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Structural optimization for 3D concrete printing

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Before you lies the Master thesis on Structural Optimization for 3D Concrete Printing. This project concludes the master Architecture, Building and Planning, specialized towards Structural Design, at the Eindhoven University of Technology.

The purpose of this research is to use the theory of structural optimization and combine it with the new and innovative manufacturing technique 3D Concrete Printing. This additive manufacturing method was introduced at the Eindhoven University of Technology in September 2015 and has been used ever since for scientific research to learn more about its possibilities and to further optimize the technique as a viable new method to be used in the building industry. Structural optimization can therefore be of great benefit, as this manufacturing technique is capable to construct concrete objects in a smart way, which can result in the application of optimized concrete structures.

This project has been carried out under the supervision of Prof. Dr. Ir. T.A.M. (Theo) Salet, Ir. R.J.M. (Rob) Wolfs and Ir. A.P.H.W. (Arjan) Habraken and in collaboration with my fellow students of the 3D Concrete Printing research group. I would like to thank them for their time and effort during the printing sessions, their criticism and insightful comments during meetings and presentations and their interest in this project. This project has certainly been elevated to a final result of which I am proud that this has finalized my master.

And finally, I would also like to thank my family, as they have supported me throughout the duration of my studies and have always shown their interest in my work within this period of time.

Jasper van Alphen

Eindhoven, February 2017
Abstract

In recent years, 3D printing has become a widely used manufacturing technique throughout the world. New developments also introduced 3D printing to the building industry as new researches have started on how to print building components with structural materials. One of these materials is concrete and therefore a 3D Concrete Printer was installed at the Eindhoven University of Technology in 2015. The aim of this extensive research is to gain more knowledge about this promising manufacturing technique and to establish it as a viable new method to be used in the building industry.

Another technique that has gained more interest than ever is structural optimization. As it is used in other industries, like the aerospace or automobile industry, the building industry still has not adopted this innovative theory on structural design. As the world demands for more sustainability and environmentally efficient structures, structural optimization can be of great importance, as this theory can be used to optimize structures in ways to reduce the amount of material used for newly built building components or to implement several disciplines of the building industry within the same design.

However, the output of structural optimization algorithms are relatively complex to manufacture with conventional manufacturing techniques. Although 3D Concrete Printing gives great freedom in design compared to traditionally casted concrete, it still needs to cope with several challenges within the manufacturing process to obtain certain designs. The combination of these two subjects is researched in this graduation project with the goal to find whether 3D Concrete Printing is applicable as a manufacturing technique for structurally optimized objects.

Since many people are unfamiliar with both topics, this thesis also aims to give further clarification for a better understanding of 3D Concrete Printing and structural optimization. An extensive explanation is given on the challenges of 3D Concrete Printing at the Eindhoven University of Technology and a literature review is presented on the widely used theories on topology optimization, as in theories on Bi-directional Evolutionary Structural Optimization (BESO), Solid Isotropic Material with Penalization (SIMP) and the Level-Set Method (LSM).

As structural optimization is traditionally used for a linear isotropic material without any constraints related to practice, multiple methods are investigated related to the material concrete and the challenges within the manufacturing process of 3D Concrete printing. The influence of casting constraints, simple orthotropic material properties and stress constraints in terms of the Von-Mises criterion and Drucker-Prager criterion are researched. Factors that influence the final results of 3D Concrete Printing are the fixed layer thicknesses due to nozzle dimensions and angle and overhang constraints on the printed fresh concrete.

Since the overhang of this material in this state is limited, support material for the ability to manufacture these optimized objects is a necessity. The 3D Concrete Printer, at this time, is only able to print with one material and with continuous print paths, resulting in a research to find new developments in this printing process to obtain support structures of this concrete. As sustainability is a main topic of the research project at the Eindhoven University of Technology, the support material can not only be used as a temporary support structure for the structural material distributed by structural optimization. This is why the stiffness of these support structures is taken into account within the optimization algorithms, as they then contribute to the overall stiffness and structural performance of the whole object.
Within these smart solutions on printing of concrete support structures, more possibilities will arise as they introduce small holes in the concrete structure, a little like cellular material does in research on topology optimization. Cavities can be used for architectural reasons, for building physics or for example the placement of different materials like reinforcement. The dimensions of these cavities can be slightly altered, according to research on the output of several print paths. This will directly influence the local density of the support structure, therefore questioning the necessity of the penalty factor, which is used in topology optimization to obtain solutions made of material with its full potential or of voids.

As some new possibilities of 3D Concrete Printing are introduced in this research, they all come with their own structural challenges for a viable use within this manufacturing process. For further application of these print techniques, more research on their structural behavior should be performed to gain a better understanding on how the concrete will behave in fresh and hardened state in practical situations, considering 3D Concrete Printing.
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1. Introduction

The building industry is looking for new ways to become more sustainable. Especially the material concrete has a large influence on the total amount of emissions from the building industry. Therefore new techniques and processes need to be developed or already existing ones need to be improved to achieve this necessary reduction of emission for more a more sustainable industry.

An already existing technique to reduce the amount of material used in structures is the use of topology optimization. It relies on a mathematical algorithm for a finite element model which considers a certain design space, along with its material properties, boundary conditions and loadings, that calculates at which elements in the model material is required for it to be used in a structural way. The user needs an objective and certain constraints as input in the model. Traditionally most structures are optimized towards a maximum stiffness with a volume constraint in order to simply reduce the amount of material while maintaining the structure with the highest stiffness.

A new innovative building technique is 3D printing with concrete. Recent developments of these systems throughout the world have shown that it is a promising technique, but that there still is a lot to learn for it to be eventually used in daily practice. The University of Technology in Eindhoven developed a 3D concrete printer in 2015 and has been used for over a year at the end of this research. It allows for the production of more complex designs of concrete due to its design freedom and the absence of formwork, while reducing the amount of labor required for this production process.

Topology optimization has not been used a lot in practice, due to its complex designs that result from the algorithm and the cost related possibilities of manufacturing techniques. Structural integrity has also not been proved a lot in tests, since most research restricts itself to development of the theory and the software, without testing it in practice. Another thing with topology optimization is that the basic theory does not consider the manufacturing technique that is required to actually produce the specific object.

A research project, prior to this graduation project, was performed to get an introduction with topology optimization and to use it in practice for a fictive project. The assignment was to design a bridge for bicycles and pedestrians with the use of topology optimization in order to reduce the amount of material used to construct this bridge. The bridge would have a length of 20 meters and a height and width of 5 meters. It would be supported at both ends at the land abutments and the total design domain also considered support conditions within an area or 5 by 5 meters directly left of the middle of the bridge span.

Figure 1.1. Side view of the optimized bridge.
The finite element modeling was done in Abaqus 6.14. It considers the design domain and the boundary conditions that come with it in terms of support conditions and loadings. For the model linear elastic material properties of steel were used for simplistic modeling of the material properties with a Young’s modulus of 210000 N/mm$^2$ and a Poisson’s ratio of 0.3. This finite element model was then used in the optimization package that comes with Abaqus, which is named Tosca. Considering this design domain, a topology optimization was performed with maximum stiffness as objective and material reduction as constraint, resulting in the outcome displayed in figure 1.1. Now this outcome was useful for the assignment, because it does reflect how the bridge was supposed to be designed with several passages underneath, but its structural integrity was not yet assured do the fact that the implemented material properties were not realistic, especially considering concrete as the structural material that is to be used.

Another problem that was found during this project was that the resulting design was still hard to manufacture with traditional formwork methods, due to the fact that its boundary conditions were not implemented in the topology optimization algorithm. This meant that the manufacturing procedure had to be chosen according to the resulting design, which lead to alterations of the model for it to be able to be casted and demolded. These alterations have then led to a less optimal solution of the optimization problem. To still showcase the result, a column was cut out of the model and scaled to half its dimensions. A CNC machine was used to create the mold required for this column in Polystyrene. By then casting the column with fiber reinforced high performance concrete it could eventually still be manufactured.

This project in a way stated the problem for topology optimization and its application in practice for the building industry. Although this technique is still relatively new and has not yet been developed enough for use in the building industry, it is still promising for the fact that the design can be manufactured and that it allows for structural optimization in several ways, which is what the building industry has been asking for. However, the optimization procedure should take the important practical factors into account, like material properties and manufacturability. For the material concrete this is of great importance, due to the fact that the material is brittle and weak in tension, while strong in compression, and that traditional casting with formwork comes with limitations in design freedom because of the necessity of demolding. To reduce the impact of these factors in the topology optimization, 3D concrete printing comes to mind, but this technique also has its limitations.
This graduation thesis aims to combine these two techniques for further knowledge into complex applications using 3D concrete printing and to research whether the fact that topology optimization can actually be properly used with this technique. It is therefore also important to always relate the outcome of topology optimization to the manufacturability of the object.

This project will then focus on an elaborate study into 3D concrete printing, topology optimization and the possibilities within 3D concrete printing at the Eindhoven University of Technology to be combined with structural optimization. This was possible to be done, because of an extensive literature study into 3D concrete printing throughout the world, especially concrete printing at the TU/e, and the research project in which the basic knowledge of topology optimization was acquired.

The second chapter will focus on the literature study of 3D concrete printing and the techniques that are mostly used at this point. Several different techniques are shortly explained, followed by an extensive research into 3D Concrete Printing at TU/e. This way, the reader can learn the basics of this manufacturing technique, and learn the possibilities and limitations of the printer that is used for this project.

Chapter three will then focus on a review into topology optimization. Several different theories will be explained and compared, with the goal to find out which technique is most suitable for use, while considering manufacturability. Because of different adaptations of the basic theory and developments throughout the years, several different topology optimization algorithms are now under control and to be used for further research.

With the knowledge obtained in the previous chapters, chapter four will focus on further elaboration of the chosen topology optimization algorithm, considering concrete as the material that is to be used. The influence of some of the material properties of concrete will be explained and researched.

Chapter five will then show the influence of 3D Concrete Printing on the final results of topology optimization. Some proof of concepts will be introduced to use 3D Concrete Printing to manufacture these optimized objects. The thesis will finalize with annexes, containing detailed information and measurements about this case study and the algorithms that are used within this graduation project and the alterations that are done to these publicly available scripts.

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*Figure 1.3. Flowchart of this graduation project.*
2. 3D Concrete Printing

The building industry of concrete is demanding a manufacturing technique to achieve more possibilities in the creation of complex designs. Until recently, concrete was mostly built in a traditional way by casting the material into rectangular moulds, creating straight-forward structural elements. But recent developments within the building industry can change the possibilities for more accessible, but complex, shapes and designs of structural elements for concrete design. This can be achieved by the implementation of new 3D printing techniques with concrete. This construction method does not require the need of formwork, while the machine takes care of the manufacturing process in a quick and controlled way. While this technique is still in an early stage, it is promising as new and advanced building process to use in practice, especially for to use for highly customized or complex designs. Several techniques have been developed now that will be explained in this chapter.

2.1. Contour crafting

This construction technique relies on the method of layered manufacturing. To create specific design, several materials can be used like; polymers, ceramic pastes and cementitious pastes. The machine consists of a moving nozzle that is used to extrude the material in such a way that a path is created of a material with a relatively high viscosity. As its name already implies, it only extrudes material at the contours of the design, resulting in a hollow or partially hollow structure. The dimensions of the nozzle and the hose are the boundary conditions of the dimensions of the particles within the paste. By smoothening the layers after extrusion with trowels, smooth surfaces can be created and the layered design disappears.

The type of contour crafting machine can be divided in two groups, as shown in figure 2.1. It can exist of gantries, supporting a movable beam that carries the nozzle. This system creates a rectangular design space, with equal dimensions of the contour crafting system. Another method requires the use of a robotic arm, commonly used in the automotive industry, which is able to carry the nozzle to extrude the material at the desired coordinates. The design space of this technique is spherical. Both processes require a material with certain workability for the ability to pump the material through the system towards the desired deposition coordinates. Several companies have started the use of this technique and created their own machine for custom manufacturing of concrete designs. Research towards implementation of this technique on extra-terrestrial territories has also started.

![Figure 2.1. Contour crafting machines: A movable contour crafting gantry (left) and robotic arm (right).](image-url)
2.2. Concrete printing

Similar to contour crafting, there is another technique called concrete printing. The setup is equal to the setup of contour crafting, but the dimensions of the nozzle are smaller. This results in a layered manufacturing technique of layers with smaller dimensions, which are deposited next to each other as well as on top of each other. This more solid design, compared to the hollow structural elements of contour crafting, will provide more freedom on a local scale due to more precise material deposition. Because of the dimensions of the nozzle, the material should only exist of fine particles and a high workability must be achieved in order to be able to extrude the material out of the nozzle.

![Figure 2.2. The difference of nozzle dimensions between concrete printing (left) and contour crafting (right).](image)

2.3. D-shape

A whole other approach of a 3D printing method is D-shape to create stone-like structures. This machine consists of a large aluminium frame supporting the printing head. This nozzle sprays a binder material on the desired location in a layer of sand. This binder will solidify the material, creating a freeform structure inside a pile of loose sand. This excess sand is therefore also used as a formwork during the solidification of the structure. Once the structure has hardened, the excess material is removed and can be used again, while the desired structure remains. This method provides even more freedom in the creation of forms and shapes with dimensions limited to the dimensions of the machine. However, this method does require the deposition of lots of extra material like sand and this excess material needs to be taken into account during the manufacturing process of the structure.

![Figure 2.3. The printing process of D-shape; CAD model (left), removal of excess sand (middle) and final design (right).](image)
2.4. 3D Concrete Printing at TU/e

A new concrete printer has been developed at the Eindhoven University of Technology in 2015. This setup is similar to the setup of contour crafting and concrete printing and is able to extrude material in a rectangular design space with x, y and z coordinates, as can be seen in figure 2.4. The frame consists of two gantries, supporting a combined beam that can move over the y-direction. This beam carries another unit that can move over the length of this beam, the x-direction. Consequently, this unit carries a vertically moving beam with the printing nozzle attached to its end which enables the nozzle to also move in z-direction. Another feature is the ability of the nozzle to rotate around the z-axis. The dimensions of the printable volume are somewhat around 5m x 9m x 3m.

![Figure 2.4. The setup of the 3D Concrete Printer at the TU/e.](image)

The procedure of the printing process is as follows:

1. The mixing device needs to be filled with cement compatible for concrete printing.
2. The Archimedes screw and the mixing chamber are installed on the mixing device and the device is connected to a water supply with a hose.
3. The cavity pump is installed and placed over the mixing chamber.
4. A sensor is placed within the cavity pump.
5. The pressure meter and the hose are connected to the bottom of the cavity pump.
6. The other end of the hose is connected to the nozzle.
7. The settings of the printing process and the printing path are loaded into the controls unit. These are determined prior to this manufacturing procedure.
8. The controls unit and the mixing machine are connected.
9. When the printing space is prepared, the printing process can start.
This setup comes with certain parameters that influence the result of the printing process:

- Amount of water supply in the mixing chamber in litres per hour.
- Rotating speed of the cavity pump.
- Pressure in the cavity pump.
- Length of the hose.
- Shape of the nozzle head.
- Radius of the corners in the printing path.
- Rotation speed of the nozzle.
- Printing speed.
- Length of the printing path.

The amount of water supply can be chosen at all times during the printing process. A graduated cylinder filled with water is connected to the mixing chamber. The operator can determine and alter the amount of water supply during the printing process at all times. The amount used during the testing procedures of this report are based on a trial and error base for proper workability of the material to be able to use it for concrete printing.

The rotating speed of the cavity pump can also be set with the mixing machine. This will influence the rotating speed of the mixing rod and the rotor in the cavity pump. This speed will cause more mixing time of the cement with the water, resulting in a cementitious paste with a lower viscosity. This speed has also been predetermined with this setup to achieve a proper workability of the material.

At the bottom of the cavity pump a stator with a rotor are connected to the mixing rod. The cementitious paste is pushed through this element, seen in figure 1.6. This rotor and stator combination can be changed, resulting in an element that will cause a different amount of friction between the material and the stator. This will influence the mixing process, as well as it will influence the temperature increase due to a changed friction. Temperature is an important parameter to take into account for the structural build-up of concrete during its hardening process, since it will change the bonding speed between particles in the mixture.
Once the material is pumped through the cavity pump, it will go into the hose. Before this happens, the pressure can be measured on the gauge and the temperature can be determined here as well. The material will be pumped through the hose towards the nozzle, but its length is of importance for the material properties. The flow of the material through the hose will create friction between the outer material and the inner surface of the hose when looked at the cross-section of the hose during a printing process. This will generate an even more increase in temperature, but another effect of this friction is that the inner material in the hose will flow quicker than the outer material. This difference in flowing speed will mix the material even more, which means that the viscosity of the material will change again.

Once the material reaches the nozzle head, it will be extruded through the opening at the bottom due to the pressure in the hose. This opening will determine what the shape of the layers will be during the printing process and therefore also the strength properties of the structure during and after this process. The operator should take this shape into account, since it is also influential for the design in the corners of a print path. The material will also be extruded at the same speed when the pressure remains equal, which could lead to more deposition in the inside of corners than at the outside when the radius of these corners is small.

When looked at the print path, the length and the speed will be influential to the material deposition, strength development and the bonding between layers. As mentioned, the amount of material extruded from the nozzle head will remain the same, which will require a certain print speed for specific material deposition at certain designs. If the printing speed is set to slow speed, more material will be extruded at those coordinates, which could result in larger dimensions of the cross-section of the layer. The opposite could be the case when the printing speed is set to a high speed, which could lead to gaps in the layer, since the material is not being extruded quickly enough. The length of the path is closely related to the speed of the nozzle, since it will determine the open time between layers. When the material hardens quickly, the open time should not be too long because of reduced bonding properties due to deposition of material on a hardened surface. It is more beneficial to the structure to extrude fresh material on the top of other fresh material. However, strength properties of fresh concrete are far less than the strength properties in hardened state. The structural capacity of the material must, at all times, be high enough for the ability to bear the loads introduced by the application of new layers. This means that a certain ratio between open time and strength development must be taken into account during the printing process.
At first it is thought by many people that the possibilities with this manufacturing method come with an infinite amount of possibilities. But it should be stated that this is also not the perfect construction method, compared to the conventional and other innovative methods. The designers still have to take these construction boundaries into account, since it is definitely not the case that every design can simply be made by using the right coordinates for the deposition of material. Gravity is still a natural phenomenon to take into account. When the material is extruded, gravity will act on the material. This means that a new print path must always be printed on an existing surface, e.g. the floor or for example an already printed layer. To create certain spans, the designer can play with the orientation of the design. Once hardened, the structural element can be rotated in such a way that a span can still be created. This method can also be used then the designer wants to create an inclined vertical surface. The layers are printed on their side on a horizontal surface and rotated afterwards. Angles can also be made by printing new layers on top of the existing layers, but slightly shifted. When this difference becomes too large, the structure will tilt and fail. This is visualized in figure 2.8. For the printing process to be able to produce inclined structures, different concrete is required with a faster hardening process or with the application of reinforcement to withstand higher tensile forces once hardened. Another possibility to obtain inclined structures is the introduction of a support material which should be used within the same printing process to carry the structural concrete that is to be printed.

Figure 2.7. Different factors to take into account for 3D Concrete printing are the print speed and the print length. Both factors relate to the time as an important factor within the printing process.

Figure 2.8. The angle constraint for 3D Concrete Printing (left). This angle can be improved by using different material (upper right) or by using support material (lower right).
Another factor to take into account for now is the incapability of the setup to create gaps in the printing path. It is not possible yet to pause the pump pressure and stop extruding the material for a moment. The yield stress and the friction in the setup are too large for the system to start the flow of the material again. This means that the design must be made from a continuous printing path for now. It also means that the layer shape and dimensions are now the same within the same printing process. Alterations of the nozzle are required for more variations in this procedure while printing. In an ideal world, the nozzle will work like the diaphragm of a camera, which makes it possible to have actively changing layer dimensions. Another possibility is taken from the way how a microscope works. The nozzle is then rotatable, resulting in the possibility to print with several layer dimensions within the same printing process. This could also mean that it is then possible to reach the ability to create discontinuous printing paths. Something that then always should be considered is that the pump pressure will keep increasing during the time that the nozzle is closed and not extruding material. With smart techniques like this, the system can be improved to create smarter designs and to obtain a more controlled printing process with much more possibilities. When the setup is changed, this might still be possible in the future.

