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Description of the data generating system utilized in “Prediction-Error Identification of LPV Systems: A Nonparametric Gaussian Regression Approach”

By

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I. INTRODUCTION

In this report, we give the exact coefficient function matrices $a_i, b_j$ of the Linear Parameter-Varying (LPV) process model utilized in [1], together with the coefficient function matrices $c_i, d_j$ of the considered noise dynamics, i.e., the corresponding full Box Jenkins (BJ) model.

II. LPV-BJ MODEL DESCRIPTION

Consider a multi-input multi-output (MIMO) data generating LPV system described in discrete-time by the following difference equations:

\begin{align}
A_0(p, k, q^{-1})\tilde{y}(k) &= B_0(p, k, q^{-1})u(k), \quad (1a) \\
D_0(p, k, q^{-1})v(k) &= C_0(p, k, q^{-1})e(k), \quad (1b) \\
y(k) &= \tilde{y}(k) + v(k), \quad (1c)
\end{align}

where $k \in \mathbb{Z}$ is the discrete time, $q$ is the forward time-shift operator, i.e., $qx(k) = x(k + 1)$, $u : \mathbb{Z} \to \mathbb{U} = \mathbb{R}^{n_u}$ is the input, $\tilde{y}, y : \mathbb{Z} \to \mathbb{Y} = \mathbb{R}^{ny}$ are the noiseless and noisy outputs respectively, $p : \mathbb{Z} \to \mathbb{P}$ is the so-called scheduling variable with compact range $\mathbb{P} \subseteq \mathbb{R}^{np}$, $v : \mathbb{Z} \to \mathbb{Y}$ is a coloured noise process, and $e : \mathbb{Z} \to \mathbb{Y}$ is a white noise process with normal (Gaussian) distribution, i.e., $e(k) \sim \mathcal{N}(0, \Sigma_e)$ with covariance $\Sigma_e \in \mathbb{R}^{ny \times ny}$. The $p$-dependent operators $A_0(p, k, q^{-1})$ and $B_0(p, k, q^{-1})$ that describe the process model (1a) are matrix polynomials in $q^{-1}$ of degree $n_a$ and $n_b$ respectively:

\begin{align}
A_0(p, k, q^{-1}) &= I_{n_y} + \sum_{i=1}^{n_a} a_i(p, k, i)q^{-i}, \quad (2a) \\
B_0(p, k, q^{-1}) &= \sum_{j=1}^{n_b} b_j(p, k, j)q^{-j}, \quad (2b)
\end{align}

where $I_{n_y}$ is the $n_y$-dimensional identity matrix and the matrix functions $a_i(p, k, i) : \mathbb{P} \times \ldots \times \mathbb{P} \to \mathbb{R}^{ny \times ny}$ and $b_j(p, k, j) : \mathbb{P} \times \ldots \times \mathbb{P} \to \mathbb{R}^{ny \times nu}$ are shorthand notations for $a_i(p, k, i) = a_i(p(k), \ldots, p(k-i))$ and $b_j(p, k, j) = b_j(p(k), \ldots, p(k-j))$. These functions are assumed to be smooth and bounded functions on $\mathbb{P}$. In a similar fashion, for the noise model (1b), the relations $D_0(p, k, q^{-1})$ and $C_0(p, k, q^{-1})$ are defined as

\begin{align}
C_0(p, k, q^{-1}) &= I_{n_y} + \sum_{i=1}^{n_c} c_i(p, k, i)q^{-i}, \quad (3a) \\
D_0(p, k, q^{-1}) &= I_{n_y} + \sum_{j=1}^{n_d} d_j(p, k, j)q^{-j}, \quad (3b)
\end{align}

where $d_j(p, k, j) : \mathbb{P} \times \ldots \times \mathbb{P} \to \mathbb{R}^{ny \times ny}$ and $c_i(p, k, i) : \mathbb{P} \times \ldots \times \mathbb{P} \to \mathbb{R}^{ny \times ny}$ are the coefficient functions matrices of the monic polynomials matrices in $q^{-1}$ of degree $n_c$ and $n_d$, respectively.

In [1, Section 5], a MIMO LPV-BJ model in the form of (1) is considered with $n_y = 2, n_u = 2, n_p = 2$. The full description of this model is given below.
III. COEFFICIENT FUNCTIONS OF THE PROCESS DYNAMICS

\[ b_0(p, k, 0) = \begin{bmatrix}
1 - \exp(-0.6p_1(k)) & 0.64 - 0.72\exp(0.7p_1(k)) \\
0.3 - 0.4p_1^2(k) + 0.5p_2(k) & 0.2 + 0.98\tan^{-1}(0.66p_2(k))
\end{bmatrix} \]

(4a)

\[ b_1(p, k, 1) = \begin{bmatrix}
0.24 - 0.32p_1^2(k) + 0.4p_2(k - 1) & 0.22\exp(0.4p_1(k - 1)) \\
0.16 + 0.9\tan^{-1}(0.63p_2(k)) & 0.22 - 0.5p_1^2(k) + 0.45p_2(k - 1)
\end{bmatrix} \]

(4b)

\[ b_2(p, k, 2) = \begin{bmatrix}
0.16 + 0.64\tan^{-1}(0.8p_2(k - 2)) & 0.14 + 0.7\tan^{-1}(0.6p_2(k - 2)) \\
0.64 - 0.64\exp(-0.6p_1(k - 1)) & 0.17 - 0.32p_1^2(k) + 0.32p_2(k - 1)
\end{bmatrix} \]

(4c)

\[ a_1(p, k, 1) = \begin{bmatrix}
0.2 + 0.12p_2^2(k - 1) & 0 \\
0 & 0.2 + 0.35\tan^{-1}(p_1(k))\cos(p_1(k - 1))
\end{bmatrix} \]

(4d)

\[ a_2(p, k, 2) = \begin{bmatrix}
0.19 + 0.15\tan^{-1}(p_1(k - 1))\cos(p_2(k - 2)) & 0 \\
0 & 0.17 + 0.11p_2^3(k - 1)
\end{bmatrix} \]

(4e)

IV. COEFFICIENT FUNCTIONS OF THE NOISE DYNAMICS

\[ d_1(p, k, 1) = \begin{bmatrix}
0.3 + 0.3\sqrt{|p_1(k)|} & 0 \\
0 & 0.45 + 0.45\sin(p_2(k))
\end{bmatrix} \]

(5a)

\[ d_2(p, k, 2) = \begin{bmatrix}
0.34 + 0.34\sin(p_2(k - 1)) & 0 \\
0 & 0.23 + 0.23\sqrt{|p_1(k - 2)|}
\end{bmatrix} \]

(5b)

\[ c_1(p, k, 1) = \begin{bmatrix}
0.3 + 0.45p_1^2(k) + 0.3p_1^2(k - 1) & 0 \\
0 & 0.3 + 0.45p_2^2(k - 1)
\end{bmatrix} \]

(5c)

\[ c_2(p, k, 2) = \begin{bmatrix}
0.24 + 0.36p_1^2(k - 1) & 0 \\
0 & 0.24 + 0.36p_2^2(k - 2) + 0.24p_2^2(k - 1)
\end{bmatrix} \]

(5d)

REFERENCES