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Position Control of a Wheeled Mobile Robot Including Tire Behavior

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Abstract—Advanced driver-assistance systems are increasingly available on road vehicles. These systems require a thorough development procedure, an important part of which consists of hardware-in-the-loop experiments in a controlled environment. To this end, a facility called Vehicle Hardware-In-the-Loop (VeHIL) is operated, aiming at testing the entire road vehicle in an artificial environment. In VeHIL, the test vehicle is placed on a roller bench, whereas other traffic participants, i.e., vehicles in the direct neighborhood of the test vehicle, are simulated using wheeled mobile robots (WMRs). To achieve a high degree of experiment reproducibility, focus is put on the design of an accurate position control system for the robots. Due to the required types of maneuvers, these robots have independently driven and steered wheels. Consequently, the robot is overactuated. Furthermore, since the robot is capable of high-dynamic maneuvers, slip effects caused by the tires can play an important role. A position controller based on feedback linearization is presented, using the so-called multicycle approach, which regards the robot as a set of identical unicycles. As a result, the WMR is position controlled, whereas each unicycle is controlled, taking weight transfer and longitudinal and lateral tire slip into account.

Index Terms—Advanced driver assistance, hardware-in-the-loop simulation, mobile robots, nonlinear control, robotics.

I. INTRODUCTION

ADVANCED driver-assistance systems, such as adaptive cruise control, collision warning, and collision mitigation, have become increasingly available on road vehicles [1]. Recently, research has also been directed toward assistance systems that aim for a common goal, such as increased throughput, safety, and/or fuel efficiency [2], [3]. To support the development of this wide variety of assistance systems, a test facility called Vehicle Hardware-In-the-Loop (VeHIL) was put into operation several years ago [4]. VeHIL enables the hardware-in-the-loop testing of entire road vehicles equipped with advanced driver-assistance functionality based on environment sensors, such as radar, lidar, or vision. The principle of VeHIL is to simulate the relative motion of other vehicles

Fig. 1. One of the wheeled mobile robots used in VeHIL (without vehicle body), which serves as a target vehicle for testing advanced driver assistance systems.

with respect to the test vehicle, allowing for efficient, safe, and reproducible testing. The neighboring vehicles are simulated by wheeled mobile robots (WMRs), one of which is shown in Fig. 1, with the vehicle body removed. This VeHIL WMR or *Moving Base* (MB) differs from most wheeled robots used in the industry in that it is a high-dynamic robot that is capable of extreme maneuvers at velocities of up to 50 km/h in all directions [5]. Consequently, the control system has to be designed such that these high-dynamics specifications are fully exploited, which is the focus of this paper.

The main control objective of the MB is to let its center track a reference trajectory consisting of the position and the orientation in the horizontal plane. This is achieved by four independently driven and steered wheels. As a consequence, the MB has eight actuators—four driving and four steering motors—whereas the control objective comprises 3 degrees of freedom only. The MB can therefore be characterized as being *overactuated*. In [6], a control method based on feedback linearization is presented, which handles the overactuatedness by regarding the MB as four independent unicycles. Although the results of this controller were promising, they can further be improved by taking tire behavior into account in the controller design. Tires introduce slip effects that compromise the position accuracy. A possible approach for counteracting this effect is to incorporate a tire model, which is well known in the field of automotive engineering [7], in a feedback linearization-based controller, as is commonly used in the field of robotics [8], [9].

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TABLE I
MOVING BASE SPECIFICATIONS

Characteristic	Specification
Vehicle mass with/without body	756/694 kg
Wheel base $\times$ track width	1.4 m $\times$ 1.4 m
Center of gravity height	0.40 m
Maximum velocity (in all directions)	50 km/h
Maximum translation acceleration	10 m/s ²
Maximum centripetal acceleration	12 m/s ²
Installed power	52 kW
Steering angle range	$[-350^\circ, 350^\circ]$

Fig. 2. Normalized longitudinal MB tire characteristic and its linear approximation around slip $\kappa = 0$.

II. MOVING BASE CHARACTERISTICS

Table I summarizes the main MB characteristics relevant to the controller design. The high acceleration levels, together with the considerable mass, as mentioned in the table, lead to a significant weight transfer, influencing the actual vertical load of the tires. The friction force that a tire can deliver is, in turn, approximately proportional to its actual vertical load [7]. Consequently, the drive-torque distribution across the four wheels should correspond to the actual vertical loads to obtain the maximum performance of the MB in terms of acceleration and maneuverability.

The tire-friction force also depends on the longitudinal slip κ and the lateral slip angle α , where κ is equal to the normalized velocity difference between the tire and the road, and α is the angle between the wheel plane and the velocity direction. This slip dependency is illustrated in Fig. 2, which shows the longitudinal tire force F_{long} , which is normalized by the vertical load F_z , as a function of the slip κ according to the Magic Formula tire model [7]. The lateral force characteristic is described by a similar function. This characteristic justifies the incorporation of tire slip into the control design. Note that Fig. 2 also shows the linear approximation of the tire characteristic, which is valid for $\kappa \ll 1$. The same type of approximation applies to the lateral characteristic.

III. CONTROL CONCEPT

The control objective of the MB is to let its center track a reference trajectory $\mathbf{s}_{\text{ref}}$, which consists of the position in the x - y plane and the orientation ψ as a function of the time t , i.e.,

$$\mathbf{s}_{\text{ref}}(t) = (x_{\text{ref}}(t) \quad y_{\text{ref}}(t) \quad \psi_{\text{ref}}(t))^T \quad (1)$$

Fig. 3. Moving Base coordinate systems, consisting of the global Cartesian coordinate system $\{G\}$ and the local Cartesian coordinate system $\{L\}$.

where $\mathbf{s}_{\text{ref}}(t)$ is a continuously differentiable feasible trajectory. Equation (1) implies that the MB must be able to move in all directions.

The controller is designed using a similar approach as that applied in [6], being inspired by the idea presented in [10], which is to decentralize the tracking problem. To this end, the reference vector $\mathbf{s}_{\text{ref}}$ is converted to reference positions $x_{\text{ref},i}$ and $y_{\text{ref},i}$, $i = 1, \dots, 4$, for the separate wheels, enumerating them in a clockwise fashion, starting with $i = 1$ for the front left wheel (see Fig. 3, which shows the MB coordinate systems). The reference positions for wheel $i = 1$ thus read

$$\begin{aligned} x_{\text{ref},1} &= x_{\text{ref}} + L_d \cos(\psi_{\text{ref}} + \arctan(W/L)) \\ y_{\text{ref},1} &= y_{\text{ref}} + L_d \sin(\psi_{\text{ref}} + \arctan(W/L)) \end{aligned} \quad (2)$$

where L and W are half the vehicle length and width respectively, and $L_d = \sqrt{L^2 + W^2}$. The reference positions for the wheels $i = 2, 3, 4$ are described with similar equations.

