller has a certain propeller efficiency at full scale. So it is in the interest of both the propeller manufacturer and the ship operator to have propeller efficiency measurable at full scale. Second, the ship operator wants to make use of the propeller at its optimal efficiency. For that the propeller should be controlled towards the operational conditions for which it functions efficiently according to the design. For this purpose on board propeller efficiency indicator for advisory purpose is needed. Third, efficient use of propellers is related to propeller maintenance. As all other underwater structures, propellers suffer from bio-fouling and damages which decrease the propeller performance. Propeller maintenance takes place periodically. Predicting the right moment when maintenance should be carried out would mean less downtime.

To end the motivation part of the thesis we explain its setting. The idea to write a thesis on propeller efficiency originates from the commitment of Wärtsilä Netherlands, a manufacturer of ship propulsion packages, to estimate propeller efficiency at full scale. Close collaboration between Wärtsilä Netherlands, VAF Instruments,
Introduction

a sensor manufacturer, and Eindhoven University of Technology exists since 2006. Full scale measurements, vital for research of this type were provided by two ship operators, whose names we are not allowed to reveal in this thesis. Results of this collaboration were several graduate projects of the program Mathematics for Industry of Eindhoven University of Technology, projects related to propeller efficiency at full scale, measurement processing, and sensor technology. This thesis is the final product of this collaboration. It is a thesis on scientific design of a system that measures efficiency of propellers at full scale.

1.2 Design objectives

The main objective of this thesis is the design of a system of algorithms based on mathematical modeling and measurements to compute propeller efficiency at full scale. In this respect its design objectives are

- **design of a mathematical environment** that accommodates the concept of propeller efficiency. This environment consists of a dynamic model for ship propulsion and a kinematic model for ship motion, algorithms to process signals, to reconcile signals, and to estimate parameters.

- **design of a measurement environment** that accommodates the concept of propeller efficiency. This environment consists of a data collection system that records the required signals and extract useful information from the raw data and test setups. The data collection system is completely specified for the signals to be measured, the measurement sample rate, measurement accuracy, and the measurements storage.

- **design of an analysis environment** that accommodates the statistical aspects of propeller efficiency. This environment consists of a data base of processed data, and statistical algorithms to analyze and interpret this data.

1.3 Design concept

To estimate propeller efficiency at full scale an algorithmic system has to be designed. Here, algorithmic system has a generic meaning. It is made of three modules, each representing a principle: the mathematical environment, the measurement environment, and the statistical environment. The concept is schematically depicted in Figure 1.1.

The measurement environment takes care of everything related to data collection and storage on board of the ship. In real life, the design of this environment comprises hardware and software elements. The sensors and the data collection system represent the main hardware elements; low level data filtering and data storing are the main software elements. In the context of the measurements, special tests were designed to measure data at special condition.
The design of the mathematical environment means creation of a virtual environment in which the information from the measured data is transformed and processed. The mathematical environment structures the entire design. Mathematical algorithms, which are practical implementations of the mathematical environment, are designed to compute unknown quantities and parameters. Part of these algorithms requires measurements obtained under well specified conditions that we refer to as propulsion tests.

Combining the output of the measurement environment and the mathematical environment we are led to the statistical environment. At this part of the design, interpretation and analysis of the results takes place.

\[ \text{Measurements} \quad \Downarrow \quad \text{Propulsion test} \quad \rightarrow \quad \text{Mathematical models} \quad \Downarrow \quad \text{Mathematical algorithms} \quad \rightarrow \quad \text{Statistics} \quad \Downarrow \quad \text{Interpretation} \]

\textbf{Figure 1.1: A tool that estimates propeller efficiency at full scale relies on three principles: measurements, models, and statistics.}

\section{1.4 Thesis layout}

The first and second chapters address the context and motivation of the thesis, the challenges to estimate propeller efficiency at full scale. The third chapter focuses on the design aspects of the design concept mentioned in the previous section. In chapters four, five, and six the mathematical environment is addressed. In chapters seven and eight the measurement environment is addressed. In chapters nine and ten the statistical environment is addressed. Chapter eleven concludes the thesis with some philosophy on propeller efficiency.

This section explains the organization of the thesis. Each chapter is briefly described in one paragraph, emphasizing the most important ideas and findings.

Chapter 2 presents the current practice and the future prospects of the propeller efficiency at full scale. The concept of propeller efficiency is introduced in the first section. Propeller is regarded as an element of the propulsion train that can be analyzed separately if input and output are measured, namely, thrust, torque, advance velocity, and angular velocity. The second section describes the current practice in calculating propeller efficiency. Traditionally, propeller efficiency is calculated from open water tests conducted with scaled propellers in model basins. The computational fluid dynamics software is a reliable alternative to model tests, that is nowadays used in the research and design phase of propellers. The views of the propeller designer and the ship operator with respect to propeller efficiency are also covered in the second section. The need for propeller efficiency at full scale is put forward in the third section. Challenges to full scale propeller efficiency
are presented in the fourth section. We identify two types of challenges: technical and interpretational. Technical challenges are related to sensor technology and mathematical modeling. Interpretational challenges are related to the effects of the surrounding environment on propeller efficiency.

Chapter 3 presents the design aspects of the propeller efficiency at full scale. The design strategy, which is presented in the first section, covers almost entirely the chapter. The main steps of the strategy are the mathematical models, the parameter estimation, the full scale measurements, and the statistical analysis of propeller efficiency. In the second section of the chapter the technical feasibility of propeller efficiency at full scale is discussed.

Chapter 4 deals with the mathematical models. The objective of the chapter is to formulate the mathematical environment that assists the estimation of propeller efficiency at full scale. The first section presents the coordinate systems in which the mathematical models are formulated. The second section presents the ship trajectory model, which is important for the correction of the measured speed-through-water. The third section presents the ship propulsion model. The design of the ship propulsion model is inspired by mathematical models used in ship navigation and control. The four physical quantities involved in the propeller efficiency formula are also states of the ship propulsion model. The ship propulsion model comprise of several model parameters that are unknown at full scale. These unknown model parameters play a central role in this thesis. In fact, propeller efficiency at full scale needs one model parameter, the wake fraction.

In Chapter 5 we analyze the ship propulsion model. The main themes of this chapter are time scale, model stability, and the qualitative validation of the ship propulsion model. The dimensionless formulation of the model is presented in the first section, where the main discussion point is related to the time scales of the propulsion model. Section two presents the stability analysis of the propulsion model. The mathematical model is harmonically disturbed around stationary states to find the eigenvalues and the eigenvectors. Real data from a model scale and a full scale vessel are used to actually calculate the eigenvalues and the eigenvectors. In the last section the ship propulsion model is qualitatively validated on basis of the same model test and full scale data.

In Chapter 6 we describe the methods to estimate the unknown model parameters. The first section describes the general approach to estimate the parameters. Specific controls are applied to the ship propulsion; consequently, the structure of the propulsion model changes into forms that allow us to calculate a particular model parameter. Three types of controls are applied: stationary, zero-thrust, and harmonic. All parameter estimation methods follow the same approach: an optimization scheme is setup such that the Euclidean distance from the full scale measurements to the model prediction is minimized with respect to certain model parameters. In successive sections we describe the methods to estimate each model parameters. Wake fraction receives additional attention due to particular challenges posed by this parameter. In the last section of the chapter we describe the
measurement scheme: the list of variables to measure at full scale, the functional specifications of the data collection system, and the propulsion tests required by the parameter estimation methods.

The topic of Chapter 7 is measurements at full scale. The data collection system as implemented in practice is described in the first section. Two systems were installed on board of two different vessels, a ferry and a bulk carrier. The measured quantities are also listed in the first section. In the second section several important measured variables are illustrated. In section three we describe features that are common to most of the measured signals, namely, trend, oscillations, and noise.

Chapter 8 deals with the analysis of the full scale measurements. The objective of the chapter is to extract reliable information from the raw signals. The chapter is divided into three sections. In the first section the signal decomposition method is presented. Using this method, the raw signal is decomposed into three constituents: trend, oscillation, and noise. The trend of the signal is the most important information. In the second section the signal reconciliation method is presented. The objective of the signal reconciliation is to make the measured signals consistent with each other using an underlying mathematical model. In particular, the signal reconciliation is used to correct the distortions found in the measurements of ship speed. The underlying mathematical model is the ship trajectory model. A practical example is given in the end of the section. In the third section the stationary state extraction algorithm is presented. The output of this algorithm is a data base of stationary states.

Chapter 9 contains the practical implementation of propeller efficiency at full scale. The first section presents the propulsion tests performed on board of one of the monitored vessels. In the second section we actually estimate the model parameters based on the propulsion tests. Here we learn that some of the model parameters are influenced by external factors. The third section deals with propeller efficiency estimation. In this section we also introduce the concepts of reference and relative propeller efficiency.

Chapter 10 presents some statistical aspects of the relative propeller efficiency at full scale. The chapter opens with a discussion on what we expect and what we observe in reality. In the second section we present the data base of stationary state to be used in this chapter. The third section presents the data structuring. From the data base we select two operational modes in which the stationary points are densely clustered; we presents some statistical aspects of these two clusters. The reference and relative propeller efficiency concepts are tried out on the stationary points of the two modes; this is done in the fourth section. In the last section of the chapter we represent another ship performance indicator, the load power curve, for the two operational modes.

Chapter 11 concludes the thesis with some philosophy on propeller efficiency at full scale. Some recommendations towards practical implementation of the methods described in the thesis are given in this chapter.
Chapter 2

Propeller efficiency: definition, current practice, and challenges at full scale

Screw propellers dominate the propulsion of ships since the mid-nineteenth century. Back then, John Ericsson in the United States and Sir Francis Pettit Smith in England came with the first practical applications (E.D. Lewis, 1989, vol. I). The advantages of the screw propeller over the propulsion systems existing at that time were obvious. With the first crossings of the Atlantic, the screw propeller dominates ship propulsion, a position held nowadays. One of the main advantages of screw propellers when compared to other propulsion solutions is efficiency of energy conversion. In this chapter we introduce the concept of propeller efficiency, a concept that is widely used in the design of marine propellers, typically in the hydrodynamic aspects. The first section describes a way to regard the ship propulsion train. The propulsion train is decomposed into elements; the input and output of elements are indicated. Then, the power and efficiency of each element of the propulsion train are defined. In this context the propeller efficiency formula is defined. In the second section we present the current practice to determine the propeller efficiency. We describe the practice of propeller manufacturer, the manner in which the ship operator uses propeller efficiency, and the methods to measure propeller efficiency at model scale. In section three we discuss the need for propeller efficiency at full scale. The estimation of propeller efficiency at full scale poses a number of challenges. These challenges are described in the fourth section of the chapter.
2.1 Power and efficiency

The ship propulsion train converts the chemical power of the fuel into mechanical work done. This mechanical work propels the vessel through the water. The propulsion train can be split into main components: engine, shaft, propeller, the hull, see Figure 2.1. The internal combustion engine converts the chemical power of the fuel, \( P_{\text{fuel}} \), into mechanical power, \( P_{\text{eng}} \). This power is transmitted from the engine to the mechanical transmission, which consists of clutches, gear box, bearings, and the propeller shaft. In this context we identify the entire transmission with the propeller shaft. Thus, the power delivered through the transmission is the shaft power, \( P_s \). The propeller is mounted at the external end of the propeller shaft and converts shaft rotative power into thrust power, \( P_t \). The effective power, \( P_e \), is related to the work done to drive the ship through water.

Figure 2.1: The schematic of ship propulsion train

In general, power is not measured directly by means of a sensor but determined from relations that are specific to the type of application. The chemical power of the fuel can be defined as the product of the calorific value, \( C_v \), and the mass flow rate of the fuel, \( \dot{m} \)

\[
P_{\text{fuel}} = C_v \cdot \dot{m} \tag{2.1}
\]

For a ship, the engine power* is defined as the product of torque produced by the engine, \( Q_{\text{eng}} \), and angular velocity of the engine shaft, \( \omega_{\text{eng}} \)

\[
P_{\text{eng}} = Q_{\text{eng}} \cdot \omega_{\text{eng}} \tag{2.2}
\]

The shaft power is the product of the shaft torque and the angular velocity of the shaft. In this thesis we refer to the shaft torque as the propeller torque, \( Q_p \), and to the angular velocity of the shaft as the propeller angular velocity, \( \omega \). Thus,

\[
P_s = Q_p \cdot \omega \tag{2.3}
\]

The propeller produces the thrust \( T_p \), the propeller thrust, and advances relatively to the water with the velocity \( u_a \), the advance velocity. The thrust power produced as such by the propeller is defined as

\[
P_t = T_p \cdot u_a \tag{2.4}
\]

The effective power is defined as the product of the ship resistance, \( R \), and the velocity of the ship relative to the surrounding water, the speed-through-water \( u \),

\[
P_e = R \cdot u \tag{2.5}
\]

* Also known as brake power
2.2. Current practice

To assess the performance of a process or system the concept of efficiency is widely used in engineering. In this context efficiency is defined as the ratio of the useful power produced in the process to the power consumed during the process

$$\eta = \frac{P_{\text{produced}}}{P_{\text{consumed}}} \quad (2.6)$$

In this light we define the efficiency for each component of the propulsion train. The efficiencies can be determined when the consumed power and the produced power are calculated. With the definitions (2.1) to (2.6) the efficiencies of the ship propulsion train components are:

- $\eta_{\text{eng}} = \frac{P_{\text{eng}}}{P_{\text{fuel}}} = \frac{Q_{\text{eng}} \cdot \omega_{\text{eng}}}{C_v \cdot m} \quad (2.7)$
- $\eta_{\text{trans}} = \frac{P_s}{P_{\text{eng}}} = \frac{Q_p \cdot \omega}{Q_{\text{eng}} \cdot \omega_{\text{eng}}} \quad (2.8)$
- $\eta_{\text{prop}} = \frac{P_t}{P_s} = \frac{T_p \cdot u_a}{Q_p \cdot \omega} \quad (2.9)$
- $\eta_{\text{hull}} = \frac{P_t}{P_t} = \frac{R \cdot u}{T_p \cdot u_a} \quad (2.10)$

In this thesis we focus on the propeller efficiency. However, this does not mean that the propeller is taken out of its physical context, the ship. Actually, as the story progresses, there will be more and more references to the influences of other factors on the propeller efficiency.

As stated in the introductory chapter, the objective is propeller efficiency at full scale. This means that the quantities involved in relation (2.9), namely, thrust, torque, advance velocity, and angular velocity, need to be measured directly on board of the ship. This is easy said but not easy done. As far as we know, little research has been carried out, as reported in this thesis, to determine propeller efficiency at full physical scale. Thus, it is a challenge to determine propeller efficiency at full scale. To describe the context of the problem we first have to present the propeller efficiency in current practice. This is the topic of the next section.

2.2 Current practice

In this section we describe the standing point in practice and in research with respect to the efficiency of marine propellers. The propeller manufacturer designs and produces the propellers that equip the vessels operated by the ship operator. Research on the efficiency of propeller is traditionally done by testing scaled models in basins. With the advance of numerical methods and powerful computers, propellers are also tested in a virtual reality. Nowadays, Computational Fluid Dynamics (CFD) software packages are used to check the performance of a propeller.
design before entering the production, or even before a model test is performed. In fact, CFD software becomes a tool that is used also by the engineer in the propeller design phase. Thus, research is done either at specialized knowledge institutes or at the R&D departments of the propeller manufacturers.

### 2.2.1 Propeller designer

The objective of propeller designer is to maximize the percentage of consumed work that is converted by the propeller into useful work.

The information needed to design a propeller is listed below:
- Effective power, estimated from model test or other available data;
- Principal dimensions, proportions, and form coefficients for ship, to estimate wake and thrust deduction factors and other propulsion information
- Engine power and rated rpm
- Ship speed
- Restrictions, such as limit for the maximum propeller diameter

The necessary information to design a propeller is presented in all reference works on marine propellers, as for instance in (E.D. Lewis, 1989, vol. II, p. 202).

In the design phase special charts are used. These charts are related to long history of methodical experiments with propeller design, as for instance the B-screw series of MARIN (the Netherlands). The effective power and the design speed are fixed while the propeller diameter, angular velocity and pitch ratio are varied according to the charts to get the best propeller efficiency. These charts are presented, for instance, in (Lewis, 1989, vol II, pp. 192-201) and (Carlton, 2007, pp. 91-93). Alternatively, the circulation theory is applied to propeller design, see (Lewis, 1989, vol II, pp. 204-213) for a thorough history of this method.

The efficiency of propeller, in the case of a fixed pitch propeller, is best for one operational point, that means, a design power, a design speed, a wake field, a ship displacement, and a sea state. In the case of controllable pitch propeller several design points can be achieved. Thus, for the propeller manufacturer the propeller efficiency resembles an *energy label*, as mostly used in the marketing of domestic appliances. In practice, the ship operator is responsible for getting the best possible efficiency out of the propeller.

### 2.2.2 Ship operator

The ship operator is interested in minimizing the costs with operating the vessel. The propeller affects these costs, especially those related to fuel and maintenance. The more efficient a propeller is or the more efficient a propeller is operated the lower fuel consumption can be achieved.

The ship operator wants to buy efficient propellers and wants to operate the propeller in efficient manner. The propeller manufacturer delivers the propeller with a certain efficiency value, achievable at the design points. Thus, the best a
ship operator can do is to take the efficiency label given by the manufacturer for
granted, and, to try to sail the ship at design points where efficiency is the highest.

In practice the ship hardly ever sails exactly at its design points. This happens mainly
due to external factors and navigation constraints. It means that the skills
of the captain and his crew mainly determine how efficient the ship is operated
under such off-design situations.

A tool to measure the propeller efficiency, as well as the efficiency of other
propulsion train elements, does not yet exist. Such a tool can be imagined as a
decision support for the ship crew. The tool would require on board measurements
of all variables connected to the propulsion train powers, see Section 2.1. In
an optimization scheme, the propulsion controls (propeller angular velocity and
propeller pitch) would be optimized with respect to the new measured propulsion
conditions. The system would advice but the decision would be made by the ship
captain.

In literature there are attempts in this direction. In (Pivano, 2008) and (Blanke,
2009) a propeller efficiency optimization scheme is designed. The scheme controls
the propeller angular velocity such that the efficiency of the propeller is kept at the
design point. Such approach could be extended to the entire propulsion system,
with the goal of optimizing the fuel consumption and reduce engine and propeller
loads.

Besides the environmental factors and navigation restrictions, there are also con-
straints imposed by the International Maritime Organization (IMO). The interna-
tional body\(^1\) adopts regulations especially with respect to safety of navigation and
the protection of maritime environment. A way to respect to environment policies
is the improvement of propeller efficiency.

### 2.2.3 Model test

The model test is the traditional method to measure the behavior of a new design
of a propeller and of a hull. In naval architecture the model test is the equivalent
of laboratory experiments in the general context of physics. In fact, most of the
knowledge in naval architecture was gathered from model tests. The tests of scaled
versions of propellers and hulls are done in model basins. "Laws of similitude" are
used to transfer the physical object, the propeller or the ship, from full scale to
model scale. The reader can find details on the laws of similitude in (Lewis, 1989,
vol II, pp. 143-145) and (Carlton, 2007, pp. 89-94).

In this section we describe the method to determine the efficiency of a propeller
using scaled models. Three types of model tests are of particular interest for the
scope of this thesis: open water test, resistance test, and self-propulsion test. The
propeller efficiency is determined from open water test measurements. Due to
their relevance in a further chapter of the thesis we also consider the resistance

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\(^1\)The scope of the International Maritime Organization is publicly available at [http://www5.imo.org/SharePoint/blastDataOnly.asp?datacid=4527/english.pdf](http://www5.imo.org/SharePoint/blastDataOnly.asp?datacid=4527/english.pdf)
Open water test

The open water tests provide most of the knowledge on the performance of propellers, (Lewis, Vol. 2, 1989). Open water condition means that the down-scaled propeller spins without the hull ahead of it. Thus, the propeller spins in an undisturbed water flow. The diameter of the scaled propeller is in the range from 0.2 to 0.3 meters, (Carlton, 2007, p. 95). The propeller is mounted on a long shaft that is connected to a small underwater housing, see Figure 2.2 for a schematic. The underwater housing contains the mechanical transmission and the sensors. The entire setup is connected to a towing carriage.

![Figure 2.2: A schematic of the open water test setup. The propeller spins with an imposed angular velocity $\omega$, and the entire setup advances through water with the advance velocity $u_a$. The thrust $T_p$ and the torque $Q_p$ are measured by dynamometers.](image)

During the open water test an angular velocity $\omega$ is imposed on the scaled propeller. During one run the angular velocity and the advance velocity $u_a$ of the towing carriage are kept constant. The propeller thrust and torque are measured by thrust and torque balances, respectively, see (Carlton, 2007, p. 306), or by dynamometers. The open water tests are performed at different advance velocities. Measurements are recorded when all monitored variables are stabilized.

The results of open water tests are presented as open water diagrams; such diagram is depicted in Figure 2.3. These diagrams depict the dimensionless thrust and torque as functions of the advance ratio, a dimensionless quantity, for fixed angular velocities. The propeller efficiency $\eta_{prop}$ is calculated according to relation (2.9). Note that both thrust and torque decrease with the increase of the advance velocity. The efficiency increases with the advance velocity until a maximum; beyond this maximum the efficiency decreases drastically (not captured in Figure 2.3).

Resistance test

The main objective of the resistance test is to measure the resistance of the hull and to determine the effective power required at full scale. These tests are conducted
2.2. Current practice

![Graph](image)

**Figure 2.3:** A typical open water diagram. $J$ is the advance ratio, $KT$ is the dimensionless thrust, and $KQ$ is the dimensionless torque. The scales are removed due to confidentiality agreement.

with scaled models of the ship hull. The length of these scaled models varies between one and ten meters.

The scaled model is attached to a towing carriage, as schematically depicted in Figure 2.4. The model is ballasted and trimmed according to the displacement of the ship at full scale. Several runs are performed at different predefined velocities of the towing carriage. During one run the velocity of the carriage is kept constant. The resistance force of the model is measured by a balance, see (Carlton, 2007, p. 305), or by a dynamometer. Measurements are collected after the predefined speed is achieved. Often, additional observations are carried out during the resistance tests, such as, images of the wave profile and stream lines along the hull.

It is important to mentioned that the hull of the model is not fitted with a propeller or any other appendages, such as, rudder and fins. Thus, the measured resistance force is also called the bare hull resistance.

**Self-propulsion test**

In the self-propulsion test the scaled propeller and all other appendages are attached to the hull. The main objective of the self-propulsion tests is to predict the propulsive performance of the full scale ship.

During the self-propulsion test the propeller is driven by inboard electric motors. As in the resistance test, the hull is attached to the towing carriage through a dynamometer. The velocity of the model ship is imposed by the towing carriage. The propeller spins with predefined angular velocity. Thus, the dynamometer measures the difference between the effective thrust produced by the propeller and the total resistance force. This difference can be positive or negative depending on the relation between the carriage velocity and propeller angular velocity.

During the self-propulsion tests both propeller and hull are observed. Propeller
Propeller efficiency: definition, current practice, and challenges at full scale

Figure 2.4: A schematic of the resistance test setup. The bare hull model is connected to a carriage. The carriage moves with a predefined velocity. The towing force is measured by a balance or a dynamometer. The resistance of the bare hull, \( R \), is equal to the towing force.

Thrust and torque are measured by dynamometers. The difference between the propeller thrust and model resistance is measured by a dynamometer. From the self-propulsion tests both propeller and hull efficiency can be determined. The full scale powering performances are predicted by well established procedures, such as the ITTC Recommended Procedures and Guidelines (ITTC, 1999).

Figure 2.5: A schematic of the self-propulsion test setup. A scaled propeller is attached to the ship model. The ship model is connected to a carriage. The carriage moves with the advance velocity \( u_a \), towing the ship with force \( F_T \). The propeller is rotated by an inboard electric engine. The propeller thrust and torque are measured by dynamometers. The towing force \( F_T \) is needed to overcome the hull resistance force which at model scale is larger than at full scale.

The self-propulsion test is the closest to full scale ship operation among all model tests. Still, there are fundamental differences between the model scale and full scale conditions. Two of them are mentioned in the remarks below.

Remarks: The benefits of model tests in the research of marine propulsion are undisputable. In relation with the full scale performance of the ship it worth men-
tioning two aspects that cannot be dealt with in the model tests:

- **Scaling problems.** At full scale, Reynolds number is in the order of $10^7 - 10^9$ for the flow of water around the ship, and in order of $10^6$ for the water flow around the propeller blade. Such values of Reynolds number cannot be achieved at model scale. Thus, the friction force is proportionally larger at model scale than at full scale. Since it is not practical to replace the water by a less viscous fluid, the researchers had to invent procedures that reduces the effects of this scaling problem. For instance, the use of boundary layer stimulation devices to induce turbulence is advocated, see (Lewis, 1989, a, p. 154).

- **Prediction of performance in operational conditions.** Model scale tests are laboratory tests, in which the influence of external conditions is generally neglected. A ship at full scale hardly experiences the conditions under which she was tested in the model basin.

### 2.2.4 CFD simulations

With the progress of computational power and reliability of numerical methods, the Computational Fluid Dynamics (CFD) software became a tool to serve the research and, later, the design of marine propellers. The ultimate goal of the CFD engineer is to model the whole ship at full scale; that means phenomena such as, propeller dynamics, propeller-hull interactions, propeller cavitation, hull-wave interactions, and ship interaction with external factors. The simulation of open water propellers to obtain the open water diagram is a common procedure during the design phase. A general description of the process is given below.

The geometry of the full scale propeller is constructed in a specialized geometry modeling software from data points given by the propeller designer. In case of an open propeller the CFD engineer models only one blade. When the propeller interacts with other structures of the ship, such as, propeller nozzle, thruster, and rudder, the complete propeller is modeled. Then, around the propeller a calculation domain is created, where the flow is analyzed. The last step in building the geometry is the mesh generation. At this step the most important is the discretization of the blade surface and the boundary layer.

A numerical routine is used to solve the mathematical model that describes the flow around the propeller blades. Generally, the Reynolds-Averaged Navier-Stokes (RANS) equations are used to model the turbulent flow around the propeller. The numerical solver evaluates the RANS equations at each node point of the mesh using the finite volume method.

The inputs of the simulation are the advance speed, which is a boundary condition, and the propeller angular velocity, using the multi-frame of reference. The CFD engineer measures the normal and shear stresses on the surface of propeller blade. The integration of these stresses on the blade gives the axial, the radial, and the angular forces. The axial force is the propeller thrust. The propeller torque is
Propeller efficiency: definition, current practice, and challenges at full scale

calculated from the angular force.

The propeller thrust and torque are presented in dimensionless form similarly to model test data. The propeller open water efficiency is determined according to formula (2.9). In case of controllable pitch propellers, the simulation process described above is repeated for several pitch angles. The computation time of one open water diagram is about five hours. The computation time increases up to several days in case of simulation of ducted propellers.

2.3 Need for propeller efficiency at full scale

Propeller efficiency at full scale would be beneficial for the same guilds that currently use the concept of propeller efficiency: the propeller manufacturers, the ship operators, and the researchers. The ship operators need, probably, more than others the propeller efficiency at full scale. No doubt, the ship operators would like to have propeller efficiency determined during ship operation, like other propulsion parameters and indicators already displayed on their monitoring screens. Propeller efficiency could be a propulsion indicator used to:

- track physical changes, such as, fouling and damages, that affect the performance of propeller. The result could be a warning system that assists the ship operator to forecast and plan the ship maintenance.
- plan the optimal route of the vessel, from fuel consumption point of view. Propeller efficiency indicator could assist the ship crew to adapt the propulsion controls to current sea conditions, in light of fuel consumption too.

There are differences between the theoretical and the real-life in-service propeller efficiency. Quantitative information of these differences could be used as feedback to the propeller designer. The design of propellers either in general or for particular cases could be improved. Improved propeller design, in turn, brings benefits to the propeller manufacturer.

To researchers in the field of marine propulsion the propeller efficiency at full scale could clarify the differences between model tests predictions and full scale measurements. If factors that influence propeller efficiency at full scale are known then these factors could be used as input in model tests and CFD simulations for thorough analysis.

The reader must be aware that many challenges at full physical scale must be overcome to get the propeller efficiency. At the moment, adequate sensor technology and reliable methods represent obstacles to propeller efficiency at full scale. In the next section we present these challenges.

2.4 Challenges at full scale

The propeller works behind the ship, in an uncontrolled environment. This statement refers to the main challenges to estimate the propeller efficiency at full scale.
In this section we describe these challenges. They are both technical and interpretational.

Similarly to model tests and CFD simulations, calculation of propeller efficiency at full scale implies that the four quantities, namely, thrust, torque, angular velocity, and advance velocity, need to be measured. With the technical solutions available nowadays only three of the four quantities are measurable at full scale; the advance velocity is not. The reason is the wake field caused by the ship.

The propeller is attached at the stern of the ship. The propeller advances through the water with a velocity that is different from the velocity of the ship with respect to the surrounding water. Put differently, the velocity of water that enters the propeller plane is different from the velocity of water at considerable distance from the ship. The cause of this effect is the wake field that develops behind a ship in motion and disturbs the flow. The velocity field inside the wake is much different from the one outside the wake. Outside the wake, the longitudinal component of the water velocity is dominant; inside the wake, no component is dominant. At the expense of the longitudinal component of the velocity, the magnitude of the other two components increase due to the recirculating flow.

At present, there are no practical means to measure the advance velocity at full scale, on board of a ship. J.S. Carlton, see (Carlton, 2007, pp. 79-85), presents a list of instrumentation that may be used in wake field measurement. The traditional way to measure the wake field is the Pitot tube. This method has been used at full scale, as mentioned in (Carlton, 2007, p. 81) and the reference mentioned therein. Two other ways to measure the wake field are the hot-wire anemometry and the laser-Doppler anemometry. For full scale wake field measurements there are no applications known to the author. Laser-Doppler anemometry has been used to measure the wake field of model scale ships, as reported in (Tukker et al., 2000).

Thrust, torque, and angular velocity are measurable on board of a ship. Measuring them, however, is not an easy job. Of the three quantities the most difficult to measure at full scale is the propeller thrust; then comes the propeller torque. The angular velocity of the shaft is relatively easy to measure when compared to the other two. In the next few paragraphs we present the state-of-art in measuring these three quantities.

The angular velocity of the propeller is measured by sensors attached to the propeller shaft that, basically, count the number of shaft revolutions per unit of time. There are many commercially available solutions for this type of measurement. Typical devices are shaft encoders, proximity sensors, and photoelectric sensors. The shaft encoders give the highest reliability and resolution.

The measurement of the twist angle of the propeller shaft is proportional to the torque of the propeller. The following formula is used to calculate the shaft torque

\[
Q_p = \frac{\pi D^4 G \cdot \Delta \theta}{32 L}
\] 

(2.11)
where, $D$ is the shaft diameter, $G$ is the shear modulus of the shaft material, $\Delta \theta$ is the measured angle of twist, and $L$ is the distance over which the twist angle is measured. For a large vessel, the order of magnitude of the shaft deformation over a length of one meter is $10^{-1}$ degrees or $10^{-4}$ meters.

There are commercially available sensors to measure the twist angle, and with this the shaft torque. These sensors need to be mounted aboard on the shaft, as close as possible to the stern tube bearing. The value of shaft torque measured as such is close to the value of propeller torque, see (Lewis, 1989, vol. II, p. 130). In (Journée, 2003a, p. 17) the losses in the stern tube are estimated at about 0.75 percent of the engine power.

Propeller thrust measurements are difficult to perform. In fact, nowadays, there are no commercially available thrust sensors. The working principle of a thrust sensor would be similar to the working principle of the existing torque sensors: the thrust force is assumed to be linearly related to the axial shaft deformation

$$T_p = \frac{\pi D^2 E \cdot \Delta x}{4L}$$

where, $D$ is the shaft diameter, $E$ is the tensile modulus of the shaft material, $\Delta x$ is the deformation along the axis of the shaft, and $L$ is the distance over which $\Delta x$ is measured. The order of magnitude of the axial shaft deformation due to thrust over a length of one meter is $10^{-5}$ meters. Thus, the axial shaft deformation due to thrust are one order of magnitude smaller than the angular shaft deformations due to torque.

There is limited number of references to propeller thrust measurements at full scale. Journée (2003a, p. 32) reports thrust measurements at full scale on board of the M.V. Hollandia using strain gauges. The author mentions that "no higher accuracy than about 10 percent of the maximum thrust - so about 300 kN - can be expected". Similarly, thrust measurements by strain gauges are reported in (Hylarides, 1974), (Ligtelijn, 1988), and (Carlton, 2007, p.372).

Another technique, developed at VAF Instruments in the past five years, in close collaboration with Wärtsilä Netherlands, uses optical sensors to measure the nanometer-scale deformations. This optical sensor is still under development. A prototype of this sensor was used in this work to measure propeller thrust and torque. It appears that the precision, resolution, and robustness of the sensor are suitable for on-board installation. This sensor solves the bottleneck of measuring propeller thrust at full scale.

Apart from technical challenges there are interpretational challenges. In fact, the reader may ask: How do we interpret the propeller efficiency at full scale? We see three issues related to this question: the influence of external factors on the propeller efficiency, the existence of a propeller efficiency time-scale, and the lack of references at full scale.

That vessel dynamics is under the influence of external factors, such as, wind, waves, sea currents, water temperature, salinity, is no new information. T. Munk
2.4. Challenges at full scale

(Munk, 2006) showed that full scale monitoring of ship performance is affected by these factors. Recently, T.A. Dinham-Peren and I.W. Dand (Dinham-Peren, 2010) in their paper on the need of full scale measurements point out that external factors are responsible for large spreads in the measurement data. The authors proposed data correction and filtering based on known information. In a work related to this thesis Y. Meninato (Meninato, 2010) concluded that the external factors heavily influence the ship propulsion and navigation variables. In Figure 2.6 we depict the speed-through-water versus shaft power at similar propulsion input; data points represent stationary excerpts extracted from different trips of the vessel. The spread in the data points is mainly associated with the influence of external factors.

![Figure 2.6: Normalized shaft power versus normalized speed-through-water. Data points represent means of stationary excerpts selected from different trips of the vessel. The stationary excerpts are collected at similar propulsion controls, i.e., similar propeller angular speed and propeller pitch angle.](image)

The time-scale of propeller efficiency becomes an issue if measurement variations and physical changes of propeller surface are taken into consideration. In the short time-scale, measurement variations are taken into formula (2.9); in the long time-scale, propeller fouling and damages determines relative decrease of propeller efficiency. The two time-scales are schematically depicted in Figure 2.7. The short time-scale (order $10^0$ to $10^1$ seconds) is relatively easy to understand. To quantify long time-scale (order $10^6$ to $10^7$ seconds, or months to years) full scale measurements must be performed for long time periods. Thus, the question Which are the relevant time scales of propeller efficiency? arises.
The third issue is related to reference propeller efficiency values at full scale, or, better said, the lack of reference. As we have seen in Section 2.2.3 the values of open water propeller efficiency from model tests are susceptible to scaling errors. The CFD simulations are not yet capable of dealing with the propeller-hull interactions at full scale. At full scale, open water tests are hardly possible to perform. Thus, we are confronted with the lack of reference values of propeller efficiency. *Should the value of the propeller efficiency be the same at full scale and at model scale?* The answer to this question is not straightforward.
Chapter 3

Design aspects of propeller efficiency at full scale

In this chapter we present the general approach and the practical realizability of our design.

3.1 Approach

The design strategy is graphically represented by Figure 3.1. The blocks represent the basic principles we rely upon in the design: mathematical models, estimation of parameters, design of measurements, and statistical analysis of data.

![Figure 3.1: The schematic of strategy. The blocks represent principles we rely on: mathematical models, parameter estimation, full scale measurements, propulsion tests, and statistics.](image)

In the previous chapter we introduced the concept of propeller efficiency and
Design aspects of propeller efficiency at full scale

the current practice in model tests and CFD simulations. At full scale the main technical challenge is the advance velocity, which cannot be directly measured by means of sensors. The alternative to direct measurement is indirect measurement from a mathematical model. This mathematical model should contain states that are directly observable at full scale.

The mathematical models we elaborate in this thesis originate from ship guidance and control applications. The mathematical models are organized into two separate models: ship trajectory and ship propulsion. Both models are systems of differential algebraic equations. The general form of the models is

\[ M(x, \dot{x}, p, c) = 0 \] (3.1)

where \( x \) denotes the vector of states, \( p \) denotes the vector of model parameters, and \( c \) denotes the vector of propulsion controls. The ship trajectory model does not contain the vector of controls. In fact, the structure and the scope of the ship trajectory model differ very much from that of ship propulsion model.

In the ship propulsion model the states are related by physical laws and empirical relationships. The advance velocity is one of the model states. In propeller hydrodynamics community the advance velocity is assumed to be equal to the ship speed-through-water up to a factor. This factor, which is a model parameter, is the wake fraction. The other three quantities involved in propeller efficiency formula, namely, thrust, torque, and angular velocity, are among the states of the model.

Given a set of realistic model parameters and the propulsion controls applied in real-life sailing, the ship propulsion model is capable to mimic the ship motion in longitudinal direction*. The performance of our ship propulsion model is presented in Section 5.4 where we use stationary model test data and full scale measurements to verify the model.

*Propeller efficiency makes sense only when the trajectory of the ship is straight. Thus, our ship propulsion model takes only this motion into account.
3.1. Approach

The propulsion model is an ill-posed problem because some of the model parameters are unknown. Thus, to determine the advance velocity we need to identify the unknown model parameters. This brings us to the next step of the strategy, namely, the parameter estimation.

Well-posed problems are sometimes referred to as forward problems. Conceptually, the forward problem can be described as follows:

\[ \text{Model} + \text{Parameters} \rightarrow \text{Data} \]

Mathematical models predict the outcome of experiments. In an inverse problem we use the outcome of the experiment to operate changes inside the mathematical model. Conceptually, the inverse problem is defined as follows:

\[ \text{Data} + \text{Model} \rightarrow \text{Parameters} \]

The way the unknown parameters are distributed inside the model represents the challenge of the parameter estimation. There are model parameters that are coupled to each other inside one term so that we cannot make difference between them. With similar effects, other model parameters are coupled to states that are not measured. Thus, we need to design methods that isolate the unknown model parameters. These methods rely on the response of the ship dynamics, in terms of speed, thrust, torque, and angular velocity, to specific controls:

- **Stationary states.** The ship is brought in a stationary state by imposing stationary propulsion controls, i.e., propeller angular velocity and propeller pitch angle. In our definition, a process is stationary if the trend is close to zero; small variations around the trend are allowed. Stationarity is defined in Section 8.3.

- **Deceleration state.** The ship is decelerated from an initial speed by keeping the propeller thrust to zero. This manoeuvre can only be done if real-time thrust measurements are available to the captain.

- **Oscillatory state.** The propulsion dynamics is brought in an oscillatory state by small, harmonic excitations introduced by the control angular velocity. The frequency, or the frequency band, of the control angular velocity is established by means of simulations before applying on board of the vessel.
The data required by the parameter estimation step need to be collected from designed full scale propulsion tests. The main specification of the propulsion tests are:

- the list of signals to be recorded, including resolution, sampling frequency, accuracy, and source-sensor, and, the data collection system
- the external conditions, such as, weather conditions, water depth, and location
- the detailed protocol for each state. For stationary state we specify the ship velocity, the number of states; for deceleration state we specify the initial velocity and the end velocity, and the number of deceleration states; for oscillatory states we specify the stationary state around which the tests are performed, and the frequency and amplitude of the control angular velocity.

In practice, the full scale measurements are continuously recorded, whether during normal ship operations or during propulsion tests. Most of the needed sensors exist on board of most of the ships. However, the same quantity can be measured by different sensors with different reliability, on board of different ships.

Parameter estimation and propeller efficiency estimation require full scale reliable data of certain format. The data processing, in which we deal with measurement reliability and formatting, consists of the following steps:

- **Signal decomposition.** We regard the raw signals as composed of three components: the trend, the oscillation, and the noise. The signals are decomposed in these three components, each component having certain uses.

- **Signal reconciliation.** The method is designed to correct signals that are related by an underlying mathematical model. Using this method we correct the vessel speed measured by the Speed log.

- **Stationary state selection.** Propeller efficiency and most of the unknown parameters are determined when ship is in a stationary state. The stationary states are automatically selected from the data base of measurements.
In the implementation we use the measurements collected from the propulsion tests to estimate the unknown model parameters. We show that some of the model parameters depend on the external factors, such as wind speed and wind direction. In the implementation we make distinction between the propeller efficiency as required by the propeller manufacturer and the propeller efficiency as needed by the ship operator. The propeller manufacturer needs the absolute value of the propeller efficiency, that is to be compared with propeller efficiency from model tests and CFD simulations. The absolute propeller efficiency need to be determined from propulsion tests. The ship operator wants propeller efficiency for monitoring and maintenance purpose. The solution for the maintenance is reference and relative propeller efficiency at similar states.

To determine the relative propeller efficiency at full scale we need to clearly define the concept of similar states. Similar states are situations when ship propulsion controls are the same and the external factors are comparable. We assume that the relative propeller efficiency is the same for similar propulsion controls. Since the external factors have a stochastic nature we want to know how they influence the relative propeller efficiency at full scale.
3.2 Realizability

How much effort does it take to implement now the design of propeller efficiency in practice? The question is important for the industrial partners as well as for the ship operators. In this section we answer this question from the perspective of technological challenges. Thus, the section can be regarded as a technical feasibility of full scale propeller efficiency.

From the technical perspective, the structure of propeller efficiency design can be separated in three classes: software, hardware, and operation. In a feasibility study the three classes need to be thoroughly analyzed. Here we give the general picture alone:

1. The hardware can be split in two elements: the sensoring system and the data collection system.
   (a) The sensoring system comprises of the sensors that measure the ship behavior at full scale. The sensoring system is critical to propeller efficiency. In fact, the entire design relies on data measured by the sensors. Two aspects of sensors are very important at technical level: reliability and precision. Reliable and precise sensors are available on the market mainly because the sensor technology is driven by these two aspects. At different level, sensor availability is an important aspect. Most of the sensors required by the propeller efficiency design are available on board of most of the vessels. Thrust and torque sensor is currently available only for research applications. The availability of such a sensor does also depend on the application it would serve; propeller efficiency is an application that requires thrust and torque sensor. When sensors are not available on board of a vessel they must be supplied according to the requirements formulated in this thesis.
   (b) The data collection system roughly represents the hardware that collects data from sensors, supports the software for data processing, and communicates the information to the users. Reliability is the most important feature of the data collection system. System functionality comes second. Various technical solutions to build reliable and functional data collection system exist on the market. Two such systems designed according to requirements formulated in this thesis are operational, see Chapter 7.

2. The software refers to data processing, mathematical algorithms for parameter estimation, simulation, statistical analysis, Human-Machine Interface (HMI), and communication. In this thesis, the mathematical algorithms work off-line. In practice, all of the mathematical algorithms can be implemented on board of a vessel. The algorithms require data collection for a predetermined period of time, depending on the task they perform. The mathematical algorithms designed and implemented in this thesis are re-
liable. In Chapter 9 we calculate the precision of some estimates of the mathematical algorithms. All algorithms, except for the HMI and communication, are described in this thesis, at conceptual level. The practical implementation of these algorithms is straightforward.

3. The operation mainly refers to the propulsion tests. The propulsion test can be performed on board of any vessel if equipped with required sensors. The frequency, duration, and location of the propulsion tests depend on each vessel.

Other operational aspects refer to periodic calibration of the measurement system, as for instance, setting the zero values for sensitive quantities.
Chapter 4

Design of mathematical models

The estimation of propeller efficiency at full scale needs mathematical modeling mainly because the advance velocity cannot be directly measured. So, we are looking for a mathematical model that relates the advance velocity to other measured states, namely, propeller thrust and torque, ship velocity, and shaft angular velocity. At generic level, the mathematical model represents a mathematical environment in which solutions to various problems can be found. In this sense, the mathematical model structures the general approach.

