Batch-to-batch rational feedforward control

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Batch-to-Batch Rational Feedforward Control: From Iterative Learning to Identification Approaches, With Application to a Wafer Stage

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Abstract—Feedforward control enables high performance for industrial motion systems that perform nonrepeating motion tasks. Recently, learning techniques have been proposed that improve both performance and flexibility to nonrepeating tasks in a batch-to-batch fashion by using a rational parameterization in feedforward control. This paper aims to unify these approaches through a single framework that provides transparent connections and clear differences between the alternatives. Experimental results on an industrial motion system confirm the theoretical findings and illustrate benefits of rational feedforward tuning in motion systems, including preactuation and postactuation.

Index Terms—Feedforward control, learning control, motion control.

I. INTRODUCTION

FEEDFORWARD control is an essential component in industrial motion systems, since it can compensate for known disturbances before these affect the system. The main performance improvement is typically achieved by using feedforward to compensate for the reference, see, e.g., [1]–[3].

Improvements in feedforward control are an important step toward meeting tightening requirements on throughput and accuracy in next-generation industrial motion systems. The following requirements are imposed on feedforward control for next-generation motion systems:

- R1) small servo error;
- R2) high flexibility to nonrepeating tasks.

Requirement R1 is a common requirement for industrial motion systems. Requirement R2 is crucial since industrial motion systems often perform nonrepeating tasks. For exam-

Fig. 1. Batch-to-batch feedforward control with parameterized feedforward combines the advantages of ILC and model-based feedforward, aiming to simultaneously satisfy R1 and R2.

ple, wafer scanners perform varying motion tasks to control the wafer height during exposure [4], [5]. In semiconductor bonding equipment, systems perform varying tasks due to small corrections required in the pick and/or place location, see, e.g., [6] and [7]. Performance deterioration due to nonrepeating tasks restricts the achievable throughput, and is, therefore, typically not acceptable in industrial applications.

A common approach to feedforward control is model-based feedforward. Typically, a parametric model of the plant is identified and inverted for use in feedforward as in [2]. In [8], the inverse system is identified directly. However, the achievable performance strongly depends on the model quality and the accuracy of the model inversion [9]. Hence, R1 is often not sufficiently satisfied. Note that model-based feedforward approaches typically achieve R2. This is illustrated in Fig. 1.

Iterative learning control (ILC) [1] can significantly improve the performance of systems subject to repeating tasks. As a result, R1 is typically achieved. In ILC, the feedforward signal is determined in a batch-to-batch fashion, see, e.g., [10]. After each task, the feedforward signal for the next task is determined based on measured data and an approximate model of the system. However, extrapolation of the learned signal to other tasks often results in a significant performance deterioration, see, e.g., [11] and [12]. Hence, R2 is not achieved by ILC, see Fig. 1.

To meet R2, a segmented approach to ILC is presented in [13]. This approach is extended in [14], where the task is divided into subtasks that are learned separately. The use of such a signal library can be restrictive since tasks are required to consist of previously learned subtasks. Instead of learning subtasks, ILC

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has been extended with basis functions in, e.g., [15] and [16]. The basis functions are used to parametrize the feedforward signal in terms of the motion task. This approach is further extended in [17], where a rational parametrization of the feedforward controller is introduced, which enables R1 and R2 for the generic class of linear rational systems. These recent developments in ILC are promising for motion control with nonrepeating tasks, yet require an approximate model of the system.

Recently, an alternative approach has been developed to batch-to-batch feedforward control that enables estimation of feedforward controllers based on measured data only, denoted iterative feedforward tuning. An approach for a rational parameterization is proposed [18], and results for a polynomial parameterization are presented in [19] and [20]. For the specific class of point-to-point motion tasks, an extension toward input shaping is presented in [21]. Interestingly, rational feedforward controllers can also be used to generate preactuation by means of stable inversion procedures, as will be experimentally demonstrated in this paper.

Although important theoretical steps have been made toward achieving R1 and R2 in batch-to-batch feedforward control, see Fig. 1, at present the connections and practical differences between various approaches are not yet fully established. The contribution of this paper is threefold. First, a unifying framework is proposed for batch-to-batch feedforward control. Second, it is shown that iterative feedforward tuning can be interpreted in the framework of norm-optimal ILC with basis functions, yet without the need for a model. Third, an experimental comparison is performed on an industrial motion system, illustrating differences in the achievable performance, and validating the proposed approach for pre-actuation. In fact, benefits of stable inversion are clearly observed in the experimental results. Related approaches include the use of such methods for decoupling, see, e.g., [22] and [23]. In contrast, the aim of this paper is on requirements R1 and R2 for feedforward control. Preliminary research on this topic is presented in [24].

This paper is organized as follows. In Section II, the notation is introduced. In Section III, a motivation is presented for the use of batch-to-batch parametrized feedforward control in view of requirements R1 and R2. In Section IV, a unifying framework is introduced for batch-to-batch feedforward control. Procedures for norm-optimal ILC and iterative feedforward tuning with rational parametrizations are proposed in Sections V and VI, respectively. In Section VII, an approach is investigated enabling preactuation and postactuation. In Section VIII, an experimental comparison is provided of the presented approaches to batch-to-batch parametrized feedforward control. Finally, conclusions are provided in Section IX.

II. PRELIMINARIES

Let $\mathbf{H}(z)$ denote a discrete-time, linear time-invariant (LTI), single-input, single-output (SISO) system. A positive definite matrix A is denoted $A \succ 0$. For a vector x , the weighted two norm is given by $\|x\|_W^2 = x^\top W x$. The i th element of x is expressed as $x[i]$. The identity matrix of size n is denoted I_n .

Signals are often assumed to be of length N . Given input and output vectors $u, y \in \mathbb{R}^{N \times 1}$. Let $h(t)$ be the impulse response

Fig. 2. In conventional ILC, the signal f^j is updated based on measured data after each task j . In contrast, in batch-to-batch parametrized feedforward control, parameters θ^j of $f^j = C_{ff}(\theta^j)r^j$ are updated after each task j .

vector of $\mathbf{H}(z)$. Then, the finite-time response of the possibly noncausal $\mathbf{H}(z)$ to input u is given by the truncated convolution $y[t] = \sum_{l=1-N}^t h(l)u[t-l]$, where $0 \leq t < N$ and zero initial and final conditions are assumed, i.e., $u(t) = 0$, $y(t) = 0$ for all $t < 0$ and $t \geq N$. The finite-time convolution is denoted as

$$\begin{bmatrix} y[0] \\ y[1] \\ \vdots \\ y[N-1] \end{bmatrix} = \underbrace{\begin{bmatrix} h(0) & h(-1) & \dots & h(1-N) \\ h(1) & h(0) & \dots & h(2-N) \\ \vdots & \vdots & \ddots & \vdots \\ h(N-1) & h(N-2) & \dots & h(0) \end{bmatrix}}_H \underbrace{\begin{bmatrix} u[0] \\ u[1] \\ \vdots \\ u[N-1] \end{bmatrix}}_u$$

with H the convolution matrix corresponding to $\mathbf{H}(z)$.