![Image](image.png)

*Figure 2.9. With the current nozzle the layer dimensions are the same throughout the printing process (left). With new developments in the nozzle more dimensions within the same process can be possible (right).*

The fact that the system uses one setup of a combination of mixer, hose and nozzle means that it is not yet possible to print several types of materials within the same printing process. The only possibility for now is to print the design from one type of material, which is concrete at the moment. And this material is also dependent on the dimensions of these parts of the setup, since the small dimension of the nozzle head for example do not allow the extrusion of large element or particles like large aggregates or fibres. If the designer wants to introduce more types of material within the same design, the setup must consist of more combinations of pump and nozzle.
Structural Optimization for 3D Concrete Printing

The material properties of the concrete that is used within this research is stated in [17]. A custom concrete mix was developed by SG Weber Beamix for use within the 3D Concrete Printing research. It was developed for optimal workability during the printing process, which is why the cement mix consists of the following:

- Portland cement (CEM I 52,5 R) for a high initial strength and relatively high final strength, which is ideal for printed concrete.
- Siliceous aggregate with an optimized particle size distribution and a maximum particle size of 1 mm. This small particle size is maintained in order for the material to be pumped through the pumping system.
- Limestone filler and specific additive for a better workability during the printing process. This means that the mixture can more easily be pumped to the nozzle where it is deposited on the specific coordinates.
- Rheology modifiers to obtain thixotropic behaviour of the fresh concrete. The thixotropic behaviour is relevant for initial strength development.
- Small polypropylene fibres to reduce the amount of crack formation during hardening and early drying.

An experimental research was conducted [49] to obtain the setting time, the initial strength capacity, the strength development and the final strength capacity of the material for a single layer. The results of this research where:

- A 28-day compressive strength of approximately 20 N/mm² and a flexural strength of approximately 5 N/mm².
- An initial setting time of approximately 3 hours and a final setting time of approximately 5 hours.

These characteristics were chosen to achieve a no-slump concrete mixture, resulting in sufficient material properties related to 3D Concrete Printing. This way, during the process, the layers will remain stable with the same dimensions once extruded, which allows for a more precise printing process. The setting time of the material is still relatively long, which means that the newly extruded layers will be able to bond with the already printed layers, since they are still in the dormant period. It is also feasible for practical reasons, since the quality of the printer can be maintained better by cleaning without lots of effort.
3. Structural optimization

This chapter presents an overview of the most common theories required for structural optimization to be performed. Structures are here considered to be optimized, when an optimal material distribution method is performed over the specific design domain, resulting in an optimum lay-out of material in a linear elastic structure. Figure 3.1. shows the design domain most frequently worked with, a simply supported two-dimensional beam with a point load in the middle of the span. Since these optimization processes require the use of finite element analysis and the optimization procedure also requires computational power, the simply supported beam is modeled with half its original length to reduce the amount of elements and therefor formulas to be computed. The support conditions have been altered accordingly for an equal structure and flow of forces throughout the structure. The supports, along with the forces, material properties and geometries are therefore fixed and they are part of the fixed design domain in which this structural optimization needs to take place.

The building industry is nowadays mostly looking to optimize the production and design processes as well as to achieve an optimized design, with for example a combination of functions within the same object or a designed object with a minimum cost or minimum amount of material for more sustainable structures. Traditionally the optimization procedure is an iterative procedure which is described by Christensen and Klarbring [14] as follows:

- A specific design is suggested.
- Requirements on the function are researched.
- If these requirements are not satisfied, the design needs to be altered.
- Eventually the design fulfills the needs and the iterative process has come to an end.

With the use of finite element modeling this process has become mathematical and automatic and an approach to the exact solution can be obtained within this optimization procedure to make the process as a whole more efficient. This means that structural optimization through mathematical computer models can be explained by a similar procedure with the following functions and variables:

- Objective function $f$: This is a function used to set the goal for the structural optimization. Most frequently this is minimization problem in order to optimize towards a minimum amount of this function, for example weight.
- Design variable \( x \): A function or vector that describes a part of the design, for example the geometry.
- State variable \( y \): This is a function that describes the design's response, for example deformation.

This means that a general structural optimization can be shown as follows:

\[
\begin{align*}
\text{SO} \quad & \min f(x, y) \\
\text{subject to} \quad & x, y \text{equilibrium}
\end{align*}
\]

The variables can also be used in terms of constraints in order to control the structural optimization and let it converge to a proper solution. These design constraints can relate for example to a fixed volume or weight reduction. Another constraint to be considered is equilibrium, since the whole structure needs to be in equilibrium for it to be solved. This is done in finite element modeling by equalizing the stiffness matrix of the whole structure and its deformations to the total force on the structure. The equilibrium constraint is:

\[
KU = F
\]

In which:
- \( K \) is the stiffness matrix of the structure.
- \( U \) is the displacement vector.
- \( F \) is the force vector.

These variables and functions are given for several features of the total structure. Depending on the desired structural optimization, this theory can be divided in three types of structural optimization [7]:

- Sizing optimization: This type of optimization is performed when \( x \) represents the thickness of the structure and this thickness needs to be locally optimized within the structure.
- Shape optimization: This type of optimization is performed when \( x \) represents the boundaries of the structural domain and these boundaries within this domain need to be optimized, meaning that the shape of for example voids within the structure will change.
- Topology optimization: This type of optimization takes the full design domain, consisting totally of elements, and dependent on the objective function and design variables, the optimization procedure calculates at which nodes material is required and where voids are allowed to be existent.

![Figure 3.2. The effect of size optimization (a), shape optimization (b) and topology optimization (c). [7]](image)
3.1. Size optimization

Size optimization is mostly performed on truss structures or shell structures for local optimization of members of the global structure. This optimization can be seen as an optimal distribution of member thicknesses throughout the whole structure, with the objective to reduce the total amount of material to be used. The size optimization problem for compliance minimization can be stated as follows:

\[
\begin{align*}
\min_x \quad & c(x) = U^T K U \\
\text{Subject to} \quad & KU = F \\
& \sum_{j=1}^n l_j x_j \leq V_{\text{max}} \\
& x_j^{\text{min}} \leq x_j \leq x_j^{\text{max}}
\end{align*}
\]

In which:
- \(n\) is the number of bars.
- \(l_j\) is the length of bar \(j\).
- \(x_j\) is the cross-sectional area of bar \(j\).
- \(V_{\text{max}}\) is the maximum allowed volume of the truss.

The sizing optimization is allowed to optimize towards cross-sectional area’s between \(x_j^{\text{min}}\) and \(x_j^{\text{max}}\) for every bar within the structural design domain.

To achieve a convex iterative procedure towards an optimized result, the Method of Moving Asymptotes (MMA) is mostly used, as proposed by Svanberg [48]. This basically means that, to achieve the minimum of the compliance within the optimization problem, the derivative of the compliance needs to be computed. This procedure is called the sensitivity analysis and must be done for all the bars within the truss. The procedure calls for an approximation of the value of the cross-sectional area \(x_j\), of which the derivatives give the maximum and minimum value. While actively changing the values of these asymptotes, the values for \(x_j\) converge to an approximated solution which results in an optimized truss frame when this procedure is performed for every member of the structure. For the exact formulation of the MMA, the reader is referred to [7] and [14].

![Figure 3.3. The initial truss (left) and several iterations of the size optimization process (right). [14]](image-url)
3.2. Shape optimization

Shape optimization will most often be used at the end of the design process, when the global shape or design is already determined and only minor alterations are still allowed. This is done, because shape optimization is mostly performed to reduce stress concentrations within the edges of the design, based on stress analysis. Due to this procedure, the surface geometry of the finite element models are then automatically improved to avoid local material failure. This minimization of stress deviation is based on the fact that stresses along the surfaces of the geometry should be fully constant. [49]

Shape optimization can be useful to consider in combination with topology optimization. The sharp edges and corners resulting from topology optimization, can be smoothened or optimized towards the material property constraints and manufacturing constraints. This means that the local stresses for example can be reduced for use with the specific material. Or that curvatures for example are increased in such a way that the can be manufactured with the process that the designer had in mind. Shape optimization should therefore be performed after topology optimization has been performed, since shape optimization relies on the fixed domain with holes that results from topology optimization. For the exact formulation of shape optimization, the reader is referred to [14], since this thesis focusses on topology optimization and this topic is only mentioned to for a complete overview regarding structural optimization.

Figure 3.4. An object that failed due to high local stresses (top left), the differences between the initial geometry and the optimized result (bottom left and right). [49]
3.3. Topology optimization

3.3.1. Density methods

In order to perform topology optimization, the finite element model is placed in a minimum compliance optimization problem, which is similar to the previous problems of different theories:

\[
\begin{align*}
\min_{x} & \quad c(x) = U^T K U = \sum_{e=1}^{N} (\rho_e)^P u_e^T k_e u_e \\
\text{Subject to} & \quad \frac{v(x)}{V_0} = f \\
& \quad K U = F \\
& \quad 0 < \rho_{\text{min}} \leq \rho \leq 1
\end{align*}
\]

Based on the early introduced homogenization method, a theory was suggested called SIMP [7], which stands for Solid Isotropic Material with Penalization. This theory states the relation between the density design variable and the material properties for isotropic material, using the power-law principle. This can be written as:

\[
E(\rho) = \rho^P E_0
\]

In which:
- \( \rho \) is the density design variable.
- \( P \) is the penalization parameter.
- \( E_0 \) is the Young’s modulus of the solid material.

The problem within this formula lies in the fact that the element stiffness will be zero when the density is zero, which is infeasible for the finite element model in terms of the fact that the stiffness matrix will become singular. An alternative SIMP interpolation scheme is therefore proposed by [3], which introduces a minimum value \( E_{\text{min}} \) for the voids. This will also allow for simple implementation of additional filters, explained later in this chapter. The alternative SIMP approach will then be:

\[
E(\rho) = E_{\text{min}} + \rho^P (E_0 - E_{\text{min}})
\]

Within this chapter, the 88 line MATLAB code [3] will be used for further explanation of the parameters and theories of the SIMP approach. This code is based on the 99 line MATLAB code by Sigmund [42] and is publically available for research purposes. These topology optimization algorithms are based on finite element modeling with square bilinear 4-node elements. The benchmark test case consists of a rectangular design domain with 120 elements in x-direction and 40 elements in y-direction. The constraint is a volume fraction of 50%, filter radius 3,5 and density filtering, resulting in topology optimization as shown in figure:

Figure 3.5. The initial design space with boundary conditions and loads (left) [42] and the optimized result (right).
3.3.1.1. Optimality Criteria method

The optimization problem can be solved with several different theories, like the MMA method explained in chapter 3.1. The SIMP method in this thesis however, will use the Optimality Criteria method to update the design density variables throughout the optimization procedure. This is done with the following formulations [3]:

\[ \rho_e^{\text{new}} = \begin{cases} \max(\rho_{\text{min}}, \rho_e - m), & \text{if } \rho_e B_e^\eta \leq \max(\rho_{\text{min}}, \rho_e - m) \\ \rho_e B_e^\eta, & \text{if } \max(\rho_{\text{min}}, \rho_e - m) < \rho_e B_e^\eta < \min(1, \rho_e + m) \\ \min(1, \rho_e + m), & \text{if } \min(1, \rho_e + m) \leq \rho_e B_e^\eta \end{cases} \]

In which:
- \( \rho_e \) is the relative density of an element.
- \( m \) is a positive move-limit with a value of 0,2, taken from experimental research.
- \( \eta \) is a numerical damping coefficient with a value of 0,5, taken from experimental research.
- \( B_e \) is found from the optimality condition as:

\[ B_e = \frac{\lambda}{\lambda^2 \partial V / \partial \rho_e} \]

In which:
- \( \lambda \) is a Lagrangian multiplier
- \( \partial c / \partial \rho_e \) is the derivative of the compliance, also known as the sensitivity of the objective function. It is formulated as:

\[ \frac{\partial c}{\partial \rho_e} = -p(\rho_e)^{p-1} u_e^T k_0 u_e \]

In which:
- \( p \) is the penalization factor.
- \( \rho_e \) is the relative density of an element.
- \( u_e \) is the element displacement vector.
- \( k_0 \) is the element stiffness matrix.

The OC-method is important within the structural optimization scheme, since it is essentially the loop of the optimization process. With these formulations, the densities of all the elements will be updated towards a more optimized solution, until the stop criterion is met. The final result is obtained when the difference in design variables of two consecutive designs becomes less than 1%.

For the algorithm to obtain stable and correct results, some other functions and theories are implemented within the algorithm. This is mostly done to get results, which are relatable to realistic object and have therefore a higher possibility to be eventually used in practice then the solutions obtained without the additional functions. These can be explained in terms of the penalization in the interpolation scheme, filtering techniques, mesh-independency and the application of other constraints and objectives. These other constraints and objectives are important to consider, since the minimization of compliance is in reality most likely not the objective that the designer wants to consider. From a structural point of view, it seems to be more important to achieve a minimization in stresses and displacement, in which the objective is subjective to a stiffness constraint. In reality it is also required to consider multiple objectives and constraints and this is why Pareto-optimality should also be considered. This is a function that results in a summation of weighted design variables and therefore considering more factors in a more realistic way.
### 3.3.1.2. Penalization

The density design variable $\rho$ is referred to as a density of material for the given fact the volume of the total structure is given by:

$$V = \sum_{\Omega} \rho(\chi) d\Omega$$

Since the design domain is created in finite element modeling and therefore consisting of elements, every element will obtain a certain value for its corresponding density. The relative density is related to the constraint in the optimization problem, stating that the value should be between $\rho_e = 0$ and $\rho_e = 1$. This means that the final result consists of elements with no density, and therefore being a void or white design, and elements with densities with values higher than zero, therefore being solid material or gray design. Fully solid material or black design is reached with $\rho_e = 1$. These are the reasons why this type of topology optimization may also be called gradient type topology optimization.

The penalization factor is introduced to ensure more realistic solutions for the stiffness of every element, due to the fact that the penalty factor forces the final result towards more 0-1 solutions. According to research, the most conventional value is $p = 3$, with $p \geq 1$. The comparison between different values for the penalty factor are shown in figure 3.6. and 3.7.. The algorithm could not come to a solution with $p \geq 5$, and figure 3.6.already shows that the boundaries are already becoming inconsistent for $p = 4$.

![Figure 3.6. The optimized results with $p = 1$ (top left), $p = 2$ (top right), $p = 3$ (bottom left) and $p = 4$ (bottom right).](image)

![Figure 3.7. Graph of the effect of the penalty factor on the elemental stiffness.](image)
3.3.1.3. Filtering techniques

Topology optimization performed on finite element models are created by the visualization of several connecting elements. The fact that the OC-method considers each element individually, means that every element can have a value for the relative density that lies between 0 and 1. This also means that neighboring elements can have the exact opposite solution, which results in an element with a relatively high density, while the next element has a relative density of zero. This is due to the fact that the compliance must be minimized and it then results in minimum and maximum values possible per each individual element, while not considering the total structure. In practice however, this is not beneficial, since elements can then be connected with nodal hinges, resulting in a checkerboard pattern. This problem can be tackled by the use of several filtering techniques, with the most common one to be sensitivity filtering.

Sensitivity filtering can be explained by modifying the design sensitivity of a specific element, based on the weighted average of the sensitivities of the elements in its direct fixed neighborhood. This modification is done with the following formula [7]:

\[
\frac{\bar{\partial}c}{\bar{\partial}\rho_e} = \frac{1}{\max(\gamma, \rho_e) \sum_{e=1}^{N} \hat{H}_{el} \rho_e} \sum_{i=1}^{N} \hat{H}_{el} \rho_i \frac{\partial c}{\partial \rho_i}
\]

In which:

- \( \gamma \) is a small positive number to avoid division by zero in this formula.
- \( N \) is the set of elements \( i \) for which the center-to-center distance \( \Delta(e, i) \) between elements is smaller than the filter radius \( r_{min} \):

\[
\{i \in N \mid \text{dist}(e,i) \leq r_{min}\}, \quad e = 1, \ldots, N
\]

- \( \hat{H}_{el} \) is the weight factor formulated as:

\[
\hat{H}_{el} = r_{min} - \text{dist}(e, i)
\]

Figure 3.8. Visualization of the filter radius [23].
In the classic SIMP approach filtering of the mesh was performed by a density filter and is formulated as:

$$\tilde{\rho}_e = \frac{1}{\sum_{e=1}^{N} H_{ei}} \sum_{i=1}^{N} H_{ei} \rho_i$$

With this technique, each element density is recalculated and defined as a weighted average of the densities in its direct fixed neighborhood. This is done before the finite element solver is called and performed. The sensitivities of the elements are calculated afterwards. The difference with the sensitivity filter is that in this case the densities are redefined prior to the finite element solver, while for the sensitivity filter the sensitivities are calculated after the finite element solver and then modified with weighted averages in its direct fixed neighborhood. This means that for density filtering, the stiffness of the elements are modified before the finite element solver, while for sensitivity filtering, they are modified afterwards [43].

Figure 3.10. on the next page shows the effect of the filter radius $r_{min}$, while the same mesh is maintained for every other optimization procedure. The value is increased with 1.0 for every topology optimization. In this figure, it is clear to see that a checkboard pattern will occur, since the filter does not have to take the surrounding elements into account. While increasing the filter radius, it can also be seen that eventually the filter radius is too large to still maintain proper black and white solutions. This is due to the fact that the volume constraint needs to be met. The neighborhood of elements is becoming so large, that they can no longer be black or white, but that they have to become grey in order to meet the requirements for this volume constraint. It seems that the filter radius should be as low as possible to obtain results with better black and white solutions, but this also means that the member of the structure will become thinner. This effect should be taken into account considering the manufacturing process.

Figure 3.9. shows the effect of the filter radius, when the input for the mesh is altered, but the value for the filter radius remains the same. In the upper row, the first figure shows the benchmark test with a halved amount of elements in both directions, while the second figure shows the benchmark test with a doubled amount of elements in both directions. It is clear to see that the filter radius takes the center-to-center distance between a specific element and the furthest elements within this radius as the neighborhood that needs to be considered. This example shows that the mesh of the structure is also important to consider, since totally different designs can occur, while the rest of the settings are the same. The bottom row shows the effect when the filter radius is modified accordingly to the modification to the total mesh. It shows that when the mesh is doubled for example, the filter radius should be doubled as well to obtain the same results, but with more precision due to this finer mesh. This does come with a higher computational cost.

Figure 3.9. Result of the influence of the mesh. Filter radius of 3.5 with a mesh of 60x20 (top left), 240x80 (top right), 120x40 (bottom left) and 240x80 with a filter radius of 7.0 (bottom right).
Figure 3.10. The influence of the filter radius on a 120x40 design domain with a value of 1.0 (top left) and increasing with 1.0 until a value of 10.0 (bottom right).

Figure 3.11. The influence of the filter radius on the compliance. The structure becomes less stiff with an increase in the filter radius.
The density and sensitivity filters are the ones that are most common and used in research. As can be seen in previous figures, the problem with these filters is that the boundaries of the final design are not clearly designed, due to the gradient method. The elements at the boundaries all gradually change from the black elements within the members of the structure to the white elements outside its boundaries. The gray elements are difficult to relate to practice and manufacturing procedure, since it is not quite possible to produce these exact boundaries with these properties. And if the designer wants to consider every element with a density higher than zero, then the volume constraint cannot be met anymore.

A solution to overcome this problem is the use of black and white filtering, instead of density or sensitivity filtering. The black and white filters are modifications to the original filters and converge to only black and white or 0-1 solutions, while the gradient procedure is maintained throughout the process for a precise final result. According to [43] there are several filters that can be used to obtain this effect, but for this research only density filtering with the Heaviside step function will be considered. This theory has been used and developed the most in other research [3], [18], [20], [21], [22], [23] and it is also possible to use it for other constraints, as will be shown in chapter 5.

