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Wien filter as energy analyzer for an ultracold electron source

Kromwijk, J.M.

Award date:
2014

Link to publication
Wien Filter as Energy Analyzer for an Ultracold Electron Source

J.M. KROMWIJK

September 12, 2014 CQT2014–08

Bachelor Thesis

Supervisors:
prof. dr. ir. O.J. LUITEN
dr. ir. E.J.D. VREDENBREGT

Applied Physics:
Coherence and Quantum Technology

Eindhoven University of Technology
Abstract

In this report a Wien filter is designed for use as energy analyzer for electron bunches extracted from an ultracold electron source in the UCP experiment. The electron source produces high-coherence bunches, which have previously been characterized in the transverse direction. In this research, a Wien filter is designed to characterize the energy (velocity) distribution in the longitudinal direction. In chapter 1, the velocity spread of the electrons is estimated to be 0.12% of the average bunch velocity.

A Wien filter is a device with perpendicular electric and magnetic fields, which will deflect the electrons of different velocities. By projecting the produced streak on a detector the energy (velocity) spread can potentially be measured. In chapter 2, an analytic investigation of electron trajectories in the Wien filter is performed, where it is concluded that they follow a periodic trajectory. To obtain the maximum energy analyzing performance the Wien filter must be designed to have a length equal to 1/4 this period.

In chapter 3, a Wien filter is designed and built, taking into consideration the constraints of the existing UCP experiment. The produced electric and magnetic fields are simulated using CST STUDIO. The magnetic field is compared to measurement and agrees remarkably good despite non-ideal testing conditions. A small asymmetry is observed, which may be the result of a defect in the magnetic shielding.

Finally, in chapter 4, the electron trajectories are simulated through the fields of the final design and are compared to the analytic trajectories. The obtained streak size of 2.55 mm is larger than the analytic expected size of 1.92 mm, which may indicate additional lensing effects by the Wien filter.
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Chapter 1

Introduction

1.1 The ultracold electron source

An ultracold electron source based on near-threshold photoionization of laser-cooled atoms has been proposed and since built for the purpose of producing high-coherence, high-charge electron bunches useful for ultrafast electron microscopy [1, 2, 3, 4]. The source consists of Rubidium atoms trapped in a Magnetic Optical Trap (MOT), which are laser cooled to temperatures as low as 230 µK [1, 5]. In the trapped atom cloud, a small volume is ionized in a two-step process by two laser beams, ionizing the atoms only just above the ionization threshold. By applying a static electric field, the electrons are accelerated away from the MOT, to be used in the experiment. In figure 1.1 an illustration of the process is given.

![Illustration of the process](image)

Figure 1.1: Extraction of electron bunches from the Magnetic Optical Trap (MOT). [(a)] Rubidium atoms are trapped using two trapping coils and are cooled by 6 laser beams. [(b)] A small volume undergoes near-threshold ionization in a two-step process. [(c)] Electrons are extracted from the MOT by a static electric field. *Images by W. Engelen* [2]

Extensive experiments characterizing the transverse energy distribution of the electrons extracted from the source have been performed [1, 2]. A similar characterization for the longitudinal spatial and energy distribution has not yet been done. In this report, a possible device for analyzing this energy distribution is designed.

1.1.1 Electron source details

The predominant cause of the longitudinal energy spread is not the excess ionization energy given to the electrons (10 – 100 meV), but instead the finite size of the initial bunch in the electric field. Electrons in the front of the bunch are accelerated to a lower energy compared to the electrons in the back, leading to a spread energies.

The bunch size is determined by the size of the ionization volume. The trapped atoms are first brought to an excited state using an excitation laser making a small angle with the axis and
are then ionized using an ionization laser from the top. The initial longitudinal size $\sigma_{zi}$ of the extracted electron bunch is given by the size of the ionization laser. In previous work its rms size was measured to be $\sigma_{\text{ion}} = 30 \, \mu\text{m}$ \cite{4}.

The acceleration field is produced by a cylindrically symmetric pair of electrodes with a voltage difference of $V_a$, up to 30 kV. These accelerate the extracted electron bunch to an average energy of \cite{1, 6, 7}:

$$U_b = 0.47 f e V_a,$$

where $e$ the elementary charge and $f$ a correction factor deviating from unity when the ionization volume is not exactly in the center of the accelerator.

The electric field strength at the center is $37 \, \text{kV/m}$ per kV of input voltage $V_a$ \cite{1, 6}. The energy spread given to the electron bunch due to its finite size is:

$$\sigma_U = (37 \, \text{m}^{-1}) \times \sigma_{zi} e V_a.$$  \hspace{1cm} (1.2)

Using (1.1) yields an expression in terms of the bunch energy $U_b$:

$$\sigma_U = \left(\frac{37}{0.47} \, \text{m}^{-1}\right) \times \sigma_{zi} U_b / f,$$

Assuming that $f = 1$ and $\sigma_{zi} = 30 \, \mu\text{m}$ this results in an electron energy spread:

$$\sigma_U \approx 0.0024 \, U_b.$$  \hspace{1cm} (1.4)

For a typical acceleration energy of $U_b = 5 \, \text{keV}$ this results in an energy spread of $12 \, \text{eV}$ given to the electron bunch extracted from the MOT.