III. MOTIVATION

In this section, theoretical and experimental results are presented to show that standard ILC approaches can typically not achieve R2. Moreover, it is shown that by extending ILC with basis functions, R1 and R2 can be achieved simultaneously. This is the key motivation for this approach. As shown in the remainder of this paper, ILC with basis functions is a particular implementation of a batch-to-batch parametrized feedforward approach. Hence, the results in this section aim to motivate the use of a batch-to-batch parametrized feedforward approach.

The considered control setup is shown in Fig. 2, and consists of a stabilizing feedback controller C_{fb} , an unknown system P , and optionally a feedforward controller C_{ff} . A sequence of finite-time tasks is performed, denoted by index $j = 0, 1, 2, \dots$. Furthermore, r^j denotes the reference, f^j the feedforward signal, y^j the measured output, v^j an unknown disturbance, and e^j the measured error in task j (resp. e^{j+1} in task $j+1$) given by

$$e^j = S r^j - S P f^j - S v^j \quad (1)$$

$$e^{j+1} = S r^{j+1} - S P f^{j+1} - S v^{j+1} \quad (2)$$

with $S = (I + P C_{fb})^{-1}$. In Sections III-A and III-B, the inflexibility of ILC to nonrepeating tasks is demonstrated. This is

the key motivation to adopt the use of parametrized feedforward approaches in Section III-C.

A. Inflexibility of ILC to Nonrepeating Tasks

In a typical ILC approach, f^{j+1} is determined by using a frequency-domain or a time-domain ILC approach, see Appendix I and [1] and [25]. Their solutions can typically be expressed in an ILC update law, see, e.g., [1], of the form

$$f^{j+1} = Q_f f^j + L_f e^j \quad (3)$$

with robustness filter Q_f and learning filter L_f . Detailed expressions for L_f and Q_f are presented in, e.g., [1]. A key assumption underlying (3) is that the system performs repeating tasks, i.e., $r^{j+1} = r^j \forall j$. Violating this assumption can significantly affect the achievable performance, as is shown next.

Example 1: Inflexibility of ILC. Suppose that $f^j = P^{-1}r^j$, $Q = I$, $v^{j+1} = v^j = 0$, and $r^{j+1} \neq r^j$. Substituting $f^j = P^{-1}r^j$ into (1) gives $e^j = 0$. By using $e^j = 0$ and $Q = I$ in (3), it follows that $f^{j+1} = f^j$, i.e., the ILC update law in (3) is converged. Then, substitution of $f^{j+1} = f^j = P^{-1}r^j$ into (2), and using $v^{j+1} = 0$, yields

$$e^{j+1} = S(r^{j+1} - r^j). \quad (4)$$

This shows that the performance in task $j+1$ can significantly deteriorate if $r^{j+1} \neq r^j$, which compromises R2. Note that the achieved performance can become worse than by using feedback only, for example, if $r^{j+1} = -r^j$.

B. Experimental Demonstration of Inflexibility of ILC

The assumption that $r^{j+1} = r^j$ for all j is often violated in motion systems. For example, nonrepeating tasks are often encountered in wafer stages [13]. Next, the importance of a feedforward approach that incorporates flexibility with respect to nonrepeating tasks is experimentally illustrated on a wafer stage as used in Section VIII, shown in Fig. 5.

Example 2: Inflexibility of ILC (Experimental—Preview on Section VIII). The two reference trajectories considered in this experimental example are depicted in Fig. 3(a). The number of tasks is 14. Reference trajectory r_1 is used in tasks $j = 0, 1, \dots, 6$, while r_2 is used in tasks $j = 7, 8, \dots, 13$. The ILC approach in Appendix I is used, where the design guidelines follow [1]. The learned feedforward signal is not reinitialized when changing the reference. The results in Fig. 3(b) experimentally confirm that the performance with standard ILC significantly deteriorates when confronted with nonrepeating tasks due to considerable learning transients. Hence, R2 is not met.

These experimental observations match with earlier observations, see, e.g., [11], [12], and [26]. That is, direct application of standard ILC algorithms can lead to a significant performance deterioration when confronted with nonrepeating tasks.

C. Flexibility Through Parametrized Feedforward

Flexibility with respect to nonrepeating tasks is a key reason to introduce a parametrization for f^{j+1} in ILC, see, e.g., [6], [12], [15], and [16]. In such an approach, f^{j+1} is parameterized

Fig. 3. Demonstration of inflexibility of standard ILC. During the first seven trials r_1 (—) is performed, while r_2 (---) is performed in the last seven trials. The performance with standard ILC (—○) significantly deteriorates after the reference change at $j = 7$. In contrast, the batch-to-batch parametrized feedforward approach (—○) is insensitive to the reference change at $j = 7$. (a) Reference signals r_1 and r_2 . (b) Sample normalized two norm of error signals e^j .

in terms of parameters θ^{j+1} , and the reference trajectory r^{j+1} . Here, the following parametrization is used

$$f^{j+1} = C_{ff}(\theta^{j+1})r^{j+1}. \quad (5)$$

Detailed expressions for the feedforward controller $C_{ff}(\theta^{j+1})$ are given in Section IV-B. Essentially, the update law (5) aims to determine parameters θ^{j+1} , instead of a signal f^{j+1} as in (3). Similar to (3), the following update law is derived:

$$\theta^{j+1} = Q\theta^j + Le^j \quad (6)$$

where Q and L are robustness and learning filters, respectively. The next example shows that flexibility against nonrepeating tasks can be obtained by using the update law (6).

Example 3: Flexibility through parametrized feedforward. Suppose that $C_{ff}(\theta^j) = P^{-1}$, $Q = I$, $v^j = 0 \forall j$, and $r^{j+1} \neq r^j$. Then, f^j parametrized as in (5) is equal to $f^j = P^{-1}r^j$. Furthermore, substituting $f^j = P^{-1}r^j$ into (1) and using $v^j = 0$ gives $e^j = 0$. Then from (6), it follows that $\theta^{j+1} = \theta^j$, i.e., (6) is converged. As a result $C_{ff}(\theta^{j+1}) = C_{ff}(\theta^j)$, which implies $f^{j+1} = P^{-1}r^{j+1}$, and hence, (2) gives

$$e^{j+1} = S r^{j+1} - S P f^{j+1} = S(r^{j+1} - r^{j+1}) = 0 \quad (7)$$

for any r^{j+1} . Comparing the result (7) with (4) shows that a parametrized ILC update law as in (6) can significantly enhance flexibility to nonrepeating tasks, hence enabling R2.

Next, the flexibility to nonrepeating tasks of a batch-to-batch parametrized feedforward approach is experimentally illustrated on the wafer stage that is used in Section VIII.

Example 4: Flexibility through parametrized feedforward (Experimental—Continued). In this experimental example, a

specific implementation of (6) is applied to the industrial motion system of Fig. 5. Similar to Example 2, implementation details are omitted for clarity of exposition. The results in Fig. 3 illustrate the following.

