Risk management at the interface of operations and finance

Risk Management at the Interface of Operations and Finance

Taimaz Soltani

Printed and cover design by Proefschriftmaken.nl || Uitgeverij BOXPress

This thesis is number D203 of the thesis series of Beta Research School for Operations Management and Logistics. Beta Research School is a joint effort of the School of Industrial Engineering and the Department of Mathematics and Computer Science at Eindhoven University of Technology, and Center for Production, Logistics and Operations Management at the University of Twente.

A calatogue record is available from the Eindhoven University of Technology Library.

ISBN: 978-94-6295-586-8

Risk Management at the Interface of Operations and Finance

PROEFSCHRIFT

ter verkrijging van de graad van doctor aan de Technische Universiteit Eindhoven, op gezag van de rector magnificus prof.dr.ir. F.P.T. Baaijens, voor een commissie aangewezen door het College voor Promoties, in het openbaar te verdedigen op dinsdag 24 januari 2017 om 16:00 uur

door

Taimaz Soltani

geboren te Zanjan, Iran

Dit proefschrift is goedgekeurd door de promotoren en de samenstelling van de promotiecommissie is als volgt:

voorzitter:	prof.dr. I.E.J. Heynderickx
1e promotor:	prof.dr.ir. J.C. Fransoo
copromotoren:	dr. A. Chockalingam
	dr. S.S. Dabadghao
leden:	prof.dr. C.Y. Lee (Hong Kong University of Science and Technology)
	prof.dr.ir. O.J. Boxma
	prof.dr. A.G. de Kok
	dr. H. Feng (Sun Yat-Sen University)

Het onderzoek dat in dit proefschrift wordt beschreven is uitgevoerd in overeenstemming met de TU/e Gedragscode Wetenschapsbeoefening.

Acknowledgements

I would like to express my deepest gratitude to my advisors, Prof. Jan Fransoo, Dr. Arun Chockalingam, and Dr. Shaunak Dabadghao, for their encouragement, guidance, inspiration, and immense knowledge. In addition to having learned a tremendous amount, I enjoyed working with them and took away a very positive experience. I could not have imagined having better advisors for my PhD studies.

Jan, I still do not know how you handle your heavy workload; I am amazed at how quickly you were always able to respond to my inquiries. Your broad view, valuable feedback, and constructive criticism substantially improved the quality of my work. I am very grateful that you were my PhD advisor, and I will always hold you in that regard even though the dissertation is complete.

Arun, I was very lucky to have the chance to be supervised by you. You are a great mentor and friend. You were with me with every step of the process and the knowledge I learned from you was invaluable. I believe you are true scientist in every sense of the word and I thank you for imparting to me how to be one. No words can truly express how grateful I am to have had you as my supervisor.

Shaunak, I believe your presence was a gift; when Arun and Jan were not here you took care of my supervision which has resulted in chapter 4 of the thesis. I still remember our three hour discussions about ”update conditions”. I appreciate the time and effort you spent reading my manuscripts and providing recommendations for its improvement.

During my PhD I had the great honor to work with Prof. Chung-Yee Lee who supervised my work during my stay at Hong Kong University of Science and Technology. Thank you for your great hospitality, guidance, and for sharing your knowledge with me. I am beholden to you, as everything that I know about ocean transportation I have learned it from you.

I am grateful to Prof. Onno Boxma, Prof. Ton de Kok, and Dr. Haolin Feng for being a part of my thesis committee. Thank you for your detailed comments and suggestions which have substantially improved the quality of this thesis.

I am grateful to all my current and former colleagues at the OPAC department whom

I enjoyed working with the past four years. A special thanks to my colleagues from the finance lab, Kasper, Matthew, Hande, Fehmi, Boray for insightful discussions concerning our field. I would also like to thank Slavic, Peng, Qianru, Joni, Baoxiang, Stefano, Chiel, Josue, Maxi, Anna, Engin, Zumbul, Joachim, Loe, Claudine, Jose, Jolanda, Christel for all the discussions and help.

I would like to thank all my friends in Eindhoven, which made my life here great. Thank you, Yousef, Shaya, Taher, Nooshin, Fardin, Maryam, Haji, Behdad, Bahareh, Mohsen, Negar, for all the good times spent together.

My special thanks go to the military service in Iran. If there was not such an obligatory program, I might have never continued my studies after high school.

I want to thank my best friend Ehsan, for always being able to count on him, no matter what. Ehsan, I am fortunate to have you as my friend. I also want to thank my friend Mahsa, who helped me whenever I needed it.

Most importantly, none would have been possible without the love and patience of my family. I would like to thank my parents in law and brother in law for believing in me and making every second of my life more pleasant. Maman Foozi, Baba, and Dash Fardad you have always been a constant source of love, concern, and support. I would like to express my deepest gratitude to my parents. Mom and dad, this thesis is our common achievement due to your constant support and for cheering me up in difficult moments. Without your unconditional love I would have given up many times when I felt I had failed. Your act of selflessness to provide the best quality of life throughout my upbringing has been the best education unmatched by any other in my life. Thanks for lifting me up, when I was down. Thank you for being my parents. I am also very grateful to my little brother who always stands beside me. Thank you Araz for being such a great brother. Finally, my beautiful wife Farnaz, your love, understanding and moral support were vital during my PhD. I could always rely on your love and encouragement. You always helped, supported, and you always knew how to cheer me up. Thank you for always being there when I needed you most. Thank you for sticking by my side, through good and bad. Thank you for making me the happiest man on the planet. You mean the world to me.

Taimaz Soltani
Eindhoven, November 2016

Contents

Chapter 1

Introduction

Operations management is inextricably linked with finance. Any operational activity in a firm affects the firms' financial position. The decision when to order materials, how much material to procure, which supplier to buy from, and how much final product to produce influences the financial position of the firm one way or the other. Consider, for example, a bicycle producer in the Netherlands. If the operations department of the firm decides to produce sufficient bicycles to meet demand, the financial department needs to obtain financing to procure the raw material and facilitate this production. On the other hand, if the financial department has difficulty raising the necessary finance, the operations department needs to scale back its production plans. The financial department will have more difficulties to finance operational plans when the raw material or the service needed is a volatile commodity. Consider, for example, an oil or electricity producer; their main inputs are oil and gas whose prices fluctuate from second to second. Such volatility substantially increases the financial risks faced by financial departments.

Modigliani & Miller (1958) show that in a perfect world, decisions made by operations and finance departments do not affect each other. However, the real world has many imperfections such as transaction costs, taxes, and informational asymmetry. Consequently, decisions made by the finance and operations departments of the firm will substantially affect each other. Under such circumstances, it is imperative for firms to make joint optimal decisions on both operations and financing to remain competitive, to protect themselves from adverse economic events, and to be able to take advantage of positive economic events. The interface of operations and finance is the domain where these optimal joint decisions are developed. This domain focuses on modelling the interaction between the financial and operations departments of firms with the aim of better understanding these interactions so that joint optimal decisions on both operations and finance can be made by firms at both the strategic and operational levels.

Research at the interface of operations and finance domain is not just limited to the investigation of how joint operational and financial decisions can be made. It also encompasses the study of risk management at the interface of operations and finance. Many firms have already been able to generate value by hedging against financial risks, for instance, by using financial derivatives such as options. According to a survey of large US non-financial firms (Bodnar et al. 1995), approximately 40% of firms routinely purchase options or futures contracts in order to hedge price risks. Firms can hedge financial risks arising from the procurement and production of commodities and the sale of the processed products. Even in the absence of specific market hedges, financial derivatives may be useful in making operational decisions. The current thesis is positioned exactly in this aspect of operations and finance interface. Strictly speaking, this thesis encompasses the study of the hedging instruments and financial techniques to find optimal hedging policies for the firms facing financial and operational risks. In the next section, we briefly introduce these instruments.

1.1 A Brief Introduction to Financial Hedging Tools

Financial hedging tools were developed in the domain of mathematical finance. The first theories in mathematical finance date back to a PhD thesis in 1900 in which Louis Bachelier modelled the stochastic process that is now called Brownian motion. Subsequently Einstein (1905)'s finding in Brownian motion and Wiener's mathematical model opened other scientific doors to stochastic models. In mathematics, the Wiener process is a continuous-time stochastic process named in honour of Norbert Wiener. It is often called standard Brownian motion, after Robert Brown as well. A few decades after Einstein (1905), Samuelson (1965) presented his reasons on why commodity price fluctuations are stochastic; this was the gate to the valuation of financial derivatives. In finance, a derivative is a contract that derives its value from the performance of an underlying entity. This underlying entity can be an asset, index, or interest rate, and is often simply called the underlying. In sections 1.1.1 and 1.1.2 we briefly introduce some financial derivatives such as futures, forward and option contracts.

1.1.1 Future and Forward Contracts

The simplest financial derivatives are futures and forward contracts. A futures contract enables the holder to buy or sell a particular quantity of a commodity over a certain time frame for a particular price. Futures contracts are negotiated on futures exchanges, which act as a marketplace for buyers and sellers. The buyer of a contract is said to be the long position holder, and the selling party is said to be the short position holder. The contract involves both parties lodging a margin of the value of the contract with a mutually trusted third party as both parties risk their counterparty walking away if the price goes against them. A forward contract is a customized contract between two parties to buy or sell an asset at a specified price on a future date. A forward contract is often used for hedging. Unlike standard futures contracts, a forward contract can be customized to any commodity, amount and delivery date. Forward contracts do not trade on a centralized exchange and are therefore regarded as over-the-counter (OTC) instruments. While their OTC nature makes it easier to customize terms, the lack of a centralized clearing-house also increases the degree of default risk.

1.1.2 Option Contracts

Options are more sophisticated financial derivatives, the valuation of which has been developed in the domain of mathematical finance. Financial options are contracts which give the right but not the obligation to their owners to buy or to sell a risky asset at a predetermined price at a specified time in the future (maturity date). Call options give the right to buy, while put options give the right to sell. European options can only be exercised at maturity, while American options can be exercised at or before maturity. The primary difference between options and futures lies in the obligation placed on the contract parties. In a futures contract, both participants are obliged to buy (or sell) the underlying asset at the specified price on the settlement day, while the option contract holder has the right but not the obligation to buy (or sell) the underlying asset. This right comes at a price in the form of a premium. As a result, in a futures contract both buyers and sellers of futures contracts face the same amount of risk, while the option buyer's risk is limited to the premium paid but his potential profit is unlimited. Pricing the option contracts was an open problem for many years, until 1973 when Black-Scholes and Merton separately worked on the option pricing and combined Samuelson theory, Brownian motion and Ito's lemma to price European options. They presented a way to convert the stochastic differential equation to a partial differential equation which is now called Black-Scholes equation. They were awarded the Nobel Prize for their work in 1997. American options price do not have any closed form solution so far; however, there are several numerical methods which can estimate the price of an American option.

1.2 Stochastic Control and Financial Derivatives

Unlike European options -which are dependent on the underlying price at maturity-American options depend on the underlying price during the whole period before and at maturity. Such financial instruments are called path-dependent and we might need stochastic control techniques to value them. Stochastic control problems in general are problems where a controller attempts to control a system governed by an evolving stochastic process with the aim of optimizing an objective function. Due to the costs involved in controlling and making changes to the underlying system, the controller needs an optimal control policy that will dictate the actions necessary to optimize the controller's objective function. The cost structure determines the type of stochastic control problem facing the controller. American options are examples of stochastic control problems where the option holder needs an optimal control policy to give the optimal exercise time.

There are several ways to tackle stochastic control problems. One is the dynamic programming principle. Using dynamic programming arguments, stochastic control problems typically reduce to solving a system of differential equations, either ordinary or partial. The domains over which these systems of equations are to be solved are unknown a priori, therefore resulting in free-boundary problems. Closed-form solutions rarely exist for such problems. Numerical solutions are then needed to determine the optimal value functions and policies. One question arising here is whether these financial instruments create value for firms, and if so, despite their tough mathematical theories involved whether we can extract simple and useful managerial strategies out of them. In the next section we discuss these questions.

1.3 Application of Financial Instruments at the Interface of Operations and Finance

Options have two sorts of applications at the interface of operations and finance. One is called real option which applies option valuation techniques to capital budgeting decisions. A real option itself, is the right but not the obligation to undertake certain business initiatives, such as deferring, abandoning, expanding, staging, or contracting a capital investment project. For example, the opportunity to invest in the expansion of a firm's factory, or alternatively to sell the factory, is a real call or put option, respectively. Real options are generally distinguished from conventional financial options in that they are not typically traded as securities, and do not usually involve decisions on an underlying asset that is traded as a financial security. A further distinction is that option holders, i.e. management, can directly influence the value of the option's underlying project; whereas this is not a consideration as regards the underlying security of a financial option. Unlike financial options, management also has to create or discover real options, and such creation and discovery process comprises an entrepreneurial or business task. Real options are most valuable when uncertainty is high; management has significant flexibility to change the course of the project in a favourable direction and is willing to exercise the options. In the literature, several papers exist that have applied real options approach to different operational circumstances. Deferment options have been studied in Ingersoll Jr & Ross (1992), Trigeorgis (1996), and Benaroch & Kauffman (2000). Stop-resume options for mining projects have been studied in Brennan & Schwartz (1985), Trigeorgis & Mason (1987), and Pindyck & Rotemberg (1988). Exploration options have been studied in Clemons & Weber (1990) and Amram & Kulatilaka (1999). Outsourcing options have been studied in Richmond & Seidman (1991), Nembhard et al. (2003), and Alvarez & Stenbacka (2007). Options to alter operating states have been studied in Copeland & Keenan (1998), Trigeorgis & Mason (1987), and Amram & Kulatilaka (1999). Options to abandon projects have been studied in Copeland & Keenan (1998) and Trigeorgis & Mason (1987).

Another application of options at the interface of operations and finance is using financial options to hedge against financial risks. The existing literature demonstrates that procurement policies utilizing derivatives such as forwards and options can significantly reduce the price risk inherent in a firm's operations. Secomandi & Kekre (2014) show that firms facing the newsvendor problem with price and demand uncertainty should procure supply partially in the forward market rather than entirely in the spot market. Like forwards and futures, options can also help firms to secure commitments for supplies and avoid over-reliance on the volatile spot markets. Unlike forward contracts which lock the buyer's position, options provide the buyer with greater flexibility by postponing procurement until price and demand uncertainties are resolved. Barnes-Schuster et al. (2002) demonstrate that option contracting between a buyer and supplier provides additional flexibility to the buyer in responding to uncertain demand. When a firm negotiates a procurement option with its supplier, it pays a premium and reserves the right to procure the commodity in the future at a pre-specified price. Barnes-Schuster et al. (2002) investigate the role of options in a buyer-supplier system in a two period setting. They showed how options provide flexibility for a buyer to respond to market changes in the second period. Fu et al. (2010) consider the value of portfolio procurement in a supply chain where a buyer can either procure parts for future demand from sellers using fixed price contracts or tap into the market for spot purchases. Kleindorfer & Wu (2003) survey the underlying theory and practice in the use of options in support of emerging business-to-business (B2B) markets. Spinler & Huchzermeier (2006) develop an analytical framework to value options on capacity for production of non-storable goods or dated services. Dong & Liu (2007) provide a potential explanation for the prevalence of bilateral supply contracts like forwards in preference to spot market deals even when spot markets are liquid and without delivery lag. Furthermore, Li et al. (2009) investigate the role of forward commitments and option contracts between a seller and a buyer in the presence of asymmetric information and uncertain demand, and identify cases where combinations of the forwards and options are dominant. Martínez-de Albéniz & Simchi-Levi (2005) develop a general framework for supply contracts in which portfolios of contracts can be analysed and optimized. They derive conditions to determine when an option is relatively attractive compared to other options or the spot market. Their experiments indicate that portfolio contracts not only increase the manufacturer's expected profit, but can also reduce its financial risk. In finance theory, a firm can create value from using financial derivatives if at least one of these conditions holds:

1. There are capital market frictions.
2. Firm managers are not risk neutral.
3. There exist arbitrage opportunities.

Without these conditions, a risk neutral firm cannot create value by trading financial derivatives under a perfect capital market (Modigliani & Miller 1958). Even though the Modigliani-Miller assumptions are based on non-real market conditions, they opened the gate for further research and new theories regarding issues stressed by those theorems. Many papers consider one of the conditions and show value creation. Some papers explore the value of futures or options contracts under a true measure while maximizing an expected concave utility function for a risk averse firm. Moschini & Lapan (1992) show that the use of options not only raises expected utility by reducing income risk, but in general also affects the firm's input decisions. Sakong et al. (1993) maximize the expected utility problem for producers with both price and production uncertainty who have access to both futures and options markets. In a similar setting, Hirshleifer (1988) shows that, when demand is inelastic, futures trading can act as a substitute for vertical integration to diversify risk for a commodity processing firm and the commodity grower, because the risk positions of growers are complementary to those of processors. Gaur & Seshadri (2005) consider a risk-averse firm that uses financial information to set optimal inventory levels and hedges demand risk with financial hedging. Anderson & Danthine (1983) explore the role of futures contracts in a multi-period setting and conclude that firms under-hedge in the futures market as a consequence of demand uncertainty. All these papers show the different applications of financial instruments in operational decisions and they all have one important key factor: using the flexibility inherent in financial instruments in order to hedge against the financial and operational risks in different settings.

In this thesis, Chapters 2 and 3 concern options as hedging tools, while chapters 4 and 5 develop models where options are used as real options.

1.4 Research Question

In the previous sections we have discussed the importance of research in the domain of operations and finance interface. We also briefly introduced some financial instruments and their role in the risk management of this domain. Now we briefly discuss the research objectives of this thesis; however at this stage of the thesis, we do not position our research objectives relative to existing literature. This is done in more detail in each chapter.

The current thesis develops a framework based on options and stochastic control techniques that allow firms to take advantage of the flexibility inherent in their decision-making processes. We consider firms that manufacture products and have the flexibility to take different operational decisions such as switching between suppliers, switching between production locations, enter or exit the market. Flexibility provides some degrees of freedom for the management to hedge the company against the future uncertainties and/or increase profit. These degrees of freedom of course provide some value for the company and the main objective of the current thesis is to value this flexibility. The flexibility valuation is important because by determining it, we obtain the optimal control strategy with which to achieve this value.

The purpose of this PhD thesis is to develop a framework for modelling and evaluating problems at the interface of operations management and finance where risk management is a major consideration. Expanding upon these approaches, we adopt option valuing and stochastic control techniques in this thesis to define and value the flexibility inherent in managerial controls. While we discuss different areas in the various chapters of this thesis, the principal question remains unchanged in the whole thesis:

What is the maximum value generated by using options for risk hedging and how can firms attain this value?

Due to the breadth of the stated research question, we will focus on four representative problems to tackle this question. We discus each problem in a separate chapter and show that stochastic control and options valuation, despite their tough mathematical background, can yield simple strategies to managers. In all four research topics we will use theoretical tools in mathematics but eventually we will offer policies that are easy to understand by managers and can be readily implemented in their firms. These efficient control policies reduce the risk and increase the firm's profit.

1.5 Contribution of the Thesis

The current thesis contributes to both methodology and application domains. From an application perspective, the thesis contributes to the literature on sourcing, offshoring and ocean freight transportation. From a methodological perspective, the thesis contributes to the literature on options and stochastic control. In this section we briefly discuss the contributions of each chapter separately.

In Chapter 2 we consider a commodity processor that procures a commodity and transports it via ocean freight to its processing plant where the commodity is converted to a final product to meet customer demand. We develop models to determine the firm's optimal hedging policy. The models allow for three sources of uncertainty: demand, commodity spot price and freight rate. Our work makes two contributions to the existing literature on commodity procurement. First, we develop a tractable and parsimonious model that combines both procurement and transportation of the commodity and integrates demand uncertainty for the final product, uncertainty in the commodity price, and uncertainty in the freight rates. Our second contribution is the demonstration of value creation through hedging for a risk-neutral firm, *even* in the absence of market frictions.

Motivated by the increasing role that freight transportation plays in dual sourcing and a firm's tendency to use financial contracts in order to hedge against price risks, Chapter 3 derives an optimal hedging policy to minimize total procurement and transportation cost. We develop a model that applies to tanker ships which cannot be partially chartered, and a model that applies to the chartering of container ships which can be partially hired. Our work makes three contributions. The first is the development of a dual-sourcing model that integrates a firm's optimal hedging decision explicitly taking into account the transportation cost and the constraint that only an integer number of ships can be hired. Second, we extend the literature on ocean container transport. Fransoo & Lee (2013) highlight four issues that need to be addressed in the domain of ocean container transport. One of these issues relates to contracting, pricing and risk management along the container supply chain. This chapter directly addresses this point. Third, we numerically demonstrate with the developed model that (1) transportation costs need to be explicitly accounted for in models on dual-sourcing strategies, (2) dual sourcing is indeed more effective than single sourcing by limiting the firm's price risk exposure and simultaneously offering flexibility to take advantage of significant price differences in the commodity, and (3) that chartering large ships, while more expensive per ship, lowers the total expected cost for the firm.

The primary contribution of chapter 4 is the development of a stochastic control-based methodology to tackle the optimal-switching problem. Specifically, we extend the Moving-Boundary method to tackle such problems. The Moving-Boundary method has been successfully applied to optimal-stopping problems. Optimal-switching problems can be thought of as sequences of optimal-stopping problems and possess complicating features, making an extension of the Moving-Boundary method to tackle such problems non-trivial. The method is then applied to problems in the sourcing and energy domains.

In Chapter 5 we tackle the offshoring problem. The literature on operational-decision making for offshoring decisions is limited. We thus seek to contribute to the literature by considering a dynamic model for making offshoring decisions that is grounded in stochastic control theory allowing us to derive optimal control policies that are easy to obtain, understand and implement. First we consider the offshoring problem as a optimal switching problem where the firm can choose the production site: domestic or offshore. The firm can switch between the locations at any point in time. The aim of this chapter is to provide an optimal switching policy to hedge the firm against uncertain profit. Then we consider this problem in a more generalized setting in which we assume the firm can offshore-backshore any proportion of its production and model it based on stochastic impulse control.

1.6 Outline of the Thesis

The thesis is organised as follows. In Chapter 2 we consider a commodity processor facing financial and operational risks and develop a hedging policy based on options. Chapter 3 extends the developed model to dual-sourcing and container ships. Chapters 2 and 3 are intended to be accessible to the audience interested in ocean transportation, freight options, and procurement policies. In Chapter 4 we develop a numerical method to solve optimal switching problems and apply the method in tolling agreements. Chapter 4 contains more methodological results and might be interesting for the readers interested in free boundary problems, switching options, and energy domain. Chapter 5 develops a operational model for offshoring strategy based on switching options and stochastic impulse control. This chapter might be interesting for the readers interested in the offshoring domain. The chapters have been set up such that they can be read independently. Finally Chapter 6 concludes with a summary and discussion.

Chapter 2

Transporting Commodities: Hedging against Price, Demand and Freight Rate Risk with Options

In this chapter we consider a commodity processor that procures a commodity and transports it via ocean freight to its processing plant. The firm faces three source of risks, two financial risks: commodity spot price and freight rate, and one operational risk: demand. Our aim is to show how the European style options can be used as hedging tools against the operational and financial risks. We show that partially procuring the commodity and its freight through option contracts, rather than entirely on the volatile spot market creates value, even for a risk-neutral firm.

2.1 Introduction

Procuring raw materials and commodities at offshore locations and transporting them to their domestic production facilities has become a common business practice among corporations. Firms procure commodities at offshore locations for a number of reasons. For example, a commodity might only be available at a certain location, such as cocoa and oil. Alternatively, there might be price differences which might prove advantageous to the firm. After procurement, the commodity is frequently transported via ocean freight. Fransoo & Lee (2013) state that nearly all intercontinental transport of goods takes place by sea. Increased globalization and the increased demand for ocean-based transportation of commodities has resulted in ocean freight becoming a highly volatile commodity in its own right (Nomikos et al. 2013). An additional cause for ocean freight volatility is the fact that ships have become larger and hence more expensive, and that the lead time to build a ship takes very long. The volatility inherent in the availability and cost of freight transportation combined with existing demand and price uncertainty for the commodity itself may have a significant impact on a firm's operations and cash flows. Motivated by the increasing role that freight transportation plays in a firm's supply chain and its interactions with other existing uncertainties in the firm's operations, we study the value created by hedging these uncertainties in the context of a firm that operates in such an environment.

As we mentioned in Chapter 2, according to a survey of large US non-financial firms (Bodnar et al. 1995), approximately 40% of firms routinely purchase options or futures contracts in order to hedge price risks. The existing literature demonstrates that procurement policies utilizing derivatives such as forwards and options can significantly reduce the price risk inherent in a firm's operations. Secomandi & Kekre (2014) show that firms facing the newsvendor problem with price and demand uncertainty should procure supply partially in the forward market rather than entirely in the spot market. Like forwards and futures, options can also help firms to secure commitments for supplies and avoid over-reliance on the volatile spot markets. Unlike forward contracts which lock the buyer's position, options provide the buyer with greater flexibility by postponing procurement until price and demand uncertainties are resolved. Barnes-Schuster et al. (2002) demonstrate that option contracting between a buyer and supplier provides additional flexibility to the buyer in responding to uncertain demand. When a firm negotiates a procurement option with its supplier, it pays a premium and reserves the right to procure the commodity in the future at a pre-specified price. In practice, the operations department decides the number of options to obtain from the supplier based solely on demand and price uncertainty of the commodity to be procured. In doing so, however, the volatility inherent in the cost of transporting the commodity is ignored by the operations department and can result in increased costs to the firm. We show that value is created for the firm (through cost reduction) either if the operations department and transportation department jointly decide on an optimal hedging policy, or if each department selects its own optimal hedging policy, independent of the other. We present scenarios under which joint hedging is optimal, and scenarios under which autonomous hedging is optimal.

In this chapter, we consider a firm, such as an oil producer, that procures a commodity and transports it via ocean freight to its processing plant where the commodity is converted to a final product to meet customer demand. Oil firms like BP source oil near the crude oil fields and conduct refining operations close to the market; a company like Cargill sources its cacao in West Africa and processes it in its plants in Amsterdam. The firm faces three types of uncertainty; (1) demand for the final product; (2) the cost of procuring the commodity on the spot market; (3) the cost of ocean transport of the commodity to its production facility. The firm can negotiate procurement options (similar in style to financial European call options) with its suppliers. These options allow the firm to hedge the uncertainty associated with the spot price of the commodity by reserving the right to procure the commodity at a pre-specified strike price. Naturally, the firm pays the supplier a premium to reserve this flexibility. If options on the commodity are traded on standardized exchanges, the firm simply procures the required options on the exchange. The firm also negotiates a similar option with the ocean shipping company and reserves the right to transport the commodity at a pre-specified strike price on the freight rate, paying the freight company a premium for this flexibility. Note that freight options are not traded on exchanges and must be negotiated with an ocean shipping company in Over-The-Counter (OTC) markets. We consider three scenarios in formulating the firm's cost minimization problem. In the first scenario, we assume that the firm need not meet all demand, and that it may forgo some demand if the cost of procuring and transporting the commodity exceeds a preset *reservation price*, the maximum price the firm is willing to pay to procure and transport the commodity. Indeed, management may set a reservation price to ensure that some target profit margin is achieved or exceeded. We further assume independence between all variable elements in the problem. We find that the operations department should take into account freight rate volatility when deciding on a hedging policy, i.e., the shipping and operations departments should jointly decide how much commodity to procure and transport.

In the second scenario, we impose a 100 % service level restriction on the firm, thereby requiring the firm to procure the commodity on the spot market to meet excess demand if necessary. We still assume independence between the variable elements in this scenario (which can be viewed as a specific case of the first scenario). With these assumptions and restrictions in place, we find that the hedging decision is separable, implying that the operations department can decide how many options on the commodity price to procure, independent of the shipping department's procurement of options on the freight rate and vice versa. In these two scenarios, we determine the optimal number of option contracts that should be procured, either jointly or independently, depending on the scenario in closed-form. In the final scenario, we relax the independence assumption and allow for correlation between commodity price and demand, and correlation between freight rate and demand for the commodity. We maintain the assumption that the firm may forgo demand if its reservation price is exceeded to preserve generality of the problem. The inclusion of correlation between the variables precludes a closed-form expression for the optimal number of options that should be procured. To further generalize the problem, we make no distributional assumptions on the demand faced by the firm, the price of the commodity and the freight rate, only doing so when performing numerical studies.

Our work is related to two streams of literature. The first stream focuses on the value created to firms by hedging. Financial hedging is defined in Van Mieghem (2003) to be the purchase and sale of commodity derivatives as a protection against loss or failure due to price fluctuations. By considering supply and demand risks in addition to price fluctuations, the definition can be extended to include operational hedging. Gaur & Seshadri (2005) consider a risk-averse firm that uses financial information to set optimal inventory levels and hedges demand risk with financial hedging. A popular view held in the literature is that financial hedging only creates value in the presence of market imperfections such as taxes (Smith & Stulz 1985), asymmetric information, (DeMarzo & Duffie 1995) and costly external capital (Froot et al. 1993). A recent paper, however, shows that value can be created by hedging even for a risk-neutral firm in the absence of the aforementioned market frictions. Turcic et al. (2015) considers a risk-neutral supply chain and shows that even in the absence of market imperfections, hedging creates value for at least some of the members of the supply chain. These papers focus on the use of financial hedging strategies which might be viewed as substitutes to operational hedging. Chod et al. (2010) shows that operational and financial hedging strategies can in fact be complements in the presence of market imperfections for firms that demonstrates risk-averse behaviour. Our approach to hedging the three sources of uncertainty (commodity price, freight rate and demand risk) by jointly determining the optimal number of commodity procurement and shipping options for a risk-neutral firm in the absence of market imperfections, and the subsequent results significantly generalize and extend this complementarity.

Our work is also related to papers concerned with the procurement of a commodity in the presence of price and demand uncertainty (Berling & Martínez-de Albéniz 2011, Devalkar et al. 2011) and the operations management literature on the procurement of a commodity in spot and forward markets. In particular, our work is closely related to Secomandi & Kekre (2014) in that we consider commodity procurement in a newsvendor-setting with price and demand uncertainty. We extend the model by considering the stochastic cost of transporting the commodity once procured and by using options to hedge the three sources of risk. Our work is also related to papers that discuss the procurement of commodities with options contracts. Goel & Tanrisever (2011) consider a firm that procures a commodity with a combination of option and spot contracts. The authors also consider how the commodity is delivered when procured in the different markets.

Our work makes two contributions to the existing literature on commodity procurement. First, we develop a tractable and parsimonious model that combines both procurement and transportation of the commodity and integrates demand uncertainty for the final product, uncertainty in the commodity price, and uncertainty in the freight rates. We highlight the necessity of integrating the procurement and transportation decisions. Our second contribution is the demonstration of value creation through hedging for a risk-neutral firm, *even* in the absence of market frictions.

The chapter is organized as follows. In Section 2.2, we present the mathematical model and central results of the research. We quantify the value created by hedging in Section 2.3 and discuss the necessary conditions for value creation. In Section 2.4, we explore numerically the influence of the underlying parameters on the firm's hedging policy and costs. Finally, we conclude in Section 2.5.

2.2 Determining the Optimal Options Position

In this section, we detail the optimization problem facing the firm with regards to the number of options contracts that it should negotiate. We focus on minimizing the firm's procurement and transportation costs and formulate the optimization problem as a classic news-vendor model with three stochastic parameters. As mentioned previously, we consider three scenarios under which the firm negotiates option contracts. We discuss the firm's optimization problem under each of these scenarios and the accompanying optimality conditions and solutions which form the central thesis of this research.

To preserve focus on the value created through the use of options, following Secomandi & Kekre (2014), we frame the firm's optimization problem in a single-period model. The firm produces a single type of product and faces stochastic demand for this product. The firm procures the commodity required to produce this product from an offshore supplier at a stochastic price. Once procured, the firm transports the commodity via ocean freight, at stochastic freight rates, to its production facility where the commodity is converted to the final product to meet demand. For simplicity, we assume that a single unit of the commodity is required to produce a single unit of the product. We further assume zero production and transportation lead times.

At the start of the period, at time $t = 0$, the operations department of the firm negotiates options with its offshore supplier to procure the commodity, taking into account expected final demand for the product. We denote the strike price of this option to be $K_s \in \mathbb{R}_+$. Thus, if the option is exercised, the firm pays K_s for a unit of the commodity, regardless of the commodity's prevailing spot price. The firm pays the offshore supplier a premium of $P_s \in \mathbb{R}_+$ for each option. If the option is procured via an exchange, the firm pays a premium of P_s per option as determined by the exchange.

Similarly, the transportation department of the firm also negotiates options on the freight rate over the counter with the freight company. We denote by $K_\lambda \in \mathbb{R}_+$ the strike price of these freight options. If the option is exercised, the firm pays the freight company K_λ per unit of the commodity being transported, regardless of the prevailing freight rate. The firm pays the freight company $P_\lambda \in \mathbb{R}_+$ for each freight option, where P_λ is negotiated with the freight company (since freight options are not traded on exchanges). The number of options the firm negotiates with each party depends on four factors; the terms of the options (strike price and premium), the firm's expectation of final demand for its product, the firm's expectation of the commodity's spot price, and the firm's expectation of the freight rate. We represent by $\phi_d(\cdot)$, $\phi_s(\cdot)$, and $\phi_\lambda(\cdot)$ the *marginal* probability density functions of final demand, the commodity's spot price and the freight rate, respectively. To simplify notation, we denote by $\mathbf{M}$ the set of these marginal distributions. Demand, commodity spot price and spot freight rate at the start of the period are known. At the end of the period, at time $t = T$, all uncertainties are resolved and the firm observes final demand $d_T \in \mathbb{R}_+$, the commodity spot price $s_T \in \mathbb{R}_+$ and freight rate $\lambda_T \in \mathbb{R}_+$. The firm then decides how many of the options procured at time 0 it will exercise.

2.2.1 Scenario 1: Forgone sales and independent variables

Under scenario 1, we assume that the firm is not obliged to meet all of its demand. The firm forgoes demand if the cost of procuring and transporting a unit of the commodity exceeds the unit reservation price L, i.e., if $s_T + \lambda_T > L$. As mentioned above, management may set a reservation price to ensure that a target profit margin is achieved or exceeded. We assume that the reservation price L is exogenous to the model and given. Recall that the reservation price is the maximum price the firm is willing to pay for each unit of commodity and its transportation cost. We further assume that the firm hires ships to transport the commodity solely on the basis of the units of commodity to be transported. For example, if the commodity in question is crude oil, the firm constructs a contract with the ocean freight company based on the number of barrels of oil to be transported, as opposed to the number of ships required to transport the commodity. This assumption implies that the firm can partially hire a ship, and is not restricted to hiring an integer number of ships. Thus, each option negotiated corresponds to a single unit of the commodity. The procurement department and transportation department could each negotiate options independent of the other, i.e., the procurement department could negotiate q_s options with the offshore commodity supplier, and the transportation department could negotiate q_λ options with the ocean freight company. Since the reservation price is compared to the cost of both procuring and transporting the commodity, the decision to forgo demand has to be jointly taken by both the procurement department and transportation department. We thus assume $q_s = q_\lambda$, and omit the subscripts in the rest of this section, denoting the number of options negotiated by each department as q. This assumption reflects the practice where the procurement department negotiates options with the commodity supplier first, then communicates this decision to the shipping department, which in turn negotiates the same number of options with the ocean freight company.