### 1.1.2 Initial velocity spread

In the non-relativistic case, the energy $U$ of an electron can be related to its velocity $v$ by $U = \frac{1}{2} m_e v^2$, where $m_e$ is the electron mass. The initial spread in electron velocity spread $\sigma_{v_i}$ can then be related to the energy spread $\sigma_U$ by:

$$v_b \pm \sigma_{v_i} = \sqrt{2(U_b \pm \sigma_U) / m_e} = v_b \sqrt{1 \pm \frac{\sigma_U}{U_b}} \simeq v_b \left(1 \pm \frac{1}{2} \frac{\sigma_U}{U_b} + \ldots\right),$$

where $v_b = \sqrt{2U_b / m_e}$ is the average bunch velocity. From this follows that:

$$\frac{\sigma_{v_i}}{v_b} \approx \frac{1}{2} \frac{\sigma_U}{U_b}$$  \hspace{1cm} (1.6)

Inserting the energy spread (1.4) yields the longitudinal velocity spread:

$$\sigma_{v_i} \approx 0.0012 \, v_b.$$  \hspace{1cm} (1.7)
Chapter 2

Theory of the Wien filter

2.1 Ideal Wien filter

A Wien filter is a device with electric and magnetic fields perpendicular to each other and to the particle beam. Charged particles traveling through it will be deflected due to a Lorentz force:

\[ \vec{F}_L = q \left[ \vec{E} + \vec{v} \times \vec{B} \right], \]  

(2.1)

with \( q \) the charge of the particle, \( \vec{v} \) its velocity and \( \vec{E} \) and \( \vec{B} \) the electric and magnetic fields, respectively. In the ideal case the fields are uniform and constant within the device, and zero elsewhere. The electric and magnetic fields are chosen to be oriented along the \( x \)- and \( y \)-axis, with the \( z \)-axis corresponding to the electron beam axis:

\[ \vec{E} = E_0 \hat{x}, \quad \vec{B} = B_0 \hat{y}. \]  

(2.2)

Because the Lorentz force only acts perpendicular to the magnetic field, particles will only experience a deflection in a single plane. In the case of the fields (2.2) the deflection is confined to the \( yz \)-plane, with no component of the Lorentz force along the \( x \)-axis:

\[ \vec{F}_L = q \left[ E_0 \hat{x} + \vec{v} \left( B_0 \hat{y} \right) \right], \]  

(2.3)

where \( \vec{v} = v_x \hat{x} + v_y \hat{y} + v_z \hat{z} \).

For particles initially traveling parallel to the \( z \)-axis with \( \vec{v} = v_z \hat{z} \), zero deflection is observed if:

\[ \vec{v} = v_0 \hat{z} = \frac{E_0}{B_0} \hat{z}. \]  

(2.4)

For the electron bunch to travel straight through the Wien filter, the electric and magnetic fields must be matched to the average bunch velocity, such that \( v_b = v_0 = E_0/B_0 \). Electrons (with \( q = -e \)) traveling with a velocity other than \( v_0 \) will be deflected in the \( +x \) direction for velocities greater than \( v_0 \) and in the \( -x \) direction for velocities less than \( v_0 \). In conventional monochromating Wien filters, a slit is placed directly behind the device, such that only electrons with the velocity \( v_b \) are selected. By omitting the slit, the device can be used as energy analyzer.

2.2 Electron trajectories

By inserting the Lorentz force (2.3), the electron charge \( q = -e \) and the electron mass \( m_e \), the (non-relativistic) equations of motion in the \( xz \)-plane become:

\[ m_e \ddot{x} = -e \left( E_0 - \dot{z} B_0 \right), \]

\[ m_e \ddot{z} = -e \dot{x} B_0, \]  

(2.5)
where the dot notation denotes derivatives with respect to time (i.e. $v_x \equiv \dot{x}$ and $v_z \equiv \dot{z}$). The electrons experience no acceleration in the $y$-direction. The longitudinal velocity of an individual electron can be expressed in terms of the straight-through bunch velocity $v_b$ and some initial velocity difference $\delta v$:

$$\dot{z}(0) = v_b + \delta v_i.$$  

(2.6a)

The transverse velocity in the $x$-direction can be expressed in terms of the angle $x'_i$ by which the particle enters the Wien filter:

$$\dot{x}(0) = \left. \frac{dx}{dt} \right|_{t=0} = \left. \frac{dx}{dz} \frac{dz}{dt} \right|_{t=0} \simeq x'(z=0)v_b = x'_iv_b,$$

(2.6b)

where the prime notation denotes derivatives with respect to $z$.

2.2.1 Co-moving frame of reference

In the frame of reference moving with the bunch at a velocity $\vec{v} = v_b \hat{z} = (E_0/B_0) \hat{z}$, the relative longitudinal motion can be described by a local coordinate $\zeta(t)$:

$$\zeta = z - v_b t,$$  

(2.7a)

$$\dot{\zeta} = \dot{z} - v_b,$$  

(2.7b)

$$\ddot{\zeta} = \ddot{z}.$$  

(2.7c)

In this frame of reference the equations of motion (2.5) reduce to:

$$m_e \ddot{x} = eB_0 \dot{\zeta},$$  

(2.8a)

$$m_e \ddot{\zeta} = -eB_0 \dot{x}.$$  

(2.8b)

These equations are equivalent with an electron traveling (in the bunch frame) in a uniform magnetic field perpendicular to the plane of motion. Their trajectories describe circular cyclotron-like orbits in the co-moving reference frame \[8\]. The angular frequency of the orbit is equal to the cyclotron frequency:

$$\omega = \frac{eB_0}{m_e}.$$  

(2.9)

The electron motion in the bunch frame is illustrated in figure 2.1.

Figure 2.1: Cyclotron motion of the electrons within the bunch.