- 1) The parametrized feedforward approach can effectively handle the change in the reference trajectory in task $j = 7$. These results indicate that a batch-to-batch parametrized feedforward approach can meet R2, as shown in Fig. 1.
- 2) The parametrized feedforward approach significantly improves performance compared to feedback only, confirming that batch-to-batch feedforward control sufficiently achieves R1, as in Fig. 1. Note that for a single specific reference, standard ILC outperforms the parametrized approach.

The presented theoretical and experimental examples indicate that a batch-to-batch parametrized feedforward approach as in (6) can achieve both R1 and R2, in contrast to standard ILC methods as in (3). It is emphasized that Q and L in (6) depend on: 1) the used batches of measured signals; 2) the parametrization for $C_{ff}(\theta^{j+1})$ in (5); and 3) the criterion used in the corresponding optimization problem. In the next section, a unifying framework is proposed to determine Q and L , and consequently θ^{j+1} , based on these three important aspects. The framework facilitates a coherent overview of the available approaches, such that a well-motivated choice can be made for the particular problem at hand.

IV. BATCH-TO-BATCH FEEDFORWARD FROM A SYSTEM IDENTIFICATION PERSPECTIVE

The goal in batch-to-batch feedforward control is to iteratively improve control performance by learning a feedforward controller C_{ff} from measured data in a batch-to-batch fashion. Essential for the achievable performance are: 1) the used batches of measured data; 2) the feedforward controller parameterization; and 3) the optimization criterion, which are the focus of Sections IV-A–IV-C, respectively. This typical system identification perspective on batch-to-batch feedforward control is illustrated in Fig. 4. On the basis of these aspects, an overview of approaches to batch-to-batch feedforward is provided in Section IV-D.

A. Measurement Data

To achieve high performance C_{ff} is optimized based on measured data. A sequence of finite-time tasks is performed under normal operating conditions. After each task, C_{ff} is updated based on batches of data before starting the next task. Here, data from a single task are used to update C_{ff} to facilitate presentation. Note that data from multiple tasks can be used, as in [27], based on similar algorithms as proposed in [28].

B. Feedforward Parameterization

To meet R2 in batch-to-batch feedforward control, f^{j+1} is parameterized in terms of parameters θ^{j+1} and r^{j+1} . As a result, nonrepeating tasks are explicitly taken into account in f^{j+1} . Many batch-to-batch feedforward approaches fit into this frame-

Fig. 4. Batch-to-batch feedforward from a system identification perspective. Based on a batch of data, parameterization C and criterion V , θ^{j+1} is determined and implemented to obtain a new batch of data.

work, including: basis tasks [13], [14], polynomial basis functions [15], [16], [20], [29], joint input shaping and feedforward [21], and rational basis functions [17], [18].

In this paper, the following rational parameterization is adopted, which encompasses rational and polynomial basis functions. The feedforward, see Fig. 2, is given by $f^{j+1} = C_{ff}(\theta^{j+1})r^{j+1}$ as in (5), where $C_{ff}(\theta^{j+1})$ is defined next.

Definition 1: The feedforward controller $C_{ff}(\theta)$ parameterized in terms of a rational basis C_{rat} is given by

$$C_{rat} = \left\{ C_{ff}(\theta) \mid C_{ff}(\theta) = B(\theta)^{-1}A(\theta), \quad \theta \in \mathbb{R}^{n_a+n_b} \right\}$$

where

$$A(\theta) = \sum_{i=1}^{n_a} \psi_i \theta[i], \quad B(\theta) = I + \sum_{i=n_a+1}^{n_a+n_b} \psi_i \theta[i].$$

Here, ψ_i is the convolution matrix corresponding to the polynomial basis function $\psi_i(z)$. The underlying transfer function $\mathbf{C}_{ff}(z, \theta)$ of $C_{ff}(\theta)$ can be constructed as $\mathbf{C}_{ff}(z, \theta) = \mathbf{B}^{-1}(z, \theta)\mathbf{A}(z, \theta)$ where $\mathbf{A}(z, \theta) = \sum_{i=1}^{n_a} \psi_i(z)\theta[i]$ and $\mathbf{B}(z, \theta) = 1 + \sum_{i=n_a+1}^{n_a+n_b} \psi_i(z)\theta[i]$. Furthermore, let $\Psi = [\Psi_A, \Psi_B]$ with $\Psi_A = [\psi_1, \dots, \psi_{n_a}]$ and $\Psi_B = [\psi_{n_a+1}, \dots, \psi_{n_a+n_b}]$.

Remark 1: Note that a polynomial parametrization C_{pol} is recovered by setting $B(\theta) = I$ in Definition 1. Furthermore, parameterization C_{rat} is a direct extension of joint input shaping and feedforward, explained in detail in [21].

C. Criterion

Based on measured data from previous tasks and a feedforward parametrization, the aim after task j of each approach to batch-to-batch feedforward control is to determine

$$\theta^{j+1} = \arg \min_{\theta^{j+1}} V(\theta^{j+1}) \quad (8)$$

where the criterion $V(\theta^{j+1})$ depends on the selected approach. Typically, $V(\theta^{j+1})$ is selected as some norm of the predicted error $\hat{e}^{j+1}$ in task $j+1$, see, e.g., [16] and [25]. Given SP, the error propagation from task j to $j+1$ is written as

$$\hat{e}^{j+1}(\theta^{j+1}) = e^j + SP(C_{ff}(\theta^j)r^j - C_{ff}(\theta^{j+1})r^{j+1}). \quad (9)$$

D. Overview of Approaches

In Sections IV-A–IV-C, the ingredients are provided for batch-to-batch feedforward in a system identification perspective, see Fig. 4. This section provides an overview of the con-

sidered approaches to determine θ^{j+1} . This paper focuses on measured data from a single task and parameterization $\mathcal{C}_{\text{rat}}$.

In Sections V and VI, norm-optimal ILC and iterative feedforward tuning approaches are presented for $\mathcal{C}_{\text{rat}}$, respectively. In each section, the corresponding criterion $V(\theta^{j+1})$ is defined and a solution algorithm for (8) is provided. The solutions for θ^{j+1} are interpreted in terms of the ILC update law as in (6) as follows:

$$\theta^{j+1} = Q\theta^j + Le^j \quad (10)$$

with robustness matrix $Q \in \mathbb{R}^{(n_a+n_b) \times (n_a+n_b)}$ and learning matrix $L \in \mathbb{R}^{(n_a+n_b) \times N}$.

V. NORM-OPTIMAL ILC WITH A RATIONAL BASIS

In Section IV, three ingredients to batch-to-batch feedforward control are defined. In this section, a norm-optimal ILC approach is pursued for rational parametrization $\mathcal{C}_{\text{rat}}$, as in [17]. This is a specific algorithm for the computation of θ^{j+1} , see Fig. 4. The corresponding criterion is defined next.

Procedure 1: Iterative scheme for rational ILC optimization.

Given a model $\widehat{SP}$ of SP and initial estimate θ^0 , set the task index $j = 0$ and perform the following sequence of steps.

