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Design Methods Based on Stiffness

APPROXIMATE DEFLECTION ANALYSIS OF NON-SYMMETRIC HIGH RISE STRUCTURES

Hans J.C.D. Hoenderkamp

An approximate hand method for estimating deflections in asymmetric multi-bent structures subjected to horizontal loading is presented. The structures may consist of combinations of couples walls, rigid frames, braced frames and wall-frames with shear walls. Results for structures that are uniform with height compare closely with results from stiffness matrix analyses. The method is developed from coupled-wall deflection theory which is expressed in non-dimensional structural parameters. It accounts for bending deformations in all individual members as well as axial deformations in the vertical members and is, therefore, more accurate for very tall structures. A closed solution of coupled differential equations for deflection and rotation gives the deflected shape along the height of the building. The deflection and rotation at the top can be obtained graphically. The proposed method of analysis offers a relatively simple and rapid means of comparing the deflection characteristics of different stability systems for a proposed tall building.

The derivation of equations for analysis shown in this paper are for uni-symmetric stability systems only but the method is also applicable to general asymmetric structures.

INTRODUCTION

Single-Bent structures

The continuous medium method of analysis for a pair of high-rise coupled shear walls has been available and used for about thirty years [1-3]. On the basis of the associated theory, the characteristic deflection equation has been shown to be [7]

\[ \frac{d^4y}{dx^4} - (k\alpha)^2 \frac{d^2y}{dx^2} = \frac{q(x) - M(x)\beta^2}{EI} \]  

Where \( y \) is the lateral deflection, \( x \) is the distance measured from the top down the structure, \( q \) represents the intensity of a general horizontal load and \( M \) is the bending moment due to load \( q \). The characteristic parameters are defined as follows

\[ \alpha^2 = \frac{GA}{EI} \quad \text{(2)} \]
\[ k^2 = \frac{EI}{EAC^2} \quad \text{(3)} \]
\[ \beta^2 = \frac{GA}{EAC^2} = \alpha^2(k^2-1) \quad \text{(4)} \]

in which a gross flexural stiffness parameter is defined as

\[ EI_k = EI + EAC^2 = EI \frac{k^2}{k^2-1} \quad \text{(5)} \]

In the analysis of the coupled walls the structure is represented by three distinct structural parameters.

(a) \( EI \) - This term expresses the flexural rigidity of all vertical members in the bent which are continuous up the total height of the structure.

(b) \( GA \) - This represents the racking shear rigidity. It is a measure of the vertical shearing stiffness of the member system connecting between the vertical components and of the complementary shearing stiffness across the bent.

(c) \( EAC^2 \) - This is a flexural stiffness parameter which expresses the contribution from the axial stiffness of the vertical members to the total moment resistance of the bent.
Taking boundary conditions of fixity at the base, zero external moment at the top, and total shear force at the base taken by the columns, the deflection equation for the bent subject to a uniformly distributed horizontal load \( w \), has been shown [8] to be

\[
y(x) = \frac{wH^4}{EI} \left[ \frac{1}{8} - \frac{1}{6} \left( \frac{x}{H} \right) + \frac{1}{24} \left( \frac{x}{H} \right)^4 + \frac{1}{k^2 - 1} \left( \frac{1 - (x/H)^2}{2(\kappa H)^2} \right) \right] \\
+ \frac{\cosh((\kappa H)(1-x/H)) - 1 - (\kappa H)(\sinh(\kappa H) - \sinh(\kappa x))}{(\kappa H)^4 \cos(\kappa H)}
\]  

(6)

where \( H \) represents the total height of the structure. It has also been shown that the continuous medium method is suitable for the deflection analysis of other single bent structures such as rigid frames [8], braced frames [13], wall frames [9], rigid frames with centrally located shear walls [12] and braced frames with multi-story bracing systems [11].

**MULTI-BENT STRUCTURES**

The floor plan of a stability system for a typical high-rise structure is shown in Fig. 1a. It comprises a combination of four bents of which only the in-plane stiffnesses are taken into account. Bents nr. 1 and nr. 3 are identical. There is one axis of symmetry on which the vertical axis is arbitrarily located. The origin is at the top of the building. The lateral load \( q \), with an arbitrary distribution up the height of the building, acts perpendicular to the z-axis at a distance, \( e \), from the x-axis.

