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Wave particle duality in Faraday waves

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Wave Particle Duality in Faraday Waves

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Abstract

When a fluid in a container is shaken vertically, so-called Faraday waves are observed. For an acceleration just below the Faraday threshold, droplets can move around the surface without merging with the same fluid below. These-so called walkers move with their own wave packet and exhibit quantum mechanical behaviour. For example the double slit experiment gives a quantum mechanics-like diffraction pattern. In this thesis the interaction with boundaries has been examined. This is done by macroscopic particle tracking and the investigation of the guiding waves produced by the droplets. The guiding waves are visualised with the use of the synthetic schlieren method which produces a height field. The cross section of this height profile gives an insight in the wave function of a walker near a boundary. In the experiments it is observed that the degree of distortion of the wave function depends on the distance from the boundary.
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Chapter 1

Introduction

Back in 1831 Michael Faraday described the strange behaviour of sand when shaken vertically. He saw that different structures occurred when a plate of sand was vibrating with different frequencies. In the appendix of this paper [7] he noticed that this phenomenon also occurs with liquids. The experiment done with liquids is now known as the Faraday experiment. The standing waves that occur in the fluid are also called after him, Faraday waves. The appearance of Faraday waves depend on various variables, the frequency, amplitude and viscosity.

When the fluid is shaken just under the point where those Faraday waves occur, it is possible to get a droplet of the same fluid bouncing on the liquid layer. When the frequency, viscosity, amplitude and dropsize are varied the bouncing droplets will show different dynamic behaviours. Bouncing, period doubling bouncing, period doubling cascade and the walking regimes will occur. This last regime, the walking regime, was first observed by J. Walker [12] on a soap layer, and later by (Couder et al.) on other liquids.

This walking droplets will behave like non-classical particles when they interact with a boundary due to its particle-wave interaction. In this thesis the dynamic behaviour of this reflection will be examined. This is done by looking at the macroscopic trajectory and the guiding waves that the walker emits. This guiding waves will be made visible by using the synthetic schliere n method. By this method it is possible to see the height of the guiding waves at every position in the liquid layer. From this height profile a cross section is taken to see what happens with a walker when it interacts with a boundary.
Chapter 2

Surface waves

When a liquid layer gets disturbed by a rock or a vertical oscillation there start to appear waves on the surface of the liquid. These waves occur on the interface of two different media. In our example the interface of an air layer and an silicon layer. The different waves that will be discussed in this chapter are capillary waves and Faraday waves. The capillary waves are observed in many places in nature. For the Faraday waves a vertical oscillation is needed.

2.1 Capillary waves

Capillary waves are the waves travelling around the surface and are dominated by the effect of surface tension. The waves are derived by the dynamics of a free surface of a frictionless fluid. By the NavierStokes equation and the assumption that there is no vorticity and no friction the velocity potential becomes $\vec{u} = \nabla \phi$. The rest is determined by the boundary conditions. The velocity field beneath the surface is frictionless and irrotational, therefore Bernoulli’s law can be used. If the NavierStokes equation, boundary conditions and Bernoulli’s law are combined this will lead to a set of linear equations which allow harmonic solutions. If all of this information is combined eventually the dispersion relation becomes:

$$\frac{\omega^2}{k} = g + \frac{\sigma}{\rho} k^2$$  \hspace{1cm} (2.1)

This dispersion relation is roughly Newton’s equation $F = ma$.

2.2 Faraday waves

When a fluid is shaken in vertical direction so-called Faraday waves will occur. This instability arises due to the modulation of the gravitational acceleration. As seen in figure 2.1 instability domains will occur. The dash-dotted lines in the first two tongues indicate the viscosity effect. This ensures that with rising $k$ the amplitude needs to be larger to excite a wave. In practice only the first two modes are taken into account.

