Microwave resonance spectroscopy of RF plasma
inspection of plasma parameters through non-invasive methods

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Microwave Resonance Spectroscopy of RF Plasma

Inspection of plasma parameters through non-invasive methods

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1. Summary

In this experiment, the electron density was inspected of an RF plasma, ignited via an AC signal, via the change in the refractive index. For this purpose, a cylindrical resonance cavity was constructed out of aluminium, measuring roughly 98 mm in diameter and 69 mm in length. The plasma was ignited from Argon. The cavity was kept in a vacuum vessel, and was equipped with an RF antenna to ignite the plasma, and two antennas inside the cavity. The two antenna’s are used to excite and measure a resonant electromagnetic mode inside the cavity. The frequency of the resonant mode is dependent on the plasma’s refractive index and the geometry of the cavity. As the geometry of the cavity remained fairly constant throughout the experiment, the refractive index was taken to be the most significant factor. Argon was used as a filling as, kept in the vacuum vessel at a pressure of 0.1 mBar.

In addition, the electron density was inspected as a function of the power used to sustain the plasma. An RF plasma typically has an electron density of $10^{16}$ m$^{-3}$ in order of magnitude. For a power input of 2 to 22 W, an electron density was found in a range from $10^{15}$ to $10^{16}$ m$^{-3}$ in orders of magnitude, with uncertainties of an order of magnitude of $10^{13}$. The electron density suggests a square root relation to the power input on a logarithmic scale.
Contents
1. Summary ................................................................................................................................. 2
2. Introduction .............................................................................................................................. 4
3. Fundamentals of the experiment ........................................................................................... 5
4. Theory and underlying principles .......................................................................................... 6
  4.1 RF plasmas and their properties ......................................................................................... 6
  4.2 Resonant electromagnetic modes ......................................................................................... 6
  4.3 Inferring Electron densities from the Refractive Index ...................................................... 7
4 Experimental set-up and procedure ......................................................................................... 10
  4.1 Equipment and set-up ......................................................................................................... 10
  Resonance Cavity .................................................................................................................... 11
  Matching Box .......................................................................................................................... 11
  Spectrum Analyser ................................................................................................................ 11
  Function generator .................................................................................................................. 12
  4.2 Experimental procedure .................................................................................................. 13
  Preparation .............................................................................................................................. 13
  Experimental measurement ..................................................................................................... 13
5 Results ..................................................................................................................................... 15
6 Discussion of Results ............................................................................................................... 16
Appendix A ................................................................................................................................. 17
Appendix B ................................................................................................................................... 18
Appendix C .................................................................................................................................. 19
2. Introduction

Nuclear fusion is an exciting new technology for the production of energy. In the physical process of fusing two light atoms into a new one, large amounts of excess energy are released which can be harnessed for the production of energy. The most promising method of applying fusion is the TOKAMAK reactor: a toroidal cavity, wherein plasma of hydrogen isotopes, deuterium and tritium, is kept. Plasma is sometimes referred to as the fourth state of matter: it consists of an ionised gas, a cloud of positive ions and electrons. The deuterium and tritium ions are used in the fusion process. In the interest of fusion, knowledge of the plasma’s state inside the TOKAMAK reactor is important for proper operation of the installation, and needs to be constantly monitored. However, the extreme conditions inside the reactor make constant plasma diagnostics difficult. One of the monitored properties of the fusion plasma is its density. A simple method used to measure plasmas densities is the Langmuir probe. This method works by inserting two or more electrodes within the plasma glow and measure the current flowing in between them as a rate for the electron density. With increasing electron temperatures, the Langmuir probe loses its functionality as the probe can only remain in the plasma for a limited time, to prevent the electrodes from being damaged or vaporized in the plasma, making constant measurement impossible. In addition, measuring electron densities with the Langmuir probe becomes increasingly unreliable as the density decreases.

An alternative method of measurement is via microwave cavity spectroscopy. This method is less intrusive than the probe, and can also be used to measure the electron density on chemically active AC plasma discharges. RF plasma is ignited in a cavity which resonates with a specific electromagnetic mode. When the cavity is filled with plasma, the resonance frequency changes due to the plasma’s refractive index. By inspecting this shift in frequency, the refractive index and electron density can be inferred. This method originates from the technology of plasma etching. Plasma etching is primarily used in the Integrated Circuit industry, where it is used to etch the microchip circuits into silicon wafers.