Fig.1. Uni-symmetric structure with lateral loading.
In order to simplify the structural analysis this load is replaced by four separate horizontal loads located at the individual bents and acting in the directions as shown in Fig. 1b. The load \( q \) is distributed in such a way that at any level

\[
q = q_1 + q_2 + q_3 + q_4 \tag{7}
\]

\[
qe = q_1\eta_1 + q_2\xi_2 = q_3\eta_3 + q_4\xi_4 \tag{8}
\]

in which \( \eta_1 \) and \( \xi_1 \) are the y- and z-coordinate of the bents.

Due to the large in-plane stiffness of the floors and the inherent different deflection behavior of each bent, the four loads will have different distributions up the height. This will cause the structure to deflect in the y-direction and to rotate about the x-axis as shown in Fig. 2. In here deflection \( y \), indicates the translation of the x-axis. The deflections of the individual bents in the y-direction due to rotation of the structure about the x-axis are given by

\[
y_i = \xi_i \cdot \theta \tag{9a}
\]

and for bents in the z-direction the deflections are

\[
z_i = \eta_i \cdot \theta \tag{9b}
\]
The individual bents in the structure can be analyzed by applying the characteristic differential equation, Eq. 1, to each individual bent. This yields for bents in the y-direction

\[ EI_y \frac{d^4(y+\xi\beta)}{dx^4} - k^2GA_y \frac{d^2(y+\xi\beta)}{dx^2} = q - (M\beta^2)_y \]

and for bents in the z-direction

\[ EI_z \frac{d^4(\eta\beta)}{dx^4} - k^2GA_z \frac{d^2(\eta\beta)}{dx^2} = q - (M\beta^2)_z \]

Shear walls

The term \( k^2GA \) for a simple shear wall is undetermined since values for \( GA \) and \( EA^2c_2 \) are effectively zero. It is suggested that the racking shear and "axial bending" stiffnesses, \( GA \) and \( EA^2c_2 \) resp., be given artificially small values e.g.

\[ GA(\text{wall}) = \Sigma GA_i, 10^{-10} \]

\[ EA^2c_2(\text{wall}) = \Sigma EA^2c_2 i, 10^{-10} \]

Where \( \Sigma GA_i \) and \( \Sigma EA^2c_2 \) represent stiffnesses of the total structure, i.e. summations for all bents. These extremely small values for the wall will not have any impact on the final outcome of the analysis.

DEFLECTION AND ROTATION EQUATIONS

A "deflection" equation for the total structure is obtained if Eqs. 10 could be summed for all bents. This can be done by assuming the values of \( \beta^2 \) to be the same for each bent. That situation occurs for example when a group of identical rigid frames is combined with shear walls. Simplifying by leaving out the summation signs, the "deflection" equation for the total structure then becomes

\[ EI_y \frac{d^4y}{dx^4} + d.EI_y \frac{d^4\theta}{dx^4} - k^2GA_y \frac{d^2y}{dx^2} - a.k^2GA_y \frac{d^2\theta}{dx^2} = q - M\beta^2 \]

in which \( EI_y \) and \( k^2GA_y \) represent respectively, the bending stiffness and the adjusted racking shear stiffness of the structure in the y-direction. The total applied bending moment is given by

\[ M = M_1 + M_2 + M_3 + M_4 \]

The distance from the centroid of the \( EI_y \)-system to the y-axis, \( d \), is defined as follows
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\[ d = \frac{\sum EI_y \xi_i}{EI_y} \]  
(15)

And the centroid of the \( k^2GA_y \)-system is defined by

\[ a = \frac{\sum k^2GA_y \xi_i}{k^2GA_y} \]  
(16)

Multiplying Eqs. 10a and 10b through by \( \xi_i \) and \( \eta_i \) respectively and summing for all the bents gives the "rotation" equation for the total structure.