The firm's optimization problem can then be written as

$$\begin{aligned}\min_{q\in\mathbb{R}_+} C_1(q,\mathbf{M},L) = \quad & \mathbb{E}_{s,d}[\min(s_T,K_s)\min(d_T,q)] + \mathbb{E}_{\lambda,d}[\min(\lambda_T,K_\lambda)\min(d_T,q)] \\ + \quad & \mathbb{E}_{s,\lambda,d}[(d_T-q)^+\min(s_T+\lambda_T,L)] + q(P_\lambda+P_s). \end{aligned} \tag{2.1}$$

The function $C_1(q,\mathbf{M},L)$ (where the subscript refers to the scenario) represents the combined expected procurement and transportation costs. The decision variable in the optimization problem is q, while $\mathbf{M}$ is the set of marginal distributions of demand, commodity price and freight rate, and L is the reservation price. The function $C_1(q,\mathbf{M},L)$ consists of three parts. The first part,

$(\min(s_T, K_s) + \min(\lambda_T, K_\lambda)) \min(d_T, q)$, reflects the cost of procuring and transporting the commodity using options. Depending on the demand realization d_T, the firm exercises at most d_T options if it negotiated a sufficient number of options at time 0. The firm then pays the strike prices, K_s and K_λ to procure and transport the commodity, respectively. The second part,$(d_T - q)^+ \min(s_T + \lambda_T, L)$, reflects the costs of procuring and transporting excess demand not covered by the options. This demand is procured at the commodity's spot price s_T and transported at the prevailing freight rate λ_T, provided that the combined cost of procuring and transporting in the spot market is less than the unit reservation price. We thus assume that $L > K_s + K_\lambda$ to avoid trivial solutions. Any excess demand not covered by the options is either completely covered in the spot market or completely lost. The firm does not partially satisfy the excess demand $(d_T - q)^+$. If either or both of the spot prices fall below their corresponding strike prices, naturally, the firm would not exercise any of its options and procure and transport all of its realized demand at the prevailing spot prices. This possibility is also captured in the first two parts of Equation (2.1). Expectations on the demand and freight rate are both taken with respect to the physical probability-measure. The third and final part, $q(P_\lambda + P_s)$, reflects the premiums paid at time 0 for the options. If the commodity and options on the commodity are traded in the financial markets, P_s is determined by the market under the risk-neutral probability-measure. If the commodity and options on the commodity are instead traded in OTC markets via direct negotiations, the option premium P_s is also negotiated and thus taken to be exogenous. Expectations on P_s are then taken with respect to the physical probability-measure. Freight rates are not traded assets. Consequently, options on freight rates are negotiated between the firm and the ocean shipping company in OTC markets. The option premium P_λ is thus also negotiated and taken to be exogenous. Expectations on P_λ are thus also taken with respect to the physical probability-measure.

As a consequence of the independence between the three underlying uncertainties, we can rewrite Equation (2.1) as follows:

$$\begin{aligned} C_1(q, \mathbf{M}, L) = \quad & \mathbb{E}_s[\min(s_T, K_s)]\mathbb{E}_d[\min(d_T, q)] + \mathbb{E}_\lambda[\min(\lambda_T, K_\lambda)]\mathbb{E}_d[\min(d_T, q)] \\ + \quad & \mathbb{E}_d[(d_T - q)^+]\, \mathbb{E}_{s,\lambda}[\min(s_T + \lambda_T, L)] + q(P_\lambda + P_s). \end{aligned}$$

We will consider dependent random variables in the third scenario. We further mention here that we have made the implicit assumption that the risk-free rate is 0. This assumption can be readily relaxed with no consequences on the subsequent results and insights.

Before discussing the number of options the firm should obtain at time 0, we introduce the following notation for expositional ease.

$$A = \mathbb{E}_s[\min(s_T, K_s)] + \mathbb{E}_\lambda[\min(\lambda_T, K_\lambda)] \quad (2.2)$$

$$B = \mathbb{E}_{s,\lambda}(\min(s_T + \lambda_T, L)) \quad (2.3)$$

$$P = P_\lambda + P_S \quad (2.4)$$

Each of these terms represents a type of cost incurred by the firm and can, naturally, be directly linked to the three parts of the cost function in Equation (2.1). The term A represents the unit cost of satisfying demand using options and links to the first part of the cost function, the cost of procuring and transporting the commodity using options. Denoting the cumulative distribution functions of s_T and λ_T as Φ_s and Φ_λ respectively, we can write A as $\mathbb{E}_s(s_T|s_T < K_s) + K_s[1 - \Phi_s(K_s)] + \mathbb{E}_\lambda(\lambda_T|\lambda_T < K_\lambda) + K_\lambda[1 - \Phi_\lambda(K_\lambda)]$. The term B represents the unit cost of satisfying or forgoing demand not covered by options and links to the second part, the costs of satisfying excess demand. The term P represents the unit cost of procuring options and links to the third part, the premiums paid by the firm for the q options. We are now in a position to derive the firm's optimal hedging decision at time 0. We first show that the cost function $C_1(q, \mathbf{M}, L)$ is convex in q.

Lemma 2.1 *The cost function $C_1(q, \mathbf{M}, L)$ is convex in q.*

PROOF: We show the convexity of $C_1(q, \mathbf{M}, L)$ by showing that $\frac{\partial^2 C_1}{\partial q^2} \geq 0$ for all $q \in \mathbb{R}_+$. We first rewrite and simplify the cost function in Equation (2.1) to yield

$$C_1(q, \mathbf{M}, L) = A\,\mathbb{E}_d(\min(d_T, q)) + B\,\mathbb{E}_d[(d_T - q)^+] + Pq. \tag{2.5}$$

Using the identities

$$\begin{aligned} \min(a, b) &= a - (a - b)^+ \text{ and} \\ (a - b)^+ &= a - b + (b - a)^+, \end{aligned}$$

we rewrite and simplify Equation (2.5) further to obtain

$$C_1(q, \mathbf{M}, L) = (A - B + P)q + B\mathbb{E}_d(d_T) - (A - B)\mathbb{E}_d[(q - d_T)^+]. \tag{2.6}$$

Differentiating Equation (2.6) with respect to q yields

$$\frac{\partial C_1}{\partial q} = A - B + P - (A - B)\frac{d}{dq}\mathbb{E}_d[(q - d_T)^+]. \tag{2.7}$$

Using Leibniz's rule, we obtain

$$\begin{aligned} \frac{d}{dq}E_d[(q - d_T)^+] &= \frac{d}{dq}\int_0^q (q - x)\phi_d(x)dx \\ &= \int_0^q \phi_d(x)dx \\ &= \Phi_d(q), \end{aligned} \tag{2.8}$$

where we have used $\Phi_d(\cdot)$ to represent the cumulative distribution function of final demand. We then arrive at

$$\frac{\partial C_1}{\partial q} = A - B + P - (A - B)\Phi_d(q). \tag{2.9}$$

Differentiating Equation (2.9) again with respect to q yields

$$\frac{\partial^2 C_1}{\partial q^2} = (B - A)\phi_d(q).$$

Since $L > K_s + K_\lambda$ by assumption, we have that $B \geq A$. Also, $\phi_d(q) \geq 0$ for all $q \in \mathbb{R}_+$, implying that the second derivative of $C_1(q, \mathbf{M}, L)$ is non-negative for all $q \in \mathbb{R}_+$. Hence, $C_1(q, \mathbf{M}, L)$ is convex. This completes the proof. □

Theorem 2.1 *The firm's optimal options position at time 0 is given by*

$$q^* = \Phi_d^{-1}\left(\frac{B - A - P}{B - A}\right), \tag{2.10}$$

with A, B, and P, as in Equations (2.2) - (2.4).

Proof: We first show that q^* is a stationary point of $C_1(q, \mathbf{M}, L)$. To obtain the first order condition for optimality, we set Equation (2.9) to 0 and solve for q to obtain Equation (2.10). Since the second derivative of $C_1(q, \mathbf{M}, L)$ is non-negative for all q as a consequence of the convexity of the cost function, the stationary point q^* minimizes $C_1(q, \mathbf{M}, L)$. □

We make two remarks about q^*. First, we have derived q^* without making any distributional assumptions on the underlying uncertainties. The specific form of q^* is heavily dependent on the choice of distributions. Deriving a closed-form expression for q^* hinges upon being able to compute the terms A and B closed-form. This in turn is dependent on whether the partial expectations of the underlying distributions can be analytically written. A closed-form expression for A can be easily obtained. Since s_T and λ_T are independent random variables, the distribution of $s_T + \lambda_T$ can be easily derived using the convolution of ϕ_s and ϕ_λ. Deriving a closed-form expression for B then depends on whether the partial expectation of this distribution can be written analytically. As an illustrative example, assume that d_T, s_T, and λ_T are normally distributed with means μ_d, μ_s, μ_λ, and standard deviations σ_d, σ_s, σ_λ respectively. Let $\mathcal{N}$ (N) denote the cumulative (probability) distribution function of the standard normal distribution. Using the properties of the truncated normal distribution, we can express A as follows:

$$\begin{aligned} A = \quad & \mu_s - \sigma_s \frac{N\left(\frac{K_s - \mu_s}{\sigma_s}\right)}{\mathcal{N}\left(\frac{K_s - \mu_s}{\sigma_s}\right)} + K_s\left(1 - \mathcal{N}\left(\frac{K_s - \mu_s}{\sigma_s}\right)\right) + \mu_\lambda - \sigma_\lambda \frac{N\left(\frac{K_\lambda - \mu_\lambda}{\sigma_\lambda}\right)}{\mathcal{N}\left(\frac{K_\lambda - \mu_\lambda}{\sigma_\lambda}\right)} \\ + \quad & K_\lambda\left(1 - \mathcal{N}\left(\frac{K_\lambda - \mu_\lambda}{\sigma_\lambda}\right)\right) \end{aligned}$$

To derive an expression for B, we define $Y = s_T + \lambda_T$. Since Y is the sum of two normal random variables, Y is normally distributed with mean $\mu_Y = \mu_s + \mu_\lambda$ and standard deviation $\sigma_Y = \sigma_s + \sigma_\lambda$. Using the properties of the truncated normal distribution again, we can express B as follows:

$$B = \mu_Y - \sigma_Y \frac{N\left(\frac{L-\mu_Y}{\sigma_Y}\right)}{\mathcal{N}\left(\frac{L-\mu_Y}{\sigma_Y}\right)} + L\left(1 - \mathcal{N}\left(\frac{L-\mu_Y}{\sigma_Y}\right)\right).$$

Using these expressions for A and B, we can then compute q^* using Equation (2.10). Naturally, if a distribution's partial expectation has no closed-form representation, the terms A and B, and consequently q^* can be computed numerically.

The second remark relates q^* to the critical fractile in the newsvendor model. The optimal stocking quantity in the newsvendor model is given by the critical fractile

$$q^* = F^{-1}\left(\frac{p-c}{p}\right),$$

where p typically refers to the unit retail price of the product, c typically refers to the unit purchase price and F is the cumulative distribution of demand. Comparing Equation (2.10) to the critical fractile in the newsvendor model, we find that the two expressions are remarkably similar. We may view $B-A$ in Equation (2.10) as p in the newsvendor critical fractile, and P in Equation (2.10) as c in the newsvendor critical fractile. Intuitively, we may view $B-A$ as the reduction in cost arising from the use of a single option.

Under the conditions presented above, the firm might be willing to forgo demand not covered by the options position q^* if the reservation price L is sufficiently low. The proportion of time that demand is completely covered could be interpreted as the service level for the firm demand is completely covered if $q^* \geq d_T$, i.e., the firm has negotiated sufficient options at time 0 to completely satisfy demand using options. If $q^* < d_T$, the firm will still completely satisfy demand provided that $s_T + \lambda_T < L$. We can then write the service level for the firm as follows:

$$\nu(L) = \mathbb{P}(d_T \leq q^*) + \mathbb{P}(d_T > q^*)\,\mathbb{P}(s_T + \lambda_T \leq L). \tag{2.11}$$

Given L and $\mathbf{M}$, we can compute q^* according to Equation (2.10) and then compute the associated service level $\nu(L)$ using Equation (2.11).

2.2.2 Scenario 2: 100 percent service level and independent variables

We now assume that the firm operates in a highly competitive environment, entailing that it must meet all demand. We still assume that the stochastic variables are independent. The difference between scenarios 1 and 2 is that the reservation price constraint ($s_T + \lambda_T > L$) is relaxed in scenario 2. Without the reservation price constraint, the departments need not jointly decide on a hedging policy. The cost function for each department can therefore be written separately as follows:

$$C_s(q_s, \mathbf{M}) = \mathbb{E}_{d,s}\left[\min(s_T, K_s)\min(d_T, q_s) + (d_T - q_s)^+ s_T\right] + q_s P_S, \tag{2.12}$$

and

$$C_\lambda(q_\lambda, \mathbf{M}) = \mathbb{E}_{d,\lambda}\left[\min(\lambda_T, K_\lambda)\min(d_T, q_\lambda) + (d_T - q_\lambda)^+ \lambda_T\right] + q_\lambda P_\lambda. \tag{2.13}$$

Naturally, Equation (2.12) refers to the procurement costs while Equation (2.13) refers to the transportation cost.

Lemma 2.2 *The cost functions $C_s(q_s, \mathbf{M})$ and $C_\lambda(q_\lambda, \mathbf{M})$ and are convex in q.*

Proof: We show the convexity of $C_s(q_s, \mathbf{M})$ by showing that $\frac{\partial^2 C_s}{\partial q_s^2} \geq 0$ for all $q_s \in \mathbb{R}_+$. We first rewrite and simplify the cost function in Equation (3.14) to yield

$$C_s(q_s, \mathbf{M}) = A_s\,\mathbb{E}_d(\min(d_T, q)) + B_s\,\mathbb{E}_d[(d_T - q)^+] + P_s q. \tag{2.14}$$

Using the identities

$$\begin{aligned} \min(a,b) &= a - (a-b)^+ \text{ and} \\ (a-b)^+ &= a - b + (b-a)^+, \end{aligned}$$

we rewrite and simplify Equation (2.5) further to obtain

$$C_s(q_s, \mathbf{M}) = (A_s - B_s + P_s)q + B_s\mathbb{E}_d(d_T) - (A_s - B_s)\mathbb{E}_d[(q_s - d_T)^+]. \tag{2.15}$$

where $A_s = \mathbb{E}_s\left[\min(s_T, K_s)\right]$ and $B_s = \mathbb{E}_s(s_T)$. Differentiating Equation (2.6) with respect to q_s yields

$$\frac{\partial C_s}{\partial q_s} = A_s - B_s + P_s - (A_s - B_s)\frac{d}{dq_s}\mathbb{E}_d[(q_s - d_T)^+]. \tag{2.16}$$

Using Leibniz's rule, we obtain

$$\begin{aligned} \frac{d}{dq_s}E_d[(q_s - d_T)^+] &= \frac{d}{dq_s}\int_0^{q_s}(q_s - x)\phi_d(x)dx \\ &= \int_0^{q_s}\phi_d(x)dx \\ &= \Phi_d(q_s), \end{aligned} \tag{2.17}$$

where we have used $\Phi_d(\cdot)$ to represent the cumulative distribution function of final demand. We then arrive at

$$\frac{\partial C_s}{\partial q_s} = A_s - B_s + P_s - (A_s - B_s)\Phi_d(q_s). \tag{2.18}$$

Differentiating Equation (2.18) again with respect to q yields

$$\frac{\partial^2 C_s}{\partial q_s^2} = (B_s - A_s)\phi_d(q_s).$$

We have that $B_s \geq A_s$. Also, $\phi_d(q) \geq 0$ for all $q_s \in \mathbb{R}_+$, implying that the second derivative of $C_s(q_s, \mathbf{M})$ is non-negative for all $q \in \mathbb{R}_+$. Hence, $C_s(q_s, \mathbf{M})$ is convex. This completes the proof. In a similar way one can show the convexity of $C_\lambda(q_\lambda, \mathbf{M})$ □

Each function therefore attains a unique minimum which we denote q_s^* and q_λ^* respectively. As a result of the separability of costs, the firm's combined cost of procurement and transportation is minimized if each of the departments selects an options position to minimize their individual costs while meeting demand.

We now derive the optimal hedging decisions for each of the departments at time 0 in the following theorem.

Theorem 2.2 *The procurement department's optimal hedging decision at time 0, q_s^* is given by*

$$q_s^* = \Phi_d^{-1}(\frac{B_s - A_s - P_s}{B_s - A_s}), \tag{2.19}$$

where $A_s = \mathbb{E}_s(s_T | s_T < K_s) + K_s\,(1 - \Phi_s(K_s))$ and $B_s = \mathbb{E}_s(S_T)$. The shipping department's optimal hedging decision at time 0, q_λ^* is given by

$$q_\lambda^* = \Phi_d^{-1}(\frac{B_\lambda - A_\lambda - P_\lambda}{B_\lambda - A_\lambda}), \tag{2.20}$$

where $A_\lambda = \mathbb{E}_\lambda(\lambda_T | \lambda_T < K_\lambda) + K_\lambda\,(1 - \Phi_\lambda(K_\lambda))$ and $B_\lambda = \mathbb{E}_\lambda(\lambda_T)$.

PROOF: Using arguments similar to those presented in the proof of Theorem 2.1 and Lemma 2.2 yields the desired results. □

As we did above, we remark that closed-form representations of Equations (2.19) and (2.20) can be obtained if the partial expectation of the convolution of ϕ_s and ϕ_λ, and the partial expectation of ϕ_d admit closed-form representations.

2.2.3 Scenario 3: 100 percent service level and dependence between demand and commodity spot price/freight rate

We have thus far assumed independence between the underlying random variables. Building on from the previous scenario, to better reflect reality, we now relax this assumption and examine the optimal options position at time 0. We assume that the commodity's spot price s_T and final demand d_T are correlated. We also assume that the freight rate λ_T and demand are correlated. This implies that rather than marginal distributions for each of the underlying uncertainties, we now have joint distributions $\phi_{s,d}$ and $\phi_{d,\lambda}$. Utilizing the setting from Scenario 2 allows us to separate the cost function and better examine the influence of each source of dependency. We still assume independence between the commodity's spot price and the freight rate, since practically, each has little or no impact on the other. The set $\mathbf{M}$ now refers to the collection of joint distributions.

The cost minimization problem the firm faces is the same as that faced in Scenario 2. Since the firm must meet all demand, the cost function can again be separated into procurement costs, $C_{3,s}(q_s, \mathbf{M})$ and transportation costs $C_{3,\lambda}(q_\lambda, \mathbf{M})$.

The underlying uncertainties are, however, no longer independent of one another, so the expectations on s_T and d_T , and the expectations on λ_T and d_T cannot be separated. Rewriting Equation (2.12) explicitly in terms of integrals yields

$$\begin{aligned} C_{3,s}(q_s, \mathbf{M}) = \quad & \int_0^{q_s} \int_0^{K_s} d \cdot s\, \phi_{s,d}\, ds\, dd + \int_{q_s}^{\infty} \int_0^{K_s} q_s \cdot s\, \phi_{s,d}\, ds\, dd \\ + \quad & \int_0^{q_s} \int_{K_s}^{\infty} d \cdot K_s\, \phi_{s,d}\, ds\, dd + \int_{q_s}^{\infty} \int_{K_s}^{\infty} q_s \cdot K_s\, \phi_{s,d}\, ds\, dd \\ + \quad & \int_{q_s}^{\infty} \int_0^{\infty} (d - q_s) \cdot s\, \phi_{s,d}\, ds\, dd + P_s q_s. \end{aligned} \tag{2.21}$$

The cost function $C_{3,\lambda}(q_\lambda, \mathbf{M})$ can be written similarly, with the freight rate λ replacing the commodity spot price s, and K_λ replacing K_s.

As in Scenarios 1 and 2, the cost function $C_{3,s}(q_s, \mathbf{M})$ and $C_{3,\lambda}(q_\lambda, \mathbf{M})$ are convex in q_s and q_λ, as shown by the following proposition.

Proposition 2.1 *The expected procurement cost function $C_{3,s}(q_s, \mathbf{M})$ under Scenario 3 is convex in q_s.*

PROOF: Differentiating Equation (2.21) with respect to q_s, we obtain

$$\frac{\partial C_{3,s}}{\partial q_s} = \int_{q_s}^{\infty} \int_0^{K_s} s\, \phi_{s,d}\, ds\, dd + \int_{q_s}^{\infty} \int_{K_s}^{\infty} K_s\, \phi_{s,d}\, ds\, dd - \int_{q_s}^{\infty} \int_0^{\infty} s\, \phi_{s,d}\, ds\, dd + P_s \tag{2.22}$$

Differentiating Equation (2.22) with respect to q_s yields

$$\frac{\partial^2 C_{3,s}}{\partial q_s^2} = -\int_0^{K_s} s\, \phi_{s,q_s}\, ds - \int_{K_s}^{\infty} K_s\, \phi_{s,q_s}\, ds + \int_0^{\infty} s\, \phi_{s,q_s}\, ds,$$

which we rewrite as

$$\frac{\partial^2 C_{3,s}}{\partial q_s^2} = -\int_0^\infty \min(s, K_s)\, \phi_{s,q_s}\, ds + \int_0^\infty s\, \phi_{s,q_s}\, ds.$$

Since $s_T \geq \min(s_T, K_s)$ the second derivative of $C_{3,s}(q_s, \mathbf{M})$, with respect to q_s is positive. □

The convexity of $C_{3,\lambda}(q_\lambda, \mathbf{M})$ in q_λ can be demonstrated using similar arguments. As above, to obtain the optimal options position at time 0, we need only set Equation (2.22) to 0 and solve for q_s. Due to its complicated form, Equation (2.22) cannot be simplified further to obtain an analytical expression for q_s^*. Given distributions for the underlying random variables, Equation (2.22) can be easily solved numerically.

2.3 Value creation

In the previous section, we have discussed the cost minimization problem facing the firm and derived analytical expressions for the optimal options positions. In this section, we discuss the creation of value for the firm by negotiating options with its suppliers and the freight company. We first demonstrate that the use of options as a hedging strategy does indeed create value for the firm. We then discuss the topic of value creation by hedging for a risk-neutral firm. We develop the exposition within the confines of Scenario 1 due to its generality and tractability. The analytical results in this section are developed independent of any distributional assumptions, as in the previous section. To numerically demonstrate and examine value creation through hedging in the subsequent analysis, we consider a commodity that is traded on an exchange and assume that the commodity spot price is lognormally distributed with $\mu_s = 0.8$ and $\sigma_s = 0.8$ being the mean and standard deviation respectively of the associated normal distribution. Similarly, we have assumed that the freight rate is lognormally distributed with $\mu_l = 0.9$ and $\sigma_l = 0.8$ being the mean of the associated normal distribution. We assume that demand is also lognormally distributed with $\mu_d = 0.7$, $\sigma_d = 0.5$ representing the mean and standard deviation of the associated normal distribution. Assuming a lognormal distribution for the commodity spot price and freight rate is equivalent to assuming that the commodity spot price and freight rate evolve according to geometric Brownian motion processes. Stochastic processes incorporating stochastic volatility, or jumps, or mean-reverting features could also be used to model the evolution of these quantities. Our results, however, are distribution independent. As such, we have opted to model commodity spot price, demand and freight rate as lognormal random variables. Assuming a lognormal distribution also allows us to compute option prices using the Black & Scholes (1973) formula. Furthermore, when considering a traded commodity, option premiums for options on the commodity would be determined by the market under the risk-neutral measure. Assuming a lognormal distribution allows us to compute the option premium using the Black & Scholes (1973) formula without having to derive the option premium as an expectation based on the process underlying spot price evolution. Using the Black & Scholes (1973) formula allows us to focus on the risk-neutral nature of the firm under consideration and does not mire us in technicalities regarding the risk preferences of the firm providing ocean freight services. The current spot price, freight rate and demand are each normalized to 1. The option strike prices are assumed to be $K_s = K_\lambda = 2$. Computing option premiums using the Black & Scholes (1973) formula yields $P = 0.2302$. Finally, the unit reservation price is set to $L = 6$.

2.3.1 Value creation by hedging with options

If the firm does not negotiate options (equivalent to selecting $q = 0$), the firm procures and transports the commodity at the commodity spot price and spot freight rate, or forgoes demand if the unit reservation price L is low enough. The cost of this strategy is naturally $C_1(0, \mathbf{M}, L) = B\mathbb{E}_d[(d_T - q)^+]$. The utilization of options creates value for the firm in terms of cost reduction. We define this reduction (and consequently the value created for the firm) as $C_R(q, \mathbf{M}, L) = C_1(0, \mathbf{M}, L) - C_1(q, \mathbf{M}, L)$. We can easily observe that the utilization of options will lead to value creation for the firm. Indeed, since $q = 0$ is a feasible solution to the firm's cost minimization problem, by the definition of optimality, at the optimal options position q^*, we must have that $C_1(0, \mathbf{M}, L) \geq C_1(q^*, \mathbf{M}, L)$, implying that the utilization of options will create value for the firm. This result is, of course, intuitive and echoes the observations in Barnes-Schuster et al. (2002). The cost reduction function is concave in q, as shown by the following proposition.

Proposition 2.2 $C_R(q, \mathbf{M}, L)$ *is concave in the number of options negotiated, q.*

PROOF: Differentiating $C_R(q, \mathbf{M}, L)$ twice with respect to q yields $\dfrac{\partial C_R^2}{\partial q^2} = -\dfrac{\partial C_1^2}{\partial q^2}$. Since $C_1(q, \mathbf{M}, L)$ is convex in q, as shown in Lemma2.1, $\dfrac{\partial C_1^2}{\partial q^2} \geq 0$, implying that $\dfrac{\partial C_R^2}{\partial q^2} \leq 0$, leaving us with the desired result. □

We can easily observe that the greatest reduction in cost is achieved at q^*. Consequently, the use of options to hedge the underlying risks leads to value creation for the firm, with the greatest cost reduction being attained when the firm negotiates q^* options at time 0. This clearly demonstrates that both the commodity spot price and freight rate need to be taken into consideration when deciding on the hedging policy and ignoring either one in the decision-making process can lead to a loss in value for the firm. Figure 2.1 illustrates this by plotting the total expected cost of hedging with options and the resulting cost savings.

The figure shows that the total cost function does indeed attain its minimum at $q = q^*$. The firm thus achieves maximum value creation at this point as well.

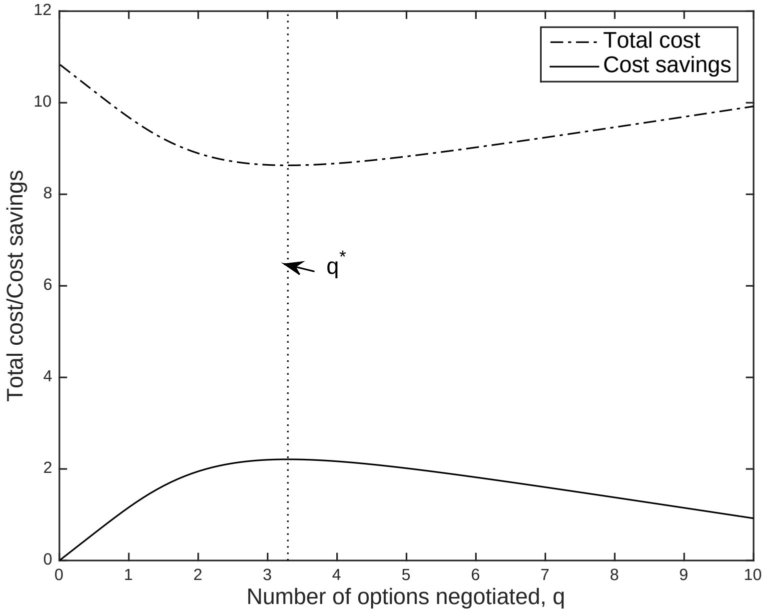


Figure 2.1 Total cost and savings as q changes under Scenario 1

A key factor affecting the amount saved by utilizing options is the premiums paid for the freight options which are typically negotiable. The result developed in this section provides a guideline for firms in terms of a bound on the maximum that they can pay as premiums for the freight options in order to realize cost savings.

Proposition 2.3 *To realize cost savings from hedging with options, the firm should pay at most $B - A$ per combined option, i.e., $P_s + P_\lambda = P < B - A$.*

PROOF: If value is created by hedging with q options, $C_R(q, \mathbf{M}, L) > 0$. Thus,

$$B\,\mathbb{E}_d(d_T) - B\,\mathbb{E}_d[(d_T - q)^+] - A\,\mathbb{E}_d[\min(d_T, q)] - Pq > 0.$$

Rearranging yields

$$Pq < B\left(\mathbb{E}_d(d_T) - \mathbb{E}_d[(d_T - q)^+]\right) - A\,\mathbb{E}_d[\min(d_T, q)].$$

Using the identity $(\min(a, b) = a - (a - b)^+$ and collecting terms results in $P < B - A$.
□

If the commodity and options on the commodity are traded assets, the option premium on each of these options, P_s is determined by the market in order to preclude arbitrage from trading in options on the commodity. This then implies that when negotiating freight options with the ocean shipping company, the firm should pay at most $B - A - P_s$ per freight option, i.e., $P_\lambda < B - A - P_s$ in order to realize cost savings. If the commodity and options on the commodity are traded over the counter, like freight options, the firm can negotiate the option premium P_s as well. Proposition 2.3 then provides a guideline on the negotiation of both P_s and P_λ.

2.3.2 Value creation through hedging for a risk-neutral firm

Thus far, we have discussed *how* value is created for the firm by hedging. We now examine *why* hedging creates value. According to literature, risk-neutral firms have no incentive to hedge. Under the assumption of perfect capital markets, the propositions in Modigliani & Miller (1958) imply that hedging should not contribute to firm value. If capital markets are perfect, the firm's shareholders themselves could undertake hedging. A consequence of these important propositions is that hedging creates value for a risk-neutral firm in the presence of market frictions such as taxes, financial distress costs, and information asymmetry. In our setup, the firm still realizes value creation even in the absence of these market frictions.

Furthermore, according to options pricing theory, the option premium for a traded option is the expected present value under the risk-neutral measure of any potential gains that could be realized from exercising the option. Thus, the premiums paid for the options by the firm should, in expectation, erode any potential gains. As mentioned earlier, however, freight rates and options on the freight rates are not traded on exchanges. Rather, they are negotiated over the counter. As such, premiums for options on the freight rate are also negotiable. Under this setup, a risk neutral firm could potentially create value by hedging with options.

By utilizing both commodity and freight rate options, the firm hedges itself against the underlying price risks, thereby providing it with insurance against commodity spot price and freight rate increases. In expectation, the option premium paid for an exchange-traded commodity would erode any potential gains from the commodity option. The freight rate option could still create value for the firm, as long as $P_\lambda < B - A - P_s$. If the commodity is not an exchange-traded asset either, then options negotiated on them could potentially create value as well. The use of options on the commodity, however, also provides the firm with some insurance in cases where the realized demand is low. In these cases, the firm need not exercise all of the procured options. This additional flexibility translates into increase the value created for the firm. Our results therefore highlight that even in the absence of market frictions, hedging does contribute to value creation for risk-neutral firms.

Since freight options are not exchange-traded assets, their price is negotiable. To examine how the option premiums influence the firm's optimal cost savings, Figure

2.2 plots the optimal cost savings realized by the firm as the price paid per freight option increases. As the figure shows, the use of options results in value creation *even* when the firm pays an option premium computed according to the Black & Scholes (1973) formula, further strengthening the argument that hedging results in value creation for risk-neutral firms. The figure further demonstrates that the firm realizes cost savings as long as Proposition 2.3 is satisfied. We may view the premium the firm is willing to pay for an option as an indication of its risk-aversion level. Thus, a risk-neutral firm will pay no more than then fair option price. A risk-averse firm would be willing to pay more than the fair price for the sake of insurance. Figure 2.2 shows that paying a freight option premium greater than the fair price can still result in value creation as long as $P_\lambda < B - A - P_s$, highlighting that hedging also creates value for risk-averse firms.

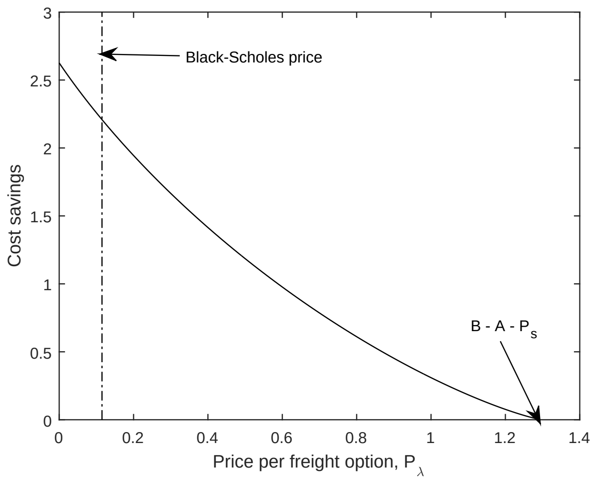


Figure 2.2 Effect of freight option premium on cost savings

2.4 Numerical studies

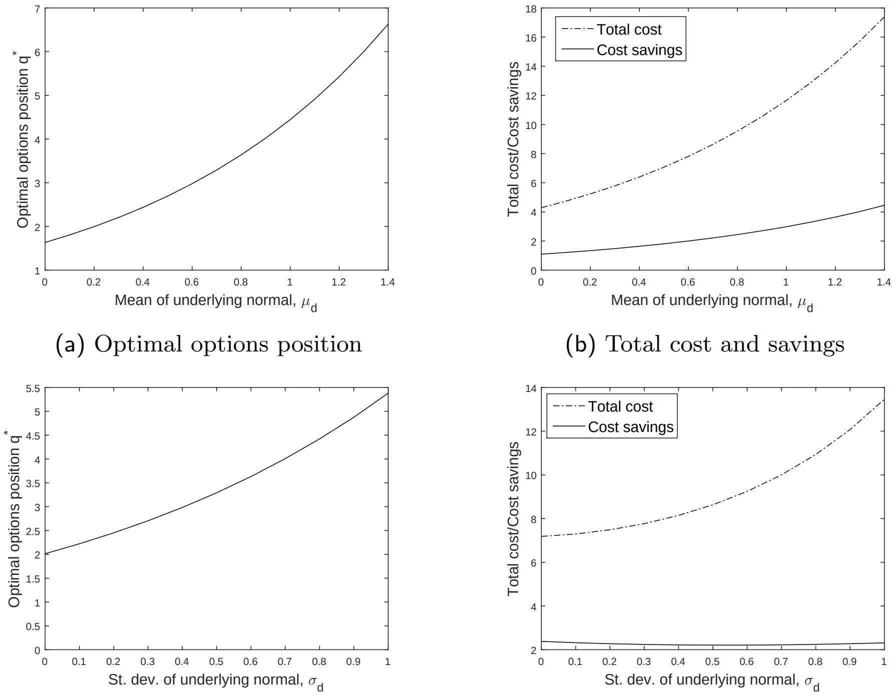
In this section, we use the models developed in Section 2.2 to study the influence of the underlying parameters on the firm's hedging policy and associated costs under the setting of Scenario 1. We then compare hedging policies and costs between Scenarios 1 and 2. As base parameters, we use the same parameters as those used in Section 2.3. We then vary one parameter at a time to examine its influence on hedging and value creation. Last part of this section summarizes the managerial insights coming from our extensive numerical studies.

2.4.1 Hedging policies and costs under Scenario 1

We first focus on the effect of demand on the firm's hedging policies and costs. Recall that demand is assumed to be lognormally distributed with parameters μ_d and σ_d. Figure 2.3(a) shows that increasing μ_d results in the firm negotiating more commodity and freight options at time 0. An increase in μ_d corresponds to an increase in the expected demand at time T since $\mathbb{E}_d(d_T) = e^{\mu_d + \frac{\sigma_d^2}{2}}$. An increase in μ_d also increases the standard deviation (SD) of realized demand at time T since $SD(d_T) = \mathbb{E}_d(d_T)\sqrt{e^{\sigma_d^2} - 1}$. Thus, as expected demand and the standard deviation of realized demand increase, the firm's optimal options position q^* increases to satisfy this increased demand cost effectively. The increased total expected cost and cost savings arising from this increased expected demand and options position are illustrated in Figure 2.3(b). As the figure shows, the total expected cost increases as μ_d increases due to increases in both $\mathbb{E}_d(d_T)$ and $SD(d_T)$. Cost savings (value) also increase as μ_d increases, due to the increased hedging position adopted by the firm in response to increased and more uncertain demand. Cost savings, however, increase at a slower rate compared to the total cost.

