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Dual sourcing in the age of near-shoring: trading off stochastic capacity limitations and long lead times

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We model a periodic review inventory system with non-stationary stochastic demand, in which a manufacturer is procuring a component from two available supply sources. The faster supply source is modeled as stochastic capacitated with immediate delivery, while the slower supply source is modeled as uncapacitated with a longer fixed lead time. The manufacturer's objective is to choose how the order should be split between the two supply sources in each period, where the slower supply source is used to compensate for the supply capacity unavailability of the faster supply source. This is different from the conventional dual sourcing problem, and motivated by the new reality of near-shoring options. We derive the optimal dynamic programming formulation that minimizes the total expected inventory holding and backorder costs over a finite planning horizon and show that the optimal policy is relatively complex. We extend our study by developing an extended myopic two-level base-stock policy and we show numerically that it provides a very close estimate of the optimal costs. Numerical results reveal the benefits of dual sourcing under near-shoring, where we point out that in most cases the manufacturer should develop a hybrid procurement strategy, taking advantage of both supply sources to minimize its expected total cost.

Keywords: inventory, stochastic inventory theory; dual sourcing; uncertain supply; myopic policy

1. Introduction

With the rise of reshoring options and growth of B2B supply platforms, the complexity of sourcing management in the supply chain is increasing. Recent studies by BCG (2014b) and others suggest that the cost of sourcing in the U.S.A. may be approaching the cost of producing in China due to matters such as the rising wages in coastal China, the increase in transportation costs from more remote regions in China, the exchange rate developments of the RMB vs the USD, and the growth in productivity in the U.S.A. due to extensive

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automation. The realization that the economics of manufacturing are swinging in favor of the U.S.A. has seen companies becoming interested in shifting manufacturing back to the U.S.A., for both goods to be sold at home and those intended for major export markets. The BCG's survey found that the number of respondents saying that their companies are already bringing production back from China to the United States had risen by 20 %, from roughly 13% to 16% in the past year (BCG 2014a). Similar developments are taking place in Europe.

Although the comparative attractiveness of the U.S.A. and other developed economies has increased, the current nearshoring trend is facing several challenges. The traditionally presumed unlimited capacity availability of nearshore manufacturing options may be decreasing due to the fact that so much capacity has been taken out of the system over the past two decades and that widespread automation is reducing the ability to respond to short-term changes in demand. Companies would have to rebuild their supply chains and identify people with the right skills to handle the increasingly sophisticated automated operations. In an executive survey by Alix Partners (2014), the respondents mention that the challenges center on the availability and capability of the local workforce, and the capability and flexibility of suppliers. In addition, with one-third of U.S. 3PLs reporting increased volumes and revenues as a result of nearshoring, their CEOs are also reporting capacity shortages across transport modes, including truckload, LTL, intermodal and rail, with the outcome that *"The capacity crunch has led to higher rates and longer transit times with 3PLs struggling to meet on-time service goals and cost targets"* (Penske Logistics 2014).

The fact that manufacturing jobs are coming back to developed economies due to reshoring, does not mean that they are not continuing to develop in Asia and other developing countries. In fact, the number of global sourcing options is still growing, with the global supply chains effectively pooling the global manufacturing capacities and making them available to companies worldwide. For instance, the rise of B2B platforms such as Alibaba, enables suppliers from further away to flexibly respond to market needs, albeit with longer (transportation) lead times.

As a consequence, the trade-offs in dual sourcing are changing. Most research in dual sourcing studies the trade-offs between a nearshoring option that is more expensive and faster against an offshoring option that is cheaper but requires a longer lead time. Yet the developments described above suggest that at least for part of the product portfolio the trade-off may be less about product cost, and nearby suppliers may be faster but not

necessarily more flexible. They may suffer from limited capacity availability.

In this paper, we focus on such a newly developing trade-off. We study the problem of a manufacturer procuring a component for the production of a finished product, where two sourcing options are considered: a faster, yet partially available source, and a slower, yet fully available source. Both sources supply at the same landed cost. In this case, it is unlikely that the faster source is always favorable. In any case, supplying from the faster nearshore source will involve uncertainties related to the current capacity availability, which could result in uncertain replenishment to the manufacturer. This uncertainty would need to be hedged using the slower, presumably uncapacitated, offshore source. When a manufacturer experiences or anticipates a supply shortage of the nearshore source, a decision has to be made concerning the extent to which the slower offshore source should also be utilized. Due to the sourcing lead time, the ordering decisions are made before demand for the finished product is realized. Thus, the decision maker is facing both, the uncertain replenishment from the fast supply source, and the uncertainty of demand for the finished product, and needs to allocate the replenishment between the two supply sources.

To study this, we model a zero-lead-time supply source that is stochastic capacitated, where the supply capacity is exogenous to the manufacturer and the actual capacity realization is only revealed upon replenishment. The reliable positive lead time supply source is modeled as uncapacitated with a fixed one-period lead time. Both the supply capacity of the nearshore source and the demand are assumed to be stochastic and non-stationary with known distributions in each time period. Unmet demand is backordered. In each period the manufacturer places the order with the fast source, the slow source, or with both sources. Our goal is to find an optimal policy that minimizes the inventory holding costs and backorder costs over a finite planning horizon. See Figure 1 for a sketch of the supply chain under study.

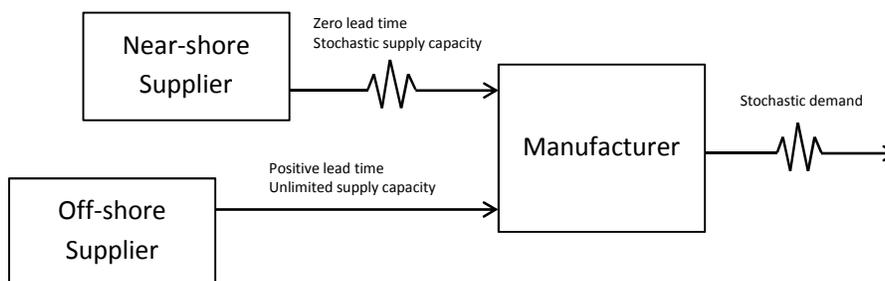


Figure 1: Sketch of the supply chain under study.

1.1 *Related literature*

We proceed with a review of the relevant literature on supply uncertainty models in a single-stage setting, where our interest lies in two research tracks: single sourcing inventory models with random capacity and dual sourcing models with suppliers that differ in their delivery times and/or supply capacity availability. The way we model the supply capacity of the faster supply source is in line with the work of Ciarallo et al. (1994), Khang & Fujiwara (2000), Iida (2002), Jakšič et al. (2011) and Jakšič & Fransoo (2015), where the random supply/production capacity determines a random upper bound on the supply availability in each period. For a finite horizon stationary inventory model they show that the optimal policy is a base-stock policy (or a modified base-stock policy if capacity is known prior to placing an order), where the optimal base-stock level is increased to account for possible, albeit uncertain, capacity shortfalls in future periods. An important observation for our work is the insight provided by Ciarallo et al. (1994) in their analysis of a single-period problem, where they show that stochastic capacity does not affect the order policy. The myopic policy of the newsvendor type is optimal to cover the demand uncertainty, meaning that the decision maker is not better off by asking for a quantity higher than that of an uncapacitated case.

For a general review of multiple supplier inventory models we refer the interested reader to Minner (2003). The review is based on the important criteria for the supplier choice, mainly the price and the supplier's service. A more focused review of multiple sourcing inventory models when supply components are uncertain by Tajbakhsh et al. (2007) reveals that most of these models consider uncertainty either in supply lead time, supply yield, or supplier availability. More specifically, the review of dual sourcing literature shows that a series of papers shares some basic modeling assumptions with our model: dual sourcing periodic review with deterministic lead times that are different for the two supply sources. These papers can be divided into two streams depending on the assumptions on supply capacity availability, where for the first stream the assumption of unconstrained suppliers holds, while for the second some sort of capacity constraint is introduced at the faster supplier or both suppliers. Most of these papers rely on the price difference between the two suppliers that stimulates the manufacturer to partially source from the cheaper, slower supplier. However, we argue that in the case where the supply capacity availability at the faster supply source is limited, variable, and potentially stochastic, the above-mentioned price incentive is not needed. Thus, in our case the incentive to find the optimal dual sourcing strategy lies in

finding the right balance between the responsiveness of the faster supply source and the reliability of the slower supply source.