\[ \frac{d^4\theta}{dx^4} + \frac{d}{dx} \left( \frac{d^3y}{dx^3} + a \cdot k^2GA_y \frac{d^2y}{dx^2} - k^2GJ \frac{d^2\theta}{dx^2} \right) = qe - M_0\beta^2 \]  
(17)

in which

\[ M_0 = M_1\eta_1 + M_2\eta_2 + M_3\eta_3 + M_4\xi_4 \]  
(18)

The warping stiffness \( EI_x \) and torsional stiffness adjusted for axial deformations \( k^2GJ \), include all bents. They are defined with respect to the x-axis and can be expressed as

\[ EI_x = \sum (EI_x \xi_i^2 + EI_{xx} \eta_i^2) \]  
(19)

\[ k^2GJ = \sum (k^2GA_y \xi_i^2 + k^2GA_{yy} \eta_i^2) \]  
(20)

Eqs. 13 and 17 are two coupled differential equations in \( y \) and \( \theta \) which define the deflection behavior of the structure when subjected to lateral load \( q \). They must be solved simultaneously.

**UNCOUPLING OF EQUATIONS**

The problem of solving the deflection and rotation equations can be simplified by choosing the x-axis at the centroid of the EI-system, i.e. setting \( d=0 \). Dividing through by \( EI_y \) and rewriting them in matrix form yields

\[
\begin{bmatrix}
\frac{d^4y}{dx^4} \\
\frac{d^3y}{dx^3} \\
\frac{d^2\theta}{dx^2} \\
\frac{d\theta}{dx} \\
r \frac{d^2\theta}{dx^2} \\
r \frac{d\theta}{dx} \\
r \frac{d^3y}{dx^3}
\end{bmatrix} = \begin{bmatrix}
(k\alpha)^2 & \left(\frac{\alpha}{r}\right)(k\alpha)^2 \\
\left(\frac{\alpha}{r}\right)(k\alpha)^2 & (k\alpha)^3 \\
\frac{d^2y}{dx^2} \\
r \frac{d^2\theta}{dx^2} \\
r \frac{d\theta}{dx} \\
r \frac{d^3y}{dx^3}
\end{bmatrix} \begin{bmatrix}
d^4y \\
d^3y \\
d^2\theta \\
d\theta \\
r d^2\theta \\
r d\theta \\
r d^3y
\end{bmatrix} = \frac{q-M_0\beta^2}{EI_y} \begin{bmatrix}
1 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0
\end{bmatrix}
\]

(21)
in which the various structural parameters have been combined as follows [14]

\[(k_\omega)^2 = \frac{k^2GA_y}{EI_y} \]  
\[(k_\omega)^2 = \frac{k^2GJ}{EI_w} \]  
\[r^2 = \frac{EI_w}{EI_y} \]

The coupled deflection and rotation equations, Eq. 21, can be solved by uncoupling, i.e. by transferring them to another coordinate system. This procedure is analogous to the mode-superposition method used for the undamped vibrations in dynamic analysis [5]. It has been applied earlier to the static analysis of high-rise structures by Rutenberg and Heidebrecht [6] and their procedure is followed here. A normal-coordinate transformation serves to change a set of coupled equations into a set of uncoupled equations which are then solved for the normal-deflections. These must then in turn be transferred back to the original coordinate system.

The solution first requires a calculation of the two eigenvalues for the homogeneous form of Eq. 21.

\[ (k_\omega)^2, (k_\omega)_w^2 = \frac{(k_\omega)^2 + (k_\omega)_w^2}{2} \pm \sqrt{\left(\frac{(k_\omega)^2 - (k_\omega)_w^2}{2}\right)^2 + \frac{a}{r}(k_\omega)^2} \]  

where \((k_\omega)_w^2\) is the smaller of the eigenvalues. By substituting them back into the equation for the eigenvalue problem associated with the homogeneous form of Eq. 21, they will yield the eigenvectors which can be written as follows

\[ \{N\} = \frac{1}{(1+f)^{\frac{1}{2}}} \begin{bmatrix} 1 & f \\ -f & 1 \end{bmatrix} \]  

in which

\[ f = \frac{(k_\omega)^2 - (k_\omega)_w^2}{(k_\omega)^2 \cdot a/r} \]

