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Radial Contour Labeling with Straight Leaders

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Abstract

In this paper we introduce a flexible and general approach for external label placement assuming a given *contour* of the figure prescribing the possible positions of the labels. While much research on external label placement aims for fast labeling procedures for interactive systems, we focus on highest-quality illustrations. We design a new efficient geometric label placement algorithm that is based only on few fundamental design criteria. Yet, other criteria can flexibly be included in the algorithm as hard or soft constraints.

1 Introduction

Atlases of human anatomy play a major role in the education of medical students and the teaching of medical terminology. Such books contain a broad spectrum of filigree and detailed drawings of the human anatomy from different cutaway views. For example, the third volume of the popular human anatomy atlas Sobotta [8] contains about 1200 figures on 384 pages. Figure 1 (labels added by our algorithm) is one of them showing a cross section of the human skull. The usefulness of the figures essentially relies on the naming of the illustrated components. In order not to spoil the readability of the figure by occluding it with text, the names are placed around the figure without overlapping it. Thin black lines, called *leaders*, connecting the features with their names accordingly guarantee that the reader can match names and features correctly. Following preceding research, we call this labeling technique *external label placement*. In this paper we present a flexible and versatile approach for external label placement in figures. We use medical drawings as running example, but occlusion-free label placements are also indispensable for the readability of other highly detailed figures as they occur, for example, in scientific publications, mechanical engineering and maintenance manuals.

Our approach bridges the gap between practical and theoretical results. While previous practical results (e.g., [1,5]) aim for fast approaches using heuristic multi-criteria optimization, previous theoretical results (e.g., [2, 3]) mostly consider simple models,

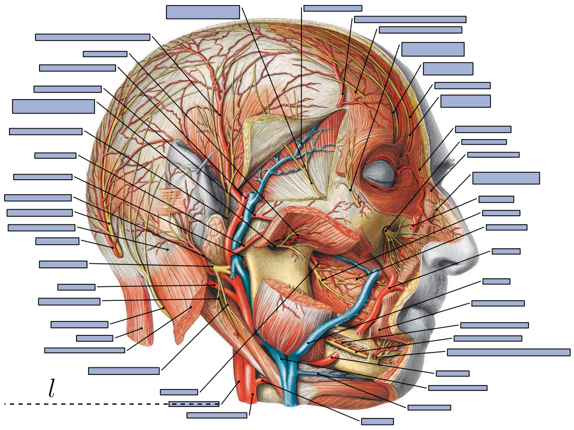

Figure 1: Medical drawing labeled by our approach. Source: Paulsen, Waschke, Sobotta Atlas Anatomie des Menschen, 23.Auflage 2010 ©Elsevier GmbH, Urban & Fischer, München.

typically with one optimization criterion, e.g., minimizing the total leader length.

Like many of the theoretical results, our approach uses a clear mathematical model to guarantee compliance with pre-defined design rules. However, in contrast to preceding research our approach is significantly more flexible and stands out by its ability to support an easy integration of specific design rules. It particularly relies on only a few key assumptions that most figures with external label placement have in common. Other rules can easily be patched in both as *hard* and *soft* constraints, where hard constraints may not be violated and the compliance of soft constraints is rated by a cost function.

Moreover, in contrast to previous work, our approach also takes costs of consecutively placed labels into account. At first glance this seems to be a small improvement, but in fact it is important to obtain an appealing labeling where, for example, labels have regular distances or the angles of consecutive labels should be similar. Further, our approach supports labels of different sizes. Indeed, for each point feature, the user can pre-define a set of different label sizes modeling formatting rules. The approach also allows to pre-define groups of labels that are placed consecutively, which is required when naming semantically related features.

We first introduce a flexible formal model for *contour labeling*, which is a generalization of boundary

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labeling (Sect. 2). This model is based on interviews with one layout artist and two editors of the human anatomy atlas Sobotta [8]. We further empirically verified the model by a semi-automatic quantitative analysis of 202 figures printed in the Sobotta [8] atlas. A detailed discussion of the interviews and the semi-automatic analysis is found in [7]; in this preprint we focus on the algorithmic core of our approach, which yields the mathematically optimal solution (Sect. 3).

The strength of our approach comes at the cost of a high asymptotic running time of $O(n^8)$, where n describes the complexity of the input instance. Recently, Keil et al. [6] presented a similar general dynamic programming approach for computing an independent set in outerstring graphs, which can be utilized to solve contour labeling in $O(n^6)$ time for a general cost function rating individual labels; however, it cannot take joint costs of two consecutive labels into account. In contrast to Fink and Suri [4] our approach is significantly faster ($O(n^8)$ instead of $O(n^{15})$) and it supports non-uniform labels and more general shapes.