It is important to consider the fact that the Heaviside function distinguishes a difference between the physical element density $\bar{\rho}_e$ and the filtered element density $\tilde{\rho}_e$. The physical element density is the density that is shown in the plots and is merely the final result, while the filter element density is the density that is used in the SIMP interpolation schemes. The Heaviside function is formulated as:

$$\tilde{\rho}_e = 1 - e^{-\beta \rho_e} + \bar{\rho}_e e^{-\beta}$$

In which:
- $\bar{\rho}_e$ is the physical element density.
- $\tilde{\rho}_e$ is the filtered element density.
- $\beta$ is the parameter for the smooth function that is increasing in this iterative process, until the final result has reached its maximum value in the use of elements.

The use of this function is that it changes the physical density to $\tilde{\rho}_e = 1$ when $\tilde{\rho}_e > 0$ and only when $\tilde{\rho}_e = 0$ then $\tilde{\rho}_e = 0$.

The implementation of this function is done in the sensitivity analysis by applying the chain rule:

$$\frac{\partial f}{\partial \tilde{\rho}_e} = \sum_{j \in N_i} \frac{\partial f}{\partial \tilde{\rho}_e} \frac{\partial \tilde{\rho}_e}{\partial \tilde{\rho}_e}$$

After which the derivative of the physical density can be obtained by:

$$\frac{\partial \bar{\rho}_e}{\partial \tilde{\rho}_e} = \beta e^{-\beta \rho_e} + e^{-\beta}$$

Figure 3.12. Comparison between the final result with a density filter (left) and Heaviside filter (right). The boundaries become precise with Heaviside filtering, compared to the gradually changing density at the boundaries for density filtering.
3.3.1.4. Volume fraction

In all the cases presented here, the objective is to minimize the compliance and therefore optimizing towards maximum stiffness, while a volume constraint is to be met. In standard procedure the volume constraint is set to 50% of the original volume, resulting in an approximation of a reduction of half of the material required. It is however not clear what the number for the compliance should be and how much material should be removed to maintain a proper structure. From a logical aspect, it seems clear that the more material is used, the stiffer the structure will be. Figure 3.13. shows the difference in design when the same settings are used as the benchmark test, but the volume constraint is reduced by steps of 10% within a range 100% and 10%. The effect of the filter radius can be seen here as well, since the structure with a volume constraint of 30% remains the same even when more volume reduction is required. The volume reduction is then taken from black elements, changing them to gray elements, instead of taking black elements and changing them to white elements in the previous topology optimization results.

![Figure 3.13. The influence of the volume constraint on a 120x40 design domain with a value of 1.0 (top left) and decreasing with 0.1 until a value of 0.1 (bottom right).](image)

Figure 3.14. shows that the compliance does increase when a higher volume constraint is introduced, therefore resulting in stiffer structures. This is obviously relatable to reality. The difference seems to be exponential and it shows that the difference in stiffness becomes relatively minor between a range of reduction between 30% and 100%. To obtain an optimal result with this procedure, the designer should first determine what the compliance should be and then figure out what volume reduction is required to still maintain this desired stiffness. That way the maximum amount of material will be removed within this optimization problem.
3.3.1.5. Flowchart of the SIMP method

This method for topology optimization can be explained as follows [7]:

- Determine a proper design domain with fixed boundary conditions, loads and material properties.
- Determine which elements of this design domain should be fixed and which can be removed or modified during the optimization procedure. Therefor the mesh is also of importance, since a finer mesh gives more precise results, but comes with a computational cost.
- Determine the optimization problem: The objective, the constraints, the type of filter to be used and the filter radius.
- Determine the factors in the Optimality Criteria method and the stop criteria accordingly.
- Start the optimization procedure. The flowchart of this procedure is shown in figure.
- Determine if the final result satisfies the requirements. If not satisfies, then determine which setting should be modified for a proper result.
- Post-processing of the results.

![Flowchart of the SIMP method of topology optimization.](image)

Figure 3.14. Graph of the influence of the volume constraint on the compliance.

Figure 3.15. Flowchart of the SIMP method of topology optimization.
3.3.2. Evolutionary methods

3.3.2.1. Evolutionary Structural Optimization (ESO)

Topology optimization performed by Evolutionary Structural Optimization (ESO) was first proposed in the early 1990s and has been further developed by Xie and Steven to solve many other topology optimization problems. It is based on the concept to continuously remove elements from the design space of the structure. This way the structure, through the iterative process will evolve towards an optimized design. The ESO technique is to be used by designers to create the design in the early stages of the process for a conceptual design and better understanding in the use of material within the structure.[56]

Analysis of the models are performed in finite element software packages. An indicator for the ESO theory is the inefficient use of material with low stress values. To maintain the same stress value in the whole structure, a rejection criterion is introduced, based on the local stress value of an element, resulting in the removal of element with low stresses. The stress level of each element is compared to the stress level in the whole structure. The following condition results in the removal of elements:

\[
\frac{\sigma_e}{\sigma_{\text{max}}} < RR_i
\]

In which:
\(\sigma_e\) is the stress value of an element.
\(\sigma_{\text{max}}\) is the stress value of the whole structure.
\(RR_i\) is the rejection ratio.

When there are no more elements deleted, an evolutionary rate (ER) is added to the rejection ratio in order to gradually remove more elements from the whole structure, due to an increase in the rejection criterion. This procedure is repeated until the desired optimum is reached. This way the stress values of the elements in the whole structure will become more uniform.

\[
RR_{i+1} = RR_i + ER
\]

To maintain maximum stiffness of the model, the theory considers minimizing the mean compliance (C), in which \(f\) is the force vector and \(u\) the displacement vector.

\[
C = \frac{1}{2} f^T u
\]

In finite element analysis, using the global stiffness matrix \(K\), when the \(i\)th element is removed, the increase in mean compliance results in:

\[
\Delta C = \frac{1}{2} u_i^T K_i u_i = a_i^e
\]

This results in the sensitivity number of the mean compliance \((a_i^e)\), so the objective function. The above equation shows that an increase in the mean compliance as a result of the removal of the \(i\)th element is the same as the strain energy of that element. To minimize the mean compliance, therefore maximizing the stiffness, due to the removal of elements, it is obvious that the most effective way is to remove the elements which have the lowest values of \(a_i\), so that the increase in the compliance will be minimized. A sensitivity number can also be derived when optimized towards a displacement objective, instead the common objective of maximum stiffness [28].
To achieve a certain volume reduction with the specific objective of compliance minimization the ESO method seems to follow a logical procedure to reduce the volume by gradually removing element in the design domain, until the constraint can no longer be satisfied. However, it is possible that the elements which are removed early within the iterative process might be useful later to be part of the optimal design. The ESO algorithm is then not able to add the elements again, once they have been deleted from the structure. This is why the final result might not be the optimal solution, although the ESO method will still come to an optimized solution compared to the original design space. The ESO algorithm is still lacking in many ways in terms of checkerboard patterns, finding the optimal solution and for example mesh-independency. In order to come to proper solutions the ESO method has been modified in such a way that these functions and theories are added to get rid of these problems. This has been introduced as the Bi-directional Evolutionary Structural Optimization method (BESO) [28].

3.3.2.2. Bi-directional Evolutionary Structural Optimization (BESO)

The Bi-directional Evolutionary Structural Optimization (BESO) method allows material to be removed and added within the same process. The optimization problem can be stated as [28]:

Minimize: $C = \frac{1}{2} f^T u$

Subjected to: $V^* - \sum_{i=1}^{N} V_i x_i = 0,
 x_i = 0 \text{ or } 1$

In which
$V^*$ is the prescribed total structural volume.
$V_i$ is the volume of an individual element.
$N$ is the total number of elements in the system.
$x_i$ declares the absence (0) of presence (1) of an element.

The difference in this problem statement with the problem statement of SIMP is that BESO is not a gradient method. The elements are either black or white, 0-1, instead of the possibility for occurrence of gray material.

The new BESO method requires a new algorithm to be convergent and stable towards finding a solution which considers this optimization problem statement. Previously, the effect of the removal of an element on the total structure was not considered with the sensitivity number. By the introduction of element strain density, the effect on the volume is taken into account.

$$\Delta C = a_i^e = e_i = \frac{1}{2} u_i^T K_i u_i$$

A filter scheme is introduced for the ability to add elements in the design domain, and with that creating the new BESO algorithm. When a continuum structure is discretized using bilinear square 4-node elements, the sensitivity numbers could become discontinuous across element boundaries, leading to checkerboard patterns and manufacturing difficulties. The filter scheme smoothens these sensitivity numbers in such a way that they remain continuous around element boundaries. Average nodal sensitivity numbers are obtained by:

$$\alpha_j^n = \sum_{i=1}^{M} w_i \alpha_i^e$$
In which:

\( M \) is the total number of elements connected to the \( j \)th node.

\( w_i \) is the weight factor of the \( i \)th element and is defined by:

\[
    w_i = \frac{1}{M-1} \left( 1 - \frac{r_{ij}}{\sum_{i=1}^{M} r_{ij}} \right)
\]

With \( r_{ij} \) being the distance between the center of the \( i \)th element and the \( j \)th node. The above sensitivity numbers of the nodes will then be converted into smoothed sensitivity numbers of all the elements. This conversion takes place due to projection of these nodal sensitivity numbers on top of the total design domain. This is done with the use of a filter. The filter has a length scale \( r_{min} \) that does not change with mesh refinement, which means that it is mesh-independent. The most important role of the variable \( r_{min} \) in the filter is to identify the nodes that will influence the sensitivity of the \( i \)th element. This can be visualized by drawing a circle of radius \( r_{min} \) with the center at an equal location as the center of the \( i \)th element, therefore generating a circular sub-domain \( \Omega_i \) in which the sensitivity numbers of all the nodes are considered. Usually the value of \( r_{min} \) should be larger than the size of one element, so that \( \Omega_i \) covers more than one element. The size of the sub-domain \( \Omega_i \) does not change with mesh size. Nodes located inside this sub-domain \( \Omega_i \) contribute to the calculation of the modified sensitivity number of the \( i \)th element with:

\[
    \alpha_i = \frac{\sum_{j=1}^{K} w(r_{ij}) \alpha_j^k}{\sum_{j=1}^{K} w(r_{ij})}
\]

In which:

\( K \) is the total number of nodes in the sub-domain.

\( w(r_{ij}) \) is the linear weight factor:

\[
    w(r_{ij}) = r_{min} - r_{ij}, \text{ with } (j = 1, 2, ..., K)
\]

With ESO/BESO methods, lots of large deviations are often seen in the iterative process of the optimization loop towards finding the optimal solution for the objective. The reason for this behavior is that the sensitivity numbers of the solid and void elements, are based on purely 0-1 solutions. Elements therefore change constantly from being solid to passive, largely influencing the result of the sensitivity functions. This makes the objective function and the topology difficult to converge. Because of this reason, the sensitivity numbers of each element are averaged over its history within the same optimization process. The simple averaging scheme is given as:

\[
    \alpha_i = \frac{\alpha_i^k + \alpha_i^{k+1}}{2}
\]

In which \( k \) is the current iteration number. By letting \( \alpha_i^k = \alpha_i \), the new value will be used for the next iteration. Therefore the modified sensitivity number includes the full history of the sensitivity information in the previous iterations of each element.

Before elements are removed from or added to the current design, the target volume for the next iteration (\( V_{k+1} \)) needs to be given first. Since the volume constraint (\( V^* \)) can be greater or smaller than the volume of the initial guess design, the target volume in each iteration may decrease or increase step by step until the constraint volume is achieved. The evolution of the volume can be formulated as:

\[
    V_{k+1} = V_k (1 \pm ER) \quad (k = 1, 2, 3, ...)
\]
In which $ER$ is the evolutionary volume ratio. Once the volume constraint is satisfied, the volume of the structure will be kept constant for the remaining iterations as:

$$V_{k+1} = V^*$$

Then the sensitivity numbers of all elements, solids and voids, are calculated as described with the explained formulas. The elements are then sorted according to the values of their sensitivity numbers from the highest to the lowest values. For solid elements with $x_i = 1$, the value will be modified to $x_i = 0$, when:

$$a_i \leq a_{ch}^{th}$$

For void elements with $x_i = 0$, the value will be modified to $x_i = 1$ if:

$$a_i > a_{add}^{th}$$

Where $a_{del}^{th}$ and $a_{add}^{th}$ are the threshold sensitivity numbers for removing and adding elements. They are determined by the following steps:

- Let $a_{add}^{th} = a_{del}^{th} = a_{ch}$, thus $a_{ch}$ can be easily determined by $V_{k+1}$.
- Calculate the volume addition ratio $AR$, which is defined as the number of added elements divided by the total number of elements in the design domain. If $AR \leq AR_{max}$ where $AR_{max}$ is a prescribed maximum volume addition, skip the next step.
- Calculate $a_{del}^{th}$ by first sorting the sensitivity number of the void elements. The number of elements to be switched from 0 to 1 will be equal to $AR_{max}$ multiplied by the total number of elements in the design domain. $a_{del}^{th}$ is then determined so that the removed volume is equal to $V_k - V_{k+1} +$ the volume of the added elements.

Along with the objective volume, the convergence criterion must be satisfied as well:

$$\text{error} = \left| \frac{\sum_{i=1}^{N} C_{k-i+1} - \sum_{i=1}^{N} C_{k-N-i+1}}{\sum_{i=1}^{N} C_{k-i+1}} \right| \leq \tau$$

The procedure for the BESO method can then be easily explained by the following steps:

- Discretize the design domain using a finite element mesh and assign initial property values (0 or 1) for the elements to construct an initial design.
- Perform finite element analysis and then calculate the elemental sensitivity number.
- Average the sensitivity number with its historical information using and then save the resulted sensitivity number for next iteration.
- Determine the target volume for the next iteration.
- Add and delete elements.
- Repeat steps 2–5 until the constraint volume $V^*$ is achieved and the convergence criterion is satisfied.
3.3.3. Level-set methods

The level-set method was first developed by Osher and Sethian in 1988 and are at this point further developed for use in research. But only since recently it is applied in combination with topology optimization to find optimal material distribution of structures, like it is performed with SIMP and BESO. In this thesis the MATLAB code developed by Challis [12] is used, which is inspired on the 99 line for topology optimization introduced by Sigmund [42]. This level-set method is not a gradient based method, it therefore only has two values for the densities, 1 for solids and 0 for voids. This way, clear boundaries will be formed to create the final result for the topology optimization problem of finding the structure with minimum compliance, subjected to a volume constraint. The topology optimization problem can then be stated as well as:

\[
\begin{align*}
\min_{\mathbf{x}} : \quad & c(\mathbf{x}) = U^T K U = \sum_{e=1}^{N} x_e u_e^T k_e u_e \\
\text{Subject to} : \quad & V(\mathbf{x}) = V_{\text{req}} \\
& K U = F \\
& x_e = 0 \text{ or } x_e = 1
\end{align*}
\]

In which:
- \( \mathbf{x} \) is the vector of element densities with values of 0 or 1.
- \( c(\mathbf{x}) \) is the compliance objective function.
- \( U \) is the global displacement vector.
- \( K \) is the global stiffness matrix.
- \( F \) is the global force vector.
- \( u_e \) is the element displacement matrix.
- \( k_e \) is the element stiffness matrix.
- \( N \) is the total number of elements in the design domain.
- \( V(\mathbf{x}) \) is the number of solid elements.
- \( V_{\text{req}} \) is the required number of solid elements, the volume constraint.

The level set formulation can be explained with figure 3.16. A specific structure is given with a solid domain \( D \), of which the boundaries are given by \( \partial D \). The problem than states to find the optimal boundary \( \partial D \) of the solid domain \( D \), for which the compliance is minimized with the volume constraint. \( \bar{D} \) is set as a reference domain larger than \( D \), in such a way that it always fully contains the structure. The level-set model is given by \( S \) and it resembles the boundaries in every dimension, every height, in figure. \( \phi(\mathbf{x}) \) is the scalar function that represents the boundaries of \( \bar{D} \). The principal guideline of the optimization process is to move the design boundary over the solid domain, until the objective function, subjected to the constraints, is met.

\[\text{Figure 3.16. Visualization of the factors in the level-set method: the design domain } D, \text{ the reference domain } \bar{D}, \text{ the level-set function } \phi \text{ and the level-set model } S [52].\]
The level-set function is then given by [50]:

\[
\begin{cases}
\phi(x) < 0 & \text{if } x \in D \\
\phi(x) = 0 & \text{if } x \in \partial D \\
\phi(x) > 0 & \text{if } x \notin D
\end{cases}
\]

In which \( x \) is any point in the given design domain.

The level set function is updated by a generalized Hamilton-Jacobi equation:

\[
\frac{\partial \phi}{\partial t} = -v|\nabla \phi| - \omega g
\]

In which:
- \( t \) is the time.
- \( v(x) \) is a scalar field over the design domain, the speed function.
- \( g(x) \) is a scalar field over the design domain, forces new holes to be created in \( \bar{D} \).
- \( \omega \) is a positive parameter that determines the influence of \( g \).

In figure 3.17, three different initial designs are chosen with holes to be used for the level-set topology optimization. It can be clearly seen here that the level-set function evolves over the reference domain \( \bar{D} \), therefore evolving the designed domain \( D \) as explained with figure 3.16. The boundaries of the holes and the domain are changing and growing. Once interior boundaries meet, they merge into a new hole and create a new design with different boundaries and a different domain. This is the way the level-set method can function as a shape optimization combined with topology optimization, until the final result is obtained. By adding more holes in the initial design, the optimization process will converge faster towards the final solution [52]. For a full review on the level-set method and its application and implementation, the reader is referred to [50].
3.3.4. Comparison of the topology optimization methods

Now that the basic theories of the most common optimization theories have been explained and that multiple scripts have been used and experimented with [3], [12], [28] and [42], a distinction must be made between all the theories, to see which is best suitable for now for use with 3D concrete printing and which is able to adapt with the manufacturing constraints that are required to take into account during the optimization process for 3D concrete printing.

BESO and SIMP seem to be very relatable at this point, since the original ESO method has been adapted in such a way, that it now uses quite similar functions as the SIMP method already used. It also determines sensitivity numbers and does the same kind of sensitivity analysis with a certain filter radius. There are however some essential differences still, which lead to different solutions for the same design spaces that are used. This comes from the fact that the SIMP method is a gradient based method, while the BESO method is only used as a hard-kill or soft-kill method. A gradient based method like SIMP, relates a relative value for the density to each element with a range of:

\[
0 < \rho \leq 1
\]

BESO however, as a hard-kill or soft-kill method, gives a sensitivity number to an element for simple absence, a void element, or presence, a solid element. This means that:

\[
x_i = 0 \text{ or } 1 \quad \text{(Hard-kill method)}
\]

\[
x_i = x_{\text{min}} \text{ or } 1 \quad \text{(Soft-kill method, e.g. } x_{\text{min}} = 0,001)
\]

The hard-kill method of BESO removes elements in a way that their values will become zero. This also means that the information that they give to the whole optimization process, in terms of for example historical information, vanishes [28]. The soft-kill method still gives a sensitivity number higher than zero to a void element, therefore still maintaining the element in the iterative process and the historical information that it provides and requires.

The BESO procedure for these same reasons does always come with a black and white, or 0-1, solution, which could be beneficial when the boundaries are important to be determined. As mentioned, the SIMP model is a gradient-based method, therefore creating gray boundaries, which makes it hard to distinguishes what the exact model should be when the designers wants to manufacture it, as shown in figure 3.18. This problem can be tackled however by using the Heaviside filter [3].

Another thing to keep in mind is that the BESO method and the SIMP method do not always come to the same solutions, due to the minor difference in formulation. In figure 3.18. this difference can be seen. Both designs are taken from a 120 x 40 mesh with a volume constraint of 50% and optimized towards maximum stiffness. The filter radius for both designs is taken as 3,5 as well. The only difference in formulation is that the BESO method works with an evolutionary rate, while the SIMP method works with a penalization factor. The MATLAB codes for these models can be found in Appendix B, C and D.