For the dispersion relation of the Faraday waves it is necessary to take a look at Mathieu’s equation. In 1954, Benjamin and Ursell showed that the linearised equation of motion take the form of Mathieu’s equation (2.2) and from this equation it is possible to extract the
Figure 2.1: A phase diagram of the harmonic $H$ and subharmonic $S$ tongues in which the solution is linear unstable. The fluid has density $\rho = 892.4 \text{kgm}^{-3}$ and surface tension $\alpha = 18.3 \cdot 10^{-3} \text{Jm}^{-2}$. The excitation frequency is $50 \text{Hz}$. The dash-dotted lines indicate the effect of viscosity ($\mu = 3.668 \cdot 10^{-6} \text{ms}^{-2}$). With the effect of viscosity the amplitude needs to be increased. Plot originally presented in [1]

dispersion relation. In the Mathieu’s equation $n_k$ is the amplitude of a wave with wave number $k$. $\Omega$ is the driving frequency, $a$ the amplitude of the excitation and $\omega_k$ the natural frequency of a wave with wavenumber $k$.

\[
\ddot{n}_k + (\omega_k^2 + ak \sin(\Omega t))n_k = 0
\] (2.2)

For a more detailed way of getting the dispersion relation take a look at [10]. Eventually the dispersion relation is given by equation (2.3)

\[
\omega^2(k) = \tanh(k h)(g k + \frac{\sigma}{\rho} k^3)
\] (2.3)

Where $h$ is the depth of the fluid layer, $g$ the acceleration of gravity, $\sigma$ the surface tension and $\rho$ the density of the fluid. In practice the $\tanh(k h)$ in equation (2.3) gets approximately equal to one with the depths used in the experiments of this thesis. When $\tanh(k h)$ becomes approximately one it is possible to divide equation (2.3) into two different parts. So the dispersion relation becomes:

\[
\omega_0^2 = g k_0 + \frac{\sigma}{\rho} k_0^3
\] (2.4)
and deviding by $\omega_0^2$ the dispersion relation gets:

$$1 = G + \Sigma$$

(2.5)

With a gravitational part $G = gk_0$ and a part due to the surface tension $\Sigma = (\sigma/\rho)k_0^3$. Where $G = 1$ corresponds to a pure gravity wave and $\Sigma = 1$ to a capillary wave. The ratio of the gravitational and capillary part shows that the capillary behaviour is more important. At the shallow zone in a boundary as explained in section §5 the height ensured that the $\tanh(kh)$ term is not negligible.

The Mathieu equation has two solutions which contain a series of frequencies. The sub-harmonic solution consist of a series of frequencies $\omega_{k,n} = (n + 1/2)\Omega$. And the harmonic solution a series of frequencies $\omega_{k,n} = n\Omega$. In figure 2.1 the sub-harmonic and harmonic solutions are shown. If the viscosity factor is absent the treshold for both harmonic and sub-harmonic solutions is at zero acceleration. As shown in figure 2.1 the solution tongues get narrower as the $k$ value increases. And with the effect of viscosity the threshold of excitation will raise. In practice, therefore, only the first sub-harmonic solutions will be excited. Therefore the Faraday frequency is equal to: $\omega_k = 1/2\Omega$
Chapter 3

Bouncing droplets

When a fluid is shaken at an acceleration below the Faraday threshold $\gamma_m^F$, it is possible to get a droplet bouncing on that fluid. This droplet of the same liquid is bouncing due to an air film between droplet and surface. At each oscillation, as the drop hits the bath, an air film is squeezed between the drop and the liquid layer. This air film has no time, approximately one fifth of the bouncing period, to break up before the drop lifts off again. For liquids with very low viscosity the bouncing drop will excite surface waves. The bouncing drop phenomenon was first observed by J. Walker [12] on an oscillating soap solution and later by (Couder et al.) [4] with a pure fluid.

3.1 Domains

The dynamic behaviour of the bouncing droplets varies when the viscosity, frequency and acceleration are changed. In figure 3.1 the different behaviours of a drop are given. The different domains are: B simple bouncing, PDB period-doubling bouncing, PDC transition to temporal chaos by a period doubling cascade, INT intermittent behaviour and W the walking domain. The phase-diagrams shown in figure 3.1 are originally presented in respectively [3] and [6], where viscosity and driving frequency are different.

If the accelerations is smaller than approximately one, the acceleration is too low to keep the drop bouncing so it will merge with the liquid below. In the regime above the Faraday instability threshold $\gamma_m^F$, standing waves or Faraday waves will occur. In this Faraday domain the drops are still bouncing on the wavy surface but there motion is very chaotic, the droplets will also merge after a short time.
Figure 3.1: Two different phase diagrams of the bouncing drop as a function of its diameter \( D \) and acceleration \( \gamma/g \). The viscosity and frequency are different, (a) has viscosity \( \mu = 50 \times 10^{-3} \text{Pa}s \) and a frequency of 50 Hz and (b) has a viscosity of \( \mu = 20 \times 10^{-3} \text{Pa}s \) and a frequency of 80 Hz.