For this experiment, a cylindrical cavity was used, measuring $97.77\times10^{-3}$ m in diameter and $69.01\times10^{-3}$ m in length and the frequency domain inspected for resonance frequency is 100kHz-3GHz.
3. Fundamentals of the experiment

The purpose of this experiment is measure the electron density of low density RF plasmas via a non-invasive method, as opposed to methods like the Langmuir probe. For this purpose, a resonance cavity is filled with an RF plasma, generated from Argon. A pair of microwave antennae is used to radiate a signal into the cavity and detect the plasma’s refractive index, and from that infer relevant parameters, such as the electron density.
4. Theory and underlying principles

4.1 RF plasmas and plasma optics

An RF plasma is ignited in a similar way to a DC plasma: a gas caught between an anode and cathode with a sufficient voltage over the gap is broken down into ions and electrons. For this experiment, Argon was used. However, as an AC signal is used to ignite and sustain the plasma, both the ions and electrons oscillate with the electromagnetic field. Given the difference in weight of the ions and the electrons, the inertia of electrons is much lower than that of the ions and thus can respond to signals of higher frequency than the ions can. For higher frequencies, the ions are subjected to the time averaged field and can be interpreted as stationary.

The electron oscillations in the plasma can be interpreted as a Plasmon: a collective oscillation of all the electrons within a plasma or metal. An important parameter of Plasmons and plasma oscillations is the plasma frequency:

\[
\omega_p = \sqrt{\frac{n_e e^2}{\varepsilon_0 m_e}}
\]

Herein, \(n_e\) is the electron density in \(\text{m}^{-3}\), \(e\) is the electron charge in C, \(\varepsilon_0\) the permittivity of vacuum in \(\text{Fm}^{-1}\) and \(m_e\) the electron mass in kg. The plasma frequency is a measure of the restoring Coulomb force in the plasma. The electron density of RF plasmas made Argon is generally \(10^{15} \text{ m}^{-3}\) or higher, by order of magnitude (1).

As light can be considered an electromagnetic wave, its propagation speed is determined by the electromagnetic properties of the medium it traverses, indicated by its refractive index. In plasma, the refractive index is dependent on the density of charged particles within the plasma glow.

4.2 Resonant electromagnetic modes

To inspect a plasma’s electron density through the refractive index, the refractive index is inferred by measuring the changes to an electromagnetic signal as it passes through the plasma. To better control the electromagnetic wave, the plasma is ignited inside a metal resonance cavity. As the electromagnetic waves still need to satisfy the Maxwell equation, a finite number of resonant modes can fit in the cavity. As the cavity employed in this experiment has a circular shape, the resonating modes will be solutions to Bessel’s differential equations. The electric field must be finite at the centre of the cavity; as such, the radial dependency of the modes goes by the Bessel functions of the first kind. The modes excited in the cavity can be transverse electric or magnetic: TE and TM respectively. In TE modes, there is no electric field in the direction of propagation and no magnetic field for TM modes. The resonance frequencies for these modes are defined as follows:
\[ f_{mnp} = \frac{c}{2\pi n_r} \sqrt{\left(\frac{x_{mn}}{R}\right)^2 + \left(\frac{p\pi}{L}\right)^2} \quad [4.2.1] \]

\[ f_{mnp} = \frac{c}{2\pi n_r} \sqrt{\left(\frac{x'_{mn}}{R}\right)^2 + \left(\frac{p\pi}{L}\right)^2} \quad [4.2.2] \]

Of these two, \([4.2.1]\) goes for TM modes, and \([4.2.2]\) goes for TE modes. Herein, \(c\) is the speed of light in m\(\cdot\)s\(^{-1}\), \(n_r\) the dimensionless refractive index, \(x_{mn}\) is the \(n\)th node of the \(m\)th Bessel function, \(x'_{mn}\) is the \(n\)th node of the derivative of the \(m\)th Bessel function, \(R\) and \(L\) are the radius and length of the cavity, respectively, both in m. \(P\) is the third integer in the notation of the mode.