![Figure 3.18. Comparison between the final results of the soft-kill method of BESO and the SIMP method.](image)
Another difference lies in the iterative process. The BESO method evolves from a so called initial full design, while the SIMP method starts optimizing from an initial guess design [28]. This means that the BESO method uses its Evolutionary Rate, ER, to gradually add or remove elements to find an optimized solution and to work towards the volume constraint that is set in the topology optimization problem. Eventually material will be removed during this process, because of this volume constraint. This also means that the stiffness of the structure will reduce, and therefore the compliance of the structure will increase gradually as well. It should be emphasized that the occasional jumps in the compliance graph come from the fact that it is not a gradient based method. Sudden changes of elements changing from solid to void and vice versa are therefore inevitable. The iterative process stops, once the mean compliance reaches a value that no longer changes anymore.

The SIMP method optimizes from an initial guess design. This means that the iterative process starts from a design that already meets the requirements for the constraints, in these cases the volume constraint. During the rest of this procedure, it will find the optimized solution by changing the design constantly, until the final design is found that has the minimum value for the compliance, while it is still subjected to the constraints.

Figure 3.19. The initial design domain (left) and the final result (right) of the SIMP method on a full MBB-beam, performed in Abaqus 6.14.

Figure 3.20. Graph of compliance and the volume fraction in the iterative process of the SIMP method.
The problem with the BESO method is the lack of proper convergence, due to the difference in stopping criteria. The algorithm stops, when the difference in in the objective function of the last 6 to 10 iterations and the last five iterations becomes smaller than the stop criterion. But this has nothing to do with convergence, since it could just as well show that the algorithm has run into an oscillating state. It could also happen that the algorithms already stops right after the volume constraint is met, since the averages of the objective are not fluctuating anymore. Another problem with this method, is the lack of implementation of multiple non-linear constraints, which could be possible for the SIMP method [46]. BESO does require less computational time and requires less iterations to come to the final design, the question is then if this is the proper final design or not. [28]
Researches into topology optimization have shown that the main theories that are used are SIMP, BESO, but since recently also the level-set method. It has already become clear that the BESO method has evolved from the ESO method in such a way, that it resembles the SIMP method quite well. But the level-set method makes use of the same information as these other methods, like filtering techniques, handling sensitivity information and mesh-independency. The difference within these techniques is the way they converge towards the final solution. As explained previously, the BESO method starts from an initial full design and gradually sets elements to being void, until the volume constraint is met. The SIMP method locates a proper initial guess design with gray elements, after which it optimizes towards minimum compliance. The level-set method uses only solid and void elements and changes the shape of the boundaries of the design domain, until proper final design is acquired. This however comes with a computational cost, compared to the SIMP method [46].

Compared to the density optimization methods, the level-set method requires additional development for it to be beneficial over the SIMP model. Some particular problems, like efficiency in terms of numbers of iteration until convergence, mesh-dependency and finding global optimum still need to be addressed. Although this is partly done in [50], it still requires mathematical programming methods for further development [46].
For further use of topology optimization within this project, the SIMP method has the preference over the other theories, even though it is a gradient-based topology optimization method, which seems to be hard to use for manufacturing processes. It is chosen for the following reasons [40]:

- It is the most common method to be used, not only in papers, but especially in commercial software like Tosca and Altair Optistruct.

- The gradient-based optimization technique provides more precise final results, even with a coarser mesh compared to non-gradient based methods. The gradients can also still be modified to black and white solution by means of post-processing or application of other interpolation schemes like the Heaviside function.

- The publicly available codes for SIMP converge just as fast or even fast then the other publicly available codes that consider other topology optimization methods.

- More additional alterations are at this point performed on the publicly available codes of the SIMP method, than on any other method. So more practical examples can be found for research, then instead inventing the method on your own again.
4. Topology optimization and concrete

Within this research, topology optimization has been considered for an isotropic linear elastic material. The only material properties that were considered until now, are the Young’s modulus and the Poisson’s ratio. This also means that there were, for example, no stress limits involved. Since the material that is to be used is concrete, new challenges arise in the implementation of its material properties within the topology optimization algorithms. As not only the material properties are important to consider, but also the manufacturing techniques to actually construct the final designs, the final results of the topology optimization process will change considerably and will be more relatable to the material concrete.

4.1. Casting constraints

The conventional manufacturing process related to concrete is casting. A formwork is required to shape the fresh concrete in such a way that the desired design can be created with this material, once it has hardened. But this formwork also needs to be removed. Figure 4.1 visualizes the removal of formwork for concrete. In this example the mold consists of two equal parts, which enwrap the casted concrete part. The removal of the molds is then performed outward from the mid plane of the concrete object. This process can only be done when the molds can freely be removed and if there is no under cut in the object. As the example on the right shows, the bottom mold cannot be removed, due to this under cut.

Another thing to consider in practice, is the pull angle that is required when molds are removed [49]. This can be seen in figure 4.2. If this pull angle is not taken into account, it is still hard to remove the mold, since the casted concrete sticks to the molds. In combination with the friction over the maximized contact surface between concrete and mold, this makes it hard to remove it. If the pull angle is considered however, the amount of space for the mold to be removed is increased, resulting in an easier process.
These casting constraints are useful and required to ensure that the concrete object is manufacturable. Topology optimization algorithms do not consider these manufacturing methods in the basic algorithm. This usually means that for an optimization problem formulation that optimizes towards maximum stiffness with a volume constraint the object becomes hollow and with undercuts. Hollow object obviously give the stiffest result since the material is then used at the outer parts of the design domain, resulting in a higher moment of inertia of the structure and therefore a higher stiffness. However, for casting, this means that there is a large undercut in the model, since the inner mold to create a hollow object cannot be removed easily.

Implementation in the topology optimization formulation is therefore required to create objects that can be casted and of which the molds can be removed after hardening. This can be introduced with a new constraint that considers the densities in the mesh of the design space and the parting direction of the mold. The object must first be discretized in elements with specific densities. The constraint must then check whether no internal voids or undercuts are present in the parting direction. This means that every next element in the parting direction must have a lower density than the previously considered element, resulting in rows of elements with the maximum value for the densities of the first element. The formulation of this constraint can be shown as follows [37]:

$$\rho_i \leq \rho_{i+1} \leq \rho_{i+2} \leq \cdots \leq \rho_{i+n}$$

In which:

- $\rho_i$ is the first element with a density value, considering the parting plane and direction.
- $\rho_{i+n}$ is every next element in the same parting direction and line of elements in that direction.

![Figure 4.2. The removal of the mold, while the pull angle is taken into account [49].](image)

![Figure 3.3. Standard design space discretization (left) and design space discretization with a casting constraint in the optimization problem (right) [19].](image)
An example has been carried out in the optimization module in Abaqus 6.14 and is shown in figure 4.4. This example is done with this software package, since the results are more logical to comprehend in a 3D visualization and the casting constraint can easily be implemented in the optimization task. A 3D beam of 150x50x50 elements has been optimized towards maximum stiffness and a volume constraint. The left surface of the beam is fixed and a line load is introduced at the top right edge of the beam. The top frames of figure 4.4. show the final result of a standard SIMP method. It is clear that the outcome is a hollow beam and this is logical considering the topology optimization problem formulation. Maximum stiffness can be achieved by using relatively the most material at the outer edges of an object to increase the moment of inertia and therefore the stiffness.

However, the bottom frame shows the final result of an equal model and optimization task with a casting constraint introduced. The mid-plane over the length direction of the model is considered and two molds are removed in z-direction. The result clearly shows an optimized object with rows of elements or rows of voids in z-direction, resulting in an object that is manufacturable by casting. Two molds can envelop this object and can be freely removed. This is how the internal voids can still be made in concrete, while the structure is optimized.

Figure 4.4. Topology optimization performed in Abaqus 6.14. on a beam, fixed on the left side with a line load on the top right edge, without casting constraint (top) and with casting constraint (bottom).
4.2. Orthotropic material properties

The material concrete that is considered until now is normally having homogeneous material properties and it is therefore an isotropic material in that sense. This is also what is considered until now for all the optimization models that are run for previous results in this report. The fact that concrete will be reinforced, means that the isotropic behavior of the material will disappear, and in a way it can become orthotropic with different material properties in x-direction and in y-direction. Also for 3D printed concrete, the bond strength between layers weakens the total structure. The strength of a single layer is therefore different than the strength of a layered material. This means that for a layered material as well, the strength will be different in x-direction than it will be in y-direction.

The SIMP method for topology optimization has been chosen to be used for this project, but SIMP stands for Solid Isotropic Material with Penalization. The MATLAB codes from [3] and [42] therefore only consider isotropic material. The calculations to form isotropic material are done in the formulation of the element stiffness matrix. This formulation has been shortened and only the final result has been implemented in the scripts for bilinear 4-node square elements. To change the script for material with orthotropic materials, the part of the formulation of the element stiffness matrix had to be rewritten. The element stiffness matrix for bilinear quadrilateral elements results from the following double integration [15] and [31]:

$$k_0 = t \int_{-1}^{1} \int_{-1}^{1} B^T DB \, d\xi d\eta$$

In which:

- $t$ is the thickness of the element. Since it is done in 2D, $t = 1$.
- $D$ is the constitutive relation given by:

$$D = \frac{E}{1-\nu^2} \begin{bmatrix} 1-\nu & \nu & 0 \\ \nu & 1-\nu & 0 \\ 0 & 0 & \frac{1-2\nu}{2} \end{bmatrix}$$

In which:

- $E$ is the Young’s modulus of the isotropic material.
- $\nu$ is the Poisson’s ratio of the isotropic material.

$B$ is the strain-displacement matrix:

$$B = \frac{1}{j} \begin{bmatrix} B_1 & B_2 & B_3 & B_4 \end{bmatrix}$$
In which:

\[
B_i = \begin{bmatrix}
\frac{\partial N_i}{\partial \xi} - b \frac{\partial N_i}{\partial \eta} & 0 \\
0 & c \frac{\partial N_i}{\partial \eta} - d \frac{\partial N_i}{\partial \xi} \\
c \frac{\partial N_i}{\partial \eta} - d \frac{\partial N_i}{\partial \xi} & a \frac{\partial N_i}{\partial \xi} - b \frac{\partial N_i}{\partial \eta}
\end{bmatrix}
\]

The parameters \(a, b, c\) and \(d\) are given by:

\[
a = \frac{1}{4} \left[ y_1(\xi - 1) + y_2(-1 - \xi) + y_3(1 + \xi) + y_4(1 - \xi) \right]
\]

\[
b = \frac{1}{4} \left[ y_1(\eta - 1) + y_2(1 - \eta) + y_3(1 + \eta) + y_4(-1 - \eta) \right]
\]

\[
c = \frac{1}{4} \left[ x_1(\eta - 1) + x_2(1 - \eta) + x_3(1 + \eta) + x_4(-1 - \eta) \right]
\]

\[
d = \frac{1}{4} \left[ x_1(\xi - 1) + x_2(-1 - \xi) + x_3(1 + \xi) + x_4(1 - \xi) \right]
\]

And the shape derivatives of \(N_i\) are given by:

\[
N_1 = \frac{1}{4} (1 - \xi)(1 - \eta)
\]

\[
N_2 = \frac{1}{4} (1 + \xi)(1 - \eta)
\]

\[
N_3 = \frac{1}{4} (1 + \xi)(1 + \eta)
\]

\[
N_4 = \frac{1}{4} (1 - \xi)(1 + \eta)
\]

\(J\) is the determinant given by:

\[
J = \frac{1}{8} \left[ x_1 \ x_2 \ x_3 \ x_4 \right] \begin{bmatrix}
0 & 1 - \eta & \eta - \xi & \xi - 1 \\
\eta - 1 & 0 & \xi + 1 & -\xi - \eta \\
\xi - \eta & -\xi - 1 & 0 & \eta + 1 \\
1 - \xi & \xi + \eta & -\eta - 1 & 0
\end{bmatrix} \begin{bmatrix}
y_1 \\
y_2 \\
y_3 \\
y_4
\end{bmatrix}
\]

According to Hoglund and Smith [26] the SOMP method [1], Solid Orthotropic Material with Penalization, can be obtained by altering the constitutive relation for plane stress to:

\[
D = \begin{bmatrix}
E_x & \nu_{yx}E_x & 0 \\
\nu_{yx}E_x & \frac{1 - \nu_{yy}}{\nu_{yx}}E_x & 0 \\
\nu_{yx}E_x & \frac{1 - \nu_{yy}}{\nu_{yx}}E_x & \frac{E_x}{2(1 + \nu_{xy})}
\end{bmatrix}
\]

In which:

\(E_x\) is the Young’s modulus of the material in x-direction.

\(E_y\) is the Young’s modulus of the material in y-direction.

\(\nu_{xy}\) is the Poisson’s ratio of the material in x-y plane.

\(\nu_{yx}\) is the Poisson’s ratio of the material in y-x plane.
Due to the required symmetry of the compliance matrix, the following relationship needs to be taken into account:

\[
\frac{E_x}{\nu_{yx}} = \frac{E_y}{\nu_{xy}}
\]

The direction of the orthogonal plane can also be rotated by adding the following matrix to the formulation that determines the element stiffness matrix:

\[
R = \begin{bmatrix}
\cos \theta & -\sin \theta & 0 \\
\sin \theta & \cos \theta & 0 \\
0 & 0 & 1
\end{bmatrix}
\]

The formulation for the element stiffness matrix for orthotropic material with the orthogonal plane rotated over an angle then changes to:

\[
k_0 = t \int_{-1}^{1} \int_{-1}^{1} B^T R D R^T B J d\xi d\eta
\]

With these last alterations in the formulation of the element stiffness matrix in the 99 line MATLAB code of Sigmund [42], simple two-dimensional topology optimization can be performed for orthotropic materials. It should be stated that the elastic orthotropic material properties and the according material orientation are defined before starting the topology optimization procedure and that the process does not optimize towards the ideal material orientation.

In figure 4.6, the effect is shown when the material has a higher Young’s modulus in y-direction. The first model is the final result for the isotropic case, with a mesh of 120 x 40 elements. The objective of the optimization problem is stated as minimization of the compliance, subjected to a volume constraint of 50%. The penalty factor is taken as \( p = 3 \) and the filter radius is taken as \( r_{min} = 1.5 \). This filter radius is taken to obtain a benchmark result with more members in the structure of the final result, so that the difference due to the influence of the orthotropic material becomes more clear. For each new optimization model, the final result is shown with an increase of 1,0 for the Young’s modulus in y-direction. It can be seen that the members of the structures in the final results become more vertical, since the material becomes stiffer in that direction and therefore the compliance will become less, compared to the previous optimization result. This can also be seen in the graph in figure 4.7.
Figure 4.6. The final results of optimized MBB-beams with different values for the Young’s modulus in x and in y-direction, with the ratio of 1,0 (top left) decreased until a value of 0,1 (bottom right).

Figure 4.7. Graph of the ratio of the Young’s moduli and the compliance.
The opposite happens when the Young’s modulus in x-direction is increased for each individual optimization problem, as can be seen in figure 4.8. The benchmark test remains the same and isotropic material properties are applied for this model. After that model, every other model has an increase of the Young’s modulus in x-direction with a value of 1.0. Since every next individual optimized result becomes stiffer in x-direction, the members of the global structure will be orientated more in horizontal direction.

Figure 4.8. The final results of optimized MBB-beams with different values for the Young’s modulus in x and in y-direction, with the ratio of 1.0 (top left) increased until a value of 10.0 (bottom right).

Figure 4.9. Graph of the ratio of the Young’s moduli and the compliance.
In the graph of figure 4.9, it can be seen that the compliance reduces for orthotropic material with a higher Young’s modulus in y-direction and a higher Young’s modulus in x-direction until the ratio of the Young’s moduli becomes 6,0, therefore making the global structure stiffer compared to the same structure with isotropic material properties. When the value for this ratio increases, due to an increase for the value in x-direction, the compliance will increase again, resulting in less stiff structures. This can be explained by the horizontal orientation of the material and the total structure in the design domain. Most of the structural members and all of the material itself are working mostly in x-direction, which is not beneficial for the stiffness of the structure for this model.

![Graph of the ratio of the Young’s moduli and the compliance.](image)

Within the formulation for orthotropic material properties, the orientation of the orthogonal plane can be rotated as well. The effect on the final result of the optimized beams can be seen in figure 4.11. The optimized beams have a ratio of the Young’s moduli of 0,2 and the orthogonal plane is rotated 0 degrees, 45 degrees, 90 degrees and 135 degrees. The final results show that the orientation of the structural members change along the rotation of the orthogonal plane. It can be used for reinforced concrete, if the orientation of the reinforcement has a specific direction. With this optimization algorithm it is possible to take this orientation into account and also optimize with material characteristics that are different from the conventional plane.

![Optimized final results of the MBB-beam with a ratio of the Young’s moduli of 0,2, rotated 0 degrees (top left), 45 degrees (top right, 90 degrees (bottom left) and 135 degrees (bottom right).](image)
4.3. Stress constraints

Topology optimization with stress constraints has a different optimization problem than the traditional problem for compliance minimization, subjected to a volume constraint. Stress constrained topology optimization, the problem can be formulated as follows from [59] and [60]:

\[
\begin{align*}
\min_{\rho} & \quad \sum_{e=1}^{N} \rho_e v_e \\
\text{Subject to} & \quad KU = F \\
& \quad \sigma_e \leq \sigma_L \\
& \quad 0 < \rho_{\text{min}} \leq \rho_e \leq 1
\end{align*}
\]

In which:

- \(N\) is the number of elements.
- \(\rho_e\) is the elemental density.
- \(v_e\) is the elemental volume.
- \(K\) is the global stiffness matrix.
- \(U\) is the displacement vector.
- \(F\) is the force vector.
- \(\sigma_e\) is the elemental stress.
- \(\sigma_L\) is the stress limit.

This problem formulation shows that the optimization task is performed to minimize the amount of elements with a relative density value, and therefore minimize the total volume or mass of the structure. The structure is subjected to the standard equilibrium equations, as well as a stress constraint for every individual element. This elemental stress is required to be equal or less than the stress limit, which is mostly given as the Von Mises stress:

\[
\sigma_{VM} = \sqrt{\sigma_x^2 + \sigma_y^2 - \sigma_x\sigma_y + 3\sigma_{xy}^2}
\]

In which the stresses are given with the stress tensor by:

\[
\sigma = \begin{pmatrix}
\sigma_x \\
\sigma_y \\
\sigma_{xy}
\end{pmatrix}
\]

And it is obtained for linear isotropic material properties for bilinear quadrilateral elements by:

\[
\sigma = DBu
\]

The code in [8] is used to optimize structures for the Von Mises stress criterion. This method is called the Proportional Topology Optimization method. It is similar to the BESO method, since it removes or adds gradient values to the elements for the relative densities in order to obtain the optimized result. For every iteration, the elemental Von Mises stress is checked and compared to the stress limit. Therefore the updating criteria is set to add or remove elemental volume, until the stop criterion is met and the structure is optimized for a minimum amount of material, related to the stress limit of the material. For every iterative step, the material amount is updated with \(0.001 \times N\). Subsequently, the new amount of material is distributed in a proportional matter to the elemental stress values by:
$\rho_e^{\text{opt}} = \frac{v_R}{\sum \sigma_e^q} \sigma_e^q$

In which:

- $\rho_e^{\text{opt}}$ is the optimized elemental density.
- $v_R$ is the remaining elemental volume to be distributed over the elements, because of the iterative updating algorithm for the material in the structure.
- $q$ is the proportion exponent, usually set to a value of 2 for a stress problem formulation.