### 3.2 Bouncers

As shown in figure 3.1 the bouncing region B starts at a threshold close to \( \gamma/g = 1 \). With \( \gamma \) the acceleration of the fluid. The non-dimensional number for the deformation of the drops the Weber number \( We = \rho V^2 R/\sigma \) is small for the drops, so the shape of the drops remains approximately spherical. For large drops the threshold \( \gamma/g \) is larger than one. This occurs because the air-film between the drop and surface becomes wide and thin. Extra energy is needed to suppress the resulting adhesion. For the smaller drops the threshold \( \gamma/g \) is smaller than one. This indicates that the energy stored in the drop due to the surface tension is lowering the energy needed to keep the drop bouncing.

Drops made of liquids with a rather low viscosity will produce surface waves which will propagate all over the surface, an example is shown in figure 3.2. This surface waves oscillate at the frequency of the bouncing drop. The wavelength of this particular image is \( \lambda = 11.4\text{mm} \). For this image the method PIV (Particle image velocimetry) is used, further explained in section §4.1.

When \( \gamma/g \) is increased, for intermediate drop sizes, the drop will enter the PDB domain. In the PDB domain the drop will get so much energy by the acceleration of the surface that the drop will get a period twice the period of the driving frequency. For real small drops the period doubling cascade will be entered where the drops bounce with two times the period but in a very chaotic motion.

### 3.3 Walkers

The most interesting phenomenon occurs when the drops of intermediate size hit the walker regime W. In this regime, which is always located just below the Faraday instability, the
drop starts moving with a constant velocity in the horizontal plane. Just like the PDB the drop bounces with twice the period. The drops thus touch the surface only once in two driving periods.

The drop will emit surface waves, or guiding waves, these guiding waves and drop together will be called a walker, which interact as a particle-wave couple. The guiding waves emitted by the drop are in agreement with the Faraday standing waves which are discussed in section §2.2. So with the wavelength of the guiding waves \( \lambda_F \) and the frequency of the bouncing drop \( f_F = f_0/2 \) with \( f_0 \) the driving frequency, the dispersion relation equation (2.3) should match.

In figure 3.3 the gradient field of the walker is shown. In this figure the guiding waves are visualised using PIVview as explained in section §4.1. The big red arrow indicates the direction of the walker. With a wavelength of the guiding waves of \( \lambda_F = 10.8mm \), a viscosity of \( \mu = 15mPas \) and a driving frequency of \( f_0 = 40Hz \) the dispersion relation equation (2.3) gives a frequency of \( f = 31Hz \).
Figure 3.3: The contour plot of a walker, the red arrow marks the direction of the walker. The picture shows a $60 \times 60\text{mm}^2$ region inside the container boundaries. The wavelength of the guiding waves is $\lambda_F = 5.4\text{mm}$, with a viscosity of $\mu = 15\text{mPas}$ and a driving frequency of $f_0 = 40\text{Hz}$. 
Chapter 4

Synthetic schlieren

To take a better look at the height of the guiding waves of a walker the synthetic schlieren method is used. A further explanation of this method is given in paper [9]. In this method two different images are correlated, the image of an undistorted situation and the image of a distorted one. In the experiments of this thesis a random dot pattern is placed behind the liquid layer. This indicates the displacement by the gradient $\nabla h(x, y)$ of the waves.

The program PIVview [8] is used to produce the velocity field which is proportional to the gradient field. The integration of this field is done by a script based on the integration method of [11, p.144] which provides a height landscape.

4.1 The gradient field

For the calculation of the gradient field PIVview [8] is used. This program provides displacements in both the $x$ and $y$ direction for every group of pixels in the picture, this is done with a sub pixel accuracy. It is possible to show a contourplot and vector velocity field of the correlated images done by PIVview. This contourplots are ideal to get an quick overall view of the guiding waves. Some examples of this contourplots are shown in figure 3.2 and figure 3.3. PIVview uses a window size of $64 \times 64$ where the stepsize is 7 pixels which corresponds with an overlap of 89%.

