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Photon radiation produced by the fusor
theoretical model and measurements

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Abstract

In this report a theoretical model is established for calculating the photon radiation power of the fusor at the University of Technology in Eindhoven. The main contribution to the radiation comes from the bremsstrahlung production of electrons that collide with the wall of the fusor. All other bremsstrahlung processes as well as characteristic X-rays have a negligible contribution.

The photon radiation power is measured for hydrogen and deuterium plasmas at different power settings of the fusor. By comparing the radiation power measured of hydrogen and deuterium it is shown that the high energy protons produced in a deuterium plasma have a negligible contribution to the radiation power. This result is according to the model. The radiation power predicted by the model is below background levels up to voltages across the fusor of 55 kV while the measured radiation power exceeds background levels starting at 40 kV. This discrepancy is not instigated by variations in the thickness of the wall of the fusor or deviations in the calculated penetration depth of the electrons. Corrections applied in the model show that these factors cannot account for the discrepancy. A more plausible explanation could be the windows mounted on the fusor. In the model the fusor is considered as a spherical vessel without any apertures but in fact 8 windows of maximum 8 cm diameter are present. To check this hypothesis experimentally measurements are performed where radiation not coming from the wall is shielded. The results show that the radiation power measured in the latter configuration is a factor $10^3$ smaller than measured in the original setup.

It can be concluded from the measurements and the model that the radiation power increases exponentially with the voltage due to absorption effects. The experimental results show that as a rule of thumb the radiation increases a factor 10 when the voltage across the fusor is raised by 5 kV in the energy range from 30 to 50 kV. The radiation power increases linearly with the current flowing through the fusor.
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1 Introduction

The Farnsworth Hirsch fusor was invented in 1964 by Philo Taylor Farnsworth. Hopes were high that this apparatus might become a source of fusion power. Unfortunately it has turned out not to be the case [Rider1995]. However the Farnsworth Hirsch Fusor is an interesting means of studying the physics of fusion in a relatively simple experimental setup.

This bachelor report aims to analyse and predict the amount of photon radiation produced by a fusor. As charged particles in the fusor are accelerated bremsstrahlung is produced to conserve the energy in the system. This phenomenon is the major cause of the radiation produced by a fusor. In the first chapter the different contributions to the radiation production will be discussed and a model will be established for calculating the amount of radiation. In order to assess the model a series of measurements is performed at the fusor of the research group Science and Technology of Nuclear Fusion of the University of Technology in Eindhoven (TU/e). Therefore the second chapter will be dedicated to the experimental setup. In the third chapter the experimental results will be presented and compared to the model. In the last chapter of this bachelor thesis the conclusions will be presented as well as an outlook.
2 Modelling the radiation produced by the fusor

In this chapter the model for calculating the amount of photon radiation produced by the fusor is established. The script of the model, which can be found in the appendix, is written in Matlab. The relevant theory is given as well as an analysis on the different contributions to the radiation power. First of all the fusor itself is briefly discussed, followed by a section on characteristic X-rays. The main part of this section is dedicated to bremsstrahlung. This is followed by the penetration depth of electrons in steel as well as the absorption of radiation in different parts of the fusor. The final part is of this chapter is dedicated the output parameter of the model: the photon radiation power.

2.1 The Farnsworth Hirsch fusor

The Farnsworth Hirsch fusor is an example of an Inertial Electrostatic Confinement (IEC) device. It consists of a hollow spherical vessel which is grounded where in the centre a nearly transparent spherical grid is placed. A voltage difference between the wall of the fusor and the grid accelerates particles to fusion-relevant velocities within the grid. For the fusion reactions to occur low pressures must be obtained in the fusor, typically a few deci-pascals. As the grid is negatively charged free electrons are accelerated towards the wall were they collide. The fusor at the University of Eindhoven is built according to the design and optimisation of E.C.G. Hermans [Hermans2013]. See latter reference for more general information about the fusor or IEC Fusion written by Miley [Miley2014]. Suffices it to say that the fusor can be operated in two different modes, star- and jet-mode. Research on this topic has been done by S. van Limpt [Limpt2013]. In the figure below pictures of star- and jet-mode are shown.

![Figure 1. Left: a picture of typical star-mode of the plasma inside the fusor. Right: a picture of a jet-mode.](image)

2.2 Characteristic X-rays

Characteristic X-rays are emitted by a material when outer-shell electrons fill a vacancy in one of the inner shells of the atom. For characteristic X-rays to occur electrons need be excited to higher shells. In the fusor the inner wall and the grid are bombarded with electrons and thus some electrons of the target material are excited. The wall is made of stainless steel type 304, which is an alloy of ferrite, nickel and chromium and the grid is made of nickel. This determines the characteristic X-rays that can be produced by the fusor. The energy of the X-rays produced by these
materials varies from 5 to 8 keV [Bearden1967]. Photons of these energies are highly absorbed by the steel wall of the fusor as is described in section 2.5. It can thus be concluded that the effect of characteristic X-rays is negligible in the production of radiation with the current configuration of the fusor.

However if the grid is changed into a tungsten grid characteristics X-rays might become important since the X-rays emitted by tungsten have energies of around 60 keV [Bearden1967]. It is then important to analyse the quantity of interactions that the electrons and ions make with the grid to estimate the number of electrons that are excited and hence the amount of radiation produced. Only when relatively high voltages are applied to the fusor particles with enough energy will be created to ionise the tungsten. The ionisation energy is at least as high as the characteristic X-ray energies of tungsten so no characteristic X-rays will be produced under 60 kV operation of the fusor with a tungsten grid.

2.3 Bremsstrahlung
Bremsstrahlung is the radiation that occurs when a charged particle is accelerated due to an electric field. The term bremsstrahlung originates from the German 'bremsen' to break and 'Strahlung' radiation. In the case of deceleration, the moving particle loses kinetic energy which is converted into a photon in accordance with the principle of energy conservation. In plasmas produced in the fusor the main contribution comes from the electrons as they are relatively free compared to the heavy ions. However also the ions and in the case of a deuterium plasma the protons contribute to the production of bremsstrahlung. The production of bremsstrahlung can be divided into four contributions. The first contribution is thermal bremsstrahlung, which occurs in plasmas upon interaction of the electrons with the other particles in the plasma. The second contribution is caused by accelerated particles travelling from or towards the grid and interacting with other gas particles along the way. The third contribution is polarisation bremsstrahlung and occurs when the target particle becomes polarised by the Coulomb field of the incoming particle. The fourth and last contribution is bremsstrahlung production by charged particles when they hit a solid material. These topics will be discussed subsequently in the next four subsections.