2.2.2 Trajectory

Solving the equations of motion (2.8) yields the trajectory of the electrons through the Wien filter:

$$x(t) = x_i + \frac{\delta v_i}{\omega} (1 - \cos(\omega t)) + \frac{x'_i v_b}{\omega} \sin(\omega t),$$  

(2.10a)

$$\zeta(t) = \zeta_i + \frac{\delta v_i}{\omega} \sin(\omega t) + \frac{x'_i v_b}{\omega} (\cos(\omega t) - 1),$$  

(2.10b)
where \( x_i \) and \( \zeta_i \) describe the position of the electron within the bunch as it enters the Wien filter at \( t = 0 \). The transverse angle \( x' \) and longitudinal relative velocity \( \delta v \) are given by:

\[
x'(t) \approx \frac{\dot{x_i}}{v_b} = \frac{\delta v_i}{v_b} \sin(\omega t) + x'_i \cos(\omega t), \tag{2.11a}
\]

\[
\delta v(t) = \dot{\zeta_i} = \delta v_i \cos(\omega t) - x'_i v_b \sin(\omega t). \tag{2.11b}
\]

For a complete derivation of (2.10) and (2.11), see appendix A. The radius of the cyclotron motion in the co-moving frame as a result of the relative velocity \( \delta v_i \) can be identified as:

\[
r_{cyc} = \frac{\delta v_i}{\omega} = m_e \delta v_i \varepsilon B_0. \tag{2.12}
\]

### 2.2.3 Use as energy analyzer

The electrons describe a periodic orbit with a single common frequency \( \omega \). After one complete orbit when \( t = 2\pi/\omega \), the entire bunch of electrons returns to its original configuration. In other words, an exact image of the bunch is formed in both the longitudinal and transverse phase-space of the bunch. The distance after the entrance plane at which this image is formed, \( L_{cyc} \), is:

\[
L_{cyc} = \frac{2\pi}{\omega} v_b = \frac{2\pi m_e v_b}{e B_0}. \tag{2.13}
\]

In figure 2.2, the electron trajectories are plotted a distance \( L_{cyc} \) through the Wien filter, for both a bunch with a velocity \( v_z = v_b \) and a bunch with \( v_z < v_b \). After one cyclotron orbit, both bunches converge in an identical (achromatic) image of the original bunch in the entrance plane.

![Figure 2.2: Trajectory of electrons in the Wien filter. An achromatic image is produced in \( z = L_{cyc} \). An inverted, chromatic image is produced in \( z = L_{cyc}/2 \). The dashed lines are for a beam with \( \delta v_i < 0 \). Image by G.H. Curtis and J. Silcox [5]](image)

At half this distance, another image is formed. This image is not exact, it is inverted and contains chromatic aberrations as the electrons are displaced a distance \( 2r \) in the image plane. An energy analyzer can be build by magnifying this image and projecting it onto a detector using electron optics [8]. However, to not degrade the energy analyzing performance, the displacements must be large compared to the beam cross section on the entrance plane. This can partially be achieved by focusing the beam at the entrance, which is ultimately limited by the (transverse) beam emittance. It can also be achieved by increasing the cyclotron radius \( r_{cyc} \), effectively requiring a longer Wien filter or a low-energy beam in order to maintain the exit plane at \( \frac{1}{2} L_{cyc} \).

In this report another approach to constructing an energy analyzer is taken. The exit plane of the Wien filter is located at \( \frac{1}{4} L_{cyc} \), resulting in a maximum angular dispersion. At the detector a streak image is formed, corresponding to the energy spread of the electrons in the bunch.
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2.3 Streak imaging

For a Wien filter of length $L_{\text{Wien}}$, the electrons exit the Wien filter at $t = L_{\text{Wien}}/v_b$. Inserting this into equations (2.10) and (2.11) yields the phase-space configuration of the electrons as they exit the filter:

$$x_1 = x_i + \frac{\delta v_i}{\omega} \left( 1 - \cos(\omega L_{\text{Wien}}/v_b) \right) + \frac{x'_i v_b}{\omega} \sin(\omega L_{\text{Wien}}/v_b),$$  \hspace{0.5cm} (2.14a)

$$x'_1 = \frac{\delta v_i}{v_b} \sin(\omega L_{\text{Wien}}/v_b) + x'_i \cos(\omega L_{\text{Wien}}/v_b),$$  \hspace{0.5cm} (2.14b)

$$\zeta_1 = \zeta_i + \frac{\delta v_i}{\omega} \sin(\omega L_{\text{Wien}}/v_b) + x'_i \frac{v_b}{\omega} \left( \cos(\omega L_{\text{Wien}}/v_b) - 1 \right),$$  \hspace{0.5cm} (2.14c)

$$\delta v_1 = \delta v_i \cos(\omega L_{\text{Wien}}/v_b) - x'_i v_b \sin(\omega L_{\text{Wien}}/v_b).$$  \hspace{0.5cm} (2.14d)

Maximum angular dispersion for the energy analyzer is obtained by maximizing the first term of (2.14b). This requires that:

$$\frac{\omega L_{\text{Wien}}}{v_b} = \frac{eB_0 L_{\text{Wien}}}{v_b m_e} = \frac{\pi}{2},$$  \hspace{0.5cm} (2.15)

from which follows that indeed:

$$L_{\text{Wien}} = \frac{\pi v_b m_e}{2eB_0} = \frac{1}{4} L_{\text{cycl}}.\quad (2.16)$$

