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Rheology of a suspension of vesicles
effect of the viscosity contrast

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Rheology of a suspension of vesicles: effect of the viscosity contrast

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Abstract

We use the lattice-Boltzmann method to solve fluid flow and couple it with vesicles (a model for red blood cells) using the immersed boundary method to simulate and study vesicle rheology in two dimensions. This study will help us to understand the relation between the microscale (dynamics) and the macroscale (rheology) of blood flow. Our work focusses on the effects of the viscosity contrast (the ratio of the viscosities of fluid inside a vesicle and outside a vesicle) on the rheology of a suspension of vesicles under shear flow. First, we study the case of a single isolated vesicle as a benchmark test for the method. We observe that the vesicle performs tank-treading motion for small values of the viscosity contrast. For larger values, the vesicle performs tumbling motion. The viscosity of the system (vesicle and suspending fluid) is observed to be a decreasing function of the viscosity contrast, up to a minimum at the tank-treading to tumbling transition, after which it becomes an increasing function. We have not observed a monotonic behaviour as was predicted in ref. [9]. Next, we study the case of higher concentrations (up to 9 vesicles in the system, corresponding with a concentration of 22.6%). It is observed that by increasing the concentration, the intrinsic viscosity of the suspension increases. The transition point between tank treading and tumbling shifts to higher values and becomes a continuous cross-over. We study the effect of the membrane rigidity on the rheology. We observe that the membrane rigidity has no effect on the rheology of the system. Finally, we study the effect the swelling degree. We observe the viscosity to increase for larger swelling degrees and the minimum in the viscosity shifts towards higher values, which is a consequence of it being harder for swollen vesicles to tumble.
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1 Introduction

The rheology of a suspension of deformable particles, for example red blood cells (RBCs) suspended in blood plasma, is not well-understood yet. Studying the rheology of blood on a macroscopic level is not sufficient: the dynamics of the RBCs at the microscale dictate the macroscopic behaviour of the RBC suspension. For example, the Fähræus-Lindqvist [3] effect in blood vessels which predicts the viscosity of blood to decrease when the vessel width becomes smaller, is caused by the RBCs moving towards the center of the vessel. This effect can only be explained at the microscale when studying the dynamics of each RBC. Studying the rheology of blood is important for both understanding the fundamentals of the rheology of deformable particles and for practical applications.

The properties and shapes of an RBC can be influenced by diseases. The shape and swelling degree (how much an RBC is swollen) are influenced by disorders like sickle cell disease where the cells take on the shape of a sickle. Ellipsocytosis and spherocytosis are disorders that alter the shape of an RBC from a concave disk to an ellipse and a sphere, respectively. Malaria is a disease that affects the cell membrane of an RBC as well as the viscosity of the hemoglobin inside the cell. Since malaria is common across a large part of Africa, South-America and Asia, understanding the effects these altered cells have on the blood flow is important to develop new techniques to identify these diseases more reliably and in an earlier stage. One way is to measure the viscosity of the blood and how this changes with the deformations of RBCs.

In this work, we focus on the effect of the viscosity contrast (the ratio of the internal RBC viscosity and the viscosity of the surrounding fluid) have on the rheology. We assume the RBC’s membrane is a uniform, massless layer with no width, neglecting the existence of proteins and a membrane skeleton to simplify the model. We model the RBC as a vesicle, which captures the relevant properties of an RBC. In our work, the vesicles may exhibit two types of motion: (1) tank-treading (TT) motion where the vesicle makes a fixed angle with the flow direction and the membrane undergoes a tank-treading like motion and (2) tumbling (TB) motion where the vesicle will not find a fixed angle but will start to rotate around its center. The viscosity contrast is the central variable we vary to study the rheology of the system. Lower viscosity contrasts cause TT motion while higher viscosity contrasts lead to TB motion.

We use the lattice-Boltzmann method (LBM) to solve the fluid flow. The advantage of the LBM procedure is that it is easy to implement and accurately recovers the Navier-Stokes equations, which describe the motion of fluids.
Over the past decade, the method is gaining in popularity very fast. In order to solve the membrane-fluid interaction, we make use of the immersed boundary method (IBM) which was originally developed to couple fluids and deformable structures. The effect of viscosity contrast on the rheology has been investigated both experimentally [14] and numerically [1]. The numerical results predicting monotonic behaviour, however do not match up with the experimental results which show non-monotonic behaviour. The goal of this work is to use an alternative model and investigate the differences between the numerical and the experimental results. We also look at how the membrane rigidity and the swelling degree affect the rheology.