The resonant mode could possibly be in the TM\(_{010}\), or in the TE\(_{210}\) modes, as these frequencies lie relatively close to the resonant frequency, in comparison of other frequencies (compare Appendix A, tables A-3 and A-4, with Appendix B, table B-1). This deviation can probably be attributed to the cavity not being ideal: The cavity is not made of an ideal conductor, and neither is the geometry an ideal cylinder. In addition, the cavity is built from segments, rather than one continuous whole, and one section of a face plate is fully isolated from the rest of the cavity, to accommodate the RF antenna used to ignite and sustain plasma within the cavity.

### 4.3 Inferring Electron densities from the Refractive Index

An important quantity of any dielectric is its refractive index:

\[ n_r = \sqrt{\varepsilon_r \mu_r} = \frac{c}{v_p} \quad [4.3.1] \]

Herein, \(c\) is the velocity of light and \(v_p\) the phase velocity in m\(\cdot\)s\(^{-1}\), and \(\varepsilon_r\) and \(\mu_r\) the relative dielectric permittivity and permeability in F\(\cdot\)m\(^{-1}\) and H\(\cdot\)m\(^{-1}\), respectively. As this experiment concerns a low-temperature plasma, the refractive index can be expressed by the Appleton-Hartree equation. The Appleton-Hartree equation expresses the refractive index for wave propagation inside “cold” plasmas, where the average electron velocity is well below \(c\):

\[ n_r^2 = 1 - \frac{X(1 - X)}{1 - X - \frac{1}{2} Y^2 \sin^2 \theta \pm \left( \frac{1}{2} Y^2 \sin^2 \theta \right)^2 + (1 - X)^2 Y^2 \cos^2 \theta} \quad [4.3.2] \]

\[ X = \frac{\omega_e^2}{\omega^2} \quad [4.3.3] \]

\[ Y = \frac{\omega_n}{\omega} \quad [4.3.4] \]
Herein, $\omega_e$ is the plasma frequency of electrons, as seen in equation [4.1.1], in $\text{rad} \cdot \text{s}^{-1}$, $\omega_H$ is the electron gyro-frequency in $\text{rad} \cdot \text{s}^{-1}$, $\omega$ is the frequency of the electromagnetic wave propagated through the plasma and $\theta$ is the angle between the magnetic field and the vector of propagation. The gyro-frequency is defined as follows:

$$\omega_H = \frac{B_0 |e|}{m_e} \quad [4.3.5]$$

Herein, $n_e$ is the electron density in $\text{m}^{-3}$, $e$ is the electron charge in $\text{C}$, $\varepsilon_0$ the permittivity of vacuum in $\text{F} \cdot \text{m}^{-1}$, $m_e$ is the electron mass in $\text{kg}$ and $B_0$ is the absolute magnitude of the magnetic field in $\text{T}$. In the context of this particular experiment, there is no external magnetic field. The Appleton-Hartree equation reduces to the following form:

$$n_e^2 = 1 - X \quad [4.3.6]$$

For an unspecified resonance cavity, one of the modes resonating in this cavity has the frequency $f_0$ with an associated wavelength $\lambda_0$. When the cavity is filled with a dielectric medium, the resonance frequency shifts to $f$, as the medium changes the propagation speed of light. These wavelengths are defined as follows:

$$\lambda_0 = \frac{c}{f_0} = \frac{2\pi c}{\omega_0} \quad [4.3.7]$$

$$\lambda = \frac{c}{n_e f} = \frac{2\pi c}{\omega} \quad [4.3.8]$$

Herein $c$ is the speed of light in vacuum in $\text{m} \cdot \text{s}^{-1}$ and $n$ is the refractive index. To maintain a specific resonant mode, the ratio of the resonator dimensions and the resonant wavelength must remain identical. Since the resonance cavity used is static, it’s dimensions cannot be altered. As such, the wavelengths in the evacuated and filled cavity are the same. Ergo, for different refractive indices, or phase velocities, the resonant mode occurs at different frequencies. By using equations [4.3.7] and [4.3.8] to create an expression for refractive index $n_e$, the reduced A-H equation, [4.3.6] can be written as follows:

$$\frac{\omega_0^2}{\omega^2} = 1 - \frac{\omega_e^2}{\omega^2} \quad [4.3.9]$$

Combining this with the equation for the plasma frequency, [4.1.1], an equation for the plasma electron density was derivated as follows:
This equation denotes the average electron density across the entire plasma glow. As the density is virtually homogenous inside the glow, with a sharp decrease near the edge, the average density can be taken close to the actual density.