- 1) Construct $C_{ff}(\theta^j)$, perform task j , and measure e^j .
 - 2) Determine θ^{j+1} with the following iterative scheme.
 - a) Set $k = 0$ and initialize $\theta_{(0)}^{j+1} = \theta^j$.
 - b) Construct $L_{\text{ILC},(k)}$ and $Q_{\text{ILC},(k)}$ with $\theta_{(k)}^{j+1}$.
 - c) Determine $\theta_{(k+1)}^{j+1} = Q_{\text{ILC},(k)}\theta^j + L_{\text{ILC},(k)}e^j$.
 - d) Until a stopping condition is met, set $k \rightarrow k+1$ and return to 2b). Else, set $\theta^{j+1} = \theta_{(k+1)}^{j+1}$.
 - 3) Set task index $j \rightarrow j+1$ and return to 1).
-

Definition 2: The criterion for norm-optimal ILC with basis functions is given by

$$V(\theta^{j+1}) = \|\hat{e}^{j+1}(\theta^{j+1})\|_W^2 \quad (11)$$

where $W \succ 0$ is a user-defined weighting matrix and $\hat{e}^{j+1}(\theta^{j+1})$ is the predicted error in task $j+1$.

This criterion can be directly extended by including weights on the feedforward signal as in, e.g., [16] and [25]. To facilitate presentation, the basic form (11) is used.

The solution to (8) for this approach with criterion (11), formulated as an ILC update law, is given by

$$\theta_{(k+1)}^{j+1} = Q_{\text{ILC},(k)}\theta^j + L_{\text{ILC},(k)}e^j \quad (12)$$

where index k denotes the k th computational iteration within the calculation of θ^{j+1} . Next, expressions are provided for $Q_{\text{ILC},(k)}$ and $L_{\text{ILC},(k)}$, and attention is given to important aspects in the

computation of θ^{j+1} . The learning matrices are

$$L_{\text{ILC},(k)} = \left[\left(\frac{\partial \hat{e}^{j+1}(\theta_{(k)}^{j+1})}{\partial \theta_{(k)}^{j+1}} \right)^\top W \Phi_{\text{ILC}}(\theta_{(k)}^{j+1}) \right]^{-1} \cdot \left(\frac{\partial \hat{e}^{j+1}(\theta_{(k)}^{j+1})}{\partial \theta_{(k)}^{j+1}} \right)^\top W B^{-1}(\theta_{(k)}^{j+1}) B^{-1}(\theta^j) \\ Q_{\text{ILC},(k)} = L_{\text{ILC},(k)} B(\theta^j) \Phi_{\text{ILC}}(\theta^j) \begin{bmatrix} I_{n_a} & 0 \\ 0 & -I_{n_b} \end{bmatrix}$$

where 0 denotes a zero matrix. A proof of (12) is provided in Appendix II. In view of (9), $\hat{e}^{j+1}(\theta^{j+1})$ is given by

$$\hat{e}^{j+1}(\theta^{j+1}) = B^{-1}(\theta^{j+1})\tilde{e}^j - \Phi_{\text{ILC}}(\theta^{j+1})\theta^{j+1} \quad (13)$$

with $\tilde{e}^j = e^j + \widehat{SP}f^j$ and regression matrix

$$\Phi_{\text{ILC}}(\theta^{j+1}) = B^{-1}(\theta^{j+1}) \left[\widehat{SP}\Psi_A r^j, -\Psi_B \tilde{e}^j \right]. \quad (14)$$

Three key observations are made. First, both $Q_{\text{ILC},(k)}$ and $L_{\text{ILC},(k)}$ use the approximate model $\widehat{SP}$ of SP . Second, if $B(\theta) = I$, i.e., $\mathcal{C}_{\text{pol}}$ is used, then (12) forms an analytic solution. Third, if $\mathcal{C}_{\text{rat}}$ is used, (11) is in general nonlinear in θ^{j+1} , requiring an iterative solution in (12). Typically, Procedure 1 is invoked, which is based on (12)–(14).

Summarizing, a solution algorithm is provided for norm-optimal ILC with a rational parametrization, achieving R1 and R2. Note that a model $\widehat{SP}$ is required to apply Procedure 1.

VI. IV-BASED ITERATIVE FEEDFORWARD TUNING WITH A RATIONAL BASIS

The developments in ILC regarding basis functions described in Section V are promising for motion control, yet require an approximate model of SP . Recently, an approach to batch-to-batch feedforward control is presented in [20] based on instrumental variables (IV), see, e.g., [30]. In this approach, the feedforward update is based on measured data only. This key difference potentially improves the convergence properties of the algorithm compared to norm-optimal ILC with basis functions, see Section VIII-C for experimental results.

In Section VI-A, the need for a model $\widehat{SP}$ is eliminated. However, the immediate and naive implementation of this approach suffers from a closed-loop identification problem. This is the main motivation to adopt IV-based approaches in iterative feedforward tuning. In Section VI-B, an IV-based approach is presented to iterative feedforward tuning for parametrization $\mathcal{C}_{\text{rat}}$ and interpreted in an ILC framework. Throughout, a standard assumption is imposed on v^j [31]: v^j is given by $v^j = H\varepsilon^j$, where H is monic and $\varepsilon^j \sim \mathcal{N}(0, \lambda_\varepsilon^2)$.

A. Eliminating the Need for an Approximate Model

In this section, the need for an approximate model of the system is eliminated by using measured data only. This relies on the observation, derived in [29], that

$$y^j = (C_{fb} + C_{ff}(\theta^j))SPr^j + Sv^j$$

is equivalent to

$$(C_{fb} + C_{ff}(\theta^j))^{-1} y^j = SP r^j \quad (15)$$

when assuming $v^j = 0$. Based on (15), measured data can be used to estimate $SP r$, eliminating the need to use $\widehat{SP}$. By replacing $\widehat{SP} r$ with (15) in Φ_{ILC} , (13) can be rewritten to

$$\hat{e}^{j+1}(\theta^{j+1}) = B^{-1}(\theta^{j+1}) \tilde{e}^j - \Phi_{IV}(\theta^{j+1}) \theta^{j+1} \quad (16)$$

with $\tilde{e}^j = e^j + C_{ff}(\theta^j) C^{-1} y^j$, $C = C_{fb} + C_{ff}(\theta^j)$, and

$$\Phi_{IV}(\theta^{j+1}) = B^{-1}(\theta^{j+1}) [\Psi_A C^{-1} y^j, -\Psi_B \tilde{e}^j]. \quad (17)$$

Comparing (17) with (14) shows that Φ_{IV} is fully based on measured data, while Φ_{ILC} contains an approximate model $\widehat{SP}$. The gradient of (16) with respect to θ^{j+1} becomes

$$\frac{\partial \hat{e}^{j+1}(\theta^{j+1})}{\partial \theta^{j+1}} = B^{-1}(\theta^{j+1}) [-\Psi_A, \Psi_B C_{ff}(\theta^{j+1})] C^{-1} y^j. \quad (18)$$

Using (17) and (18) enables the estimation of rational feedforward controllers, as in (12), based on measured data only. However, a detailed analysis reveals that the presence of noise, i.e., v^j , in fact introduces a closed-loop identification problem, see, e.g., [20] and [32]. As a consequence, biased estimates are obtained. To eliminate the closed-loop identification problem, a connection is proposed in [20] between IV identification techniques, see, e.g., [30], and iterative feedforward tuning.