2.3.1 Streak size

In the most basic setup the electrons pass through the Wien filter and are allowed to drift until finally reaching the detector. After a drift length $d_{\text{drift}}$, the final position of the electrons on the detector can be written as:

$$x_f = x_i + x'_i d_{\text{drift}} = x_i + x'_i \left[ \frac{v_b}{\omega} \sin(\omega L_{\text{Wien}}/v_b) + d_{\text{drift}} \cos(\omega L_{\text{Wien}}/v_b) \right]
+ \frac{\delta v_i}{v_b} \left[ \frac{v_b}{\omega} \left( 1 - \cos(\omega L_{\text{Wien}}/v_b) \right) + d_{\text{drift}} \sin(\omega L_{\text{Wien}}/v_b) \right].$$  \hspace{0.5cm} (2.17)

By adjusting the magnetic field strength to satisfy (2.16), this simplifies to:

$$x_f = x_i + x'_i v_b \frac{d_{\text{drift}}}{\omega} + \frac{\delta v_i}{v_b} \left( \frac{v_b}{\omega} + d_{\text{drift}} \right).$$  \hspace{0.5cm} (2.18)

Here the distribution of the electron positions (i.e. the image on the detector) depends on the initial transverse $x_i$-$x'_i$ phase-space distribution of the bunch and the initial longitudinal velocity spread $\delta v_i$. The energy analyzing ability of the setup is the result of the last term in (2.18), which results in a streak image. Electrons with a positive initial velocity error $\delta v_i$ will be displaced in the $+x$ direction on the detector. In the UCP setup, a drift length of 80 cm between the Wien filter and detector can be realized. For a velocity spread $\delta v_i/v_b \approx 0.0012$ in both directions, this results in a streak size of approximately 1.92 mm on the detector.

The first two terms in the right-hand side of (2.18) are dependent only on the transverse shape of the initial bunch, resulting in an additional broadening of the streak image. For the best energy analyzing performance, this spot size must be small in comparison to the streak size.
Chapter 3

Design of the Wien filter

3.1 Design considerations

Many approaches to designing a cross-field energy analyzer are possible. The electrostatic field can be generated by two opposite electrodes positioned perpendicular to a static magnetic field. Because of the relatively high electric fields required these electrodes can only be placed inside the vacuum with the rest of the beam line. The strength of the required electric and magnetic field can be estimated from the electron bunch velocity \( v_b \) and the desired length of the device \( L_{\text{wien}} \). Requiring the condition (2.16) and using (2.4), the perpendicular electric and magnetic field strengths in the Wien filter are obtained. In terms of the MOT acceleration energy \( U_b \) with \( v_b = \sqrt{2U_b/m_e} \), these are:

\[
B_0 = \frac{\pi}{eL_{\text{Wien}}} \sqrt{\frac{1}{2}m_e U_b}, \quad E_0 = v_b B_0 = \frac{\pi}{eL_{\text{Wien}}} U_b.
\]  

(3.1)

Inserting a MOT acceleration energy \( U_b = 10 \text{ keV} \) and a filter length of \( L_{\text{wien}} = 40 \text{ mm} \) yields:

\[
B_0 \approx 13 \text{ mT}, \quad E_0 \approx 800 \text{ kV/m}.
\]  

(3.2)

For electrodes spaced 10 mm apart, this corresponds to voltage difference of 8000 V between the electrodes. At a lower acceleration energy of \( U_b = 3 \text{ keV} \) this corresponds to:

\[
B_0 \approx 7 \text{ mT}, \quad E_0 \approx 240 \text{ kV/m},
\]  

(3.3)

corresponding to a voltage of 2400 V between the electrodes.

For the magnetic field, a number of constructions are possible. Although compact, the use of permanent magnets was not desirable for this design. This was due to the limited time available for construction and also the inability to tune the field strength or turning it off completely for nearby sensitive experiments. Ultimately, a pair of electromagnets (solenoids) was chosen to produce the magnetic field. While a magnetic yoke is desirable for producing a uniform and powerful field, it was not possible to include it in this design due to time and size constraints.

3.1.1 Construction

The Wien filter energy analyzer is to be positioned in the current setup, without large modifications to the beam line. For this purpose, the Wien filter will be inserted in an existing CF63 (ConFlat) vacuum cube situated approx. 70 cm after the electron source (the MOT). The cube has a side-length of 114 mm and has a 63 mm diameter bore drilled through it on all three axes. Two of the sides are connected to the beam line tube, the other four are sealed by CF63 flanges. The Wien filter energy analyzer will be built on top of a single CF63 flange, such that it can be inserted easily.

Due to the limited space inside the cube a compact design is required. In this design, both the electrostatic and magneto-static components will be contained inside the vacuum cube. The construction is based on a single aluminium tube mounted perpendicular to the CF63 flange. It
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will house both the solenoids (wound around the tube) and electrodes (suspended inside the tube), see figure 3.3, 3.4 and 3.5.

3.1.2 $\mu$-Metal Magnetic shielding

In order to maximize Wien filter performance, the field profiles of the electric and magnetic field must overlap with minimal differences. This is not possible to achieve perfectly using a simple solenoid field and two capacitor plates for the magnetic and electric field, respectively. An effort is made to minimize fringe fields and external fields by adjusting the dimensions of the capacitor plates.

The electric field of the two capacitor plates will be mostly uniform in between and has a sharp fall-off near the edges if the plates are positioned close enough together. In the final design, a distance of 8 mm is chosen. Outside the Wien filter a small dipole field will exist, but it will be mostly shielded by the grounded aluminium tube and vacuum cube. Using CST STUDIO the electric field profile on the electron beam axis has been simulated. For the electrodes, a length of 36 mm was chosen to match the profile of the magnetic field as closely as possible (figure 3.1 and 3.2).