In the full paper [7], we show in a detailed experimental evaluation on a large set of real-world instances that with some engineering we can solve realistically sized instances in adequate time and high layout quality. Considering different speed-up techniques, the variants of our approaches need between 7 seconds and 346 seconds on average. While the slow variants are optimal, the fast variants achieve near-optimal solutions. The domain experts assessed our algorithm to be a tool of great use that could reduce the working load of a designer significantly.

2 Formal Model

We now describe a model for contour labeling. Let $\mathbb{F}$ be a simple polygon that describes the contour of the figure and contains n points to be labeled, which we call *sites*. We denote the set of the sites by $\mathbb{S}$ and assume that the sites are in general position, i.e., no three sites are collinear. For each site $s \in \mathbb{S}$ we describe its *label*[1] ℓ by a rectangle r and an oriented line segment λ that starts at s and ends on the boundary of r. We call λ the *leader* of ℓ, r the *text box* of ℓ, and the endpoint of λ on r the *port* of ℓ. The other endpoint is the site of ℓ.

A set $\mathcal{L}$ of labels over $\mathbb{S}$ is called an *external labeling* of $(\mathbb{F}, \mathbb{S})$, if (1) $|\mathcal{L}| = |\mathbb{S}|$, (2) for each site $s \in \mathbb{S}$ there is exactly one label in $\mathcal{L}$ that belongs to s, and (3) every text box of a label in $\mathcal{L}$ lies outside of $\mathbb{F}$. If no two labels in $\mathcal{L}$ intersect each other, $\mathcal{L}$ is *planar*. Traversing the figure's boundary in clockwise order starting from the boundary's topmost point defines an ordering on the labels; we call this the radial ordering of $\mathcal{L}$ (in case that a leader intersects the figure's boundary multiple times, we regard the intersection point closest to the port). Two labels are *consecutive* in $\mathcal{L}$ if one directly follows the other in the radial ordering.

Let $\mathcal{L}$ be a planar labeling. Let $\ell_1, \dots, \ell_n$ be the labels of $\mathcal{L}$ in the radial ordering. For simplicity we define $\ell_{n+1} := \ell_1$. The *cost* c of a labeling $\mathcal{L}$ is defined as $c(\mathcal{L}) = \sum_{i=1}^{n} c_1(\ell_i) + c_2(\ell_i, \ell_{i+1})$, where c_1 is a function assigning a cost to a single label ℓ_i and c_2 is a function assigning a cost to two consecutive labels ℓ_i and ℓ_{i+1}. We note that in contrast to previous research the cost function also supports rating two consecutive labels, which is crucial to assess labels in relation to each other. Given the cost function c, the problem ExternalLabeling then asks for a planar labeling $\mathcal{L}$ of $(\mathbb{F}, \mathbb{S})$ that has minimum cost with respect to c, i.e., for any other planar labeling $\mathcal{L}'$ of $(\mathbb{F}, \mathbb{S})$ it holds that $c(\mathcal{L}) \leq c(\mathcal{L}')$.

We consider the special case that the ports of the labels lie on a common *contour* enclosing $\mathbb{F}$. The contour schematizes the shape of the figure with a certain offset and describes the common silhouette formed by the labels. It thus generalizes the typically rectangular figures studied in boundary labeling [2, 3]. We assume that the contour is given as a simple polygon $\mathbb{C}$ enclosing $\mathbb{F}$. An external labeling $\mathcal{L}$ is called a *contour labeling* if for every label of $\mathcal{L}$ its leader lies inside $\mathbb{C}$ and its port lies on the boundary $\partial\mathbb{C}$ of $\mathbb{C}$. Since not every part of $\mathbb{C}$'s boundary may be suitable for the placement of labels, we require that the ports of the labels are contained in a given subset $\mathbb{P} \subseteq \partial\mathbb{C}$ of candidate ports. If $\mathbb{P}$ is finite, the input instance has *fixed ports* and otherwise *sliding ports*.

A tuple $\mathbb{I} = (\mathbb{C}, \mathbb{S}, \mathbb{P})$ is called an *instance* of contour labeling. The *region* of $\mathbb{I}$ is the region enclosed by $\mathbb{C}$. We restrict ourselves to convex contours and clearly separated sites and text boxes, i.e., we assume that the contour $\mathbb{C}$ is convex and no text box of any label intersects the convex hull of $\mathbb{S}$. Further, we are only interested in *staircase labelings*, i.e., for each label ℓ there is a horizontal half-line l that emanates from the port of ℓ through the text box of ℓ such that no other label intersects l; see Fig. 1. In the full version [7] of this paper, we give empirical evidence that these assumptions are reasonable.