2.3.1 Thermal bremsstrahlung
Thermal bremsstrahlung is caused by electrons interacting with ions and other electrons in the plasma. The term thermal bremsstrahlung is often used to describe two different radiative phenomena together. The first one is free-free transition in which the finale state of the accelerated electron is free. The second process in a free-bound transition in which the electron in captured by the ion. The latter process is often called recombination radiation. Electron-electron collisions have a negligible contribution to bremsstrahlung in plasmas. In binary collision between two identical particles there is no net acceleration of the centre of mass or centre of charge. Hence the radiated fields from the two particles are equal and opposite and cancel each other. Hence free-free and recombination radiation are considered in calculating the radiation power of the thermal bremsstrahlung. The formula used for this calculation is given on the next page.

Because free electrons have a continuum of possible energy states, bremsstrahlung produces a continue radiation spectrum. The maximum energy of the photons in the spectrum is determined by the Duane-Hunt law [DuHu1915]. The Duane-Hunt law states that the energy of the photons is always less than the kinetic energy of the electrons, which determines the maximum of the radiation spectrum.
Emission spectrum

The electrons in a plasma in thermal equilibrium have a Maxwell-Boltzmann velocity distribution. For such a plasma the following expression holds for the emission per unit volume per unit solid angle per unit frequency ($\nu$): [Hutch2002]

$$\epsilon(\nu) = n_e n_i Z^2 \left( \frac{e^2}{4\pi \varepsilon_0} \right)^3 \frac{8\pi}{3\sqrt{3}} m_e^{3/2} \left( \frac{2m_e}{\pi \varepsilon_0} \right)^{1/2} e^{-\frac{hv}{T_e}}$$

$$\times \left[ \frac{1}{\tilde{g}_{ff}} + G_n \sum_{i=1}^{\infty} \frac{\chi_i}{T_e} e^{-\frac{\chi_i}{T_e}} + \frac{Z^2}{\tilde{g}_{ff}} \frac{1}{T_e} e^{-\frac{Z^2\chi_h}{T_e}} \right] \text{ [W m}^{-3} \text{ sr}^{-1} \text{ Hz}^{-1}] \tag{2.1}$$

The first term in square brackets is the free-free contribution, the second the recombination to the lowest unfilled shell ($n$) and the third to all other shells. The expression above is a classical (non-relativistic) formula with quantum mechanical corrections. The quantum mechanical and some classical effects are described by the Gaunt factor.

The meaning of the symbols is as follows: $n_e$ is the density of the electrons, $n_i$ is the density of the ions in charge state $Z$, $e$ the electron charge, $\varepsilon_0$ the permittivity of vacuum, $m_e$ the electron mass, $c$ the speed of light, $T_e$ the electron temperature, $h$ the Planck constant, $\tilde{g}_{ff}$ the Maxwell averaged free-free Gaunt factor, $\chi_i$ the number of unfilled positions in the lowest shell, $\chi$ the ionisation potential of the ion, $\chi_h$ the Rydberg energy (13.6 eV) and $G_l$ the Gaunt factor which is 0 for $h\nu < Z^2\chi_h/n^2$ since no recombination processes are then possible. Evaluating the fundamental constants in formula 2.1 gives:

$$\epsilon(\nu) = 5.03 \cdot 10^{-54} n_e n_i Z^2 \frac{1}{\sqrt{T_e}} e^{-\frac{h\nu}{T_e}}$$

$$\times \left[ \frac{1}{\tilde{g}_{ff}} + G_n \sum_{i=1}^{\infty} \frac{\chi_i}{T_e} e^{-\frac{\chi_i}{T_e}} + \frac{Z^2}{\tilde{g}_{ff}} \frac{1}{T_e} e^{-\frac{Z^2\chi_h}{T_e}} \right] \text{ [W m}^{-3} \text{ sr}^{-1} \text{ Hz}^{-1}] \tag{2.2}$$

**Thermal radiation power of the plasma in the fusor**

It is assumed that the plasma in the fusor is in thermal equilibrium. Strictly speaking this assumption is not correct since the fast ions and electrons pass through the plasma therefore making it a non-thermal plasma. In this report the contribution of the fast particles is considered as a superposition on the thermal plasma. The amount of bremsstrahlung produced by the plasma in the grid of the fusor is estimated by using equation 2.2 without taking into account recombination bremsstrahlung. Neglecting the recombination contribution to the bremsstrahlung is acceptable as the major contribution to the thermal radiation power comes from free-free radiation. Recombination processes produce narrow peaks in the emission spectrum and hence have a small power contribution. This is clearly shown in the graph on the next page, which represents the emissivity $j(\omega)$ of a plasma in which $T_e = Z^2\chi_h$ [Hutch2002].

An upper limit for the density of electrons in a plasma of the fusor is $1 \cdot 10^{20}$ m$^{-3}$ [Spijkers2013]. It will be shown later on that even in this upper limit the thermal bremsstrahlung can be neglected. Assuming single ionisation of gas atoms, the ion density of the plasma is of the same order. The electron temperature of the plasma inside the fusor is assumed to be order eV. The Maxwell averaged free-free Gaunt factor can be approximated by 1, since it deviates roughly from 0.2 to 1.5 for different frequencies and electron temperatures [KarLat1960].
To calculate the radiation power produced per volume per steradian $p$ equation 2.2 is evaluated for the given values of the constants above and integrated over the X-ray frequency range. The energy X-ray range starts at 100 eV and ends at 100 keV.

$$p = \int_{0.1keV}^{100keV} 5.03 \cdot 10^{-54} n_e n_i Z^2 \frac{\sqrt{Z}}{T_e} e^{\frac{hv}{T_e}} \cdot dv \quad [W \cdot m^{-3} \cdot sr^{-1}]$$ \hspace{1cm} (2.3)

The thermal radiation power is then given by multiplication of $p$ with the volume of the plasma. It is assumed that there is only plasma within the grid. Hence the plasma volume is a sphere which contains $4\pi$ steradians. The radius of the grid is about 5 cm. In the figure below the thermal radiation power $P$ is plotted against the electron temperature $T_e$ together with the background radiation level (see section 2.6). It is clear from this graph that the thermal bremsstrahlung contribution is negligible. This is plausible result since the electrons have a random kinetic energy in the order of eV. It is therefore unlikely for the electrons to produce radiation in the X-ray energy regime.