The final step of the updating scheme is to update the elemental densities with the new values, also taking historical information into account, by the following formulation:

$$\rho_e^{\text{new}} = \alpha \rho_e^{\text{prev}} + (1 - \alpha) \rho_e^{\text{opt}}$$

In which:

- $\rho_e^{\text{new}}$ is the new elemental density.
- $\rho_e^{\text{prev}}$ is the elemental density in the previous iterative step.
- $\alpha$ is the history coefficient that checks multiple previous elemental densities, usually set to a value of 0 for a stress problem formulation.

The procedure to finding the optimized final result, regarding stress constrained topology optimization, is done by first optimizing the model for compliance minimization in order to obtain the minimum Von Mises stress and to subsequently use it as input for the compliance constraint in the stress constrained topology optimization problem. In the example in figure 4.12. the final results are shown for both optimization problems and the output of the compliance minimization is used as input for the volume minimization. The design space also consists of 120x40 elements, with equal boundary conditions and loads as previously used in this research.

The Von Mises stress limit was set to 0.92 and the volume fraction in the stress constrained topology optimization was reduced from 50% to 43%, therefore increasing the compliance from 204 to 230. The topology of both structures is similar, with a slight difference in the structural members.
The Von Mises yield criterion is used to determine when a material starts to yield when the critical value of the summation of stresses in multiple dimensions is reached. However, this criterion is mostly used for ductile materials or materials with plastic deformation. Concrete is a brittle material, which means that there is hardly any plastic deformation. It also has a high compressive strength, in relation to its tensile strength. These material properties state that the Von Mises criterion is not properly useful when the material concrete is taken into account.

The stress criterion should therefore be changed from the Von Mises stress criterion to the Drucker-Prager stress criterion. According to Drucker and Prager [61] the equivalent stress can then be written as [62]:

$$
\sigma^{eq} = \frac{s + 1}{2s} \sqrt{3J_{2D}} + \frac{s - 1}{2s} J_1
$$

In which:

- $J_1$ is the first invariant of the stress tensor:
  $$
  J_1 = \sigma_{11} + \sigma_{22}
  $$

- $J_{2D}$ is the second invariant of the stress deviator:
  $$
  3J_{2D} = \sigma_{11}^2 + \sigma_{22}^2 - \sigma_{11}\sigma_{22} + 3\sigma_{12}^2
  $$

- $s$ is the ratio between the compressive strength and the tensile strength:
  $$
  s = \frac{\sigma_{LC}}{\sigma_{Lt}}
  $$

The effect of the ratio in the criterion is shown in figure 4.13. For $s = 1$ the criterion is equal to the Von Mises stress criterion. It changes however when $s \neq 1$, since it then results in material properties with unequal behavior in compression and tension.

![Figure 4.13. Drucker-Prager failure functions for several ratios between the compressive stress limit and tensile stress limit [61].](image-url)
5. Topology optimization and 3D printed concrete

With the theory explained of implementing some properties of the material concrete and traditional manufacturing techniques that are used for casting concrete, it is now important to see the possibilities of 3D Concrete Printing and how the boundary conditions of this manufacturing technique can be used in combination with topology optimization. As chapter 2.4 already states, some important factors to take into account when considering 3D Concrete Printing are the angles, the layer dimensions and the use of material, whether it will be used as structural material or support material. All these factors are significantly influencing the final results of topology optimization, but they can be taken into account during the optimization process.

5.1. The effect of nozzles on the concrete layers

Every structure created by 3D Concrete Printing is constructed of a layer-wise built up of concrete layers. The shape of these layers have a significant influence on the total structure, since the dimensions and the bond strength of the structure are directly related to the dimensions of the layers, which are extruded from the nozzle. These dimensions determine what the contact surface between layers is, as well as the width and height of a layer that influence the stability of the structure.

This is why the last year there has been experimented with several layer dimensions and shapes with the use of aluminum and 3D printed plastic nozzles. Figure 5.1. shows several examples of nozzles and the influence on the shape of a layer. The 3D Concrete Printing project started off with an aluminum nozzle that extruded concrete layers with a circular cross-section, but this was not beneficial, due to the small contact surface between layers. This is why the nozzles have been further developed for being able to create layers with square or rectangular cross-sections. The nozzles that extrudes layers with a rectangular cross-section is used now. The high surface area to height ratio, gives more stability but slower built up of the total structure. This slower built up, due to the reduced layer height, is beneficial for the hardening time of the concrete. Combined with the higher contact surface, these properties make the printing process more controlled because of better stability properties.

![Figure 4.1. Two different nozzles used for 3D Concrete Printing. A nozzle that creates square layers (left) and a nozzle that creates rectangular layers (right).]
But the disadvantage of these fixed nozzles, and therefore fixed layer dimensions during the printing process, is that a certain minimum construction thickness has to be taken into accounted, while designing the products, which are to be printed as can be seen in figure 5.2. This can be done with the filtering techniques discussed in chapter 3.3.1.3. Especially the Heaviside filter can be of use to obtain a final result that considers the manufacturing constraints of 3D Concrete Printing.

Figure 5.2. The fixed layer dimensions during the printing process.

The Heaviside filter, as suggested by Guest [23], solves two issues considering manufacturability, which results obtain black and white solutions and it imposes a minimum length scale through a projection method. The resemblance of projection of the minimum length scale on the elements with the use of a filter radius with density or sensitivity filtering is that it makes the structure mesh-independent. As seen in figure 5.3. the value for the filter radius is independent on the mesh, as the value needs to be altered in the case of mesh refinement when the designer wants to obtain an equal result. However, mesh refinement is important for the boundaries of the final design, as the transitions from black to white will eventually vanish due to the preciseness in the mesh. This does come with a computational cost.

Figure 5.3. Final result of topology optimization of a coarse mesh with density filter (left) and of a fine mesh with Heaviside filter (right).
With this filter a method to consider certain minimum thickness has been introduced with clear boundaries, but it does not consider maximum thickness of the structural members of the final result. This means that during the printing process, thicker structural members need to be obtained by coupling newly printed paths with the already printed ones. The coordinates of the newly printed layers are required to be offset a certain distance from already printed layers, therefore creating an overlap and resulting in thicker members. This can be seen in figure 5.4. The lower path in the figure has a certain overlap with the upper path, as the center to center distance of the two paths is less than the width of one layer. Because 3D Concrete Printing works with concrete in a fresh state, it is able to merge the two layers into one, resulting in wider structural members. This does mean that excess waste material will form on the outside of the nozzle and this should at some point be removed during the printing process to prevent it from making the print less fine.

*Figure 5.4. A close up view of the overlap of concrete layers.*
5.2. Angle and overhang constraints

Concrete is a material with a relative high density and with a low strength capacity directly after it is extruded from the nozzle. This means that the overhang of the material becomes minimal and therefore it is almost impossible to print structures with an angle in the global structure. Figure 5.5 shows a sketch of a situation, in which the maximum overhang is reached and shows what the angle then can be of the total structure, but this situation has already been exaggerated when compared to 3D Concrete Printing. However, in this situation the total geometry has not been taken into account, since in this situation only straight lines on top of each other have been printed with a slight increase in printing length. When the geometry of the total structure is taken into account, structures with other angles are possible, as can be seen in figure 5.5. In this situation, the weight and bond strength of the previously printed concrete within the same layer makes it possible that the overhang of the structure can become larger and higher, so that the angle that is possible for 3D Concrete Printing will become sharper or a structure with the same angle can become higher.

If a situation is reached, in which the structure is starting to tilt and collapse, the angle should be reversed, so that the self-weight of the structure corrects the eccentricity caused by its own weight. With this process it is then also possible to print higher structures that will not collapse during the printing process, due to the low strength properties of the fresh concrete. This effect can be seen in figure 5.5. After a certain construction height is reached with the initial angle, the rotation of the total geometry is reversed, therefore creating an angle in the opposite direction and correcting the instability with it the self-weight of the newly printed material.

Figure 5.5. Overhang constraint in 3D Concrete Printing. A visualization of the constraint (top left), a printed structure with overhang (top right), a geometry required for increased overhangs (bottom left) and a close up of the overhang of printed structures (bottom right).
Research has already been performed to combine topology optimization with additive manufacturing. However, the printing techniques considered here are different from 3D Concrete Printing, since they mostly consider additive manufacturing techniques that use plastics as the structural material and within these processes a support material is mostly applied as well. These manufacturing techniques still come with certain constraints in terms of the possible angle in which a structure can be printed.

A method to consider the angle constraint is proposed by Leary [34] for combining topology optimization with Fused Deposition Modeling. This is a manufacturing process that also involves the layer-wise built of structures from bottom to top with an overhang constraint, therefore also creating an angle constraint. This angle constraint is determined first and then implemented in the optimization algorithm. The optimization process then runs normally, but after post-processing the critical members of the structure are determined in terms of which members need additional support material. Consequently, additional support material is added to the structure, until the maximum allowed overhang constraints and angles are met and the structure is considered to be manufacturable for Fused Deposition Modeling.

However, if the support material is added afterwards in the post-processing step when 3D Concrete Printing is considered, the structure will be filled with support material, due to the angle and overhang constraints that result in more infeasible members in the optimized structure. For every added member of support material, another member is required to bear the weight of that member and so on. This means that eventually a fully solid structure will be created as the final design when this technique is used for 3D Concrete Printing. This process can however be used, once the 3D concrete printer is able to print multiple materials. This way a different kind of material can be used to bear the structural concrete that has more beneficial material properties for when this technique is considered.

Another method can be used as proposed by Gaynor [18], who combines topology optimization with Selective Laser Melting as the manufacturing technique that also makes used of an upwards, layer-wise built up. This technique has a maximum overhang angle of 45°, which needs to be taken into account in the topology optimization process if the final design will be made with Selective Laser Melting. In this research, the angle constraint is imposed through a projection method, instead of taking the constraint as a design variable in the optimization problem formulation. It therefore creates final results without the use of support material, by considering the boundary conditions of the used structural material. This is also done with the Heaviside Projection Method, like it is done in the theory in chapter 3 to create black and white solution, but another factor is added in the chain rule of the sensitivity analysis. This means that within the optimization process, for every iterative step, it is checked whether these

![Figure 5.6. Method to use additive manufacturing as manufacturing technique for optimized objects. The infeasible overhangs will be supported by additionally added material in the post-processing step [34].](image)
elements may exist or not considering the angle constraint, whether the black elements are properly supported or not. If this is not the case, then a new design will be created, that eventually fulfills the objective and constraints of the topology optimization problem.

This technique seems to be more feasible for 3D Concrete Printing, since the exact angle constraint can be implemented and is taken into account during the optimization process. However, the fact that the material and 3D Concrete Printing as the manufacturing process, still have an angle constraint with an angle of almost $90^\circ$, it would still mean that almost the full design domain will be used as structural material with black elements. What could also happen is that the volume constraint cannot be met and that the optimization process will not converge to a proper final solution. So this technique can only be feasible if a new material is introduced within the printing process with which structures can be built with a sharper angle constraint than with the material that is applicable at this point. Also, if support material will be used within the printing process, the angle constraint could change to a solution that is more beneficial, but this support material is not taken into account within this theory and will only be applied to bear the weight of the structural material.

With the use of these researches, it can be possible to implement an angle or overhang constraint in the topology optimization problem formulation. However, for 3D Concrete Printing it seems unlikely to do this at this point, since the angle constraint will be almost $90^\circ$, dependent on the total geometry of the printed structure. Therefore a support material will be required to eventually being able to create optimized objects with this manufacturing technique. A new method will be explained in the next chapter to obtain optimized concrete printed structures that result from considering two material structures within the optimization process. This way, the angle constraint can be put aside for now and optimized structures can still be made with this manufacturing technique.
5.3. 3D Concrete Printing with support material

At this point it is only possible to use one material within the 3D Concrete Printing process and therefore it does not seem logical to consider the use of two or even more materials for this manufacturing process. However, it is possible to change the settings of the print paths or the types of nozzles in a smart way, resulting in different designs than the standard layer-wise built up that is usually done. With the application of these techniques, a new type of material can be introduced that is still made of the same concrete, but it is then applied in a different way with the objective to still obtain a reduction in weight and material usage. This can be translated to a topology optimization problem with the objective to achieve minimum compliance subjected to a certain volume, or weight, reduction as the constraint.

In order to optimize structures considering the use of two materials, the 88 line MATLAB code of Andreassen [3], which can be seen in appendix C, can be used to optimize a design domain in which two materials are used. The final result will then exist of a structural material, the black elements, and a support material, consisting of the white elements. This means that the same interpolation scheme can be used, which is formulated as:

\[
E(\rho) = E_{\text{min}} + \rho^p (E_0 - E_{\text{min}})
\]

And rewritten as:

\[
E(\rho) = \rho^p E_0 + (1 - \rho^p)E_{\text{min}}
\]

In which:

- \(E_0\) is the Young’s modulus of the solid material.
- \(E_{\text{min}}\) is the Young’s modulus of the voids.

According to Bendsøe [6], this interpolation scheme can also be used for topology optimization with two materials when the following is considered:

- \(E_0\) is the Young’s modulus of the structural material.
- \(E_{\text{min}}\) is the Young’s modulus of the support material.

For which the following relation should always be taken into account:

\[
E_0 \geq E_{\text{min}}
\]

This still means that the total volume of the structure is a summation of the densities of the structural material and the support material and is still formulated as:

\[
V = \sum_{\Omega} \rho(x) d\Omega
\]

In figure 5.8, it can be seen what the effect is in the final results of the topology optimization algorithms. First the benchmark test is given again, while the other final results are given with different values for the Young’s modulus of the support material, \(E_{\text{min}}\). The graph in figure 5.9 gives the difference in compliance of the final models. Again, it should be emphasized that the black elements are considered to be structural material with a stiffness \(E_0\) and the white elements are considered to be support material with a stiffness \(E_{\text{min}}\).
The final results all show where structural material is required and where the secondary support material is required. It is clear here as well that the stiffest material is distributed at the elements where in reality the bending moment or displacement will be the highest and at the supports where material is required to resist the shear forces. The graph in figure 5.9, therefore also shows the expected output. The total structure will have a decrease in compliance, meaning it becomes stiffer, when the support material becomes stiffer related to the structural material.
From the theory on topology optimization explained in chapter 3, a penalty factor is introduced in the interpolation scheme to obtain solutions for the final designs with mostly black and white elements. This factor is used to obtain solutions that are manufacturable in practice, since used material only has the full density and not a fraction of it. However, considering the possibilities for 3D Concrete Printing, it is possible to manufacture objects in a way that results in structures with different densities, in relation to according printer settings. In several researches like [28], a cellular material is introduced to use within the topology optimization algorithm, as can be seen in figure 5.10. It can be used as a support material for optimized concrete structures, as the necessity is explained in the previous chapter.

A similar approach can be used with 3D Concrete Printing, which will be explained in chapter 6. The secondary structure can achieve multiple densities, since the physical densities can be varied. This means that gradient structures can be maintained as final results and that the penalty factor or Heaviside function is no longer required. Figure 5.11. shows the final results of structures with stiffness for the structural material with $E_0 = 1$ and stiffness for the support materials with $E_{\text{min}} = 0.001$, $E_{\text{min}} = 0.1$, $E_{\text{min}} = 0.25$, $E_{\text{min}} = 0.5$, $E_{\text{min}} = 0.75$ and $E_{\text{min}} = 0.9$ respectively. Every final result also had an input for the penalty factor of $p = 1$. Compared to the final results of the optimized structures with a penalty factor of $p = 3$, these structures are gradient like, while the previous results showed more black and white solutions, as expected due to the penalty factor.

So in order to obtain optimized structures the support material with related densities and stiffness need to be developed for 3D Concrete Printing. This will allow for manufacturing of concrete structures with an optimized material distribution, according to the material structures that can be created with this technique. It should be emphasized at this point that it is only possibly to create continuous structures with the same concrete as material for every element. Several proof of concepts for this theory will be explained in chapter 6.
Figure 5.11. Final results of the topology optimization with two materials of nonzero stiffness with a penalty factor of \( p = 1 \).
The benchmark test is given with \( E_{\text{min}} = 0.001 \) and this value is increased to \( E_{\text{min}} = 0.1 \) (top right), \( E_{\text{min}} = 0.25 \) (middle left), \( E_{\text{min}} = 0.5 \) (middle right), \( E_{\text{min}} = 0.75 \) (bottom left) and \( E_{\text{min}} = 0.90 \) (bottom right).
5.4. Comparison of optimized results

As the term optimization can be interpreted in many different ways, it is relevant to determine whether the structural optimization process has a significant influence on the final outcome. Most literature about structural optimization is about finding an optimal material distribution for a specific design domain. However, the problem can also be altered to find an optimized design while considering a specific manufacturing technique. Therefore it can also be of relevance to consider other optimization problems than structural behavior, as the construction process, the design freedom, construction costs or for example self-weight could be considered as well.

To establish a benchmark for comparative reasons, a casted concrete beam is considered, as shown in figure 5.12. It is a statically determinate beam with a rectangular cross-section, \( b \times h \), and a length of 2\( L \). A concentrated force \( F \) is applied in the middle of the span at distance \( L \) from the supports. The considered material is 3D printed concrete, of which the material properties are explained in [50]. The tensile strength of this material is taken as 1.9 N/mm\(^2\) and it has a density of 2100 kg/m\(^3\). The concrete structures in this chapter are assumed not to be reinforced with steel rebar or any other kind of material. The results from the optimization processes are implemented in a finite element method to consider its structural behavior in terms of strength capacity and stress distribution.

As the structures are not reinforced, they will behave as brittle structures, meaning that the cracking moment of the concrete may not be exceeded. Also no safety factors will be taken into account, as this research is performed to determine the structural capacity in practice until failure.

![Figure 5.12. Scheme of a benchmark test case for a simply supported beam.](image)

To compare the structures given by the topology optimization process, a situation is assumed in which the cross-section of the solid beam has dimensions of \( b = 100 \text{ mm} \), \( h = 500 \text{ mm} \) and \( L = 1500 \text{ mm} \), giving the beam a total length of 3000 mm. The length to height ratio is equal to the ratio of elements in horizontal direction and elements in vertical direction used as input within the topology optimization process. With this information, the maximum allowed point load \( F \) can be determined, thereby giving the maximum force the structure can take before failure.

Due to self-weight, a line load is acting on the beam of \( 0.5 \times 0.1 \times 21 = 1.05 \text{ kN/m} \).

The bending moment due to the self-weight is \( M_{\text{self}} = \frac{1}{8} q L^2 = \frac{1}{8} \times 1.05 \times 3^2 = 1.18 \text{ kNm} \).

Research at the TU/e shows that the tensile strength of the concrete is 1.9 N/mm\(^2\). This gives a cracking moment of:

\[
M_r = \frac{1}{6} f_{ctm} \times bh^2 \times 10^{-6} = \frac{1}{6} \times 1.9 \times 100 \times 500^2 \times 10^{-6} = 7.92 \text{ kNm}.
\]

The maximum bending moment caused by the point load before the structure starts to crack is:

\[
M_{Ed} = 7.92 - 1.18 = 6.74 \text{ kNm}.
\]
The bending moment caused by the point load is:

\[ M_{\text{point}} = \frac{1}{2} FL = 6.74 \, \text{kNm}. \]

This means that the maximum value of the concentrated force on the structure may be:

\[ F_{Ed} = \frac{6.74}{0.5 + 1.5} = 8.99 \, \text{kN}. \]

As a check to see if FEM works as well to come to the same solution, the same structure is implemented as a plate. A structural material is added with linear elastic properties, a density of 2100 kg/m\(^3\) and a Young’s modulus of 20000 N/mm\(^2\), taken from material research at the TU/e [50]. A point load of -8.99 kN in y-direction is applied in the middle on top of the plate. The final result of stresses in x-direction due to the combination of self-weight and the point load can be seen in figure 5.13. It is clear that the maximum tensile stress in the element with the highest bending moment is 1.83 N/mm\(^2\), which is almost equal to the maximum tensile stress. The difference can be explained with the fact that finite element method is always an approximation of the exact solution related to the size and amount of elements that are used to model the structure. As more elements are used, the more precise the calculation will be.