The orthonormalized mode-shape matrix \(\{N\}\) is used to transform the geometric coordinates to the generalized coordinates such that

\[ \{y_{10}\} = \{N\} \{\nu\} \]
It can be shown that substituting Eq. 25 into Eq. 21 and multiplying through by \( \{N\} \) will lead to

\[
\frac{d^4u}{dx^4} - (\kappa\alpha)_u^2 \frac{d^2u}{dx^2} = p_u \cdot \frac{q-M\beta^2}{El_y}
\]

(29)

\[
\frac{d^4v}{dx^4} - (\kappa\alpha)_v^2 \frac{d^2v}{dx^2} = p_v \cdot \frac{q-M\beta^2}{El_y}
\]

(30)

These are two fourth order differential equations representing two separate orthogonal bents with their normalized characteristic parameters. They are of the same form as the generic deflection equation for a single bent as given in Eq. 1.

The load factors \( p_u \) and \( p_v \) are the equivalent of the modal participation factors in dynamic analysis and are given by

\[
p_u = \frac{1-fe/r}{(1+f^2)^N}
\]

(31)

\[
p_v = \frac{f+e/r}{(1+f^2)^N}
\]

(32)

When the structure is subjected to a uniformly distributed lateral load \( w \), instead of the arbitrary load \( q \), the solutions to Eqs. 29 and 30 are given by Eq. 6 since the boundary conditions of the normalized bents are the same as for the actual structure.

Dividing through by the participation factors the deflections \( u \) and \( v \) can be expressed as

\[
u(x) = \frac{p_u WH^4}{EI_y} \left[ \frac{1}{8} - \frac{1}{6} (\frac{x}{H}) + \frac{1}{24} \left( \frac{x}{H} \right)^2 + \frac{1}{2 k^2 - 1} \left( \frac{1-(x/H)^2}{2(\kappa\alpha H)^2} \right) + \frac{\cosh((\kappa\alpha H)_u (1-x/H)) - (\kappa\alpha H)_u (\sinh(\kappa\alpha H)_u - \sinh(\kappa\alpha x)_u)}{(\kappa\alpha H)_u \cosh(\kappa\alpha H)_u} \right]
\]

(33)

and

\[
v(x) = \frac{p_v WH^4}{EI_y} \left[ \frac{1}{8} - \frac{1}{6} (\frac{x}{H}) + \frac{1}{24} \left( \frac{x}{H} \right)^4 + \frac{1}{2 k^2 - 1} \left( \frac{1-(x/H)^2}{2(\kappa\alpha H)^2} \right) + \frac{\cosh((\kappa\alpha H)_u (1-x/H)) - (\kappa\alpha H)_u (\sinh(\kappa\alpha H)_u - \sinh(\kappa\alpha x)_u)}{(\kappa\alpha H)_u \cosh(\kappa\alpha H)_u} \right]
\]

(34)

The terms are identical to those in the deflection equation for a single bent with
the exception that the characteristic parameters are normalized functions of lateral and rotational stiffnesses. Analogous to Eq. 4, in the normalized system

$$\beta^2 = \alpha_2^2(k_u^2 - 1)$$

$$\beta^2 = \alpha_2^2(k_r^2 - 1)$$

from which expressions for $k_u^2$ and $k_r^2$ can be obtained

$$k_u^2 = \frac{(k\alpha)^2}{(k\alpha)^2 - \beta^2}$$

$$k_r^2 = \frac{(k\alpha)^2}{(k\alpha)^2 - \beta^2}$$

and as in Eq. 5 the modified gross moments of inertia for the normalized structure multiplied by the elastic modulus then become

$$EI_u = EI_y \frac{k_u^2}{k_u^2 - 1}$$

$$EI_r = EI_y \frac{k_r^2}{k_r^2 - 1}$$

It is now possible to calculate the complete deflection profiles up the height of the two normalized bents.