The next part of the report covers a brief description of the method. After that we introduce the vesicle model and how we apply it with the IBM procedure to compute the coupling between fluid and membrane. Finally we discuss the simulation results.

2 The lattice Boltzmann method

There is a wide variety of methods to simulate fluid dynamics on different scales. In order to study the fluid dynamics of a suspension of several RBCs, we use the lattice Boltzmann method (LBM). One of the key advantages of the LBM is its simplicity in implementing the method. Furthermore, the LBM recovers the Navier-Stokes equations, which describe the motion of fluids for small Mach and Knudsen numbers, with good accuracy. The Navier-Stokes equations are given by

\[ \rho \left( \frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} \right) = -\nabla p + \eta \nabla^2 \mathbf{u} + \mathbf{F}, \]

\[ \nabla \cdot \mathbf{u} = 0, \]

where \( \rho \) is the mass density, \( \eta \) the dynamic viscosity, \( \mathbf{u} \) the velocity field and \( p \) the pressure field. The bulk force \( \mathbf{F} \) is the total force acting on the system, such as gravity.

2.1 Flow solver

As the name suggests, the LBM models the fluid on a lattice. The main quantity in the LBM procedure is the distribution function given by \( f_i(r, t) \). This function represents the probability of finding a pseudoparticle at position \( r \) and time \( t \) in the \( i \)’th velocity direction. The possible velocities \( c_i \) are determined by the choice of the lattice. In our model, we use the D2Q9 lattice. The D2Q9 lattice corresponds to a 2 dimensional (D2) lattice grid with
9 discrete velocity vectors (Q9). The evolution in time of the distribution function is given by the LB equation

\[ f_i(r + c_i \Delta t, t + \Delta t) - f_i(r, t) = \Delta t (\mathcal{L}_i + F_i) \quad (i = 0 \ldots 8), \]

where \( f_i(r, t) \) is the distribution function of the previous time step, \( f_i(r + c_i \Delta t, t + \Delta t) \) is the updated distribution function in the current time step.

In this model, the time step \( \Delta t \) is set to 1. \( F_i \) corresponds to an external applied force and \( \mathcal{L}_i \) is the collision operator. The left hand side of the equation describes the free flow of the fluid. The right hand side describes the interaction between the particles (i.e. collision).

For this model, the collision operator is the Bhatnagar-Gross-Krook (BGK) [2] approximation given by

\[ \mathcal{L}_i = -\frac{1}{\tau}[f_i(r, t) - f_i^{eq}(r, t)], \]

which describes the relaxation of the system towards an equilibrium state \( f_i^{eq}(r, t) \). This approximation describes the relaxation of \( f_i(r, t) \) towards an equilibrium distribution with relaxation time \( \tau \). \( \tau \) describes the time scale on which the flow will recover its equilibrium state. The relaxation time is related to the viscosity according to \( \eta = \rho c_s^2 (\tau - 1/2) \Delta x^2 / \Delta t \), where \( c_s \) is the speed of sound. For a D2Q9 lattice, this is given by \( c_s = 1 / \sqrt{3} \). With this speed of sound, the viscosity reduces to

\[ \eta_{out} = \rho \frac{2\tau - 1}{6} \]

The equilibrium distribution function is given by:

\[ f_i^{eq}(r, t) = \omega_i \rho(r, t) [c_1 + c_2 (c_i \cdot u) + c_3 (c_i \cdot u)^2 + c_4 (u \cdot u)] \]

that is a second order Taylor expansion of the Maxwell-Boltzmann distribution. The constants \( c_1, c_2, c_3 \) and \( c_4 \) are lattice constants and depend on the lattice type and the lattice speed of sound. In the case of a D2Q9 lattice, these are given by \( c_1 = 1, c_2 = 1/c_s^2, c_3 = 1/2c_s^4 \) and \( c_4 = -1/c_s^2 \). The \( \omega_i \)'s are the weight factors for the velocity vectors. In this lattice type, the weight vectors are 4/9 for the 0 velocity vector, 1/9 for the horizontal and vertical velocity vectors and 1/36 for the diagonal velocity vectors.

