Lp string stability of cascaded systems : application to vehicle platooning

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$\mathcal{L}_p$ String Stability of Cascaded Systems: Application to Vehicle Platooning

Jeroen Ploeg, Nathan van de Wouw, and Henk Nijmeijer, *Fellow, IEEE*

***Abstract*—Nowadays, throughput has become a limiting factor in road transport. An effective means to increase the road throughput is to employ a small intervehicle time gap using automatic vehicle-following control systems. String stability, i.e., the disturbance attenuation along the vehicle string, is considered an essential requirement for the design of those systems. However, the formal notion of string stability is not unambiguous in literature, since both stability and performance interpretations exist. Therefore, a novel definition for string stability of nonlinear cascaded systems is proposed, using input–output properties. This definition is shown to result in well-known string stability conditions for linear cascaded systems. The theoretical results are experimentally validated using a platoon of six passenger vehicles equipped with cooperative adaptive cruise control.**

***Index Terms*—Cascaded systems, cooperative adaptive cruise control (CACC), input–output stability, string stability, vehicle platoons.**

I. Introduction

LIMITED highway capacity causes traffic jams, which tend to increase over the years with respect to both the number of traffic jams and their length. An effective means to increase road capacity is to decrease the intervehicle distance. As this would be unsafe in case of human drivers, longitudinal automation will be required. To this end, cooperative adaptive cruise control (CACC) can be employed as an automatic vehicle-following system based on intervehicle data exchange through wireless communications, in addition to the data obtained by radar or lidar [1], [2]. CACC is known to allow for time gaps significantly less than 1 s, being the standardized minimum value for currently available adaptive cruise control (ACC) systems [3]. Thus, an increase in traffic throughput is expected [4], [5]. In addition, the aerodynamic drag is reduced, especially for heavy-duty vehicles, thereby decreasing fuel consumption [2], [6].

A leading objective in the design of CACC systems is to prevent disturbance amplification in upstream direction, for instance induced by velocity variations of the lead vehicle, which would compromise throughput and safety. The disturbance propagation along interconnected systems, such as a vehicle platoon, is covered by the notion of string stability of which a vast amount of literature is available. In [7]–[16], for instance, several types of string stability definitions are given, focusing on various aspects of cascaded systems. In addition, publications that focus on controller design tend to interpret string stability as a performance criterion, rather than a stability property [17]–[25]. As a result, the notion of string stability has become rather ambiguous over the years. This brief, therefore, first aims to formally define string stability, providing a rigourous basis for often-used string stability criteria for linear systems, thus including and generalizing existing results. Second, using a test set-up of six vehicles, it is shown that, using these criteria, controller design for string stability is not only theoretically, but also practically feasible.

This brief is organized as follows. Section II summarizes existing string stability concepts. Section III derives a platoon model that forms the basis for the definition of string stability in Section IV and the analysis thereof for vehicle platoons in Section V. Section VI presents experimental results of a vehicle platoon specifically developed for this purpose. Section VII summarizes the main conclusions.

II. String Stability Review

As opposed to conventional stability notions for dynamical systems, that are basically concerned with the evolution of system states over time, string stability focuses on the propagation of system responses along a cascade of systems. Several approaches exist regarding string stability, as reviewed below.

Probably the most formal approach is based on Lyapunov stability, of which [7] provides an early description, comprehensively formalized in [8]. In this approach, the notion of Lyapunov stability is employed, focusing on initial condition perturbations. Consequently, string stability is interpreted as asymptotic stability of interconnected systems [9]. Recently, new results appeared in [10], regarding a one-vehicle look-ahead topology in a homogeneous vehicle platoon. In [10], the response to an initial condition perturbation of a single vehicle in the platoon is considered, thereby conserving the disturbance-propagation idea behind string stability. The drawback of this approach, however, is that only this special case is regarded, ignoring the effect of initial condition perturbations of other vehicles in the platoon, as well as the effect of external disturbances to the interconnected system. Consequently, the

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J. Ploeg is with TNO, Integrated Vehicle Safety Department, 5700 AT Helmond, The Netherlands (e-mail: jeroen.ploeg@tno.nl)

N. van de Wouw and H. Nijmeijer are with Eindhoven University of Technology, Mechanical Engineering Department, 5600 MB Eindhoven, The Netherlands (e-mail: n.v.d.wouw@tue.nl; h.nijmeijer@tue.nl)

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practical relevance of this approach is limited, since external disturbances, such as velocity variations of the first vehicle in a platoon, are of utmost importance in practice.

The perspective of infinite-length strings of interconnected systems [16] also gave rise to a notion of string stability, described in [11] in the context of a centralized control scheme and in [12] for a decentralized controller. Various applications regarding interconnected systems are reported in [13] and [14], whereas [15] and [16] provide extensive analyzes of the system properties. In this approach, the system model is formulated in the state space and subsequently transformed using the bilateral Z-transform. The Z-transform is executed over the vehicle index instead of over (discrete) time, resulting in a model formulated in the "discrete spatial frequency" domain [15], related to the subsystem index, as well as in the continuous-time domain. String stability can then be assessed by inspecting the eigenvalues of the resulting state matrix as a function of the spatial frequency. Unfortunately, the stability properties of finite-length strings, being practically relevant, might not converge to those of infinite-length strings as length increases. This can be understood intuitively by recognizing that in a finite-length platoon, there will always be a first and a last vehicle, whose dynamics may significantly differ from those of the other vehicles in the platoon, depending on the controller topology. Consequently, the infinite-length platoon model does not always serve as a useful paradigm for a finite-length platoon as it becomes increasingly long [16].