In the first of the two above-mentioned streams that assumes unconstrained suppliers, the search for the best dual sourcing strategy revolves around the dilemma of when to use a faster and necessarily more expensive supplier to compensate for a slow response by a cheaper supplier. Several papers discuss the setting in which the lead times of the two suppliers differ by a fixed number of periods (Daniel 1963, Fukuda 1964, Bulinskaya 1964, Whittmore & Saunders 1977). More specifically, Fukuda (1964) shows the optimality of the two-level base-stock policy for the case where the two suppliers' lead times differ by one period (so-called consecutive lead times). The policy instructs that first the order with the fast supply source is placed so that the inventory position is raised to the first base-stock level, and then the slow supply source is used to raise the inventory position to the second base-stock level. For general, nonconsecutive, lead times, Whittmore & Saunders (1977) found the optimal policy to be quite complex and lacking structure. To tackle this problem, Veeraraghavan & Scheller-Wolf (2008) propose the so-called Dual-Index order-up-to Policy (DIP), and show that it performs close to the optimal policy. Arts et al. (2011) extend the analysis of DIP to accommodate stochastic regular lead times, and provide an efficient optimization algorithm based on the separability result. An extension to DIP is suggested by Arts & Kiesmüller (2013), where the policy has three base-stock levels. Allon & Van Mieghem (2010) and Janakiraman et al. (2014) study Tailored Base-Surge policy where the quantity procured from the slower regular supplier is constant, and the faster supplier is used to manage demand surges.

Within the second stream of papers different approaches are used to model the supply capacity constraints: fixed capacity limits, random yield, Bernoulli all-or-nothing type supply disruptions, and random capacity limits. The first to assume a capacitated faster supplier is Daniel (1963). He showed that a modified two-level base-stock policy is also optimal in this case, where ordering with the faster supplier is up to the first base-stock level if the fixed capacity limit allows it. Yazlali & Erhun (2009) impose minimum and maximum fixed capacity limits on the availability of the two supply sources, and again show that the two-level base-stock policy is optimal when lead times are consecutive. Similarly, Xu (2011) assumes fixed capacity bounds on the cumulative order quantity to the two suppliers. He proves the optimality of a myopic ordering policy for a setting where the faster supply mode provides instant delivery, while the slower delivers an order one period later.

Despite the fact that random yield models essentially differ from the uncertain capacity models even for the simplest single-period single-supplier inventory model, there are relevant insights to be learned from studying the random yield dual sourcing models. Chen et al. (2013) consider price-dependent demand model where sourcing is done from two unreliable suppliers with random yield and negligible lead times. They show that the reorder point exists for each supplier such that an order is placed for almost every inventory level below the reorder point. They also characterize the conditions under which the optimal order quantities are decreasing in the inventory level, and a strict reorder point would apply. Ju et al. (2015) extend this work for general lead times, and develop a heuristic based on the before-mentioned dual-index order-up-to policy approach.

Gong et al. (2014) and Zhu (2015) studied a special case of random yield in the form of all-or-nothing supply disruptions. Both models assume a dual sourcing setting equal to ours in terms of lead time assumption; with the immediate delivery from the faster supplier, and the delay of one period in the replenishment from the slower supplier. The first paper assumes random supply disruptions that follow a Markov process, and prove that the optimal policy is a reorder type policy for both suppliers, where the target inventory positions are increasing in the starting inventory level. Due to Markovian disruptions, reorder points also depend on both suppliers' delivery capabilities in the previous period. Zhu (2015) study additional scenarios that differ depending on which supplier is facing supply disruptions, and whether supply disruption information is available at the time the orders are placed. They confirm that the rather complex optimal policy is of reorder type for both suppliers in the case where faster supplier is the only one facing disruptions. When the information is available, the optimal policy is a base-stock type policy. Observe that all-or-nothing supply disruptions can also be considered as a special case of random capacity limits.

To our knowledge, so far Yang et al. (2005) are the only one who considered dual sourcing model with random supply capacity that is consistent with the approach by Ciarallo et al. (1994) in a single-source setting. They consider a Markovian capacity constraint on the fast supply mode, however the available capacity can be observed prior to the time orders are placed with the two supply channels. With some additional restrictive assumptions, this leads to a reduction in complexity of the optimal policy compared to the two above mentioned supply disruption models (despite the more elaborate supply capacity process). For the case where no fixed ordering costs are assumed, they show that the optimal policy is the capacity-dependent modified-base stock policy for both suppliers. For the stochastically

monotone Markov process (and consequently in the case of deterministic capacities) both optimal base-stock levels decrease in the current capacity level. In our model, we assume that both ordering decisions in a particular period are made with no insight into current supply capacity availability and demand. Thus, the optimal policy has a more complex structure, which also led us to consider deriving a simplified approximate policy. We provide a more elaborate comment on how the differences in the modeling assumptions affect the structure of the optimal policy in Section 3.

1.2 Statement of contribution

The contributions of this study are threefold. First, this study is the first to consider dual sourcing inventory problem with random supply capacity at the faster supplier, where the supply capacity is not observed prior to placing orders to both supply sources in a particular period. We show that the optimal policy can be characterized as: a reorder type policy for the faster supply source, where the target inventory is increasing in the inventory position before ordering; a base-stock type policy for ordering with the slower supply source, where the optimal base-stock level depends on the size of the order placed with the faster supplier. Second, due to the complexity of the optimal policy, we develop an extended myopic policy of a base-stock type. The order with the faster supplier is placed up to the inventory target that corresponds to the solution of the uncapacitated newsvendor problem. We assess the accuracy of the myopic heuristic by means of numerical analysis where the results show that the costs of the myopic policy are very close to the optimal costs. Lastly, to understand the effect of the system parameters on the benefits of a dual sourcing option, we perform a numerical analysis. We show that even if the faster supply source is not fully utilized, the slower supply source is used extensively to improve the reliability of the supply process, which leads to lower costs.

The remainder of the paper is organized as follows. We present the model formulation in Section 2. In Section 3, the general structure of the optimal policy is characterized and the myopic policy is developed. In Section 4, we present the results of a numerical study to determine the optimal costs of dual sourcing, to study the order allocation between the supply modes and quantify the benefits of dual sourcing over single sourcing. Finally, we summarize our findings and suggest possible extensions in Section 5.

2. Model formulation

In this section, we give the notation and the model description. The faster, zero-lead-time, supply source is stochastic capacitated where the supply capacity is exogenous to the manufacturer and the actual capacity realization is only revealed upon replenishment. The slower supply source is modeled as uncapacitated with a fixed one period lead time. The demand and supply capacity of the faster supply source are assumed to be stochastic non-stationary with known distributions in each time period, although independent from period to period. In each period, the customer places an order with either an unreliable, or a reliable supply mode, or both.

Presuming that unmet demand is fully backordered, the goal is to find the optimal policy that would minimize the inventory holding costs and backorder costs over finite planning horizon T . We intentionally do not consider any product unit price difference and fixed ordering costs as we are chiefly interested in studying the trade-off between the capacity uncertainty associated with ordering from a faster supply source and the delay in the replenishment from a slower source. Any fixed costs would make the dual sourcing strategy less favorable, and the difference in the fixed costs related to any of the two ordering channels would result in a relative preference of one channel over the other. The notation used throughout the paper is summarized in Table 1 and some is introduced when needed.

We assume the following sequence of events. (1) At the start of the period, the manager reviews the inventory position before ordering x_t , where $x_t = \tilde{x}_t + v_{t-1}$ is the sum of the net inventory \tilde{x}_t and places order v_{t-1} with a slower supply source made in the previous period. (2) Order z_t is placed with a faster supply source and order v_t with a slower supply source. For the purpose of the subsequent analysis, we define two inventory positions after the order placement. First, after placing order z_t the inventory position is raised to y_t , $y_t = x_t + z_t$, and subsequently, after order v_t is placed, the inventory position is raised to w_t , $w_t = x_t + z_t + v_t$. Observe that it makes no difference in which sequence the orders are actually placed as long as both are placed before the current period's capacity of the fast supply source q_t and demand d_t are revealed. (3) The order with the slower supply source from the previous period v_{t-1} and the current period's order z_t are replenished. The inventory position can now be corrected according to the actual supply capacity realization $w_t - (z_t - q_t)^+ = x_t + \min(z_t, q_t) + v_t$, where $(z_t - q_t)^+ = \max(z_t - q_t, 0)$. (4) At the end of the period, demand d_t is observed and satisfied through on-hand inventory; otherwise, it is

Table 1: Summary of the notation.