Although (2) uniquely defines the MB position and orientation, reference wheel orientation angles $\psi_{\text{ref},i}$ are also needed, because the MB should be able to move in all directions, i.e., the MB must be fully controllable. These angles are kinematically calculated according to

$$\psi_{\text{ref},i} = \arctan\left(\frac{\dot{y}_{\text{ref},i}}{\dot{x}_{\text{ref},i}}\right) \quad (3)$$

where the velocities $\dot{x}_{\text{ref},i}$ and $\dot{y}_{\text{ref},i}$ are determined by differentiation of (2). Note that (3) introduces a necessary condition: The reference wheel velocity must be unequal to zero, i.e.,

$$\sqrt{\dot{x}_{\text{ref},i}^2 + \dot{y}_{\text{ref},i}^2} > 0. \quad (4)$$

The MB is thus regarded as a set of four identical subsystems, which are called *unicycles*, that are single wheels that can be steered and driven. All four unicycles have their specific continuously differentiable reference trajectory

$$\mathbf{s}_{\text{ref},i}(t) = (x_{\text{ref},i}(t) \quad y_{\text{ref},i}(t) \quad \psi_{\text{ref},i}(t))^T \quad (5)$$

corresponding to the reference trajectory of the MB. Consequently, four identical tracking problems effectively remain.

As mentioned in Section II, the mechanical coupling between the unicycles results in weight transfer when accelerating, directly influencing the actual vertical load on each wheel. Recalling that the tire characteristic (see Fig. 2) is approximately proportional to the actual vertical load, each tire will be operated with approximately the same amount of longitudinal slip if the drive-torque distribution equals the vertical-load distribution. This approach effectively yields optimal use of the tires, preventing the situation that some tires are operated far beyond the peak in their characteristic, whereas others are operated with a very low slip at the same time. The drive-torque distribution requirement is met by introducing a fictitious equivalent unicycle mass $\tilde{m}_i$, which is being determined by the nominal mass and the weight transfer at each instant according to Newton's second law:

$$\begin{aligned}\tilde{m}_1 &= \frac{m}{4} - \frac{hm}{4Lg}(\dot{u} - v\dot{\psi}) - \frac{hm}{4Wg}(\dot{v} + u\dot{\psi}) \\ \tilde{m}_2 &= \frac{m}{4} - \frac{hm}{4Lg}(\dot{u} - v\dot{\psi}) + \frac{hm}{4Wg}(\dot{v} + u\dot{\psi}) \\ \tilde{m}_3 &= \frac{m}{4} + \frac{hm}{4Lg}(\dot{u} - v\dot{\psi}) - \frac{hm}{4Wg}(\dot{v} + u\dot{\psi}) \\ \tilde{m}_4 &= \frac{m}{4} + \frac{hm}{4Lg}(\dot{u} - v\dot{\psi}) + \frac{hm}{4Wg}(\dot{v} + u\dot{\psi})\end{aligned}\quad (6)$$

where m is the total MB mass, h is the center-of-gravity height, g is the gravitational constant, and u and v are the velocity components expressed in the local MB coordinate frame $\{L\}$, as shown in Fig. 3. $\dot{u} - v\dot{\psi}$ and $\dot{v} + u\dot{\psi}$ are thus the longitudinal and lateral MB accelerations, respectively. These accelerations affect the actual vertical wheel forces, which, when divided by g , result in the equivalent masses $\tilde{m}_i$ according to (6). The equivalent masses are now considered to be the “inertial masses” of the unicycles. Note that the hyperstaticity of the MB is solved by assuming a perfectly flat floor and a uniformly distributed MB mass, such that the nominal mass of each unicycle is equal to the total MB mass divided by the number of wheels.

It should be mentioned that the mechanical coupling between the unicycles is likely to cause disturbances. It is however assumed that these disturbances are small and rather well damped due to the tire compliance. This assumption is justified by the practical experiments (Section VII).

The next step is to design a position controller for each unicycle. If tire slip is neglected, the robotics theory based on motion constraints [11] could be applied to formulate a unicycle model and subsequently design a feedback linearizing controller [8], [9]. This method, as applied in [6], appears to yield rather acceptable results. One might however expect that taking tire slip into account will improve the characteristics of the controlled unicycles and, consequently, the MB, with respect to accuracy. Along this line of thinking, [12] and [13] provide a solution, based on the fact that tire dynamics are generally significantly faster than WMR dynamics, leading to a so-called *singular perturbation* model. Using this model, the feedback-linearization procedure is essentially straightforward, even though it is mathematically complicated. An explicit slip measurement appears not to be required, which is an advantage of the proposed controller. The resulting controller, however,

Fig. 4. Unicycle model, top view (left) and side view (right).

has a rather complicated structure, providing limited insight. Moreover, the singular perturbation model incorporates the linearized tire characteristics, whereas extension to the nonlinear characteristics (see Fig. 2) is far from straightforward. The next section therefore explores a different approach, based on a unicycle model taken from the field of automotive engineering and feedback linearization in a master-slave structure.

IV. UNICYCLE MODELING AND CONTROL

Before developing a controller for the MB, this section first focuses on the modeling and control design for a unicycle.

A. Modeling

Based on the physical description of a tire, as commonly used in the field of automotive engineering [7], this section will derive a unicycle model, including a linear tire model with first-order dynamics. Note that longitudinal and lateral slip, and the resulting forces are assumed to be independent, i.e., combined slip effects are ignored.

The equations of motion of unicycle $i = 1, \dots, 4$ in the horizontal plane are

$$\tilde{m}_i(\dot{u}_i - v_i\dot{\psi}_i) = F_{\text{long},i} \quad (7a)$$

$$\tilde{m}_i(\dot{v}_i + u_i\dot{\psi}_i) = F_{\text{lat},i} \quad (7b)$$

$$I_s\ddot{\psi}_i = T_{s,i} \quad (7c)$$

with longitudinal velocity u_i , lateral velocity v_i , heading angle ψ_i , longitudinal force $F_{\text{long},i}$, and lateral force $F_{\text{lat},i}$. I_s is the lumped inertia of the steering system, and $T_{s,i}$ is the steer torque. According to (7c), the steering system is modeled as a second-order system without damping. This can be motivated by the fact that the wheels of the MB have center point steering. Moreover, the self-aligning torque [7] is very small, compared with the maximum steering torque. Finally, the friction in the steering system is minimized through careful mechanical design. Note that the subscript i is omitted for those parameters that are identical for all unicycles. See Fig. 4 for a schematic of the unicycle model (7).