The magnetic field of the two solenoids will also form a relatively uniform field inside the Wien filter if they are positioned in a so-called Helmholtz configuration, where the radius of the coil is equal to the distance between them. In the final design the radius of the coils is 20 mm.

An approximate analytic field profile on the axis can be derived for such a configuration. The derivation is given in Appendix B, with the result re-stated here:

$$B_y(z) = \frac{\mu_0 n I}{\pi \alpha \beta} \left[ \left(\frac{3}{4} \alpha^2 - z^2\right) E(m) + \alpha^2 K(m) \right],$$

(3.4)

where $K(m)$ and $E(m)$ are the complete elliptic integrals of the first and second kind and the following substitutions were used:

$$\alpha^2 = \frac{5}{4} \alpha^2 + z^2 - 2az, \quad \beta^2 = \frac{5}{4} \alpha^2 + z^2 + 2az, \quad m = 1 - \frac{\alpha^2}{\beta^2}.$$

The approximate field profile is plotted along with the simulated electric field profile in figure 3.1.

Outside the Wien filter the magnetic field will form a dipole-field, but unlike the electric dipole field, it is opposite in sign to the internal field and large compared relatively to the electric field outside. Because this is undesirable, a magnetic shielding made out of $\mu$-Metal is proposed. This material has a very high magnetic permeability and effectively "traps" the magnetic field, shielding the external field from the inside of the Wien filter. Using CST STUDIO a simulation was done using a $\mu$-Metal rolled cylinder around the Wien filter (see figure 3.2). The $\mu$-Metal cylinder has a diameter of 54 mm, is 80 mm long and was rolled from a 0.2 mm plate, as was used in the final design. See figure 3.3, 3.4 and 3.5.

![Figure 3.1: Field profiles without magnetic shielding](image.png)

![Figure 3.2: Field profiles with magnetic shielding](image.png)
3.1.3 Vacuum

Because the vacuum cube is not pumped directly, care must be taken in order to achieve the high vacuum required (approximately $10^{-7}$ mbar). The magnetic shielding can not enclose the inner construction entirely, nor can the tube extend the length of the vacuum cube. The magnetic shield will be made from a rolled cylinder of $\mu$-Metal, which is mounted around the aluminum tube. In the final design, an outer tube diameter of 45 mm and magnetic shielding diameter of 54 mm is chosen, allowing for 4.5 mm of clearance with the tube on the inside and 4.5 mm clearance with the 63 mm diameter bore in the vacuum cube on the outside. This provides a good compromise between maximizing the length of the Wien filter and achieving high vacuum. Additionally, the length of the shielding tube is chosen to be 80 mm, allowing 17 mm clearance with the flange on either side. See figure 3.3, 3.4 and 3.5.

3.2 Final design

The final design of the Wien filter is illustrated in figure 3.3, 3.4 and 3.5. It is based on two coils wound on an aluminium tube mounted on top of a single CF63 flange. This allows the filter to be inserted as a single unit in the vacuum cube.

The aluminium tube consists of a 45 mm diameter cylinder with a 31 mm bore. The bore diameter tapers outwards near the end to allow for clearance between the tube (which is grounded) and the high-voltage feed-through connector welded into the CF63 flange. The tube has a flange such that it can be mounted on the CF63 flange by four screws (screws not illustrated). At a distance of 57 mm from the CF63 flange, in the center of the vacuum cube, two rectangular holes of size $13 \times 23$ mm with rounded corners are drilled on each side of the tube, see figure 3.4.

The two coils are wound from $\varnothing 0.6$ mm insulated copper wire with $n = 54$ turns on each coil. The four wires (not illustrated) are connected to the 8-pin feed-through connector on the CF63 flange.

The high-voltage electrodes are suspended from two “half-moon” PEEK insulator wedges (figure 3.5), joined by a non-conducting Nylon screw to the aluminium tube. The electrodes are separated by a gap of 8 mm, are 8 mm wide (figure 3.4) and have a length of 36 mm, extending partially beyond the inner diameter of the tube and in the rectangular holes (figure 3.5). Both electrodes are connected by insulated copper wire to two high-voltage feed-throughs welded in the CF63 flange.

A 54 mm diameter tube rolled from 0.2 mm $\mu$-Metal plate provides the magnetic shielding. It has a length of 80 mm (figure 3.4) and is held by three spacer screws (figure 3.5) in place around the aluminium tube. Two holes, approximately 10 mm in diameter, are cut on each side for the electrons to pass through.
Figure 3.3: Construction of the Wien filter with wedge-cut. The electrons traverse the filter along the $z$-axis (blue line).
3.2. FINAL DESIGN

Figure 3.4: Front view (along the z-axis) cross-section of the Wien filter. Not illustrated: two high-voltage insulated copper wires connecting the electrodes to the HV connectors and 4 copper wires connecting the coils to pins on the 8-pin connector. The coils and their connecting wires are wound using of ∅0.6 mm enameled copper wire for each coil in a 5 × 5 mm profile.
Figure 3.5: Side view (along the $y$-axis) cross-section of the Wien filter.
3.3 Measured field profiles

After construction, the magnetic field profile of the Wien filter was measured to verify if the μ-Metal magnetic shielding worked as intended in reducing the fringe fields. The measurement was performed using a handheld gauss meter with transverse probe, clamped on a linear translation stage. The Wien filter was clamped in a stationary position with the probe moving along the \( z \)-axis, measuring the field strength in the transverse direction on the \( y \)-axis.

