Coupled-cavity quantum electrodynamics for the optical control of the quality factor of photonic crystal nanocavities

Westendorp, S.

Award date:
2012

Disclaimer
This document contains a student thesis (bachelor's or master's), as authored by a student at Eindhoven University of Technology. Student theses are made available in the TU/e repository upon obtaining the required degree. The grade received is not published on the document as presented in the repository. The required complexity or quality of research of student theses may vary by program, and the required minimum study period may vary in duration.

General rights
Copyright and moral rights for the publications made accessible in the public portal are retained by the authors and/or other copyright owners and it is a condition of accessing publications that users recognise and abide by the legal requirements associated with these rights.

• Users may download and print one copy of any publication from the public portal for the purpose of private study or research.
• You may not further distribute the material or use it for any profit-making activity or commercial gain
Coupled-cavity quantum electrodynamics for the optical control of the quality factor of photonic crystal nanocavities

S. Westendorp
July 2012
Coupled-cavity quantum electrodynamics for the optical control of the quality factor of photonic crystal nanocavities

S. Westendorp
June 2012

Research group: Photonics and Semiconductor Nanophysics (PSN)
Technische Universiteit Eindhoven
Supervisor: Dr. C. Y. Jin
Supervising professor: Prof. dr. A. Fiore
Abstract

Photonic crystals are periodic nanostructures that strongly modify the propagation of electromagnetic waves. Photonic crystals can have a three-dimensional bandgap which makes it possible to confine photons inside a defect by breaking the periodicity within a small region which makes a photonic crystal cavity. The behavior of the photons inside the photonic crystal cavities needs to be studied for the application in for example all optical systems. One of the interesting topics is how to capture and release photons all-optically. This requires that the optical confinement of the photonic crystal cavities has to be manipulated dynamically.

In this project two coupled cavities in photonic crystal structures were studied with coupled mode theory and with experiments. One of the defects, the control cavity, was used to control the confinement of the other defect, the target cavity.

In the experiments an increase in the Q-factor of the target cavity mode of around 80% was found when the target cavity mode was shifted out of resonance with the control cavity mode. This shows the possibility of large optical confinement switching (Q-switching). A study with the coupled mode theory predicts the highest Q-switching in the boundary region between weak and strong coupling.
# Table of contents

1 Introduction.................................................................................................................................................. 1
   1.1 The history of photonics......................................................................................................................... 1
   1.2 Photonic crystals ...................................................................................................................................... 1
   1.3 Cavity quantum electrodynamics .......................................................................................................... 1
   1.4 The goal of this experiment .................................................................................................................. 2
2 Theory ............................................................................................................................................................ 3
   2.1 Coupled-cavity quantum electrodynamics ............................................................................................... 3
      2.1.1 The modes of the system .................................................................................................................... 3
      2.1.2 Resonance between the target cavity and FP mode ........................................................................... 4
      2.1.3 The spontaneous emission enhancement .......................................................................................... 5
   2.2 Simulations of the theory ......................................................................................................................... 6
      2.2.1 FP mode wavelength detuning ........................................................................................................... 6
      2.2.2 Simulation of the system for a range of coupling rates ..................................................................... 7
      2.2.3 Finding the optimal Q factor for the FP cavity ................................................................................ 8
3 Experimental set-up ........................................................................................................................................... 10
   3.1 The sample structures ............................................................................................................................... 10
   3.2 The Micro-PL setup .................................................................................................................................. 12
4 Results and discussion ..................................................................................................................................... 14
   4.1 Right barrier of 10 periods ....................................................................................................................... 14
   4.2 Right barrier of 8 periods .......................................................................................................................... 16
   4.3 Right barrier of 20 periods ....................................................................................................................... 18
5 Conclusion ....................................................................................................................................................... 20
   5.1 Recommendations ..................................................................................................................................... 20
6 Literature ............................................................................................................................................................ 21
Appendix A: Target cavity coupled with FP cavity ......................................................................................... 22
Appendix B: Linewidth separation of two strong coupled modes ...................................................................... 24
1 Introduction

1.1 The history of photonics

Photonics is a field of research that is dedicated to the practical application of photons in for example all optical systems. Photonics is much like electronics, which is a field of research that specializes in the use of electrons in electrical systems. The field of photonics emerged around 1960 after the invention of the first semiconductor lasers \[1\]. The term became more commonly used in the 1980s, when the first telecommunications network was constituted using fiber optics for data transmission. Around the same time the research on photonic crystals was growing at a high rate and in 1996 the first two-dimensional photonic crystal at optical wavelengths was created \[2\].

1.2 Photonic crystals

Photonic crystals are periodic nanostructures that affect the propagation of electromagnetic waves. The principle of photonic crystals is similar to the periodic potential in semiconductor crystals that affects the motion of electrons. Due to the wavelength-sized periodicity of the nanostructure standing waves can arise between the edges where the dielectric constant changes. This gives rise to a photonic bandgap that will only allow specific wavelengths of light to propagate. A complete photonic bandgap blocks light in all directions within a certain range of wavelength.

A defect in the structure of a photonic crystal can locally change the propagation of electromagnetic waves. For example a point defect cavity can confine a photon and a line defect waveguide can guide a photon in a specific direction. This ability to guide and manipulate photons is required to make an all optical circuit. Another possible application with photonic crystals is all-optical switching \[3\]. This would be particularly useful in optical routers, where a lot of energy and time is wasted on the conversion of photons to electrons and electrons to photons in the present day internet. Another advantage of all optical circuits is that quantum encryption of the information can be applied, because the information carriers of individual photons are quantum particles.