Finally, a performance-oriented approach for string stability is frequently adopted, since this appears to directly offer tools for controller design for linear cascaded systems. This approach is employed for the control of a vehicle platoon with and without lead vehicle information in [17], whereas [18] and [19] apply inter-vehicle communication to obtain information of the preceding vehicle. In [20], a decentralized optimal controller is designed by decoupling the interconnected systems using the so-called inclusion principle, and in [21], optimal decentralized control is pursued by means of nonidentical controllers. Furthermore, [22] extensively investigates the limitations on performance, whereas in [23], a controller design methodology is presented. Finally, in [24] the performance-oriented approach is adopted to investigate a warning system for preventing head-tail collisions in mixed traffic. In the performance-oriented approach, string stability is characterized by the amplification in upstream direction of either distance error, velocity, or acceleration, the specific choice depending on the design requirements at hand. Let the signal of interest be denoted by y_i for vehicle i, and let $\Gamma_i(j\omega)$ denote the frequency response function describing the relation between the scalar output y_{i-1} of a preceding vehicle $i-1$ and the scalar output y_i of the follower vehicle i. Then the interconnected system is considered string stable if

$$\sup_{\omega} |\Gamma_i(j\omega)| \le 1, \quad 2 \le i \le m \tag{1}$$

where m is the string length; the supremum of $|\Gamma_i(j\omega)|$ equals the scalar version of the $\mathcal{H}_\infty$ norm. Since the $\mathcal{H}_\infty$ norm is induced by the $\mathcal{L}_2$ norms of the respective signals, this approach requires the $\mathcal{L}_2$ norm $\|y_i(t)\|_{\mathcal{L}_2}$ to be nonincreasing for increasing index i. Because of its convenient mathematical properties, the $\mathcal{L}_2$ gain is mostly adopted; nevertheless, approaches that employ the induced $\mathcal{L}_\infty$ norm are also reported [25]. Regardless of the specific norm that is employed, the major limitation of the performance-oriented approach is that only linear systems are considered, usually without considering the effect of nonzero initial conditions.

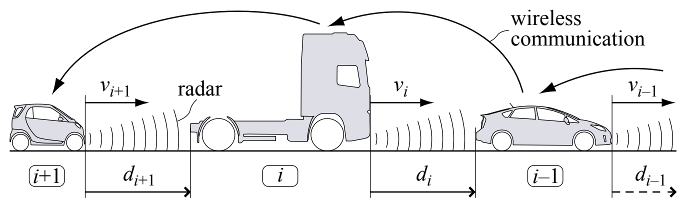

Fig. 1. CACC-equipped vehicle platoon.

Summarizing, string stability appears to be defined in various ways, focusing on specific properties. Building on these earlier results, a new generic definition of string stability is proposed. To this end, the next section will first introduce a model of a homogeneous vehicle platoon, which motivates the formal definition of string stability as proposed in Section IV.

III. Platoon Dynamics

Consider a platoon of m vehicles, schematically depicted in Fig. 1, with d_i being the distance between vehicle i and its preceding vehicle $i-1$, and v_i its velocity. The objective of each vehicle is to follow the preceding vehicle at a desired distance $d_{r,i}$ according to

$$d_{r,i}(t) = r_i + hv_i(t), \quad i \in S_m \tag{2}$$

where h is referred to as the time headway, and r_i is the standstill distance. $S_m = \{i \in \mathbb{N} \mid 1 \le i \le m\}$ is the set of all vehicles in a platoon of length $m \in \mathbb{N}$. The spacing policy (2) is known to improve string stability [18], [19] and safety [26]. A homogeneous platoon is assumed, so h is the same for all i. The spacing error $e_i(t)$ is then defined as

$$\begin{aligned} e_i(t) &= d_i(t) - d_{r,i}(t) \\ &= (q_{i-1}(t) - q_i(t) - L_i) - (r_i + hv_i(t)) \end{aligned} \tag{3}$$

with q_i being the rear-bumper position of vehicle i and L_i its length. The control problem now encompasses two requirements: the vehicle-following objective $\lim_{t\to\infty} e_i(t) = 0 \; \forall\, i \in S_m$, and the string stability requirement.

As a basis for controller design, the following vehicle model is adopted [27], omitting the time argument t for readability:

$$\begin{pmatrix} \dot{d}_i \\ \dot{v}_i \\ \dot{a}_i \end{pmatrix} = \begin{pmatrix} v_{i-1} - v_i \\ a_i \\ -\frac{1}{\tau}a_i + \frac{1}{\tau}u_i \end{pmatrix}, \quad i \in S_m \tag{4}$$

where a_i is the acceleration of vehicle i, u_i the external input (desired acceleration), and τ a time constant representing driveline dynamics, the latter being vehicle-independent because of the homogeneity assumption. With different types of vehicles, as suggested by Fig. 1, homogeneity may be obtained by low-level acceleration controllers so as to arrive at identical vehicle behavior according to (4).

Next, the controller as described in [27] is adopted. This controller defines a new input ξ_i such that

$$h\dot{u}_i = -u_i + \xi_i \tag{5}$$

upon which the control law for ξ_i is chosen as

$$\xi_i = K \begin{pmatrix} e_i \\ \dot{e}_i \\ \ddot{e}_i \end{pmatrix} + u_{i-1}, \quad i \in S_m \tag{6}$$

with $K = (k_p \; k_d \; k_{dd})$. The feedforward term u_{i-1} is obtained through wireless communication with the preceding vehicle.

Since string stability is commonly evaluated by analyzing the amplification in upstream direction of either distance error, velocity, and/or acceleration, a platoon model is formulated in terms of these state variables. Using (3)–(6), the following homogeneous platoon model is thus obtained:

$$\begin{pmatrix} \dot{e}_i \\ \dot{v}_i \\ \dot{a}_i \\ \dot{u}_i \end{pmatrix} = \begin{pmatrix} 0 & -1 & -h & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -\frac{1}{\tau} & \frac{1}{\tau} \\ \frac{k_p}{h} & -\frac{k_d}{h} & -k_d - \frac{k_{dd}(\tau-h)}{h\tau} & -\frac{k_{dd}h+\tau}{h\tau} \end{pmatrix} \begin{pmatrix} e_i \\ v_i \\ a_i \\ u_i \end{pmatrix} + \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & \frac{k_d}{h} & \frac{k_{dd}}{h} & \frac{1}{h} \end{pmatrix} \begin{pmatrix} e_{i-1} \\ v_{i-1} \\ a_{i-1} \\ u_{i-1} \end{pmatrix} \tag{7}$$

or, in short,

$$\dot{x}_i = A_0 x_i + A_1 x_{i-1}, \quad i \in S_m \tag{8}$$

with state vector $x_i = \begin{pmatrix} e_i & v_i & a_i & u_i \end{pmatrix}^{\mathrm{T}}$, and the matrices A_0 and A_1 defined accordingly.