After a long derivation the relation between $\nabla h$ and $\delta \vec{r}$ can be obtained as shown in [9]. Finally the linear relation gets:

$$\nabla h = -\frac{\delta \vec{r}}{h^*}, \quad \text{with} \quad \frac{1}{h^*} = \frac{1}{\alpha h_p} - \frac{1}{H} > 0. \quad (4.1)$$

Where $H$ the height of the camera, $\alpha$ a constant $\alpha = 1 - n/n'$ with $n$ and $n'$ the refractive index of the two media. $\alpha \simeq 0.24$ for an air-water interface and $h_p$ the average height of the fluid layer. In practice, with a camera far away from the fluid, $h^*$ gets $h^* \simeq \alpha h_p$. And with this simple linear relation the gradient of the height of the fluid seems proportional with the displacement field.
4.2 Random dot pattern

Because the program PIVview correlates two different pictures a dot pattern is used. The ideal case would be a dot size of $3 \times 3$ pixels big. With this dot size it is possible to make sub-pixel correlations which will provide more accurate gradients. The random dot pattern as used in the experiment is shown in figure 4.1 and has a dot size of about 10 pixels. This rather big dots will cause a more inaccurate resolution. But the high particle density will correct for this. The picture given in figure 4.1 covers a subsection of the container used in the experiments.

![Figure 4.1: A subsection of the container with a boundary present at the top, with the random dot pattern that has been used in all of the experiments. This particular dot pattern has a black white ratio of 50%. These dots are numerically generated with MATLAB.](image)

4.3 Calculating the height

As shown in figure 4.2[a] it is possible to get the surface height profile by looking at the shifted dot pattern beneath the liquid layer. This shifted dot pattern can be approximated as the gradient which is proven in [9]. To provide a function of the height of the liquid surface $h(x, y)$, the gradient of all pixels need to be integrated. So $h(x, y) = h_p + \nabla^{-1}\xi$ with $\xi$ the measured surface gradient and $h_p$ the average height of the fluid. This integrating is done by integrating along different integrating paths from one center grid point to the outside. The integrating pattern is given in figure 4.2[b] where the process starts at the red dot and propagates via the given steps [11, p.144]. The integration is done in the $x$-direction and in the $y$-direction and in grid points where the $x$- and $y$-direction intersect the values are averaged. The $x$-direction gets:

$$h(x + \Delta x, y) = h(x, y) + \Delta x \frac{h_x(x + \Delta x, y) + h_x(x, y)}{2}$$

(4.2)
where $\Delta x$ is the grid spacing in the x-direction, $h_x$ is the partial derivative of $h(x, y)$ with respect to $x$ and $h(x, y)$ is the height on the grid point $(x, y)$. The y-direction gets:

$$h(x, y + \Delta y) = h(x, y) + \Delta y \frac{h_y(x, y + \Delta y) + h_y(x, y)}{2}$$

(4.3)

where $\Delta y$ is the grid spacing in the y-direction and $h_y$ is the partial derivative of $h(x, y)$ with respect to $y$. For the grid points where the x- and y-direction get together the integrating is averaged. This will reduce the measurement errors in both $h_x$ and $h_y$.

Figure 4.2: (a) Three-dimensional ray geometry for an arbitrary deformed interface. A ray coming from $M$ appears to come from $M'$. With the camera $C$ and normal $\hat{n}$. Figure originally presented in [9] (b) The integrating path from a local gridpoint (the red dot) the horizontal integration paths use the horizontal velocity $u$ and the horizontal integration paths use the vertical velocity $v$. At grid points where a vertical and horizontal integration path come together, both values are averaged. By doing this the chance of mistakes is the smallest.
Chapter 5

Boundary’s

The walkers created in the container eventually get to the edge of the container where they have to bounce back. A special method is needed to make the walkers reflect without merging with the fluid below. The property of the walker threshold, Faraday threshold and depth of the liquid layer is used to make a proper boundary. As observed earlier in this report the dispersion relation (equation (2.3)) depends on the depth of the liquid layer. Because walkers can only live with there guiding waves this indicates that the absence of a guiding wave automatically means a shallow zone for a walker.