This chapter is devoted to the development of a mathematical model for ship kinematics and ship dynamics. In the first section we define two coordinate systems, a local and a global system, to which the mathematical models are related. In the second section we present the ship trajectory model. This model fuses ship position, ship orientation, ship speed, and ship angular motions. The ship trajectory model creates a mathematical environment that is used in another chapter to correct the navigation measurements. In the third section we describe the mathematical model of ship propulsion. This mathematical model is inspired by literature on ship control, see (Blanke et al., 2000) and (Fossen, 2002). The ship propulsion model relates the advance velocity to other variables, such as, propeller thrust and torque, propeller angular velocity, and ship velocity. With the ship propulsion model we create a mathematical environment that facilitates a solution of the advance velocity problem. In the fourth section we discuss the model parameters introduced by the ship propulsion model.
4.1 Coordinate systems

The mathematical models are formulated with respect to two coordinate systems: the global North-East-Down (NED) coordinate system and the local body-fixed coordinate system. Both are suggested in Figure 4.1. The NED coordinate system is the curvilinear coordinate system defined with respect to the Earth surface. From this point onwards this system is denoted by the labels \((X, Y, Z)\). The positive \(X\)-coordinate indicates North, the positive \(Y\)-coordinate indicates East, and positive \(Z\)-coordinate points downwards normal to the Earth surface. We assume that the trajectory sailed by the ship is that short that the curvature of the Earth surface can be neglected; thus, the global coordinate system can be regarded as fixed. The global NED system is described by the frame \([\vec{E}_X, \vec{E}_Y, \vec{E}_Z]\).

We assume that the body-fixed frame has its origin in the ship’s center of gravity, \(C\); the location of the center of gravity depends on the loading condition of the ship; for simplification we assume that the center of gravity coincides with the point where the ship planes of symmetry intersect. The body-fixed frame is attached to the ship, thus moves with respect to the NED coordinate system. Its \(x\)-coordinate is along the longitudinal axis of the ship, pointing to the bow; its \(y\)-coordinate points to the starboard; and its \(z\)-coordinate points downwards. The body-fixed coordinate system is described by the frame \([\vec{e}_x, \vec{e}_y, \vec{e}_z]\).

A vessel has six degrees of freedom. In 1950, the Society of Naval Architects and Marine Engineers (SNAME) agreed on the terminology to be used for these six degrees of freedom, see (SNAME, 1950). The six variables are listed in Table 4.1, and, depicted in Figure 4.2.

The transformation from the body-fixed frame to the global-frame is done by
4.1 Coordinate systems

Table 4.1: List of ship positions and velocities adopted by SNAME in 1950

<table>
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<th>name</th>
<th>position</th>
<th>linear velocity</th>
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<td>surge</td>
<td>x</td>
<td>u</td>
</tr>
<tr>
<td>in y-direction</td>
<td>sway</td>
<td>y</td>
<td>v</td>
</tr>
<tr>
<td>in z-direction</td>
<td>heave</td>
<td>z</td>
<td>w</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>rotation</th>
<th>name</th>
<th>Euler angle</th>
<th>angular velocity</th>
</tr>
</thead>
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<td>about x-axis</td>
<td>roll</td>
<td>φ</td>
<td>p</td>
</tr>
<tr>
<td>about y-axis</td>
<td>pitch</td>
<td>θ</td>
<td>q</td>
</tr>
<tr>
<td>about z-axis</td>
<td>yaw</td>
<td>ψ</td>
<td>r</td>
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</tbody>
</table>

the rotation matrix, \( R(\Theta) \). The matrix \( R(\Theta) \) is called Euler angle rotation matrix, with vector \( \Theta = [\phi, \theta, \psi] \). In (SNAME, 1950) the orientation of the body-frame relative to the global-frame is described by the \( zyx \)-convention. This convention is described as follows: translate the global-frame \( XYZ \) until its origin coincides with the origin of the body-frame, \( xyz \). Let this be the initial orientation, \( x_0y_0z_0 \). Rotate the body-frame about the \( z_0 \)-direction by the angle \( \psi \) such that the axes \( x_0 \) and \( y_0 \) assume the intermediate position \( x_i \) and \( y_i \). Rotate the body-frame about the new axis \( y_i \) by an angle \( \theta \), such that \( z_0 \) becomes \( z_i \) and \( x_i \) becomes \( x \). Rotate the body-frame about the \( x \)-axis by an angle \( \phi \) such that the axes \( z_i \) and \( y_i \) become \( z \) and \( y \), the final positions. At each step the rotation matrices

\[
\begin{align*}
R(\psi) &= \begin{pmatrix}
cos \psi & -sin \psi & 0 \\
sin \psi & cos \psi & 0 \\
0 & 0 & 1
\end{pmatrix} \\
R(\theta) &= \begin{pmatrix}
cos \theta & 0 & sin \theta \\
0 & 1 & 0 \\
-sin \theta & 0 & cos \theta
\end{pmatrix} \\
R(\phi) &= \begin{pmatrix}
1 & 0 & 0 \\
0 & cos \phi & -sin \phi \\
0 & sin \phi & cos \phi
\end{pmatrix}
\end{align*}
\]

are used.

Thus, for the \( zyx \)-convention, the rotation sequence is mathematically expressed by the rotation matrix \( R(\Theta) \)

\[
R(\Theta) := R(\psi) \ R(\theta) \ R(\phi)
\]

or, in expanded form,

\[
R(\Theta) =
\begin{pmatrix}
cos \psi \cos \theta & -sin \psi \cos \phi + \cos \psi \sin \theta \sin \phi & \sin \psi \sin \phi + \cos \psi \cos \phi \sin \theta \\
sin \psi \cos \theta & cos \psi \cos \phi + sin \psi \sin \theta \sin \phi & \cos \psi \sin \phi + \sin \psi \cos \phi \sin \theta \\
-sin \theta & cos \psi \sin \phi & \cos \theta \sin \phi
\end{pmatrix}
\]
The transformation from local-frame to global-frame is carried out according to
\[
\begin{pmatrix}
    \mathbf{E}_X \\
    \mathbf{E}_Y \\
    \mathbf{E}_Z
\end{pmatrix}
= \mathbf{R}(\Theta)
\begin{pmatrix}
    \mathbf{e}_x \\
    \mathbf{e}_y \\
    \mathbf{e}_z
\end{pmatrix}
\tag{4.6}
\]

The transformation from global-frame to local-frame is done according to
\[
\begin{pmatrix}
    \mathbf{e}_x \\
    \mathbf{e}_y \\
    \mathbf{e}_z
\end{pmatrix}
= \mathbf{R}(\Theta)^{-1}
\begin{pmatrix}
    \mathbf{E}_X \\
    \mathbf{E}_Y \\
    \mathbf{E}_Z
\end{pmatrix}
\tag{4.7}
\]
where, \( \mathbf{R}(\Theta)^{-1} = \mathbf{R}(\Theta)^T \).

**Approximation for surface vessels**

The orientation of surface vessels can be approximated with three degrees-of-freedom, namely, two translations, surge and sway, and one rotation, yaw. This implies that heave motion and pitch and roll angles are considered relatively small and are ignored completely. This is a useful simplification of relation (4.6); it is generally assumed that such relation holds for most of the surface vessels in calm water, see (Fossen, 2002, p. 29) and (Lewis, 1989, Vol. 3, p. 193).

For the three degrees-of-freedom motion the rotation matrix \( \mathbf{R}(\Theta) \) is replaced by \( \mathbf{R}(\psi) \), see relation (4.1), where \( \psi \) is the angle between \( \mathbf{e}_z \) and \( \mathbf{E}_X \) directions, see Figure 4.3; \( \psi \) is called heading. The transformation from local to global-frame is carried out according to
\[
\begin{align*}
 \mathbf{E}_X &= \mathbf{e}_x \cos \psi - \mathbf{e}_y \sin \psi \\
 \mathbf{E}_Y &= \mathbf{e}_x \sin \psi + \mathbf{e}_y \cos \psi \\
 \mathbf{E}_Z &= \mathbf{e}_z
\end{align*}
\tag{4.8}
\]
4.2 Ship trajectory

In the NED coordinate system the position of the ship’s center of gravity is described by the position vector $\mathbf{C}$ with

$$\mathbf{P}_C = X_C \mathbf{E}_X + Y_C \mathbf{E}_Y$$  \hspace{1cm} (4.9)

where $(X_C, Y_C)$ are the coordinates of ship’s center of gravity in the global-frame.

![Figure 4.3: The 3 DOF approximation for surface vessels. The heading is the angle between the $X$-coordinate and the $x$-coordinate. The position vector $\mathbf{P}_C$ represents the position of ship center of gravity (COG) in global-frame.](image)

### 4.2 Ship trajectory

**Trajectory of the center of gravity**

Once the position is defined the idea of motion follows naturally. In classical mechanics the velocity is defined as the rate of change of the position with respect to time. In our particular case, the position of the ship in global coordinate system is represented by $\mathbf{P}_C$; thus, the velocity of the ship is

$$\mathbf{u}_g = \frac{d}{dt} \mathbf{P}_C$$  \hspace{1cm} (4.10)

where the components of the position vector are defined in (4.9).

The ship velocity $\mathbf{u}_g$ is called the velocity-over-ground, due to the fact that it is measured with respect to Earth. Its magnitude is called the speed-over-ground. Velocity-over-ground in the global frame is defined by

$$\mathbf{u}_g = U_g \mathbf{E}_X + V_g \mathbf{E}_Y$$  \hspace{1cm} (4.11)

where the coordinates $(U_g, V_g)$ are the components of the velocity-over-ground in the global-frame.

In the local-frame, the velocity-over-ground is defined by

$$\mathbf{u}_g = u_g \mathbf{e}_x + v_g \mathbf{e}_y$$  \hspace{1cm} (4.12)
where the coordinates \((u_g, v_g)\) are the components of the vector in the local-frame.

If we denote the velocity-over-ground in the global-frame by \(u_g^g\), and the velocity-over-ground in local-frame by \(u_g^l\), then they satisfy the following relation

\[
u_g^g = R(\psi) \ u_g^l \tag{4.13}\]

In general, the ship position and the ship velocity-over-ground are measured by two different sensors. What is more important, ship position is measured in global-frame and velocity-over-ground is measured in local-frame by these sensors. Thus, we write that

\[
\frac{d}{dt} P_C = R(\psi) \ u_g
\]

which we call the equation of ship trajectory.

The ship trajectory is often expressed in discrete time by the corresponding difference equation, see (Fossen, 2002, p. 26)

\[
P_C(k+1) = P_C(k) + T_s \ R(\psi(k)) \ u_g(k) \tag{4.15}\]

where \(T_s\) denotes the sampling time and \(k\) denotes the sampling index.

**Ship motions and measurements**

Our intention is to use the ship trajectory Equation (4.14) as a mathematical model in a method to correct the measured speed-over-ground. In this method, which is described in Section 8.2, direct measurements of ship position, velocity-over-ground, and Euler angles are input. The GPS antenna, the Speed log, and the Gyrocompass are not installed in the center of gravity of the ship and not even on the same position. For large vessels the distance from the center of gravity to the point where position and speed are measured is considerable, see Figure 4.4 for a schematic of situation on board of the vessel. Under external influences the angular motions increase to the point where their influence on the measurements cannot be neglected. These influences are presented in Chapter 7.

We consider the particular case of ship position measured by the GPS receiver that is located on top of the bridge. For large vessels, the distance from the center of gravity to the GPS receiver can be 100 meters, or more. Given the accuracies\(^*\) and resolutions\(^\dagger\) of the state-of-art GPS receivers, the motion of the antenna is picked up by the sensor. When the vessel sails in rough water the amplitude of the angular motions (roll, pitch, and yaw) significantly increase, and, thus the surface

\(^*\)The accuracy of a marine GPS receiver is in range from 1 to 5 meters. Sub-meter accuracies are achieved by the DGPS (Differential GPS) when aided by ground-based reference stations. Such ground-based stations are the Wide Area Augmentation System (WAAS) and the European Geostationary Navigation Overlay Service (EGNOS).

\(^\dagger\)In general, the GPS receiver reads the geographical position every second.
vessel assumption is not valid anymore. When the deviations of the antenna are in the same order of magnitude as the accuracy of the GPS receiver then the position measurements are affected. In Figure 4.5 we suggest the influence of the angular motion on the measured ship trajectory. Due to the motion of the GPS receiver the trajectory of the GPS antenna is longer than the trajectory of the center of gravity. Additionally, due to this GPS motion, velocity-over-ground calculated from the geographical position, according to Equation (4.10), shows large and unnatural variations. To overcome this practical problem we must take into account the position of the sensors on board of the ship.

Figure 4.5: The apparent trajectory (dashed line) compared to the real trajectory (solid line). The height of the GPS antenna and the ship motions are exaggerated for better schematic representation.
Trajectory model

In practice, the ship position measurements are much more affected by external distortions than the speed and heading measurements. This is enforced by the fact that modern GPS receivers are able to measure the position of the ship accurately. We assume that

- the yaw motion has no influence on the ship position, which means that the GPS is located above the center of gravity
- the Speed log and the Gyrocompass are located in the center of gravity of the ship
- the roll and pitch angular motions are small, but are not ignored as in the surface vessel approximation; instead we replace $\cos(\phi)$ and $\cos(\theta)$ by 1, and $\sin(\phi)$ and $\sin(\theta)$ by $\phi$ and $\theta$, respectively.

Figure 4.6 schematically depicts the position of the GPS antenna under the approximation of the surface vessel motion. The position of the center of gravity in global frame is $P_C$. The position of the GPS antenna in global frame measured by the GPS receiver is $P_A$. The relative position of the GPS antenna with respect to the center of gravity, in global frame, is $P_{CA}$. The relationship between the three position vectors is

$$ P_C = P_A - P_{CA} \quad (4.16) $$

where $P_A$ and $P_{CA}$ are expressed in global frame as

$$ P_A = X_A \mathbf{E}_X + Y_A \mathbf{E}_Y \quad (4.17) $$

and

$$ P_{CA} = X_{CA} \mathbf{E}_X + Y_{CA} \mathbf{E}_Y \quad (4.18) $$

respectively. With $P_A$ directly measured by the GPS receiver, we need to express the relative position of the antenna, $P_{CA}$, in order to calculate the position of the center of gravity, required in Equation (4.14).

The position vector $P_{CA}$ is affected by ship pitch and roll angular motions. Figure 4.7 schematically depicts the effect of these motions on the GPS antenna. Given the distance, $h_A$, of the GPS antenna from the center of gravity, the relative position vector $P_{CA}$ can be written as

$$ P_{CA} = \begin{pmatrix} X_{CA} \\ Y_{CA} \end{pmatrix} = R(\psi) \begin{pmatrix} x_{CA} \\ y_{CA} \end{pmatrix} = R(\psi) h_A \begin{pmatrix} \theta \\ \phi \end{pmatrix} \quad (4.19) $$

where $x_{CA}$ and $y_{CA}$ are the projections of the position vector $P_{CA}$ on the $x$- and $y$-axis of the local frame; $\theta$ is ship pitch angle, and $\phi$ is the roll angle. Thus, the position of the center of gravity is calculated from the relation

$$ \begin{pmatrix} X_C \\ Y_C \end{pmatrix} = \begin{pmatrix} X_A \\ Y_A \end{pmatrix} - R(\psi) h_A \begin{pmatrix} \theta \\ \phi \end{pmatrix} \quad (4.20) $$
4.3 Ship propulsion

The main elements of the ship propulsion train are presented in the diagram of Figure 4.8, which we reproduced from (Izadi-Zamanabadi and Blanke, 1999). The ship propulsion is controlled by the angular velocity of the shaft, \( \omega_{\text{set}} \), fixed by the captain. If the vessel is fitted with a controllable pitch propeller then the propeller pitch, \( \theta_p \), is controlled in a separate loop. The captain can choose to set the angular velocity or to set the pitch angle. The engine governor regulates the rotational speed of the engine by controlling the amount of fuel, \( Y \), fed into the engine. In this thesis we do not consider the engine governor. The engine produces a torque, \( Q_{\text{eng}} \), that rotates the shaft with an angular velocity, \( \omega \). The losses in the mechanical transmission due to friction are accounted by the friction torque \( Q_f \). The shaft rotates the propeller with an effective torque, \( Q_p \); due to its particular geometry the propeller generates a force in the normal direction to the
propeller plane, \( T_p \). The thrust \( T_p \) pushes the hull through the water with velocity \( u \).

\[ Q_{\text{net}} = Q_{\text{eng}} - Q_f - Q_p = 0 \]  

(4.21)

During transient states the net torque applied to the shaft determines the rate of change of the angular momentum, \( L = L \mathbf{e}_s \), where \( \mathbf{e}_s \) is in the direction of the shaft

\[ Q_{\text{net}} = \frac{dL}{dt} \]  

(4.22)

Figure 4.8: Schematic of the ship propulsion train.

In this section we present the equations of the ship propulsion model. The model is taken from literature in the field of ship propulsion control; see for example (Yoerger et al, 1991), (Healey et al, 1995), (Izadi-Zamanabadi and Blanke, 1999), (Blanke et al, 2000), (Fossen and Blanke, 2000), and the literature mentioned therein. In light of the goal of this thesis, which is propeller efficiency at full scale, we do not model the governor separately as done in the literature; also, it is combined with the engine dynamics. The engine dynamics is not a scope of this thesis, but we need to include it because of its relationship with the angular velocity, \( \omega \), and the control, \( \omega_{\text{set}} \). The elements of the propulsion train are modeled by first order ordinary differential equations. We obtain a closed system by adding algebraic relations to the system. The result is a system of differential-algebraic equations, where the algebraic equations are explicit. The four ordinary differential equations describe the dynamics of the propulsion elements. The three algebraic equations represent constitutive relations for propeller thrust and torque, and, for the advance velocity. In the following subsections we present these seven equations as mentioned.

4.3.1 Shaft

The power generated by the engine is transmitted to the propeller via a mechanical transmission that consists of shafts, gears, and clutches, and that varies from ship to ship. We simplify the mechanical transmission to one shaft that is subject to engine torque, \( Q_{\text{eng}} \), propeller torque, \( Q_p \), and friction torque, \( Q_f \). At equilibrium the net torque, \( Q_{\text{net}} \), is zero

\[ Q_{\text{net}} = Q_{\text{eng}} - Q_f - Q_p = 0 \]  

(4.21)

During transient states the net torque applied to the shaft determines the rate of change of the angular momentum, \( L = L \mathbf{e}_s \), where \( \mathbf{e}_s \) is in the direction of the shaft

\[ Q_{\text{net}} = \frac{dL}{dt} \]  

(4.22)
4.3. Ship propulsion

For a rigid body rotating around its axis of symmetry the angular momentum is defined as \( L = I \omega \), where the \( I \) represents the total moment of inertia and \( \omega \) is the shaft angular velocity. Replacing the net torque from Equation (4.21) in Equation (4.22) we get

\[
I \frac{d\omega}{dt} = Q_{\text{eng}} - Q_f - Q_p
\]  

(4.23)

The total moment of inertia, \( I \), includes the moment of inertia of the propeller shaft, of the propeller, and of the gearbox. The additional inertia due to propeller added mass is not taken into account; see Wereldsma (1965) for more details on propeller added mass.

Engine torque and propeller torque from Equation (4.23) are measurable at full scale. Due to the challenges posed by direct measurements at full scale, see Section 2.4, only propeller torque is directly measured. A model of the engine torque is introduced in Section 4.3.2; a constitutive model for the propeller torque is introduced in Section 4.3.5.

The friction torque, \( Q_f \), can be indirectly measured from specific controlled tests. During such tests, the angular velocity of the shaft, \( \omega \), is imposed and engine and propeller torque are directly measured. The friction torque is then computed from Equation (4.23). In (Pivano, 2008) the author sets up a model scale test to determine the friction torque as aforementioned. L. Pivano relates the friction torque computed as such to shaft angular velocity, \( \omega \), in a mathematical model. A similar friction torque model is presented in (Kim et al., 2005).

In this thesis we do not measure the friction torque. The friction torque is modeled by a general model as the static friction (Coulomb friction) plus the viscous friction. The viscous friction is a function of the angular velocity, \( \omega \), and has two components: a nonlinear component, which acts at low angular velocities \( \omega \), and a linear component for other angular velocities. In this thesis the nonlinear component of the friction is disregarded. The mathematical model for friction torque that we use in this thesis is thus given by

\[
Q_f = Q_0 + q_s \omega
\]  

(4.24)

where \( Q_0 \) is the startup torque and \( q_s \) is the linear friction coefficient. For a survey of friction models we suggest (Armstrong-Hélouvry et al., 1994). Literature sources, such as (Pivano et al., 2009), suggest that the losses due to friction torque are less significant at full scale than at model scale. In our analysis we use \( Q_0 \) and \( q_s \) as fit parameters.

In practice, the shaft angular velocity is measured in revolutions per second, \( \text{rps} \), or per minute, \( \text{rpm} \). To keep the mathematical model consistent with the measurements the angular velocity, \( \omega \), is translated into rotations per second, \( n \),

\[
2\pi I \frac{dn}{dt} = Q_{\text{eng}} - 2\pi q_s n - Q_0 - Q_p
\]  

(4.25)
4.3.2 Engine

Mathematical modeling of the control and dynamics of marine diesel engines can be as complex as desired, see (Izadi-Zamanabadi and Blanke, 1999) and (Schulten, 2005) for two notable examples. In light of the objectives of this thesis we opted for a simple model of the diesel engine. We looked for a simple model that relates the engine torque, $Q_{\text{eng}}$, to the difference $\Delta \omega = \omega_{\text{set}} - \omega$, where $\omega_{\text{set}}$ is the desired shaft angular velocity and $\omega$ is the resulting shaft angular velocity. We decided to model the control by

$$Q_{\text{eng}} = k_{\text{eng}}(\omega_{\text{set}} - \omega)$$  \hspace{1cm} (4.26)

where $k_{\text{eng}}$ is the proportional gain. In literature such control is referred to as proportional control. More enhanced control models are the PID controllers, where derivatives and/or integrals of the difference $\Delta \omega$ are introduced. The full scale measurements carried out during this project indicate that in stationary state the difference between the $\omega_{\text{set}}$ and $\omega$ range between 0.5 and 1 percent of $\omega_{\text{set}}$; in practice we take $\Delta \omega = 0.01 \omega_{\text{set}}$.

For interpretational reasons we factor the model parameter $k_{\text{eng}}$ into two model parameters, $t_{\text{eng}}$ and $q_{\text{eng}}$, such that

$$\frac{1}{t_{\text{eng}}} Q_{\text{eng}} = q_{\text{eng}}(\omega_{\text{set}} - \omega)$$  \hspace{1cm} (4.27)

where $t_{\text{eng}}$ represents a time constant and $q_{\text{eng}}$ represents a proportionality constant, dimensionally equivalent to the torque.

When the desired angular velocity changes, the actual angular velocity needs to react in a specified time interval. In the transient state we introduce the engine model as

$$\frac{dQ_{\text{eng}}}{dt} + \frac{1}{t_{\text{eng}}} Q_{\text{eng}} = q_{\text{eng}}(\omega_{\text{set}} - \omega)$$  \hspace{1cm} (4.28)

The reaction time is controlled by the time constant $t_{\text{eng}}$; the larger the values of $t_{\text{eng}}$ the longer the time to reach the stationary state. Note that Equation (4.28) can be explicitly solved.

---

‡ Mathematical models that come closer to real processes that happen in engine controls and dynamics are proposed by Izadi-Zamanabadi and Blanke in (Izadi-Zamanabadi and Blanke, 1999). The authors model the engine governor and the engine dynamics separately. An extra variable, the fuel index, is introduced. The fuel index represents the amount of fuel the governor lets in the engine. The authors pay special attention to the limitations in the fuel index to avoid overloading of the diesel engine. The engine dynamics is modeled as a transfer function that accounts for the time between the cylinder firings and the slow build up of cylinder pressure. This engine modeling approach is used also in the thesis of Ø. Smogeli and L. Pivano, see (Smogeli, 2006) and (Pivano, 2008), respectively.

§ A detailed model of a marine diesel engine is described in the thesis of P. Schulten, see (Schulten, 2005). The author models in detail the processes that happen in a marine diesel engine, as for instance, the cylinder processes, the gas exchanges at the cylinder, the fuel pumps, the air filter, the air compressor, the turbine, and many more. A similar approach can be found in (Vrijdag, 2009).
To be consistent with the shaft model, see Equation (4.25), we replace the angular velocity $\omega$ by rotational speed, $n$, measured in rotations per second. The engine model becomes

$$\frac{dQ_{\text{eng}}}{dt} = -\frac{1}{t_{\text{eng}}}Q_{\text{eng}} + 2\pi q_{\text{eng}}(n_{\text{set}} - n) \tag{4.29}$$

### 4.3.3 Propeller

Ship propeller develops thrust by accelerating the water in its control volume, (Lewis, 1989, Vol. 2). In 1889 Robert Edmund Froude introduced the concept of propeller actuator disc, see (Froude, 1889). Froude’s theory was a continuation of the momentum theory of William J.M. Rankine, see (Rankine, 1865). The simplifying assumptions of the momentum theory refer to inviscid, incompressible, and irrotational flow; and, velocity and static pressure are uniform over each cross-section of the actuator disc and the stream tube.

Let us consider the fluid domain schematically depicted in Figure 4.9. The water enters the fluid domain through section 1 with velocity $u_a$, passes through actuator disc, section 2, with velocity $u_p$, and exits at section 3 with velocity $u_w$. The static pressures at inlet and outlet are equal, $p_1 = p_3$. According to Froude’s theory, the propeller is a mechanism capable of inducing a sudden increase of pressure to the fluid passing through it, $\Delta p$, as suggested in Figure 4.9. Integrating the pressure $\Delta p$ over the actuator disc area, $A_p$, gives the thrust

$$T_p = A_p (p_{2,d} - p_{2,u}) \tag{4.30}$$

where $p_{2,u}$ and $p_{2,d}$ are the static pressures on the front side and on the back side of the disc, respectively.

The time-independent linear momentum theorem applied on the fluid domain yields

$$T_p = \rho Q_w u_w - \rho Q_a u_a \tag{4.31}$$

where $\rho$ is the mass density of water, and $Q_a$ and $Q_w$ are the flow rates in section $a$ and $w$, respectively. For an incompressible flow the continuity equation states that the flow rates at various sections in the considered fluid domain are equal, $Q_a = Q_w = Q$. The flow rate in the actuator disc is $Q = A_p u_p$. Thus, Equation (4.31) becomes

$$T_p = \rho A_p u_p(u_w - u_a) \tag{4.32}$$

Equating relations (4.30) and (4.32) gives

$$p_2 - p_1 = \rho u_p(u_w - u_a) \tag{4.33}$$
Figure 4.9: Schematic of the general momentum theory of propellers. The propeller is represented by an actuator disc that induces a sudden increase of pressure in the fluid. The motion equation of the water mass in the propeller control volume is derived from the momentum theorem and Bernoulli principle.

The simplifying assumptions allow us to apply Bernoulli’s principle upstream of the actuator disc

\[ \frac{1}{2} \rho u_a^2 + p_1 = \frac{1}{2} \rho u_p^2 + p_{2,u} \]  \hspace{1cm} (4.34)

and downstream of the actuator disc

\[ \frac{1}{2} \rho u_w^2 + p_3 = \frac{1}{2} \rho u_p^2 + p_{2,d} \]  \hspace{1cm} (4.35)

Subtracting (4.34) from (4.35) and taking into account that \( p_1 = p_2 \), yields

\[ p_{2,d} - p_{2,u} = \frac{1}{2} \rho (u_w^2 - u_a^2) \]  \hspace{1cm} (4.36)

The result (4.36) is used in Equation (4.33) to derive the relationship between the velocities in the three sections

\[ u_p = \frac{1}{2} (u_w + u_a) \]  \hspace{1cm} (4.37)

In (Carlton, 2007), the author expresses the velocities \( u_p \) and \( u_w \) in terms of velocity \( u_a \), \( u_p = (1 + a) u_a \) and \( u_w = (1 + a_1) u_a \), with \( a \) and \( a_1 \) the axial flow parameters at the propeller disc and at the downstream section \( w \). Plugging these
two expressions in the result (4.37) results in \( a_1 = 2a \). This means that half the acceleration of the flow takes place before the actuator disc and the remaining half after the actuator disc. In (Fossen, 2002) this result is used to define the steady state ratio \( \bar{a}_a/\bar{a}_p = 1/(1 + a) \), which means that in a time-independent situation \( u_p \) and \( u_a \) are interchangeable, given the axial flow parameter \( a \).

Expressing \( u_w \) from (4.37) in terms of \( u_p \) and \( u_a \) and plugging the result in the momentum Equation (4.32) yields

\[
T_p = 2\rho A_p u_p(u_p - u_a)
\]

(4.38)

This is the stationary state propeller equation used in this thesis.

During transient regimes, i.e., change in thrust \( T_p \) or change in the income velocity \( u_a \), the inertia of the mass of water in the control volume is added

\[
m_w \frac{du_p}{dt} = T_p - 2\rho A_p u_p(u_p - u_a)
\]

(4.39)

where \( m_w \) denotes the mass of water in the control volume. In (Whitcomb et al, 1999) and (Fossen et al, 2000) the mass of water \( m_w \) is expressed as \( \gamma \rho A_p L \), with \( \gamma \) as the coefficient of the added mass, which is determined empirically, and \( L \) a characteristic length. In this thesis the characteristic length is taken as the length of an imaginary duct that is fitted to the propeller, see Figure 4.10 for a schematic. In practice, \( L \) is half the propeller diameter. The water mass is modeled according to

\[
m_w = \frac{\pi}{8} \rho D^3
\]

(4.40)

where the coefficient \( \gamma \) is neglected. The control volume of a propeller is schematically depicted in Figure 4.10.

![Figure 4.10: The propeller control volume is defined based on the dimensions of the nozzle of a ducted propeller. The control volume is the volume of the cylinder of diameter \( D \) and length \( D/2 \) less the volume of the propeller itself.](image)
The Equation 4.39 concerns the dynamics of water around the propeller, thus, it is of great importance for the propeller efficiency calculation. Two of the quantities involved in propeller formula (2.9), the propeller thrust \( T_p \) and the advance velocity \( u_a \), are present in this equation. In literature, Equation (4.39) is used in the control of the underwater and the surface vessels, see (Yoerger et al., 1990), (Cody, 1992), (Healey et al., 1995), and (Blanke and Fossen, 2000)\(^4\).

### 4.3.4 Surge

The ship moves through water due to the thrust generated by the propeller. At stationary state, the propeller thrust, \( T_p \), is balanced by the resistance force, \( R \), in the longitudinal direction

\[
T_p = R \quad (4.41)
\]

The ship motion in longitudinal direction is called surge.

If the ship is fitted with multiple propellers the total thrust is the sum of the forces generated by each propeller, \( T_i = \sum T_{p,i}, i = 1, ..., N \), where \( N \) is the number of propellers. In case of ducted propellers the nozzle produces a significant amount of thrust, \( T_n \), see (Carlton, 2007, pp. 15-17). The total thrust produced by the ducted propeller is the sum \( T_t = T_p + T_n \).

Modeling the resistance of a ship is complex due to numerous components into which resistance can be separated. In (Carlton, 2007) the components of the resistance force are grouped into calm water resistance, air resistance, propeller-augmented resistance, rough water resistance, and restricted water resistance. The last two components of the resistance force represent special situations and are not taken into account, in light of full scale propeller efficiency estimation.

The calm water resistance is the dominant resistance term. The main components of the calm water resistance are the naked hull skin friction resistance, the appendage skin friction resistance, and the wave making resistance, see (Carlton, 2007). In Section 2.2.3 we described the resistance tests as a well-established tests procedure in the model basins; in fact, it is the oldest form of model testing. The problem with full scale ship resistance is that it cannot be directly measured. Thus, we need to rely on mathematical models.

We are looking for a resistance model that:

\(^4\)The equation was refined by M. Blanke and T.I. Fossen in (Blanke and Fossen, 2000). Their equation is design to take into account velocities \( u_p \) and \( u_a \) of opposite direction. Additionally, M. Blanke and T.I. Fossen include a linear term in \( u_p \) that ensures the convergence of the solution of the equation at low speed, as in

\[
mw \frac{d u_p}{dt} = T_p - df_0 \ u_p - df \ |u_p| (u_p - u_a)
\]

where \( df = 2 \rho A_p \) and \( df_0 = mw / T_f \) are regarded as design parameters, with \( T_f \) a time constant. This version of equation is used in other references, see (Kim et al., 2005), (Smogeli, 2006), and (Pivano, 2008).
4.3. Ship propulsion

- incorporates full scale measured quantities,
- is generic,
- and, is reliable.

The resistance model that fulfills the three conditions stated above happens to be the oldest mentioned one. In 1889 R.E. Froude published a famous paper in which he proposed a relation between the total resistance force and the ship velocity. The relation, which was empirically deduced from testing planks of various length and surface finishes, stated that

\[ R = fSV^n \]  \hspace{1cm} (4.42)

where, \( V \) is the velocity of the model, \( S \) is the wetted area of the model, \( f \) is a coefficient that varies with length and roughness of the model, and the index \( n \) had the constant value 1.825.

The resistance model we design is inspired by Froude’s model. The resistance \( R \) is modeled as a power function of the ship velocity

\[ R = p \frac{u}{u_0}^{\gamma-1} u \]  \hspace{1cm} (4.43)

where \( u \) is the ship velocity, \( u_0 \) is a characteristic velocity, necessary for the consistency of dimensions, and \( p \) and \( \gamma \) are the model parameters. The resistance model (4.43) is a generic in the sense that it models the total resistance force of the ship.

In practice, the ship experiences various external conditions, and, thus, the total resistance force varies. The variation in resistance force is accounted by the model parameters \( p \) and \( \gamma \). Figure 4.11 depicts two situations that could be encountered in real life. Let us define a reference state with a corresponding reference resistance force (which is not the ideal situation). The ship is likely to experience external conditions that are rougher than the conditions of a reference state. Naturally, rougher external conditions mean higher total resistance force (dotted line). The mathematical model (4.43) deals with the increased resistance force by changing the model parameters \( p \) and \( \gamma \), such that a new resistance curve results. The opposite effect takes place when the external conditions are calmer (dashed line) than the conditions of the reference state.

The resistance model (4.43) is also known as the bare hull resistance, i.e., the hull without the propeller. When the hull is towed a high-pressure area forms at the stern of the ship. When the propeller rotates behind the ship the water at the stern of the hull is accelerated and, thus, the high-pressure area is disturbed. Lower pressure in the area behind the ship means additional resistance force. This additional resistance force is called the propeller-augmented resistance, and, it is modeled as fraction of the propeller thrust

\[ R_a = t_d T_p \]  \hspace{1cm} (4.44)

where, \( t_d \) is called the thrust deduction fraction.
Figure 4.11: The resistance model accounting for states with external conditions different than the conditions of a reference state.

Summing up the resistance models (4.43) and (4.44) and plugging into the stationary surge equation, see Equation (4.45), we get that

\[
(1 - t_d) \ T_p = p \ \frac{u}{u_0}^{\gamma-1} u
\]

(4.45)

where \(1 - t_d\) is called the thrust deduction factor, see (Lewis, Vol. 2, 1989). The term \(1 - t_d\) \(T_p\) is called the effective thrust, \(T_E\).

During the acceleration and deceleration intervals the ship inertia comes into play

\[
(m_s + m_a) \ \frac{du}{dt} = (1 - t_d) \ T_p - p \ \frac{u}{u_0}^{\gamma-1} u
\]

(4.46)

where \(u\) is the ship velocity in the longitudinal direction, which is called the speed-through-water; \(m_s\) is the mass of the ship, and \(m_a\) is the added mass. The concept of added mass is introduced due to the fact that the "force required to accelerate a body in a fluid is always larger than the product of the actual mass of the body times its acceleration", see (Lewis, 1989, vol. 3, p. 198). In fact, the added force is the hydrodynamic force that appears due to the acceleration of the body in the fluid. The concept of added mass is detailed in (Fossen, 2002, pp. 65-70).

The Equation (4.46) is used in the present work to model the motion of the ship in longitudinal direction. The surge models that are used in literature are similar to the one we design. Faltinsen and Sortland in (Faltinsen and Sortland, 1987) model the resistance force as the sum of the nonlinear quadratic drag and the linear laminar skin friction. In (Blanke et al., 2000) this approach is used to model the resistance of small underwater vehicles at low velocities.
4.3. Thrust and torque

The propeller torque $Q_p$ present in the shaft model, see Equation (4.25), and the propeller thrust $T_p$ present in the propeller model, see Equation (4.39), are state variables. Thus, the system made of Equations (4.29), (4.25), (4.39), and (4.46) is underdetermined. Separate models for $T_p$ and $Q_p$ need to be introduced.

Introduction

The blade-element theory, and the more general circulation theory, explain the mechanisms by which propeller thrust and torque are generated. These two theories are detailed, for instance, in (Lewis, Vol. 2, 1989) and (Carlton, 2007). Results of theories of propeller action are impossible to apply for practical problems as, for instance, to determine propeller efficiency at full scale. For this type of practical problems empirical thrust and torque models are best suitable. The empirical thrust and torque models are derived from model tests called open water tests.

Open water tests represent the main source of information on the performance of propellers, as already mentioned in Section 2.2.3. During such a test, constant angular velocity and the advance velocity are imposed upon the propeller. Thrust and torque are directly measured. The test is repeated for different advance velocities, keeping the angular velocity constant. Such tests reveal certain relationships between the thrust, torque, angular velocity, and the advance velocity. Changes in propeller geometry and in the physical properties of the water also determine changes in thrust and torque.

By now, open water test is the traditional method to study propellers. As with any tradition there are usual practices associated with open water tests. A practice connected to the empirical model is the dimensionless representation of the thrust and torque.

Dimensional analysis

The dimensional analysis of propeller thrust and torque is a topic of every book on marine propellers, as, for instance, in (Lewis, Vol. 2, 1989) and (Carlton, 2007). In this subsection the main ideas of the dimensional analysis, as described in (Lewis, Vol. 2, 1989), are mentioned.

Propeller thrust is function of water density, propeller diameter, advance velocity, gravitational acceleration, rotational speed, viscosity, and pressure

$$T_p = f(\rho, D, \alpha, \omega, \rho_f, \mu)$$

(4.47)

The relation is written using proper dimensions as following

$$\frac{ML}{T^2} = \left(\frac{M}{L^3}\right)^a \left(\frac{L}{T^3}\right)^b \left(\frac{L}{T^2}\right)^c \left(\frac{1}{T}\right)^d \left(\frac{M}{LT^2}\right)^e \left(\frac{M}{LT}\right)^f$$

(4.48)
The exponents of both sides of relation (4.48) are equated and represented as follows

$$
\begin{align*}
    a &= 1 - f - g \\
    b &= 2 + d + e - g \\
    c &= 2 - 2d - e - 2f - g
\end{align*}
$$

(4.49)

The exponents $a$, $b$, and $c$ are plugged in relation (4.47) to obtain

$$
T_p = \rho D^2 u_a^2 f \left( \left( \frac{gD}{u_a^2} \right)^d, \left( \frac{nD}{u_a} \right)^e, \left( \frac{\rho u_a^2}{\rho D u_a} \right)^f, \left( \frac{\mu}{\rho D u_a} \right)^g \right)
$$

(4.50)

This relation is dimensionless

$$
\frac{T_p}{\frac{1}{2} \rho D^2 u_a^2} = f \left( Fr, J, \sigma, Re \right)
$$

(4.51)

where the arguments of the function are dimensionless numbers: Froude’s number, the advance ratio, the cavitation number, and Reynolds number. This relation states that if the dimensionless numbers have the same values for two geometrically different propellers then the ratio $T_p/\frac{1}{2} \rho D^2 u_a^2$ is the same. This statement is important in the case of model scale testing. Cavitation number and Reynolds number are difficult to scale down since it is difficult to scale the atmospheric pressure and the water viscosity, respectively. For regular model testing these numbers are neglected.

The dimensionless ratio

$$
c_T = \frac{T_p}{\frac{1}{2} \rho D^2 u_a^2}
$$

(4.52)

is called thrust coefficient. A similar approach for propeller torque leads to torque coefficient

$$
c_Q = \frac{Q_p}{\frac{1}{2} \rho D^3 u_a^2}
$$

(4.53)

The advance velocity can be equal to zero during bollard pull tests and during startup of the ship; thus, coefficients $c_T$ and $c_Q$ tend to infinity. These two particular situations restrict the use of thrust and torque coefficients. The problem is solved if the equivalent dimensionless representations are used

$$
K_T = \frac{T_p}{\rho n^2 D^4}
$$

(4.54)

$$
K_Q = \frac{Q_p}{\rho n^2 D^5}
$$

(4.55)
The coefficients $K_T$ and $K_Q$ are the most used dimensionless representation of thrust and torque. For instance, the results of the open water tests are always presented as $K_T$ and $K_Q$ curves. If the cavitation and viscous effects are neglected in relation (4.51) then the dimensionless coefficients $K_T$ and $K_Q$ are functions of the advance ratio $J$

$$K_T = f(J), \quad K_Q = g(J)$$

(4.56)

where

$$J = \frac{u_a}{nD}$$

(4.57)

The advance ratio is interpreted as the distance the propeller travels for one complete revolution.

**Constitutive model**

The results of the open water tests are represented in terms of dimensionless thrust and torque as functions of the advance ratio. These are called open water diagrams, see Figure 4.12.

![Typical open water diagram of a fixed pitch propeller](courtesy of Wärtsilä Netherlands).

In the first quadrant, which is positive thrust and torque and positive advance ratio, the measured $K_T$ and $K_Q$ data points are very close to linear behavior. Naturally, the empirical thrust and torque constitutive equations are linear

$$K_T = \alpha_1 J + \alpha_0$$

(4.58)

$$K_Q = \beta_1 J + \beta_0$$

(4.59)

where, $\alpha_0 > 0$, $\alpha_1 < 0$, $\beta_0 > 0$, and $\beta_1 < 0$ are constant parameters. The values of the constant parameters are found from the regression of the measured data points. At bollard pull condition, that is when the advance velocity is zero, thus
Design of mathematical models

\( J = 0 \), the propeller thrust and torque reach the maximum values, \( K_{T, \text{bollard}} = \alpha_0 \) and \( K_{Q, \text{bollard}} = \beta_0 \). \( K_T \) and \( K_Q \) decrease with the increase in \( J \).

In full dimensions the thrust and torque constitutive equations are

\[
T_p = \rho D^3 \alpha_1 n u_a + \rho D^4 \alpha_0 n^2
\]

\( (4.60) \)

\[
Q_p = \rho D^4 \beta_1 n u_a + \rho D^5 \beta_0 n^2
\]

\( (4.61) \)

Remark: The propeller efficiency can be expressed in terms of dimensionless quantities as follows

\[
\eta_{\text{prop}} = \frac{J}{2\pi} \frac{K_T}{K_Q}
\]

\( (4.62) \)

Constitutive model of a CPP

The acronym CPP stands for controllable pitch propeller. The blades of a CPP rotate around their long axis to vary the pitch angle. The variable pitch is used as another control of the ship propulsion system, besides the shaft speed. The advantage of ship fitted with a controllable pitch propeller over a ship with a fixed pitch propeller is improved manoeuvrability due to fine tuning of propeller thrust. Nowadays, about 35% of the operational ships are fitted with CPP, see (Carlton, 2007, p. 21). The ships mentioned in Chapter 7 are fitted with controllable pitch propellers.

The thrust and torque of a CPP are functions of the advance ratio and the pitch angle

\[
K_T = f(J, \theta_p), \quad K_Q = g(J, \theta_p)
\]

\( (4.63) \)

The open water diagram of controllable pitch propeller is a surface, see Figure 4.13. The thrust and torque constitutive equations for a CPP are

\[
K_T = (\alpha_{11} \theta_p + \alpha_{12}) J + (\alpha_{01} \theta_p + \alpha_{02})
\]

\( (4.64) \)

\[
K_Q = (\beta_{11} \theta_p + \beta_{12}) J + (\beta_{01} \theta_p + \beta_{02})
\]

\( (4.65) \)

where \( \theta_p \) is the pitch angle, and, \( \alpha_{ij} \) and \( \beta_{ij} \) are constant parameters, satisfying \( (\alpha_{11} \theta_p + \alpha_{12}) < 0, (\beta_{11} \theta_p + \beta_{12}) < 0, (\alpha_{01} \theta_p + \alpha_{02}) > 0, \) and \( (\beta_{01} \theta_p + \beta_{02}) > 0 \).

Sometimes, pitch deflection \( \Delta \theta \), which is the change in pitch angle relative to a reference value, is used instead the absolute pitch angle.