**DEFLECTION AND ROTATION OF BUILDING**

The deflections must be transferred back to the geometric coordinate system according to Eq. 28. This will yield the following equations for the deflection and rotation in the actual uni-symmetric multi-bent structure

$$y(x) = \frac{u(x) + f_v(x)}{(1 + f^2)^{\alpha}}$$

$$r\theta(x) = \frac{-f.u(x) + v(x)}{(1 + f^2)^{\alpha}}$$

These equations allow the calculation of the translation at any point in the
The terms inside the brackets of Eqs. 33 and 34 are identical to those in the deflection equation shown in Eq. 6. This makes it possible to use earlier devised diagrams [7] to obtain sway indices for the individual bents. The sway index is defined as the deflection at the top of the structure divided by the total height. The maximum sway index will occur in one of the exterior bents of the building. The sway index for bent "i" in the y-direction is a combination of the deflection and the rotation at the top of the structure

\[
\left( \frac{y(0)}{H} \right)_i = \left( \frac{y(0)}{H} \right)_s + \left( \frac{y(0)}{H} \right)_r
\]

where \( \left( \frac{y(0)}{H} \right)_s \) is the sway index of the x-axis in the structure and \( \left( \frac{y(0)}{H} \right)_r \) is the additional sway index due to rotation.

For bents in the z-direction the sway index is a function of rotation only.

\[
\left( \frac{z(0)}{H} \right)_i = \left( \frac{z(0)}{H} \right)_s
\]

Substituting Eqs. 9, 41 and 42 into Eqs. 43 gives the sway indices for bents in the y-direction or the z-direction. For a bent in the y-direction the sway index is expressed as

\[
\left( \frac{y(0)}{H} \right)_i = \left( \frac{u(0)}{H} \right) \left( 1 - \frac{f_\xi}{r} \right) + \left( \frac{v(0)}{H} \right) \left( f + \frac{\xi}{r} \right)
\]

and for a bent in the z-direction the sway index becomes

\[
\left( \frac{z(0)}{H} \right)_i = \left( - \frac{u(0)}{H} \right) \frac{\tau}{r} + \left( \frac{v(0)}{H} \right) \frac{\eta}{r}
\]

The sway components for the normalized bents, \( \frac{u(0)}{H} \) and \( \frac{v(0)}{H} \) are obtained from Eqs. 33 and 34 by setting \( x = 0 \) and dividing through by \( H \). This yields

\[
u(0) = \frac{P_u WH^3}{EI_{yy}} \left[ \frac{1}{8} + \frac{1}{k_x^2 - 1} \frac{1}{2(\kappa\alpha H)_u} \left( \cosh(\kappa\alpha H)_u - 1 - (\kappa\alpha H)_u \sinh(\kappa\alpha H)_u \right) \right]
\]

\[
v(0) = \frac{P_v WH^3}{EI_{yy}} \left[ \frac{1}{8} + \frac{1}{k_y^2 - 1} \frac{1}{2(\kappa\alpha H)_v} \left( \cosh(\kappa\alpha H)_v - 1 - (\kappa\alpha H)_v \sinh(\kappa\alpha H)_v \right) \right]
\]
Fig. 3. Total sway factor $K_1$, uniformly distributed load
These can be written as

\[ u(0) = \frac{P_w H^3}{EI_p} (K_m) \]  

(47)

where \( K_m \) can be obtained directly from the diagram in Fig. 3 for practical ranges of \((\alpha H)_a\) and \(k_2^2\), also

\[ v(0) = \frac{P_w H^3}{EI_p} (K_m) \]  

(48)

for which \( K_m \) is plotted in the same diagram as a function of \((\alpha H)_a\) and \(k_2^2\). Eqs. 47 and 48 allow the deflection as well as the rotation at the top of the structure to be calculated by using one diagram only.

**ACCURACY**

An approximation exists in the stage of combining a set of bents into a single structure represented by only four properties: \( E_{ly}, E_{iw}, k'GA_y \) and \( k'_GA_j \). Eqs. 13 and 17 are mathematically correct only for structures comprising single shear walls and other types of bents with identical values for \( \beta^2 \). A rigorous analysis of bents with non-identical values is extremely complex and unsuitable for a graphical solution [11].