This example shows that post-processing of the structure with finite element method can be performed to check the stress distribution. By obtaining a structure with maximum tensile stress to be similar to the tensile strength of the material, the maximum allowed point load on the structure can be obtained, resulting in a comparison to see what structural capacity the structures have.

The solid beam of figures 5.12. and 5.13. can easily be printed. Given is a print speed set to 5000 mm per minute and dimensions of the beam of 3000 x 500 x 100 mm with the nozzle dimensions of 40 x 10 mm. The structure requires 3 times the width of the layer with some overlap taken into account to obtain the 100 mm thickness. This means that one layer of the structure has a length of 9000 mm. The total structure requires 50 layers, resulting in the total length of the print path of 450000 mm. In total it would then take 90 minutes to print the total structure.

Finally, the weight of the beam will be 2100 * 0.500 * 0.100 * 3000 = 315 kg as it is a solid concrete beam with these specific dimensions and the given density of 2100 kg/m\(^3\).

Now from the optimization process the final result of the benchmark optimization process is shown in figure 5.14. This can be translated to a finite element model by considering an equal plate model with openings added to the design. This way the model will behave structurally as one object as will be the case for a printed truss frame. The same boundary conditions will be started with, meaning that the length of the beam is L = 3000 mm, a height h = 500 mm and width b = 100 mm with a point load of 10 kN in the middle of the span.
Considering the fact that the stresses will be equally changed when the force is modified due to simple linear elastic calculation, the force is modified to such a value that the tensile stresses in the material will become approximately 1.9 N/mm². This process is performed to see what the maximum concentrated force can be until cracking and therefore failure of the structure. This way the different structures can be compared to see which structure has the highest structural capacity.

The result of the stress distribution for stresses in x-direction is shown in figure 5.15. The outer fibers of the member in tension, due to the highest bending moment, have stress values of approximately 1.9 N/mm². The concentrated force acting on the beam is 7.30 kN, while a material reduction is obtained of 50%. Since material is removed, it is inevitable that the structural capacity will decrease. The amount of force that the structure can take however is only reduced by 19%. This means that the structural capacity is relatively decreasing less than the amount of material is being removed.

In order to print this model, a continuous path must be created in which all the structural members are implemented. Consider the fact that the nozzle has specific dimensions and that the path must be continuous. A print path must be created that covers every element from the topology optimization result and some overlap of layers must be required to gain the required thickness of these structural members.

Assume that the print speed is set to 5000 mm per minute and that the structure from figure 5.14 requires a print path of approximately 12000 mm for it to be continuous and looping. This means that the printer is able to print 83 mm per second, meaning that each layer of the total structure takes 144 seconds. For the finite element analysis a thickness of the structure is taken of 100 mm, which means that 10 layers must be printed for it to be completed. The time required to print this structure will then be 1440 seconds, which is equal to 24 minutes.
The total weight of the structure will be approximately half of the solid beam, as the topology optimization process has removed 50% of the material due to the constraints in volume reduction. This means that the mass of this optimized beam will be approximately 158 kg.

It must be emphasized however, that modeling this structure for it to be printable will require some additional time. In addition, the bond strength between the sides of layers that are connecting is an important variable to consider. In the case of poor connection between the sides of the material within the same layer, the total structure can fail when it is flipped for use in practice, as the printed members will detach from each other due to stresses in the connection. The bond strength between sides of layers is unknown, but the amount of overlap is crucial for this bond strength.

The optimization process can also be altered by considering two materials with different stiffnesses, as explained in the previous chapter. This means that a second material must be implemented in the manufacturing process as well. The result of implementing a secondary material with a stiffness that is 10% of the primary material is shown again in figure 5.16. Here the black elements have the primary material properties with 100% stiffness and the white elements have the secondary material properties with 10% stiffness. The gray elements have a stiffness that lies within this range.

A simple way to create this structure, is by using 3DCP to create the parts of the structure with black elements and therefore producing the part of the primary material with 3DCP. In its essence, this means that a concrete formwork is obtained with this process, which can be filled with the secondary material with a stiffness between 10 and 100% of the printed concrete. By using materials with for example lower densities, the total mass of the structure can be reduced, while the volume remains the same. Also other material properties can be taken into account, as for example transparency or insulation properties could be of relevance. It must be emphasized that this situation does assume that the connection between these two materials is able to fully transfer the forces as it is intended in the finite element model and that it does not show any effects like detachment.

Finite element analysis is again used to obtain the maximum value of the concentrated force for the structure to reach the 1.9 N/mm² tensile stress. Again a plate model is used with equal dimensions as the previous cases. A linear elastic structural material is used with a density of 2100 kg/m³ and a modulus of elasticity of 25000 N/mm². Within the same model, another region is specified in which different material properties can be implemented. This way the structure will still work as one object, while the material properties are different. For this secondary material a density is given of 800 kg/m³ and a modulus of elasticity of 2500 N/mm². The final result of stresses in x-direction due to the combination of self-weight and the point load can be seen in figure 5.17. It is clear that the maximum tensile stress in the element with the highest bending moment is 1.9 N/mm², which is equal to the maximum tensile stress.
The concentrated force acting on the beam is 8,00 kN, while a material ratio between structural and support material is obtained of 50/50. The maximum value for the concentrated force can be little higher than the truss frame, as the structure remains solid and in theory no material is removed. The reduced stiffness however due to the secondary material, results in a structural capacity that is lower than the benchmark model, as the maximum allowed concentrated force is lower.

In its essence, the model described above behaves similar to a sandwich structure, with in this case printed concrete as the outer skin material and supporting material as the core. However, opposed to sandwich structures in which the shear force is transferred by the core material, the shear forces here are transferred by the concrete. This is possible due to the fact that the outer material is a closed shape and shaped in such a way that the outer material uses its relatively high stiffness to transfer the forces to the supports. The support material is then only required to increase the stiffness of the structure, opposed to a structure that only contains the printed concrete outer skin. This effect is shown in figure 5.18.

In order to print the outer so called skin, two layers are required with different amounts of overlap in the x-direction to obtain a structure similar to the structure composed of black elements of the topology optimization process. This way the variable thickness of the members can be achieved. Therefore, a total length of one layer is estimated to be 14000 mm. Again a thickness is required of 100 mm to obtain the same model, so 10 layers must be stacked again. This gives a total length of the print path of approximately 140000 mm. A print speed of 5000 mm per minute, gives a total print time of 28 minutes. During or after hardening, the structure can be filled with a different material, preferably with a low density and a useful value for a modulus of elasticity. The above example gives a total weight of:

\[
\frac{315}{2} + \frac{800 \cdot 0.1 + 0.5 \cdot 3}{2} = 217.5 \text{ kg.}
\]

A comparison is made between the specified printing techniques mentioned previously. A distinction is made between a solid beam and final results of topology optimization in terms of the truss and the beam with two material properties. The subjects which are compared here are structural capacity, compliance, weight, volume and build speed. The values are normalized, meaning that the technique with the best performance regarding a subject has the highest value in the chart.
Structural Optimization for 3D Concrete Printing

Structural capacity considers the maximum allowed of concentrated force in the middle of the span of the beam. The compliance is then a relative term taken as the stiffness of the object and is determined by the result of the optimization process. The weight, volume and build speed are all related to each other as all these subjects rely on the amount of concrete that is used for the structure. A distinction is still made to see their individual effects:

<table>
<thead>
<tr>
<th></th>
<th>Structural capacity [kN]</th>
<th>Compliance</th>
<th>Weight [kg]</th>
<th>Volume [m³]</th>
<th>Build speed [minutes]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Solid</td>
<td>9</td>
<td>128,35</td>
<td>315</td>
<td>0,15</td>
<td>90</td>
</tr>
<tr>
<td>Truss</td>
<td>7,3</td>
<td>209,95</td>
<td>157,5</td>
<td>0,075</td>
<td>24</td>
</tr>
<tr>
<td>Sandwich</td>
<td>8</td>
<td>189,18</td>
<td>217,5</td>
<td>0,15</td>
<td>28</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Structural capacity [kN²]</th>
<th>Compliance</th>
<th>Weight [kg]</th>
<th>Volume [m³]</th>
<th>Build speed [minutes]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Solid</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>Truss</td>
<td>81,1</td>
<td>163,57</td>
<td>50</td>
<td>50</td>
<td>26,67</td>
</tr>
<tr>
<td>Sandwich</td>
<td>88,9</td>
<td>147,39</td>
<td>69,04</td>
<td>100</td>
<td>31,11</td>
</tr>
</tbody>
</table>

Clearly, the weight and build speed are reduced compared to a solid beam that is printed. This is due to the volume constraint in the topology optimization process. The compliance of both optimized objects is also higher, meaning that the stiffness is reduced, because of removal of material. However, the structural capacity is relatively less reduced compared to the other factors, meaning that the influence of the optimization process has a positive effect on the ratio between structural capacity and material usage. A positive result can be seen in the figures on the next page. Here the final results of the optimization process with a volume reduction respectively 30 and 70% is used in FEM analysis. The maximum point loads are -5,13 kN and -8,20 kN accordingly. It shows that the relationship between the volume fraction and the structural capacity is degressive. The compliance of both results explained in chapter 3 resembles the stiffness of the structure and therefore the deflection. So to conclude, the relationship between the structural capacity and volume is degressive, while the relative reduced amount of material is more than the relative reduced amount of structural capacity.

Figure 5.19. Comparison of the results after topology optimization and FEM analysis of a solid beam with the optimized truss frame and with an optimized beam with two different material properties.
Topology optimization and 3D printed concrete

Figure 5.20. FEM analysis of an optimized beam with a volume reduction of 70% and a point load of -5.14 kN.

Figure 5.21. FEM analysis of an optimized beam with a volume reduction of 30% and a point load of -8.20 kN.

Figure 5.22. Graph that shows the relationship between the volume fraction and the maximum value for the point load.
Another way to create these structures can be done by implementing the design within the print path. As the previous example is done by printing the formwork, consequently adding the secondary material and then flipping the structure so it works as it is modeled, this technique does not require the structure to be flipped once it is hardened. By altering the printer settings, the shape of individual layers can be altered of the layer-wise built up of the structure can be changed in such a way that the density of the structure can be reduced. Several techniques for this process are shown in the next chapter as a proof of concept.

An example is shown here to see the influence on print speed and therefor construction time of the object. The final result from the optimization process is taken where a secondary material is taken into account with a stiffness of 10% of the primary material, as shown in figure 5.16. This means that the white or grey elements can be printed with an alternative method with a different print speed and the black elements are printed in the conventional way. The layers of these two methods have the following properties:

- Conventional 3DCP technique:
  - Speed: 5000 mm/min
  - Height: 10 mm/layer

- Secondary structure:
  - Speed: 2500 mm/min
  - Height: 30 mm/layer
  - Speed: 5000 mm/min
  - Height: 10 mm/layer

When an object is considered with a length of 1 meter and is continuously printed, with every next layer printed in opposite direction, then the differences in speed of the layer wise built up can be considered. The object will be built until it reaches a total height of 1000 mm. It must be emphasized that this comparison does not take vertical deformation and stability of the layers into account.

Conventional 3DCP technique:
- Required amount of layers: 100
- Total length to be printed: 100 m
- Duration: 20 minutes

Secondary structure:
- Required amount of layers: 25
  - 25 layers of secondary structure
  - 25 layers of intermediate layers
- Total length to be printed: 100 m
- Duration: \((25 \times 24 + 25 \times 12) / 60 = 15\) minutes

This calculation shows that a certain height of a structure can be achieved more quickly with the alternative method, in comparison to the conventional 3DCP technique. Since the material is still in its dormant period during this construction phase, the material will deform quicker as well, and might therefore not be suitable to produce large elements within the same printing process or with a short print path. In hardened state, the structural behavior will also not be beneficial as the layers within the structure are all connected at small locations. This will result in high stress concentrations and therefore early failure of the material and consequently failure of the structure. Finally, the design of the print path also requires more time, as the coordinates must be specified for each layer individually and therefore the benefit of increased building speed could be taken away. Due to these reasons, this method is not further taken into account in the comparison, but is only mentioned and shown as a proof of concept in the next chapter to show that complex printing technique could be used to alter the structure and can be of use when for example other material is used.
6. Proof of concepts

This chapter will explain several proof of concepts that will introduce ways to use the concrete as support material for the concrete that will be used as structural material. These support structures need to be made as efficient as possible within the print setup, which is used at this time at the Eindhoven University of Technology. First a benchmark print will be shown, as this concept shows that the beams can easily be printed on its side. Subsequently, the beams need to be able to be printed without rotating them afterwards, as this will not be possible with objects that are different in every dimension like double curved shells. This subject will give the necessity of the support material.

6.1. Initial print MBB beam

As an initial test, the benchmark result of the optimization problem stated in chapter 3, will be printed first. Figure 6.1. shows the print path of this print procedure. The printer setup as of today consists of a system which is only able to print continuous layers, so therefore the print path must be designed in such a way that it is a closed path. This means that certain overlaps of layers are inevitable if these continuous paths are always required. Therefore, another important factor to consider is the width of the structural members in relation to the dimensions of the layers. Since the width of the nozzle is 40 mm, some overlap of layers is required to obtain a difference between the thickness of every individual structural member.

Another point to consider is the radius of the corners of the structure. While printing, when the nozzle is at the coordinates of a corner, it will rotated over its own z-axis. This means that a straight angle will be formed, while stopping the motion of the printer arm. Concrete will continue to flow, resulting in a lump of material at the corners. To avoid this from happening a radius is introduced to have a smooth motion of the printer arm through the whole print path. Material will be distributed equally, since the speed of the printer will not change. The corners will also be rounded, resulting in a more precise and fine finish of the product.

Figure 5.1. Print path of the initial print of the MBB beam.
Figure 6.2. Printing the initial MBB beam. The overlap of layers can be clearly seen, as the excess material is pushed away by the nozzle.

Figure 6.3. The initial print of the MBB beam once it has hardened. The overlap of the layers can still be clearly seen. These overlaps determine the thickness of structural members.
It is clear that these kind of models can easily be printed and that the relevant factors of 3D Concrete Printing need to be taken into account while modeling the print path. This will create a layer-wise built up of the object in a conventional way with this manufacturing technique. Every layer in z-direction will remain the same and can be printed on top of the previous one, until the fresh concrete can no longer bear the self-weight of the material on top. The casting constraint, which is explained in chapter 4.1, can therefore be used for concrete printing as well, since the same kind of models are created this way. The casting constraint, normally used to implement the constraints for conventional casting of concrete, results in models with equal rows of elements that are either void or material. When used for 3D Concrete printing, these rows can be seen as the layers in z-direction, creating an object with equal layers over it’s the height of the total structure. By then using this technique, the object can be rotated 90 degrees to obtain the object as it is supposed to be.

By using this technique, some other consequences need to be taken into account. The layers will be rotated as well, meaning that all the properties belonging to the layer-wise built up of the concrete will be rotated along with it. The contact surface between the layers will work in the same plane as the plane of the object itself. The self-weight of every individual layer will cause shear force within the cross-section of the structure, compared to a surface load introduced on every layer in the conventional built up. The bond strength in the overlap of layers should be considered for the bending moment. This bending moment introduces shear forces in the overlap in the bottom of the structure, which the bond strength of these layers needs to withstand.

Research [50] also showed that the strength capacity of a single layer is different in every dimension. As the initial design of the model of the topology optimization problem shows, the point load will be applied on the side of the layer in this rotated situation, meaning that the strength capacity in this direction needs to be taken into account.

The manufactured beam is designed to withstand a point load, causing bending in the beam. Since concrete is a material which is stronger in compression than it is in tension, reinforcement should be applied in the material. The position of the reinforcement should therefore be considered as well, since this will also be rotated along with the total structure. Another factor to consider due to bending in this situation, is the curvature of the corners at the point where the overlap will start. The bending will cause stresses similar to peel stresses, which are relevant in laminated glass structures. The flow of forces needs to be considered here as well.

From these conclusions it can be seen that 3D Concrete Printing can be used for optimized structures. The rotation of the printed object however, is not beneficial for its structural performance. Also, when considering objects which are different in every dimension, the rotation of the total object is not possible, since every subsequent layer still needs to be different than the previous one. This will cause overhangs and angles in the structure, which is not possible for fresh concrete as explained in chapter 5.2. Support material is therefore required and this needs to be manufactured within the same printing process and with the same material.
6.2. Cellular support structure

A cellular support structure, as introduced in chapter 5.3., can be manufactured with the 3D Concrete Printing process. It requires a repetitive structure with small cavities that can be created by altering the print settings of the print path. The conventional way of printing is done by setting the nozzle height close to the surface on which it will print with a certain print speed that has proven to work with this height. The reason for this is to increase the bond strength between layers, since the material will be pushed on the print surface. By altering these settings, the material will not be pushed on the print surface anymore, but instead will fall on it by its self-weight. Since the printer settings remain constant throughout this print path and the printing process is fully automated, the output will remain the same at all times. The nozzle height will give the material some space to let it find the way it will fall on the print surface, creating a constant pattern over the length of the print path.

Figure 6.4. shows the way that this procedure is performed. The movement of the nozzle is towards the right and the print speed and nozzle height are set according to the desired output. As the extrusion of material is fast related to this print speed and nozzle height, eventually too much concrete will hang between the nozzle and the print surface. This means that the concrete layer will flip in the length direction, as can be seen in the middle frame of figure 6.4. Since the printer arm will continue to print, this way, a pattern is formed with constant distances between each individual fold.

Figure 6.5. shows the output of a single layer for several different settings of nozzle height and print speed. For every loop, the print speed is reduced every 500 mm with a constant nozzle height. The nozzle height is therefore different between every loop, resulting in several different patterns with different dimensions. Also the occurrence of the patterns is therefore variable, as they can be print close or far away of each other. The bottom three layers show the effect of rotation of the nozzle. This required a different print speed and the nozzle height is set the same as in the three previously printed loops. Appendix A shows the exact output and dimensions of these layers. The results show that the height, width and distance between each fold remains constant as well, showing that the process can be controlled and used for further research.

![Figure 6.4](image.png)
In order to create a support structure with this technique, the layers must be able to stack on top of each other to be used for 3D Concrete Printing. The distance between each fold in the print path in figure 6.5. is determined and used to see which settings can be used for the layered built up of structures. This distance will be same as the distance the upcoming layer needs to span and it is therefore an important parameter, as the overhang of fresh concrete is limited.

From appendix A, the settings of line numbers 7, 8, 15 and 16 are used for further research into the stacking over layers. As the dimensions are constant and the span distance will be sufficiently low, while there will still be cavities in the structure for the volume reduction of structural optimization, the settings seem proper. In order to stack them however, the next layer will be printed in the conventional way. This is done every time to create a horizontal print surface for the cellular structure for every new layer. As the cellular structure will have cavities in the top of the layer, the newly printed cellular structure cannot fall into this cavity and therefore a base layer is required at all times.

![Figure 6.5. Results of the print of a single layer of the cellular structure. The nozzle height and print speed are varied over the length of the print path. The bottom three layers shows the results when the nozzle is rotated.](image1)

![Figure 6.6. Final results of the stacked layers of cellular structures. The difference in the printer settings can be seen as the pattern is changed with every beam as well as the size of the cavities.](image2)
The beams with cellular structures and a beam that is printed in the conventional way are sawn to obtain the same length for every beam. Therefore the dimensions in height and in length are same and the weight can be compared. The weight of the bars in figure 6.6. from top to bottom are:

1. 8,452 kg
2. 9,843 kg
3. 8,338 kg
4. 9,647 kg
5. 12,185 kg

The difference is significant, as the cellular structures are between 30% and 19% lighter than the benchmark test beam. This can however also be explained by the width of every beam, since the width of the cellular structures is the exact width of the width of the nozzle, due to the fact that the material is not pushed on the print surface, but falls on it instead, resulting in the fact that the layers are not flattened. The layers of the benchmark test beam are flattened, due to the small nozzle height during printing. This is done for better bond strength, but increases the width of the structure to 50 mm, compared to 40 mm.