After linearization with respect to the longitudinal wheel slip κ_i , the longitudinal force $F_{\text{long},i}$ can be expressed as

$$F_{\text{long},i} = K(F_{z,i})\kappa_i \quad (8)$$

with longitudinal slip stiffness $K(F_{z,i})$ where $F_{z,i}$ is the actual vertical force acting on wheel i . $K(F_{z,i})$ is approximately

proportional to the vertical load $F_{z,i} = \tilde{m}_i g$. Consequently, (8) can be rewritten as

$$F_{\text{long},i} = K_n \tilde{m}_i \kappa_i \quad (9)$$

with normalized longitudinal slip stiffness K_n . Tire relaxation effects are represented by the following first-order differential equation for the longitudinal wheel slip:

$$\sigma_\kappa \dot{\kappa}_i + u_i \kappa_i = R \omega_i - u_i \quad (10)$$

where σ_κ is the longitudinal relaxation length, R is the wheel radius, and ω_i the rotation velocity. Note that, for steady state, (10) yields $\kappa_i = (R \omega_i - u_i)/u_i$, i.e., the normalized velocity difference between the tire and the road. The longitudinal tire model is completed by the dynamics due to the inertia I_d of the tire/wheel/drive combination according to

$$I_d \dot{\omega}_i = T_{d,i} - R F_{\text{long},i} \quad (11)$$

where $T_{d,i}$ is the drive torque.

Similarly, the lateral tire force is approximated by a linear function of the slip angle α_i :

$$F_{\text{lat},i} = -C_n \tilde{m}_i \alpha_i \quad (12)$$

with normalized cornering slip stiffness C_n . Introducing the lateral relaxation length σ_α , α_i is described by

$$\sigma_\alpha \dot{\alpha}_i + u_i \alpha_i = v_i \quad (13)$$

where $\tan \alpha_i$ is approximated by α_i , assuming small slip angles (not to be confused with possible large *steering* angles). For steady state, (13) yields $\alpha_i = v_i/u_i$, which is, indeed, the tangent of the angle of the wheel velocity with respect to the wheel plane.

In summary, the complete unicycle model reads

$$\dot{x}_i = u_i \cos \psi_i - v_i \sin \psi_i \quad (14a)$$

$$\dot{y}_i = K_n \kappa_i + v_i \dot{\psi}_i \quad (14b)$$

$$\dot{\kappa}_i = \frac{R \omega_i - u_i - u_i \kappa_i}{\sigma_\kappa} \quad (14c)$$

$$\dot{\omega}_i = \frac{T_{d,i} - R K_n \tilde{m}_i \kappa_i}{I_d} \quad (14d)$$

$$\dot{y}_i = u_i \sin \psi_i + v_i \cos \psi_i \quad (14e)$$

$$\dot{v}_i = -C_n \alpha_i - u_i \dot{\psi}_i \quad (14f)$$

$$\dot{\alpha}_i = \frac{v_i - u_i \alpha_i}{\sigma_\alpha} \quad (14g)$$

$$\ddot{\psi}_i = \frac{T_{s,i}}{I_s} \quad (14h)$$

where (x_i, y_i) is the position of the center of gravity (see Fig. 4). The ninth-order unicycle model (14) has two external inputs, i.e., the drive torque $T_{d,i}$ and the steer torque $T_{s,i}$.

B. Control Design

The unicycle controller will be based on input–output linearization by time-invariant state feedback [14], with the advantage of this approach being that it (partly) linearizes the system and, at the same time, decouples a multi-input multi-output system. A necessary condition for input–output linearization is that the system must be square. Consequently,

Fig. 5. The virtual control point $V_{\text{cp},i}$, being the unicycle point that is actually position controlled.

two outputs have to be defined. A possible choice for the unicycle output function $\mathbf{z}_{1,i}$ is

$$\mathbf{z}_{1,i} = \begin{pmatrix} z_{1,i,1} \\ z_{1,i,2} \end{pmatrix} = \begin{pmatrix} x_i + l_{\text{cp}} \cos \psi_i \\ y_i + l_{\text{cp}} \sin \psi_i \end{pmatrix} \quad (15)$$

with $l_{\text{cp}} > 0$ being a constant parameter. This choice can be motivated from a physical point of view: Instead of controlling the position and the heading of the center of gravity of the unicycle, as indicated by (5), the position of a virtual control point $V_{\text{cp},i}$ is controlled. This control point is located at distance l_{cp} in front of the center of gravity (see Fig. 5), which guarantees that not only the position (x_i, y_i) , but, in addition, the heading ψ_i converge to their reference value as long as the forward velocity u_i is nonzero and the controlled system is stable. l_{cp} is, in fact, a tuning parameter, primarily influencing the damping of the controlled system.

Input–output linearization is basically performed by differentiating the outputs with respect to time until both inputs “appear” and then inverting the input–output relation. The number of differentiations of output $z_{1,i,k}$, $k = 1, 2$, that are necessary for at least one input to appear is called the *relative degree* r_k . For the unicycle model, r_1 and r_2 are both equal to 2. It appears, however, that only the second input $T_{s,i}$ is then visible in both outputs, which renders the system nonlinearizable by state feedback¹. The solution adopted here is to reduce the model by taking κ_i , instead of $T_{d,i}$, as input, thereby removing (14c) and (14d) from the model (14). Consequently, the model order reduces to $n = 7$. The resulting model can be written as

$$\dot{\mathbf{q}}_i = \mathbf{f}(\mathbf{q}_i) + \mathbf{G}(\mathbf{q}_i) \mathbf{u}_i \quad (16)$$

with state vector $\mathbf{q}_i$ and input vector $\mathbf{u}_i$ according to

$$\mathbf{q}_i = (x_i \ u_i \ y_i \ v_i \ \alpha_i \ \psi_i \ \dot{\psi}_i)^T \quad (17a)$$

$$\mathbf{u}_i = (\kappa_i \ T_{s,i})^T. \quad (17b)$$

The functions $\mathbf{f} : \mathbb{R}^n \rightarrow \mathbb{R}^n$ and $\mathbf{G} : \mathbb{R}^n \rightarrow \mathbb{R}^{n \times l}$, with $l = 2$ being the number of inputs, follow from (14). However, because the real input of the unicycle remains $T_{d,i}$, a slave controller that controls κ_i using $T_{d,i}$ is needed. In

¹Only after four differentiations does the input $T_{d,i}$ appear in the outputs. Consequently, the second time derivative $\ddot{T}_{s,i}$ then also appears in the outputs. Defining $\ddot{T}_{s,i}$ as a new input would provide a solution, which is known as *dynamic extension*. This would however increase the system order, complicating the feedback control design.

the remainder of this section, first, the position controller for the reduced unicycle model is designed, after which the κ controller will be described.