![Figure 5.1](image)

Figure 5.1: (a) The measured threshold for the Faraday instability $\gamma_{Fm}$ (triangles) and for the walking instability of $\gamma_{Wm}$ of a drop of diameter $D = 0.78$ mm (filled circles) as a function of the depth $h$ of the liquid bath. (b) in cells formed of two regions of depths $h_0 = 4.1$ mm and $h_1 = 1.1$ mm, when the system is tuned to a value $\gamma_{m}^{W}(h_0) > \gamma_{m} > \gamma_{m}^{W}(h_1)$ (e.g., $\gamma_{m}/g = 3.75$), a drop forms a walker in the regions of large depth but remains motionless in the shallow zone. The continuous lines are simple interpolations. This image was originally presented in [5]

As shown in figure 5.1, for a height of $h = 1.5mm$ or less, the continuous lines of the Faraday threshold $\gamma_{Fm}$ and the walker threshold $\gamma_{Wm}$ are the same. The area above the Faraday
threshold and the area below the Walking threshold are both walker shallow zones. Meaning that between the continuous Faraday threshold line and the continuous walker line a walker can exist. So with a depth of $h = 1.5mm$ or less a walker can’t exist. This can be used to make walls where walkers are not allowed to go.

Because a walker can be seen as a wave-particle interaction there is no simple refraction law. The interaction with boundaries is not as straightforward as a classical system. In a classical system the angle of incidence is equal to the angle of refraction. But as shown in [5, FIG. 4] the trajectory of a walker inside a square container wouldn’t give any classical results. As shown in previous experiments by (Couder et al.) the double slit experiment will give a diffraction pattern for the histogram of many successive crossings. Which indicates a quantum-mechanic like behaviour.
Chapter 6

Experimental setup

6.1 Setup

For all of the experiments done in this thesis the same sort of setup is used. The setup consists of a square container with dimensions $90mm \times 90mm$. This container is filled with silicon oil (Tegiloxan 3 and 2000) with respectively viscosities $\mu = 3cSt$ and $\mu = 2000cSt$ which can be mixed to every viscosity in between. The reason for using silicon oil in stead of other liquids like plain water is the insensitivity of surfactant effects [13, page 57]. Different viscosities of 15cSt, 20cSt, 30cSt, 40cSt and 50cSt have been made, to optimise the walking domain. The density of the different mixtures is $\rho = 892.4kgm^{-3}$ and surface tension $\sigma = \alpha = 18.3 \times 10^{-3}Jm^{-2}$

This whole setup is shaken vertically by a vibrating exciter (Brüel & Kjaer Type 4808) which frequency can range from 5Hz to 10kHz and can handle a force of $F = 112N$. Because the vibrating exciter is limited to a certain force it is important to get the total weight of the setup as low as possible.

The setup is being captured with a high speed camera, the Phanton v5.1 which is triggered at half the driving frequency $f_0$. By doing this the walker is captured at the same point in the bouncing period. This ensures that the guiding waves can be viewed at the same point in a period. The camera is located at 22cm from the.

To get a macroscopic view on the refraction of the walkers the open source program [2] is used to track the walkers. With this program it is possible to determine the position of the walker with an accuracy of 0.5mm.

6.2 Alignment of the setup

The alignment of the setup can be done as described in [9]. This is done by looking at the Faraday waves that occur when the silicon oil is shaken vertically with an acceleration inside one of the Faraday instability tongues as shown in figure 2.1. The Faraday waves that occur will start at the walls of the container. If the setup is aligned properly the Faraday waves should occur at the same time at all of the walls. Another method to look if the setup is properly aligned is by creating a group of bouncers and see if the bouncers stay at the same position. Both methods are used in this experiment to achieve the most accurate alignment.
Figure 6.1: The setup as used in all of the experiments. To block the incoming 50Hz light coming from the lights there is placed a cap around the camera.

6.3 Gradient field lens

A lens with known dimensions is used to check if the integrating program that is used for the high-profile gives the right values. The measured gradient field of a lens with radius $r = 5.0\,\text{cm}$ is shown in figure 6.2 on the left. On the right is the integrated gradient field
which results in the $h(x, y)$ field. The height difference of the base of the lens and the top of the lens is $3.40\, mm$ and the osculating circle is $R = 24.7\, mm$. The height $h(x, y)$ of the integrated field is not corresponding with the measured height but does have the right shape. So the values need to be multiplied by a certain factor $k = \alpha$, the $\alpha$ obtained in section §4.1. In figure 6.2 the contour plot of the gradient field (left) and the integrated field (right) are shown.