Some systematic errors may exist due to stray fields from the slightly magnetized table and possibly the clamps as well. These errors were estimated to be as large as 0.1 – 0.2 mT in some locations around the experiment. Another factor that may contribute is slight misalignments of the flexible probe along the Wien filter axis.

In figure 3.6, the measured on-axis field strengths are shown together with the analytic expression (3.4) and the CST STUDIO [9] simulated fields. The fields were measured while applying an electric current of \( I = 3 \) A. The analytic profile was calculated using a coil diameter of 40 mm and distance 20 mm.

The field strength at the center of the Wien filter according to the CST STUDIO simulation is \( B_0 = 8.05 \) mT. In the measured profile the maximum field was \( B_0 = (7.87 \pm 0.02) \) mT. In figure 3.6, the simulated profile has been rescaled to compare the profile shape.

The measured field without μ-Metal agrees remarkably well with the analytical result, despite the non-ideal testing conditions. Similarly, the measured field with μ-Metal also agrees very well with the CST STUDIO simulation, although it has a small asymmetry near \( z = -30 \) mm. The
deviation may be the result of reduced shielding capability of the \( \mu \)-Metal material, because of local defects or internal stresses in the material. It may also have been the result of the probe being nearer to the slightly magnetized clamp by which the Wien filter was held, on that side of the translation table.

The reduction in the fringe fields around the Wien filter due to the \( \mu \)-Metal shielding is remarkable good. On the right-hand side of figure 3.6 the reduction is better than 91\% everywhere and on the left-hand side it is still better than 70\%.
Chapter 4

GPT Simulation results

4.1 Effective filter length

The Wien filter was designed with a coil diameter of 40 mm. However, the field profiles (as calculated and measured in section 3.1.2 and 3.3) do not extend the full 40 mm. Because of the non-uniform field strength, the Wien filter length (2.16) must be corrected to take into account that the field strength is not everywhere equal to its value at the center \( B_0 \).

The width of the streak for an ideal bunch emitted from a point-source with no transverse momentum can be approximated from equation (2.17) as:

\[
\Delta x = 2d_{\text{drift}} \frac{\delta v_i}{v_b} \sin \left( \frac{eB_0 L_{\text{Wien}}}{m_e v_b} \right),
\]

(4.1)

where it is assumed that \( d_{\text{drift}} \gg v_b/\omega = \frac{2}{\pi} L_{\text{Wien}} \).

Using the General Particle Tracer [10] simulation package, the expected streak sizes were simulated for both an ideal Wien filter using analytic (“square”) fields and the designed Wien filter using fields simulated in CST STUDIO [9]. This was done for a number of different center field strengths \( B_0 \), with the maximum streak size located at \( B_0 = B_{\text{max}} \). The acceleration energy used in the simulation was \( U_b = 4 \text{ keV} \) for an ideal bunch with an energy spread of \( \delta v_i/v_b = 0.0012 \) in both directions.

The effective Wien filter length of the design is then found using the equivalent of equation (2.16):

\[
L_{\text{Wien,eff}} = \frac{\pi v_b m_e}{2eB_{\text{max}}},
\]

(4.2)

In figure 4.1, the simulated streak sizes are plotted along with the analytic approximation (4.1). The maximum streak size which matches the designed filter closest occurs at \( B_{\text{max}} = 10.76 \text{ mT} \), indicated by a dotted line in figure 4.1. From equation (4.2) this results in an effective Wien filter length of \( L_{\text{Wien,eff}} \approx 31.13 \text{ mm} \).

The maximum streak size for the designed Wien filter is considerably larger than for the ideal equivalent filter. This may be the result of the non-uniform electric and magnetic fields acting as an additional defocussing lens.
Figure 4.1: Streak sizes $\Delta x$ on the detector as function of the magnetic field in the center $B_0$. The maximum streak size is observed at $B_0 = B_{\text{max}} \approx 10.76 \text{ mT}$ (dotted line). The ideal filter and approximation use an effective length of $L_{\text{Wien,eff}} \approx 31.13 \text{ mm}$.

Figure 4.2: Electron trajectories simulated using GPT. Above the trajectories for an ideal Wien filter with uniform fields, below the designed Wien filter with CST STUDIO simulated fields. The streak size produced by the ideal Wien filter is $1.92 \text{ mm}$, the streak from the simulated fields is $2.55 \text{ mm}$ in size.
4.2 Electron trajectories

Next, the complete electron trajectories through the Wien filter are simulated using GPT, for both an ideal filter (with length $L_{\text{Wien,eff}}$) and the simulated fields of the final design from CST STUDIO. The electrons are released in the MOT at $z = 0$ cm, which is approximated as a single point source. The electrons are given an initial velocity spread of $\frac{\Delta v}{v_b} = 0.0012$ in both directions, at an acceleration energy of $U_b = 4$ keV. For the purpose of this simulation only the longitudinal velocity spread is considered. The electrons are released with zero transverse momentum. The electrons pass through the Wien filter at a distance $z = 70$ cm from the MOT. Finally they are collected by the detector at $z = 150$ mm.

In figure 4.2 the trajectories of the electrons in the $y$-$z$ plane are shown for both the ideal Wien filter and the final design. In both Wien filters, the electrons are given a similar angular dispersion as a result of their energy spread. The trajectories in the Wien filter are very different. In the final design, the electrons are deflected off-axis due to stray fields around the Wien filter. Inside the filter an oscillation is observed corresponding with locations where the electric and magnetic field profiles are not properly matched as in equation (2.4), resulting in a net deflection of the entire bunch (compare with figure 3.2).
Chapter 5

Conclusion

In this report, a Wien filter with the purpose of analyzing the energy spread of electrons extracted from an ultracold electron source has been designed and implemented. First it was determined that in a Wien filter the electrons follow a periodic trajectory. For the best performance as an energy analyzer, the length and magnetic field strength Wien filter had to be matched to obtain a maximal angular dispersion.