T	: number of periods in the finite planning horizon
c_h	: inventory holding cost per unit per period
c_b	: backorder cost per unit per period
α	: discount factor ($0 \leq \alpha \leq 1$)
\tilde{x}_t	: net inventory before ordering in period t
x_t	: inventory position before ordering in period t
y_t	: inventory position after ordering from a faster capacitated supply source in period t
w_t	: inventory position after ordering from a slower uncapacitated supply source in period t
z_t	: order placed with the faster supply source in period t
v_t	: order placed with the slower supply source in period t
d_t, D_t	: actual realization and random variable denoting demand in period t
$g_t(d_t)$: probability density function of demand in period t
$G_t(d_t)$: cumulative distribution function of demand in period t
q_t, Q_t	: actual realization and random variable denoting the available supply capacity of the faster capacitated supply source in period t
$r_t(q_t)$: probability density function of the supply capacity of the faster supply source in period t
$R_t(q_t)$: cumulative distribution function of the supply capacity of the faster supply in period t

backordered. Inventory holding and backorder costs are incurred based on the end-of-period net inventory, $\tilde{x}_{t+1} = y_t - (z_t - q_t)^+ - d_t$. Correspondingly, the expected single-period cost function is defined as $C_t(x_t, z_t) = \alpha E_{Q_t, D_t} \tilde{C}_t(\tilde{x}_{t+1}) = \alpha E_{Q_t, D_t} \tilde{C}_t(x_t + \min(z_t, Q_t) - D_t)$, where $\tilde{C}_t(\tilde{x}_{t+1}) = c_h(\tilde{x}_{t+1})^+ + c_b(-\tilde{x}_{t+1})^+$. The minimal discounted expected cost function that optimizes the cost over a finite planning horizon T from period t onward, starting in the initial state x_t , can be written as:

$$f_t(x_t) = \min_{z_t \geq 0, v_t \geq 0} \{C_t(x_t, z_t) + \alpha E_{Q_t, D_t} f_{t+1}(x_t + \min(z_t, Q_t) + v_t - D_t)\}, \text{ for } 1 \leq t \leq T \quad (1)$$

and the ending condition is defined as $f_{T+1}(\cdot) \equiv 0$.

3. Characterization of the near-optimal myopic policy

In this section, we first characterize the optimal policy for the non-stationary demand and supply capacity setting. We show that the structure of the optimal policy is relatively complex, where the order with the faster supply source depends on the inventory position before ordering, while the order with the slower supply source is placed up to a state-dependent base-stock level. We continue by studying the stationary setting by introducing the myopic policy where the myopic orders are the solutions to the extended single period problem, and show the properties of the two myopic base-stock levels¹. We conduct the

numerical analysis to show that the costs of the myopic policy provide a very accurate estimate of the optimal costs. See the Appendix for proofs of the following propositions.

In the literature review, we refer to a series of papers studying the dual sourcing inventory problem with consecutive lead times. The two-level base-stock policy characterizes the structure of the optimal policy in all of them, both in the case of uncapacitated supply sources and in the case where one or both supply sources exhibit the fixed capacity limit. However, when studying the properties of the cost functions given in (2)-(4), we provide an example, for which it turns out that the optimal cost function is not convex. We also show that the optimal inventory position after ordering with the faster supplier \hat{y}_t is not independent of the inventory position before ordering x_t , and therefore cannot be characterized as the optimal base-stock level.

As single-period costs C_t in period t are not influenced by order v_t , we can rewrite (1) in the following way:

$$f_t(x_t) = \min_{z_t \geq 0} \left\{ C_t(x_t, z_t) + \min_{v_t \geq 0} \alpha E_{Q_t, D_t} f_{t+1}(x_t + \min(z_t, Q_t) + v_t - D_t) \right\}, \text{ for } 1 \leq t \leq T, \quad (2)$$

which now enables us to introduce auxiliary cost functions $J_t(x_t, z_t)$ and $H_t(x_t, z_t, v_t)$:

$$J_t(x_t, z_t) = C_t(x_t, z_t) + \min_{v_t \geq 0} \alpha E_{Q_t, D_t} f_{t+1}(x_t + \min(z_t, Q_t) + v_t - D_t), \text{ for } 1 \leq t \leq T \quad (3)$$

$$H_t(x_t, z_t, v_t) = \alpha E_{Q_t, D_t} f_{t+1}(x_t + \min(z_t, Q_t) + v_t - D_t), \text{ for } 1 \leq t \leq T \quad (4)$$

For the order placed with the faster supplier, we define \hat{z}_t as the minimizer of $J_t(x_t, z_t)$, and correspondingly \hat{y}_t as the optimal inventory position that \hat{z}_t is placed up to. Similarly, we define the optimal order with the slower supplier \hat{v}_t that minimizes $H_t(x_t, z_t, v_t)$, and \hat{w}_t as the optimal inventory position that \hat{v}_t is placed up to. Based on the above, we give the structure of the optimal policy under stochastic non-stationary demand and supply capacity in the following proposition:

Proposition 1. *The following holds for all t :*

1. $\hat{y}_t(x_t)$ depends on x_t .
2. $\hat{w}_t(z_t)$ is a state-dependent base-stock level.
3. Under the optimal policy, the inventory position $w_t(x_t)$ after placement of optimal orders \hat{z}_t and \hat{v}_t is:

$$w_t(x_t) = \begin{cases} x_t, & \hat{w}_t(0) \leq x_t, & \hat{z}_t = 0, \hat{v}_t = 0, \\ \hat{w}_t(0), & \hat{y}_t(x_t) \leq x_t < \hat{w}_t(0), & \hat{z}_t = 0, \hat{v}_t = \hat{w}_t(0) - x_t, \\ \hat{w}_t(\hat{z}_t), & x_t < \hat{y}_t(x_t), & \hat{z}_t = \hat{y}_t(x_t) - x_t, \hat{v}_t = \hat{w}_t(\hat{z}_t) - \hat{y}_t(x_t). \end{cases} \quad (5)$$

Part 1 states that the optimal policy from the faster supplier is not of a base-stock type, but rather a reorder policy where the target inventory position $\hat{y}_t(x_t)$ depends on the inventory position before ordering x_t . In Part 2, $\hat{w}_t(z_t)$ is a base-stock level, state-dependent on the order z_t placed with the faster supplier, and the optimal policy from the slower supplier is a state-dependent base-stock policy.

The observation of the dependency of \hat{y}_t on x_t is based on the study of the numerical example presented in Figure 2 and in Table 2. For this example, we show in the Proof of Proposition 1 in the Appendix that this implies that the optimal cost function f_t is not convex.

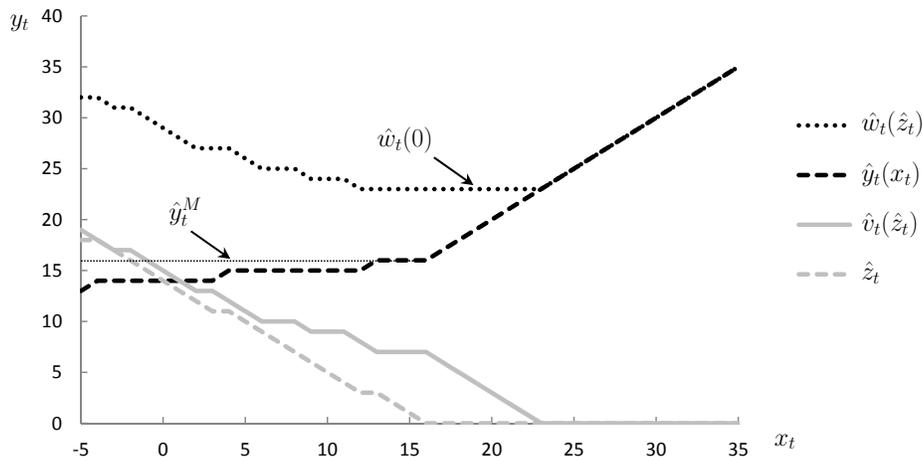


Figure 2: The optimal inventory positions after ordering and the optimal orders.

For a clearer representation of the optimal policy as given in (5), we depict the optimal order sizes depending on the initial inventory position x_t and the corresponding inventory positions after ordering in Figure 2. For comparison, we also plot the myopic base-stock level \hat{y}_t^M that optimizes the single period cost function C_t , defined as:

$$\hat{y}_t^M = G_t^{-1} \left(\frac{c_b}{c_b + c_h} \right). \quad (6)$$

Observe that \hat{y}_t^M corresponds to the solution of the uncapacitated single period newsvendor problem. The details are provided in Part 3 of Lemma 1 in Appendix.

For $x_t < \hat{y}_t^M$ we order up to \hat{y}_t^M for small optimal orders with a faster supplier \hat{z}_t , while for higher \hat{z}_t , $\hat{y}_t(x_t)$ lies below \hat{y}_t^M . For $x_t \geq \hat{y}_t^M$ no \hat{z}_t is placed. Similarly, looking at the optimal order with the slower supply source \hat{v}_t , it is only placed if $x_t \leq \hat{w}_t(0)$, and at $\hat{y}_t(x_t) \leq x_t < \hat{w}_t(0)$ the inventory position x_t is increased to a constant base-stock level

$\hat{w}_t(0)$. For $x_t < \hat{y}_t(x_t)$ the size of the order with the slower supply source \hat{v}_t depends on the anticipated supply shortage of the faster supply source, thus the state-dependent base-stock level $\hat{w}_t(\hat{z}_t)$ depends on the size of the order \hat{z}_t placed with the faster supply source.

Based on the above results, the optimal policy instructs that the order with the slower supply source should compensate for the probable supply shortage of the faster supply source. The goal here is to bring the next period's starting inventory position to the optimal level (where the demand uncertainty also needs to be taken into account). In the next period, this enables the decision-maker to place the order with the faster supply source in such a way that the period's inventory costs are minimized given the supply capacity uncertainty and demand uncertainty.