The first vehicle in the platoon, not having a preceding vehicle, will follow a so-called virtual reference vehicle ($i = 0$), allowing the lead vehicle to employ the same controller as the other platoon vehicles. Using the above state definition, the virtual reference vehicle model may be formulated as

$$\begin{pmatrix} \dot{e}_0 \\ \dot{v}_0 \\ \dot{a}_0 \\ \dot{u}_0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -\frac{1}{\tau} & \frac{1}{\tau} \\ 0 & 0 & 0 & -\frac{1}{h} \end{pmatrix} \begin{pmatrix} e_0 \\ v_0 \\ a_0 \\ u_0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 0 \\ \frac{1}{h} \end{pmatrix} \xi_0 \tag{9}$$

or, in short,

$$\dot{x}_0 = A_r x_0 + B_r u_r \tag{10}$$

with state vector $x_0 = \begin{pmatrix} e_0 & v_0 & a_0 & u_0 \end{pmatrix}^{\mathrm{T}}$, external platoon input $u_r = \xi_0$, and the matrices A_r and B_r defined accordingly. Consequently, (9) represents a nonminimal realization, in which $e_0(t) = e_0(0)$ is a dummy state, having no further influence since the first column of both A_r and A_1 equals zero. In the remainder of this brief, $e_0(0) = 0$ is chosen.

The equilibrium state of (9) equals $\bar{x}_0 = \begin{pmatrix} 0 & \bar{v}_0 & 0 & 0 \end{pmatrix}^{\mathrm{T}}$ for $u_r = 0$, where $\bar{v}_0$ is a constant velocity. This equilibrium is only marginally stable since the virtual reference vehicle is in fact an uncontrolled vehicle model. Returning to the homogeneous platoon model (7), it can be easily established that $x_i = \bar{x}_0$, with $i = 1, 2, \ldots, m$, is an equilibrium of the vehicle platoon for $x_0 = \bar{x}_0$ and $u_r = 0$; in other words, the platoon equilibrium is characterized by a constant velocity $\bar{v}_0$ of all vehicles. Applying the Routh-Hurwitz stability criterion, it follows that this equilibrium is asymptotically stable for any time headway $h > 0$, and with any choice for $k_p, k_d > 0$, $k_{dd} > -1$, such that $(1 + k_{dd})k_d > k_p\tau$, thereby fulfilling the vehicle-following control objective. The second objective, being string stability, will be addressed in the next section.

IV. String Stability

In this section, the platoon model is generalized to a nonlinear cascaded state-space system, upon which a new string stability definition is proposed. This definition appears to serve as a rigourous basis for $\mathcal{L}_2$ and $\mathcal{L}_\infty$ string stability conditions commonly used in the performance-oriented approach, and the relation to the other string stability notions is briefly discussed.

A. $\mathcal{L}_p$ String Stability

The homogeneous platoon model (8), (10) is a special, linear case of the following cascaded state-space system:

$$\dot{x}_0 = f_r(x_0, u_r) \tag{11a}$$

$$\dot{x}_i = f_i(x_i, x_{i-1}), \quad i \in S_m \tag{11b}$$

$$y_i = h(x_i), \quad i \in S_m \tag{11c}$$

representing a general, possibly nonlinear, heterogeneous interconnected system with the same interconnection structure as (8), (10). Here, $u_r \in \mathbb{R}^q$ is the external input, $x_i \in \mathbb{R}^n$, $i \in \{0, S_m\}$, is the state vector, and $y_i \in \mathbb{R}^\ell$, $i \in S_m$, is the output. Moreover, $f_r : \mathbb{R}^n \times \mathbb{R}^q \mapsto \mathbb{R}^n$, $f_i : \mathbb{R}^n \times \mathbb{R}^n \mapsto \mathbb{R}^n$, $i \in S_m$, and $h : \mathbb{R}^n \mapsto \mathbb{R}^\ell$. In the scope of vehicle platooning, the state is typically defined as $x_i = \begin{pmatrix} e_i & v_i & a_i & \ldots \end{pmatrix}^{\mathrm{T}}$, $i \in \{0, S_m\}$, indicating a possible extension with additional states, for instance due to controller dynamics, as in Section III. Note that heterogenous strings may arise due to nonidentical (decentralized) controllers [21]. Using the model (11), the following string stability definition is now proposed.

Definition 1 ($\mathcal{L}_p$ string stability)**.** *Consider the interconnected system* (11)*. Let* $x = \begin{pmatrix} x_0^{\mathrm{T}} & x_1^{\mathrm{T}} & \ldots & x_m^{\mathrm{T}} \end{pmatrix}^{\mathrm{T}}$ *be the lumped state vector and let* $\bar{x} = \begin{pmatrix} \bar{x}_0^{\mathrm{T}} & \bar{x}_0^{\mathrm{T}} & \ldots & \bar{x}_0^{\mathrm{T}} \end{pmatrix}^{\mathrm{T}}$ *denote the constant equilibrium solution of* (11) *for* $u_r = 0$*. The system* (11) *is* $\mathcal{L}_p$ string stable *if there exist class* $\mathcal{K}$ *functions*[1] α *and* β *such that, for any initial state* $x(0) \in \mathbb{R}^{(m+1)n}$ *and any* $u_r \in \mathcal{L}_p^q$,

$$\|y_i(t) - h(\bar{x}_0)\|_{\mathcal{L}_p} \le \alpha(\|u_r(t)\|_{\mathcal{L}_p}) + \beta(\|x(0) - \bar{x}\|), \quad \forall\, i \in S_m \text{ and } \forall\, m \in \mathbb{N}.$$

If, in addition, with $x(0) = \bar{x}$ *it also holds that*

$$\|y_i(t) - h(\bar{x}_0)\|_{\mathcal{L}_p} \le \|y_{i-1}(t) - h(\bar{x}_0)\|_{\mathcal{L}_p}, \quad \forall\, i \in S_m\backslash\{1\} \text{ and } \forall\, m \in \mathbb{N}\backslash\{1\}$$

the system (11) *is* strictly $\mathcal{L}_p$ string stable *with respect to its input* $u_r(t)$.

[1] A continuous function $\alpha : [0, a) \mapsto [0, \infty)$ is said to belong to class $\mathcal{K}$ if it is strictly increasing and $\alpha(0) = 0$.

Remark 1. Without principal consequences for Definition 1, (11) could be further generalized with respect to the interconnection structure (or "topology"), so as to include multiple-vehicle look-ahead or bidirectional interconnections.

Clearly, Definition 1 takes the external disturbance u_r into account, imposed by the virtual reference vehicle, through the class $\mathcal{K}$ function $\alpha(\|u_r(t)\|_{\mathcal{L}_p})$, as well as initial condition perturbations, through the class $\mathcal{K}$ function $\beta(\|x(0)-\bar{x}\|)$, where $\|\cdot\|_{\mathcal{L}_p}$ denotes the signal p-norm [28] and $\|\cdot\|$ denotes any vector norm. It should be mentioned that only initial condition perturbations are considered for which the norm $\|x(0)-\bar{x}\|$ exists, which limits the allowable class of perturbations in view of the fact that x will be infinite-dimensional for $m\to\infty$. Furthermore, Definition 1 obviously applies to both linear and nonlinear systems, and homogeneous as well as heterogeneous strings are included, providing, for instance, a rigourous basis for the string stability analysis of heterogeneous strings pursued in [29].