B. IV-Based Rational Iterative Feedforward Tuning

The goal in instrumental variable-based iterative feedforward tuning is to obtain unbiased estimates θ^{j+1} for C_{ff} , using only measured data. To this purpose, a modification is proposed to criterion (11). It turns out that the minimizing argument to the modified criterion in fact minimizes (11). The criterion for IV-based feedforward tuning is defined next.

Definition 3: The criterion in instrumental variable-based iterative feedforward tuning is given by

$$V(\theta^{j+1}) = \|Z \hat{e}^{j+1}(\theta^{j+1})\|_W^2 \quad (19)$$

with instrumental variables $Z \in \mathbb{R}^{N \times (n_a + n_b)}$, $W \succ 0$ a user-defined weighting matrix, and $\hat{e}^{j+1}(\theta^{j+1})$ as in (16).

The proposed solution, recast to the form of (10), is

$$\theta_{(k+1)}^{j+1} = Q_{IV,(k)} \theta^j + L_{IV,(k)} e^j \quad (20)$$

where

$$L_{IV,(k)} = \left[ZW \Phi_{IV}(\theta_{(k)}^{j+1}) \right]^{-1} ZW B^{-1}(\theta_{(k)}^{j+1}) B^{-1}(\theta^j)$$

$$Q_{IV,(k)} = L_{IV,(k)} B(\theta^j) \Phi_{IV}(\theta^j) \begin{bmatrix} I_{n_a} & 0 \\ 0 & -I_{n_b} \end{bmatrix}$$

with $\Phi_{IV}(\theta^{j+1})$ defined in (17). A proof of (20) is provided in Appendix III. Three key observations are made. First, both $Q_{ILC,(k)}$ and $L_{ILC,(k)}$ do not require a model $\widehat{SP}$. Second, if $B(\theta) = I$, i.e., C_{pol} is used, then (20) is an analytic solution. Third, if C_{rat} is used, (19) is in general nonlinear in θ^{j+1} , requiring an iterative solution in (20).

With solution (20), unbiased estimates are obtained when Z is chosen correlated with r and uncorrelated with v^j . The remaining freedom in the design of Z can be used to achieve optimal accuracy of the estimates, see [18] for a detailed analysis. The following instruments are proposed:

$$Z^\top(\theta_{(k)}^{j+1}) = C_{(k)}^{-1} \begin{bmatrix} \Psi_A r^j \\ -C_{ff}(\theta_{(k)}^{j+1}) \Psi_B r^j \end{bmatrix} \quad (21)$$

with $C_{(k)} = C_{fb} + C_{ff}(\theta_{(k)}^{j+1})$. Similar to $\Phi_{IV}(\theta_{(k)}^{j+1})$, (21) depends on $\theta_{(k)}^{j+1}$. An iterative scheme is invoked to refine (21), as in, e.g., [33], which can be straightforwardly included in the iterative scheme that is already required to determine θ^{j+1} for C_{rat} . The iterative scheme is presented in Procedure 2. For C_{pol} , i.e., $B(\theta) = I$, the scheme provided in [20] is recovered.

Procedure 2: Iterative scheme for rational IV-based feedforward optimization.

Given an initial estimate θ^0 , set the task index $j = 0$ and perform the following sequence of steps.

- 1) Construct $C_{ff}(\theta^j)$, perform task j , and measure e^j, y^j .
 - 2) Determine θ^{j+1} with the following iterative scheme:
 - a) Set $k = 0$ and initialize $\theta_{(0)}^{j+1} = \theta^j$.
 - b) Construct $Z(\theta_{(k)}^{j+1})$, $L_{IV,(k)}$ and $Q_{IV,(k)}$ with $\theta_{(k)}^{j+1}$.
 - c) Determine $\theta_{(k+1)}^{j+1} = Q_{IV,(k)} \theta^j + L_{IV,(k)} e^j$.
 - d) Until a stopping condition is met, set $k \rightarrow k + 1$ and return to 2b). Else, set $\theta^{j+1} = \theta_{(k+1)}^{j+1}$.
 - 3) Set task index $j \rightarrow j + 1$ and return to 1).
-

Interestingly, it is shown in [33] that, when using instruments (21), the minimizing argument to (19) approximately minimizes (11) upon convergence of the iterative procedure. In other words, the obtained estimates minimize the two norm of the predicted error, which motivates the use of (19) in combination with (21) in batch-to-batch feedforward control.

In Sections V and VI, approaches are presented to construct rational feedforward controllers. In the next section, a procedure is proposed that essentially enables preactuation by means of rational feedforward controllers.

VII. STABLE INVERSION

In this section, a stable inversion approach is presented to determine f^{j+1} for a rational feedforward parametrization. This approach relies on the assumption that r is known beforehand and is bounded, which is a typical assumption in motion systems. Let θ^{j+1} be determined according to update laws (12) or (20). Then, $C_{ff}(\theta^{j+1})$ follows from Definition 1 together with the underlying transfer function $\mathbf{C}_{ff}(z, \theta^{j+1})$.

If $\mathbf{C}_{ff}(z, \theta^{j+1})$ is unstable, a bounded f^{j+1} cannot be constructed when filtering forward in time. Next, a stable inversion approach is investigated to deal with unstable $\mathbf{C}_{ff}(z, \theta^{j+1})$. Start by decomposing f^{j+1} into

$$f^{j+1} = \mathbf{C}_{ff}^{\text{st}}(z, \theta^{j+1}) r^{j+1} + \mathbf{C}_{ff}^{\text{un}}(z, \theta^{j+1}) r^{j+1}$$

where $\mathbf{C}_{ff}^{\text{st}}(z, \theta^{j+1})$ and $\mathbf{C}_{ff}^{\text{un}}(z, \theta^{j+1})$ contain the stable and unstable dynamics, respectively. By means of stable inversion

the unstable dynamics C_{ff}^{un} are filtered in backward time with suitable boundary conditions. Detailed state-space expressions for C_{ff}^{st} and C_{ff}^{un} are given in [20, Appendix A], see also [34] for further details. As a result, a bounded f^{j+1} is computed.

The key benefit of stable inversion for feedforward is that preactuation is possible through C_{ff}^{un} , i.e., the feedforward starts before the motion task. Simulation results have indicated that preactuation can potentially improve performance, see, e.g., [34]–[36]. In this paper, the benefits of stable inversion are demonstrated on the motion system in Fig. 5. Preactuation and postactuation are clearly observed in Fig. 8.

VIII. EXPERIMENTAL COMPARISON

In this section, the batch-to-batch feedforward approaches of Sections V and VI and the stable inversion procedure in Section VII are demonstrated on the industrial motion system, which is introduced in Section VIII-A. Table I lists the approaches that are experimentally compared. The following aspects are experimentally investigated.