Observe that in the case of Yang et al. (2005), in addition to knowing the current period's capacity limits, they assume that demand is realized before the order with a slower supplier is placed. The single period cost function is thus convex, and due to time-separate realizations of capacity and demand levels within a period, the optimal policy is simplified to the two-level modified base-stock policy. For the case of deterministic capacities they further assume that the slower supply source is more expensive than the faster source, which means that an order is not placed with the slower supplier unless capacity unavailability is observed at the faster supplier. As they point out, intuitively this result does not hold when random capacities would be assumed. In a numerical experiment in Section 4.2, we show that the even if the faster supply source is not fully utilized, the slower supply source is used extensively to improve the reliability of the supply process, which leads to lower costs.

Next, we focus on a stationary demand and supply capacity setting. Based on the insights obtained studying the optimal inventory policy, one might suggest that the myopic base-stock level \hat{y}_t^M is a good approximation for $\hat{y}_t(x_t)$. To study this, we introduce the notion of the extended myopic policy, which optimizes the so-called extended single period problem in every period. In the following proposition, we show that the proposed extended myopic policy can be characterized as the two-level base-stock policy, where the myopic order \hat{z}_t^M is placed up to the first base-stock level \hat{y}_t^M , and the myopic order $\hat{v}_t^M(\hat{z}_t^M)$ is placed up to the second state-dependent base-stock level $\hat{w}_t^M(\hat{z}_t^M)$ in period t .

The sequence of ordering decisions and resulting events associated with the extended myopic policy are presented in Figure 3. After incurring the relevant inventory costs at the end of period $t - 1$, we start period t at the net inventory position before ordering \tilde{x}_t . The goal is to optimize the expected single period costs $E_{Q_t, D_t} \tilde{C}_t(\tilde{x}_{t+1})$ in period t , where

period t by searching for the optimal orders \hat{v}_{t-1}^M and \hat{z}_t^M that directly affect the costs in this period. Due to its myopic nature, the policy disregards the influence of the ordering decision on the costs in the subsequent future periods.

The myopic equivalent to the optimal cost function as given in (2) can be written in the following way:

$$f_t^M(x_t) = C(x_t, \hat{z}_t^M) + \alpha E_{Q_t, D_t} f_{t+1}^M(x_t + \min(\hat{z}_t^M, Q_t) + \hat{v}_t^M - D_t), \text{ for } 1 \leq t \leq T, \quad (8)$$

with the ending condition $f_{T+1}^M(\cdot) \equiv 0$.

In Proposition 3 we derive some monotonicity properties of the state-dependent myopic base-stock level $\hat{w}^M(\hat{z}^M)$. Part 1 suggests that for the pair of optimal orders $\hat{z}^M = \{\hat{z}_1^M, \hat{z}_2^M\}$ placed with the faster supply source in any period, the decision-maker has to raise the base-stock level $\hat{w}^M(\hat{z}_1^M)$ above $\hat{w}^M(\hat{z}_2^M)$, when $\hat{z}_1^M \geq \hat{z}_2^M$. From this $\hat{v}^M(\hat{z}_1^M) \geq \hat{v}^M(\hat{z}_2^M)$ follows directly given the fact that \hat{z}_1^M and \hat{z}_2^M were placed up to the constant \hat{y}^M . With an increase in order \hat{z} , the probability of a supply shortage at the faster supply source increases. To compensate for this, it is optimal that a higher order \hat{v}^M is placed with the slower supply source. In Part 2, we show that the level of compensation to account for the additional supply uncertainty (due to the higher \hat{z}^M placed) should at most be equal to the difference between the optimal order sizes η .

Proposition 3. *The following holds for all t :*

1. For any pair $\hat{z}^M = \{\hat{z}_1^M, \hat{z}_2^M\}$, where $\hat{z}_1^M \geq \hat{z}_2^M$: $\hat{w}^M(\hat{z}_1^M) \geq \hat{w}^M(\hat{z}_2^M)$.
2. For $\eta \geq 0$: $\hat{w}^M(\hat{z}^M + \eta) - \hat{w}^M(\hat{z}^M) \leq \eta$.

Observe that the constant base-stock level \hat{y}^M is the solution to the multiperiod uncapacitated single source inventory model. For the capacitated single supplier model, Ciarallo et al. (1994) show that while \hat{y}^M optimizes a single period problem, it is far from optimal in a multiperiod setting. However, we show that in the dual sourcing model under consideration the appropriate combination of the two myopic base-stock levels provides a very good substitute for the optimal target inventory levels.

In Table 2, we present the optimal and the myopic inventory positions for a chosen system setting, described with the following set of parameters: $T = 12$, $c_h = 1$, $c_b = 20$, $\alpha = 0.99$, the utilization² of the faster supply source $Util = 1$, discrete uniform distribution was used to model stochastic demand and supply capacity with the coefficients of variation $CV_D = 0.49$

and $CV_Q = 0.61$. Numbers in bold are used to denote the cases, where the myopic inventory positions and orders differ from the optimal ones. The results for $\hat{y}(x)$ confirm that \hat{y} is a function of x . We see that with increasing x , $\hat{y}(x)$ is increasing, approaching the myopic \hat{y}^M . For lower x , the optimal policy suggests that it is not optimal anymore to place \hat{z} up to \hat{y}^M . This can be attributed to the increased uncertainty about the replenishment of \hat{z} . To compensate for the relatively smaller \hat{z} placed, the optimal decision is to rely more heavily on the slower supplier, by increasing \hat{v} above \hat{v}^M .

Table 2: The optimal and the myopic inventory positions and orders.

x	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23
$\hat{y}(x)$	14	14	14	14	15	15	15	15	15	15	15	15	15	16	16	16	16							
\hat{y}^M	16	16	16	16	16																			
\hat{z}	14	13	12	11	11	10	9	8	7	6	5	4	3	3	2	1	0							
\hat{z}^M	16	15	14	13	12	11	10	9	8	7	6	5	4	3	2	1	0							
$\hat{w}(\hat{z})$	29	28	27	27	27	26	25	25	25	24	24	24	23	23	23	23	23	23	23	23	23	23	23	23
$\hat{w}^M(\hat{z}^M)$	31	30	29	28	27	27	26	25	25	25	24	24	24	23	23	23	23	23	23	23	23	23	23	23
$\hat{v}(\hat{z})$	15	14	13	13	12	11	10	10	10	9	9	9	8	7	7	7	7	6	5	4	3	2	1	0
$\hat{v}^M(\hat{z}^M)$	15	14	13	12	11	11	10	9	9	9	8	8	8	7	7	7	7	6	5	4	3	2	1	0

Despite the fact that the myopic base-stock levels generally differ from the optimal inventory positions, we now proceed to show that the costs of the myopic policy provide a very close estimate of the optimal costs.

We carried out a numerical experiment by solving the optimal dynamic programming formulation given in (1) and its myopic counterpart given in (8), and compared their performance. We used the following set of input parameters: $T = 12$, $c_h = 1$, $\alpha = 0.99$; discrete uniform distribution and truncated normal distribution is used to model stochastic demand and supply capacity. Throughout the experiments we varied the utilization of the faster supply source, $Util = \{0, 0.5, 0.67, 1, 2, \infty\}$, the per unit backorder cost $c_b = \{5, 20, 100\}$, and the coefficient of variation of demand, $CV_D = \{0, 0.14, 0.26, 0.37, 0.49, 0.61, 0.80, 1.00\}$, and the supply capacity of the faster supply source, $CV_Q = \{0, 0.14, 0.26, 0.37, 0.49, 0.61, 0.80, 1.00\}$, where the CV s do not change over time³. The accuracy of the proposed myopic policy was therefore tested on 1200 scenarios.

In Figure 4, we provide the histogram of the relative differences in costs of the optimal and myopic policy. Additionally, we provide the relationships between the main input parameters and the performance of the myopic policy.