It is important to note that Definition 1 closely resembles the common input–output or $\mathcal{L}_p$ stability definition as far as (nonstrict) $\mathcal{L}_p$ string stability is concerned, except for the fact that the norm requirements must hold *for all string lengths* $m\geq 1$. This is essential to string stability, indicating that a string-stable system is scalable [9].

The notion of strict $\mathcal{L}_p$ string stability, for which not only the first but also the second inequality in Definition 1 must hold, has been introduced to accommodate the common requirement of upstream disturbance attenuation. Note that $i=1$ has been excluded in the requirement for strict string stability since the virtual reference system (11a) does not have an output associated with it, which would be practically irrelevant.

B. String Stability Conditions for Linear Systems

In order to derive string stability conditions for linear systems, the linear, homogeneous version of (11) is considered, which, in lumped form, can be denoted by

$$\begin{pmatrix}\dot{x}_0\\ \dot{x}_1\\ \vdots\\ \dot{x}_m\end{pmatrix}=\begin{pmatrix}A_r & & & \mathrm{O}\\ A_1 & A_0 & & \\ & \ddots & \ddots & \\ \mathrm{O} & & A_1 & A_0\end{pmatrix}\begin{pmatrix}x_0\\ x_1\\ \vdots\\ x_m\end{pmatrix}+\begin{pmatrix}B_r\\ 0\\ \vdots\\ 0\end{pmatrix}u_r \quad (12)$$

or, in short,

$$\dot{x}=Ax+Bu_r \quad (13)$$

with $x=\begin{pmatrix}x_0^{\mathrm{T}} & x_1^{\mathrm{T}} & \dots & x_m^{\mathrm{T}}\end{pmatrix}^{\mathrm{T}}$, and the matrices A and B defined accordingly. The matrices A_0, A_1, A_r, and B_r can, e.g., be chosen identical to those used in (8) and (10). In addition, consider linear output functions according to

$$y_i=C_i x,\quad i\in S_m \quad (14)$$

with output matrices C_i. The model (13), (14) can then be formulated in the Laplace domain as follows:

$$\hat{y}_i(s)=P_i(s)\hat{u}_r(s)+O_i(s)x(0),\quad i\in S_m \quad (15)$$

with outputs $y_i(t)\in\mathbb{R}^{\ell}$ and exogenous input $u_r(t)\in\mathbb{R}^{\ell}$, whose Laplace transforms are denoted by $\hat{y}_i(s)$ and $\hat{u}_r(s)$, with $s\in\mathbb{C}$, respectively. $x(0)\in\mathbb{R}^{(m+1)n}$ denotes the initial condition, whereas $P_i(s)=C_i(sI-A)^{-1}B$ and $O_i(s)=C_i(sI-A)^{-1}$. In view of the upcoming analysis, $P_i(s)$ is thus assumed to be square, having ℓ inputs and ℓ outputs. Also, without loss of generality, the equilibrium state $\bar{x}=\begin{pmatrix}\bar{x}_0^{\mathrm{T}} & \bar{x}_0^{\mathrm{T}} & \dots & \bar{x}_0^{\mathrm{T}}\end{pmatrix}^{\mathrm{T}}=0$ is chosen, hence $h(\bar{x}_0)=C_i\bar{x}=0$.

Since (12) describes a controlled system, the matrix A_0 is typically Hurwitz. However, this may not be the case for the matrix A_r, related to the virtual reference vehicle in case of vehicle following. As indicated by (9), for instance, A_r has a marginally stable mode associated with v_0 (besides the mode associated with the dummy state e_0). Hence, the system matrix A in (13) is not Hurwitz. In the remainder of this section, however, it is assumed that the pair (C_i,A) is such that unstable (including marginally stable) modes are unobservable by a specific choice of C_i. Consequently, it suffices to only analyze the output response to the external input in view of string stability (or, equivalently, to assume $x(0)=\bar{x}=0$), in accordance with the following remark.

Remark 2. Consider the system (11a), (11b), but with a single output vector $y_k=h(x_k)$, $1\leq k\leq m$. Then this system is $\mathcal{L}_p$ stable if

$$\|y_k(t)-h(\bar{x}_0)\|_{\mathcal{L}_p}\leq\alpha_k(\|u_r(t)\|_{\mathcal{L}_p})+\beta_k(\|x(0)-\bar{x}\|)$$

with class $\mathcal{K}$ functions α_k and β_k. When (11a) and (11b) represent a linear system, the existence of α_k implies that β_k exists, provided that unstable and marginally stable modes are unobservable [30]. Since this statement holds for any $k\in S_m$, it also applies to α and β in Definition 1.

Adopting the $\mathcal{L}_2$ signal norm for string stability, it follows from (15), that, with $x(0)=0$,

$$\begin{aligned}\|y_i(t)\|_{\mathcal{L}_2}&\leq\|P_i(j\omega)\|_{\mathcal{H}_\infty}\|u_r(t)\|_{\mathcal{L}_2}\\ &\leq\max_{i\in S_m}\|P_i(j\omega)\|_{\mathcal{H}_\infty}\|u_r(t)\|_{\mathcal{L}_2},\quad\forall\, i\in S_m\end{aligned} \quad (16)$$

using the fact that the $\mathcal{H}_\infty$ norm (or $\mathcal{L}_2$ gain) $\|P_i(j\omega)\|_{\mathcal{H}_\infty}$ is induced by the $\mathcal{L}_2$ norm on inputs and outputs. It is important to note that (16) is not conservative, in the sense that there is always a subsystem $i\in S_m$ and a specific signal $u_r(t)$ for which the equality holds [28]. According to Definition 1, $\mathcal{L}_2$ string stability of the interconnected system (13), (14) thus requires $\max_{i\in S_m}\|P_i(j\omega)\|_{\mathcal{H}_\infty}$ to exist for all $m\in\mathbb{N}$, being a necessary and sufficient condition.

For further analysis, a specific type of interconnection topology will be adopted, as mentioned in the following remark.

Remark 3. In the case of a look-ahead topology, such as described by (12), the interconnection is unidirectional, from which it directly follows that if the infinite-length string has a bounded output response to a bounded input, then all finite-length strings as a subset thereof have a bounded response as well. Therefore, it suffices to only regard $m\to\infty$ for string stability assessment, implying that the sets $i\in S_m$, $m\in\mathbb{N}$, can be reduced to a single set $i\in\mathbb{N}$.