The dimension full thrust and torque constitutive equations are

\[
T_p = \rho D^3 (\alpha_{11} \theta_p + \alpha_{12}) n u_a + \rho D^4 (\alpha_{01} \theta_p + \alpha_{02}) n^2
\]

\( (4.66) \)

\[
Q_p = \rho D^4 (\beta_{11} \theta_p + \beta_{12}) n u_a + \rho D^5 (\beta_{01} \theta_p + \beta_{02}) n^2
\]

\( (4.67) \)
4.3. Ship propulsion

The open water diagram of a CPP. The tuples $[K_T, J, \Delta \theta]$ and $[K_Q, J, \Delta \theta]$ represent three dimensional surfaces.

4.3.6 Advance velocity

The advance velocity is closely related to the concept of wake field. In fact, the advance velocity constitutive relation results from one of the approximations of the wake field.

Wake field

The wake field occurs behind a solid object when there is a relative motion between the object and the fluid. The flow in the wake of the object has a complex structure due to fluid recirculation zones. An important aspect in case of propellers working in the wake field of the hull is that the longitudinal component of the wake velocity is somewhat smaller than the longitudinal velocity of the ship. The longitudinal component of the wake velocity is the advance velocity.

The wake field has major implications upon the performance of the propeller. Lower advance velocity means lower efficiency, according to definition (2.9). It is mainly due to the wake field that full scale propeller efficiency is a challenge.

In ship hydrodynamics community, the wake field is separated into two components: the nominal wake that is the wake field in the absence of the propeller, and, the effective wake that is the wake field when the propeller is present and produces thrust. The nominal wake is, in turn, separated into potential wake, frictional wake, and wave-induced wake. Model scale measurements indicate that the wake field encountered by the propeller has a mean component and a fluctuating component, see (Carlton, 2007). The mean component is an important aspect in propeller design. Based on historical records of wake field from model scale measurements several regression models were designed. These regression models take into account various geometrical aspects of the hull form. For instance, the regression model designed by K.E. Schoenherr takes into account dozen of ship...
Design of mathematical models

coefficients

\[ \dot{w} = 0.10 + 4.5 \frac{C_{pv}C_{ph}B}{(7 - C_{pv})(2.8 - 1.8C_{ph})} + \frac{1}{2} \left( \frac{E}{T} - \frac{D}{B} - k\eta \right) \]  

(4.68)

where \( L \) is the ship length, \( B \) is the ship breadth, \( T \) is the ship draught, \( D \) is the propeller diameter, \( E \) is the height of the propeller shaft above the keel, \( C_{pv} \) is the vertical prismatic coefficient of the ship, \( C_{ph} \) is the horizontal prismatic coefficient of the ship, \( k \) is a coefficient of the stern shape, and \( \eta \) is the rake angle of the propeller. Several other regression models, more or less extensive compared with the example above, are summarized in (Carlton, 2007).

In calculations related to propeller efficiency the wake field influence is represented by a parameter called wake fraction, \( w \). R.E. Froude and D.W. Taylor proposed two models to estimate the wake fraction. In both models the advance velocity and the ship velocity are present. Both models regard the wake fraction as the ratio of two velocities. The difference between the two models is the reference velocity. Froude’s model takes the advance velocity \( u_a \) as the reference

\[ w_F = \frac{u - u_a}{u_a} \]  

(4.69)

while Taylor’s model takes the ship velocity \( u \) as the reference

\[ w_T = \frac{u - u_a}{u} \]  

(4.70)

The model proposed by D.W. Taylor is preferred in the ship hydrodynamics community. In this thesis the model proposed by D.W. Taylor is used to retrieve the advance velocity constitutive relation.

Constitutive relation

The constitutive equation for the advance velocity is retrieved from Taylor’s definition of the wake fraction

\[ u_a = (1 - w) \ u \]  

(4.71)

This relation states that the advance velocity is a fraction of the ship velocity. The wake fraction number, \( w \), strongly depends on the shape of the hull. Thus, every ship has a unique wake field. In the field of ship design there are several regression models to estimate the wake fraction of a particular ship based on hull coefficients, see (Carlton, 2007, pp. 69-71).

The wake field, and, thus, the wake fraction, are different from one ship to another, as indicated by the regression formula (4.68). In case of a particular ship several questions could be posed: Is the wake fraction a constant parameter or a time-dependent variable? In this thesis the wake fraction is assumed constant for stationary ship velocity. Model test results indicate that the wake fraction changes with ship velocity.
4.4 Summary of the ship propulsion model

The equations of the ship propulsion model are summarized below:

\[
\frac{dQ_{\text{eng}}}{dt} = -\frac{1}{t_{\text{eng}}} Q_{\text{eng}} + 2\pi q_{\text{eng}} (n_{\text{set}} - n) \tag{4.72}
\]

\[
2\pi I_t \frac{dn}{dt} = Q_{\text{eng}} - 2\pi q_s n - Q_0 - Q_p \tag{4.73}
\]

\[
m_w \frac{du_p}{dt} = T_p - 2\rho A_p (u_p - u_a) \tag{4.74}
\]

\[
(m_s + m_a) \frac{du}{dt} = (1 - t_d) T_p - p \left| \frac{u}{u_0} \right|^{\gamma - 1} u \tag{4.75}
\]

\[
0 = -T_p + \rho D^3 \alpha_1 n u_a + \rho D^4 \alpha_0 n^2 \tag{4.76}
\]

\[
0 = -Q_p + \rho D^4 \beta_1 n u_a + \rho D^5 \beta_0 n^2 \tag{4.77}
\]

\[
0 = -u_a + (1 - w) u \tag{4.78}
\]

The ship propulsion model is a system of differential-algebraic equations (DAE), with four differential equations and three algebraic equations. The model has seven states, \(Q_{\text{eng}}, n, u_p, u, T_p, Q_p, u_a\), and 18 parameters, which are listed in Table 4.2.

All quantities involved in the propeller efficiency formula (2.9), namely, \(n, u_a, T_p, Q_p\), are also states of the ship propulsion model. \(T_p\) is related to \(u_a\) in a differential equation, and, to \(u_a\) and \(n\) by a constitutive equation. Similarly, \(Q_p\) is related to \(n\) by a differential equation, and to \(u_a\) and \(n\) by a constitutive equation. The thrust and torque constitutive relations have the same structure, which indicates a linear relationship between the two, see (Smogeli, 2006).

The advance velocity \(u_a\) and the axial flow velocity \(u_p\) are coupled in one term of the propeller Equation (4.74). The fact that the quantity we are looking for, namely \(u_a\), is linked to a quantity that cannot be directly measured at full scale represents a complication. The advance velocity constitutive Equation (4.71) does not solve this problem but transforms it: the advance velocity is replaced by the ship velocity, which is measurable at full scale, and the wake fraction, which is unknown at full scale. This fact, however, has a fundamental impact on the overall strategy. The propeller efficiency formula (2.9) becomes

\[
\eta_{\text{prop}} = \frac{T_p u (1 - w)}{2\pi Q n} \tag{4.79}
\]

in which the wake fraction appears explicitly. So, to determine propeller efficiency we need to estimate the wake fraction at full scale.

The wake fraction is one of the model parameters listed in Table 4.2. The model parameters can be analyzed from several perspectives. There are engine and shaft parameters, namely, \(t_{\text{eng}}, q_{\text{eng}}, I_t, q_s, \) and \(Q_0\); propeller parameters, namely, \(m_w, \rho, A_p, w, \alpha_0, \alpha_1, \beta_0, \) and \(\beta_1\); and ship parameters, namely, \(m_s, m_a, t_d, p, \) and
Design of mathematical models

γ. Most important, there are known\(^\|$ parameters and unknown parameters. The unknown parameters at full scale are \(w, t_d, p, \gamma, \alpha_0, \alpha_1, \beta_0, \) and \(\beta_1\). These model parameters and the methods to estimate them are the main focus of the coming chapter of the thesis.

Table 4.2: List of model parameters

<table>
<thead>
<tr>
<th>parameter</th>
<th>name</th>
<th>dimension</th>
<th>status</th>
</tr>
</thead>
<tbody>
<tr>
<td>(t_{eng})</td>
<td>time constant</td>
<td>[s]</td>
<td>known</td>
</tr>
<tr>
<td>(q_{eng})</td>
<td>proportionality constant</td>
<td>[N · m]</td>
<td>known</td>
</tr>
<tr>
<td>(I_t)</td>
<td>total moment of inertia</td>
<td>[kg · m(^2)]</td>
<td>known</td>
</tr>
<tr>
<td>(Q_0)</td>
<td>startup torque</td>
<td>[N · m]</td>
<td>known</td>
</tr>
<tr>
<td>(q_s)</td>
<td>linear friction coefficient</td>
<td>[kg · m(^2) · s(^{-1})]</td>
<td>known</td>
</tr>
<tr>
<td>(m_w)</td>
<td>mass of water in propeller control volume</td>
<td>[kg]</td>
<td>known</td>
</tr>
<tr>
<td>(\rho)</td>
<td>water density</td>
<td>[kg · m(^{-3})]</td>
<td>known</td>
</tr>
<tr>
<td>(A_p)</td>
<td>propeller disc area</td>
<td>[m(^2)]</td>
<td>known</td>
</tr>
<tr>
<td>(m_a)</td>
<td>ship mass</td>
<td>[kg]</td>
<td>known</td>
</tr>
<tr>
<td>(t_d)</td>
<td>thrust deduction</td>
<td>--</td>
<td>unknown</td>
</tr>
<tr>
<td>(p)</td>
<td>resistance coefficient</td>
<td>[kg · s(^{-1})]</td>
<td>unknown</td>
</tr>
<tr>
<td>(\gamma)</td>
<td>resistance coefficient</td>
<td>--</td>
<td>unknown</td>
</tr>
<tr>
<td>(\alpha_0, \alpha_1, \beta_0, \beta_1)</td>
<td>propeller characteristics</td>
<td>--</td>
<td>unknown</td>
</tr>
<tr>
<td>(w)</td>
<td>wake fraction</td>
<td>--</td>
<td>unknown</td>
</tr>
</tbody>
</table>

The methods to estimate the unknown parameters are the subjects of Chapter 6. Before the parameter estimation we analyze the propulsion model. The analysis of the ship propulsion model is the topic of the next chapter. There, we dimensionalize the mathematical model, we calculate the stationary solution of the model, and we analyze the time scales of the model.

\(^\|$Actually, we assume they are known. The parameters with certain values are \(A_p\), due to the fact that the diameter of propeller is known, and the water density, \(\rho\). Limited information is known about \(m_a, m_w, I_t, \) and \(m_w\). The parameters \(t_{eng}, q_{eng}, q_s, \) and \(Q_0\) are assumed such that they fit the measurements.
Chapter 5

Analysis of the ship propulsion model

In this chapter, we focus on the ship propulsion model presented in Section 4.4. The ship propulsion model is a system of differential-algebraic equations (DAE), and can be written as

\[
\frac{dx_1}{dt} = f(x_1, x_2, p, c) \quad (5.1)
\]

\[
x_2 = g(x_1, p)
\]

where the states are \(x_1 = [Q_{eng}, n, u, u]\) and \(x_2 = [T_p, Q_p, u]\), the model parameters are \(p = [\text{eng}, q_{eng}, I, Q_0, q_s, m_w, p, A_p, m_s, m_a, t_d, p, \gamma, \alpha_0, \beta_0, \beta_1, \omega]\), and the control is \(c = n_{set}\). The DAE system (5.1) is a semi-explicit DAE of index 1; it is a system of ordinary differential equations (ODE) with constraints.

For a general DAE, the index provides information about the structure of the equation and potential complications in the solution of the DAE. The index is the minimum number of differentiations of the DAE system in order to obtain an ODE system. In fact, the index measures the distance from the the DAE to its related ODE; thus, the smaller the index of the DAE system the closer to an ODE system. Details on differential-algebraic systems, their indices, and ways to solve them are available in (Kunkel, 2006). The index of the system (4.72) to (4.78) is 1 because \(\partial g/\partial x_2\) is nonsingular, i.e., \(\det(\partial g/\partial x_2) \neq 0\).

Plugging equation \(x_2\) in the ordinary differential part of the DAE system (5.1) results in

\[
\frac{dx_1}{dt} = h(x_1, p, c) \quad (5.2)
\]

which is an ODE system.

In this chapter we analyze the ship propulsion model from the perspective of time scales. In the first section we dimensionalize the mathematical model. There
we learn that the mathematical model can be dimensionalized with respect to two fundamentally different time scales. In the second section we compute the stationary state of the ship propulsion model; we show that there is only one feasible solution of the stationary model. In the end of the second section we compare the solutions of the mathematical model with direct measurements from model scale tests. In the third section we analyze the time scales of the mathematical model when a step change in the control is imposed. The computed eigenvalues indicate the time it takes the system to reach the equilibrium state. In the fourth section we validate the ship propulsion model from qualitative point of view. The full scale measurements are the starting point. We simulate the acceleration of the ship from an initial constant speed. The computed time-dependent solutions are compared to the full scale measurements.

5.1 Dimensionalization

To derive a dimensionless form of the ship propulsion model we introduce independent and dependent dimensionless variables

\[ \hat{t} = \frac{1}{t_c} t; \quad \hat{n} = \frac{1}{n_c} n; \quad \hat{u}_p = \frac{1}{n_c D} u_p; \quad \hat{u} = \frac{1}{n_c D} u; \quad \hat{u}_0 = \frac{1}{n_c D} u_0; \quad \hat{u}_a = \frac{1}{n_c D} u_a; \]

\[ \hat{T}_p = \frac{1}{\rho D^4 n_c^2} T_p; \quad \hat{Q}_p = \frac{1}{\rho D^5 n_c^2} Q_p; \quad \text{and} \quad \hat{Q}_{eng} = \frac{1}{\rho D^5 n_c^2} Q_{eng} \]

The scales \( n_c \) and \( t_c \) are a characteristic rotational speed and a characteristic time, respectively. We fix a characteristic rotational speed \( n_c \). The characteristic time, \( t_c \), is still free to choose. The scales for the other variables are related to a characteristic speed, \( n_c D \), water mass density, \( \rho \), and propeller diameter, \( D \). Notice that the scaling of speed, thrust, and torque is similar to the scaling that is being used in the open water diagrams, see Section 4.3.5.

A dimensionless formulation of our mathematical model can be presented with respect to two time scales:

1. a time scale that is proportional to the water mass flow through the propeller,

\[ t_c = \frac{m_w}{\rho D^4 n_c} \]

2. a time scale that is proportional to the mass of water displaced by the ship,

\[ t_c = \frac{m_s + m_a}{\rho D^4 n_c} \]
5.1. Dimensionalization

$10^2 - 10^3$ times larger than the first time scale, introduced by (5.4). The time scales represent time windows through which the dynamics of the ship propulsion system is viewed. Setting the first time scale means that we look at the ship dynamics through a narrow time window; only the phenomena that are characteristic for this time scale are visible. Phenomena that have a long time scale characteristic are considered stationary for a short time scale window. For the second time scale we get the opposite effect. Short time scale effects are noise (high frequency variations) in the long time horizon of ship propulsion dynamics.

The two time scales are related as follows

$$t_{c,\text{long}} = \mu \cdot t_{c,\text{short}}$$  \hspace{1cm} (5.6)

where the scaling factor $\mu$ is the ratio of the water masses,

$$\mu = \frac{m_s + m_a}{m_w}$$  \hspace{1cm} (5.7)

Typical values for the scaling factor are between 200 and 800; for instance, for the container ship and the ferry mentioned in Chapter 7, $\mu$ is 720 and 330, respectively.

The following dimensionless model parameters result from the dimensionalization

$$\tau_{\text{eng}} = \frac{t_c}{t_{\text{eng}}}; \quad \zeta_{\text{eng}} = \frac{2\pi q_{\text{eng}} t_c}{\rho D^5 n_c}; \quad \zeta_q = \frac{D^5 n_c}{2\pi I t}; \quad \zeta_s = \frac{q_s t_c}{T}; \quad \varphi = \frac{p}{\rho D^3 n_c \hat{u} \gamma}$$  \hspace{1cm} (5.8)

Parameters $\tau_{\text{eng}}, \zeta_{\text{eng}}, \zeta_q$, and $\zeta_s$ are dependent of the time scale $t_c$; parameter $\varphi$ is independent of $t_c$.

Remark: Model parameters such as wake fraction, $w$, thrust deduction, $t_d$, resistance parameters, $\gamma$ and $\varphi$, and propeller characteristics, $[\alpha_0, \alpha_1, \beta_0, \beta_1]$, are subject to parameter estimation methods in Chapter 6 and Chapter 9. Knowing that parameter estimation methods are sensitive to scale, we selected the scales such that these parameters are order 1. The scales were verified by simulations.

Short time-scale representation

With the short time scale, the time-dependent part of the mathematical model becomes

$$\frac{d\hat{Q}_{\text{eng}}}{dt} = -\tau_{\text{eng}} \hat{Q}_{\text{eng}} - \zeta_{\text{eng}} \hat{u} + \zeta_{\text{eng}} \hat{n}_{\text{set}}$$  \hspace{1cm} (5.9)

$$\frac{d\hat{n}}{dt} = \zeta_q \hat{Q}_{\text{eng}} - \zeta_q \hat{n} - \zeta_q \hat{Q}_a - \zeta_q \hat{Q}_p$$  \hspace{1cm} (5.10)

$$\frac{d\hat{u}_p}{dt} = \hat{T}_p - \frac{1}{2} \pi \hat{u}_p (\hat{u}_p - \hat{u}_a)$$  \hspace{1cm} (5.11)

$$\frac{d\hat{u}}{dt} = \frac{1}{\mu} \left[ (1 - t_d) \hat{T}_p - \varphi \hat{u} \gamma \right]$$  \hspace{1cm} (5.12)
The first three model equations are represented in the short time-scale. The right hand side of the fourth model equation (5.12) is weighted by the scaling factor $\mu$, and, thus, the ship speed, $\hat{u}$ appears stationary when visualized through the short time-scale.

**Long time-scale representation**

With the long time-scale the time-dependent part of the mathematical model becomes

$$\frac{1}{\mu} \frac{dQ_{eng}}{dt} = -\tau_{eng} \hat{Q}_{eng} - \zeta_{eng} \hat{n} + \zeta_{eng} \hat{n}_{set} \tag{5.13}$$

$$\frac{1}{\mu} \frac{dn}{dt} = \zeta_{Q} \hat{Q}_{eng} - \zeta_{s} \hat{n} - \zeta_{Q} \hat{Q}_{0} - \zeta_{Q} \hat{Q}_{p} \tag{5.14}$$

$$\frac{1}{\mu} \frac{du_{p}}{dt} = \hat{T}_{p} - \frac{1}{2} \pi \hat{u}_{p} (\hat{u}_{p} - \hat{u}_{a}) \tag{5.15}$$

$$\frac{d\hat{u}}{dt} = (1 - \eta) \hat{T}_{p} \varphi \hat{u} \tag{5.16}$$

In the long time scale representation of the mathematical model the magnitude of the differential terms $dQ_{eng}/dt$, $dn/dt$, and $du_{p}/dt$ is weighted down by $1/\mu$, so, these terms only contribute if the variations in $Q_{eng}$, $n$, and $u_{p}$ are very high. Only the long time scale phenomena, such as ship acceleration and deceleration, are visible in this time window; the other variables are merely noise.

From mathematical perspective the constitutive relations

$$0 = -\hat{T}_{p} + \alpha_{1} \hat{n} \hat{u}_{a} + \alpha_{0} \hat{n}^{2} \tag{5.17}$$

$$0 = -\hat{Q}_{p} + \beta_{1} \hat{n} \hat{u}_{a} + \beta_{0} \hat{n}^{2} \tag{5.18}$$

$$0 = -\hat{u}_{a} + (1 - w) \hat{u} \tag{5.19}$$

do not change when the time scale changes. In practice, this is an open question.

The choice of the time-scale has impact on the setup of full scale measurements. Practically, the choice of time scale dictates the choice of sampling frequency of the full scale measurements. From the point of view of data processing, the short time scale is required. Short time scale means that measurements are collected at high sampling rate. This choice provides the complete picture of the ship propulsion dynamics. Long time-scale phenomena become visible when the measurements are collected for periods of time that are longer than the long time scale. This requires possibility of data storage. In applications, the data measured at high sampling frequency are stored parametrically. Of course, when the measurement sampling frequency is selected according to the long time-scale the short time-scale phenomena become invisible and irretrievable.
5.2 The stationary state model

Note: From this point onwards we only use the dimensionless formulation of the ship propulsion model. For ease of reading we skip the "hat" notation on top of the dimensionless state variables. The eventual use of dimension full notations is mentioned.

5.2 The stationary state model

For a stationary control $c = \bar{c}$ the ship propulsion model (5.1) has a stationary state given by

$$0 = f(\bar{x}_1, \bar{x}_2, p, \bar{c}) \quad (5.20)$$
$$\bar{x}_2 = g(\bar{x}_1, p)$$

In explicit form, the stationary state model is

$$0 = -\tau_{eng} \bar{Q}_{eng} - \zeta_{eng} \bar{n} + \zeta_{eng} \bar{n}_{set} \quad (5.21)$$
$$0 = \zeta_{q} \bar{Q}_{eng} - \zeta_{a} \bar{n} - \zeta_{q} \bar{Q}_a - \zeta_{q} \bar{Q}_p \quad (5.22)$$
$$0 = \bar{T}_p - \frac{1}{2} \pi \bar{u}_p (\bar{u}_p - \bar{u}_a) \quad (5.23)$$
$$0 = (1 - t_d) \bar{T}_p - \varphi \bar{n} \bar{\gamma} \quad (5.24)$$
$$0 = -\bar{T}_p + \alpha_1 \bar{n} \bar{u}_a + \alpha_0 \bar{n}^2 \quad (5.25)$$
$$0 = -\bar{Q}_p + \beta_1 \bar{n} \bar{u}_a + \beta_0 \bar{n}^2 \quad (5.26)$$
$$0 = -\bar{u}_a + (1 - w) \bar{\bar{n}} \quad (5.27)$$

This section is devoted to the stationary state. First, we show that the system of nonlinear algebraic equations (5.21) to (5.27) has a unique solution in the practical feasibility region. Second, we calculate the stationary state for a set of model test data; the solutions are compared to the data from direct measurements.

5.2.1 Solutions of the stationary state model

The system of Equations (5.21) to (5.27) can be reduced to a system of two nonlinear algebraic equations. To begin with, we do some substitutions. From Equation (5.21) we write $Q_{eng}$ as a function of $\bar{n}$, and replace $\bar{Q}_{eng}$ in Equation (5.22). From torque constitutive model, Equation (5.26), we write $\bar{Q}_p$ as a function of $\bar{n}$ and $\bar{u}_a$, and replace it in Equation (5.22). From thrust constitutive model, Equation (5.25), we write $\bar{T}_p$ as a function of $\bar{n}$ and $\bar{u}_a$, and replace it in Equations (5.23) and (5.24). From advance velocity constitutive model, Equation (5.27), we write $\bar{u}_a$ as a function of $\bar{u}$, and replace it in Equations (5.22), (5.23), and (5.24). These
substitutions result in the following system of three nonlinear algebraic equations

\[ 0 = a_1 \bar{n}^2 + (a_2 - a_3 \bar{u})\bar{n} - a_4 \tag{5.28} \]
\[ 0 = b_1 \bar{u}_p^2 - b_2 \bar{u}_p - b_3 \bar{n} - b_4 \bar{n}^2 \tag{5.29} \]
\[ 0 = c_2 \bar{u} - c_3 \bar{n} - c_3 \bar{n}^2 \tag{5.30} \]

with parameters

\[ a_1 = \zeta_{eng} \beta_0; \quad a_2 = \left( \frac{\zeta_q \zeta_{eng}}{\tau_{eng}} + \zeta_\lambda \right); \quad a_3 = \zeta_q \beta_1 (1 - w); \quad a_4 = \frac{\zeta_q \zeta_{eng}}{\tau_{eng}} \bar{n}_{set} - \zeta_q Q_0; \tag{5.31} \]
\[ b_1 = \frac{1}{2}\pi; \quad b_2 = \frac{1}{2}\pi (1 - w); \quad b_3 = \alpha_1 (1 - w); \quad b_4 = \alpha_0; \]
\[ c_1 = \varphi; \quad c_2 = \alpha_1 (1 - w)(1 - t_d); \quad c_3 = \alpha_0 (1 - t_d) \]

The system of Equations (5.28) to (5.30) is further simplified. Propeller rotational speed, \( \bar{n} \), is solved from Equation (5.28) as function of the stationary speed, \( \bar{u} \),

\[ \bar{n}_{1,2} = \frac{-(a_2 - a_3 \bar{u}) \pm \sqrt{(a_2 - a_3 \bar{u})^2 + 4a_4a_4}}{2a_1} \tag{5.32} \]

Feasible solution means that propeller rotational speed, \( \bar{n} \), is between 0 and 1. The terms \( (a_2 - a_3 \bar{u}) \) and \( (a_2 - a_3 \bar{u})^2 + 4a_4a_4 \) are positive since all parameters \( a_1, a_2, \) and \( a_4 \) are positive, and \( a_3 \) is negative due to \( \beta_1 < 0 \). Thus, the feasible solution of \( \bar{n} \) is

\[ \bar{n} = \frac{-(a_2 - a_3 \bar{u}) + \sqrt{(a_2 - a_3 \bar{u})^2 + 4a_4a_4}}{2a_1} \tag{5.33} \]

The solution can also be checked by using the model parameters listed in Table 5.3 and assigning the speed \( \bar{u} \) a value between 0 and 1.

Substituting the solution (5.33) in equations (5.29) and (5.30) we obtain a system of two nonlinear algebraic equations,

\[ 0 = k_1(\bar{u}_p, \bar{u}, p) \tag{5.34} \]
\[ 0 = k_2(\bar{u}_p, \bar{u}, p) \]

with \( \bar{u}_p \) and \( \bar{u} \) the unknowns. Each equation represents a surface in a three-dimensional space. The solution of the system of nonlinear equations lies on the three-dimensional curve formed by the intersection of the two surfaces. The solution for a stationary state is graphically depicted in Figure 5.1.
5.2. The stationary state model

Figure 5.1: The two nonlinear equations represent three-dimensional surfaces that intersect, forming a three-dimensional curve. The intersection (the white dot) of this three-dimensional curve with the horizontal plane \((\bar{u}_p, \bar{u}, 0)\) represents the solution of the stationary model.

Figure 5.1 shows that the stationary state system (5.21) to (5.27) has a unique feasible solution, which means for stationary states between 0 and 1. Thus, solving the system of nonlinear algebraic equations by numerical schemes poses no problems; the numerical schemes will always converge to the right solution.

Remark: In practice, the ship speed, \(\bar{u}\), is measured, so, it can be considered as a known quantity. Thus, the system can also be used to calculate \(\bar{u}_p\), which cannot be directly measured, and one model parameter.

5.2.2 Model test data

In this section we compute the stationary state solution for a model test data and compare the solutions with the direct measurements of speed, thrust and torque.

During the self-propulsion tests at model scale the speed of the model ship, the shaft rotational speed, the propeller thrust, the propeller torque, and the resistance force are directly measured. Additionally, the characteristics of the propeller are determined from open water tests. The wake fraction and thrust deduction are determined at model scale and predicted at full scale by specific procedures. The model test data are listed in Table 5.1. The measurements were collected during eight stationary states performed at different ship model velocities, see Section 2.2.3 for details on self-propulsion tests with scaled ship models. The values of the variables are averaged over the stationary time interval. The data is
Analysis of the ship propulsion model provided by Wärtsilä Netherlands, and, due to confidentiality reasons, is presented in dimensionless form.

An additional towing force, \( F_t \), is applied to the model ship during the self-propulsion tests to achieve the desired speed. With the additional force the surge equation for the model ship is

\[
0 = (1 - t_d)T_p + F_t - \varphi \bar{u}^\gamma
\]  

(5.35)

The presence of the towing force is explained with more details in Section 2.2.3.

**Table 5.1:** Model scale data in dimensionless form

<table>
<thead>
<tr>
<th>speed ( u )</th>
<th>resistance ( R )</th>
<th>towing force ( F_t )</th>
<th>thrust ( T_p )</th>
<th>torque ( Q_p )</th>
<th>rotational thrust deduction ( t_d )</th>
<th>wake fraction ( w )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.422</td>
<td>0.078</td>
<td>0.022</td>
<td>0.069</td>
<td>0.0098</td>
<td>0.605</td>
<td>0.187</td>
</tr>
<tr>
<td>0.465</td>
<td>0.097</td>
<td>0.026</td>
<td>0.087</td>
<td>0.0122</td>
<td>0.673</td>
<td>0.183</td>
</tr>
<tr>
<td>0.507</td>
<td>0.116</td>
<td>0.030</td>
<td>0.109</td>
<td>0.0152</td>
<td>0.749</td>
<td>0.209</td>
</tr>
<tr>
<td>0.549</td>
<td>0.138</td>
<td>0.035</td>
<td>0.129</td>
<td>0.0179</td>
<td>0.815</td>
<td>0.202</td>
</tr>
<tr>
<td>0.570</td>
<td>0.151</td>
<td>0.037</td>
<td>0.142</td>
<td>0.0196</td>
<td>0.859</td>
<td>0.199</td>
</tr>
<tr>
<td>0.591</td>
<td>0.165</td>
<td>0.039</td>
<td>0.158</td>
<td>0.0217</td>
<td>0.894</td>
<td>0.199</td>
</tr>
<tr>
<td>0.612</td>
<td>0.184</td>
<td>0.042</td>
<td>0.178</td>
<td>0.0233</td>
<td>0.944</td>
<td>0.204</td>
</tr>
<tr>
<td>0.634</td>
<td>0.206</td>
<td>0.044</td>
<td>0.204</td>
<td>0.0279</td>
<td>1.000</td>
<td>0.207</td>
</tr>
</tbody>
</table>

As mentioned earlier, the most important model parameters, namely, the wake fraction, thrust deduction, propeller characteristics, and the resistance parameters, are retrievable at model scale. The first two parameters, wake fraction and thrust deduction, are directly determined by the model basin, see Table 5.1. The propeller characteristics and the resistance parameters are retrieved from the open water tests and from the resistance force, respectively.

The open water diagram of the propeller used during the self-propulsion tests is depicted in Figure 5.2a. The thrust and torque models are empirical models as function of the advance ratio, \( J \); see Section 4.3.5. Figure 5.2a shows that dimensionless thrust and torque are linearly related to the advance ratio, \( J \). The values of the propeller parameters, \([\alpha_0, \alpha_1, \beta_0, \beta_1]\), are calculated by regressing the open water data points, see Figure 5.2a, using the thrust and torque constitutive relations, (5.25) and (5.25), respectively. Due to confidentiality reasons we do not reveal their values.

The hydrodynamic resistance of the ship model is measured during the model test, see Table 5.1. We model the resistance force as a power function with respect to speed, see Section 4.3.4. The resistance parameters \( \varphi \) and \( \gamma \) are found by fitting the model to the measured resistance, see Figure (5.2b). The values of the resistance parameters are: \( \varphi = 0.1406 \) and \( \gamma = 2.4193 \).

The model parameters of the engine and shaft equations are not available, so, we assign them conveniently. In their dimensionless form the engine and shaft model parameters are: \( \tau_{eng} = 0.3009 \), \( \zeta_{eng} = 0.8816 \), \( \zeta_f = 5.3684 \), and \( \zeta_s = 0.0094 \).

The solutions of the stationary propulsion model are listed in Table 5.2. Along with the numerical solutions we list the model test measurements of speed, thrust,
5.2. The stationary state model

and torque, and, the relative differences between the numerical solutions and the measurements. The relative difference, $\Delta$, between the directly measured speed and the computed speed is smaller than 2% for all tests. The relative difference is slightly larger in cases of propeller thrust and torque; for propeller thrust, the relative difference ranges from 2.3% to 5.4%; for propeller torque the relative difference ranges from 4.5% to 8.7%. Finding the sources of the relative differences between the direct measurements at model scale and the solutions of the mathematical model is not trivial. Most likely, the relative differences are linked to the fact that we use open water propeller characteristics to simulate the propeller rotating behind the hull. Sources of the relative difference can also be related to the model scale measurements. The conditions during the model tests are unknown to the author. The results of the model test are processed according to procedures that are specific to the model basin.

Table 5.2: Comparison between the model test data and the solutions of the mathematical model

<table>
<thead>
<tr>
<th>Speed, $u$, [-]</th>
<th>Thrust, $T_p$, [-]</th>
<th>Torque, $Q_p$, [-]</th>
</tr>
</thead>
<tbody>
<tr>
<td>model test</td>
<td>math. model</td>
<td>$\Delta$ [%]</td>
</tr>
<tr>
<td>0.422</td>
<td>0.423</td>
<td>0.2</td>
</tr>
<tr>
<td>0.463</td>
<td>0.463</td>
<td>0.2</td>
</tr>
<tr>
<td>0.507</td>
<td>0.501</td>
<td>1.0</td>
</tr>
<tr>
<td>0.549</td>
<td>0.559</td>
<td>1.7</td>
</tr>
<tr>
<td>0.579</td>
<td>0.559</td>
<td>1.9</td>
</tr>
<tr>
<td>0.601</td>
<td>0.584</td>
<td>1.5</td>
</tr>
<tr>
<td>0.612</td>
<td>0.606</td>
<td>0.9</td>
</tr>
<tr>
<td>0.634</td>
<td>0.633</td>
<td>0.1</td>
</tr>
</tbody>
</table>
5.3 Time scale analysis

In Section 5.1 two different time scales resulted from model dimensionalization. In this section we are interested in the behavior of the dynamic system when the control changes from value \( c_0 \) to value \( c_1 \) at time \( t_0 \). How long does it take the system to adapt to the new control? This idea is schematically depicted in Figure 5.3. In this section we use the dimensionless formulation of the system with respect to the short time-scale (5.4).

![Figure 5.3: Schematic of the response of the state \( x_1 \) to a sudden, small change in control \( c \). We are looking for the time scale in which \( x_1 \) adapts to the new control, \( c_1 \).](image)

For the control \( c_0 \) the stationary state is given by

\[
0 = f(\bar{x}_1, \bar{x}_2, p, c_0) \\
\bar{x}_2 = g(\bar{x}_1, p)
\]

where \( \bar{x}_1 = [Q_{eng}, n, u_p, u] \) and \( \bar{x}_2 = [T_p, Q_p, u_a] \) are the stationary states. The solutions of the propulsion model for a stationary control were presented in Section 5.2 for a model scale test.

At time \( t_0 \) the control is suddenly changed from \( c_0 \) to \( c_1 \). It means that at \( t = t_0 \) the system is initialized by \( \bar{x}(p, c_0) \). Since functions \( f \) and \( g \) that describe the dynamics of the system are linearly depending on the control \( c \) the solutions of the system depend smoothly on the control. If the change in control, \( \Delta c = c_1 - c_0 \) is small then the corresponding states, \( \bar{x}(p, c_0) \) and \( \bar{x}_1(p, c_1) \), are close. To analyze the behavior of the solution we are allowed to linearize the system that is initialized by \( \bar{x}(p, c_0) \). We write \( x(t, p, c_1) = \bar{x}(p, c_1) + \tilde{x}(t, p) \) and linearize according to

\[
f(\bar{x} + \tilde{x}, p, c_1) = f(\bar{x}, p, c_1) + \frac{\partial f}{\partial \bar{x}} \tilde{x}
\]

where \( f(\bar{x}, p, c_1) = 0 \) and \( \partial f/\partial \bar{x} \) is the Jacobian matrix.
After linearization we obtain the following system

\[
\frac{d\tilde{x}_1}{dt} = A_{11}\tilde{x}_1 + A_{12}\tilde{x}_2
\]

(5.38)

\[0 = A_{21}\tilde{x}_1 + A_{22}\tilde{x}_2\]

where \(A_{11}, A_{12}, A_{21}\) and \(A_{22}\) are the Jacobian matrices

\[
A_{11} = \begin{pmatrix}
-\tau_{\text{eng}} & -\zeta_{\text{eng}} & 0 & 0 \\
\zeta_q & -\zeta_s & 0 & 0 \\
0 & 0 & \pi\bar{u}_p + \frac{1}{2}\pi(1-w)\bar{u} & 0 \\
0 & 0 & 0 & -\frac{\gamma}{\mu}\bar{u}^{\gamma-1}
\end{pmatrix},
\]

(5.39)

\[
A_{12} = \begin{pmatrix}
0 & 0 & 0 & 0 \\
0 & -\zeta_q & 0 & 0 \\
1 & 0 & \frac{1}{2}\pi\bar{u}_p & 0 \\
\frac{1-\mu}{\mu} & 0 & 0 & 0
\end{pmatrix},
\]

(5.40)

\[
A_{21} = \begin{pmatrix}
0 & \alpha_1(1-w)\bar{u} + 2\alpha_0\bar{n} & 0 & 0 \\
0 & \beta_1(1-w)\bar{u} + 2\beta_0\bar{n} & 0 & 0 \\
0 & 0 & 0 & (1-w)
\end{pmatrix},
\]

(5.41)

\[
A_{22} = \begin{pmatrix}
-1 & 0 & \alpha_1\bar{n} \\
0 & -1 & \beta_1\bar{n} \\
0 & 0 & -1
\end{pmatrix}
\]

Note that in the element (3,3) of the matrix \(A_{11}\), and in the elements (1,2) and (2,2) of the matrix \(A_{21}\) we replaced \(\bar{u}_a\) by \((1-w)\bar{u}\).

Plugging \(\tilde{x}_2\) from the algebraic part into the differential part of the linearized system (5.38) we get

\[
\frac{d\tilde{x}_1}{dt} = A_{11.2}\tilde{x}_1
\]

(5.42)

where \(A_{11.2} = A_{11} - A_{12}A_{22}^{-1}A_{21}\). The matrix \(A_{11.2}\) is

\[
A_{11.2} = \begin{pmatrix}
-\tau_{\text{eng}} & -\zeta_{\text{eng}} & 0 & 0 \\
\zeta_q & -\zeta_s & 0 & -\zeta_q\beta_1(1-w) \\
0 & \alpha_1\bar{u}(1-w) + 2\alpha_0\bar{n} & \frac{1}{2}\pi\bar{u}(1-w) - \pi\bar{u}_p & (1-w)(\alpha_1\bar{n} + \frac{1}{2}\pi\bar{u}_p) \\
0 & \frac{1}{2}(1-t_d)(\alpha_1\bar{u}(1-w) + 2\alpha_0\bar{n}) & 0 & \frac{1}{2}(1-t_d)(1-w)(\alpha_1\bar{n} - \varphi\bar{u}^{\gamma-1})
\end{pmatrix}
\]

(5.43)

The solution of equation (5.40) can be written as

\[
\tilde{x}_1(t) = C_1e^{\lambda_1 t}\mathbf{v}_1 + C_2e^{\lambda_2 t}\mathbf{v}_2 + C_3e^{\lambda_3 t}\mathbf{v}_3 + C_4e^{\lambda_4 t}\mathbf{v}_4
\]

(5.44)
where $\lambda_1...\lambda_4$ are the eigenvalues of matrix $A_{11,2}$, and $v_1...v_4$ the corresponding eigenvectors; $C_1...C_4$ are constants.

The eigenvalues are the solutions of the characteristic equation

$$\det \left( \lambda I - A_{11,2} \right) = 0 \quad (5.43)$$

From the extended form of the characteristic equation

$$\begin{align*}
\left( \lambda - \frac{1}{2} \pi \bar{u}(1 - w) + \pi \bar{u}_p \right) \\
\left( \lambda + \tau_{eng} \right) \frac{1}{\mu} \left( \frac{1}{2} (1 + a) \right) \\
\left( \lambda - \frac{1}{\mu} \right) (1 - w) \alpha_1 \bar{n} + \frac{1}{\mu} \bar{v} \gamma \bar{u}^{-1} \\
\left( \zeta_{eng} \zeta_q + \left( \lambda + \tau_{eng} \right) \left( \lambda + \zeta_s + \zeta_q (\beta_1 \bar{u}(1 - w) + 2 \beta_0 \bar{n}) \right) \right) = 0
\end{align*}$$

we identify one real root

$$\lambda_3 = \frac{1}{2} \pi \bar{u}(1 - w) - \pi \bar{u}_p \quad (5.45)$$

To assess the sign of $\lambda_3$ we recall that $\bar{u}(1 - w) = \bar{u}_a$, see Section 4.3.6, and that from momentum theory $u_a$ is proportional to $u_p$, $u_p = u_a(1 + a)$, $a > 0$, in which $a$ is called the axial flow parameter, see Section 4.3.3. Using the result of momentum theory the solution for $\lambda_3$ becomes

$$\lambda_3 = -\pi \bar{u}_p \left( 1 - \frac{1}{2(1 + a)} \right) \quad (5.46)$$

The result (5.46) is always negative since only positive axial flow velocities $u_p$ are regarded in this thesis, and the axial flow parameter $a$ is a positive number. One real eigenvalue means there is at least one more real eigenvalue. We find the other three roots of the characteristic equation by solving the equation numerically, using the model test data presented in Section 5.2.2.

**Eigenvalues and eigenvectors**

To actually calculate the eigenvalues and eigenvectors we use the model scale data presented in Section 5.2. The values of the model parameters are mentioned in Section 5.2.2; a model parameter that is not mentioned there is the scaling factor, $\mu$, whose value, in this particular case, is 526. We use the solutions of the eighth stationary test, see Tables 5.1 and 5.2. The values of the stationary states and the model parameters are listed once more in Table 5.3.
### Table 5.3: The values used to compute the eigenvalues and eigenvectors

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \bar{n} )</td>
<td>0.9901</td>
</tr>
<tr>
<td>( u_p )</td>
<td>0.6458</td>
</tr>
<tr>
<td>( \bar{u} )</td>
<td>0.634</td>
</tr>
<tr>
<td>( \tau_{eng} )</td>
<td>0.3099</td>
</tr>
<tr>
<td>( \zeta_q )</td>
<td>0.6816</td>
</tr>
<tr>
<td>( \zeta_s )</td>
<td>5.3684</td>
</tr>
<tr>
<td>( \zeta_{eng} )</td>
<td>0.0094</td>
</tr>
<tr>
<td>( w )</td>
<td>0.287</td>
</tr>
</tbody>
</table>

The eigenvalues of the matrix \( A_{11.2} \) are the solutions of the characteristic equation (5.43). The values of the eigenvalues, \( \lambda \), for the model scale data are listed in Table 5.4. Negative real part of the eigenvalues indicate that the system is damped. The ship dynamics in real life confirms this aspect. There are two complex eigenvalues that correspond to \( Q_{eng} \) and \( n \); the real eigenvalues correspond to \( u_p \) and \( u \).

### Table 5.4: Eigenvalues of model ship

| \( \lambda_1 \) | \(-0.36 + 2.17i\) |
| \( \lambda_2 \) | \(-0.36 - 2.17i\) |
| \( \lambda_3 \) | \(-1.31\) |
| \( \lambda_4 \) | \(-0.002\) |

The complex eigenvalues can be written as

\[ \lambda_j = \delta_j + \omega_j i \]  

(5.47)

where \( \delta_j = -1/\tau_j \) with \( \tau_j \) the relaxation time; \( \omega_j = 2\pi f_j \) is the complex frequency.

The relaxation time represents a characteristic time of the system; it says how fast a system returns to equilibrium. Table 5.5 lists the relaxation time of the scaled model. The first three state variables, namely, \( Q_{eng}, n, \) and \( u_p \), have similar relaxation time. As expected, the relaxation time of the fourth state, the speed \( u \), is order \( 10^3 \) times larger than the former three states. The difference in relaxation time is controlled by the parameter \( \mu \), see the dimensionless parameters in Section 5.1.

### Table 5.5: Relaxation time

<table>
<thead>
<tr>
<th>( \tau_j ) (seconds)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \tau_1 )</td>
</tr>
<tr>
<td>( \tau_2 )</td>
</tr>
<tr>
<td>( \tau_3 )</td>
</tr>
<tr>
<td>( \tau_4 )</td>
</tr>
</tbody>
</table>

The eigenvectors \( \mathbf{v} \) associated to the eigenvalues \( \Lambda = [\lambda_1, \lambda_2, \lambda_3, \lambda_4]^T \) are the
solutions of the system of linear equations

\[(\Lambda I_4 - A_{11.2}) \mathbf{v} = 0 \quad (5.48)\]

The four normalized eigenvectors determined for the model test data are

\[
\mathbf{v}_1 = \mathbf{v}_2 = \begin{pmatrix} -1.6 \cdot 10^{-3} \\ 1.12 \\ 0.12 \\ -6.46 \cdot 10^{-5} \end{pmatrix} + i \begin{pmatrix} 0.45 \\ 0.03 \\ -0.26 \\ -4.7 \cdot 10^{-4} \end{pmatrix}; \quad (5.49)
\]

\[
\mathbf{v}_3 = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}; \quad \mathbf{v}_4 = \begin{pmatrix} -0.031 \\ 0.010 \\ 0.281 \\ 0.959 \end{pmatrix}
\]

In Figures 5.4 to 5.6 we depict the solutions

\[
\ddot{x}_1(t) = A_j e^{\delta_j t} \cos(\omega_j t + \varphi_j) \quad (5.50)
\]

of the linearized system, where \(\varphi_j, j = 1.4\) is the phase angle. The plots capture all characteristics of the solutions, namely, the relaxation time, the oscillations, and the phase. The relaxation time is dimensionless; the amplitude of the oscillations is normalized.

