Description of the data generating system utilized in “prediction-error identification of LPV systems: a nonparametric gaussian regression approach”

Darwish, M.A.H.; Cox, P.B.; Proimadis, I.; Pillonetto, G.; Toth, R.

Published: 10/01/2017

Document Version
Publisher’s PDF, also known as Version of Record (includes final page, issue and volume numbers)

Please check the document version of this publication:

- A submitted manuscript is the author's version of the article upon submission and before peer-review. There can be important differences between the submitted version and the official published version of record. People interested in the research are advised to contact the author for the final version of the publication, or visit the DOI to the publisher’s website.
- The final author version and the galley proof are versions of the publication after peer review.
- The final published version features the final layout of the paper including the volume, issue and page numbers.

Link to publication

Citation for published version (APA):
Description of the data generating system utilized in “Prediction-Error Identification of LPV Systems: A Nonparametric Gaussian Regression Approach”

By

Mohamed A. H. Darwish, Pepijn B. Cox, Ioannis Proimadis, Gianluigi Pillonetto, Roland Toth
Description of the data generating system utilized in “Prediction-Error Identification of LPV Systems: A Nonparametric Gaussian Regression Approach”

Mohamed A. H. Darwish, Pepijn B. Cox, Ioannis Proimadis, Gianluigi Pillonetto, Roland Tóth

I. INTRODUCTION

In this report, we give the exact coefficient function matrices \(a_i, b_j\) of the Linear Parameter-Varying (LPV) process model utilized in [1], together with the coefficient function matrices \(c_i, d_j\) of the considered noise dynamics, i.e., the corresponding full Box Jenkins (BJ) model.

II. LPV-BJ MODEL DESCRIPTION

Consider a multi-input multi-output (MIMO) data generating LPV system described in discrete-time by the following difference equations:

\[
A_0(p, k, q^{-1})\hat{y}(k) = B_0(p, k, q^{-1})u(k), \quad (1a)
\]

\[
D_0(p, k, q^{-1})v(k) = C_0(p, k, q^{-1})e(k), \quad (1b)
\]

\[
y(k) = \hat{y}(k) + v(k), \quad (1c)
\]

where \(k \in \mathbb{Z}\) is the discrete time, \(q\) is the forward time-shift operator, i.e., \(qx(k) = x(k+1)\), \(u : \mathbb{Z} \rightarrow U = \mathbb{R}^{n_u}\) is the input, \(\hat{y}, y : \mathbb{Z} \rightarrow Y = \mathbb{R}^{n_y}\) are the noiseless and noisy outputs respectively, \(p : \mathbb{Z} \rightarrow P\) is the so-called scheduling variable with compact range \(P \subseteq \mathbb{R}^{n_p}\), \(v : \mathbb{Z} \rightarrow \mathbb{Y}\) is a coloured noise process, and \(e : \mathbb{Z} \rightarrow \mathbb{Y}\) is a white noise process with normal (Gaussian) distribution, i.e., \(e(k) \sim \mathcal{N}(0, \Sigma_e)\) with covariance \(\Sigma_e \in \mathbb{R}^{n_y \times n_y}\). The \(p\)-dependent operators \(A_0(p, k, q^{-1})\) and \(B_0(p, k, q^{-1})\) that describe the process model (1a) are matrix polynomials in \(q^{-1}\) of degree \(n_a\) and \(n_b\) respectively:

\[
A_0(p, k, q^{-1}) = I_{n_y} + \sum_{i=1}^{n_a} a_i(p, k, i)q^{-i}, \quad (2a)
\]

\[
B_0(p, k, q^{-1}) = \sum_{j=0}^{n_b} b_j(p, k, j)q^{-j}, \quad (2b)
\]

where \(I_{n_y}\) is the \(n_y\)-dimensional identity matrix and the matrix functions \(a_i(p, k, i) : \mathbb{P} \times \ldots \times \mathbb{P} \rightarrow \mathbb{R}^{n_y \times n_y}\) and \(b_j(p, k, j) : \mathbb{P} \times \ldots \times \mathbb{P} \rightarrow \mathbb{R}^{n_y \times n_u}\) are shorthand notations for \(a_i(p, k, i) = a_i(p(k), \ldots, p(k-i))\) and \(b_j(p, k, j) = b_j(p(k), \ldots, p(k-j))\). These functions are assumed to be smooth and bounded functions on \(\mathbb{P}\). In a similar fashion, for the noise model (1b), the relations \(D_0(p, k, q^{-1})\) and \(C_0(p, k, q^{-1})\) are defined as