The effects of increased demand volatility are illustrated in Figures 2.3(c) and 2.3(d). Figure 2.3(c)shows that an increase in σ_d results in a higher q^*. An increase in σ_d increases both $\mathbb{E}_d(d_T)$ as well as $SD(d_T)$. Increasing σ_d, however, has a greater impact on $SD(d_T)$ than increasing μ_d does. Again, to satisfy the increased and more uncertain cost effectively, the firm needs to adopt a larger hedging position. Consequently, the total expected cost also increases, as shown in Figure 2.3(d). In contrast to the earlier analysis on μ_d, Figure 2.3(d) shows that as σ_d increases, the cost savings for the firm remain unchanged (even decreasing slightly for large σ_d). In fact, if we compare Figure 2.3(a) to Figure 2.3(c) and Figure 2.3(b) to Figure 2.3(d), we find that μ_d has a greater influence on the hedging policies and total costs than σ_d does. Typically, volatility has a greater impact on costs/profits than the mean does, so our observation is somewhat contradictory, but can be explained by noting that the negotiation/purchase of options *already* serves to provide a hedge against the underlying uncertainties. As such, the firm should focus more on μ_d as opposed to σ_d.

We now examine the influence of the commodity price parameters on the firm's hedging policies and costs. As before, the commodity spot price is assumed to be lognormally distributed with parameters μ_s and σ_s. Figure 2.4(a) shows that increasing μ_s results in the firm taking a larger hedge position. Again, as above, an increase in μ_s results in an increase in $\mathbb{E}_s(s_T)$ and $SD(s_T)$. To mitigate the increased price and uncertainty, the firm needs to negotiate more options with its supplier. Naturally, this increased hedge, increased expected prices and uncertainty result in a larger total cost, as demonstrated by Figure 2.4(b). In contrast to our analysis on μ_d, we find that the total cost and cost savings increase at a similar rate. The observation can be explained by noting that the commodity price options provide a direct hedge



(a) Optimal options position

(b) Total cost and savings

(c) Optimal options position

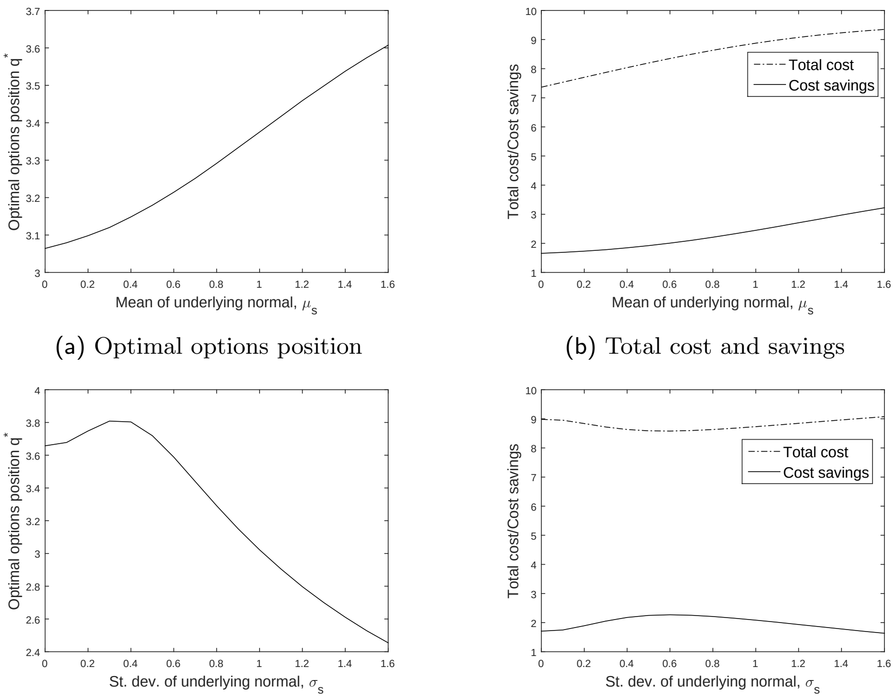
(d) Total cost and savings

Figure 2.3 Influence of demand parameters on q^* and costs

for the uncertainty underlying the commodity spot price.

Increasing the volatility of the commodity spot price, σ_s, first results in the firm taking a larger hedging position. As σ_s continues to increase, the firm reduces its hedging position. Again, following the arguments above, increasing σ_s increases both $\mathbb{E}_s(s_T)$ as well as $SD(s_T)$. Naturally, σ_s has a significantly greater impact on $SD(s_T)$ than μ_s does. When σ_s starts to increase lower levels of volatility, the firm negotiates more options with its supplier to mitigate the increased price and uncertainty. As σ_s continues to increase, however, the cost of procuring the commodity on the spot market increases. The option premiums continue to increase as well (since option prices increase as volatility increases). The combined increase in spot and option price will result in the firm choosing to forgo demand at higher levels of σ_s, thereby negotiating fewer options. Figure 2.5 plots the changes in $\nu(L)$ as σ_s varies and demonstrates that at higher levels of σ_s, service level does indeed decrease, owing to the reduced options position and increased spot procurement and transportation costs.

Figure 2.4(d) illustrates the effect of σ_s on the total cost function $C_1(q^*, \mathbf{M}, L)$



(a) Optimal options position (b) Total cost and savings

(c) Optimal options position (d) Total cost and savings

Figure 2.4 Influence of commodity spot price parameters on q^* and costs

and the cost savings function $C_R(q^*, \mathbf{M}, L)$. Following our observation that q^* first increases then decreases as σ_s increases, we see in Figure 2.4(d) that the total cost first decreases, then increases. Similarly the value created by hedging also increases, then decreases. We observe, however, that the total cost function and cost savings function remain largely unchanged as σ_s changes, each changing value in a narrow range. We can thus conclude that the influence of σ_s on the firm's hedging policy and costs is minimal compared to the influence of μ_s. The minimal influence of σ_s on the firm's cost savings when hedging with options can be explained by noting that the fair price or premium paid by the firm for each option is also a function of σ_s. Thus, as σ_s increases the option premium increases as well. Now suppose that the firm agrees a fixed premium for the commodity option. The resulting optimal options position, total expected cost and cost savings are shown in Figure 2.6. We now see that as σ_s increases, q^* increases as well. The subsequent value creation through hedging increases as well. We can thus conclude that if the firm were able to negotiate a fixed option premium, the influence of σ_s on total cost and cost savings is substantial.

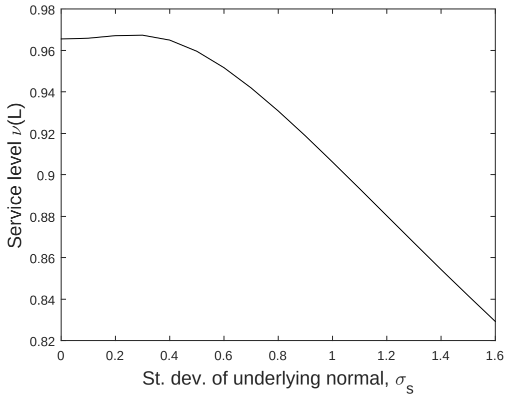
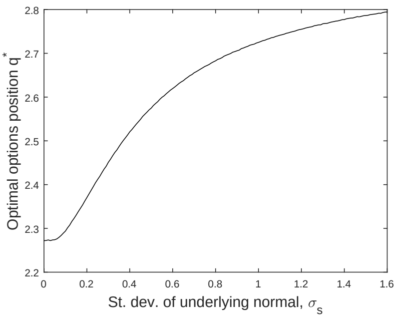
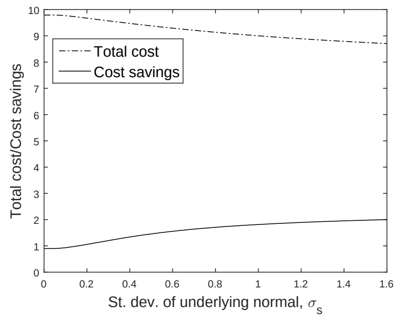


Figure 2.5 Influence of commodity spot price volatility on service level $\nu(L)$



(a) Optimal options position

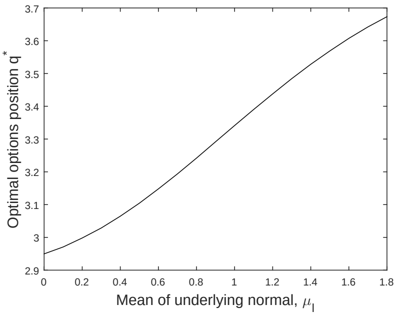


(b) Total cost and savings

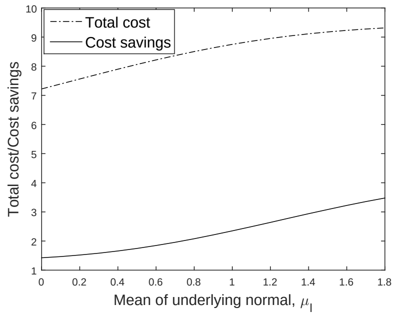
Figure 2.6 Influence of commodity spot price parameters on q^* and costs when option price is fixed

Next, we consider the effect of the freight rate parameters (which are lognormally distributed with parameters μ_l and σ_l) on the firm's hedging policy and costs. Figures 2.7(a) - 2.7(d) illustrate how q^*, $C(q^*), \mathbf{M}, L)$ and $C_R(q^*), \mathbf{M}, L)$ vary as μ_l and σ_l increase. The figures indicate that the freight rate affects the firm's hedging policies and costs the same way the commodity spot price affects them, implying that with regards to hedging and costs, the two are substitutes.

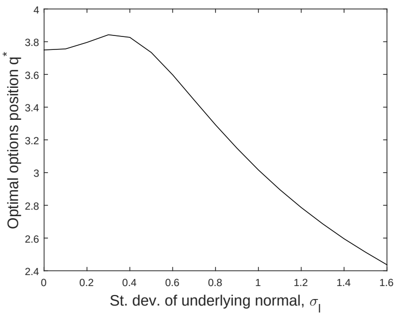
Under Scenario 1, the firm can choose to forgo demand if the unit reservation price L is low enough (compared to the cost of procuring and transporting the commodity at spot prices). We thus explore the impact of L on the firm's hedging policies and costs. Figure 2.8(c) illustrates how q^* changes as L varies. The figure shows that as



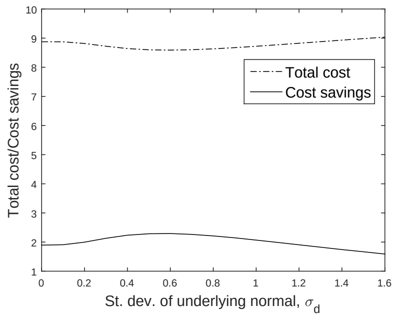
(a) Optimal options position



(b) Total cost and savings



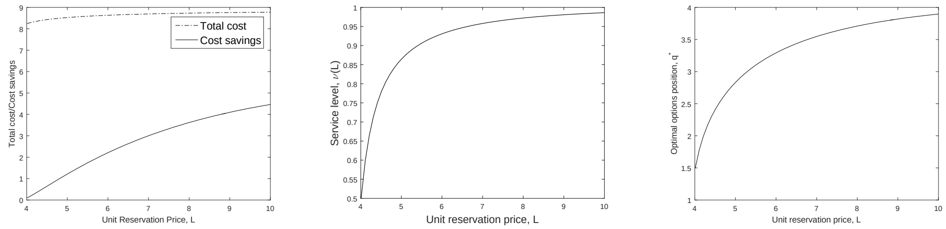
(c) Optimal options position



(d) Total cost and savings

Figure 2.7 Influence of freight rate parameters on q^* and costs

L increases, q^* increases. As L increases, the cost of forgoing demand increases. To limit exposure to drastic increases in the commodity spot price and freight rate, while still meeting as much demand as is cost effective, the firm would opt to negotiate a greater number of options. At lower levels of L, the increase in q^* is steeper. As L increases, q^* increases less quickly and eventually starts to level off. The reason for this behaviour can be ascertained by looking at Figure 2.8(b) which plots service level $\nu(L)$ as L varies. First, observing the figure, we see that as L increases $\nu(L)$ increases, partially as a result of the larger hedging position, and partially as a result of the costs associated with forgoing demand. The figure shows that as L increases, $\nu(L)$ tends to 1. Now, coming back to q^*, at low levels of L, the firm forgoes a larger portion of demand, resulting in a low service level. To increase the service level rapidly, the firm needs to negotiate more options. At higher levels of L, the service level $\nu(L)$ is quite high and close to 1. The firm thus will require fewer additional options to increase or maintain the service level. Thus, at higher levels of L, q^* increases less quickly. The increased reservation price naturally results in increased total costs to the firm, as illustrated in Figure 2.8(a). The figure also shows that increasing L results in increased cost savings as a result of the larger hedging position adopted by



(a) Total cost and savings (b) Service level (c) Optimal options position

Figure 2.8 Influence of unit reservation price on q^*, costs and service level

the firm. The firm's total cost increases much slower than the amount saved by the firm through hedging, implying that hedging can substantially reduce the amount of demand that the firm forgoes.

2.4.2 Comparing costs under Scenarios 1 and 2

Recall that under Scenario 2, the firm sets a 100% service level, either due to the nature of the industry it operates in, or perhaps the firm is a new player in the market and needs to establish itself. Under such a scenario, we showed in Section 2.2 that unlike Scenario 1, where the procurement and transportation departments should jointly decide on a hedging policy, the procurement and transportation departments could decide on their hedging policies independent of one another, due to the separability of the cost function, thus resulting in options positions q_s^* and q_l^* for the procurement department and transportation department respectively. The insights gained in Section 2.4.1 by varying parameters still hold under the current setting. We thus focus on comparing hedging decisions and the resulting costs between the two scenarios.

Using the same parameters as in Section 2.4.1, we solve Equations (2.19) and (2.20) to obtain $q_s^* = 4.4045$ and $q_l^* = 4.5246$. We denote by C_2 the total optimal cost for the firm from adopting these positions. We denote by C_1 the total optimal cost for the firm under Scenario 1. Figure 2.9 plots $C_2 - C_1$ as L varies. We see that at lower levels of L, $C_2 > C_1$, implying that jointly deciding on the hedging policy is beneficial to the firm. As L increases, however, C_1 tends to C_2. In fact, at large levels of L, C_1 is marginally lower than C_2, by about 0.2%. Keeping in mind that we have normalized all prices and rates to 1, this difference could become quite significant. Thus, if we view L as a measure of market competitiveness, this leads us to conjecture that as a market becomes more competitive, procurement and transportation departments should decide hedging policies independently.

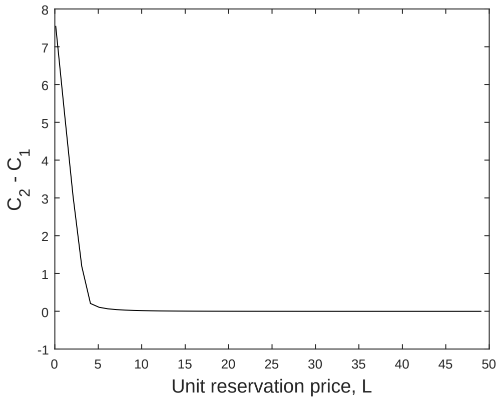
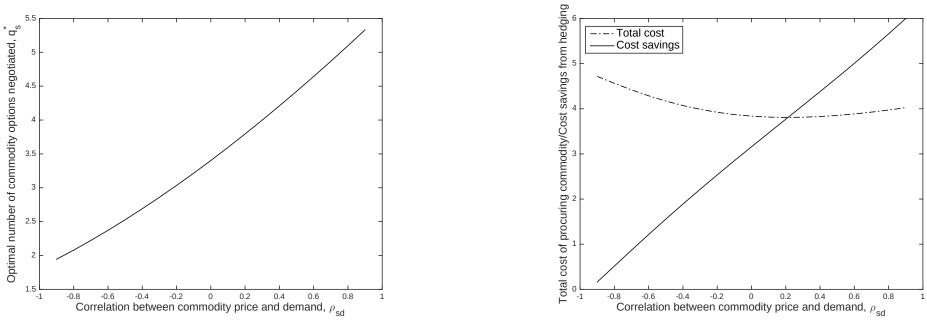


Figure 2.9 Differences in total optimal costs between Scenarios 1 and 2

2.4.3 The impact of correlation under Scenario 3

The previous numerical analyses focused on the cases where the underlying uncertainties were independent of one another. Under Scenario 3, we relaxed this assumption and derived the necessary condition for optimality, assuming dependence between commodity spot price and demand, and freight rate and demand. Dependence between the uncertainties is modelled by way of correlation. We assume a bivariate lognormal distribution for the joint distribution of commodity spot price and demand, and freight rate and demand. The results and insights gained from varying parameters in earlier analysis still hold under Scenario 3. We thus restrict our focus in this section to the numerical exploration of the influence of correlation on the firm's hedging policies and costs. We present results for the correlation between the commodity spot price and demand, as similar results are obtained when varying the correlation between freight rate and demand. This is unsurprising, given our observations in Scenario 1 on the similarities between the impact of commodity spot price on hedging and costs, and the impact of freight rate on hedging and costs.

Figure 2.10(a) shows that as the correlation between commodity spot price and demand, ρ_{sd}, increases, q_s^* increases. To explain this observation, we draw an analogue from portfolio theory. In a portfolio of risky assets, if the correlation between the returns of the assets increases, the portfolio is exposed to greater risk. Similarly, in our case, if we consider the commodity spot price and demand as risky assets, increasing



(a) Optimal options position

(b) Total cost and savings

Figure 2.10 Influence of correlation between spot price and demand on on q_s^* and costs

correlation between them results in increased risk of larger procurement costs. To mitigate this increased risk, the firm needs to adopt a larger hedging position.

Figure 2.10(b) illustrates that as ρ_{sd} increases, the optimal cost of procuring the commodity first decreases, then increases. The cost function does not attain its minimum at either end of the correlation spectrum. Instead, the minimum occurs near the middle of the correlation spectrum. This is somewhat counterintuitive at first when drawing analogues from portfolio theory, where strong negative correlation is preferred to independence. In our case though, with negative correlation, if demand were to decrease, the spot price would increase, also driving up procurement costs. As a result, less dependence between the two is preferable if the focus is on minimizing costs. This phenomenon can also be explained by noting that the increasing optimal options position, corresponding to an increase in the correlation, results in the firm procuring and transporting less of the commodity on the spot market, leading first to a decrease in the total cost. As correlation continues to increase, the costs associated with negotiating and procuring with options increases, causing the total cost to increase as well. When looking at the cost savings, however, increasing ρ_{sd} results in greater cost savings. This is because with positive correlation, increasing demand entails increasing commodity spot prices. The larger options position though helps to mitigate the increased cost of procuring the commodity by allowing the firm to procure the commodity at the strike price K_s. Table 2.1 collects and summarizes the impact of changing the various parameters on the optimal options position q^*, total cost, and the cost savings to the firm.

2.4.4 Managerial insights

Our extensive numerical studies yield four managerial insights. First, our results show that if demand is expected to grow, the firm should negotiate more commodity

Parameters	μ_d	σ_d	μ_s	σ_s	μ_l	σ_l	L	ρ_{sd}
q^*	+	+	+	+	+	+	+	+
Total cost	+	+	+	-	+	-	+	-
Cost saving	+	No effect	+	+	+	+	+	+

Table 2.1 Summary of influence of different parameters on q^*, total cost and cost saving

and freight options. The flexibility afforded by the additional options in the face of increased expected demand offsets the cost of negotiating these additional options. The same holds true if freight rates and/or commodity prices are expected to increase.

Second, when the costs to procure and transport the commodity are highly volatile, the firm should take a larger options position to offset the increased volatility. This result also applies to erratic market conditions, under which spot prices and freight rates would become more volatile. The protection against fluctuations in the spot prices and freight rates leads to value creation for the firm.

Our third insight relates to positively correlated commodity spot prices/freight rates and demand. Our results show that the firm should take a larger options position in such situations. This insight is somehow surprising, because at the first glance, predicting the effect of correlation on the firm's option position is not trivial; however thanks to portfolio theory we are able to give an intuitive explanation for this phenomenon. We know form portfolio theory that higher correlation between risky assets increases the risk of a portfolio. A similar effect can be explained here: when the correlation between demand and commodity price increases, the firm faces higher risk and consequently needs a larger option position to hedge against such risk. Indeed, when procuring and transporting commodities such as crude oil, the cost of procuring the barrels of oil and transporting them via ocean freight is positively correlated with demand. Our numerical studies indicate that under such market conditions, the increased options position can lead to greater value creation for the firm.

Our fourth and final managerial insight relates the firm's reservation price to value creation. Increasing the reservation price naturally leads to higher service levels. Doing so, however, also leads to greater value creation for the firm as the firm takes a larger options position to meet the increased service level. As above, the increased flexibility, combined with the mitigation against volatility, offsets the increased premium paid for the options and results in greater value created for the firm. Thus, as market competitiveness increases, the firm should hedge with options to realize value.

2.5 Conclusion

Freight transportation is playing an ever increasing role in the supply chain of corporations. Motivated by this increased role and interactions with other existing uncertainties in the firms operations, we consider the problem of minimizing the cost of procuring and transporting the commodity by hedging with options. We develop a tractable and parsimonious model that allows us to compute the optimal number of options that the firm should negotiate with both its commodity supplier and the ocean shipping company, closed-form. We consider three scenarios under which the firm could operate. Under the first scenario, the firm can experience lost sales, while under the second scenario, the firm is required to meet all of its demand. These two scenarios assume that the commodity spot price, freight rate and final demand are independent of one another. We show that under the first scenario, the procurement and transportation departments should collaborate on deciding a hedging policy. Even with three sources of uncertainty, we have derived an elegant expression for the optimal options position, in the form of the newsvendor critical fractile. Under the second scenario, we show that the 100% service level requirement results in separability of the cost function and each department independently selecting a hedging policy so as to be able to completely satisfy demand. The optimal options position for each department is again given by the newsvendor critical fractile. In both of these scenarios, no distributional assumption is made on the underlying random variables. In the third scenario, we relax the independence assumption and derive the necessary optimality condition. Our results show that hedging with options results in cost reduction for the firm. Our results make two important contributions. First, we demonstrate that hedging with options does in fact result in cost savings for the firm. Our second contribution is the demonstration of value creation through hedging for a risk-neutral firm. In perfect capital markets, hedging is a zero-sum game, such that the cost of hedging eradicates gains from hedging. In literature, hedging is shown to create value for a risk-neutral firm in the presence of market frictions, or for firms that demonstrate risk-averse behaviour. We have shown that hedging creates value for a risk-neutral firm, without assuming any market frictions. We have shown this to be the case even when the firm pays the fair price for the negotiated option contracts.

Chapter 3

Dual Sourcing: Optimal Procurement Policy with Option Hedging against Freight Rate Risk

In the previous chapter, we consider a commodity processor who sources from a single supplier and show that the European style options can hedge the firm against operational and financial risks. In this chapter, we consider a commodity processor who employs a dual sourcing strategy. In addition to that, this chapter has three more significant differences. First, Chapter 2 allows the firms to negotiate options on both the freight rate and the commodity spot price while the current chapter restricts attention to the influence of transportation costs but not commodity prices. Second, Chapter 2 considers only cargo ships while the current chapter considers both cargo ships and container ships. Finally, Chapter 2 allows the partial hire of ships by allowing the firm to negotiate a non-negative, non-integer number of options while the current chapter imposes an integer constraint on option negotiation.

3.1 Introduction

Studies show that sourcing from two suppliers can improve lead-time performance and reduce total inventory cost (Tyworth & Ruiz-Torres 2000). With increased globalization, raw materials may be procured either domestically or from offshore locations. The sourcing decision essentially boils down to a question of cost. From a firm's perspective, raw materials typically cost less to procure in offshore markets as opposed to domestic markets. For example, oil producers have agreements amongst themselves to sell crude oil to each other domestically at higher prices in case of shortage. Though less expensive, raw materials procured in offshore markets need to be transported to a firm's domestic processing plants. Thus, when deciding on a dual-sourcing strategy, a firm must also take into account the relevant transportation costs. Studies in the literature incorporate transportation costs implicitly as an element of the cost of placing an order (Tyworth & Ruiz-Torres 2000). This treatment of transportation costs overlooks the volatility inherent in the cost of some forms of transportation, such as ocean freight, which has evolved to become the main form of global transportation. Nomikos et al. (2013) state that increased globalization and the increased demand for ocean-based transportation of commodities has resulted in ocean freight becoming a highly volatile commodity in its own right. Hence, a firm with two sourcing choices needs to consider not only the price difference in the cost of acquiring the raw material, but also the volatile freight rate. Financial contracts such as options, however, can help the firm to hedge against the risk of volatile freight rates. According to a survey of large US non-financial firms (Bodnar et al. 1995), approximately 40% of firms routinely purchase options or futures contracts in order to hedge price risks. Motivated by the increasing role that freight transportation plays in dual sourcing and a firm's tendency to use financial contracts in order to hedge against price risks, the objective of the current research is to minimize the total expected procurement and transportation costs of a firm that employs a dual-sourcing strategy. The minimization of these costs first involves determining the optimal number of options to negotiate, and deriving an optimal hedging policy conditional on the number of options negotiated.

We consider a commodity processor, such as an oil producer or flour miller, that requires commodities such as oil or wheat as inputs for its processing plant, where the commodity is converted to a final product. The firm can procure the commodity from two sources. First, it can buy from an offshore supplier and transport it via ocean freight to its processing plant. Second, the firm can buy the commodity from a local supplier close to its processing plant at a higher price. Consequently, the firm does not pay for ocean transportation. In the setting that we consider,the firm faces three types of uncertainty: (1) demand for the final product, (2) the cost of procuring the commodity on the spot market, and (3) the cost of ocean transport of the commodity to its production facility. The firm can negotiate procurement options (similar in style to financial European call options) with an ocean shipping company. These options allow the firm to hedge the uncertainty associated with the freight rate by reserving the right to charter the ship at a pre-specified strike price. The firm pays the shipping company a premium to reserve this flexibility.

Lee et al. (2015) point out that some shippers are reluctant to purchase freight services from carriers in advance due to the volatility inherent in the freight rates unless the carrier is willing to partially match the realized spot freight rate. Options on the freight rate would circumvent these issues by allowing shippers to reserve capacity in advance with options and giving them the flexibility to exercise the options based on spot price realizations, thereby not requiring partial price matching guarantees from carriers. From the carrier's perspective, in addition to receivng a premium for the options, information about capacity requirements will also be revealed. These freight options, however, are not standardised contracts that are traded on exchanges. These are Over-the-counter (OTC) instruments that are negotiated by both the commodity processor and the ocean shipping company. As such, market forces do not determine prices of these options. To preserve focus on the influence of transportation, we do not consider options on the required commodity. If the underlying commodity is traded on an exchange, options on the commodity would be traded on an exchange as well. Market forces would determine the premiums to be paid for these options. To preclude arbitrage, these prices would be determined based on the expected gain from optimally exercising the option. As has been established in the literature, a risk-neutral firm would therefore not take a position in options on the commodity. Since we are focusing on exchange-traded commodities, such as oil and wheat, we consider only options on the freight rate and assume that the firm procures the necessary amount of the raw material on the spot market.

We consider two types of cargo ships: tankers and container ships. Tankers are cargo ships that are typically used to transport fluids such as crude oil. Container ships are cargo ships that carry their entire load in truck-size containers standardized at 20 and 40 feet, with fixed dimensions and properties Fransoo & Lee (2013). Oil firms typically charter tankers, while flour mills might either charter a container ship with its all containers, or rent one or more containers instead. When hiring tankers, a firm can only hire an entire ship. Partial hire of a tanker is not possible. Container ships, on the other hand, can be partially hired since the containers on the ship are hired, as opposed to the entire ship itself. In this work, we consider both categories and develop optimal hedging policies for both.

The research makes three contributions. The first is the development of a dual-sourcing model that integrates a firm's optimal hedging decision explicitly taking into account the transportation cost and the constraint that only an integer number of ships can be hired. Second, we extend the literature on ocean container transport. Fransoo & Lee (2013) highlight four issues that need to be addressed in the domain of ocean container transport. One of these issues relates to contracting, pricing and risk management along the container supply chain. This chapter directly addresses this point. Third, we numerically demonstrate with the developed model that (1) transportation costs need to be explicitly accounted for in models on dual-sourcing strategies, (2) dual sourcing is indeed more effective than single sourcing by limiting the firm's price risk exposure and simultaneously offering flexibility to take advantage of significant price differences in the commodity, and (3) that chartering large ships, while more expensive per ship, lowers the total expected cost for the firm.

The chapter is structured as follows. Section 3.2 reviews the related literature. Section 3.3 develops the mathematical model for minimizing the total procurement and transportation cost for a firm that charters tankers. Section 3.4 presents the model for firms that utilize container shipping. Section 3.5 provides some numerical examples and experiments, and finally Section 3.6 concludes with a summary and discussion.

3.2 Related Literature

Our work is related to three streams of literature. The first stream focuses on dual sourcing, the second focuses on ocean freight, and the third stream focuses on option hedging.

The literature on dual-sourcing dates back to 1961. Barankin (1961) studies an inventory model in which there is a fixed time-lag of one period for delivery of orders, but in which there is also defined an emergency situation. When this situation occurs, an order is taken by immediate delivery with higher cost. Multi-sourcing strategies can be used for several purposes. Burke et al. (2007), Tomlin & Wang (2005), Yang et al. (2012) and Federgruen & Yang (2009) study multi-sourcing in the context of hedging against supplier unreliability. These papers focus on multi-sourcing strategies under supply chain risks. Several papers study sourcing and inventory related costs. Allon & Van Mieghem (2010) considers a firm that has access to a responsive nearshore source (e.g., Mexico) and a low-cost offshore source (e.g., China). The firm must determine an inventory sourcing policy to satisfy random demand over time. Janssen & de Kok (1999) provide an analysis of the two supplier model. They develop an algorithm for the determination of the decision parameters S and Q such that the long-run expected average costs per time unit are minimized subject to a service level constraint. Moinzadeh & Nahmias (1988) consider two supply sources, with one having a shorter lead time and propose a reasonable extension of the standard (Q, R) policy to allow for two different lot sizes, and two different reorder levels, comparing the average annual cost with and without emergency ordering. Under the assumption of relatively small demand and fixed ordering costs (compared to the holding cost) Moinzadeh & Schmidt (1991) propose a more sophisticated policy in which real time supply information on the age of all outstanding orders and the inventory level is used. Song & Zipkin (2009) extend Moinzadeh & Schmidt (1991) by considering a system with multiple supply sources under Poisson demand and stochastic lead times. The literature on dual-sourcing has started focusing on the differences in lead time and cost between nearshore sources and offshore sources. These factors become even more important when the cost of shipping goods around the world started to rise sharply and goods spent weeks in transit (Boute & Van Mieghem 2014).

Another stream of literature which our work relates to is ocean shipping. As a result of offshoring in the last decade, the global container trade has been growing excessively (Fransoo & Lee 2013). Nearly 90 percent of international trade involves ocean transportation (Yercan & Yildiz 2012). Fransoo & Lee (2013) investigates the critical role of ocean shipping in global supply chain performance, and conclude that, although ocean transportation is a well-studied research area in maritime economics, many supply chain management issues arising from ocean freight are not well understood. The impact of ocean container transport on supply chain performance and supply chain decision making remains significantly understudied compared to the impact of other aspects such as inventory management or road transport. Lee et al. (2015) state that unlike the commercial airline industry that has a well-established process to manage dynamic pricing and overbooking, the ocean transportation industry lacks a systematic approach for examining pricing and contracting issues. They state that the issue of pricing and contracting arising from ocean freight is an emergent topic that practitioners have begun to examine. The supply chain contracting literature is mostly about contracts with an inventory risk (Cachon 2003). There is literature about other industries related to logistics, including, for example, airline ticket contracts for large corporations (Pachon et al. 2007), bus service contracts (Hensher & Houghton 2004, Hensher & Stanley 2003), and contracts and regulations for road franchising (Tan et al. 2010). However, there is the little literature studying ocean shipping contracts between different parties. Garrido (2007) studies an electronic bidding system in which the shippers contract each shipment to a single carrier following an open auction, and a shipper selects the carrier based on the best bidding price. Lim et al. (2008) studied a shipper's transportation procurement model, in which the shipper gives assurances, through volume guarantees negotiated with the transportation companies, that shipments made in nonpeak periods will be commensurate with shipments in peak periods. The shipper uses the model in an auction process, in which the transportation companies bid for routes giving prices and capacity limits, to procure freight services from the companies, which minimizes its total transportation costs. Kavussanos & Visvikis (2004) investigates the impact of the introduction of Forward Freight Agreement (FFA) trading on spot market price volatility in two panamax and two panamax Pacific (2 and 2A) trading routes of the dry-bulk shipping industry. Koekebakker et al. (2007) studies the freight derivatives market, setting up the theoretical framework for the valuation of the Asian-style options traded in the freight derivatives market. Our research studies ocean transportation from a price and contracting aspect. The uncertain freight rate, however, is a potential risk in global sourcing, it can be hedged with option contracts.

Our work is also related to papers considering the use of derivatives in supply chain contracting. We have extensively discussed this stream of literature in Chapter 1.

3.3 Model for Hiring Tankers

In this section, we detail the optimization problem facing the firm with regards to the number of freight options contracts that it should negotiate when hiring tankers. Recall that tankers are primarily used to transport fluids such as crude oil and cannot be partially hired, implying that a firm must hire an integer number of tankers. We focus on minimizing the firm's procurement and transportation costs with three stochastic parameters: demand, commodity spot price, and freight rate.

We frame the firm's optimization problem in a single-period model. The firm produces a single type of product and faces stochastic demand for this product type. The firm can procure the commodity required to produce this product from an offshore supplier at a stochastic price. Once procured, the firm transports the commodity via ocean freight, at stochastic freight rates, to its production facility where the commodity is converted to the final product to meet demand. The firm can also procure the commodity at a higher price in the local market, with no ocean transportation needed. For simplicity, we assume that a single unit of the commodity is required to produce a single unit of the product. We further assume zero production and transportation lead times. Each tanker has a capacity of β, where the unit depends on the commodity. For example, if the commodity is a fluid like crude oil, β is measured in gallons. Since partially hiring a tanker is not possible, the firm must hire an entire tanker, even if only part of the tanker is needed. This constraint reflects practice, where firms, especially oil producers, generally charter the tankers.

At the start of the period, at time $t = 0$, the firm negotiates options with the freight company to hire the tanker, taking into account the expected final demand for the product. We denote the strike price of this option to be $K_\lambda \in \mathbb{R}_+$. Thus, if the option is exercised, the firm pays K_λ for each tanker, regardless of the tanker's negotiated spot freight rate. The firm pays the shipping company a premium of $P_\lambda \in \mathbb{R}_+$ for each option. The number of options the firm negotiates with each party depends on four factors; the terms of the options (strike price and premium), the firm's expectation of final demand for its product, the firm's expectation of the commodity's spot price, and the firm's expectation of the freight rate. We represent by $\phi_d(\cdot)$, $\phi_s(\cdot)$, and $\phi_\lambda(\cdot)$ the *marginal* probability density functions of final demand, the commodity's spot price and the freight rate, respectively. To simplify notation, we denote by $\mathbf{M}$ the set of these marginal distributions. At the end of the period, at time $t = T$, all uncertainties are resolved and the firm observes final demand $d_T \in \mathbb{R}_+$, the commodity spot price $s_T \in \mathbb{R}_+$ and freight rate $\lambda_T \in \mathbb{R}_+$. We assume that the firm operates in a competitive market and therefore has a 100 percent service level, meaning that the firm will meet all demand at time $t = T$. The firm decides how much of the required commodity should be procured from the offshore supplier. The amount procured from the offshore supplier is then transported by tanker. The firm needs to decide whether to exercise the freight options or to hire tankers on the spot market. The firm decides at time T how many of the freight options procured at time 0 it will exercise at the strike price K_λ, and how many tankers will be hired on the spot market at price λ_T. We denote by s_T the unit spot price of the commodity at time T. Procuring a unit of the commodity in the offshore market entails a unit cost of s_T for the firm. Procuring a unit of the commodity in the local market entails a higher cost of $(1+\alpha)s_T$, where $\alpha > 0$ denotes the markup in the domestic market. Naturally, units of the commodity procured in the domestic market need not be transported. The firm can simultaneously procure in both the offshore and domestic markets to satisfy demand.