The differences in densities can be used to print the gradients in the optimized beams by structural optimization. As the densities can now be controlled within a certain range, the penalty factor is no longer required to obtain black and white solutions. The fact that the elemental density will then not be rounded towards 0 or 1 solutions, means that the structural optimization becomes more precise, as the penalty factor is removed. Therefore the initial formula is used to obtain the relative densities of the elements, resulting in a gradient based topology optimization, as can be seen in figure 6.7.

In order to print the result of figure 6.7. A continuous path needs to be created that takes the print speed, nozzle height and dimensions of the layers of the cellular structure and the straight layers into account. The straight layers need to be used at the black elements, while the cellular structure can be used at gray elements. Figure 6.8. shows the how this beam can be printed with cellular support material, taking these parameters into account.
The print shows potential, but still requires further research for further application on optimized structures by 3D Concrete Printing. As the black on the right side in figure 6.7. are all connected, the print in figure 6.8. is not at the right side. Some overlap is required to maintain the flow of forces in the beam as illustrated in figure 6.7. through structural material (black elements).

The strength capacity of the support material is smaller, which can be seen on the right side of the print in figure 6.8. as well. The stacking of layers on a local scale, deforms the structural material in ways that the height will be reduced and that therefore the nozzle height will increase. This is also the reason why the support material shows differences in the patterns of each layer of support material.

The beam collapsed after further printing. The reasons for this can be explained by the strength capacity of the cellular support material in its fresh state and the deformations that come along with it. Also, the geometry of the total structure is not beneficial, as the width is relatively small compared to the length and height of the total structure. Imperfections in the print will cause it to fail faster, due to second order effects in the fresh concrete.

However, the technique shows potential. The stacking could initially be performed in the first three layers of the support material. The amount of imperfections in the printing machine, increasing the strength capacity of the support material and considering the total geometry of the printed structure are challenges yet to be tackled in future research. This can be done by doing test on structural performance on the support material in hardened, but especially in fresh state, to gain knowledge on its structural behavior. Also the print setup will be constantly further developed to obtain a printing process with constant output in terms of material and manufacturing properties. The geometry of the total structure can easily be changed by altering the print paths in such ways that the influence of the second order effect in the fresh material becomes less significant.
6.3. Open support structure

Inspired by the ‘Programmed Wall’ project by ETH Zurich, support structures can also be made by leaving gaps between layers and then stacking the new layers on the center line of the gap. A visualization is shown in figure 6.9. It shows a cross-section of a potential open support structure where four layers are stacked upon each other and subsequently a shift in the position of the layers takes place to form a pyramid-like structure. By altering the amount of layers, the inclination is altered and therefore the ability to print support structures with different inclinations.

To obtain the maximum span distance between layers, a simple test procedure is performed with varying distances between layers, as shown in figure 6.10. The distances between layers is varied between 5 mm and 30 mm with an increase of 5 mm every other distance. Results showed that the span distance of a single layer of fresh concrete on top of the corners of two other fresh layers has a maximum value of around 15 mm. Therefore, the print of open support structures as shown in figure 6.9 can have a maximum distance between layers of 15 mm.
Subsequently, an open support structure is printed as visualized in figure 6.9. with stacks of four layers between every shift of layers and with an internal distance between the layers of 10 mm. Figure 6.11. shows the open support structure during printing. The programming cost of this technique is relatively high to the other prints, since the exact coordinates need to be calculated for every shift of layers. The figure also shows the fact that a continuous print path needs to be created at all times. This comes with the fact that lots of material needs to be removed at the left and right part of the concrete object in this figure.

Figure 6.12. shows the cross-section of the open support structure, once hardened. The deformation of the layers is clear here as well, as the cavities are less open due to flattening of the layers at the bottom of the structure. It does however show the preciseness of the printer setup, as the exact coordinates are maintain and the cavities have a width of around 10 mm.

The potential in this technique lies in the fact that no alterations have to be made within the printing process on the nozzle height and the print speed. Only the print direction is of relevance, since this needs to be perpendicular to the required print direction to obtain an inclined support structure against which the structural material can be printed. A top view of a visualization of this technique can be seen in figure 6.11. The cavities are parallel to the print direction of the open support structure, therefore creating possibilities in the openings for other objects or architectural reasons with lighting. These cavities can be seen in figure 6.13.
Figure 6.12. Side view during a print of the open support structure. The internal distance between layers is set to 10 mm.

Figure 6.13. Cross-section of the print of the open support structure with an internal distance between layers of 10 mm. The deformation of the material causes the gaps to fill with concrete, resulting in a less open structure as initially desired.
6.4. Printing hollow layers

The previously introduced techniques to obtain efficient support structures within this printer setup and material use are done by alterations in the print path and therefore creating cavities or lighter structures. But by modifying the nozzle on the printer setup, the amount of material per layer can be reduced in another way. Figure 6.14. shows a visualization of how to print with hollow nozzles and what its effect will be. In order to print with hollow nozzles, the print strategy must slightly be altered, as the conventional print strategy will not result in hollow layer with this nozzle design. When the usual print direction is maintained, the nozzle should extrude material from the side of the nozzle, instead of the bottom, to till create hollow layers.

Another way to print with a nozzle that creates hollow layers is inspired by the ‘Smart Dynamic Casting’ project of ETH Zurich. The nozzle can print small columns with a hollow cross-section by changing the print direction from horizontal to vertical. However, this is not beneficial to implement within the printing process, since two different print strategies should then be taken into account, instead of one.

A new nozzle has to be created in order to print hollow layers. The technique of the middle figure of figure 6.14. is adapted and the test of the open support structure to determine the maximum span distance is considered as well. The hollow square cross-section of figure 6.14. has been modified to obtain U-shaped cross-section for every layer. This is done to reduce more amount of material, since the stacking of layers will now result in the same cross-section of the total structure, while there is more space for a void, due to the fact that a horizontal member of the square cross-section has been left out. Because the same print strategy is maintained, and therefore the same pump pressure and flow of concrete, the U-shaped opening of the nozzle needs to be of the same area as the nozzle with rectangular opening, which is normally used. The dimensions are calculated for a span of again 10 mm within the layer and the removed area of the cross-section is added to the vertical parts of the U-shape. This resulted in a new nozzle, which can be seen in figure 6.15. It clearly shows the flow of concrete through the nozzle for extrusion out of the side of the nozzle.
The potential of this print procedure lies in the fact that the print strategy does not have to be altered to achieve proper results for concrete printing. However, it does involve a new nozzle and layer dimension, that are different from the conventional way of printing with the nozzle that prints layers with rectangular cross-section.

Figure 6.16. and 6.17. show the printed result, once the material has hardened. Cavities in the cross-section are clear to see and can be seen through the length of the whole print path. This means that cavities will occur, similar to the cavities of the open support structure, but in a smaller scale. These cavities can be used as voids to ensure material reduction, but can also be used for other reasons. For example, different material than concrete can be printed in this cavity within the same printing process, so obtain better insulating properties. These cavities can also be used as space for reinforcement for example or electric wiring. There are many possibilities to which this type of nozzle can be extended for future use within 3D Concrete Printing process, to optimized structures in different ways than only material reduction.

However, further research must be done to control this printing process better, as the material still deformed and reduced the size of the cavities. Another challenge to always consider is that newly printed concrete is not printed within the cavity of the previously printed layer. Therefore, sufficient nozzle height is recommended.
Figure 6.16. Cross-section of a printed object with hollow layers. The cavities can be found throughout the whole length of the print path. It is clear to see that the material deformed in the vertical brackets of the cross-section, resulting in a smaller cavity size.

Figure 6.17. Front view of the printed object with hollow layers (left) and the side view (right).
7. Conclusion

3D Concrete Printing shows potential as a manufacturing technique to be used for complex objects, like it is in the case of structurally optimized objects. However, since this method involves working with concrete in its fresh state, many challenges arise. The material has a low strength capacity and will deform during printing, making it harder to withstand loads form subsequently added layers. Especially overhangs and inclinations occur regularly in optimized structures, resulting in asymmetrical loadings on layers, which cause the printed structure to tilt and collapse. The established casting constraints in topology optimization software can be used as a constraint for 3D Concrete Printing as well, to create object with repetitive print paths of every layer, but this procedure requires rotation of the concrete design, which comes with challenges in terms of structural behavior and logistics due to the high self-weight.

However, in the case of an optimized object that is different in every dimension, this procedure will not be sufficient to obtain the desired result with this manufacturing technique. For example, this is the case for double curved shell structures. The need of support material will become inevitable, but the printer setup of today is only able to print objects of one material. Another limitation lies in the fact that the print paths for these objects need to be continuous, since the printing process cannot be paused once started.

Therefore, a printing strategy must be obtained that results in efficient material distribution. By using the constraints in the optimization algorithm and 3D Concrete Printing, several strategies can be used. The solutions in this research lie in either changing the printing settings or by changing the material properties in the algorithm. The common result in research on topology optimization, the truss beam, can be printed by creating the print path while considering the printer constraints and consequently rotating the object once hardened. Another way is to use the stiffness of a secondary material and implement it in the algorithms. This way, the mass of the object could still be reduced, while the structural capacity is relatively reduced less. FEM analysis acknowledges that there is a positive relationship between the material reduction and the strength reduction for uncracked concrete. Relatively, more material can be removed than the relative amount of strength capacity is reduced.

However, it is inconvenient to rotate the objects once hardened, so a solution can also be found by manipulating the printing settings. Several proof of concepts are introduced that involve material reduction within the printing setup at the Eindhoven University of Technology. These concepts are related to modifications in the conventional print paths and by using a modified nozzle. With these procedures, cavities will be created in the printed concrete structure, resulting in a reduction of material usage. These cavities can also be used for other reasons like application of reinforcement of for example electric wiring. They also present new architectural features to the concrete structure, as the repetitive patterns can be used for lighting or esthetic reasons as well.

However, the difference in material usage is minor, as the manufacturing process involves fresh concrete and the strength capacity and stiffness of this material is not ideal during the printing process. The self-weight of the subsequently printed layers will deform the structure more, compared to the conventional way of concrete printing. This way, the volume of the initially created cavities will reduce and the structures will become more solid than intended to be. These proof of concepts still show that there are possibilities in smart ways of 3D Concrete Printing to either reduce the amount of material that is used or to combine multiple disciplines of the building industry.
8. Recommendations

To create optimized objects with 3D Concrete Printing, the designer must at all times weigh the advantages of this method against the disadvantages when used with this manufacturing technique. Optimized objects can be produced by using the many possibilities in freeform design. However, it does require smart thinking in manufacturing or application of this theory. When the conventional printing technique is used as it is used nowadays, the optimization algorithms can only be combined by applying the demolding constraints to create objects that have the same cross-section in one direction. This way the designer does not have to consider inclined structures, which are difficult to create working with fresh concrete. If an object is made with a secondary material that is used to support the inclined structures, other advantages and disadvantages arise. The designer must take care of the fact that both materials will work together and that local failure of the structure is not an issue. Also the secondary material must have certain properties in the relationship between stiffness and density, to make sure that the final result is still an optimized object. For some materials it could be the case that the final result might still have an equal mass or that the structural capacity is reduced more than intended.

The proof of concepts have shown that efficient support material can be printed, but that the used concrete shows too much deformation to obtain the desired result for material reduction. From previous research [50] it is concluded what the setting time and strength capacity and strength development of the material was, taking into account the influence of the printing process. Compared to other researches on 3D printing with concrete, the material used at the Eindhoven University of Technology has a long setting time, while the strength capacity remains relatively low throughout the hardening process. Modifications in the concrete mixture can be one of the solutions.

As the material is already no-slump concrete, it still deforms under loading. Therefore the initial strength capacity needs to be increased or the strength development of the material needs to be quicker. By applying alterations to the cement mix, for example using type III cement for higher initial strength or adding accelerators for faster strength gain, these factors can be improved for use within the printing process. While these factors are of importance of the buildability of the printed structures, the workability of the material should always be proper for concrete printing, as the fresh material needs to be pumped through the system.

Also, the mixture can be modified for alterations in the material properties in terms of the internal friction angle and the cohesive strength. This can be beneficial to the bond strength and the stress limits of the material for use with the Drucker-Prager criterion. Reinforcement of the material can also be beneficial for the stiffness of the material, as long as the cohesion between reinforcement and concrete is proper and the reinforcement is not pulled out of the material due to deformations and applied forces.

An extreme measure can be taken for the printing of support material by developing a system which can be used to print another material within the same printing process that is able to carry the structural concrete during printing. This, however, requires lots of modifications to the printing process for it to work properly with the automated system that is used at this time.
Once the material is modified or the material that is to be used is confirmed for further use, structural tests of the support material should be performed to gain insight in its structural behavior in fresh state, as well as hardened state. The stress limits and stiffness need to be determined, so that they can be taken into account in the structural optimization process. Also the structural behavior is of relevance, since the shape and contact surfaces for bonding will be changed, compared to the conventional way of printing. Especially when considering three dimensional objects, the bond strength in horizontal direction will be influential to the strength and stiffness of the total concrete object. This effect is also of importance when concrete objects are rotated once hardened, after printing.

Just by these reasons it is clear that more research is required towards the material properties of printable concrete, the structural behavior of the material in fresh and hardened state and the structural behavior of layered structures, whether they are printed in the conventional way or not, as in terms of the support material introduced in this project.


Appendix A

Appendix A shows the results of the measurements performed on single layer cellular support structure. 36 lines with different printer settings are printed and measured for their output to determine which printer settings are beneficial for the use of these support structures.

### AVERAGE RESULTS

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<th>Mass [kg]</th>
<th>[kg/m]</th>
<th>Distance A [mm]</th>
<th>Distance B [mm]</th>
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### Structural Optimization for 3D Concrete Printing

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Appendix A
Structural Optimization for 3D Concrete Printing
Figure A.1. Line numbers from top to bottom: 6, 7, 8, 14, 15, 16.
Figure A.2. Line numbers from top to bottom: 22, 23, 24, 28, 29, 30, 31, 32.
Figure A.3. Line numbers from top to bottom: 31, 32, 33, 34, 35, 36.
Figure A.4. Line numbers from right to left: 6, 7, 8, 14, 15, 16, 22, 23, 24. These lines are printed with the nozzle set at 0 degrees.

Figure A.5. Line numbers from right to left: 27, 28, 29, 30, 31, 32, 33, 34, 35, 36. These lines are printed with the nozzle set at 22.5 degrees, 45 degrees and 67.5 degrees.
Appendix B

%%% A 99 LINE TOPOLOGY OPTIMIZATION CODE BY OLE SIGMUND, JANUARY 2000 %%%
%%% A 142 LINE CODE MODIFIED FOR USE WITH ORTHOTROPIC MATERIALS %%%

function top(nelx,nely,volfrac,penal,rmin);
% INITIALIZE
x(1:nely,1:nelx) = volfrac;
loop = 0;
change = 1.0;
% START ITERATION
while change > 0.01
loop = loop + 1;
xold = x;
% FE-ANALYSIS
[U]=FE(nelx,nely,x,penal);
% OBJECTIVE FUNCTION AND SENSITIVITY ANALYSIS
[KE] = lk;
c = 0.0;
for ely = 1:nely
for elx = 1:nelx
n1 = (nely+1)*(elx-1)+ely;
n2 = (nely+1)*elx   +ely;
Ue = U([2*n1-1;2*n1; 2*n2-1;2*n2; 2*n2+1;2*n2+2; 2*n1+1;2*n1+2],1);
c = c + x(ely,elx)^penal*Ue'*KE*Ue;
dc(ely,elx) = -penal*x(ely,elx)^(penal-1)*Ue'*KE*Ue;
end
end
% FILTERING OF SENSITIVITIES
[dc] = check(nelx,nely,rmin,x,dc);
% DESIGN UPDATE BY THE OPTIMALITY CRITERIA METHOD
[x] = OC(nelx,nely,x,volfrac,dc);
% PRINT RESULTS
change = max(max(abs(x-xold)));
disp(["It.: " sprintf('%4i',loop) ' Obj.: ' sprintf('%10.4f',c) ...'
' ' Vol.: ' sprintf('%6.3f',sum(sum(x))/nelx*nely) ...'
' ' ch.: ' sprintf('%6.3f',change )])
% PLOT DENSITIES
colormap(gray); imagesc(-x); axis equal; axis tight; axis off;pause(1e-6);
end

%%%%%%%%%% OPTIMALITY CRITERIA UPDATE
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
function [xnew]=OC(nelx,nely,x,volfrac,dc)
l1 = 0; l2 = 100000; move = 0.2;
while (l2-l1 > 1e-4)
lmid = 0.5*(l2+l1);
xnew = max(0.001,max(x-move,min(1.0,x+move,x.*sqrt(-dc./lmid))));
if sum(sum(xnew)) - volfrac*nelx*nely > 0;
l1 = lmid;
else
l2 = lmid;
end
end

%%% MESH-INDEPENDENCY FILTER
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
function [dcn]=check(nelx,nely,rmin,x,dc)
dcn=zeros(nely,nelx);
for i = 1:nelx
    for j = 1:nely
        sum=0.0;
        for k = max(i-floor(rmin),1):min(i+floor(rmin),nelx)
            for l = max(j-floor(rmin),1):min(j+floor(rmin),nely)
                fac = rmin-sqrt((i-k)^2+(j-l)^2);
                sum = sum+max(0,fac);
                dcn(j,i) = dcn(j,i) + max(0,fac)*x(l,k)*dc(l,k);
            end
        end
        dcn(j,i) = dcn(j,i)/(x(j,i)*sum);
    end
end

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% FE-ANALYSIS
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
function [U]=FE(nelx,nely,x,penal)
[K] = lk;
K = sparse(2*(nelx+1)*(nely+1), 2*(nelx+1)*(nely+1));
F = sparse(2*(nely+1)*(nelx+1),1); U = zeros(2*(nely+1)*(nelx+1),1);
for elx = 1:nelx
    for ely = 1:nely
        n1 = (nely+1)*(elx-1)+ely;
        n2 = (nely+1)*elx   +ely;
        edof = [2*n1-1; 2*n1; 2*n2-1; 2*n2; 2*n2+1; 2*n2+2; 2*n1+1; 2*n1+2];
        K(edof,edof) = K(edof,edof) + x(ely,elx)^penal*KE;
    end
end

% DEFINE LOADS AND SUPPORTS (HALF MBB-BEAM)
F(2,1) = -1;
fixeddofs = union([1:2:2*(nely+1)],[2*(nelx+1)*(nely+1)]);
alldofs = [1:2*(nely+1)*(nelx+1)];
freedofs = setdiff(alldofs,fixeddofs);

% SOLVING
U(freedofs,:) = K(freedofs,freedofs) \ F(freedofs,:);
U(fixeddofs,:) = 0;