\[
n_e = \frac{\varepsilon_0 m_e (\omega^2 - \omega_0^2)}{e^2}
\]
5  Experimental set-up and procedure

5.1  Equipment and set-up

List of equipment used in the experimental set-up:

- Resonance Cavity
- Vacuum vessel
- HAMEG HMS3010 TG Network Analyser
- ISO-TECH GFG2120 function generator
- ENI A150 RF amplifier
- Matching box
- B/S NAP power reflection meter
- Pfeifer Vacuum HiCube vacuum pump
- Pfeifer Vacuum D-35614 Asslar EVN 116 valve
- Thyraccont VD 85 Pressure meter

![Figure 5.1-1 Schematic of the set-up used to ignite the plasma and measure its density](image)
**Resonance Cavity**

The resonance cavity used in this experiment has a cylindrical shape, with a diameter of roughly $98 \times 10^3$ m and a length of roughly $69 \times 10^{-3}$ m. The cavity is made of aluminium; the walls and face plates are kept together by brass bolted joints. One the end faces of the cavity contains a plate, isolated from the rest of the cavity, which functions as an RF antenna to ignite the plasma, as can be seen in Figure 5.1-1. The other end plate is perforated with circular holes. The wall is constructed of a thin, curved plate with square perforations.

The standing electromagnetic modes inside the cavity are emitted and received by a pair of loop antennas on the end of the cavity opposite to the RF antenna. These antennas are placed at opposite ends of the end plate’s diameter.

**Matching Box**

The use of a matching box is imperative for the proper operation of the experiment. Because an AC signal is used, we can attribute a wavelength to the electricity inside the circuit. As the signal reaches the interface surface of the antenna, it can be reflected back into the circuit. This can result in standing waves within the circuit, causing energy to be caught in the circuit, rather than being projected into the cavity. Aside from diminishing the effectiveness of the experiment, it can also overheat the circuit and pose a potential fire hazard. The matching box is used to adjust the impedance of the circuit, to minimize reflections.

![Electric circuit inside the matching box](image)

*Figure 5.1-2* Electric circuit inside the matching box

The schematic in *figure4.1-2* is a representation of the matching box’s internal circuitry. The left side is the input, and the right side the output. The lower half of the circuit is grounded. Both the capacitances and inductance of each element can be changed, by changing the capacitor surface and number of windings, respectively. The capacitance, of the two left most capacitors, is inversely linked: increasing one decreases the other.

**Network Analyser**

The network analyser that is used measures the spectrum of an electric signal over a range of 0.1 MHz to 3 GHz. The spectrum is displayed as the intensity, measured in dBm and plotted against frequency. The unit dBm is a specialized form of the decibel scale. 1 dBm corresponds to 1 mW; from there on, it behaves like the decibel scale. An important feature of the analyser, is that it can generate a signal itself. This is mostly used in network analysis, where a network’s properties are determined by measuring it’s response to the signal generated by the analyser. This signal is nearly flat along the entire
range of the device. This feature is important to the measurements of this experiment. Instead of an electric circuit, the signal is injected into plasma-filled cavity, and the refractive index inferred from the frequency shift from the empty cavity.

Function generator
The function generator is used to generate the signal which ignites the plasma inside the cavity. The frequency used is 13.56 MHz. This frequency was used, because it’s a so-called “free frequency”: it is not used as a carrying frequency by any radio broadcast station, wireless networks or cellular phones.

Experimental Setup
The following image shows the experimental setup, and the components used.

![Experimental Setup Image](image-url)

5.2 Experimental procedure

Experimental measurement
When the network analyser is radiating a signal into the cavity, the spectral curve shown on the network analyser several sharp dips, corresponding to the cavity’s resonant frequencies. For the purpose of this experiment, only one resonant mode needs to be tracked. First a reference measurement is taken, determining the frequency of a resonant mode at vacuum. Next, the plasma is ignited. The amplifier is activated, and the power output of the function generator increased, until plasma ignition occurs. After the plasma is ignited, the dip of the resonant modes shifts to a new frequency. By employing equation [4.3.10], the plasma density can be obtained from the new resonant frequency. After this the power output of the function generator was varied, to inspect a possible relation between the power used to sustain the plasma and its electron density.