As a result, the interconnected system (13), (14) is $\mathcal{L}_2$ string stable if and only if $\sup_{i\in\mathbb{N}}\|P_i(j\omega)\|_{\mathcal{H}_\infty}$ exists. The class $\mathcal{K}$ function α in Definition 1 can then be chosen as

$$\alpha(\|u_r(t)\|_{\mathcal{L}_2})=\left(\sup_{i\in\mathbb{N}}\|P_i(j\omega)\|_{\mathcal{H}_\infty}\right)\|u_r(t)\|_{\mathcal{L}_2}. \quad (17)$$

Because of the linear form of α in (17), this type of string stability may be referred to as finite-gain $\mathcal{L}_2$ string stability, similar to the notion of finite-gain $\mathcal{L}_2$ stability.

The existence of the supremum of the $\mathcal{L}_2$ gain can be further analyzed by factorization, leading to the theorem below. As a preliminary to this theorem, the *string stability complementary sensitivity* is introduced first. From (15), it directly follows that (with $x(0) = 0$)

$$\hat{y}_i(s) = \Gamma_i(s)\hat{y}_{i-1}(s) \tag{18}$$

with the string stability complementary sensitivity

$$\Gamma_i(s) = P_i(s)P_{i-1}^{-1}(s) \tag{19}$$

assuming functional controllability of (15), i.e., $P_{i-1}^{-1}(s)$ exists. The following theorem can now be stated.

Theorem 1. *Let* (13), (14) *represent a linear unidirectionally interconnected system for which the input–output behavior is described by* (15)*. Assume that the pair (C_i, A) is such that unstable and marginally stable modes are unobservable and that $P_i(s)$ is square and nonsingular, for all $i \in \mathbb{N}$. Then the system* (13), (14) *is $\mathcal{L}_2$ string stable if*

1) $\|P_1(j\omega)\|_{\mathcal{H}_\infty}$ *exists;*
2) $\|\Gamma_i(j\omega)\|_{\mathcal{H}_\infty} \leq 1, \quad \forall\, i \in \mathbb{N}\backslash\{1\}$*;*

with $\Gamma_i(s)$ as in (19)*. Moreover, the system is strictly $\mathcal{L}_2$ string stable if and only if conditions 1 and 2 hold.*

Proof: Using (15), (18), and (19), the input–output relation for a specific subsystem $i \geq 2$ can be formulated as

$$\hat{y}_i(s) = P_i(s)\hat{u}_r(s) = \left(\prod_{k=2}^{i} \Gamma_k(s)\right) P_1(s)\hat{u}_r(s). \tag{20}$$

Having factorized $P_i(s)$ in this way, the submultiplicative property dictates that

$$\|P_i(j\omega)\|_{\mathcal{H}_\infty} \leq \left(\prod_{k=2}^{i} \|\Gamma_k(j\omega)\|_{\mathcal{H}_\infty}\right) \|P_1(j\omega)\|_{\mathcal{H}_\infty}. \tag{21}$$

Consequently, under the conditions 1 and 2 in Theorem 1, $\sup_{i\in\mathbb{N}} \|P_i(j\omega)\|_{\mathcal{H}_\infty}$ exists. Because it is also assumed that unstable and marginally stable modes are unobservable for all $i \in \mathbb{N}$, the linear system is $\mathcal{L}_2$ string stable, according to Definition 1 and Remark 2, while using (17). Moreover, from (18) and condition 2, it follows that

$$\|y_i(t)\|_{\mathcal{L}_2} \leq \|y_{i-1}(t)\|_{\mathcal{L}_2}, \quad \forall\, i \in \mathbb{N}\backslash\{1\} \tag{22}$$

which yields the interconnected system strictly $\mathcal{L}_2$ string stable. The necessity of the conditions 1 and 2 for strict $\mathcal{L}_2$ string stability is immediate. ■

It is noted that condition 2 closely resembles the well-known string stability criterion (1). As such, Definition 1 together with Theorem 1 provide a rigorous basis for this criterion. The fact that Theorem 1 only yields sufficient conditions for $\mathcal{L}_2$ string stability is basically due to the submultiplicative property. In specific cases, however, the conditions become also necessary, as shown below.

Remark 4. When $u_r \in \mathbb{R}$ and $y_i \in \mathbb{R}$, $i \in \mathbb{N}\backslash\{1\}$, and $\Gamma(s) = P_i(s)P_{i-1}^{-1}(s)$, $i \in \mathbb{N}\backslash\{1\}$, is independent of i, then

$$\|P_i(j\omega)\|_{\mathcal{H}_\infty} = \sup_\omega \left\{|\Gamma(j\omega)|^{i-1}|P_1(j\omega)|\right\} \tag{23}$$

due to (20). Consequently, $\|P_i(j\omega)\|_{\mathcal{H}_\infty}$ exists for all $i \in \mathbb{N}\backslash\{1\}$, if and only if $|P_1(j\omega)| < \infty$ and $|\Gamma(j\omega)| \leq 1$ for all ω, rendering the interconnected system strictly $\mathcal{L}_2$ string stable. Note that the necessity of these conditions only holds in the absence of poles of $P_1(s)$ on the imaginary axis, being canceled by zeros of $\Gamma(s)$ since, in that case, $\sup_\omega |P_1(j\omega)|$ is unbounded whereas $\sup_\omega |\Gamma(j\omega)^{i-1}P_1(j\omega)|$ may not be.

It thus follows that for linear unidirectionally coupled homogeneous systems with scalar input and output, $\mathcal{L}_2$ string stability and strict $\mathcal{L}_2$ string stability are equivalent.

Until now, only $\mathcal{L}_2$ string stability has been considered. Physically, this can be motivated by the requirement of energy dissipation along the string. Obviously, the induced $\mathcal{L}_\infty$ norm can be used instead. In the scope of vehicle following, the motivation for using this norm would be traffic safety, since the $\mathcal{L}_\infty$ norm is directly related to maximum overshoot. The conditions for $\mathcal{L}_\infty$ string stability can be derived as follows. Let $p_i(t)$ denote the impulse response matrix, corresponding to the transfer function $P_i(s)$. Then, from linear system theory

$$\|p_i(t)\|_{\mathcal{L}_1} = \max_{u_r \neq 0} \frac{\|y_i(t)\|_{\mathcal{L}_\infty}}{\|u_r(t)\|_{\mathcal{L}_\infty}}. \tag{24}$$

Consequently, the interconnected system is $\mathcal{L}_\infty$ string stable if and only if $\sup_{i\in\mathbb{N}} \|p_i(t)\|_{\mathcal{L}_1}$ exists. The class $\mathcal{K}$ function α in Definition 1 can then be chosen as

$$\alpha(\|u_r(t)\|_{\mathcal{L}_\infty}) = \left(\sup_{i\in\mathbb{N}} \|p_i(t)\|_{\mathcal{L}_1}\right) \|u_r(t)\|_{\mathcal{L}_\infty}. \tag{25}$$

This leads to the following theorem.

