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On The Rate of Constrained Arrays

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Abstract

Sudokus are nowadays very popular puzzles and they are studied for their mathematical structure. Binary Puzzles are also interesting puzzles with certain rules. A solved Binary Puzzle is an $n \times n$ binary array satisfying: (i) there are no three consecutive ones and also no three consecutive zeros in each row and each column, (ii) the number of ones and zeros must be equal in each row and in each column, and (iii) every two rows and every two columns must be distinct.

Binary Puzzles can be seen as constrained arrays and can be used for modulation purposes. In [2], we outlined some problems related to Binary Puzzles such as (1) rate of these code, (2) erasure decoding probability, (3) decoding algorithms and their complexity.

In this paper, we focus on the first problem, that is finding the rate of a code based on the Binary Puzzle.

The computation of the number of $n \times n$ Binary Puzzles is a very difficult problem, and so far we were only able to obtain the values for small $n$, by brute force.

Since a Binary Puzzle has to satisfy the conditions (1), (2) and (3), we consider these conditions separately and split the computation in three different part, where each part corresponds to one condition.

That means we consider the following collections of $n \times n$ binary arrays that are constrained:

\[
A_{n \times n} = \{ X \in F_2^{n \times n} \mid X \text{ satisfies (i) } \} ; \\
B_{n \times n} = \{ X \in F_2^{n \times n} \mid X \text{ satisfies (ii) } \} ; \\
C_{n \times n} = \{ X \in F_2^{n \times n} \mid X \text{ satisfies (iii) } \} ; \\
D_{n \times n} = \{ X \in F_2^{n \times n} \mid X \text{ satisfies (i), (ii) and (iii) } \},
\]

where $F_2^{n \times n}$ is the set of all $n \times n$ binary arrays

Although the exact size of $A_{n \times n}, B_{n \times n}$ and $D_{n \times n}$ is still an open problem, we provide the lower and upper bound of their size, and also of the asymptotic rates. The exact value for $|C_{n \times n}|$ by means of a recursive formula.

Keywords: Binary Puzzle, rate of a code, a constrained arrays.

References

http://www.w-i-c.org/proceedings/proceedings_SITB2015.pdf