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% ELEMENT STIFFNESS MATRIX
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
function [KE]=lk
% BilinearQuadElementStiffness This function returns the element
% stiffness matrix for a bilinear quadrilateral element with modulus
% of elasticity E, Poisson’s ratio nu, thickness h, coordinates of
% node 1 (x1,y1), coordinates node 3 (x3,y3), and coordinates of
% node 4 (x4,y4). Use ps = 1 for cases with isotropic material, and ps = 2 for
% cases with orthotropic material. The size of the element stiffness matrix is 8 x 8.
Ex = 1;
Ey = 5;
nuxy = 0.3;
nuxy = 0.3;
h = 1;
x1 = -1;
y1 = -1;
x2 = 1;
y2 = -1;
x3 = 1;
y3 = 1;
x4 = -1;
\( y_4 = 1; \)
\( ps = 2; \)
\( T = 0.3.141592654; \)

```matlab
syms s t;
ap = (y1*(s-1)+y2*(-1-s)+y3*(1+s)+y4*(1-s))/4;
b = (y1*(t-1)+y2*(1-t)+y3*(-1-t)+y4*(1+t))/4;
c = (x1*(t-1)+x2*(1-t)+x3*(1+t)+x4*(-1-t))/4;
d = (x1*(s-1)+x2*(-1-s)+x3*(1+s)+x4*(-1-s))/4;
B1 = [a*(t-1)/4-b*(s-1)/4 0 ; 0 c*(s-1)/4-d*(t-1)/4 ;
     c*(s-1)/4-d*(t-1)/4 a*(t-1)/4-b*(s-1)/4];
B2 = [a*(1-t)/4-b*(-1-s)/4 0 ; 0 c*(-1-s)/4-d*(1-t)/4 ;
     c*(-1-s)/4-d*(1-t)/4 a*(1-t)/4-b*(-1-s)/4];
B3 = [a*(t+1)/4-b*(s+1)/4 0 ; 0 c*(s+1)/4-d*(t+1)/4 ;
     c*(s+1)/4-d*(t+1)/4 a*(t+1)/4-b*(s+1)/4];
B4 = [a*(-1-t)/4-b*(-1-s)/4 0 ; 0 c*(1-s)/4-d*(-1-t)/4 ;
     c*(1-s)/4-d*(-1-t)/4 a*(-1-t)/4-b*(-1-s)/4];
Bfirst = [B1 B2 B3 B4];
Jfirst = [0 1-t t-s s-1 ; t-1 0 s+1 -s-t ;
     s-t -s-1 0 t+1 ; 1-s s+t -t-1 0];
J = [x1 x2 x3 x4]*Jfirst*[y1 ; y2 ; y3 ; y4]/8;
B = Bfirst/J;
R = [ cos(T) -sin(T) 0
     sin(T) cos(T) 0
     0 0 1 ];
if ps == 1
    D = (Ex/(1-nuxy*nuxy))*[1, nuxy, 0 ; nuxy, 1, 0 ; 0, 0, (1-nuxy)/2];
elseif ps == 2
    D = [ Ex/(1-(nuxy*nuyx)) (nuyx*Ex)/(1-(nuxy*nuyx)) 0
          (nuyx*Ex)/(1-(nuxy*nuyx)) Ey/(1-(nuxy*nuyx)) 0
          0 0 Ex/(2*(1+nuxy)) ];
end
BD = J*transpose(B)*R*D*transpose(R)*B;
r = int(int(BD, t, -1, 1), s, -1, 1);
z = h*r;
w = double(z);
KE = w;
```
Appendix C

%%%% AN 88 LINE TOPOLOGY OPTIMIZATION CODE Nov, 2010 %%%%
function top88(nelx,nely,volfrac,penal,rmin,ft)

%%% MATERIAL PROPERTIES
E0 = 1;
Emin = 0.9;
nu = 0.3;

%%% PREPARE FINITE ELEMENT ANALYSIS
A11 = [12 3 -6 -3; 3 12 0 -6; -6 3 12 -3; -3 -6 0 12];
A12 = [-6 -3 0 3; -3 -6 -3 0; 0 -6 3 -3; 3 -6 -3 -6];
B11 = [-4 3 -2 9; 3 -4 -9 4; -2 -9 -4 -3; 9 4 -3 -4];
B12 = [2 -3 4 -9; -3 -2 9 -2; 4 9 2 3; -9 -2 3 2];
KE = 1/(1-ν^2)/24*([A11 A12;A12' A11]+ν*[B11 B12;B12' B11]);
nodenss = reshape(1:(1+nelx)*(1+nely),1+nely,1+nelx);
edofVec = reshape(2*nodenrs(1:end-1,1:end-1)+1,nelx*nely,1);
edofMat = repmat(edofVec,1,8)+repmat([0 1 2*nely+[2 3 0 1] -2 -1],nelx*nely,1);
iK = reshape(kron(edofMat,ones(8,1)),64*nelx*nely,1);
jK = reshape(kron(edofMat,ones(1,8)),64*nelx*nely,1);

%%% DEFINE LOADS AND SUPPORTS (HALF MBB-BEAM)
F = sparse(2,1,-1,2*(nely+1)*(nelx+1),1);
U = zeros(2*(nely+1)*(nelx+1),1);
fixeddofs = union([1:2:2*(nely+1)], [2*(nelx+1)*(nely+1)]);
alldofs = [1:2*(nely+1)*(nelx+1)];
freedofs = setdiff(alldofs,fixeddofs);

%%% PREPARE FILTER
iH = ones(nelx*nely*(2*(ceil(rmin)-1)+1)^2,1);
jH = ones(size(iH));
sH = zeros(size(iH));
k = 0;
for i1 = 1:nelx
  for j1 = 1:nely
    e1 = (i1-1)*nely+j1;
    for i2 = max(1-(ceil(rmin)-1),1):min(i1+(ceil(rmin)-1),nelx)
      for j2 = max(1-(ceil(rmin)-1),1):min(j1+(ceil(rmin)-1),nely)
        e2 = (i2-1)*nely+j2;
        k = k+1;
        iH(k) = e1;
        jH(k) = e2;
        sH(k) = max(0,rmin-sqrt((i1-i2)^2+(j1-j2)^2));
      end
    end
  end
end
H = sparse(iH,jH,sH);
Hs = sum(H,2);

%%% INITIALIZE ITERATION
x = repmat(volfrac,nely,nelx);
xPhys = x;
loop = 0;
change = 1;

%%% START ITERATION
while change > 0.01
  loop = loop + 1;
  change = 1;
  %% FE-ANALYSIS
  sK = reshape(KE(:)*(Emin+xPhys(:).^penal*(E0-Emin)),64*nelx*nely,1);
K = sparse(iK,jK,sK); K = (K+K')/2;
U(freedofs) = K(freedofs,freedofs)
%% OBJECTIVE FUNCTION AND SENSITIVITY ANALYSIS
ce = reshape(sum((U(edofMat).*KE).*U(edofMat),2),nely,nelx);
c = sum(sum((Emin+xPhys.*penal*(E0-Emin)).*ce));
dc = -penal*(E0-Emin)*xPhys.^penal-1.*ce;
dv = ones(nely,nelx);
%% FILTERING/MODIFICATION OF SENSITIVITIES
if ft == 1
dc(:) = H*(x(:).*dc(:))./Hs./max(1e-3,x(:));
elseif ft == 2
dc(:) = H*(dc(:)/Hs);
dv(:) = H*(dv(:)/Hs);
end
%% OPTIMALITY CRITERIA UPDATE OF DESIGN VARIABLES AND PHYSICAL DENSITIES
l1 = 0; l2 = 1e9; move = 0.2;
while (l2-l1)/(l1+l2) > 1e-3
lmid = 0.5*(l2+l1);
xnew = max(0,max(x-move,min(1,min(x+move,x.*sqrt(-dc./dv/lmid)))));
if ft == 1
xPhys = xnew;
elseif ft == 2
xPhys(:) = H*xnew(:))/Hs;
end
if sum(xPhys(:)) > volfrac*nelx*nely, l1 = lmid; else l2 = lmid; end
end
change = max(abs(xnew(:)-x(:))); x = xnew;
%% PRINT RESULTS
fprintf(' It: %5i Obj: %.11f Vol: %.7.3f ch: %.7.3f
', loop, c, mean(xPhys(:)), change);
%% PLOT DENSITIES
colormap(gray); imagesc(1-xPhys); caxis([-1 1]); axis equal; axis off; drawnow;
end
Appendix D

%%% A SOFT-KILL BESO CODE BY X. HUANG and Y.M. Xie

function softbeso(nelx,nely,volfrac,er,rmin);
% INITIALIZE
x(1:nely,1:nelx) = 1.; vol=1.; i = 0; change = 1.; penal = 3.;
% START ITH ITERATION
while change > 0.001
  i = i + 1; vol = max(vol*(1-er),volfrac);
  if i >1; olddc = dc; end
% FE-ANALYSIS
[U]=FE(nelx,nely,x,penal);
% OBJECTIVE FUNCTION AND SENSITIVITY ANALYSIS
[KE] = lk;
c(i) = 0.;
for ely = 1:nely
  for elx = 1:nelx
    n1 = (nely+1)*(elx-1)+ely;
    n2 = (nely+1)* elx   +ely;
    Ue = U([2*n1-1;2*n1; 2*n2-1;2*n2; 2*n2+1;2*n2+2; 2*n1+1;2*n1+2],1);
    c(i) = c(i) + 0.5*x(ely,elx)^penal*Ue'*KE*Ue;
    dc(ely,elx) = 0.5*x(ely,elx)^(penal-1)*Ue'*KE*Ue;
  end
end
% FILTERING OF SENSITIVITIES
[dc] = check(nelx,nely,rmin,x,dc);
% STABLIZATION OF EVOLUTIONARY PROCESS
if i > 1; dc = (dc+olddc)/2.; end
% BESO DESIGN UPDATE
[x] = ADDDEL(nelx,nely,vol,dc,x);
% PRINT RESULTS
if i>10;
  change=abs(sum(c(i-9:i-5))-sum(c(i-4:i)))/sum(c(i-4:i));
end
disp([ ' It.: ' sprintf('%4i',i) ' Obj.: ' sprintf('%10.4f',c(i)) ...
         ' Vol.: ' sprintf('%6.3f',sum(sum(x))/(nelx*nely)) ...
         ' ch.: ' sprintf('%6.3f',change )])
% PLOT DENSITIES
colormap(gray); imagesc(-x); axis equal; axis tight; axis off; pause(1e-6);
end

%%% OPTIMALITY CRITERIA UPDATE
function [x]=ADDDEL(nelx,nely,volfra,dc,x)
l1 = min(min(dc)); l2 = max(max(dc));
while (((l2-l1)/l2 > 1.0e-5)
  th = (l1+l2)/2.0;
  x = max(0.001,sign(dc-th));
  if sum(sum(x))-volfra*(nelx*nely) > 0;
    l1 = th;
  else
    l2 = th;
  end
end

%%% MESH-INDEPENDENCY FILTER
function [dcf]=check(nelx,nely,rmmin,x,dc)
Appendix D

dcf=zeros(nely,nelx);
for i = 1:nelx
  for j = 1:nely
    sum=0.0;
    for k = max(i-floor(rmin),1):min(i+floor(rmin),nelx)
      for l = max(j-floor(rmin),1):min(j+floor(rmin),nely)
        fac = rmin-sqrt((i-k)^2+(j-l)^2);
        sum = sum+max(0,fac);
        dcf(j,i) = dcf(j,i) + max(0,fac)*dc(l,k);
      end
    end
    dcf(j,i) = dcf(j,i)/sum;
  end
end

%%%%%%%%%%%%%%%%%%%%% FE-ANALYSIS
function [U]=FE(nelx,nely,x,penal)
    [KE] = lk;
    K = sparse(2*(nelx+1)*(nely+1), 2*(nelx+1)*(nely+1));
    F = sparse(2*(nely+1)*(nelx+1),1);
    U = zeros(2*(nely+1)*(nelx+1),1);
    for elx = 1:nelx
      for ely = 1:nely
        n1 = (nely+1)*(elx-1)+ely;
        n2 = (nely+1)* elx +ely;
        edof = [2*n1-1; 2*n1; 2*n2-1; 2*n2; 2*n2+1; 2*n2+2; 2*n1+1; 2*n1+2];
        K(edof,edof) = K(edof,edof) + x(ely,elx)^penal*KE;
      end
    end
    % DEFINE LOADS AND SUPPORTS (Cantilever)
    F = sparse([2,1,-1,2*(nely+1)]*(nelx+1),1);
    fixeddofs = union([1:2:2*(nely+1)],[(nelx+1)*(nely+1)]);
    alldofs = [1:2*(nely+1)*(nelx+1)];
    freedofs = setdiff(alldofs,fixeddofs);
    % SOLVING
    U(freedofs,:) = K(freedofs,freedofs) \ F(freedofs,:);
    U(fixeddofs,:)= 0;
end

%% ELEMENT STIFFNESS MATRIX
function [KE]=lk
    E = 1.0; nu = 0.3;
    k= 1/2-nu/6 1/8+nu/8 -1/4-nu/12 -1/8+3*nu/8 ...
    -1/4+nu/12 -1/8-nu/8 nu/6 1/8-3*nu/8];
    KE = E/(1-nu^2)*[ k(1) k(2) k(3) k(4) k(5) k(6) k(7) k(8)
    k(2) k(1) k(8) k(7) k(6) k(5) k(4) k(3)
    k(3) k(8) k(1) k(6) k(7) k(4) k(5) k(2)
    k(4) k(7) k(6) k(1) k(8) k(3) k(2) k(5)
    k(5) k(6) k(7) k(8) k(1) k(2) k(3) k(4)
    k(6) k(5) k(4) k(3) k(2) k(1) k(8) k(7)
    k(7) k(4) k(5) k(2) k(3) k(8) k(1) k(6)
    k(8) k(3) k(2) k(5) k(4) k(7) k(6) k(1)];
% TOPOLOGY OPTIMIZATION USING THE LEVEL-SET METHOD, VIVIEN J. CHALLIS 2009

function [struc] = top_levelset(nelx,nely,volReq,stepLength,numReinit,topWeight)

% Initialization
struc = ones(nely,nelx);
[lsf] = reinit(struc);
shapeSens = zeros(nely,nelx); topSens = zeros(nely,nelx);
[KE,KTr,lambda,mu] = materialInfo();

% Main loop:
for iterNum = 1:200
% FE-analysis, calculate sensitivities
[U] = FE(struc,KE);
for ely = 1:nely
    for elx = 1:nelx
        n1 = (nely+1)*(elx-1)+ely;
        n2 = (nely+1)*elx  +ely;
        Ue = U([2*n1-1;2*n1; 2*n2-1;2*n2; 2*n2+1;2*n2+2; 2*n1+1;2*n1+2],1);
        shapeSens(ely,elx) = -max(struc(ely,elx),0.0001)*Ue'*KE*Ue;
        topSens(ely,elx) = struc(ely,elx)*pi/2*(lambda+2*mu)/mu/(lambda+mu)*
        (4*mu*Ue'*KE*Ue+(lambda-mu)*Ue'*KTr*Ue);
    end
end
% Store data, print & plot information
objective(iterNum) = -sum(shapeSens(:));
volCurr = sum(struc(:))/(nelx*nely);
disp([' It.: ' num2str(iterNum) ' Compl.: ' sprintf('%10.4f',objective(iterNum)) ...
       ' Vol.: ' sprintf('%6.3f',volCurr)]);
end
% Check for convergence
if iterNum > 5 && ( abs(volCurr-volReq) < 0.005 ) && ...
    all( abs(objective(end)-objective(end-5:end-1) ) < 0.01*abs(objective(end)) )
    return;
end
% Set augmented Lagrangian parameters
if iterNum == 1
    la = -0.01; La = 1000; alpha = 0.9;
else
    la = la - 1/La * (volCurr - volReq); La = alpha * La;
end
% Include volume sensitivities
shapeSens = shapeSens - la + 1/La*(volCurr-volReq);
topSens = topSens + pi*(la - 1/La*(volCurr-volReq));

% Design update
[struc,lsf] = updateStep(lsf,shapeSens,topSens,stepLength,topWeight);
% Reinitialize level-set function
if ~mod(iterNum,numReinit)
    [lsf] = reinit(struc);
end
end

%%---- REINITIALIZATION OF LEVEL-SET FUNCTION ----
function [lsf] = reinit(struc)
strucFull = zeros(size(struc)+2); strucFull(2:end-1,2:end-1) = struc;

% Use "bwdist" (Image Processing Toolbox)
lsf = (~strucFull).*bwdist(strucFull-0.5) - strucFull.*bwdist(strucFull-1)-0.5;

%%----- DESIGN UPDATE ----
function [struc, lsf] = updateStep(lsf, shapeSens, topSens, stepLength, topWeight)

% Smooth the sensitivities
[shapeSens] = conv2(padarray_vjc(shapeSens), 1/6*[0 1 0; 1 2 1; 0 1 0], 'valid');
[topSens] = conv2(padarray_vjc(topSens), 1/6*[0 1 0; 1 2 1; 0 1 0], 'valid');

% Load bearing pixels must remain solid - Bridge:
shapeSens(end, [1, round(end/2):round(end/2+1), end]) = 0;
topSens(end, [1, round(end/2):round(end/2+1), end]) = 0;

% Design update via evolution
[struct, lsf] = evolve(-shapeSens, topSens.*(lsf(2:end-1,2:end-1)<0), lsf, stepLength, topWeight);

%%---- EVOLUTION OF LEVEL-SET FUNCTION ----
function [struc, lsf] = evolve(v, g, lsf, stepLength, w)

% Extend sensitivities using a zero border
vFull = zeros(size(v)+2); vFull(2:end-1,2:end-1) = v;
gFull = zeros(size(g)+2); gFull(2:end-1,2:end-1) = g;

% Choose time step for evolution based on CFL value
dt = 0.1/max(abs(v(:)));

% Evolve for total time stepLength * CFL value:
for i = 1:(10*stepLength)

% Calculate derivatives on the grid
dpx = circshift(lsf,[0,-1])-lsf;
dmx = lsf - circshift(lsf,[0,1]);
dpy = circshift(lsf,[-1,0]) - lsf;
dmy = lsf - circshift(lsf,[1,0]);

% Update level set function using an upwind scheme
lsf = lsf - dt * min(vFull,0).* ...
    sqrt( min(dmx,0).^2+max(dpx,0).^2+min(dmy,0).^2+max(dpy,0).^2 ) ...
    - dt * max(vFull,0).* ...
    sqrt( max(dmx,0).^2+min(dpx,0).^2+max(dmy,0).^2+min(dpy,0).^2 ) ...
    - w*dt*gFull;
end

% New structure obtained from lsf
strucFull = (lsf<0); struc = strucFull(2:end-1,2:end-1);

%%---- FINITE ELEMENT ANALYSIS ----
function [U] = FE(struc, KE)

[nely, nelx] = size(struc);
K = sparse(2*(nelx+1)*(nely+1), 2*(nelx+1)*(nely+1));
F = sparse(2*(nely+1)*(nelx+1),1); U = zeros(2*(nely+1)*(nelx+1),1);
for elx = 1:nelx
    for ely = 1:nely
        n1 = (nely+1)*(elx-1)+ely;
        n2 = (nely+1)* elx +ely;
        edof = [2*n1-1; 2*n1; 2*n2-1; 2*n2; 2*n2+1; 2*n2+2; 2*n1+1; 2*n1+2];
        K(edof,edof) = K(edof,edof) + max(struc(ely,elx),0.0001)*KE;
    end
end

% Define loads and supports - Half MBB-beam:
F = sparse(2,1,-1,2*(nely+1)*(nelx+1),1);
fixeddofs = union([1:2:2*(nely+1)],[2*(nelx+1)*(nely+1)]);
alldofs = [1:2*(nely+1)+nelx+1];
freedofs = setdiff(alldofs,fixeddofs);
U(freedofs,:) = K(freedofs,freedofs) \ F(freedofs,:);

%%---- MATERIAL INFORMATION ----
function [KE, KTr, lambda, mu] = materialInfo()

% Set material parameters, find Lame values
E = 1.; nu = 0.3;
lambda = E*nu/((1+nu)*(1-nu)); mu = E/(2*(1+nu));

% Find stiffness matrix "KE"
k = [1/2-\nu/6   1/8+\nu/8 -1/4-\nu/12 -1/8+3\nu/8 ...
-1/4+\nu/12 -1/8-\nu/8  \nu/6  1/8-3\nu/8];
KE = E/(1-\nu^2)*stiffnessMatrix(k);
% Find "trace" matrix "KTr"
k = [1/3 1/4 -1/3 1/4 -1/6 -1/4 1/6 -1/4];
KTr = E/(1-\nu)*stiffnessMatrix(k);
%%---- ELEMENT STIFFNESS MATRIX ----
function [K] = stiffnessMatrix(k)
% Forms stiffness matrix from first row
K=[k(1) k(2) k(3) k(4) k(5) k(6) k(7) k(8)
k(2) k(1) k(8) k(7) k(6) k(5) k(4) k(3)
k(3) k(8) k(1) k(6) k(7) k(4) k(5) k(2)
k(4) k(7) k(6) k(1) k(8) k(3) k(2) k(5)
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k(7) k(4) k(5) k(2) k(3) k(8) k(1) k(6)
k(8) k(3) k(2) k(5) k(4) k(7) k(6) k(1)];