![Figure 5.4: Solutions for \(\lambda_1\)](image-url)
5.4 Qualitative validation of the model

In this section we compare the solution of the ship propulsion model with the full scale measurements. To compute the solution of the mathematical model for a full scale ship we need realistic model parameters. We use the resistance parameters, $\varphi$ and $\gamma$, and the thrust deduction, $t_d$, estimated in Chapter 9; we use the propeller coefficients, $\alpha_0$, $\alpha_1$, $\beta_0$, and $\beta_1$, estimated from CFD simulations; and, we conveniently select the rest of model parameters, namely, $w$, $\tau_{eng}$, $\zeta_{eng}$, $\zeta_q$, and $\zeta_s$.

We simulate a real-life ship acceleration from a constant initial speed. In this particular case the control of the ship propulsion is done by changing the propeller pitch angle, since the ship is fitted with a controllable pitch propeller. Changing the propeller pitch angle, $\theta_p$, the characteristics of the propeller change according to the constitutive relations (4.66) and (4.67). The change in propeller pitch is
depicted in Figure 5.7 in dimensionless form. The propeller pitch increases linearly in time, from the initial state 0.6 to the final state 0.84. To have an idea about the time scale of such a manoeuver, the change happens in about one minute.

Figure 5.7: The measured propeller pitch angle is the input of the mathematical model.

The solutions of the mathematical model in terms of speed-through-water, propeller thrust, and propeller torque, and their measured counterparts are depicted in Figure 5.8. The general observation is that the solution of the mathematical model follows closely the full scale measurements. Of course, in our mathematical model we do not introduce any disturbances, so the solutions represented in Figure 5.8 are smooth. The offset between the measurements and the solutions can be due to unappropriate model parameters for full scale use.

The solutions of the mathematical model take into account both short and long time scales. The computed speed-through-water slowly increases, along the measured speed-through-water. We have seen in Section 5.3 that ship speed operates on the long time scale. To get an idea about the time scale of ship speed dynamics, the ship takes about 200 seconds to accelerate from the initial speed, 0.46, to 0.55 (the final speed is 0.58), under the prescribed control depicted in Figure 5.7.

According to their constitutive equations, propeller thrust and torque have both short time scale from propeller rotational speed, \( n \), and long time scale from the ship speed, \( u \). The effects of these two opposite time scales are visible in Figures 5.8b and 5.8c. \( T_p \) and \( Q_p \) increase fast with the changes in propeller pitch angle. After the propeller pitch achieves the maximum set value, \( \theta_p = 0.84 \), propeller thrust and torque decrease. This decrease happens due to the fact that term that involves the ship speed bears the negative sign due of \( \alpha_1 \) and \( \beta_1 \), respectively; see the constitutive equations (4.60) and (4.60). As soon as the ship speed achieves the equilibrium, thrust and torque settle.
5.4. Qualitative validation of the model

Figure 5.8: The solutions of the propulsion model compared to the measured speed (a), the measured propeller thrust (b), and the measured propeller torque (c). The mathematical model yields good results, given appropriate set of model parameters. Any offset between the mathematical solutions and the full scale measurements is due to model parameters.
Chapter 6

Estimation of model parameters

In the mathematical model we introduced a set of model parameters, see Section 4.4. Some of the model parameters, namely, wake fraction, \( w \), thrust deduction, \( t_d \), resistance parameters, \( \varphi \) and \( \gamma \), and propeller characteristics, \( \alpha_0 \), \( \alpha_1 \), \( \beta_0 \), and \( \beta_1 \), are unknown at full scale. To determine propeller efficiency at full scale means to estimate these parameters at full scale. Methods to estimate the unknown parameters are the topic of this chapter.

The parameter estimation methods described in this chapter have the same fundamental approach: use of specific controls, real-life controls and model controls, to force the ship propulsion system and its mathematical model into desired states. It is implicitly assumed that, in real life, the ship propulsion system would respond as anticipated by the mathematical model. The response of the propulsion system is modeled. The modeled response is compared with the response of the ship propulsion system, measured at full scale. Fitting the model response to the real response of the ship propulsion yield the unknown model parameters. The general idea is schematically depicted in Figure 6.1.

![Figure 6.1: The general approach of parameter estimation. Specific control is applied to ship propulsion; the mathematical model transforms accordingly; the response of the ship is directly measured; parameters are estimated on base of the transformed mathematical model and the direct measurements by means of least squares methods.](image)

75
The use of specific controls to enforce a dynamic behavior on ship propulsion is common practice in the maritime community. For instance, stationary control is applied at ship commissioning to compare the performance of the ship at full scale with the designed performance, see (ITTC, 2003). Turning circle and zig-zag control are standard procedures in vessel maneuvering, see (ITTC, 2011). The Kalman filter, or its general form, the extended Kalman filter, is a method often used to estimate state variables and model parameters in ship maneuvering applications. The mathematical models describing the motion of the ship with several degrees of freedom contain many terms and model parameters that account for the hydrodynamic forces that influence ship maneuvering, see (Triantafyllou and Hover, 2003) for instance. Since only a fraction of the state variables are directly measured at full scale, powerful tools, such as the extended Kalman filter, are employed. In (Hwang, 1980), the author identifies the hydrodynamic coefficients of ship maneuvering model with an extended Kalman filter. A similar approach is used in (Liu, 1988) to identify ship resistance coefficients.

General aspects of parameter estimation

The classical way to estimate model parameters is by the least squares method. The method dates back to 1809, when C.F. Gauss published a work on celestial mechanics. For a nice overview on least squares methods for models we refer to the work of J. Kallrath, see (Kallrath, 2009). As described by the author, the least squares problem requires the following input:

- model,
- data,
- variances associated with the data,
- measure of goodness, e.g., the Euclidean norm.

Assume we are given \( N \) measured data points, \( y_1, \ldots, y_N \), that are related according to a model, \( M(\cdot, \mathbf{p}) \), such that

\[
y_k = M(t_k, \mathbf{p}), \quad k = 1, \ldots, N
\]

(6.1)

where \( \mathbf{p} \) denotes a parameter tuple. Then, the classical approach is to minimize the sum of squared residuals, as in

\[
\min_{\mathbf{p} \in \mathcal{P}} \sum_{k=1}^{N} |y_k - M(t_k, \mathbf{p})|^2
\]

(6.2)

with respect to a feasible parameter region, \( \mathcal{P} \). Since measurements are endowed with errors, the variance of the data points, \( \sigma_k^2 \), are introduced and a weighting factor, \( \beta \), adjusting the sum of squares to

\[
\sum_{k=1}^{N} \frac{\beta}{\sigma_k^2} |y_k - M(t_k, \mathbf{p})|^2
\]

(6.3)
Popular methods to solve the minimization problem (6.2) are the Levenberg-Marquardt algorithms, see (Levenberg, 1944) and (Marquardt, 1963). If parameters should be estimated from a system of differential equations with state variable $x$,

$$\frac{d}{dt}x = f(t, x, p)$$

the usual approach is to identify time points, $t_1, ..., t_N$, the number of observed values $y_{ik}$ of the state variable, $N_k$, at time point $t_k$, with $i = 1, ..., N_k$, and the observed relation

$$y_{ik} = g_i(x(t_k), p) + \varepsilon_{ik}$$

where $\varepsilon_{ik}$ are the measurement errors with mean 0 and variance $\sigma_{ik}$. The results is the least squares functional

$$l(p) = \sum_{k=1}^{N} \sum_{i=1}^{N_k} \frac{1}{\sigma_{ik}^2}|\varepsilon_{ik}|^2 = \sum_{k=1}^{N} \sum_{i=1}^{N_k} \frac{1}{\sigma_{ik}^2}|y_{ik} - g_i(x(t_k), p)|^2$$

that is minimized with respect to a feasible parameter set.

In case of an autonomous system, $f(t, x, p) = f(x, p)$, observations can be taken when the state is stationary, $f(\bar{x}, p) = 0$, so that in formula (6.6) $x(t_k) = \bar{x}$ and $y_i = g_i(x(t_k), p) + \varepsilon_i$, simplifying the optimization.

In this thesis we did not adopt these classical parameter estimation methods. The measured state indicated by $y$, measured at time points $t_k$, is decomposed into three components: the stationary, the oscillatory, and the noise component, see Section 8.1. With this method we avoid the use of statistics into the parameter estimation. In notation, by $\bar{y}$ we denote the stationary component, by $Y$ we denote the complex amplitude of the oscillatory component at frequency $\omega$, and by $\varepsilon$ we denote the noise component as extracted from the data. Then, for all measured time points we have

$$y(t_k) = \bar{y} + Ye^{j\omega t_k} + \varepsilon(t_k)$$

If the system is in stationary state, i.e., $Y = 0$, we obtain the equations

$$0 = f(\bar{x}, p)$$
$$\bar{y}_i = g_i(\bar{x}, p)$$

To include the oscillatory part, we assume that $|Y|$ is small with respect to $|\bar{y}|$ so that linearization of $f(x, p)$ is allowed, i.e.,

$$f(x, p) \cong f(\bar{x}, p) + \frac{d}{dx}f(\bar{x}, p)(x - \bar{x}) = A(p)(x - \bar{x})$$

$$g_i(x, p) \cong g_i(\bar{x}, p) + \frac{d}{dx}g_i(\bar{x}, p) \cdot (x - \bar{x}) = g_i(x, p) + \gamma_i(p) \cdot (x - \bar{x})$$
Under the assumption of fast damping, the state variable is approximated by

\[ x(t) \cong \bar{x} + X e^{j\omega t} \quad (6.11) \]

The resulting set of equations is given by

\[ 0 = f(\bar{x}, p) \quad (6.12) \]
\[ y_i = g_i(\bar{x}, p) \]
\[ 0 = (j\omega - A(p))X \]
\[ Y = \gamma_i(p)\cdot X \]

This approach is detailed in the coming sections. In the first section we describe at generic level the types of control and response of the system, namely, the stationary, the zero-thrust, and the harmonic control. Then, in subsequent sections we detail the estimation methods for each unknown parameter. All sections contain real-life examples, except for the one with the harmonic control where simulated measurements are used. The last section contains requirements from the full scale measurements. These requirements are formulated on basis of the parameter estimation methods and include the list of signals to be measured, the architecture of the measurements system, and the propulsion tests.

6.1 Control and response

The controls we describe in this section are:

- stationary control,
- zero-thrust control,
- harmonic control.

6.1.1 Stationary control

If a stationary control is applied, \( c = \bar{c} \), then a stationary state, \( x = \bar{x} \), is achieved, after a time period equal to the longest relaxation time of the system. The stationary state is observed, thus, \( y_{model} = \bar{y}_{model} \). The stationary model and the equation of observable states become

\[ 0 = f(\bar{x}, \bar{c}; p) \quad (6.13) \]
\[ 0 = g(\bar{x}; p) \quad (6.14) \]
\[ \bar{y}_{model} = h(\bar{x}) \quad (6.15) \]

The components of the state vector \( \bar{x} \) are \([Q_{eng}, \bar{n}, \bar{u}_p, \bar{u}, \bar{T}_p, \bar{Q}_p, \bar{u}_a]^T \); the observed states are \( y_{model} = [\bar{n}, \bar{u}, \bar{T}_p, \bar{Q}_p]^T \).
Let us take a close look at the expanded stationary model

\begin{align}
0 &= -\tau_{\text{eng}} Q_{\text{eng}} - \zeta_{\text{eng}} \tilde{n} + \zeta_{\text{eng}} \tilde{n}_{\text{set}} \tag{6.16} \\
0 &= \zeta_q Q_{\text{eng}} - \zeta_s \tilde{n} - \zeta_q Q_0 - \zeta_q \bar{Q}_p \tag{6.17} \\
0 &= \bar{T}_p - \frac{1}{2} \pi \bar{u}_p (\bar{u}_p - \bar{u}_a) \tag{6.18} \\
0 &= (1 - t_d) \bar{T}_p - \varphi \bar{u} \tag{6.19} \\
0 &= -\bar{T}_p + \alpha_1 \bar{n} \bar{u}_a + \alpha_0 \bar{n}^2 \tag{6.20} \\
0 &= -\bar{Q}_p + \beta_1 \bar{n} \bar{u}_a + \beta_0 \bar{n}^2 \tag{6.21} \\
0 &= -\bar{u}_a + (1 - w) \bar{u} \tag{6.22}
\end{align}

The unknown quantities of direct interest in this thesis are \(w, t_d, \phi, \gamma, \alpha_1, \alpha_0, \beta_1,\) and \(\beta_0\). To the unknown parameters we add three unmeasured states, \(\bar{Q}_{\text{eng}}, \bar{u}_a\) and \(\bar{u}_p\). Obviously, the number of unknowns is way more that the system of equations can handle. Yet, more problematic is the way the unknowns are distributed inside the mathematical model. For instance, Equation (6.19) representing surge motion contains three unknown parameters that are only found in this equation. Another example is the wake fraction, \(w\), which is related to two unmeasured states, \(\bar{u}_p\) and \(\bar{u}_a\) in Equations (6.18) and (6.22), respectively. When we plug \(\bar{u}_a\) from Equation (6.22) in Equations (6.18), (6.20), and (6.21), we see that \(w\) cannot be separated from \(\bar{u}_p, \alpha_1,\) and \(\beta_1\). In the following paragraphs we elaborate on the situation at stationary state model.

**Engine and shaft**

The engine and shaft equations

\begin{align}
0 &= -\tau_{\text{eng}} Q_{\text{eng}} - \zeta_{\text{eng}} \bar{n} + \zeta_{\text{eng}} \bar{n}_{\text{set}} \tag{6.23} \\
0 &= \zeta_q Q_{\text{eng}} - \zeta_s \bar{n} - \zeta_q Q_0 - \zeta_q \bar{Q}_p \tag{6.24}
\end{align}

contain two measurable states, namely, \(\bar{n}\) and \(\bar{Q}_p\), and the control \(\bar{n}_{\text{set}}\), which is also measured. Parameter \(\zeta_q\) is known. The model state \(Q_{\text{eng}}\) and the model parameters \(\tau_{\text{eng}}, \zeta_{\text{eng}},\) and \(\zeta_s\) are unknown. However, estimation of these model parameters is not the goal of this thesis; thus, we fit appropriate model parameters to the measured states, \(\bar{y}\).

**Propeller**

The stationary propeller equation

\[0 = \bar{T}_p - \frac{1}{2} \pi \bar{u}_p (\bar{u}_p - \bar{u}_a) \tag{6.25}\]
Estimation of model parameters

contains two unknowns: the axial flow velocity \( \bar{u}_p \) and the advance velocity \( \bar{u}_a \). The only measured state is the propeller thrust \( \bar{T}_p \). The solution of the quadratic equation is

\[
\bar{u}_{p,1,2} = \frac{1}{2\pi} \bar{u}_a \pm \sqrt{\frac{1}{4\pi^2 \bar{u}_a^2 + 2\pi \bar{T}_p}}
\] (6.26)

All quantities are positive and larger than zero, thus, the following relation between \( \bar{u}_p \) and \( \bar{u}_a \) results

\[
\bar{u}_p = \frac{1}{2} \bar{u}_a + \frac{1}{2} \sqrt{\frac{\bar{u}_a^2 + \frac{8}{\pi} \bar{T}_p}{\pi}}
\] (6.27)

The axial flow velocity can be expressed in terms of the advance velocity, and vice versa. Due to the fact that both quantities are unmeasurable, this result has limited relevance. By replacing the advance velocity constitutive relation in (6.28)

\[
\bar{u}_p = \frac{1}{2} (1 - w) \bar{u} + \frac{1}{2} \sqrt{(1 - w)^2 \bar{u}^2 + \frac{8}{\pi} \bar{T}_p}
\] (6.28)

we link the axial flow velocity to the thrust and torque constitutive relations. However, this action has little relevance too, as will be seen later in this chapter.

Surge

Three unknowns, namely, \( t_d, \varphi, \) and \( \gamma \), are only found in the surge equation

\[
0 = (1 - t_d) \bar{T}_p - \varphi \bar{u}^\gamma
\] (6.29)

Due to the fact that \( \bar{T}_p \) and \( \bar{u} \) are measured, this equation detaches from the stationary model. Obviously, this is an underdetermined problem if parameters \( t_d, \varphi, \) and \( \gamma \) need to be calculated.

The only way to solve the parameters of the surge equation is by considering multiple stationary states. Consequently, the underdetermined problem may be transformed in an overdetermined problem by considering pairs of stationary states \([\bar{T}_p, \bar{u}]\). The overdetermined problem is solved by least squares method. Note that the surge equation is transformed into linear equations before applying the fore mentioned procedure.

Using the stationary control only one parameter can be directly estimated, namely, the resistance exponent \( \gamma \). For the thrust deduction \( t_d \) and the resistance factor \( \varphi \) a relation is found. The procedure is detailed in Section 6.2.

If the resistance factor \( \varphi \) is known then the stationary states can be used to estimate the thrust deduction. This procedure is presented in Section 6.4.
6.1. Control and response

Constitutive relations

Let us consider the constitutive relations

\begin{align*}
0 &= -\bar{T}_p + \alpha_1 \bar{n} \bar{u}_a + \alpha_0 \bar{n}^2 \\
0 &= -\bar{Q}_p + \beta_1 \bar{n} \bar{u}_a + \beta_0 \bar{n}^2 \\
0 &= -\bar{u}_0 + (1 - w) \bar{u}
\end{align*}

(6.30) (6.31) (6.32)

as one group.

If the advance velocity \(\bar{u}_a\) from Equation (6.32) is replaced in thrust and torque constitutive equations we get

\begin{align*}
0 &= -\bar{T}_p + \alpha_1 (1 - w) \bar{n} \bar{u} + \alpha_0 \bar{n}^2 \\
0 &= -\bar{Q}_p + \beta_1 (1 - w) \bar{n} \bar{u} + \beta_0 \bar{n}^2
\end{align*}

(6.33) (6.34)

In this system of equations \(\bar{T}_p, \bar{Q}_p, \bar{n}, \text{ and } \bar{u}\) are directly measurable. If the open water characteristics \(\alpha_0, \alpha_1, \beta_0, \text{ and } \beta_1\) are considered to be known from the model tests or from CFD simulations then the only unknown of the system is the wake fraction \(w\). The overdetermined system of equations can be solved by least squares method to find the best fitting \(w\). For reliable results, several stationary states should be considered. But, are propeller parameters \(\alpha_0, \alpha_1, \beta_0, \text{ and } \beta_1\) the same at model scale and full scale? And, do they have the same values when propeller rotates behind the hull as they have from open water tests? In this thesis the propeller parameters determined from open water tests at model scale and from CFD simulations are only used as instructive information.

6.1.2 Zero-thrust control

Zero-thrust is a special type of control. In practice, this control can only be carried out if propeller thrust measurements are available real-time, on-board of the ship. The purpose of the zero-thrust control is to separate the unknown parameters \(t_d\) and \(\varphi\).

By setting propeller thrust to zero the ship decelerates due to the resistance force, \(-\varphi u^\gamma\). Due to zero-thrust control the surge equation can be analytically solved. The solution \(u(t)\) is a time-dependent nonlinear function. Consequently, the solution can be transformed into a first order equation by assuming that \(\gamma\) is known. The parameter \(\varphi\) is left as the sole unknown of the equation. The first order equation holds for all measured time-points. Parameter \(\varphi\) is estimated in a fitting-like approach. The entire procedure is detailed in Section 6.3.

Note that zero-thrust control does not mean that set propeller angular speed, and, if applicable, the propeller pitch angle, is zero. This type of control is illustrated in Section 6.7.3.
6.1.3 Harmonic control

In harmonic control we take the control a superposition of a harmonic component and a stationary component, where the harmonic component has a fixed frequency and an amplitude that is small compared to the stationary component. The frequency is related to the short time scale, $t_c = n_w / (\rho D^3 n_s)$, so the frequency is high. These high frequencies are filtered from the long time dynamics so it is as if the speed-through-water, $u$, is stationary. Due to this, surge equation becomes stationary and the ship speed becomes a parameter. We still obtain the same mathematical model

\[
\frac{d}{dt} x = f(x, c; p) \quad (6.35)
\]

\[
0 = g(x; p) \quad (6.36)
\]

\[
y_{\text{model}} = h(x) \quad (6.37)
\]

The approach is the following. We first calculate the stationary state given the stationary component of the control, implicitly as functions of the unknown parameters. Then we linearize the equations with respect to the stationary state and the stationary control. The harmonic component of the control acts linearly on the observed model state. Similarly, the measured state can be split into a stationary component (trend) and a harmonic component with the selected frequency. Then, to estimate the parameters both stationary component and harmonic component of the measured state are compared to the stationary components and harmonic component of the observed model state.

In formula we write $c(t) = \hat{c} + \tilde{c}(t)$ and $x(t) = \bar{x} + \tilde{x}(t)$. We define $\tilde{y}_{\text{model}}$ as the stationary observed model state and its harmonic component. Then, $f(x, c; p) = 0$, $g(x; p) = 0$, and $\tilde{y}_{\text{model}} = h(x, c; p)$. As for stationary control we consider the difference between $\tilde{y}_{\text{model}}$ and the $\tilde{y}_{\text{meas}}$ to be minimized for a given parameter collection.

Linearization of the Equations (6.35) to (6.37) lead to

\[
\frac{d}{dt} \tilde{x}_1 = \frac{\partial}{\partial x_1} f(\bar{x}, \hat{c}; p) \tilde{x}_1 + \frac{\partial}{\partial x_2} f(\bar{x}, \hat{c}; p) \tilde{x}_2 + \frac{\partial}{\partial c} f(\bar{x}, \hat{c}; p) \hat{c} \quad (6.38)
\]

\[
0 = \frac{\partial}{\partial x_1} g(\bar{x}; p) \tilde{x}_1 + \frac{\partial}{\partial x_2} g(\bar{x}; p) \tilde{x}_2 \quad (6.39)
\]

\[
\tilde{y}_{\text{model}} = \frac{\partial}{\partial \bar{x}} h(\bar{x}) \tilde{x} \quad (6.40)
\]

where $\partial f/\partial x_1$, $\partial f/\partial x_2$, $\partial f/\partial c$, $\partial g/\partial x_1$, $\partial g/\partial x_2$, and $\partial h/\partial \tilde{x}$ are the respective Jacobian matrices. In these equations the stationary terms vanish due to results (6.13) to (6.15).

In the harmonic control $\hat{c} = C e^{j\omega t}$. Seen the time constants as calculated in the Chapter 5 the system reacts on the harmonic control within the time scale. Thus,
we may assume that both $\tilde{x}$ and $\tilde{y}_{\text{model}}$ are purely harmonic:
\begin{align*}
x &= \tilde{x} + X e^{j\omega t} \\
y_{\text{model}} &= \tilde{y}_{\text{model}} + Y_{\text{model}} e^{j\omega t}
\end{align*}  
(6.41)  
(6.42)
We get the equations
\begin{align*}
j\omega X_1 &= A_1(p)X_1 + A_2(p)X_2 + B(p)C \quad (6.43) \\
0 &= D_1(p)X_1 + D_2(p)X_2 \\
Y_{\text{model}} &= E(p)X
\end{align*}
(6.44)
(6.45)
From Equation (6.44) we write
\begin{align*}
X_2 &= -D_2^{-1}(p)D_1(p)X_1 \quad (6.46)
\end{align*}
and substitute in Equation (6.43) to get
\begin{align*}
j\omega X_1 &= A_1(p)X_1 - A_2(p)D_2^{-1}(p)D_1(p)X_1 + B(p)C \\
\text{The solution of Equation (6.47) is}
\end{align*}
\begin{align*}
X_1 &= (j\omega I - A(p))^{-1}B(p)C \\
\text{where } A(p) &= A_1(p) - A_2(p)D_2^{-1}(p)D_1(p).
\end{align*}  
(6.47)  
(6.48)
Plugging the solution (6.48) into Equation (6.45) the following expression is obtained for the observed states
\begin{align*}
Y_{\text{model}}(\omega) &= E(p)(j\omega I - A(p))^{-1}B(p)C \\
\text{At full scale we measure } y_{\text{meas}}(t) \text{ for the imposed harmonic control, } c(t) = \tilde{c} + \tilde{c}(t). \text{ The measured signal is decomposed into three components: the trend, } \tilde{y}_{\text{meas}}, \text{ the harmonic component, } y_{\text{meas}}, \text{ and the noise, which is discarded. The signal decomposition method is presented in Section 8.1. Full scale propulsion tests show that the frequency of the imposed harmonic control is found in the measured signals. Figure 6.6 depicts the harmonic control, propeller pitch angle, and one measured state, propeller thrust. Thus, in formula we write}
\end{align*}
\begin{align*}
y_{\text{meas}} &= \tilde{y}_{\text{meas}} + Y_{\text{meas}} e^{j\omega t} \quad (6.50)
\end{align*}
To estimate the sought model parameters we compare the model, $y_{\text{model}}(p)$, and the measurements, $y_{\text{meas}}$, in terms of the stationary states, $\tilde{y}_{\text{model}}$ and $\tilde{y}_{\text{meas}}$, and the harmonic dynamic state, $Y_{\text{model}}(\omega)$ and $Y_{\text{meas}}$. The following cost function
\begin{align*}
\min_{p, \omega \in \Omega} ||y_{\text{meas}} - \tilde{y}_{\text{model}}(p)||^2 + ||Y_{\text{meas}} - E(p)(j\omega I - A(p))^{-1}B(p)C||^2 \\
\text{is minimized with respect to the sought model parameters and the imposed frequencies. Note that } \tilde{y}_{\text{meas}} \text{ and } Y_{\text{meas}} \text{ are vectors with } n \text{ components; } \tilde{y}_{\text{model}} \text{ and the term } E(p)(j\omega I - A(p))^{-1}B(p)C \text{ is a } n \times m \text{ matrix.} \\
\end{align*}  
(6.51)  
6.2 Resistance exponent from stationary control

The resistance exponent $\gamma$ is an unknown parameter of the surge equation, along with thrust deduction $t_d$ and resistance factor $\varphi$

$$\frac{du}{dt} = (1 - t_d)T_p - \varphi u^\gamma$$ (6.52)

Parameter $\gamma$ is isolated from parameters $t_d$ and $\varphi$ in two steps. In the first step, a stationary control is imposed. The stationary surge equation is the balance between the effective thrust and the resistance force

$$(1 - t_d)\bar{T}_p = \varphi \bar{u}^\gamma$$ (6.53)

In the second step, the logarithm of both sides of the equality (6.53) is taken. The result can be conveniently arranged into

$$\log \bar{T}_p = \gamma \log \bar{u} + \log \frac{\varphi}{1 - t_d}$$ (6.54)

Let us denote $\log \bar{u}$ by $\bar{x}_{\text{model}}$, $\log \bar{T}_p$ by $\bar{y}_{\text{model}}$, and $\log \varphi/(1 - t_d)$ by $\delta_{\text{stat}}$. The following equation results

$$\bar{y}_{\text{model}} = \gamma \bar{x}_{\text{model}} + \delta_{\text{stat}}$$ (6.55)

Equation (6.55) suggests that there is a linear relation between $\log \bar{T}_p$ and $\log \bar{u}$. In the linear representation of the stationary surge equation the parameter $\gamma$ is separated from parameters $t_d$ and $\varphi$. Parameter $\gamma$ represents the slope of the linear function $\bar{y}_{\text{model}} = f(\bar{x}_{\text{model}})$, and $\delta_{\text{stat}} = \log \varphi/(1 - t_d)$ represents the offset.

At full scale, pairs of stationary data points, $(\bar{x}_{\text{meas}}, \bar{y}_{\text{meas}})$, are collected. The linear model applied to the measured stationary points yield the equation

$$\bar{y}_{\text{meas}} = \gamma \bar{x}_{\text{meas}} + \delta_{\text{stat}} + \varepsilon$$ (6.56)

where $\varepsilon$ is the vector of errors.

Parameters $\gamma$ and $\delta$ are found by regressing the stationary data points. The sum of the squared residuals, $\varepsilon_i$, is minimized with respect to $\gamma$ and $\delta_{\text{stat}}$

$$\min_{\gamma, \delta_{\text{stat}}} \sum_{i=1}^{N} \varepsilon_i^2 = \min_{\gamma, \delta_{\text{stat}}} \|\bar{y}_{\text{meas}} - (\gamma \bar{x}_{\text{meas}} + \delta_{\text{stat}})\|$$ (6.57)

The offset $\delta_{\text{stat}}$ gives a linear relation between parameters $\varphi$ and $t_d$

$$t_d = 1 - e^{-\delta_{\text{stat}} \varphi}$$ (6.58)

which can be used to retrieve either one of them.
Example

Real-life measurements exemplify the estimation method. The stationary data points are extracted from full scale measurements performed on board of a vessel. Detailed information on the measurements is presented in Chapter 7. The data points are listed in Figure 6.2. Due to external conditions, the data points are categorized with respect to wind direction.

As illustrated in Figure 6.2, the linearity of the stationary data points \((x_i, y_i)\) is obvious. The slope of the regression line represents the resistance exponent \(\gamma\), as indicated by the model (6.55).

Before making the next observation with respect to the results illustrated in Figure 6.2 we briefly recall Section 4.3.4 where the surge equation was presented. The surge equation does not contain any variable that accounts for the external forces. As stated there, the effects of the external factors are actually taken into the model by the resistance parameters \(p\) and \(\gamma\). It means that \(p\) and \(\gamma\) change with the external factors, as, for instance, can be seen in Figure 6.2.

The clear separation in the stationary data points \((\bar{x}_{meas}, \bar{y}_{meas})\) is due to the external factors. Part of the measurements collected were the wind speed and direction. The correlation between the direction of the wind with respect to the ship orientation and the ship dynamics is obvious. Also the slope of the regression line, which is the parameter \(\gamma\), changes with wind direction. For aft wind, that is wind blowing with the ship, the value of \(\gamma\) is slightly larger than for head wind situation, that is wind blowing against the ship. Particularly, in this case, \(\gamma_{aw} = 2.502\) and \(\gamma_{hw} = 2.267\). The relation between the model parameters and the external factors is a topic of the Chapter 9.

\[
\begin{array}{c|c|c|c|c|c}
\text{Aft wind} & \text{Head wind} \\
\hline
u & T_p & u & T_p \\
0.646 & 0.272 & 0.616 & 0.296 \\
0.616 & 0.234 & 0.587 & 0.264 \\
0.594 & 0.217 & 0.566 & 0.237 \\
0.572 & 0.196 & 0.539 & 0.213 \\
0.542 & 0.178 & 0.512 & 0.185 \\
0.496 & 0.135 & 0.458 & 0.144 \\
0.440 & 0.103 & 0.399 & 0.112 \\
\end{array}
\]

Figure 6.2: The stationary data points and the regression line. The slope of the regression line is \(\gamma\).
6.3 Resistance factor from zero-thrust control

Let us recall the surge equation during a dynamic state

$$\frac{du}{dt} = (1 - t_d)T_p - \varphi u^\gamma$$  \hspace{1cm} (6.59)

The method to estimate the resistance factor, $\varphi$, uses the propeller thrust as control. Imagine the following scenario: the ship is sailing with constant speed $u_{init}$ due to a stationary control, $T_p$. At time point $t_{init}$ the control is switched to $T_p = 0$. The ship leaps from the stationary state into a dynamic state. In this dynamic state the ship decelerates, due to friction forces, from the initial speed $u(t_{init}) = u_{init}$ to zero, if the zero-thrust control is kept long enough. Note that, in practice, zero propeller thrust does not mean that propeller angular velocity or propeller pitch angle are zero; this practical aspect is discussed in Section 6.7. During the zero-thrust control the surge equation becomes

$$\frac{du}{dt} = -\varphi u^\gamma$$  \hspace{1cm} (6.60)

Equation (6.60) has analytical solution of the form

$$u(t) = (1 - \gamma)[-\varphi(t - t_{init}) + u_{init}]^{\frac{1}{1-\gamma}}$$  \hspace{1cm} (6.61)

where $u_{init}$ is the initial vessel speed.

The solution (6.61) is conveniently arranged as

$$\frac{u(t)^{1-\gamma}}{1-\gamma} = -\varphi t + \frac{u_{init}^{1-\gamma}}{1-\gamma}$$  \hspace{1cm} (6.62)

where $t_{init} = 0$ for convenience.

Let us change the variable in Equation (6.62)

$$y_{model} = -\varphi t + \delta_{dyn}$$  \hspace{1cm} (6.63)

where $y_{model} = u(t)^{1-\gamma}/(1-\gamma)$ is the new time-dependent variable, and $\delta_{dyn} = u_{init}^{1-\gamma}/(1-\gamma)$ is the constant term. The Equation (6.63) suggests a linear relation between variables $y_{model}$ and $t$; the slope of the linear function is $\varphi$; the offset is $\delta_{dyn}$.

At full scale the variable $y_{meas}(t)$ is measured. The measured ship speed $u$ is regarded as discrete-time signal, with samples collected at equal time-intervals, $T_s$. Hence, speed and time can be represented as $u = u(kT_s)$ and $t = t(kT_s)$, respectively, with $k \in \mathbb{N}$ and $T_s$ the sampling time. The equation of observable states is

$$y_{meas}(k) = -\varphi t(k) + \delta_{dyn} + \varepsilon(k)$$  \hspace{1cm} (6.64)
6.3. Resistance factor from zero-thrust control

where \( \varepsilon(k) \) is the vector of errors. The solutions of the overdetermined linear system, the slope \( \varphi \) and the offset \( \delta_{dyn} \), are approximated by the least squares method

\[
\min_{\varphi, \delta_{dyn}} \| y_{meas}(k) - (-\varphi t(k) + \delta_{dyn}) \|^2
\]  
(6.65)

Example

To actually estimate the resistance factor \( \varphi \) we use the full scale measurements mentioned in the previous section. The resistance factor needs deceleration with zero-thrust control. The measurements recorded during the deceleration tests are thoroughly described in Section 7.2 and Chapter 9. The measured data points \((t(k), y_{meas}(k))\) are depicted in Figure 6.3. Note that \((t(k), y_{meas}(k))\) data set contains much more points than the stationary data set used in the previous section. The variable \( y_{meas}(k) \) is actually the speed of the ship measured with sampling time \( T_s \), and transformed according to (6.62).

![Figure 6.3: The data points and the regression line. The data points represent the transformed vessel speed \( y = u(t)^{1-\gamma}/(1 - \gamma) \), for a given \( \gamma \). The zero-thrust control test is performed for aft wind and head wind conditions.](image)

The measured points \((t(k), y_{meas}(k))\) have the linear dependency, predicted by the model (6.63). The small variations of \( y_{meas}(k) \) around the regression line are due to the fact that the ship becomes unstable during deceleration with zero-thrust; this fact is explained in Section 9.2.2. However, the linear trend of the data points is obvious. The slope of the regression line is the the parameter sought, \( \varphi \).

The resistance factor changes with the external factors. Similarly to \( \gamma \), Figure 6.3 illustrate two different data sets, differentiated by wind direction with respect to the ship. The values of \( \varphi \) are 0.719 for aft wind case, and 0.777 for head wind case. These results and the impact of the external factors is thoroughly discussed in Section 9.
6.4 Thrust deduction from stationary control

In a stationary state the surge equation is the balance between the effective thrust and the resistance force

\[ T_p = \frac{1}{1-t_d} \bar{R} \]  

(6.66)

where the resistance force model, in dimensionless formulation, is \( \bar{R}_f = \varphi \bar{u} \gamma \), see Section 4.3.4.

Clearly, Equation (6.66) represents a linear equation of the form

\[ \bar{y}_{model} = \frac{1}{1-t_d} \bar{x}_{model} \]  

(6.67)

where \( \bar{y}_{model} \) and \( \bar{x}_{model} \) are independent variables, and \( 1/(1-t_d) \) is the slope of the linear function. Note that the offset term is missing from the linear function. At equilibrium state, zero propeller thrust means zero resistance force, which means the ship is at rest.

At full scale, propeller thrust is directly measured and resistance force is calculated from direct ship speed measurements; thus, the measurement tuples \( (\bar{x}_{meas}, \bar{y}_{meas}) \) result. If the model (6.67) is applied to a collection of stationary states then an overdetermined system of linear equations is obtained.

\[ \bar{y}_{meas} = \frac{1}{1-t_d} \bar{x}_{meas} + \varepsilon \]  

(6.68)

where \( \varepsilon \) is the vector of errors.

The overdetermined system is solved by least squares method

\[ \min_{t_d} \left\| \bar{y}_{meas} - \frac{1}{1-t_d} \bar{x}_{meas} \right\|^2 \]  

(6.69)

Example

For a practical example we use the stationary data points used in the example of parameter \( \gamma \). The stationary data points \( (\bar{x}_{meas}, \bar{y}_{meas}) \) are listed in Figure 6.4 as function of the wind direction. The linear trend in the data points \( (\bar{x}_{meas}, \bar{y}_{meas}) \) is evident. It is also evident that the external factors do not influence the value of the thrust deduction. This means that the external factors are accounted by the resistance parameters, \( \gamma \) and \( \varphi \). The value of the thrust deduction estimated from these data points is 0.14.
6.5 Wake fraction from stationary control

The wake fraction is an obscure quantity. Ship hydrodynamics literature abounds of formulas to calculate wake fraction. The complexity of these formulas varies greatly. There are sophisticated formulas that take into account ship geometry, as the one presented in Section 4.3.6. There are also unsophisticated ones, as, for instance, the formula used in this thesis

\[ 0 = -u_a + (1 - w) u \]  

(6.70)

which was invented by Taylor. In fact, we use this formula to explain the advance velocity, \( u_a \), another obscure quantity that cannot be directly measured at full scale.

The wake fraction relates to other measured quantities of the mathematical model. Replace the advance velocity according to (6.70) in propeller equation, and thrust and torque constitutive relations to get the following system

\[ 0 = \bar{T}_p - \frac{1}{2} \pi \bar{u}_p (\bar{u}_p - (1 - w)\bar{u}) \]  

(6.71)

\[ 0 = -\bar{T}_p + \alpha_1 (1 - w) \bar{n} \bar{u} + \alpha_0 \bar{n}^2 \]  

(6.72)

\[ 0 = -\bar{Q}_p + \beta_1 (1 - w) \bar{n} \bar{u} + \beta_0 \bar{n}^2 \]  

(6.73)

The system of three algebraic equations contains four measurable quantities, namely, \( \bar{T}_p, \bar{Q}_p, \bar{u}, \) and \( \bar{n} \), one unmeasurable state, the axial flow velocity \( \bar{u}_p \), and five unknown parameters, namely, \( w, \alpha_0, \alpha_1, \beta_0, \) and \( \beta_1 \). That makes six unknowns with three equations to solve. The problem cannot be solved if only one stationary state is used.

![Figure 6.4: The stationary data points and the regression line.](image-url)
Table 6.1: Model scale data

<table>
<thead>
<tr>
<th>speed</th>
<th>thrust</th>
<th>torque</th>
<th>rotational speed</th>
<th>wake fraction</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.422</td>
<td>0.069</td>
<td>0.0098</td>
<td>0.605</td>
<td>0.307</td>
</tr>
<tr>
<td>0.469</td>
<td>0.087</td>
<td>0.0122</td>
<td>0.613</td>
<td>0.309</td>
</tr>
<tr>
<td>0.507</td>
<td>0.109</td>
<td>0.0132</td>
<td>0.749</td>
<td>0.304</td>
</tr>
<tr>
<td>0.549</td>
<td>0.129</td>
<td>0.0173</td>
<td>0.815</td>
<td>0.302</td>
</tr>
<tr>
<td>0.570</td>
<td>0.142</td>
<td>0.0196</td>
<td>0.850</td>
<td>0.301</td>
</tr>
<tr>
<td>0.591</td>
<td>0.158</td>
<td>0.0217</td>
<td>0.883</td>
<td>0.298</td>
</tr>
<tr>
<td>0.612</td>
<td>0.178</td>
<td>0.0245</td>
<td>0.944</td>
<td>0.293</td>
</tr>
<tr>
<td>0.634</td>
<td>0.204</td>
<td>0.0279</td>
<td>1.000</td>
<td>0.287</td>
</tr>
</tbody>
</table>

Let us rewrite the equations (6.71) to (6.73) as follows

\[ u_{p,j} = \frac{1}{2} (1 - w) u_j + \frac{1}{2} \sqrt{(1 - w)^2 u_j^2 + \frac{8}{\pi} T_{p,j}} \]  

\[ T_{p,j} = \alpha_1 (1 - w) n_j u_j + \alpha_0 n_j^2 \]  

\[ Q_{p,j} = \beta_1 (1 - w) n_j u_j + \beta_0 n_j^2 \]

where \( u_{p,j} \) is now expressed in terms of wake fraction, \( w \), using the result (6.28). The subscript \( j \) represents the stationary state.

The unknowns of the problem are \( w, \alpha_1, \alpha_0, \beta_1, \beta_0, \) and \( u_{p,j} \). We consider the case of a fixed pitch propeller, thus, the propeller parameters are fixed. We assume that the wake fraction is constant. The minimum number of stationary states we have to consider is \( 3 \cdot j \geq 5 + j \Rightarrow j \geq 3 \). The model tests data we use have eight stationary states; this means we have an overdetermined system of 24 equations with 13 unknowns.

In reality the system of equations offers little perspectives to solve the unknowns, even when considering more stationary states. There are two main drawbacks when we try to solve this system of equations. First, the axial flow velocity \( u_p \) is directly determined by the wake fraction. Second, parameters \( \alpha_1 \) and \( \beta_1 \) cannot be separated from \( w \).

Thrust-torque relation

A linear relationship between propeller thrust and torque is documented in literature, see (Lerbs, 1951) and (Smogeli, 2006), and can be observed from measurements

\[ T_p = \gamma_1 Q_p + \gamma_0 \]

where \( \gamma_1 \) and \( \gamma_0 \) are constants for a fixed pitch propeller.

Figure 6.5 depicts the thrust and torque measured from the self-propulsion tests at model scale, see Table 6.1. Each data point represents a stationary state. The solid line is the regression line through the data points. The slope of the regression
line is 7.25 in this case. Parameter $\gamma_1$ can be also found at full scale from stationary states. The offset, $\gamma_0$, is -0.0048, and for this particular case is neglected.

\[ \text{Figure 6.5: Thrust and torque linear relationship.} \]

The linear relation between thrust and torque offers little help. If we approximate Equation (6.77) with $T_p = \gamma_1 Q_p$, then the following relationships result

\[ \alpha_1 = \gamma_1 \beta_1; \quad \alpha_0 = \gamma_1 \beta_0 \quad (6.78) \]

With these extra knowledge the system of equations becomes

\[
\begin{align*}
\frac{1}{N} \sum_j T_{p,j} &= \gamma_1 \beta_1 (1 - w) u_j + \gamma_1 \beta_0 \frac{8}{\pi} T_{p,j} \\
\frac{1}{N} \sum_j Q_{p,j} &= \beta_1 (1 - w) u_j + \beta_0 \frac{n_j^2}{N} 
\end{align*}
\]

This system of equations has $3 + j$ unknowns, namely, $w, \beta_0, \beta_1, \text{ and } u_{p,j}, \text{ and } 3j$ available equations. To solve this problem we need at least two stationary states.

Let us fix $w, \alpha_1, \text{ and } \beta_1$. We want to get $u_{p,j}, \alpha_0, \text{ and } \beta_0$ in terms of the fixed parameters. The axial flow $u_{p,j}$ is already expressed in terms of $w$, see Equation (6.74), so we do not have to repeat it.