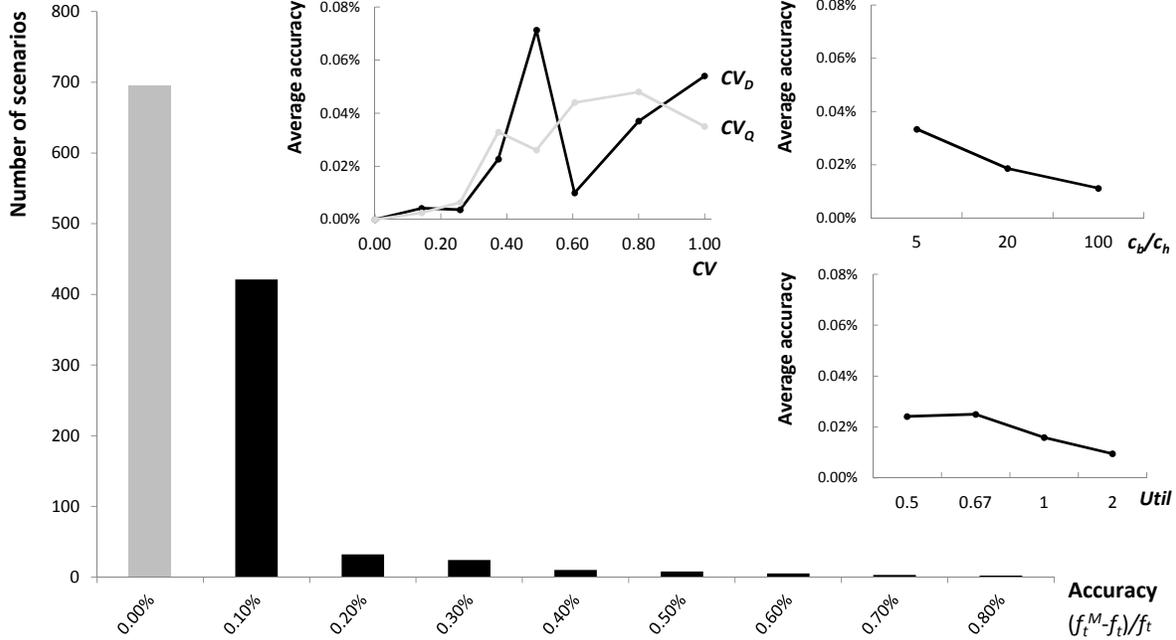


Figure 4: The relative difference in costs between the optimal and the myopic policy.

Observe first that in 58% of the analyzed scenarios the costs of the myopic policy are equal to the optimal costs (represented with a lightly shaded bar). The highest observed relative cost difference across all scenarios is 0.81%. A careful study of suboptimal scenarios has not revealed a clear pattern that would point out the characteristics of these scenarios. The average accuracy is in general decreasing with increasing demand and supply capacity uncertainty, which is not unexpected. The average accuracy is higher when the utilization of the system is lower, however the change is minor. The effect of the change in backorder costs relative to holding costs turns out to be the highest, where the average accuracy of the myopic policy is increasing with increasing backorder costs. Studying the above mentioned relationships, we have not observed any relevant differences between the scenarios where uniform and normal distribution was used to model stochastic demand and supply capacity. Finally, although we pointed out the general characteristics observed when studying the average accuracy figures, the accuracy of an individual scenario might still greatly differ from these, thus it is hard to predict the accuracy of the myopic policy in a particular scenario.

We conclude this section with the discussion on the optimality of the myopic policy. The high accuracy of the myopic policy might suggest that myopic policy could be optimal under certain assumptions. Due to myopic nature of the proposed approximate policy, the policy

will generally not perform well in a non-stationary demand and supply capacity setting. The reason for this is that the myopic policy cannot account for the expected mismatches between the demand and the available supply capacity in the future periods. While this is possible for the anticipated mismatch in the following period through a proper placement of order v_t , the mismatches in the remaining periods cannot be accounted for.

Despite the close-to-optimal performance of the myopic policy in a stationary setting, the policy does not exhibit the necessary properties of the myopic optimality. Heyman & Sobel (1984) point out the following two conditions (among others) for the myopic policy to be optimal: single period cost function needs to be additively separable on the state and action, and that the myopic policy guarantees that the set of consistent states (from which the optimal action can be taken) are visited in the next period. It is clear that the single period cost function C_t does not satisfy the first condition, as it does not depend additively on x_t and y_t . While for \hat{w}^M it does hold that the optimal action will again be feasible in the next period, this is not the case for \hat{y}^M as the system can easily end up in $x_t > \hat{y}^M$ from which the optimal action is infeasible. Despite the fact that the myopic base-stock levels differ from the optimal inventory positions after ordering, the costs of the myopic policy are very close to the optimal costs. The accuracy can be attributed to the possibility of balancing the orders placed with the two supply sources, which in most situations leads to the optimal costs.

4. Value of dual sourcing

In this section, we present the results of the numerical analysis, where we investigate the effect of different system parameters on: (1) the optimal costs of dual sourcing; (2) the relative utilization of the two supply sources; and (3) the benefits of dual sourcing compared to single sourcing from either supply source. Numerical calculations were carried out with the use of the near-optimal myopic policy given in (8). We used the same set of input parameters as given in Section 3, and a discrete uniform distribution to model stochastic demand and supply capacity.

4.1 *Optimal costs of dual sourcing*

We start by studying the optimal costs under different system parameters and by comparing the performance of the dual sourcing model with the two base cases: the worst case in which the supply is only available through the slower supply source (the faster supply source

is unavailable, $Util = \infty$), and the best case in which the faster supply source is assumed to be uncapacitated, and thus becomes fully available ($Util = 0$). The results are presented in Figure 5 and Table 3.

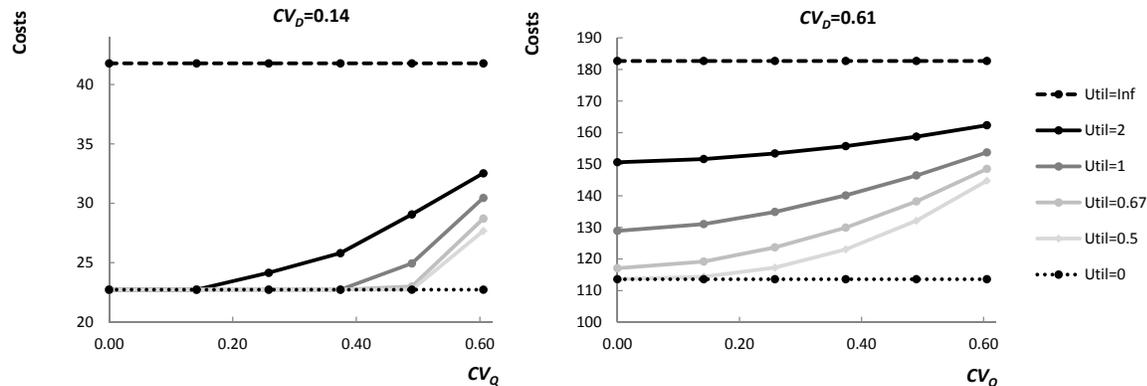


Figure 5: Optimal system costs.

As expected, the system costs rise with increasing demand and capacity uncertainty, as well as with increasing utilization of the faster supply source. The sensitivity of costs is highest in the case of increasing CV_D . Moreover, it is at a high CV_D where the costs are the most sensitive to an increase in CV_Q and $Util$. For relatively low CV_D and up to moderate CV_Q , the system works with minimal costs even for the cases of high utilization. In this setting, the capacity and demand realizations are easy to anticipate, and sourcing from the slower supply source to compensate for the lack of supply capacity availability with the faster supply source does not increase the costs compared to the best case. However, for a high CV_D , due to the longer lead time, ordering with the slower supply source becomes riskier as the exposure to demand uncertainty increases. This leads to a relatively high increase in costs already for moderate utilizations.

4.2 Relative utilization of the two supply sources

To investigate to what extent either of the two supply sources is utilized under different system parameters, we present the calculations for the share of inventories replenished from a faster capacitated supply source in Figure 6. The higher the CV_Q and $Util$, the lower the share of inventories replenished from the faster supply source. A high CV_Q leads to more probable shortages in replenishment from the faster source and there is thus a greater need to compensate for these shortages by placing a bigger share of orders with the slower source. However, as an order with the slower supply source needs to be placed one period earlier, one

cannot take advantage of learning about the demand realization in the current period. The accuracy of this compensation strategy suffers when CV_D is increasing, which is reflected in a high increase in costs.

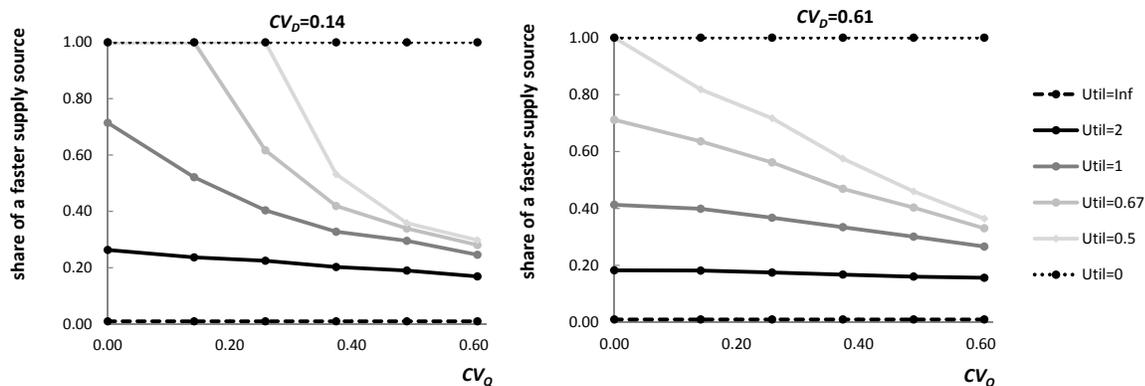


Figure 6: The share of inventories replenished from the faster capacitated supply source.

