A note on maximal covering location games

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A note on Maximal Covering Location Games

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Abstract

In this note we introduce and analyse maximal covering location games. As the core may be empty, several sufficient conditions for core non-emptiness are presented. For each condition we provide an example showing that when the condition is not satisfied, core non-emptiness is not guaranteed.

Keywords : maximal covering location problem; cooperative game; core.

1 Introduction

In the maximal covering location problem (Church and ReVelle [1]) a single decision maker has to position a predetermined number of resources in order to maximize profit of the *covered* demand points, where a demand point is covered if a resource is positioned within a certain radius. This well-known location model has proven to be useful in many settings, e.g., for positioning of emergency vehicles (Li et al. [4]), cell towers (Lee and Murray [3]) and retail stores (Plastria and Vanhaverbeke [5]). Another interesting setting is the one with several small-sized regions, e.g., villages or municipalities, that each may or may not own a *single* resource to cover their region completely. When those regions pool their resources a maximal covering location problem arises. Typically, additional coverage, and so additional profit, can be realized and a *joint profit allocation* issue arises amongst the collaborating regions. In this note, we will investigate this allocation aspect. We introduce a maximal covering location (MCL) situation wherein regions are represented by *single* demand points that may or may not keep a *single* resource. For such an MCL situation, an associated MCL game

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is introduced. For this game, we provide several sufficient conditions (in terms of the number of players, the number of resources, and the underlying integer linear program) for core non-emptiness. For each condition we provide an example showing that when the condition is not satisfied, core non-emptiness is not guaranteed.

The outline of this note is as follows. We start in Section 2 with preliminaries on cooperative game theory. In Section 3, we introduce MCL situations, subsequently we introduce MCL games, and finally we present our results.

2 Preliminaries

In this section, we provide some basic elements of cooperative game theory. Consider a finite set $N = \{1, 2, \dots, n\}$ of *players* and a function $v : 2^N \rightarrow \mathbb{R}$ called the *characteristic function*, with $v(\emptyset) = 0$. The pair (N, v) is a *cooperative game with transferable utility*, shortly *game*. A subset $S \subseteq N$ is a *coalition* and $v(S)$ is the worth that coalition S can obtain by itself. The worth can be transferred freely among the players. The set N is the *grand coalition*. A game (N, v) is *superadditive* if the value of the union of any two disjoint coalitions is larger than or equal to the sum of the values of these disjoint coalitions, i.e., $v(S) + v(T) \leq v(S \cup T)$ for all $S, T \subseteq N$ with $S \cap T = \emptyset$ and *monotonic* if the value of every coalition is at least the value of any of its subcoalitions, i.e., $v(S) \leq v(T)$ for all $S, T \subseteq N$ with $S \subseteq T$. An allocation for a game (N, v) is an n -dimensional vector $x \in \mathbb{R}^N$ where player $i \in N$ receives x_i . An allocation is *efficient* if $\sum_{i \in N} x_i = v(N)$. This implies that all worth is divided among the players of the grand coalition N . An allocation is *stable* if no group of players has an incentive to leave the grand coalition N , i.e., $\sum_{i \in S} x_i \geq v(S)$ for all $S \subseteq N$. The set of efficient and stable allocations of (N, v) is the *core* of (N, v) and denoted by $C(N, v)$.

3 Model

In this section, we introduce maximal covering location situations and define the associated games, called maximal covering location games. Finally, we present properties of maximal covering location games.