A physical design for the Wien filter has been developed in this report, taking into consideration the constraints of the UCP experiment setup and its performance as energy analyzer. For this purpose, magnetic shielding was required in the form of a $\mu$-Metal cylinder. Using CST STUDIO the magnetic and electric fields of the final design have been simulated.

In order to validate the simulated fields, the magnetic field was measured on the axis the electrons travel, with and without shielding. Compared to an analytic expression without magnetic shielding the measurement agreed remarkably well, despite non-ideal testing conditions. When the measurement with magnetic shielding was compared to the CST STUDIO fields a similar agreement was found, although a small deviation might indicate reduced shielding near one of the openings in the $\mu$-Metal.

For the estimated rms energy spread the source of $\sigma_v \approx 0.0012v_b$, the device is expected to produce a streak of 1.92 mm. This was verified by simulation of the electron trajectories through the device, using the solved electric and magnetic fields. Here, an ideal bunch from a point source with only a longitudinal energy spread was used. The streak size produced was 2.55 mm, which may indicate that the Wien filter acts as a lens. Further research is needed to characterize the effects of the Wien filter on bunches with a finite transverse and longitudinal emittance.

Outlook

Currently, the Wien filter has been built following the design outlined in this report and has been installed in the UCP experiment. Due to time constraints it was unfortunately not possible to perform experiments with the Wien filter in operation. Further research is now required to validate the simulated energy analyzing ability of the design and using it to measure the energy spread of the ultracold electron source.
References


Appendix A

Derivation of electron trajectory

The equations of motion in the co-moving reference frame (2.8) can be written as a matrix differential equation:

\[
\begin{bmatrix}
\dot{x} \\
\dot{\zeta}
\end{bmatrix} = \frac{eB_0}{mc} \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} x \\
\zeta
\end{bmatrix}. 
\tag{A.1}
\]

Identifying the cyclotron frequency (2.9), the eigenvalues and corresponding eigenvectors of the matrix A are:

\[
\lambda_1 = i\omega, \quad \lambda_2 = -i\omega, \tag{A.2a}
\]

\[
\vec{u}_1 = \begin{bmatrix} -i \\ 1 \end{bmatrix}, \quad \vec{u}_2 = \begin{bmatrix} i \\ 1 \end{bmatrix}. \tag{A.2b}
\]

The general solutions to (A.1) are given by:

\[
\begin{bmatrix}
\dot{x} \\
\dot{\zeta}
\end{bmatrix} = C_1 e^{\lambda_1 t} \vec{u}_1 + C_2 e^{\lambda_2 t} \vec{u}_2. \tag{A.3}
\]

The unknown coefficients \(C_1\) and \(C_2\) can be written in terms of two new coefficients \(A\) and \(B\):

\[
\begin{align*}
\begin{bmatrix}
\dot{x} \\
\dot{\zeta}
\end{bmatrix} &= \frac{A + iB}{2} e^{i\omega t} \begin{bmatrix} -i \\ 1 \end{bmatrix} + \frac{A - iB}{2} e^{-i\omega t} \begin{bmatrix} i \\ 1 \end{bmatrix} \\
&= \frac{A}{2} \left( e^{i\omega t} \begin{bmatrix} -i \\ 1 \end{bmatrix} + e^{-i\omega t} \begin{bmatrix} i \\ 1 \end{bmatrix} \right) + \frac{iB}{2} \left( e^{i\omega t} \begin{bmatrix} -i \\ 1 \end{bmatrix} - e^{-i\omega t} \begin{bmatrix} i \\ 1 \end{bmatrix} \right) \\
&= \begin{bmatrix}
\frac{A}{2} \left( e^{i\omega t} - e^{-i\omega t} \right) + \frac{B}{2} \left( e^{i\omega t} + e^{-i\omega t} \right) \\
\frac{A}{2} \left( e^{i\omega t} + e^{-i\omega t} \right) - \frac{B}{2i} \left( e^{i\omega t} - e^{-i\omega t} \right)
\end{bmatrix} \\
&= \begin{bmatrix}
A \sin(\omega t) + B \cos(\omega t) \\
A \cos(\omega t) - B \sin(\omega t)
\end{bmatrix}. \tag{A.4}
\end{align*}
\]

Using the initial values (2.6), an expression for the unknown coefficients \(A\) and \(B\) are found:

\[
B = \dot{x}(0) = x_v' v_b, \quad A = \dot{\zeta}(0) = \dot{z}(0) - v_b = \delta v_i. \tag{A.5}
\]

Inserting this into (A.4), the velocities \(\dot{x}\) and \(\dot{\zeta}\) become:

\[
\begin{align*}
\dot{x}(t) &= \delta v_i \sin(\omega t) + x_v' v_b \cos(\omega t), \tag{A.6a} \\
\dot{\zeta}(t) &= \delta v_i \cos(\omega t) - x_v' v_b \sin(\omega t). \tag{A.6b}
\end{align*}
\]
Integrating the above equations with respect to time yields the trajectory of the electron in the co-moving frame:

\[ x(t) = x_i + \int_0^t \dot{x}(\tau) \, d\tau = x_i + \frac{\delta v_i}{\omega} (1 - \cos(\omega t)) + \frac{x'_b v_b}{\omega} \sin(\omega t), \]  
(A.7a)

\[ \zeta(t) = \zeta_i + \int_0^t \dot{\zeta}(\tau) \, d\tau = \zeta_i + \frac{\delta v_i}{\omega} \sin(\omega t) + \frac{x'_b v_b}{\omega} (\cos(\omega t) - 1). \]  
(A.7b)
Appendix B

Magnetic field profile

B.1 Magnetic field of a circular current loop

The magnetic field of a (short) solenoid with \( n \) turns and carrying a current \( I \) can be approximated by the magnetic field of a single circular wire loop carrying a current \( nI \). Such a wire loop is illustrated in figure B.1.