![Figure 6.2: (a) A PIVview contour plot of a lens of diameter $d = 50\, mm$ and Osculating circle $R = 24.7\, mm$. All of the dimensions are known and therefore the height profile of the integrated field can be calibrated. (b) The contour plot of the integrated gradient field. The heightfield is uncallibrated.](image)

### 6.4 Recipe for walkers

Making a walker is not easy. There are a lot of parameters which can be changed and just a few combinations eventually will lead to the walking domain. As shown in figure 3.1 a different viscosity and frequency will eventually lead to different phase diagrams. Because this particular phase diagrams are tuned to the setups used this phase diagrams are only for a general thought in the experiments of this thesis. To optimise this walking domain different viscosities of 15, 20, 30, 40 and 50cSt.

As seen in figure 3.1 the walking domain is just below the Faraday threshold. Therefore the amplitude of the vibrating exciter is then tuned just below the amplitude where surface waves will occur. Then for a certain value of the viscosity a large amount of drops, of various size, are created. This is done by picking a drop out of the surface with a stick and dropping it on the surface. The drops in the walking regime are kept and the drops that are in the bouncing regime are carefully removed by picking them up with the stick. It is also possible that none of the bouncing drops are behaving like walkers, because the walking regime in the phase diagram has vanished for a certain value of viscosity and frequency.
Chapter 7

Results

7.1 The integrated lens

As described in section §6.3 it is necessity to check the height profile by a lens with known dimensions. The lens with a height difference of 3.40\,mm between center and edge is shown in figure 7.1 with its original integrated lens with a height of 11.44\,mm. The correction factor needs to be $k = 0.30$ to get the correct height. For an air-PMMA interface $\alpha = 0.33$. So the uncertainty is $0.1 \times h$. Because the lens is put in the liquid layer the capillarity and in-continuity causes the increasing lines around the lens. For the silicon-air interface an correction factor $k = \alpha = 0.28$ is used.

![Figure 7.1: A plot of the integrated lens with the dimensions of the integrated lens and in the same plot the dimensions of the corrected integrated lens. The uncorrected lens has a height of 11.4\,mm and the real lens has a height of 3.4\,mm](image)

7.2 Walker trajectories

For a macroscopic and fast view on the reflection of a walker with a boundary the trajectory is tracked in the horizontal plane. In figure 7.2 three of those trajectories, with each one or
two interactions, are shown. To see if the process repeats itself the reflections are all done at the same wall. A stick is used to push the walker in the right direction. The capillary effect of the stick ensures that the walker goes away from the stick and thus can be controlled. As seen in figure 7.2 the reflections are not classical, therefore the effect of the guiding waves is further examined in section §7.3. The three collisions of figure 7.2 also show that a collision of a walker with a boundary is not linearly dependent. For collisions with the same angle of incidence the angle of reflection can be different.

![Figure 7.2: three collisions with a wall, it is clear that the reflection of a walker with a wall is not a classical reflection. The angle of incidence is not the angle of reflection. And even no correlation between the three reflections.](image)

### 7.3 Guiding waves

To get a better knowledge about the guiding waves of a walker the trajectory is being captured. The synthetic schlieren method is then applied to reveal the guiding waves. In figure 7.3 six of those images are shown, but only the gradient field of it.

A simple bouncing droplet will produces the same guiding waves as a rock thrown in a pond, capillary waves. The form of this capillary waves are Bessel functions. The guiding waves of a walker differ not much from the guiding waves of a bouncer, the difference is the deformation of the Bessel function. In the direction opposite of the walking direction the guiding waves will disappear due to the interference of the guiding waves made in every bouncing period.

Walkers will follow a continuous and non-classical reflection. The velocity of a walker interacting with a boundary will never be zero, therefore it is continuous. The angle of reflection is not the same as the angle of incidence thus the collision is non-classical. This is clearly seen in figure 7.3.
Figure 7.3: Six contour plots showing the motion and gradient field of a walker and his guiding waves. With forcing frequency $f_0 = 40\text{Hz}$ and viscosity $\mu = 15 \times 10^{-3}\text{Pas}$. The amplitude is just below the faraday threshold $\gamma^F$.