Given a cost function c, the problem ContourLabeling then asks for a (cost) optimal, planar staircase contour labeling $\mathcal{L}$ of $(\mathbb{C}, \mathbb{S}, \mathbb{P})$ with respect to c, i.e., for any other planar staircase contour labeling $\mathcal{L}'$ of $(\mathbb{C}, \mathbb{S}, \mathbb{P})$ it holds that $c(\mathcal{L}) \leq c(\mathcal{L}')$.

3 Algorithm

In this section we describe how to construct the optimal labeling $\mathcal{L}$ of a given instance $(\mathbb{C}, \mathbb{S}, \mathbb{P})$ with respect to a given cost function c. To that end we apply a dynamic programming approach. The ba-

[1] To ease presentation we define that the leader is a component of the label. In preceding research only the rectangle r is called label.

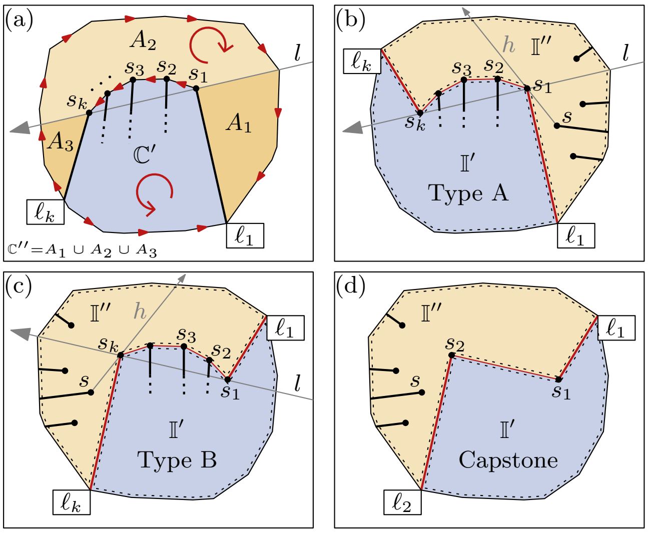

Figure 2: Decomposition in convex instance $\mathbb{I}'$ (blue) and concave instance $\mathbb{I}''$ (orange). (a) Basic definitions. (b) Type A instance with $k > 2$. (c) Type B instance with $k > 2$. (d) Capstone instance.

sic idea is that any optimal contour labeling can be recursively decomposed into a set of sub-labelings inducing disjoint sub-instances. As we show later, these sub-instances are of a special form; we call them *convex sub-instances*. We further show that any such sub-instance can be described by a constant number of parameters over $\mathbb{S}$ and $\mathbb{P}$. Hence, enumerating all choices of these parameters, we enumerate in polynomial time all possible convex sub-instances that an optimal labeling may consist of. For each such sub-instance we compute the cost of an optimal labeling reusing the results of already computed values of smaller sub-instances. In this way we obtain the value of the optimal labeling for $(\mathbb{C}, \mathbb{S}, \mathbb{P})$.

We now sketch the decomposition of a planar labeling $\mathcal{L}$ into a finite set of sub-instances of three types. We describe a sub-instance by a simple polygon that consists of two polylines. One polyline is part of the original contour $\mathbb{C}$ and the other polyline consists of a convex chain of sites and two leaders; see Fig. 2(a). More precisely, assume that we are given a convex chain $K = (s_1, \ldots, s_k)$ of sites with $k \geq 2$ and the two non-intersecting labels ℓ_1 and ℓ_k of s_1 and s_k, respectively. The directed polyline $K' = (p_1, s_1, \ldots, s_k, p_k)$ splits the polygon $\mathbb{C}$ into two polygons $\mathbb{C}'$ and $\mathbb{C}''$, where p_1 and p_k are the ports of ℓ_1 and ℓ_k, respectively. We consider the order of the sites such that we meet p_1 before p_k when going along the contour of $\mathbb{C}$ in clockwise-order starting at the top of $\mathbb{C}$. Further, going along K' we denote the sub-polygon to the left of K' by $\mathbb{C}'$ and to the right of K' by $\mathbb{C}''$. With respect to the direction of K', the sub-polygon $\mathbb{C}'$ is counter-clockwise oriented, while $\mathbb{C}''$ is clockwise oriented. Further, $\mathbb{C}''$ contains the top point of $\mathbb{C}$. We define that $\mathbb{C}'$ contains the sites $s_2, \ldots, s_{k-1}$, while $\mathbb{C}''$ does not.