The macroscopic quantities of the flow are given by the following equations:

The local mass density

\[ \rho(r, t) = \sum_{i=0}^{8} f_i(r, t), \]
the local fluid velocity
\[
\mathbf{u}(r, t) = \frac{1}{\rho(r, t)} \sum_{i=0}^{8} f_i(r, t) \mathbf{c}_i, \tag{8}
\]
and the local fluid pressure
\[
p(r, t) = \rho(r, t) c_s^2. \tag{9}
\]

### 2.2 Boundary conditions

We consider walls with no-slip boundary conditions, which may move freely in the x-direction. The no-slip boundary condition for moving walls is achieved by using the bounce-back with moving wall boundary condition
\[
f_{-i}(r, t + \Delta t) = f_i(r, t) + 2 \frac{\rho \omega_i}{c_s^2} (\mathbf{u}_{\text{wall}} \cdot \mathbf{c}) \tag{10}
\]
The domain of the lattice is a rectangle with the the origin in the center. The width of the channel is \(2n_y\) (y-coordinate) and the length is \(2n_x\) (x-coordinate). At the in- and outlet of the lattice, periodic boundary conditions are imposed. Particles leaving the channel on the right hand side reappear on the left hand side. Allowing the walls to move at a velocity \(u_{\text{wall}}\) allows shear flow to be established.

### 2.3 Rheology

In order to obtain the effective viscosity of the system (the vesicles and the suspending fluid), we make use of the average hydrodynamic stress on the bounding walls. The average stress on the walls is given by
\[
\langle \sigma_{xy}(t) \rangle = \frac{1}{4n_x} \int_{-n_x}^{n_x} \sigma_{xy}(x) dx \tag{11}
\]
where the factor \(1/(4n_x)\) comes from averaging over two walls both with length \(2n_x\). The effective viscosity \(\eta\) is then calculated from the average stress on the walls and the applied shear rate \(\dot{\gamma}\) according to
\[
\eta(t) = \frac{\langle \sigma_{xy}(t) \rangle}{\dot{\gamma}}. \tag{12}
\]
The intrinsic viscosity \(\eta_I(t)\) is given by
\[
\eta_I(t) = \frac{\eta(t) - \eta_{\text{out}}}{\eta_{\text{out}} \phi} \tag{13}
\]
where $\eta_{out}$ is the viscosity of the external fluid and $\phi$ the concentration of vesicles. The viscosity obtained is the viscosity at a specific point in time. We average the viscosity over time once the system has reached a quasi-steady state and obtain:

$$\eta_l = \frac{1}{t_f - t_s} \int_{t_s}^{t_f} \eta_l(t) dt,$$

where $t_s$ is the starting time and $t_f$ the ending time of the considered interval.

3 The vesicle model

Vesicle membranes generally consist of phospholipid molecules. Such molecules consists of two parts: a hydrophilic phosphate head and a hydrophobic fatty acid tail. These molecules arrange themselves in such a way that the hydrophilic heads are in contact with the fluid in which they are suspended, while the hydrophobic tails band together away from the fluid. This arrangement creates a membrane which is nearly impenetrable for the surrounding fluid.

The diameter of a typical vesicle/RBC is of the order of $10\mu m$, while the thickness of the phospholipid bilayer is of the order of $5nm$. The width and mass of the membrane are neglected in our model for simplicity.

Vesicles have two important properties that closely resemble the RBC. The first property of a vesicle is that the area (2D) inside the vesicle is conserved in time. An RBC has a complex exterior separating the internal hemoglobin from the external blood plasma. However, even though fluid may enter and exit the RBC, this effect is reasonably small such that the area can be considered to be constant. The second property is the conservation of the perimeter. The membrane length of an RBC does not change in time. Combining these two important properties, the vesicle is a good way to model an RBC.

3.1 The membrane force

The membrane resists to bending and stretching. So it exerts a restorative force given by (in 2D)

$$f(r_m) = \left[ \kappa_B \left( \frac{\partial^2 h}{\partial s^2} + \frac{h^3}{2}\right) - h \zeta \right] n + \frac{\partial \zeta}{\partial s} t$$

where $h$ is the local membrane curvature, $\kappa_B$ is the bending modulus or membrane rigidity, $s$ is the arclength coordinate along the membrane and $n$ and
\( \mathbf{t} \) are the normal and tangential unit vectors, respectively. \( \zeta \) is a Lagrange multiplier field that enforces local length conservation. A detailed derivation of this force can be found in [6]. In order to preserve the enclosed area of the vesicle, an additional term \( \kappa_A (A - A_0) \mathbf{n} \) is included in equation 15, where \( A \) is the current area and \( A_0 \) is the initial area taken as a reference. \( \kappa_A \) is an arbitrary parameter.