1.3 Cavity quantum electrodynamics

Cavity quantum electrodynamics (CQED) is a research field to study the atom-photon interaction within the cavity \[4\]. It can sometimes be described as a combination of uncoupled cases with a perturbation theory. If the photon and the two-level system within the atom interact, then they become a coupled system. This interaction is divided into the weak coupling regime and the strong coupling regime. If the system is in weak coupling, then the spontaneous emission of the two-level system can be enhanced or reduced. In strong coupling the system will oscillate between an excited two-level system and a two-level system in ground state plus a photon. The coupled system can be described by three parameters: the coupling strength between the two-level system and the cavity $g$, the loss rate of the two-level system $\kappa_0$ and the loss rate of the cavity $\kappa_{\text{cav}}$. 
The spontaneous emission rate of this two-level system can be modified by changing the local density of states (LDOS) at the position of two-level system. This effect was discovered by Purcell \[5\] and is therefore called the Purcell effect. If the LDOS is increased, then the spontaneous emission rate should increase as well. The LDOS can be increased if the cavity mode is spectrally aligned with the emission frequency of the two-level system \[6\]. This is because the density of states (DOS) in the cavity is at its maximum at the resonant frequency of the cavity mode. Now if the cavity mode is spectrally aligned with the emission frequency of the two-level system, then the LDOS can also be changed by modifying the quality factor or the mode volume of the cavity. An increase of the quality factor will narrow the distribution of the DOS, while increasing the maximum DOS; therefore the spontaneous emission rate will increase. Decreasing the mode volume will give a narrower spatial distribution of the electric field, which increases the LDOS at the center of the cavity. The enhancement of the spontaneous emission due to the cavity properties is given by the Purcell factor \[5\]:

\[
F_{\text{Purcell}} = \frac{\gamma_{SE}}{\gamma_0} = \frac{3}{4\pi^2} \left( \frac{\lambda_c}{n} \right)^3 \frac{Q}{V}
\]

where \(\gamma_{SE}\) and \(\gamma_0\) are the spontaneous emission rates in respectively the cavity and in free space, \((\lambda_c/n)\) is the cavity wavelength inside the material, \(Q\) is the quality factor of the cavity and \(V\) is the mode volume of the cavity.

1.4 The goal of this experiment

The goal of this project is to apply coupled mode theory on two coupled cavities and compare it with measurements on a coupled system. A second goal is to research the ideal conditions for high Q-switching and to demonstrate it in the experiment.
2 Theory

2.1 Coupled-cavity quantum electrodynamics

Coupled-Cavity quantum electrodynamics (CCQED) has been proposed by Hughes [7] to describe the interaction between two coupled cavities. Two cavities can couple if their electrical fields have spatial overlap, and accordingly changing the conditions for one cavity will induce changes in the other cavity. This principle makes it possible to use one cavity as control cavity for Q-switching in the target cavity. Other approaches for Q-switching have been reported previously [8][9]. If a quantum dot is embedded in the target cavity, then the spontaneous emission can be enhanced or reduced by shifting the modes of the two cavities in and out of resonance.

2.1.1 The modes of the system

The system in this project consists of a target cavity, a Fabry-Perot (FP) cavity and a two level quantum dot. It is assumed that the quantum dot does not couple with the FP mode, because the electric field of the FP mode has a negligible amplitude at the position of the quantum dot. The coupling between the components of this system can then be described with the non-Hermitian Hamiltonian $H$:

$$H = \hbar \begin{bmatrix} \omega_{QD} + i\kappa_0 & g & 0 \\ g & \omega_N + i\kappa_N & \eta \\ 0 & \eta & \omega_{FP} + i\kappa_{FP} \end{bmatrix},$$

where $\omega_{QD}$, $\omega_N$ and $\omega_{FP}$ are respectively the angular frequencies of the quantum dot, the target mode and the FP mode. $\kappa_0$, $\kappa_N$ and $\kappa_{FP}$ are the loss rates for the exponential decay for each corresponding component. Finally $g$ is the coupling rate between the quantum dot and the target cavity mode and $\eta$ is the coupling rate between the target cavity mode and FP mode. The eigenvalues $\tilde{\omega}_{1,2}$ of the coupled-cavity system, described by a 2x2 sub-matrix of eq. 2.1, can be obtained by solving the eigenvalue problem of the sub-matrix $H_{cc}$:

$$H_{cc} = \hbar \begin{bmatrix} \tilde{\omega}_N & \eta \\ \eta & \tilde{\omega}_{FP} \end{bmatrix},$$

$$\tilde{\omega}_{1,2} = \frac{1}{2} \left( \tilde{\omega}_N + \tilde{\omega}_{FP} \pm \sqrt{(\tilde{\omega}_N - \tilde{\omega}_{FP})^2 + 4\eta^2} \right),$$

where $\tilde{\omega}_N = \omega_N + i\kappa_N$ and $\tilde{\omega}_{FP} = \omega_{FP} + i\kappa_{FP}$ denote the complex frequency for each cavity. For a derivation of eq. 2.3 see Appendix A. Each eigenvalue $\tilde{\omega}_i$ corresponds to a mode of the coupled system and the real and imaginary parts of the eigenvalue are respectively the frequency $\omega_i$ and the loss rate $\kappa_i$ of this mode. The quality factor $Q_{1,2}$ of the two new modes can be determined from the new frequencies and loss rates:

$$Q_{1,2} = \frac{\text{Re}(\tilde{\omega}_{1,2})}{2\text{Im}(\tilde{\omega}_{1,2})} = \frac{\omega_{1,2}}{2\kappa_{1,2}}.$$
2.1.2 Resonance between the target cavity and FP mode

If the FP mode is brought into resonance with the target cavity mode \((\omega_{FP} = \omega_N)\), then equation (2.2) can be simplified to:

\[
\tilde{\omega}_{1,2} = \omega_N + \frac{i}{2}(\kappa_N + \kappa_{FP}) \pm \frac{1}{2}\sqrt{4\eta^2 - (\kappa_N - \kappa_{FP})^2}. \tag{2.5}
\]

These two eigenvalues are equal and therefore indistinguishable if the square root term is equal to zero. The coupling rate at which this happens is:

\[
\eta_b = \frac{|\kappa_N - \kappa_{FP}|}{2}. \tag{2.6}
\]

For coupling rates greater than this value the frequencies of the two modes are different, but the loss rates are the same. But for coupling rates smaller than this value the frequencies are the same, while the loss rates are different for each mode. So the characteristics of the coupling between the two cavities are different for coupling rates greater or less than this value. This coupling rate is therefore the mathematical border between the weak and strong coupling regimes.