$$\text{Figure 2. Graph of the thermal emissivity } j(\omega) \text{ of a plasma in which } T_e = Z^2 \chi_h, \text{ showing the free-free radiation and recombination radiation [Hutch2002].}$$

$$\text{Figure 3. Logarithmic plot of the thermal radiation power as a function of the electron temperature in electrons volts. The assumptions made in the calculation are stated in the paragraph above the figure.}$$
2.3.2 Bremsstrahlung produced by fast particles interacting with gas

The bremsstrahlung produced by fast particles interacting with gas can be neglected. To prove this statement the radiation power produced by fast particles upon collision with gas $P_{\text{gas}}$ is compared to the radiation power produced when particles hit the wall $P_{\text{wall}}$. $P_i$ is proportional to the collision rate $\xi_i$ times the typical length $l_i$. $l_{\text{gas}}$ is the radius of the fusor which is 0.25 m while $l_{\text{wall}}$ is the penetration depth of the electrons in steel which is of the order of $10^{-5}$ m for electrons of 50 keV (see section 2.4). The fraction $P_{\text{gas}}/P_{\text{wall}}$ is considered in the formula below.

$$\frac{P_{\text{gas}}}{P_{\text{wall}}} \sim \frac{\xi_{\text{gas}} l_{\text{gas}}}{\xi_{\text{wall}} l_{\text{wall}}} = \frac{n_{\text{gas}} \nu \sigma_{\text{gas}}}{n_{\text{wall}} \nu \sigma_{\text{wall}}} = \frac{n_{\text{gas}} l_{\text{gas}}}{n_{\text{wall}} l_{\text{wall}}}$$

(2.4)

Where $n_i$ is the density (number of particles per m$^3$), $\nu$ the velocity of the particles and $\sigma$ the cross section for collision. Both $\nu$ and $\sigma$ are assumed to be of the same order for both the gas and wall interactions. The density of steel type 304 can be calculated straightforwardly by dividing the density in kg/m$^3$ by the molar mass and multiplying by Avogadro’s number, yielding $9 \cdot 10^{28}$ particles per m$^3$. The density of the gas surrounding the grid can be estimated by ideal gas law assuming that the temperature of the gas is equal to room temperature. Performing this calculation yields a density of $3 \cdot 10^{18}$ particles per m$^3$.

$$\frac{P_{\text{gas}}}{P_{\text{wall}}} \approx \frac{n_{\text{gas}} l_{\text{gas}}}{n_{\text{wall}} l_{\text{wall}}} \approx 10^{-7}$$

(2.5)

Hence the bremsstrahlung produced by fast particles interacting with gas can be neglected.

2.3.3 Polarisation bremsstrahlung

Polarisation bremsstrahlung occurs when the target particle becomes polarised by the Coulomb field of the incoming particle. Formula 2.1 does not take polarisation bremsstrahlung into account. However in a free electron gas as target material like a plasma polarisation bremsstrahlung is observed [WillQuar2008]. According to San Portillo and C.A. Quarles the contribution of polarisation bremsstrahlung is only dominating below energies of several keV at least for the rare gasses used in their experiment [PortQuar2003]. On this basis the contribution of polarisation bremsstrahlung is neglected since this low energies are always absorbed by materials around the fusor. Polarisation bremsstrahlung is also negligible in the production of bremsstrahlung when solid materials are bombarded with electrons [WillQuar2008].

2.3.4 Bremsstrahlung in solid materials

When charged particles interact with solid materials the acceleration is usually relatively high compared to the interactions with gas. It is therefore expected that the radiation power due to collisions of charged particles with solid material is high compared to bremsstrahlung produced by particles interacting with gas.

The positively charged ions created in the plasma will be accelerated towards the grid which has a high transparency and therefore relatively few interactions will take place. Hence this contribution is neglected. The electrons will be accelerated towards the wall of the fusor. A significant amount of the electrons will hit the wall. (Some electrons will not reach the wall because of recombination and absorption processes.) The number of electrons reaching the wall can be estimated by the current in the fusor. When fusion reactions occur in a deuterium plasma 3.02 MeV protons are produced. These protons will hit the wall of the fusor as their energy is too high be stopped by the voltage across the fusor. Hence there are two contributions: the electrons and the protons which will be discussed separately in the paragraphs below.
Bremsstrahlung production by electrons interacting with the wall of the fusor
In the fusor at the University of Technology in Eindhoven the electrons interact with stainless steel type 304, which is the material the wall of the fusor consists of. The radiation power $P_{\text{x-ray}}$ produced in the steel wall of the fusor can be calculated by the following empirical relation: [Beatty1913]

$$P_{\text{x-ray}} = P_{\text{electrons}} l Z V$$

(2.6)

Where $P_{\text{electrons}}$ is the power from the electrons hitting the wall of the fusor, $Z$ the atomic number of the target material, $l$ an empirical constant and $V$ the voltage across the fusor. It is assumed in this relationship that all electrons have maximum energy which is not always the case in the fusor. In this way the model serves as an upper limit. The constant $l$ is determined by several authors [CompAll1954]. The method of Nicholas is used in the calculations here as it is considered to be the most accurate since it takes backscattering electrons into account [Nicholas1930]. The radiation power according to Nicolas is:

$$P_{\text{Nicholas}} = 1.1 \cdot 10^{-9} P_{\text{electrons}} Z V$$

(2.7)

This approximation is probably correct within about 20% [CompAll1954]. The steel wall of the fusor has an average atomic number $Z$ of 25.8 amu as it consist of 74% Fe, 18% Cr and 8% Ni. $P_{\text{electrons}}$ is determined by $V$ and the current density. Since the fusor has a radius $r_{\text{fusor}}$ of 25 cm the current density is approximately the measured current in the fusor divided by $4\pi r_{\text{fusor}}^2 \approx 0.78$ m$^2$. The radiation output power is then in W m$^{-2}$. The radiation power per square meter is calculated using the relation above for different voltages across the fusor and different currents. The results are shown in the graph below.