### 3.2 Viscosity contrast

The internal fluid of an RBC consists of the protein hemoglobin (which has a red colour due to the iron it contains) and various other proteins to maintain themselves. The viscosity of the fluid inside of the RBC changes depending on the age of the cell. The values for the viscosity of the fluid encapsulated by the vesicle (\( \eta_{\text{int}} \)), primarily hemoglobin, varies depending on the state of the RBC. The viscosity ranges between seven and thirteen times as viscous as the viscosity of the surrounding fluids (\( \eta_{\text{ext}} \)), the blood plasma, for healthy RBCs.

We define the viscosity contrast \( \Lambda \) as the ratio between the internal and external viscosities according to \( \Lambda = \frac{\eta_{\text{int}}}{\eta_{\text{ext}}} \). In our work, we are interested in \( \Lambda \neq 1 \). In order to achieve this numerically, the relaxation time, which depends on the viscosity, is adjusted for fluid nodes within the vesicle membrane (\( \Omega_{\text{int}} \)) and outside the vesicle (\( \Omega_{\text{out}} \)). In order to determine which fluid nodes are part of a vesicle, we need to determine how often the vesicle membrane intersects the line segment \( L \) from the lattice node to the top or bottom wall (an odd amount of times indicates the node is inside a vesicle, an even amount indicates that the node is outside the vesicle). This method is known as the even-odd rule [10]. We chose an \( L \) that connects the lattice node to the top wall, such that \( L \) is perpendicular to the top wall. After finding whether the fluid node is inside or outside the vesicle, we adjust the relaxation time according to:

\[
\tau(r, t) = \begin{cases} 
\tau_{\text{int}} & \text{if } r \in \Omega_{\text{int}} \\
\tau_{\text{ext}} & \text{if } r \in \Omega_{\text{ext}} 
\end{cases}
\]

\( \tau_{\text{ext}} \) is set to unity and \( \tau_{\text{int}} \) is calculated based on \( \tau_{\text{ext}} \) and \( \Lambda \):

\[
\tau_{\text{int}} = \Lambda (\tau_{\text{ext}} - 1/2) + 1/2.
\]

### 4 Vesicle-fluid two-way coupling

With LBM as a fluid solver and the vesicle as a model for the RBCs, we need a way to couple the two together. Peskin developed a model that could
handle these variable boundaries between two fluids and bounding walls: the Immersed Boundary Method (IBM) [11]. The method was originally developed to model blood flow in the heart. The basis of the method is to compute the velocity and pressure fields on an Eulerian regular fixed mesh while the deformable bodies are modeled as a Lagrangian moving mesh. The lagrangian mesh is then ‘immersed’ in the Eulerian mesh.

4.1 Action: fluid-membrane one-way coupling

The boundary separating the internal and the external fluid (the membrane) of the vesicle is discretized into points. These points are interconnected by elastic ”springs”. The first step is to solve the fluid flow, as if the membrane does not exist. After, the advection of the membrane by the fluid is computed and the position of the membrane is updated.

The interaction between the fluid and the membrane is short range. It assumes the form of a $\delta$-function around every membrane node on the moving mesh. Peskin suggested to use a so called discrete $\Delta$-function. This function has non-zero values only around a membrane point. The $\Delta$-function is given by [12]

$$\Delta(x, y) = \begin{cases} \frac{1}{16} (1 + \cos(\pi x))(1 + \cos(\pi y)) & |x|, |y| \leq 2 \\ 0 & |x|, |y| > 2, \end{cases} \quad (17)$$

where $x$ and $y$ are taken relative to the coordinates of the membrane node.

The advection of the interface is given by interpolating the local fluid velocities with the membrane node such that the membrane moves at the same velocity of the surrounding fluid. Using the $\Delta$-function we obtain

$$\mathbf{u}(\mathbf{r}_m) = \sum_{f} \Delta(\mathbf{r}_f - \mathbf{r}_m) \mathbf{u}(\mathbf{r}_f), \quad (18)$$

where the fluid velocity $\mathbf{u}(\mathbf{r}_f)$ is obtained from the LBM procedure. We can now update the position of the membrane nodes using an Euler scheme. We assume that our membrane nodes are massless and at this point in the time iteration behave as tracer-like particles. The Euler scheme is given as follows:

$$\mathbf{r}_m(t + \Delta t) = \mathbf{r}_m(t) + \mathbf{u}(\mathbf{r}_m(t)) \quad (19)$$