Thus, the polyline K' partitions the instance $(\mathbb{C}, \mathbb{S}, \mathbb{P})$ into two sub-instances $\mathbb{I}' = (\mathbb{C}', \mathbb{S}', \mathbb{P}')$ and $\mathbb{I}'' = (\mathbb{C}'', \mathbb{S}'', \mathbb{P}'')$ such that

(1) $\mathbb{S}' \cup \mathbb{S}'' = \mathbb{S} \setminus \{s_1, s_k\}$ and $\mathbb{P}' \cup \mathbb{P}'' = \mathbb{P} \setminus \{p_1, p_k\}$,
(2) the sites of $\mathbb{S}'$ lie in $\mathbb{C}'$ or on K and the sites of $\mathbb{S}''$ lie in the interior of $\mathbb{C}''$,
(3) the ports of $\mathbb{P}'$ lie on the boundary of $\mathbb{C}'$ and the ports of $\mathbb{P}''$ lie on the boundary of $\mathbb{C}''$.

Note that the sites s_1, s_k and the ports p_1, p_k neither belong to $\mathbb{I}'$ nor to $\mathbb{I}''$, because they are already used by the fixed labels ℓ_1 and ℓ_k. We call (ℓ_1, ℓ_k, K), which defines the polyline K', the *separator* of $\mathbb{C}'$ and $\mathbb{C}''$.

In the following, we only consider sub-instances, in which the convex chain K lies to the right of the line l through s_1 and s_k pointing towards s_k from s_1; we will show that these are sufficient for decomposing any instance. Put differently, the chain K is a convex part of the boundary of $\mathbb{C}'$ and a concave part of the boundary of $\mathbb{C}''$. We call $\mathbb{I}'$ a *convex* sub-instance and $\mathbb{I}''$ a *concave* sub-instance.

The line l splits $\mathbb{C}''$ into three regions A_1, A_2 and A_3; see Fig. 2(a). Let A_2 be the region to the right of l and let A_1 and A_3 be the regions to the left of l such that A_1 is adjacent to the leader of ℓ_1 and A_3 is adjacent to the leader of ℓ_k. Depending on the choice of ℓ_1 and ℓ_k, the regions A_1 and A_3 may or may not exist. We distinguish the following convex instances. A convex instance has type A (type B) if there is a site $s \in A_1$ ($s \in A_3$) such that ℓ_1 (ℓ_k) and the half-line h emanating from s through s_1 (s_k) separates K from the sites in $\mathbb{C}''$; see Fig. 2(b) and Fig. 2(c).

For both types the chain K is uniquely defined by the choice of ℓ_1, ℓ_k and s, because h separates the sites of $\mathbb{I}'$ from the sites of $\mathbb{I}''$. Thus, type A and type B instances are uniquely defined by ℓ_1, ℓ_k and s; we denote these instances by $\mathbb{I}_\mathrm{A}[\ell_1, \ell_k, s]$ and $\mathbb{I}_\mathrm{B}[\ell_1, \ell_k, s]$, respectively. We call s the *support point* of the instance. In case that $\mathbb{C}''$ is empty, the chain K is already uniquely defined by ℓ_1 and ℓ_k and we write $\mathbb{I}_\mathrm{A}[\ell_1, \ell_k, \perp]$ and $\mathbb{I}_\mathrm{B}[\ell_1, \ell_k, \perp]$. Hence, we can enumerate all such instances by enumerating all possible triples consisting of two labels and one site. Since each label is defined by one port and one site, we obtain $O(|\mathbb{S}|^3|\mathbb{P}|^2)$ instances in total.

For $k = 2$ the chain consists of the sites s_1 and s_2 and the support point is superfluous; such an instance is solely defined by the labels ℓ_1 and ℓ_2 of s_1 and s_2, respectively. We call these instances *capstone instances* and denote them by $\mathbb{I}_\mathrm{C}[\ell_1, \ell_2]$; see Fig. 2(d).

The next lemma implies that any labeling of any instance $\mathbb{I}$ is a type A instance; see [7] for the proof.

Lemma 1 *Let $\mathbb{I}$ be an instance of* ContourLabeling *and let $\mathcal{L}$ be a planar labeling of $\mathbb{I}$. The first leader ℓ and the last leader ℓ' in the radial ordering of*

$\mathcal{L}$ define a type A instance $\mathbb{I}' = \mathbb{I}_{\mathrm{A}}[\ell, \ell', \perp]$ such that the exterior of $\mathbb{I}'$ is empty.

Hence, optimizing over all choices of first and last labels we find a type A instance that corresponds to an optimal labeling. It remains to show how to solve such an instance. To that end we show that any labeling of that instance can be decomposed into type A, type B and capstone instances recursively.