Theorem 2. *Let* (13), (14) *represent a linear unidirectionally interconnected system for which the input–output behavior is described by* (15)*. Assume that the pair (C_i, A) is such that unstable and marginally stable modes are unobservable and that $P_i(s)$ is square and nonsingular, for all $i \in \mathbb{N}$. Then the system* (13), (14) *is $\mathcal{L}_\infty$ string stable if*

1) $\|p_1(t)\|_{\mathcal{L}_1}$ *exists;*
2) $\|\gamma_i(t)\|_{\mathcal{L}_1} \leq 1, \quad \forall\, i \in \mathbb{N}\backslash\{1\}$*;*

where $p_1(t)$ and $\gamma_i(t)$ are the impulse responses corresponding to $P_1(s)$ and $\Gamma_i(s)$, respectively, with $\Gamma_i(s)$ according to (19)*. Moreover, the system is strictly $\mathcal{L}_\infty$ string stable if and only if conditions 1 and 2 hold.*

Proof: Applying Young's inequality for convolutions, the proof is similar to that of Theorem 1. ■

Again, Theorem 2 only provides sufficient conditions for $\mathcal{L}_\infty$ string stability. Note that, using a Lyapunov-stability approach for linear systems, [31] discusses the relation between $\mathcal{L}_\infty$ and $\mathcal{L}_2$ string stability, the main results of which can be extended to the new framework by defining the output y_i to contain all states x_i (subject to the remark in the next subsection), while focussing on initial condition perturbations.

C. Discussion

From the previous section, it is clear that the performance-oriented approach to string stability [17]–[25] is captured by Definition 1 as a special case for linear, unidirectionally interconnected systems. In addition, the Lyapunov-stability approach [7]–[10] is captured as well by the inclusion of initial condition perturbations in the definition. An apparent difference, however, is that the Lyapunov-stability approach focusses on the system states, whereas Definition 1 regards the outputs. Nevertheless, there is no essential limitation in choosing the output so as to include all states, albeit that the string stability complementary sensitivity can no longer be computed using (19). In the framework of infinite-length interconnected systems [11]–[16], string stability requires the states to (exponentially) decay both over time and system index as a result of initial condition perturbations. As such, it can be argued that such behavior corresponds to strict string stability as in Definition 1. Summarizing, while the existing definitions are adequate, Definition 1 encompasses those.

V. String Stability of Vehicle Platoons

In order to analyze $\mathcal{L}_2$ string stability of the platoon model (7), (9), $P_1(j\omega)$ and $\Gamma_i(j\omega)$ need to be determined. To this end, the frequency-domain model of a controlled platoon vehicle is formulated first by introducing the vehicle transfer function $G(s) = \hat{q}_i(s)/\hat{u}_i(s)$, according to:

$$G(s) = \frac{1}{s^2(\tau s + 1)} \tag{26}$$

which follows from $\dddot{q}_i = -\frac{1}{\tau}\ddot{q}_i + \frac{1}{\tau}u_i$, see (4), the *spacing policy transfer function* $H(s) = \hat{\xi}_i(s)/\hat{u}_i(s)$ derived from (5):

$$H(s) = hs + 1 \tag{27}$$

and the feedback law $K(s)$ with input $\hat{e}_i(s)$, defined in (6):

$$K(s) = k_p + k_d s + k_{dd} s^2. \tag{28}$$

A controlled vehicle i is then represented by the block scheme as shown in Fig. 2. The occurrence of $H(s)$ in the feedback loop can be readily explained by considering $\tilde{q}_i$ as depicted in the block scheme, which equals, using (27),

$$\tilde{q}_i(t) = L_i + r_i + q_i(t) + hv_i(t). \tag{29}$$

Consequently, $\tilde{q}_i$ can be interpreted as the "virtual control point" of vehicle i, that must converge to the actual position q_{i-1} of the preceding vehicle $i-1$. Furthermore, since the frequency-domain approach allows for the inclusion of a latency θ induced by the wireless communication network, the block scheme also includes a time delay $D(s) = e^{-\theta s}$.

The virtual reference vehicle (9) is now described by the series connection of $H^{-1}(s)$ and $G(s)$, indicated in Fig. 2 by "vehicle 0," with $u_r(t) = \xi_0(t)$ as external input. Consequently, the vehicle platoon has a scalar input u_r, upon which a scalar output needs to be selected according to Theorem 1. To this end, $y_i(t) = a_i(t)$ is chosen since the acceleration is physically relevant on the one hand, and guarantees the existence of $\|P_1(j\omega)\|_{\mathcal{H}_\infty}$ on the other, as will be shown later. Moreover, with this output, the marginally stable mode associated with

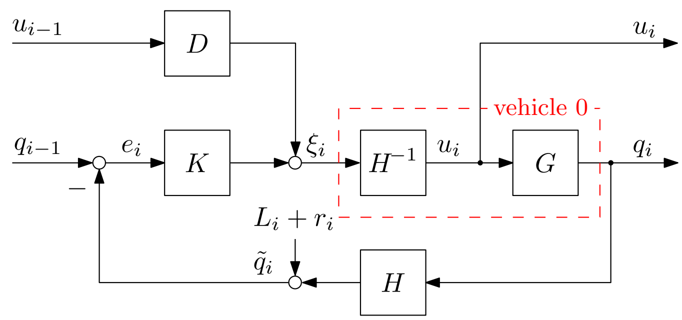

Fig. 2. Block scheme of a controlled platoon vehicle $i \geq 1$ and of the virtual reference vehicle, indicated by the index 0.

the reference vehicle appears to be unobservable. Using the block scheme in Fig. 2, the string stability complementary sensitivity $\Gamma_i(s) = \Gamma(s)$ (independent of i) now satisfies

$$\Gamma(s) = \hat{a}_i(s)/\hat{a}_{i-1}(s) = \frac{1}{H(s)} \frac{K(s)G(s) + D(s)}{1 + K(s)G(s)}. \tag{30}$$