- 1) The achievable performance with rational and polynomial feedforward is compared, see Section VIII-B.
- 2) The benefit of preactuation and postactuation for motion systems by means of stable inversion is investigated, see Section VIII-B.
- 3) The achievable performance of norm-optimal ILC and IV-based iterative feedforward tuning with rational bases is compared, see Section VIII-C.
- 4) The influence of the design of the instrumental variables on the achievable performance is demonstrated for IV-based iterative feedforward tuning, see Section VIII-D.

A. Experimental Setup

The considered next-generation wafer stage, see, e.g., [5], is shown in Fig. 5. The stage is controlled in all six motion degrees of freedom (DOFs), i.e., three translations and three rotations. The system is equipped with moving-coil permanent magnet planar motors that enable contactless operation, see [37] for the underlying principle. Laser interferometers enable subnanometer accuracy position measurements in all DOFs. All signals operate in discrete time with a sample time $T_s = 1/2500$ s. A stabilizing multivariable feedback controller is determined by means of sequential loop closing. The proposed feedforward approaches are applied to the main translational direction of motion. An identified frequency response function (FRF) of the corresponding system is shown in Fig. 5(c).

The control goal for the considered motion system is to minimize $\|e^j\|^2$, where y^j is the measured position of the system, and r is a fourth-order motion task with $N = 1001$, similar to those shown in Fig. 3(a) but with different constraints on the time derivatives. Unless stated otherwise, the basis functions used in polynomial parametrization C_{pol} are given by

$$\Psi_A(z) = \left[\frac{z-1}{zT_s}, \left(\frac{z-1}{zT_s} \right)^2, \left(\frac{z-1}{zT_s} \right)^3, \left(\frac{z-1}{zT_s} \right)^4, \left(\frac{z-1}{zT_s} \right)^5 \right]$$

Fig. 5. Experimental setup. ①: light beam, ②: reticle, ③: reticle stage, ④: lens, ⑤: wafer, ⑥: wafer stage, ⑦: metrology frame, ⑧: mover, ⑨: airmount. (a) Schematic illustration of a wafer scanner. (b) Photograph of experimental wafer stage. (c) FRF (—) of the plant P and model (---) used by ILC.

TABLE I
OVERVIEW OF APPROACHES COMPARED IN SECTION VIII

Approach	Basis	Instruments Z	VIII-B	VIII-C	VIII-D
IV-based FF, Sec. VI-B	C_{rat}	optimal, (21)	✓	✓	
	C_{pol}	optimal, (21)	✓		✓
	C_{pol}	suboptimal, (23) [19]			✓
N-O ILC, Sec. V	C_{rat}	-		✓	

corresponding with first- to fifth-order differentiators, as in, e.g., [20] and [38]. The corresponding parameters are denoted $\theta = [\theta_A^v, \theta_A^a, \theta_A^j, \theta_A^s, \theta_A^c]^\top$. The rational parameterization C_{rat} in Definition 1 is given by

$$\Psi_A(z) = \left[\frac{z-1}{zT_s}, \left(\frac{z-1}{zT_s} \right)^2, \left(\frac{z-1}{zT_s} \right)^4, \left(\frac{z-1}{zT_s} \right)^5 \right]$$

$$\Psi_B(z) = \left[\frac{z-1}{zT_s}, \left(\frac{z-1}{zT_s} \right)^2, \left(\frac{z-1}{zT_s} \right)^3 \right]$$

with $\theta = [\theta_A^v, \theta_A^a, \theta_A^s, \theta_A^c, \theta_B^v, \theta_B^a, \theta_B^j]^\top$. The initial parameters are set to zero, except for $\theta_A^a = 25$.

B. Rational Versus Polynomial Feedforward

In this section, the influence is investigated of the feedforward parameterization on the performance. To this purpose,

Fig. 6. Servo performance with iterative feedforward tuning improves by 50% with C_{rat} (—○) compared to C_{pol} (—○), see (a), (b). The dominant performance improvement is obtained around the resonance frequency of the plant at approximately 220 Hz, see (c). The convergence speed of ILC with C_{rat} (—○) is compromised compared to feedforward tuning due to the model quality of $\hat{S}P$, see (a). After convergence, similar performance is obtained. (a) Sample normalized two norm of error signals of iterative feedforward tuning with C_{rat} (—○), C_{pol} (—○), and ILC with C_{rat} (—○). (b) Time-domain error signals e^5 with C_{rat} (—) and C_{pol} (—). (c) Cumulative power spectral densities (CPS) of error signals e^5 with C_{rat} (—) and C_{pol} (—).

parametrizations C_{rat} and C_{pol} are compared for the IV-based iterative feedforward tuning approach in Section VI-B. Similar experimental results and conclusions for norm-optimal ILC with basis functions are omitted for brevity. The results are shown in Figs. 6–8, where the following observations are made.

- 1) The rational parametrization improves performance by 50% compared to polynomial feedforward in terms of $\|e^j\|^2$, see Fig. 6(a).
- 2) Both procedures converge in a single task, see Fig. 6(a). This validates the use of (15) in update law (20).

Fig. 7. Parameter estimates θ_v^A , θ_s^A , θ_c^A and θ_v^B as function of iterations for rational feedforward (—○) and polynomial feedforward (—○).

Fig. 8. Rational parameterization (—) enables preactuation (left inset) and postactuation (right inset) of the system, in contrast to the polynomial parametrization (—). The start and end times of the motion task are indicated by black dashed lines.

The performance enhancement of rational feedforward contributes to the following effects.

- 1) The dominant performance improvement of rational feedforward compared to polynomial feedforward is achieved in the frequency range 180–220 Hz, see Fig. 6(c). The enhanced flexibility of the rational basis enables compensation for the first flexible mode of the plant, whereas this is not achieved by the polynomial basis. This high-frequency difference can also be observed in the estimates corresponding to higher order basis functions, see θ_A^s and θ_A^c in Fig. 7.
- 2) Fig. 8 demonstrates preactuation, used to prevent transient errors at the start of the motion task, and postactuation, used to reduce residual vibrations in the system. This feature is key for the potential performance improvement

of rational feedforward compared to polynomial feedforward. Resulting effects can be observed in Fig. 6(b).

To further improve performance, the following is proposed.

- 1) Fig. 6(b) and 6(c) indicates that a low frequent contribution up to 60 Hz is not compensated for by the feedforward. This is attributed to nonlinearities in the system and the cable slab between the fixed world and the stage, acting as a low-frequency disturbance. Note that these disturbances are reproducible, as is shown in Section III, and can thus be compensated for. The difference in low-frequency contribution is related to the parameters corresponding to low-order basis functions, see, e.g., θ_A^v in Fig. 7.

C. Rational Feedforward: IV-Based Feedforward Tuning Versus Norm-Optimal ILC

In this section, norm-optimal ILC and IV-based feedforward tuning with rational bases are compared, see Sections V and VI. Fig. 5(c) depicts the model $\hat{P}$ for ILC. The results are shown in Fig. 6(a). The following observations are made.