In the cylindrical coordinate system \(( r_1, \varphi_1, z_1 )\) with \( z_1 \) the axis of symmetry of the loop (see figure B.1), the magnetic field components are \( B_{r_1}, B_{\varphi_1} \) and \( B_{z_1} \). Due to symmetry, the field has no tangential component (i.e. \( B_{\varphi_1} = 0 \)) and is only a function of \( r_1 \) and \( z_1 \). It can be shown that the magnetic field of such a current loop can be written in terms of complete elliptical integrals of the first and second kind \([11, 12]\). In the parameter \( m \) the first and second integral are respectively:

\[
K(m) = \int_0^{\pi/2} \frac{1}{\sqrt{1 - m \sin^2 \theta}} d\theta,
\]

\[
E(m) = \int_0^{\pi/2} \sqrt{1 - m \sin^2 \theta} d\theta.
\]

An alternative notation often encountered is the integrals given in the modulus \( k \) or the modular angle \( \alpha \), whereby \( m = k^2 = \sin^2 \alpha \). For a circular current loop carrying a current \( nI \) and having a radius \( a \) the following substitutions are introduced:

\[
\alpha^2 = a^2 + r_1^2 + z_1^2 - 2ar_1, \quad \beta^2 = a^2 + r_1^2 + z_1^2 + 2ar_1, \quad m = 1 - \alpha^2/\beta^2.
\]

The magnetic field components are \([12]\):

\[
B_{r_1, \text{loop}}(r_1, z_1) = \frac{\mu_0 nI}{2\pi a^2 \beta} \frac{z_1}{r_1} \left[ (a^2 + r_1^2 + z_1^2) E(m) - \alpha^2 K(m) \right],
\]

(B.1a)

\[
B_{z_1, \text{loop}}(r_1, z_1) = \frac{\mu_0 nI}{2\pi a^2 \beta} \left[ (a^2 - r_1^2 - z_1^2) E(m) + \alpha^2 K(m) \right].
\]

(B.1b)

Figure B.1: A single circular current loop.

Figure B.2: Two current loops placed in Helmholtz configuration.
B.2 On-axis field profile

In the Wien filter, a magnetic field is produced by placing two solenoid coils in the so-called Helmholtz configuration. The coils are placed a distance equal to their radii $a$ apart on the same axis. This is illustrated in figure B.2, where both coils are approximated as single current loops. The electrons pass on an axis $r_1$ perpendicular to the two coils, see figure B.2.

Note that in equation (B.1) the radial component $B_{r_1}(r_1, z_1)$ is anti-symmetric with respect to $z_1$ as it can be written in the form $z_1 f(z_1^2)$ for some function $f$. In the case of two current loops placed at $z_1 = a/2$ and $z_1 = -a/2$, both carrying the same current, this implies that there is no $B_{r_1}$ component along any axis in the plane $z_1 = 0$. All that remains is the profile of the magnetic field component $B_{z_1}$, which is perpendicular to the electron trajectory.

The $B_{z_1}$ component is only a function of $z_1^2$, and is therefore symmetric with respect to $z_1$. The contribution of both coils is equal, which yields an on-axis profile of twice the magnetic field of a single loop (B.1b) at a distance $z = a/2$:

$$B_{z_1, \text{axis}}(r_1) = 2 B_{z_1, \text{loop}}(r_1, a/2) = \frac{\mu_0 n I}{\pi a^2 \beta} \left[ \left( \frac{3}{4} a^2 - r_1^2 \right) E(m) + \alpha^2 K(m) \right],$$  \hspace{1cm} (B.2)

where $z = a/2$ is used for the substitutions, such that:

$$\alpha^2 = \frac{5}{4} a^2 + r_1^2 - 2a r_1, \quad \beta^2 = \frac{5}{4} a^2 + r_1^2 + 2a r_1, \quad m = 1 - \alpha^2 / \beta^2.$$

Transforming from the local coordinate system $(r_1, \varphi_1, z_1)$ to the Wien filter coordinate system $(x, y, z)$ by using $r_1 \rightarrow z$ and $z_1 \rightarrow y$ yields equation (3.4) used in section 3.1.2.

B.3 Field strength at center

Inserting $r_1 = 0$ into equation (B.2) yields the magnetic field strength at the centerpoint of the Wien filter:

$$B_0 = B_{z_1, \text{axis}}(0) = \frac{\mu_0 n I}{\pi \frac{5}{4} a^2 \sqrt{\frac{5}{3} a^2}} \left[ \frac{3}{4} a^2 E(0) + \frac{5}{4} a^2 K(0) \right] = \left( \frac{8}{5 \sqrt{3}} \right) \frac{\mu_0 n I}{a} \frac{1}{a}.$$  \hspace{1cm} (B.3)

where the identity $E(0) = K(0) = \frac{\pi}{2}$ was used.