Because of the specific choice for the virtual reference vehicle model, $\Gamma(s)$ also equals the transfer function from $\hat{a}_0(s)$ to $\hat{a}_1(s)$. It therefore follows that, using (9),

$$\begin{aligned} \hat{a}_1(s) = \Gamma(s)\hat{a}_0(s) &= \frac{\Gamma(s)}{H(s)} \frac{1}{\tau s + 1} \hat{u}_r(s) \\ &:= P_1(s)\hat{u}_r(s) \end{aligned} \tag{31}$$

from which it directly follows that $\|P_1(j\omega)\|_{\mathcal{H}_\infty}$ exists when $\|\Gamma(j\omega)\|_{\mathcal{H}_\infty}$ exists, due to the submultiplicative property of the $\mathcal{H}_\infty$ norm and the fact that $H^{-1}(s)(\tau s + 1)^{-1}$ is a stable transfer function (provided that $h \geq 0$). It follows from (30) that without delay ($D(s) = 1$), $\|\Gamma(j\omega)\|_{\mathcal{H}_\infty}$ exists, since $\|\Gamma(j\omega)\|_{\mathcal{H}_\infty} = \sup_\omega |H^{-1}(j\omega)| = 1$. According to Theorem 1, the system without delay is thus strictly $\mathcal{L}_2$ string stable for any choice of controller gains and time headway.

Note that, if $u_r = 0$ and the virtual reference vehicle has a constant velocity $\bar{v}_0$, the states of all vehicles in the platoon will asymptotically converge to the equilibrium state $\bar{x}_0 = \begin{pmatrix} 0 & \bar{v}_0 & 0 & 0 \end{pmatrix}^{\mathrm{T}}$; see Section III. In other words, $\lim_{\omega \to 0} (\hat{v}_i(j\omega) - \hat{v}_{i-1}(j\omega)) = 0$. Hence,

$$\lim_{\omega \to 0} |\Gamma(j\omega)| = 1 \Rightarrow \|\Gamma(j\omega)\|_{\mathcal{H}_\infty} \geq 1 \tag{32}$$

which is why the strict $\mathcal{L}_p$ string stability inequality in Definition 1 includes the equality. As a consequence, however, string stability robustness with respect to, e.g., model uncertainties, may be poor in case these uncertainties cause $|\Gamma(j\omega)|$ to increase in the lower frequency region.

As already mentioned, wireless communications exhibit latency, which in general increases with increasing communication load. This time delay compromises string stability [32], as illustrated in Fig. 3(a), showing the gain $|\Gamma(j\omega)|$ for various values of the time delay θ. Here, $\tau = 0.1$ s, $k_p = 0.2$, $k_d = 0.7$, $k_{dd} = 0$, and $h = 0.5$ s, yielding asymptotic stability of the platoon; see also Section VI.

Fig. 3(b) illustrates the influence of the time headway on string stability in the presence of a communication delay, showing the maximum communication delay $\theta_{\max}$ that yields

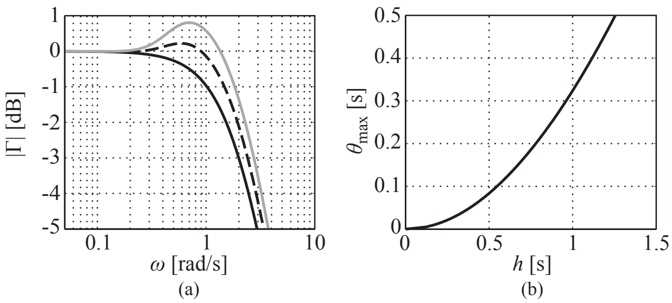

Fig. 3. $\mathcal{L}_2$ string stability properties: (a) string stability complementary sensitivity magnitude $|\Gamma(j\omega)|$ for communication delay (solid black) $\theta = 0\,s$, (dashed black) $\theta = 0.15\,s$, (grey) $\theta = 0.3\,s$, and (b) maximum delay $\theta_{\max}$ as a function of headway h.

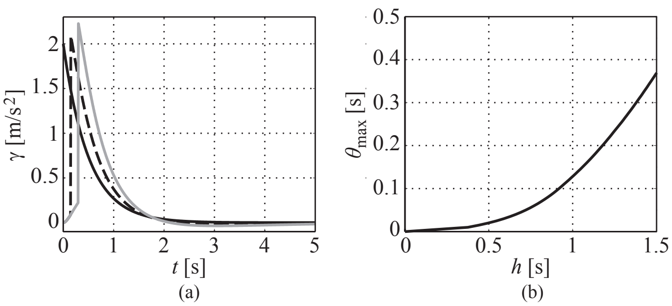

Fig. 4. $\mathcal{L}_\infty$ string stability properties: (a) impulse response $\gamma(t)$ for communication delay (solid black) $\theta = 0\,s$, (dashed black) $\theta = 0.15\,s$, (grey) $\theta = 0.3\,s$, and (b) maximum delay $\theta_{\max}$ as a function of headway h.

$|\Gamma(j\omega)| \leq 1$, as a function of time headway h. This result is calculated by taking a fixed value for θ and then searching for the smallest value of h such that $\|\Gamma(j\omega)\|_{\mathcal{H}_\infty} = 1$. The given system falls into the category as mentioned in Remark 4, since, first, $y_i, u_r \in \mathbb{R}\ \forall i \in \mathbb{N}$, second, $\Gamma(j\omega)$ does not depend on the vehicle index i, and, third, it follows from (31) that $P_1(s)$ does not have poles on the imaginary axis that might be canceled by zeros in the product $\Gamma(s)P_1(s)$. Consequently, $\|P_1(j\omega)\|_{\mathcal{H}_\infty} < \infty$ and $|\Gamma(j\omega)| \leq 1$ together form necessary and sufficient conditions for (strict) $\mathcal{L}_2$ string stability.

Regarding $\mathcal{L}_\infty$ string stability, applying the inverse Laplace transform to (31) to obtain the impulse response, and subsequently using Young's inequality for convolutions, it can be shown that $\|p_1(t)\|_{\mathcal{L}_1}$ exists when $\|\gamma(t)\|_{\mathcal{L}_1}$ exists, similar to the result obtained for the existence of $\|P_1(j\omega)\|_{\mathcal{H}_\infty}$. Since $\Gamma(s) = H^{-1}(s)$ without communication delay, the impulse response equals $\gamma(t) = h^{-1}e^{-t/h}$. Hence $\|\gamma(t)\|_{\mathcal{L}_1} = 1$, rendering the system strictly $\mathcal{L}_\infty$ string stable for all time headways and controller parameters according to Theorem 2.