In this step the membrane is advected and deformed and the fluid flow acting on the membrane is computed.
4.2 Reaction: membrane-fluid one-way coupling

The next step is to calculate the force the membrane exerts on the fluid:

\[ F(r_f) = \sum_m f(r_m) \Delta(r_f - r_m), \quad (20) \]

where \( f(r_m) \) is the force of the individual membrane nodes, as given by equation 15 and \( m \) goes over every membrane node. The \( \delta \)-function is again the discrete \( \Delta \)-function given by equation 17.

The fluid experiences a force due to the presence of the membrane. At the same time, the membrane itself is advected by the surrounding fluid flow. These two interactions provide a fluid-structure two-way coupling causing a disturbance and modification of the flow.

5 Physical and numerical parameters

The vesicle dynamics is governed by the following physical and numerical parameters:

**The domain of the simulation box:** length \( L \) and width \( W \). The domain is set to \( L = W = 200 \).

**The vesicle effective radius** \( R_0 \). In our work we set \( R_0 = 20 \).

**The shear rate** \( \gamma \). \( \gamma \) is set to 0.0002083 in all simulations.

**The Reynolds number**, \( Re = \frac{\rho \gamma R_0^2}{\eta \omega} \), describes the importance of intertial forces versus viscous forces. Our simulations are done for small values of \( Re \) to avoid inertia effects. We set \( Re = 0.5 \) in all simulations.

**The membrane rigidity** \( \kappa_B \) controls the membrane deformability. A higher value for \( \kappa_B \) makes the membrane more resistant to bending.

**The capillary number**, \( Ca = \frac{\eta \gamma R_0^3}{\kappa_B} \), measures the ratio between viscous forces and bending forces. It is the ratio between the characteristic time \( (\frac{\eta R_0^3}{\kappa_B}) \) needed by a vesicle to recover its equilibrium shape after flow cessation and the shear time \( (1/\gamma) \). The deformability of the vesicle is controlled by
this parameter. Larger Ca lead to larger deformations.

**The swelling degree (2D),** \(\Delta = \frac{A}{A_c} = \frac{4\pi A}{P^2}\), quantifies how much a vesicle is swollen. It is the ratio of the vesicle area \(A\) and the area of a circle \(A_c\) with the same perimeter \(P\) as the vesicle. For a circular vesicle, \(\Delta\) equals 1, for a deflated vesicle, \(\Delta\) is less than 1.

**The viscosity contrast,** \(\Lambda = \frac{\eta_{\text{int}}}{\eta_{\text{ext}}}\), gives the ratio of the viscosity inside the vesicle to the viscosity outside the vesicle. For small \(\Lambda\), vesicles perform TT motion, while for larger \(\Lambda\) vesicles act as solid particles and perform TB motion.

**The degree of confinement,** \(\chi = \frac{2R_0}{W}\), describes the ratio of the vesicle’s size versus the channel height. Large confinements lead to more TT motion while small confinements allow vesicles to perform TB motion more easily. \(\chi\) is kept constant at 0.2 \((R_0 = 20, W = 200)\).

**The vesicle perimeter \(\kappa_P\) and area \(\kappa_A\) conservation parameter.** The perimeter conservation parameter is used to calculate the Lagrange multiplier field in equation 15 according to \(\zeta = \kappa_P(ds(t) - ds(t_0))\), where \(ds(t)\) is the distance between two adjacent membrane nodes at time \(t\) and \(ds(t_0)\) is the distance between the two membrane nodes at a reference time \(t_0\). The area conservation parameter is imposed by adding the term \(\kappa_A(A - A_0)n\) in equation 15. We set \(\kappa_P\) to 3 and \(\kappa_A\) to 0.01 in order to fulfill the perimeter and area conservation constraints while keeping the code stable.