Although utilization of the faster supply source represents a hard limit on the size of possible replenishment, the actual utilization falls well below its maximum (theoretical) availability. For instance, for $Util = 1$, the share of inventories sourced from the faster source is at 71%, and decreases further to 41% as demand uncertainty increases (even when $CV_Q = 0$). Still, observe that up to moderate $Util$ and low CV_Q the sourcing is done almost exclusively through the faster source (100% for $CV_D = 0.14$), which suggests that the faster supply source is actually sufficiently available in this setting. While we would expect increasing demand uncertainty to make sourcing from the slower supply source less favorable, somewhat counter-intuitively the opposite holds when CV_Q is relatively low. In a setting with high CV_D more safety stock is required to avoid backorders. To keep inventories at this relatively higher level, occasionally a large order with the faster supply source needs to be placed as a response to the high demand realization (which is more probable due to the high CV_D). However, this might lead to a supply shortage even for relatively low system utilizations, and consequently to costly backorders. It therefore makes sense to replenish some inventories from the slower supply source, so that we start the next period with a higher starting inventory position. While this can lead to higher inventory holding costs, it is still less costly than incurring backorder costs.

Next, we study the effect of the cost structure on the actual utilization of the two supply sources. When backorder costs increase relative to holding costs, increasing the share of sourcing from the slower supply source is optimal (Figure 7). This is-in-line with the above

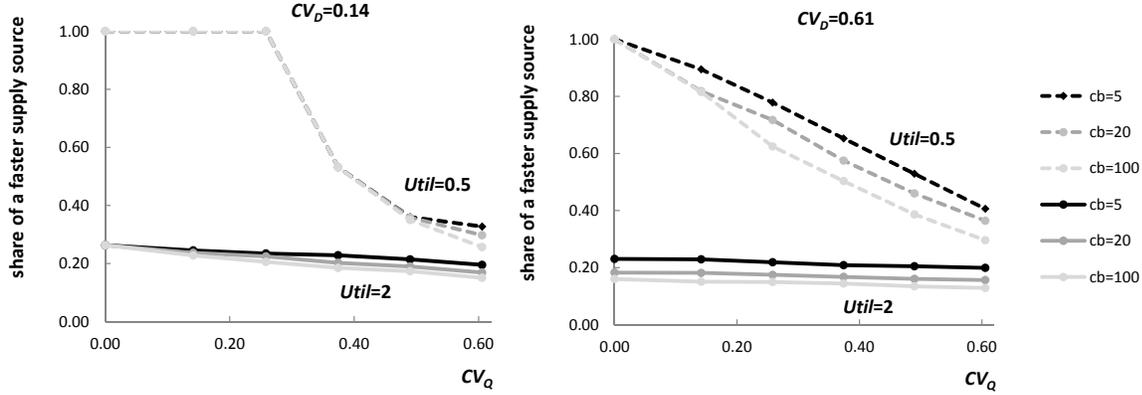


Figure 7: The share of inventories replenished from the faster capacitated supply source.

reasoning where, through sourcing from the slower supplier, one can avoid backorders while incurring higher inventory holding costs instead. The share of inventories replenished from a slower supplier also increases relatively more for higher CV_Q , particularly when CV_D is also high, as the need for a more reliable supply source increases. This effect is higher for lower utilizations, which can be attributed to the fact that at high $Util$ ordering through the slower source is already extensively used (around 80% of inventories is replenished from the slower source) and despite the increased backordering costs, increasing the exposure to the slower supply source further only marginally increases the benefits.

4.3 Benefits of dual sourcing

Lastly, the benefits of dual sourcing are assessed relative to the performance of the two single sourcing cases. To quantify the benefits of dual sourcing, we define the *relative value* of dual sourcing, $\%V_{DS}$, as the relative cost savings over a single-source setting in which either a faster (FS) or slower (SS) supply source is used:

$$\%V_{DS/j} = \frac{f_t^j - f_t^{DS}}{f_t^j}, \quad (9)$$

where $j \in \{FS, SS\}$, and f_t^{FS} , f_t^{SS} and f_t^{DS} represent the corresponding cost functions from (1) that apply to a chosen decision policy.

We present the results on the relative value of dual sourcing compared to single sourcing in Table 3. In the table, numbers in bold are used to denote the single supply source with the lower costs in each of the settings.

Looking at the benefits of dual sourcing over sourcing from a faster capacitated supply source, it is expected that its utilization largely influences the extent of the savings that

Table 3: The optimal costs and the relative value of dual sourcing.

$Util$	CV_Q	CV_D	Costs (f_t^{DS})				% $V_{DS/FS}$				% $V_{DS/SS}$			
			0.00	0.14	0.37	0.61	0.00	0.14	0.37	0.61	0.00	0.14	0.37	0.61
∞			0.0	41.8	112.1	182.7	100.0	93.8	85.3	79.1	-	0.0	0.0	0.0
2	0.00		0.0	22.7	82.9	150.6	100.0	93.6	81.6	73.0	-	45.6	26.1	17.6
2	0.14		0.0	22.7	84.1	151.6	100.0	93.8	81.5	72.9	-	45.6	25.0	17.0
2	0.37		0.0	25.8	89.5	155.7	100.0	93.4	80.9	72.7	-	38.2	20.1	14.8
2	0.61		0.0	32.5	96.8	162.3	100.0	92.4	80.4	72.5	-	22.2	13.6	11.1
1	0.00		0.0	22.7	69.5	128.9	-	66.8	62.0	56.5	-	45.6	38.1	29.4
1	0.14		0.0	22.7	71.3	131.1	100.0	75.7	63.3	56.8	-	45.6	36.5	28.3
1	0.37		0.0	22.7	78.3	140.1	100.0	87.2	68.5	59.1	-	45.6	30.2	23.3
1	0.61		0.0	30.5	91.1	153.7	100.0	88.8	71.9	61.9	-	27.1	18.8	15.8
0.67	0.00		0.0	22.7	68.2	117.1	-	0.0	1.0	15.3	-	45.6	39.2	35.9
0.67	0.14		0.0	22.7	68.2	119.2	-	0.0	8.5	19.6	-	45.6	39.2	34.8
0.67	0.37		0.0	22.7	72.2	129.9	100.0	68.7	44.8	36.5	-	45.6	35.6	28.9
0.67	0.61		0.0	28.7	87.6	148.5	100.0	84.7	62.1	49.8	-	31.3	21.9	18.7
0.5	0.00		0.0	22.7	68.2	113.6	-	0.0	0.0	0.0	-	45.6	39.2	37.8
0.5	0.14		0.0	22.7	68.2	114.4	-	0.0	0.0	0.2	-	45.6	39.2	37.4
0.5	0.37		0.0	22.7	69.4	123.1	100.0	37.1	20.5	16.5	-	45.6	38.1	32.6
0.5	0.61		0.0	27.7	85.2	144.8	100.0	81.4	54.7	40.7	-	33.8	24.0	20.7
0			0.0	22.7	68.2	113.6	0.0	0.0	0.0	0.0	-	45.6	39.2	37.8

can be achieved through the dual sourcing. For an overutilized system, % $V_{DS/FS}$ approaches 100%, while for moderate utilizations the relative savings still range from 20% to over 50%. The lack of availability and reliability of supply from a faster source inherently exposes the system to supply shortages, and leads to substantial backorder costs. For up to moderate $Util$ with high CV_Q , the average supply availability is sufficient, although the system is exposed to supply shortages due to the highly variable supply capacity of the faster supply source. This again results in considerable benefits from utilizing the slower uncapacitated supply source as an alternative. It is the settings with up to moderate $Util$ and low CV_Q where supply solely through the faster supply source is adequate to run the system at close to minimum costs.

Sourcing exclusively through the slower supply source is optimal in the setting where there is no demand uncertainty (for $CV_D = 0$, the cases denoted with "-" represent the settings in which the costs of using a single supply mode are zero), and obviously when the faster supply source is completely unavailable ($Util = \infty$). For the remaining settings, we

observe that $\%V_{DS/SS}$ ranges from 15% to 45%, and is less sensitive to the changes in $Util$ and CV_Q than in the case of sourcing solely through the faster supply source, as expected. While utilizing the slower supplier successfully resolves the CV_Q and $Util$ related problems in the settings where the faster supplier's replenishment is inadequate (when $\%V_{DS/FS}$ is high), we see that considerable savings are still to be gained by partially sourcing through the faster supply source.