To get $\alpha_0$ and $\beta_0$ we write the constitutive equations as follows

\[
\begin{align*}
\frac{1}{N} \sum_j T_{p,j} &= \gamma_1 \beta_1 (1 - w) \frac{1}{N} \sum_j n_j u_j + \gamma_1 \beta_0 \frac{1}{N} \sum_j n_j^2 \\
\frac{1}{N} \sum_j Q_{p,j} &= \beta_1 (1 - w) \frac{1}{N} \sum_j n_j u_j + \beta_0 \frac{1}{N} \sum_j n_j^2
\end{align*}
\]

where $N$ is the number of stationary states available. This system is equivalent to

\[
\begin{align*}
\overline{T}_p &= \gamma_1 \beta_1 (1 - w) \overline{u} + \gamma_1 \beta_0 \overline{n^2} \\
\overline{Q}_p &= \beta_1 (1 - w) \overline{u} + \beta_0 \overline{n^2}
\end{align*}
\]
Estimation of model parameters

where \( \overline{T_p} \) denotes the average of the quantity.

From equations (6.84) and (6.85) we get \( \beta_0 \) as

\[
\beta_0 = \frac{1}{\delta} \frac{\overline{T_p}}{n^2} - \beta_1 (1 - w) \frac{\overline{nu}}{n^2} \quad (6.86)
\]

\[
\beta_0 = \frac{\overline{Q_p}}{n^2} - \beta_1 (1 - w) \frac{\overline{nu}}{n^2} \quad (6.87)
\]

We plug \( \beta_0 \) from Equation (6.86) and (6.87), respectively, in Equations (6.80) and (6.81) and rearrange the terms to get

\[
T_{p,j} - \frac{\overline{T_p}}{n^2} n_j^2 = \delta \beta_1 (1 - w) \left[ n_j u_j + \frac{\overline{nu}}{n^2} n_j^2 \right] \quad (6.88)
\]

\[
Q_{p,j} - \frac{\overline{Q_p}}{n^2} n_j^2 = \beta_1 (1 - w) \left[ n_j u_j + \frac{\overline{nu}}{n^2} n_j^2 \right] \quad (6.89)
\]

Introducing the notations

\[
T_{p,j} - \frac{\overline{T_p}}{n^2} n_j^2 = U_j; \quad Q_{p,j} - \frac{\overline{Q_p}}{n^2} n_j^2 = V_j; \quad n_j u_j + \frac{\overline{nu}}{n^2} n_j^2 = W_j
\]

we get the following equations

\[
U_j = \delta \beta_1 (1 - w) W_j \quad (6.90)
\]

\[
V_j = \beta_1 (1 - w) W_j \quad (6.91)
\]

where \( U_j, V_j, \) and \( W_j \) are directly measured. These equations are undefined for \( \beta_1 = 0 \) and \( w = 1 \). In fact, there is no improvement compared with the initial problem, Equations (6.71) to (6.73). We still cannot separate the wake fraction from the propeller characteristics.

Example

The unknown parameters, \( \beta_1, \beta_0, w, \) and \( u_{p,j} \), can be solved from a constrained optimization scheme

\[
\min_p \| \mathbf{y}_{meas} - \mathbf{y}_{model}(p) \|^2 + \| p_{ref} - p \|^2 \quad (6.92)
\]

where \( p \) represents the unknown model parameters, \( \mathbf{y}_{meas} \) are the measurements, \( \mathbf{y}_{model}(p) \) are the model predictions, and \( p_{ref} \) are reference model parameters used as constraints. The reference model parameters can be parameters retrieved from model scale tests and/or CFD simulations.

To calculate the model parameters using the constrained optimization scheme (6.92) we use model scale measurements of \( T_p, Q_p, n, \) and \( u \). From model scale
data we know the values of the model parameters, \( w, \alpha_0, \alpha_1, \beta_0, \) and \( \beta_1, \) so they are used as reference values. For the axial flow velocity, \( u_p, \) there are no reference values. The value of \( \gamma_1 \) from open water tests is 10.28, which significantly differs from the value retrieved from self-propulsion tests.

The results of the constrained optimization problem are listed in Table 6.2. The relative difference from the model test values are also listed. The calculated values of the propeller characteristics are left out due to confidentiality agreement. The optimization is initiated from values that are very close to the model scale values. The relative difference of parameters \( \alpha_0 \) and \( \alpha_1 \) are much higher than the ones of parameters \( \beta_0 \) and \( \beta_1. \) This happens due to parameter \( \gamma_1; \) its value measured from self-propulsion tests is 7.25 while its value calculated from open water tests is 10.28. The problem is also sensitive to initial guess. In Table 6.3 we list the results when the initial guess is changed by 10 %. Parameters \( \beta_1 \) and \( w \) change the most; the axial flow velocity, \( u_p, \) changes due to \( w. \) This example indicates that the method is basically useless at full scale, where no reference values exist since the model scale figures are doubtful at full scale.

### Table 6.2: Results

<table>
<thead>
<tr>
<th>unknown quantity</th>
<th>calculated ([-,])</th>
<th>relative difference from model test ([]%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \alpha_1 )</td>
<td>###</td>
<td>32</td>
</tr>
<tr>
<td>( \alpha_0 )</td>
<td>###</td>
<td>16</td>
</tr>
<tr>
<td>( \beta_1 )</td>
<td>###</td>
<td>4</td>
</tr>
<tr>
<td>( \beta_0 )</td>
<td>###</td>
<td>1</td>
</tr>
<tr>
<td>( w )</td>
<td>0.295</td>
<td>4</td>
</tr>
<tr>
<td>( u_p,1 )</td>
<td>0.405</td>
<td>-</td>
</tr>
<tr>
<td>( u_p,2 )</td>
<td>0.450</td>
<td>-</td>
</tr>
<tr>
<td>( u_p,3 )</td>
<td>0.496</td>
<td>-</td>
</tr>
<tr>
<td>( u_p,4 )</td>
<td>0.539</td>
<td>-</td>
</tr>
<tr>
<td>( u_p,5 )</td>
<td>0.562</td>
<td>-</td>
</tr>
<tr>
<td>( u_p,6 )</td>
<td>0.587</td>
<td>-</td>
</tr>
<tr>
<td>( u_p,7 )</td>
<td>0.615</td>
<td>-</td>
</tr>
<tr>
<td>( u_p,8 )</td>
<td>0.647</td>
<td>-</td>
</tr>
</tbody>
</table>

### Table 6.3: Results with 10% off initial guess

<table>
<thead>
<tr>
<th>unknown quantity</th>
<th>calculated ([-,])</th>
<th>relative difference from model test ([]%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \alpha_1 )</td>
<td>###</td>
<td>34</td>
</tr>
<tr>
<td>( \alpha_0 )</td>
<td>###</td>
<td>16</td>
</tr>
<tr>
<td>( \beta_1 )</td>
<td>###</td>
<td>7</td>
</tr>
<tr>
<td>( \beta_0 )</td>
<td>###</td>
<td>1</td>
</tr>
<tr>
<td>( w )</td>
<td>0.271</td>
<td>11</td>
</tr>
<tr>
<td>( u_p,1 )</td>
<td>0.413</td>
<td>-</td>
</tr>
<tr>
<td>( u_p,2 )</td>
<td>0.458</td>
<td>-</td>
</tr>
<tr>
<td>( u_p,3 )</td>
<td>0.506</td>
<td>-</td>
</tr>
<tr>
<td>( u_p,4 )</td>
<td>0.549</td>
<td>-</td>
</tr>
<tr>
<td>( u_p,5 )</td>
<td>0.572</td>
<td>-</td>
</tr>
<tr>
<td>( u_p,6 )</td>
<td>0.598</td>
<td>-</td>
</tr>
<tr>
<td>( u_p,7 )</td>
<td>0.627</td>
<td>-</td>
</tr>
<tr>
<td>( u_p,8 )</td>
<td>0.658</td>
<td>-</td>
</tr>
</tbody>
</table>

The axial inflow factor

At stationary state the advance velocity is proportional to the axial flow velocity according to

\[
\bar{u}_a = \frac{1}{1 + \alpha} u_p \tag{6.93}
\]

where parameter \( \alpha \) is called axial inflow factor and is unknown. This result is derived from the momentum theory applied to the propeller actuator disc, see Section 4.3.3.
Let us plug this result in the system of equations (6.74) to (6.76)

\[
\bar{u}_{p,j} = \frac{1}{2} (1 - w) \bar{u}_j + \frac{1}{2} \sqrt{(1 - w)^2 \bar{u}_j^2 + \frac{8}{\pi} \bar{T}_{p,j}} \tag{6.94}
\]

\[
\bar{T}_{p,j} = a_1 \bar{n}_j \bar{u}_{p,j} + a_0 \bar{n}_j^2 \tag{6.95}
\]

\[
\bar{Q}_{p,j} = b_1 \bar{n}_j \bar{u}_{p,j} + b_0 \bar{n}_j^2 \tag{6.96}
\]

where \(a_1 = \alpha_1/(1 + \alpha)\), \(a_0 = \alpha_0\), \(b_1 = \beta_1/(1 + \alpha)\) and \(b_0 = \beta_0\).

Basically, there is no improvement compared to the initial problem. The number of unknowns is the same; \(u_p\) cannot be distinguished from \(a_1\) and \(b_1\), and \(w\) is calculated for any \(u_p\).

Remarks

To conclude this section, we showed that wake fraction and propeller characteristics cannot be calculated from the stationary states of the ship propulsion model. The main difficulty is the fact that unknown wake fraction cannot be separated by the unknown propeller parameters and the unknown axial flow velocity. Using information from model scale tests or CFD simulations is not appropriate in this case. It appears that the ship propulsion model needs more relationships that connect the measured states at full scale and the unknown model parameters, namely, the wake fraction and the propeller parameters. We do not undertake this challenge in this thesis. But we do alter the ship propulsion model and use a dynamic control to invent a method that estimates the wake fraction and propeller parameters. This method is presented in the next section.

6.6 Wake fraction from harmonic control

The conclusion of the previous section was that the mathematical model we use is not suitable to estimate the wake fraction from stationary states alone. From previous section we keep the idea of representing the thrust and torque constitutive relations with respect to axial flow velocity but we change the control. The stationary state is complemented by the dynamic state. The dynamic state is achieved by imposing small oscillations around a stationary state by controlling the set rotational speed. The response of the ship propulsion to the harmonic control enables us to incorporate more information in the parameter estimation scheme. In the end of the section we give the proof of principle that wake fraction can be estimated from this method.
6.6. Wake fraction from harmonic control

6.6.1 Preliminaries

Harmonic control means small-amplitude, short-time oscillations around the stationary state. The oscillations are introduced by the set angular velocity, \( n_{\text{set}} \). The small oscillations do not affect the ship speed due to immense inertia. In our mathematical model the term \( \mu \) takes care that the ship speed does not react fast to the short-time oscillations. Advance speed is related to ship speed by the constitutive Equation (4.71), and, thus, has the same dynamic behavior as the ship speed. Under these conditions the ship speed and the advance speed are regarded as constants; thus, they are denoted \( \bar{u} \) and \( \bar{u}_a \), respectively. The surge equation and the advance speed constitutive equation are skipped from the mathematical model.

Let us assume that the advance velocity, \( u_a \), in thrust and torque constitutive relations can be replaced by the axial flow velocity, \( u_p \). The axial flow velocity should reflect more accurately the changes in thrust and torque when the propeller angular velocity changes. This change affects the propeller parameters. New parameters \( a_0, a_1, b_0, \) and \( b_1 \) replace the old ones \( \alpha_0, \alpha_1, \beta_0, \) and \( \beta_1 \), respectively. The new parameters are unknown.

With the transformations above the mathematical model becomes

\[
\begin{align*}
\frac{dQ_{\text{eng}}}{dt} &= -\tau_{\text{eng}} Q_{\text{eng}} - \zeta_{\text{eng}} n + \zeta_{\text{eng}} n_{\text{set}} \\
\frac{dn}{dt} &= \zeta_{q} Q_{\text{eng}} - \zeta_{s} n - \zeta_{q} Q_0 - \zeta_{q} Q_p \\
\frac{du_p}{dt} &= T_p - \frac{1}{2} \pi u_p (u_p - \bar{u}(1 - w)) \\
0 &= -T_p + a_1 u_p n + a_0 n^2 \\
0 &= -Q_p + b_1 u_p n + b_0 n^2
\end{align*}
\]

where \( \bar{u}(1 - w) \) replaces the advance velocity \( \bar{u}_a \) in the propeller equation, see Equation (6.99). Thus, the wake fraction appears explicitly in the mathematical model.

The mathematical model contains five state variables, \( Q_{\text{eng}}, n, u_p, T_p, \) and \( Q_p \). The state variable \( u_p \) can be written as function of \( n \) and \( T_p \). The following intrinsic relation is obtained

\[
u_p = \zeta_1 x_1 + \zeta_2 x_2
\]

where the newly introduced quantities are \( x_1 = T_p/n, \ x_2 = n, \ \zeta_1 = 1/a_1, \) and \( \zeta_2 = -a_0/a_1 \).

The intrinsic formula (6.102) is used in torque constitutive equation to get the

*In practice oscillations can be introduced in the propulsion system by the propeller pitch angle too.
Estimation of model parameters following result

\[ Q_p = \zeta_3 x_1 x_2 + \zeta_4 x_2^2 \]  

(6.103)

where \( \zeta_3 = b_1/a_1 \) and \( \zeta_4 = b_0 - b_1 a_0/a_1 \).

With \( u_p \) and \( Q_p \) expressed in terms of \( x_1 \) and \( x_2 \) the mathematical model becomes

\[
\begin{align*}
\zeta_1 \frac{dx_1}{dt} + \zeta_2 \frac{dx_2}{dt} &= x_1 x_2 - \frac{1}{2} \pi \zeta_1^2 x_1^2 - \frac{1}{2} \pi \zeta_2^2 x_2^2 - \pi \zeta_1 \zeta_2 x_1 x_2 + \\
&+ \frac{1}{2} \pi (1 - \zeta_5) \zeta_1 \bar{u} x_1 + \frac{1}{2} \pi (1 - \zeta_5) \zeta_2 \bar{u} x_2 \\
\frac{dx_2}{dt} &= \zeta_q x_3 - \zeta_s x_2 - \zeta_q Q_0 - \zeta_3 \zeta_q x_1 x_2 - \zeta_4 \zeta_q x_2^2 \\
\frac{dx_3}{dt} &= -\tau_{eng} x_3 - \zeta_{eng} x_2 + \zeta_{eng} \bar{n}_{set}
\end{align*}
\]

(6.104)

(6.105)

(6.106)

Let us summarize the new model states

\[ x_1 = \frac{T_p}{n}; \quad x_2 = n; \quad x_3 = Q_{eng} \]  

(6.107)

and the new unknown parameters

\[ \zeta_1 = \frac{1}{a_1}; \quad \zeta_2 = -\frac{a_0}{a_1}; \quad \zeta_3 = b_1/a_1; \quad \zeta_4 = b_0 - b_1 a_0/a_1; \quad \zeta_5 = w \]  

(6.108)

There are six unknowns, the parameters \( \zeta_i, \ i = 1..5 \) and the state \( x_3 \), and only three equations to solve them. To derive extra equations from the mathematical model the propulsion system is controlled in a special way: small harmonic variations around a stationary state are introduced. Let us proceed with the stationary state.

### 6.6.2 Stationary control

Given a stationary control, \( n_{set} = \bar{n}_{set} \), the propulsion system operates in a stationary state \( \bar{x} = [\bar{x}_1, \bar{x}_2, \bar{x}_3] \). The stationary mathematical model is

\[
\begin{align*}
0 &= \bar{x}_1 \bar{x}_2 - \frac{1}{2} \pi \zeta_1^2 \bar{x}_1^2 - \frac{1}{2} \pi \zeta_2^2 \bar{x}_2^2 - \pi \zeta_1 \zeta_2 \bar{x}_1 \bar{x}_2 + \\
&+ \frac{1}{2} \pi (1 - \zeta_5) \zeta_1 \bar{u} \bar{x}_1 + \frac{1}{2} \pi (1 - \zeta_5) \zeta_2 \bar{u} \bar{x}_2 \\
0 &= \zeta_q \bar{x}_3 - \zeta_s \bar{x}_2 - \zeta_q Q_0 - \zeta_3 \zeta_q \bar{x}_1 \bar{x}_2 - \zeta_4 \zeta_q \bar{x}_2^2 \\
0 &= -\tau_{eng} \bar{x}_3 - \zeta_{eng} \bar{x}_2 + \zeta_{eng} \bar{n}_{set}
\end{align*}
\]

(6.109)

(6.110)

(6.111)

The states \( y_{meas} = [\bar{y}_{1,meas}, \bar{y}_{2,meas}, \bar{y}_{3,meas}] \) are measured at full scale.
The stationary model has five unknowns, namely, $\zeta_1$ to $\zeta_5$. The unknowns form two groups. The first group of unknowns, $\zeta_1$, $\zeta_2$, and $\zeta_5$, is located in Equation (6.109). The second group of unknowns, $\zeta_3$ and $\zeta_4$, is located in equations (6.110). Obviously, the five unknowns cannot be solved from the stationary state model alone. Our intention is to make use of a specific control to force the model into a state that provides additional equations for parameter estimation.

### 6.6.3 Harmonic control

The general form of the system of equations (6.109) to (6.111) is

$$M \frac{d}{dt} \mathbf{x} = f(\mathbf{x}, \mathbf{c}; \mathbf{p}) \quad (6.112)$$

and the observed states are

$$\mathbf{y}_{\text{model}} = h(\mathbf{x}) \quad (6.113)$$

In this section we impose small harmonic oscillations on the system (6.112), and, based on the response of the system we retrieve extra relations that support the parameter estimation.

Let us consider a control that is the superposition of a harmonic component on top of a stationary component, $\mathbf{c}(t) = \bar{\mathbf{c}} + \tilde{\mathbf{c}}(t)$. The amplitude of the harmonic component is small relatively to the stationary component. The state variables are of the form $\mathbf{x}(t; \mathbf{p}) = \bar{\mathbf{x}}(\mathbf{p}) + \tilde{\mathbf{x}}(t; \mathbf{p})$. The harmonic control acts linearly on the observed states, thus we write $\mathbf{y}_{\text{model}}(t) = \bar{\mathbf{y}}_{\text{model}} + \tilde{\mathbf{y}}_{\text{model}}(t)$. The states measured at full scale, $\mathbf{y}_{\text{meas}}(t)$ are split into a stationary component, $\bar{\mathbf{y}}_{\text{meas}}$, which we refer as the trend, and a harmonic component, $\tilde{\mathbf{y}}_{\text{meas}}(t)$.

Small amplitude oscillations means that we are allowed to linearize the mathematical model around stationary states

$$M(\mathbf{p}) \frac{d}{dt} \mathbf{x} = f(\bar{\mathbf{x}}, \bar{\mathbf{c}}, \mathbf{p}) + \frac{\partial}{\partial \mathbf{x}} f(\bar{\mathbf{x}}, \bar{\mathbf{c}}, \mathbf{p}) \tilde{\mathbf{x}} + \frac{\partial}{\partial \mathbf{c}} f(\bar{\mathbf{x}}, \bar{\mathbf{c}}, \mathbf{p}) \tilde{\mathbf{c}} \quad (6.114)$$

$$\tilde{\mathbf{y}}_{\text{model}} = \frac{\partial}{\partial \mathbf{x}} h(\bar{\mathbf{x}}) \tilde{\mathbf{x}}$$

where $\frac{\partial f}{\partial \mathbf{x}}$, $\frac{\partial f}{\partial \mathbf{c}}$, and $\frac{\partial h}{\partial \mathbf{x}}$ are the Jacobian matrices. The stationary state term vanishes due to Equations (6.109) to (6.111). The linearized mathematical model becomes

$$M(\mathbf{p}) \frac{d}{dt} \mathbf{x} = \mathbf{A}(\mathbf{p}) \tilde{\mathbf{x}} + \mathbf{B}(\mathbf{p}) \tilde{\mathbf{c}} \quad (6.115)$$

$$\tilde{\mathbf{y}}_{\text{model}} = \mathbf{E} \tilde{\mathbf{x}}$$

where the Jacobian matrices $M(\mathbf{p})$, $\mathbf{A}(\mathbf{p})$, $\mathbf{B}(\mathbf{p})$, and $\mathbf{E}$ are
Estimation of model parameters

\[ A(p) = \begin{pmatrix}
\pi \zeta_2 \dot{x}_1 - \pi \zeta_1 \dot{x}_2 + \frac{1}{2} \pi (1 - \zeta_5) \ddot{u} & \pi \zeta_1 \dot{x}_1 + \frac{1}{2} \pi (1 - \zeta_5) \dot{u} & 0 \\
\zeta_5 \dot{x}_2 & -\zeta_5 \dot{x}_1 - 2 \zeta_4 \dot{x}_2 & -\zeta_{eng} \\
0 & 0 & -\zeta_{eng}
\end{pmatrix} \tag{6.116} \]

\[ M(p) = \begin{pmatrix}
\zeta_1 & \zeta_2 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{pmatrix}; \quad B = \begin{pmatrix}
0 \\
0 \\
\zeta_{eng}
\end{pmatrix}; \quad E = \begin{pmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{pmatrix} \tag{6.117} \]

We write the harmonic component of the control as \( \tilde{c}(t) = C e^{j\omega t} \), where \( \omega \) is the frequency of the imposed harmonic control. The same frequency is found in the state \( \tilde{x}(t; p) = X(p)e^{j\omega t} \), and is observed in \( \tilde{y}_{model}(t) = Y_{model}(e^{j\omega t}) \). The linearized dynamic model becomes

\[ j\omega M(p) X(p) = A(p) X + B C \tag{6.118} \]
\[ Y_{model}(p) = E X(p) \tag{6.119} \]

The solution of the linearized model is

\[ X(p) = (j\omega M(p) - A(p))^{-1} B C \tag{6.120} \]

We use the solution for \( X(p) \) in Equation (6.119) to get

\[ Y_{model}(\omega, p) = E (j\omega M(p) - A(p))^{-1} B C \tag{6.121} \]

At full scale, we measure \( y_{meas}(t) \). The signals are decomposed in trend, a harmonic component, and noise, using the signal decomposition method described in Section 8.1. From full scale propulsion tests we learn that the imposed frequency, \( \omega \), is found in the measured signals. Figure 6.6 depicts an excerpt of the propulsion tests conducted on a container vessel. The control in this case is the propeller pitch angle, \( \theta_p \). The pitch angle has a harmonic component superimposed on a stationary component. The imposed frequency, \( \omega \), is observed in the measured propeller thrust. Note that there is no phase shift between the pitch angle and thrust oscillations. The measurements are depicted dimensionless; the period of one oscillation is about 20 seconds. The measured quantities are scaled to point out the resemblances.
Figure 6.6: Imposed harmonic oscillation at full scale. The oscillation is introduced by changing the propeller pitch angle, $\theta_p$. The measured propeller thrust, $T_p$, oscillates with the imposed frequency. Both thrust and propeller pitch are depicted dimensionless and are scaled. The period of one oscillation is about 20 seconds.

This example proves our point. The measured states are decomposed as in

$$y_{\text{meas}} = \bar{y}_{\text{meas}} + Y_{\text{meas}}e^{j\omega t}$$

where $\bar{y}_{\text{meas}}$ is the trend, and $Y_{\text{meas}}e^{j\omega t}$ is the harmonic component; the noise is discarded. The use of harmonic dynamics instead of other type of ship dynamics, as, for instance, the zero-thrust control in Section 6.3, has to do with the number of equations. The harmonic control enables us to double the amount of equations for the dynamic state by considering the amplitude and the phase of the control.

### 6.6.4 The parameter estimation scheme

To estimate the sought model parameters, $p = [\zeta_1, \zeta_2, \zeta_3, \zeta_4, \zeta_5]$, we minimize the difference between the model and measurements

$$\min_{p, \omega} \|y_{\text{meas}} - y_{\text{model}}(\omega, p)\|^2$$

with respect to the model parameters and the frequency of the control.

Since the observed states and the measured states are decomposed in trend and harmonic components, as in

$$y_{\text{model}}(\omega, p) = y_{\text{model}}(p) + Y_{\text{model}}(\omega, p)e^{j\omega t}$$

$$y_{\text{meas}} = \bar{y}_{\text{meas}} + Y_{\text{meas}}e^{j\omega t}$$

the optimization problem (6.123) can be written as

$$\min_{\omega, p} \|y_{\text{meas}} - y_{\text{model}}(p)\|^2 + \|Y_{\text{meas}} - E(j\omega M(p) - A(p))^{-1}B\|^2$$
The stationary state provides three equations for the optimization problem, see Equations (6.111) to (6.109). Note that Equation (6.109) does not contain any of the unknown parameters. The harmonic dynamic state has three equations. By considering the amplitude and phase information of the harmonic dynamic state the number of equations doubles to six. Thus, we rely on nine equations to estimate the five unknown model parameters.

6.6.5 Proof of principle

The parameter estimation method presented in this section requires full scale measurements with harmonic control of the form $c = \bar{c} + C e^{j\omega t}$, with a predefined frequency $\omega$. Full scale measurements with imposed harmonic control are not available in this thesis.

In this section we present the proof of principle of the parameter estimation based on harmonic control. The method is verified by using simulated measurements; these measurements are actually the solutions of the ship propulsion model. The ships we simulate in this section are a full scale ship and a model scale ship; both exist in real-life. The sought model parameters, namely, $w, a_1, a_0, b_1,$ and $b_0$, are fixed and input in the simulation; their values are the true ones. The simulated measurements are processed and fed to the parameter estimation scheme. The unknown parameters, $[w, a_1, a_0, b_1, b_0]$, are estimated in the optimization procedure and their values are compared to the input values. In this section we use a simplified version of the parameter estimation scheme. In this version only $w, a_1,$ and $a_0$ are assumed unknown, thus, significantly reducing the problem. The schematic of the procedure followed in this section is depicted in Figure 6.7.

The simulation

A simulation of ship dynamics is setup on basis of the ship propulsion model

\[
\frac{dQ_{\text{eng}}}{dt} = -\tau_{\text{eng}} Q_{\text{eng}} - \zeta_{\text{eng}} n + \zeta_{\text{eng}} n_{\text{set}} \quad (6.126)
\]

\[
\frac{dn}{dt} = \zeta_q Q_{\text{eng}} - \zeta_s n - \zeta_q Q_0 - \zeta_q Q_p \quad (6.127)
\]

\[
\frac{d\psi}{dt} = \frac{1}{\mu} \left( (1-t_d)T_p - \varphi u \gamma \right) \quad (6.129)
\]

\[
0 = -T_p + a_1 n u_p + a_0 n^2 \quad (6.130)
\]

\[
0 = -Q_p + b_1 n u_p + b_0 n^2 \quad (6.131)
\]

\[
0 = -u_a + (1-w) u \quad (6.132)
\]

Note that in this version of the ship propulsion model the thrust and torque constitutive relations are represented with respect to the axial flow velocity, $u_p$. 

The stationsary state provides three equations for the optimization problem, see Equations (6.111) to (6.109). Note that Equation (6.109) does not contain any of the unknown parameters. The harmonic dynamic state has three equations. By considering the amplitude and phase information of the harmonic dynamic state the number of equations doubles to six. Thus, we rely on nine equations to estimate the five unknown model parameters.
6.6. Wake fraction from harmonic control

![Figure 6.7](image)

Figure 6.7: The $\tilde{\theta}_{\text{true}}$ values of $[w, a_0, a_1, b_0, b_1]$ are preset; they are conveniently chosen such that the stationary solution is close to the measurements. The ship propulsion model is solved with this set of parameters and for a harmonic control with an initial frequency. The solutions of the model represent our measurements; they are simulated measurements. The measurements are input to the parameter estimation method. The estimated parameters are compared with the $\tilde{\theta}_{\text{true}}$ values. If the estimated values are within the preset range the algorithm stops. If the estimated values are far from the $\tilde{\theta}_{\text{true}}$ ones the control of the ship propulsion model is changed, namely, the frequency of control.

as opposed to their original form, see Equations (5.17) to (5.19). This change in approach was introduced in the beginning of Section 6.6.

We solve the mathematical model for two real-life ships: a full scale ship and a model scale ship. The full scale ship is one of the ships we performed full scale measurements; see Chapter 7. The model scale ship is mentioned in the context of stationary state solutions of the ship propulsion model, see Section 5.2.2.

The input values of the sought model parameters are listed in Table 6.4. With these input values of the model parameters the solutions of the mathematical model, Equations (6.126) to (6.132), are computed. The results are listed in Table 6.5 and compared to the direct measurements.

Table 6.4: The input values of the sought parameters

<table>
<thead>
<tr>
<th>ship</th>
<th>$w$</th>
<th>$a_1$</th>
<th>$a_0$</th>
<th>$b_1$</th>
<th>$b_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>model scale</td>
<td>0.287</td>
<td>-1.17</td>
<td>0.96</td>
<td>-0.11</td>
<td>0.10</td>
</tr>
<tr>
<td>full scale</td>
<td>0.3</td>
<td>-1.1</td>
<td>0.9</td>
<td>-0.11</td>
<td>0.105</td>
</tr>
</tbody>
</table>

Table 6.5: The calibration of the simulation model

<table>
<thead>
<tr>
<th>ship</th>
<th>simulated [-]</th>
<th>measured [-]</th>
<th>relative difference [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$n$ $\tilde{n}$</td>
<td>$\tilde{u}$ $u$</td>
<td>$\tilde{f}_p$ $f_p$</td>
</tr>
<tr>
<td>model scale</td>
<td>0.98 0.98</td>
<td>0.19 0.20</td>
<td>0.026 0.027</td>
</tr>
<tr>
<td>full scale</td>
<td>0.99 0.99</td>
<td>0.20 0.20</td>
<td>0.033 0.036</td>
</tr>
</tbody>
</table>
The parameter estimation scheme requires an imposed control of the form

\[ n_{\text{set}} = \bar{n}_{\text{set}} + N_{\text{set}}e^{2\pi j f_{\text{set}}t} \]  

(6.133)

where \( f_{\text{set}} \) is the frequency of the imposed control imposed.

The solution of the parameter estimation scheme depends on the mean value of the control, \( \bar{n}_{\text{set}} \), the amplitude of the control, \( N_{\text{set}} \), and the imposed frequency, \( f_{\text{set}} \). In the following we test the estimation scheme with respect to these three control parameters.

![Figure 6.8: The parameters of the imposed harmonic control.](image)

**Figure 6.8: The parameters of the imposed harmonic control.**

**Estimated parameters as function of the frequency of the control, \( f_{\text{set}} \)**

Let us fix the mean value and the amplitude of the imposed control to \( \bar{n}_{\text{set}} = 1 \) and \( N_{\text{set}} = 0.05 \cdot \bar{n}_{\text{set}} \). The frequency \( f_{\text{set}} \) of the harmonic control is varied from 0.005 Hz to 5 Hz, in case of the full scale ship, and 30 Hz in case of the model scale ship. The results of the parameter estimation scheme are depicted in Figure 6.9 as the relative difference between the true parameters and the estimated parameters, \( 100 \cdot \frac{|p_{\text{sought}} - p_{\text{est}}|}{p_{\text{sought}}} \). The relative difference is plotted against the dimensionless frequency of the harmonic control. Note that all plots from this point onward are represented in logarithmic scale.

The frequency of the harmonic control has clear influence on the parameter estimation. The results of this test indicate that parameters are estimated accurately for a specific range of frequencies. For both full scale and model scale ships the relative differences decrease with increase of the frequency. When the input frequency is from 0.05 to 0.1 the relative differences decrease below 1%. For frequencies higher than 0.1 the solutions are generally good, but with particular differences. We note that the relative differences of parameter \( a_0 \) are on average smaller than those of parameter \( a_1 \) and \( w \). This tendency is observed for both full scale and model scale ships. As with the further increase in the frequency of the harmonic control the solutions of the parameter estimation scheme tend to worsen. For frequencies above 0.6 the relative differences of the parameters at full scale are 5% and more. At model scale only the values of parameter \( w \) display a similar tendency; \( a_1 \) and, especially, \( a_0 \) remain under 1% relative difference up to the maximum frequency tested.
In dimensionless formulation, the behavior of the solutions of the parameter estimation scheme with respect to the frequency of the harmonic control are similar at full scale and model scale. This means, of course, that in full dimension formulation the behavior is different. Figure 6.10 depict the solutions of the parameter estimation scheme as function of the dimension full frequency of the harmonic control. The plots reveal a fundamental difference between full scale and model scale solutions. Let us look at the lowest control frequency for which the relative difference drops below 1%. At full scale, the control frequency ranges from 0.3 Hz for parameter $a_0$ until 0.6 Hz for parameter $w$. At model scale, the control frequency ranges from 2 Hz for parameter $a_0$ until 3.5 Hz for parameter $w$. A discussion on the influence of the control frequency on the solution of the parameter estimation scheme is reserved for Section 6.6.6.
Estimated parameters as function of the amplitude of the control, $N_{set}$

Let us fix the dimensionless control frequency, $f_{set}$, to 0.07 and the mean value $n_{set} = 1$. We test the influence of the solutions of parameter estimation scheme with respect to the amplitude of the harmonic control. We vary $N_{set}$ between 0.5% and 50% of the mean value. The behavior of the solution in terms of the relative difference is depicted in Figure 6.11.

As general observation, the changes in control amplitude does not have the dramatic influence observed in case of the changes in control frequency. At extreme values of the amplitude the relative difference is at most 10%. The solution behavior with respect to amplitude changes in similar manner at full scale and model scale. Again, the relative difference for parameter $a_0$ is lower than the relative differences of $a_1$ and $w$. There is an exception in the neighborhood of 0.15 when the quality of parameter $w$ improves dramatically. It is safe to conclude that the overall solution is at its best for amplitudes ranging from 5% to 10% of the mean.
value.

\begin{figure}
\centering
\includegraphics[width=\textwidth]{figure6_11}
\caption{The estimated parameters as function of the amplitude $\tilde{n}_{set}$ of the imposed harmonic control.}
\end{figure}

Estimated parameters as function of the mean value of the control, $\bar{n}_{set}$

Let us fix the parameters of the amplitude and the frequency of the imposed harmonic oscillation to $N_{set} = 0.05$ and $f_{set} = 0.07$, respectively. The mean value, $\bar{n}_{set}$, of the imposed control varies from 0.5 to 1. The behavior of the solution of parameter estimation scheme is depicted in Figure 6.12.

The behavior of the solutions at full scale and model scale are similar, except for the parameter $w$ at full scale. The relative differences are lower than 1% when $\bar{n}_{set}$ is between 0.9 and 1. In case of $w$ at full scale it appears that the lower $\bar{n}_{set}$ the lower the relative difference.
6.6.6 Discussion

In this section we show that, theoretically, it is possible to estimate the wake fraction parameter as introduced by the ship propulsion model. Yet, this parameter estimation scheme requires the response of the ship to a specific control. The imposed harmonic control, which is quite exotic for a ship’s propulsion system, enables us to transform the mathematical model into a parameter estimation scheme for wake fraction. The change in thrust and torque constitutive relation contributed decisively to the outcome.

The simulated experiments reveal that the performance of the estimation scheme largely depend on the frequency of the harmonic control. Most probably, this aspect has a major impact on the practical implementation of the method at full scale.

Harmonic control has been performed on a full scale ship by means of varying the propeller pitch. The measurements indicated that propeller thrust and torque

Figure 6.12: The estimated parameters as function of the mean value $\bar{n}_{\text{set}}$ of the imposed harmonic control.
respond to this type of control with the same frequency. Moreover, no phase shift between control and response is observed. To include pitch control in the parameter estimation scheme implies to change the mathematical model accordingly. This change has not been attempted in this thesis.

Full scale measurements show that oscillations occur in propeller thrust and torque, ship angular motions (roll, pitch, and yaw), and propeller pitch angle, see Section 7.3.2. There are reasons to believe that these oscillations are the results of the influence of the external factors on ship dynamics. These oscillations could replace the imposed ones; of course, the mathematical model, and perhaps, the estimation method, have to change.

6.7 Design of measurement scheme

Full-scale measurements are prerequisites for the objective of this thesis. In this section we present the details of the full scale measurements as needed by the mathematical models and the parameter estimation methods. The structure of this section is as follows: in the first subsection the variables to be measured are listed, mentioning their resolution and sampling frequency. In the second subsection the general requirements for the data collection system are stated. Details on how the propulsion tests should be performed are presented in the third section.

6.7.1 List of variables

The variables that need to be measured at full scale are listed in Table 6.6. The structure of the list is mainly driven by parameter estimation. Note that the axial flow velocity and the advance velocity are not listed since it is assumed that they cannot be measured at full scale in regular ship operations. Variables that are not directly connected to parameter estimation are required for signal reconciliation. The signal reconciliation is a special method designed in this work to correct the full scale measurements. The method is presented in Section 8.2.

Each variable is recorded as digital signal. If there are analogue signals these are converted into digital signals. Due to reliability aspects of the signal processing it is important that the signals have certain resolutions and certain sampling frequencies. The resolution and the sampling frequency of each variable is mentioned in Table 6.6.

Obviously, reliable sensors are desired, in terms of accuracy and precision. In practice most of the sensors are already installed on board of the ship and little can be done to improve the quality of the signals. This thesis deals with such aspects of the full scale measurements. Some of the observed signal distortions are presented in Section 7.3. In this thesis the measured ship speed receives special attention. In Section 8.2 a data fusion method to correct the ship speed is presented.
Table 6.6: List of variables that need to be measured at full scale

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Resolution</th>
<th>Sampling frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>Speed-through-water longitudinal</td>
<td>0.01 [kn]</td>
<td>10 [Hz]</td>
</tr>
<tr>
<td>Speed-through-water transversal</td>
<td>0.01 [kn]</td>
<td>10 [Hz]</td>
</tr>
<tr>
<td>Speed-over-ground longitudinal</td>
<td>0.01 [kn]</td>
<td>10 [Hz]</td>
</tr>
<tr>
<td>Speed-over-ground transversal</td>
<td>0.01 [kn]</td>
<td>10 [Hz]</td>
</tr>
<tr>
<td>Propeller thrust</td>
<td>0.001 [kN]</td>
<td>10 [Hz]</td>
</tr>
<tr>
<td>Engine torque</td>
<td>0.001 [kNm]</td>
<td>10 [Hz]</td>
</tr>
<tr>
<td>Actual propeller angular velocity</td>
<td>0.1 [s⁻¹]</td>
<td>10 [Hz]</td>
</tr>
<tr>
<td>Set propeller angular velocity</td>
<td>0.1 [°]</td>
<td>10 [Hz]</td>
</tr>
<tr>
<td>Actual propeller pitch angle</td>
<td>0.1 [°]</td>
<td>10 [Hz]</td>
</tr>
<tr>
<td>Set propeller pitch angle</td>
<td>0.1 [°]</td>
<td>10 [Hz]</td>
</tr>
<tr>
<td>Latitude and Longitude</td>
<td></td>
<td>10 [Hz]</td>
</tr>
<tr>
<td>Ship angular motions (yaw, pitch,</td>
<td>0.1 [°]</td>
<td>10 [Hz]</td>
</tr>
<tr>
<td>roll)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Heading</td>
<td>0.1 [°]</td>
<td>10 [Hz]</td>
</tr>
<tr>
<td>Wind speed</td>
<td>1 [kn]</td>
<td>1 [Hz]</td>
</tr>
<tr>
<td>Wind direction</td>
<td>1 [°]</td>
<td>1 [Hz]</td>
</tr>
<tr>
<td>Displacement, draft trim</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Water depth</td>
<td>1 [m]</td>
<td>1 [Hz]</td>
</tr>
</tbody>
</table>

6.7.2 Data collection system

Full-scale measurements require a data collection system (DCS). The DCS is a collection of hardware that performs the following tasks:

- collect analogue and digital signals from the sensors,
- convert analogue signals into digital,
- filter the digital signals for unnecessary information,
- scale the digital signals,
- store the digital signals into log files,
- process the digital signals,
- display the processed signals and the propeller efficiency for the crew members,
- send the raw and processed data to a home base.
The functions listed above are fulfilled by elements of the DCS. The main elements of the data collection system are the programmable logic controller (PLC), the industrial computer (iPC), the data transfer unit (telemetry), and a local area network.

A schematic layout of the data collection system is proposed in Figure 6.13. The analogue or digital signals are collected from the sensors by the programmable logic computers. The PLC is a digital computer with limited computational power. At the PLC the analogue signals are digitized. Digital signals are filtered from unnecessary information. For instance, digital signals that come from sensors are strings of standardized information into NMEA protocol. Most of the information contained in these strings is discarded by the PLC and only the actual value of the variable is kept. At the PLC the scaling of the digital signals can also be done. The digital signals are transferred from the PLC to the industrial computer through a local area network (LAN). The industrial computer (iPC) is a key component of the system. In the iPC, the raw signals are stored into log files and dedicated software run mathematical algorithms that process the measurements. Modern iPC have touch screens to display and to input information. Raw signals and processed information is sent over the telemetry connection to a home base on shore. At the home base the data from several ships is stored, and is further processed. The data collection system allows information to travel both ways. From home base the operator can monitor the entire data collection system. The telemetry connection enables the operator to update the software running on the iPC and the PLC, and to identify malfunctions of the system.
6.7.3 Propulsion tests

Parameter estimation methods require full scale measurements performed under certain sailing conditions. Due to this reason we talk about propulsion tests. There are three types of propulsion tests required by the parameter estimation method: stationary control test, zero-thrust control test, and harmonic control test. Beside the controlled states there are uncontrolled states that need to be taken into account during the propulsion tests, such as, weather state and sea state.

Stationary control

We define a process to be stationary when the trend of the process is zero (or close to zero). We allow small harmonic oscillations around the trend. When the process is stationary we can, under this definition, replace the time-dependent signal by one single value, which can be the average value.

In practice, a ship sails mostly in stationary states. This is the reason why propeller efficiency is only defined for a stationary state, see formula (2.9) introduced in Section 2.1. Nowadays, stationary propulsion tests are carried out every time a ship is commissioned. The objective of these propulsion tests is to map the power (shaft power) versus speed curve of the ship. In this sense, the International Towing Tank Conference (ITTC) issued a set of standardized procedures, which are periodically updated, see (ITTC, 2003). These standardized procedures are also valid in case of stationary states required by the parameter estimation methods.

The most important specifications of the ITTC procedures with respect to conducting stationary control tests refer to duration, location, and track. The minimum advised duration for a stationary state is 10 minutes. Obviously, the more data is recorded during the stationary state the more reliable the averaged value. The location of the tests is important since it is well known that shallow waters negatively affect the overall performances of the ship. It is common practice to carry out two runs in opposite directions for each stationary control state. It is assumed that the environmental effects, e.g., wind, waves, currents, are canceled out.

A sample of stationary control states is given in Figure 6.14. These stationary states are presented in details in Chapter 9. The control applied to the ship propulsion during the stationary states is depicted in Figures 6.14a and 6.14b. The ship has one controllable pitch propeller; during the stationary states the control was done by propeller pitch angle. The response of the ship in terms of propeller thrust, propeller torque, speed-through-water, and heading, is depicted in Figures 6.14c, 6.14d, 6.14e, and 6.14f, respectively. Note the small oscillations around the stationary state that are allowed by definition.

†Here reliable may be understood in terms of statistical reliability. Since it is difficult to set a reference value, statistical reliability may be interpreted as precision rather than accuracy.
Additionally to ITTC specifications, we are interested in the minimum number of individual stationary control tests to be performed. As seen in Sections 6.2 and 6.4 the parameter estimation methods are essentially fitting problems; see equations (6.57) and (6.69). The statistical reliability of the calculation depends on the number of stationary data points available. A hindrance is of course the total time available for such full scale propulsion tests.

**Zero-thrust control**

Zero-thrust control tests are special propulsion tests. Essentially, this kind of test simulates the situation when propeller is removed from the ship and the ship
decelerates from an initial speed. The speed of the ship measured during such
test is required to estimate one resistance parameter in the surge equation, see
Section 6.3. To the knowledge of the author there are no such tests performed at
full scale. One important reason is that real-time propeller thrust measurements
are required.

The main idea of the zero-thrust control test is to “trick” the ship to behave
as if the propeller were missing. In this sense the propeller thrust is a control.
The zero-thrust control test starts from a stationary state. At a certain moment
the propeller rotational speed or the propeller pitch angle is control such that the
propeller thrust is zero. In Figure 6.15 is depicted such a situation. The ship has a
controllable pitch angle, thus, the control was done by pitch angle. The pitch angle
is depicted in Figure 6.15b. Changes in pitch angle directly affect the propeller
thrust as suggested by the constitutive relation (4.64). The moment the propeller
thrust is zero the ship decelerates. Note that the propeller pitch angle is updated
as the ship velocity decreases. Lowering the ship velocity means increasing the
propeller thrust keeping the same pitch angle. The velocity of the ship is depicted
in Figure 6.15e. The propeller torque, depicted in Figure 6.15d, is not zero since
the propeller still rotates with constant angular velocity, see Figure 6.15a. The
heading of the ship need to be constant without much control of the rudder; rudder
changes add extra resistance forces.