Figure 5.2-1 Profile of the resonant mode, with the cavity filled argon at a pressure of 0.1 mbar, and filled with plasma.
Figure 5.2-1 shows a close-up of the spectrum of the resonant modes inside the cavity. It shows a very clear resonant frequency just above 2.84 GHz without plasma, and shifted to roughly 2.857 GHz with the introduction of plasma.

**Preparation**
The conditions in the vacuum vessel must be good for the ignition of plasma: the vacuum vessel needs to be filled with Argon, at a pressure of 0.1 mbar.

First, the cavity is evacuated with the vacuum pump. During the evacuation, the Argon valve is opened, to flush the vessel and ensure relatively high gas purity. During the experiment, the cavity is continuously flushed by having the Argon valve opened and the valve to the vacuum pump nearly closed. This ensures a constant pressure inside the vessel.

To reduce the loading on the power source and the amplifier, the matching box needs to be properly adjusted to the impedance of the plasma-filled cavity. Therefore, the impedance of the matching box is set up while there is plasma in the cavity. The process is entirely empirical: the capacitance and inductance are changed at small increments until the reflected power reaches a minimum. The required settings of the matching box vary with the frequency of the excitation signal.
6 Results

The resulting plasma electron densities have been plotted against power in Error! Reference source not found. below. The blue plot represents the density plotted against the total power generated and the red plot represents the densities plotted against the net power radiated into the cavity (total power minus power reflected).

![Figure 6-1](image-url)  
Figure 6-1 Graphic of the measured electron densities for both the total and net power used to sustain the plasma, plotted in a linear scale.
7 Discussion of Results

The inspected resonance frequency of the cavity approaches the TM$_{010}$ and TE$_{210}$ modes. The difference can most likely be attributed to the cavity not being ideal. Possibly, the mode resonates with the vacuum vessel instead of the cavity.

The calculated electron densities fall between $10^{15}$ m$^{-3}$ and $10^{16}$ m$^{-3}$ in orders of magnitude. This is within the expected range.

The graphic of the density dependency shows a higher electron density for the net power, than the total power. Mostly this is due to a horizontal displacement of the graph, as the reflected power is subtracted from the measured power. As the power increases, the electron density for net power increases faster, than for total power, due to more power being reflected back into the circuit, resulting in a greater horizontal displacement. There were no measurements for an input power of 21 Watt or higher, due to the resonant frequency increasing beyond the reach of the Network analyser.

The plots of the electron density for both the net power and the total power show a linear relation to the electron density, with the electron density having a steeper inclination for net power than total power. Fitting the results with MATLAB’s polyfit function, yielded the following equations for the electron density as a function of the input power:

\[
 n_{e,\text{tot}} = -2.40 \cdot 10^{14} + 5.84 \cdot 10^{14} \cdot P \\
 n_{e,\text{net}} = -5.46 \cdot 10^{14} + 6.59 \cdot 10^{14} \cdot P
\]

As can be seen, neither of these functions passes through the origin, with a deviation well into $10^{14}$ by order of magnitude. This deviation, especially for the electron density plotted against net power, can be attributed to uncertainty in the net power inserted into the cavity: the reflected power was constantly fluctuating.
### Appendix A

**Table A-1** $n^{th}$ nodes of Bessel functions of the first kind and $m^{th}$ order

<table>
<thead>
<tr>
<th>$n\backslash m$</th>
<th>$J_0$</th>
<th>$J_1$</th>
<th>$J_2$</th>
<th>$J_3$</th>
<th>$J_4$</th>
<th>$J_5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2.4048</td>
<td>3.8317</td>
<td>5.1356</td>
<td>6.3802</td>
<td>7.5883</td>
<td>8.7715</td>
</tr>
<tr>
<td>2</td>
<td>5.5201</td>
<td>7.0156</td>
<td>8.4172</td>
<td>9.761</td>
<td>11.0647</td>
<td>12.3386</td>
</tr>
</tbody>
</table>