Under the given setting and notation, the firm's optimization problem can then be written as

$$\begin{aligned}\min_{q\in\mathbb{Z}_+} C_1(q,\mathbf{M}) = \quad & \mathbb{E}_{d,s,\lambda}\left[(\min(\lambda_T, K_\lambda, \beta\alpha s_T))\min(z,q)\right] \\ + \quad & \mathbb{E}_{d,s,\lambda}\left[(z-q)^+\min(\lambda_T,\beta\alpha s_T)\right] \\ + \quad & \mathbb{E}_{d,s,\lambda}\left[\min\left((q-z)^+,1\right)\min(\theta\alpha s_T,\lambda_T,K_\lambda)\right] \\ + \quad & \mathbb{E}_{d,s,\lambda}\left[\min\left((z-q+1)^+,1\right) min(\theta\alpha s_T,\lambda_T)\right] \\ + \quad & \mathbb{E}_{d,s}\left[d_T s_T\right] + qP_\lambda.\end{aligned} \tag{3.1}$$

where

$$z = \left\lfloor \frac{d_T}{\beta} \right\rfloor \quad \text{and,} \tag{3.2}$$

$$\theta = d_T - z\beta. \tag{3.3}$$

The variable z denotes the number of full tankers that could *potentially* fulfill all or some of the demand. The decision to charter tankers also depends on factors such as realized commodity spot price and spot freight rate. The variable θ defined in Equation (3.3) denotes the portion of demand which might not be fulfilled by chartering z full tankers. The amount procured would be less than the capacity of a tanker, i.e., $0 \leq \theta < \beta$. This portion of the demand could be procured in both markets, but if the firm procures it in the offshore market, the firm has to hire a tanker even though $\beta - \theta$ of the tanker's capacity would not be utilized. The function $C_1(q,\mathbf{M})$ represents the combined expected procurement and transportation costs and consists of four parts. The first part reflects the expected cost of transporting the commodity procured in the offshore market at time T. The firm exercises at most z options if it negotiated a sufficient number of options at time 0, and the strike price for each tanker, K_λ, is less than both the spot market price λ_T, and the extra local market cost $\beta\alpha s_T$. The firm pays the strike price K_λ for each option (each option is equal to one tanker). Otherwise the firm either hires the tanker on the spot market λ_T, or buys the commodity on the local market and pays the extra local cost $\beta\alpha s_T$. The term $(z-q)^+$ reflects demand not covered by the options due to the insufficient number of available options. In order to meet the demand, the firm can only choose between hiring a tanker on the spot market or buy the commodity in the local market. The second part reflects the expected cost of transporting the portion of demand not covered by $z\beta$. For $0 < \theta < \beta$, the firm has three options. If the firm still has unexercised options, the firm can compare the option strike price K_λ with the spot market price λ_T and the local market premium $\theta\alpha s_T$ to choose the most cost-effective alternative. If the firm has exercised all options, it can choose only between hiring one tanker on the spot market, or buy θ units of the commodity on the local market. If the firm decides to use ocean transportation for the θ part of demand, the remaining capacity of the tanker would be unutilized. The third part $\mathbb{E}_{d,s}\left[d_T s_T\right]$ reflects the total procurement cost. In any possible scenario at time T, the firm must pay $d_T s_T$ for the procurement cost in order to meet all demand, due to the 100 percent service level requirement. The fourth part qP_λ reflects the premium paid at time 0 for the options. This premium is negotiable between the firm and its ocean service provider and is thus taken to be exogenous to our model.

3.3.1 Determining the Optimal Options Position at Time 0

In this section we solve the firm's optimization problem with respect to q. As mentioned in the previous section, the firm cannot partially hire a tanker. Mathematically this implies that $q^* \in \mathbb{Z}^+$, i.e., the firm can only negotiate a non-negative integer number of options. We need to show that the objective function is convex, and consequently, that it has just one global minimum. To show this, we use the definition of convexity in integer functions from Van Houtum & Kranenburg (2015). Based on this definition, we need to show that $\Delta^2 C_1(q) = \Delta C_1(q+1) - \Delta C_1(q)$ is positive for all q, and then the minimum point would be the smallest q such that $\Delta C_1(q) = C_1(q+1) - C_1(q) > 0$.

$$q* = \min_{q \in \mathbb{Z}^+} \Delta C_1(q) > 0 \tag{3.4}$$

Lemma 3.1 *The cost function $C_1(q, \mathbf{M})$ is convex in q.*

Proof: We show the convexity of $C_1(q, \mathbf{M})$ by showing that $\Delta^2 C(q) \geq 0$ for all $q \in \mathbb{Z}_+$. Since s_T, λ_T, and d_T are independent, we rewrite Equation (3.1) as follows:

$$\begin{aligned} C_1(q) = \quad & \mathbb{E}_d\left[\min(z,q)\right]\mathbb{E}_{s,\lambda}\left[\min(\lambda_T, K_\lambda, \beta\alpha s_T)\right] \\ + \quad & \mathbb{E}_d\left[(z-q)^+\right]\mathbb{E}_{s,\lambda}\left[\min(\lambda_T, \beta\alpha s_T)\right] \\ + \quad & \mathbb{E}_d\left[\min\left((q-z)^+, 1\right)\right]\mathbb{E}_{s,\lambda}\left[\min(\theta\alpha s_T, \lambda_T, K_\lambda)\right] \\ + \quad & \mathbb{E}_d\left[\min\left((z-q+1)^+, 1\right)\right]\mathbb{E}_{s,\lambda}\left[min(\theta\alpha s_T, \lambda_T)\right] \\ + \quad & \mathbb{E}_{d,s}\left[d_T s_T\right] + qP_\lambda. \end{aligned}$$

Then, $\Delta C_1(q) = C(q+1) - C(q)$

$$\begin{aligned} \Delta C_1(q) = \quad & \mathbb{E}_d\left[\min(z,q+1) - \min(z,q)\right]\mathbb{E}_{s,\lambda}\left[\min(\lambda_T, K_\lambda, \beta\alpha s_T)\right] \\ + \quad & \mathbb{E}_d\left[(z-q-1)^+ - (z-q)^+\right]\mathbb{E}_{s,\lambda}\left[\min(\lambda_T, \beta\alpha s_T)\right] \\ + \quad & \mathbb{E}_d\left[\min\left((q+1-z)^+, 1\right) - \min\left((q-z)^+, 1\right)\right]\mathbb{E}_{s,\lambda}\left[\min(\theta\alpha s_T, \lambda_T, K_\lambda)\right] \\ + \quad & \mathbb{E}_d\left[\min\left((z-q)^+, 1\right) - \min\left((z-q+1)^+, 1\right)\right]\mathbb{E}_{s,\lambda}\left[min(\theta\alpha s_T, \lambda_T)\right] \\ + \quad & P_\lambda. \end{aligned} \tag{3.5}$$

Using the demand probability distribution function ϕ_d and finding the expectation on the demand, we obtain

$$\begin{aligned}\mathbb{E}_d\left[\min(z, q+1) - \min(z, q)\right] &= \phi_d(z > q),\\ \mathbb{E}_d\left[(z-q-1)^+ - (z-q)^+\right] &= -\phi_d(z > q),\\ \mathbb{E}_d\left[\min\left((q+1-z)^+, 1\right) - \min\left((q-z)^+, 1\right)\right] &= \phi_d(z = q),\\ \mathbb{E}_d\left[\min\left((z-q)^+, 1\right) - \min\left((z-q+1)^+, 1\right)\right] &= -\phi_d(z = q).\end{aligned}$$

We introduce the following notation for the expectations with respect to s and λ for expositional ease.

$$A = \mathbb{E}_{s,\lambda}\left[\min(\lambda_T, K_\lambda, \beta\alpha s_T)\right] \quad (3.6)$$

$$B = \mathbb{E}_{s,\lambda}\left[\min(\lambda_T, \beta\alpha s_T)\right] \quad (3.7)$$

$$C = \mathbb{E}_{s,\lambda}\left[\min(\theta\alpha s_T, \lambda_T, K_\lambda)\right] \quad (3.8)$$

$$D = \mathbb{E}_{s,\lambda}\left[min(\theta\alpha s_T, \lambda_T)\right]. \quad (3.9)$$

Simplifying and rewriting Equation (3.5), we obtain

$$\Delta C_1(q) = (C-D)\phi_d(z = q) + (A-B)\phi_d(z > q) + P_\lambda. \quad (3.10)$$

Denoting by $\Phi_d(x)$ the cumulative distribution function for demand, we can rewrite and simplify Equation (3.10) to obtain

$$\begin{aligned}\Delta C_1(q) = &\quad (C-D-A+B)\Phi_d(\beta(q+1))\\ &- \quad (C-D)\Phi_d(\beta q) + A - B + P_\lambda.\end{aligned} \quad (3.11)$$

Since $\Delta^2 C(q) = \Delta C(q+1) - \Delta C(q)$, using Equation (3.11) we have

$$\begin{aligned}\Delta^2 C_1(q) = &\quad (C-D-A+B)[\Phi_d(\beta(q+2)) - \Phi_d(\beta q+1)]\\ &+ \quad (D-C)[\Phi_d(\beta(q+1)) - \Phi_d(\beta q)].\end{aligned} \quad (3.12)$$

If we examine $\Delta^2 C_1(q)$ in Equation (3.12) component by component, we obtain

$$\begin{aligned}C-D-A+B = \mathbb{E}_{s,\lambda}\left[\min(\theta\alpha s_T, \lambda_T, K_\lambda)\right] - \mathbb{E}_{s,\lambda}\left[min(\theta\alpha s_T, \lambda_T)\right],&\\ -\mathbb{E}_{s,\lambda}\left[\min(\lambda_T, K_\lambda, \beta\alpha s_T)\right] + \mathbb{E}_{s,\lambda}\left[\min(\lambda_T, \beta\alpha s_T)\right] &\geq 0,\\ D-C = \mathbb{E}_{s,\lambda}\left[min(\theta\alpha s_T, \lambda_T)\right] - \mathbb{E}_{s,\lambda}\left[\min(\theta\alpha s_T, \lambda_T, K_\lambda)\right] &\geq 0,\\ F_d(\beta(q+2)) - F_d(\beta q+1) &\geq 0,\\ F_d(\beta(q+1)) - F_d(\beta q) &\geq 0.\end{aligned}$$

This implies that $\Delta^2 C(q, \mathbf{M})$ is non-negative for all $q \in \mathbb{Z}+$. Hence, $C(q, \mathbf{M})$ is convex and completes the proof. □

Theorem 3.1 *The firm's optimal options position at time 0, q^* is given by*

$$q^* = \min_{q \in \mathbb{Z}} (C - D - A + B)\Phi_d(\beta(q+1)) - (C - D)\Phi_d(\beta q) + A - B + P_\lambda > 0 \quad (3.13)$$

with A, B, C, and D, as in Equations (3.6) - (3.9).

Proof: We first show that the smallest q satisfying the inequality in Equation (3.13), denoted by q^*, is a stationary point of $C_1(q, \mathbf{M})$. To obtain the first order condition for optimality, we set Equation (3.11) greater than 0 and solve for the smallest q such that the inequality in Equation (3.13) holds. Since $\Delta^2 C_1(q, \mathbf{M})$ is non-negative for all q as a consequence of the convexity of the cost function, the stationary point q^* minimizes $C_1(q, \mathbf{M})$. □

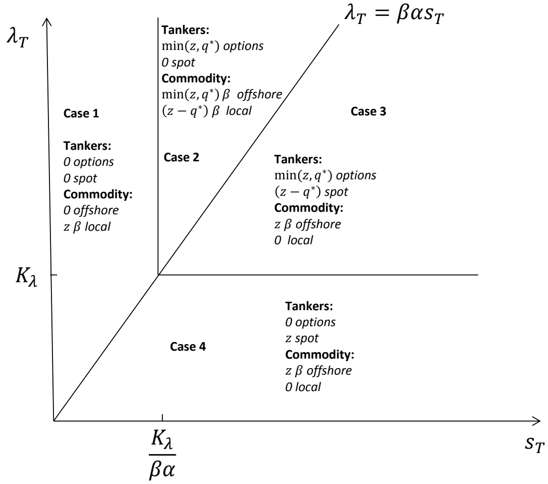
3.3.2 Optimal Policy at Time T

In this section, we characterize the optimal policy for exercising options, hiring tankers on the spot market, and procuring commodity to meet realized demand. In the previous section we solved an optimization problem which gave us the optimal option position at time 0 (q^*). In this section, we assume that q^* is given and describe the optimal policy at time T. At time T, the firm observes demand d_T, and needs to meet all demand. The firm needs to make 4 decisions: 1) How many options to exercise. 2) How many tankers to hire on the spot market?3) How much of the commodity to procure in the offshore market and 4) How much of the commodity to procure in the domestic market. As in the previous section, the firm needs to divide the total demand into two parts $z\beta$ (offshore procurement with z tankers) and θ (excess demand not covered by z tankers) and find the optimal policy based on these two variables. In addition to demand, two other stochastic variables s_T and λ_T are realized at time T and will influence the firm's decisions. We consider all eventual scenarios at time T, and describe the optimal actions that the firm needs to take in each scenario. The optimal policy dictates how many options the firm should exercise, how many tankers to hire at the spot freight rate, how much of the commodity to procure in the offshore market and how much to procure in the domestic market. To this end, in order to characterize the optimal policy at time T we define the following variables:

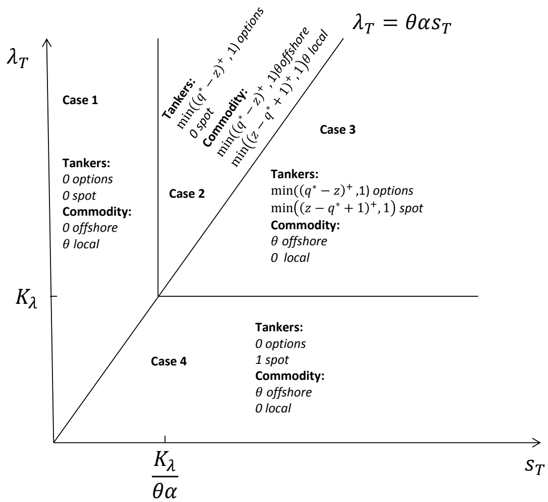
O_{Tz}: The number of options the firm should exercise in order to meet the demand that can be fulfilled with z tankers.

$O_{T\theta}$: The number of options the firm should exercise in order to meet the excess demand that cannot be fulfilled with z tankers.

O_T: $O_{Tz} + O_{T\theta}$



(a) Optimal policy for hiring full tankers, z



(b) Optimal policy for procuring excess demand, θ

Figure 3.1 Optimal policy for procuring demand

We similarly define the variables F_{Tz}, $F_{T\theta}$, F_T as the number of tankers that the firm has to hire at the spot freight rate, a_{Tz}, $a_{T\theta}$, a_T as the units of commodity that the firm has to procure in the offshore market, and b_{Tz}, $b_{T\theta}$, b_T as the units of commodity that the firm has to procure in the domestic market. With these variables defined, we are now in a position to present the optimal policy for exercising options, hiring tankers in the spot market, and procuring demand.

Theorem 3.2 *Given the solution to Equation(3.1), q^*,*

(a) and z as defined in Equation(3.2),

- *(i) if $\beta\alpha s_T \leq \lambda_T$ and $\beta\alpha s_T \leq K_\lambda$ then $O_{Tz} = 0$, $F_{Tz} = 0$, $a_{Tz} = 0$, and $b_{Tz} = z\beta$,*
- *(ii) if $\beta\alpha s_T \leq \lambda_T$ and $\beta\alpha s_T > K_\lambda$ then $O_{Tz} = \min(z, q^*)$, $F_{Tz} = 0$, $a_{Tz} = \min(z, q^*)\beta$, and $b_{Tz} = (z - q^*)^+\beta$,*
- *(iii) if $\beta\alpha s_T > \lambda_T$ and $K_\lambda \leq \lambda_T$ then $O_{Tz} = \min(z, q^*)$, $F_{Tz} = (z - q^*)^+$, $a_{Tz} = z\beta$, and $b_{Tz} = 0$,*
- *(iv) if $\beta\alpha s_T > \lambda_T$ and $\lambda_T < K_\lambda$ then $O_{Tz} = 0$, $F_{Tz} = z$, $a_{Tz} = z\beta$, and $b_{Tz} = 0$,*

(b) and θ as defined in Equation(3.3),

- *(i) if $\theta\alpha s_T \leq \lambda_T$ and $\theta\alpha s_T \leq K_\lambda$ then $O_{T\theta} = 0$, $F_{T\theta} = 0$, $a_{T\theta} = 0$, and $b_{T\theta} = \theta$.*
- *(ii) if $\theta\alpha s_T \leq \lambda_T$ and $\theta\alpha s_T > K_\lambda$ then $O_{T\theta} = \min((q^* - z)^+, 1)$, $F_{T\theta} = 0$, $a_{T\theta} = \min((q^* - z)^+, 1)\theta$, and $b_{T\theta} = \min((z - q^* + 1), 1)\theta$.*
- *(iii) if $\theta\alpha s_T > \lambda_T$ and $K_\lambda \leq \lambda_T$ then $O_{T\theta} = \min((q^* - z)^+, 1)$, $F_{T\theta} = \min((z - q^* + 1), 1)$, $a_{T\theta} = \theta$, and $b_{T\theta} = 0$.*
- *(iv) if $\theta\alpha s_T > \lambda_T$ and $\lambda_T < K_\lambda$ then $O_{T\theta} = 0$, $F_{T\theta} = 1$, $a_{T\theta} = \theta$, and $b_{T\theta} = 0$.*

PROOF: We prove each assertion sequentially. The cases are also highlighted in Figure 3.1.

Case 1: $\beta\alpha s_T \leq \lambda_T$ and $\beta\alpha s_T \leq K_\lambda$
procuring in the domestic market is the more cost-effective option. The firm should procure $z\beta$ units of the commodity domestically. Thus, the firm should not exercise any options, yielding $O_{Tz} = 0$. As a consequence of domestic procurement, the firm need not charter any tankers,thus $F_{Tz} = 0$. Since $z\beta$ units of the commodity are procured domestically, $a_{Tz} = 0$, and $b_{Tz} = z\beta$.

Case 2: $\beta\alpha s_T \leq \lambda_T$ and $\beta\alpha s_T > K_\lambda$
Exercising the options is the most-effective option, so $O_{Tz} = \min(z, q^*)$. The portion of demand that cannot be met using options should be procured in the domestic

market rather than on the offshore market. Thus, $F_{Tz} = 0$, $a_{Tz} = \min(z, q^*)\beta$, and $b_{Tz} = (z - q^*)^+\beta$.

Case 3: $\beta\alpha s_T > \lambda_T$ and $K_\lambda \leq \lambda_T$
Exercising the options is the most cost-effective option. The number of available options is, however, limited to q^*. Furthermore, the firm should hire the tankers on the spot market for the remaining part, $(z - q^*)\beta$. This implies that $O_{Tz} = \min(z, q^*)$ and $F_{Tz} = (z - q^*)^+$. On the other hand, since $\beta\alpha s_T > \lambda_T$, the commodity should be procured in the offshore market. Thus, $a_{Tz} = z\beta$, and $b_{Tz} = 0$.

Case 4: $\beta\alpha s_T > \lambda_T$ and $\lambda_T < K_\lambda$
Chartering tankers on the spot market is the most cost-effective option. The firm should charter the required tankers on the spot market rather than excising the available options, implying that $O_{T\theta} = 0$, $F_{T\theta} = 0$. And since $\beta\alpha s_T > \lambda_T$, $z\beta$ units of the commodity should be procured in the offshore market. Thus $a_{Tz} = z\beta$, and $b_{Tz} = 0$.

One can do the same case-based analysis for the part of demand that cannot be covered by z full tankers, the θ part, to find $O_{T\theta}, F_{T\theta}, a_{T\theta}$, and $b_{T\theta}$. The only difference would arise in the number of tankers. For this portion of demand, the firm would need to charter at most one tanker for the transportation. Finally, we obtain the total number of the options to exercise, the total number of tankers to charter, and can determine where the commodity should be sourced from, yielding $O_T = O_{Tz} + O_{T\theta}$, $F_T = F_{Tz} + F_{T\theta}$, $a_T = a_{Tz} + a_{T\theta}$, and $b_T = b_{Tz} + b_{T\theta}$. This completes the proof. □

Theorem 3.2 characterises the optimal policy by considering all potential scenarios at time T, and choosing the action with the lowest cost. Figure 3.1 illustrates this policy in two parts. Figure 3.1(a) shows the optimal policy for the part of demand that can be fulfilled with z full tankers. Procuring the commodity in the domestic market entails an extra cost of $\beta\alpha s_T$. When deciding between offshore and domestic procurement, this extra cost must be compared against the transportation cost λ_T. The diagonal line $\lambda_T = \beta\alpha s_T$ in Figure3.1(a) reflects this. On this line, the cost of hiring a full tanker is equal to procuring β units of the commodity in the domestic market. The diagonal line characterizes the optimal policy based on cases 1-4 which have been explained in the proof Theorem 3.2. Similarly Figure 3.1(b) shows the policy for the part of demand that cannot be fulfilled with z full tankers.

3.4 Model for Hiring Container Ships

Many commodities like wheat are transported by containers. For these types of commodities, the firm will charter a ship and all the containers on it if it can fill the ship's capacity. Otherwise the firm will partially charter the ship, meaning that it will rent some of the containers on the ship. In this section we consider the other major form of ocean freight transportation and investigate the role of option contracts in container shipping. We still focus on minimizing the firm's procurement and transportation costs and formulate the optimization problem as a classic newsvendor model with three stochastic parameters. In this section, chartering a ship, means chartering a ship with all the containers on it.

The assumptions made in the previous section hold. A few changes are required to accommodate the context of container shipping. We assume each ship has γ containers. The firm will charter/hire an entire ship if it needs to use all γ containers. Otherwise, the firm rents the required number of containers. We assume that the cost of renting a container is fixed at Γ. Naturally, chartering a ship would be more cost effective than renting all γ containers at Γ per container. We thus assume that Γ is such that $\gamma\Gamma \geq \lambda_T$.

In the previous sections, demand and commodity procurement were in terms of units of the commodity. In this section, to reflect practice, we assume that demand and procurement are in terms of numbers of containers. This assumption reflects practice since the capacity of any container is small enough that the cost associated with not entirely filling the capacity of a container is negligible.

As in Section 3.3, at the start of the period, time $t = 0$, the firm negotiates options with the freight company to charter a ship. The firm, however, cannot procure options on individual containers. We again denote the strike price of this option to be K_λ. The option premium is also denoted as P_λ. At the end of the period, at time $t = T$, all uncertainties are resolved and the firm observes final demand d_T and the commodity spot price s_T. Recall that demand and procurement are now in terms of numbers of containers. In a slight abuse of notation, we denote by d_T and s_T the realized demand and commodity spot price per container, respectively. Thus d_T now refers to the numbers of containers demanded. Similarly, s_T refers to the spot price of the commodity per container of the commodity. Finally, the firm observes the spot freight rate, λ_T, per ship (with γ containers). As stated above, the rental price of each container is fixed at Γ where $\lambda_T < \gamma\Gamma$. We still assume that the firm achieves a service level of 100 percent, implying that the firm will meet all realized demand at time $t = T$. The firm then decides where to procure the commodity from and how to transport commodity procured in offshore markets about the procurement and the transportation policy. The firm can either procure the commodity in the offshore market and utilize ocean transportation with some or all the containers on a ship, or procure the commodity in the domestic market without the need for ocean transportation. If procuring in and transporting from an offshore market, the firm could either utilize options, or pay the spot freight rate.

The firm's optimization problem with regards to container shipping can now be written as

$$\begin{aligned}\min_{q\in\mathbb{Z}_+} C_2(q,\mathbf{M}) = \quad & \mathbb{E}_{d,s,\lambda}\left[(\min(\lambda_T, K_\lambda, \gamma\alpha s_T))\min(z',q)\right] \\ + \quad & \mathbb{E}_{d,s,\lambda}\left[(z'-q)^+\min(\lambda_T,\gamma\alpha s_T)\right] \\ + \quad & \mathbb{E}_{d,s,\lambda}\left[\min\left((q-z')^+,1\right)\min(\theta'\alpha s_T, K_\lambda, \theta'\Gamma, \lambda_T)\right] \\ + \quad & \mathbb{E}_{d,s,\lambda}\left[\min\left((z'-q+1)^+,1\right)\min(\theta'\alpha s_T, \theta'\Gamma, \lambda_T)\right] \\ + \quad & \mathbb{E}_{d,s}\left[d_T s_T\right] + qP_\lambda,\end{aligned} \tag{3.14}$$

where $z' = \left\lfloor \frac{d_T}{\gamma} \right\rfloor$, and $\theta' = d_T - z'\gamma$.

As in the model for tankers, we divide the total demand into two components, but this time based on container units z' and θ'. The variable z' represents the number of ships with γ containers that could potentially fulfill the realized demand d_T. The variable θ' consequently represents the excess demand that cannot be met with z' ships. Furthermore, θ' is always less than γ. The function $C_2(q,\mathbf{M})$ represents the combined expected procurement and transportation costs and consists of four parts. The first part $\mathbb{E}_{d,s,\lambda}\left[(\min(\lambda_T, K_\lambda, \gamma\alpha s_T))\min(z',q) + (z'-q)^+\min(\lambda_T,\gamma\alpha s_T)\right]$ reflects the expected cost of chartering the z' ships according to the corresponding optimal policy at time T. The firm exercises at most z' options if it negotiated a sufficient number of options at time 0, and the strike price for one ship,K_λ, is less than both the spot market price λ_T, and the extra domestic market cost αs_T. Otherwise, the firm either hires the ship on the spot market for λ_T, or buys the commodity in the domestic market and pays the extra local cost $\gamma\alpha s_T$. The term $(z'-q)^+$ reflects the demand not covered by the options due to insufficient number of options negotiated at time 0. In order to meet this demand, the firm can only choose between hiring a ship on the spot market or buying the commodity in the local market. The third term $\mathbb{E}_{d,s}\left[d_T s_T\right]$, and the fourth term qP_λ are as in Equation (3.1).

The terms

$$\begin{aligned}& \mathbb{E}_{d,s,\lambda}\left[\min\left((q-z')^+,1\right)\min(\theta'\alpha s_T, K_\lambda, \theta'\Gamma, \lambda_T)\right] \\ + \quad & \mathbb{E}_{d,s,\lambda}\left[\min\left((z'-q+1)^+,1\right)\min(\theta'\alpha s_T, \theta'\Gamma, \lambda_T)\right]\end{aligned}$$

reflect the expected cost of transporting the excess demand, θ' according to the optimal policy at time T for procuring and transporting excess demand. For $0 < \theta' < \gamma$, the firm has three choices. If the firm still has an option remaining, the firm compares the option strike price K_λ with renting θ' containers at Γ per container and the extra local cost αs_T and selects the most cost efficient choice. Otherwise, the firm chooses only between hiring θ' containers or procuring the required amount of the commodity on the local market.

3.4.1 Determining the Optimal Options Position at Time 0 for Hiring Containers

Lemma 3.1 and Theorem 3.1 can be readily extended to accommodate the context of container shipping. We need to substitute A, B, C, and D with A', B', C' and D' as follows

$$
\begin{aligned}
A' &= \mathbb{E}_{s,\lambda}\left[\min(\lambda_T, K_\lambda, \gamma\alpha s_T)\right] \\
B' &= \mathbb{E}_{s,\lambda}\left[\min(\lambda_T, \gamma\alpha s_T)\right] \\
C' &= \mathbb{E}_{s,\lambda}\left[\min(\theta'\alpha s_T, K_\lambda, \theta'\Gamma, \lambda_T)\right] \\
D' &= \mathbb{E}_{s,\lambda}\left[min(\theta'\alpha s_T, \theta'\Gamma, \lambda_T)\right]
\end{aligned}
$$

Then with an approach similar to the one employed in the proofs of Lemma 3.1 and Theorem 3.1 we can show the convexity of the objective function, and then determine the optimal options positions at time zero.

3.4.2 Optimal Policy at Time T

Given the optimal options position at time 0, q^*, the firm must decide at time T, upon realization of d_T, s_T, and λ_T, how many options to exercise, how many ships to charter at the spot rate, how many containers to rent, and how the commodity should be procured. Figure 3.2 shows the optimal policy for the part of demand that can be fulfilled with z' ships. The x-axis shows the commodity price at time T, and the y-axis shows the freight rate at time T. Figures 3.4 and 3.3 show the optimal policy for the excess demand, θ'. In order to avoid the complexity of 3D graphs, we show the optimal policy for this demand in two figures based on the realization of demand and consequently θ'. Figure 3.4 shows the optimal policy when $\theta'\Gamma > K_\lambda$, and Figure 3.3 shows the optimal policy when $\theta'\Gamma < K_\lambda$. The term $\theta'\Gamma$ represents the cost of renting θ' containers on a ship. When renting θ' containers, the firm should always compare the cost of renting these containers to the cost of chartering a ship with all containers on it using an option, should one remain. Thus, the optimal policy for containers distinguishes between these two cases. After this trade-off has been resolved, as with chartering tankers, the firm must decide between procuring the commodity in the offshore market and the domestic market. The diagonal lines $\lambda_T = \gamma\alpha s_T$ and $\lambda_T = \theta'\alpha s_T$ represent this trade off. The structure of the optimal policy for the part of demand that could be fulfilled by z' ships with containers is identical to the structure of the optimal policy for the demand that could be fulfilled with z tankers in Section 3.3, with both policies consisting of four parts.The optimal policy for the excess demand that could be fulfilled with θ' containers is structurally similar to the optimal policy for the excess demand that could be fulfilled with θ tankers, with the latter policy again consisting of four parts. The optimal policy relating to θ' consists of five parts, instead of four. The extra part arises as a consequence of the divisibility of containers. In order to formalize the optimal policy, we modify the definition of some variables used to formally characterize the optimal policy in the context of tankers.

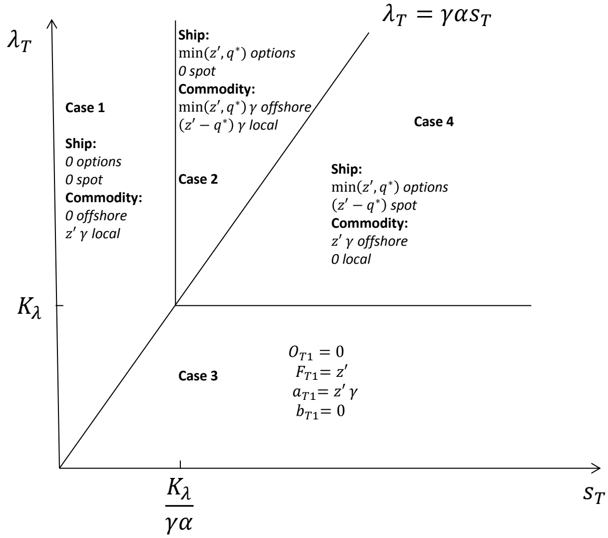


Figure 3.2 Optimal policy for z' part of demand

O_{Tz}: The number of options the firm should exercise in order to meet the demand that can be fulfilled with z' container ships.

$O_{T\theta}$: The number of options the firm should exercise in order to meet the excess demand that cannot be fulfilled with z' container ships.

O_T: $O_{Tz} + O_{T\theta}$

As we did before, we similarly define the variables F_{Tz}, $F_{T\theta}$, F_T as the number of container ships that the firm has to hire at the spot freight rate, a_{Tz}, $a_{T\theta}$, a_T as the number of containers of commodity that the firm has to procure in the offshore market, and b_{Tz}, $b_{T\theta}$, b_T as the units of commodity (in terms of numbers of containers) that

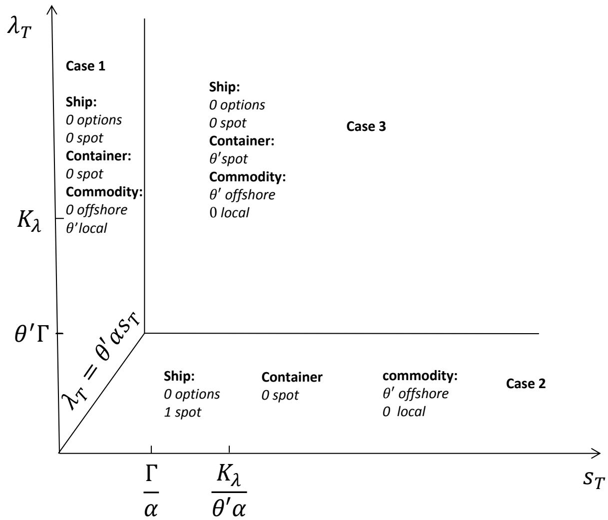


Figure 3.3 Optimal policy for θ' part of demand if $\theta'\Gamma < K_\lambda$

the firm has to procure in the domestic market. Theorem 3.3 now characterizes the associated optimal policy

Theorem 3.3 *Given the q^*, and z', (Figure 3.2)*

(i) if $\gamma\alpha s_T \leq \lambda_T$ and $\gamma\alpha s_T \leq K_\lambda$ then $O_{Tz} = 0$, $F_{Tz} = 0$, $a_{Tz} = 0$, and $b_{Tz} = z'\gamma$,

(ii) if $\gamma\alpha s_T \leq \lambda_T$ and $\gamma\alpha s_T > K_\lambda$ then $O_{Tz} = \min(z', q^*)$, $F_{Tz} = 0$, $a_{Tz} = \min(z', q^*)\gamma$, and $b_{Tz} = (z' - q^*)^+\gamma$,

(iii) if $\gamma\alpha s_T > \lambda_T$ and $\lambda_T < K_\lambda$ then $O_{Tz} = 0$, $F_{Tz} = z'$, $a_{Tz} = z'\gamma$, and $b_{Tz} = 0$,

(iv) if $\gamma\alpha s_T > \lambda_T$ and $K_\lambda \leq \lambda_T$ then $O_{Tz} = \min(z', q^*)$, $F_{Tz} = (z' - q^*)^+$, $a_{Tz} = z'\gamma$, and $b_{Tz} = 0$,

given θ' where $\theta'\Gamma < K_\lambda$ meaning that the containers are cheaper than the freight options, and we need to just compare the price of θ' containers with the price of one extra full container ship and local market price. (Figure 3.3)

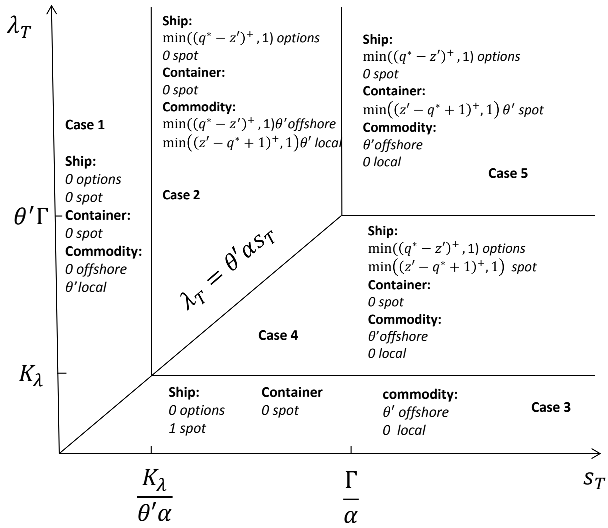


Figure 3.4 Optimal policy for θ' part of demand if $\theta'\Gamma > K_\lambda$

(i) *if* $\theta'\alpha s_T \leq \lambda_T$ *and* $\alpha s_T \leq \Gamma$ *then* $O_{T\theta} = 0$, $F_{T\theta} = 0$, , *container* $= 0$, $a_{T\theta} = 0$, *and* $b_{T\theta} = \theta'$.