In the region with coupling rates slightly larger than \(\eta_b\) the separation of the resonating modes will not be observable. The reason for this is that the Lorentzian peaks of the modes have a non-zero linewidth. Since the resonating modes have the same linewidth in the strong coupling regime, the Lorentzian peaks will have to be separated at least by a distance equal to their linewidths for them to be observable as two peaks instead of one peak. The linewidth in strong coupling is equal to the average of the linewidths of the two uncoupled modes, because of the symmetrical splitting of the eigenvalues. The coupling rate, where the two peaks are separated by a distance equal to the linewidth \(\Delta \lambda\), is given by:

\[
\eta_{b,2} = \frac{(\kappa_N - \kappa_{FP})^2}{4} + \frac{4\pi^2c^2}{\Delta \lambda^2} \left(-1 + \sqrt{\frac{\Delta \lambda^2}{\lambda_N^2} + 1}\right)^2, \tag{2.7}
\]

where \(\lambda_N\) is the wavelength at which the two modes start resonating, \(\Delta \lambda\) is the linewidth of the resonating modes in strong coupling and \(c\) is the speed of light. A derivation of this coupling rate can be found in Appendix B. The region between \(\eta_b\) and \(\eta_{b,2}\), where the two modes are in strong coupling but look like they are in weak coupling, will be called the transition region in the remaining part of the report.

In real experiments they will have to be separated by a larger distance to be visible, because of noisy signals and finite resolutions. Also with fitting algorithms it should be possible two extract the two original Lorentzian curves from the single peak.
2.1.3 The spontaneous emission enhancement

The two-level system can couple to both modes of the system. The coupling strength to these modes depends on the fractions of the original electrical fields inside the target cavity. The enhancement of spontaneous emission $\gamma_{SE}/\gamma_0$ of the quantum dot depends on the detuning of the mode from resonant frequency $\omega_{QD}$ of the quantum dot $^{[6]}$: 

$$\gamma_{SE} = \frac{3}{4\pi^2} \left( \frac{\lambda_{QD}}{n_r} \right)^3 \left[ \frac{\kappa_1^2 |\hat{E}_1(\mathbf{r}_N)|^2 q_1}{\kappa_1^2 + 4(\omega_1 - \omega_{QD})^2} + \frac{\kappa_2^2 |\hat{E}_2(\mathbf{r}_N)|^2 q_2}{\kappa_2^2 + 4(\omega_2 - \omega_{QD})^2} \right]$$

(2.8)

where $\lambda_{QD}$ is the resonant frequency of the quantum dot, $n_r$ is the refractive index of the material, $\hat{E}_1(\mathbf{r})$ and $\hat{E}_2(\mathbf{r})$ are the normalized amplitudes of electric field of the modes at the center $\mathbf{r}_N$ of the target cavity, $\kappa_1$ and $\kappa_2$ are the loss rates of the modes and $\gamma_0$ is the decay rate of the quantum dot in free space. Here it is assumed that the electrical fields of the modes have a Lorentzian distribution.
2.2 Simulations of the theory

The model of the coupled cavity system has been simulated in Matlab. In these simulations the wavelength and the Q factor of the target mode are chosen to be $\lambda_N = 1550$ nm and $Q_N = 20000$.

2.2.1 FP mode wavelength detuning

First the model has been solved as function of the FP mode wavelength for both weak and strong coupling. The FP mode wavelength was detuned from the resonant wavelength to a maximum of 2 nm, which is limited by the finite space between FP modes. In this simulation the Q factor of the uncoupled FP mode was set to $Q_{FP} = 2000$. The results of this simulation are shown in figure 2.1.

Figure 2.1a shows the behavior of the frequencies of the modes for both weak and strong coupling as result of the FP mode wavelength detuning. The two modes of the system never cross, not even in the weak coupling case. This is because the eigenvalues of the Hamiltonian are solved numerically and the eigenvalue with the positive square root will always be greater than or equal to the eigenvalue with the negative square root.

Figure 1b shows the change in the Q factor as result of the FP mode wavelength detuning. The change of Q in the weak coupling regime is smaller than in the strong coupling regime. The highest change in Q is achieved if the FP mode wavelength is detuned as far away as possible from the resonant wavelength. But because the FP has a periodic structure with periodic modes, shifting the FP detuning too much will cause the target cavity to go in resonance with another FP mode. This is also the reason why the FP mode was detuned a maximum of 2 nm.

Figure 2.1: Results of the FP mode wavelength detuning simulation for weak coupling (dashed lines) and strong coupling (solid lines) with a coupling rate of respectively of $\eta = \kappa_{FP}/10$ and $\eta = 10\kappa_{FP}$. Figure (a) shows the relative shift of the wavelength of the modes as function of the relative detuning of the FP mode wavelength. Figure (b) shows the change in the Q factor as function of the relative detuning of the FP mode wavelength.
2.2.2 Simulation of the system for a range of coupling rates

The system has also been solved as function of the coupling rate. In this case the FP mode wavelength is shifted 2.2 nm out of resonance in order to achieve a change in Q. The Q factor of the uncoupled FP mode was set to $Q_{FP} = 2000$. The results of this simulation are shown in figure 2.2.

In figure 2.2 the eigenvalues of the resonating modes are solved as function of the coupling strength between the two modes. Figures 2.2(a) and (b) show that both the wavelengths and linewidths of the modes split at $\eta = |\kappa_N - \kappa_{FP}| / 2$ as predicted in eq. (2.5). Figure 2.2(a) also shows the boundary with the strong coupling regime at $\eta = \kappa_{FP} + \kappa_N$. The separation between the two peaks is equal to 0.78 nm, which is equal to the linewidth of the FP mode: $\Delta\lambda_{FP} = \lambda_{FP} / Q_{FP} \approx 0.78$ nm.

Figure 2.2(b) shows that the linewidths of the modes are constant and the same in the strong coupling regime. This also means that the Q factors of the resonating modes are the same in the strong coupling regime.