As shown in figure 7.3 the walker will not hit the wall at any time, it will stay at a finite distance from the wall. This is due to the guiding waves that will vanish in the shallow zone. As explained earlier in this report, the walls of the container consist of a shallow. In this shallow the guiding waves of the walker can not exist so there is no possibility for the drop, which lives with his wave, to exist.

### 7.4 Cross section walker

A cross section of the height profile is viewed to get a better look on the symmetry of the walker. In figure 7.4 (top) a cross section of a walker near a boundary is shown, this walker moves parallel to the boundary. For the shape of the walker in the walking direction figure 7.4 (bottom) is shown. The cross section of the top graph indicate that indeed the wave can not live in the shallow zone and will decrease fast to zero amplitude, where the other side will oscillate and decrease quadratic. As seen in the bottom graph the front of the walker consist of a high "leading front". This "leading front" is the wave which hits the shallow zone first and decreases when the walker gets closer to the boundary. Both of the cross sections where taken out of a 3D-plot like figure 7.6.
Figure 7.4: Two graphs of the cross sections of the guiding waves of a walker near a boundary. The graph at the top is a cross section perpendicular on the wall, the striped rectangle suggests the wall. The red line indicates the fitted function. The bottom graph is a cross section of the waves in the walking direction of the walker which moves freely around the surface.

As shown in the top graph of figure 7.4 the decrease $A_R/A_L$, of a walker which is 9mm away from a boundary, is $A_R/A_L = 0.69$. Where $A_L$ is the amplitude of the left top close to the walker and $A_R$ is the amplitude of the top right of the walker. For walkers with different distances to the wall the height difference of the left and right top around the walker have been calculated and are shown in figure 7.5. It is clear that droplets that are further away from the boundary have less interaction with it, so that the wave functions are more symmetric. The great variety of decreases $A_R/A_L$ is due to the shape of the walker and the direction the walker is going compared with the wall. The cross sections of all the measured decreases are done perpendicular to the wall.

The damped oscillation on the left side of the walker is compared with the standard function of a damped wave $y = y_0 + A \exp(-x/t_0) \cdot \sin(\pi(x - x_c)/\omega))$. Where the value of the damping proved to be $t_0 = 8.5 \pm 0.4$. In the top graph of figure 7.4 the red line indicates the standard function. It can be seen that the oscillation on the left side of the walker is indeed a damped wave.
Figure 7.5: The decrease $A_R/A_L$ of the guiding waves near a boundary as a function of the distance to that boundary. As suspected the walkers that are further away from the wall are less influenced, so the left and right top near the droplet are more equal.

Figure 7.6: A 3D-plot of the guiding waves of a walker, at the right rear plane is the boundary and the walker is walking to the right front plane.
Chapter 8

Conclusion and Discussion

The interaction of a walker with a shallow boundary is examined in this thesis. This is done by tracking of the macroscopic trajectory of the walker and by looking at the structure of the guiding waves of the walker. These guiding waves can be observed by the synthetic schlieren method which produces a height profile for all coordinates.

The walking trajectories have been captured where the angle of incidence found to be different from the angle of reflection. This indicates that the walkers does not follow the classical collision law. For three different collisions it is also been found that the collisions can not be considered as linear.

For a walker the guiding waves consist of a leading front wave which decreases when a walker approaches a boundary. The degree of decrease depends on the distance to the boundary. This decrease of the wave has to do with the shallow boundary where the waves are not allowed to be. The complex interaction between waves of previous bounces will make the prediction of a reflection angle very difficult by looking at the guiding waves. It is clear that there is more work needed to understand the true relationship between a walker and a boundary.

There are also a lot of things to say about the collision of walkers with boundaries for instance the analogy with the quantum mechanics, in particular the tunnelling in an infinite well. Where the shallow boundary can be compared with the wall of the infinite well. Some part of the wave function will tunnel trough the wall and so does the wave in the shallow zone.

It is also interesting to get a relationship between the dispersion relation on the shallow and the dispersion relation in the container with an explanation about the connection of both of this dispersion relations. Because the dispersion relation depends on the height the driving amplitude has to be different for shallow zone and the container.

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Bibliography