![Graph of the radiation power produced by bremsstrahlung when electrons hit stainless steel. The radiation power per square meter is plotted as a function of the voltage across the fusor.](image)

Figure 4. Graph of the radiation power produced by bremsstrahlung when electrons hit stainless steel. The radiation power per square meter is plotted as a function of the voltage across the fusor.
Kramers’ law
In order to be able to calculate the absorbed amount of radiation power by the wall of the fusor one needs to know the spectrum as the absorption is dependent on the energy of the photons (see section 2.5). The bremsstrahlung intensity spectrum derived by H.A. Kramers for complete absorption of an electron of energy $E_e$ is given by: \[ I_{Kramer} = KZ \frac{(E_e-E)}{E} \text{ [s}^{-1}\text{ m}^{-2}] \] (2.8)
Where $K$ is a constant of proportionality, $Z$ the average atomic number of the target material, $E_e$ the energy of the incident electrons and $E$ the energy of the emitted photons. As the total power is calculated using Nicolas’ relation the constant $K$ can be derived and hence an approximation of the bremsstrahlung spectrum can be obtained. The integral over the energy of $I_{Kramer}$ represents the total radiation power produced by bremsstrahlung upon interaction with the wall of the fusor. A typical intensity spectrum is shown in the graph below for a current of 40 mA and a voltage of 60 kV.

Figure 5. Logarithmic plot of the un-attenuated intensity spectrum produced by a fusor operating at 40 mA and 60 kV.

Bremsstrahlung production by protons interacting with the wall of the fusor
Besides the electrons, high energetic protons can contribute to the amount of bremsstrahlung produced. Protons are created in fusion reactions in a deuterium plasma. If two deuterium particles happen to fuse there are two possible reactions that can occur.

$D + D \rightarrow 1.01 \text{ MeV} \; T + 3.02 \text{ MeV} \; p$

$D + D \rightarrow 0.82 \text{ MeV} \; \text{He}3 + 2.45 \text{ MeV} \; n$
Where $T$ is tritium the isotope of hydrogen, $p$ a proton and $n$ a neutron. Both reactions above have an equal probability of occurrence. Hence if one can measure the number of neutrons per second the number of protons is known as well. The fusor at the University of Technology in Eindhoven generates maximum $10^7$ neutrons per second [Wijnen2014]. This is a relatively low number compared to the amount of electrons generated, which is typically $10^{17}$ for electron currents between 20 and 40 mA. The electrons have an average energy of 50 keV whereas the protons have an energy of 3.02 MeV, roughly 10^2 higher than the electrons. Hence the power in the neutrons is of the order of $10^{20}$ eV/s whereas the power in the protons is roughly $10^{10}$ eV/s. Therefore the contribution of protons is negligible. The calculations on the amount of radiation produced by the baser are solely based on the electrons that hit the steel wall of the fusor.

2.4 Penetration depth electrons in steel

The penetration depth of electrons in steel is determined by inelastic scattering of the incident electron with the target atoms. The primary electron loses energy due to excitation and ionisation of the target atoms. Thus the electron slows down through the target material until it reaches the Fermi energy and flows from the specimen to earth. The relationship of Bethe and Ashkin for the stopping power based on a continuously slowing down approximation is used to calculate the penetration depth [BethAshkin1953]. The stopping power $\frac{dE}{dx}$ is the rate of loss of energy per unit path length and is given by:

$$\frac{dE}{dx} = -78500 \frac{Z \rho}{A} \ln \left( \frac{E_0}{J} \right)$$

(2.9)

Where $Z$ is the average atomic number of the target material, $\rho$ the density in g/cm$^3$, $A$ the atomic mass in amu, $E_0$ the energy of the incident electron in eV and $J$ the mean ionisation energy of the target material. $J$ in eV is approximated by the following empirical relationship: [BrunRiv1983].

$$J = 9.76 Z + 58.8 Z^{-0.19}$$

(2.10)

To calculate the penetration depth $d$ of the electrons the inverse of the stopping is integrated over the X-ray energy range up to the incident energy of the electron.

$$d = \int_{E_0}^{E_{1.166}} \frac{1}{\frac{dE}{dx}} dE$$

(2.11)

For the lower cut off energy $J/1.166$ is taken since the relationship of Bethe and Ashkin is not valid for energies below $J/1.166$ as according to their relationship the energy will increase upon collision. Nonetheless provides their formula a means to estimate the penetration depth of the electrons as a function of the energy. The results are shown in the graph on the next page.

It is clear from the graph that the penetration depth is relatively small compared to the thickness of the wall of the fusor at the University of Technology in Eindhoven. In the next section the effect of photon absorption will be discussed. To calculate the effective thickness of the steel the penetration depth will be taken into account.
2.5 Absorption
The photons that are produced by bremsstrahlung and characteristic X-rays in the fusor are attenuated by the wall of the fusor and by the lead mounted against the outer side of the wall. The wall of the fusor at the University of Technology in Eindhoven consists of 2 mm of stainless steel type 304, 1 cm of water to cool the vessel and another 2 mm of stainless steel. Due to the way the vessel of the fusor is created, it is forced into the spherical shape, some variations in thickness might be present. On the outside of this wall 0.5 mm of lead is mounted. Absorption in the water is neglected since absorption is most effective in heavy atomic substances like lead. The intensity of the radiation after absorption $I/I_0$ can be calculated by the following formula:

$$I/I_0 = e^{-(\mu/\rho)l}$$  \hspace{1cm} (2.12)

Where $\mu/\rho$ is the mass attenuation coefficient which is material dependent, $\rho$ the density of the target material and $l$ the thickness. The formula above can be considered as a form of Beer-Lambert's law [Swinehart1962]. In this report experimental X-ray mass attenuation coefficients have been used from the database of the National Institute of Standards and Technology (NIST) [HubSeltz1996]. The mass attenuation coefficient for steel type 304 has been obtained from the NIST database by combining the coefficients for ferrite, chromium and nickel. Stainless steel type 304 consists for 74% of ferrite, 18% of chromium and 8% of nickel. The obtained values are in accordance with experimental work of the Oak Ridge National Laboratory [FosEvans1964]. To extent the number of data points of the mass attenuation coefficients of both lead and steel a piecewise cubic interpolation is applied. See the appendix for the data and interpolation.
The total absorption is calculated by taking the penetration depth of electrons in stainless steel is taken into account, since this reduces the effective thickness of the target material. The intensity of the radiation after absorption $I/I_0$ is smaller than $10^{-10}$ for X-ray photons below 40 keV. Therefore the characteristic X-rays are neglected in calculating the total radiation produced by the fusor (see section 2.2). A logarithmic plot of the intensity after absorption in the energy range from 20 keV to 100 keV is shown in the graph below.