![Figure 2.2](image)

*Figure 2.2: The eigenvalues of the modes are calculated for a range of coupling rates, when the two modes are in resonance. The detuning from the resonance mode was 2 nm. Figure (a) shows the wavelengths of the modes and figure (b) shows the linewidths of the modes. These figures show that the frequencies and loss rates split at $\eta_{b}$ of eq. 2.6 (black dotted line). The red dotted line shows the coupling rate $\eta_{b,2}$ of eq. 2.7, where the two modes are separated by their linewidth.*

In figure 2.3 the change in Q of each mode is plotted as function of the coupling rate between the two modes. The largest Q change is achieved in the transition region between weak and strong coupling. The region of largest Q change also depends on the detuning. If one would be able to infinitely detune the target mode away from the FP mode, then the change in Q would be constant in the strong coupling regime. This is simply because the modes would return to their uncoupled state.
2.2.3 Finding the optimal Q factor for the FP cavity

In this simulation the system is solved for different Q factors for the FP cavity, while the coupling rate is chosen to be the coupling rate at the boundary between weak and strong coupling $\eta_b$ (eq. 2.6). In this way the parameters for the maximum Q tuning can be approximated for a given target cavity. The results of this simulation are shown in figure 2.4. As expected the change in Q of the modes is zero when the FP and target mode have the same Q factor. Figure 2.4 shows that the maximum Q change is achieved when the Q factor of the FP mode is 20 times smaller than the Q factor of the target mode.

The region where the highest change in Q is achieved can also be determined from an intensity map scanned over different values for the coupling rate $\eta$ and the FP mode loss rate $\kappa_{FP}$. Figure 2.5 shows this map for a quality factor of the target mode of $Q_N = 2 \cdot 10^4$. The left side of the white dotted line is called the weak coupling regime and the right side of the line is called the strong coupling regime. As can be seen the maximum change in Q is achieved in the transition region between weak and strong coupling.

Figure 2.3: The Q factor of each mode is calculated as function of the coupling rate for a detuning of 2 nm. The largest change in Q is for the coupling rates just above $\eta_b/\kappa_{FP}$ (black dotted line) and below $\eta_{b,2}/\kappa_{FP}$ (red dotted line) in the transition area between weak and strong coupling.
Figure 2.4: The relative change in $Q$ factor of the modes calculated as function of the loss rate of the FP mode for a detuning of 2 nm. The black dotted line shows the $Q$ factor of the FP mode that is used in previous simulations. As can be seen this value corresponds to the maximum ratio in figure 3.

Figure 2.5: The change in the $Q$ factor of the first mode as function of the loss rate of the FP mode and the coupling strength between the two cavities for a detuning of 2 nm. The white dotted line shows the curve $\eta_b$ (eq. 2.6) and the black dotted shows the curve $\eta_{b,2}$ (eq. 2.7). As can be seen the largest $Q$ change is achieved in the transition region between weak and strong coupling.
3 Experimental set-up

For optical measurements in this project a micro-photoluminescence (micro-PL) setup was used. In this project two kinds of sample devices were used, both with a different structure. The samples were fabricated in an InGaAsP membrane with InAs dot emitting around 1570 nm at room temperature. This membrane had a thickness of 220 nm.

3.1 The sample structures

The first kind of sample device contained only one defect, which was created by increasing the lattice constant with 3% for two periods long. The lattice constant of the crystal was 480 nm and the hole radius was 160 nm. The area around the defect, where the lattice constant is normal, is called the barrier. Both barriers were 20 periods long. Figure 3.1(a) shows a schematic drawing of this structure. This double heterostructure (DHS) cavity has been proposed by Noda’s group[10] as a design to create a cavity with a high Q factor. This cavity will be used as the target cavity. This device with a single cavity will be used to check the position of the target cavity mode, such that in the measurements the correct mode was measured.

A PL-spectrum of a typical DHS cavity is shown in figure 3.1(b), where the target mode is shown in blue and with a wavelength around $\lambda_N = 1566$ nm, around the optical fiber communication wavelength. Figure 3.1(c) shows an example of a DHS cavity mode which is fitted with a Lorentzian curve with a quality factor of $Q_N = (1.34 \pm 0.02) \cdot 10^4$.

![Figure 3.1: (a) A schematic drawing of the structure of the first type of sample. The structure contains a DHS cavity in the centre with a barrier on each side. (b) Example of a PL-spectrum of a single DHS cavity. The blue mode is the target cavity mode, the green modes are the modes of the bandgap edges of the crystal, the yellow mode is the $x$-polarized cavity mode and the red modes are the barrier modes. (c) The spectrum around the target mode. This target mode has a $Q$ of $Q_N = (1.34 \pm 0.02) \cdot 10^4$.](image-url)
The second type of sample was a waveguide with a lattice constant of 480 nm. In this waveguide two defects were created by changing the lattice constant with 3%. The first defect is a DHS cavity like in the first type of sample. The second defect is a cavity with Fabry-Pérot (FP) like resonance modes. This cavity will therefore be referred to as the FP cavity. Both the cavities are separated by a barrier with a length of 4.5 periods. The barrier at the left side of the cavities is 20 periods wide and the barrier at the right side is 5 to 20 barriers wide. These two barriers will be called the left and right barrier throughout the rest of the report. Figure 3.2(a) shows this structure. Figure 3.2(b) shows the FP modes and the target cavity mode for a typical structure. This structure will be used for the measurements to compare with the coupled mode theory.

![Figure 3.2: (a) A schematic drawing of the structure of second type of sample. (b) The periodic FP modes (green) and the target mode (blue) of a typical coupled-cavity structure. The target mode is coupled to the center FP mode, as can be seen by the small offset in the center of the two modes.](image-url)
3.2 The Micro-PL setup

The micro-PL setup can be used to measure the photoluminescence of a microscopic sample. In this project the microscopic sample will be a photonic crystal device as described above.

The setup basically consists of a continuous wave (CW) laser, a cryostat with a piezoelectric sample holder, an objective and a monochromator with an InGaAs detector cooled by liquid nitrogen. Two one-way mirrors guide the light emitted by the sample away from the path of the laser beam to the monochromator and a camera. Figure 3.3 shows a schematic drawing of the micro-PL setup.