Since $r_k = 2$, $k = 1, 2$, the output vector $\mathbf{z}_{1,i}$ has to be differentiated twice to arrive at the linearized model. To this end, first, the output function (15) is denoted by

$$\mathbf{z}_{1,i} = \mathbf{h}(\mathbf{q}_i) = \begin{pmatrix} h_1(\mathbf{q}_i) \\ h_2(\mathbf{q}_i) \end{pmatrix} \quad (18)$$

with $\mathbf{h} : \mathbb{R}^n \rightarrow \mathbb{R}^l$. Introducing the Lie derivative $L_f h_k(\mathbf{q}_i)$ defined by

$$L_f h_k(\mathbf{q}_i) := \frac{\partial h_k(\mathbf{q}_i)}{\partial \mathbf{q}_i} \mathbf{f}(\mathbf{q}_i) \quad (19)$$

the first derivative $\dot{\mathbf{z}}_{1,i}$ can then be formulated as

$$\begin{aligned} \dot{\mathbf{z}}_{1,i} &= \begin{pmatrix} L_f h_1(\mathbf{q}_i) \\ L_f h_2(\mathbf{q}_i) \end{pmatrix} \\ &= \begin{pmatrix} u_i \cos \psi_i - (v_i + l_{cp} \dot{\psi}_i) \sin \psi_i \\ u_i \sin \psi_i + (v_i + l_{cp} \dot{\psi}_i) \cos \psi_i \end{pmatrix}. \end{aligned} \quad (20)$$

Introducing a state vector $\mathbf{z}_{2,i} = \dot{\mathbf{z}}_{1,i}$, the second derivative is equal to

$$\dot{\mathbf{z}}_{2,i} = \mathbf{b}(\mathbf{q}_i) + \mathbf{H}(\mathbf{q}_i) \mathbf{u}_i \quad (21)$$

with

$$\begin{aligned} \mathbf{b}(\mathbf{q}_i) &= \begin{pmatrix} L_f^2 h_1(\mathbf{q}_i) \\ L_f^2 h_2(\mathbf{q}_i) \end{pmatrix} \\ &= \begin{pmatrix} C_n \alpha_i \sin \psi_i - l_{cp} \dot{\psi}_i^2 \cos \psi_i \\ -C_n \alpha_i \cos \psi_i - l_{cp} \dot{\psi}_i^2 \sin \psi_i \end{pmatrix} \\ \mathbf{H}(\mathbf{q}_i) &= \begin{pmatrix} L_{g_1} L_f h_1(\mathbf{q}_i) & L_{g_2} L_f h_1(\mathbf{q}_i) \\ L_{g_1} L_f h_2(\mathbf{q}_i) & L_{g_2} L_f h_2(\mathbf{q}_i) \end{pmatrix} \\ &= \begin{pmatrix} K_n \cos \psi_i & -\frac{l_{cp}}{I_s} \sin \psi_i \\ K_n \sin \psi_i & \frac{l_{cp}}{I_s} \cos \psi_i \end{pmatrix} \end{aligned} \quad (22)$$

where g_1 and g_2 indicate the first and second columns of the matrix function $\mathbf{G}$ in (16), respectively. This clearly shows that the inputs appear in the differential equation after two differentiations. Note that the determinant $|\mathbf{H}| = K_n l_{cp} / I_s$ must be nonzero, because the inverse $\mathbf{H}^{-1}$ will be applied in the design. Consequently, l_{cp} must be nonzero, which can readily be understood because the wheel orientation would be undefined if $V_{cp,i}$ is located in the wheel center.

The differential equations (20) and (21), in fact, provide a new description of the linearizable part of the reduced unicycle model. The order of this subsystem is equal to $\sum_{k=1}^l r_k = 4$. Since the order of the reduced model is equal to 7, a subsystem of order 3 remains. Denoting the state of this subsystem by $\mathbf{z}_{3,i}$, a possible choice for this state is $\mathbf{z}_{3,i}^T = (v_i \quad \alpha_i \quad \dot{\psi}_i)$, which, after differentiation, results in an expression of the form

$$\dot{\mathbf{z}}_{3,i} = \mathbf{r}(\mathbf{z}_{1,i}, \mathbf{z}_{2,i}, \mathbf{z}_{3,i}, \mathbf{u}_i) \quad (23)$$

with $\mathbf{r} : \mathbb{R}^2 \times \mathbb{R}^2 \times \mathbb{R}^3 \times \mathbb{R}^2 \rightarrow \mathbb{R}^3$ being a nonlinear function of the system input $\mathbf{u}_i$ and the states $\mathbf{z}_{j,i}$, $j = 1, 2, 3$. This expression cannot be linearized using input–output linearization, because the state $\mathbf{z}_{3,i}$ is not “visible” in the output function (15). With (20), (21), and (23), the reduced unicycle model is now rewritten in the so-called *normal form*.

The actual feedback linearization is obtained by choosing the input $\mathbf{u}_i$ according to the following feedback law:

$$\mathbf{u}_i = \mathbf{H}^{-1}(\mathbf{q}_i) (\mathbf{v}_i - \mathbf{b}(\mathbf{q}_i)) \quad (24)$$

which finally results in the unicycle model

$$\dot{\mathbf{z}}_{1,i} = \mathbf{z}_{2,i} \quad (25a)$$

$$\dot{\mathbf{z}}_{2,i} = \mathbf{v}_i \quad (25b)$$

$$\dot{\mathbf{z}}_{3,i} = \tilde{\mathbf{r}}(\mathbf{z}_{1,i}, \mathbf{z}_{2,i}, \mathbf{z}_{3,i}, \mathbf{v}_i) \quad (25c)$$

with new external input $\mathbf{v}_i$. The input $\mathbf{u}_i$ in the nonlinear state equation (25c) is also replaced by the new input $\mathbf{v}_i$, resulting in an adapted nonlinear function $\tilde{\mathbf{r}}$. The model (25) shows that the dynamics of the reduced unicycle model have now been decomposed into a linear decoupled input–output part with states $\mathbf{z}_{1,i}$ and $\mathbf{z}_{2,i}$, and a nonlinear “unobservable” part with state $\mathbf{z}_{3,i}$, which is generally referred to as the *internal dynamics*.

Tracking behavior of the linear input–output dynamics is obtained by a regular proportional–differential (PD) controller with feedforward

$$\mathbf{v}_i = \ddot{\mathbf{z}}_{1,i,\text{ref}} + \mathbf{K}_d (\dot{\mathbf{z}}_{1,i,\text{ref}} - \dot{\mathbf{z}}_{1,i}) + \mathbf{K}_p (\mathbf{z}_{1,i,\text{ref}} - \mathbf{z}_{1,i}) \quad (26)$$

where $\mathbf{K}_p$ and $\mathbf{K}_d$ are diagonal 2×2 matrices containing the proportional and differential gains, respectively. To obtain equal dynamic behavior in the longitudinal and lateral directions, the elements of $\mathbf{K}_p$ and $\mathbf{K}_d$ that correspond to the longitudinal direction are chosen equal to those relating to the lateral direction. The desired output $\mathbf{z}_{1,i,\text{ref}}$ is calculated by substituting the reference trajectories (2) and (3) into (15). The resulting expression can subsequently be differentiated to obtain $\dot{\mathbf{z}}_{1,i,\text{ref}}$ and $\ddot{\mathbf{z}}_{1,i,\text{ref}}$.