3.1 Maximal covering location situation

We consider an environment with a finite set $N = \{1, 2, \dots, n\}$ of players and a finite set $L = \{n + 1, n + 2, \dots, n + l\}$ of possible resource locations. The distance between

player $i \in N$ and resource location $j \in L$ is denoted by $d_{ij} \in \mathbb{R}_+$. For any player $i \in N$, we introduce $r_i \in \{0, 1\}$, where $r_i = 1$ indicates that player $i \in N$ owns a resource, and $r_i = 0$ indicates that player $i \in N$ does *not* own a resource. Every player $i \in N$ with $r_i = 1$ positions its resource at any resource location $j \in L$ and if $d_{ij} \leq \mathcal{D} \in \mathbb{R}_+$, i.e., if the player is covered by the resource, a profit of $p_i \in \mathbb{R}_+$ is obtained. To analyse this setting, we define a *maximal covering location (MCL) situation* as a tuple $(N, L, p, r, d, \mathcal{D})$ with $N, L, p = (p_i)_{i \in N}, r = (r_i)_{i \in N}, d = (d_{ij})_{i \in N, j \in L}$ and $\mathcal{D}$ as described above. For short, we will use θ to refer to such an MCL situation $\theta = (N, L, p, r, d, \mathcal{D})$ and Θ for the set of MCL situations. In addition, for all $\theta \in \Theta$, we define $N_j = \{i \in N | d_{ij} \leq \mathcal{D}\}$ for all $j \in L$, $L_i = \{j \in L | d_{ij} \leq \mathcal{D}\}$ for all $i \in N$, and construct a corresponding (bipartite) graph $\mathcal{G} = (N, L, E)$ with N and L the sets of nodes and $E = \{(i, j)_{i \in N, j \in L_i}\}$ the set of edges. Note that an edge between player $i \in N$ and resource location $j \in L$ indicates that the distance between these nodes is no more than $\mathcal{D}$, implying that player i is covered when a resource is positioned at this location j .

3.2 Maximal covering location game

As some players may not own a resource, additional profit can be realized when resources are pooled amongst the players. In line with Church and ReVelle [1], we assume for any coalition $S \subseteq N$ that coverage of any player $i \in S$ by one (or possibly multiple) resource(s) results into a profit of p_i . As a consequence, any $S \subseteq N$ faces the joint problem of where to position the resources such that the sum of the individual profits (of coalition S) is maximized. For every MCL situation $\theta \in \Theta$ and all $S \subseteq N$ this corresponding MCL problem can be formulated as

$$\begin{aligned}
MCL^\theta(S) : \quad & \max \quad \sum_{i \in S} p_i \cdot y_i \\
\text{s.t.} \quad & y_i - \sum_{j \in L_i} x_j \leq 0 \quad \forall i \in S \\
& \sum_{j \in L} x_j \leq \sum_{i \in S} r_i \\
& x_j \in \{0, 1\} \quad \forall j \in L \\
& y_i \in \{0, 1\} \quad \forall i \in S.
\end{aligned}$$

The first constraint ensures that the profit of player $i \in S$ is obtained only if at least one resource of coalition S is positioned within distance $\mathcal{D}$. The second constraint ensures that the total number of resources used does not exceed the number of available resources of coalition S . The third and fourth constraint enforce integrality of

the variables. Note that a solution of the MCL problem indicates at which resource locations a resource is positioned and which players obtain a profit. In particular, if a resource is positioned at resource location $j \in L$, then $x_j = 1$ and otherwise $x_j = 0$. Similarly, if player $i \in S$ obtains profit p_i , then $y_i = 1$, otherwise $y_i = 0$.

In the remainder of this paper, we denote for every MCL situation $\theta \in \Theta$ and all $S \subseteq N$ the optimal value of $MCL^\theta(S)$ by $\mathbf{opt}(MCL^\theta(S))$.

Example 1. Let $\theta \in \Theta$ be an MCL situation with $N = \{1, 2, 3\}$, $L = \{4, 5\}$, $p = (1, 2, 3)$, $r = (1, 0, 0)$, $d_{14} = d_{24} = d_{25} = d_{35} = 1$, $d_{15} = d_{34} = 2$ and $\mathcal{D} = 1$. Observe that $L_1 = \{4\}$, $L_2 = \{4, 5\}$, and $L_3 = \{5\}$. The corresponding graph $\mathcal{G} = (N, L, E)$ with $E = \{(1, 4), (2, 4), (2, 5), (3, 5)\}$ is represented in Figure 1.