The method presented here allows the assumption of equal values of \( \beta^2 \) for non-identical bents in a structure. This simplifies the calculations but imposes an additional restraint. Eq. 16 determines the centroid of the \( k'GA_y \)-system assuming the centroids of the \( GA_y \) and \( EA_c \)-systems to have the same location which is only the case for bents with identical values for \( \beta^2 \). It causes the rotation of the bents in the vertical plane, due to axial deformations in the columns, to be constrained at each floor level, i.e. the slope of the floor at each bent is proportional to the distance of its principal coordinate \( \xi \) or \( \eta \). This is shown in Fig. 4. It should be noted that this is not identical to the slope of the deflection profile of the bent.

For a structure comprising a group of identical bents and a number of single shear walls the solution is mathematically correct and the \( \beta \)-value of one of the bents can be used in the computations. In cases where the stability system consists of non-identical bents with or without single shear walls it is suggested that a weighted average value for \( \beta^2 \) be used,

\[ \beta^2 = \frac{\sum GA_i}{\sum EA_c^2} \]  

(49)

in which the summation terms include all lateral load resisting bents in the structure. Eq. 49 can of course be used for identical bents as well.
A study has been made of various structures of different height with a wide range of values for the dimensionless parameters. It has been found that for ratios of $\beta_{\text{max}}/\beta_{\text{min}} < 2.0$, 80% of the additional horizontal deflection as a result of axial deformations in the columns are taken into account [11]. Combinations of non-identical bents of the same type, e.g. several rigid frames with similar $\beta$-values, fall for the majority of structures within this range.

**EXAMPLE**

A computation procedure for the analysis of asymmetric tall building structures subject to horizontal load is given in Appendix I. The structural floor plan of a 16 storey, 48 m. high building in Fig. 5 shows a uni-symmetric arrangement of six single shear walls and five identical rigid frames. It was first analyzed by Gluck [4]. The geometrical properties of the structural members are given in Table 1. The building is subjected to a uniformly distributed lateral load of 40 kN/m. acting at the center of the structure in a direction parallel to the y-axis. The modulus of elasticity is taken as $20 \times 10^6$ kN/m$^2$. The principal stiffness parameters for the bents are given in Table 2. They are obtained from equations...
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given in earlier publications [8-12]. Using Eq. 49, \( \beta^2 = 3.769 \times 10^4 \text{m}^2 \). The basic characteristic parameters are \((k\alpha)^2 = 1.25 \times 10^3 \text{m}^2\) and \((k\alpha)^2 = 6.91 \times 10^4 \text{m}^2\). The resulting eigenvalues are \((k\alpha)^2 = 4.83 \times 10^4 \text{m}^2\) and \((k\alpha)^2 = 1.46 \times 10^3 \text{m}^2\). The characteristic parameters and normalized structural stiffnesses necessary for the calculation of deflections and rotations up the height of the structure are given in Table 3.

![Fig. 5 Structural floor plan](image)

### Table 1. Principal Member Properties

<table>
<thead>
<tr>
<th>Single Shear Walls</th>
<th>Moment of inertia (\text{m}^4)</th>
<th>Rigid Frames 7-11</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Element</td>
</tr>
<tr>
<td>1,4</td>
<td>5.7166</td>
<td>Int. col.</td>
</tr>
<tr>
<td>2,3</td>
<td>2.0834</td>
<td>Ext. col.</td>
</tr>
<tr>
<td>5,6</td>
<td>1.2348</td>
<td>Beam</td>
</tr>
</tbody>
</table>
Table 2. Principal Stiffness Parameters for Bents

<table>
<thead>
<tr>
<th>Bent</th>
<th>EI ( \times 10^6 ) kNm²</th>
<th>GA_i kN</th>
<th>EAc_i² kNm²</th>
<th>k²GA_i kN x 10^3</th>
</tr>
</thead>
<tbody>
<tr>
<td>1,4</td>
<td>114.332</td>
<td>2.726x10⁻⁵</td>
<td>7.233x10⁻²</td>
<td>43.090</td>
</tr>
<tr>
<td>2,3</td>
<td>41.668</td>
<td>2.726x10⁻⁵</td>
<td>7.233x10⁻²</td>
<td>15.704</td>
</tr>
<tr>
<td>5,7</td>
<td>24.696</td>
<td>2.726x10⁻⁵</td>
<td>7.233x10⁻²</td>
<td>9.308</td>
</tr>
<tr>
<td>7-11</td>
<td>0.213</td>
<td>5.452x10³</td>
<td>1.447x10⁶</td>
<td>54.601</td>
</tr>
</tbody>
</table>