![Logarithmic plot](image)

*Figure 7. Logarithmic plot of the intensity of the radiation after absorption by 4 mm of steel type 304 and 0.5 mm lead as a function of the photon energy.*

The absorption as described above is applied to the spectra calculated via Kramer's algorithm. A typical attenuated spectrum is shown in the figure below.

![Attenuated spectrum](image)

*Figure 8. Attenuated spectrum produced by a fusor operating at 40 mA and 60 kV.*
The graph on the previous page shows that photons with energies below 45 keV are heavily absorbed by the wall and the lead mounted on the outside of the wall. The graph also shows the Duane-Hunt limit as discussed in section 2.3.1 [DuHu1915]. There are no photons with higher energies than the maximum energy of the electrons.

2.6 The radiation power according to the model

The final output of the model is the radiation power per square meter that is produced by the fusor. This value is obtained by integrating the attenuated spectra over the energy. In the figures below the radiation power is shown for the fusor operating at 40 mA for different voltages.

The background radiation on earth is of the order of $10^{-8}$ Sv per hour [UNSC2010]. 1 Sievert is equivalent to 1 J/kg. The conversion from radiation power to Sievert includes a biological factor that accounts for the absorbed amount of radiation by the human body. Suffices it to say that radiation power below $10^{-7}$ is below background levels of radiation which is shown by calculating the radiation power out of background spectra. In the graph on the next page the radiation power is again plotted against the voltage but this time on a non-logarithmic scale in the energy range from 50 to 80 keV.

The graph above shows that the fusor does not produce any radiation above background radiation levels for operation below 55 keV according to the model established in this section. These results will be compared to the experimental data in chapter 4 of the report. In the next chapter the experimental setup will be discussed.
Figure 10. Plot of the radiation power produced by the fusor operating at 40 mA as a function of the voltage.
3 Experimental setup

In this chapter the experimental setup is described to perform measurements to assess the model established. The specifications of the detector are discussed as well as the setup of the fusor at the plasma group of the University of Technology in Eindhoven. Followed by a description of the measurements performed. The last section of the chapter is dedicated to the data analysis.

3.1 Detector: Canberra Inspector 1000

The radiation power that is produced by the fusor at the University of Technology in Eindhoven is measured with a Canberra Inspector 1000 using a IPROS-2 stabilised probe, which consist of a 50 mm x 50 mm NaI crystal. The Inspector 1000 is a scintillation detector. An X-ray photon upon detection generates a light flash in the crystal, mainly due to photoelectric absorption. The multichannel analyser (MCA) converts this light flash to a digital signal. The number of channels determines the resolution of the detector as it limits the step size in the energy spectrum. The Inspector 1000 is used in 512 channel mode with an maximum energy detection of about 400 keV. The minimum energy that can be detected is determined by the implementation of the NaI crystal and will be specified in the paragraph below. The implementation of the crystal also determines the efficiency of the detector. The efficiency is defined as the probability that a photon will interact with the detector and produces a count. Both the energy and the efficiency calibration will be discussed in the paragraphs below.

Energy Calibration

The energy calibration is obtained by measuring the spectrum of gamma emitters in the energy range up to 400 keV. The channel number, where detection occurred, is plotted against the energy to obtain the calibration curve. The radioactive substances americium-241, jodium-129, cesium-137 and barium-133 were used to perform the calibration. A spectrum measured with the Inspector 1000 of barium-133 is shown in the graph below. The graph is made after the energy calibration was applied.

Figure 11. Spectrum from barium-133 measured with Canberra Inspector 1000.
It is clear from the graph on the previous page that the width of the peaks in the barium spectrum is relatively wide especially for energies around 300 keV. This is a limitation of the detector. Since the expected energy of the photons is maximum 100 keV the width of the extremely wide peaks around 300 keV are not a problem. The typical full width at half maximum (FWHM) is approximately 8 keV in the energy range up to 100 keV. This limitation will not be an issue since assuming that the peaks are symmetrical the average energy in the peak will be exactly the centre. Thus the integrated spectra will represent the actual amount of energy in the spectrum. The graph also shows the minimum energy that is detected by the probe, which is 36 keV.

Similar spectra are measured of cesium-137, ameritium-241 and jodium-127. The database of the ICRP is used to find the energies of the photons emitted by the radioactive substances [ICRP1983]. The uncertainty in the energy is estimated by taking half of the FWHM. The results are plotted in the graph below as a function of the channel number. The linear fit is used as an energy calibration.

![Graph](image)

*Figure 12. Graph of the energy as a function of the channel number for measurements performed with the Canberra Inspector 1000.*

**Efficiency calibration**

The efficiency curve used is the standard calibration performed by Canberra, the manufacturer of the detector. Unfortunately it has not been possible to find out the configurations and limitations of this calibration. Suffices it to say that the calibration curve used is given by the formula below:

\[
\text{Efficiency} = -9.034 + 1.504 x - 1.473 \cdot 10^{-1} x^2 + 1.641 \cdot 10^{-2} x^3 - 6.163 \cdot 10^{-2} x^4 + 5.737 \cdot 10^{-3} x^5
\]

Where \(x = \ln(3.53 \cdot 10^3/E)\) with \(E\) in keV.
3.2 Setup of the fusor at the fusion group of the TU/e

The fusor at the fusion group of the University of Technology in Eindhoven is situated in the TNO building room number 2.208. A schematic picture of the setup is shown in the figure on the next page. The fusor is powered by a Heinzinger high voltage supply which can generate maximum 120 kV and 100 mA. However the maximum power is limited by the feed through which can stand a maximum voltage of 80 kV. A 210 kΩ resistant is connected in series with the fusor to limit the current flowing through the circuit. Hence if one wants to have a constant voltage across the fusor for different currents the voltage across the Heinzinger has to be adapted. More information about the setup and safety issues can be found in the safety assessment of the fusor [OostJas2014]. The grid in the fusor is made out of nickel. This limits the maximum current that can flow through the circuit. The maximum current is 40 mA.

![Diagram of the fusor setup](image)

Figure 13. Schematic setup of the fusor installation present at the fusion group of the University of Technology in Eindhoven.

The radiation power of the fusor is measured using a Canberra Inspector 1000 as is described in the previous section. The detector is placed at the left side of the fusor at two different positions. Two pictures of the first setup position are depicted in the figure on the next page.