**Number of particles, \(np\), and concentration, \(\phi\).** The number of particles determines the concentration of vesicles in the suspension. We vary the number of particles between 1 \(\phi = \frac{npA_v}{LW} = 2.5\%\) and 9 \(\phi = 22.6\%\) for vesicles with \(R_0 = 20\) and \(\Delta = 0.8\).
6 Results and Discussion

6.1 Benchmarking the method

6.1.1 Single vesicle

In order to validate the code, we study the limit of a very dilute suspension: the case of a single isolated vesicle. This case offers a good benchmarking test to verify if the model is working correctly, as this case has been studied in detail [7, 8, 15, 4]. We use a vesicle with $\Delta = 0.8$. For the confinement $\chi = 0.2$, the transition from TT to TB is expected to occur at $\Lambda = 7$ [7]. The angle of the long-axis of the vesicle is an indication of the state in which the vesicle resides. Figure 1 shows the inclination angle $\theta$ of a vesicle in the TT regime ($\Lambda = 4$) and in the TB regime ($\Lambda = 16$). A tank-treading vesicle has a steady inclination angle. The angle of a tumbling vesicle is periodic.

Next, we compute the effective viscosity on the walls from equation 12 as it evolves in time. We consider the same situation as in figure 1. Figure 2 shows the effective viscosity $\eta$ as a function of time. The vesicle in the TT regime performs a steady motion. Due to this, the hydrodynamic stress it
Figure 2: The evolution of the viscosity $\eta$ in time of a vesicle in the TT regime ($\Lambda = 4$) and in the TB regime ($\Lambda = 16$). A vesicle in the TT regime performs steady motion and there are only small fluctuations in the viscosity. A TB vesicle performs unsteady motion and causes large fluctuations in the viscosity.

causes on the walls remains constant in time. The tumbling vesicle causes the hydrodynamic stress to change periodically with the same frequency as the inclination angle.

In figure 3 we show the intrinsic viscosity, as computed by equation 14, versus the viscosity contrast. The intrinsic viscosity decreases with the viscosity contrast in the TT regime. This is expected as the inclination angle decreases with viscosity contrast, as shown in Ref. [8], which causes the vesicle to exert less resistance to the flow and thus less stress on the wall, and hence a decrease in viscosity. At the transition point from TT- to TB- motion, the viscosity drops down to a minimum. After the transition point the viscosity increases again with $\Lambda$ being in the TB regime. When the vesicle tumbles, the forces exerted on the surrounding fluid increase and as such also the stresses on the walls. This increases the viscosity of the suspension. Error bars are taken as the standard deviation from the evolution of the viscosity in time. The error in unsteady motion is significantly larger than that of steady motion. The results agree qualitatively with the numerical results [4] performed with the phase-field and the boundary integral method.
6.1.2 Area and perimeter conservation constraints

We look at how the properties of area and perimeter conservation change over time to check whether the model imposes the constraints on the vesicle correctly. We also look at the conservation of the swelling degree ∆ (how much the vesicle is swollen). Figure 4 shows how the various parameters change over time for a simulation run with 9 vesicles (φ = 22.6%), with ∆ = 0.8. The constraints on the area and perimeter are shown to be preserved in time. The swelling degree of the vesicle is also observed to be conserved in time.

6.2 Effect of the concentration

We increase the concentration of the vesicles in the suspension by adding more vesicles to the suspension, such that there are 3, 6 and 9 vesicles, corresponding to φ = 7.5%, 15.1% and 22.6%, respectively. We again look at the effective viscosity over time. Figure 5 shows the effective viscosity for φ = 15.1% for Λ = 4 and 16. Both of the cases considered exhibit unsteady motion. Interaction between the vesicles in the TT regime causes the stress
Figure 4: The area variation, perimeter variation and the variation in the swelling degree variance in time for a simulation with 9 vesicles ($Re = 0.5, \Lambda = 4, \phi = 22.6\%, \Delta = 0.8, Ca = 10$). The area, perimeter and the swelling degree are shown to be preserved in time, indicating that the constraints imposed on the vesicle are satisfied.
Figure 5: The viscosity $\eta$ as a function of time for 6 vesicles ($\phi = 15.1\%$) in the TT regime ($\Lambda = 4$) and the TB regime ($\Lambda = 16$). The viscosity in the TT regime now varies in time, as the relative positions of the vesicles change due to the applied shear flow. The viscosity in the TB regime varies in time and no periodicity is observed due to unsteady motion of the whole suspension.