The controller (26) stabilizes the input–output dynamics. To prevent undesirable phenomena, the internal dynamics, however, must also be stable or, in other words, the system should be minimum phase in the nonlinear sense. An example of such a phenomenon is the lateral oscillation of the unicycle wheel while the control point $V_{cp,i}$ “perfectly” tracks the reference trajectory. Due to the nonlinearity, the stability has to be checked for each reference trajectory. In case the unicycle is driving along a straight line with a constant forward velocity, the internal dynamics appear to be asymptotically stable for $u_i > 0$ and $l_{cp} > \sigma_\alpha = 0.22$ m. Remarkably, the same stability requirement is also found regarding the effect of landing-gear shimmy for aircraft [15]. The requirement $u_i > 0$ can easily be understood from a physical point of view: When the unicycle is standing still, the heading angle ψ_i does not converge to the reference value. It should be noted that the stability proof is not pursued for a large number of trajectory types. In practice, however, the controlled MB appears to be stable, regardless of the specific trajectory, as illustrated in Section VII.

Finally, the slip controller is designed. The dynamics between the drive torque $T_{d,i}$ and the longitudinal slip κ_i are described by (14c) and (14d), where the forward velocity u_i is regarded as a relatively slowly varying parameter, which is indicated by the time argument t in the remainder of this section. Note that the longitudinal tire dynamics are thus assumed to be significantly faster than the MB dynamics,

which is the same fundamental idea as that used in [13]. For this subsystem, it is possible to again apply input–output linearization using the exact same procedure as previously described. To this end, the model (14c) and (14d) is written as

$$\dot{\mathbf{q}}_{\kappa,i} = \mathbf{f}_{\kappa}(\mathbf{q}_{\kappa,i}) + \mathbf{g}_{\kappa}(\mathbf{q}_{\kappa,i})u_{\kappa,i} \quad (27)$$

with vector functions $\mathbf{f}_{\kappa} : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ and $\mathbf{g}_{\kappa} : \mathbb{R}^2 \rightarrow \mathbb{R}^2$. The states $\mathbf{q}_{\kappa,i}$ and input $u_{\kappa,i}$ are defined as

$$\begin{aligned} \mathbf{q}_{\kappa,i} &= (\kappa_i \quad \omega_i)^T \\ u_{\kappa,i} &= T_{d,i}. \end{aligned} \quad (28)$$

Choosing the controlled output $z_{\kappa,1,i}$ according to

$$z_{\kappa,1,i} = h_{\kappa}(\mathbf{q}_{\kappa,i}) = \kappa_i \quad (29)$$

with $h_{\kappa} : \mathbb{R}^2 \rightarrow \mathbb{R}$ being the output function, now leads to the second-order linear system

$$\begin{aligned} \dot{z}_{\kappa,1,i} &= z_{\kappa,2,i} \\ \dot{z}_{\kappa,2,i} &= \nu_{\kappa,i} \end{aligned} \quad (30)$$

with states $z_{\kappa,1,i}$ and $z_{\kappa,2,i}$, and new input $\nu_{\kappa,i}$. Note that there are no internal dynamics because the relative degree is equal to the order of the system. This system can be controlled using the following regular PD controller:

$$\nu_{\kappa,i} = K_{d,\kappa}(\dot{z}_{\kappa,1,i,\text{ref}} - \dot{z}_{\kappa,1,i}) + K_{p,\kappa}(z_{\kappa,1,i,\text{ref}} - z_{\kappa,1,i}) \quad (31)$$

with differential gain $K_{d,\kappa}$ and proportional gain $K_{p,\kappa}$. $z_{\kappa,1,i,\text{ref}} = \kappa_{i,\text{ref}}$ is the longitudinal slip reference generated by the linearizing feedback law (24) of the “master” position controller. The first element of u_i in (17b) is thus regarded as the desired slip $\kappa_{i,\text{ref}}$ rather than the actual slip κ_i . Note that (31) does not include a feedforward term that is similar to $\ddot{z}_{1,i,\text{ref}}$ in (26), because double differentiation of the position controller output is not considered feasible.

Referring to the equivalent unicycle masses $\tilde{m}_i$ defined by (6), it appears that these are explicitly contained in the slip controller. This can be shown by calculating the linearizing feedback law, which is similar to (24), resulting in

$$\begin{aligned} u_{\kappa,i} &= L_{g_{\kappa}} L_{f_{\kappa}} h_{\kappa}(\mathbf{q}_{\kappa,i})^{-1} (\nu_{\kappa,i} - L_{f_{\kappa}}^2 h_{\kappa}(\mathbf{q}_{\kappa,i})) \\ &= \frac{\sigma_{\kappa} I_{\omega}}{R} \left\{ \nu_{\kappa,i} + u_i(t) \frac{R\omega_i - u_i(t) - u_i(t)\kappa_i}{\sigma_{\kappa}^2} \right\} \\ &\quad + RK_n \tilde{m}_i \kappa_i. \end{aligned} \quad (32)$$

Consequently, the slip controller (32) actually implements the drive torque distribution requirement, as stated in Section III.

In summary, the unicycle controller developed in this section consists of a slave slip controller and a master position controller, both of which are designed using input–output linearization involving time-invariant state feedback. Fig. 6 shows the resulting block scheme, providing an overview of the controller structure.

V. UNICYCLE OBSERVERS

The feedback linearizing controller requires all unicycle states to be available. To this end, a linear observer is described that estimates the position and the velocity. Next, two additional nonlinear observers are described that estimate the lateral and longitudinal slip.

Fig. 6. Block scheme of the controlled unicycle.

A. Motion Observer

Some MB sensors, among them an accelerometer and a gyroscope, are not repeatedly installed for each separate unicycle because of cost considerations. Instead, a single sensor is mounted on the MB frame. For this reason, the motion estimation, i.e., position and velocity, is implemented for the MB as a whole, instead of using a separate observer for each unicycle. Based on the estimated MB motion, the unicycle motion is then kinematically calculated.

The motion observer utilizes the following MB model:

$$\begin{aligned} \dot{\mathbf{x}} &= \mathbf{A}_m \mathbf{x} + \mathbf{B}_m \mathbf{u} + \mathbf{v} \\ \mathbf{y} &= \mathbf{C}_m \mathbf{x} + \mathbf{w} \end{aligned} \quad (33)$$

with states $\mathbf{x} = (x \ y \ \dot{x} \ \dot{y})^T$ where (x, y) and $(\dot{x}, \dot{y})$ are the positions and velocities of the MB center in the global coordinate system $\{G\}$ (see Fig. 3). The inputs are the measured accelerations a_x and a_y in x - and y -direction respectively, i.e., $\mathbf{u} = (a_x \ a_y)^T$, and the outputs $\mathbf{y}$ are equal to the states. From these definitions, the matrices $\mathbf{A}_m$, $\mathbf{B}_m$, and $\mathbf{C}_m$ directly follow. Finally, $\mathbf{v}$ is the process noise and $\mathbf{w}$ the output noise, both assumed to be Gaussian white noise with zero mean, essentially representing model and measurement uncertainties.