Again, communication delay compromises $\mathcal{L}_\infty$ string stability. Using the same parameter values as before, Fig. 4(a) shows the impulse response $\gamma(t)$ for various values of the delay θ, calculated using the inverse Fourier transform of $\Gamma(j\omega)$ from (30). It appears that $\|\gamma(t)\|_{\mathcal{L}_1}$ is an increasing function of θ, the effect of which is illustrated in Fig. 4(b), showing the maximum communication delay $\theta_{\max}$ that yields $\|\gamma(t)\|_{\mathcal{L}_1} \leq 1$, as a function of the time headway h. Apparently, $\|\gamma(t)\|_{\mathcal{L}_1} \leq 1$ is a more stringent criterion than $\|\Gamma(j\omega)\|_{\mathcal{H}_\infty} \leq 1$, requiring a significantly larger time headway. This could be expected

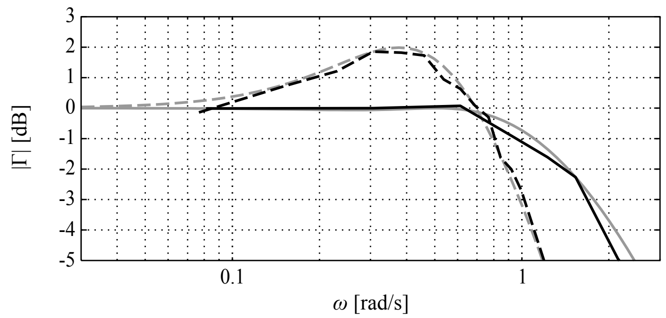

Fig. 5. String stability complementary sensitivity magnitude $|\Gamma(j\omega)|$: (solid black) CACC measured, (solid grey) CACC theoretical, (dashed black) ACC measured, (dashed grey) ACC theoretical.

since, from linear system theory, $\|\Gamma(j\omega)\|_{\mathcal{H}_\infty} \leq \|\gamma(t)\|_{\mathcal{L}_1}$.

VI. Experimental Validation

To validate the theoretical results and to demonstrate its technical feasibility, CACC has been implemented in six passenger vehicles [27], equipped with IEEE 802.11p-based wireless communication, allowing for communication of the desired vehicle acceleration at an update rate of 10 Hz.

The test vehicle model has been identified as [27]:

$$G(s) = \frac{1}{s^2(\tau s + 1)} e^{-\phi s} \tag{33}$$

with $\tau = 0.1\,s$ and $\phi = 0.2\,s$, the latter leading to an adaptation of (26) so as to include this time delay. Considering stability of the dynamics (7), speed of response, and comfort, suitable controller gains were found to be $k_p = 0.2$ and $k_d = 0.7$, with $k_{dd} = 0$ to avoid feedback of the jerk, which is in practice unfeasible. The communication delay appeared to be $\theta \approx 0.15$ s. Using the analysis presented in Section V, $h = 0.7$ s is chosen, just achieving strict $\mathcal{L}_2$ string stability.

Focussing on $\mathcal{L}_2$ string stability, a test is carried out using a prescribed acceleration profile $a_0(t)$, as described in [27], based on which the input $u_r(t)$ has been calculated through differentiation, employing the dynamic inverse of (9). The measured response has subsequently been used to determine $|\Gamma(j\omega)|$, employing Welch's averaged periodogram method [33]. The result is depicted in Fig. 5 for two cases: *with* the communicated desired acceleration u_{i-1} of the preceding vehicle and *without* (i.e., $u_{i-1} = 0$), referred to as CACC and ACC, respectively. Also the theoretical gain (30) is shown. The benefit of CACC in view of $\mathcal{L}_2$ string stability is clearly illustrated, validating the theoretical analysis in Section IV. Another important observation is that string stability may not always be easy to assess in practice, since $|\Gamma(j\omega)|$ will be close to 1 for low frequencies, see (32), as a result of which estimation inaccuracy may compromise the second string stability criterion from Theorem 1. In this experiment, estimation errors also cause the impulse response function, computed using the inverse Fourier transformation, to be inaccurate. Therefore, $\mathcal{L}_\infty$ string stability is not further investigated here.

In addition to the time responses shown in [27], Fig. 6 shows the velocity response to a constant acceleration of the lead vehicle for both ACC and CACC, which clearly illustrates

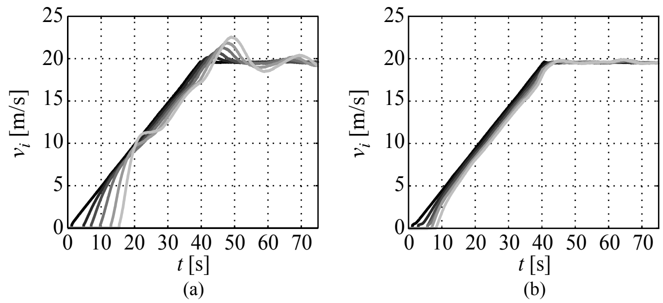

Fig. 6. Measured velocity response at startup (black–light grey: vehicle 1–6): (a) ACC and (b) CACC.

string stability in case of CACC and the lack thereof for ACC. Noteworthy is the observation that with ACC, the last vehicle starts to accelerate after 15.5 s, whereas with CACC, this is already after 8.5 s, showing that CACC may also be effective at traffic lights.

VII. Conclusion

A novel string stability definition was proposed, on the basis of the notion of $\mathcal{L}_p$ stability, which applies to both linear and nonlinear systems, while accommodating initial condition perturbations as well as external disturbances, independent of the interconnection topology. The definition appeared to provide a rigorous basis for well-known $\mathcal{L}_2$ and $\mathcal{L}_\infty$ string stability conditions for linear, unidirectionally interconnected systems.

Next, the string stability properties of CACC for vehicle platoons were analyzed, showing that time gaps well below 1 s were admissible. To assess string stability in practice and to demonstrate the technical feasibility of CACC, experiments were conducted using a test fleet of six passenger vehicles. As a result, a time headway of 0.7 s appeared to yield strict $\mathcal{L}_2$ string-stable behavior, in accordance with the theoretical analysis, which also indicated that time gaps down to 0.3 s are feasible when minimizing the latency of the wireless link. Such small time headways, however, will require insight into the string stability margins in the presence of uncertainties or unknown disturbances, which will be the subject of further research.

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