The interaction between vesicles is displayed in figure 6. The snapshots from (a) are taken for $\Lambda = 4$, corresponding to the TT regime. The vesicles pass each other without breaking the TT motion or affecting each others inclination angle. The relative position of the vesicles only has an effect on the stresses on the walls. (b) shows the same TT regime, but for a higher concentration. Vesicles have less space and the interaction between vesicles causes the inclination angle to increase as the vesicles try to pass each other. Vesicles which have no neighbours have the same inclination angle as those shown in (a). The snapshots from (c) are taken for $\Lambda = 16$ in the TB regime. The vesicles with sufficient free space perform tumbling motion. Vesicles that are close to each other prevent tumbling from occuring and pass each other. (d) shows the same TB regime for a higher concentration of vesicles. The free space the vesicles have is decreased dramatically, only allowing a single vesicle to tumble in the displayed time frame.

Figure 7 shows the intrinsic viscosity $\eta_I$ as a function of the viscosity contrast $\Lambda$ for $\phi = 7.5\%, 15.1\%$ and 22.6%. By increasing the concentration, the minimum of the viscosity shifts to higher values of $\Lambda$. It is observed that...
Figure 6: Snapshots of (a) tank-treading motion (Λ = 4, φ = 15.1%), (b) tank-treading motion (Λ = 4, φ = 22.6%), (c) tumbling motion (Λ = 16, φ = 15.1%) and (d) tumbling motion (Λ = 16, φ = 22.6%) as they evolve in time from left to right. (a) shows the vesicles performing steady motion with little interaction with each other. (b) shows tank-treading in a denser solution where interacting vesicles have an increased inclination angle as they pass each other. (c) shows tumbling motion for vesicles that find themselves in free space. Vesicles near each other are blocked from performing tumbling motion. (d) shows a denser suspension where there is only a single vesicle able to perform tumbling motion.
Figure 7: Intrinsic viscosity $\eta_I$ as a function of viscosity contrast $\Lambda$ for different concentrations $\phi$ with $\Delta = 0.8$ and $\chi = 0.2$. For larger concentrations, the viscosity $\eta$ increases. The critical value of $\Lambda$ shifts to the right, indicating that TB-motion will occur at larger values of $\Lambda$. The cross-over from TT to TB becomes continuous for denser solutions.

by increasing the concentration, the viscosity of the system increases. This is explained by having more vesicles that give resistance to the flow. We observe that a continuous cross-over from TT to TB occurs for higher concentrations ($\phi = 22.6\%$). This is caused by the interaction between multiple vesicles: there is less chance to perform tumbling motion as in the case shown in figure 6.

### 6.3 Effect of the membrane rigidity

The capillary number $Ca$ controls the deformability of the vesicles. We change the membrane rigidity $\kappa_B$ in order to adjust the capillary number. This has the advantage of not changing the shear rate which introduces additional effects such as changing the Reynolds number which also depends on the shear rate. Rigid vesicles ($Ca < 1$) with a large viscosity contrast are comparable to solid particles which perform tumbling motion in shear flow.
Figure 8: The intrinsic viscosity $\eta_I$ versus the viscosity contrast $\Lambda$ for different capillary numbers ($Ca = 1, 5$ and $10$). The deformability of the vesicles does not have any notable effect on the macroscopic viscosity of the system. ($\Delta = 0.8$, $\chi = 0.2$ and $\phi = 22.6\%$)

The goal of this simulation is to study what happens for deformable particles ($Ca$ large) and what the effect is on the macroscopic viscosity. In figure 9 we show snapshots for the TT ($\Lambda = 4$) and TB ($\Lambda = 16$) regime for $Ca = 1$. Figure 6 (b) and (d) show the same situations but for $Ca = 10$. The dynamics of the vesicles are unchanged by varying the membrane rigidity. The rigid vesicles have similar inclination angles in the TT regime as the deformable vesicles. The TB regime for solid vesicles shows that vesicles only tumble when they are given enough space, in the same way as was the case for deformable vesicles.

Figure 8 shows the macroscopic viscosity as a function of the viscosity contrast for $Ca$ 1, 5 and 10. The deformability of the vesicles does not appear to have an effect on the rheology of the suspension. The deformable vesicles start to tumble in a similar manner as the rigid vesicles. This causes similar effects on the macroscopic viscosity regardless of the deformability of the vesicles. Tumbling occurs when a vesicle has enough free space around the vesicle to make a tumbling motion. The amount of free space for tumbling to occur does not change when the rigidity of the membrane is changed. Since
the deformable vesicles perform TB-motion just as much as rigid vesicles, the viscosity of the system remains unchanged and as such is independent of the capillary number.