![Figure 3.3: A schematic drawing of the micro-PL setup that was used in this project. The laser heated and excited the optically active sample, while the monochromator was used to create a spectrum of the photoluminescence collected from the sample.](image)

The objective had a magnification of 50 times and it was used to focus the laser beam onto an individual cavity and to collect the photoluminescence from the sample.

The sample holder in the cryostat could be moved by piezoelectric parts. The cryostat itself could be cooled down with liquid nitrogen, but it could also be cooled down with liquid helium.

The monochromator had two different gratings, one with a resolution of 0.016 nm and the other one with a resolution of 0.08 nm. A high resolution is required to measure cavities with a high Q factor. If the resolution is larger than the peak width, then a measurement will only show a partial peak with a wrong Q factor. Therefore the grating with the largest resolution should be used for measuring high Q cavities. For this spectrometer the highest measurable Q factor was around 90000.

The camera was used to see the position of the laser spot on the sample. This way the laser spot could be positioned exactly on top of the cavity and not on a barrier. The
camera was also used to focus the objective onto the sample, since the camera will capture a sharp image if the objective is at the correct distance of the sample.

The laser that was used in this project has a wavelength of 780 nm. Since the area of interest was around 1560 nm, the reflections of the laser beam on the sample could be filtered out with a long pass filter. The calibration of the laser is shown in figure 3.4 (Note: the laser power calibrated here is the power before the objective lens. At least half of the power is lost before it is injected into the sample.). In the experiments the current through the laser should always be greater than the threshold current of roughly 30 mA.

![Figure 3.4: Calibration of the laser power as function of the current. The slope of the linear fit is (44.63 ± 0.25) µW/mA and a threshold current of (29.69 ± 0.41) mA. The current through the laser should therefore always be larger than 30 mA.](image-url)
4 Results and discussion

In these experiments measurements are done on sample devices with both the target and FP cavity. The spot of the CW laser was moved onto the target cavity to heat and excite it. This way the target cavity mode could be moved in and out of resonance with a FP mode by a thermal shift. Only the samples where the target cavity could be shifted past a FP mode are further investigated with simulations. The parameters for these simulations are taken from the measurements on the cavities.

The resonant frequency could be estimated by taking the wavelength at the center of the two peaks of the coupled modes. The quality factors of the two modes could be roughly estimated from the graph of Q versus the detuning. At an infinite shift the quality factors should be equal to their uncoupled Q factors. The shift of the FP modes could be measured directly from another uncoupled FP mode, since the distance between the FP modes should almost keep the same.

4.1 Right barrier of 10 periods

The first sample that was investigated had a right barrier of 10 periods. The quality factor and loss rate that were found for the uncoupled target cavity mode are $Q_N = (15.6 \pm 0.2) \cdot 10^3$ and $\kappa_N = (3.87 \pm 0.05) \cdot 10^{10}$ s$^{-1}$. The quality factor and loss rate that were found for the uncoupled FP mode are $Q_{FP} = (5.8 \pm 0.1) \cdot 10^3$ and $\kappa_{FP} = (1.04 \pm 0.02) \cdot 10^{11}$ s$^{-1}$. The coupling rate between them was found to be $\eta = (1.7 \pm 0.1) \cdot 10^{11}$ s$^{-1} \approx (1.6 \pm 0.1) \cdot \kappa_{FP}$, which indicates strong coupling.

Figure 4.1 shows the summarized results of the measurement on this sample compared with a simulation based on the theory. Figure 4.1(a) shows the measured spectra for different laser powers. In figure 4.1(b) the peak wavelengths of the coupled modes are plotted as function of the detuning of the target mode. This figure shows clear anticrossing between the two coupled modes, which means that they are in strong coupling. Due to this strong coupling, the target mode will change into the FP mode when it is shifted in and out of resonance. At the same time the FP mode changes into the target mode. The figure also shows that both the FP mode and target mode are shifted due to heating. In the ideal case the FP modes would not shift.

Figure 4.1(c) shows the quality factor of both the coupled modes as function of the detuning of the target mode. The simulation does not fit through all of the points, because of the finite precision of the measurements and also the instability caused by the thermal heating. There is also an error in the fit, which is shown with the errors bars. During the experiment the laser spot could be shifted a little bit, which would change the heat distribution and as result the shift of the modes. Another explanation can be found in the estimation of the detuning of the target mode. The exact detuning can’t be determined from the measurements, therefore it is estimated by a linear interpolation. This approximation can be justified by the linearity of current dependence of the laser power. The uncertainty in the detuning is estimated to be 0.2 nm.

Another thing that can be observed in figure 4.1(b) is the large change in the Q factor when the target mode is shifted out of resonance. At a detuning of -1.8 nm the Q factor of the high Q mode is roughly equal to 15500, while in resonance the Q factor of the high Q mode is around 8500. This is an increase of a factor 1.8, which is very interesting if one wants to control the spontaneous emission of a quantum dot by Q-switching.
Figure 4.1: One beam measurement on a sample with a FP barrier of 10 periods. (a) The measured spectra for this sample. (b) The wavelengths of the modes obtained from the spectra. (c) The calculated Q factors for both modes. The dots show the results of the measurements and the solid curves show the coupled mode theory simulations for this sample. The coupling rate between the two modes is estimated to be $1.6 \cdot \kappa_{FP}$. The error bars correspond to the fitting error of the Lorentzian curves.
4.2 Right barrier of 8 periods

The next sample had a right barrier of 8 periods. Since the barrier next to the FP cavity is smaller when compared to the previous sample, the loss rate of the FP cavity in this sample should be larger than the loss rate in the previous sample. The quality factor and loss rate that were found for the uncoupled target mode are $Q_N = (12.0 \pm 0.1) \cdot 10^3$ and $\kappa_N = (5.04 \pm 0.04) \cdot 10^{10} \text{s}^{-1}$. The quality factor and loss rate that were found for the uncoupled FP mode are $Q_{FP} = (3.9 \pm 0.1) \cdot 10^3$ and $\kappa_{FP} = (1.57 \pm 0.04) \cdot 10^{11} \text{s}^{-1}$. The coupling rate between them was found to be $\eta = (3.3 \pm 0.2) \cdot 10^{11} \text{s}^{-1} \approx (2.1 \pm 0.1) \cdot \kappa_{FP}$, which indicates strong coupling.