The detector is placed in front of a part of the wall of the fusor where no window is present as is shown on the left picture on the next page. Both pictures represent the first setup position. In the second position the detector is placed in front of the window straight above the probe in the left picture of figure 14. The distances from parts of the fusor with respect to the detector for both setup situations are shown in the figures below.
3.3 Measurements
The fusor is operated in star-mode (see section 2.1) with two different gasses: hydrogen and deuterium. For both gasses the radiation power is measured as a function of the voltage across the fusor for a constant current of 40 mA. The goal of these measurement series is to check if protons contribute to the radiation power and to assess the radiation power predicted by the model. For deuterium the current dependency of the radiation power is checked by performing a series of measurements at constant voltage of 41 kV across the fusor. Finally the probe of the detector is covered with a cylinder of lead to exclude radiation not passing through the wall of the fusor. This setup also excludes the possibility that the detector has an over response for non-frontal detection of photons. The lead used to make the cylinder is 2 mm thick. In this configuration the radiation power is measured as a function of the voltage for deuterium plasma and a constant current of 40 mA. A picture of the probe covered with lead is shown in the figure on the next page.
Figure 17. Picture of the measuring configuration where the probe of the detector is covered by a cylinder of lead. The cylinder is wrapped in duct tape as lead is poisonous.

### 3.4 Data analysis

The spectra measured with the Canberra Inspector 1000 are obtained in a measuring time of 1 minute. At the beginning of each measuring series a background spectrum is gauged. All other spectra are corrected for the background radiation. For each configuration 4 spectra are measured which are averaged to obtain a more accurate result. A typical spectrum for setup position 1 is depicted in the figure below.

![Typical spectrum measured in setup position 1 with the fusor operating at 43 kV and 44 mA.](image)

Figure 18. Typical spectrum measured in setup position 1 with the fusor operating at 43 kV and 44 mA.
As the detector cannot detect photons below 36 keV the full spectrum cannot be measured. However the second peak provide a means to measure the first peak of the radiation spectrum. The second peak is created by combined detection of two photons. This phenomenon only occurs at relatively high dose rates. Thus the middle of the second peak is twice the energy of the first peak in the spectrum which is only partly measured. For the configuration above it can be concluded that most of the photons emitted have an energy of about 32 keV. The Duane-Hunt limit is not visible in the spectrum above. The minimum in between the first and the second peak occurs at 48 keV which is more than the maximum energy of the electrons. This deviation is probably due to the relatively wide peaks that are measured in the spectrum for a single photon energy as is described in section 3.1. A typical spectrum measured in setup position 2 is shown in the figure below.

It is clear from the graph in figure 19 that total saturation has occurred in the detector since the Duane-Hunt limit is violated to a large extent. Therefore it is plausible that the windows mounted on the fusor pass through more radiation than through the wall. This will be discussed in more detail in the next chapter. Suffices it to say that the measurements performed at position 2 are not analysed any further.

![Figure 19. Typical spectrum measured in setup position 2 with the fusor operating at 40 kV and 40 mA.](image)

The spectra measured in position 1 are multiplied by the energy and integrated over the channel number to obtain the radiation energy in the spectrum. To calculate the radiation power in the detector the radiation energy is divided by the live time. The live time is defined as the time that the detector is able to detect photons and not busy processing previous detections. Finally the radiation power per square meter is calculated by dividing the radiation power in the detector by the frontal surface of the NaI crystal in the measuring probe.
It should be noted that because the detector cannot measure photons with energy lower than 36 keV, the actual radiation power is higher. Especially for the low voltage measurements a relatively large part of spectrum is not measured. Since the voltage across the fusor determines the energy of the electrons colliding with the wall and thus the photon energy. In the graph below two normalized spectra are shown to illustrate this.

Figure 20. Normalised radiation spectra for the fusor operating at 40 mA. Normalisation is performed by dividing both spectra by the maximum numbers of counts in that specific spectrum.

Figure 20 shows two spectra. The red spectrum is measured at 34 kV and represents the worst case measuring scenario. The two photon peak occurs at 55 keV thus it is expected that the first peak occurs at 27.5 keV which is outside the measuring range of the detector. The black spectrum is measured at 50 kV and represents the best measuring scenario. The two photon peak occurs in this spectrum at 70 keV and thus the first peak is at 35 keV. Even in this configuration about 50% of the first peak is not measured. The two photon peak in the spectrum provides a means to measure part of the low energy photons. It is not possible to tell how many low energy photons are measured this way and which part is not as it strongly depends on the count rate which differs in each configuration of the fusor. In this report no correction has been applied for this limitation of the detector. It is recommended to do so if any measurements might be carried out in the future. The live time is important to consider in the constructing of a correction method cause it is crucial in calculating the radiation power out of the radiation energy in a spectrum.
4 Results and discussion

A series of measurements is performed in setup position 1 as described in the previous section for both hydrogen and deuterium plasmas at different voltages. The current is kept constant at 40 mA. The separate radiation power measurements from single measurements have been averaged as stated in section 3.4. The standard deviation in these averages is at maximum 20% of the average itself. It is therefore save to assume that the measured values are correct within 20% inaccuracy. This relatively high uncertainty is probably instigated by the instability in the plasmas. It is almost impossible to maintain a plasma at the exact same voltage, current and pressure for a period of 1 minute. There are always small oscillations in either the voltage or the current depending on which of the two is locked by the Heinzinger. These oscillations are within the read accuracy of the Heinzinger display, which is about 1 kV or 1 mA. There is also an opportunity that the temperature dependence of the NaI crystal in the detector effects the radiation power. The operating temperature range of the detector is -10°C to +55°C. However the absolute temperature dependence is not specified by the manufacturer of the detector. The measured radiation power of both deuterium and hydrogen is shown in the graph below.

![Graph showing logarithmic plot of measured radiation power as a function of voltage for both deuterium and hydrogen plasmas.](image)

*Figure 21. Logarithmic plot of the measured radiation power as a function of the voltage for both deuterium and hydrogen plasmas. The current is held constant at 40 mA.*

It is clear from the graph above that both the deuterium and hydrogen curve overlap to a large extent. Therefore it can be concluded that the fusion protons in the deuterium plasma have a negligible contribution to the radiation power produced. This is in accordance with the model (see section 2.3.4). The graph also shows that the radiation power increases exponentially with the voltage. As a rule of thumb: the radiation power increases a factor 10 every 5 kV in this energy range.
For deuterium plasmas the current dependency of the radiation power is checked at constant voltage of 41 kV. It is expected that the radiation power increases proportional to the current (see section 2.3.4). The results are not in accordance with the theory as is shown in the graph below. Deviations could have been caused by the factors discussed on the previous when the uncertainty in the measured values is addressed.