As already noted, demand uncertainty negatively affects the accuracy of using the slower yet reliable supply source to compensate for potential supply shortages of the faster source. It is therefore somewhat surprising that $\%V_{DS/SS}$ are decreasing with increasing CV_D . However, the opposite holds when looking at the absolute cost savings. These are increasing considerably with an increase of CV_D , particularly for up to moderate $Util$ and low CV_Q . This is in-line with our previous observations where such a set of parameter values characterizes the settings in which sourcing is performed almost exclusively through the faster supply source. $\%V_{DS/FS}$ is in general also decreasing with increasing CV_D , which is in agreement with our initial observation that a slower supply source is inherently more exposed to the risk of demand uncertainty. Here the absolute cost savings exhibit non-monotonic behavior with increasing CV_D . For settings with high CV_Q the slower reliable source is already extensively used and the absolute savings are decreasing with CV_D (more so for lower $Util$). However, this is not the case for low CV_Q (particularly at moderate $Util$), where the absolute savings are increasing with CV_D . This is the setting we already addressed in which limited sourcing from the slower supply source is beneficial for tackling instances of supply unavailability due to the placement of big orders in response to large demand variations. Obviously, the absolute cost reduction is significant in the case where we need to compensate for a general lack of supply availability (high $Util$ and high CV_Q), and smaller when the system only faces occasional supply shortages due to large demand uncertainty.

Finally, we may conclude with the observation that, besides the setting in which the faster supply source is close to being fully available and essentially reliable, the relative cost savings are considerable, ranging from just above 10% to close to 50%. Based on these results, we can conclude that dual sourcing can successfully resolve the issues related to an overutilized and uncertain availability of a faster supply source. In addition, a smart decision on how to split the orders between the two supply sources depending on the system setting, also substantially reduces the costs of demand uncertainty.

5. Conclusions

In this paper, we analyze a sourcing situation in which both sourcing from a nearshore and offshore supply source may be used. As the product purchase costs of sourcing from the two sources are more or less similar in various industry sectors nowadays, we argue that the trade-off is leaning towards finding the right balance between the limited supply availability of the nearshore supply options and the longer lead times of offshore sourcing.

Accordingly, we model a dual sourcing inventory model with a stochastic capacitated, zero-lead-time nearshore supplier and an uncapacitated offshore supplier with a longer lead time. We derive the dynamic programming formulation for the optimal inventory holding and backorder costs, where we show that the structure of the optimal inventory policy is not a two-level base-stock policy as it is the case in both unlimited and fixed capacity dual sourcing models studied in the literature. The optimal inventory position to which we order by placing an order with the stochastic capacitated faster supplier depends on the inventory position before ordering. To reduce the complexity of the policy we study the myopic policy, which proves to be a two-level base-stock policy, where the first myopic base-stock level corresponds to the solution of the single period newsvendor problem. We show that the myopic policy is not optimal, however it still provides a very accurate approximation of the optimal costs. For the majority of the instances studied the myopic policy turns out to be exact, while the largest relative error observed was 0.81%.

By means of a numerical analysis we reveal several managerial insights. The results show that the costs increase substantially when the capacity of the faster supply source is stochastic (already for moderate utilization of the faster supply source). The extent to which we rely on the faster supply source will therefore depend on the trade-off between its current availability and the benefits we obtain by delaying the order decision, and thus taking advantage of the latest demand information. However, somewhat counter-intuitively, we observe that in some settings the increasing demand uncertainty leads to an increase in sourcing from the slower supplier. We attribute this to the high costs related to incurring backorders, which results in a strategy primarily oriented to avoiding supply shortages, namely a strategy that instructs the decision-maker to prebuild inventories by utilizing the slower supply source.

In addition, we show that the relative cost savings of dual sourcing over single sourcing range from 10% to close to 50%. Exceptions to this are cases where the faster supply source is only moderately utilized and the demand uncertainty is low. This shows that, despite

the lack of a price incentive for placing orders with the slower offshore supplier (which is a common assumption in the literature), the offshore supply option is used to obtain considerable operational cost savings. Given the limitations and uncertainties that the move towards nearshoring brings in terms of supply availability, we show that a mixed strategy which allocates the orders between the two sourcing options is optimal.

Our results provide a potential justification for continuing to use offshore sources following the commencement of nearshoring operations, despite the fact that the costs are similar and the lead times are longer. Managers will need to take the capacity flexibility of the nearshore option into account when moving an operation nearshore.

Future research could relax the assumptions about the two delivery lead times. However, assuming a non-zero-lead time of the faster supply source greatly increases the complexity of the model as the inventory position becomes a function of multiple uncertain pipeline orders. Accordingly, also the myopic policy could be very complex (state-dependent first base-stock level) and probably less accurate, even in the case where the lead times of the two supply sources differ by one period.

Appendix

In Lemma 1 we provide the convexity results and the optimal solution to a single period cost function $C_t(x_t, z_t)$. We elect to suppress subscript t in the state variables for clarity reasons.

Lemma 1. *Let \hat{y}^M be the smallest minimizer of $C(x, z)$ to which the optimal order \hat{z}^M is placed, where $\hat{z}^M = \hat{y}^M - x$:*

1. $C(x, z)$ is convex in x .
2. $C(x, z)$ is quasiconvex in z , where:

$$\begin{aligned} \frac{\partial^2}{\partial z^2} C(x, z) &\geq 0, \text{ for } z \leq \hat{z}^M, \\ \frac{\partial}{\partial z} C(x, z) &\geq 0, \text{ for } z > \hat{z}^M. \end{aligned}$$

3. \hat{y}^M is the myopic base-stock level, where $\hat{y}^M = G^{-1} \left(\frac{c_b}{c_b + c_h} \right)$.

Proof: We first rewrite the single period cost function $C(x, z)$ as:

$$\begin{aligned}
C(x, z) &= \alpha(1 - R(z)) \left[c_b \int_{x+z}^{\infty} (d - x - z)g(d)dd + c_h \int_0^{x+z} (x + z - d)g(d)dd \right] \\
&+ \alpha c_b \int_0^z \int_{x+q}^{\infty} (d - x - q)g(d)ddr(q)dq \\
&+ \alpha c_h \int_0^z \int_0^{x+q} (x + q - d)g(d)ddr(q)dq.
\end{aligned} \tag{A1}$$

To prove Part 1, we derive the first partial derivative of (A1) with respect to x :

$$\begin{aligned}
\frac{\partial}{\partial x} C(x, z) &= \alpha(c_b + c_h)(1 - R(z)) \left(G(x + z) - \frac{c_b}{c_b + c_h} \right) \\
&+ \alpha(c_b + c_h) \int_0^z G(x + q)r(q)dq - \alpha c_b R(z),
\end{aligned} \tag{A2}$$

and the second partial derivative:

$$\frac{\partial^2}{\partial x^2} C(x, z) = \alpha(c_b + c_h) \left[(1 - R(z))g(x + z) + \int_0^z g(x + q)r(q)dq \right]. \tag{A3}$$

Since all terms in (A3) are nonnegative, Part 1 holds.

For Part 2, we obtain the first two partial derivatives of $C(x, z)$ with respect to z :

$$\frac{\partial}{\partial z} C(x, z) = \alpha(c_b + c_h)(1 - R(z)) \left(G(x + z) - \frac{c_b}{c_b + c_h} \right), \tag{A4}$$

$$\frac{\partial^2}{\partial z^2} C(x, z) = \alpha(c_b + c_h) \left[(1 - R(z))g(x + z) - r(z) \left(G(x + z) - \frac{c_b}{c_b + c_h} \right) \right]. \tag{A5}$$

Observe first that setting (A4) to 0 proves Part 3. For $z \leq \hat{z}^M$, $G(x + z) \leq \frac{c_b}{c_b + c_h}$ holds in the second term of (A5). Since the first term is always nonnegative, the function $C(x, z)$ is convex on the respected interval. For $z > \hat{z}^M$ this does not hold, although we see from (A4) that due to $G(x + z) > \frac{c_b}{c_b + c_h}$ and $1 - R(z) \rightarrow 0$ with $z \rightarrow \infty$, the function $C(x, z)$ is nondecreasing, which proves Part 2. Due to this, the $C(x, z)$ has a quasiconvex form, which is sufficient for \hat{y}^M to be its global minimizer. \square

Proof of Proposition 1:

Part 1 of Proposition 1 is based on the study of the numerical example presented in Figure 2 and in Table 2. Observe that for $x_t < \hat{y}_t^M$, it holds that \hat{y}_t depends on x_t . In fact, based on the example, we can see that $0 \leq d\hat{y}_t/dx_t \leq 1$ (or equivalently $-1 \leq d\hat{z}_t/dx_t \leq 0$) holds.

While we are not able to show the above result analytically, this observation has some implications for the properties of the optimal policy and the optimal cost function f . As

$\hat{y}_t(x_t)$ is generally not independent of x_t , ordering with the faster supplier is not done in the manner of a base-stock policy.