- 1) The ILC procedure requires multiple tasks to converge, whereas the IV-based procedure converges in a single task. This is due to the imperfect model $\hat{S}\hat{P}$ used in (12). Consequently, the performance after one task is not yet optimal and needs to be further improved.
- 2) After convergence, both approaches achieve similar performance. This confirms the equivalence of criteria (11) and (19), as is argued in Section VI-B.

D. Design of Instrumental Variables

In Sections VIII-B and VIII-C, the optimal instrumental variables (21) are used in the IV-based iterative feedforward tuning approaches. It is argued in Section VI-B that these instruments minimize $\|e\|_2$. In this section, it is demonstrated that the design of instrumental variables is indeed essential for the achievable performance. For clarity of exposition, the IV-based iterative feedforward tuning approach with parametrization C_{pol} is considered, see Section VI-B. A comparison is provided between optimal instruments Z_o (21) and suboptimal instruments Z_s , that are initially used in [19] as

$$Z_o^\top(\theta_{(k)}^{j+1}) = C_{(k)}^{-1} \Psi_A r \quad (22)$$

$$Z_s^\top = \Psi_A r. \quad (23)$$

A fourth-order motion task r is performed with $N = 1001$, similar to those shown in Fig. 3(a) but with different constraints on the time derivatives. To emphasize the influence of Z on the achievable performance, the motion profile is less aggressive than r_3 , as used in Sections VIII-B and VIII-C. This motivates the selection of basis functions

$$\Psi_A(z) = \left[\frac{z-1}{zT_s}, \left(\frac{z-1}{zT_s} \right)^2, \left(\frac{z-1}{zT_s} \right)^3, \left(\frac{z-1}{zT_s} \right)^4 \right].$$

The following observations are made.

Fig. 9. Servo performance is significantly improved by the proposed instruments Z_o (—○—, —) compared to Z_s (—○—, —). The dominant performance improvement is achieved at frequencies where Z_o is relatively large, i.e., where $C_{(k)}^{-1}$ (—) is large. For optimal performance, $C_{(k)}^{-1}$ should approximate SP (—) at frequencies important for the servo error. (a) Two-norm of error signals e^j . (b) Time-domain error signals e^5 . (c) CPS of error signals e^5 (top), and Bode plot of $C_{<k>}^{-1}$ and SP (bottom).

- 1) The proposed instruments Z_o improve performance by 50% compared to Z_s in terms of $\|e^j\|_2$, see Fig. 9(a) and 9(b);
- 2) The procedures require multiple tasks to converge, see Fig. 9(a).

The performance enhancement of Z_o and the multistep convergence are contributed to the following effects.

- 1) The compared to Z_s , the instruments Z_o emphasize the servo error at frequencies where $C_{(k)}^{-1}$ is relatively large, see (22) and (23). In Fig. 9(c), it is observed that the dominant performance improvement is indeed achieved in this frequency range. For optimal performance, $C_{(k)}^{-1}$ should approximate SP at frequencies important for the servo error, see [18, Sec. IV] and [19, Sec. III].
- 2) The procedures require multiple tasks to converge due to a lack of excitation by r . For the results in Section VIII-B, a more aggressive motion profile is performed leading to convergence in a single task.

Interestingly, the design of Z influences the resulting parameter estimates, which can be observed in the error signals.

- 1) The different frequency weighting by Z_o and Z_s , see Fig. 9(c), result in different estimates θ . In Fig. 9(b), significant peaks are observed in the error signal for Z_s that correlate with the jerk, i.e., the third time derivative, of r .

IX. CONCLUSION

In this paper, a framework is introduced for batch-to-batch feedforward control from a system identification perspective. Two approaches are investigated and compared for batch-to-batch learning of rational feedforward controllers: 1) norm-optimal ILC using a model and 2) IV-based iterative feedforward tuning using only measured data. Both approaches enable high performance and extrapolation capabilities for nonrepeating tasks. Experimental results on an industrial motion system validate the approaches, and illustrate benefits of rational feedforward: 1) improved performance with respect to polynomial feedforward and 2) possible preactuation and postactuation of the system by means of stable inversion.

APPENDIX I

RELATIONS WITH CONVENTIONAL FREQUENCY-DOMAIN AND TIME-DOMAIN ILC DESIGNS

In this appendix, conventional ILC schemes are described, see, e.g., [1] and [25], in both time and frequency domain, see, e.g., [39], and direct relations are provided with the proposed framework to batch-to-batch feedforward.

A. Conventional Frequency- and Time-Domain ILC Schemes

In frequency-domain ILC schemes, see, e.g., [1], [39], the following general ILC algorithm is typically invoked:

$$f^{j+1} = \mathbf{Q}(z)f^j + \mathbf{L}(z)e^j \quad (24)$$

with robustness and learning filters $\mathbf{Q}(z)$, $\mathbf{L}(z)$. For the design of $\mathbf{Q}(z)$, $\mathbf{L}(z)$ and convergence of (24), see, e.g., [1] and [39].

A direct relation exists between conventional frequency- and time-domain ILC schemes, see [39]. In addition, very similar performance and robustness design issues are present, though handled through different design parameters, see, e.g., [40]. Consider the frequency-domain filters $\mathbf{Q}(z)$, $\mathbf{L}(z)$ in (24), which can be described in matrix form as in Section II using the impulse responses. The update law (24) can be written as

$$f^{j+1} = Qf^j + Le^j \quad (25)$$

with Q and $L \in \mathbb{R}$ convolution matrices of appropriate dimensions corresponding to $\mathbf{Q}(z)$ and $\mathbf{L}(z)$. Theoretically, the Fourier transform bijectively connects the time-domain and frequency-domain designs. In that sense, conventional time-domain ILC schemes can be designed which are equivalent to conventional ILC schemes based on frequency-domain designs, see [39].

B. Relation of Conventional ILC With the Proposed Framework

Next, the relation is pointed out of conventional time- and frequency-domain ILC schemes with the proposed framework of Section IV, where solutions are expressed in the update law

$$\theta^{j+1} = Q\theta^j + Le^j$$

with robustness matrix $Q \in \mathbb{R}^{(n_a+n_b) \times (n_a+n_b)}$ and learning matrix $L \in \mathbb{R}^{(n_a+n_b) \times N}$. Conventional ILC schemes, see, e.g., [1], [25] and (24), (25), consider the vector f^{j+1} as the optimization variable. This can directly be accommodated in the proposed framework by considering the elements of $f^{j+1} \in \mathbb{R}^{N \times 1}$ as optimization variables. This corresponds to

$$f^{j+1} = \Psi\theta^{j+1}$$

where $\theta^{j+1} \in \mathbb{R}^{N \times 1}$ and $\Psi = I_N$. Note that here flexibility to nonrepeating tasks is in fact sacrificed for performance. That is, conventional frequency- and time-domain ILC schemes do not achieve R2, since f^{j+1} is not an explicit function of r^{j+1} .