Figure 22. Graph of the radiation power as a function of the current flowing through the fusor. The voltage is kept constant at 41 kV.

A more thoroughly check of the current dependency could have been carried out if more measurements were performed. However the number of measuring points was limited by the instability of the plasmas. In the graph below the experimental results on the radiation power, presented earlier on logarithmic scale in figure 20, is plotted together with the predicted radiation power according to the model.

Figure 23. Graph of the radiation power as function of the voltage. Both the experimental and predicted values according to the model are plotted.
It is clear from the graph on the previous page that there is a discrepancy between the predicted and the measured radiation power. The radiation power is below background level up to 55 kV according to the model, while the measurements show significant amounts starting at 40 kV. The thickness of the steel walls could be subject to some variations as is described in section 2.5. To analyse the effect this could cause, the radiation power is calculated for a total thickness of the steel walls of 2 mm instead of 4 mm. The penetration depth is calculated due to Bethe’s relation which is based on a continuous slowing down approximation. It could be that some deviation from the actual penetration is present in the calculated value. To assess this influence the model is run for a 10 times bigger penetration depth compared Bethe’s relation. Both contributions are depicted in the graph below.

![Graph of the radiation power as a function of the voltage. The experimental values are plotted as well as values predicted by the model for different parameters.](image)

Figure 24. Graph of the radiation power as a function of the voltage. The experimental values are plotted as well as values predicted by the model for different parameters.

The graph shows that the two contributions discussed above cannot be the primary cause of the discrepancy. 1 mm steel walls are almost impossible to make. In this way the extra modelled curves are an upper boundary. It is more likely that the discrepancy is instigated by the windows in the fusor. In the model the fusor is considered as a spherical vessel without any apertures. However there are in total 4 windows with a diameter of 8 cm and 4 windows with a diameter of 5 cm present. This hypothesis could also explain why saturation of the detector occurred in setup position 2 as described in section 3.4. To investigate this hypothesis the probe of the detector is covered with a cylinder of lead and some measurements on the radiation power are performed. In this way radiation coming from the wall is most directly measured and all other radiation is highly attenuated. The results from these measurements are shown in the graph on the next page.
It is clear from the graph above that the radiation power measured with the lead cylinder present is about $10^3$ times smaller than without the lead. This result makes the hypothesis that the windows are responsible for most of the radiation plausible. However to verify it more measurements should be performed and the cylinder should be attached to the wall of the fusor. In the setup used to perform the measurements which results are shown above some space was present between the cylinder and the wall. The cylinder itself was also not perfectly shaped. There was a small gap along the long axis where the lead was bended together. Also the scattering of photons could contribute to the deviation. Besides that the efficiency curve specified in formula 3.1 is only valid for an unshielded configuration. A new calibration should be performed for this setup taking into account the angular dependency of the detector. These contributions might account for the deviation from the curve according to the model and measured radiation power with the lead cylinder attached to the probe.

The radiation in the fusor is produced when the electrons hit the steel wall as described in chapter 2. Thus the source of the radiation is some distance away from the probe as specified in figure 15. The model however calculates the radiation power at the fusor wall. Since the fusor cannot be considered as a radiation point source a correction for this distance is rather complex. Therefore this correction is not applied. It would have been better to place the probe as close to the wall from this point of view. However in that configuration saturation might occur in the detector. Hence it is recommended for future measurements to place the probe on the wall and to shield the frontal surface with some extra lead if needed to avoid saturation.
5 Conclusion and outlook

The radiation power is calculated and measured for hydrogen and deuterium plasmas at different power settings of the fusor. The probe of the detector is placed in two different setup positions. In the first configuration the probe was situated roughly 18 cm away from a bare part of the fusor wall in the second setup the probe was placed in front of an aperture of the fusor. The second position resulted in saturation of the detector.

It can be concluded from the measurements and the model that the radiation power increases exponentially with the voltage due to absorption effects. The experimental results show that as a rule of thumb the radiation increases a factor 10 when the voltage across the fusor is raised by 5 kV. The radiation power increases linearly with the current flowing through the fusor.

By comparing the radiation power measured in setup position 1 of hydrogen and deuterium it is shown that the high energy protons produced in a deuterium plasma have a negligible contribution to the radiation power. This result is according to the model. The radiation power predicted by the model is below background levels up to voltages across the fusor of 55 kV while the measured radiation power exceeds background levels starting at 40 kV. This discrepancy is not instigated by variations in the thickness of the wall of the fusor or deviations in the calculated penetration depth of the electrons. Corrections applied in the model show that these factors cannot account for the discrepancy. A more plausible explanation could be the windows mounted on the fusor. In the model the fusor is considered as a spherical vessel without any apertures but in fact 8 windows of maximum 8 cm diameter are present. This could also explain the saturation of the detector in setup position 2. To check this hypothesis experimentally measurements are performed with a lead cylinder mounted on the probe of the detector. In this way only radiation from the wall of the fusor is measured. The results show that the radiation power measured in the latter configuration is a factor $10^3$ smaller than measured in the original setup. However the measured values with lead present still differ with the values according to the model. This could be instigated by the imperfections in the cylinder and by scattering of the photons.

If more measurements are to be done on this topic it is recommended to set up a configuration where the probe is perfectly connected to the wall of the fusor to exclude any radiation not coming from the wall. It is also recommended to place the probe as close the wall as possible to avoid complicated corrections to the radiation power for the distance in between the source and the detector. It could be that the frontal part of the probe has to be shielded to prevent saturation to occur in the detector.
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References


Appendix: Script of model

\( k = 1.1 \times 10^{-9} \); \% empirical constant Beatty algorithm
\( A = 4 \pi \times 0.25 \times 0.25 \); \% area inside fusor which has a radius of 0.25 m
\( Z_{av} = (0.74 \times 26 + 0.18 \times 24 + 0.08 \times 28) \); \% average atomic number steel type 304
\( A_{av} = (0.74 \times 56 + 0.18 \times 52 + 0.08 \times (58 + 60)) \); \% average atomic mass steel type 304
\( E_{min} = 0.1 \times 10^{3} \); \% X-ray energy range starts at 100 eV
\( e = 1.60217665 \times 10^{-19} \); \% charge of an electron
\( \rho = 0.75 \times 7.860 + 0.18 \times 7.190 + 0.08 \times 8.902 \); \% density steel type 304 in g/cm³
\( J = 9.76 \times Z_{av} + 58.5 / Z_{av}^{0.19} \); \% mean ionisation energy in eV of steel