Figure 1: Graph corresponding to MCL situation.

For coalition $S = \{1, 3\}$, the maximization problem boils down to a trade off between a profit of 1, when the resource is positioned at location 4 and a profit of 3 when the resource is positioned at location 5. Hence, $\mathbf{opt}(MCL^\theta(\{1, 3\})) = 3$. $\diamond$

We proceed with associating an MCL game to any MCL situation.

Definition 1. For every MCL situation $\theta \in \Theta$, we call the game (N, v^θ) with

$$v^\theta(S) = \begin{cases} 0 & \text{if } S = \emptyset; \\ \mathbf{opt}(MCL^\theta(S)) & \text{if } S \subseteq N, S \neq \emptyset, \end{cases}$$

the associated MCL game.

Now, we present an example of an MCL game.

Example 2. Consider the situation of Example 1. The associated MCL game is presented in Table 1. $\diamond$

Table 1: Corresponding profit per coalition of MCL game.								
S	$\emptyset$	$\{1\}$	$\{2\}$	$\{3\}$	$\{1, 2\}$	$\{1, 3\}$	$\{2, 3\}$	$\{1, 2, 3\}$
$v^\theta(S)$	0	1	0	0	3	3	0	5

3.3 Properties of maximal covering location games

In this section, we present general properties of MCL games. We start by showing that the value of the union of two disjoint coalitions is larger than or equal to the sum of the values of the disjoint subcoalitions and that the value of every coalition is at least the value of any of its subcoalitions.

Proposition 1. *Every MCL game is superadditive and monotonic.*

The proof of Proposition 1 is relegated to Appendix A. We describe the intuition behind the proof. For superadditivity, optimal solutions of two disjoint coalitions are combined into a feasible solution for their union. For monotonicity, we use superadditivity in combination with non-negativity of the coalitional values.

A natural next step is to investigate whether the cores of MCL games are non-empty. The following example illustrates that this is not the case in general.

Example 3. *Let $\theta \in \Theta$ be an MCL situation with $N = \{1, 2, 3, 4\}$, $L = \{5, 6, 7, 8\}$, $p = (1, 1, 1, 1)$, $r = (1, 1, 0, 0)$, $d_{15} = d_{17} = d_{25} = d_{26} = d_{36} = d_{37} = d_{48} = 1$, $d_{16} = d_{18} = d_{27} = d_{28} = d_{35} = d_{38} = d_{45} = d_{46} = d_{47} = 2$ and $\mathcal{D} = 1$. Then $L_1 = \{5, 7\}$, $L_2 = \{5, 6\}$, $L_3 = \{6, 7\}$ and $L_4 = \{8\}$. The corresponding graph $\mathcal{G} = (N, L, E)$ with $E = \{(1, 5), (1, 7), (2, 5), (2, 6), (3, 6), (3, 7), (4, 8)\}$ is represented in Figure 2.*

Figure 2: Graph corresponding to MCL situation.

Now, observe that $v^\theta(N) = 3$, $v^\theta(N \setminus \{i\}) = 2$ for $i \in \{1, 2\}$, and $v^\theta(N \setminus \{i\}) = 3$ for $i \in \{3, 4\}$. Suppose the core is non-empty. Let $x \in C(N, v^\theta)$. As $x_i = \sum_{j \in N} x_j - \sum_{j \in N \setminus \{i\}} x_j = v^\theta(N) - \sum_{j \in N \setminus \{i\}} x_j \leq v^\theta(N) - v^\theta(N \setminus \{i\})$ for all $i \in N$, we obtain $x_1 \leq 1$, $x_2 \leq 1$, $x_3 \leq 0$, and $x_4 \leq 0$. This conflicts with efficiency, i.e., $\sum_{i \in N} x_i \leq 2 < 3 = v^\theta(N)$. Hence, we conclude that the core is empty. $\diamond$

The graph corresponding to the MCL situation of Example 3 contains a cycle. In some other cooperative games related to problems in combinatorial optimization non-emptiness of the core is guaranteed when cycles are not present in the corresponding graph (Deng et al. [2]). For instance, for cost covering games this has been studied by Tamir [7]. One may wonder whether this holds for MCL games as well. The following 6-person MCL situation with a corresponding graph without cycles illustrates that this is not the case in general.