(ii) *if* $\theta'\alpha s_T > \lambda_T$ *and* $\lambda_T < \theta'\Gamma$ *then* $O_{T\theta} = 0$, $F_{T\theta} = 1$, *container* $= 0$, $a_{T\theta} = \theta'$, *and* $b_{T\theta} = 0$.

(iii) *if* $\lambda_T > \theta'\Gamma$ *and* $s_T > \frac{\Gamma}{\alpha}$ *then* $O_{T\theta} = 0$, $F_{T\theta} = 0$, *container* $= \theta'$, $a_{T\theta} = \theta'$, *and* $b_{T\theta} = 0$.

and given θ' *where* $\theta'\Gamma > K_\lambda$ *meaning that we need to compare all choices:* θ' *freight options, local market price, one extra full container ship, and* θ' *containers. (Figure 3.4)*

(i) *if* $\theta'\alpha s_T \leq \lambda_T$ *and* $\theta'\alpha s_T \leq K_\lambda$ *then* $O_{T\theta} = 0$, $F_{T\theta} = 0$, *container* $= 0$, $a_{T\theta} = 0$, *and* $b_{T\theta} = \theta'$.

(ii) *if* $\theta'\alpha s_T \leq \lambda_T$ *and* $\frac{K_\lambda}{\theta'\alpha} < s_T < \frac{\Gamma}{\alpha}$ *then* $O_{T\theta} = \min((q^* - z')^+, 1)$, $F_{T\theta} = 0$, *container* $= 0$, $a_{T\theta} = \min((q^* - z')^+, 1)\theta'$, *and* $b_{T\theta} = \min((z' - q^* + 1), 1)\theta'$.

(iii) *if* $\theta'\alpha s_T > \lambda_T$ *and* $\lambda_T < K_\lambda$ *then* $O_{T\theta} = 0$, $F_{T\theta} = 1$, *container* $= 0$, $a_{T\theta} = \theta'$, *and* $b_{T\theta} = 0$.

(iv) *if* $\theta'\alpha s_T > \lambda_T$ *and* $K_\lambda \leq \lambda_T \leq \theta'\Gamma$ *then* $O_{T\theta} = \min((q^* - z')^+, 1)$, $F_{T\theta} = \min((z' - q^* + 1), 1)$, *container* $= 0$, $a_{T\theta} = \theta'$, *and* $b_{T\theta} = 0$.

(v) *if* $\lambda_T > \theta'\Gamma$ *and* $s_T > \frac{\Gamma}{\alpha}$ *then* $O_{T\theta} = \min((q^* - z')^+, 1)$, $F_{T\theta} = 0$, *container* $= \min((z' - q^* + 1), 1)\theta'$, $a_{T\theta} = \theta'$, *and* $b_{T\theta} = 0$.

Proof: As with the optimal policy for tankers, the optimality of the policy for containers can be shown by considering each scenario and determining the optimal action for that scenario. Again, the cases are highlighted in Figures 3.2, 3.3, and 3.4. □

3.5 Numerical Studies

In Section 3.3 we have discussed the cost minimization problem facing the firm when chartering tankers. In Section 3.4, we have discussed the optimization problem facing the firm when chartering ships and containers. In this section, we use the developed models to study the influence of the underlying parameters on the firm's hedging policy and associated costs. The section consists of two parts. In the first part, we consider a firm that charters tankers. In the second part, we consider a firm that charters ships and containers.

The models presented above are distribution-independent. These models, however, preclude analytical results for the optimal options position. Consequently, they must be solved numerically. To this end, we assume that demand for the final product, d_T, is lognormally distributed. We further assume that the commodity spot price and freight rate are also lognormally distributed. This assumption implies that the commodity price and freight rate, in addition to final demand, can be modeled as a Geometric Brownian Motion in continuous time. A consequence of this assumption is that we may compute the option premium using the Black & Scholes (1973) formula for the numerical studies. Recall, however, that these options are not exchange-traded and are negotiated over-the-counter between the firm and the ocean freight provider. One could naturally assume other stochastic processes and models for the underlying variables with no impact on the nature of the results.

We set the commodity spot price to be lognormally distributed with $\mu_s = 0.8$ and $\sigma_s = 0.6$ being the mean and standard deviation respectively of the associated normal distribution. Similarly, we set the freight rate to be lognormally distributed with $\mu_l = 0.8$ and $\sigma_l = 0.8$ being the mean and standard deviation of the associated normal distribution. We set the demand to be also lognormally distributed with $\mu_d = 0.4$, $\sigma_d = 0.4$ representing the mean and standard deviation of the associated normal distribution. We normalize the current commodity spot price to 1 per unit. We set the current freight rate to 100 per tanker and the current demand to 500 commodity units. The option strike price is set to be $K_\lambda = 150$. Since the freight rate is lognormally distributed, we can compute the option premium using the Black & Scholes (1973) formula to yield $P_\lambda = 18.4089$. Finally, we set one ship to have a capacity of $\beta = 100$ commodity units, and the extra margin for domestic procurement is $\alpha = 0.7$ per commodity unit. Table 3.1 summarizes the parameter setting. We use this setting in the following numerical experiments.

μ_s	σ_s	μ_l	σ_l	μ_d	σ_d	K_λ	P	β	α
0.8	0.6	0.8	0.8	0.4	0.4	150	18.4089	100	0.7

Table 3.1 Parameter setting

3.5.1 Effect of Tanker Size on Total Cost

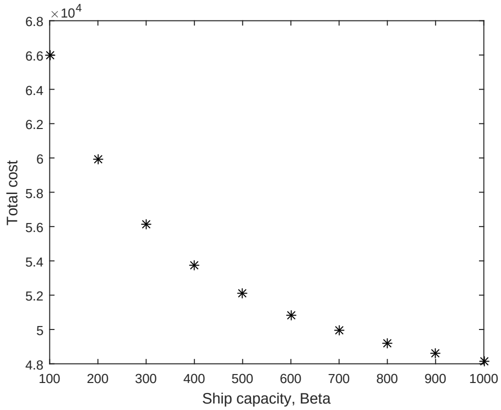


Figure 3.5 Total cost as the size of the tanker changes

We now investigate the effect of tanker size on the firm's total cost. Advances in engineering have facilitated the building of larger ships that can transport larger volumes of goods. We use the same parameter setting as above with the exception of setting initial demand (at time 0) to $d = 10000$, in order to have a large enough demand for larger ships. Figure 3.5 plots total cost to the firm against ship capacity β. Naturally, as the capacity of the ship increases, the cost of chartering it also increases. Economies of scale, however, imply that the rate of increase of the charter cost increases slower than the rate of increase of ship capacity. As the charter cost increases, naturally, the strike price would increase as well. In Figure 3.5, the charter cost increases at half the rate at which the ship capacity increases, while the option strike price increases at 75%. We achieve these rates of increase in the numerical study by controlling the step sizes. The figure shows that increasing β results in lower costs to the firm. The increased cost of chartering these ships is offset by the economies of scale gained. So-called 'mega ships' have gained attention in both literature and practice recently. These are very large container ships that can hold much more containers than traditional container ships. The Maersk Triple E Class of container ships, for example, has a capacity of more than 18,000 TEU. The study carried out in this section can be repeated here by changing γ, the number of containers per ship, rather than β to yield similar results, i.e., as γ increases, the total cost incurred by the firm decreases even though the cost chartering these 'mega ships' is higher. This result also explains the attractiveness of 'mega ships' to firms.

3.5.2 Effect of Dual-Sourcing Strategy on Total Cost

Having examined the transportation aspect of procuring the commodity in offshore markets, we now turn our attention to the dual sourcing strategy itself. A key driver of the strategy is the extra cost a firm incurs when procuring in domestic markets. The margin α represents this additional cost. We thus increase the variable α and study the effect of the dual sourcing strategy on total cost. Again, we use the same parameter settings in Table 3.1. Figure 3.6 plots the change in total cost as α increases and shows, somewhat surprisingly, that as α increases, the total cost to the firm increases. If α is low, the firm would choose to procure the commodity in the domestic market, saving itself the cost of transporting the commodity in the offshore market. As α increases, the firm starts to procure more of the commodity in the offshore market. Doing so incurs transportation costs. Thus, as α increases, even though the firm procures more of the required commodity in the offshore market, the transportation costs drive up total cost. Nonetheless, a dual sourcing strategy is beneficial to the firm. For large values of α, Figure 3.6 shows that at some α, the total cost stabilizes. At this point, the firm procures all the required commodity in the offshore market. Consequently, a change in the local market price has no impact on total cost. The dual-sourcing strategy stabilizes the cost incurred by the firm and limits its price exposure. On the other hand, if α is low, the firm can choose to procure as much of the commodity as needed in the domestic market and save itself the cost of transporting the commodity. The figure highlights the importance of explicitly considering transportation costs, and also demonstrates that the existence of a domestic market can reduce a firm's total expected procurement and transportation costs.

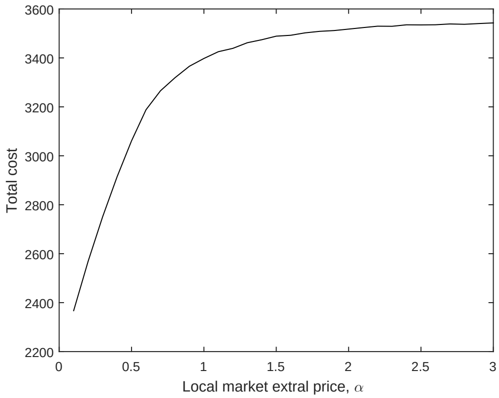
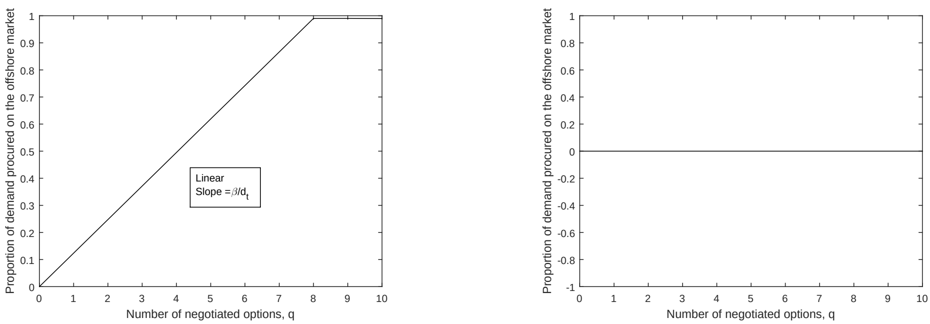


Figure 3.6 Total cost as the local extra price changes



(a) Freight option strike is relatively low

(b) Freight option strike is relatively high

Figure 3.7 Offshoring proportion as number of negotiated options changes

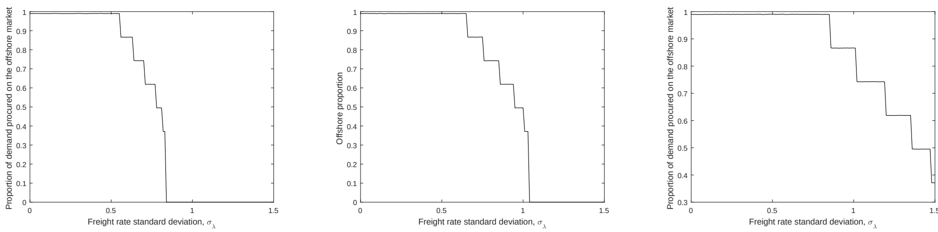
3.5.3 Effect of Options on Dual Sourcing Strategy

We now investigate the effect of freight options on the dual sourcing strategy under two scenarios. Figure 3.7 plots the proportion of demand procured on the offshore market as the number of negotiated options changes. We examine the effect under two scenarios. In the first scenario, the optimal policy for the firm is to exercise all the negotiated options. Figure 3.7(a) shows that the offshoring proportion increases as the number of options negotiated changes. This increase is linear with slope $\frac{\beta}{d_T}$, the proportion of demand that a single ship could fulfill. The reason is that if the firm has one additional option, the firm would exercise the option, and consequently the proportion of demand procured on the offshore market would increase by the capacity of one more ship over the whole demand ($\frac{\beta}{d_T}$). This process will continue until all demand is satisfied ($q = 8$). After that, increasing the number of options has no effect on the offshoring proportion.

In the second scenario (Figure 3.7(b)), the optimal policy is to procure the whole demand on the local market rather than on the offshore market. This implies that the firm should not exercise the negotiated options at time T, and consequently increasing the number of negotiated options has no effect on the proportion of procured commodity on the offshore market.

The main difference between Figure 3.7(a) and Figure 3.7(b) is in the option strike price K_λ. We have shown in Theorem 3.2 that by the realization of the commodity price and the freight rate at time T, the firm should compare them with the strike price and decide whether to exercise the options or not. Theorem 3.2 shows that if the options strike price K_λ is relatively low, the firm should exercise the options. Figure 3.7(a) shows such cases where the firm exercises the options; consequently, a change in the number of the procured options influences the sourcing decision. On the other hand, there are some cases in Theorem 3.2 in which the options strike price K_λ is relatively high compared to the local market and/or freight spot market. In those cases the firm - regardless of the number of options - should not exercise any of them; consequently, a change in the number of procured options has no effect on the sourcing decision. Figure 3.7(b) highlights such cases.

3.5.4 Effect of Freight Rate Volatility on Dual Sourcing Strategy



(a) $\alpha = 0.5$ (b) $\alpha = 0.7$ (c) $\alpha = 0.9$

Figure 3.8 Influence of volatility on optimal policy

Given the volatile nature of freight rates, we study the impact of freight rate volatility on the proportion of demand procured on the offshore market. Figure 3.8(b) plots the change in offshore proportion as freight rate volatility increases for three different levels of α. As Figure 3.9 shows, when σ_λ increases, the offshoring proportion decreases, but when α is higher i.e. $\alpha = 0.9$, this decrease happens at a higher level of freight rate volatility. This implies that the effect of the freight volatility on the offshoring proportion can be offset by the domestic margin α.

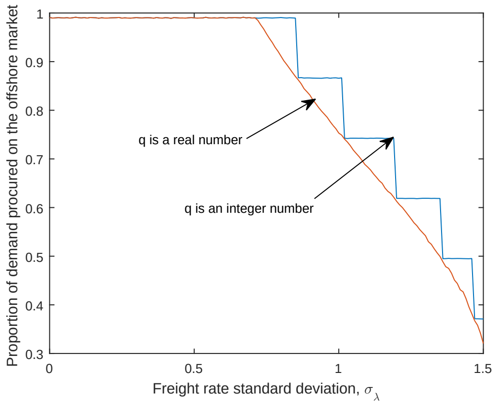


Figure 3.9 Influence of volatility on optimal policy with and without integer constraint, $\alpha = 0.9$

Another interesting observation in Figure 3.8 is that the graph is discontinuous at some points. Such discontinuities occur because we impose an integer constraint on q. For completeness, we relax this contraint in Figure 3.9 and obtain a continuous graph. Considering the integer constraint on q, Figure 3.8 shows that for many levels of volatility the sourcing decision should remain unchanged; however in some levels the offshoring proportion would dramatically decrease. Thus, ignoring reality and relaxing the integer constraint would result in a solution that is suboptimal which could consequently lead to dramatically higher costs.

3.5.5 Effect of Commodity Price volatility on Dual Sourcing Strategy

Having examined the freight rate volatility effect on the sourcing strategy, we now seek to study the effect of another important volatility on the sourcing decision: volatility of the commodity price. Intuitively one would think that it does not influence the sourcing decision, since the commodity price has the same process in both locations, but Figure 3.10 illustrates a different insight. It shows that the offshore proportion increases as the volatility of the commodity price increases, implying that the firm should procure more on the offshore market when the volatility of commodity price is too high. The reason behind this insight is the extra cost that the firm incurs when procuring on the domestic market (α). This extra cost α is a multiplier and intensifies the change in the volatility like a booster: when the volatility of the commodity price increases, the local market becomes riskier than the offshore market. Again, the graph here is not strictly increasing, and in some levels of standard deviation, the offshore proportion should remain unchanged, while in some levels, it should dramatically

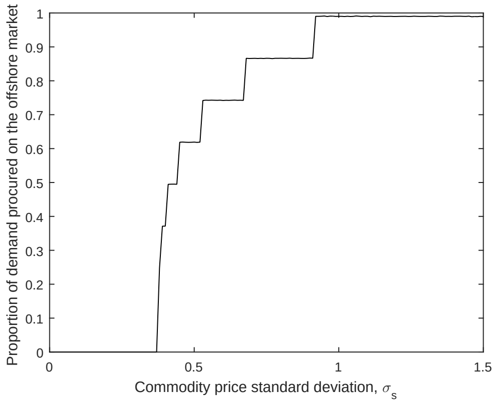


Figure 3.10 Offshoring proportion as volatility of commodity price changes

increase. Like the previous experiment, again the reason is the integer constraint on q.

3.6 Conclusion

Firms frequently employ dual-sourcing strategies to take advantage of price differences. Goods procured in offshore markets as a consequence of the dual-sourcing strategy need to be transported, typically via ocean freight. With increased globalization and demand for ocean freight services, coupled with long lead times to build ships, ocean freight rates have become highly volatile. Motivated by these developments, we investigate the role of options in hedging against ocean freight rates for a commodity processor that employs a dual-sourcing strategy. We develop mathematical models of the firm's total expected procurement and transportation costs, explicitly taking into account the transportation cost by allowing for stochastic freight rates, in addition to stochastic demand and commodity spot price, and determine the optimal number of options the firm should negotiate. Subsequently, we derive the the optimal policy the firm should follow once all uncertainties are resolved. We develop models and derive optimal policies for the two main categories of ocean transport: tankers and container ships. Our numerical studies based on the developed models highlight the importance of explicitly taking into account transportation costs. The results also show that hedging against freight rate risk with options can lead to cost reduction for a firm that employs a dual-sourcing strategy. Furthermore, our numerical studies also highlight that dual-sourcing strategies limit price risk exposure and offer the firm flexibility by allowing the firm to take advantage of low domestic costs. Our numerical studies demonstrate that cost savings may be achieved by chartering larger tankers and container ships even though they may cost more to charter, shedding some light on the popularity of ‘mega ships’. Our numerical studies further show that procuring freight options may increase the offshore proportion, and finally, the results show that the firm should procure more on the domestic market when the volatility of freight rate is too high, while the firm should buy more on the offshore market when the volatility of commodity price is too high.

Chapter 4

Valuing Optimal Switching Options with the Moving-Boundary Method

The previous two chapters focused on freight options that are similar in nature to European call options. We have shown the value of freight options as hedging tools against the operational and financial risks. We have considered the European style options and formulate the problem in one period setting. In this chapter we study a different kind of options which are called switching options. A switching option is a well-known example of a real option, and as we mentioned in Chapter 1 a real option references a tangible asset instead of financial instrument. In the two previous chapters we have focused on options as hedging instruments, while in the next two chapters we develop models in which options are defined on operations like shutting down a firm or switching a production place. Furthermore, unlike previous chapters where we had a one-period setting, in the next two chapters we formulate the problem in a continuous time horizon. Particularly in this chapter, we develop a numerical method based on stochastic control to value switching options.

4.1 Introduction

The Moving-Boundary method has been applied to several stochastic control problems. Stochastic control problems are problems where a controller attempts to control a system governed by an evolving stochastic process with the aim of optimizing an objective function. Due to the costs involved in controlling and making changes to the underlying system, the controller needs an optimal control policy that will dictate the actions necessary to optimize the controller's objective function. The cost structure determines the type of stochastic control problem facing the controller. There are several types of stochastic control problems such as optimal stopping problems, singular control problems, and impulse control problems. The theory of optimal stopping is concerned with the problem of choosing a time to take a particular action- like exercising an American option-, in order to maximise an expected reward or minimise an expected cost. Singular stochastic control problems are those problems in which the system controller can change the state of the system instantaneously by paying just the proportional cost, while in impulse control problems there is a fixed cost associated with each state change. Hence, when the control has proportional cost, the problem is called singular control wherein the optimal control only makes infinitesimal changes in states, but when the cost contains fixed component as well, the problem is called impulse control, and it is usually optimal for the controller to bring about non-infinitesimal changes to the state.

Using dynamic programming arguments, stochastic control problems typically reduce to solving a system of differential equations, either ordinary or partial. The domains over which these systems of equations are to be solved are unknown a priori, therefore resulting in free-boundary problems. Closed-form solutions rarely exist for such problems. Numerical solutions are then needed to determine the optimal value functions and policies. The Moving-Boundary method is a numerical scheme that can be used to solve a certain class of such free-boundary problems. The method transforms the free-boundary problem into a sequence of fixed-boundary problems that are easier to solve.

Kumar & Muthuraman (2004) applied Moving-Boundary method for the first time to solve singular control problems. They combine finite element methods that numerically solve partial differential equations with a policy update procedure based on the principle of smooth pasting to iteratively solve Hamilton-Jacobi-Bellman equations associated with the stochastic control problem. Following this, Muthuraman & Kumar (2006) provide a computational study of the problem of how to optimally allocate wealth among multiple stocks and a bank account to maximize the infinite horizon discounted utility of consumption. They consider the situation where the transfer of wealth from one asset to another involves transaction costs that are proportional to the amount of wealth transferred. Their model allows for correlation between the price processes, which in turn gives rise to interesting hedging strategies. This results in a stochastic control problem with both drift-rate and singular controls, which can be recast as a free boundary problem in partial differential equations. Adapting the finite element method and using an iterative procedure that converts the free boundary problem into a sequence of fixed boundary problems, Kumar & Muthuraman (2004) provide an efficient numerical method for solving this problem. Muthuraman (2008) develops the variant of their method for optimal stopping problems. Interestingly, at that time he showed that the Moving-Boundary idea also worked for optimal stopping problems, in particular for pricing American options. Apart from providing an efficient methodology to solve the free boundary problem in American option pricing, he extends the Moving-Boundary approach to solve an optimal stopping problem, thereby taking a modest step in making the Moving-Boundary method more general. Chockalingam & Muthuraman (2011) apply the Moving-Boundary method to price American options under stochastic volatility. First, they develop a transformation procedure to compute the optimal-exercise policy and option price and provide theoretical guarantees for convergence. Second, using this computational tool, they explore a variety of questions that seek insights into the dependence of option prices, exercise policies, implied volatilities on the market price of volatility risk and correlation between the asset and stochastic volatility. They compare the speed and accuracy of the procedure against existing methods as well. Feng & Muthuraman (2010) extend the Moving-Boundary method in stochastic impulse control problems. They develop a methodology that converts the free-boundary problem into a sequence of fixed boundary problems. They show that the arising sequence has monotonically improving solutions and that the sequence converges. Provided the converged solution is C^1, they show that it is the optimal solution and that the optimal policy takes the form of a control band policy which has a simple and intuitive representation. They also provide an ϵ-optimality result that provides an upper bound on the error when the sequence is terminated after convergence to within a tolerance.

Switching options are another kind of stochastic control problems which have found diverse applications in operations management and finance such as Tolling agreements. Ludkovski (2005) defines a tolling agreement as any temporary contract between the permanent owner and another agent that allows the latter to claim ownership and management of the output, subject to pre-specified exercise rules. The holder of the tolling agreement can gain the benefits or suffer the losses depending on commodity prices and operational costs. Limited operational flexibility is also a feature of tolling agreements such that the manager may not be allowed to scale production up or down in a way that is most advantageous. This is where optimal switching is of particular interest in that it addresses the question when it is optimal for an investor to enter, or exit the market or adjust the investment. The pioneering work of Brekke & Øksendal (1994) considers the problem of finding the optimal sequence of opening and closing times of a multi-activity production process, given the costs of opening, running, and closing the activities and assuming that the state of the economic system is a stochastic process. They formulated the problem as an extended impulse control problem over an infinite time horizon and solved using stochastic calculus. Duckworth & Zervos (2001) address the problem of determining in an optimal way the sequence of times at which a firm can enter or exit an economic activity. They consider an investment model which involves production scheduling as well as a sequence of entry and exit decisions over an infinite time horizon. The pricing of an investment using this model gives rise to a stochastic impulse control problem that they explicitly solve. They do not assume additional costs associated with scaling production up or down, as long as this does not involve a complete shut-down. They assume the profit function is upper semi-continuous however, and impose further constraints to justify their analysis. Ly Vath & Pham (2007) consider the problem of determining the optimal sequence of stopping times for a diffusion process subject to regime switching decisions. They use a viscosity solution approach combined with the smooth-fit property, and explicitly solve the problem in the two-regime case when the state process is Geometric Brownian in nature. The results of their analysis take several qualitatively different forms, depending on model parameter values. They focus on two cases: one when the underlying dynamics are unaffected by the switching regime, but the profit functions are different; and another when the profit functions are identical, but the underlying dynamics depend on the switching regime. Djehiche et al. (2009) consider a finite time horizon and two production regimes. They use backward stochastic differential equations and Snell envelopes to solve completely the starting and stopping problem when the dynamics of the system are a general adapted stochastic process. Ludkovski (2005) allow multiple production regimes. However, the number of allowed switches is limited. They propose a new method of numerical solution based on Monte Carlo regressions. The scheme uses dynamic programming to simultaneously approximate the optimal switching times along all the simulated paths.

Our contribution in this research is to develop a computational method based on the Moving-Boundary that can solve the switching problem in two regimes over a finite time horizon. We provide the necessary theoretical guarantees for our proposed computational method. The model formulation is presented in Section 4.2 where we show the quasi variational equalities. In Section 4.3 we develop a variant of the Moving-Boundary which is used to solve optimal switching problems. In Section 4.4 we provide some numerical studies. In Section 4.5 we introduce two applications of switching options in the real world, and finally in Section 4.6 we make some concluding remarks and suggest a future research track.

4.2 Quasi Variational Inequalities

We consider a firm which has two regimes: ON or OFF. The firm can switch between the operating regimes at any point in time over a finite time horizon $0 < T < \infty$. The firm produces a commodity like electricity. We let X_t denote the commodity market price at time t as Markov process whose dynamics are described by

$$dX_t = \mu(t, X_t, m_t)dt + \sigma(t, X_t, m_t)dW_t,$$

where W_t is a one-dimensional Brownian motion, and $m_t = \{0, 1\}$ represents the current regime. When the system is in ON state $m_t = 1$, and when the system is in OFF state $m_t = 0$. The function $h(t, X_t, m_t)$ represents the instantaneous profit generated at time t by producing the commodity in regime m_t and selling it at price

X_t. Let K_{01} denote the cost of switching from OFF to ON, and K_{10} denote the cost of switching from ON to OFF. Let $0 \leq \tau_1 \leq \tau_2 \leq \cdots \leq \tau_i \leq \cdots$ be a sequence of stopping times such that only a finite number of τ_i will occur in any bounded interval with probability one. A switching strategy ν is defined as

$$\nu = (\tau_1, \zeta_1; \tau_2, \zeta_2; \cdots; \tau_i, \zeta_i; \cdots) \tag{4.1}$$

The policy implies that at time τ_i, the firm should take the action ζ_i which is switching the operating regime, either from OFF to ON or from ON to OFF. For a given policy the expected profit over the time horizon $[t, T]$ is

$$\mathcal{J}_x(\nu) = \mathbb{E}\left[\int_t^T h(s, X_s, m_s)ds - \Sigma_n A(K)\right], \tag{4.2}$$

where

$$A(K) = \begin{cases} K_{01} & \text{if the firm switches from OFF to ON} \\ K_{10} & \text{if the firm switches from ON to OFF} \\ 0 & \text{Otherwise} \end{cases}$$

and

$$X(t) = x$$

The objective is to maximize the expected profit over the time horizon $[0, T]$ and we thus arrive at the following stochastic control problem:

$$V(x) = \sup_{\nu} \mathcal{J}_x(\nu) \tag{4.3}$$

By using dynamic programming along with Ito's lemma, we convert stochastic control formulations to the differential equation problem. The idea is to study the parabolic partial differential equation resulting from applying Bellman's principle to the process, conditional on no control on $[t, t + dt)$, together with the equation corresponding to applying optimal control at t. By dynamic programming principle if at time t the firm is in ON state, then one possible strategy is to run it in the same regime for a short time up to a time $\hat{t}$ and then continue optimally. Let $V(t, x, m)$ represent the value function at time t when the system is in $m = \{0, 1\}$ state and $X(t) = x$. This strategy is at most optimal, hence

$$V(t, x, 1) \geq \mathbb{E}\left[\int_t^{\hat{t}} h(s, X_s, m_s)ds + V(\hat{t}, X_{\hat{t}}, 1)|X_t = x\right] \tag{4.4}$$

Letting $\hat{t} \to t$, assuming sufficient smoothness of $V(t, x, i)$ and applying Ito's formula to obtain

$$\mathcal{L}V(t, x, 1) + h(t, x, 1) \leq 0 \tag{4.5}$$

where

$$\mathcal{L}V(t,x,1) = \frac{\partial V(t,x,1)}{\partial t} + \mu\frac{\partial V(t,x,1)}{\partial x} + \frac{1}{2}\sigma^2\frac{\partial^2 V(t,x,1)}{\partial x^2}$$

is the space time generator.

Next, say we choose to instantaneously switch the operating regime at time t, and then follow the optimal policy thereafter, we will have

$$V(t,x,1) \geq V(t,x,0) - K_{10} \tag{4.6}$$

We can do the same steps to obtain similar inequalities to 4.5 and 4.6 when the firm is in OFF state. Finally we have

$$\begin{aligned} \frac{\partial V(t,x,1)}{\partial t} + \mu\frac{\partial V(t,x,1)}{\partial x} + \frac{1}{2}\sigma^2\frac{\partial^2 V(t,x,1)}{\partial x^2} + h(t,x,1) \leq 0 \\ V(t,x,1) \geq V(t,x,0) - K_{10} \end{aligned}$$

$$\begin{aligned} \frac{\partial V(t,x,0)}{\partial t} + \mu\frac{\partial V(t,x,0)}{\partial x} + \frac{1}{2}\sigma^2\frac{\partial^2 V(t,x,0)}{\partial x^2} + h(t,x,0) \leq 0 \\ V(t,x,0) \geq V(t,x,1) - K_{01} \end{aligned}$$

The differential equation problem is called the Hamilton-Jacobi-Bellman (HJB) equation which takes the form of quasi-variational inequalities(QVI). At least one of the two inequalities for each state must hold tightly at any point in time. Solving the quasi-variational inequalities directly gives the value function which can then be used to easily obtain the optimal control. The value function $V(t,x,m)$ is Lipschitz in x if $h(t,x,m)$ is Lipschitz in x, and $V(t,x,m)$ is Lipschitz continuous in t if $h(t,x,m)$ and the process X_t are time-homogeneous and and $\mathbb{E}\left[\sup_s |h_t(X_s)|\right] < \infty$. Now we need the verification theorem which states that a smooth solution of the quasi-variational inequality (QVI) is in fact the value function of the switching control problem. The value function properties and the verification theorem for a switching control problem are proved in Ludkovski (2005) and Carmona & Ludkovski (2010).

We now have one set of QVI for each regime including one PDE for continuation in that regime, and one inequality for switching the operating regime. In the continuation region, the firm should continue in that regime, and in the action region, the firm should switch to the other operating regime. In order to characterize the continuation and action regions we need to first characterize the shape of optimal policy. The optimal exercise policy can be represented by two continuous boundaries $d(t)$, $u(t)$, $0 \leq t < \infty$ (Ludkovski 2005). If the system is in the ON state, then it is optimal to switch if the underlying $x \leq d(t)$ and continue if $x > d(t)$, while if the system is in the OFF state, then it is optimal to switch if the underlying $x \geq u(t)$ and continue if $x < u(t)$. Hence, we define two continuation regions

$$\begin{aligned} C_{ON} &= \{(t,x) \in (0,T) \times (0,\infty); x > d(t)\} \\ C_{OFF} &= \{(t,x) \in (0,T) \times (0,\infty); x < u(t)\} \end{aligned}$$

and two exercise regions

$$\begin{aligned} I_{ON} &= \{(t,x) \in (0,T) \times (0,\infty); x \leq d(t)\} \\ I_{OFF} &= \{(t,x) \in (0,T) \times (0,\infty); x \geq u(t)\} \end{aligned}$$

Figure 4.1 shows the shape of policy schematically.

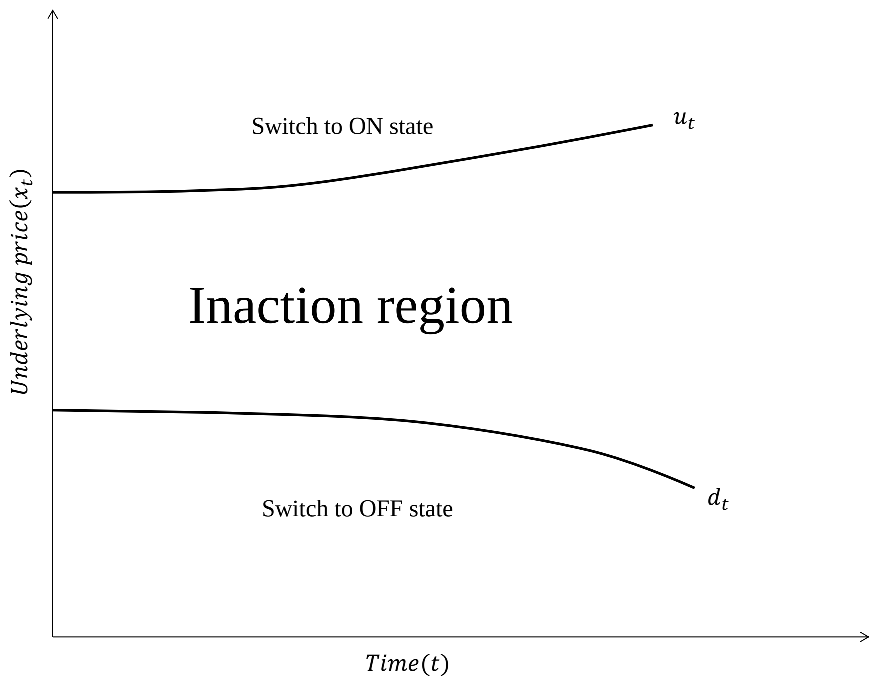


Figure 4.1 Shape of policy

When the underlying $X(t)$ is too high and the firm is in the ON state, the firm will never switch to the OFF state, and consequently the value of switching option is zero $\lim_{x\to\infty} V_x(t,x,1) = 0$. On the other hand when the underlying $X(t)$ is too low and the firm is in the OFF state, the firm will never switch to the ON state, and consequently the value of switching option is again zero $\lim_{x\to 0} V_x(t,x,0) = 0$. We also assume the firm will shut down at time T meaning that $V(T,x,0) = V(T,x,1) = 0$. Taking into account all the assumptions $V(t,x,1)$ is the unique solution to the free boundary problem

$$\mathcal{L}V(ON) + h(ON) = 0, \text{ in } C_{ON} \tag{4.7}$$
$$V(t,x,1) = V(t,x,0) - K_{10} \text{ in } I_{ON} \tag{4.8}$$
$$V(T,x,1) = 0 \tag{4.9}$$
$$\lim_{x\to\infty} V_x(t,x,1) = 0 \tag{4.10}$$

and $V(t,x,0)$ is the unique solution to the free boundary problem:

$$\mathcal{L}V(OFF) + h(OFF) = 0, \text{ in } C_{OFF} \tag{4.11}$$
$$V(t,x,0) = V(t,x,1) - K_{01} \text{ in } I_{OFF} \tag{4.12}$$
$$V(T,x,0) = 0 \tag{4.13}$$
$$\lim_{x\to 0} V_x(t,x,0) = 0 \tag{4.14}$$

4.3 Moving Boundary Method

In this section we propose a methodology that converts the free boundary problem associated with optimal switching to a sequence of fixed boundary problems. We also present and prove the necessary theoretical guarantees. As we mentioned in the previous section, we characterize the policy by a two-tuple (d,u). For any given policy (d_n,u_n), the associated value function $V^n(ON)$ is the solution to the set of differential equations 4.7 - 4.10 , and the associated value function $V^n(OFF)$ is the solution to the set of differential equations 4.11-4.14 respectively. To begin our search for the optimal policy, we begin with an initial guess and compute its associated value function. Using the guess and associated value function, we would like to construct an improved guess in a fashion that, on iteration, is guaranteed to converge to the optimal policy and value function. It also turns out that such an iteration can be constructed to provide a monotonically decreasing sequence of inaction region, helping to establish convergence and to improve the speed of convergence.