We also show that this implies that f is generally not convex. If we differentiate the first-order optimality condition for \hat{z}_t with respect to x_t , and apply it again to simplify the formulation, we obtain:

$$\begin{aligned} & \left(1 + \frac{d\hat{z}_t}{dx_t}\right) (1 - R_t(\hat{z}_t))(c_b + c_h)g(x_t + \hat{z}_t) \\ & + \left(1 + \frac{d\hat{z}_t}{dx_t}\right) (1 - R_t(\hat{z}_t)) \left(1 + \frac{\partial\hat{v}_t(\hat{z}_t)}{\partial z_t}\right) \int_0^\infty f''_{t+1}(x_t + \hat{z}_t + \hat{v}_t(\hat{z}_t) - d_t)r_t(q_t)dq_tg_t(d_t)dd_t = 0. \end{aligned} \quad (\text{A6})$$

As $(1 - R_t(\hat{z}_t))(c_b + c_h)g(x_t + \hat{z}_t)$ is nonnegative, and in the case of our example, $(1 - R_t(\hat{z}_t))(1 + \partial\hat{v}_t(\hat{z}_t)/\partial z_t)$ is also nonnegative, we see that (A6) can only be satisfied if $f''_t(x_t) < 0$ for the cases where $d\hat{z}_t/dx_t = -1$ does not hold. Thus, in the case of the chosen example, we have shown that f is not convex.

To prove Part 2 we need to show that $d\hat{w}_t(z_t)/dy_t = 0$ holds for any z_t , or equivalently $d\hat{v}_t(z_t)/dy_t = -1$. To show this, we first need to differentiate (4) with respect to v_t to obtain the first-order optimality condition, $\frac{\partial}{\partial v_t}H_t(x_t, z_t, v_t) \equiv 0$, in period t :

$$\begin{aligned} & (1 - R_t(z_t)) \int_0^\infty f'_{t+1}(x_t + z_t + \hat{v}_t(z_t) - d_t)r_t(q_t)dq_tg_t(d_t)dd_t \\ & + \int_0^\infty \int_0^{z_t} f'_{t+1}(x_t + q_t + \hat{v}_t(z_t) - d_t)r_t(q_t)dq_tg_t(d_t)dd_t = 0. \end{aligned} \quad (\text{A7})$$

Differentiating (A7) with respect to y_t , where $y_t = x_t + z_t$, yields:

$$\left(1 + \frac{d\hat{v}_t(z_t)}{dy_t}\right) (1 - R_t(z_t)) \int_0^\infty f''_{t+1}(x_t + z_t + \hat{v}_t(z_t) - d_t)r_t(q_t)dq_tg_t(d_t)dd_t = 0, \quad (\text{A8})$$

where we see that $d\hat{v}_t(z_t)/dy_t = -1$ satisfies (A8), which proves Part 2.

The above results imply the proposed structure of the optimal policy in Part 3. \square

Proof of Proposition 2: Part 1 holds directly from convexity results in Lemma 1. More specifically, from Part 3 of Lemma 1 we see that \hat{y}^M is independent of x_t and ordering up to \hat{y}^M is of a base-stock type.

From the definition in Part 2 (second line in (7)) we see that $\hat{w}^M(z_t^M)$ depends on x_t and z_t^M . To show that \hat{w}^M is a state-dependent base-stock level, we need to show that it does not depend on $y_t = x_t + z_t^M$. We follow the same steps as in Part 2 of the Proposition 1, where instead of f_{t+1} we now have $C_{t+1} = E_{Q_t, D_t}\tilde{C}(\tilde{x}_{t+1})$. The result for the myopic \hat{w}^M is stronger as in (A8) as C is convex in x due to Part 1 in Lemma 1, therefore $d\hat{w}^M/dy_t = -1$ needs to hold. \square

Lemma 2. *Let $f(x)$ and $g(x)$ be convex, and x_f and x_g be their minimizers. If $f'(x) \leq g'(x)$ for all x , then $x_f \geq x_g$.*

Proof: Following Heyman & Sobel (1984), let's assume for a contradiction that $x_f < x_g$, therefore we can write $0 < \delta \leq x_g - x_f$. Then $f'(x_g - \delta) \geq 0$ holds due to convexity of f , and due to the initial assumption $f'(x) \leq g'(x)$, we have $0 \leq f'(x_g - \delta) \leq g'(x_g - \delta)$. As g is convex, $g'(x_g - \delta) \geq 0$ contradicts the definition of x_g being minimizer of $g(\cdot)$, hence $x_f \geq x_g$ needs to hold. \square

Proof of Proposition 3: We first define the myopic equivalent to H_t defined in (4) as $H_t(x_t, z_t, v_t) = \alpha E_{Q_t, D_t} C_{t+1}(x_t + \min(z_t, Q_t) + v_t - D_t)$. We can rewrite H_t as a function of w_t : $H_t(w_t, z_t) = \alpha E_{Q_t, D_t} C_{t+1}(w_t - (z_t - Q_t)^+ - D_t)$, where $w_t - (z_t - Q_t)^+ = x_t + \min(z_t, Q_t) + v_t$.

To prove Part 1, we need to show that for the first derivative of H_t with respect to w_t , $H'_t(w_t, \hat{z}_t^M)$, it holds that $H'_t(w_t, \hat{z}_{1,t}^M) \leq H'_t(w_t, \hat{z}_{2,t}^M)$ for $\hat{z}_{1,t}^M \geq \hat{z}_{2,t}^M$ for all t :

$$\begin{aligned}
H'_t(w_t, \hat{z}_{1,t}^M) &= \alpha E_{Q_t, D_t} C'_{t+1}(x_{1,t+1}) \\
&= \alpha E_{Q_t, D_t} C'_{t+1}(x_{1,t} + \min(\hat{z}_{1,t}^M, Q_t) + v_t - D_t) \\
&\leq \alpha E_{Q_t, D_t} C'_{t+1}(x_{2,t} + \min(\hat{z}_{2,t}^M, Q_t) + v_t - D_t) \\
&= \alpha E_{Q_t, D_t} C'_{t+1}(x_{2,t+1}) \\
&= H'_t(w_t, \hat{z}_{2,t}^M).
\end{aligned}$$

The first equality is due to the definition of H_t in (4), applied to Part 2 of the Proposition 2. The second equality is due to the state update $x_{t+1} = x_t + \min(\hat{z}_t^M, Q_t) + v_t - D_t$, where $w_t = x_t + \hat{z}_t^M + v_t^M$. Due to $\hat{z}_{1,t}^M \geq \hat{z}_{2,t}^M$, and the fact that \hat{z}_t^M is placed up to \hat{y}^M (Part 1 of the Proposition 2), it follows that $x_{1,t+1} \leq x_{2,t+1}$, while v_t is placed in both cases up to w_t . Consequently, $x_{1,t} + \min(\hat{z}_{1,t}^M, Q_t) \leq x_{2,t} + \min(\hat{z}_{2,t}^M, Q_t)$ holds, and the inequality is due to convexity of C in x (Part 1 of Lemma 1) and the fact that taking expectation over Q_t and D_t preserves the inequality sign. Thus, we have shown that $H'_t(w_t, \hat{z}_{1,t}^M) \leq H'_t(w_t, \hat{z}_{2,t}^M)$ holds. Due to Lemma 2, this proves Part 1.

To prove Part 2, we write:

$$\begin{aligned}
H'_t(w_t, \hat{z}_t^M + \eta) &= \alpha E_{Q_t, D_t} C'_{t+1}(x_t + \min(\hat{z}_t^M + \eta, Q_t) + v_t - D_t) \\
&\geq \alpha E_{Q_t, D_t} C'_{t+1}(x_t + \min(\hat{z}_t^M, Q_t) - \eta + v_t - D_t) \\
&= H'_t(w_t - \eta, \hat{z}_t^M).
\end{aligned}$$

The first equality is due to the definition of H_t in (4), applied to Part 2 of the Proposition 2, and the state update. It holds that $\min(\hat{z}^M + \eta, Q_t) \geq \min(\hat{z}^M, Q_t) - \eta$, and the inequality is due to convexity of C in x and the fact that taking expectation over Q_t and D_t preserves the inequality sign. Thus, we have shown that $H'_t(w_t, \hat{z}_t^M + \eta) \geq H'_t(w_t - \eta, \hat{z}_t^M)$ holds. Observe that the minimizer of $H_t(w_t - \eta, \hat{z}_t^M)$ is $\hat{w}_t^M(\hat{z}_t^M) + \eta$, and together with Lemma 2, this implies $\hat{w}_t^M(\hat{z}_t^M + \eta) \leq \hat{w}_t^M(\hat{z}_t^M) + \eta$, which proves Part 2. \square

Notes

¹With the phrase “myopic policy” we denote the policy that optimizes the single period problem. As we later demonstrate that the myopic policy is not optimal in a multiperiod setting, we avoid using the phrase “optimal myopic policy” to avoid confusion.

²Defined as the expected demand over the available capacity of the faster supply source.

³We give the approximate average CVs for demand and supply capacity distributions since it is impossible to come up with the exact same CVs for discrete uniform distributions with different means. Demand and supply capacity distributions with CVs 0.80 and 1.00 were only modeled with normal distribution, as uniform distribution cannot attain such high CVs.

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