**Table A-2** $n^{th}$ nodes of derivatives of the Bessel functions of the first kind and $m^{th}$ order

<table>
<thead>
<tr>
<th>$n\backslash m$</th>
<th>$J'_0$</th>
<th>$J'_1$</th>
<th>$J'_2$</th>
<th>$J'_3$</th>
<th>$J'_4$</th>
<th>$J'_5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3.8317</td>
<td>1.8412</td>
<td>3.0542</td>
<td>4.2012</td>
<td>5.3175</td>
<td>6.4156</td>
</tr>
<tr>
<td>2</td>
<td>7.0156</td>
<td>5.3314</td>
<td>6.7061</td>
<td>8.0152</td>
<td>9.2824</td>
<td>10.5199</td>
</tr>
</tbody>
</table>

**Table A-3** Resonance frequencies of the set of $TM_{mn0}$ modes

<table>
<thead>
<tr>
<th>$n\backslash \omega_m$</th>
<th>$f_0$</th>
<th>$f_1$</th>
<th>$f_2$</th>
<th>$f_3$</th>
<th>$f_4$</th>
<th>$f_5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2.35·10^9</td>
<td>3.74·10^9</td>
<td>5.01·10^9</td>
<td>6.23·10^9</td>
<td>7.41·10^9</td>
<td>8.56·10^9</td>
</tr>
<tr>
<td>2</td>
<td>5.39·10^9</td>
<td>6.85·10^9</td>
<td>8.22·10^9</td>
<td>9.53·10^9</td>
<td>1.08·10^10</td>
<td>1.20·10^10</td>
</tr>
<tr>
<td>3</td>
<td>8.45·10^9</td>
<td>9.93·10^9</td>
<td>1.13·10^10</td>
<td>1.27·10^10</td>
<td>1.40·10^10</td>
<td>1.53·10^10</td>
</tr>
<tr>
<td>4</td>
<td>1.15·10^10</td>
<td>1.30·10^10</td>
<td>1.44·10^10</td>
<td>1.58·10^10</td>
<td>1.72·10^10</td>
<td>1.85·10^10</td>
</tr>
<tr>
<td>5</td>
<td>1.46·10^10</td>
<td>1.61·10^10</td>
<td>1.75·10^10</td>
<td>1.89·10^10</td>
<td>2.03·10^10</td>
<td>2.17·10^10</td>
</tr>
</tbody>
</table>

**Table A-4** Resonance frequencies of the set of $TE_{mn0}$ modes

<table>
<thead>
<tr>
<th>$n\backslash \omega_m$</th>
<th>$f_0$</th>
<th>$f_1$</th>
<th>$f_2$</th>
<th>$f_3$</th>
<th>$f_4$</th>
<th>$f_5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3.74·10^9</td>
<td>1.80·10^9</td>
<td>2.98·10^9</td>
<td>4.10·10^9</td>
<td>5.19·10^9</td>
<td>6.26·10^9</td>
</tr>
<tr>
<td>2</td>
<td>6.85·10^9</td>
<td>5.20·10^9</td>
<td>6.55·10^9</td>
<td>7.82·10^9</td>
<td>9.06·10^9</td>
<td>1.03·10^10</td>
</tr>
<tr>
<td>3</td>
<td>9.93·10^9</td>
<td>8.33·10^9</td>
<td>9.73·10^9</td>
<td>1.11·10^10</td>
<td>1.24·10^10</td>
<td>1.37·10^10</td>
</tr>
<tr>
<td>4</td>
<td>1.30·10^10</td>
<td>1.14·10^10</td>
<td>1.29·10^10</td>
<td>1.42·10^10</td>
<td>1.56·10^10</td>
<td>1.69·10^10</td>
</tr>
<tr>
<td>5</td>
<td>1.61·10^10</td>
<td>1.45·10^10</td>
<td>1.60·10^10</td>
<td>1.74·10^10</td>
<td>1.87·10^10</td>
<td>2.01·10^10</td>
</tr>
</tbody>
</table>
### Appendix B

**Tabel B-1** Frequencies of signals used

<table>
<thead>
<tr>
<th>Resonant frequency at vacuum (Hz)</th>
<th>Plasma ignition frequency (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$2.834 \times 10^9$</td>
<td>$13.56 \times 10^6$</td>
</tr>
</tbody>
</table>