%% Calculating the empirical constant of Kramer's algorithm for I and V
\( P_z \) = zeros(10, 51);
\( V_0 \) = zeros(1, 51);
\( k_m \) = zeros(1, 51);
l = 1;
for \( I = 0.01:0.01:0.1 \) \% current variation 10 to 100 mA
\( I_e = I / A \);
i = 1;
for \( V = 30 \times 10^{3}:1 \times 10^{3}:80 \times 10^{3} \) \% voltage variation 30 to 80 kV
\( V_0(1, i) = V \);
\( P(l, i) = I_e \times V^2 \times Z_{av} \times k \); \% Beatty's algorithm for calculating power
\( k_m(l, i) = (V^2 \times k) / (V \times \log(V / E_{min}) - (V + E_{min})) \); \% Kramer integrated
i = i + 1;
end
l = l + 1;
end

%% Calculation of the penetration depth of the electrons in steel
\( de_{rvs} = \delta(E) = 1/(78500 \times Z_{av} \times \rho \times (A_{av} \times e)) \times \log(1.166 \times e \times J) \times 1 \times 10^{-5} \); \% inverse stopping power according to Bethe units: mm/eV
\( X_{RVS} \) = zeros(1, 1000);
\( \text{Energy} \) = zeros(1, 1000); \% Energy steps
x = 1;
for \( i = 0.1:0.1:100 \)
\( \text{Energy}(1, x) = i \);
x = x + 1;
end
for \( i = 1:1:1000 \)
\( \text{min} = 283 \); \% minimum energy which Bethe's relation is valid otherwise the % stopping power is positive which would imply an energy gain upon % collision
\( \text{max} = \text{Energy}(1, i) \times 1 \times 10^{3} \);
\( X_{RVS}(1, i) = \text{integral}(de_{rvs}, \text{min}, \text{max}) \); \% penetration depth in mm
end

%% The mass attenuation coefficients
\( \text{Ener} = [1 1.5 2 2.484 2.53429 2.5856 3 3.0664 3.3013 3.5542 3.69948 3.8507 4 5 6 8 10 13.0352 15 15.2 15.5269 15.8608 20 30 40 50 60 80 100] \);
\% Energy points of NIST mass attenuation coefficients of lead
\( \text{E}_{Pb} = [1E^{-3} 0.0015 0.002 0.003 0.004 0.005 0.006 0.00712 0.00712 0.008 0.01 0.015 0.02 0.03 0.04 0.05 0.06 0.08 0.1] \times 10^{3} \);
\% energy points of ferrite
\( \text{E}_{Ni} = [1E^{-3} 0.00100404 0.001008 0.0010082 0.001005 0.002 0.003 0.004 0.005 0.006 0.008 0.01 0.015 0.02 0.03 0.04 0.05 0.06 0.08 0.1] \times 10^{3} \);
\% energy points of nickel
\( \text{E}_{Cr} = [1E^{-3} 0.0015 0.002 0.003 0.004 0.005 0.0059892 0.0059893 0.006 0.008 0.01 0.015 0.02 0.03 0.04 0.05 0.06 0.08 0.1] \times 10^{3} \);
\% energy points of chromium
\( \text{Pb} = [5210 2356 1285 800.6 1726 1944 1965 1857 1796 1496 14421311 1251 730.4 467.2 228.7 130.6 67.01 111.6 107.8 141.6 134.4 86.36 30.32 14.36 8.041 5.021 2.419 1.976] \); \% mass attenuation coefficients of lead, units: cm²/g
Fe=[9085 3399 1626 557.6 256.7 139.8 84.84 53.19 407.6 305.6 170.6 57.08 25.68 8.176 3.629 1.958 1.205 0.5952 0.3717];
%mass attenuation coefficients of ferrite, units: cm^2/g
Ni=[9855 9753 9654 10990 4234 2049 709.4 328.2 179.3 109 49.52 44.28 329.4 209 70.81 32.2 10.34 4.6 2.474 1.512 0.7306 0.444];
%mass attenuation coefficients of nickel, units: cm^2/g
Cr=[7405 2694 1277 433.9 198.8 108 65.74 32.2 10.34 4.6 2.474 1.512 0.7306 1.205 0.5952 0.3717];
%mass attenuation coefficients of chromium, units: cm^2/g

%interpolation of the mass attenuation coefficients data sets
Int_Pb = interp1(Ener,Pb,Energy,'pchip');
Int_Fe=interp1(E_Fe,Fe,Energy,'pchip');
Int_Ni=interp1(E_Ni,Ni,Energy,'pchip');
Int_Cr=interp1(E_Cr,Cr,Energy,'pchip');

%calculation of the mass attenuation coefficients of steel type 304 using %the composition of the material
Int_rvs=(0.74.*Int_Fe+0.18.*Int_Ni+0.08.*Int_Cr);

% Absorption as a function of energy corrected for the penetration depth
I_rvs=zeros(1,1000);
for g=1:1:1000
    I_rvs(1,g)=exp(-Int_rvs(1,g)*(0.4-(X_RVS(1,g)/10))*7.8302);
    %thickness of steel is 4 mm corrected with the penetration depth
g=g+1;
end
I_pb=exp(-Int_Pb.*0.05.*11.34);
Absor=I_rvs.*I_pb;%total absorption after the steel and lead as a function
% of energy

%% Generating Kramers spectrum and calculating attenuated spectrum for 40mA
I_kramer=zeros(51,1000);
I_at=zeros(51,1000);
Ne=(0.04/A)/Q;
for q=1:1:51
    z=1;
    Emax=(29+q)*1e3;
    for z=1:1:1000
        Temp=k_m(4,q)*Z_av*Ne*(Emax-(Energy(1,z)*1e3))/(Energy(1,z)*1e3);
        if Temp>0
            I_kramer(q,z)=Temp; %unit of I_kramer is counts per second
        else
            I_kramer(q,z)=0;
        end
    end
    for w=1:1:1000
        I_at(q,w)=I_kramer(q,w)*Absor(w);
    end
end

%% Calculating radiation power as a function of the voltage for I=40mA
Power=zeros(1,51);
E_kev=zeros(1,51);
for c=1:1:51
    Power(1,c)=trapz(Energy,I_at(c,:))*Q*1e3/10; %numerical integration of spectrum, conversion from keV to joule and %correction by a factor 10 which appeared after applying trapz to the %original kramer’s spectrum
    E_kev(1,c)=(29+c);
end