Based on the model (33), a common Kalman filter is formulated, which is described by

$$\begin{aligned} \dot{\hat{\mathbf{x}}} &= \mathbf{A}_m \hat{\mathbf{x}} + \mathbf{B}_m \mathbf{u} + \mathbf{L}_m (\mathbf{y} - \hat{\mathbf{y}}) \\ \hat{\mathbf{y}} &= \mathbf{C}_m \hat{\mathbf{x}} \end{aligned} \quad (34)$$

where $\hat{\mathbf{x}}$ and $\hat{\mathbf{y}}$ are the estimated states and outputs, respectively; $\mathbf{u}$ contains the measured input accelerations, and $\mathbf{y}$ contains the measured position and velocity. The Kalman gain matrix $\mathbf{L}_m$ is calculated to minimize the covariance of the error $\mathbf{e}_m = \mathbf{x} - \hat{\mathbf{x}}$, the result of which depends on (and can be tuned with) the noise covariance $\mathbf{R}_v$ of $\mathbf{v}$ and $\mathbf{R}_w$ of $\mathbf{w}$. Note that $\mathbf{L}_m$ guarantees stable error dynamics

$$\dot{\mathbf{e}}_m = (\mathbf{A}_m - \mathbf{L}_m \mathbf{C}_m) \mathbf{e}_m. \quad (35)$$

The observer (34) relies on measurements of the acceleration, velocity, and position. None of these is, however, directly measured. For example, the accelerations a_{long} and a_{lat} with respect to the local MB coordinate system $\{L\}$ are measured; the accelerations a_x and a_y , which are defined with respect to the global coordinate system $\{G\}$, are not. It is however easily possible to convert a_{long} and a_{lat} to the global

accelerations using the measured yaw angle from the onboard gyroscope. Also the velocity $(\dot{x}, \dot{y})$ is not directly measured. Instead, the MB velocity is kinematically calculated using the measured wheel velocities and steering angles, i.e., an odometric calculation. Finally, the position (x, y) is measured using magnets mounted in the road surface in a regularly spaced grid. These magnets are detected by linear transducers (magnetostrictive wave guides) mounted on each side of the MB. Because the transducers only measure the position of a magnet along the transducer, some additional calculation steps are required—involving a look-up procedure to determine the position of the magnet being detected—to calculate the MB center position. In addition to these preprocessing steps, there is also some postprocessing needed: From the estimated state $\hat{x}$, the velocity (u_i, v_i) and the position (x_i, y_i) of the unicycle, being part of the unicycle states (17a), are kinematically calculated.

The aforementioned (nonlinear) pre- and postprocessing steps have deliberately been excluded from the model equations, resulting in a simple linear observer. This approach, however, appears to be only possible if the longitudinal and lateral slip are excluded from the state estimation; this is the main reason to design separate observers for the tire slip.

B. Slip Observers

To estimate the lateral and longitudinal wheel slip α_i and κ_i , respectively, two reduced-order observers are designed, using the same technique. To this end, the approach used to design an observer with linear error dynamics [16] is described first.

Consider a nonlinear single-input single-output system with input u , output y , and states $\mathbf{q}$ that is generally formulated as

$$\begin{aligned}\dot{\mathbf{q}} &= \mathbf{f}(\mathbf{q}, u) \\ y &= h(\mathbf{q})\end{aligned}\quad (36)$$

with $\mathbf{f} : \mathbb{R}^n \times \mathbb{R} \rightarrow \mathbb{R}^n$ and $h : \mathbb{R}^n \rightarrow \mathbb{R}$ where n is the number of states. If (36) can be written in the *observer form*

$$\dot{\mathbf{z}} = \mathbf{A}\mathbf{z} + \mathbf{p}(\mathbf{C}\mathbf{z}, u) \quad (37a)$$

$$y = \mathbf{C}\mathbf{z} \quad (37b)$$

with $\mathbf{z}$ being the (possibly) redefined states, $\mathbf{A}$ and $\mathbf{C}$ being the state and output matrices, respectively, and $\mathbf{p} : \mathbb{R} \times \mathbb{R} \rightarrow \mathbb{R}^n$ being a nonlinear function of the input and the output, then the following observer can be formulated:

$$\begin{aligned}\dot{\hat{\mathbf{z}}} &= \mathbf{A}\hat{\mathbf{z}} + \mathbf{p}(y, u) + \mathbf{L}(y - \hat{y}) \\ \hat{y} &= \mathbf{C}\hat{\mathbf{z}}\end{aligned}\quad (38)$$

where $\hat{\mathbf{z}}$ is the estimated state, $\hat{y}$ is the estimated output, and $\mathbf{L}$ is the $n \times 1$ observer gain. Introducing the observer error $\mathbf{e} = \mathbf{z} - \hat{\mathbf{z}}$, the resulting error dynamics appear to be linear due to cancelation of the nonlinear term $\mathbf{p}(y, u)$, i.e.,

$$\dot{\mathbf{e}} = (\mathbf{A} - \mathbf{L}\mathbf{C})\mathbf{e}. \quad (39)$$

Note the similarity with the error dynamics (35) of the linear motion observer. Furthermore, if the pair $(\mathbf{A}, \mathbf{C})$ is observable, the eigenvalues of $\mathbf{A} - \mathbf{L}\mathbf{C}$ can be placed at any desired location (within physical limits obviously), by appropriately

choosing $\mathbf{L}$. Also note that the linearity and, consequently, the stability of the error dynamics (39) are subject to a sufficient level of robustness with respect to model uncertainties particularly since $\mathbf{p}(\mathbf{C}\mathbf{z}, u)$ typically cannot exactly be known.

Based on the aforementioned approach, a longitudinal slip observer with linear error dynamics is designed. The objective of this observer is to estimate the longitudinal unicycle slip κ_i using the available states from the motion observer, as described in Section V-A, and some direct measurements. To this end, (14c) and (14d), describing part of the unicycle, can be applied. Taking $u = T_{d,i}$ as input and $y = \omega_i$ as output, and choosing the state vector according to $\mathbf{z} = (\kappa_i \ \omega_i)^\top$, (14c) and (14d) can be rewritten in the observer form (37) with

$$\mathbf{A}_{\kappa,i} = \begin{pmatrix} -\frac{u_i(t)}{R\sigma_\kappa} & \frac{R}{\sigma_\kappa} \\ -\frac{\sigma_\kappa}{T_{d,i}} & 0 \end{pmatrix} \quad (40a)$$

$$\mathbf{p}_{\kappa,i}(y, u) = \begin{pmatrix} -\frac{u_i(t)}{T_{d,i}} \\ \frac{\sigma_\kappa}{T_{d,i}} \end{pmatrix} \quad (40b)$$

$$\mathbf{C}_\kappa = \begin{pmatrix} 0 & 1 \end{pmatrix}. \quad (40c)$$

The system matrix $\mathbf{A}$ in (37a) is substituted by $\mathbf{A}_{\kappa,i}$, and the nonlinear function $\mathbf{p}$ is substituted by $\mathbf{p}_{\kappa,i}$, where the subscript i indicates the specific unicycle. Similarly, the output matrix $\mathbf{C}$ in (37b) is substituted by $\mathbf{C}_\kappa$.