APPENDIX II

DERIVATION OF (12) AND (13)

By using C_{rat} in Definition 1, and $B(\theta^{j+1}) = 1 + \Psi_B\theta_B^{j+1}$, $\hat{e}^{j+1}$ in (11) can be written as

$$\hat{e}^{j+1}(\theta^{j+1}) = B^{-1}(\theta^{j+1})\tilde{e}^j - \Phi_{\text{ILC}}(\theta^{j+1})\theta^{j+1} \quad (26)$$

where $\tilde{e}^j = e^j + C_{ff}(\theta^j)\widehat{SP}r$, and

$$\Phi_{\text{ILC}}(\theta^{j+1}) = B^{-1}(\theta^{j+1})[\Psi_A\widehat{SP}r, -\Psi_B\tilde{e}^j].$$

Expression (26) can be used to solve (8). Any solution θ^{j+1} to optimization problem (8) necessarily satisfies $\frac{\partial V(\theta^{j+1})}{\partial \theta^{j+1}} = 0$. Evaluating this condition for criterion (11) with (26) gives

$$\left(\frac{\partial \hat{e}^{j+1}(\theta^{j+1})}{\partial \theta^{j+1}} \right)^\top W (B^{-1}(\theta^{j+1})\tilde{e}^j - \Phi_{\text{ILC}}(\theta^{j+1})\theta^{j+1}) = 0 \quad (27)$$

where the gradient of $\tilde{e}^{j+1}(\theta^{j+1})$ is given by

$$\frac{\partial \hat{e}^{j+1}(\theta^{j+1})}{\partial \theta^{j+1}} = \left[\frac{\partial \hat{e}^{j+1}(\theta^{j+1})}{\partial \theta_A^{j+1}}, \frac{\partial \hat{e}^{j+1}(\theta^{j+1})}{\partial \theta_B^{j+1}} \right]$$

with $\frac{\partial \hat{e}^{j+1}(\theta^{j+1})}{\partial \theta_A^{j+1}} = -B^{-1}(\theta^{j+1})\Psi_A\widehat{SP}r$, and $\frac{\partial \hat{e}^{j+1}(\theta^{j+1})}{\partial \theta_B^{j+1}}$ given in (28).

$$\begin{aligned} \frac{\partial \hat{e}^{j+1}(\theta^{j+1})}{\partial \theta_B^{j+1}} &= B^{-2}(\theta^{j+1}) \left[-\Psi_B\tilde{e}^j + (\Psi_B B(\theta^{j+1}) \right. \\ &\quad \left. - \Psi_B^2 \theta_B^{j+1})\tilde{e}^j + \Psi_B A(\theta^{j+1})\widehat{SP}r \right] \\ &= B^{-2}(\theta^{j+1})\Psi_B A(\theta^{j+1})\widehat{SP}r \end{aligned} \quad (28)$$

From (27), it then follows

$$\begin{aligned} \theta^{j+1} &= \left[\left(\frac{\partial \hat{e}^{j+1}(\theta^{j+1})}{\partial \theta^{j+1}} \right)^\top W \Phi_{\text{ILC}}(\theta^{j+1}) \right]^{-1} \\ &\quad \cdot \left(\frac{\partial \hat{e}^{j+1}(\theta^{j+1})}{\partial \theta^{j+1}} \right)^\top W B^{-1}(\theta^{j+1})\tilde{e}^j \end{aligned} \quad (29)$$

where it is assumed that the inverse exists. It remains to write θ^{j+1} as an explicit function of e^j and θ^j . Similar to (26), it can be derived that

$$\tilde{e}^j = B^{-1}(\theta^j)e^j + \Phi_{\text{ILC}}(\theta^j) \begin{bmatrix} I_{n_a} & 0 \\ 0 & -I_{n_b} \end{bmatrix} \theta^j \quad (30)$$

where $\Phi_{\text{ILC}}(\theta)$ is defined in (14) and 0 denotes the zero matrix of appropriate dimensions. Combining (30) with (29) yields

$$\theta^{j+1} = Q_{\text{ILC}}\theta^j + L_{\text{ILC}}e^j$$

with

$$L_{\text{ILC}} = \left[\left(\frac{\partial \hat{e}^{j+1}(\theta^{j+1})}{\partial \theta^{j+1}} \right)^\top W \Phi_{\text{ILC}}(\theta^{j+1}) \right]^{-1} \cdot \left(\frac{\partial \hat{e}^{j+1}(\theta^{j+1})}{\partial \theta^{j+1}} \right)^\top W B^{-1}(\theta^{j+1}) B^{-1}(\theta^j)$$

$$Q_{\text{ILC}} = L_{\text{ILC}} B(\theta^j) \Phi_{\text{ILC}}(\theta^j) \begin{bmatrix} I_{n_a} & 0 \\ 0 & -I_{n_b} \end{bmatrix}.$$

APPENDIX III DERIVATION OF (20)

Any solution θ^{j+1} to optimization problem (8) satisfies the necessary condition for optimality $\frac{\partial V(\theta^{j+1})}{\partial \theta^{j+1}} = 0$. Evaluating this expression for criterion (19) with predicted error (16) gives

$$\left(Z \frac{\partial \hat{e}^{j+1}(\theta^{j+1})}{\partial \theta^{j+1}} \right)^\top W Z (B^{-1}(\theta^{j+1}) \tilde{e}_m^j - \Phi_{\text{IV}}(\theta^{j+1}) \theta^{j+1}) = 0.$$

Note that since $Z \in \mathbb{R}^{n_\theta \times N}$, the matrix $Z \frac{\partial \hat{e}^{j+1}(\theta^{j+1})}{\partial \theta^{j+1}}$ is square. Therefore, the parameters θ^{j+1} are characterized by

$$\theta^{j+1} = [ZW\Phi_{\text{IV}}(\theta^{j+1})]^{-1} ZWB^{-1}(\theta^{j+1}) \tilde{e}_m^j \quad (31)$$

where it is assumed that the inverse exists. It remains to write θ^{j+1} as an explicit function of e^j and θ^j . By using $B(\theta^j) = 1 + \Psi_B \theta_B^j$, it holds

$$\tilde{e}_m^j = B^{-1}(\theta^j) e^j + \Phi_{\text{IV}}(\theta^j) \begin{bmatrix} I_{n_a} & 0 \\ 0 & -I_{n_b} \end{bmatrix} \theta^j \quad (32)$$

where $\Phi_{\text{IV}}(\theta)$ is defined in (17) and 0 denotes the zero matrix of appropriate dimensions. Combining (32) with (31) yields

$$\theta^{j+1} = Q_{\text{IV}}\theta^j + L_{\text{IV}}e_m^j$$

with

$$L_{\text{IV}} = [ZW\Phi_{\text{IV}}(\theta^{j+1})]^{-1} ZWB^{-1}(\theta^{j+1}) B^{-1}(\theta^j)$$

$$Q_{\text{IV}} = L_{\text{IV}} B(\theta^j) \Phi_{\text{IV}}(\theta^j) \begin{bmatrix} I_{n_a} & 0 \\ 0 & -I_{n_b} \end{bmatrix}.$$

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