Table 3. Parameters of Normalized Structure

| f    | -1.9174 | r      | 14.526 m |
| k_r² | 4.5354  | k_2    | 1.3494   |
| (kαH)_u | 1.0555 | (kαH)_v | 1.8313   |
| EI_u | 4.02x10⁶ kNm² | EI_v | 1.21x10⁶ kNm² |

The deflections and rotations about the x-axis are shown in Fig. 6 together with results from computer stiffness matrix analyses. The differences between the results from the method of analysis presented here and the computer results are less than 1.0% at all storey levels.

The curves showing the results for the case in which the columns are taken as axially rigid demonstrate clearly that the inclusion of the cross-sectional areas of the columns in the analysis can be quite important. If axial deformations are allowed to take place the reversal of rotation along the height of the structure, as shown in Fig. 6b, does not take place. Also, the deflection of wall 1 has increased by 17%. Both these results are confirmed by computer analyses.

CONCLUSIONS

A generalized hand method for deflection analysis is presented for structures with non-symmetrically arranged stability systems. The lateral load resisting elements may include combinations of shear walls, rigid frames, braced frames, coupled walls and wallframes (punched walls). The method allows an assessment of a structure's adequacy in sway resistance due to horizontal loading for different structural proposals. It requires calculation of the characteristic parameters \((αH)_u\), \((αH)_v\), \(k_r²\) and \(k_2\) which characterize the performance of the structure. These parameters may be substituted into formulae to yield deflections and rotations along the height of the building, or used to refer to graphs, to obtain solutions for the deflected shape of a particular bent, i.e. the total sway index.

The theory is based on the assumption of, and therefore is accurate only for, structures that are uniform through their height. It may be used, however, to
obtain the approximate deflection and comparison of sway between practical structures whose properties vary with height.

Fig.6. Deflections and rotations

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APPENDIX I

Computation Procedure

1. Structural parameters

Single Bents - For each individual bent calculate EI, GA and EAc². Details for these parameters were given earlier [8-12]. Compute values for k²GA using Eq. 3, for single shear walls use Eqs. 11 and 12.

Total structure - Determine values for ΣGA, and ΣEAc² by summing for all bents in the structure. Compute β² using Eq. 49. By summing for the bents in the y-direction obtain EI, and k²GA. Locate x-axis at centroid of EI-components using Eq. 15 and determine centroid of k²GA-components from Eq. 16. Using all bents calculate parameters for warping, EI, and torsion, k²GJ from Eqs. 19 and 20 resp.
2. Characteristic parameters

Multi-bent Structure - Compute \((k\alpha)^2\), \((k\alpha)_p^2\), and \(r^2\) given by Eqs. 22, 23 and 24 resp.

Normalized Structure - Calculate the eigenvalues \((k\alpha)^2\) and \((k\alpha)_p^2\) using Eq. 25. Determine factor, \(f\), from Eq. 27 and compute the load factors \(p_1\) and \(p_2\) by using Eqs. 31 and 32. Parameters \(k_p^2\), \(k_v^2\), \(E_{lp}\) and \(E_{lv}\) are given by Eqs. 37-40.

3. Deflections and rotations

Using Formulae - For translations up the height of the building first calculate the normalized deflection components \(u(x)\) and \(v(x)\) given by Eqs. 34 and 35 resp. The deflections of the actual structure in the y-direction and its rotations about the x-axis are then obtained from Eqs. 41 and 42 resp.

Using Diagrams - For sway indices of selected bents, enter the diagram in Fig. 3 with \((\alpha H)_v\) and \(k_v^2\) to obtain \(K_v\) which is then substituted into Eq. 47 to yield the normalized drift component \(u(0)/H\). Enter the same diagram again with \((\alpha H)_p\) and \(k_p^2\) to get \(K_p\) and calculate \(v(0)/H\) using Eq. 48. The drift index for bent "i" is then given by Eq. 44.