The longitudinal velocity $u_i(t)$, appearing in (40a) and (40b), is estimated by the motion observer, whereas the wheel speed ω_i and the drive torque $T_{d,i}$ are directly measured. Similar to the design of the longitudinal slip controller in Section IV-B, $u_i(t)$ is again regarded as a relatively slowly time-varying parameter, as indicated by the time argument t . As a consequence, however, both $\mathbf{A}_{\kappa,i}$ and $\mathbf{p}_{\kappa,i}$ are now time dependent.

The observability of this system can be checked by calculating the observability matrix $\mathcal{O}_\kappa$, i.e.,

$$\mathcal{O}_\kappa = \begin{pmatrix} \mathbf{C}_\kappa \\ \mathbf{C}_\kappa \mathbf{A}_{\kappa,i} \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -\frac{R\sigma_\kappa}{T_{d,i}} & 0 \end{pmatrix}. \quad (41)$$

Apparently, $\mathcal{O}_\kappa$ has full rank, yielding the system fully observable. Denoting the observer gain vector as $\mathbf{L} = (l_{\kappa,1} \ l_{\kappa,2})^\top$, the observer error dynamics (39) are finally described by

$$\begin{pmatrix} \dot{e}_{\kappa,1,i} \\ \dot{e}_{\kappa,2,i} \end{pmatrix} = \begin{pmatrix} -\frac{u_i(t)}{R\sigma_\kappa} & \frac{R}{\sigma_\kappa} - l_{\kappa,1} \\ -\frac{\sigma_\kappa}{T_{d,i}} & -l_{\kappa,2} \end{pmatrix} \begin{pmatrix} e_{\kappa,1,i} \\ e_{\kappa,2,i} \end{pmatrix} \quad (42)$$

with $e_{\kappa,1,i} = \kappa_i - \hat{\kappa}_i$ and $e_{\kappa,2,i} = \omega_i - \hat{\omega}_i$. Using Lyapunov's direct method, it can be shown that the error dynamics are actually asymptotically stable, even for a varying velocity, under the following conditions:

$$\begin{aligned}l_{\kappa,1} - R/\sigma_\kappa &< 0 \\ l_{\kappa,2} &> 0 \\ u_i(t) &> 0.\end{aligned}\quad (43)$$

The observer for the lateral slip α_i is designed in a similar way, now based on (14f) and (14g). With input $u = \dot{\psi}_i$, output $y = v_i$, and states $\mathbf{z} = (v_i \ \alpha_i)^\top$, (14f) and (14g) can be

rewritten in the observer form (37) with

$$\begin{aligned} \mathbf{A}_{\alpha,i} &= \begin{pmatrix} 0 & -C_n \\ \frac{1}{\sigma_\alpha} & -\frac{u_i(t)}{\sigma_\alpha} \end{pmatrix} \\ \mathbf{p}_{\alpha,i}(y, u) &= \begin{pmatrix} -u_i(t)\dot{\psi}_i \\ 0 \end{pmatrix} \\ \mathbf{C}_\alpha &= (1 \quad 0) \end{aligned} \quad (44)$$

substituting $\mathbf{A}$, $\mathbf{p}(y, u)$, and $\mathbf{C}$ in (37) by $\mathbf{A}_{\alpha,i}$, $\mathbf{p}_{\alpha,i}(y, u)$, and $\mathbf{C}_\alpha$, respectively. The longitudinal velocity u_i and the lateral velocity v_i are estimated by the motion observer, whereas the unicycle yaw rate $\dot{\psi}_i$ is determined by adding the measured MB yaw rate and the measured steering velocity. u_i is again regarded as a relatively slowly time-varying parameter, as indicated by the time argument t , allowing for the specific observer form. Similar to the longitudinal slip observer, the observability matrix $\mathcal{O}_\alpha$ appears to have full rank as well. Using Lyapunov's direct method, the error dynamics can be proven asymptotically stable if

$$\begin{aligned} l_{\alpha,1} &> 0 \\ l_{\alpha,2} - 1/\sigma_\alpha &< 0 \\ u_i(t) &> 0 \end{aligned} \quad (45)$$

where $\mathbf{L} = (l_{\alpha,1} \quad l_{\alpha,2})^T$ is the observer gain.

Summarizing this section, a motion observer for the MB position and slip observers for the longitudinal and lateral wheel slip have been developed. Referring to the block scheme of Fig. 6, these observers should be added after the unicycle block (14), resulting in the estimated state vectors $\hat{\mathbf{q}}_i$ and $\hat{\mathbf{q}}_{\kappa,i}$.

VI. MULTICYCLE CONTROL DESIGN

Having designed a unicycle controller, the MB controller can now be established. As explained in Section III, the MB controller consists of multiple identical unicycle controllers. It is therefore called the *multicycle* controller. One small adaptation, however, must be made, because the wheels have to be steered relative to the MB body only.

The unicycle steering torque $T_{s,i}$ is equal to $I_s\ddot{\psi}_i$ for an ideal (frictionless) unicycle. Because, however, the MB-body also rotates around its vertical axis with angular acceleration $\ddot{\psi}$, the net required steering torque $\tilde{T}_{s,i}$ for the multicycle is

$$\tilde{T}_{s,i} = I_s\ddot{\delta}_i = I_s(\ddot{\psi}_i - \ddot{\psi}) = T_{s,i} - I_s\ddot{\psi} \quad (46)$$

where δ_i is the steering angle, i.e., the wheel orientation with respect to the MB body. As a consequence, a compensation term $-I_s\ddot{\psi}$ has to be added to $T_{s,i}$ to obtain the multicycle steering torque. The MB controller is now fully determined.

VII. EXPERIMENTS

After having tuned the designed controller on a simulation level, the final step in the design entails testing the controller in reality. To this end, first, the observer performance is assessed using the position-controlled MB. For this test, a position controller is applied, which differs from that designed here in that it does not incorporate tire behavior and therefore does not need the slip to be estimated. The slip observers are implemented “in parallel”, i.e., not in the closed control loop,

Fig. 7. Reference position trajectory in the x - y plane for the observer experiment.

which allows the slip observers to be evaluated independently of the position controller. The motion observer, however, is actively used in the position feedback control, because this is the only means of reliable position and velocity measurement.

A circular track is applied as desired trajectory, as shown in Fig. 7. The desired velocity tangential to the track is increased from 0 to 30 km/h with a maximum acceleration of 7 m/s^2 , then kept constant, and finally decreased to zero again with an acceleration of -7 m/s^2 . The track radius is such that the centripetal acceleration² is equal to 7 m/s^2 at maximum velocity. The desired MB orientation is directed tangentially to the track. This test trajectory is applied to a simulation model of the controlled MB and to the real MB. The simulation model comprises a comprehensive physical model of the MB, which is implemented together with the controller in MATLAB/Simulink.