The results of the measurements on this sample are shown in figure 4.2. Figure 4.2(a) shows the measured spectra for different laser powers. In figure 4.2(b) the peak wavelengths of the coupled modes are plotted as function of the detuning of the target mode. The simulation shows a small mismatch with the experimental results at low and high laser powers. This could also be a result of a shifted laser spot or the estimation of the detuning.

Figure 4.2(c) shows the quality factor of both the coupled modes as function of the detuning of the target mode. The simulation shows a mismatch of 10% with one of the measured points. The large mismatch is not the result of a wrong simulation, since the coupling rate would have to be a lot larger to fit this point and then it would completely mismatch the mode wavelengths in figure 4.2(b). The error could be caused by heating of the sample or shifting of the laser spot. The previously estimated error in the detuning of 0.2 nm would make this measured point correspond with the simulation.

At a detuning of -0.8 nm the Q factor of the high Q mode is roughly equal to 8500, while in resonance the Q factor of the high Q mode is around 5800. This is an increase of a factor 1.5, which could be bigger if the target mode could be shifted further away. In this case the shift was limited by the laser threshold and the maximum laser intensity on the sample (to avoid destruction of the sample).
Figure 4.2: One beam measurement on a sample with a FP barrier of 8 periods. (a) The measured spectra for this sample. (b) The wavelengths of the modes obtained from the spectra. (c) The calculated $Q$ factors for both modes. The dots show the results of the measurements and the solid curves show the coupled mode theory simulations for this sample. The coupling rate between the two modes is estimated to be $2.1 \cdot \kappa_{FP}$. The error bars correspond to the fitting error of the Lorentzian curves.
4.3 Right barrier of 20 periods

The last sample had a right barrier of 20 periods. The quality factor and loss rate that were estimated for the uncoupled target mode are $Q_N = (13.6 \pm 0.3) \cdot 10^3$ and $\kappa_N = (4.5 \pm 0.1) \cdot 10^{10} \text{ s}^{-1}$. The quality factor and loss rate that were estimated for the uncoupled FP mode are $Q_{FP} = (3.0 \pm 0.5) \cdot 10^3$ and $\kappa_{FP} = (2.0 \pm 0.3) \cdot 10^{11} \text{ s}^{-1}$. The coupling rate between them was found to be $\eta = (0.8 \pm 0.2) \cdot 10^{11} \text{ s}^{-1} \approx (0.4 \pm 0.1) \cdot \kappa_{FP}$, which indicates weak coupling. The loss rate of this sample is larger than the previous two, which is strange since the barrier is actually larger; therefore it should have a better confinement. This sample was fabricated at the same time and on the same wafer as the other two samples. The only difference is that this sample was on another piece of the wafer, after it was cut into pieces. The structures of the samples should be exactly the same, apart from the right barrier lengths.

Figure 4.3 shows the results of the measurements on this sample. Figure 4.3(a) shows the measured spectra for different laser powers. As can be seen the shift of the target mode is almost linear. In figure 4.3(b) the peak wavelengths of the coupled modes are plotted as function of the detuning of the target mode. The target mode in this figure clearly crosses the FP mode, therefore these modes are in weak coupling.

Figure 4.3(c) shows the quality factor of both the coupled modes as function of the detuning of the target mode. As can be seen in this figure the measured points of the FP mode don’t match the simulation. The reason for this is that the FP modes have a really small and broad peak; therefore the quality factor obtained from a fit will have a large error. A second reason is that the two modes are not distinguishable near the resonant point. The only way to obtain their characteristics is to do a double Lorentzian fit on the single visible peak. The simulation matches the calculated Q factors quite well, apart from the leftmost point. At this point the spectrum is noisy, because the laser is close to its threshold. The error for this point should therefore be larger. Also the values for the detuning have a certain error, as previously discussed.

The Q change of the simulation for this sample is quite large. The measurements of the Q factor are not so reliable; therefore the current information on the sample is not enough to say if it is suitable for Q-switching.
Figure 4.3: One beam measurement on a sample with a FP barrier of 20 periods. (a) The measured spectra for this sample. (b) The wavelengths of the modes obtained from the spectra. (c) The calculated $Q$ factors for both modes. The dots show the results of the measurements and the solid curves show the coupled mode theory simulations for this sample. The black points correspond to the target mode and the red points to the FP mode. The coupling rate between the two modes is estimated to be $0.4 \cdot \kappa_{FP}$. The error bars correspond to the fitting error of the Lorentzian curves.
5 Conclusion

The simulations of the coupled mode theory largely match with the measurements on actual coupled cavities. The coupled mode theory described in this report is therefore useful for predicting the characteristics of a real coupled cavity system. Two cases of strong coupling and one case of weak coupling have been used for the investigation of the coupled mode theory.

The coupled mode theory predicts that the largest Q-switching is achieved near the transition region from weak to strong coupling. This region starts at the theoretical border between weak and strong coupling with a coupling rate of \( \eta = |\kappa_N - \kappa_{FP}|/2 \). The region ends at the coupling rate where the two modes are separated more than the linewidth of the resonating modes. The separation of the modes will not be observable in the transition region, because the separation is smaller than the linewidths of the modes.

In the experiments an increase in the Q-factor of a target mode of around 80\% has been found when the target mode was shifted out of resonance with the FP mode. This change could also be predicted with the coupled mode theory.

5.1 Recommendations

The boundary region between weak and strong coupling should be investigated, since the theory predicts the largest Q change in this region.