Say our initial guess policy is (d_0,u_0) and the associated value function is V^0. The following condition, assures us that the guess continuation region is a superset of the optimal continuation region:

$$V^0(t,d_0+,1) - V^0(t,d_0,1) < V^0(t,d_0+,0) - V^0(t,d_0,1) - K_{10}$$
$$V^0(t,u_0+,0) - V^0(t,u_0,0) < V^0(t,u_0+,1) - V^0(t,u_0,0) - K_{01}$$

and hereafter, we say that a value function V^n associated with (d_n, u_n) policy satisfies the superset condition if

$$V^n(t, d_0+, 1) - V^n(t, d_0, 1) < V^n(t, d_0+, 0) - V^n(t, d_0, 1) - K_{10} \tag{4.15}$$

$$V^n(t, u_0+, 0) - V^n(t, u_0, 0) < V^n(t, u_0+, 1) - V^n(t, u_0, 0) - K_{01} \tag{4.16}$$

Obviously, if a given guess does not satisfy the above conditions, we need to start with smaller d_0 and/or a larger u_0. After solving the fixed boundary problem to compute V^0, we begin our iteration with $n = 0$, (d_n, u_n) and V^n.

We define d_{n+1} by

$$\begin{aligned} d^{n+1}(t) = \quad & \sup\{x \in (d^n(t), \infty); \\ & V^n(t, x_0+, 1) - V^n(t, x_0, 1) < V^n(t, x_0+, 0) - V^n(t, x_0, 1) - K_{10}\} \\ & \forall x_0 \in [d^n(t), x)] \end{aligned} \tag{4.17}$$

and u_{n+1} by

$$\begin{aligned} u^{n+1}(t) = \quad & \inf\{x \in (0, u^n(t)); \\ & V^n(t, x_0+, 0) - V^n(t, x_0, 0) < V^n(t, x_0+, 1) - V^n(t, x_0, 0) - K_{01} \\ & \forall x_0 \in (x, u^n(t)]\} \end{aligned} \tag{4.18}$$

Then we need to solve the fixed boundary problems 4.7 - 4.10 and 4.11 - 4.14 with the policy (d_{n+1}, u_{n+1}) to obtain the associated value function V^{n+1}. To show that the above procedure converges to the value function and optimal policy, we need to show several aspects. First we need to ensure that the policy (d_{n+1}, u_{n+1}) is better than or equal to (d_n, u_n). That is, $V^{n+1} \geq V^n$. Next, to be able to use the iterative argument we will also need to ensure that conditions 4.15 and 4.16 that hold true for V^n also hold true for V^{n+1}. This will allow for repetitive improvements. Now note that this iteration produces a monotone sequence of continuation regions, and since the space is closed the convergence is inevitable. However, we will need to make sure that the converged policy actually is the optimal policy.

Before we establish the necessary theoretical guarantees we provide some intuition on why the proposed procedure works. First consider the update of d. The update procedure looks for the biggest x such that switch to the OFF regime increases the value more than continuing the ON regime; however the new continuation region still remains a superset of the optimal continuation region providing the necessary superset condition 4.15 to iterate. Similarly consider the update of u. The update procedure looks for the smallest x such that switch to the ON regime increases the value more than continuing the OFF regime. Again the new continuation region still remains a superset of the optimal continuation region providing the necessary superset condition 4.16 to iterate. Figure 4.2 shows the Moving-Boundary updates schematically.

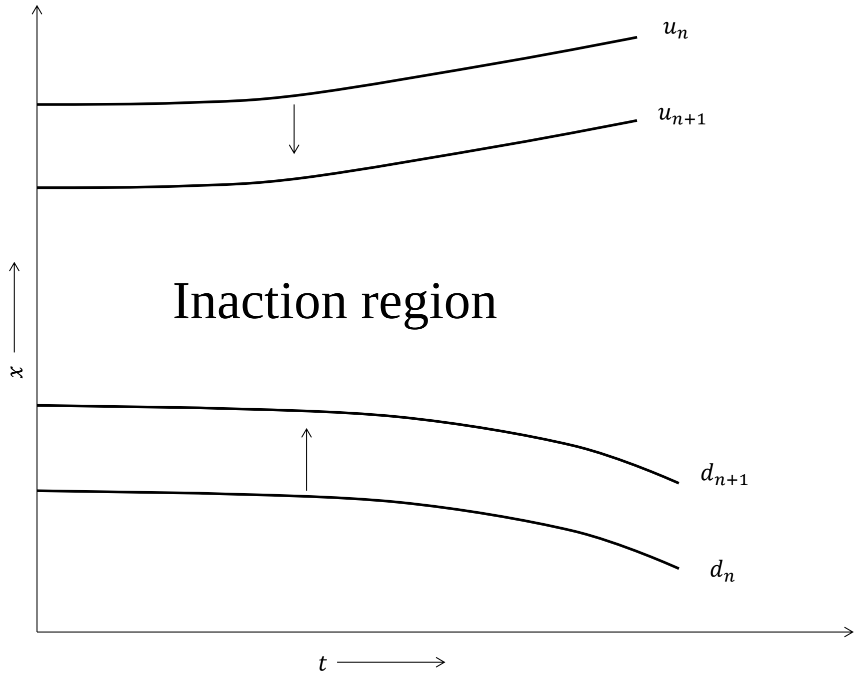


Figure 4.2 Moving Boundary

The rest of this section establishes the theoretical guarantees for the method. First we prove Lemma 4.1 and Lemma 4.2 which are used in the following theorems. Theorem 4.1 shows that the solutions to 4.7-4.10 and 4.11-4.14 are unique. Theorem 4.2 shows that the optimal condition can be violated by superset conditions, and Theorem 4.3 shows the improvement of value at each iteration and also shows the convergence of the method.

Lemma 4.1 *For a given G(t), continuous c(t) and a* $T' \in (0, \infty)$ *define*

$$C_{T'} = \{(t, x \in (0, T') \times (0, \infty); x > c(t)\}$$

and let ∂O *represent the boundary* $(0, x)$ *for* $x \in [c(0), \infty)$ *and let* ∂C *represent the* *boundary* $(t, c(t))$ *for* $t \in [0, T']$. *Let* f *be the solution to*

$$\begin{aligned} &\mathcal{L}f = 0, \;\; \textit{in } C_{T'} \\ &f(t, d(t)) = G(t), \;\; t \in [0, T') \\ &f(T', x) = 0 \\ &\lim_{x \to \infty} f_x(t, x) = 0 \end{aligned}$$

where

$$\mathcal{L}f = \frac{\partial f}{\partial t} + \mu \frac{\partial f}{\partial x} + \frac{1}{2}\sigma^2 \frac{\partial^2 f}{\partial x^2} - \beta f$$

If $G(t) > 0$ *for all* t *then the maxima of* f *is attained only on the boundary* ∂C *and the minimum value of* f *is 0 and is attained on* (T', x). *If* $G(t) = 0$ *for all* t *then* $f(t, x) = 0$ *for all* $(t, x) \in C_{T'}$.

PROOF: For $G(t) > 0$ say the maxima of f is attained at the point $(t, x) \in C_{T'}$. We have $f(t, x) \geq \max_{t \in [0, T')} G(t) > 0$ and from the required conditions for the maxima, we have $f_x = 0$, $f_t = 0$, and $f_{xx} \leq 0$. Substitutions in 4.19 yields

$$f(t, x) = \frac{1}{2\beta}\sigma^2 f_{xx} \leq 0$$

a contradiction. Next, say the maxima is attained at the point $(0, x) \in \partial O$. Again we have $f(0, x) \geq \max_{t \in [0, T')} G(t) > 0$ and with $f_x = 0$, $f_t \leq 0$, and $f_{xx} \leq 0$. Substitution in 4.19 yields

$$f(t, x) = \frac{1}{2\beta}\sigma^2 x^2 f_{xx} + \frac{1}{\beta} f_t \leq 0$$

a contradiction. Hence the maxima is attained on ∂C.
Now say the minima is attained at the point $(t, x) \in C_{T'}$ and $f(t, x) < 0$. We also have $f_x = 0, f_t = 0$ and $f_{xx} \geq 0$. Similarly as in the above, substitutions in 4.19 yields

$$f(t, x) = \frac{1}{2\beta}\sigma^2 f_{xx} \geq 0$$

a contradiction. Next, say the minima is attained on $(0, x) \in \partial O$ and $f(0, x) < 0$. Substitution in 4.19 yields a contradiction. Hence the minimum value of f is zero and is attained on (T', x). When $G(t) = 0$ for all t, the above arguments directly show that $f(t, x) = 0$ for all $(t, x) \in C_{T'}$. □

Lemma 4.2 *For a given G(t), continuous c(t) and a* $T' \in (0,\infty)$ *define*

$$C_{T'} = \{(t, x \in (0,T') \times (0,\infty); x < c(t)\}$$

and let ∂O *represent the boundary* $(0,x)$ *for* $x \in [c(0),\infty)$ *and let* ∂C *represent the boundary* $(t, c(t))$ *for* $t \in [0,T']$*. Let* f *be the solution to*

$$\begin{aligned}
&\mathcal{L}f = 0, \;\; \text{in } C_{T'}\\
&f(t,d(t)) = G(t), \;\; t \in [0,T')\\
&f(T',x) = 0\\
&\lim_{x\to 0} f_x(t,x) = 0
\end{aligned}$$

If $G(t) > 0$ *for all* t *then the maxima of* f *is attained only on the boundary* ∂C *and the minimum value of* f *is 0 and is attained on* (T',x)*. If* $G(t) = 0$ *for all* t *then* $f(t,x) = 0$ *for all* $(t,x) \in C_{T'}$*.*

Proof: For $G(t) > 0$ say the maxima of f is attained at the point $(t,x) \in C_{T'}$. We have $f(t,x) \geq \max_{t\in[0,T')} G(t) > 0$ and from the required conditions for the maxima, we have $f_x = 0$, $f_t = 0$, and $f_{xx} \leq 0$. Substitutions in 4.19 yields

$$f(t,x) = \frac{1}{2\beta}\sigma^2 f_{xx} \leq 0$$

a contradiction. Next, say the maxima is attained at the point $(0,x) \in \partial O$. Again we have $f(0,x) \geq \max_{t\in[0,T')} G(t) > 0$ and with $f_x = 0$, $f_t \leq 0$, and $f_{xx} \leq 0$. Substitution in 4.19 yields

$$f(t,x) = \frac{1}{2\beta}\sigma^2 x^2 f_{xx} + \frac{1}{\beta} f_t \leq 0$$

a contradiction. Hence the maxima is attained on ∂C.
Now say the minima is attained at the point $(t,x) \in C_{T'}$ and $f(t,x) < 0$. We also have $f_x = 0, f_t = 0$ and $f_{xx} \geq 0$. Similarly as in the above, substitutions in 4.19 yields

$$f(t,x) = \frac{1}{2\beta}\sigma^2 f_{xx} \geq 0$$

a contradiction. Next, say the minima is attained on $(0,x) \in \partial O$ and $f(0,x) < 0$. Substitution in 4.19 yields a contradiction. Hence the minimum value of f is zero and is attained on (T',x). When $G(t) = 0$ for all t, the above arguments directly show that $f(t,x) = 0$ for all $(t,x) \in C_{T'}$. □

Following theorem shows that the solutions to 4.7-4.10 and 4.11-4.14 are unique.

Theorem 4.1 *The solution to the initial value problem*

$$\begin{aligned}
&\mathcal{L}V^n(ON) + h(ON) = 0, \ \textit{in } C^n_{ON} \\
&V^n(t, d(t), 1) = V^n(t, d(t), 0) - K_{10}, \ t \in [0, T) \\
&V^n(T, x, 1) = 0 \\
&\lim_{x \to \infty} V_x(t, x, 1) = 0
\end{aligned}$$

$$\begin{aligned}
&\mathcal{L}V^n(OFF) + h(OFF) = 0, \ \textit{in } C^n_{OFF} \\
&V^n(t, d(t), 0) = V^n(t, d(t), 1) - K_{01}, \ t \in [0, T) \\
&V^n(T, x, 0) = 0 \\
&\lim_{x \to 0} V_x(t, x, 0) = 0
\end{aligned}$$

is unique.

Proof: Suppose not and that there exist two solutions f_1 and f_0 for $V^n(ON)$. Then $\bar{V} = f_1 - f_0$ solves

$$\begin{aligned}
&\mathcal{L}\bar{V} = 0, \ \text{in } C_{ON} \\
&\bar{V}(t, d(t), 1) = 0, \ t \in [0, T') \\
&\bar{V}(T', x, 1) = 0 \\
&\lim_{x \to \infty} \bar{V}_x(t, x, 1) = 0
\end{aligned}$$

Now from Theorem 4.1, $\bar{V}$ is uniformly zero in C_{ON} and the results follow. One can easily show the same for $V^n(OFF)$. □

The following theorem shows that the optimal condition can be violated by superset conditions. This is very important for the initial policy, because the algorithm should just start from a point in which the superset conditions 4.15 and 4.16 are violated.

Theorem 4.2 *If $d^0(t)- < d(t)$ for all $t \in (0, T)$, then*

$$V^0(t, d^0(t)+, 1) - V^0(t, d^0(t), 1) < V(t, d^0(t)+, 0) - V^0(t, d^0(t), 1) - K_{10}$$

and if $u^0(t)- > u(t)$ for all $t \in (0, T)$, then

$$V^0(t, u^0(t)+, 0) - V^0(t, u^0(t), 0) < V(t, u^0(t)+, 1) - V^0(t, u^0(t), 1) - K_{01}$$

Proof: Consider the region $C^0_{ON} \backslash C_{ON} = \{(t, x) : d^0(t) < x < d(t)\}$. This region indicates the region in which it is strictly optimal to exercise but the policy d^0 chooses not to. Therefore for any x and $x+$ in this region

$$V^0(t, x, 1) < V(t, x, 1) = V(t, x, 0) - K_{10}$$

and

$$V^0(t, x+, 1) < V(t, x+, 1) = V(t, x+, 0) - K_{10}$$

means that at any point in $C^0_{ON} \backslash C_{ON}$, we have

$$V^0(t, x+, 1) - V^0(t, x, 1) < V(t, x+, 0) - V^0(t, x, 1) - K_{10}$$
$$V^0(t, d^0(t)+, 1) - V^0(t, d^0(t), 1) < V(t, d^0(t)+, 0) - V^0(t, d^0(t), 1) - K_{10}$$

giving us our results. One again can easily show the similar proof for u.
□

Finally following theorems show the improvement of value at each iteration and also show the convergence of the method. Indeed we need to show that the value function is monotonically increasing in each iteration and eventually converges to the maximum level.

Theorem 4.3 *If $V^n(ON) \in C^{1,2}$ is the solution to the initial value problem*

$$\begin{aligned}
&\mathcal{L}V^n(ON) + h(ON) = 0, \;\; \textit{in } C^n_{ON} \\
&V^n(t, d^n(t)-, 1) = V(t, d^n(t)-, 0) - K_{10}, \;\; t \in [0, T) \\
&V^n(T, x, 1) = 0 \\
&\lim_{x\to\infty} V^n_x(t, x, 1) = 0
\end{aligned}$$

and

$$V^n(t, d^n(t)+, 1) - V^n(t, d^n(t), 1) < V(t, d^n(t)+, 0) - V^n(t, d^n(t), 1) - K_{10}$$

for all $t \in (0, T)$ then $d^{n+1}-$ defined by 4.17 is well defined. Moreover, $V^{n+1}(ON)$ (the solution to initial value problem with boundary $d^{n+1}-$) is such that

$$V^{n+1}(ON) > V^n(ON)$$

and

$V^{n+1}(t, d^{n+1}(t)+, 1) - V^{n+1}(t, d^{n+1}(t), 1) < V(t, d^{n+1}(t)+, 0) - V^{n+1}(t, d^{n+1}(t), 1) - K_{10}$ *for all $t \in (0, T)$.*

Proof: Since

$$V^n(t, d^n(t)+, 1) - V^n(t, d^n(t), 1) < V(t, d^n(t)+, 0) - V^n(t, d^n(t), 1) - K_{10}$$

and $\lim_{x\to\infty} V^n_x(t, x, 1) = 0$ we have from continuity of $V^n(ON)$ in C^n_{ON}, the existence of $d^{n+1}-$. Moreover since

$$V^n(t, x+, 1) - V^n(t, x, 1) < V(t, x+, 0) - V^n(t, x, 1) - K_{10}$$

for

$$x \in (d^n-, d^{n+1}(t)-)$$

then

$V^n(t, d^{n+1}(t)-, 1) - V^n(t, d^n(t)-, 1) < V(t, d^{n+1}-(t), 0) - V^n(t, d^n(t)-, 1) - K_{10}$

which implies

$$V^n(t, d^{n+1}(t)-, 1) < V(t, d^{n+1}-(t), 0) - K_{10}$$

$$V^n(t, d^{n+1}(t)-, 1) < V^{n+1}(t, d^{n+1}(t)-, 1)$$

Now consider the function $P(ON) = V^{n+1}(ON) - V^n(ON)$ in the region C^{n+1}_{ON}. We have

$$\begin{aligned}
&\mathcal{L}P(ON) = 0\\
&P(t, d^{n+1}(t)-, 1) > 0\\
&P(T, x, 1) = 0\\
&\lim_{x\to\infty} P_x(t, x, 1) = 0
\end{aligned}$$

Now directly from Theorem 4.1, $P(ON)$ is positive in C^{n+1}_{ON}. Next we need to show that

$V^{n+1}(t, d^{n+1}(t)+, 1) - V^{n+1}(t, d^{n+1}(t), 1) < V(t, d^{n+1}(t)+, 0) - V^{n+1}(t, d^{n+1}(t), 1) - K_{10}$.

To do this, it would be sufficient to show that $P_x(ON) < 0$ for all $t \in (0, T)$. Suppose not, and say $P_x(t_0, d^{n+1}(t_0), 1) \geq 0$ for some t_0. Now since $P(t_0, d^{n+1}(t_0), 1) > 0$ and $\lim_{x\to\infty} P(t_0, x, 1) = 0$ this implies that a maxima along x is attained in the interior of C^{n+1}_{ON} contradicting Theorem 4.1. Hence $P_x(t_0, d^{n+1}(t_0), 1) < 0$, that is

$$V^{n+1}_x(t, d^{n+1}(t), 1) < V_x(t, d^{n+1}(t), 1)$$

implies that

$V^{n+1}(t, d^{n+1}(t)+, 1) - V^{n+1}(t, d^{n+1}(t), 1) < V^n(t, d^{n+1}(t)+, 1) - V^n(t, d^{n+1}(t), 1)$

and since

$V^n(t, d^{n+1}(t)+, 1) - V^n(t, d^{n+1}(t), 1) = V(t, d^{n+1}(t)+, 0) - V^{n+1}(t, d^{n+1}(t), 1) - K_{10}$

we conclude that

$V^{n+1}(t, d^{n+1}(t)+, 1) - V^{n+1}(t, d^{n+1}(t), 1) < V(t, d^{n+1}(t)+, 0) - V^{n+1}(t, d^{n+1}(t), 1) - K_{10}$.

□

Theorem 4.4 *If* $V^n(OFF) \in C^{1,2}$ *is the solution to the initial value problem*

$$\begin{aligned}
&\mathcal{L}V^n(OFF) + h(OFF) = 0, \text{ in } C^n_{OFF} \\
&V^n(t, u^n(t)+, 0) = V(t, u^n(t)+, 1) - K_{01}, \ t \in [0, T) \\
&V^n(T, x, 1) = 0 \\
&\lim_{x \to 0} V^n_x(t, x, 1) = 0
\end{aligned}$$

and

$$V^n(t, u^n(t), 0) - V^n(t, u^n(t)-, 0) < V(t, u^n(t), 1) - V^n(t, u^n(t)-, 0) - K_{01}$$

for all $t \in (0, T)$ *then* $u^{n+1}+$ *defined by 4.17 is well defined. Moreover,* $V^{n+1}(OFF)$ *(the solution to initial value problem with boundary* $u^{n+1}+$*) is such that*

$$V^{n+1}(OFF) > V^n(OFF)$$

and

$V^{n+1}(t, u^{n+1}(t), 0) - V^{n+1}(t, u^{n+1}(t)-, 0) < V(t, u^{n+1}(t), 1) - V^{n+1}(t, u^{n+1}(t)-, 0) - K_{01}$ *for all* $t \in (0, T)$.

PROOF: Similar to Theorem 4.3 □

4.4 Numerical Studies

In this section we illustrate the Moving-Boundary approach with a numerical example. Before we move on to show the example, we make a brief note about the implementation. In order to solve the fixed boundary problems there are some standard methods such as the finite difference method. In this example we use the finite difference method. As with other numerical methods, we need to choose a finite domain and impose boundary conditions on the finite boundaries. As our time horizon is finite, the truncation of the time axis has no problems; however the truncation of axis for X_t brings in an approximation, since the truncation would be imposed at a finite boundary rather than at infinity. To reduce this problem, we choose an X which is sufficiently large.

We assume X_t follows a simple one-dimensional Brownian Motion driving process:

$$dX_t = 0.5d_t + 1.4dW_t$$

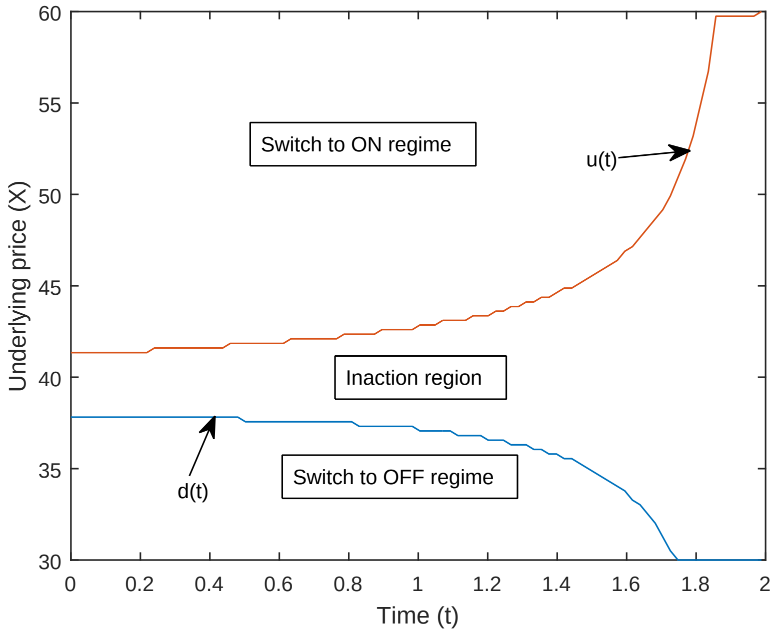


Figure 4.3 Optimal Policy

$X_0 = 30$ with time horizon $T = 2$. We have two regimes with continuous reward rates of $h(x, t, 0) = 0$ and $h(x, t, 1) = x - 42$, and the switching cost between them is $K_{01} = K_{10} = 10$

Our aim in this study is to demonstrate how the iterative procedure presented above computes the optimal policy and value function. In the Figure 4.3, the area above the blue curve $u(t)$ is the set of prices where the firm should switch the production on. Conversely, the area below the blue curve $d(t)$ encompasses all the prices where the firm should switch the production off. The continuation region is the region between the $d(t)$ and $u(t)$. In the continuation region the regime should not change irrespective of whether the current firm's regime is ON or OFF. We see that towards the end of the time horizon the operating regime should not change as fast as the beginning of the period, because the time may not be sufficient to compensate the switching costs.

Figure 4.4 shows the value function when the firm starts in ON state $V(x, t, 1)$, and Figure 4.5 shows the value function when the firm starts in OFF state $V(x, t, 0)$.

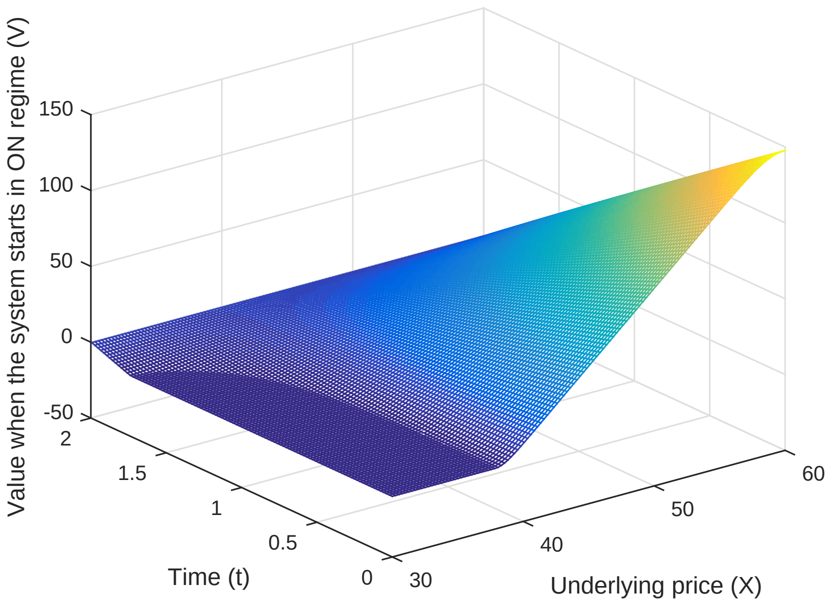


Figure 4.4 Value of switching option when the firm is in ON state

4.4.1 Convergence in Different Iterations

In Section 4.3 we have presented the theoretical guarantees for this method, and as we mentioned there, Moving-Boundary method converts the free boundary problem into a sequence of fixed boundary problems which can be solved numerically. This sequence monotonically improves the value function and converges to an optimal policy where the value is maximized. Now we show some iterations to get a better understanding, how the policy and value function changes in different iterations. Figure 4.6 plots the policy in three different iterations. Figure 4.6(a) is the first iteration after the initial policy. The initial policy should hold the superset condition where the inaction region is large enough to start the Moving-Boundary algorithm. As we see from Figure 4.6(a) to Figure 4.6(c), the inaction region becomes smaller, and the policy converges to the optimal.

Figure 4.7 plots the ON state value function convergence in the different iterations. Since the value is a function of two variables - time and underlying price- in order to better see the convergence, the time is fixed at $t = 3$. Figure 4.7(a) shows the value function associated with the initial policy at time $t = 3$. As we see in the last

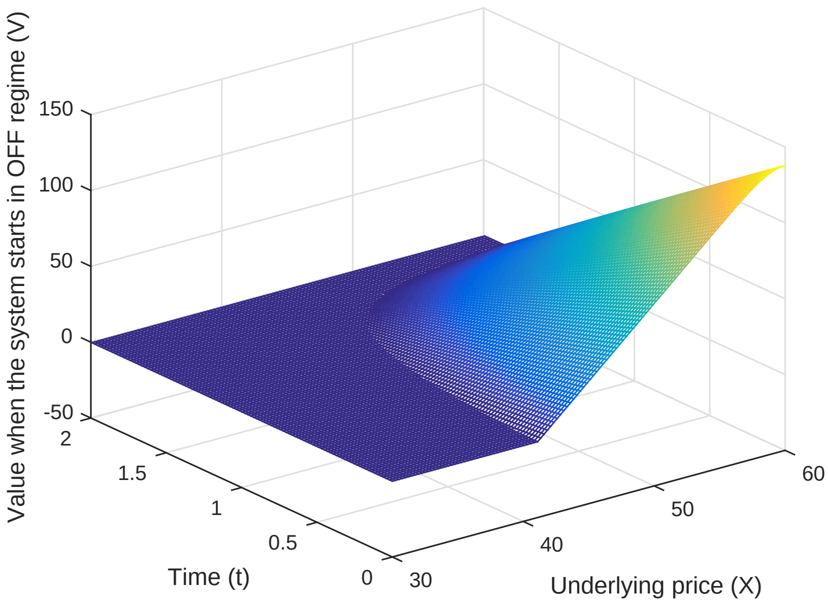
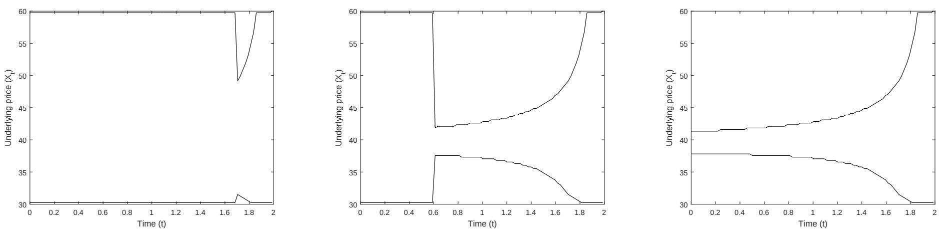


Figure 4.5 Value of switching option when the firm is in OFF state

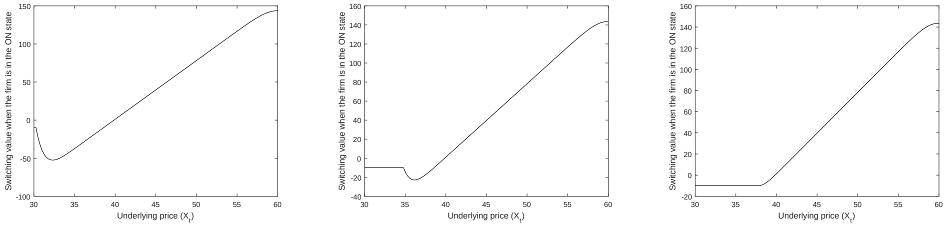


(a) First iteration (b) Second iteration (c) Last iteration

Figure 4.6 Policy convergence in different iterations

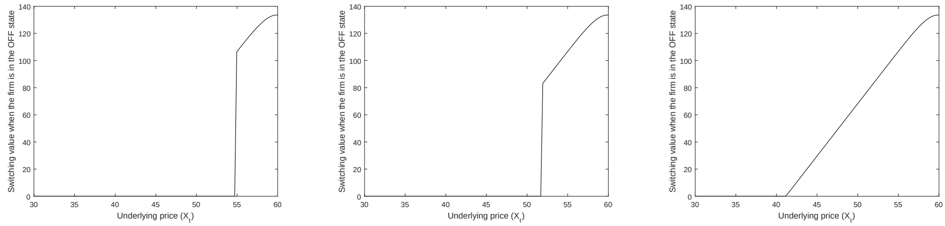
iteration (Figure 4.7(a)) the value function is quite smooth, and from 4.7(a) to 4.7(c), the value function monotonically increases.

Figure 4.8 plots the OFF state value function in the different iteration. We fix the time at $t = 3$, and show three iterations. Again from 4.8(a) to 4.8(a), the value function monotonically increases, and in 4.8(c), value functions are completely smooth.



(a) First iteration (b) Second iteration (c) Last iteration

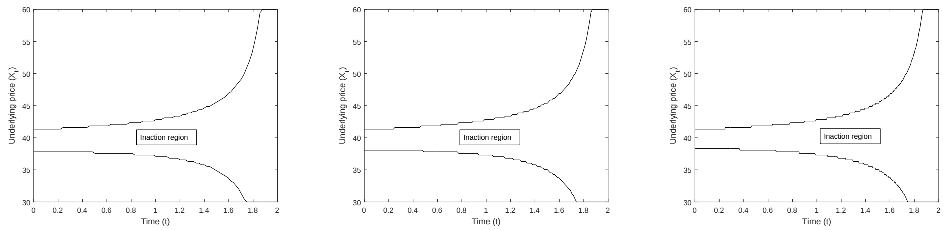
Figure 4.7 ON state value convergence for $t = 3$



(a) First iteration (b) Second iteration (c) Last iteration

Figure 4.8 OFF state value convergence for $t = 3$

4.4.2 Effect of Volatility on Optimal Policy



(a) $\sigma = 0.5$ (b) $\sigma = 1$ (c) $\sigma = 1.5$

Figure 4.9 Influence of volatility on optimal policy

Volatility is an import risk measure which can affect the policy. We thus seek to investigate how this effect looks like. Figure 4.9 plots the optimal policy for different levels of volatility. It shows that the inaction region becomes smaller as volatility increases, meaning that the firm should switch more frequently when the volatility is relatively high; however the change is not significant. The reason is that the most important factor in the switching frequency is the switching cost. Figure 4.10 plots the optimal policy when we decrease the switching cost to 1. Now we see that the size of inaction region decreases substantially.

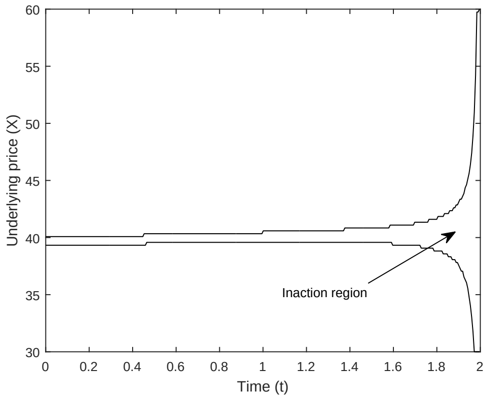


Figure 4.10 $K_{01} = K_{10} = 10, \sigma = 0.5$

4.5 Applications

As we mentioned in the introduction, tolling agreements are well-known examples of switching options. Project costs in the energy industry tend to be high. A typical fossil fuel power plant or an oil refinery costs hundreds of millions of dollars and may take three to five years to build Ludkovski (2005). Due to the capital intensive nature of the industry a few large companies own these power plants and lease them to operators. The operators, therefore, circumvent the capital intensive side of the business. The lease agreements between the operator and the owner are termed tolling agreements. The key decision facing the lessee is how and when the power plant should be operated, taking into consideration the spread between the input and out prices. Limited operational flexibility in tolling agreements prevents scaling the production up or down in all proportions of capacity. Hence, the only choices for the manager are either to enter the market with full capacity or to exit the market completely, and shut the firm down. The firm procures gas at price G_t, and converts it to electricity for sell at price P_t. The revenue from running the plant is then given by $X_t = (P_t - G_t)$ which pays the difference between the market price of the power and the market price of gas needed to produce the power. As we mentioned above, due to limited operational flexibility in tolling agreements managers have the option just either to run the plant (mode ON) or to shut it down (mode OFF). Strictly speaking the tolling agreement problem can be formulated in the optimal switching settings.

Another application of switching options is offshoring in which firms could locate its production plant in two different locations, and consequently switch between the locations as time passes. In the next chapter we present the associated switching model for the offshoring problem.

4.6 Conclusion

In this chapter we have developed a computational method based on Moving-Boundary that can solve the optimal switching problem in two regimes over a finite time horizon. The idea is to convert of a free boundary problem to a sequence of fixed boundary problems as was first developed for singular control problems in Kumar & Muthuraman (2004), and then extended to optimal stopping and impulse control. The optimal switching problem can be thought of as sequence of optimal stopping problems and posses complicating features, making an extension of the Moving-Boundary method to tackle such problems non-trivial. In this research we have demonstrated that the idea of converting to a sequence of fixed boundary problems is still possible for free boundary problems arising from switching problems, the specific method, however, is different. We further established the theoretical guarantees for the method. We have shown that the solutions to QVIs are unique. We have further shown that the optimal condition can be violated by superset conditions, and starting the Moving-Boundary iterations from violated region, the value monotonically increases in the next iteration. We have finally shown that our proposed algorithm converges to the optimal policy. A future research could focus on this method in more than two regimes.