Let $\mathbb{I} = \mathbb{I}_{\mathrm{A}}[\ell_1, \ell_k, s]$ be a type A instance with support point s, and let $\mathcal{L}$ be a planar labeling of $\mathbb{I}$. By the reasoning above this instance implies a unique convex chain $K = (s_1, \ldots, s_k)$ such that s_1 is the site of ℓ_1 and s_k is the site of ℓ_k; see Fig. 3.

First assume that $\mathbb{I}$ is not a capstone instance, i.e., $k > 2$. We show that $\mathcal{L}$ can be partitioned into a type A instance and a capstone instance as shown in Fig. 3.

To see that, let $\ell_2 \in \mathcal{L}$ be the label of s_2. Since ℓ_2 connects two points of $\mathbb{C}$'s boundary, it partitions $\mathbb{I}$ into two sub-instances $\mathbb{I}'$ and $\mathbb{I}''$ with labelings $\mathcal{L}|_{\mathbb{I}'}$ and $\mathcal{L}|_{\mathbb{I}''}$ such that any label of $\mathcal{L} \setminus \{\ell_2\}$ either is contained in $\mathcal{L}|_{\mathbb{I}'}$ or $\mathcal{L}|_{\mathbb{I}''}$. Let $\mathbb{I}'$ be the instance containing s_1 and $\mathbb{I}''$ the other one. Obviously, $\mathbb{I}'$ forms the capstone instance $\mathbb{I}_{\mathrm{C}}[\ell_1, \ell_2]$. We now show that $\mathbb{I}''$ forms the instance $\mathbb{I}'' = \mathbb{I}_{\mathrm{A}}[\ell_2, \ell_k, s_1]$ of type A.

By definition of $\mathbb{I}$ the label ℓ_1 and the half-line h emanating from s through s_1 separate the convex chain K of $\mathbb{I}$ from the sites in the exterior of $\mathbb{I}$. Because of the convexity of K, the half-line h' emanating from s_1 through s_2 and the label ℓ_2 separate the convex chain $K' = (s_2, \ldots, s_k)$ from the sites in the exterior of $\mathbb{I}''$. Hence, $\mathbb{I}'' = \mathbb{I}_{\mathrm{A}}[\ell_2, \ell_k, s_1]$ has type A.

If $\mathbb{I}$ is a capstone instance, i.e., $k = 2$, the labeling can be decomposed into smaller type A, type B and capstone instances using more intricate arguments. For type B instances we can argue symmetrically to type A instances. Further, the costs of $\mathcal{L}$ can be composed by the costs of the constructed sub-labelings. The details are given in [7].

Based on these results the dynamic programming approach works as follows, where an instance is called *valid* if the two labels ℓ_1 and ℓ_k defining the separator do not intersect and comply with the criterion of a staircase labeling.

Step 1. We compute all valid instances of type A and type B, and all valid capstone instances.

Step 2. We compute the optimal costs for all convex sub-instances. Let $\mathbb{I}$ be the currently considered instance of size $i \geq 0$ with separator $(\ell_1, \ell_k, K = (s_1, \cdots, s_k))$, where the *size* of $\mathbb{I}$ is the number of sites contained in $\mathbb{I}$; recall that s_1 and s_k do not belong to $\mathbb{I}$. Considering the instances in non-decreasing order of their sizes, we can assume that we have already computed the optimal costs for all convex instances with size less than i. Hence, we compute the optimal costs of $\mathbb{I}$ by systematically exploring all decompositions of $\mathbb{I}$ into smaller convex instances.

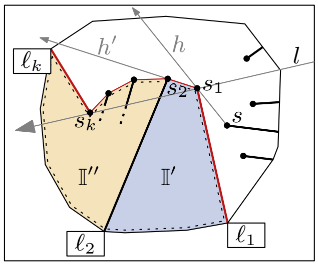

Figure 3: Decomposition of a convex type A instance $\mathbb{I}$ (dashed polygon) into a capstone instance $\mathbb{I}'$ and type A instance $\mathbb{I}''$. The site s is the support point of $\mathbb{I}$ and s_1 is the support point of I''.

Step 3. We explore all choices of first and last labels in the radial ordering. By Lemma 1 one of the choices defines a type A instance that corresponds to an optimal labeling. In the previous steps we have computed the optimal costs for that instance.

In [7] we formally prove that this approach yields a planar staircase labeling in $O(\mathbb{S}^4 \cdot \mathbb{P}^4)$ time.

Theorem 2 ContourLabeling *with fixed ports can be solved in $O(\mathbb{S}^4 \cdot \mathbb{P}^4)$ time.*

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