Some of the ITTC procedures can be applied in case of zero-thrust tests. The
location of the tests is important due to the fact that shallow waters affect the
overall performance of the ship. The ship should sail the same track twice in
opposite directions to cancel out the environmental effects.

Similarly to the stationary control tests, the number of zero-thrust control tests
is important. Actually, the resistance parameter \( p \) can be estimated from a single
zero-thrust test. The repeatability of the test is important for the reliability of the
result. A good balance between the number of zero-thrust control tests and the
available time for such tests has to be found.

**Harmonic control**

Harmonic control means oscillating propeller rotational speed. This type of control
is required by the parameter estimation scheme described in Section 6.1.3. The
author is not familiar with the details of the ship propulsion control. Due to
this reason, the harmonic control is not presented here at the same level as the
stationary and zero-thrust controls.

Surely, the most challenging aspect of the harmonic control is the frequency of
the imposed oscillations. Marine engines are not designed to vary their rotational
speed with the frequencies required by our parameter estimation. In fact, their
complex control systems are designed and tuned to eliminate oscillating loads due
to external factors that might damage the engine.

The parameter estimation based on harmonic control should first be experi-
6.7. Design of measurement scheme

...mented at model scale, where such controls are comparably easier to achieve. The observations made at model scale could then be used as feedback to the parameter estimation scheme for purpose of validation.

![Graphs showing various signals](image)

Figure 6.15: Example of signals measured during zero-thrust control.

External conditions

As already suggested in this chapter, external conditions influence the estimated parameters. External conditions cannot be controlled. At best, they can be thoroughly monitored and measured. By external conditions we mean weather related factors, such as wind, waves, and sea currents, and location related factors, such as water depth, water temperature, and traffic constraints.
General recommendations on the external conditions are included in the procedures and guidelines established by the International Towing Tank Conference (ITTC) regarding the sea trials at full scale, see (ITTC, 2003). ITTC recommends to measured wind speed and direction, wave height, period, and direction, and the sea currents. Figure 6.16 depicts the wind speed and direction measured during a propulsion test at full scale.

Figure 6.16: Wind speed and direction during a test at full scale.
The topic of this chapter is full scale measurements. This research is supported by measurements collected on board of two vessels: a large ferry and a mid-sized container ship. The two vessels are different from points of view of design, propulsion train, and operational profile.

In the first section we describe the data collection system as implemented on board of the two vessels. The data collection systems were designed and manufactured at Wärtsilä Netherlands. The data collection systems were installed on board of the ferry early 2009 and on board of the container ship mid 2010. The systems operate continuously ever since. The list of the recorded signals is presented in the same section. One of the most important component of the data collection system is the sensor that measures propeller thrust and torque, manufactured at VAF Instruments, in collaboration with Wärtsilä Netherlands.

In the second section we illustrate some of the raw signals. In this section we select signals recorded during normal ship operation for the ferry, and the same signals recorded during propulsion tests for the container ship. A thorough investigation of the raw signals reveals that most of them are distorted. We identify three types of signal distortions: the offsets that affect the trend (low-frequency range); the oscillations due to ship motions (mid-frequency range); and the noise (high-frequency range).

7.1 *In situ* data collection system

In Section 6.7 we describe, at conceptual level, what signals need to be collected and how the data collection system should look like. In this section we describe the data collection system as installed on board of the two vessels and list the collected signals.

In Figure 7.1 we depict the schematic of the data collection system installed on board of the vessels. The signals are collected by two programmable logic
controllers (PLC). The PLC that collects the navigation signals is located on the bridge. The PLC that collects the propulsion signals is located in the engine control room (ECR). At the level of PLC all the signals are sampled at 10 Hz, the NMEA 0183 strings are filtered, and the analogue signals are digitized. From PLC the signals are sent to the local area network (LAN). The communication between the bridge and engine room is done through an optical fiber that allows high-speed data transfer both directions. Digitized signals from PLC are sent to the industrial PC (iPC), located in the engine control room. The iPC has two main functions: to store data in daily log files, and to display the measured information in real time for the crew members. The data collection system is connected to a home base located on shore via a remote access connection. The High-Speed Downlink Packet Access (HSDPA) technology allows communication in both directions. Daily measurements are automatically transferred from the vessel to the home base. On demand, the software of the data collection system is updated by the home base operator.

![Figure 7.1: The schematic of data collection system installed on board of the monitored vessels.](image)

The recorded signals, their sources, and their sampling rates are listed in Table 7.1. The same signals were collected on board of both vessels, except for few cases\(^*\). All sensors, except for the accelerometer and the propeller thrust and torque sensor, are standard sensors on board of a ship.

\(^*\)The Doppler log sensor installed on board of the container ship measures the speed-through-water in longitudinal direction; the transversal component of the speed-through-water and the speed-over-ground in both directions are not measured.
Table 7.1: List of measurements collected on board of the ferry and the container ship

<table>
<thead>
<tr>
<th>Signal</th>
<th>Source</th>
<th>Sampling rate [Hz]</th>
<th>Vessel</th>
</tr>
</thead>
<tbody>
<tr>
<td>UTC</td>
<td>GPS</td>
<td>1</td>
<td>yes</td>
</tr>
<tr>
<td>Latitude</td>
<td>GPS</td>
<td>1</td>
<td>yes</td>
</tr>
<tr>
<td>Longitude</td>
<td>GPS</td>
<td>1</td>
<td>yes</td>
</tr>
<tr>
<td>Speed-over-ground</td>
<td>Doppler log</td>
<td>2</td>
<td>yes</td>
</tr>
<tr>
<td>(longitudinal and transversal direction)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Speed-through-water</td>
<td>Doppler log</td>
<td>2</td>
<td>yes</td>
</tr>
<tr>
<td>(longitudinal and only)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ship angular motions (pitch, roll, yaw)</td>
<td>Accelerometer</td>
<td>10</td>
<td>yes</td>
</tr>
<tr>
<td>(except yaw)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Heading</td>
<td>Gyrocompass</td>
<td>10</td>
<td>yes</td>
</tr>
<tr>
<td>Depth</td>
<td>Echo-sounder</td>
<td>1</td>
<td>yes</td>
</tr>
<tr>
<td>Wind speed and direction</td>
<td>Anemometer</td>
<td>1</td>
<td>yes</td>
</tr>
<tr>
<td>Rudder angle</td>
<td>Control panel</td>
<td>10</td>
<td>yes</td>
</tr>
<tr>
<td>Propeller angular velocity (actual and demand)</td>
<td>Control panel</td>
<td>10</td>
<td>yes</td>
</tr>
<tr>
<td>Propeller pitch angle (actual and demand)</td>
<td>Control panel</td>
<td>10</td>
<td>yes</td>
</tr>
<tr>
<td>Fuel index (all engines)</td>
<td>Control panel</td>
<td>10</td>
<td>yes</td>
</tr>
<tr>
<td>Propeller thrust</td>
<td>OTT</td>
<td>10</td>
<td>yes</td>
</tr>
<tr>
<td>Propeller torque</td>
<td>OTT</td>
<td>10</td>
<td>yes</td>
</tr>
<tr>
<td>Power take-off</td>
<td>Control panel</td>
<td>10</td>
<td>no</td>
</tr>
</tbody>
</table>

7.2 Illustrations of raw signals

The data collection system on board of the ferry became operational 1st of March 2009; the data collection system on board of the container ship became operational 15th September 2010. Both systems record measurements continuously ever since, yielding in huge high-resolution data bases. For instance, at the end of 2009 the ferry measurement data base consisted of some 2500 individual trips. The statistical analysis of such amount of information has been initiated in (Galizca, 2009) and (Meninato, 2010).

Measurements from both vessels are used in this thesis. The reasons are technical and conjectural. The propulsion tests, as designed in Section 6.7, were performed only on board of the container ship. The data base of the ferry is far larger and consists of trips with similar operational profile and external conditions that fit well into the scope of statistical analysis.
In this section we illustrate some of the measured signals. A selection of raw signals is depicted in Figures 7.2, 7.3, and 7.4. Similar signals measured on board of the ferry and the container ship are depicted in parallel. For the ferry, we plot measurements collected in normal operation. For the container ship we plot measurements collected during the stationary propulsion tests. Due to confidentiality reasons all signals are depicted in dimensionless form.

Figures 7.2a and 7.2b depict the trajectories of the vessels. The vessel trajectory is reconstructed from the latitude and longitude signals collected from the GPS receiver. In fact, the position of the GPS antenna is being recorded. Vessel trajectory is important in this work. In Section 4.2 the mathematical model of the ship trajectory is presented. In Section 8.2 the ship trajectory is a component in the signal reconciliation method that corrects the navigation data. The trajectory of the ferry is recorded in service. For most of its sailing time the vessel follows a straight path. The trajectory of the container ship represents the route of the ship during the stationary propulsion tests. Notice the runs in opposite direction as mentioned in Section 6.7.3.

Figures 7.2c and 7.2d depict the speed-through-water in longitudinal direction. The speed-through-water is the speed of the ship relative to the surrounding water. This speed is measured by the Speed log in two orthogonal directions, the longitudinal and the transversal, as in the case of the ferry, or just in one direction, as in case of the container ship. The speed-through-water is a state variable of the ship propulsion model, see Section 4.4. Some Speed logs measure also the ship speed with respect to the sea floor. This is called speed-over-ground. The Speed log installed on the ferry measures also the speed-over-ground.

Figures 7.2e and 7.2f depict the speed-over-ground measured from the GPS receiver. In fact, the GPS receiver calculates the geographical position of the ship at fixed time intervals. The speed-over-ground is calculated by differentiating the position over the fixed time interval. The speed-over-ground measured by the GPS receiver represents the magnitude of the velocity vector.

Comparing speed-through-water to speed-over-ground we notice differences. In case of the ferry the main reason of the difference is the sea current. The speed-over-ground, which is the true speed by which the ship moves between two earth-bound points, is the sum of the speed-through-water and the speed of the sea current. Technical reasons may also account for the difference between speed-through-water and speed-over-ground. The difference between speed-through-water in Figure 7.2d and speed-over-ground in Figure 7.2f is, in fact, mainly due to poor accuracy of the Speed log. The Speed log speed is important in this thesis since the speed-through-water is directly related to propeller efficiency by formula (4.79). In Section 8.2 a method to correct the Doppler log speed is presented.

Figures 7.2g and 7.2h depict ship heading. The heading is the angle between the $x$-axis of the local frame and the $X$-axis of the global frame as depicted in Figure 4.3. Heading is one of the most important information for ship navigation. Heading is also important for the ship trajectory model, where it is used in the
illustrations of raw signals

rotational matrix; see Section 4.2.

Figure 7.3 depicts the propulsion controls, the propeller rotational speed and propeller pitch angle, and two of the most important measurements for propeller efficiency, the propeller thrust and torque. The propeller rotational speed is depicted in Figures 7.3a and 7.3b. The ferry has two controllable pitch propellers; the container ship has one controllable pitch propeller. The propeller pitch angle is depicted in Figures 7.3c and 7.3d. Note the difference in control strategies. To change the propulsion settings of the ferry both rotational speed and pitch angle are controlled according to the combinator curves. In case of the container ship the propulsion settings were changed by controlling the pitch angle, keeping the rotational speed constant.

The propeller thrust and torque are measured by a prototype sensor manufactured at VAF Instruments. Actually, the sensor measures the axial and torsional deformations of the propeller shaft and translates them into thrust and torque, respectively. Remark the negative thrust in Figure 7.3e. This happens due to the fact that positive thrust corresponds to shaft compression and negative thrust to shaft elongation. When ship sails forward the thrust is positive. When the ship sails backwards, or the propeller is stopped while the ship is sailing forward, the thrust is negative. The propeller torque is always positive. A strong correlation between the thrust and torque is observed. A proportional relation between thrust and torque is known to exist, see (Zhinkin, 1989) and (Pivano, 2006).

The rudder controls the direction of ship. The measured rudder angle is depicted in Figures 7.4a and 7.4b. The wind is the only environmental factor that is commonly measured on board of a vessel. Wind is measured as speed, Figures 7.4c and 7.4d, and direction, Figures 7.4e and 7.4f. Indirectly, the wind is a measure of the waves height, by use of the Beaufort scale. As will be seen in Chapter 9 the wind influences the results of the parameter estimation. The wind is indirectly accounted in the ship propulsion model by the resistance term, see equation (4.46). In fact, the parameters $\gamma$ and $p$ depend on the wind, among other external factors.

Indirect measures of external factors are the ship angular motions. Figures 7.4g and 7.4h depict the roll angle. Note the obvious oscillatory behavior in roll motion. For a large ship, roll periods of 10 to 15 seconds are typical. In the ship trajectory model the roll angle is used to correct the trajectory of the ship; see Section 4.2.

A thorough inspection of the raw signals reveals that most of them are affected by distortions. These distortions are related to sensor errors and ship motions due to internal and external factors. We categorize them, as functions of frequency, into: offsets, oscillations, and noise. These features of raw signals are described in the next section.
Figure 7.2: Illustrations of ship kinematics variables.
Figure 7.3: Illustrations of ship propulsion variables.
Figure 7.4: Illustrations of rudder angle, wind speed and direction, and roll angle.
7.3 Features of raw signals

In this section we identify three features in the measured signals: trend, oscillation, and noise. The three features are important in the data processing part, see Chapter 8.

7.3.1 Trend

Among the three types of distortions we identify, the offset is the most problematic. The offset can only be identified with respect to a reference. In the existing collection of signals only one quantity is measured redundantly: the speed-over-ground. The speed-over-ground is the ship speed with respect to the Earth surface. The speed-over-ground is measured by the Speed log in local coordinate system, and by the GPS receiver in the global coordinate system. In Figure 7.5 we compare the Speed log and GPS speed-over-ground. Due to the fact that GPS receivers are very reliable and accurate, we take speed-over-ground by GPS as the reference. The difference between the two signals is about 0.2 knots for the ferry, and anywhere between 1 and 2 knots for the container ship. Bear in mind that the Speed log on board of the container ship does not measure the speed-over-ground.

Figure 7.5: Offset in Speed log measurements. In the case of the ferry (a) there is an offset about 0.15 knots between the Speed log and GPS measurements. In the case of the container ship (b) the offset between the Speed log and GPS is anywhere from 1 to 2 knots.

Speed-over-ground is important for the reliability of propeller efficiency estimate. We assume that speed-over-ground and speed-through-water share the same distortions since they are measured by the same sensor using the same physical principle. Due to this relative offset part of the research effort was put in the direction of correcting the Speed log signals. In Section 8.2 we present the signal reconciliation method that eliminates the offset in speed-over-ground.

†In modern navigation systems the GPS receiver is the main navigational sensor. The position is measured with high accuracy. The GPS signals are used in other navigation systems, such as, the electronic charts and autopilots. Due to this reason we take the GPS signals as reference.
Besides the offset, there is a time lag between the Speed log and the GPS receiver installed on board of the ferry. Early in this research, R. Hindriks (Hindriks, 2006, p.61) observed a time delay in the Speed log that varied with every trip from 13 to 17 seconds. For the same vessel, P. Galicza (Galicza, 2010, p. 66) calculated the average time lag of 14.4 seconds, on basis of 4000 individual trips. Signal filtering could be the origin of the time lag.

### 7.3.2 Oscillation

In our definition, the oscillations are harmonic variations with frequencies bounded to the natural frequencies of the ship propulsion and ship motions. In case of the ferry and the container ship the observed natural frequencies\(^\ddagger\) are in the interval [0.05, 0.2] Hz. The signals with a pronounced oscillatory behavior are propeller thrust and torque, propeller pitch angle, and ship angular motions (pitch, roll, and yaw).

\[ \theta_p, \tau_p, \psi_p, \ldots \]

\[ f [\text{Hz}] \]

\[ A [-] \]

\[ t [\text{s}] \]

\[ \text{Figure 7.6: Oscillations in propeller pitch angle (a) induces oscillations in propeller thrust (b) with the same frequencies (c); in the frequency domain the values of } T_p, \text{ and } \theta_p \text{ are scaled.} \]

The main causes of the oscillations in the measured signals are the external factors. Waves, wind and current determine the ship to pitch, roll, and yaw at certain frequencies. For instance, typical frequencies of large vessels in roll motion are between 0.05 and 0.1 Hz, see (Lewis, 1989, vol. 3, p. 79). Oscillations in propulsion variables can be backtracked to external factors too. For instance, propeller thrust changes with changes in propeller pitch and propeller angular

\[^\ddagger\] These natural frequencies were observed in ship angular motions, propeller pitch, thrust, torque, rudder angle, and fuel index.
velocity, or when the flow around the propeller is disturbed. A rigid propeller pitch controller may introduce small oscillations when trying to compensate for sudden loads. In Figure 7.6 propeller pitch oscillations induce oscillations in the propeller thrust. In frequency domain we observe that the dominant frequency has the same value. Similarly, the autopilot tries to keep the ship course on the designated path by controlling the rudder angle. In rough weather, the amplitude of the rudder angle determines changes in the ship heading. It also disturbs the flow around the propeller, causing oscillations in propeller thrust. Such a situation is depicted in Figure 7.7. The heading and thrust oscillate with the same frequency, induced by rudder angle actions.

![Figure 7.7: Oscillations in ship’s heading (a) induce oscillations in propeller thrust (b) at the same frequencies (c). The oscillations in heading are due to changes in rudder angle. In the frequency domain the values of $T_p$ and $\psi$ are scaled.](image)

Oscillations in the measured signals are important in this research. The parameter estimation method for the wake fraction relies on imposed oscillations in the ship propulsion system. These tests are not readily available, so oscillations due to external factors might be used instead.

### 7.3.3 Noise

All measured signals contain the noise component. In this thesis only the high frequency distortions, above 0.2 Hz, are referred to as noise. The sources of noise can be the sensors, the ship vibrations, and the external factors. In Figure 7.8 the noise in thrust signal is depicted. The high-frequency distortions are removed by decomposing the signal into three frequency ranges: low frequency range (trend), mid frequency range (oscillations), and the high frequency range (noise).
**Figure 7.8:** The propeller thrust measurement is affected by noise (high frequency range).
Chapter 8

Processing of measurements

In this chapter we present the steps from raw signals to processed data:

- **Remove the high frequency components.** In the first section we describe the signal decomposition method that separates the low frequency component of the signal, the trend, from the high frequency components, the oscillation and the noise.

- **Remove the offset in the signals.** The signal reconciliation method that removes the offset distortions from the signal trend is presented in the second section. The method is exemplified in the same section with full scale navigation measurements collected from the ferry. The goal is to reconcile the ship velocity from the Speed log with the ship position and heading.

- **Extract stationary data.** Most of the parameter estimation methods we presented in Chapter 6 require stationary data. Similarly, propeller efficiency is determined only for stationary states. The definition of stationarity and the method to extract stationary state data from the time signals are presented in the third section.

8.1 Signal decomposition

The goal of the signal decomposition is to split a signal into three components: trend (low frequency range), oscillation (mid frequency range), and noise (high frequency range). In practice the signal decomposition method is used as a low-pass filter, to remove the oscillation and noise components from the raw signal.

The trend is the most important component since it is often associated with the real ship behavior. For instance, the gradient of ship velocity cannot be large

*For the signal decomposition method I am grateful to the contribution of Pal Galicza. Parts of this chapter are reproduced from his report *Ship sensor integration. A signal reconciliation method*, see (Galicza, 2010)
Processing of measurements

for short time intervals due to huge ship inertia. Other signals have oscillations superimposed on the trend. This phenomenon is observed in full scale measurements as well. For instance, propeller thrust and torque during a stationary state may oscillate, as showed in Section 7.3.2. The oscillation component is used in the spectral analysis of the signals. In a particular parameter estimation method, the information carried by the oscillation component is used to estimate the wake fraction, see Section 6.6. The noise component is generally discarded from the raw signal. In the signal reconciliation method the noise component is used to tune the level of allowed modifications of the trend.

Figure 8.1 depicts the signal decomposition idea with an artificially generated signal. The three components are identified from the frequency domain; the lower frequency (2 Hz) peak represents the trend; the higher frequency (10 Hz) peak represents the oscillation; the rest is white noise.

The trend sought for has two characteristics: it is close to the original signal, and, it is continuous and smooth. The first characteristic emphasizes the importance given to the measured signal. According to the second requirement, the trend conforms to expectations. Take speed-through-water in longitudinal direction for instance; ship speed cannot suddenly change due to the huge inertia of the ship that tends to smoothen the signal.

8.1.1 General formulation

The main idea of the signal decomposition is to regard the trend, $\bar{y} \in \mathbb{R}^n$, as an orthogonal projection, $\Psi$, of signal $y \in \mathbb{R}^n$ in a subspace $\mathcal{P}$. To find the trend means to find the element of $\mathcal{P}$ that is closest to the signal $y$; in formula

$$\Psi(y) = \min_{y \in \mathcal{P}} \|y - \bar{y}\|^2 \quad (8.1)$$

A suitable subspace $\mathcal{P}$ in which the cleaned signals are represented must be
found. In this case, a suitable subspace is given through a basis function, $B$. The advantage of using basis functions is that the calculation of the orthogonal projection is computationally fast. A suitable subspace means that the basis functions of this subspace satisfy the continuity and smoothness conditions. For instance, all piecewise linear functions with the same resolution points form a subspace. The resolution points are points where the function changes its slope. If the smoothness condition is not satisfied, the piecewise linear function is replaced by Gaussian functions.

Assume that a basis function is given as $B = \{B_k, k = 1, 2, ..., N\}$. Then the expression

$$\min_{\alpha_k} \left\| \sum_{k=1}^{N} \alpha_k B_k - y \right\|^2$$

is minimized to find the orthogonal projection $\Psi(y)$; $\alpha_k$ are real coefficients of the basis $B_k$. To find the unknown coefficients $\alpha_k$ we expand the expression (8.2) into

$$\sum_{k=1}^{N} \alpha_k^2 \langle B_k, B_k \rangle - 2 \sum_{k=1}^{N} \alpha_k \langle B_k, y \rangle + \|y\|^2; \quad k = 1, 2, ..., N$$

(8.3)

take the partial derivative with respect to coefficients $\alpha_k$, and set the expression to zero; a system of linear equations is obtained. In matrix form, the coefficients of projection are given by

$$A = G(B)^{-1} \langle B, y \rangle$$

(8.4)

where, $A$ is the vector of coefficients of projection and $G(B) = \langle B_k, B_l \rangle$ is the Grammian matrix of $B$. Relation (8.4) shows that to minimize the norm in (8.2) is equivalent to solving a linear system. The size of the linear system is the dimension of the basis $B$.

So far the generic method was presented. To have a fully workable signal decomposition method we need to specify the basis that describes the subspace $P$ in which the trend is represented. We seek a basis that has the following properties:

1. Every time point has approximately the same weight.
2. The Grammian is sparse. The sparser the Grammian the faster the computational time of the projection.
3. The spanned subspace is regular. This means that the basis functions are continuous and smooth.
4. The low frequency information is well captured.

The two types of basis that satisfy most of the conditions listed above, namely, the piecewise linear and Gaussian functions, are discussed below.
Piecewise linear basis

A scaling function $f$ is used to build the piecewise linear basis

$$f(t) = \begin{cases} 
1 + t, & \text{if } -1 < t \leq 0 \\
1 - t, & \text{if } 0 < t \leq 1 \\
0, & \text{otherwise}
\end{cases} \quad (8.5)$$

The basis function is a translated version of the scaling function

$$B = \left\{ B_k(t) = f(t - k), \ k = 0, 1, \ldots, T - 1 \right\} \quad (8.6)$$

on the time interval $[0, T]$; parameter $k$ is the translation step. Figure 8.2a depicts few piecewise linear basis functions.

The basis introduced by (8.6) represents the trend of the signal for a time interval of length 1. Thus, the resolution of the basis is 1 Hz. In practice, the 1 Hz resolution might be too high to represent the trend. For instance, ship speed cannot change significantly during one second. The trend of ship velocity is realistically represented in the order of $10^1$ to $10^2$ seconds. The resolution of the basis function is adapted by changing the parameter $r$ in the scaling function $f(t)$

$$B = \left\{ B_k(t) = f\left(\frac{1}{r}(t - k)\right), \ k = 0, 1, \ldots, (T - 1)/r \right\} \quad (8.7)$$

The piecewise linear basis is continuous but might not always be smooth enough. The slope jumps at each resolution point. Whether this is a problem or not depends on the particularities of each signal.

The fourth condition listed above might not be fulfilled in case of a highly dynamic signal due to the fixed resolution points. This problem is solved when the resolution points are adapted to the signal. The issue of adaptive resolution points is discussed later in this section.

Smooth basis

The problem of smoothness of the piecewise linear basis is solved if a smooth basis is used. The smooth basis functions are translated and dilated probability density functions

$$B = \left\{ B_k(t) = \frac{1}{r\sqrt{2\pi}} \exp\left(-\frac{(r(t - k))^2}{2}\right), \ k = 0, 1, \ldots, (T - 1)/r \right\} \quad (8.8)$$

where $r$ is the resolution. Few translated basis functions are depicted in Figure 8.2b.

Strictly speaking, the Gaussian function is not sparse, since its support is infinite. However, the function decays fast so, in practice, the Gaussian function
can be used on an effective support. The computation time is slightly higher than for a piecewise linear basis of the same size. Qualitatively, the smooth basis gives comparable results with the piecewise linear basis with the advantage of that the trend is smooth.

Adaptive resolution points

In practice, a basis with fixed resolution points does not give satisfactory results for signals with dynamic behavior. This situation changes when the resolution points adapt to the particularities of the signal. The key idea of the adaptive resolution points algorithm is to find the resolution points where there is a change in the trend. This means that there is a particular basis for each signal, depending on the signal particularities.

The construction of the basis from resolution points is a straightforward generalization of the uniform case (8.7)

$$B_k(t) = \begin{cases} \frac{t - r_{k-1}}{r_k - r_{k-1}}, & \text{if } r_{k-1} < t \leq r_k \\ \frac{r_k - t}{r_{k+1} - r_k}, & \text{if } r_k < t \leq r_{k+1} \\ 0, & \text{otherwise} \end{cases}$$

(8.9)

The algorithm that adapts the resolution points to the signal follows an iterative scheme. The iteration starts with a coarse resolution. With each step additional resolution points are added and a stopping criterion is verified. The algorithm finds the most significant local maxima and minima of the residual function, i.e., the difference between the raw signal and the projected signal, and include them as resolution points. The idea behind is that if there is a peak in the residual the peak can be canceled by adding a resolution point at the place of the peak. If the stopping criterion is satisfied the algorithm stops.

Two stopping criteria work simultaneously to ensure that adequate resolution points are fixed. One stopping criterion is based on the relative amount of low
Trend, oscillation, and noise

Technically speaking, signal decomposition method is about finding the trend $\bar{y}$. Once the trend is established, the other two components, namely, oscillation and noise, follow naturally as the residual $\tilde{y} = y - \bar{y}$, i.e., the difference between the raw signal and the trend.

The separation of the oscillatory component from the noise is done by running a Discrete Fourier Transform over the time interval between two consecutive resolution points. If any dominant frequencies exist, these are extracted from the residual signal $\tilde{y}$ and stored. This step is not absolutely necessary. In fact, not all signals have significant oscillatory component. Additionally, the oscillations may change their amplitude from one time interval to another, as the external and operational conditions change. The oscillatory component is of greatest importance in the method to estimate wake fraction from imposed harmonic dynamics, see Section 6.6. The information on noise component is used in the signal reconciliation to determine the level of allowed modifications of the trend.

In the last part of this section we use full scale measurements to show the different aspects of the signal decomposition method.

8.1.2 Practical implementation

The heading signal is used to illustrate the trending method. Figure 8.3 shows the differences between the fixed resolution points and adaptive resolution points. To better illustrate the difference between the options, two excerpts with different dynamic behavior are selected: a stationary state, see Figure 8.3a, and a dynamic state, see Figure 8.3b. Obviously, the adaptive resolution points option gives better results, particularly for the excerpts with large variations. The advantage of the adaptive resolution over the fixed resolution is particularly visible in the residuals, see Figure 8.4. The computational time for the adaptive resolution is somewhat larger; given the better quality this is of little trouble.

In Figure 8.5 we compare the piecewise linear trend with the Gaussian trend. Overall, there is little difference between the two methods. Small differences are observed for excerpts with small amplitude variations, see Figure 8.5a, when the trend with Gaussian basis tends to smoothen the signal.

In Figure 8.6 we represent the raw heading and the detrended heading in the
frequency domain. The trend in the raw signal is represented by frequencies lower
than 0.05 Hz. There is no middle-range dominant frequency to be associated
with an oscillatory component; the detrended signal contains only non-relevant
information.

![Figure 8.3: Fixed versus adaptive resolution. Subplot (a) depicts a stationary state; subplot (b) depicts a dynamic state.](image)

![Figure 8.4: The residuals of fixed resolution points and adaptive resolution points.](image)

![Figure 8.5: Piecewise linear versus Gaussian basis. Subplot (a) depicts a stationary state; subplot (b) depicts a dynamic state.](image)
8.2 Signal reconciliation

The scope of signal reconciliation\footnote{For the signal reconciliation method I am grateful to the contribution of Pal Galicza, see (Galicza, 2010). It was during his graduation project at Mathematics for Industry (Eindhoven University of Technology) that the generic signal reconciliation method was completed to the level described here.} is twofold:

- at generic level, to make the measurements consistent with each other,
- and, in particular, to remove the offset distortion in the raw signal.

The signal distortions were mentioned in Section 7.3. The high frequency distortions were removed in Section 8.1 by decomposing the signal into trend, oscillation, and noise. This section deals with the offset distortions in the low frequency range. The elaboration of the signal reconciliation method was triggered by the offset distortions found in the ship velocity measurements by the Speed log. Given the importance of the Speed log measurements for propeller efficiency estimation, this method is absolutely necessary.

At the generic level, the signal reconciliation addresses the problem of measurements consistency. Measurements collected from different sources are not consistent when represented in an accepted frame. For instance, the speed-over-ground measured by the Speed log is different than speed-over-ground measured by the GPS receiver. \textit{Which source is right?} Two answers are possible: either we select the right source and use it as reference, or we take the middle road and reconcile the measurements. The signal reconciliation can do both.

The signal reconciliation method is an adaptation of the dynamic data reconciliation (DDR) developed in the chemical process industry. This method is used to correct low frequency distortions (biases) in real-life process measurements based on proven relationships between the measured quantities. These relationships are mostly mass balance and energy equations. The models are set as strict constraints and the distance from the measurements to the model prediction need to be minimized. That is to say, the data reconciliation is formulated as a constraint...
8.2. Signal reconciliation

minimization problem. For general description of the dynamic data reconciliation we recommend (Crowe, 1996) and (Soderstrom et al., 2000).

8.2.1 General formulation of signal reconciliation

The main idea of method is that the reconciled signals are at the same time close to the raw measurements and to the mathematical model that we impose upon them. These two requirements are formalized as an optimization problem. We are looking for a multi-dimensional signal \( \mathbf{x} : [0, T] \rightarrow \mathbb{R}^n \) such that

\[
\min_{\mathbf{x}} \lambda \| \mathbf{y} - \mathbf{x} \|^2 + \| M(\mathbf{x}, \dot{\mathbf{x}}, \mathbf{p}) \|^2
\]

(8.10)

where \( \mathbf{y} \) represents the raw measurements, and \( M \) represents the mathematical model with \( \mathbf{p} \in \mathbb{R}^m \) the vector of model parameters; \( \dot{\mathbf{x}} \) is the time derivative \( dx/dt \). Minimizing the first norm means that the signal \( \mathbf{x} \) stays close to the measurements; minimizing the second norm means that \( \mathbf{x} \) satisfies the imposed mathematical model \( M \). The vector \( \lambda \) weights between the two criteria; as \( \lambda \) increases staying close to the measurements becomes more important than the adherence to the model.

In the general formulation (8.10) the multi-dimensional signals \( \mathbf{x} \) and \( \mathbf{y} \) are vector-valued functions on a discrete time interval \([0, T]\). It means that at every time point, say every second, their values change. This is not how we would like to see the signal reconciliation working in practice, and, especially, in the particular case of ship dynamics. Thus, the raw measurements are replaced by their trends, \( \bar{\mathbf{y}} \). The trend represents the link to the signal decomposition, see Section 8.1. In practice, we regard signal decomposition as part of the signal reconciliation method.

The choice for signal trends has an impact on the second term of expression (8.10). The mathematical model we consider is nonlinear. To keep the optimization problem solvable the model need to be linearized around the trend.

Linearization

Assuming the mathematical model is differentiable, the model is approximated in the neighborhood of the trend \( \bar{x} \) by the Taylor expansion up to the first order term

\[
M(\mathbf{x}) \approx M(\bar{\mathbf{x}}) + \frac{\partial M(\bar{\mathbf{x}})}{\partial \mathbf{x}} (\mathbf{x} - \bar{\mathbf{x}})
\]

(8.11)

The Jacobian gives a reliable representation only if the trend of the signal is already close to the model prediction. Thus, it is important to have a reliable trend from the signal decomposition method. In this linearized scheme the approximation \( \bar{x} \) is fixed and the minimization is done in terms of residuals. If \( \mathbf{x} = \bar{\mathbf{x}} + \delta \),
the optimization problem becomes

$$\min_{\delta} \lambda \| \delta - (\bar{y} - \bar{x}) \|^2 + \left\| M(\bar{x}) + \frac{\partial M(\bar{x})}{\partial \bar{x}} \delta \right\|^2$$  \hspace{1cm} (8.12)

where $\delta$ represents the vector of residuals.

**Reconciliation in a subspace**

The signal reconciliation method follows the same idea as the signal decomposition: the minimization problem is solved in a particular subspace. The solution of the optimization is given via a basis,

$$\mathbf{B} = \{ \mathbf{B}_k : [0, T] \rightarrow \mathbb{R}^n, k = 1, 2, ...N \}.$$  \hspace{1cm} (8.13)

A basis function is a multi-dimensional signal with the same dimension as the set of raw signals. Consequently, the mathematical model $M$ is applied onto each basis function

$$\min_{\beta} \lambda \left\| \sum_{k}^{N} \beta_k \mathbf{B}_k - (y - \bar{x}) \right\|^2 + \left\| M(\bar{x}) + \frac{\partial M(\bar{x})}{\partial \bar{x}} \sum_{k}^{N} \beta_k \mathbf{B}_k \right\|^2$$  \hspace{1cm} (8.14)

where $\beta_k$ represents the coefficients of the basis functions. Since the Jacobian is a linear operator, it is applied directly to the basis functions

$$\min_{\beta} \lambda \left\| \sum_{k}^{N} \beta_k \mathbf{B}_k - (y - \bar{x}) \right\|^2 + \left\| M(\bar{x}) + \sum_{k}^{N} \beta_k J_M \mathbf{B}_k \right\|^2$$  \hspace{1cm} (8.15)

where $J_M$ is the Jacobian.

The two terms of the optimization problem have similar structures. Both terms can be interpreted as the distance from a multi-variate signal, i.e., $(\bar{y} - \bar{x})$ and $M(\bar{x})$, to a linear combination of basis functions, i.e., $\mathbf{B}_k$ and $J_M \mathbf{B}_k$. Thus, the solution of the reconciliation scheme is equivalent to a linear system of equations. By taking the partial derivative with respect to coefficients $\beta_k$ and setting the expression to zero we obtain

$$\beta_k = \left( \lambda G(\mathbf{B}) + G(J_M \mathbf{B}) \right)^{-1} \left( \lambda \langle \mathbf{B}, \bar{y} - \bar{x} \rangle - \langle M(\bar{x}), J_M \mathbf{B} \rangle \right)$$  \hspace{1cm} (8.16)

**Basis functions**

In the signal reconciliation problem we use the same types of basis functions as discussed in Section 8.1. Technically speaking, the difference between the basis functions of the two methods is that for the signal reconciliation the basis functions are vector-valued functions that run through the time interval $[0, T]$ and all the variables

$$\mathbf{B} = \{ \mathbf{B}_n^k, k = 0, 1, 2, ...T; n = 1, 2, ...N \}.$$  \hspace{1cm} (8.16)
where $N$ is the number of states of the mathematical model and $B^i_k$ is

$$B^i_k(t, i) = \begin{cases} f\left(\frac{t}{r}(t - k)\right), & \text{if } i = n \\ 0, & \text{otherwise} \end{cases} \quad (8.17)$$

The basis used for signal reconciliation is an extension of the vector space used in trending. In other words, we use the same type of basis function for signal decomposition and signal reconciliation. Else, the outcome of the reconciliation procedure might not be regular enough.

**Representing differential**

Representing the time derivative of the signal, $\dot{x}$, as the finite differences might introduce noise in the end result of the signal reconciliation. To avoid this, the differentiated signal is represented as a new time-dependent signal, $x_{diff}$, that is added to the state space. The general formulation of the reconciliation problem becomes

$$\min_x \lambda \|y - x\|^2 + \|M(x, x_{diff}, p)\|^2$$

subject to $x_{diff} = \dot{x}$.

For a signal of the form $(x, x_{diff})^T$ the basis can be written as a concatenation of two vectors, one for the original states and one for the new states, the differentials,

$$B = \{(B^x_k, B^{y}_k)^T : [0, T] \rightarrow \mathbb{R}^n, k = 1, 2, ..., N\} \quad (8.19)$$

Discrete difference is the straightforward approach to define the basis of the differentials, $B^y_k$,

$$B^y_k(t) = \dot{B}_k^x(t) = r \left( B^x_k \left( \frac{t}{r} \right) - B^x_k \left( \frac{t - 1}{r} \right) \right), \quad t \in [0, T] \quad (8.20)$$

When applying the discrete difference to piecewise linear basis functions the differentials are piecewise constant functions. Piecewise constant basis functions are discontinuous and, thus, might corrupt the entire signal reconciliation scheme. The following differential basis solves the problem of discontinuous basis

$$\dot{f}(t) = \begin{cases} t + 1, & \text{if } -1 < t < -\frac{1}{2} \\ -t, & \text{if } \frac{1}{2} \leq t < \frac{1}{2} \\ t - 1, & \text{if } \frac{1}{2} \leq t < 1 \\ 0, & \text{otherwise} \end{cases} \quad (8.21)$$
8.2.2 Reconciliation of navigation signals

The signal reconciliation method was triggered by the distortions found in speed-through-water measured by the Speed log. In this section we actually reconcile the navigation signals and correct the speed-through-water. We use the measurements collected on board of the ferry.

The background

The speed-through-water is directly involved in the propeller efficiency formula,

\[ \eta_{\text{prop}} = \frac{T_p u (1 - w)}{2\pi Q_p n} \]

This means that the reliability of speed-through-water measurements directly affects the calculation of propeller efficiency. In the practice of ship navigation it is well known that the reliability of the Speed log (Doppler log and Electromagnetic log) suffers due to technical aspects. To measure the speed relative to the surrounding water the Speed log measures the time lag between the emitted acoustic or electromagnetic waves and their reflections from layers of water. Some Speed logs can measure the speed relative to the ground by replacing the water layers with the sea bottom as reflective medium. When speed-over-ground is measured by the Speed log and by the GPS receiver the two signals can be compared.

In early stages of this work, R. Hindriks (Hindriks, 2006) found that there is an offset and a time lag between the speed-over-ground measured by the Speed log and the speed-over-ground measured by the GPS receiver. Figures 8.7a and 8.7b depict the offset and the time lag found in the measurements collected on board of the ferry. It must be specified that both GPS receiver and the Speed log installed on board of the ferry are top-of-the-line products. The offset is about 1% over the entire trip and the time lag is from 13 seconds to 17 seconds, with 14.4 seconds the average time lag based on 4000 individual trips.

\[ (a) \text{ Offset} \]

\[ (b) \text{ Time lag} \]

Figure 8.7: The comparison between the speed-over-ground measured by GPS receiver and by Speed log yields into an offset (a) and a time lag (b).
8.2. Signal reconciliation

The Speed log measures both speed-over-ground and speed-through-water. Thus, any distortion that affects the speed-over-ground signal is applied to the speed-through-water. Following this assumption, if the distorted speed-over-ground signal is corrected, the same correction is applied to the speed-through-water signal.

Navigation signals

The signals involved in the reconciliation method are listed in Table 8.1. As already stated in the general description of the signal reconciliation, we do not want to reconcile the signal point by point but rather reconcile the trend. The trend is found by the signal decomposition method; see Section 8.1. Figures 8.8a and 8.8b depict the trends of longitudinal and transversal speed-over-ground measured by the Speed log.

<table>
<thead>
<tr>
<th>Signal</th>
<th>Symbol</th>
<th>Sensor</th>
</tr>
</thead>
<tbody>
<tr>
<td>Latitude</td>
<td>$P_x$</td>
<td>GPS receiver</td>
</tr>
<tr>
<td>Longitude</td>
<td>$P_y$</td>
<td>GPS receiver</td>
</tr>
<tr>
<td>Longitudinal speed-over-ground</td>
<td>$u_g$</td>
<td>Speed log</td>
</tr>
<tr>
<td>Transversal speed-over-ground</td>
<td>$v_g$</td>
<td>Speed log</td>
</tr>
<tr>
<td>Heading</td>
<td>$\psi$</td>
<td>Gyrocompass</td>
</tr>
<tr>
<td>Pitch</td>
<td>$\theta$</td>
<td>Inclinometer</td>
</tr>
<tr>
<td>Roll</td>
<td>$\phi$</td>
<td>Inclinometer</td>
</tr>
</tbody>
</table>

Table 8.1: Signals involved in the signal reconciliation

![Graph](a) Longitudinal speed-over-ground by Speed log

![Graph](b) Transversal speed-over-ground by Speed log

Figure 8.8: The trends of the components of speed-over-ground measured by the Speed log.

Mathematical model

Let us recall the ship trajectory model described in Section 4.2

$$\frac{d}{dt} P_A = R(\psi) u_g$$  \hspace{1cm} (8.22)
where \( \mathbf{P}_C = (P_X, P_Y)^T \) is ship position, \( \mathbf{u}_g = (u_g, v_g)^T \) is speed-over-ground in local frame, \( \mathbf{R}(\psi) \) is rotation matrix, and \( \psi \) the heading. The mathematical model can be written using Euler integration

\[
\mathbf{P}_C(k + 1) = \mathbf{P}_C(k) + h \mathbf{R}(\psi(k)) \mathbf{u}_g(k) \tag{8.23}
\]

where \( h \) denotes the sampling time and \( k \) denotes the sampling index.

Predictably, the raw signals (actually their trends) do not satisfy the mathematical model. Figure 8.9 depicts the model adherence. Leaving out the beginning and the end of the time interval, which correspond to ship maneuvers in harbors, we notice that the measurements are not in agreement with the mathematical model.

![Figure 8.9: The adherence of the raw signals to the mathematical model along x- and y-directions.](image)

**Results**

We correct the speed-over-ground measured by the Speed log keeping close to the GPS position and gyrocompass heading measurements. This is done by setting the weights in the vector \( \mathbf{\lambda} \). The inputs in the reconciliation scheme are the trends of the raw signals, see Figure 8.8. The result of reconciliation scheme is depicted in Figure 8.10 as the compliance to the mathematical model. Obviously, the reconciled signals satisfy the mathematical model much better than the raw signals. The beginning and the end of the time interval correspond to ship maneuvers in the harbors, thus, the method should not be evaluated based on these specific situations.

In Figures 8.11a and 8.11b the raw speed-over-ground is compared to the reconciled speed-over-ground componentwise. Note the offset that is corrected by the reconciliation method. The offset is particularly large for the transversal component of speed-over-ground.
8.2. Signal reconciliation

Figure 8.10: The adherence of the raw signals and the reconciled signals to the mathematical model. The beginning and the end of the time interval represents the ship maneuvers in the harbors. Thus, the performance of the method should not be evaluated based on this data.

Figure 8.11: Comparison between raw and reconciled speed-over-ground by Speed log. Notice the large offset in transversal components that is removed by reconciliation.

The magnitude of the reconciled speed-over-ground vector is compared to the magnitude of speed-over-ground vector measured by the GPS receiver in Figure 8.12. Notice that the offset and the time lag, which are depicted in Figure 8.7, are removed. There is a very good agreement between the two quantities.