(a) $\Lambda = 4, \phi = 22.6\%, Ca = 1$

(b) $\Lambda = 16, \phi = 22.6\%, Ca = 1$

Figure 9: Snapshots of (a) tank-treading motion ($\Lambda = 4, \phi = 22.6\%$), (b) tank-treading motion ($\Lambda = 4, \phi = 22.6\%$) for low capillary numbers ($Ca = 1$) as they involve in time from left to right. (a) shows interacting vesicles to have altered inclination angles when passing each other. (b) shows only vesicles with enough free space to perform TB motion.

6.4 Effect of the swelling degree

Next, we investigate how the swelling degree $\Delta$ of the vesicles affects the rheology. We consider a suspension with 6 vesicles ($\phi = 15.1\%$) and $Ca = 0.5$. Figure 10 shows how the viscosity changes with the viscosity contrast for various swelling degrees $\Delta$. We observe that the viscosity increases when the vesicles become more swollen. Vesicles with a large $\Delta$ are much less deformable: as $\Delta$ increases, the vesicle shape gets an almost circular shape (or a spherical shape in three dimensions). The vesicles give more resistance to the flow and that causes the viscosity to go up when $\Delta$ increases. This is in agreement with [1]. We further observe the TT to TB transition to shift towards the right: deflated vesicles are more subject to tumbling motion than
swollen vesicles around the transition.
Figure 11 shows snapshots for deflated vesicles ($\Delta = 0.7$) and swollen vesicles ($\Delta = 0.9$) for the TT and TB regimes. The vesicles in (a) and (b) both perform steady motion. We observe the inclination angle of the deflated vesicles to differ from the inclination angle of the swollen vesicles. The larger inclination angle of the swollen vesicles leads to more resistance to the flow and a larger intrinsic viscosity. More swollen vesicles are expected to have a further increased inclination angle and resist the flow more. The transition point from TT to TB motion is also explained to shift to higher values for higher $\Delta$, as the inclination angle goes towards 0 at the transition point (See figure 1 in [8]).
In the TB regime the swollen vesicles tumble more easily than deflated vesicles. This is caused by swollen vesicles displacing less fluid during the tumbling motion due to them more closely resembling a sphere. Deflated vesicles are long and thin and tumbling around their center of mass means a lot of fluid needs to be displaced. This leads to an increase in viscosity for swollen vesicles.
Figure 11: Snapshots of (a) tank-treading motion ($\Lambda = 4$, $\Delta = 0.7$), (b) tank-treading motion ($\Lambda = 4$, $\Delta = 0.9$), (c) tumbling motion ($\Lambda = 16$, $\Delta = 0.7$) and (d) tumbling motion ($\Lambda = 16$, $\Delta = 0.9$) as they evolve in time from left to right. In the TT regime, (a) and (b) behave similarly where the inclination angle is larger for the swollen vesicles in (b). In the TB regime, it is harder for the swollen vesicles in (d) to perform tumbling motion than the deflates ones in (c).
7 Conclusions

We used two-dimensional lattice-Boltzmann with immersed boundary simulations to study vesicle rheology under shear flow with varying viscosity contrasts. We benchmarked our method with the case of a single vesicle. We identify two modes of motion: tank-treading motion at low viscosity contrast and tumbling motion at high viscosity contrast. The time evolution of the effective viscosity and the inclination angle indicate that tank-treading motion is steady and tumbling motion is unsteady and periodic. A minimum of the intrinsic viscosity is observed at the transition point from TT to TB motion. We did not find any indication of monotonic behaviour as was found numerically in [9]. We find this minimum to shift to higher viscosity contrasts when the concentration of the vesicles is increased. The transition between TT and TB becomes a continuous crossover at high concentrations. The increased concentration causes vesicle-vesicle interaction which make it impossible for vesicles near other vesicles to perform tumbling motion. The viscosity increases as the swelling degree of the vesicles is increased. This is attributed to less deformability when the swelling degree approaches unity and the shapes of the vesicles become more like circles. We also observe more swollen vesicles only start to tumble at larger viscosity contrasts. We found no indication that the membrane rigidity has an effect on the rheology of the suspension.

The results shown in this report are based on a single simulation run. It is therefore of importance to run additional simulation runs with different seeds to reduce any statistical noise and reduce the reported error bars. Investigating the effect of initial position of the vesicles in the suspension is important in determining when a steady state has been reached, which is of particular importance in dilute and semi-dilute suspensions of vesicles. Furthermore, more investigation is needed to study the effect of the swelling degree in both more dilute and denser suspensions as what we reported for.

References