Example 4. Let $\theta \in \Theta$ be an MCL situation with $N = \{1, 2, 3, 4, 5, 6\}$, $L = \{7, 8, 9, 10\}$, $p = (1, 1, 1, 1, 1, 1)$, $r = (1, 0, 0, 0, 0, 1)$, $d_{i7} = 1$ for $i \in \{1, 2\}$, $d_{i7} = 2$ for $i \in \{3, 4, 5, 6\}$, $d_{i8} = 1$ for $i \in \{2, 3, 5\}$, $d_{i8} = 2$ for $i \in \{1, 4, 6\}$, $d_{i9} = 1$ for $i \in \{3, 4\}$, $d_{i9} = 2$ for $i \in \{1, 2, 5, 6\}$, $d_{i10} = 1$ for $i \in \{5, 6\}$, $d_{i10} = 2$ for $i \in \{1, 2, 3, 4\}$, and $\mathcal{D} = 1$. Then, $L_1 = \{7\}$, $L_2 = \{7, 8\}$, $L_3 = \{8, 9\}$, $L_4 = \{9\}$, $L_5 = \{8, 10\}$ and $L_6 = \{10\}$. The corresponding graph $\mathcal{G} = (N, L, E)$ with $E = \{(1, 7), (2, 7), (2, 8), (3, 8), (3, 9), (4, 9), (5, 8), (5, 10), (6, 10)\}$ is represented in Figure 3.

Figure 3: Graph corresponding to MCL situation.

Now, observe that $v^\theta(N) = 4$ by positioning the two resources at any two resource locations. In addition, $v^\theta(N \setminus \{i\}) = 4$ for $i \in \{2, 3, 4, 5\}$, and $v^\theta(N \setminus \{i\}) = 3$ for $i \in \{1, 6\}$. Suppose the core is non-empty. Let $x \in C(N, v^\theta)$. As $x_i \leq v^\theta(N) - v^\theta(N \setminus \{i\})$ for all $i \in N$, we obtain $x_1 \leq 1, x_6 \leq 1$ and $x_i \leq 0$ for $i \in N \setminus \{1, 6\}$. This conflicts with efficiency, i.e., $\sum_{i \in N} x_i \leq 2 < 4 = v^\theta(N)$. Hence, the core is empty. $\diamond$

Example 3 demonstrates that the core may be empty from 4-person games on. In addition, Example 4 shows that under the assumption that the corresponding graph contains no cycles the core may be empty from 6-person games on. With all this in mind we address the following two questions in the remainder of this paper; (i) is the core non-empty up to 3-person games in general and (ii) is the core non-empty up to 5-person games when the corresponding graph (of the MCL situation) contains no cycles? By addressing those issues, some other interesting results come along.

Proposition 2. *For every MCL situation $\theta \in \Theta$ and associated MCL game (N, v^θ) , the core is non-empty if k players with $k \in \{0, 1, n-1, n\}$ own a resource.*

Proof: See Appendix A.

Remark 1. If the condition of Proposition 2 is not satisfied, core non-emptiness is not guaranteed. In Example 3 with four players and two resources the core is empty.

As a direct consequence of Proposition 2, we can conclude that the core of every k -person MCL game with $k \in \{1, 2, 3\}$ is non-empty.

