Description of the data generating system utilized in “prediction-error identification of LPV systems: a nonparametric gaussian regression approach”
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By

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I. INTRODUCTION

In this report, we give the exact coefficient function matrices \( a_i, b_j \) of the Linear Parameter-Varying (LPV) process model utilized in [1], together with the coefficient function matrices \( c_i, d_j \) of the considered noise dynamics, i.e., the corresponding full Box Jenkins (BJ) model.

II. LPV-BJ MODEL DESCRIPTION

Consider a multi-input multi-output (MIMO) data generating LPV system described in discrete-time by the following difference equations:

\[
\begin{align*}
A_0(p, k, q^{-1})\hat{y}(k) &= B_0(p, k, q^{-1})u(k), \\
D_0(p, k, q^{-1})v(k) &= C_0(p, k, q^{-1})e(k), \\
y(k) &= \hat{y}(k) + v(k),
\end{align*}
\]

where \( k \in \mathbb{Z} \) is the discrete time, \( q \) is the forward time-shift operator, i.e., \( qx(k) = x(k + 1) \), \( u : \mathbb{Z} \to \mathbb{U} = \mathbb{R}^{n_u} \) is the input, \( \hat{y}, y : \mathbb{Z} \to \mathbb{Y} = \mathbb{R}^{n_y} \) are the noiseless and noisy outputs respectively, \( p : \mathbb{Z} \to \mathbb{P} \) is the so-called scheduling variable with compact range \( \mathbb{P} \subseteq \mathbb{R}^{n_p}, p : \mathbb{Z} \to \mathbb{P} \) is a coloured noise process, and \( e : \mathbb{Z} \to \mathbb{Y} \) is a white noise process with normal (Gaussian) distribution, i.e., \( e(k) \sim \mathcal{N}(0, \Sigma_c) \) with covariance \( \Sigma_c \in \mathbb{R}^{n_y \times n_y} \). The \( p \)-dependent operators \( A_0(p, k, q^{-1}) \) and \( B_0(p, k, q^{-1}) \) that describe the process model (1a) are matrix polynomials in \( q^{-1} \) of degree \( n_a \) and \( n_b \) respectively:

\[
\begin{align*}
A_0(p, k, q^{-1}) &= I_{n_y} + \sum_{i=1}^{n_a} a_i(p, k, i)q^{-i}, \\
B_0(p, k, q^{-1}) &= \sum_{j=1}^{n_b} b_j(p, k, j)q^{-j},
\end{align*}
\]

where \( I_{n_y} \) is the \( n_y \)-dimensional identity matrix and the matrix functions \( a_i(p, k, i) : \mathbb{P} \times \ldots \times \mathbb{P} \to \mathbb{R}^{n_y \times n_y} \) and \( b_j(p, k, j) : \mathbb{P} \times \ldots \times \mathbb{P} \to \mathbb{R}^{n_y \times n_u} \) are shorthand notations for \( a_i(p, k, i) = a_i(p(k), \ldots, p(k - i)) \) and \( b_j(p, k, j) = b_j(p(k), \ldots, p(k - j)) \). These functions are assumed to be smooth and bounded functions on \( \mathbb{P} \). In a similar fashion, for the noise model (1b), the relations \( D_0(p, k, q^{-1}) \) and \( C_0(p, k, q^{-1}) \) are defined as

\[
\begin{align*}
C_0(p, k, q^{-1}) &= I_{n_y} + \sum_{i=1}^{n_c} c_i(p, k, i)q^{-i}, \\
D_0(p, k, q^{-1}) &= I_{n_y} + \sum_{j=1}^{n_d} d_j(p, k, j)q^{-j},
\end{align*}
\]

where \( d_j(p, k, j) : \mathbb{P} \times \ldots \times \mathbb{P} \to \mathbb{R}^{n_y \times n_u} \) and \( c_i(p, k, i) : \mathbb{P} \times \ldots \times \mathbb{P} \to \mathbb{R}^{n_y \times n_y} \) are the coefficient functions matrices of the monic polynomials matrices in \( q^{-1} \) of degree \( n_c \) and \( n_d \), respectively.

In [1, Section 5], a MIMO LPV-BJ model in the form of (1) is considered with \( n_y = 2, n_u = 2, n_p = 2 \). The full description of this model is given below.
### III. COEFFICIENT FUNCTIONS OF THE PROCESS DYNAMICS

\[ b_0(p, k, 0) = \begin{bmatrix} 1 - \exp(-0.6p_1(k)) & 0.64 - 0.72 \exp(0.7p_1(k)) \\ 0.3 - 0.4p_1^2(k) + 0.5p_2(k) & 0.2 + 0.98 \tan^{-1}(0.66p_2(k)) \end{bmatrix} \] (4a)

\[ b_1(p, k, 1) = \begin{bmatrix} 0.24 - 0.32p_1^2(k) + 0.4p_2(k - 1) & 0.22 \exp(0.4p_1(k - 1)) \\ 0.16 + 0.9 \tan^{-1}(0.63p_2(k)) & 0.22 - 0.5p_1^2(k) + 0.45p_2(k - 1) \end{bmatrix} \] (4b)

\[ b_2(p, k, 2) = \begin{bmatrix} 0.16 + 0.64 \tan^{-1}(0.8p_2(k - 2)) & 0.14 + 0.7 \tan^{-1}(0.6p_2(k - 2)) \\ 0.64 - 0.64 \exp(-0.6p_1(k - 1)) & 0.17 - 0.32p_1^2(k) + 0.32p_2(k - 1) \end{bmatrix} \] (4c)

\[ a_1(p, k, 1) = \begin{bmatrix} 0.2 + 0.12p_2^2(k - 1) \\ 0 & 0.2 + 0.35 \tan^{-1}(p_1(k)) \cos(p_1(k - 1)) \end{bmatrix} \] (4d)

\[ a_2(p, k, 2) = \begin{bmatrix} 0.19 + 0.15 \tan^{-1}(p_1(k - 1)) \cos(p_2(k - 2)) \\ 0 & 0.17 + 0.11p_2^2(k - 1) \end{bmatrix} \] . (4e)

### IV. COEFFICIENT FUNCTIONS OF THE NOISE DYNAMICS

\[ d_1(p, k, 1) = \begin{bmatrix} 0.3 + 0.3 \sqrt{|p_1(k)|} \\ 0 & 0.45 + 0.45 \sin(p_2(k)) \end{bmatrix} \] (5a)

\[ d_2(p, k, 2) = \begin{bmatrix} 0.34 + 0.34 \sin(p_2(k - 1)) \\ 0 & 0.23 + 0.23 \sqrt{|p_1(k - 2)|} \end{bmatrix} \] (5b)

\[ c_1(p, k, 1) = \begin{bmatrix} 0.3 + 0.45p_1^2(k) + 0.3p_2^2(k - 1) \\ 0 & 0.3 + 0.45p_2^2(k - 1) \end{bmatrix} \] (5c)

\[ c_2(p, k, 2) = \begin{bmatrix} 0.24 + 0.36p_1^2(k - 1) \\ 0 & 0.24 + 0.36p_2^2(k - 2) + 0.24p_2^2(k - 1) \end{bmatrix} \] . (5d)

### REFERENCES