APPENDIX II

Alternative Loading Cases

The following deflection formulae correspond to Eq. 6 for uniformly distributed loading. They are to be used for calculating the deflection components \(u\) and \(v\) of the normalized structure.

a) A concentrated horizontal load at top of structure parallel to the y-axis.

\[
y = \frac{PH^3}{EI_y} \left[ \frac{1}{3} - \frac{1}{2} \left( \frac{x}{H} \right) + \frac{1}{6} \left( \frac{x}{H} \right)^3 + \frac{1}{k^2 - 1} \left( \frac{1 - x/H}{(k\alpha H)^2} \right) \right.
\]

\[
+ \frac{\sinh(k\alpha x) - \sinh(k\alpha H))}{(k\alpha H)^2 \cosh(k\alpha H)} \right]
\]

\[(50)\]

The deflection at the top can be obtained by setting \(x=0\). When using the deflection diagram shown in Fig. 7 the formula becomes

\[
y(0) = \frac{PH^3}{EI_y} (K_v)
\]

\[(51)\]
Fig. 7. Total sway factor $K_t$, concentrated top load
Fig. 8. Total sway factor $K_t$, triangularly distributed load
b) A triangularly distributed load with intensity \( w_1 \) at the top down to zero at the base of the structure

\[
y = \frac{w_1 H^4}{E I_s} \left[ \frac{11}{120} - \frac{1}{8} \left( \frac{x}{H} \right) + \frac{1}{24} \left( \frac{x}{H} \right)^4 - \frac{1}{120} \left( \frac{x}{H} \right)^5 \right] \\
+ \frac{1}{k^2-1} \left( \frac{1}{(k \alpha H)^2} \left( \frac{1}{3} - \frac{1}{2} \left( \frac{x}{H} \right)^2 + \frac{1}{6} \left( \frac{x}{H} \right)^3 - \frac{1-x/H}{(k \alpha H)^2} \right) \\
+ \frac{\cosh((k \alpha H)(1-x/H)) - 1 + (1/(k \alpha H) - (k \alpha H)/2)(\sinh(k \alpha H) - \sinh(k \alpha x))}{(k \alpha H)^4 \cosh(k \alpha H)} \right]
\]

Substituting for \( x = 0 \) will yield the deflection at the top. A shortened formula can be used if the diagram in Fig. 8 is employed,

\[
y(0) = \frac{w_1 H^3}{E I_s} (K_s)
\]

**NOTATION**

\( a \) = centroid of \( k^2 G A y \)-system;  
\( A \) = section area;  
\( d \) = centroid of \( E I_y \)-system;  
\( e \) = eccentricity of lateral load;  
\( E \) = modulus of elasticity;  
\( E A c^2 \) = axial stiffness parameter;  
\( f \) = characteristic parameter;  
\( G A \) = parameter indicating racking shear rigidity;  
\( h \) = storey height;  
\( H \) = total height of structure;  
\( I \) = moment of inertia;  
\( I_s \) = gross moment of inertia;  
\( I_w \) = warping constant;  
\( J \) = torsional constant;  
\( k \) = characteristic parameter;  
\( K_s \) = sway factor;  
\( M \) = bending moment;  
\( N \) = transformation matrix;  
\( p \) = load factor;  
\( P \) = concentrated horizontal load at top of structure;
\( q \) = general horizontal load;
\( r \) = ratio of structural parameters;
\( u \) = deflection component of normalized structure;
\( v \) = deflection component of normalized structure;
\( w \) = uniformly distributed horizontal load of intensity \( w \);
\( w_1 \) = triangularly distributed horizontal load with maximum intensity \( w_1 \) at the top and zero at the base;
\( x \) = distance measured from top of structure;
\( y \) = horizontal deflection;
\( \alpha \) = characteristic parameter;
\( \beta \) = characteristic parameter;
\( \theta \) = rotation of structure in horizontal plane;
\( \eta \) = y-coordinate of bent in z-direction and
\( \xi \) = z-coordinate of bent in y-direction.

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