Chapter 5

A Stochastic Control Approach to Operationalizing Offshore Production Decisions

In chapter 4 we have developed a numerical method based on the Moving-Boundary to value switching options. In this chapter, we apply this method to an offshoring problem. We consider a firm that can produce in two locations: either domestic or offshore. The firm has an uncertain offshore profit margin which makes the offshoring strategy risky. Using switching options we propose an optimal hedging policy that tells the manager when the firm should produce in the offshore facility and when it should produce in the domestic one. We then extend the policy from bang-bang to the proportional policy in which we determine what proportion of the firm's production should be offshored at any point in time. To do this we formulate the problem based on stochastic impulse control.

5.1 Introduction

Increasing globalization has seen an increase in offshoring of production by firms so as to increase profit by exploiting differences between raw material prices and labour rates in various locations. Cost motives are often considered to be the most important driver for offshoring. Stratman (2008) states that the development of cheap and robust communications technologies has lowered the cost of conducting business transactions across international boarders and has opened up low wage rate global labour markets to firms facing demand for cheap and efficient service delivery. Dachs et al. (2012) argue that expected labour cost reductions explain offshoring to the EU-

12, Asia and China. Aksin & Masini (2008) carried out an empirical investigation to find effective strategies for internal outsourcing and offshoring of business services in order to reduce the cost and improve efficiency. They also mention that cost motives are very important for offshoring.

However, there are also papers in the literature showing that cost motives are not the only driver for offshoring. Bunyaratavej et al. (2007) investigate the factors that contribute to the location choices for services offshoring, including wage differentials between the home and host countries. They found that contrary to conventional expectations, a country is more likely to be a destination of services offshoring as the average wage of a country increases. They also found that education level and cultural similarity are significant drivers of offshoring location choices. Lewin & Peeters (2006) show that, although firms use offshoring to improve the efficiency of the innovation process, labour arbitrage is less important than other forms of cost savings with respect to product development activities. Dachs et al. (2012) argue that in contrast to the EU-12, where the offshoring decision is solely dominated by potential labour costs, vicinity to customers and market expansion follow as a motive with a wide margin for offshoring activities to Asia and China.

Recent trends in the global business show the importance of the offshoring strategy. The nature of international trade is changing. For centuries, trade mostly entailed an exchange of goods. Now it increasingly involves bits of value being added in many different locations, or what might be called trade in tasks Grossman & Rossi-Hansberg (2008). In February 2004, when N. Gregory Mankiw, a Harvard professor then serving as chairman of the White House Council of Economic Advisers, caused a national uproar with a textbook statement about trade, economists rushed to his defense. Mankiw was commenting on the phenomenon that has been clumsily dubbed offshoring - the migration of jobs, but not the people who perform them, from rich countries to poor ones. Offshoring, Mankiw said, is only the latest manifestation of the gains from trade that economists have talked about at least since Adam Smith (Blinder 2006). During the past three decades, domestic manufacturing employment of U.S.-based multinationals has fallen steadily. Between 1982 and 1999, affiliate foreign employment as a share of total employment of these U.S. multinationals increased, climbing from 30% to nearly 44% of their labor force. These parallel developments have led critics of globalization to conclude that U.S. firms are cutting employment at home and shifting employment abroad Harrison & McMillan (2011). Currently, eighty percent of world trade is now conducted via global supply chains, and firms and industries with the highest-paying jobs in the United States depend on offshoring (Oldenski et al. 2016).

The offshoring strategy is just one side of the story. Nowadays many companies have started a back-shoring strategy. They note that, although improving cost was an initial reason, the risks and difficulties of transferring production to low-wage countries were frequently underestimated. Kinkel & Maloca (2009) show that the firms which have offshored portions of their production after some time adapt their strategies and implement back-shore production. The Mississippi Business Journal on 5/9/2014 wrote that "The tallies increasingly show it is as cost effective to manufacture in south-east U.S as in China". The Financial Times also wrote on 5/22/2014 that "Panasonic considers bringing production back to Japan." These examples show that back-shoring has become a considerable strategy for some companies. The main reason behind this is the dynamic price change. The price difference between countries is not static. Dewhurst & Meeker (2004) shows that China had the lowest labor rates in the world, but Grünig & Morschett (2012) show that increased global competition has driven down the labour rates in many countries. In another study Dachs et al. (2012) show that the share of all surveyed companies which offshored production activities to the EU-12 decreased sharply from the middle of 2004 to the middle of 2009, and the main reason was the sharp rise in wages in some industrial regions of Poland, the Czech Republic, Hungary and Slovakia. Farrell (2005) argues that the potential cost savings for companies from offshoring appear impressive, much of the potential gains from offshoring go unrealized. Such a high volatility in low-wage countries is another important feature of offshoring. Bergin et al. (2011) claim that offshoring industries in low wage countries such as Mexico experience fluctuations in employment that are twice as large as in high wage countries such as the United States. As Liu & Nagurney (2011) show exchange rate risk is another important issue in offshoring-backshoring strategy.

We discussed that an offshoring-backshoring strategy could create value for firms by decreasing the cost of production; however we did not discuss that relocation incurs substantial costs. Dachs et al. (2012) explain some of these costs. They note the relocation of production is often followed by the relocation of certain R&D tasks as well, in order to reduce the physical distance between R&D engineers and the operation, and to ensure that problems are solved at source as they arise. Similarly, sooner or later, it is deemed sensible to shift the testing of products or materials, the training of workers and various other tasks, and to co-locate them with production.

So far, we elaborate the drawbacks and benefits of offshoring and backshoring strategies. Most of the factors we mentioned above are operational factors influencing a strategic offshoring-backshoring decision. We believe an operational approach can help the firm to make this strategic decision, even-though there are not many papers with operational approach in the literature. These include Nembhard et al. (2003), who view the offshoring decision-making problem as a switching option with no switching costs and use simulation to determine the switching policy. The authors state that although switching costs are important and need to be considered, such consideration will further complicate the problem, and require dynamic programming concepts to yield a solution. Alvarez & Stenbacka (2007) adopt a real options approach to finding the optimal proportion of partial outsourcing in order to maximize the profit. They assume that a one-time organizational redesign cost associated with outsourcing a proportion of the production is a convex function of this proportion. They further assume that this organizational redesign is irreversible. Finally Huchzermeier & Cohen (1996) develop dynamic programming models to study the value of operational flexibility of global supply chains under just exchange rate risk. They model exchange rate as diffusion process. The literature on operational-decision making for offshoring decisions is limited.

We seek to contribute to the literature by presenting a model which operationalizes the offshoring decision. We present a dynamic model for making offshoring decisions that is grounded in stochastic control theory allowing us to derive optimal control policies that are easy to obtain, understand and implement. First we consider this problem as a optimal switching problem where the firm can choose the production place between two locations: domestic or offshore. Our approach differs from Nembhard et al. (2003) in that we consider the switching costs. Then we consider this problem in a more generalized setting in which we assume the firm can offshore-backshore any proportion of its production and model it based on stochastic impulse control.

The chapter is organized as follows. In Section 5.2 we model the offshoring problem as an optimal switching problem, and present the associated mathematical model. In Section 5.4 we present the impulse control model for offshoring problem. In Section 5.3 we present a numerical example to show the optimal offshoring-backshoring policy, and finally we conclude in Section 5.6.

5.2 Switching Model

In this section we model the offshoring problem as a optimal switching problem. We consider a firm that can produce in two locations: domestic or offshore. The firm has a manager who can switch the production place at any point in time over a finite time horizon $0 < T < \infty$. Let α_h denote the firm's profit margin for goods produced domestically, we assume it is constant. Similarly, we denote by α_o the profit margin for goods produced at the firm's offshore facility. Given the uncertainties involved with offshore production, we assume that the evolution of α_o is governed by a Brownian motion process, i.e.,

$$d\alpha_o = \mu dt + \sigma dW_t, \tag{5.1}$$

where W_t is a one-dimensional Brownian motion. One can assume the drift μ is zero, because a non-zero drift means in the long run offshoring has an increasing margin over time; however, in this section in order to keep the generality we assume the drift can be any real number including zero. We further assume that the firm's domestic margin is constant, otherwise if we assume the demotic margin is stochastic, then we have to solve a two-dimensional stochastic control problem which makes the problem very complicated. Therefore, to avoid solving a two-dimensional stochastic control problem, we assume just the offshore margin is stochastic, and the domestic margin is constant. We also assume that the firm faces constant demand D for the product. If $u \in \{0,1\}$ denotes the domestic production, the firm's instantaneous profit, Π, is given by

$$\Pi(t,u,\alpha_h,\alpha_o;D) = uD\alpha_h + (1-u)D\alpha_o. \tag{5.2}$$

.

We let K_{oh} denote the cost of switching from the offshore plant to the domestic one, and K_{ho} denote the cost of switching from the domestic plant to the offshore one. The manager seeks to maximise the expected profit over the time horizon $[0,T]$ and we thus arrive at the following stochastic control problem:

$$V(t,\Pi,u) = \sup_{\nu}\{\mathbb{E}[\int_0^T \Pi(s,u,\alpha_h,\alpha_o)ds - \Sigma_n A(K)]\}, \tag{5.3}$$

where ν is all admissible strategies for the time horizon $[0,T]$ as we defined in the previous chapter.

$$A(K) = \begin{cases} K_{ho} & \text{if the firm switches from domestic to offshore} \\ K_{oh} & \text{if the firm switches from offshore to domestic} \\ 0 & \text{Otherwise} \end{cases}$$

By using dynamic programming along with Ito's lemma, we convert stochastic control formulations to the differential equation problem. The differential equation problem is called the Hamilton-Jacobi-Bellman (HJB) equation which takes the form of quasi-variational inequalities(QVI). Solving the quasi-variational inequalities directly gives the value function and the associated optimal control. We have derived the QVI for switching options in the previous chapter. We now show them for two regimes (offshore and domestic) in a form that is convenient for our exposition in the current chapter.

$$\begin{aligned} \frac{\partial V(t,\Pi,0)}{\partial t} + \mu\frac{\partial V(t,\Pi,0)}{\partial \Pi} + \frac{1}{2}\sigma^2\frac{\partial^2 V(t,\Pi,0)}{\partial \Pi^2} - \beta V(t,\Pi,0)) + \Pi(t,0,\alpha_h,\alpha_o) \leq 0 \\ V(t,\Pi,0) \geq V(t,\Pi,1) - K_{oh} \end{aligned}$$

$$\begin{aligned} \frac{\partial V(t,\Pi,1)}{\partial t} + \mu\frac{\partial V(t,\Pi,1)}{\partial \Pi} + \frac{1}{2}\sigma^2\frac{\partial^2 V(t,\Pi,h)}{\partial \Pi^2} - \beta V(t,\Pi,1)) + \Pi(t,1,\alpha_h,\alpha_o) \leq 0 \\ V(t,\Pi,1) \geq V(t,\Pi,0) - K_{ho} \end{aligned}$$

We have one set of QVI for each production facility including one PDE for continuation in that facility, and one inequality for switching to the other one. In the continuation region, the manager should let the firm continue on that location, and in the action region, the manager should intervene and move the production to the other location. We characterize the continuation and action region in the next section by finding the optimal policy. Following inequalities show the QVI separately for each location.

We assume the firm has a life period and will shut down at time T. This means that $V(T,\Pi,1) = V(T,\Pi,0) = 0$. We also have $\lim_{\Pi\to 0} V_\Pi(t,\Pi,0) = \lim_{\Pi\to\infty} V_\Pi(t,\Pi,1) = 0$, because when the profit margin is too high in the offshore facility, the firm can make a lot of profit by producing there (domestic profit is constant), and consequently there is no need to switch the location which means the switching option has no value. On the other hand, very small profit margin in the offshore facility pushes the firm to produce in the domestic plant, and consequently no switches to offshore location is needed. This also implies that the value of the switching option in the domestic place is zero. Taking into account all the assumptions $V(t,\Pi,1)$ is the unique solution to the free boundary problem:

$$\mathcal{L}V(t,\Pi,1) + \Pi(t,1,\alpha_h,\alpha_o) = 0, \text{ in } C_h \tag{5.4}$$
$$V(t,\Pi,1) = V(t,\Pi,0) - K_{ho} \text{ in } I_h \tag{5.5}$$
$$V(T,\Pi,1) = 0 \tag{5.6}$$
$$\lim_{\Pi\to\infty} V_\Pi(t,\Pi,1) = 0 \tag{5.7}$$

and $V(t,\Pi,0)$ is the unique solution to the free boundary problem:

$$\mathcal{L}V(t,\Pi,0) + \Pi(t,0,\alpha_h,\alpha_o) = 0, \text{ in } C_o \tag{5.8}$$
$$V(t,\Pi,0) = V(t,\Pi,1) - K_{oh} \text{ in } I_o \tag{5.9}$$
$$V(T,\Pi,0) = 0 \tag{5.10}$$
$$\lim_{\Pi\to 0} V_\Pi(t,\Pi,0) = 0 \tag{5.11}$$

where C_h and C_o are continuation regions for domestic and offshore plants, and I_h and I_o are switching regions for them.

We now need to solve the QVI to obtain the optimal offshoring policy in the switching setting. To do so, we use the Moving-boundary method from the previous chapter. In Section 5.3 we solve the QVI in an example, and implement several numerical studies to obtain managerial insights.

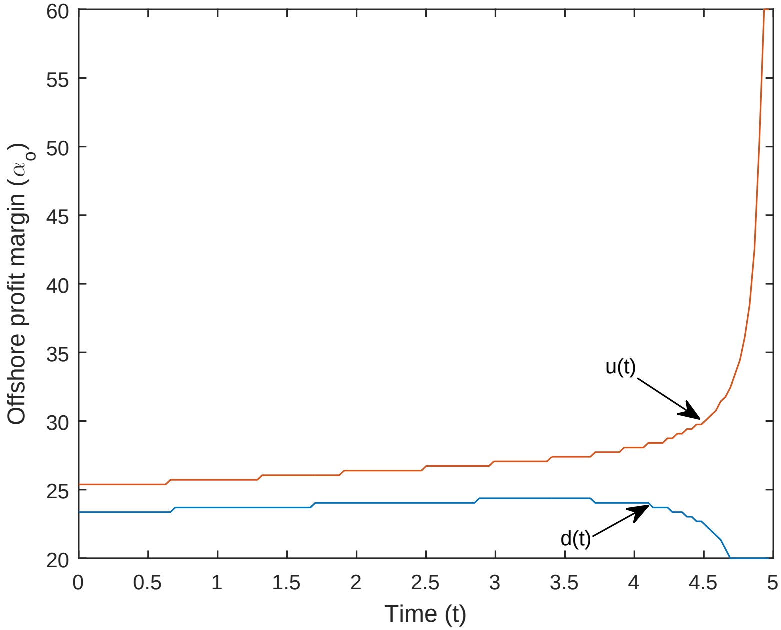


Figure 5.1 Optimal policy

5.3 Numerical Studies

We use the moving boundary method that we developed in the previous chapter to solve the following offshoring problem. Suppose we have a firm which can produce in two locations: domestic and offshore. The profit margin for goods produced at the firm's domestic facility is $\alpha_h = 20$, and he profit margin for goods produced at the firm's offshore facility is governed by a Brownian motion

$$d\alpha_o = 0.5dt + 1.5dW_t, \tag{5.12}$$

with $\alpha_o(0) = 20$. We assume that relocation cost between facilities is $K_{oh} = K_{ho} = 5$, and the firm's life period is 5 years.

In the Figure 5.1 the area above the red curve $u(t)$ is the set of offshore profit margin where the manager should offshore the production. Conversely, the area below the blue curve $d(t)$ encompasses all the offshore profit margins where the manager should backshore the production. The continuation region is the region between the blue and red curves where the manager should not change production place. We see that towards the end of the time horizon the manager becomes more cautious about relocations, because the time may not be sufficient to compensate the relocation

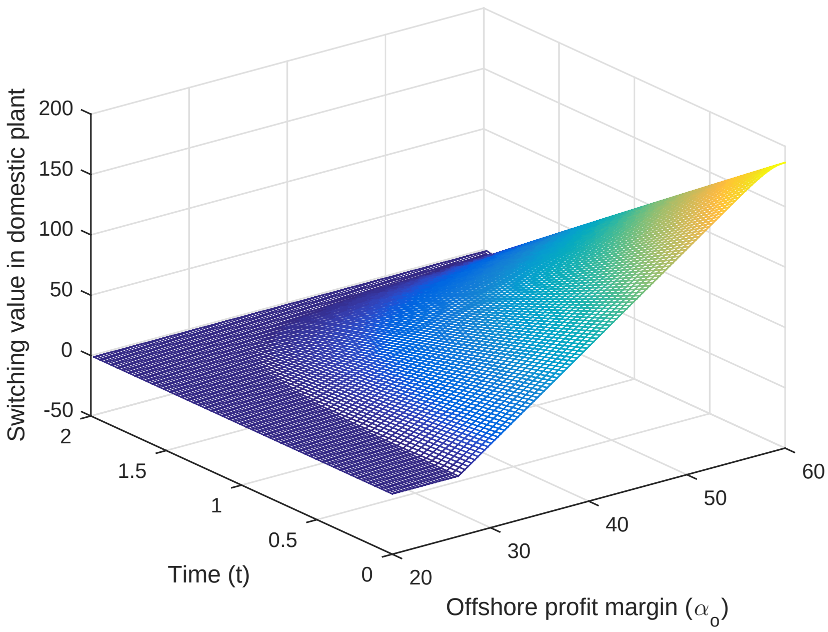


Figure 5.2 Switching value when the firm produces in domestic plant

costs. Figure 5.2 shows the value of switching option when the firm is producing in the domestic plant, and Figure 5.3 shows the value of relocation when the firm is producing in the offshore facility. In the following sections we investigate the effect of different parameters on offshoring policy.

5.3.1 Effect of Switching Cost on Switching Frequency

As we mentioned in the introduction Nembhard et al. (2003) apply switching options to obtain offshoring policies. They assume switching between in-house production and offshoring is costless. However, they mention the importance of considering such cost in the model. Our model considers the switching cost, and now we want to see how this cost can influence the offshoring policy. To see that, we run the model with three different levels of switching cost. Figure 5.4 consists of three graphs showing the optimal offshoring policy in different switching costs. As we see, when the switching cost is zero, there is no inaction region, and as the switching cost increases, the inaction region becomes larger. This implies that the firm should switch more frequently when the switching cost is relatively low. Hence, we conclude that there is an inverse relationship between switching cost and switching frequency. This insight is pretty

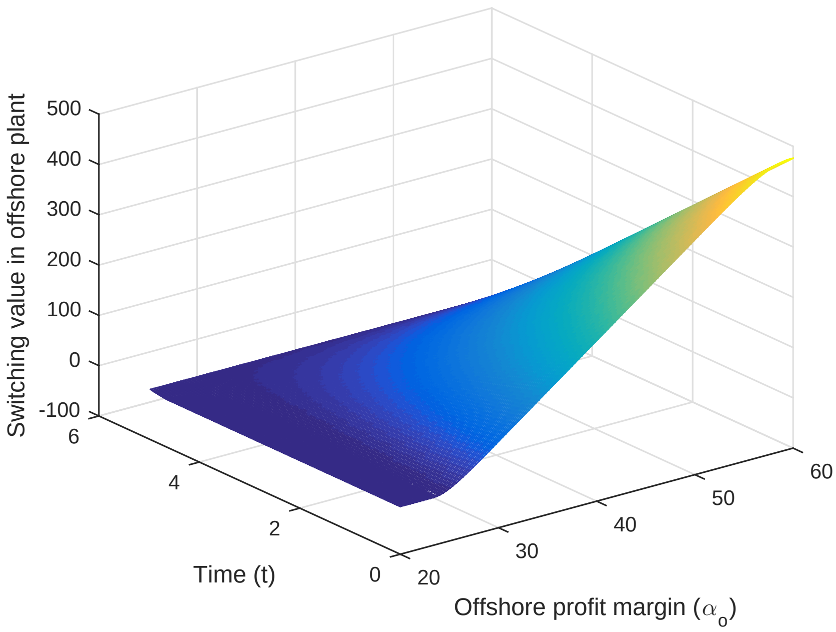
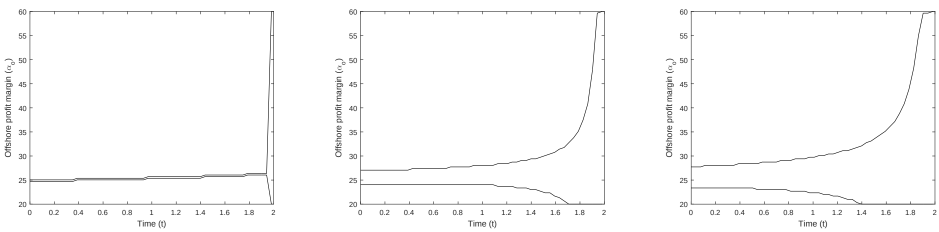


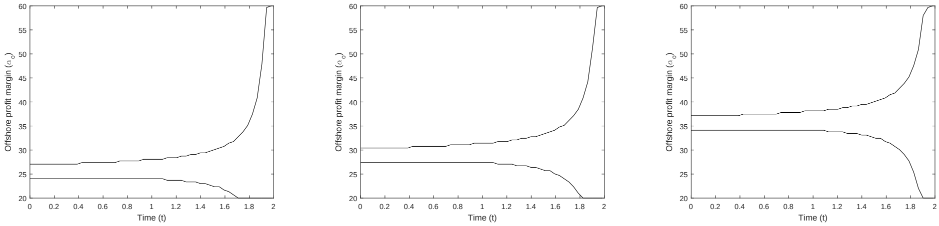
Figure 5.3 Switching value when the firm produces in offshore plant



(a) Switching cost = 0 (b) Switching cost =5 (c) Switching cost = 10

Figure 5.4 Influence of switching cost on offshoring policy

much in line with the intuition, but the reason we present it, is to show that ignoring such cost in the model can substantially change the optimal policy and make the model far off the real world.



(a) $\alpha_h = 20$ (b) $\alpha_h = 30$ (c) $\alpha_h = 40$

Figure 5.5 Influence of domestic margin on offshoring policy

5.3.2 Effect of Domestic Margin on Offshoring Policy

We now examine the effect of domestic margin on the offshoring policy. Figure 5.5 plots the offshoring policy when domestic margin has different values. We see that as α_h increases the policy shifts to the higher levels of offshore profit margin. This implies that the manager should offshore the production facility only when the expected offshore profit margin is large enough.

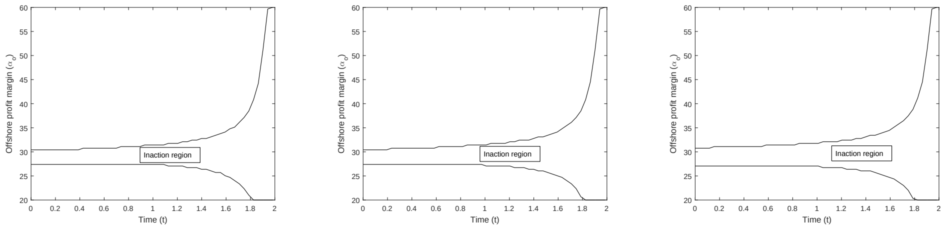
Another interesting observation in this experiment is about the impact of the profit difference between the domestic and offshore facilities on the offshoring policy. In this experiment the parameters of the offshore profit margin are fixed, consequently, by changing the α_h the profit difference between the domestic and offshore facilities also changes, and somewhat surprisingly, it does not affect the inaction area. By intuition we expect when the profit difference between the domestic and the offshore facility is low, the firm switches less frequently, but this experiment shows that the important factor affecting the frequency of switching between locations is the switching cost, not the profit difference between the domestic and offshore locations.

5.3.3 Effect of Discount Rate on Offshoring Policy

In this section we study the effect of discount rate on the offshoring policy. As we see the graphs in Figure 5.6, discount rate influences the inaction region such that as β increases the inaction region becomes larger, the size of change is however not substantial. The study shows that the manager should switch between the locations less frequently when the discount rate is high.

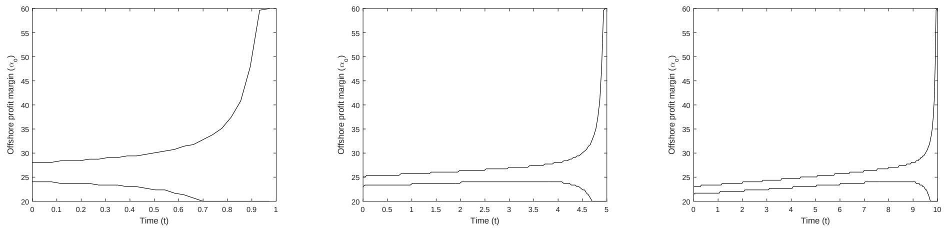
5.3.4 Effect of Time Horizon on Offshoring Policy

Companies have different time horizons, some have short, and some others have longer. In this section we seek to investigate the effect of time horizon on the offshoring policy.



(a) $\beta = 0$ (b) $\beta = 0.1$ (c) $\beta = 0.5$

Figure 5.6 Influence of discount rate on offshoring policy



(a) Time horizon = 1 year (b) Time horizon = 5 years (c) Time horizon = 10 years

Figure 5.7 Influence of time horizon on offshoring policy

Figure 5.7 plots the optimal policy in different time horizons. We see that the inaction region is substantially influenced by the time horizon such that as the time horizon becomes longer the inaction region becomes smaller. In other words, when the time horizon is relatively long, the manager should switch between the in-house production and offshoring more frequently. The reason behind this insight is that when the time horizon is relatively long, the switching option has enough time to compensate the relocation cost, and this gives the manager more flexibility to switch between the locations without much concern about the switching cost.

5.4 Stochastic Impulse Control Model

In Section 5.2 we have modelled the offshoring problem in the optimal switching setting, where the firm could choose between two locations, and no partial offshoring policy was allowed. However, in the real world, companies sometimes offshore just part of their productions. For instance, Apple produces some products in the US and some in China, the same policy for Philips that produces the medical products in the Netherlands. Alvarez & Stenbacka (2007) highlight the importance of partial offshoring, by presenting a real options model to find the optimal proportion of partial outsourcing, eventhough their model was static and does not capture the dynamic aspect of the offshoring strategy. In this section we consider the offshoring problem in a more generalized setting. We now assume the firm can dynamically choose any proportion of its production to offshore or backshore.

Let α_h denote the firm's profit margin for goods produced domestically (constant). Similarly, we denote by α_o the profit margin for goods produced at the firm's offshore facility. Given the uncertainties involved with offshore production, we assume that the evolution of α_o is governed by an arithmetic Brownian motion process, i.e.,

$$\alpha_o(t) = \alpha_o(0) + \sigma_o W_t, \tag{5.13}$$

where $\alpha_o(0) \in \mathbb{R}^+$ represent the initial production margin, and $\sigma_o \in \mathbb{R}^+$ is the volatility of the production margin, with W_t referring to a standard Brownian motion. We assume that the firm faces constant demand D for the product. If $u \in [0, 1]$ denotes the proportion of demanded goods produced domestically, the firm's instantaneous profit, Π, is given by

$$\Pi(t, u, \alpha_h, \alpha_o; D) = uD\alpha_h + (1 - u)D\alpha_o. \tag{5.14}$$

Without loss of generality, we let $D = 1$. The firm can increase or decrease u at any time. Changing u incurs both fixed and proportional costs. Denote by $K \in \mathbb{R}^+$ and $k \in \mathbb{R}^+$ the fixed and proportional costs of increasing u (i.e., increasing domestic production or reshoring). When the firm increases u by ξ, it incurs a total cost of $K + k\xi$. Similarly, denote by $L \in \mathbb{R}^+$ and $l \in \mathbb{R}^+$ the fixed and proportional costs of decreasing u (i.e., decreasing domestic production or offshoring). The total cost of decreasing u by ξ is then given by $L+l\xi$. Given some impulse control $\nu = \{(\tau_i, \xi_i)\}_{i=1}^{\infty}$ for $i = 1 \ldots n$, and starting at α_h and α_o, the total discounted expected profit is given by

$$\mathcal{J}_\nu(u_0, \alpha_o(0); \alpha_h) = \mathbb{E}\left[\int_0^\infty e^{-\beta s}[u_s\alpha_h + (1 - u_s)\alpha_o(s)]ds - \sum_{i=1}^{\infty} e^{-\beta\tau_i} B(\xi_i)\right], \tag{5.15}$$

where $B(\xi) = (K+k\cdot\xi)\mathbf{1}_{\{\xi>0\}}+(L+l\cdot|\xi|)\mathbf{1}_{\{\xi<0\}}$. Define the state $X(t) \equiv (u_t, \alpha_o(t))$, and $x = X(0) = (u_0, \alpha_o(0))$. Also, define $H(u_t, \alpha_o(t); \alpha_h) \equiv H(X_t; \alpha_h) \equiv u_t\alpha_h + (1 - u_t)\alpha_o(t)$. Hereafter, we omit the dependence on α_h since it is a pre-specified parameter, just as β, K, L, k and l are. The optimal value function is then defined as

$$V(x) = \sup_\nu \mathcal{J}_\nu(x) \tag{5.16}$$

Proceeding this way, the QVI can be derived using the usual techniques. If in $[0, \Delta t]$ we place no control at all, and starting from $\Delta t+$ we follow the optimal policy, we have

$$V(x) \geq [u_0\alpha_h + (1 - u_0)\alpha_o(0)]\frac{1 - e^{-\beta\Delta t}}{\beta} + \mathbb{E}\left[e^{-\beta\Delta t}V(X(\Delta t))|X(0) = x\right] \tag{5.17}$$

which is equivalent to

$$[u_0\alpha_h + (1-u_0)\alpha_o(0)]\frac{1-e^{-\beta\Delta t}}{\beta\Delta t} + \frac{1}{\Delta t}\mathbb{E}\left[e^{-\beta\Delta t}V(X(\Delta t)) - V(X(0))|X(0)=x\right] \le 0. \tag{5.18}$$

Defining an auxiliary function $g(s,y;u) = e^{-\beta s}V(u,y)$, we have $g(t,\alpha_o(t);u_0) = e^{-\beta t}V(u_0,\alpha_o(t)) \equiv e^{-\beta t}V(X(t))$ for $t\in[0,\Delta t]$, since $u_t \equiv u_0$ for $t\in[0,\Delta t]$ by assumption. Thus, (5.18) becomes

$$[u_0\alpha_h + (1-u_0)\alpha_o(0)]\frac{1-e^{-\beta\Delta t}}{\beta\Delta t} + \frac{1}{\Delta t}\mathbb{E}\left[g(\Delta t,\alpha_o(\Delta t);u_0) - g(0,\alpha_o(0);u_0)\right] \le 0. \tag{5.19}$$

Let Δt go to 0^+. Assuming sufficient smoothness of g and applying the appropriate version of Itô's formula to (5.19), we have

$$[u_0\alpha_h + (1-u_0)\alpha_o(0)] - \beta V(u_0,\alpha_o(0)) + \frac{\sigma_o^2}{2}V_{yy}(u_0,\alpha_o(0)) \le 0. \tag{5.20}$$

On the other hand, if at time equal to 0+ we choose to re-allocate the proportion of domestic production by adjusting u by amount ξ, we have

$$V(u_0,\alpha_o(0)) \ge V(u_0+\xi,\alpha_o(0)) - B(\xi). \tag{5.21}$$

Define the following notation

$$\begin{aligned}
\mathcal{L}V \equiv & \quad [u_0\alpha_h + (1-u_0)\alpha_o(0)] - \beta V(u_0,\alpha_o(0)) + \frac{\sigma_o^2}{2}V_{yy}(u_0,\alpha_o(0))\\
mV(u_0,\alpha_o(0)) \equiv & \quad \sup\{V(u_0+\xi,\alpha_o(0)) - (K+k\cdot\xi)|0<\xi\le 1-u_0\}\\
- & \quad V(u_0,\alpha_o(0))\\
\mathcal{M}V(u_0,\alpha_o(0)) \equiv & \quad \sup\{V(u_0-\xi,\alpha_o(0)) - (L+l\cdot\xi)|0<\xi\le u_0\}\\
- & \quad V(u_0,\alpha_o(0))
\end{aligned}$$

We have

$$\begin{aligned}
\mathcal{L}V(u_0,\alpha_o(0)) &\le 0\\
mV(u_0,\alpha_o(0)) &\le 0\\
\mathcal{M}V(u_0,\alpha_o(0)) &\le 0
\end{aligned}$$

Given a fixed boundary problem, where the no-action (continuation) region is Ω whose boundary is denoted C_Ω. The value function v corresponding to such a policy will solve the Ordinary-Differential-Equation (ODE) $\mathcal{L}v(u_0,\alpha_o(0)) = 0$ for any $(u_0,\alpha_o(0)) \in \Omega$ subjected to some boundary conditions. We now derive the boundary conditions. As $y = \alpha_o(0) \to +\infty$, and if $u_0 = 0$ (which implies that the initial allocation is to use offshore production to fully meet the demand), any 'reasonable' policy will keep the initial allocation unchanged, since the offshore production yields a profit margin goes to infinity. This yields:

$$
\begin{aligned}
\lim_{\alpha_o(0)\to+\infty} \frac{v(0,\alpha_o(0))}{\alpha_o(0)} &= \lim_{\alpha_o(0)\to+\infty} \frac{\mathcal{J}_\nu(0,\alpha_o(0);\alpha_h)}{\alpha_o(0)} \\
&= \lim_{\alpha_o(0)\to+\infty} \mathbb{E}\left[\frac{[\int_0^\infty e^{-\beta s}[u_s\alpha_h+(1-u_s)\alpha_o(s)]ds]}{\alpha_o(0)}\right] \\
&- \lim_{\alpha_o(0)\to+\infty} \mathbb{E}\left[\frac{[\sum_{i=1}^\infty e^{-\beta\tau_i}B(\xi_i)]}{\alpha_o(0)}\right] \\
&= \lim_{\alpha_o(0)\to+\infty} \mathbb{E}\left[\frac{\int_0^\infty e^{-\beta s}\alpha_o(s)ds}{\alpha_o(0)}\right] \\
&= \lim_{\alpha_o(0)\to+\infty} \mathbb{E}\left[\frac{\int_0^\infty e^{-\beta s}(\alpha_o(0)+\sigma_o W_s)ds}{\alpha_o(0)}\right] \\
&= \lim_{\alpha_o(0)\to+\infty} \mathbb{E}\left[\int_0^\infty e^{-\beta s}ds\right] \\
&= \frac{1}{\beta}. \qquad (5.22)
\end{aligned}
$$

If $y=\alpha_o(0)\to+\infty$, and $u_0\neq 0$, any 'reasonable' policy will immediately re-allocate to use offshore production to fully meet the demand, and to keep the allocation unchanged thereafter. Thus

$$
\lim_{\alpha_o(0)\to+\infty} \frac{v(u_0,\alpha_o(0))}{\alpha_o(0)} = \lim_{\alpha_o(0)\to+\infty} \frac{v(0,\alpha_o(0))-L-l\cdot u_0}{\alpha_o(0)} = \frac{1}{\beta} \qquad (5.23)
$$

Similarly, when $y=\alpha_o(0)\to-\infty$, and if $u_0=1$ (which implies that the initial allocation is to use home production to fully meet the demand), any 'reasonable' policy will keep the initial allocation unchanged, since the offshore production yields a profit margin goes to infinity. This yields:

$$
\begin{aligned}
\lim_{\alpha_o(0)\to-\infty} v(1,\alpha_o(0)) &= \lim_{\alpha_o(0)\to-\infty} \mathcal{J}_\nu(1,\alpha_o(0);\alpha_h), \\
&= \lim_{\alpha_o(0)\to-\infty} \mathbb{E}\left[\int_0^\infty e^{-\beta s}[u_s\alpha_h+(1-u_s)\alpha_o(s)]ds\right] \\
&- \lim_{\alpha_o(0)\to-\infty} \mathbb{E}\left[\sum_{i=1}^\infty e^{-\beta\tau_i}B(\xi_i)\right], \\
&= \lim_{\alpha_o(0)\to-\infty} \mathbb{E}[\int_0^\infty e^{-\beta s}\alpha_h ds] = \frac{\alpha_h}{\beta}. \qquad (5.24)
\end{aligned}
$$

If $y=\alpha_o(0)\to-\infty$, and $u_0\neq 1$, any 'reasonable' policy will immediately re-allocate to use home production to fully meet the demand, and to keep the allocation unchanged thereafter. Thus

$$\lim_{\alpha_o(0)\to-\infty} v(u_0,\alpha_o(0)) = \lim_{\alpha_o(0)\to-\infty} v(1,\alpha_o(0)) - K - k\cdot(1-u_0) = \frac{\alpha_h}{\beta} - K - k\cdot(1-u_0) \tag{5.25}$$

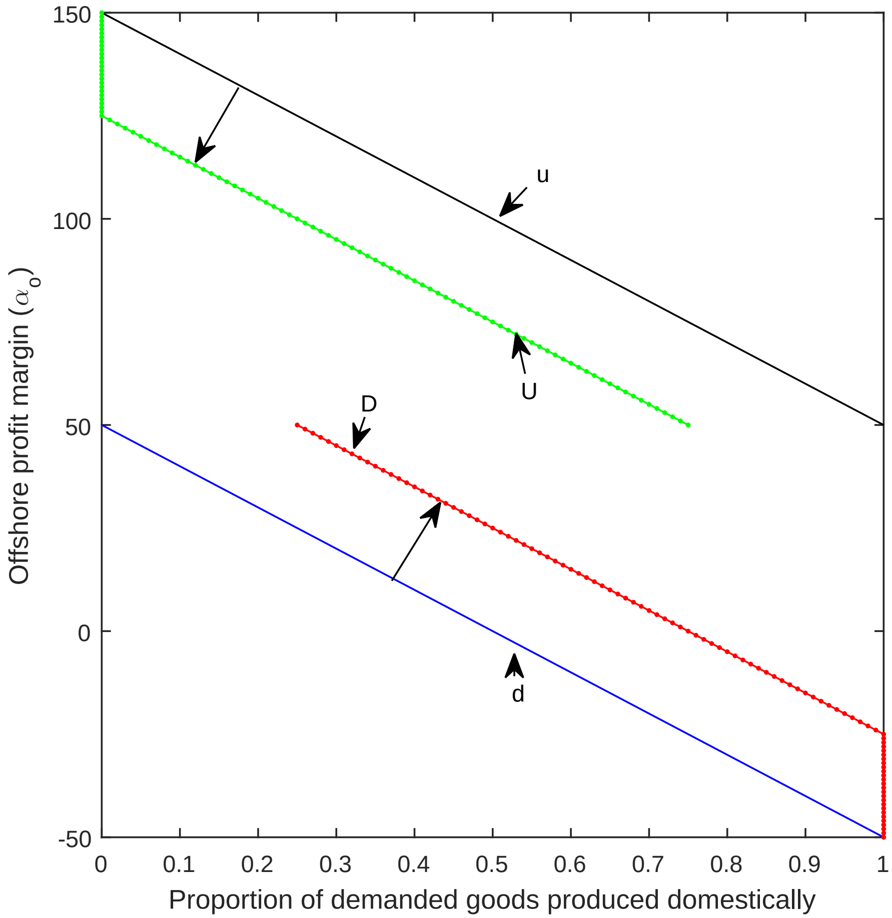


Figure 5.8 Guessed d-D-U-u policy

Now we have QVI and the boundary conditions. Our next step in the future is to use the variant of Moving-Boundary method to solve the QVI. To do so, we have some limitations that we discuss it in the next section.