A completely different output is obtained when the weight is tuned such that the reconciliation scheme stays close to the raw speed-over-ground by Speed log. Figure 8.13 depicts the result of this experiment as the raw heading versus the reconciled heading. An offset between 1.5° and 3° has been observed for all trips in the data base. It is well known that the gyrocompass and the GPS receiver are the main navigational sensors. Therefore, a consistent error of the gyrocompass is unlikely in practice. A possible explanation, and the only one found by the author, is a calibration error in the Speed log.
Processing of measurements

![Graph showing speed-over-ground GPS vs. reconciled speed-over-ground reconciled for two excerpts of the same data set.](a) GPS vs. Speed log

![Graph showing GPS vs. Speed log.](b) GPS vs. Speed log

**Figure 8.12**: Comparison between speed-over-ground measured by GPS receiver and the reconciled speed-over-ground measured by the Speed log at two different excerpts of the same data set. The relative offset and time lag has been removed by signal reconciliation.

![Graph showing raw heading and reconciled heading.](a) Raw heading vs. Reconciled heading

**Figure 8.13**: The raw and reconciled heading when the signal reconciliation scheme stays close to the raw measured speed-over-ground by Speed log.

**Speed-through-water correction**

The practical objective of signal reconciliation is to correct the speed-through-water measured by the Speed log. This step is important because speed-through-water is directly involved in calculating the propeller efficiency at full scale. The assumption we made in the beginning of Section 8.2.2 is that the speed-over-ground and the speed-through-water measured by the Speed log suffer the same type of errors, namely, an offset and a time lag. Figure 8.14 depicts the raw speed-through-water measured by the Speed log and the corrected speed-through-water.
8.2. Signal reconciliation

In Section 6.5 we have seen that wake fraction cannot be estimated using stationary control. The reason is that wake fraction cannot be distinguished from other unknown quantities, namely, \( \bar{u}, \alpha_1, \) and \( \beta_1 \). To solve this problem we imposed a harmonic control in the radically transformed mathematical model, see Section 6.6.

We believe that the concept of signal reconciliation could be extended to include also parameter estimation. The parameters would play a similar role as the variables of the optimization scheme of the signal reconciliation. The difference would be that parameters are assumed to be constant, at least for the time window considered in the reconciliation scheme.

The unknown parameters would require a first approximation, \( p_0 \), similarly to the reconciled variables. This should not be a problem since there is plenty of knowledge on the model parameters under discussion (\( w, t_d, \alpha_1, \alpha_0, \beta_1, \beta_0 \)). For instance, CFD simulations could provide the first approximation for the propeller parameters, \( \alpha_1, \alpha_0, \beta_1, \) and \( \beta_0 \); model tests could provide the first approximations for wake fraction and thrust deduction.

The general scheme of the extended reconciliation scheme that includes parameter estimation would be

\[
\min \beta, \delta p \quad \lambda \left\| \sum_k \beta_k B_k - (\bar{y} - \bar{x}) \right\|^2 + \xi \left\| \delta p \right\|^2
\]

(8.24)

where \( \delta p = p - p_0 \) is the change in model parameters that need to be calculated, and \( \xi \) is a weight by which we control how far we get from the initial values of the parameters. The scope of \( \xi \) is similar to \( \lambda \). Here, the mathematical model \( M \) represents the ship propulsion model and the ship trajectory model. The fact that
the solutions of ship propulsion model are close to the full scale measurements has been discussed in Section 5.4; the topic is discussed also in Section 9.2.5.

8.3 Stationary state extraction

In this section the stationary state extraction is described. The section begins with the stationary state definition. Then we describe the harmonic criterion that automatically decides whether an excerpt of signal is stationary or not. The overview of the algorithm is presented afterwards. The output of the stationary state extraction algorithm is a data base of stationary states. Illustrations of stationary states are presented in the last part of the section. The stationary states are extracted from the data base of raw measurements collected from the ferry.

Stationarity

Propeller efficiency, as defined in this thesis, see formula (2.9), is intended for stationary states. Thus, the quantities involved in this formula, namely thrust, $T_p$, torque, $Q_p$, rotational speed, $n$, and ship speed, $u$, are stationary. A supporting argument in this direction is given by the time-scale analysis carried out in Chapter 5: $T_p$, $Q_p$, and $n$ react on the short time scale, while $u$ reacts on the long time scale, making the propeller efficiency formula incompatible with transient states.

The signals recorded at full scale vary in time due to variations in their physical counterparts and due to distortions introduced by the ship motions, the external factors, and the measurements system itself, see Figures 7.2, 7.3, and 7.4 for the raw measurements, and Section 7.3 for the discussion on the signal distortions. Thus, we need to specify what stationarity means in this case.

In our definition, a signal is stationary when its trend is zero (or close to zero). Small oscillations around the trend are allowed.

The ship is represented in our virtual reality by a collection of uncorrelated signals. The ship is said to be in a stationary state when the following signals are stationary: propeller thrust, propeller torque, speed-through-water, and heading. This means that the ship has to sail straight and with constant speed. The stationary state can be enlarged by taking into account the environmental factors, in this case wind speed and direction.

Harmonic criterion

The stationary states are automatically recognized based on a harmonic criterion. The main idea behind the criterion is to calculate the power spectral density, $P_s(\omega)$,

$$P(\omega) = |Y(\omega)|^2$$

(8.25)
of the signal \( y(t) \), where \( \omega \) denotes the complex frequency and \( Y(\omega) \) is the Fourier transform of the signal \( y(t) \).

When the signal is white noise, the power spectral density is constant. If there is a non-zero low-frequency component in the signal \( y(t) \) then the power spectral density is higher in the low frequency region than in the high frequency region. In practice we hardly encounter white noise where the Discrete Fourier Transform (DFT) of the signal is constant. Therefore we have to give the criterion a degree of freedom.

In the harmonic criterion the frequency domain is divided into two regions: a low-frequency region, in which the trend is expected, and a high-frequency region, in which the non-relevant information is found. The border between the two regions is subjective to each signal and is determined based on past experience or expertise knowledge.

The harmonic criterion compares the power spectral density in the predefined low-frequency region, \( P_{\text{low}} \), and the total power spectral density, \( P_{\text{total}} \) of the signal. If \( P_{\text{low}} > P_{\text{total}} \), it means that there is a significant trend component that must be removed. Conversely, if \( P_{\text{low}} \leq P_{\text{total}} \), then there is no trend component in the low-frequency region, or that the trend has been removed. In formula we write

\[
\frac{P_{\text{low}}}{P_{\text{total}}} \leq \frac{\hat{\omega}}{2r}
\]  

where \( \hat{\omega} \) is the preset boundary between the low- and high-frequency regions, and \( r \) is the sampling frequency of the signal.

The algorithm

An automatic algorithm to extract stationary states has been designed. In practice, the algorithm has been used to extract the stationary states from the data base of measurements collected on board of the ferry. Alternatively, the algorithm can be installed on the data collection system and work in real-time on board of the ship. The main steps of the algorithm are described below:

- Select the signals that are analyzed by the algorithm, namely, speed-through-water, heading, thrust, and torque. Additionally, wind speed and direction can be included. Set the threshold frequency \( \hat{\omega} \).
- Select the length of the time window. The algorithm spans the signals with a time window. The length of the time window is \( 2^k \) for better performances of the DFT algorithm. The time window has to be long enough to identify the trend. However, if the time window is too long a trend might always be found.
- Differentiate the signals. For each time window the discrete difference of the signal is calculated.
• Check if the harmonic criterion is satisfied. The algorithm checks the harmonic criterion (8.26) based on the relative amount of energy in the low-frequency range of the differentiated signals.
• If the harmonic criterion is satisfied select the middle part of the time window. In this step the first and the last quarter of data points are discarded from each time window. This way we avoid the situation in which a trend starts at the edges of the time window. The stationary data point is calculated as the average of the signal for the middle part of the time window. The stationary data point is stored together with an information label.
• Slide the time window with a predefined number of time points.

Stationary states

The stationary state extraction algorithm is run for a data base of measurements collected on board of the ferry. The data base contains more than 4000 individual trips recorded during one year. About four stationary states per trip are found, resulting in a data base of more than 17000 different stationary states. Each stationary state is replaced by data points representing the average of the signal. The stationary data points are labeled with information such as the date and time of the trip, and the beginning and the end of the time interval. Figures 8.15 to 8.17 depict the stationary data points of several important variables.

The data base of stationary states is particularly important for the statistical analysis of ship-related variables and indices, which, in turn, contributes to the interpretation of ship performance aspects. This analysis is done in Chapter 10; here, we give a short preview.

Figure 8.15 depicts the stationary data points extracted from the longitudinal speed-through-water, the heading, the propeller rotational speed, and propeller pitch angle signals. All data points are represented against their index in the data base of stationary states. Note that all depicted quantities behave as random variables.

A common feature of the plots are the narrow bands that contain most of the stationary data points. This indicate that the vessel tends to sail with similar operational modes. For instance, majority of speed-through-water stationary points are between 0.8 and 0.9. Some data points cluster at higher values, around 1, indicating a second operational mode. The two operational modes are due to the propulsion control, propeller rotational speed and propeller pitch angle. Figure 8.17a compares the histograms of propeller pitch angle and the longitudinal speed-through-water. Note the clear Gaussian distribution of both quantities.
8.3. Stationary state extraction

Figure 8.15: Stationary data points of longitudinal speed-through-water, heading, propeller angular speed, and propeller pitch angle. The index represents the position of the stationary data point in the database.

Figure 8.16 depicts the thrust and torque stationary data points; here, thrust and torque data points correspond to two months of measurements. In Figure 8.17b the thrust is depicted against the torque. The data points form tow clusters, each representing an operational mode.

The illustrations of stationary data points of several variables is meant to open the topic of statistical analysis of full scale measurements. In Chapter 9 we learn that propeller efficiency inherits the random variable behavior of the direct measurements. The topic of statistical analysis of full scale measurements and propeller efficiency is taken up and developed in Chapter 10.
Figure 8.16: Stationary thrust and torque data points

Figure 8.17: The histogram of propeller pitch angle and speed-through-water (a), and the relation between thrust and torque (b)
Chapter 9

Implementation of design

In this chapter we put together the parameter estimation methods and the measurements from propulsion tests. The objective is to estimate the model parameters and the propeller efficiency at full scale. In the first section we present the propulsion tests carried out on board of the container ship. In the second section the model parameters are estimated using the methods described in Chapter 6. For each model parameter we emphasize the reliability of estimation. The influence of the external factors is emphasized in this chapter. We already mentioned in Section 6.6.5 that there are no measurements with harmonic control that could support the estimation of wake fraction. Therefore, in this section we limit ourselves to discuss the application of the signal reconciliation to include the estimation of model parameters. In the third section we discuss propeller efficiency at full scale, assuming a fixed wake fraction. The concepts of reference and relative propeller efficiency, as immediate practical application, are introduced in the same section.

9.1 Propulsion tests

September 2010 propulsion tests were carried out on board of the container ship under the supervision of a Wärtsilä Netherlands team*. The data collection system was manufactured at Wärtsilä Netherlands. The system included a prototype thrust and torque sensor manufactured by VAF Instruments. The design of the propulsion tests is described in Section 6.7.3; the data collection system and the list of recorded signals are presented in Section 7.1; and, snapshots of the signals are illustrated in Section 7.2.

The propulsion tests were performed according to ITTC guidelines; see Section 6.7.3. The trajectory of the ship during the stationary control tests is de-

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*Maarten Nijland, then at Wärtsilä Netherlands, organized and supervised the propulsion tests.
picted in Figure 9.1. For each propulsion set-point the ship effectuated a double run, according to ITTC guidelines, see (ITTC, 2003).

![Figure 9.1: The ship trajectory during the stationary control.](image)

**Figure 9.1:** The ship trajectory during the stationary control.

**Stationary control**

The vessel performed four double-runs with stationary control. For each double-run the vessel was operated at two different propulsion set-points. Figures 9.2a and 9.2b depict the propulsion controls, namely, the propeller rotational speed and propeller pitch angle. The propeller angular speed was kept constant while the propeller pitch angle was varied. Thus, pitch angle was the propulsion control during the stationary states. The duration of a stationary state was about 20 minutes.

The pitch angle varies from 1, which is the maximum value, to 0.6, following the sequence: 1, 0.95, 0.9, 0.85, 0.8, 0.7, 0.6. Consequently, thrust and torque change as illustrated in Figures 9.2c and 9.2d. Speed-through-water, measured by a Speed log, is depicted in Figure 9.2e. A visual correlation indicates that for the same propeller pitch angle the thrust, torque, and speed-through-water is different for the opposite run. Figures 9.2f and 9.2h clarify the reason for this difference: during one run the ship sails against the wind. Note that the wind direction is measured as the direction from which it originates. Thus, if ship heading and wind direction have the same value then the ship sails against the ship; a 180° difference between the heading and wind direction means that the ship sails with the wind. In this thesis the following terms are used to describe these situations: wind blowing against the ship is called *head wind*, wind blowing with the ship is called *aft wind*, and the wind blowing from the lateral of the ship is called *cross wind*. The wind direction plays an important role in estimation of model parameters, as indicated in the sections to come.

The stationary states are automatically identified by the stationary state extraction algorithm described in Section 8.3. The stationary data points are listed in Table 9.1.
9.1. Propulsion tests

Figure 9.2: Illustrations of signals recorded during stationary control. The excerpts that are automatically extracted by the algorithm are colored in red.
Table 9.1: Stationary data points

<table>
<thead>
<tr>
<th>stationary state</th>
<th>pitch angle</th>
<th>rpm</th>
<th>speed</th>
<th>thrust</th>
<th>torque</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0.646</td>
<td>0.272</td>
<td>0.0366</td>
</tr>
<tr>
<td>2</td>
<td>0.95</td>
<td>1</td>
<td>0.615</td>
<td>0.234</td>
<td>0.0305</td>
</tr>
<tr>
<td>3</td>
<td>0.95</td>
<td>1</td>
<td>0.616</td>
<td>0.299</td>
<td>0.0380</td>
</tr>
<tr>
<td>4</td>
<td>0.9</td>
<td>1</td>
<td>0.587</td>
<td>0.263</td>
<td>0.0327</td>
</tr>
<tr>
<td>5</td>
<td>0.9</td>
<td>1</td>
<td>0.594</td>
<td>0.217</td>
<td>0.0270</td>
</tr>
<tr>
<td>6</td>
<td>0.85</td>
<td>1</td>
<td>0.572</td>
<td>0.196</td>
<td>0.0240</td>
</tr>
<tr>
<td>7</td>
<td>0.9</td>
<td>1</td>
<td>0.566</td>
<td>0.237</td>
<td>0.0283</td>
</tr>
<tr>
<td>8</td>
<td>0.85</td>
<td>1</td>
<td>0.539</td>
<td>0.213</td>
<td>0.0255</td>
</tr>
<tr>
<td>9</td>
<td>0.8</td>
<td>1</td>
<td>0.542</td>
<td>0.178</td>
<td>0.0211</td>
</tr>
<tr>
<td>10</td>
<td>0.7</td>
<td>1</td>
<td>0.496</td>
<td>0.135</td>
<td>0.0158</td>
</tr>
<tr>
<td>11</td>
<td>0.8</td>
<td>1</td>
<td>0.512</td>
<td>0.185</td>
<td>0.0216</td>
</tr>
<tr>
<td>12</td>
<td>0.7</td>
<td>1</td>
<td>0.458</td>
<td>0.144</td>
<td>0.0164</td>
</tr>
<tr>
<td>13</td>
<td>0.6</td>
<td>1</td>
<td>0.399</td>
<td>0.112</td>
<td>0.0127</td>
</tr>
<tr>
<td>14</td>
<td>0.6</td>
<td>1</td>
<td>0.440</td>
<td>0.103</td>
<td>0.0116</td>
</tr>
</tbody>
</table>

Zero-thrust control

The vessel performed four tests with zero-thrust control during one double-run. The signals measured during these tests are illustrated in Figure 9.3. The ship is decelerated from the same initial speed-through-water by controlling propeller pitch angle such that propeller thrust is zero.

Figure 9.3c depicts the propeller thrust. The thrust was kept to zero by manually adjusting the propeller pitch angle as depicted in Figure 9.3b. The propeller rotational speed was kept constant, to its maximum value. The purpose of this test was to mimic the deceleration of the ship in absence of the propeller. The zero-thrust test is only possible when real-time thrust measurements are available.

The speed-through-water is depicted in Figure 9.3e. The ship is decelerated from the same initial speed. For data processing purposes we select the time excerpts corresponding to the dimensionless speed-through-water larger than 0.4, which means about 8 knots, in consonance with the stationary states.

The heading, which is depicted in Figure 9.3f, was kept constant. The difference between the heading and the wind direction indicates that two decelerations were performed with aft wind and two with headwind. The wind direction is depicted in Figure 9.3h.

Zero-thrust control is used to estimate one model parameter, namely, the resistance factor. The time excerpts that are used in the estimation method are colored in red in Figure 9.3.
9.1. Propulsion tests

Figure 9.3: Illustrations of signals recorded during zero-thrust control. The time excerpts to be used in parameter estimation are colored in red.
9.2 Model parameters at full scale

In this section the model parameters are actually estimated using the methods described in Chapter 6 and the propulsion tests presented in Section 9.1.

All parameter estimation methods are basically least square methods. The difference between the measurements, $y_{\text{meas}}$, and the prediction of a model, $y_{\text{model}}$, is minimized with respect to the unknown parameters, $p$

$$\min_p \|y_{\text{meas}} - y_{\text{model}}(p)\|^2$$  \hspace{1cm} (9.1)

To determine the reliability of the parameter estimation method we use the well-known coefficient of determination, $R^2$,

$$R^2 = 1 - \frac{SS_{\text{err}}}{SS_{\text{tot}}}$$  \hspace{1cm} (9.2)

where $SS_{\text{err}}$ is the sum of squared residuals

$$SS_{\text{err}} = \sum_{k=1}^{N} (y_{\text{meas}}(k) - y_{\text{model}})^2$$  \hspace{1cm} (9.3)

and $SS_{\text{tot}}$ is the total sum of squares

$$SS_{\text{tot}} = \sum_{k=1}^{N} (y_{\text{meas}}(k) - y_{\text{avg}})^2$$  \hspace{1cm} (9.4)

where $y_{\text{meas}}$ represents the measurements, $y_{\text{model}}$ represents the model prediction, and $y_{\text{avg}}$ represents the mean of the measured data. The coefficient of determination has value between 0 and 1 for linear regression cases. $R^2 = 0$ indicates that the measurements and the model are not related by a linear model. $R^2 = 1$ indicates that the model explains all variability in the measurements. $R^2$ between 0 and 1 indicates, with approximation, the percentage of variation in the measurements that is explained by the linear model. The rest up to 100% is inherent variation, which cannot be explained by the linear model.

Note that the sum of squared residuals, $SS_{\text{err}}$, is actually the quantity being minimized in the least squared method. So, inherently, in the parameter estimation methods the coefficient of determination is maximized.

9.2.1 The actual resistance exponent

The resistance exponent is part of the surge equation. Let us recall the surge equation in its dimensionless form

$$\frac{du}{dt} = (1 - t_d)T_p - \varphi u^\gamma$$  \hspace{1cm} (9.5)
Parameter $\gamma$ is part of the resistance force term, alongside with $\varphi$. The method to estimate $\gamma$ is described in Section 6.2. Briefly, the method relies on stationary state data; the equation is transformed into a linear relation by taking the logarithm of the stationary state equation; finally, $\gamma$, which is the slope of the linear relation is found by least squares method. The optimization problem for $\gamma$ is

$$
\min_{\gamma, \delta} \| \bar{y}_{meas} - (\gamma \bar{x}_{meas} + \delta_{stat}) \|^2
$$

(9.6)

where $x_{meas} = \log u$ and $y_{meas} = \log T_p$. The tuples of stationary data points are listed in Table 9.2. Wind direction and propeller pitch angle complement the list.

**Table 9.2: Stationary data points**

<table>
<thead>
<tr>
<th>stationary state</th>
<th>log $u$</th>
<th>log $T_p$</th>
<th>wind</th>
<th>pitch</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-0.436</td>
<td>-1.300</td>
<td>aft</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>-0.485</td>
<td>-1.449</td>
<td>aft</td>
<td>0.95</td>
</tr>
<tr>
<td>3</td>
<td>-0.483</td>
<td>-1.207</td>
<td>head</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>-0.534</td>
<td>-1.553</td>
<td>head</td>
<td>0.95</td>
</tr>
<tr>
<td>5</td>
<td>-0.520</td>
<td>-1.527</td>
<td>aft</td>
<td>0.9</td>
</tr>
<tr>
<td>6</td>
<td>-0.557</td>
<td>-1.628</td>
<td>aft</td>
<td>0.85</td>
</tr>
<tr>
<td>7</td>
<td>-0.568</td>
<td>-1.438</td>
<td>head</td>
<td>0.9</td>
</tr>
<tr>
<td>8</td>
<td>-0.617</td>
<td>-1.546</td>
<td>head</td>
<td>0.85</td>
</tr>
<tr>
<td>9</td>
<td>-0.612</td>
<td>-1.723</td>
<td>aft</td>
<td>0.8</td>
</tr>
<tr>
<td>10</td>
<td>-0.700</td>
<td>-1.995</td>
<td>aft</td>
<td>0.7</td>
</tr>
<tr>
<td>11</td>
<td>-0.668</td>
<td>-1.686</td>
<td>head</td>
<td>0.8</td>
</tr>
<tr>
<td>12</td>
<td>-0.778</td>
<td>-1.934</td>
<td>head</td>
<td>0.7</td>
</tr>
<tr>
<td>13</td>
<td>-0.918</td>
<td>-2.186</td>
<td>head</td>
<td>0.6</td>
</tr>
<tr>
<td>14</td>
<td>-0.820</td>
<td>-2.267</td>
<td>aft</td>
<td>0.6</td>
</tr>
</tbody>
</table>

Figure 9.4 depicts the measured stationary data points, $(\bar{x}, \bar{y})$. Figure and table reveal two features. Firstly, the stationary data points are separated in two clusters according to the wind direction. Secondly, the data clusters show a strong linear relationship between the measured quantities $\bar{x}_{meas}$ and $\bar{y}_{meas}$. This means that the measurements are precise and that the linear model is a natural choice.

To find parameter $\gamma$ our linear regression is applied to the stationary data points. The regression line is depicted in Figure 9.4. The slope of the regression line is $\gamma$; the offset is $\delta_{stat}$. The values are listed in Table 9.3. Of course the values of coefficient of determination, $R^2$, indicate an almost perfect linear fit. Indeed, $x_{meas}$ and $y_{meas}$ are linearly related.

**Table 9.3: Parameter $\gamma$**

<table>
<thead>
<tr>
<th>data</th>
<th>$\gamma$</th>
<th>$\delta_{stat}$</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>aft wind</td>
<td>2.5022</td>
<td>-0.2218</td>
<td>0.9972</td>
</tr>
<tr>
<td>head wind</td>
<td>2.2671</td>
<td>-0.1402</td>
<td>0.9944</td>
</tr>
</tbody>
</table>

Parameter $\gamma$ is function of the wind direction. In the design of the surge equa-
tion, see Section 4.3.4, it is mentioned that the resistance force model is as generic as possible. That means that any changes in the resistance force is taken in the mathematical model by parameters $\gamma$ and $p$ ($\varphi$ in dimensionless form). The actual $\gamma$ validates the design.

The offset $\delta_{\text{stat}}$ is used in the linear relation between the resistance factor, $\varphi$, and thrust deduction, $t_d$,

$$
\begin{align*}
\begin{aligned}
t_{d,\text{aft}} &= 1 - 1.2464 \varphi_{\text{aft}} \\
t_{d,\text{head}} &= 1 - 1.1494 \varphi_{\text{head}}
\end{aligned}
\end{align*}
$$

(9.7)

Figure 9.4: The stationary data points and the regression line. The slope of the regression line is $\gamma$.

9.2.2 The actual resistance factor

The resistance factor $\varphi$ is part of the resistance term of the surge equation

$$
\frac{du}{dt} = (1 - t_d) T_p - \varphi u \gamma
$$

(9.8)

The method to estimate $\varphi$ uses ship dynamics in deceleration. We mimic the removal of the propeller by controlling the propeller pitch angle such that the measured thrust is zero. The design of the procedure is described in Section 6.7.3; the actual procedure and the measurements are presented in Section 9.1. If propeller thrust, $T_p$, is zero then the equation has analytical solution of the form

$$
u(t) = (1 - \gamma) \left[ -\varphi t + u_{\text{init}} \right]^{\frac{1}{\gamma}}
$$

(9.9)

When $\gamma$ is known the solution can be arranged into a linear relation, see equation (6.63) in Section 6.3. Parameter $\varphi$ is the slope of the linear model. A least squares method is used to find $\varphi$

$$
\min_{\varphi, \delta_{\text{dyn}}} \| y_{\text{meas}}(k) - (-\varphi t(k) + \delta_{\text{dyn}}) \|^2
$$

(9.10)
9.2. Model parameters at full scale

where the measured quantity $y_{\text{meas}}$ is actually the speed-through-water transformed according to $u_{\text{meas}}(t)^{1-\gamma}/(1 - \gamma)$, and $t$ is the vector of time points.

The measured quantities, $(t, y_{\text{meas}})$, are depicted in Figure 9.5. Figures 9.5a and 9.5b depict the tests with aft wind; Figures 9.5c and 9.5d depict the tests with head wind. The measured data points are arranged around an obvious linear trend. The small variations of the data points are due to the fact that the ship becomes unstable during decelerations. Then, the autopilot, which was not disconnected during the tests, slightly adjusts the direction of the ship by small changes in the rudder angle, causing the variations in speed, thrust, and torque measurements.

The linear regression is depicted in Figure 9.5. Parameter $\varphi$ is the slope of the linear model. The values of the parameters $\varphi$ and $\delta_{\text{dyn}}$ are listed in Table 9.4. The values of coefficient of determination indicate that the linear model is representative for the measured data points. Similar to resistance exponent $\gamma$, the resistant factor $\varphi$ depends on the wind direction. To calculate the values of $\varphi_{\text{aft}}$ and $\varphi_{\text{head}}$ we take the average of the two zero-thrust tests with aft wind and two zero-thrust tests with head wind condition, respectively. The difference between aft wind and head wind conditions are plain when the values are brought to full dimensions, using the formulation (5.8). Actually, the ratio $p_{\text{aft}}/p_{\text{head}}$ is $0.53^\dagger$.

![Graphs of data points and regression lines for different wind conditions.](image)

Figure 9.5: The data points are the vessel speed transformed according to $u^{1-\gamma}/(1 - \gamma)$.

$^\dagger$Dimension full values are not presented due to confidentiality aspects.
The actual resistance force

With parameters $\gamma$ and $p$ estimated at full scale we calculate the total resistance force using the mathematical model

$$R = p \left| \frac{u}{u_0} \right|^{\gamma-1} u$$

(9.11)

The resistance force, calculated as such, is depicted in Figure 9.6, in dimensionless form. Expectedly, ship resistance force is larger for head wind than for the aft wind conditions. Using the resistance mathematical model we can also predict the ship resistance force for vessel speed that are smaller or larger than the speed during the tests. Of course, the external factors must be similar to the external factors during the propulsion tests. The significance of the similar external factors will be fully revealed in Section 9.2.4, where we predict the resistance force outside the propulsion tests.

![Figure 9.6: The actual resistance force. The resistance force can be calculated when the speed-through-water is measured and parameter $\gamma$ and $p$ are known. The quantities are dimensionless.](image)

### 9.2.3 The actual thrust deduction

Thrust deduction is one of the most important model parameters for the ship motion equation. The parameter originates from the assumption that the additional
9.2. Model parameters at full scale

Resistance force due to propeller action is proportional to the propeller thrust. The origin of $t_d$ is described in details in Section 4.3.4.

Thrust deduction is estimated from stationary state data. The method to estimate $t_d$ relies on the transformation of the stationary surge equation

$$0 = (1 - t_d) \hat{T}_p - \varphi \bar{u}^\gamma$$

into a linear relationship of the form

$$\bar{y}_{\text{model}} = \frac{1}{1 - t_d} \bar{x}_{\text{model}}$$

where $\bar{y}_{\text{model}} = \hat{T}_p$ and $\bar{x}_{\text{model}} = \varphi \bar{u}^\gamma = R_f$.

The parameter estimation method is a least squares method. The difference between measurements and prediction is minimized with respect to the parameter $t_d$

$$\min_{t_d} \left\| \bar{y}_{\text{meas}} - \frac{1}{1 - t_d} \bar{x}_{\text{meas}} \right\|^2$$

The measured quantities ($\bar{x}_{\text{meas}}; \bar{y}_{\text{meas}}$) are listed in Table 9.5. Wind relative direction and the propeller pitch angle are also listed. Figure 9.7 depicts the measured data points. There is no apparent relationship between the data points and the wind relative direction, as observed in case of $\gamma$ and $\varphi$. Indeed, applying the estimation method separately for head wind and aft wind we get values of thrust deduction that only differ 0.3% from each other. The conclusion is that $t_d$ is not sensitive to external factors. All external factors are taken into account by the resistance parameters $\gamma$ and $\varphi$.

**Table 9.5: Stationary data points**

<table>
<thead>
<tr>
<th>Stationary state</th>
<th>x</th>
<th>y</th>
<th>Wind</th>
<th>Pitch</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.230</td>
<td>0.272</td>
<td>aft</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>0.203</td>
<td>0.234</td>
<td>aft</td>
<td>0.95</td>
</tr>
<tr>
<td>3</td>
<td>0.249</td>
<td>0.299</td>
<td>head</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>0.223</td>
<td>0.263</td>
<td>head</td>
<td>0.95</td>
</tr>
<tr>
<td>5</td>
<td>0.186</td>
<td>0.217</td>
<td>aft</td>
<td>0.9</td>
</tr>
<tr>
<td>6</td>
<td>0.170</td>
<td>0.196</td>
<td>aft</td>
<td>0.85</td>
</tr>
<tr>
<td>7</td>
<td>0.205</td>
<td>0.237</td>
<td>head</td>
<td>0.9</td>
</tr>
<tr>
<td>8</td>
<td>0.184</td>
<td>0.213</td>
<td>head</td>
<td>0.85</td>
</tr>
<tr>
<td>9</td>
<td>0.148</td>
<td>0.178</td>
<td>aft</td>
<td>0.8</td>
</tr>
<tr>
<td>10</td>
<td>0.118</td>
<td>0.135</td>
<td>aft</td>
<td>0.7</td>
</tr>
<tr>
<td>11</td>
<td>0.164</td>
<td>0.185</td>
<td>head</td>
<td>0.8</td>
</tr>
<tr>
<td>12</td>
<td>0.127</td>
<td>0.144</td>
<td>head</td>
<td>0.7</td>
</tr>
<tr>
<td>13</td>
<td>0.095</td>
<td>0.112</td>
<td>head</td>
<td>0.6</td>
</tr>
<tr>
<td>14</td>
<td>0.087</td>
<td>0.103</td>
<td>aft</td>
<td>0.6</td>
</tr>
</tbody>
</table>
The value of thrust deduction estimated from the stationary states is 0.14. The coefficient of determination is 0.9967 which indicates that more than 99% of the measurements \((\bar{x}_{\text{meas}}, \bar{y}_{\text{meas}})\) are described by the proposed linear model.

![Figure 9.7: The stationary data points and the regression line.](image)

### 9.2.4 Qualitative validation of surge parameters

The model parameters of the surge equation were estimated in the previous three sections. From propulsion tests we found that the model parameters depend on the wind direction. The two sets of parameters are listed in Table 9.6 with respect to wind direction relative to ship motion, namely, *aft wind* and *head wind*. In this section we want to validate the estimated model parameters for measurements collected outside the propulsion tests.

**Table 9.6:** Summary of model parameters in surge equation

<table>
<thead>
<tr>
<th>parameter</th>
<th>aft wind</th>
<th>head wind</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\gamma)</td>
<td>2.5022</td>
<td>2.2671</td>
</tr>
<tr>
<td>(\varphi)</td>
<td>0.6853</td>
<td>0.7462</td>
</tr>
<tr>
<td>(t_d)</td>
<td>0.14</td>
<td>0.14</td>
</tr>
</tbody>
</table>

The propulsion tests performed by the vessel were carried out during a weeklong trip. We use the measurements collected during the entire trip. We extract the stationary states using the algorithm presented in Section 8.3. The duration of a stationary state is about ten minutes. The stationary data points are depicted in Figure 9.8b in form of dimensionless propeller pitch and dimensionless speed-through-water. There are 173 stationary data points spread over the six-day trip. The propulsion tests were carried out during the third day of the trip, as seen in Figure 9.8b. We notice that outside the propulsion tests the control of the propulsion system and the speed of the vessel is relatively constant.
9.2. Model parameters at full scale

The validation procedure is as follows. We check how well the stationary surge equation

\[ 0 = (1 - t_d)(T_p + T_n) - \varphi u^\gamma \]  

is satisfied. The term \( T_n \) is the thrust developed by the propeller nozzle, which is known from CFD simulations\(^\dagger\). The propeller thrust, \( T_p \), and the speed-through-water, \( u \), are directly measured. Parameters \( t_d \), \( \varphi \), and \( \gamma \) are estimated for the propulsion tests. The residuals are quantified in terms of relative percent error

\[ \varepsilon = \frac{|T_E - R_f|}{T_E + R_f} \cdot 100 \, [\%] \]  

where \( T_E = (1 - t_d)(T_p + T_n) \) is the effective thrust and \( R_f = \varphi u^\gamma \) is the total resistance force.

The external conditions outside of the propulsion tests are different from the external conditions experienced during the propulsion tests. From the propulsion tests we only have two sets of model parameters as function of external conditions. These two sets of parameters are assigned as follows. The stationary data points are split into four groups according to the relative wind direction: head wind, head-cross wind, aft-cross wind, and aft wind. The four categories are depicted in Figure 9.9. The 35° angle for head wind and aft wind is a convenient choice that takes into account the wind relative direction during the propulsion tests and the number of data points per group. Due to the fact that we only have two sets of model parameters, for aft wind and head wind, we have to extend their use also to aft-cross wind and head-cross wind.

The relative percent error, \( \varepsilon \), is depicted in Figure 9.10a along the time line of the measurements. There is large spread in the values of \( \varepsilon \). Expectedly, the values

\(^\dagger\)The CFD simulations of the ducted propeller were performed at Wärtsilä Netherlands.
of $\varepsilon$ are the lowest ($0.2\% < \varepsilon < 3.5\%$) during day three, the day the propulsion tests were performed. Outside the day three, the values of $\varepsilon$ reach as much as 35%. The distribution of the data points according to the group they belong is not as expected. We would expect that the data points with *aft wind* and *head wind* would have an $\varepsilon$ on average smaller than the data points with *aft-cross wind* and *head-cross wind*. The plot in Figure 9.10a does not support this expectation. We see that during day one and day two there are many *aft wind* data points with $\varepsilon$ larger than 10%, among the numerous *aft-cross wind* data points. The same holds for the *head wind* data points during day four. Most likely, there are other external factors besides the wind relative direction that are not taken into the account by the estimated model parameters. The sea state (wave height, wave period, and wave direction) could be one of the external factors that influence the stationary data points.

**Figure 9.9:** The split in relative wind direction. The $35^\circ$ angle that defines the head wind and aft wind is conveniently chosen.

In the list of collected signals, wind speed and wind direction are the only measured external factors. Alternatively, we can retrieve qualitative information about the external factors by recording the ship behavior, in particular the angular motions of the ship, namely, pitch, yaw, and roll, see (Lewis, Vol. 3, 1989, pp. 41-83). Let us use the roll angle as the measure of the other external factor. The larger the influence of the external factor the larger the amplitude of the roll angle.

We quantify the amplitude of the roll angle by calculating the standard deviation of the roll angle for each stationary state

$$\sigma_{roll} = \sqrt{\frac{1}{N} \sum_{k=1}^{N} [(y_{meas}(k) - \mu)^2]}$$  (9.17)

where $y_{meas}(k), k \in [1, N)$ represents the measured signal and $\mu$ its mean value.

The standard deviation of roll angle is represented versus $\varepsilon$ in Figure 9.10c and along the measurement time line in Figure 9.10b. These two figures show direct proportionality between the relative percent error and the standard deviation of
roll angle. For most of the data points, large $\varepsilon$ corresponds to large $\sigma_{\text{roll}}$. The few exceptions from this rule support the importance of the external factors in estimating the model parameters.

![Plot (a)](image1.png)

![Plot (b)](image2.png)

![Plot (c)](image3.png)

**Figure 9.10:** The relative percent error (a) is directly proportional to the standard deviation of the roll angle (b), as it is indicated by subplot (c).

The conclusion of the qualitative validation is that the estimated model parameters are only applicable for external conditions that are *similar* to the external
Implementation of design conditions during the propulsion tests. For instance, $\sigma_{\text{roll}}$ of the stationary states collected during day four and five is similar to $\sigma_{\text{roll}}$ of the stationary propulsion tests. All these stationary states have similar $\epsilon$, usually lower than 8%; see Figure 9.10a.

### 9.2.5 Wake fraction

A parameter estimation method for wake fraction is described in Section 6.6. However, the specific control required by the method, namely, harmonic $n_{\text{set}}$, was not tested, and, thus, measurements are not available. During the propulsion test session referred in Section 9.1, harmonic control was imposed on propeller pitch angle. A short excerpt of the tests is presented in Figure 6.6 of Section 6.6. This test revealed that, indeed, the ship propulsion responds to such test. Moreover, there is no phase shift between the measured propeller pitch and the propeller thrust. No changes in speed-through-water was observed during repetitive changes in propeller pitch angle. These observations build up confidence in this parameter estimation method.

In Section 8.2.3 we briefly discussed the possibility of using the signal reconciliation method to estimate unknown parameters. This idea is not further investigated in this thesis. Yet, this kind of approach, if successful, has a major advantage over the method that requires imposed harmonic dynamics: it does not require specific tests, except for stationary states during regular ship operations. An indication that parameter estimation from signal reconciliation might work is given by the ship propulsion model itself.

Let us simulate the stationary propulsion tests. For this purpose we use the estimated $\gamma$, $\varphi$, and $t_d$, see Section 9.2. The model parameters in the engine and shaft equations are assumed known. The propeller parameters, $a_0$, $a_1$, $\beta_0$, and $\beta_1$, estimated from CFD simulations are used. We choose a convenient wake fraction. The wake fraction is assumed to be a constant. The control of the virtual ship (the mathematical model) is the measured propeller pitch angle, which is the control of the real ship during the propulsion tests.

Figure 9.11 depicts the results of the simulation in comparison with the full scale measurements of speed-through-water in longitudinal direction, propeller thrust, and propeller torque. The value of wake fraction used in the simulation is 0.18, which is a convenient choice. The plots indicate a good qualitative agreement between the full scale measurements and the solutions of the mathematical model. This means that our choice for the wake fraction, $w = 0.18$, is not far from the measured reality. Moreover, the assumption that wake fraction is a constant stands.
9.2. Model parameters at full scale

Figure 9.11: Measurements during the stationary tests and the solutions of the ship propulsion model. The solutions are obtained for given wake fraction, $w = 0.18$, and given propeller parameters (CFD simulations). Estimated $\gamma$, $p$, and $t_d$ are used.

The qualitative differences between the measurements and simulations, namely, the sharp peaks between stationary states that are observed especially in speed-
Implementation of design

through-water and thrust, are due to the fact that the simulation does not take into account the lateral accelerations that appear when the ship turns, see Figure 9.1 for the ship trajectory during the stationary propulsion tests. The sources of the quantitative differences can be found both in mathematical model and full scale measurements. On one hand-side, unappropriate propeller parameters, $\alpha_0$, $\alpha_1$, $\beta_0$, and $\beta_1$, could cause the offsets. On the other hand-side, measurements errors could as well cause the relative differences. In the present context it is impossible to identify the sources, even part of them.

9.3 Propeller efficiency at full scale

The two parties that are interested in the performance of propeller have been mentioned in the introduction of the thesis. Let us recall them. Propeller efficiency at full scale is equally important for the propeller manufacturer and for the ship operator. Their view on the usage of propeller efficiency might diverge at certain point.

Propeller manufacturer wants to design efficient propellers. Prior to their effective production the propeller are tested as scaled models in model basins or as virtual propellers in CFD software. All propellers bear the label of propeller efficiency. When propeller is installed on a vessel the manufacturer wants to show that the propeller efficiency at full scale coincides with the values given by the model basin and the CFD simulations. In other words, the manufacturer wants the validation of the propeller design at full scale.

The ship operator wants to buy propellers with the optimal efficiency for the design and operational profile of their ships. The interest of the ship operator lies beyond the decision of buying the best propeller on the market. The ship operator wants to operate its ships for many years. Thus, the ship operator is interested to keep the propeller efficiency at the initial value, when it is mounted on the ship. This is done through propeller maintenance, as part of general ship maintenance.

In first subsection we calculate propeller efficiency from full scale propulsion tests. An important conclusion we draw is that propeller efficiency at full scale is influenced by external factors. In the second subsection we introduce the concepts of reference and relative propeller efficiency. The reference propeller efficiency is a subjective definition that takes into account the operational condition and the external factors. The relative propeller efficiency is compared to the reference propeller efficiency for similar operational and external conditions.

9.3.1 Propeller efficiency from propulsion tests

To calculate propeller efficiency at full scale form the propulsion tests we need a value for the wake fraction. Following the discussion in Section 9.2.5 we use the value 0.18 for wake fraction, which brought the solution of the mathematical model close to the measurements.
When all necessary information is available according to the prescriptions, to calculate propeller efficiency is a formality

\[
\eta_{\text{prop}} = \frac{T_p u (1 - w)}{2\pi Q_p n} \quad (9.18)
\]

The stationary data points of speed-through-water, thrust, torque, and propeller angular speed are given in Table 9.1. The values of propeller efficiency for the stationary states are listed in Table 9.7. A discussion on the absolute values of the propeller efficiency is irrelevant in this context. A slightly different wake fraction changes all listed values. A discussion from the qualitative point of view is far more relevant here.

Table 9.7 contains qualitative information on the wind relative direction to the ship, which in this context represents the external factors. Figure 9.12 depicts the values of propeller efficiency against the propeller pitch angle. A distinction is made between stationary data points with aft wind condition and the data points with head wind condition. For similar \( \theta_p \) the values of propeller efficiency with aft wind are always larger than the values with head wind.

**Table 9.7:** Propeller efficiency for the stationary states

<table>
<thead>
<tr>
<th>stationary state</th>
<th>( \eta_{\text{prop}} )</th>
<th>pitch angle</th>
<th>wind direction</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.4536</td>
<td>1</td>
<td>aft</td>
</tr>
<tr>
<td>2</td>
<td>0.4596</td>
<td>0.95</td>
<td>aft</td>
</tr>
<tr>
<td>3</td>
<td>0.4507</td>
<td>1</td>
<td>head</td>
</tr>
<tr>
<td>4</td>
<td>0.4541</td>
<td>0.95</td>
<td>head</td>
</tr>
<tr>
<td>5</td>
<td>0.4803</td>
<td>0.9</td>
<td>aft</td>
</tr>
<tr>
<td>6</td>
<td>0.4838</td>
<td>0.85</td>
<td>aft</td>
</tr>
<tr>
<td>7</td>
<td>0.4627</td>
<td>0.9</td>
<td>head</td>
</tr>
<tr>
<td>8</td>
<td>0.4566</td>
<td>0.85</td>
<td>head</td>
</tr>
<tr>
<td>9</td>
<td>0.4801</td>
<td>0.8</td>
<td>aft</td>
</tr>
<tr>
<td>10</td>
<td>0.4636</td>
<td>0.7</td>
<td>aft</td>
</tr>
<tr>
<td>11</td>
<td>0.4527</td>
<td>0.8</td>
<td>head</td>
</tr>
<tr>
<td>12</td>
<td>0.4354</td>
<td>0.7</td>
<td>head</td>
</tr>
<tr>
<td>13</td>
<td>0.4002</td>
<td>0.6</td>
<td>head</td>
</tr>
<tr>
<td>14</td>
<td>0.3469</td>
<td>0.6</td>
<td>aft</td>
</tr>
</tbody>
</table>

For a better understanding of the effects of the external factors on the propeller efficiency we split the formula (9.18) as follows

\[
\eta_{\text{prop}} = \left[ \frac{T_p}{2\pi Q_p} \right] \left[ \frac{u (1 - w)}{n} \right] \quad (9.19)
\]
Figure 9.12: Propeller efficiency during the stationary state tests as function of wind relative direction. Propeller efficiency is calculated for a given wake fraction, \( w = 0.18 \).