The coupled mode theory should be modified to support a transformation of the Hamiltonian to the modes of a coupled cavity system. This way the Purcell factor could also be estimated from the theory.
6 Literature


Appendix A: Target cavity coupled with FP cavity

In this chapter we will derive the eigenvalues and eigenstates of a system that consists of a target cavity coupled with a FP cavity. The Hamiltonian that describes this system is given by:

$$H = \hbar \begin{bmatrix} \tilde{\omega}_N & \eta \\ \eta & \tilde{\omega}_{FP} \end{bmatrix}$$ \hspace{1cm} (A.1)

where $\tilde{\omega}_N = \omega_N + i\kappa_N$ and $\tilde{\omega}_{FP} = \omega_{FP} + i\kappa_{FP}$ are the complex frequencies of respectively the target and FP mode. The eigenvalues of this Hamiltonian are obtained by solving the eigenvalue problem:

$$\det(H - \hbar \tilde{\omega} I) = (\tilde{\omega}_N - \tilde{\omega})(\tilde{\omega}_{FP} - \tilde{\omega}) - \eta^2 = 0$$

$$\tilde{\omega}_{1,2} = \frac{1}{2} \left( \tilde{\omega}_N + \tilde{\omega}_{FP} \pm \sqrt{(\tilde{\omega}_N - \tilde{\omega}_{FP})^2 + 4\eta^2} \right).$$ \hspace{1cm} (A.2)

Now that the eigenvalues are known, the eigenstates of the system can be derived:

$$H \begin{bmatrix} \alpha \\ \beta \end{bmatrix} = \hbar \tilde{\omega} \begin{bmatrix} \alpha \\ \beta \end{bmatrix}$$

$$\alpha \tilde{\omega}_N + \beta \eta = \alpha \tilde{\omega} \rightarrow \frac{\alpha}{\beta} = \frac{\eta}{\tilde{\omega} - \tilde{\omega}_N}$$

$$\beta \tilde{\omega}_{FP} + \alpha \eta = \beta \tilde{\omega} \rightarrow \frac{\alpha}{\beta} = \frac{\tilde{\omega}_N - \tilde{\omega}_{FP}}{\eta}$$

Because the eigenstates should be normalized ($|\alpha|^2 + |\beta|^2 = 1$), the eigenstates for this system become:

$$\begin{bmatrix} \alpha_1 \\ \beta_1 \end{bmatrix} = \frac{1}{\sqrt{|\eta|^2 + |\tilde{\omega}_1 - \tilde{\omega}_N|^2}} \begin{bmatrix} \eta \\ \tilde{\omega}_1 - \tilde{\omega}_N \end{bmatrix},$$ \hspace{1cm} (A.3)

$$\begin{bmatrix} \alpha_2 \\ \beta_2 \end{bmatrix} = \frac{1}{\sqrt{|\eta|^2 + |\tilde{\omega}_2 - \tilde{\omega}_N|^2}} \begin{bmatrix} \eta \\ \tilde{\omega}_2 - \tilde{\omega}_N \end{bmatrix}.$$ \hspace{1cm} (A.4)

We wish to decompose the matrix of this coupled system (A.2) to its eigenvalues and eigenstates:

$$H' = P^{-1}HP = \hbar \begin{bmatrix} \tilde{\omega}_1 & 0 \\ 0 & \tilde{\omega}_2 \end{bmatrix},$$ \hspace{1cm} (A.5)

where $P$ is a matrix with the eigenstates as columns:

$$P = \begin{bmatrix} \alpha & -\beta \\ \beta & \alpha \end{bmatrix} \rightarrow P^{-1} = \frac{1}{\alpha_1 \beta_2 - \alpha_2 \beta_1} \begin{bmatrix} \beta_2 & -\alpha_2 \\ -\beta_1 & \alpha_1 \end{bmatrix}.$$ 

One should note that the matrix $P^{-1}$ can become singular when $\alpha_1 \beta_2 = \alpha_2 \beta_1$. In this case the matrix $P$ is not invertible, which makes decomposition impossible. If we assume this does not happen then we can solve the matrix $P^{-1}HP$ directly:

$$P^{-1}HP = \frac{1}{\alpha_1 \beta_2 - \alpha_2 \beta_1} \begin{bmatrix} \beta_2 & -\alpha_2 \\ -\beta_1 & \alpha_1 \end{bmatrix} \begin{bmatrix} \tilde{\omega}_1 & \eta \\ \eta & \tilde{\omega}_{FP} \end{bmatrix} \begin{bmatrix} \alpha_1 & \alpha_2 \\ \beta_1 & \beta_2 \end{bmatrix}$$

$$= \frac{1}{\alpha_1 \beta_2 - \alpha_2 \beta_1} \begin{bmatrix} \beta_2 & -\alpha_2 \\ -\beta_1 & \alpha_1 \end{bmatrix} \begin{bmatrix} \alpha_1 \tilde{\omega}_N + \beta_1 \eta & \alpha_2 \tilde{\omega}_N + \beta_2 \eta \\ \beta_1 \tilde{\omega}_{FP} + \alpha_1 \eta & \beta_2 \tilde{\omega}_{FP} + \alpha_2 \eta \end{bmatrix}.$$
This last matrix can be filled into the expression of (B.5) to obtain new expressions for \( \tilde{\omega}_1 \) and \( \tilde{\omega}_2 \):

\[
\begin{align*}
\tilde{\omega}_1 &= \frac{\alpha_1 \beta_2 \tilde{\omega}_N + (\beta_1 \beta_2 - \alpha_1 \alpha_2) \eta - \alpha_2 \beta_1 \tilde{\omega}_{FP}}{\alpha_1 \beta_2 - \alpha_2 \beta_1} \\
\tilde{\omega}_2 &= \frac{-\alpha_2 \beta_1 \tilde{\omega}_N + (\alpha_1 \alpha_2 - \beta_1 \beta_2) \eta + \alpha_1 \beta_2 \tilde{\omega}_{FP}}{\alpha_1 \beta_2 - \alpha_2 \beta_1}
\end{align*}
\]

These expressions show us how the original modes are distributed over the new modes and they also show that there is a conservation of energy:

\[
\hbar \tilde{\omega}_1 + \hbar \tilde{\omega}_2 = \hbar \tilde{\omega}_N + \hbar \tilde{\omega}_{FP}
\]