Theorem 1. *Every k -person MCL game with $k \in \{1, 2, 3\}$ has a non-empty core.*

We continue by addressing our second question of interest. For this, we introduce some definitions and present a proposition and two lemmas which are of interest by themselves as well. For every MCL situation $\theta \in \Theta$ and all $S \subseteq N$ we define $RMCL^\theta(S)$ as a relaxation of $MCL^\theta(S)$ where $x_j \geq 0$ for all $j \in L$ and $0 \leq y_i \leq 1$ for all $i \in S$. Note that $x_j \leq 1$ for all $j \in L$ is not taken into consideration. Based on this relaxation, we formulate a sufficient condition for non-emptiness of the core.

Proposition 3. *For any $\theta \in \Theta$ it holds that if $\mathbf{opt}(RMCL^\theta(N)) = \mathbf{opt}(MCL^\theta(N))$, the core of the associated MCL game (N, v^θ) is non-empty.*

Proof: See Appendix A.

Remark 2. Introducing a relaxation of an integer programming problem in order to find (sufficient) conditions for core non-emptiness (for different classes of situations) has been used by others as well (see, e.g., Deng et al. [2]). Moreover, it turns out that if the condition of Proposition 3 is not satisfied, core non-emptiness cannot be guaranteed. For instance, in Example 4 it holds that $\mathbf{opt}(MCL^\theta(N)) = 4$, $\mathbf{opt}(RMCL^\theta(N)) = 4.5$ (resulting from $x_j = \frac{1}{2}$ for all $j \in L$), and $C(N, v^\theta) = \emptyset$. Finally, it turns out that the condition of Proposition 3 is not necessary. For instance, in Example 4 with $r_2 = r_3 = 1$ and $r_i = 0$ for all $i \in N \setminus \{2, 3\}$, it holds that $C(N, v^\theta) = \{(0, 2, 2, 0, 0, 0, 0)\} \neq \emptyset$.

A square submatrix of a matrix $A \in \mathbb{R}^{w \times z}$ (where w is the number of rows and z is the number of columns) is a matrix $A' \in \mathbb{R}^{q \times q}$ formed by selecting q rows and q columns from the matrix A . Moreover, a matrix is *totally unimodular* if every square submatrix of A has determinant equal to $+1, -1$ or 0 .

Lemma 1. *Let A be a totally unimodular $w \times z$ matrix and let $b \in \mathbb{Z}^w$ and $c \in \mathbb{R}^z$. Then the linear programming problem $\max\{cx | x \geq 0, Ax \leq b\}$ has integer optimal solutions, whenever it has a finite optimum.*

Proof: See e.g. Wolsey [8, p.40].

For all $\theta = (N, L, p, r, d, \mathcal{D}) \in \Theta$ and all $j \in L$, we define $\theta^{-j} = (N, L^{-j}, p, r, d^{-j}, \mathcal{D})$, where $L^{-j} = L \setminus \{j\}$ and $d^{-j} = (d_{ij})_{i \in N, j \in L \setminus \{j\}}$. In addition, for every $\theta \in \Theta$, resource location $j \in L$ is called *obsolete* if there exists $k \in L \setminus \{j\}$ for which $N_j \subseteq N_k$.

Lemma 2. *For any $\theta \in \Theta$ with an obsolete resource location $j \in L$, it holds that*

$$v^\theta(M) = v^{\theta^{-j}}(M) \text{ for all } M \subseteq N.$$

Note that the result of Lemma 2 follows directly, as (for every $M \subseteq N$) there exists a $k \in L \setminus \{j\}$ with $N_j \subseteq N_k$ which makes k superfluous. Now, we are able to affirmatively answer our second question of interest. We provide a sketch of the proof here. The complete proof is relegated to Appendix A.