**Table B-2** Measurements of the resonance cavity

<table>
<thead>
<tr>
<th>Diameter (m)</th>
<th>Height (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$97.77 \times 10^{-3}$</td>
<td>$69.01 \times 10^{-3}$</td>
</tr>
</tbody>
</table>

**Table B-3** Measurement values expanded

<table>
<thead>
<tr>
<th>Total power (Watt)</th>
<th>Reflected power (Watt)</th>
<th>Total power inserted (Watt)</th>
<th>Resonant frequency (Hz)</th>
<th>Electron density (m$^{-3}$)</th>
<th>Electron density error (m$^{-3}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.59</td>
<td>0.2</td>
<td>2.39</td>
<td>$2.85 \times 10^8$</td>
<td>$1.20 \times 10^{15}$</td>
<td>$\pm 7.07 \times 10^{13}$</td>
</tr>
<tr>
<td>5.32</td>
<td>0.24</td>
<td>5.08</td>
<td>$2.88 \times 10^8$</td>
<td>$2.98 \times 10^{15}$</td>
<td>$\pm 7.14 \times 10^{13}$</td>
</tr>
<tr>
<td>7.17</td>
<td>0.32</td>
<td>6.85</td>
<td>$2.89 \times 10^8$</td>
<td>$4.05 \times 10^{15}$</td>
<td>$\pm 7.17 \times 10^{13}$</td>
</tr>
<tr>
<td>9.4</td>
<td>0.42</td>
<td>8.98</td>
<td>$2.91 \times 10^8$</td>
<td>$5.06 \times 10^{15}$</td>
<td>$\pm 7.21 \times 10^{13}$</td>
</tr>
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<td>10.5</td>
<td>0.51</td>
<td>9.99</td>
<td>$2.92 \times 10^8$</td>
<td>$5.99 \times 10^{15}$</td>
<td>$\pm 7.24 \times 10^{13}$</td>
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<td>11.57</td>
<td>$2.93 \times 10^8$</td>
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<td>14.22</td>
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<td>13.33</td>
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<td>$7.96 \times 10^{15}$</td>
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<td>15.7</td>
<td>1</td>
<td>14.7</td>
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<tr>
<td>18.51</td>
<td>1.93</td>
<td>16.58</td>
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<td>$10.6 \times 10^{15}$</td>
<td>$\pm 7.40 \times 10^{13}$</td>
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<tr>
<td>20.7</td>
<td>2.36</td>
<td>18.34</td>
<td>$3.00 \times 10^8$</td>
<td>$11.9 \times 10^{15}$</td>
<td>$\pm 7.44 \times 10^{13}$</td>
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Appendix C

C-1 Relevant properties of the HAMEG Network analyser

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Specification</th>
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<tbody>
<tr>
<td>Frequency Range</td>
<td>100 kHz...3 GHz</td>
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<tr>
<td>Resolution</td>
<td>1 Hz</td>
</tr>
<tr>
<td>Sweep time</td>
<td></td>
</tr>
<tr>
<td>Span = 0 Hz</td>
<td>20 ms... 100s</td>
</tr>
<tr>
<td>Span &gt; 0 Hz</td>
<td>20 ms... 100s, minimum 20 ms/600 MHz</td>
</tr>
<tr>
<td>Resolution Bandwidths</td>
<td>-3dB</td>
</tr>
<tr>
<td></td>
<td>100 Hz...1 MHz</td>
</tr>
<tr>
<td></td>
<td>-6dB</td>
</tr>
<tr>
<td></td>
<td>200 Hz, 9 kHz, 120 kHz, 1 MHz</td>
</tr>
<tr>
<td>Video Bandwidths</td>
<td>10 Hz... 1 MHz</td>
</tr>
<tr>
<td>Amplitude range</td>
<td>-114 dBm... +20 dBm</td>
</tr>
</tbody>
</table>
References

Equations
- Informative conversation: [4.3.2], [4.3.3], [4.3.4], [4.3.5], [4.3.6]
- Deduction: [4.3.7], [4.3.8], [4.3.9], [4.3.10]

Bibliography
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(2) Electrons, Ions and Dust in a Radio-Frequency Discharge, Eva Stoffels-Adamowicz, 1994
(3) Measurement of Electron Densities by a Microwave Cavity Method in 13.56 Hz RF Plasmas of Ar, CF₄, C₂F₆, and CHF₃, M Haverlag, 1990