We now extend the system of (A.1) by embedding a two-level system into the target cavity. It is assumed that the FP mode does not couple with the two-level system; therefore we can write the Hamiltonian of this system in terms of the two cavity subsystem of matrix (A.1):

\[
H = \hbar \begin{bmatrix}
\omega_{QD} + i \kappa_{QD} & g & 0 \\
g & \tilde{\omega}_N & \eta \\
0 & \eta & \tilde{\omega}_{FP}
\end{bmatrix}, \quad (A.6)
\]

where \( \omega_{QD} \) and \( \kappa_{QD} \) are respectively the frequency and loss rate of the two-level system and \( g \) is the coupling rate of the two-level system with the target mode. This Hamiltonian can also be transformed to the eigenvalues of the coupled-cavity system. The transformation matrix is then given by:

\[
P = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \alpha_1 & \alpha_2 \\ 0 & \beta_1 & \beta_2 \end{bmatrix}, \quad P^{-1} = \frac{1}{\alpha_1 \beta_2 - \alpha_2 \beta_1} \begin{bmatrix} \alpha_1 \beta_2 - \alpha_2 \beta_1 & 0 & 0 \\ 0 & \beta_2 & -\alpha_2 \\ 0 & -\beta_1 & \alpha_1 \end{bmatrix}
\]

Then the transformation \( H' = P^{-1}HP \) gives the transformed Hamiltonian:

\[
H' = P^{-1}HP = \begin{bmatrix}
\omega_{QD} + i \kappa_{QD} & \frac{\alpha_1 g}{\alpha_1 \beta_2 - \alpha_2 \beta_1} & \frac{\alpha_2 g}{\alpha_1 \beta_2 - \alpha_2 \beta_1} \\
\frac{\beta_2 g}{\alpha_1 \beta_2 - \alpha_2 \beta_1} & \omega_1 + i \kappa_1 & 0 \\
\frac{-\beta_1 g}{\alpha_1 \beta_2 - \alpha_2 \beta_1} & 0 & \omega_2 + i \kappa_1
\end{bmatrix}
\]

This Hamiltonian is not Hermitian, not even if the imaginary parts on the diagonal are negated. If it was Hermitian, then the off-diagonal terms should

\[
\alpha^* = \frac{\beta_2}{\alpha_1 \beta_2 - \alpha_2 \beta_1} = \left( \frac{\alpha_1}{\alpha_1 \beta_2 - \alpha_2 \beta_1} \right)^*, \quad \beta = \frac{\alpha_2}{\alpha_1 \beta_2 - \alpha_2 \beta_1} = -\left( \frac{\beta_1}{\alpha_1 \beta_2 - \alpha_2 \beta_1} \right)^*
\]

Simulations of this model show that these conditions are not satisfied.
Appendix B: Linewidth separation of two strong coupled modes.

The separation between two resonating modes in strong coupling can be calculated with the eigenfrequencies in eq. A.2:

$$\lambda_1 - \lambda_2 = 2\pi c \left( \frac{1}{\omega_1} - \frac{1}{\omega_2} \right) = 2\pi c \left( \frac{\omega_2 - \omega_1}{\omega_1 \omega_2} \right)$$

The coupling rate at which the two modes are separated by a distance $\Delta \lambda = (\Delta \lambda_N + \Delta \lambda_{FP})/2$ can be calculated by solving $\lambda_1 - \lambda_2 = \Delta \lambda$:

$$\lambda_1 - \lambda_2 = 2\pi c \left( \frac{2\sqrt{\eta^2 - 1/4(\kappa_N - \kappa_{FP})^2}}{\omega_N^2 - \eta^2 - 1/4(\kappa_N - \kappa_{FP})^2} \right) = \Delta \lambda$$

We make a substitution of $u = \sqrt{\eta^2 - 1/4(\kappa_N - \kappa_{FP})^2}$ to simplify this problem.

$$2\pi c \left( \frac{2u}{\omega_N^2 - u^2} \right) = \Delta \lambda$$

$$\Delta \lambda u^2 + \frac{4\pi c}{\Delta \lambda} u = \Delta \lambda \omega_N^2$$

$$\left( u + \frac{2\pi c}{\Delta \lambda} \right)^2 = \omega_N^2 + \frac{4\pi^2 c^2}{\Delta \lambda^2}$$

$$\rightarrow u = -\frac{2\pi c}{\Delta \lambda} \pm \sqrt{\omega_N^2 + \frac{4\pi^2 c^2}{\Delta \lambda^2}}$$

Only the positive root should be taken, since $u$ cannot be negative in the strong coupling regime. Now we fill in $u$ to solve this equation for $\eta$ and we write the resonance frequency $\omega_N$ in terms of the resonance wavelength $\lambda_N$ for convenience:

$$\sqrt{\eta^2 - 1/4(\kappa_N - \kappa_{FP})^2} = -\frac{2\pi c}{\Delta \lambda} \pm \sqrt{\frac{4\pi^2 c^2}{\omega_N^2} + \frac{4\pi^2 c^2}{\Delta \lambda^2}}$$

$$\eta^2 = \frac{(\kappa_N - \kappa_{FP})^2}{4} + \frac{4\pi^2 c^2}{\Delta \lambda^2} \left( -1 + \sqrt{\frac{\Delta \lambda^2}{\lambda_N^2} + 1} \right)^2$$

The coupling rate at which the two peaks are separated by the linewidth $\Delta \lambda$ is thus equal to:

$$\eta = \frac{(\kappa_N - \kappa_{FP})^2}{4} + \frac{4\pi^2 c^2}{\Delta \lambda^2} \left( -1 + \sqrt{\frac{\Delta \lambda^2}{\lambda_N^2} + 1} \right)^2 \tag{B.1}$$

Or in terms of the linewidths of the uncoupled modes:

$$\eta = \frac{(\kappa_N - \kappa_{FP})^2}{4} + \frac{\pi^2 c^2}{(\Delta \lambda_N + \Delta \lambda_{FP})^2} \left( -1 + \sqrt{(\Delta \lambda_N + \Delta \lambda_{FP})^2 / 4\lambda_N^2 + 1} \right)^2$$