5.5 Limitations of Our Impulse Control Model

At this stage, we need to prove the verification theorem. Richard (1977) has proved the verification theorem for the standard impulse control problem. However, our problem setting differs form the standard problem. In the standard impulse control problem, the controller controls the stochastic process directly. Consider for example the problem of maintaining an optimal level of cash on hand in a firm that is subjected to various cash inflows and outflows. In such problem, the cash inflow/outflow is both the stochastic state variable and the control parameter at the same time, but in our problem the control parameter is the offshoring proportion, while the stochastic state variable is the offshore profit margin. We are not able to provide the proof of the verification theorem in the current thesis. However, we can give an initial guess for the shape of optimal policy. The guess is illustrated in Figure 5.8. The policy says when the offshore profit margin α_o is between d and u, the manager should not change the proportion of demanded goods produced domestically, but as soon as the offshore profit margin α_o exceeds d the manager should change the proportion to line D, and as soon as offshore profit margin α_o goes above u the manager should change the proportion to line U. Given the guessed policy we are able to obtain the value function for a given policy. Then we need to find an update condition such that it improves the value function monotonically and converges to the optimal policy.

5.6 Conclusion

Limited literature on the operational aspect of the offshoring-backshoring strategy motivated us to contribute to the literature by considering a dynamic model for making offshoring decisions that is grounded in stochastic control theory. We have shown that using stochastic control we are able to derive optimal control policies which are easy to obtain, understand and implement. First we have built a mathematical model in which offshoring-backshoring problem considered as an optimal switching problem. The firm could choose the production place between two locations: domestic or offshore. Our approach differs from Nembhard et al. (2003) in that we considered the switching costs. We have presented an example to show the optimal policy. Then we considered the offshoring-backshoring strategy in a more generalized setting in which we assume the firm can offshore-backshore any proportion of its production. We have modelled the new approach based on stochastic impulse control. We also have presented the quasi variational inequalities, and the guessed policy.

We have investigated the effect of several parameters on the offshoring policy to obtain managerial insights. Our study shows that the relocation cost has an important role in the offshoring policy such that high relocation costs result in large inaction regions, and consequently the frequency of switching between the locations should significantly decrease by the manager. Our study shows that discount rate does not have a significant impact on the offshoring decision. Our study further shows that the profit difference between the locations does not change the inaction region either. Our experiments demonstrate that the time horizon is an important factor in the offshoring decisions and show that the manager should switch between the domestic and offshoring plants more frequently when the time horizon is relatively long.

There are a number of questions and issues left for further exploration. One is to investigate the effect of the proportional relocation cost on the offshoring policy. However, to do so, we need to solve the QVI associated with the impulse control model to obtain impulse control optimal policies. Then we are also able to see the different aspects of proportional offshoring policy versus bang-bang policy.

Chapter 6

Conclusion

Like options on stocks, options on commodities and options on business activities provide firms with protection against financial risks. Both are routinely used by non-financial firms; however the former is an OTC contract between the buyers and suppliers used to hedge financial risks arising from the procurement and production of commodities, while the latter are called real options and are used for certain business activities such as switching by utilizing the inherent flexibility to maximize the value of firms and minimize the risk they face. The current thesis addresses both in four chapters. Chapters 2 and 3 focus on the OTC option contracts, while Chapters 4 and 5 concern real options. However, in the all four chapters we adhere to our main research question:

What is the maximum value generated by using options - either OTC contracts or real options - for risk hedging and how can firms attain this value?

This chapter concerns the investigation of the research question in the previous four chapters, and consists of three sections. In the first section we present our main findings from each chapter, then in the second section we talk about the impact of flexibility on decision making, and finally in the last section we discuss the future research directions.

6.1 Summary of Results

In Chapter 2 we study the role of OTC options in the hedging policy of a commodity processor. Motivated by the increased role of freight transportation in the supply chain of corporations, we consider the problem of minimizing the cost of procuring and transporting the commodity by hedging with options. We consider a firm that faces three sources of uncertainty and develop a model which computes the optimal number of options that the firm should negotiate with the commodity supplier and the ocean carrier. We show that the optimal options position for each department is given by an elegant expression in the form of newsvendor critical fractile. We also compare the cases with and without correlation between the stochastic variables and derive the necessary optimal conditions. Our results show that that hedging with options results in cost reduction for a risk-neutral firm, and consequently create value for the firm, with assuming there are no market frictions. Our extensive numerical studies in Chapter 2 yield five managerial insights.

First, we show that if the firm's required policy is to meet the whole demand, the procurement and transportation departments can independently select the number of options to negotiate; but if the 100% service level is not a required condition, considering a reservation price, the procurement and transportation departments should collaborate on deciding a hedging policy.

Second, our results show that if demand is expected to grow, the firm should negotiate more commodity and freight options.

Third, our study demonstrates that when the costs to procure and transport the commodity are highly volatile, the firm should take a larger options position to offset the increased volatility.

Fourth, we further show that the firm should take a larger options position when there exists a positive correlation between commodity spot prices/freight rates and demand.

Finally our results show that a higher reservation price not only leads to higher service levels, but also leads to greater value creation for the firm as the firm takes a larger options position to meet the increased service level.

In chapter 3 we consider a commodity processor who employs a dual sourcing strategy to take advantage of price differences. Materials procured on offshore markets as a consequence of the dual-sourcing strategy need to be transported, typically via ocean freight. Freight rates, however, are highly volatile because of increased globalization and demand for ocean freight services, coupled with long lead times to build ships. Motivated by these considerations, we investigate the role of options in hedging against ocean freight rates. We develop mathematical models of the firm's total expected procurement and transportation costs, explicitly taking into account the transportation cost by allowing for stochastic freight rates, in addition to stochastic demand and commodity spot price, and determine the optimal number of options the firm should negotiate. We develop models and derive optimal policies for the two main categories of ocean transport: tankers and container ships. We derive the optimal policy the firm should follow once all uncertainties are resolved. Our numerical studies based on the developed models highlight the importance of explicitly taking into account transportation costs. The results also show that hedging against freight rate risk with options can lead to cost reduction for a firm that employs a dual-sourcing strategy. Furthermore, our numerical studies in chapter three yield several managerial insights.

First, our numerical studies demonstrate that dual-sourcing strategies limit price risk exposure and offer the firm flexibility by allowing the firm to take advantage of low domestic costs.

Second, our investigation shows that cost savings may be achieved by chartering larger tankers and container ships even though they may cost more to charter.

Third, our numerical studies further show that buying freight options may increase the proportion of demand procured on the offshore market.

Fourth, the results show that when the volatility of freight rate is relatively high the firm should procure more on the domestic market.

Finally, our study demonstrates that the firm should buy more on the offshore market when the volatility of commodity price is relatively high.

In Chapter 4 we move from OTC options to real options. In Chapter 4 we consider switching options as an approach which can protect the firm against adverse price changes. The chapter is a pure methodological research, although it can be applied in many real life problems such as tolling agreements. In the chapter we develop a computational method based on Moving-Boundary to solve the optimal switching problem in two regimes over a finite time horizon. The idea is to convert a free boundary problem into a sequence of fixed boundary problems as was first developed for singular control problems in Kumar & Muthuraman (2004). In this study we demonstrate that the idea of converting to a sequence of fixed boundary problems is still possible for free boundary problems arising from switching problems; the specific method, however, is different. We further establish the theoretical guarantees for the method by proving that the solution to associated Quasi Variational Inequalities is unique, and that the optimal condition can be violated by superset conditions. Consequently, by the Moving-Boundary iterations from a violated region, the value monotonically increases in the next iteration and converges to the true value function. Then the optimal switching policy can be easily obtained.

To show that our algorithm works properly we solved a numerical example. In the example we present the shape of optimal policy and show how the policy and the value function converges to the optimum. We also study the impact of volatility on the optimal policy. Our study shows that the inaction region becomes smaller as the volatility increases. This implies that the firm should switch more frequently when the underlying is more risky.

Chapter 5 concerns operationalizing the offshoring strategy by using switching options. Limited literature on the operational aspect of the offshoring-backshoring strategy motivated us to contribute to the literature by considering a dynamic model for making offshoring decisions. We show that by using switching options, we are able to derive optimal control policies which are easy to obtain, understand and implement. Our model considers a firm that could choose two locations for the site of production: domestic or offshore. The firms has a stochastic profit margin in the offshore facility, while its domestic profit margin is deterministic. Our aim is to find an optimal policy for the firm to maximize its profit over a finite time horizon. To do so, we propose a model based on switching options. Our model differs from Nembhard et al. (2003) in that we consider the switching costs. We implemented several numerical experiments and obtained interesting managerial insights.

First, our study shows that the relocation cost has an important role in the offshoring policy such that high relocation costs result in large inaction regions, and consequently the frequency of switching between the locations should significantly decrease.

Second, our numerical experiments demonstrate that the discount rate does not have a significant effect on the offshoring decision; however, as the discount rate increases substantially, the inaction region becomes slightly larger, and consequently the firm should switch less frequently.

Third, our study shows that the profit difference between the domestic and the offshore location does not affect the frequency of switching between them; however, it obviously shifts the policy towards the offshore facility as the profit margin decreases in the domestic location.

Finally, our experiments demonstrate that the time horizon is an important factor in the offshoring decisions such that the manager should switch between the domestic and offshoring plants more frequently when the time horizon is relatively long.

6.2 Impact of Flexibility on Decision Making

In today's world managers are faced with an increasing number of alternatives in order to make an optimal decision. This obviously leads to an increase in flexibility for the manager and better management of risks faced by the firm; however, it also leads to an increase in complexity of the problem to solve. In this thesis we have taken into account the alternatives available to the manager making decisions in the setting of operational efficiency and financial risk management. In the current global environment, firms are able to source their supplies from multiple locations in the world and transport them even by different means back to their factory. Firms also have the ability to engage in OTC option contracts on commodities as well as freight rates. Further, firms can have multiple manufacturing locations.

In terms of the simple problem of purchasing raw materials the firm faces price fluctuations. This risk can be managed by purchasing option contracts. However, in the transportation of raw materials from the source to the factory managers face two kinds of price risks, in the commodity as well as the transportation. It is possible to engage in option contracts for both at the same time. Chapter 2 shows how such flexibility offered by these option contracts helps the manager to better hedge against the price risk. Another level of flexibility afforded to the managers is the choice between multiple sources for the procurement as well as multiple ways to transport them. Chapter 3 takes this into account and shows how to optimally procure goods from multiple sources and transport them to the factory.

Chapters 2 and 3 talk about optimizing procurement strategies, but upper management also faces different issues in order to maintain profit margins. The first of these issues is the decision whether or not to procure a particular item, i.e. buy or do not buy decision. This is the same problem as keeping the factory running or shutting it down. Also, analogous to this is the problem of offshoring in which the manager has to decide in which of the firm's multiple factories production should take place. In Chapters 4 and 5 we tackle these issues by providing the managers with an optimal policy to make this decision.

6.3 Future Research Directions

In this section we briefly discuss some future research directions regarding the underlying concepts presented in this thesis.

In Chapters 2 and 3 we have presented an optimal policy to hedge against the freight rate risk. One interesting extension could be to include a shipping lead time in the model. We may consider a stochastic lead time, then the firm faces another important risk. Another challenging extension could be to add the inventory aspect to the model. To do so we need to extend the model to at least two periods. Subsequently, it might be interesting to extend the model to a multi-period one, something that would make the problem very complex to solve. One more extension could be considering multi products instead of one product and see how the firm can manage their transportation when they can be carried in one type of ships.

In chapter 4 we proposed a Moving-Boundary approach to solve the switching options. Our method works in the two regimes setting. Our next step would be to extend the method for more than two regimes. To do so we might use simulation based methods.

In chapter 5 we studied a switching based model for offshoring problem. Then we introduced a model based on impulse control; however, we did not propose a solution for that. Our future work will aim to solve the impulse control model, and subsequently investigate the effect of the proportional relocation cost on the offshoring policy. To do so, our plan is to apply the idea of Moving-Boundary method.

Besides future directions for each chapter, we can aim a future direction for this area of research at large. In this thesis our approach is modelling the real world problems by using mathematical techniques. Strictly speaking, we have a model-based study in this thesis. However, we believe there is still much space to examine the models with real data to see how these models fit to the real world. In other words, an interesting future direction could be empirical research to test models and policies developed in this thesis. We believe by doing so, we can find additional nice managerial insights. For example, in Chapters 2 and 3 we build mathematical models to obtain hedging policies against the freight risk. However, by using available public data for freight rates we might be able to examine our models in the real world and find out the real value created for the firms by using freight options.

Bibliography

Aksin, O. Z. & Masini, A. (2008), 'Effective strategies for internal outsourcing and offshoring of business services: An empirical investigation', *Journal of Operations Management* **26**(2), 239–256.

Allon, G. & Van Mieghem, J. A. (2010), 'Global dual sourcing: Tailored base-surge allocation to near-and offshore production', *Management Science* **56**(1), 110–124.

Alvarez, L. H. & Stenbacka, R. (2007), 'Partial outsourcing: A real options perspective', *International Journal of Industrial Organization* **25**(1), 91–102.

Amram, M. & Kulatilaka, N. (1999), 'Real options'.

Anderson, R. W. & Danthine, J.-P. (1983), 'The time pattern of hedging and the volatility of futures prices', *The Review of Economic Studies* pp. 249–266.

Barankin, E. (1961), 'A delivery-lag inventory model with an emergency provision (the single-period case)', *Naval Research Logistics Quarterly* **8**(3), 285–311.

Barnes-Schuster, D., Bassok, Y. & Anupindi, R. (2002), 'Coordination and flexibility in supply contracts with options', *Manufacturing & Service Operations Management* **4**(3), 171–207.

Benaroch, M. & Kauffman, R. J. (2000), 'Justifying electronic banking network expansion using real options analysis', *MIS quarterly* pp. 197–225.

Bergin, P. R., Feenstra, R. C. & Hanson, G. H. (2011), 'Volatility due to offshoring: Theory and evidence', *Journal of International Economics* **85**(2), 163–173.

Berling, P. & Martínez-de Albéniz, V. (2011), 'Optimal inventory policies when purchase price and demand are stochastic', *Operations research* **59**(1), 109–124.

Black, F. & Scholes, M. (1973), 'The pricing of options and corporate liabilities', *The journal of political economy* pp. 637–654.

Blinder, A. S. (2006), 'Offshoring: the next industrial revolution?', *Foreign affairs* pp. 113–128.

Bodnar, G. M., Hayt, G. S., Marston, R. C. & Smithson, C. W. (1995), 'Wharton survey of derivatives usage by us non-financial firms', *Financial management* **24**(2), 104–114.

Boute, R. N. & Van Mieghem, J. A. (2014), 'Global dual sourcing and order smoothing: The impact of capacity and lead times', *Management Science* **61**(9), 2080–2099.

Brekke, K. A. & Øksendal, B. (1994), 'Optimal switching in an economic activity under uncertainty', *SIAM Journal on Control and Optimization* **32**(4), 1021–1036.

Brennan, M. J. & Schwartz, E. S. (1985), 'Evaluating natural resource investments', *Journal of business* pp. 135–157.

Bunyaratavej, K., Hahn, E. D. & Doh, J. P. (2007), 'International offshoring of services: A parity study', *Journal of International Management* **13**(1), 7–21.

Burke, G. J., Carrillo, J. E. & Vakharia, A. J. (2007), 'Single versus multiple supplier sourcing strategies', *European journal of operational research* **182**(1), 95–112.

Cachon, G. P. (2003), 'Supply chain coordination with contracts', *Handbooks in operations research and management science* **11**, 227–339.

Carmona, R. & Ludkovski, M. (2010), 'Valuation of energy storage: an optimal switching approach', *Quantitative Finance* **10**(4), 359–374.

Chockalingam, A. & Muthuraman, K. (2011), 'American options under stochastic volatility', *Operations research* **59**(4), 793–809.

Chod, J., Rudi, N. & Van Mieghem, J. A. (2010), 'Operational flexibility and financial hedging: Complements or substitutes?', *Management Science* **56**(6), 1030–1045.

Clemons, E. K. & Weber, B. W. (1990), 'Strategic information technology investments: guidelines for decision making', *Journal of Management Information Systems* **7**(2), 9–28.

Copeland, T. E. & Keenan, P. T. (1998), 'Making real options real', *The McKinsey Quarterly* **1**(3), 128–142.

Dachs, B., Borowiecki, M., Kinkel, S. & Schmall, T. C. (2012), 'The offshoring of production activities in european manufacturing'.

DeMarzo, P. M. & Duffie, D. (1995), 'Corporate incentives for hedging and hedge accounting', *Review of Financial Studies* **8**(3), 743–771.

Devalkar, S. K., Anupindi, R. & Sinha, A. (2011), 'Integrated optimization of procurement, processing, and trade of commodities', *Operations Research* **59**(6), 1369–1381.

Dewhurst, N. & Meeker, D. (2004), 'Improved product design practices would make us manufacturing more cost effective', *A case to consider before outsourcing to China. Boothroyd Dewhurst Inc* .

Djehiche, B., Hamadene, S. & Popier, A. (2009), 'A finite horizon optimal multiple switching problem', *SIAM Journal on Control and Optimization* **48**(4), 2751–2770.

Dong, L. & Liu, H. (2007), 'Equilibrium forward contracts on nonstorable commodities in the presence of market power', *Operations Research* **55**(1), 128–145.

Duckworth, K. & Zervos, M. (2001), 'A model for investment decisions with switching costs', *Annals of Applied probability* pp. 239–260.

Einstein, A. (1905), 'The theory of the brownian movement', *Ann. der Physik* **17**, 549.

Farrell, D. (2005), 'Offshoring: Value creation through economic change', *Journal of Management Studies* **42**(3), 675–683.

Federgruen, A. & Yang, N. (2009), 'Optimal supply diversification under general supply risks', *Operations Research* **57**(6), 1451–1468.

Feng, H. & Muthuraman, K. (2010), 'A computational method for stochastic impulse control problems', *Mathematics of Operations Research* **35**(4), 830–850.

Fransoo, J. C. & Lee, C.-Y. (2013), 'The critical role of ocean container transport in global supply chain performance', *Production and Operations Management* **22**(2), 253–268.

Froot, K. A., Scharfstein, D. S. & Stein, J. C. (1993), 'Risk management: Coordinating corporate investment and financing policies', *the Journal of Finance* **48**(5), 1629–1658.

Fu, Q., Lee, C.-Y. & Teo, C.-P. (2010), 'Procurement management using option contracts: random spot price and the portfolio effect', *IIE transactions* **42**(11), 793–811.

Garrido, R. A. (2007), 'Procurement of transportation services in spot markets under a double-auction scheme with elastic demand', *Transportation Research Part B: Methodological* **41**(9), 1067–1078.

Gaur, V. & Seshadri, S. (2005), 'Hedging inventory risk through market instruments', *Manufacturing & Service Operations Management* **7**(2), 103–120.

Goel, A. & Tanrisever, F. (2011), 'Integrated options and spot procurement for commodity processors', *Available at SSRN 1898866* .

Grossman, G. M. & Rossi-Hansberg, E. (2008), 'Trading tasks: A simple theory of offshoring', *The American Economic Review* **98**(5), 1978–1997.

Grünig, R. & Morschett, D. (2012), Evaluating new production and sourcing locations, *in* 'Developing International Strategies', Springer, pp. 167–186.

Harrison, A. & McMillan, M. (2011), 'Offshoring jobs? multinationals and us manufacturing employment', *Review of Economics and Statistics* **93**(3), 857–875.

Hensher, D. A. & Houghton, E. (2004), 'Performance-based quality contracts for the bus sector: delivering social and commercial value for money', *Transportation Research Part B: Methodological* **38**(2), 123–146.

Hensher, D. A. & Stanley, J. (2003), 'Performance-based quality contracts in bus service provision', *Transportation Research Part A: Policy and Practice* **37**(6), 519–538.

Hirshleifer, D. (1988), 'Residual risk, trading costs, and commodity futures risk premia', *Review of Financial Studies* **1**(2), 173–193.

Huchzermeier, A. & Cohen, M. A. (1996), 'Valuing operational flexibility under exchange rate risk', *Operations research* **44**(1), 100–113.

Ingersoll Jr, J. E. & Ross, S. A. (1992), 'Waiting to invest: Investment and uncertainty', *Journal of Business* pp. 1–29.

Janssen, F. & de Kok, T. (1999), 'A two-supplier inventory model', *International Journal of Production Economics* **59**(1), 395–403.

Kavussanos, M. G. & Visvikis, I. D. (2004), 'Market interactions in returns and volatilities between spot and forward shipping freight markets', *Journal of Banking & Finance* **28**(8), 2015–2049.

Kinkel, S. & Maloca, S. (2009), 'Drivers and antecedents of manufacturing offshoring and backshoringa german perspective', *Journal of Purchasing and Supply Management* **15**(3), 154–165.

Kleindorfer, P. R. & Wu, D. J. (2003), 'Integrating long-and short-term contracting via business-to-business exchanges for capital-intensive industries', *Management Science* **49**(11), 1597–1615.

Koekebakker, S., Adland, R. & Sødal, S. (2007), 'Pricing freight rate options', *Transportation Research Part E: Logistics and Transportation Review* **43**(5), 535–548.

Kumar, S. & Muthuraman, K. (2004), 'A numerical method for solving singular stochastic control problems', *Operations Research* **52**(4), 563–582.

Lee, C.-Y., Tang, C. S., Yin, R. & An, J. (2015), 'Fractional price matching policies arising from the ocean freight service industry', *Production and Operations Management* **24**(7), 1118–1134.

Lewin, A. Y. & Peeters, C. (2006), 'Offshoring work: business hype or the onset of fundamental transformation?', *Long Range Planning* **39**(3), 221–239.

Li, H., Ritchken, P. & Wang, Y. (2009), 'Option and forward contracting with asymmetric information: Valuation issues in supply chains', *European Journal of Operational Research* **197**(1), 134–148.

Lim, A., Rodrigues, B. & Xu, Z. (2008), 'Transportation procurement with seasonally varying shipper demand and volume guarantees', *Operations research* **56**(3), 758–771.

Liu, Z. & Nagurney, A. (2011), 'Supply chain outsourcing under exchange rate risk and competition', *Omega* **39**(5), 539–549.

Ludkovski, M. (2005), *Optimal switching with applications to energy tolling agreements*, University Microfilms.

Ly Vath, V. & Pham, H. (2007), 'Explicit solution to an optimal switching problem in the two-regime case', *SIAM Journal on Control and Optimization* **46**(2), 395–426.

Martínez-de Albéniz, V. & Simchi-Levi, D. (2005), 'A portfolio approach to procurement contracts', *Production and Operations Management* **14**(1), 90–114.

Modigliani, F. & Miller, M. H. (1958), 'The cost of capital, corporation finance and the theory of investment', *The American economic review* **48**(3), 261–297.

Moinzadeh, K. & Nahmias, S. (1988), 'A continuous review model for an inventory system with two supply modes', *Management Science* **34**(6), 761–773.

Moinzadeh, K. & Schmidt, C. P. (1991), 'An (s- 1, s) inventory system with emergency orders', *Operations Research* **39**(2), 308–321.

Moschini, G. & Lapan, H. (1992), 'Hedging price risk with options and futures for the competitive firm with production flexibility', *International Economic Review* pp. 607–618.

Muthuraman, K. (2008), 'A moving boundary approach to american option pricing', *Journal of Economic Dynamics and Control* **32**(11), 3520–3537.

Muthuraman, K. & Kumar, S. (2006), 'Multidimensional portfolio optimization with proportional transaction costs', *Mathematical Finance* **16**(2), 301–335.

Nembhard, H. B., Shi, L. & Aktan, M. (2003), 'A real options design for product outsourcing', *The Engineering Economist* **48**(3), 199–217.

Nomikos, N. K., Kyriakou, I., Papapostolou, N. C. & Pouliasis, P. K. (2013), 'Freight options: Price modelling and empirical analysis', *Transportation Research Part E: Logistics and Transportation Review* **51**, 82–94.

Oldenski, L. et al. (2016), How offshoring and global supply chains enhance the us economy, Technical report.

Pachon, J., Erkoc, M. & Iakovou, E. (2007), 'Contract optimization with front-end fare discounts for airline corporate deals', *Transportation Research Part E: Logistics and Transportation Review* **43**(4), 425–441.

Pindyck, R. S. & Rotemberg, J. J. (1988), 'The excess co-movement of commodity prices'.

Richard, S. F. (1977), 'Optimal impulse control of a diffusion process with both fixed and proportional costs of control', *SIAM Journal on Control and Optimization* **15**(1), 79–91.

Richmond, W. & Seidman, A. (1991), Outsourcing and contractual issues in the systems development life, Technical report, Working Paper.

Sakong, Y., Hayes, D. J. & Hallam, A. (1993), 'Hedging production risk with options', *American Journal of Agricultural Economics* **75**(2), 408–415.

Samuelson, P. A. (1965), 'Proof that properly anticipated prices fluctuate randomly', *Industrial Management Review* **6**(2), 41.

Secomandi, N. & Kekre, S. (2014), 'Optimal energy procurement in spot and forward markets', *Manufacturing & Service Operations Management* **16**(2), 270–282.

Smith, C. W. & Stulz, R. M. (1985), 'The determinants of firms' hedging policies', *Journal of financial and quantitative analysis* **20**(04), 391–405.

Song, J.-S. & Zipkin, P. (2009), 'Inventories with multiple supply sources and networks of queues with overflow bypasses', *Management Science* **55**(3), 362–372.

Spinler, S. & Huchzermeier, A. (2006), 'The valuation of options on capacity with cost and demand uncertainty', *European Journal of Operational Research* **171**(3), 915–934.

Stratman, J. K. (2008), 'Facilitating offshoring with enterprise technologies: Reducing operational friction in the governance and production of services', *Journal of Operations Management* **26**(2), 275–287.

Tan, Z., Yang, H. & Guo, X. (2010), 'Properties of pareto-efficient contracts and regulations for road franchising', *Transportation Research Part B: Methodological* **44**(4), 415–433.

Tomlin, B. & Wang, Y. (2005), 'On the value of mix flexibility and dual sourcing in unreliable newsvendor networks', *Manufacturing & Service Operations Management* **7**(1), 37–57.

Trigeorgis, L. (1996), *Real options: Managerial flexibility and strategy in resource allocation*, MIT press.

Trigeorgis, L. & Mason, S. P. (1987), 'Valuing managerial flexibility', *Midland corporate finance journal* **5**(1), 14–21.

Turcic, D., Kouvelis, P. & Bolandifar, E. (2015), 'Hedging commodity procurement in a bilateral supply chain', *Manufacturing & Service Operations Management* **17**(2), 221–235.

Tyworth, J. E. & Ruiz-Torres, A. (2000), 'Transportation's role in the sole-versus dual-sourcing decision', *International Journal of Physical Distribution & Logistics Management* **30**(2), 128–144.

Van Houtum, G.-J. & Kranenburg, B. (2015), *Spare parts inventory control under system availability constraints*, Vol. 227, Springer.

Van Mieghem, J. A. (2003), 'Commissioned paper: Capacity management, investment, and hedging: Review and recent developments', *Manufacturing & Service Operations Management* **5**(4), 269–302.

Yang, Z., Aydin, G., Babich, V. & Beil, D. R. (2012), 'Using a dual-sourcing option in the presence of asymmetric information about supplier reliability: Competition vs. diversification', *Manufacturing & Service Operations Management* **14**(2), 202–217.

Yercan, F. & Yildiz, T. (2012), '03 international maritime trade and logistics', *Maritime Logistics: A Complete Guide to Effective Shipping and Port Management* p. 23.

Summary

Risk Management at the Interface of Operations and Finance

Flexibility is inherent in the decision-making process of all firms. This flexibility, if utilized optimally, can generate value for the firm and its supply chain partners. This research develops models to value operational flexibility in a number of scenarios. The scenarios considered cover a spectrum of decision-making problems facing firms, including dual sourcing, offshoring/reshoring production, transporting commodities and switching between production states. We utilize options theory and stochastic control theory to develop models that allow for the valuation of operational flexibility in the considered scenarios. Valuing this flexibility consequently yields the optimal strategies/policies that the firm should adhere to in order to realize this value. This research contributes to the literature on flexibility and risk management at the interface of operations and finance. From an application perspective, the thesis contributes to the literature on sourcing, offshoring and ocean freight transportation. From a methodological perspective, the thesis also contributes to the literature on stochastic control and options valuation. The thesis consists of four main chapters (excluding introduction and conclusion chapters), each a working paper tackling a different scenario. The following paragraphs summarize each of the chapters.

Transporting Commodities: Hedging against Price, Demand and Freight Rate Risk with Options

Like options on stocks, options on commodities provide firms with protection against adverse price movements. Many firms procure a commodity at offshore locations and transport it via ocean freight. Increased globalization and increased demand for ocean-based transportation has resulted in ocean freight itself becoming a volatile commodity. In this chapter, we consider a commodity processor and develop models to determine the firms optimal hedging policy. The models allow for three sources of uncertainty; demand, commodity spot price and freight rate. The optimal hedging

policies are variants of the classical newsvendor critical fractile. We show that partially procuring the commodity and its freight through option contracts, rather than entirely on the volatile spot market creates value, even for a risk-neutral firm. We then perform extensive numerical experiments to study the influence of the underlying parameters on the optimal hedging policies and value creation.

Dual Sourcing: Optimal Procurement Policy with Option Hedging against Freight Rate Risk

This chapter investigates the role of options in providing protection against volatile ocean freight rates for a commodity processor that employs a dual-sourcing strategy. Procuring the commodity offshore results in lower procurement costs when compared to procuring the commodity in the domestic market, but entails a transportation cost. Transportation of goods from offshore markets typically involves ocean freight. Ocean freight rates have become highly volatile. We develop models that integrate the firm's optimal options position and optimal sourcing decision, taking into account volatile ocean transportation costs. We develop a model for a commodity processor that hires tankers, restricting the firm to charter an integer number of tankers to reflect reality. We also develop a model for a commodity processor that charters container ships. Tankers are cargo ships that are typically used to transport fluids such as crude oil, while container ships are cargo ships that carry their entire load in truck-size containers. We determine the firm's optimal options position for each model, and conduct numerical studies. Our studies highlight the importance of explicitly accounting for the cost. Our studies demonstrate the cost-effectiveness of utilizing options on the freight rate and a dual-sourcing strategy. Our studies also shed some light on the popularity of 'mega ships'.

Valuing Optimal Switching Options with the Moving-boundary Method

The primary contribution of this chapter is the development of a stochastic control-based methodology to tackle the optimal-switching problem. Specifically, we extend the Moving Boundary Method to tackle such problems. The Moving Boundary Method has been successfully applied to optimal-stopping problems. Optimal-switching problems can be thought of as sequences of optimal-stopping problems and possess complicating features, making an extension of the Moving-Boundary Method to tackle such problems non-trivial. The method is then applied to problems in the sourcing and energy domains.

A Stochastic Control Approach to Operationalizing Offshore Production Decisions

As a result of increased globalization, firms tend to offshore parts of their production to benefit from raw material price and labor cost differences across various locations, increasing the flexibility of their supply chains. These price differences, however, are not static. For example, in the past China had the lowest labor rates in the world, but increased global competition has driven drown labor rates in many countries. In this chapter, we consider a firm that can produce in two locations: either domestic or offshore. The firm has an uncertain offshore profit margin which makes the offshoring strategy risky. Using switching options we propose an optimal hedging policy that tells the manager when the firm should produce in the offshore facility and when it should produce in the domestic one. We then extend the policy from bang-bang to the proportional policy in which we determine what proportion of the firm's production should be offshored at any point in time. To do this we formulate the problem based on stochastic impulse control.

About the author

Taimaz Soltani was born on 14 August, 1986 in Zanjan, Iran. In 2004, he completed his secondary education in National Organization for Development of Exceptional Talents in Iran. Thereafter, he was admitted to Sharif University of Technology, from which he obtained his Bachelor of Science (BSc). After finishing his BSc in 2009, he studied Master of Financial Engineering at École Polytechnique Fédérale de Lausanne (EPFL) in Switzerland.

Just after finishing his MSc, he started a PhD project at Eindhoven University of Technology in the Netherlands under the supervision of Jan Fransoo, Arun Chockalingam and Shaunak Dabadghao. During his PhD he cooperated with Chung-Yee Lee from Hong Kong University of Science and Technology and Haolin Feng from Sun Yat-Sen University in China. The results of Taimaz's PhD study are presented in this dissertation.