Fig. 8 shows the results of both the simulation and the practical experiment, displaying the longitudinal velocity of the MB center and the longitudinal slip, as well as the lateral slip of the front right tire. Fig. 8 shows the following three types of signals:

- the *simulated* signals, i.e., the simulated longitudinal velocity u , longitudinal slip κ_2 , and lateral slip α_2 ;
- the *simulated estimated* signals, i.e., the velocity $\hat{u}$ and the slip $\hat{\kappa}_2$ and $\hat{\alpha}_2$, as estimated in the simulation by the motion observer and the slip observers, respectively;
- the *measured estimated* signals, i.e., the observer results $\hat{u}_m$, $\hat{\kappa}_{2,m}$, and $\hat{\alpha}_{2,m}$ from the practical experiment.

First, it can be concluded that the longitudinal MB velocity u is rather accurately estimated in the simulation. Moreover, the estimated velocity $\hat{u}_m$ during the practical experiment cannot be distinguished from the simulated signal $\hat{u}$. Furthermore, the simulated estimate $\hat{\kappa}_2$ of the longitudinal slip appears to be a little smaller (in absolute sense) than the simulated value κ_2 . This is caused by the fact that the tire behavior is described by a linearized characteristic, yielding smaller slip values at the same tire force (see Fig. 2). Nevertheless, the measured longitudinal slip estimate $\hat{\kappa}_{2,m}$ shows a very high correlation with the simulated estimate $\hat{\kappa}_2$. The same observations hold for the lateral slip α_2 , although there is a bigger difference between the simulated signal α_2 and the estimated signals $\hat{\alpha}_2$

²The term “centripetal” acceleration is used, instead of the more commonly used “lateral” acceleration to avoid ambiguity. Because the MB is an all-wheel steered robot, lateral acceleration could also be interpreted as a sideways translational acceleration.

Fig. 8. Observer results: (a) the longitudinal MB velocity u , (b) the longitudinal front right wheel slip κ_2 , and (c) the corresponding lateral slip α_2 . Each figure shows (dotted black) the simulated signals, (grey) the simulated estimated signals, and (solid black) the measured estimated signals.

and $\hat{\alpha}_{2,m}$. This effect is again caused by the linearized tire characteristic. Furthermore, the lateral slip shows a significant higher noise level in the lower frequency region, compared with the longitudinal slip, as clearly shown by the simulated lateral slip. In addition, an increased noise level in the higher frequency region exists, particularly regarding the estimated lateral slip in the practical experiment. The low-frequency noise is actually caused by the motion observer, which updates the estimated position each time that a magnet is encountered. The high-frequency noise is caused by measurement noise of the gyroscope, to which the lateral slip observer is more sensitive than the longitudinal observer, because the former explicitly uses the measured yaw rate.

The aforementioned results for the front right tire also apply to the other tires, indicating that the estimates of the velocity and the slip are accurate, or at least show a high correlation with the real signals. It should however be noted that the experimental results do not provide certainty with respect

Fig. 9. MB trajectory: (dotted) Reference position trajectory and (solid) the experimentally measured trajectory in the x - y plane for the overall experiment.

to the observer performance, because the estimated signals cannot be compared with direct measurements since the latter are not available.

To test the overall system, i.e., the MB with unicycle observers and feedback-linearizing controllers, an eight-shaped test trajectory is applied, as shown in Fig. 9. This figure shows both the reference trajectory and the resulting measured trajectory from the practical experiment. The centripetal acceleration during cornering is 9 m/s^2 , illustrating the MB behavior at a very high lateral acceleration level. The velocity tangential to the track is increased from 0 to 20 km/h with an acceleration of 5 m/s^2 , then kept constant, and finally decreased to zero again with 5 m/s^2 deceleration. The desired orientation is directed tangentially to the track. The test trajectory is again applied to the simulation model and the real MB.

From Fig. 9, it can be observed that the position error is small, compared with the actual size of the trajectory. Furthermore, it appears that the measured trajectory is always outside the reference trajectory during cornering, indicating stable but not asymptotically stable internal dynamics. The simulated trajectory, which is not shown in Fig. 9, leads to the same observations, albeit with a smaller position error.

Fig. 10 displays the x -position error $e_x = x_{\text{ref}} - \hat{x}$, the y -position error $e_y = y_{\text{ref}} - \hat{y}$, and the orientation error $e_\psi = \psi_{\text{ref}} - \psi$ for the MB center as a function of time. Both the simulated and the measured error signals are shown, which were calculated using the positions estimated by the motion observer and the directly measured yaw angle.

The noticeable differences between the simulated and the measured errors are due to model uncertainties and simplifications, particularly with respect to the tire characteristics and the floor flatness. Nevertheless, the simulation and the experiment show corresponding tendencies.

From Fig. 10 it can be concluded that the position error in the practical experiment is reasonable ($|e_x| < 0.3 \text{ m}$, $|e_y| < 0.3 \text{ m}$, and $|e_\psi| \leq 0.1 \text{ rad}$), given the high centripetal acceleration during the cornering part of the trajectory. The controller gains in (26) and (31) have, however, yet to be optimized with respect to these errors; further improvement might therefore be expected. Finally, the steady state errors in the simulation and in reality should be noted. These are caused by the fact that the internal dynamics are stable but not asymptotically stable for $u_i = 0$, allowing for a final

Fig. 10. Controller results: the MB error in (a) x -position, (b) y -position, and (c) yaw angle; (grey) simulated signals and (black) experiment measurements.

nonzero value of each wheel orientation ψ_i . Because, however, $z_{1,i}$ does converge to $z_{1,i,\text{ref}}$, i.e., the virtual control points converge to their desired values, the unicycle orientation errors inherently cause a corresponding position error of the MB center.

VIII. CONCLUSIONS

The multicycle controller, based on input–output linearization by time-invariant state feedback incorporating a linearized tire slip characteristic, successfully controls the WMR, even in high-dynamic trajectories. In the multicycle approach, the overactuatedness of the robot is employed to determine the drive torque distribution across the wheels such that all tires have approximately the same amount of slip. The resulting controller can easily be adapted to other platform configurations. It is however necessary to have the longitudinal and lateral slip available, which requires a slip observer. The current approach allows for a relatively straightforward extension to the nonlinear tire characteristic.

A possible improvement of the multicycle approach lies in the fact that the tire slip is still neglected on the multicycle level, i.e., at the kinematic determination of the reference steering angles. Furthermore, the nonlinear tire characteristic needs to be incorporated to achieve a more accurate behavior at high longitudinal and lateral accelerations. Finally, a thorough evaluation of the designed controller, compared with the controller without tire slip designed earlier, is desired to determine the level of improvement. These issues will be the subject of further research.

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