In Figure 9.13 we represent the two terms of the formula, \( T_p/2\pi Q_p \) and \( u(1 - w)/n \), against propeller pitch angle, \( \theta_p \). For reasons of clarity, the term \( u(1 - w)/n \) is scaled to similar values as the term \( T_p/2\pi Q_p \). In both plots we separate data points with \textit{aft wind} from data points \textit{head wind}. It becomes clear that the external effects are brought in propeller efficiency formula by the term \( u(1 - w)/n \). Note that for this particular case wake fraction, \( w \), and propeller rotational speed, \( n \), are constant. Thus, speed-through-water alone is responsible for the differences between \textit{aft wind} and \textit{head wind} conditions. Small differences between \textit{aft wind} and \textit{head wind} conditions exist also in the term \( T_p/2\pi Q_p \), especially at high \( \theta_p \). As \( \theta_p \) decreases the difference diminishes.

Figure 9.13: The influence of external factors, namely wind direction, on the two terms of the propeller efficiency formula, \( u(1 - w)/n \) and \( T_p/2\pi Q_p \). Note that the term \( u(1 - w)/n \) is scaled to similar values as the term \( T_p/2\pi Q_p \).

Under the assumption that wake fraction is a constant and that the measurements are reliable, it appears that propeller efficiency at full scale is not simply a \textit{label} but it changes with external conditions. This fact has important impact on understanding propeller efficiency at full scale and on further developing its practical applications. In the next subsection we introduce the concepts of \textit{reference}
and relative propeller efficiency.

### 9.3.2 On reference and relative propeller efficiency

Propeller efficiency does not exist as a physical quantity; it is defined as an index. The definition we used so far is the generally accepted one in the hydrodynamic community. This definition comes mainly from the open water tests with scaled propellers where wake fraction does not exist. Aspects related to model testing are presented in Section 2.2.3. The situation at full scale is fundamentally different. Throughout this thesis we designed mathematical models and mathematical algorithms, and, full scale measurements and data processing methods; these designs give an answer the question: What is propeller efficiency at full scale? Several challenges have been overcame, but more challenges are left, including the wake fraction at full scale. We dedicate this section to describing the concepts of reference and relative propeller efficiency. These two concepts have immediate practical applications in performance monitoring of ship propellers.

Propeller efficiency can be represented as a function in the \( N \)-dimensional space

\[
\eta_{\text{prop}} = \eta_{\text{prop}}(x_1, x_2, \ldots, x_N) \tag{9.20}
\]

where \( x_i, i = 1, \ldots, N \) represent measured quantities that define propeller efficiency at full scale. Explicitly, propeller efficiency can be represented as function of the measured ship variables \( T_p, Q_p, u, n, \) and \( \theta_p \), the wake fraction \( w \), and, the measured external factors, \( E_1, \ldots, E_n, n \in \mathbb{N} \)

\[
\eta_{\text{prop}} = \eta_{\text{prop}}(T_p, Q_p, u, n, w, \theta_p, E_1, E_2, \ldots, E_n) \tag{9.21}
\]

External factors are weather conditions, sea state, water properties, and other factors that might influence propeller efficiency.

**Reference** propeller efficiency is calculated by formula

\[
\eta_{\text{ref}} = \eta_{\text{ref}}(V_{\text{ref}}, V_0) \tag{9.22}
\]

where \( V_{\text{ref}} = (n, \theta_p, E_1, \ldots, E_n) \) is the vector of similar states and \( V_0 = (T_p, Q_p, u) \) is the vector of initial states. For practical reasons we scale the initial states to 1.

**Relative** propeller efficiency is calculated for the similar states \( V_{\text{ref}} \)

\[
\eta_{\text{rel}}(i) = \eta_{\text{rel}}(V_{\text{ref}}, V_i), \quad i = 1, \ldots, N \tag{9.23}
\]

where \( V_i = (T_p(i), Q_p(i), u(i)) \) represents the measured states \( T_p, Q_p, \) and \( u \) at a certain moment. The advantage of reference and relative propeller efficiency is that wake fraction is not required. It is assumed that wake fraction is constant for similar states. Explicitly, the relative propeller efficiency formula is

\[
\eta_{\text{rel}}(i) = \frac{T_p(i) u(i)}{Q_p(i) n_{\text{ref}}}, \quad n_{\text{ref}} \in V_{\text{ref}} \tag{9.24}
\]
under the constraint \((\theta_p, E_1, ..., E_n) \in V_{ref}\).

The concept of similar states is important. Similar states mean stationary states in which ship propulsion controls, namely, propeller rotational speed and propeller pitch angle, and external factors, namely, weather, sea state, water properties, constraints, are similar. Ideally, propeller efficiency for two similar states should be the same. In practice, we expect variations and, possibly, trends on long term.

In practice, similar states are actually bands in which we allow a particular variable to vary. For instance, we may allow the dimensionless propeller angular speed to vary within \([0.9, 0.95]\), and the dimensionless wind speed within \([1.5, 2]\). The variations in relative propeller efficiency are due to these preset bands. Long-term variations, or trends, may occur due to propeller damages and fouling. To identify these instances is the practical application of relative propeller efficiency.

In Figure 9.14 we schematically depict the reference and relative propeller efficiency. A representative state is chosen based on ship propulsion controls, \(n\) and \(\theta_p\), and the external conditions, \(E_1, ..., E_n\). At moment \(t_{init}\) propeller efficiency is calibrated by setting the reference value to 1. In ship regular operations, the relative propeller efficiency is calculated every time a similar state is encountered.

![Figure 9.14: Schematic of reference and relative propeller efficiency for similar states. The choice of reference propeller efficiency is subjectively done and is equivalent to calibration. Relative propeller efficiency is calculated for similar states and are represented with respect to the reference value.](image)

The concept of reference and relative propeller efficiency is put in practice in the next chapter. There, we use the data base of measurements collected on board of a ferry to represent propeller efficiency.

### 9.3.3 Discussion

Absolute propeller efficiency at full scale is the objective of the propeller manufacturer. Absolute propeller efficiency is a blend of mathematical modeling and full scale measurements. In this thesis we undertook the challenge of describing this blend. The ingredients of this blend are equally important. The mathematical model provides the structure of the approach. The mathematical model poses
9.3. Propeller efficiency at full scale

itself challenges when projecting to real world. Of course, these challenges are the model parameters. The full scale measurements bring the real-life information into the mathematical environment through proper data processing. As we have seen, to estimate the values of the model parameters at full scale we need to conveniently transform the mathematical model and to perform appropriate full scale propulsion tests. Full scale propulsion tests are not readily available. For this reason the wake fraction at full scale still needs appropriate propulsion tests. When propulsion tests are available the model parameter are reliably estimated, as we have seen in case of the surge parameters.

The fact that model parameters require full scale propulsion tests means that the absolute propeller efficiency is feasible only for these propulsion tests, under controlled propulsion conditions. To investigate the influence of the external factors on the model parameters, and, ultimately, on the propeller efficiency, remains a challenge.
Chapter 10

Statistical aspects of propeller efficiency at full scale

In this chapter we implement the concept of reference and relative propeller efficiency at full scale. For this purpose we use the measurements collected on board of the ferry. The stationary state extraction algorithm, presented in Section 8.3, generates a data base of stationary states. This data base is the starting point of the investigations related to reference and relative propeller efficiency.

There is a wide variety in the stationary states of the data base. The most representative stationary states must be identified. In this chapter we structure the data base of stationary points according to the ship propulsion control. We identify two operational modes with the highest density of stationary data points. The response of the ship is far from being concentrated in the same manner as the propulsion control. In fact, we show that the response of the ship is normally distributed, a characteristic of random variables. This is of course passed on to propeller efficiency, which becomes a random variable at full scale, with a mean value and a normal distribution around the mean.

We split the stationary data points of the two modes into equally weighted groups. The groups of data points are arranged according to their time labels. The first group of data points is set as the reference. The remaining groups are then represented relatively to the reference group. This is the implementation of the reference and relative propeller efficiency. We show that the relative propeller efficiency is seldom close to the reference propeller efficiency.
10.1 Reality against expectations

Studies that address the full scale, in-service, ship measurements are insufficient in literature. One of the few such papers, T.A. Dinham-Peren and I.W. Dand analyze a set of full scale measurements collected in-service conditions, see (Dinham-Peren and Dand, 2010). The goal of their study is to extract the actual ship performance, in terms of load power curve (shaft power versus vessel speed), see Figure 10.1b for a schematic. A data base of measurements collected every 5 minutes on board of an ocean going vessel is analyzed in this study. When data is represented in the load power curve the authors are confronted with a large scatter. This scatter can be explained if extra dimensions are added to the representation. The authors reduce the dimensionality of the problem to two dimensions, power and speed, by sequentially filtering the data base. For instance, several variables, such as propeller angular speed, speed-over-ground, and wind speed, are constrained to specific intervals; the data points are assigned to buckets based on external factors. At the end of the filtering process the authors are left with 2.1% of the initial data base.

![Figure 10.1: Schematic of a propeller efficiency curve (a) and a load power curve (b).](image)

The paper of T.A. Dinham-Peren and I.W. Dand comes closest to our goals in this chapter. Nevertheless, there are fundamental differences between our approach and the approach of T.A. Dinham-Peren and I.W. Dand. Probably the most fundamental difference has to do with the measurement approach. The authors collect measurements with a sampling period of five minutes. In this thesis the measurements are collected with sampling period between 0.1 and 1 second, depending on the sensor. Then, we extract the time excerpts where ship sails in a stationary state. The concept of stationarity is important in our work, and, thus, we allocated a section to defining and extracting the stationary states, see Section 8.3. Another difference has to do with expectations. T.A. Dinham-Peren and I.W. Dand have at their disposal the theoretical load power curve of the propulsion system and they use this curve as reference throughout their study. This can bias the data processing approach. The idea of reference and relative propeller efficiency does not need any theoretical preconception, such as the propeller efficiency curve schematically depicted in Figure 10.1a. The measurements are
their own preconception. Of course, this adds more stress on the importance of reliability of measurements (precise measurements, reliable sensors, reliable data collection system).

10.2 Data base

In this chapter we use the measurements recorded on board of the ferry. A data base of stationary states is generated by the stationary state extraction algorithm, presented in Section 8.3. The data base created as such contains approximately 14,000 stationary states. The duration of each stationary state is at least five minutes. In the data base, each stationary state is represented by the average values of the stationary signals; additionally, the time stamps of the stationary state is stored. The time line of the data base stretches for one year, except for the thrust and torque measurements, which stretch for half of year.

The ferry is a modern vessel that sails ten times per day between two harbors, following the same route. Due to its operational profile the ferry is the ideal candidate for a statistical analysis of the recorded measurements.

Figure 10.2 depicts the stationary data points of the speed-through-water in longitudinal direction. The $x$-axis represents the timeline of the measurements. The plot indicates that an underlying structure exists in the data base. The data appears to be structured on several layers. There is a compact layer between 0.8 and 0.9 where most of the data is located. The upper layer, higher than 0.95, appears completely separated from the previous one. Below 0.8, the data gradually grows faint, keeping a relatively denser structure between 0.8 and 0.6. The reason for the structure in the data is mainly the ship operational profile. The data structuring with respect to ship’s operational profile is done in the next section.

![Figure 10.2: The stationary data points of speed-through-water in longitudinal direction extracted from the processed signals. The plot contains all extracted stationary data points, some 14,000 data points.](image)
10.3 Data structuring

Data structuring we present in this section is, in fact, data clustering with respect to known information. The goal of this section is to identify sets of similar stationary states in the data base. The stationary data points inside a set have common characteristics, such as, similar propeller angular speed, propeller pitch angle, speed-through-water, weather conditions (wind, waves), and ship displacement. An experienced marine engineer would probably add more characteristics to this list. In this section we structure the data base with respect to the ship propulsion control mode, which is a combination of propeller rotational speed and propeller pitch angle.

The propulsion system of the ferry has several control modes. A control mode is a predefined lookup table that consists of pairs of propeller rotational speed and propeller pitch angle that correspond to certain positions of the command lever. The control modes are dedicated to specific ship operations, such as, sea going, maneuvering, emergency, and manual control.

Figure 10.3a depicts the stationary data points of propeller pitch angle versus the propeller angular velocity. The plot indicates that the stationary data points in the data base are structured in three control modes. We name these modes Mode A, Mode B, and Mode C, see Figure 10.3a. Figure 10.3b depicts the stationary speed-through-water data points corresponding to the three modes. The two plots indicate that ship speed is relatively higher when in Mode A; Mode B corresponds to a mid speed range, while Mode C corresponds to a somewhat lower average speed-through-water. Important to mention is the number of data points each set contains: Mode A contains 9% of the total data points, Mode B contains 81%, and Mode C contains 10%. Obviously, Mode B is the preferred control mode. Due to wide spread in the data points of Mode C, we do not include this mode in the statistical analysis. Further, we focus on Mode A and Mode B.

A close look into data set Mode B, see Figure 10.4, reveals that a significant
10.3. Data structuring

number of data points are situated either above or below the designated line. This phenomenon happens only close to maximum propeller angular velocity. We do not know the reason of the large spread of data in that region. We do know that 19\% (blue points in Figure 10.4) of the total number of data points reside above the line of Mode B (red points in Figure 10.4), and only 3\% (light blue points in Figure 10.4) below the line, leaving 59\% of data points in the line itself. Thus, we separate Mode B into three subsets according to Figure 10.4.

Figure 10.4: Mode B can be separated into three sets: Mode B2 that consists of the data points that belong to the line itself, Mode B1 that consists of data points located above the line, and Mode B3 that consists of data points located below the line.

A yet closer look to Mode B1 reveals a comet-like structure, see Figure 10.5b. Some 18\% of the total amount of data points form the core of this structure while only 1\% form the trail. Note that the data points in the core of the structure are selected on base of the propeller rotational speed, 0.995 < n < 1. This criterion results in propeller pitch angle, $\theta_p$, between 0.758 and 0.778. Whether the variations in n and $\theta_p$ are due to the control itself, the imprecise measurements, or external factors we do not know. Given the situation we separate the core from the trail and, thus, form two new clusters that we name Mode B11 and Mode B12. For further analysis we keep only Mode B11.

The data points that belong to Mode A are in a somehow similar situation as the data points in Mode B. There are no data points under the line that form Mode A itself, but there is a core of data points located slightly above the line and at the far end of the interval, see Figure 10.5a. Here, we apply the same criterion on propeller rotational speed as above, 0.995 < n < 1, to separate a dense core from the rest of the data. The newly formed cluster, Mode A11 contains 6\% of the total amount of data points; Mode A12 contains 3\%. Propeller pitch angle in Mode A11 varies some 4\%, from 0.95 to 0.99, see in Figure 10.5a.
Summary of data structuring

The data sets created so far are listed in Table 10.1. For further analysis we select the data sets Mode A11 and Mode B11. Data points that belong to Mode A12, Mode B12, and Mode B3 are not representative. Mode B2, which contains most of the data points, differs from Mode A11 and Mode B11 in the sense that propeller controls change, and, thus, speed, thrust and torque change. The data set Mode B2 appears very well structured but requires an approach that is different than only bucketing the data points. Data points in Mode C are widely dispersed and, thus, are irrelevant for the purpose of this analysis.

Table 10.1: Percentage of data points in the identified structures

<table>
<thead>
<tr>
<th>Structure</th>
<th>Percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mode A</td>
<td></td>
</tr>
<tr>
<td>Mode A11</td>
<td>6</td>
</tr>
<tr>
<td>Mode A12</td>
<td>3</td>
</tr>
<tr>
<td>Mode B</td>
<td></td>
</tr>
<tr>
<td>Mode B11</td>
<td>18</td>
</tr>
<tr>
<td>Mode B12</td>
<td>1</td>
</tr>
<tr>
<td>Mode B2</td>
<td>59</td>
</tr>
<tr>
<td>Mode B3</td>
<td>3</td>
</tr>
<tr>
<td>Mode C</td>
<td>10</td>
</tr>
</tbody>
</table>

Statistics of the data clusters

The two data sets, Mode A11 and Mode B11, are selected based on the ship propulsion controls, namely, propeller rotational speed and pitch angle. The intention is to generate compact clusters of data points. For the selected clusters the propeller rotational speed varies between 0.995 and 1, and propeller pitch angle varies between 0.758 and 0.778 for Mode B11, and between 0.95 and 0.99 for Mode A11. The response of the ship to the consistent control is, probably, unexpected.
10.3. Data structuring

Figure 10.6: Speed-through-water, thrust, and torque measured at full scale are random variables, characterized by a mean value, a standard deviation, and a distribution.

We measure the response of the ship as speed-through-water, thrust, and torque. Figure 10.6 depicts the histograms of these quantities for both Mode A11 and Mode B11. All quantities are scaled by setting their average values to the unit. A visual inspection of the histograms indicate that the response of the ship is rather distributed over a wide interval, suggesting a normal distribution. The measurements are compared with the normal distribution function

$$F(y_{\text{meas}}, \mu, \sigma) = \frac{1}{\sigma \sqrt{2\pi}} e^{-\frac{(y_{\text{meas}}-\mu)^2}{2\sigma^2}}$$

where $y_{\text{meas}}$ is the measured data, $\mu$ is the mean value, and $\sigma$ is the standard
deviation. The values of these parameters are listed in Table 10.2.

There is good agreement between the measurements and the normal density function fitted to the measurements, as seen in Figure 10.6. The standard deviation and variance of the distributions are listed in Table 10.2. The parameters of the normal distribution show that speed-through-water, thrust, and torque have the same distribution for the two modes. The same parameters show that speed and thrust have similar standard deviation, while torque has relatively lower standard deviation. This aspect is visualized in Figure 10.6, where torque distribution appears comparatively narrower than speed and thrust distributions.

Table 10.2: Parameters of the normal and logistic distributions fitted to the data points

<table>
<thead>
<tr>
<th>Mode</th>
<th>Quantity</th>
<th>$\sigma$</th>
<th>$\sigma^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mode A11</td>
<td>$u$</td>
<td>0.032</td>
<td>0.001</td>
</tr>
<tr>
<td></td>
<td>$T_p$</td>
<td>0.048</td>
<td>0.0023</td>
</tr>
<tr>
<td></td>
<td>$Q_p$</td>
<td>0.0095</td>
<td>9.1 \cdot 10^{-5}</td>
</tr>
<tr>
<td>Mode B11</td>
<td>$u$</td>
<td>0.027</td>
<td>0.00076</td>
</tr>
<tr>
<td></td>
<td>$T_p$</td>
<td>0.0355</td>
<td>0.0004</td>
</tr>
<tr>
<td></td>
<td>$Q_p$</td>
<td>0.014</td>
<td>2.1 \cdot 10^{-4}</td>
</tr>
</tbody>
</table>

10.4 Reference and relative propeller efficiency

Two relevant modes were identified in the large database. Mode A11 contains stationary data points that correspond to higher speed regimes; some 6% of the total amount of data points belong to this set. Mode B11 is a very dense cluster of data points, containing some 18% of the total amount of data points. The time line of the two sets is about 6 months\(^*\). These are the data sets to which we apply the reference and relative propeller efficiency concept, see Section 9.3.2.

As seen in the previous section the variables, even when structured in two dense clusters, behave as random variables. This means that there are many other dimensions along which we should structure the data. Perhaps, more structure will not bring reduce the randomness of the variables. In this section we introduce an extra step as compared to the procedure described in Section 9.3.2. We clip the data sets into time excerpts. Measured variables and propeller efficiency are represented as normally distributed around a mean value.

The two data sets are clipped into time excerpts. Mode B11 is clipped into nine time excerpts, each of approximately 20 days. Mode A11 is clipped into six time excerpts that cover somewhat longer periods due to scarcity of data points in this mode. The first time excerpts from each modes is set as the reference state. The measured states that belong to this time excerpt, $(T_p, Q_p, u, n)$ are scaled to 1. Thus, the mean value of the reference propeller efficiency is 1.

\(^*\)The time line of the data base is one year, but thrust and torque measurements are available only for the second half of the period.
The relative propeller efficiency is calculated by the formula

$$\eta_{rel}(i) = \frac{T_p(i) \ u(i)}{Q_p(i) \ n_{ref}}$$ (10.2)

where \(i\) denotes the time excerpt, \(T_p(i), Q_p(i),\) and \(u(i)\) are scaled according to

$$\hat{x} = \frac{1}{\mu_{ref}} \ x$$ (10.3)

where \(\hat{x}\) is the scaled quantity, \(x\) is the quantity to be scaled, and \(\mu_{ref}\) is the scaling factor. The scaling formula is applied per data point.

Figure 10.7 depicts the reference and relative propeller efficiency as normal distributions. In Figure 10.8 the relative propeller efficiency is compared to the reference propeller efficiency from a different perspective. The bottom of each box represents the first quartile, which means the 25th percentile; the top of the box is the third quartile, which means the 75th percentile; thus, each box covers 50% of the data points. The dot inside each box represents the median. The ends of the whiskers correspond roughly to \(\pm 2.7\sigma\); thus, 99.3% of the normally distributed data points are between the ends of the whisker. Table 10.3 lists the mean and the standard deviation of the reference and relative propeller efficiency for each time excerpt.

As seen from the figures the relative propeller efficiency is always different from the reference propeller efficiency. The reference propeller efficiency is, in fact, the propeller efficiency for the first 20 days after the installation of the thrust and torque sensor. Incidentally, the reference propeller efficiency is the maximum value. The relative propeller efficiency is always lower than this reference, for the data base we have at our disposal.

![Figure 10.7: Reference propeller efficiency and relative propeller efficiency represented as normal distributions.](image-url)
Figure 10.8: Reference propeller efficiency and the changes in relative propeller efficiency. The x-axis represents a pseudo-time axis. The bottom and top of each box represent the first and the third quartile, respectively. Thus, 50% of the data points fall inside the box. The ends of the whisker is $+/2.7\sigma$, or 99.3% of the normally distributed data points. The dot inside the box represents the mean value.

The mean relative propeller efficiency in Mode B11 shows large variations, down to 0.88 for excerpts three and four. After these two periods the efficiency increases up to 0.95 for excerpt 7; in the last excerpt the efficiency again drops. The relative propeller efficiency in Mode A11 shows a relatively smaller variation. In this mode we notice a variation of the type we see in the second half of the Mode B11. Note that excerpt 1 in Mode A11 is not the same as excerpt 1 in Mode B11.

The data base we have at our disposal does not allow us to conclude whether long term changes in propeller efficiency are noticeable or not. For this purpose measurements should be collected continuously for at least one or two years. If possible, events such as ship maintenance and propeller maintenance should be covered in the measurements period.

Table 10.3: mean values and standard deviations of Mode A11 and Mode B11

<table>
<thead>
<tr>
<th>Mode A11</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>time excerpt</td>
<td>$\mu$</td>
<td>$\sigma$</td>
</tr>
<tr>
<td>reference</td>
<td>0.93</td>
<td>0.0189</td>
</tr>
<tr>
<td>1</td>
<td>0.93</td>
<td>0.0162</td>
</tr>
<tr>
<td>2</td>
<td>0.98</td>
<td>0.0156</td>
</tr>
<tr>
<td>3</td>
<td>0.96</td>
<td>0.0221</td>
</tr>
<tr>
<td>5</td>
<td>0.93</td>
<td>0.0176</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Mode B11</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>time excerpt</td>
<td>$\mu$</td>
<td>$\sigma$</td>
</tr>
<tr>
<td>reference</td>
<td>0.93</td>
<td>0.0119</td>
</tr>
<tr>
<td>1</td>
<td>0.99</td>
<td>0.0138</td>
</tr>
<tr>
<td>2</td>
<td>0.94</td>
<td>0.0085</td>
</tr>
<tr>
<td>3</td>
<td>0.88</td>
<td>0.0140</td>
</tr>
<tr>
<td>4</td>
<td>0.88</td>
<td>0.0156</td>
</tr>
<tr>
<td>5</td>
<td>0.91</td>
<td>0.0193</td>
</tr>
<tr>
<td>6</td>
<td>0.95</td>
<td>0.0207</td>
</tr>
<tr>
<td>7</td>
<td>0.95</td>
<td>0.0213</td>
</tr>
<tr>
<td>8</td>
<td>0.91</td>
<td>0.0151</td>
</tr>
</tbody>
</table>
10.5 Discussion

In their paper, T.A. Dinham-Peren and I.W. Dand conclude that "the matter of reducing in-service data to determine the actual ship performance is not trivial. The potential benefits of being able to do this are large, giving the prospect of being able to determine when best to clean a vessel and what actual speed or power margins are required for a given level of in-service performance", see (Dinham-Peren and Dand, 2010). Their conclusion is relevant in this chapter.

Monitoring propeller performance by looking at the relative propeller efficiency is realizable. The ship owner is the most interested party in propeller performance. The ship owner wants to decrease costs with ship operations and ship maintenance. Both ship operations and ship maintenance are assisted by the relative propeller efficiency concept. This thesis represents the groundwork of such a project.

Relative propeller efficiency for propeller maintenance requires real-life validation. We have reasons to believe that propeller efficiency is a variable, similar to hull efficiency, due to the effects of damages and fouling. However, we do not know the time scale of this variable. Is the time scale in order of months? Is it in order of years? The answer to this question is conditioned by extensive measurement campaigns. The full scale measurements should extend over several years and they should capture at least one propeller maintenance. Such extensive data base of stationary states should provide enough information for a statistical model of the relative propeller efficiency.

10.6 Addendum: speed and shaft power

In Section 10.1 we mentioned the load power curve, a popular way to represent the ship performance. T.A. Dinham-Peren and I.W. Dand in their study, see (Dinham-Peren and Dand, 2010), use the same representation of ship performance to test their filtering method.

Figure 10.10 depicts the stationary data points represented along the dimensionless shaft power, $P$, and the dimensionless speed directions. The profile of the expected theoretical curve is noticeable. The large spread in the data point indicate that there are much more dimensions along which the data points should be represented. When data points are structured according to the approach described in Section 10.3 two dense clusters pop up. Predictably, the data points inside the clusters are normally distributed, as shown by the histograms on the sides of the plots. To obtain a load power curve more dimensions must be eliminated from the data points.
Statistical aspects of propeller efficiency at full scale

Figure 10.9: Shaft power versus speed-through-water. The stationary data points show a large spread.

Figure 10.10: Shaft power versus speed per mode. Along these two dimensions the stationary data points form normally distributed clusters, as seen in the histograms on the sides.
Chapter 11

Conclusions and recommendations

What is propeller efficiency at full scale? is the question which answer we quested in this thesis. In our quest we actually did more than trying to find the value of propeller efficiency at full scale. According to the generally accepted definition, propeller efficiency is an index calculated as the ratio of the power produced to the power consumed. In this thesis, we looked beyond this definition and wanted to see the practical use of this index. Propeller efficiency serves two parties: the propeller manufacturer who wants to know the absolute propeller efficiency at full scale and compare it with the absolute propeller efficiency stated in the design, and the ship operator who wants to get the best out of the propeller, in terms of efficiency. Knowledge on the absolute propeller efficiency helps the propeller manufacturer optimize the ship propulsion. For the ship operator, propeller maintenance and the ship operation are crucial aspects. In this thesis the concepts of reference and relative propeller efficiency are introduced in relation to operating conditions. The conclusion of the thesis is that the route to absolute propeller efficiency and the route to relative and reference propeller efficiency are different in content but equal in challenges. Absolute propeller efficiency is one value attached to the propeller, relative propeller efficiency is a random variable that is dictated by external and internal operational conditions that are not incorporated in the notion of absolute propeller efficiency. In the case of absolute propeller efficiency we suggest to apply in practice the wake fraction estimation method of base of harmonic control. In the case of relative propeller efficiency, we suggest the design of a statistical mathematical model to accommodate this concept. Extensive full scale measurements are precondition to relative propeller efficiency. For practical applications of full scale propeller efficiency we recommend a closer interaction between the ship operator and the industrial partners.

To estimate propeller efficiency at full scale, be it reference or relative, requires
full scale measurements and knowledge of the reality of ship operation. Reliable
data collection system and sensors are prerequisite to trustworthy measurements. Additionally, long term measurements provide the statistically reliable information for monitoring relative propeller efficiency. This is another important conclusion of this thesis. As a side conclusion we showed that the thrust and torque sensor that for the first time was applied in such a grand setup measurement system really provide the necessary measurement information to estimate reliably propeller efficiency. These conclusions were based on measurements carried out at two vessels: a ferry and a container vessel. They are completely different in operation conditions. In light of these conclusions, full scale measurements should be diversified in terms of type of vessels and operational conditions. In fact, we recommend that full scale measurements become common practice on board of vessels. There is room to improve the sensor and data collection system proposed in this thesis. In the context of ship performance, measurements of fuel consumption are essential. In the same context, we suggest that thrust and torque direct measurements become a standard, as they provide essential ship propulsion information.

The measurements system as proposed in this thesis results in signals that need to be processed. Part of the thesis is devoted to signal processing. In particular, we designed the signal decomposition method that decomposes the signal into trend, a harmonic, and noise component. This method is used to extract stationary states where the definition of stationary state is related to the amount of "energy" left in the harmonic and noise components. Processing is also related to reconciliation of signals that are correlated by physical relations. As a conclusion, we found that application of the signal reconciliation method to the navigation data, namely, speed-over-ground, position, and heading, results in reliable vessel speed and trajectory. We suggest the extension of signal reconciliation to ship propulsion data. With this step, parameter estimation becomes part of the signal reconciliation. In light of continuous full scale monitoring, the signal decomposition and signal reconciliation could be used to reduce the amount of data by parameterizing the signals.

In this thesis two mathematical models are introduced, the ship trajectory model relating speed-over-ground, position, heading, pitch, and roll, and the ship propulsion model relating physical quantities as thrust, torque, shaft rotational speed, speed-through-water, and advance speed. A conclusion of the thesis is that the ship propulsion model is able to simulate the longitudinal dynamics of a real ship, given a set of reliable model parameters.

Absolute propeller efficiency can be estimated if one of the parameters in the ship propulsion model, the wake fraction, is identified. The conclusion of the thesis is that without a priori information, such as, open water diagrams obtained from CFD simulation and model tests, the wake fraction cannot be determined reliably. Moreover, at full scale the model parameters are very sensitive to external conditions, such as wind and waves, indirectly influencing the value of the wake fraction. Thus, we conclude that absolute propeller efficiency as a label attached
to a manufactured propeller is a virtual concept. In agreement with literature we suggest to introduce the reference and the relative propeller efficiency. External conditions reflect in reference states and their statistics. We showed applicability of the concept of relative propeller efficiency but our conclusions are based on measurements for two ships only. To validate applicability we suggest measuring more ships for extensive periods of time such that a variety of operational conditions are captured.

The measurement system and the models render the possibility to estimate model parameters. In this thesis we show how parameters as thrust deduction, hull resistance, wake fraction, and certain propeller characteristics can be estimated by connecting model and measurements. For that we introduced three typical controls: stationary control, zero-thrust control, and harmonic control. Applying these controls lead to optimization problems where minimization of the cost function means estimation of parameters. The stationary control and zero-thrust control estimation methods are validated with the full scale measurements. The estimation method based on harmonic control is in a prototype phase. The method is verified in a simulated environment, but should be validated in practice.
Bibliography


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Summary

What is propeller efficiency at full scale? This question is asked equally by ship operators and by propeller and propulsion system manufacturers. The question reflects the need to measure propeller efficiency at full physical scale and during regular operation of the vessel. The question has a context: the ship operator wants to reduce the fuel consumption and forecast maintenance; and, the manufacturer wants to improve the design of the propeller and optimize the propulsion system.

In this thesis, we work towards an answer by designing methods to determine propeller efficiency at full scale. In accordance to the hydrodynamics community, we define propeller efficiency as the ratio of the power produced to the power consumed. The power produced and the power consumed are the product of thrust and advance velocity, and, of torque and angular velocity, respectively. Thus, propeller efficiency at full physical scale requires full scale measurements of thrust, torque, angular velocity, and advance velocity. Up-to-date, thrust and advance velocity are thought to be the bottlenecks in estimating propeller efficiency at full scale. Recently, a prototype thrust sensor has been developed that seems to have solved the problem of propeller thrust measurements. The remaining bottleneck, the measurement of advance velocity, is a main topic of this thesis. The advance velocity cannot be directly measured on board of a vessel with the current sensor technology. We replace the advance velocity by ship’s speed-through-water up to a factor. This factor is known as the wake fraction. Since the speed-through-water is measurable at full scale, we need to find the wake fraction. For that we apply mathematical modeling in combination with full scale measurements during rigorously defined conditions.

The method to determine propeller efficiency at full scale relies on five principles: mathematical models, design of measurements, estimation of parameters, statistical analysis of data, and identification of stationary states.

Propeller efficiency at full scale can only be defined when the ship is in a stationary state; also in the practice of propeller model tests and CFD simulations the propeller efficiency is determined under stationary conditions. First, we developed a concept of stationarity of a process; it means that the trend is close to zero, and small oscillations about the trend are allowed. Second, we developed a method that automatically extracts the stationary excerpts from the full scale
measurements.

The mathematical models that we elaborate on in this thesis are inspired by the classical mathematical models for guidance and control of vessels. In this mathematical environment, relationships between advance velocity, thrust, torque, speed-through-water, and angular velocity are defined on basis of physical laws and empirically established relationships. The mathematical models introduce parameters that are unknown at full physical scale. One of these parameters is the wake fraction. For realistic values of parameters the model mimics the ship behavior. From the measurements we want to obtain the model parameters by estimation. For that, we construct a regression kind of method that minimizes the difference between the measurements data and the output of the mathematical model when we feed it by full scale measurements.

Our measurement design includes the functional specification for a data collection system, a list of signals with details on sampling frequency and resolution, a design of propulsion tests, and processing of measurements. The result of that processing is the selection of stationary states and their characteristics. A central piece of the data collection system is the propeller thrust sensor. In fact, this thesis would not have been written as it is without the availability of the thrust sensor. The speed-through-water measured by the speed log is equally important. It is generally known that speed logs are not reliable. Thus, we had to focus on getting a reliable speed-through-water. For this purpose, we designed an algorithm that reconciles the speed-through-water with other measured navigation signals.

The combination of mathematical models and propulsion test measurements yields the model parameters. The value of each model parameter is calculated by a mathematical algorithm that needs measurements of stationary or dynamic states as input. The mathematical models structure these algorithms. It is at this stage that the external factors come into play. External factors, for instance, wind, waves, and sea currents, are not explicitly modeled so that their influence is noticeable in the model parameters. For instance, the parameters of the ship resistance force account also for the wind-induced resistance. Consequently, the estimated propeller efficiency at full scale is influenced by the external factors. If our findings were put into practice, the best would be to create a ship-tailored lookup table of reference model parameters and reference propeller efficiency as function of external conditions that would represent the reference states. At pre-defined time intervals, the reference state would be updated by new propulsion tests. During ship operations the reference model parameters would be used to retrieve the propeller efficiency relative to the selected reference states.

At sea, vessels have typical operational profiles that are concentrated at one or two predefined velocities. As a result, the relative propeller efficiency forms clusters of data points. Propeller efficiency is function of state variables connected to ship propulsion and external factors. Under the influence of the state variables the data clusters are more or less densely packed. We use statistical tools to analyze these data clusters. To evaluate the data clusters we introduce the concept of
precision that, in our definition, is the normalized standard deviation of the data clusters. Filtering the data clusters based on propulsion state variables increase the precision, i.e., the clusters become more densely packed. The clusters of data points indicate that the external factors account for most of the variations in propeller efficiency. We expect that this is the de facto situation of propeller efficiency at sea.

Thus, *what is propeller efficiency at full scale?* Propeller efficiency is an indicator with values between 0 and 1. This indicator is measured by a sensor: a mathematical algorithm structured by a mathematical model and fed with full scale measurements. Under specified conditions, this sensor measures the propeller efficiency with high precision. The ship owners and the propeller and propulsion system manufacturers may use the full scale propeller efficiency to compare different propeller designs at similar conditions, to track long-term changes in propeller efficiency for the maintenance forecast, to optimize fuel consumption as function of external conditions, or to plan the optimal route of the vessel.

Tiberiu Muntean
Eindhoven, December 2011
Samenvatting

Wat is schroefefficiëntie op volle schaal? Deze vraag wordt evenzeer door reders gesteld als wel door fabrikanten van schroef- en aandrijfsystemen. De vraag weerspiegelt de noodzaak om de schroefefficiëntie op volle fysische schaal te meten tijdens de reguliere operatie van het schip. De vraag wordt gesteld binnen een context: de reder wil de kosten voor het brandstofverbruik en het verwachte onderhoud minimaliseren en de fabrikant wil het ontwerp van de schroef verbeteren en het aandrijfsysteem optimaliseren.

In dit proefontwerp hebben we aan een antwoord op deze vraag gewerkt door het ontwerpen van methoden om de efficiëntie van de schroef op volle schaal te bepalen. In overeenstemming met de gangbare definitie in de hydrodynamische gemeenschap, definieren we de schroefefficiëntie als de verhouding van de geproduceerde energie en de verbruikte energie. De geproduceerde energie en de verbruikte energie zijn, respectievelijk, het product van de stuwkracht en stuwsnelheid, en van het koppel en de hoeksnelheid. Dus, schroefefficiëntie op de volle fysische schaal vereist volle schaalmetingen van stuwkracht, koppel, hoeksnelheid, en stuwsnelheid. Tot voor kort, dacht men dat stuwkracht en stuwsnelheid de werkelijke knelpunten waren bij de bepaling van de efficiëntie van schroeven op volle schaal. Recent is een prototype van een stuwkrachtsensor ontwikkeld die het probleem van de stuwkrachtmetingen lijkt te hebben opgelost. Het resterende knelpunt, het meten van stuwsnelheid, is een belangrijk onderwerp van dit proefschrift. De stuwsnelheid kan niet direct aan boord van een schip met de huidige sensor technologie worden gemeten. We veronderstellen dat de stuwsnelheid evenredig is met de snelheid-door-het-water. De evenredigheidsfactor staat bekend als het volgstroomgetal. Omdat de snelheid-door-het-water meetbaar is op volle schaal, rest ons ‘slechts” het volgstroomgetal te bepalen. Daartoe passen we wiskundige modellen toe in combinatie met volle schaalmetingen onder helder omschreven randvoorwaarden.

De voorgestelde methode om de efficiëntie van schroeven op de volle schaal te bepalen is gebaseerd op vijf principes: ontwerp van wiskundige modellen, ontwerp van de metingen, identificatie van parameters, statistische analyse van data, en identificatie van stationaire toestanden.

Schroefefficiëntie op de volle schaal kan alleen worden gedefinieerd wanneer het schip zich in een zekere stationaire toestand bevindt, ook in de huidige praktijk.
van schroefmodelproeven en CFD-simulaties wordt de schroef efficiëntie bepaald onder stationaire randvoorwaarden. We hebben, op de eerste plaats, een concept
van stationariteit van een proces ontwikkeld; in dit concept betekent stationariteit
dat de trend dicht bij nul is en dat kleine oscillaties rondom deze trend zijn toeges-
taan. Op de tweede plaats hebben we een methode ontwikkeld die automatisch de
stationaire fragmenten uit de volle schaalmetingen detecteert.

De wiskundige modellen die we in dit proefschrift beschrijven zijn geïnspireerd
op de klassieke wiskundige modellen voor geleiding en besturing van schepen. In
deze wiskundige omgeving, zijn relaties tussen de stuw snelheid, stuw kracht, kop-
pel, snelheid-door-het-water en de hoeksnelheid gedefinieerd op basis van fysische
wetten en empirisch vastgestelde relaties. De wiskundige modellen introduceren
parameters die onbekend zijn op volle fysische schaal. Een van deze parameters
is het volgstroomgetal. Voor realistische waarden van de parameters simuleert
het model het gedrag van het schip. Uit de metingen worden de model param-
eters verkregen door schatting. Daarvoor hebben we een regressieachtige meth-
ode ontwikkeld waarin het verschil tussen de meetgegevens en de uitvoer van het
wiskundig model minimaliseert als we de methode toepassen op de meetdata.

Het voorgestelde meetprotocol omvat de functionele specificatie van het meet-
systeem voor het verzamelen van data, een lijst van signalen met details over de
samplefrequentie en de resolutie, een ontwerp van voortstuwingstests en de verwer-
king van de metingen. Het resultaat van de dataverwerking is de selectie van
stationaire toestanden en hun kenmerken. Een centraal onderdeel van het sys-
teem van dataverzameling is de schroefstuwkrachtsensor. In feite, zou dit proef-
script niet geschreven zijn, zoals het geschreven, zonder de beschikbaarheid van
de stuwkracht sensor. De snelheid-door-het-water die door de snelheiddopplerlog
gemeten wordt is van even groot belangrijk. Het is algemeen bekend dat deze snel-
heidimeters niet betrouwbaar zijn. Dus besteedt dit proefschrift aandacht aan het
verkrijgen van betrouwbare snelheid-door-het-water-data. Hiertoe is een algoritme
ontwikkeld dat de snelheid-door-het-water met andere gemeten navigatiesignalen
in overeenstemming brengt.

De combinatie van wiskundige modellen en voortstuwingstestmetingen levert
de modelparameters. Elke modelparameter wordt berekend met behulp van een
wiskundig algoritme dat metingen van stationaire of dynamische toestanden als
invoer nodig heeft. De wiskundige modellen bepalen de structuur van deze algo-
ritmen. In deze fase gaan externe factoren, zoals wind, golven en zeestroming, een
rol spelen. Deze factoren zijn niet zo expliciet gemodelleerd dat hun invloed op
het model parameters is merkbaar. Bijvoorbeeld, de parameters die de weerstand
beschrijven die het schip ondervindt van het water zijn ook gerelateerd aan de
weerstand die het schip ondervindt van de wind. Bijgevolg is de geschatte schroef-
efficiëntie beïnvloed door externe factoren. Als de bevindingen van dit proefschrift
in praktijk zouden worden gebracht, zou het het beste zijn om een opzoektable-
op-maat te ontwikkelen van de referentimodelparameters en de referentieschroe-
fefficiëntie als functie van de externe omstandigheden die de referentietoestanden
representeren. Op vooraf bepaalde tijdsintervallen, zou de referentietoestand worden bijgewerkt door nieuwe aandrijvingstesten uit te voeren. Tijdens de operatie van het schip zou van de referentiemodelparameters gebruikt kunnen worden om de schroefefficiëntie ten opzichte van de geselecteerde referentietoestanden te bepalen.

Op zee hebben schepen typische operationele profielen die op één of twee vooraf gedefinieerde snelheden zijn geconcentreerd. Als gevolg hiervan vormen de relatieve schroefefficiëntiedata clusters. Schroefefficiëntie is een functie van de toestandsvariabelen verbonden met de schipvoortstuwing en externe factoren. Onder invloed van de toestandsvariabelen zijn de dataclusters meer of minder dicht op elkaar gepakt. Met statistische technieken worden deze dataclusters geanalyseerd. Voor evaluatie van de clusters wordt een concept van de precisie gedefinieerd dat de genormaliseerde standaarddeviatie van de data clusters beoogd te zijn. Het filteren van de dataclusters op basis van voortstuwingstoestandsvariabelen vergroot de precisie, dat wil zeggen, de clusters liggen dichter bij elkaar. De clusters van de datapunten geven aan dat voornamelijk externe factoren verantwoordelijk zijn voor de geobserveerde variaties in de schroefefficiëntie. Het is de verwachting dat dit de feitelijke situatie van de schroefefficiëntie op zee zeer wel weerspiegelt.

Dus, wat is schroefefficiëntie op volle schaal? Schroefefficiëntie is een indicator met waarden tussen 0 en 1. Deze indicator wordt door een sensor gemeten: een wiskundig algoritme gestructureerd door een wiskundig model en gevoed met metingen. Onder bepaalde voorwaarden meet deze sensor de efficiëntie met hoge precisie. Reders en fabrikanten van schroef- en aandrijfsystems kunnen gebruik maken van de sensor om verschillende schroefontwerpen onder vergelijkbare omstandigheden te vergelijken, om langzame veranderingen in de schroefefficiëntie voor onderhoudsprognose op te sporen, om het brandstoffebruik als functie van de externe omstandigheden te optimaliseren, of om de optimale route van het schip plannen.

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