\[
C_0(p, k, q^{-1}) = I_{n_y} + \sum_{i=1}^{n_c} c_i(p, k, i)q^{-i}, \quad (3a)
\]

\[
D_0(p, k, q^{-1}) = I_{n_y} + \sum_{j=1}^{n_d} d_j(p, k, j)q^{-j}, \quad (3b)
\]

where \(d_j(p, k, j) : \mathbb{P} \times \ldots \times \mathbb{P} \rightarrow \mathbb{R}^{n_y \times n_y}\) and \(c_i(p, k, i) : \mathbb{P} \times \ldots \times \mathbb{P} \rightarrow \mathbb{R}^{n_y \times n_y}\) are the coefficient functions matrices of the monic polynomials matrices in \(q^{-1}\) of degree \(n_c\) and \(n_d\), respectively.

In [1, Section 5], a MIMO LPV-BJ model in the form of (1) is considered with \(n_y = 2, n_u = 2, n_p = 2\). The full description of this model is given below.
III. Coefficient functions of the process dynamics

\begin{align*}
b_0(p, k, 0) &= \begin{bmatrix} 1 - \exp(-0.6p_1(k)) & 0.64 - 0.72 \exp(0.7p_1(k)) \\ 0.3 - 0.4p_1^2(k) + 0.5p_2(k) & 0.2 + 0.98 \tan^{-1}(0.66p_2(k)) \end{bmatrix} \\
b_1(p, k, 1) &= \begin{bmatrix} 0.24 - 0.32p_1^2(k) + 0.4p_2(k - 1) & 0.22 \exp(0.4p_1(k - 1)) \\ 0.16 + 0.9 \tan^{-1}(0.63p_2(k)) & 0.22 - 0.5p_1^2(k) + 0.45p_2(k - 1) \end{bmatrix} \\
b_2(p, k, 2) &= \begin{bmatrix} 0.16 + 0.64 \tan^{-1}(0.8p_2(k - 2)) & 0.14 + 0.7 \tan^{-1}(0.6p_2(k - 2)) \\ 0.64 - 0.64 \exp(-0.6p_1(k - 1)) & 0.17 - 0.32p_1^2(k) + 0.32p_2(k - 1) \end{bmatrix} \\
a_1(p, k, 1) &= \begin{bmatrix} 0.2 + 0.12p_2^2(k - 1) \\ 0.2 + 0.35 \tan^{-1}(p_1(k)) \cos(p_1(k - 1)) \end{bmatrix} \\
a_2(p, k, 2) &= \begin{bmatrix} 0.19 + 0.15 \tan^{-1}(p_1(k - 1)) \cos(p_2(k - 2)) \\ 0.17 + 0.11p_2^2(k - 1) \end{bmatrix}
\end{align*}

IV. Coefficient functions of the noise dynamics

\begin{align*}d_1(p, k, 1) &= \begin{bmatrix} 0.3 + 0.3 \sqrt{|p_1(k)|} \\ 0.45 + 0.45 \sin(p_2(k)) \end{bmatrix} \\
d_2(p, k, 2) &= \begin{bmatrix} 0.34 + 0.34 \sin(p_2(k - 1)) \\ 0.23 + 0.23 \sqrt{|p_1(k - 2)|} \end{bmatrix} \\
c_1(p, k, 1) &= \begin{bmatrix} 0.3 + 0.45p_1^2(k) + 0.3p_1^2(k - 1) \\ 0.3 + 0.45p_2^2(k - 1) \end{bmatrix} \\
c_2(p, k, 2) &= \begin{bmatrix} 0.24 + 0.36p_1^2(k - 1) \\ 0.24 + 0.36p_2^2(k - 2) + 0.24p_1^2(k - 1) \end{bmatrix}
\end{align*}

REFERENCES