As for every MCL situation with $0, 1, |N| - 1$ and $|N|$ resources the core of the associated MCL game is non-empty (Proposition 2), it suffices to focus on the following MCL situations only; (i) $|N| = 4$ with two resources, (ii) $|N| = 5$ with two resources, and (iii) $|N| = 5$ with three resources. For all cases, it holds that if the number of resource locations is less than or equal to the number of resources, the core is non-empty. This holds as allocating p_i to all players $i \in N$ with $L_i \neq \emptyset$, i.e., for which there exists a resource location within distance $\mathcal{D}$, results into a core element. In addition, for every MCL situation with an obsolete resource location there exists another MCL situation without this obsolete resource location such that the coalitional values of the MCL games resulting from those MCL situations coincide (Lemma 2). Hence, if the core is non-empty for the game corresponding to the MCL situation without the obsolete resource location, it is the case for the other game as well. So, it suffices to consider the three cases with the additional constraint that (i) $|L| \geq 3$ and (ii) the corresponding graphs are free of obsolete resource locations and free of cycles. In Figure 4, the possible corresponding graphs for $|N| = 4$ with $|L| \geq 3$ are presented.

Figure 4: Possible corresponding graphs with $|N| = 4$ and $|L| \geq 3$.

Note that duplicates of the graphs of Figure 4 due to relabeling of the player set and resource set are removed. Similarly, one can present the possible corresponding graphs (with no obsolete resource locations nor cycles) for $|N| = 5$ with $|L| \geq 3$. These graphs are presented in Figure 5. Note that duplicates of the graphs of Figure 5 due to relabeling of the player set and resource set are removed (again).

Figure 5: Possible corresponding graphs with $|N| = 5$ and $|L| \geq 3$.

For all remaining MCL situations, which *all* have a corresponding graph as presented in Figure 4 or Figure 5 (possibly after some relabeling of the player set and the resource location set), we can (re)formulate the relaxation of the MCL problem in standard LP-

form, i.e., in matrix form $Ax \leq b$ with $x \geq 0$.

Example 5. Let $\theta \in \Theta$ be an MCL situation with corresponding graph (e) of Figure 4. Then, for the relaxation of the MCL problem, we obtain the following A and b .

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}, \quad b = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 2 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}.$$

Note that vector x stands for $(y_1, y_2, y_3, y_4, x_5, x_6, x_7)$ where x_5, x_6 and x_7 represent successively resource locations 5, 6 and 7. $\diamond$

For all these MCL situations, we can show that matrix A (in standard LP-form) is totally unimodular. One can check, for instance via standard software packages, that every submatrix of A (of all 28 graphs of Figure 4 and Figure 5) has determinant 1, 0 or -1. Moreover, vector b (of the standard LP-form) has all integer entries. In addition, it holds that $p_i \in \mathbb{R}_+$ for all $i \in N$ and so, we can conclude that the relaxation of the MCL problem has an integer optimal solution (Lemma 1) that can straightforwardly be shown to be a binary optimal solution. Hence, the optimal value of the MCL problem coincides with the optimal value of the relaxation of the MCL problem and thus, the core is non-empty (Proposition 3). This leads to our final theorem.

Theorem 2. Every k -person MCL game with $k \in \{1, 2, 3, 4, 5\}$ and no cycles in the corresponding graph has a non-empty core.

Remark 3. Despite the fact that some MCL games may have empty cores, one can always partition the grand coalition N into coalitions with at most three players, for which one can construct corresponding MCL situations for which the related MCL games have non-empty cores (Theorem 1). Multiple options for constructing these

MCL situations exist. For instance, regarding the accessibility of the resource locations, one can distinguish between the situation wherein each resource location is accessible for (i) all players (of all coalitions) and (ii) players in *one* coalition only.

Remark 4. The more general case in which each player may own *several* resources to cover *multiple* demand points can be deducted from our situation easily by merging several players (with and without a resource) into one (super)player. All sufficient conditions can be translated to this situation easily, and will be in terms of settings with players that can be seen as forming such a superplayer.

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Appendix A. Supplementary material

Supplementary material related to this note can be found online at <http://home.ieis.tue.nl/lshlicher/supplementarymaterialMCLgamesORI2016.pdf>.

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